French and Italian National Research Councils

Stability Measurement of 3 CSOs with Tracking DDSs and Two-Sample COV

C. E. Calosso[1], F. Vernotte[2], V. Giordano[2,3], C. Fluhr[4], B. Dubois[4], E. Rubiola[1,2,3]

1 INRIM, Torino, Italy, 2 Besançon Observatory, France
3 FEMTO-ST Institute, France, 4 FEMTO Engineering, France

http://rubiola.org

Motivations and Outline

10 GHz CSOs
- $2\times10^{-16}\ldots10^{-15}$ ADEV at $1\ldots10^5$ s
- Not tested at 100 MHz

TDDS
- $2\times10^{-14}/\tau$ ADEV at 100 MHz
- Statistical limit?

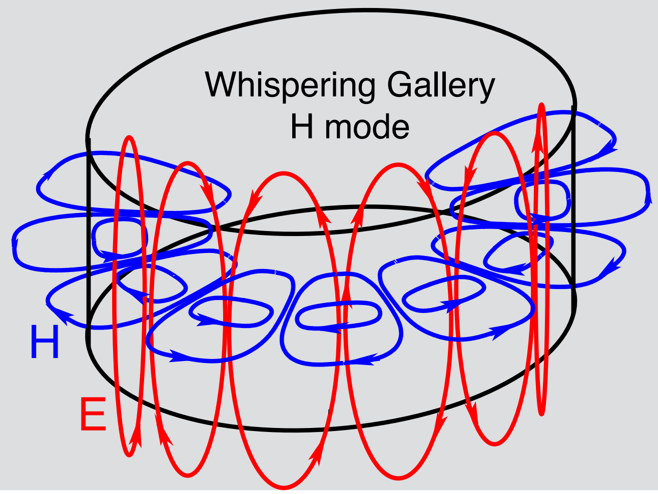
Statistics
- Scalable
- Challenging instrument and oscillators

Let’s put all things together and play

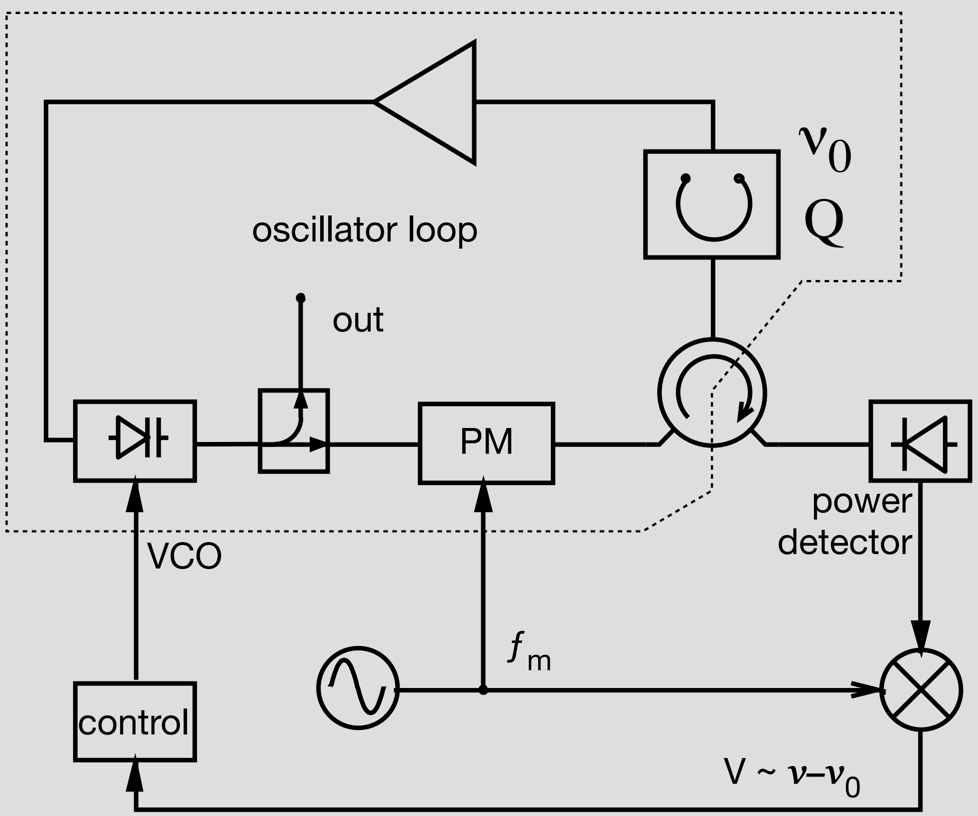
Liquid-He Sapphire Oscillator

Cr^{3+} Fe^{3+} doped
Al_2O_3 mono crystal
$\phi \approx 5$ cm, H ≈ 3 cm

10 GHz resonance
Q ≈ 2×10⁹ at 5–7 K

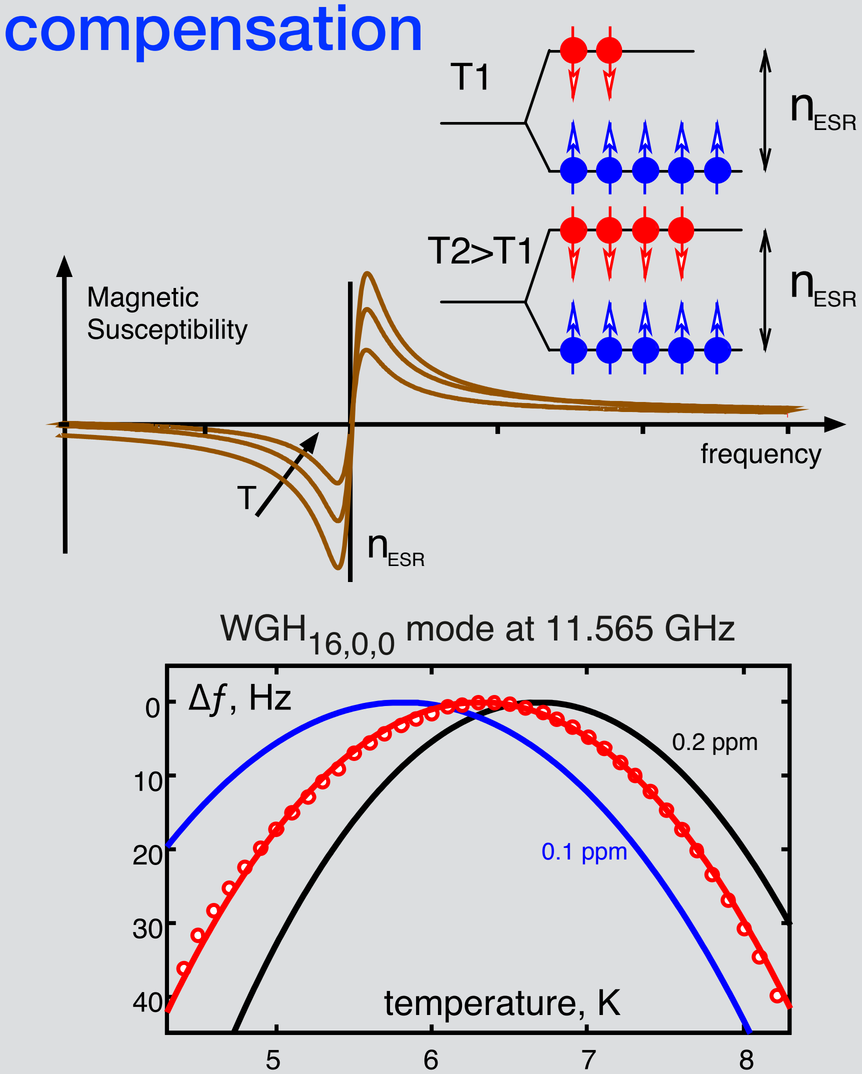


Pound-Galani Oscillator



- Pound frequency lock to the cavity
- The same cavity is used in the VCO

Paramagnetic temperature compensation

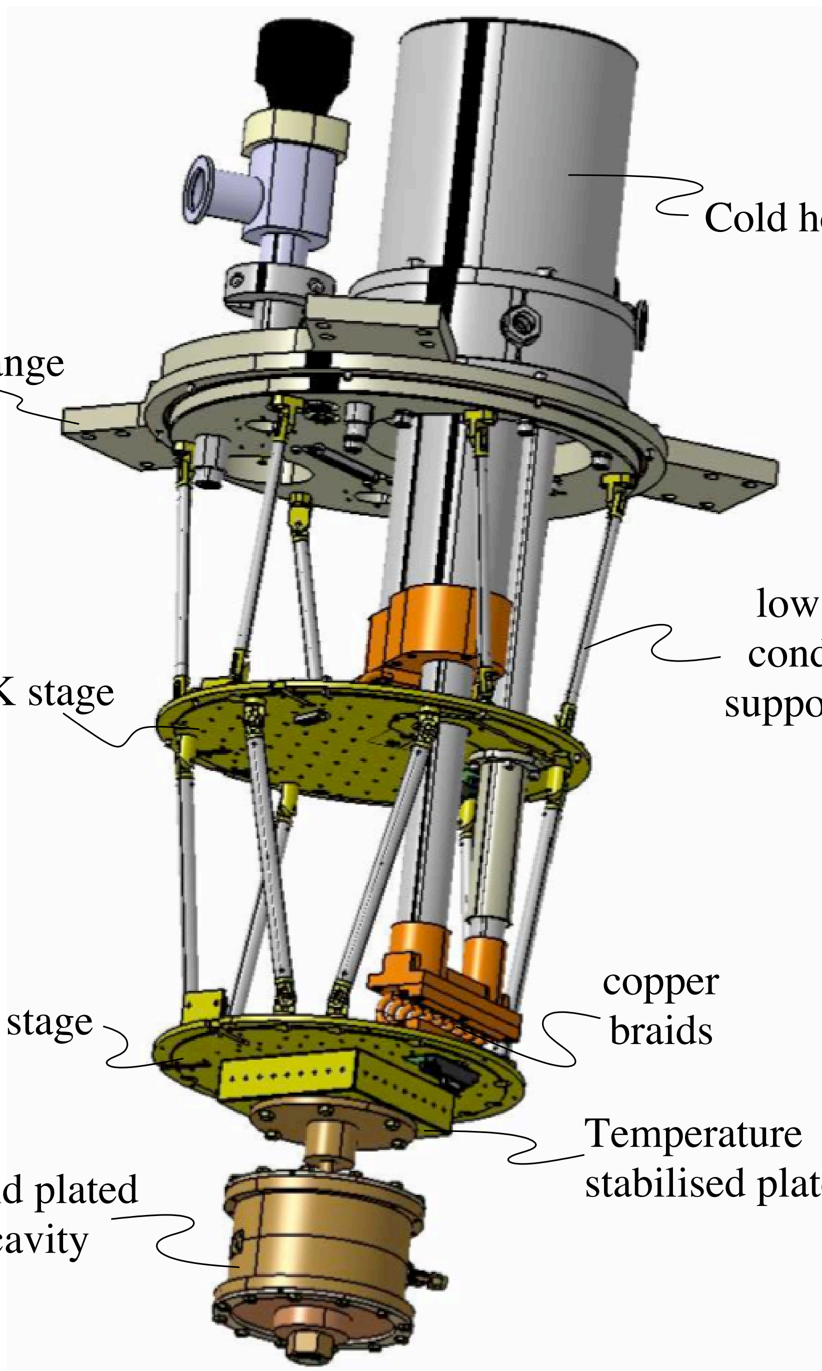


- 3 units operational
  - Transportable unit –> stability & noise validated after roundtrip
- 1 unit in progress
- μHz-resolution synthesis

(2 more units delivered to other labs)

Mechanical & Thermal Engineering

Low-vibe cryogenerator
< 2 µm displacement @ 1Hz



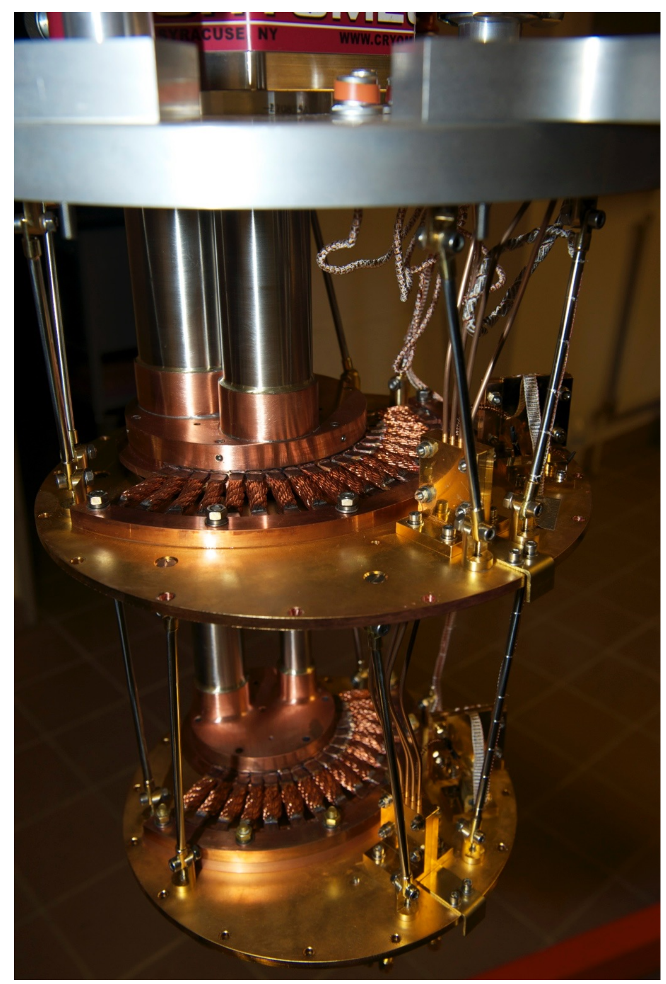
First generation: 6kW three-phase
Current generation: 3 kW mono-phase

Low acceleration sensitivity

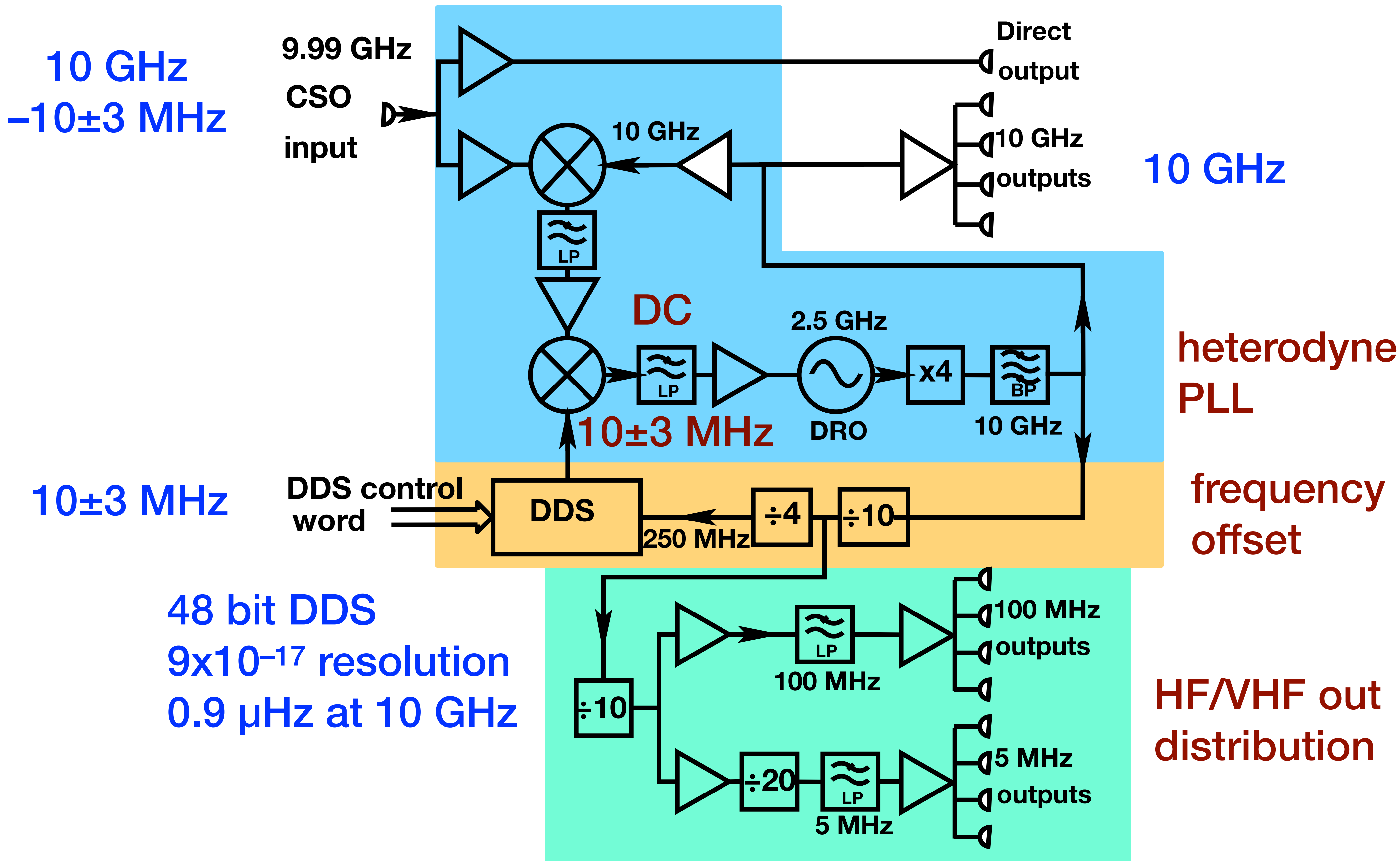
$$\frac{1}{\gamma}\frac{\Delta\nu}{\nu} \sim 3.2 \times 10^{-10}\,/g$$

Thermal ballast

Cold finger temperature stability 100 mK pk

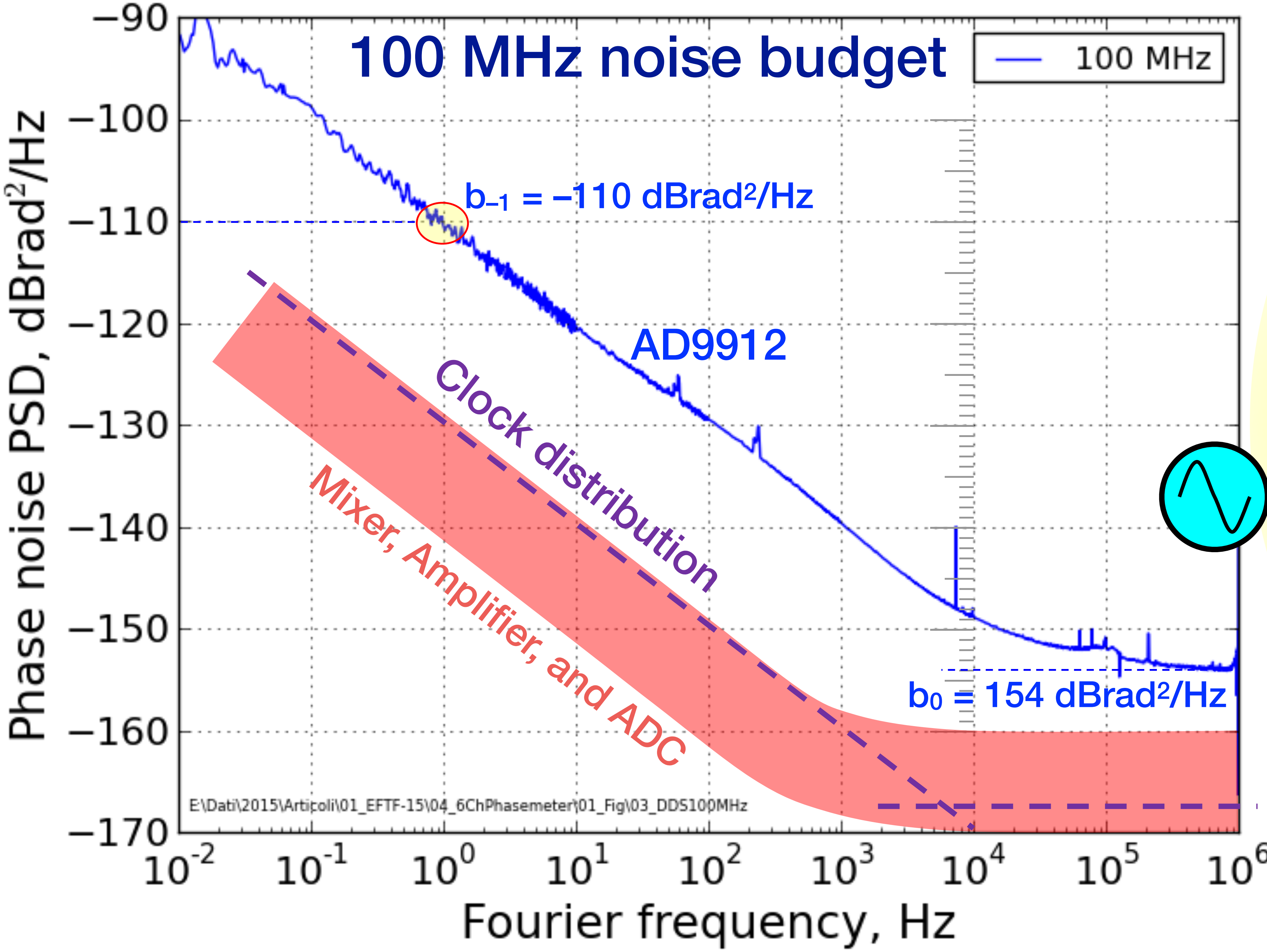


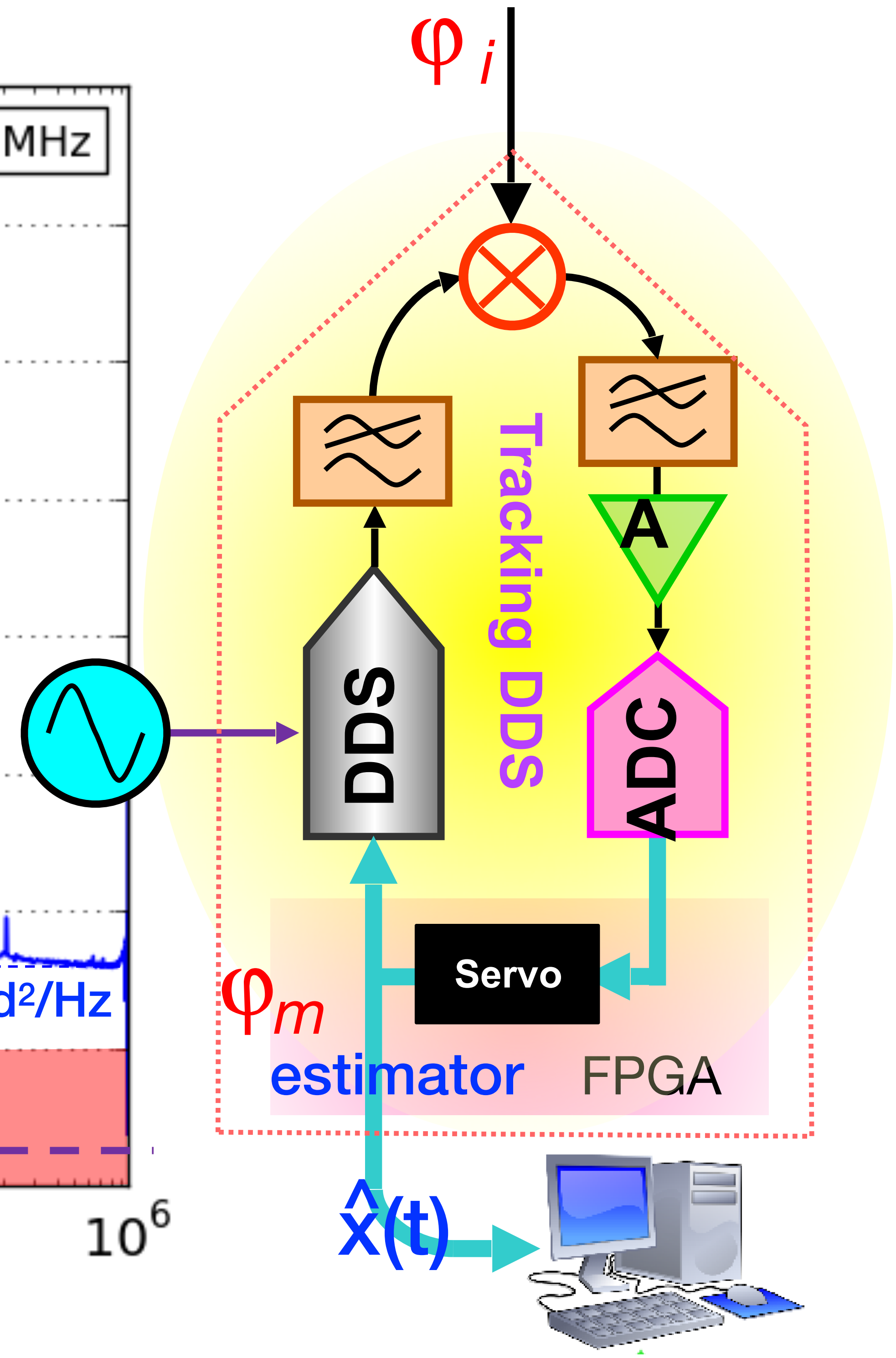
Frequency Synthesis



- Resonator engineering —> 10 GHz – 10 MHz ±3 MHz
- Small frequency offset —> DDS is OK
- Uneven frequencies —> No crosstalk

Tracking DDS —> Digital PLL



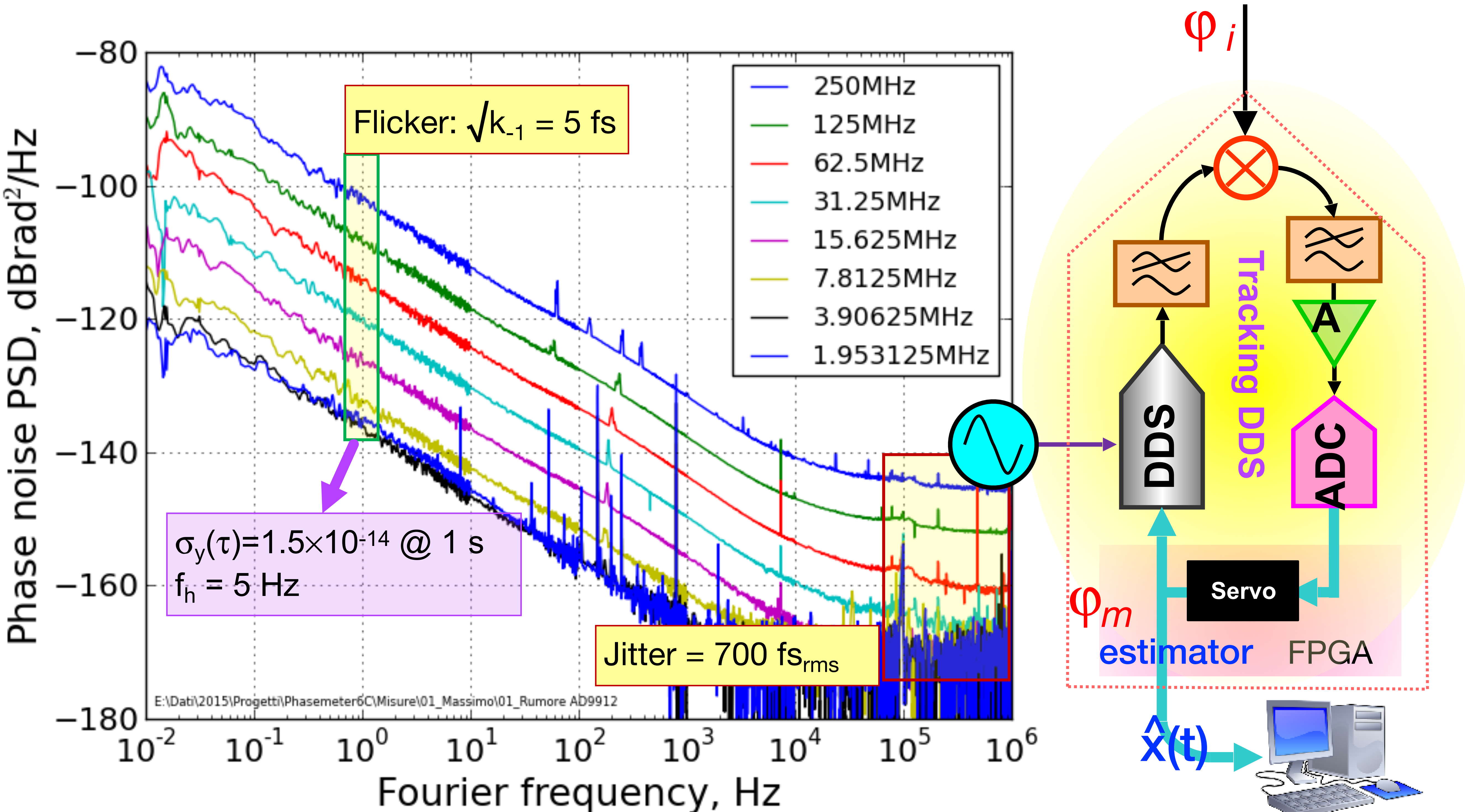


DDS Noise —> C. E. Calosso, Y. Gruson, E. Rubiola, Proc 2012 IFCS p.777-782

TDDS —> M. Calligaris, G. A. Costanzo, C. E. Calosso, Proc 2015 IFCS pp.681-683

AD9912 —> Time-PM Noise

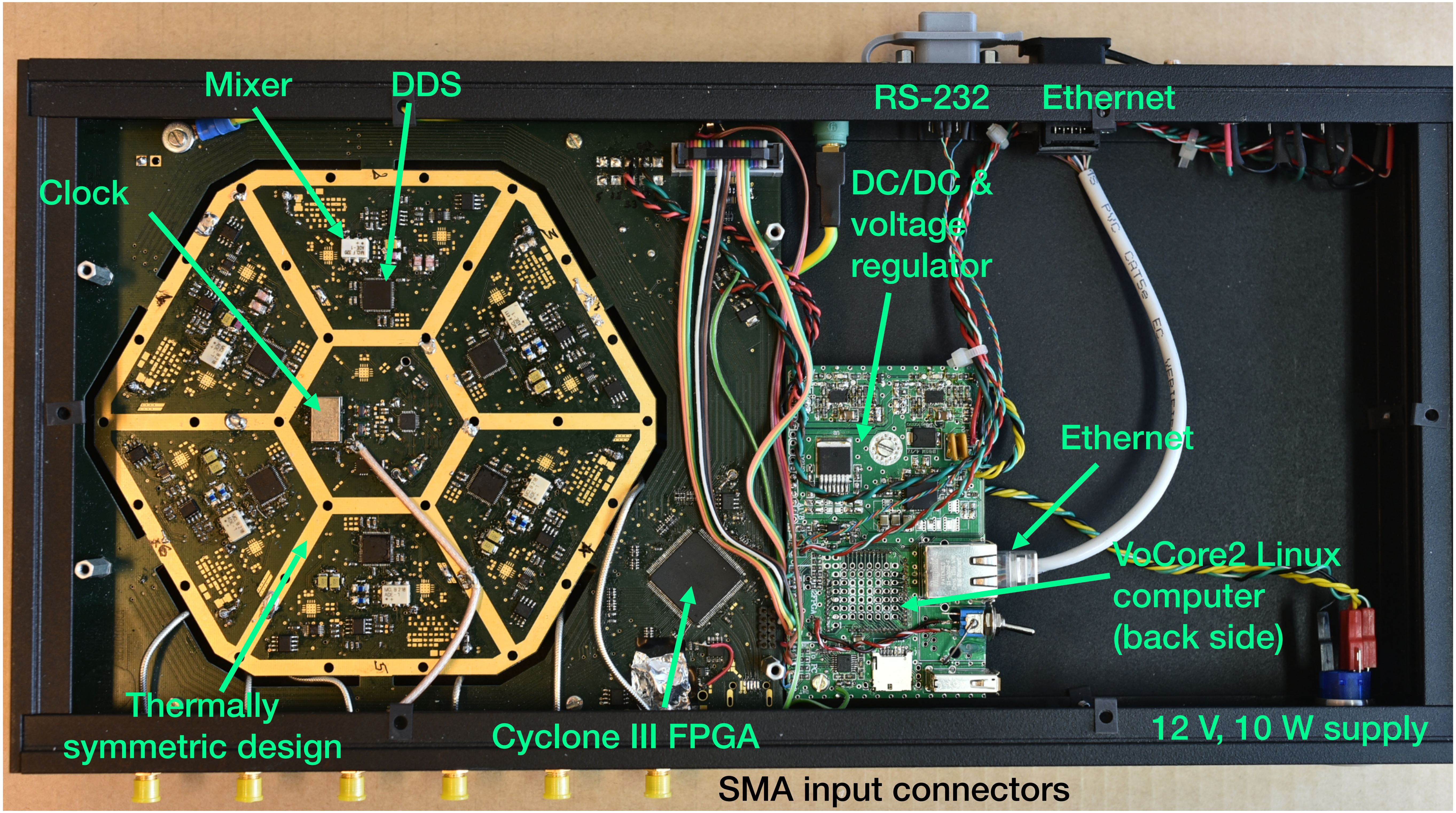
at ≥5 MHz



DDS Noise —> C. E. Calosso, Y. Gruson, E. Rubiola, Proc 2012 IFCS p.777-782

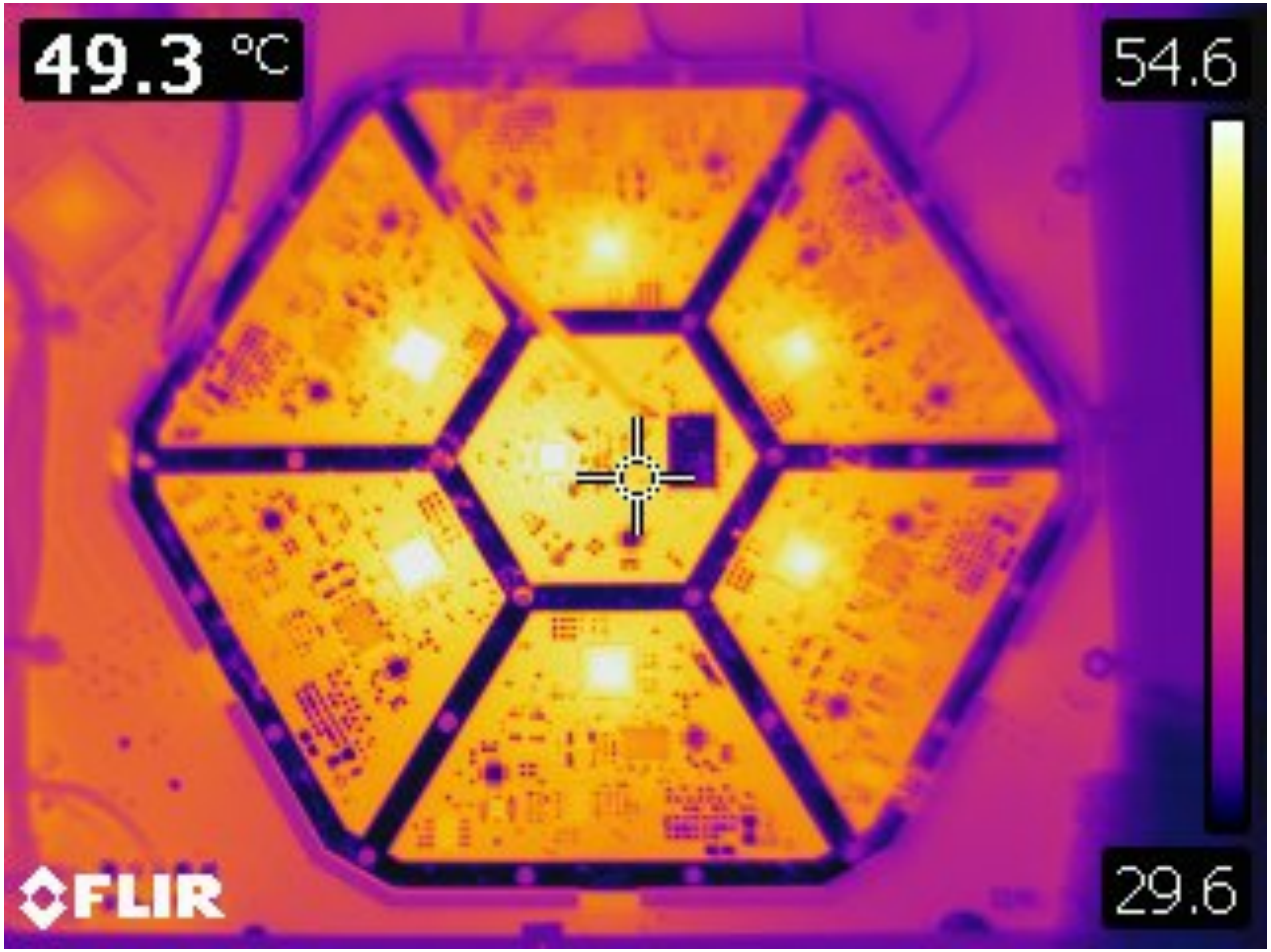
C. E. Calosso, E. Rubiola, Phase Noise and Jitter in Digital Electronics, arXiv:1701.00094 [physics.ins-det]

The 6-Channel TDDS



6 TDDSs, control unit, and interface in a small instrument

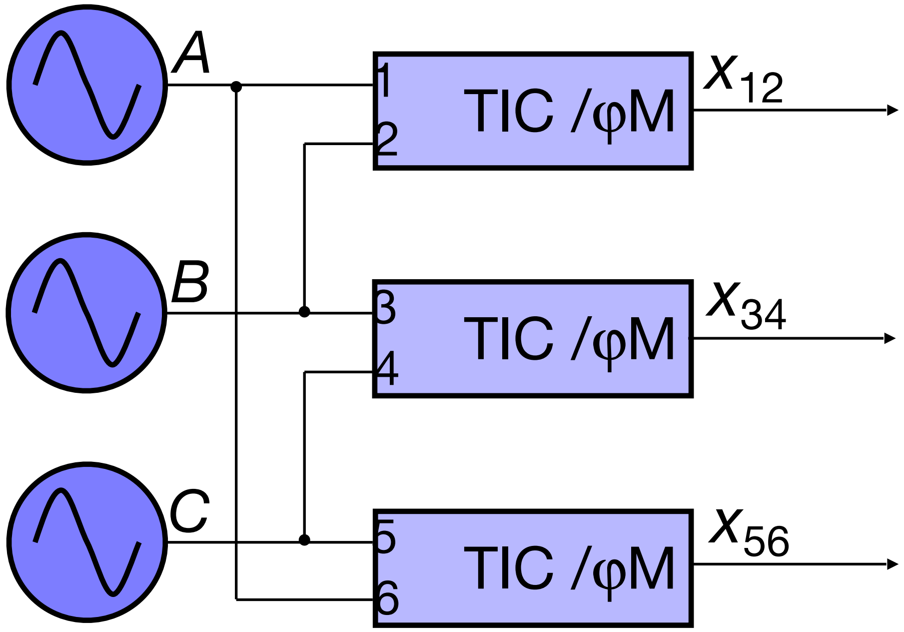
Thermal Image



Small dissipation and thermal symmetry improve phase stability

Statistics

Time Interval Counters



$$x(t) = \frac{\varphi(t)}{2\pi\upsilon_0}$$

$$y(t) = \frac{dx(t)}{dt}$$

$$z(t,\tau) \equiv \frac{\bar{y}(t) - \bar{y}(t-\tau)}{\sqrt{2}}$$

$$z = \frac{x_2 - 2x_1 + x_0}{\sqrt{2}\,\tau}$$

AVAR

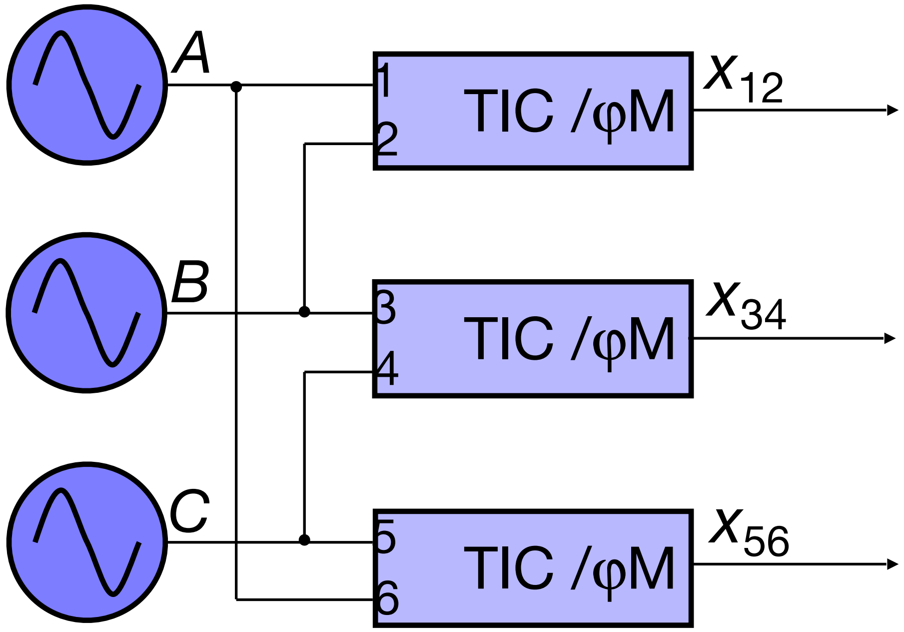
$$\sigma_y^2(\tau) = E[z^2]$$

2-Sample COV

$$\sigma_{y_A,y_B}(\tau) = E[z_B z_A]$$

Oscillator		Instrument		
		Output		Noise
Single	Delta	Single	Delta	
x_B	x_{BA}	x_2	x_{21}	x_{n21}
y_B	y_{BA}	y_2	y_{21}	y_{n21}
z_B	z_{BA}	z_2	z_{21}	z_{n21}
σ_B^2	σ_{BA}^2	σ_1^2	σ_{21}^2	σ_{n21}^2
$\sigma_{B,A}$				

Statistical Tools



3-cornered hat with noise-free instruments

$$\begin{cases} \sigma^2_{AB} = E[z^2_{12}] \\ \sigma^2_{BC} = E[z^2_{34}] \\ \sigma^2_{CA} = E[z^2_{56}] \end{cases} \Rightarrow \sigma^2_B = \frac{1}{2} E[z^2_{12} + z^2_{34} - z^2_{56}]$$

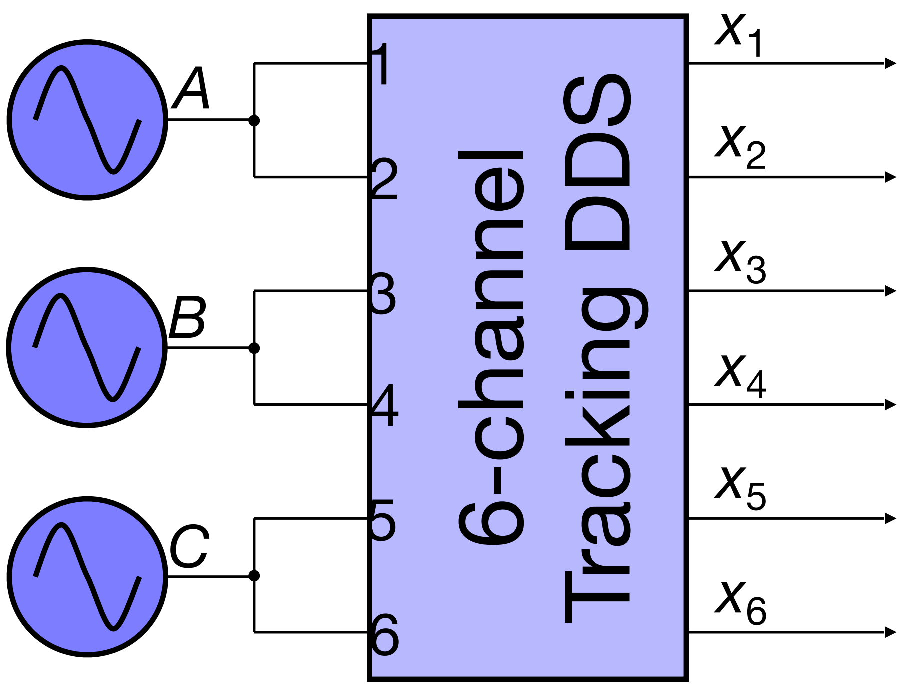
Noisy Instruments

3-Cornered Hat $\frac{1}{2} E[z^2_{12} + z^2_{34} - z^2_{56}] = \sigma^2_B + \frac{1}{2}[\sigma^2_{n12} + \sigma^2_{n34} - \sigma^2_{n56}]$

2-Sample COV $E[z_{21} z_{34}] = \sigma^2_B$ background noise —> 0

At 100 MHz the Time Interval Counter is not an option

2-Sample COV with TDDS



Channels remapping

$$\sigma_B^2 = \mathrm{E}[z_{32} z_{45}]$$

- Expand all terms
- Look at convergence laws
- Room for improvement

First improvement

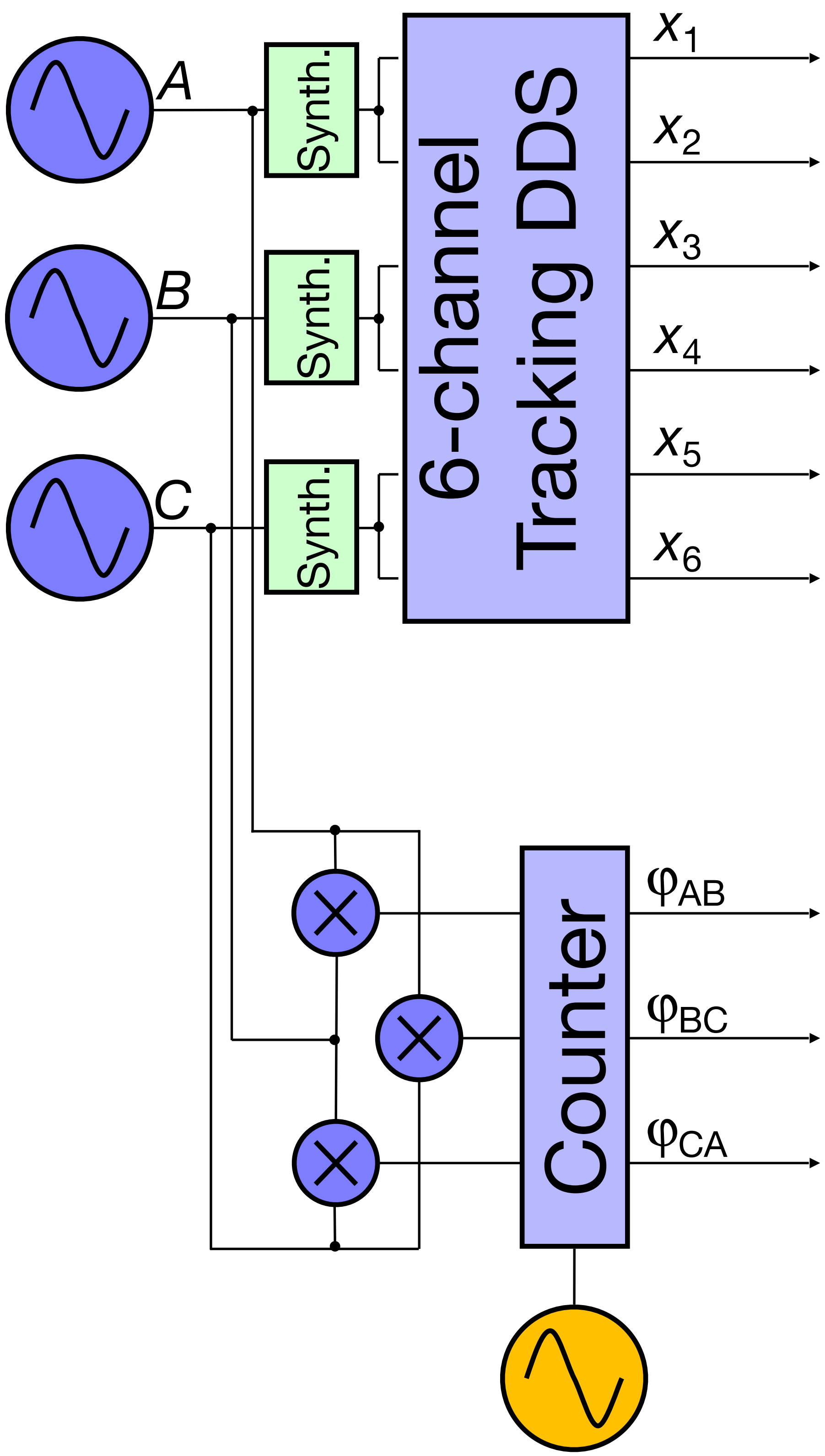
$$\sigma_B^2 = \mathrm{E}[z_{3\langle 12\rangle} z_{4\langle 56\rangle}] \quad \leftarrow \quad z_{i\langle jk\rangle} = z_i - \frac{z_j + z_k}{2}$$

Second improvement

We use this —>

$$\sigma_B^2 = \frac{1}{2}\mathrm{E}[z_{3\langle 12\rangle} z_{4\langle 56\rangle} + z_{4\langle 12\rangle} z_{3\langle 56\rangle}]$$

Experiment

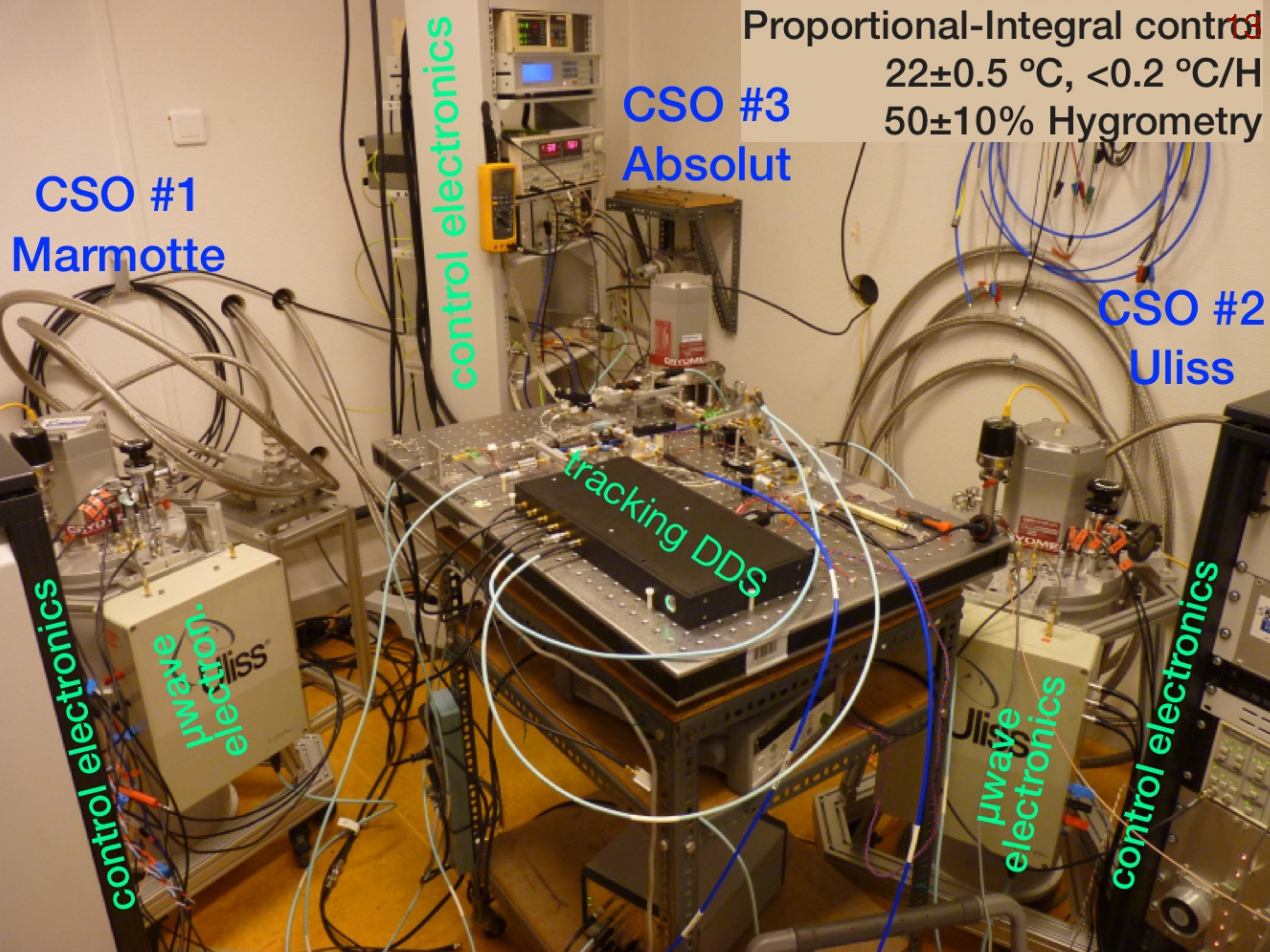


2-sample COV

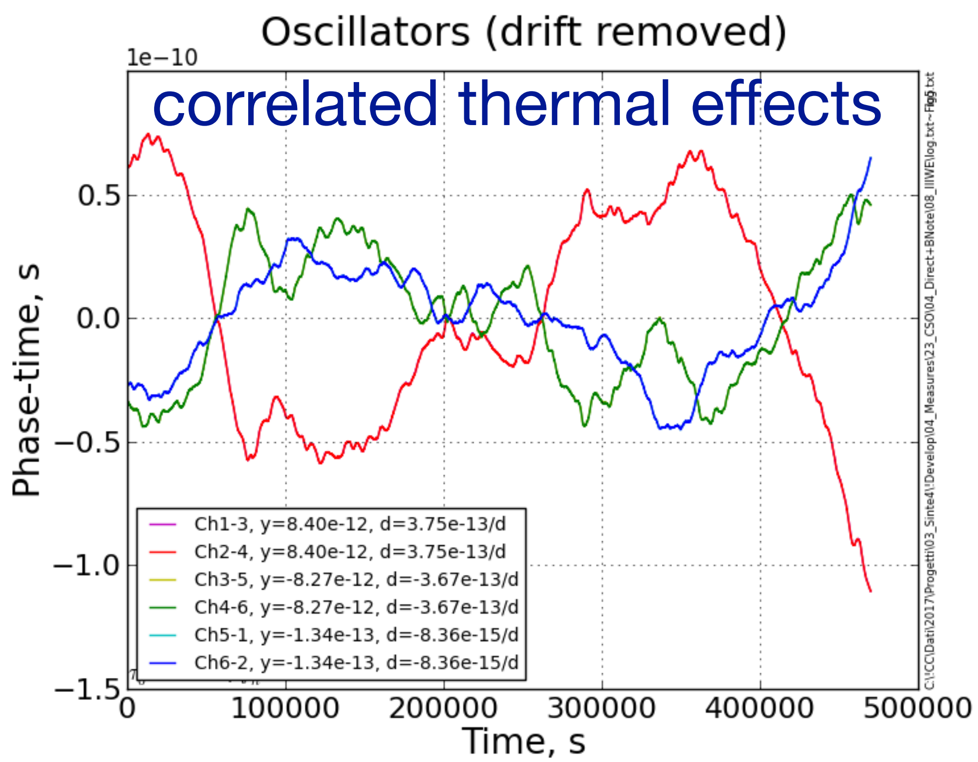
- INRIM 6-Ch TDDS
- 100 MHz outputs
- 2-sample covariance

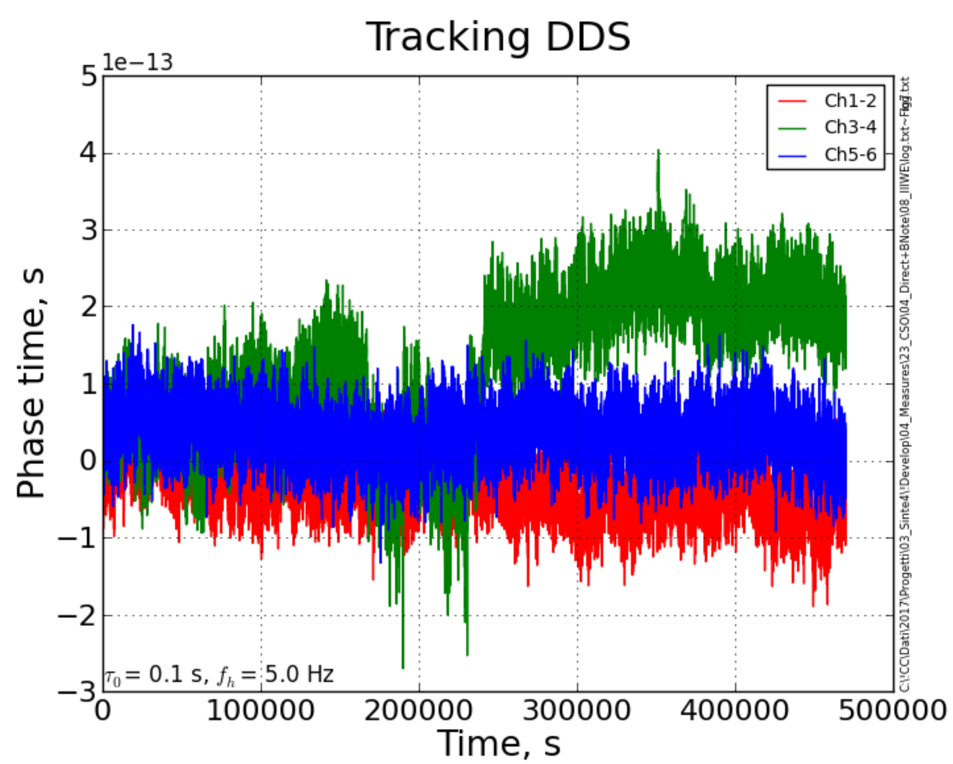
3-cornered hat

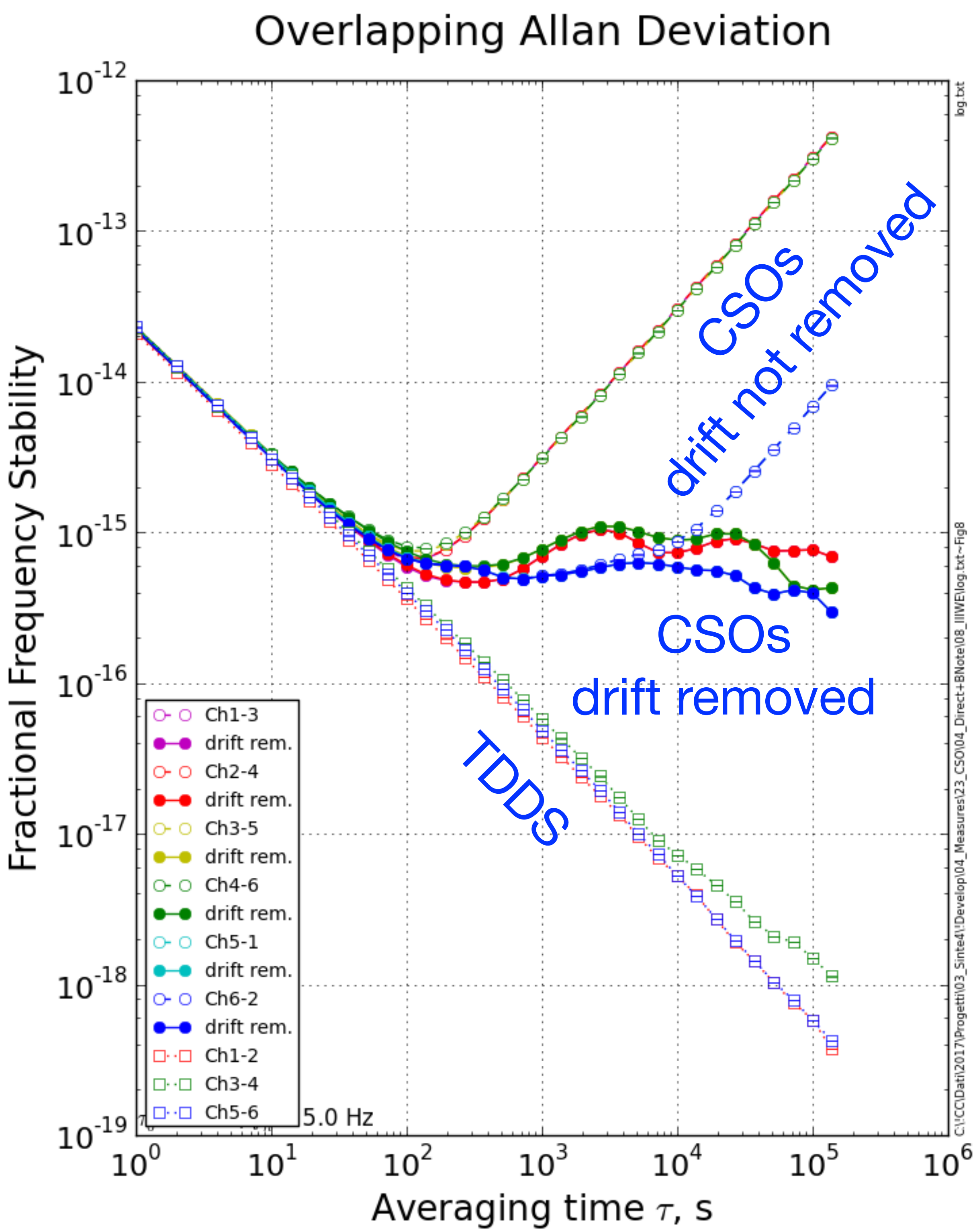
- Lange / K&K counters
- 10 GHz outputs
- Different beat notes prevent crosstalk


Proportional-Integral control
22±0.5 °C, <0.2 °C/H
50±10% Hygrometry
CSO #1
Marmotte
control electronics
CSO #3
Absolut
CSO #2
Uliss
tracking DDS
µwave electron.
control electronics
µwave electronics
control electronics

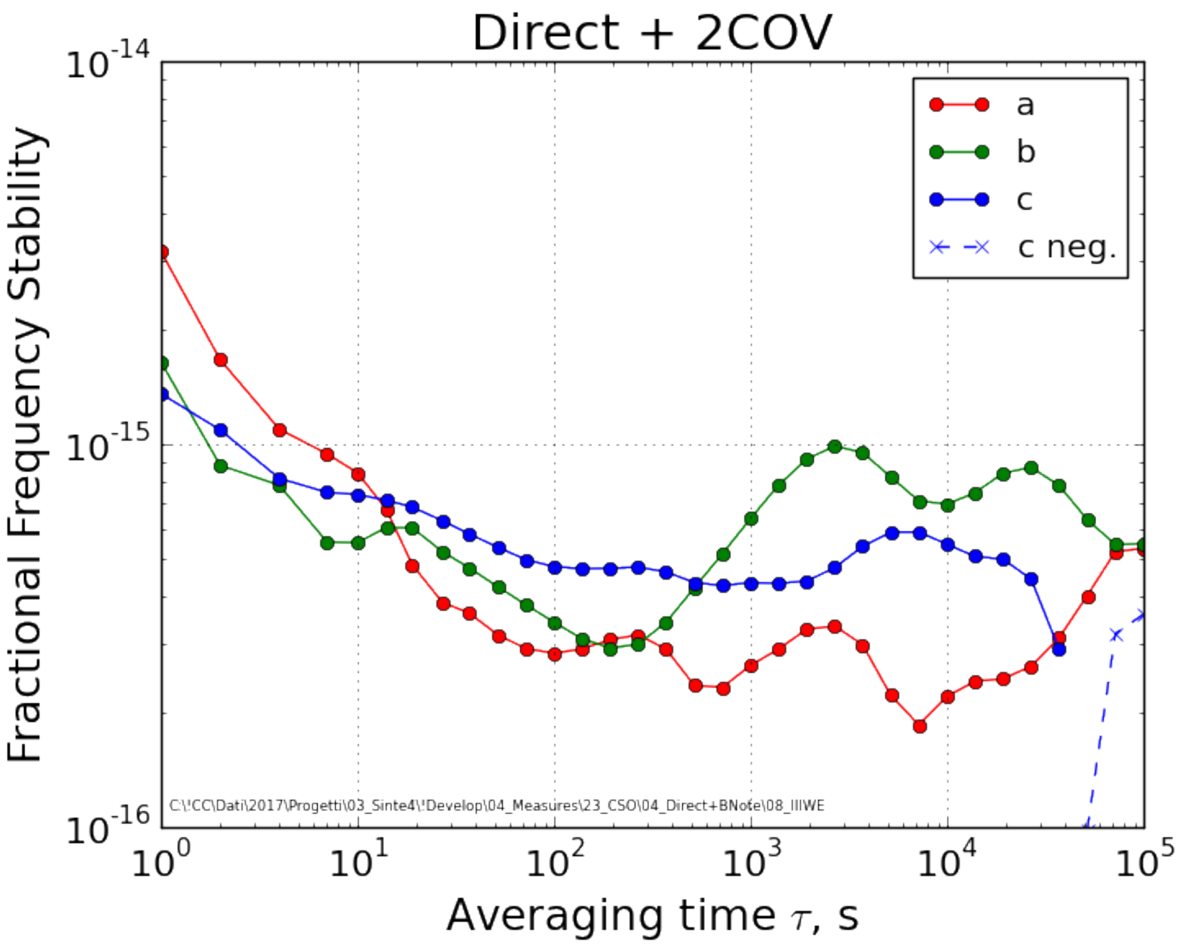
Time Domain and ADEV



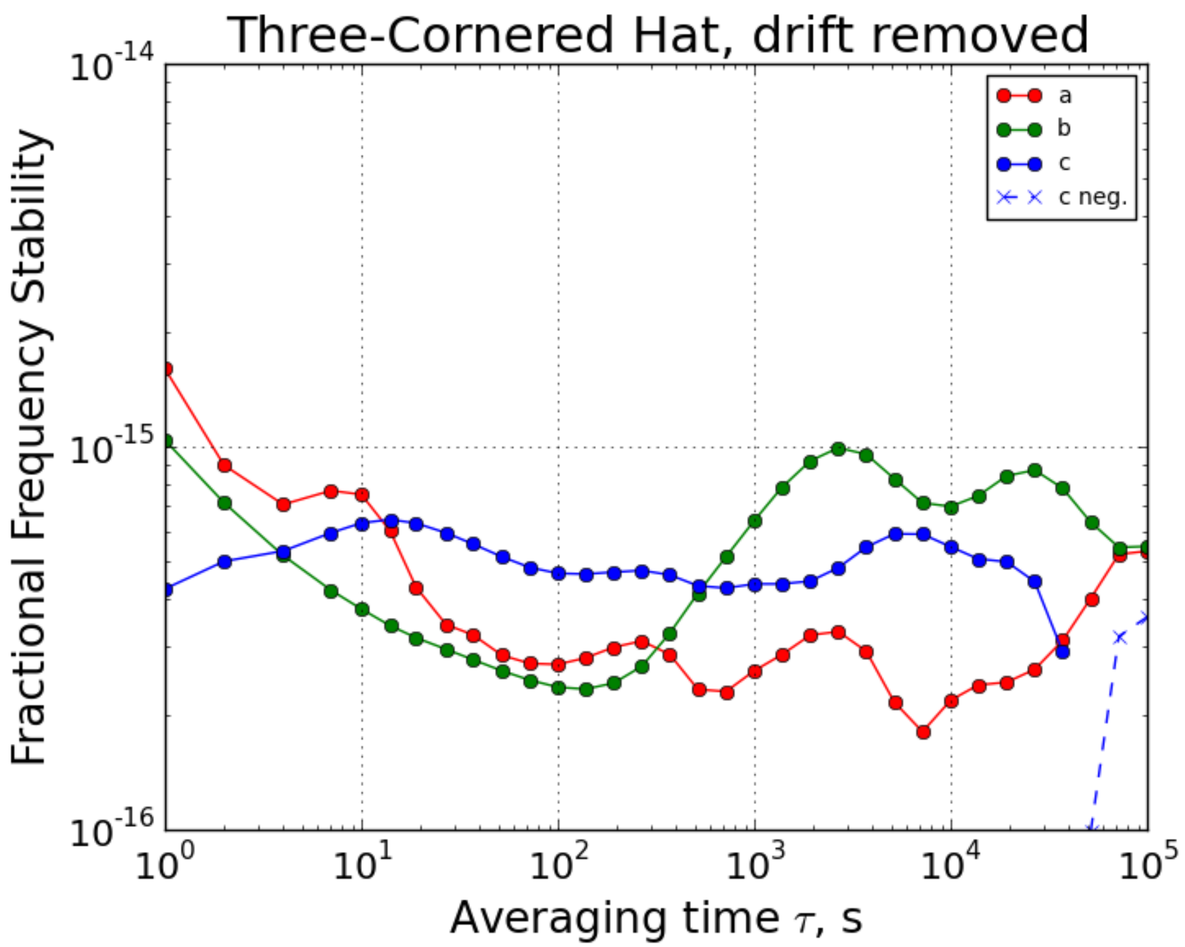




2-COV vs 3-CH



100 MHz outputs
6-Ch TDDS
2COV Algorithm

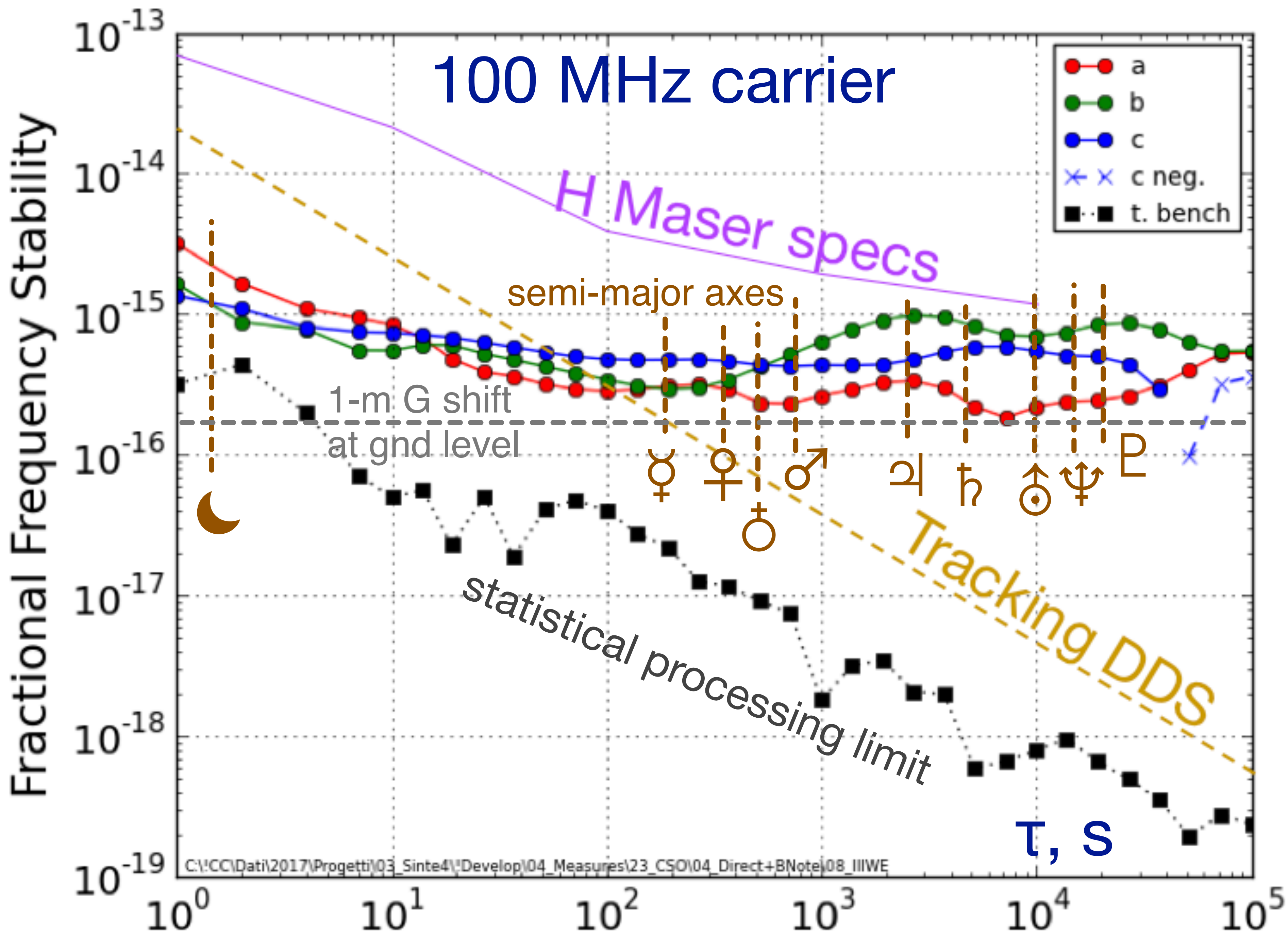


≈10 GHz outputs
HF beat notes (1…10 MHz)
Lange/K&K Counters
3CH Algorithm

10GHz —> 100 MHz synthesizer affects short-term (≤100 s) stability

2COV algorithm and 3CH give the same result

Conclusions



- Full validation of the 100 MHz output
- 5–400 MHz TDDS range
- **Next step: composite clock, $2{\times}10^{-14}/\tau$ DDS limit**