INRiM ISTITUTO NAZIONALE DI RICERCA METROLOGICA

femto-st SCIENCES & TECHNOLOGIES

Phase-Noise and Amplitude-Noise Measurement of DACs and DDSs

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Extended version

Outline

- Introduction
- Method
- Experimental details
- Results

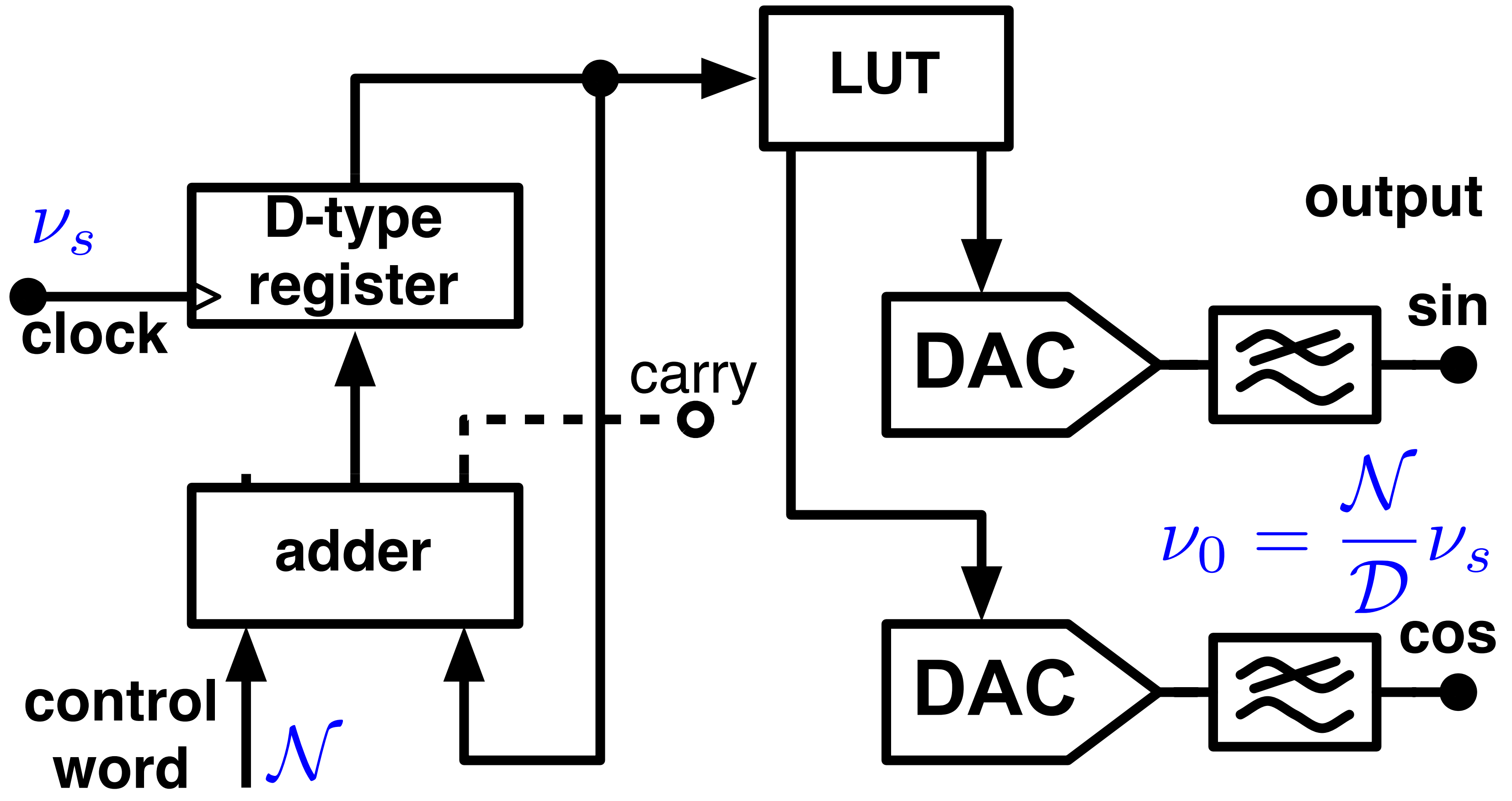
home page http://rubiola.org

Basic DDS Scheme

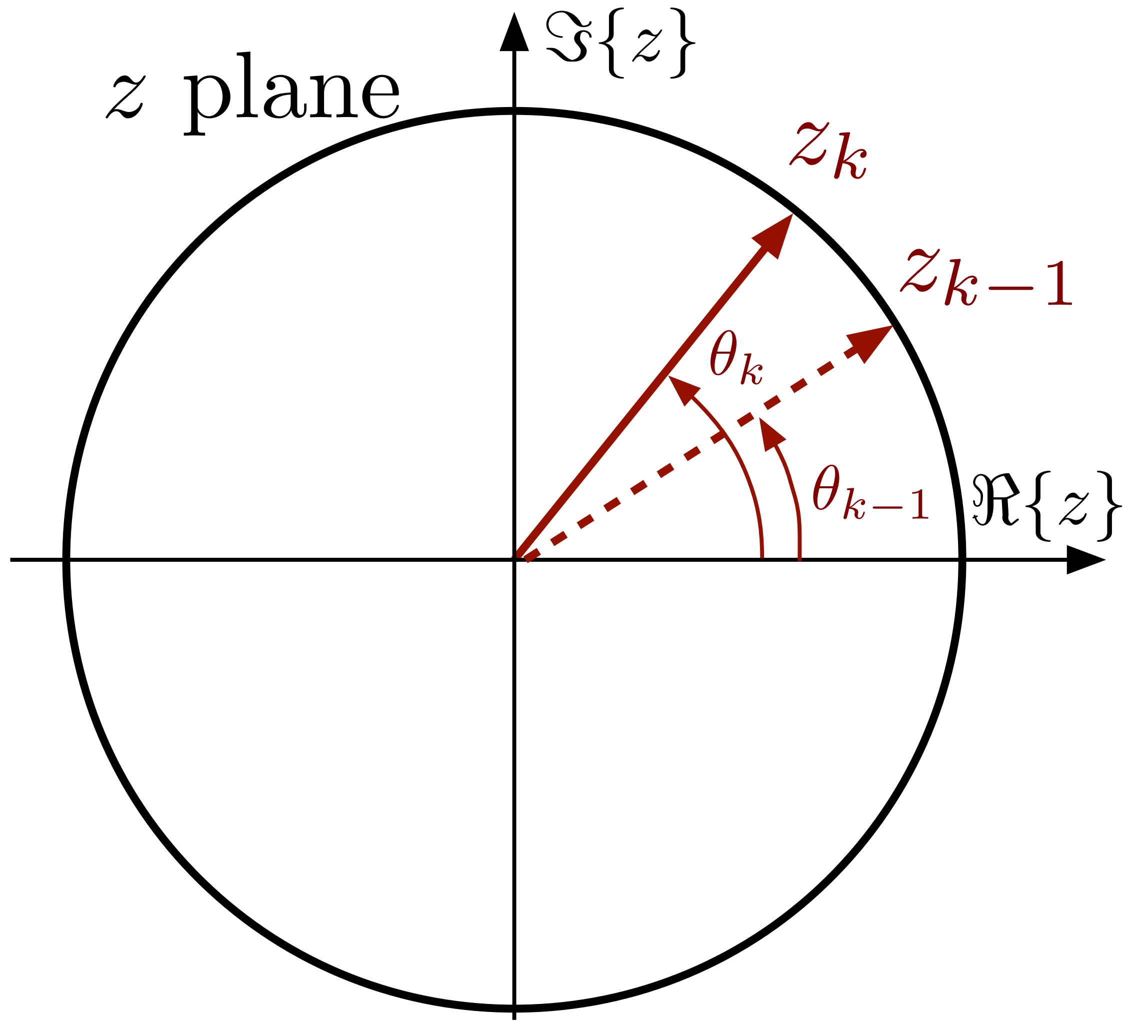
integer: $n_k = (n_{k-1} + \mathcal{N}) \bmod \mathcal{D}$

complex: $z_k = z_{k-1} \exp(j\eta)$

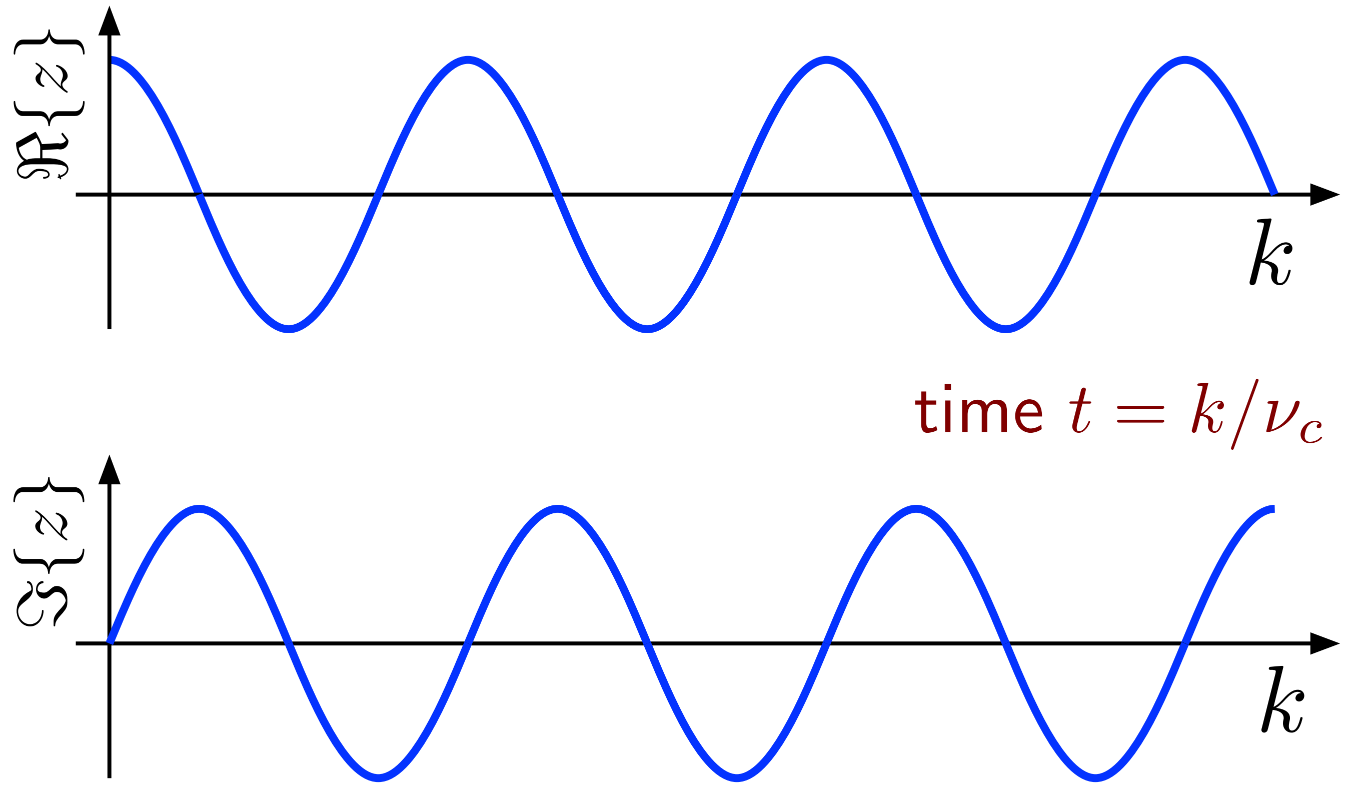
phase: $\theta_k = (\theta_{k-1} + \eta) \bmod 2\pi$



quantity	digital	analog
state variable	n	$\theta = 2\pi \frac{n}{\mathcal{D}}$
assoc. complex		$z = e^{j\theta}$
modulo	$\mathcal{D} = 2^m$	2π
increment	$\mathcal{N}$	$\eta = 2\pi \frac{\mathcal{N}}{\mathcal{D}}$
time	$k, \quad 0, 1, 2, \ldots$	$t = k/\nu_s$
clock freq. ν_s	output freq. $\nu_0 = \frac{\mathcal{N}}{\mathcal{D}}\nu_s$	



The contents *n* of the *m*-bit register is interpreted as a complex number



High resolution

$D = 2^{48}$

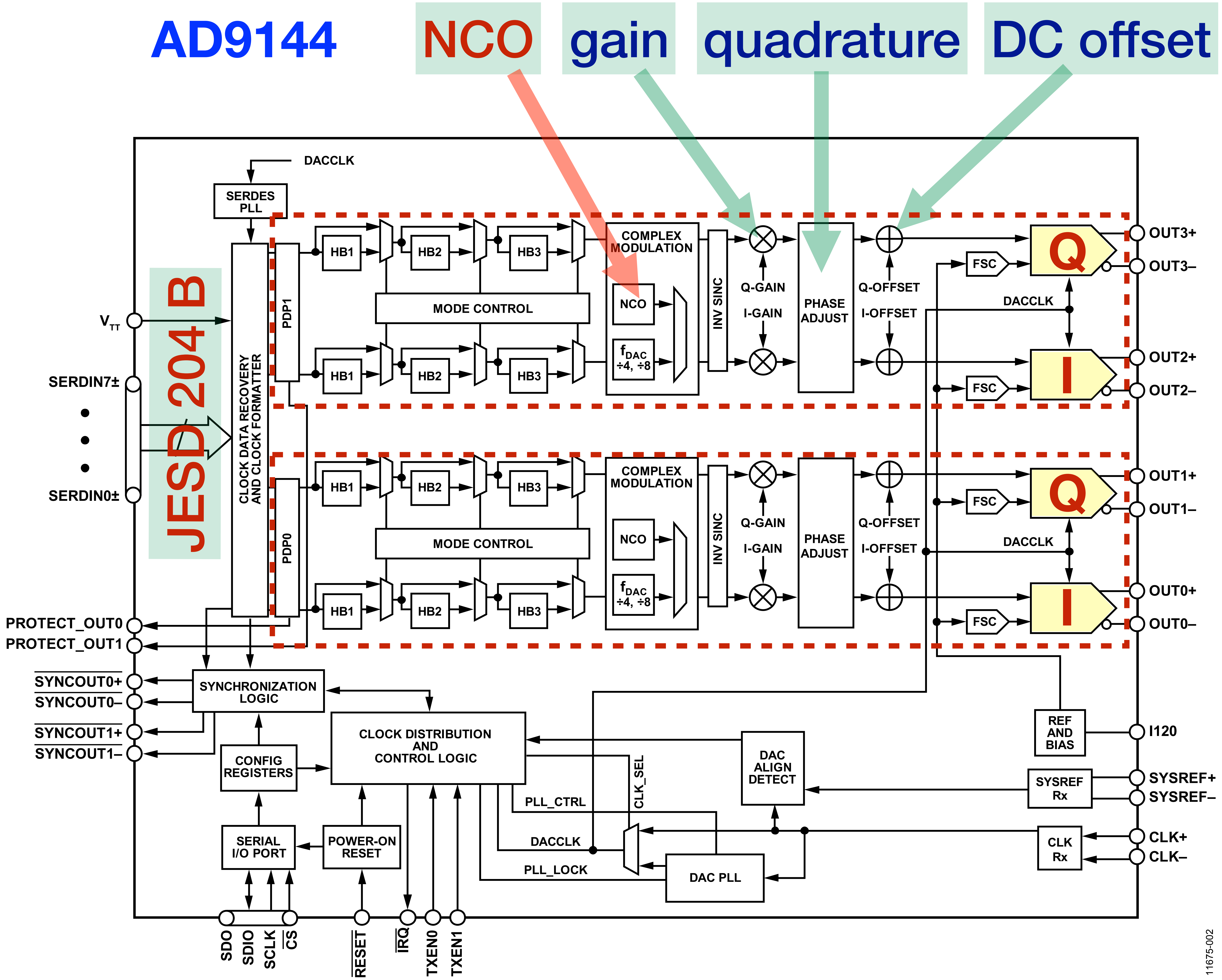
$\nu_s = 1$ GHz

$\Delta\nu = 3.55$ µHz

High-Speed DACs Have DDSs Inside

Analog Devices, AD9144 data sheet

AD9144

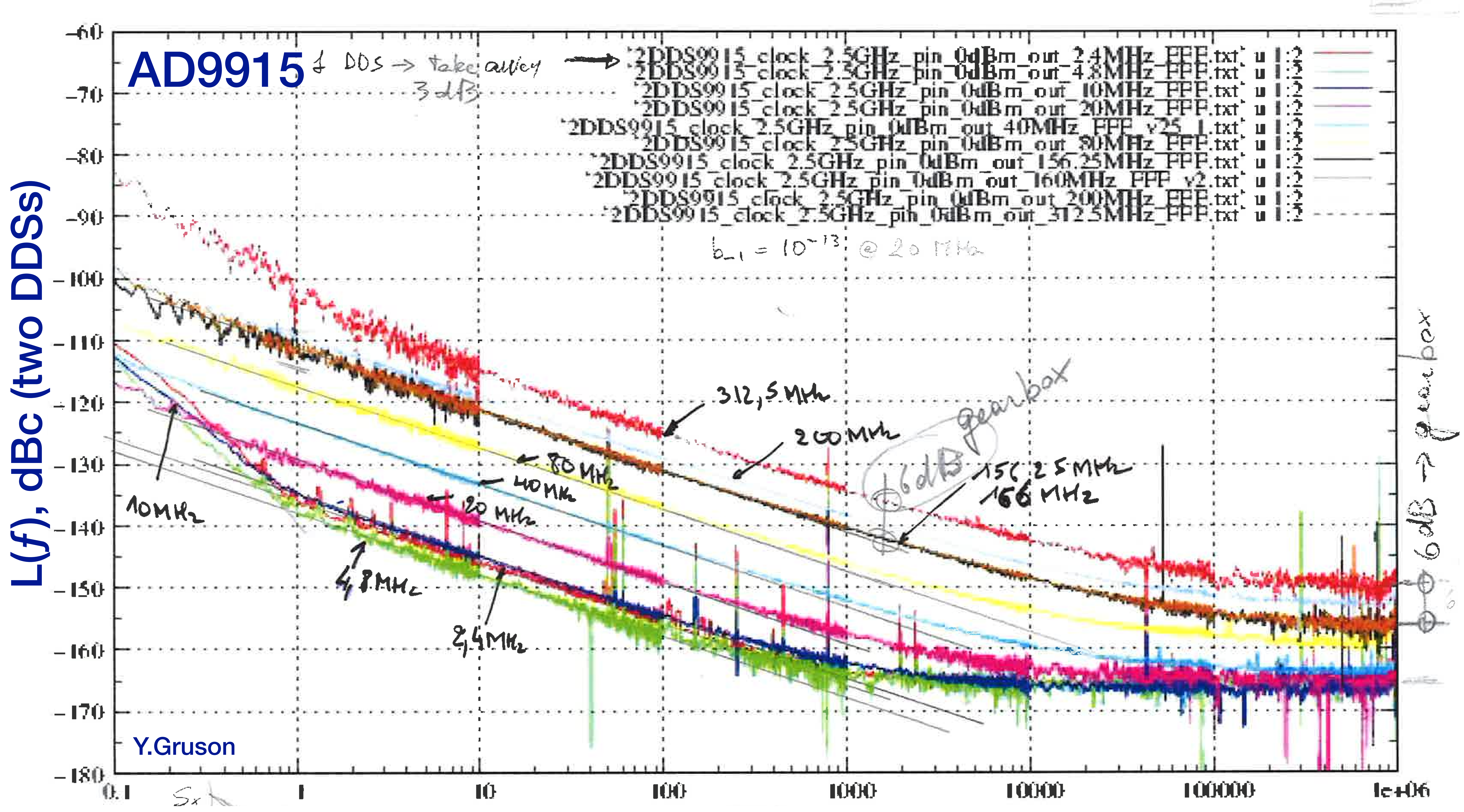


If no internal NCO

- Implement the NCO in FPGA
- The brute force of the JESD204B suffices
- IP NCOs available
- Minimal NCO not difficult to implement

The Beast to Kill

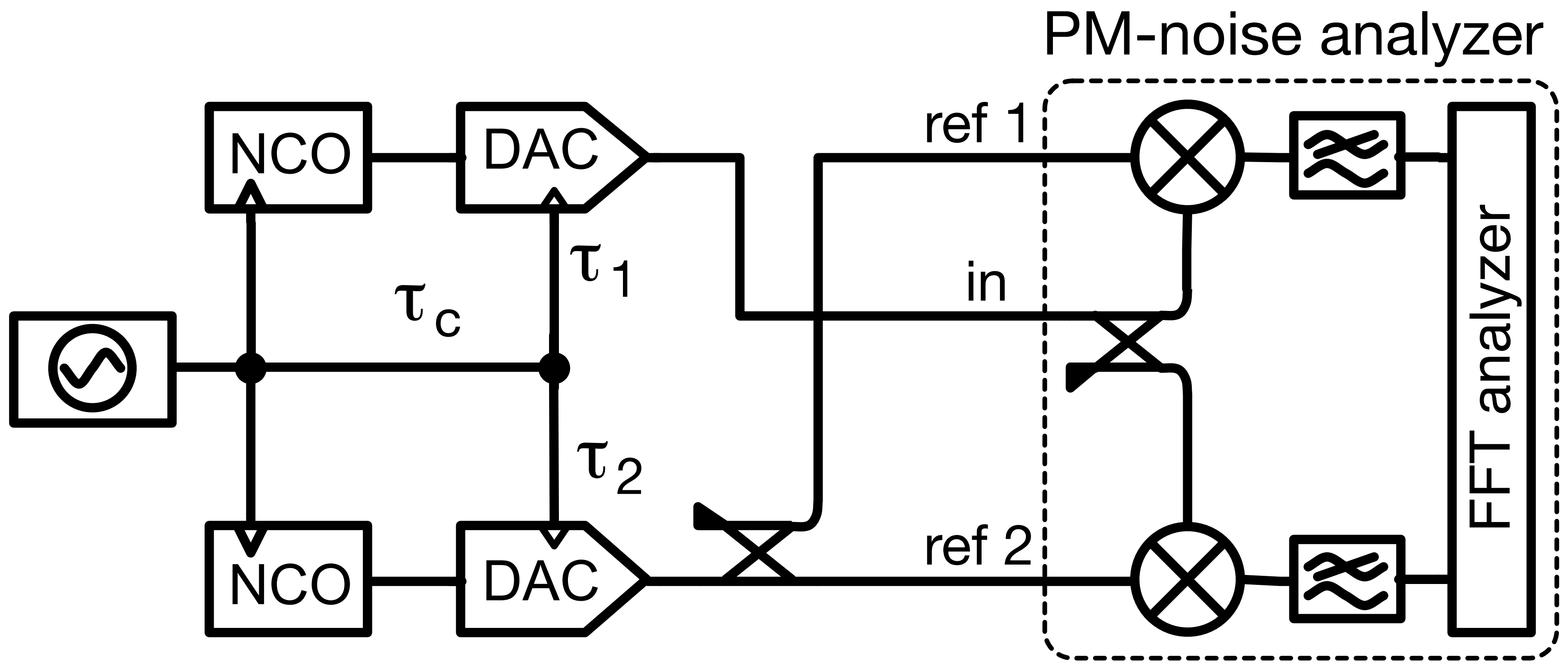
Aka, the lowest noise we have seen in a DDS



Not a real challenge, but low enough to spend attention

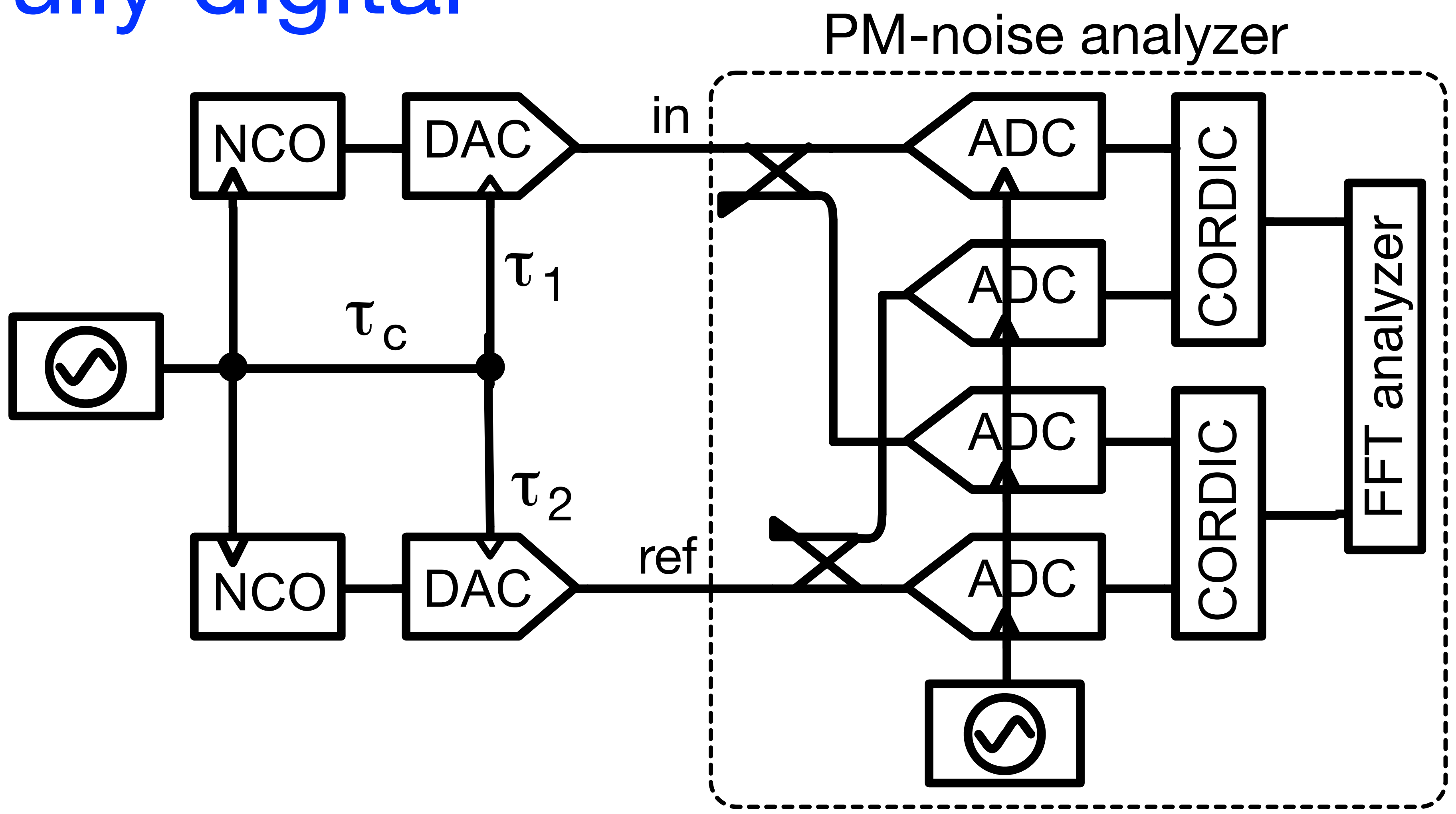
Traditional Measurement Methods

Saturated mixer



- Low output power, ≈0 dBm, the mixer takes ≈10 dBm —> amplifier
- Some commercial instruments have only one input (FSWP, E5052A), ref 1 and ref 2 come from internal synthesizers

Fully digital

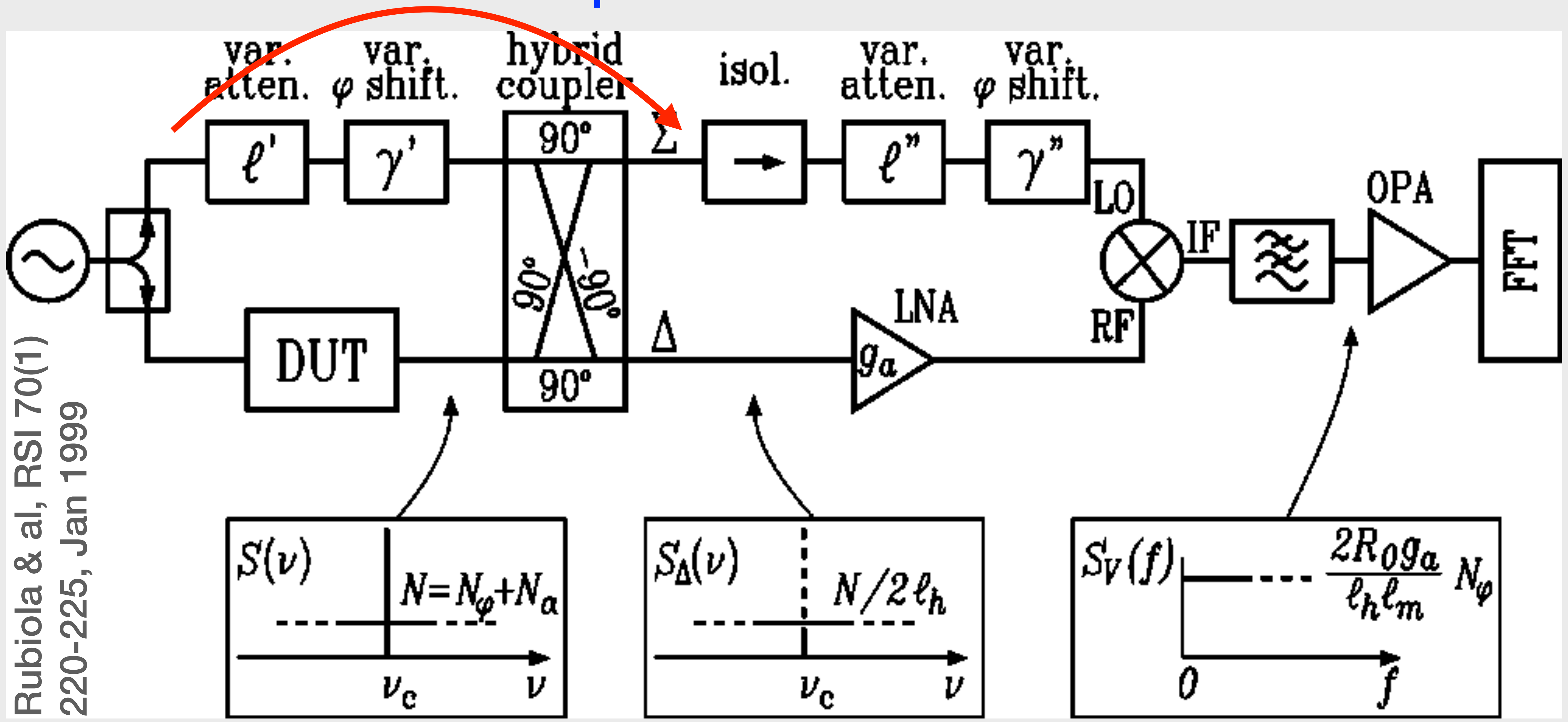


- DACs and DDSs have higher resolution and clock frequency than ADCs
- At least 100 averages for 10 dB noise rejection, 1000 for 15 dB
- How long does it take going down to ≤ 1 mHz?

Cross-spectrum and averaging —> known problems **Alternative?**

Bridge (Interferometric) AN-PN Measurement

alternate LO path



- Suppress the carrier
- Amplify and detect the noise sidebands
- PN results from (detected noise) / carrier

Rubiola & al, RSI 70(1) 220-225, Jan 1999
Also:
Ivanov, IEEE T UFFC 45(6) 1526-1536, Nov 1998
Rubiola, RSI 73(6) 2445-2457, Jun 2002

Noisy signal

$$v(t) = V_0 \cos(\omega t) + x \sin(\omega t) - y \sin(\omega t)$$

$$\varphi = x/V_0 \qquad \alpha = y/V_0$$

Suppressed carrier

$$v_\Delta(t) = x \sin(\omega t) - y \sin(\omega t)$$

Synchronous detection

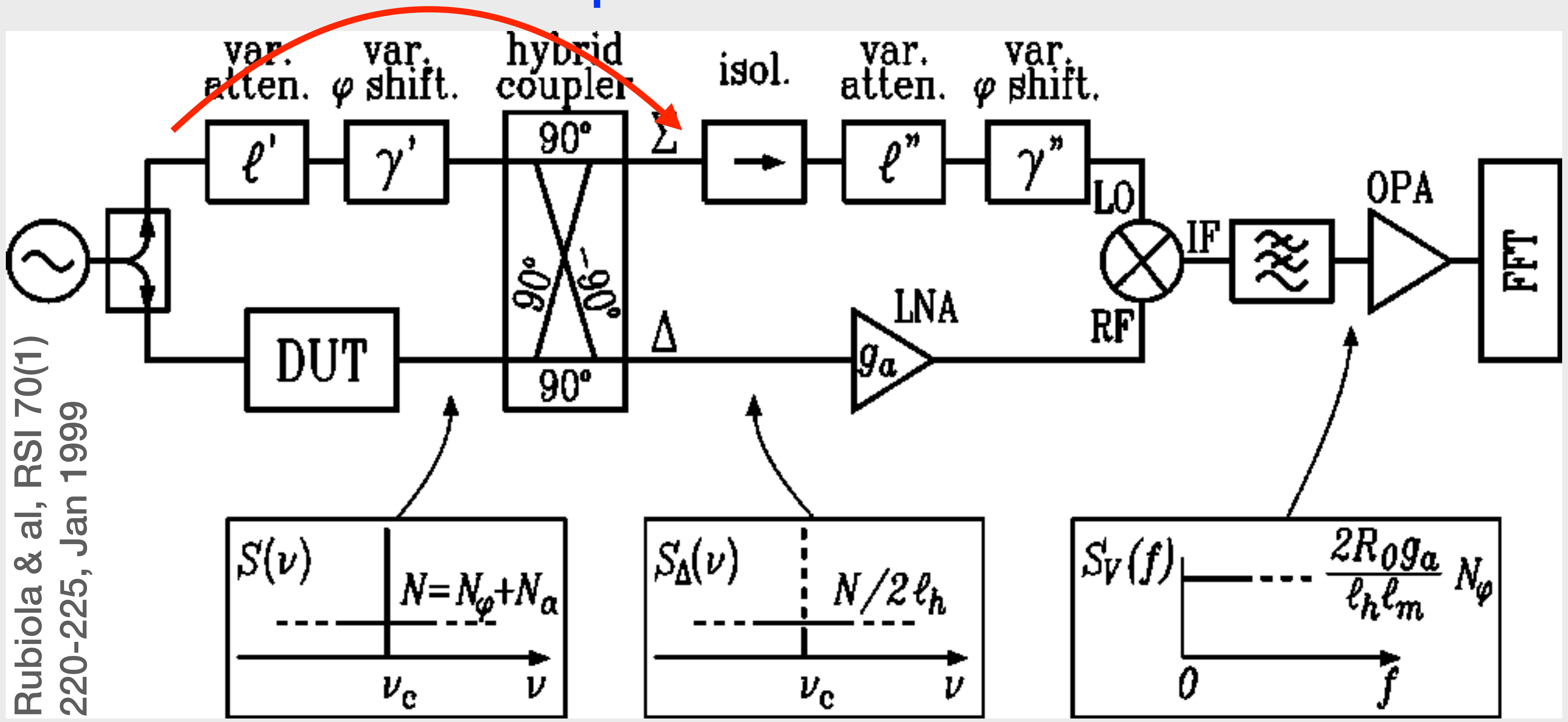
$$[v_\Delta\, 2\cos(\omega T)] * h_{lp} = x$$

$$[v_\Delta\, 2\sin(\omega T)] * h_{lp} = y$$

Use the detected x and y to estimate AM and PM noise

Bridge (Interferometric) AN-PN Measurement

alternate LO path



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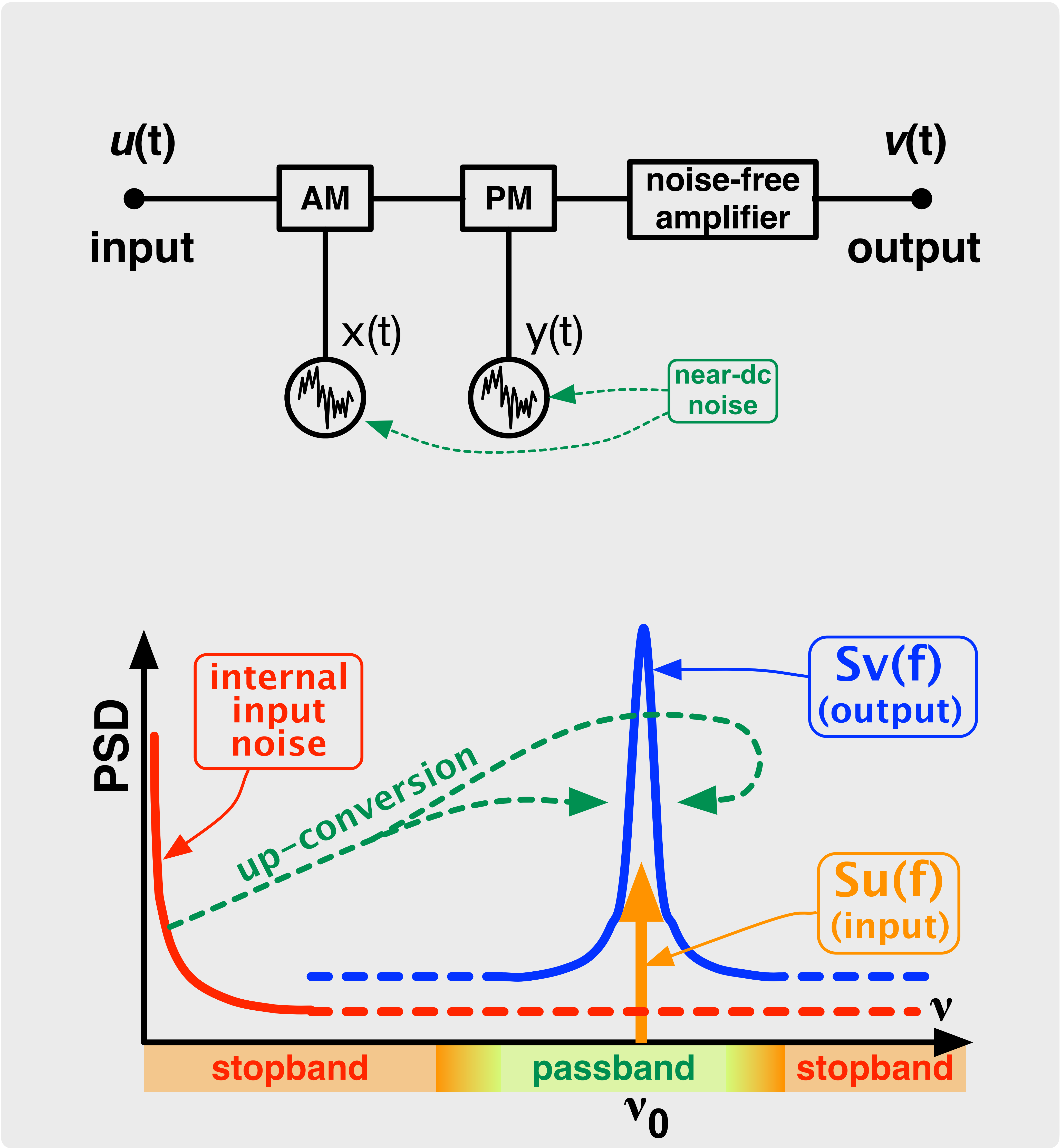
Benefits

- Lowest background noise
  - Amplifier NF —> white noise
  - No up-conversion of near-dc $1/f$ with high carrier rejection
- Low 50-60 Hz pickup due to Microwave gain before detection
- The noise in the LO arm is rejected (amplification allowed)
- No AM noise pickup, as in the saturated mixer

Annoying

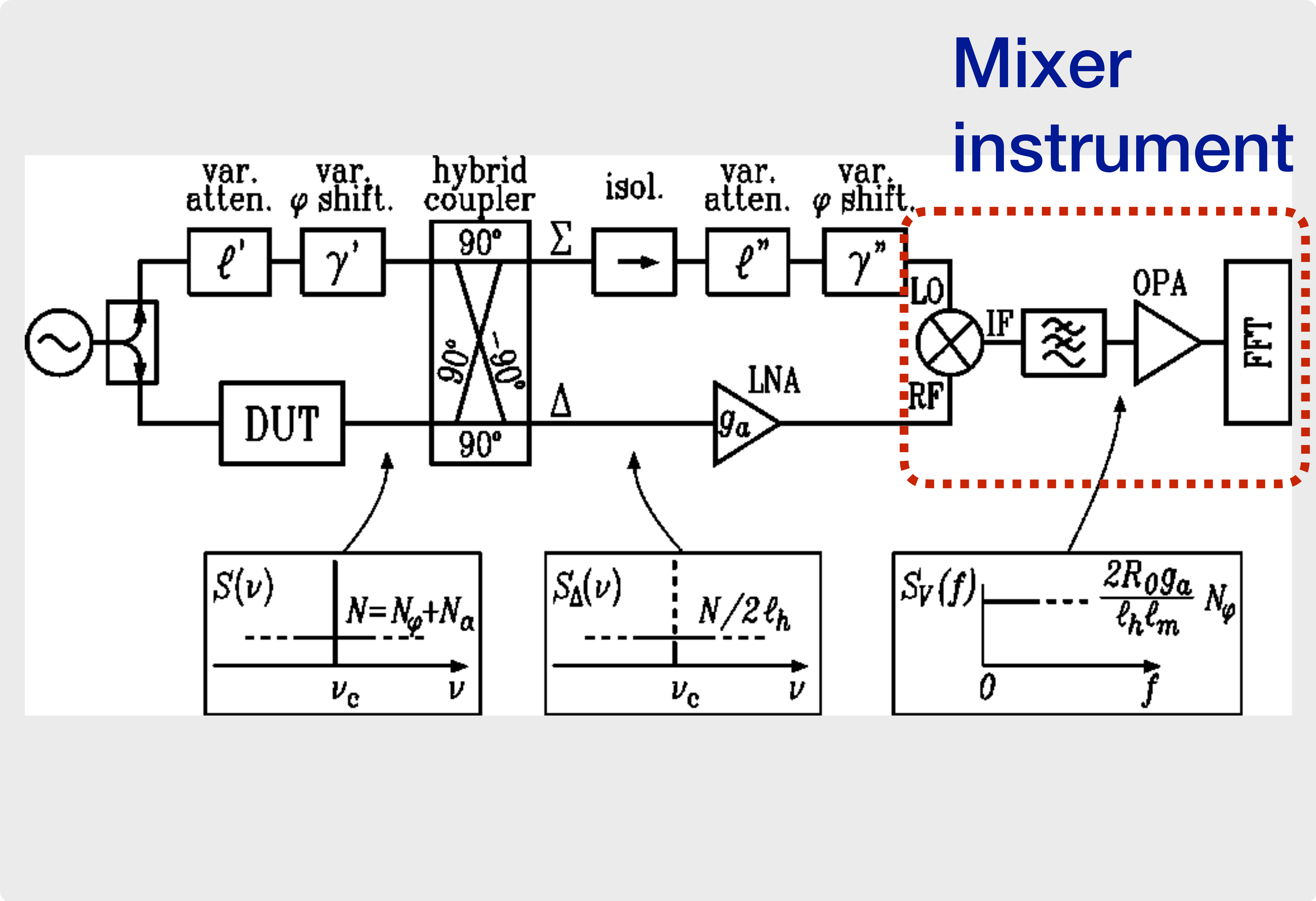
- Difficult alignment
- Narrow band setup

How Amplifier's Flicker Works



- No carrier
  - White noise only in the RF region
  - Flicker in the near DC region
  - No RF flicker
- RF noise sidebands result from up-conversion
- AM and PM noise ≈ independent of carrier power

Bridge with a Commercial Instrument?



Saturated-mixer instruments

- Only some give full access to the mixer
- Modern instruments use two channels and correlation
    - One channel cannot be used alone

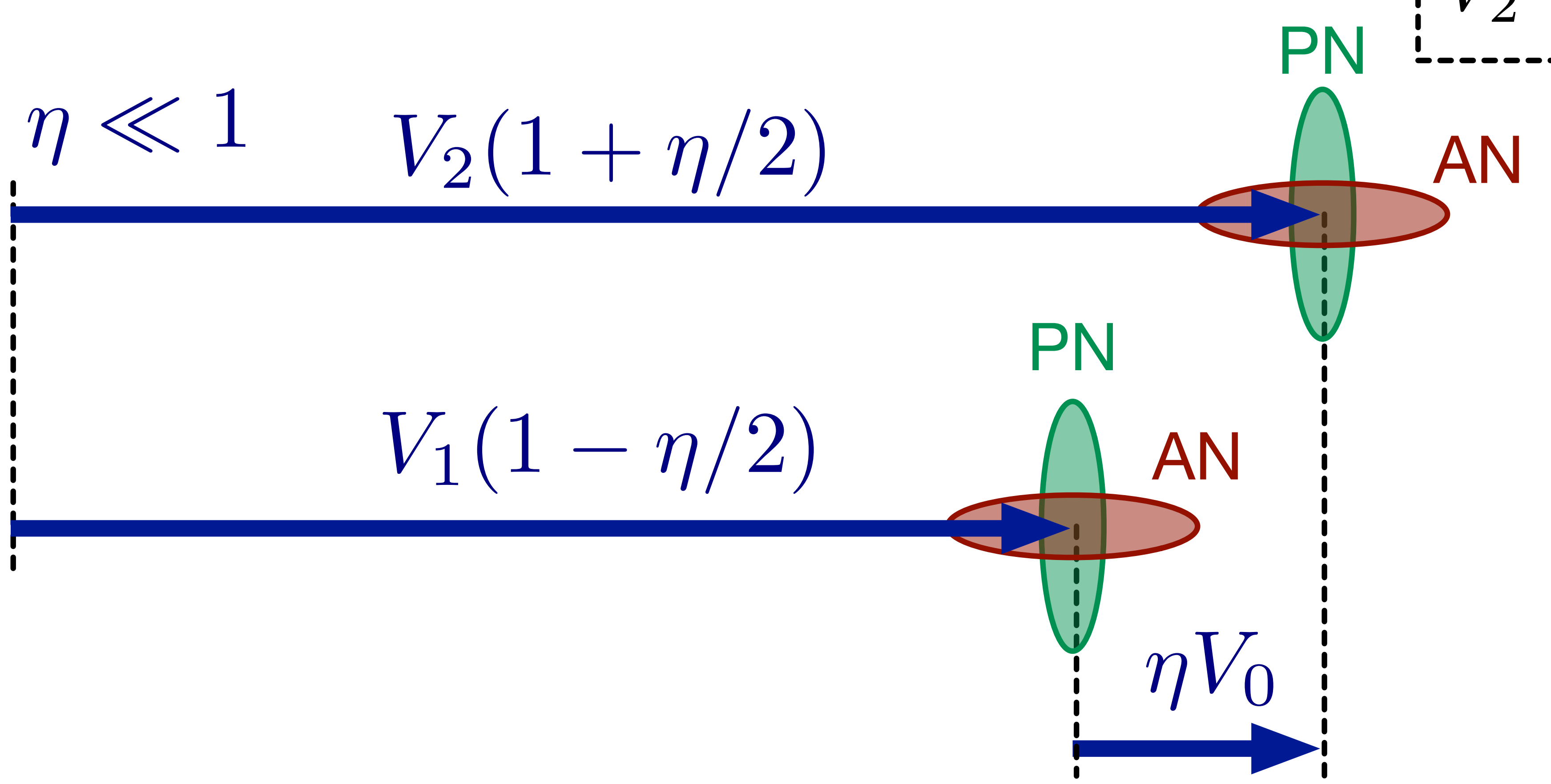
Digital instruments

- Nope, extracting the phase requires a carrier

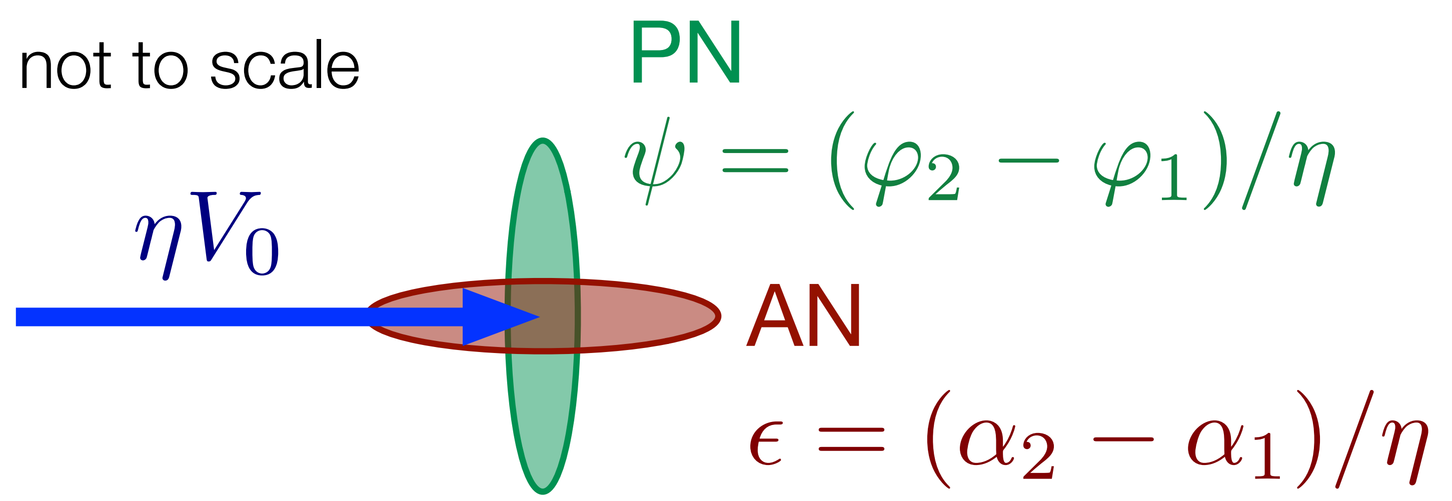
Controlled Amount of Residual Carrier

$$V_1 = V_0 e^{\alpha_1 + j\varphi_1}$$
$$V_2 = V_0 e^{\alpha_2 + j\varphi_2}$$

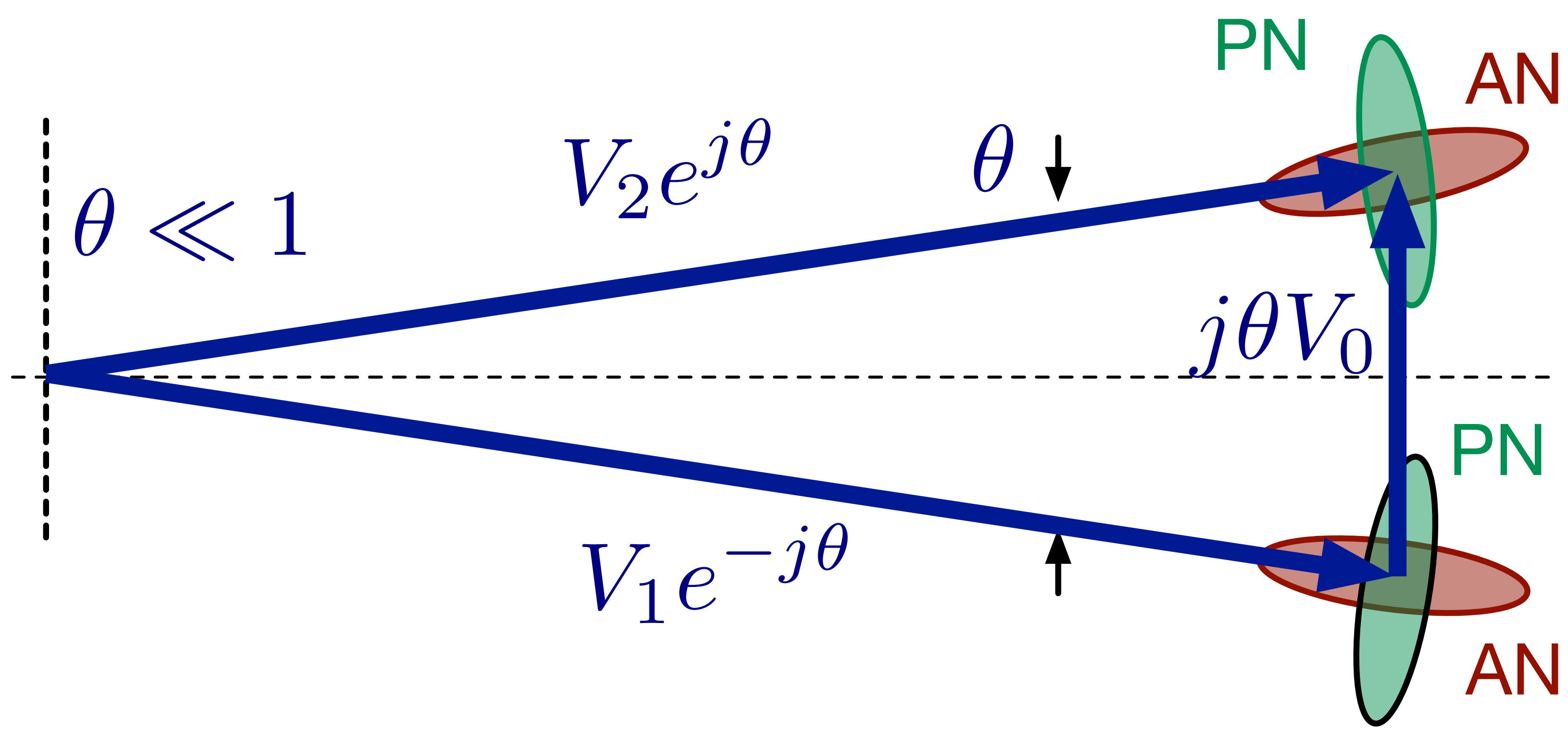
(A) AM and PM amplification

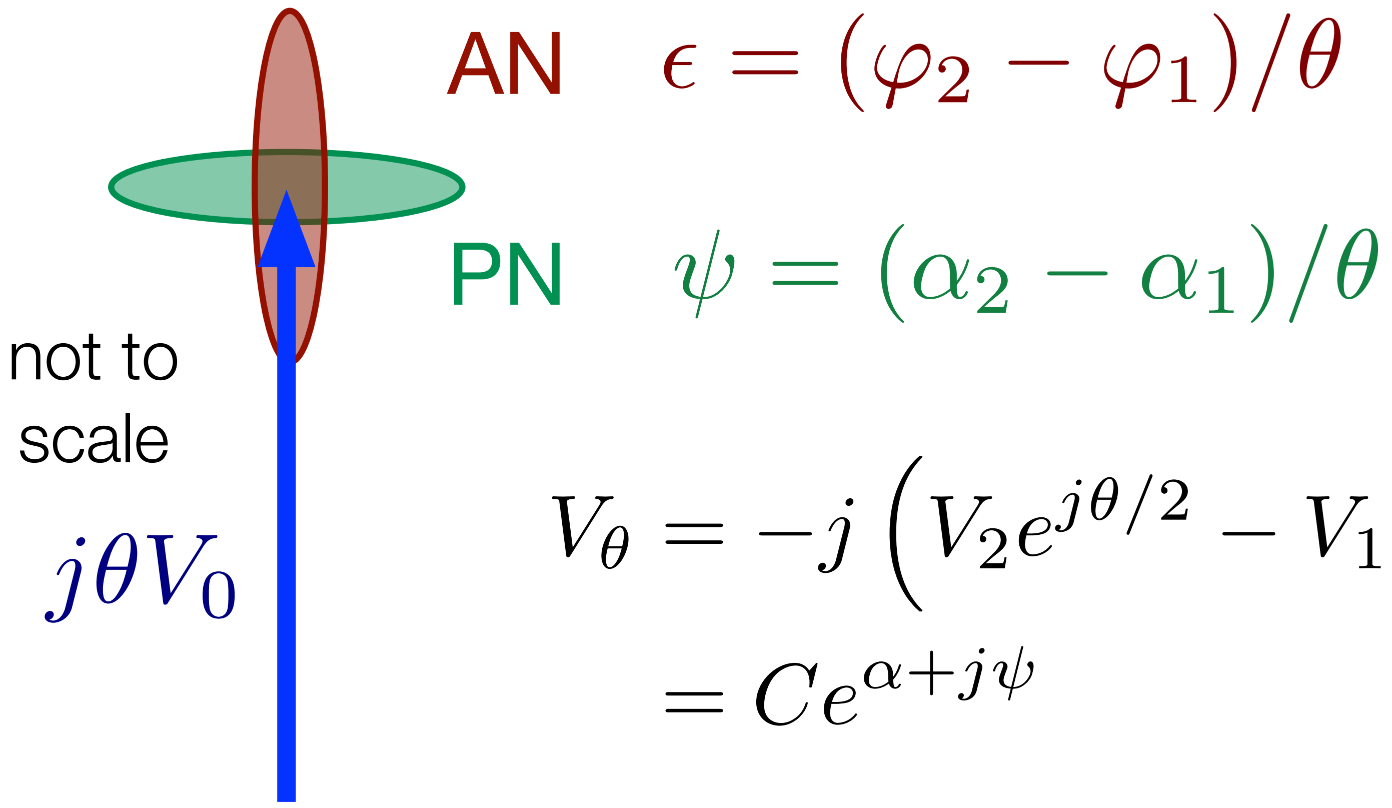


$$V_\eta = V_2(1+\eta/2) - V_1(1-\eta/2) = Ce^{\alpha + j\psi}$$



(B) AM and PM cross-amplification

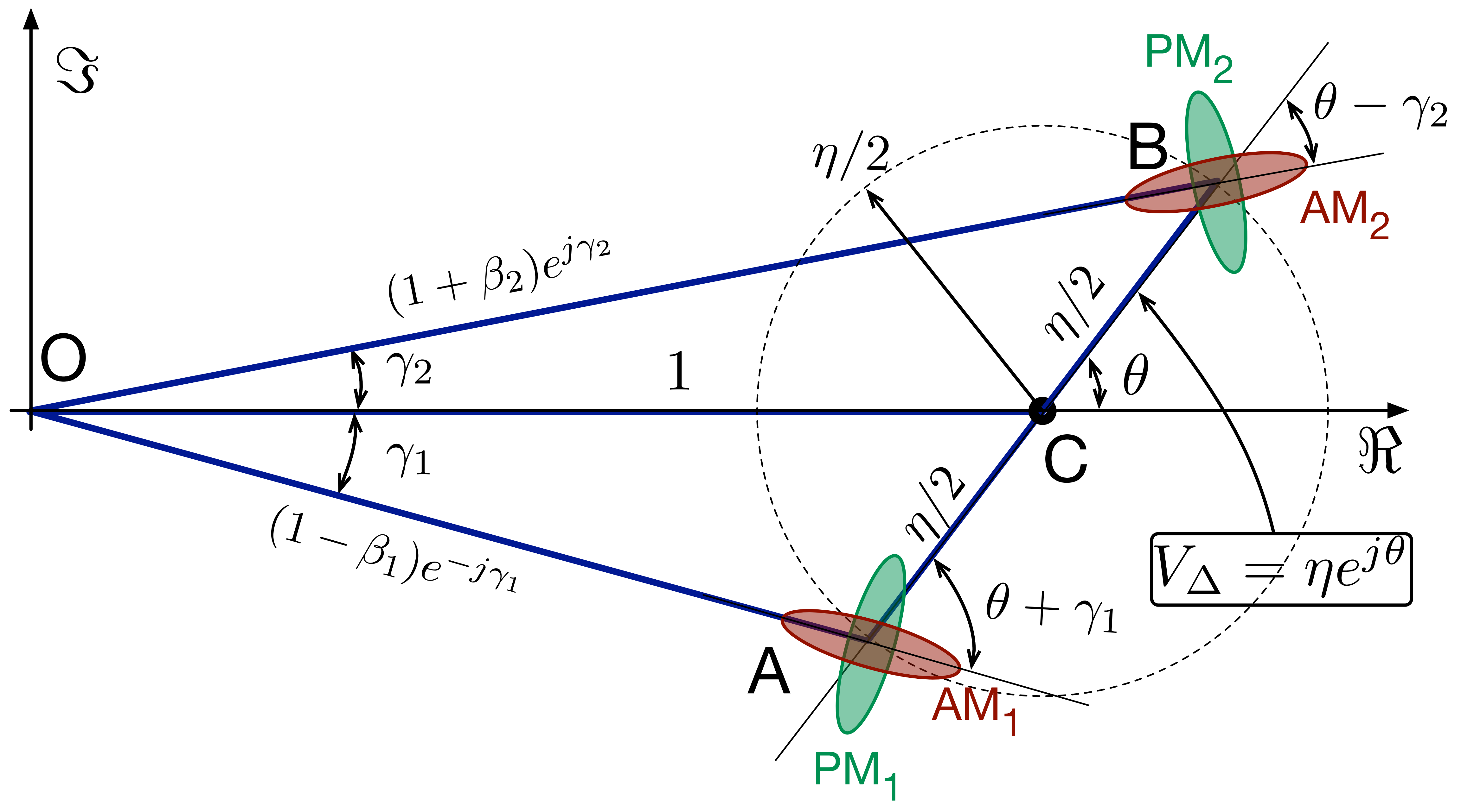




$$V_\theta = -j\left(V_2 e^{j\theta/2} - V_1 e^{-j\theta/2}\right) = Ce^{\alpha + j\psi}$$

$$V_\theta = j\left(V_2 e^{j\theta/2} - V_1 e^{-j\theta/2}\right) = Ce^{\alpha + j\psi}$$

Generalization, for Small η

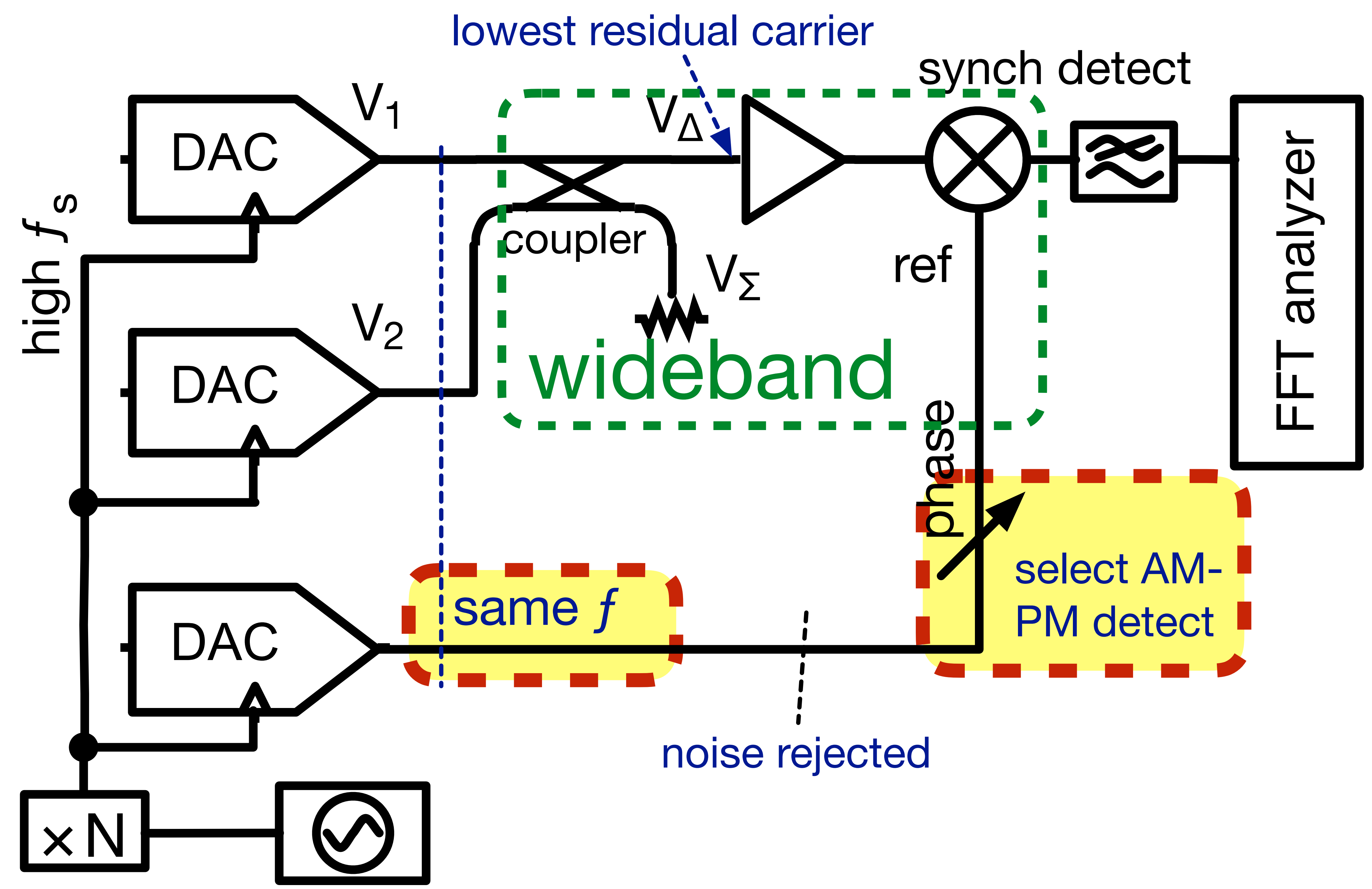


$$\begin{bmatrix}\epsilon\\ \psi\end{bmatrix} = \frac{1}{\eta}\begin{bmatrix}\cos(\theta) & \sin(\theta)\\ -\sin(\theta) & \cos(\theta)\end{bmatrix}\begin{bmatrix}\alpha_2-\alpha_1\\ \varphi_2-\varphi_1\end{bmatrix}$$

$$\begin{bmatrix}S_\epsilon\\ S_\psi\end{bmatrix} = \frac{1}{\eta^2}\begin{bmatrix}\cos^2(\theta) & \sin^2(\theta)\\ \sin^2(\theta) & \cos^2(\theta)\end{bmatrix}\begin{bmatrix}S_{\alpha 2}+S_{\alpha 2}\\ S_{\varphi 2}+S_{\varphi 2}\end{bmatrix}$$

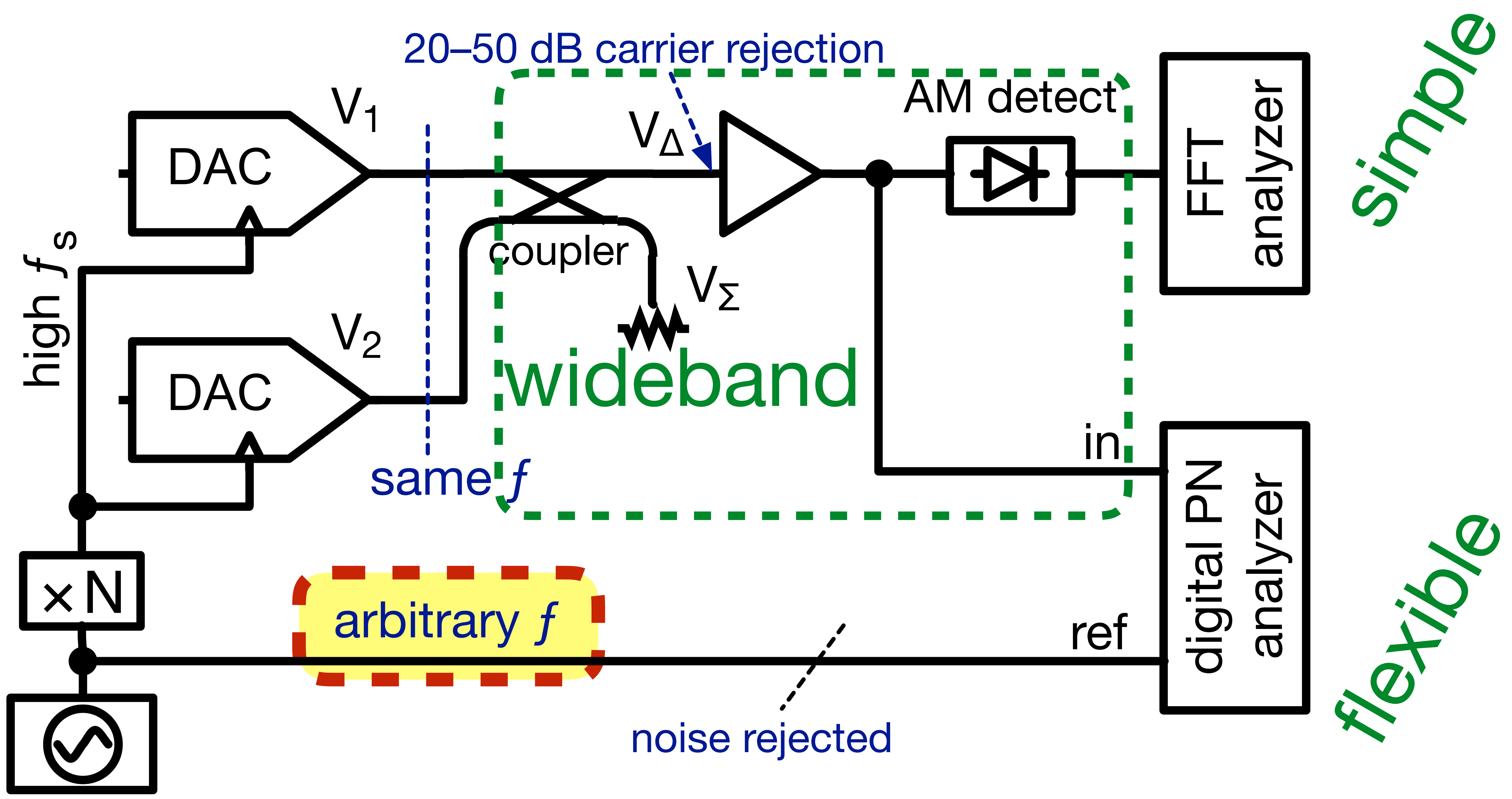
Bridge vs Modulation-Index Amplification

Bridge (Interferometer)



- Reference has the same frequency
- Reference phase selects AM/PM detection

Modulation-Index Amplification



- Arbitrary reference frequency
- Residual carrier selects AM/PM detection
- Either AM or PM detector
- Still low W & $1/f$ noise

Adjust amplitude and phase with the DDS control word

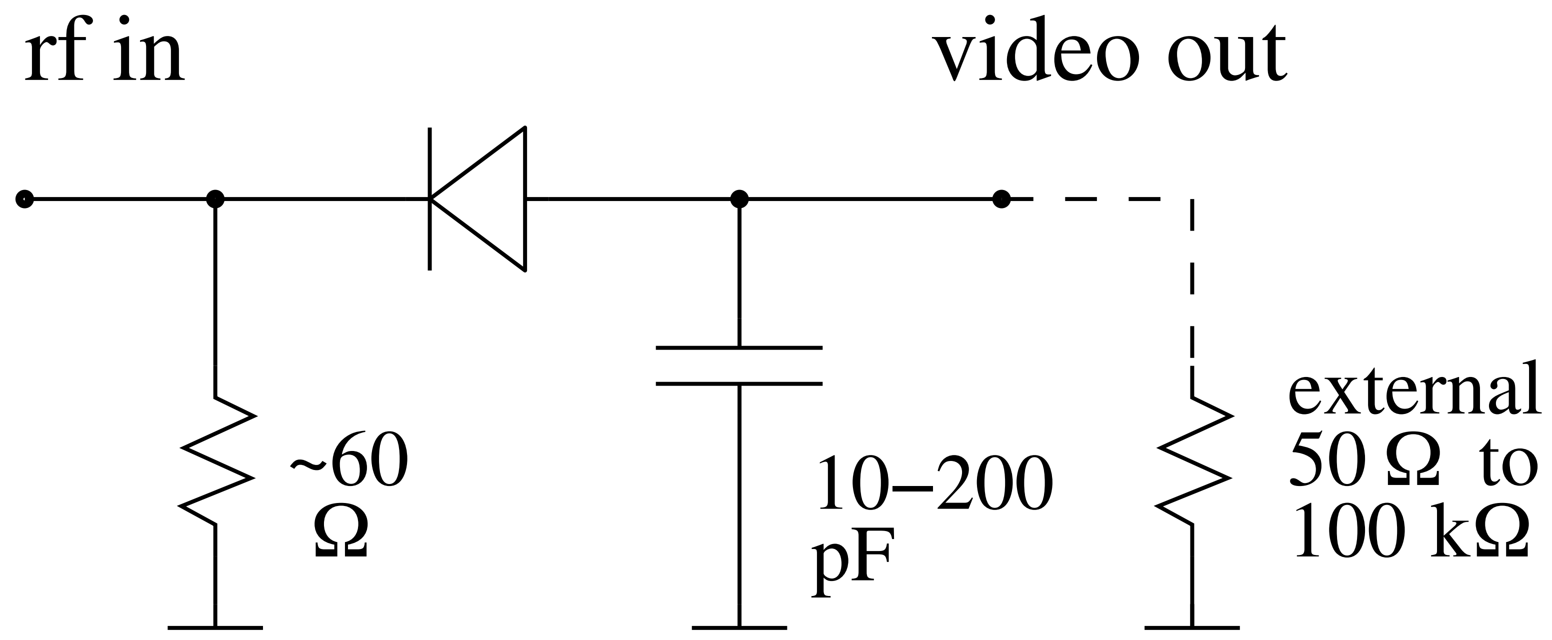
Detectors

All-digital phase meters

- Direct digitization of the RF signal
- High background noise
  - Heavy correlation and averaging
  - AVG only partially trusted
- Flexible, $f_{in} \neq f_{ref}$
- Laboratory (FEMTO, Holmes, Miles…)
- Brand (Jackson Lab, Symmetricom)

Simple amplitude detectors

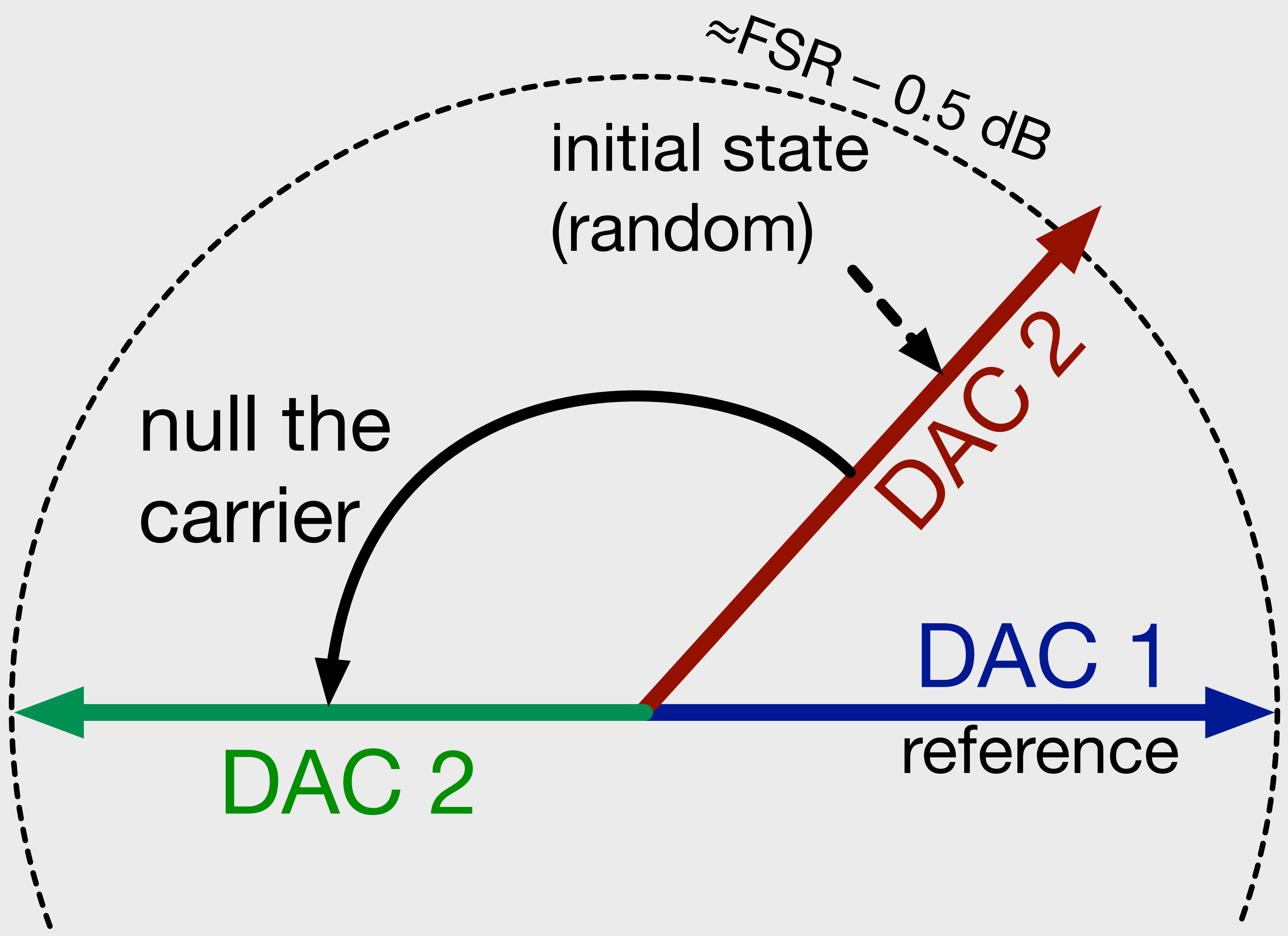
- Power detector & spectrum analyzer
- Simple lab implementation
- Fair background noise
  - No AN —> PN pollution
  - Optional correlation and averaging
- No known commercial instrument



E. Rubiola, arXiv:physics/0512082v1, Dec 2005

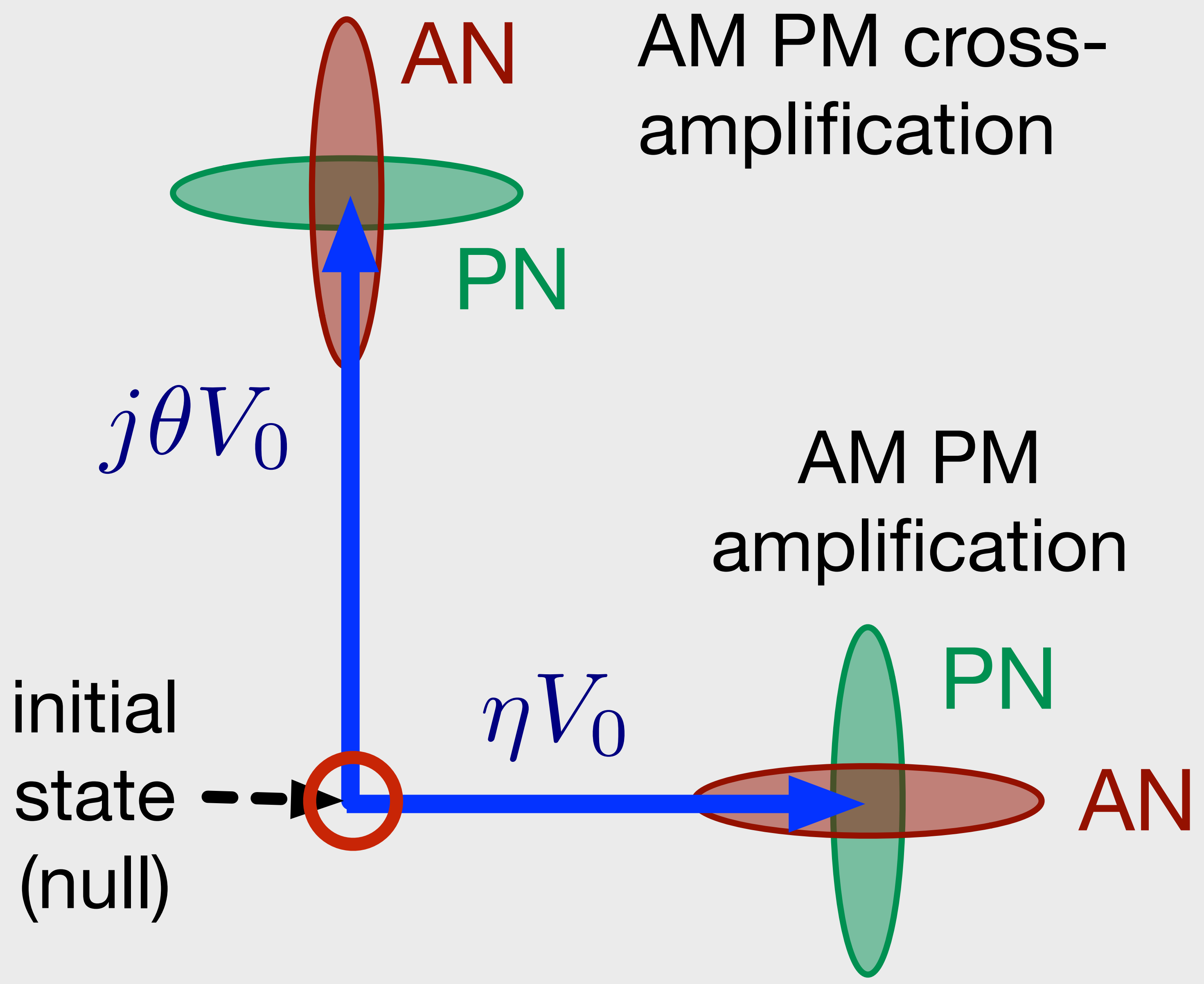
Alignment

(1) Full carrier suppression



- Sub-binary search by inspection with a spectrum analyzer
- Start from phase
- Alternate phase and amplitude

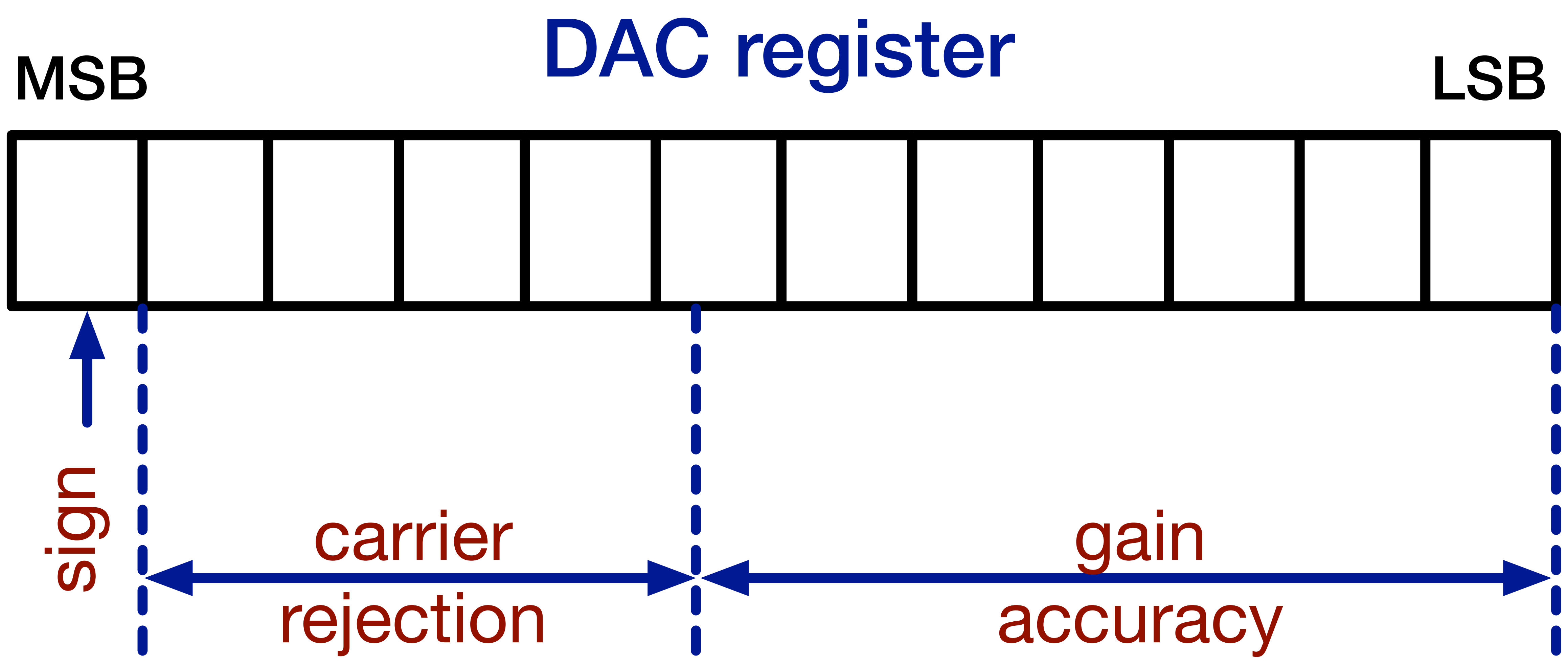
(2) Add a small carrier



- Real: modulation index amplification
- Imaginary —> AM-PM interchange and model-index ampli
- Best results with –20 to –40 dB

Accuracy

Trade accuracy vs carrier (and 1/*f*) rejection



- The upper bits are spent for carrier rejection
- Lower bits determine the accuracy of the residual carrier

Example, ENOB = 12

20 dB rejection

0.1 —> 3.33 bits

Accuracy 12–1–3.33 = 7.67 bits

0.5% —> 0.04 dB

Probably not needed for AN / PN

40 dB rejection

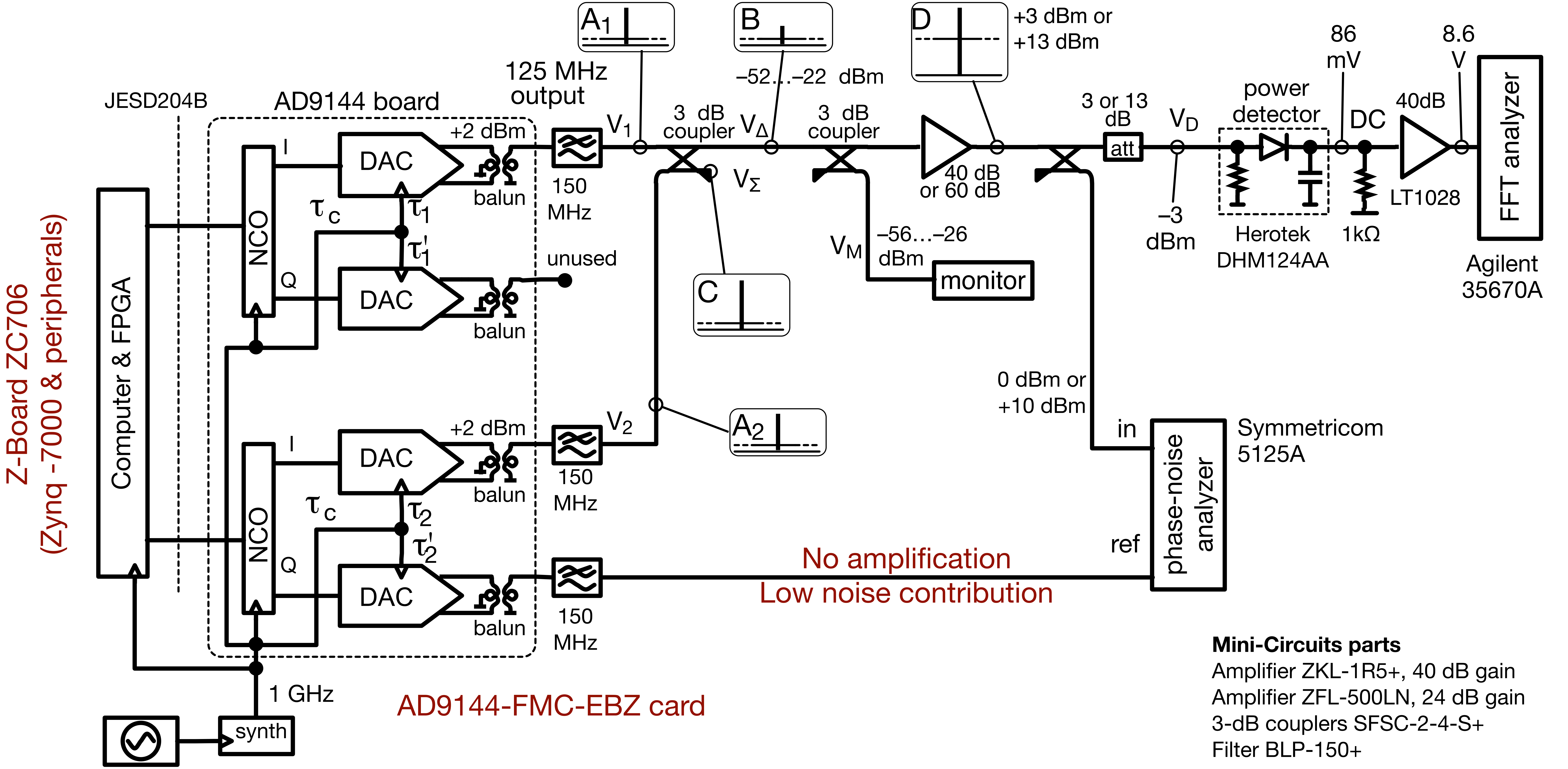
0.01 —> 6.67 bits

Accuracy 12–1–6.67 = 4.33 bits

5% —> 0.4 dB

Still OK for AN / PN

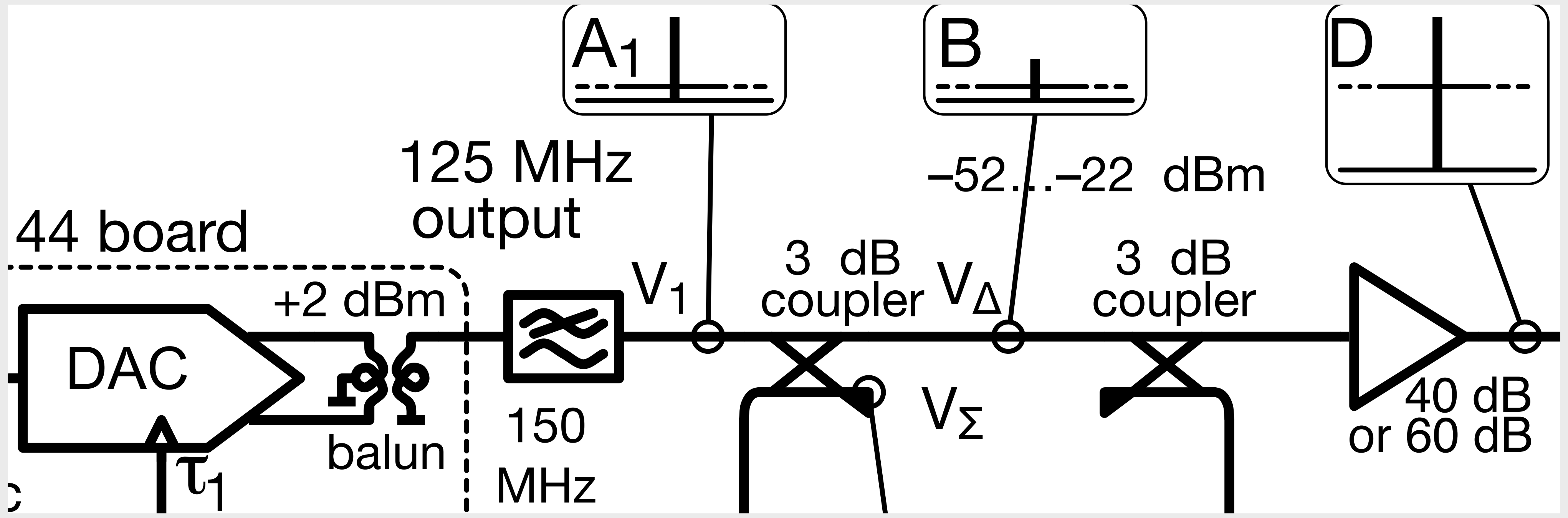
The Full Scheme



Mini-Circuits parts
Amplifier ZKL-1R5+, 40 dB gain
Amplifier ZFL-500LN, 24 dB gain
3-dB couplers SFSC-2-4-S+
Filter BLP-150+

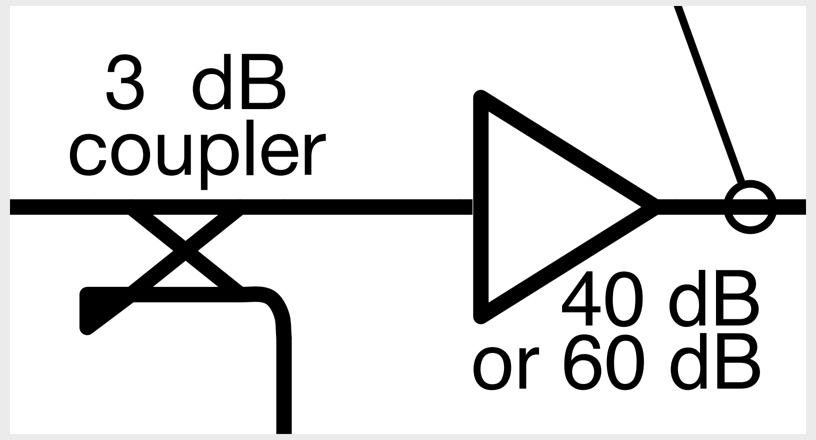
Background Noise

White PM Noise



- DAC output +2 dBm
- Amplifier input –5 dBm (7 dB loss DAC —> amplifier)
- Thermal energy –174 dBm/Hz
- Ampli noise figure F = 2 dB
- $S\varphi = S\alpha = -167$ dBrad²/Hz (–174+5+2)

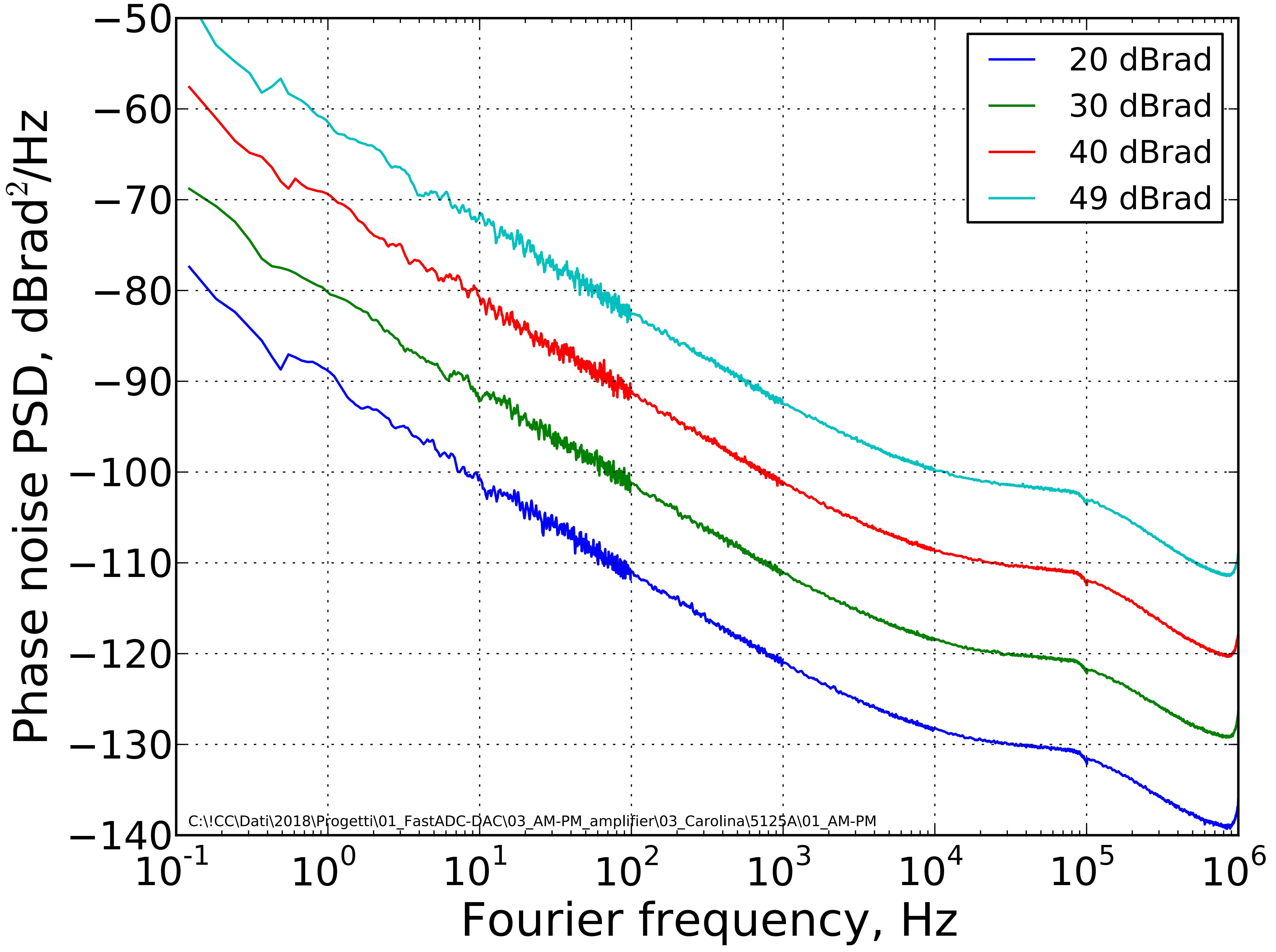
Flicker PM Noise



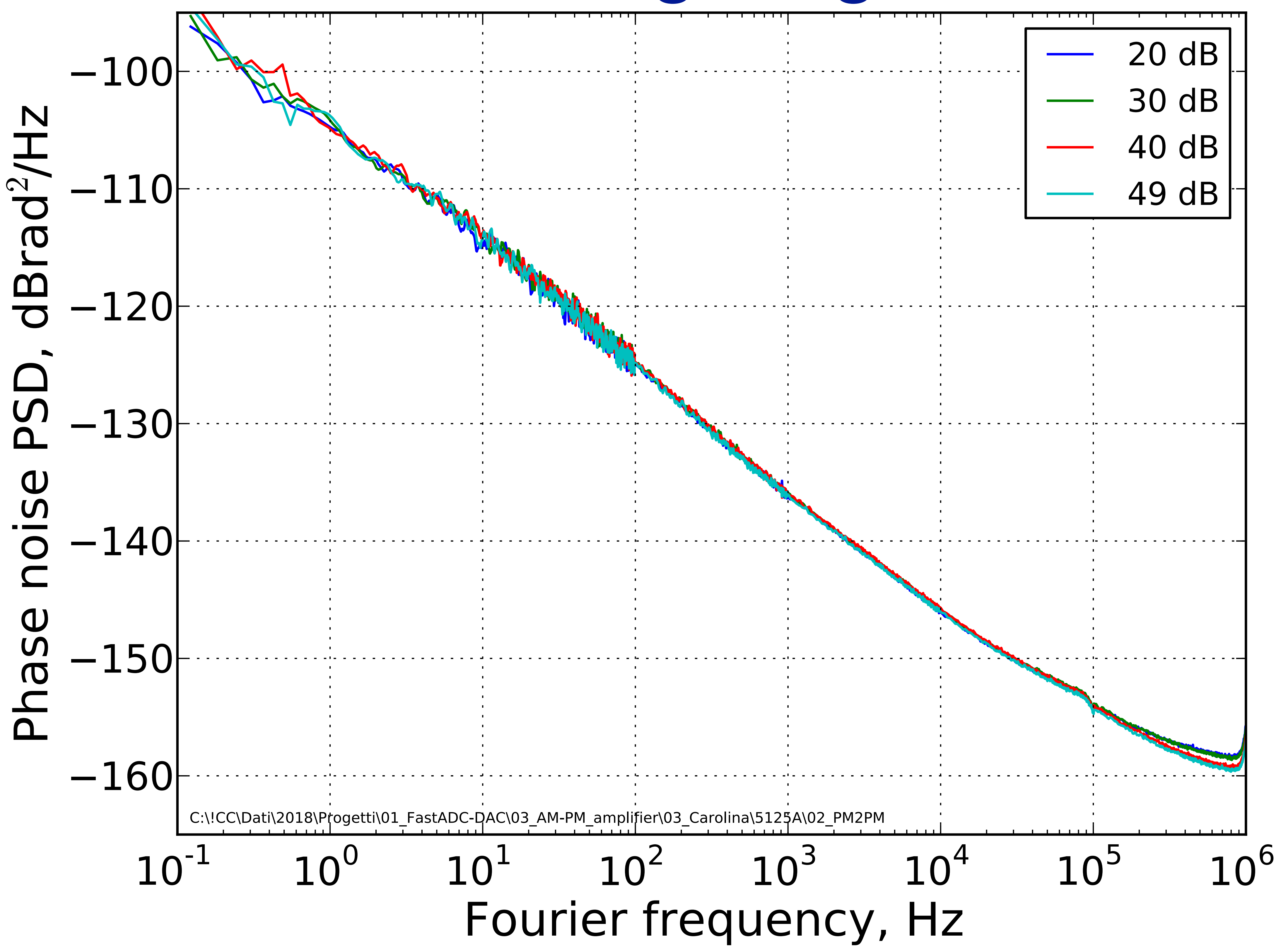
- Ampli flicker –130 dB at 1 Hz (conservative)
- Carrier rejection 30 dB
- Expected –160 dBrad²/Hz at 1 Hz
- More severe limitation from the noise analyzer

PN Measured with the Phase Detector

Raw data



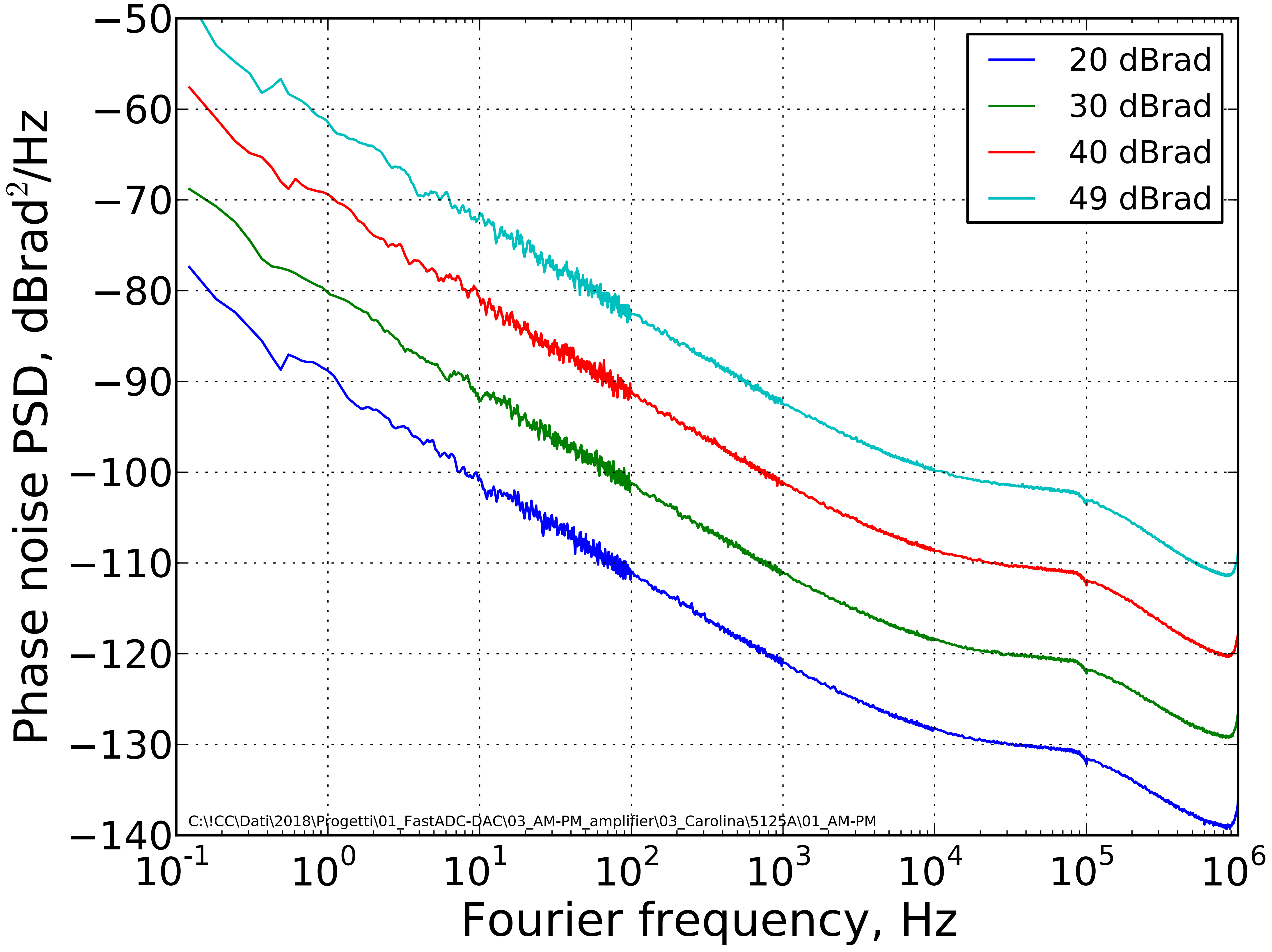
Accounting for gain



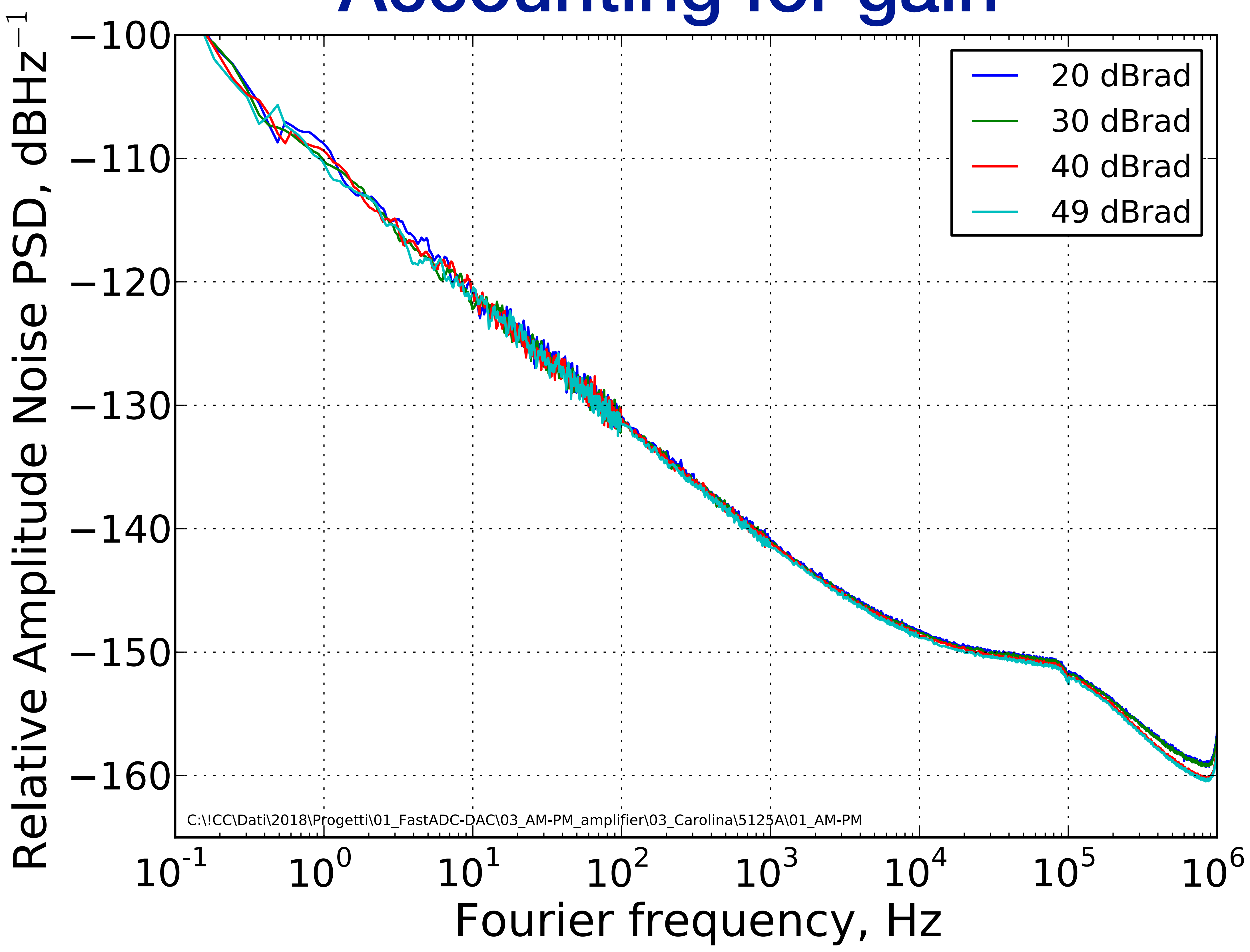
Phase noise measured with the phase meter (same information)

AN Measured with the Phase Detector

Raw data



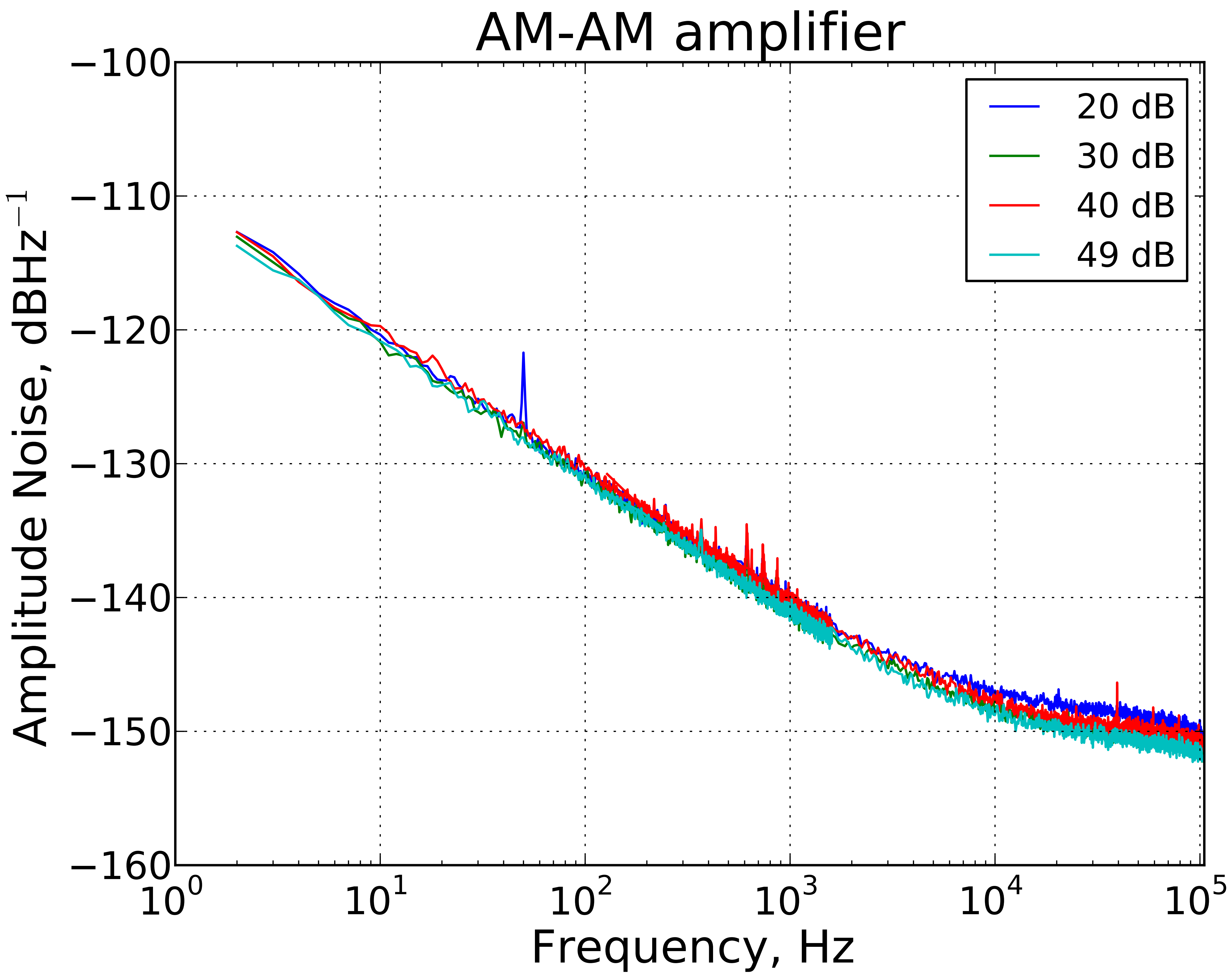
Accounting for gain



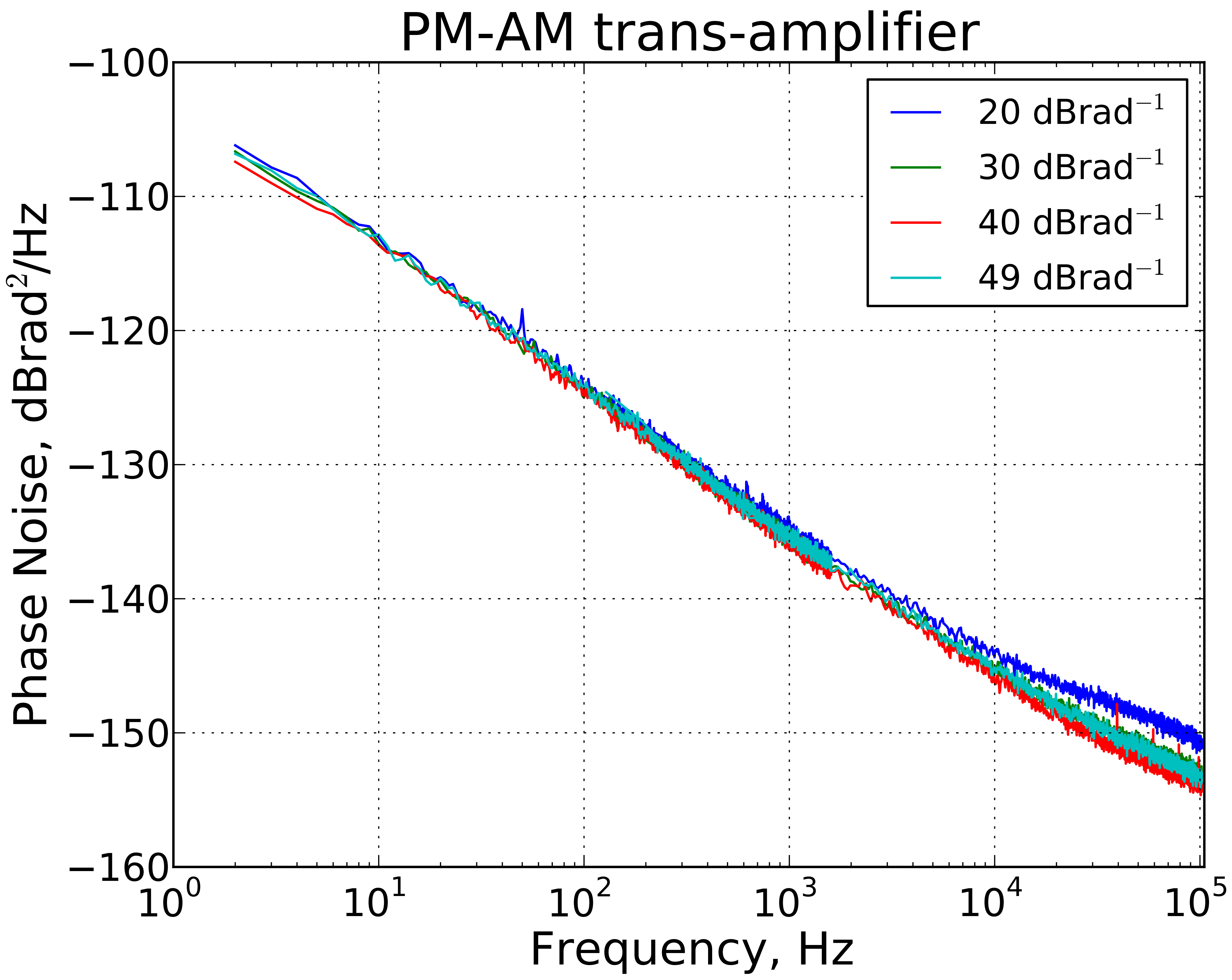
Amplitude noise measured with the phase meter (same information)

AN & PN Measured with Power Detector

AM Noise

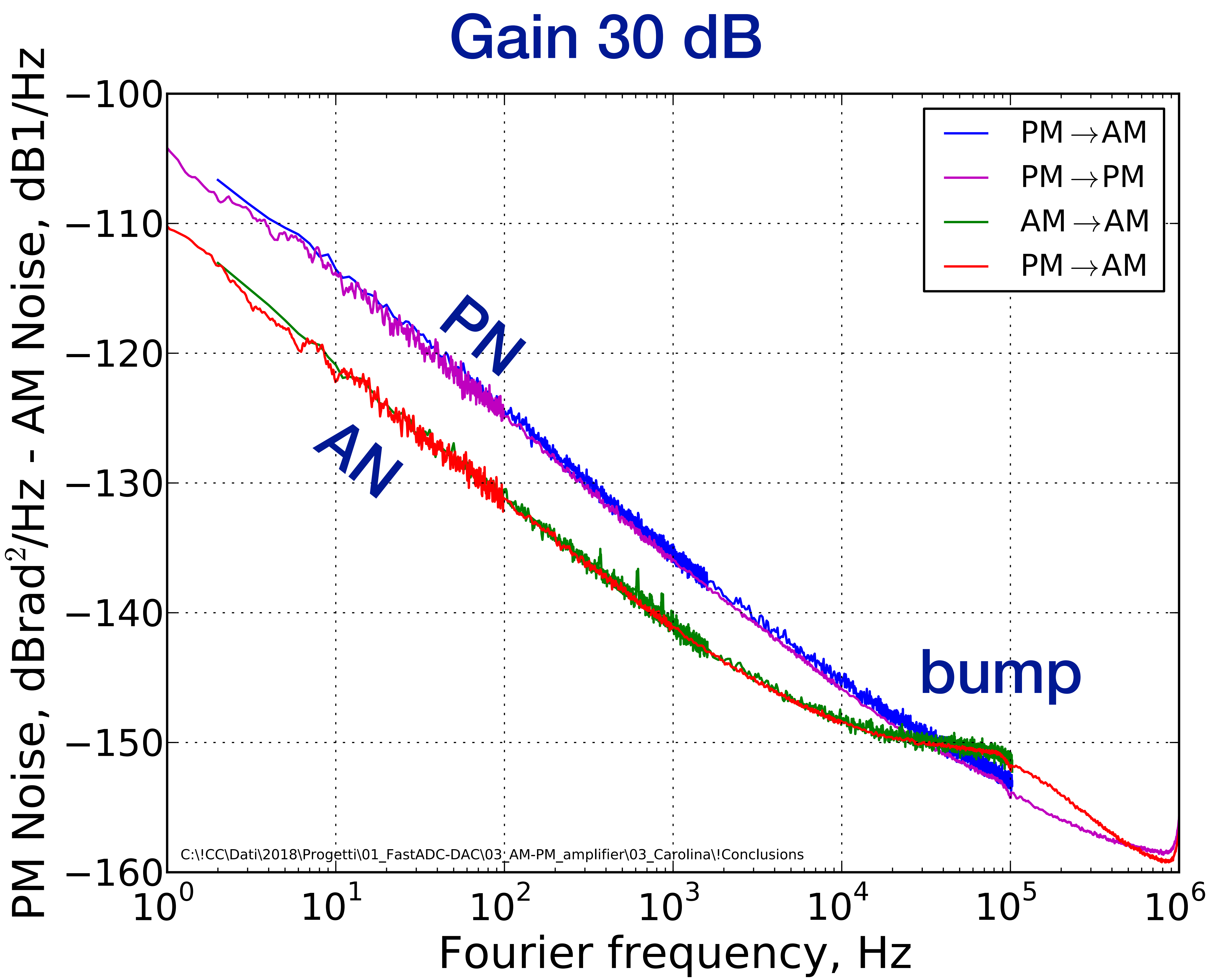


PM Noise

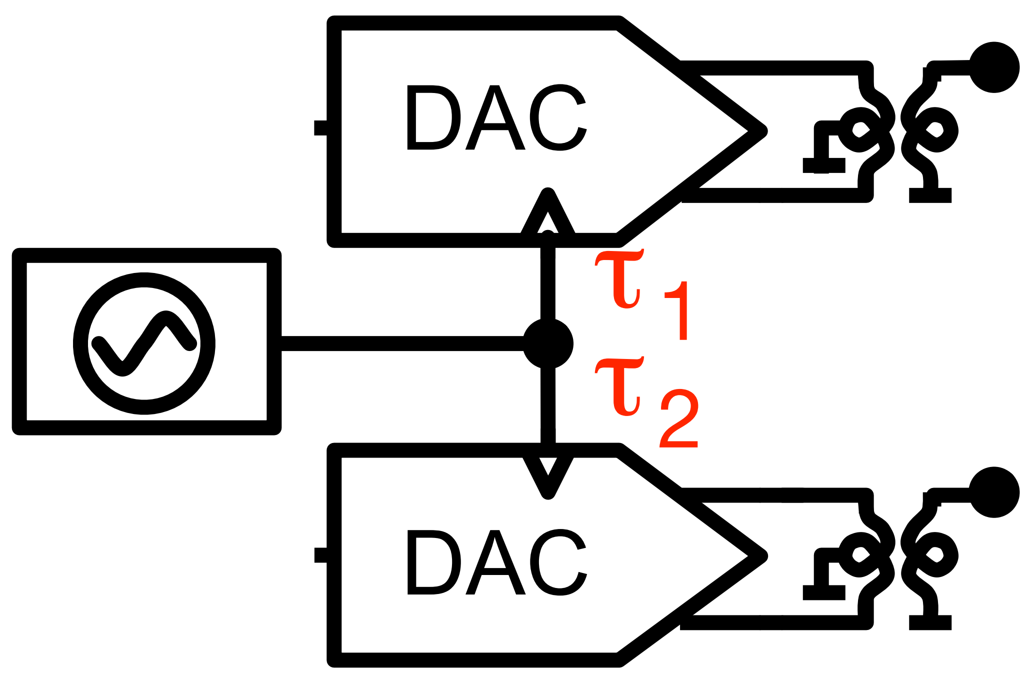


The dispersion of results will be fixed

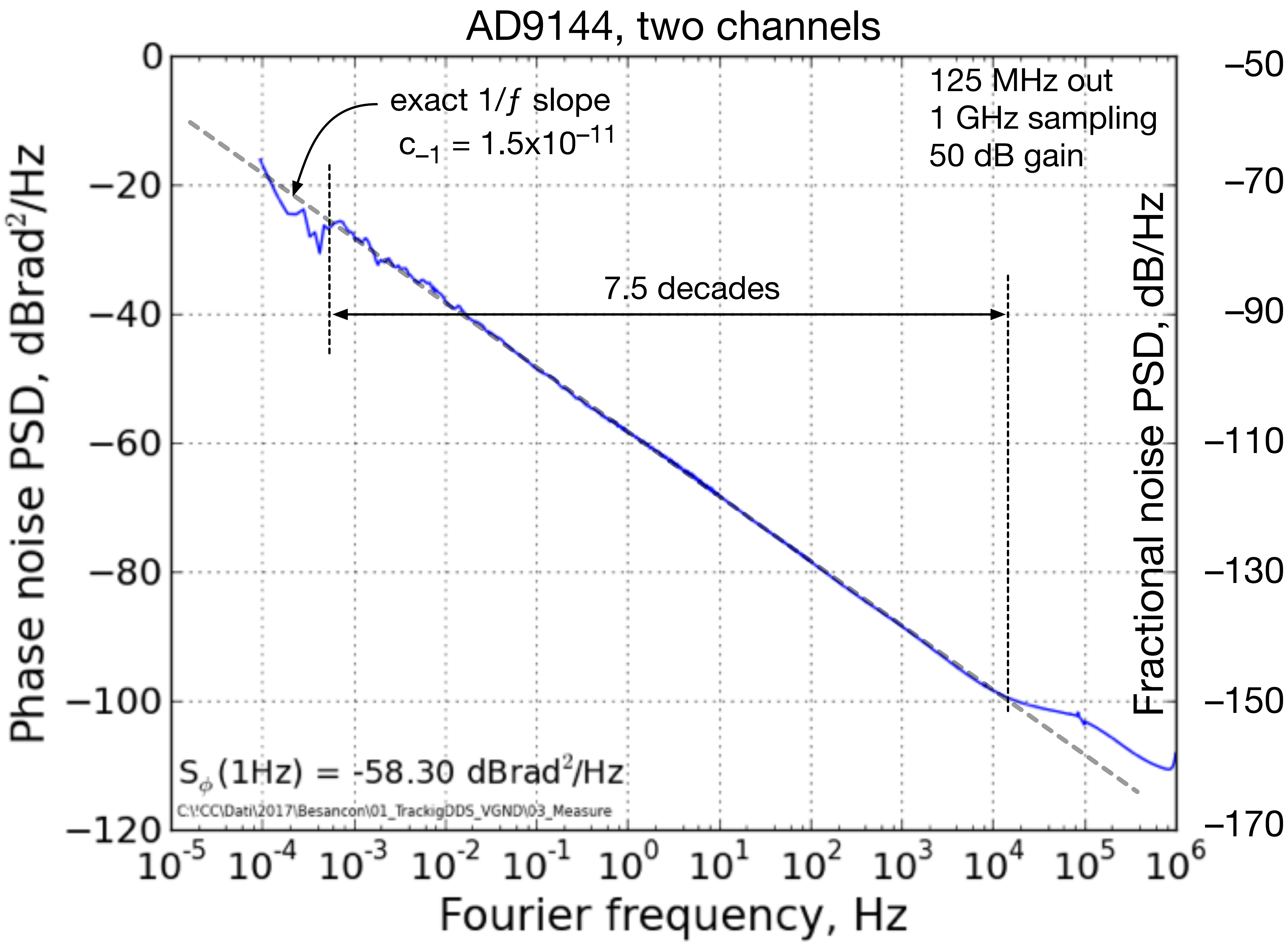
Comparison



- Best results with 30 dB gain
- The results with the two methods match perfectly
- Clock jitter contributes to the difference, PN > AN



Exact 1/ƒ PN Over 7.5 Decades



- PN between two outputs (Microsemi 5125A)
- Unfortunately, the axes were misaligned (θ≈60º)
- Exact 1/ƒ slope, ±1 dB over 7.5 decades
- Insufficient averaging below 0.7 mHz

How We Solved the Angle

For small η

$$\begin{bmatrix} S_\epsilon \\ S_\psi \end{bmatrix} = \frac{1}{\eta^2} \begin{bmatrix} \cos^2(\theta) & \sin^2(\theta) \\ \sin^2(\theta) & \cos^2(\theta) \end{bmatrix} \begin{bmatrix} S_{\alpha 2} + S_{\alpha 2} \\ S_{\varphi 2} + S_{\varphi 2} \end{bmatrix}$$

- Use trusted $1/f$ levels for calibration
- Gain 50 dB
  - Reasonably trusted
  - Consistent lowest-noise point

Trusted, from the "comparison"

$$\eta^2 S_\psi = c_{-1}/f \text{ with } c_{-1} = 1.48 \times 10^{-11}$$

$$h_{-1} = 8 \times 10^{-12} \text{ and } b_{-1} = 3.4 \times 10^{-11}$$

Solve

$$c_1 = h_{-1}(1 - \cos^2\theta) + b_{-1}\cos^2\theta$$

Result

$$\theta = 1.03 \text{ rad } (59^\circ)$$

$$\eta^2 S_\psi = \left[0.74\, S_\alpha + 0.26\, S_\varphi\right]$$


Cyclone III, voltage supply
Noise Mechanism
Some
Facts
THORLABS

Conclusions

- Original method
    - Simple & highly suitable to DACs
    - Can be automated
- RF gain —> little/no need of correlation
- Results are consistent
    - Modulation-index amplification
    - AM <—> PM conversion
- Flicker observed over a wide span
    - Exact $1/f$, <1 dB discrepancy / 7.5 decades