

# Phase Noise and Jitter in Digital Electronics

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## Outline

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- Basics
- FPGAs — Mechanisms / Examples / Facts
- ADCs — Basics / Examples
- DDSs — Basics / Advanced / Examples
- Dividers —  $\Pi$  and  $\Lambda$  / Microwave

home page <http://rubiola.org>

# Acknowledgments

This tutorial gathers a wealth of  
(mostly unpublished) material developed by

**The Go Digital Team@ FEMTO-ST, Besancon, F**

chiefly

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**Gwenhael “Gwen” Goavec-Merou**

**Jean-Michel Friedt**

**Yannick Gruson**

**Claudio Calosso, INRIM, Torino, Italy**

**...and myself**

**Caveat**

**Only a fraction of this can be taught at a 1–2 H session**

# 1 – Basics

# Jitter

$$v(t) = V_0 [1 + \alpha(t)] \cos [2\pi\nu_0 t + \varphi(t)] \quad x(t) = \frac{\varphi(t)}{2\pi\nu_0}$$

- ITU defines jitter as the variations in the significant instants of a clock or data signal, vs a “perfect” clock
- Jitter —> Usually fast phase changes  $f >$  a few tens of Hz
- Wander —> Usually slower phase changes (due to temperature, voltage, etc)
- Designers first care about consistency of logic functions,
  - First, maximum timing error
  - Sometimes RMS value and probability distribution
- Time and Frequency community focuses on
  - PM noise spectra
  - Delay spectra
  - Two-sample variances (ADEV, TDEV, etc.)

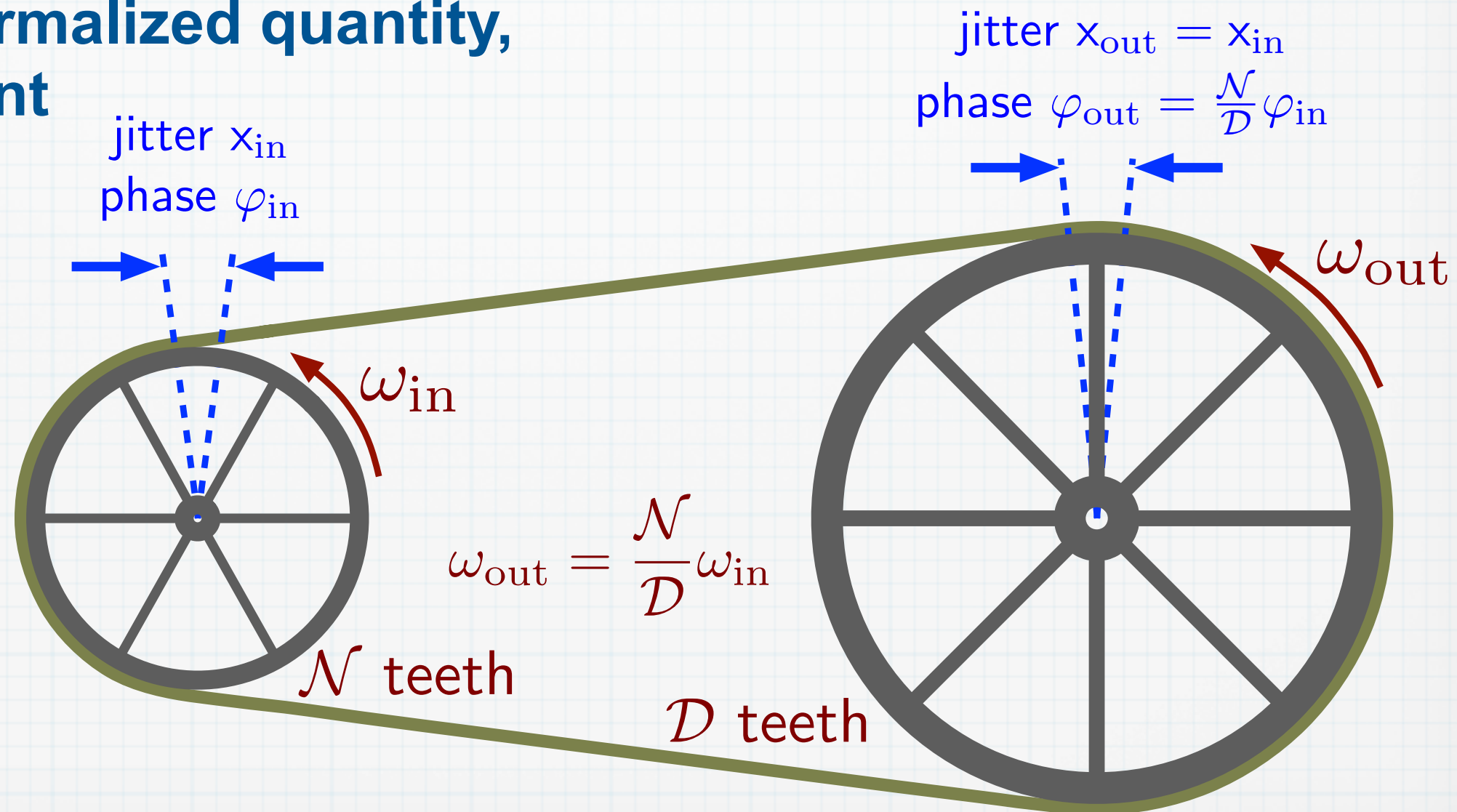


# Phase Time $x(t)$ — or Jitter

- Let's allow  $\varphi(t)$  to exceed  $\pm\pi$ , and count the no of turns
- This is easily seen by scaling  $\omega$  down (up) to  $\omega = 1$  rad/s using a noise-free gear work
- The phase-time fluctuation associated to  $\varphi(t)$  is

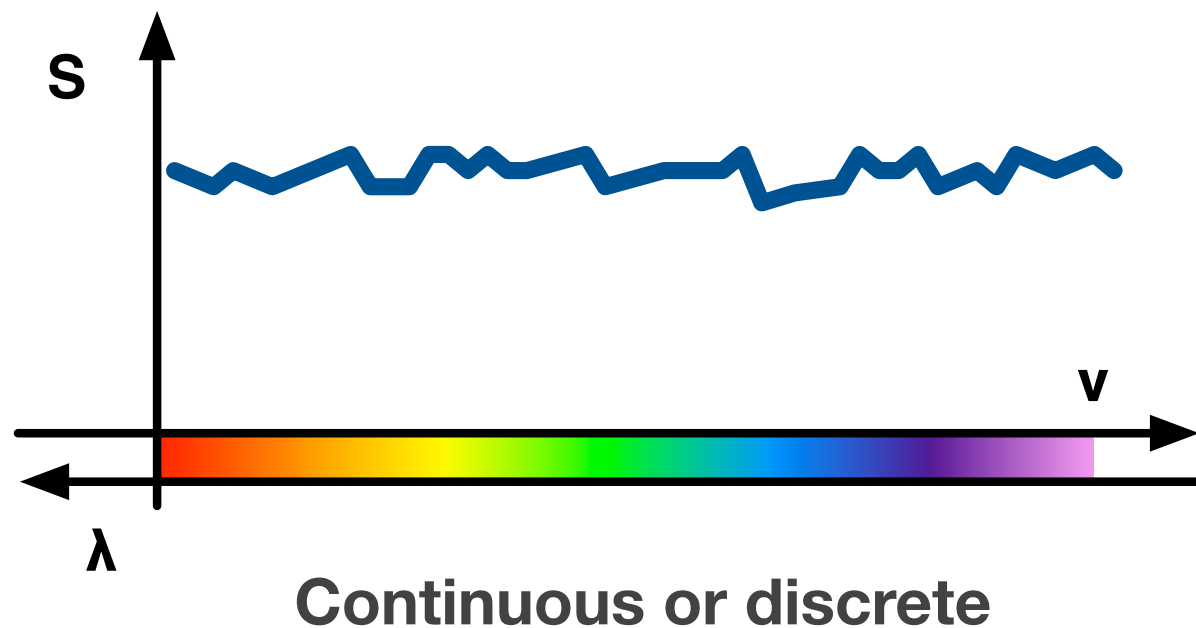
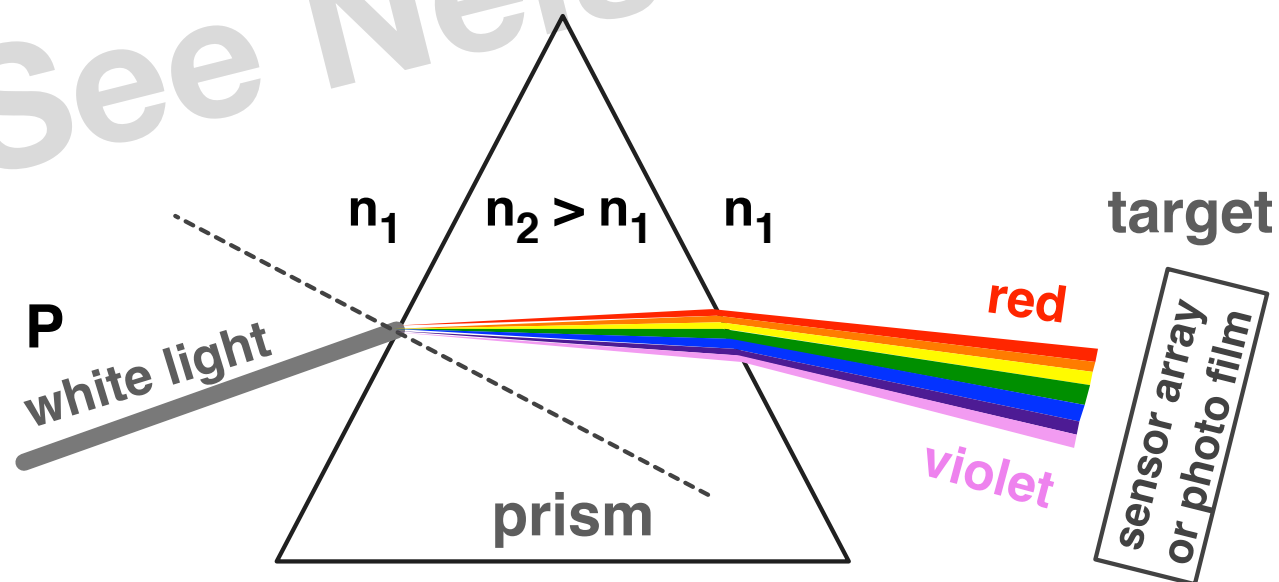
$$x(t) = \varphi(t) / \omega_0$$

- $x(t)$  is a normalized quantity, independent of  $\omega_0$ .



# Physical Concept of PSD $S(f)$

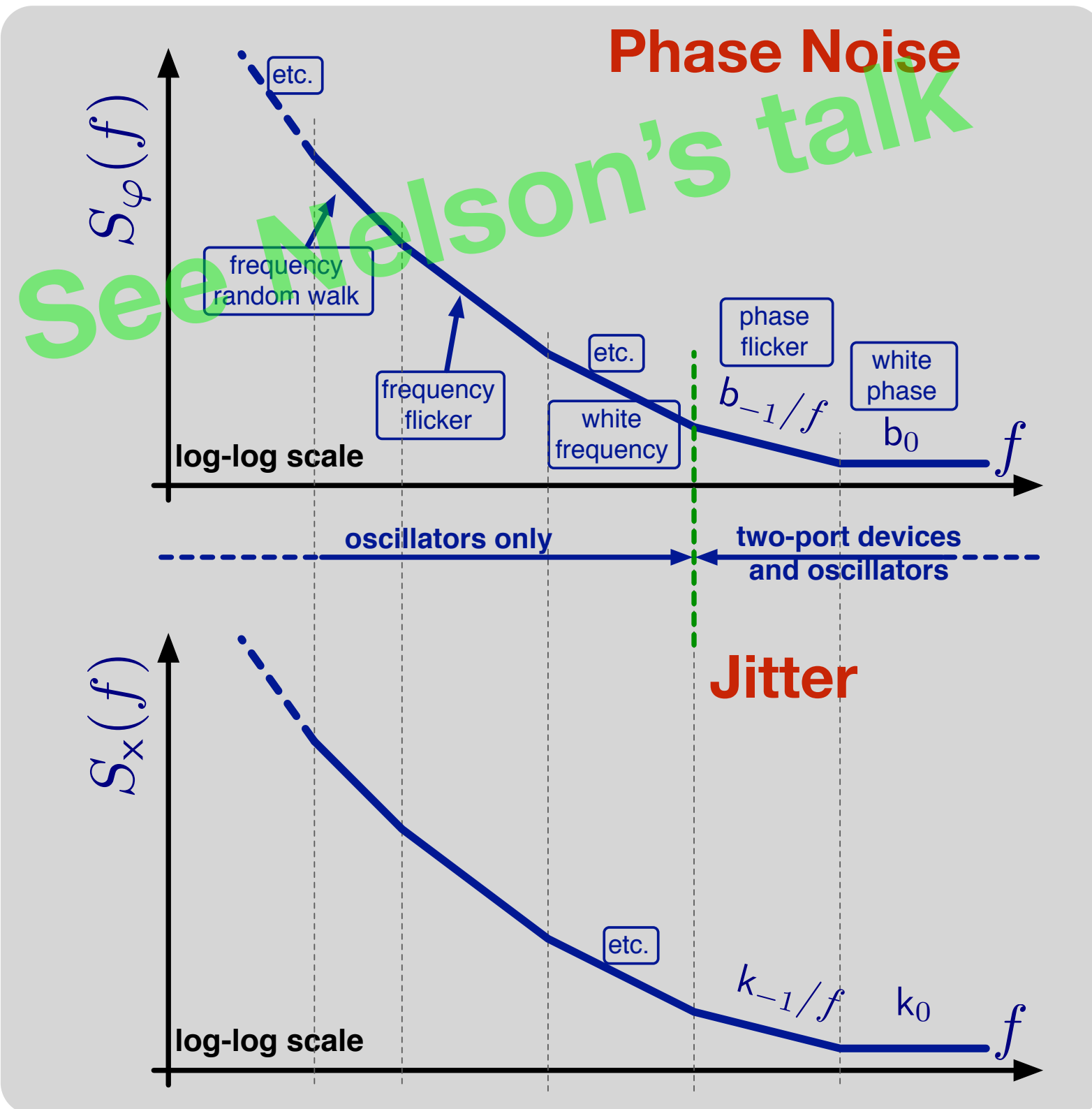
## Power Spectral Density



- The **PSD** is the **distribution of power** vs. frequency (power in 1-Hz bandwidth)
- The **PS** is the **distribution of energy** vs. frequency (energy in 1-Hz bandwidth)
- Frequency can be continuous or discrete (histogram),
- In mathematics,
  - the power is a square quantity
  - the energy is power integrated in time
- Power (energy) in physics is a square (integrated) quantity
  - PSD  $\rightarrow$  W/Hz (or  $V^2/\text{Hz}$ ,  $A^2/\text{Hz}$ , etc.)
  - PS  $\rightarrow$  J/Hz

# The Polynomial Law

$$v(t) = V_0 [1 + \alpha(t)] \cos [2\pi\nu_0 t + \varphi(t)]$$



## Phase Noise PSD

$$S_\varphi(f) = \sum_{i \leq -4}^0 b_i f^i$$

## Jitter (phase-time) PSD

$$x(t) = \frac{\varphi(t)}{2\pi\nu_0}$$

$$S_x(f) = \sum_{i \leq -4}^0 k_i f^i$$

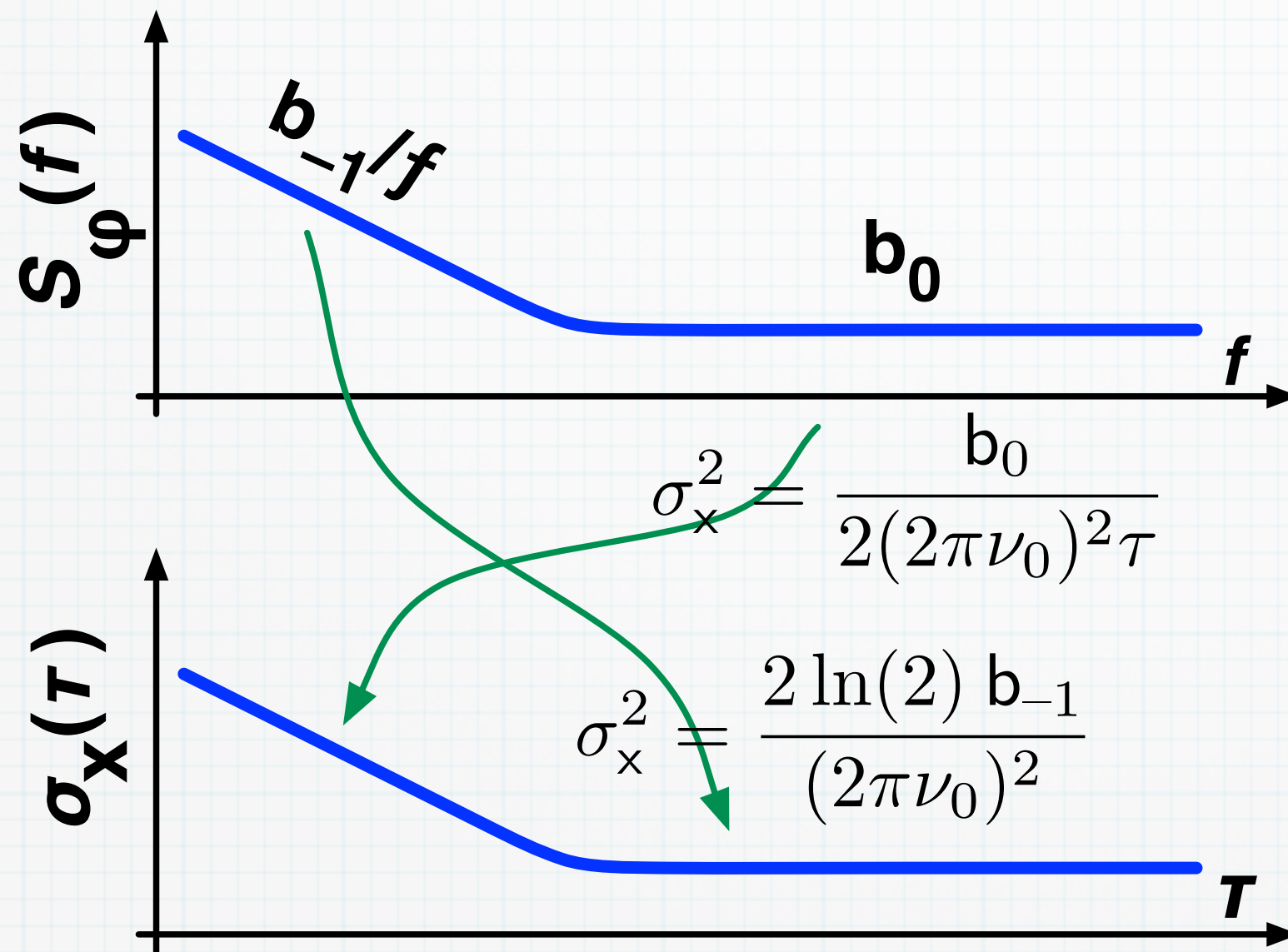
## Fractional Frequency PSD

$$y(t) = \dot{x}(t) = \frac{\dot{\varphi}(t)}{2\pi\nu_0}$$

$$S_y(f) = \sum_{i \leq -2}^2 h_i f^i$$

# Converting PM noise to TDEV

8



random phase

$$\varphi(t)$$

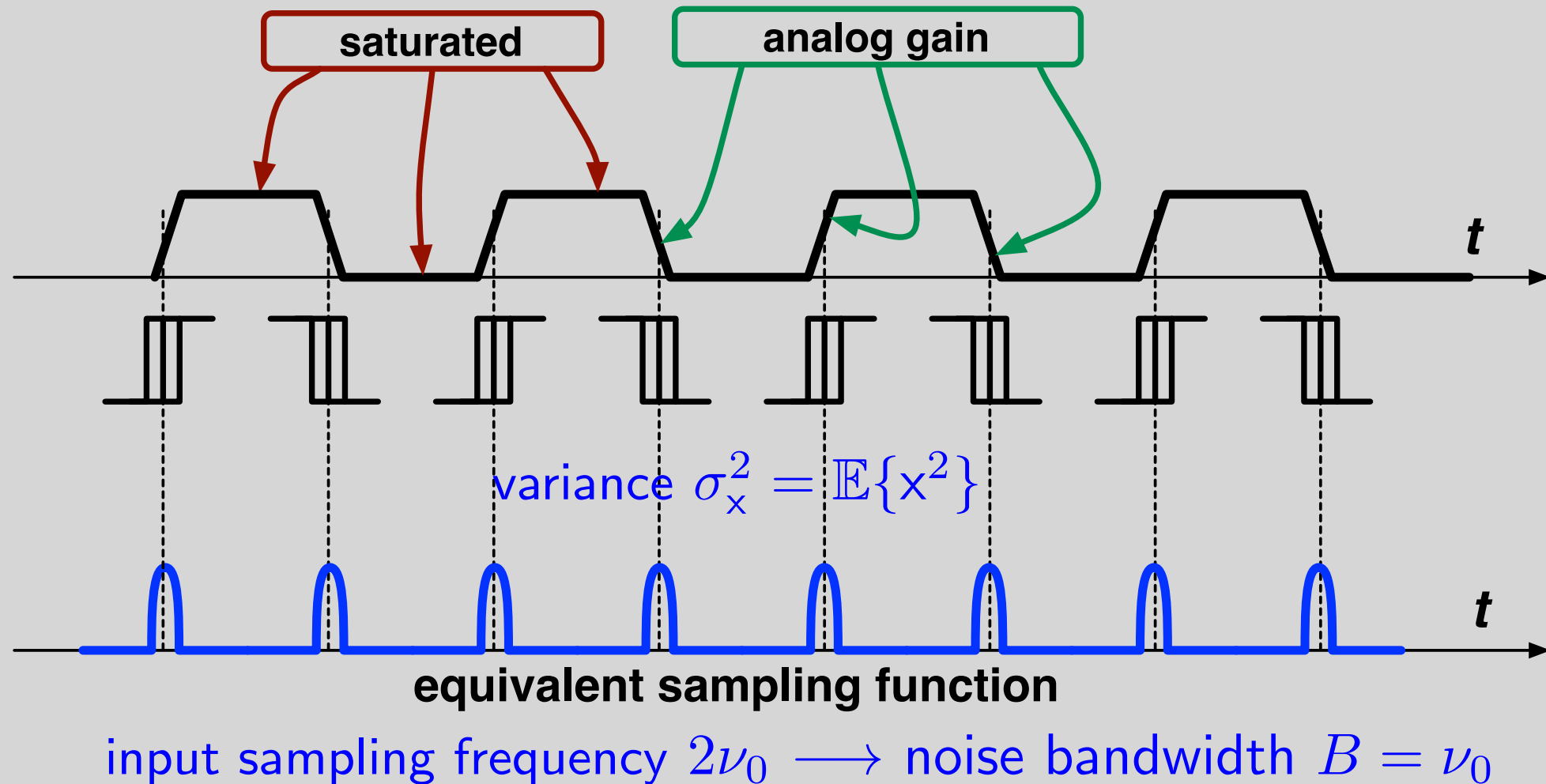
phase-time  
(fluctuation)

$$x(t) = \frac{\varphi(t)}{2\pi\nu_0}$$

**TDEV  $\sigma_x(\tau)$**   
same as **ADEV**,  
but we use  
 **$x(t)$**  instead of  **$y(t)$**

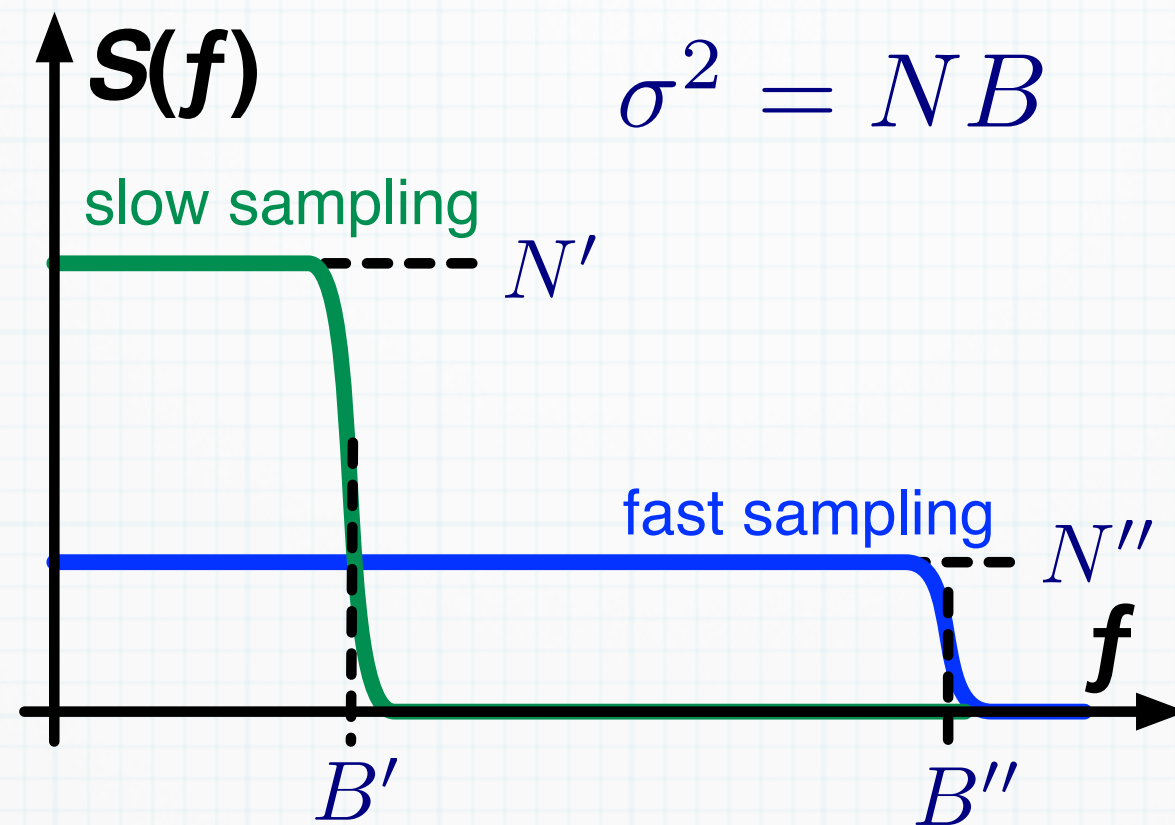
You may be more familiar to  $\sigma_y^2(\tau) = h_0/2\tau + 2\ln(2)h_{-1}$

# Phase Noise Sampling



- **Sampling occurs at the edges**
  - (in some cases, only at rising or falling edges)
- **Square wave signals need analog bandwidth at least  $3 \nu_{\max} \dots 4 \nu_{\max}$**
- **Aliasing is expected**

# Aliasing Over-Simplified



- The Parseval Theorem states that the total energy (or power) calculated in the time domain and in the frequency domain is the same
- Ergodicity allows to interchange time domain and statistical ensemble

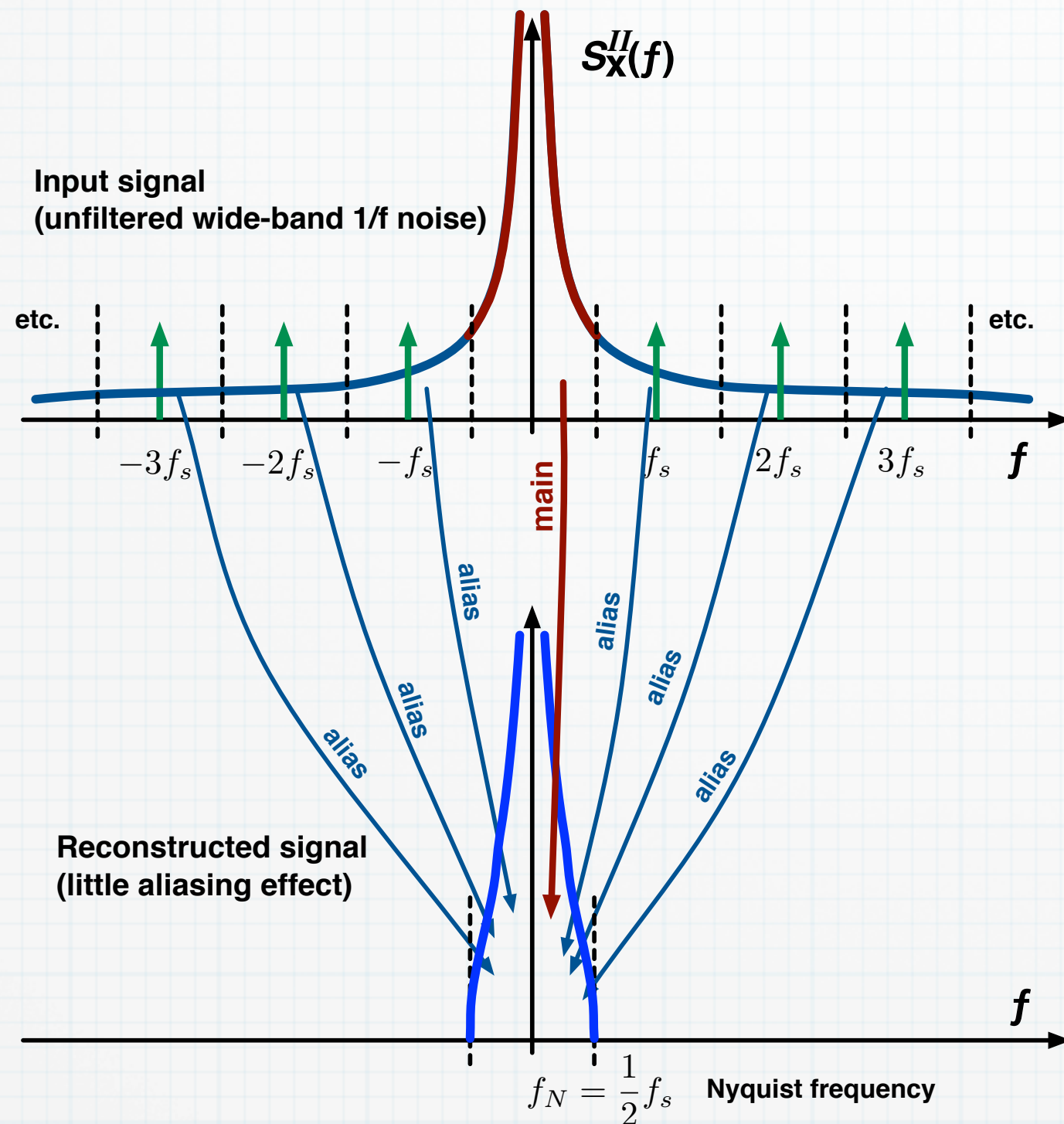
$$N'B' = N''B''$$

...and PM noise scales up with the reciprocal of the carrier frequency





# Aliasing and $1/f$ Noise



Low power in the  
high- $f$  aliases

Little or no effect on  
the noise spectrum

And virtually no effect with  $1/f^2$ ,  $1/f^3$ ,  $1/f^4$  ...

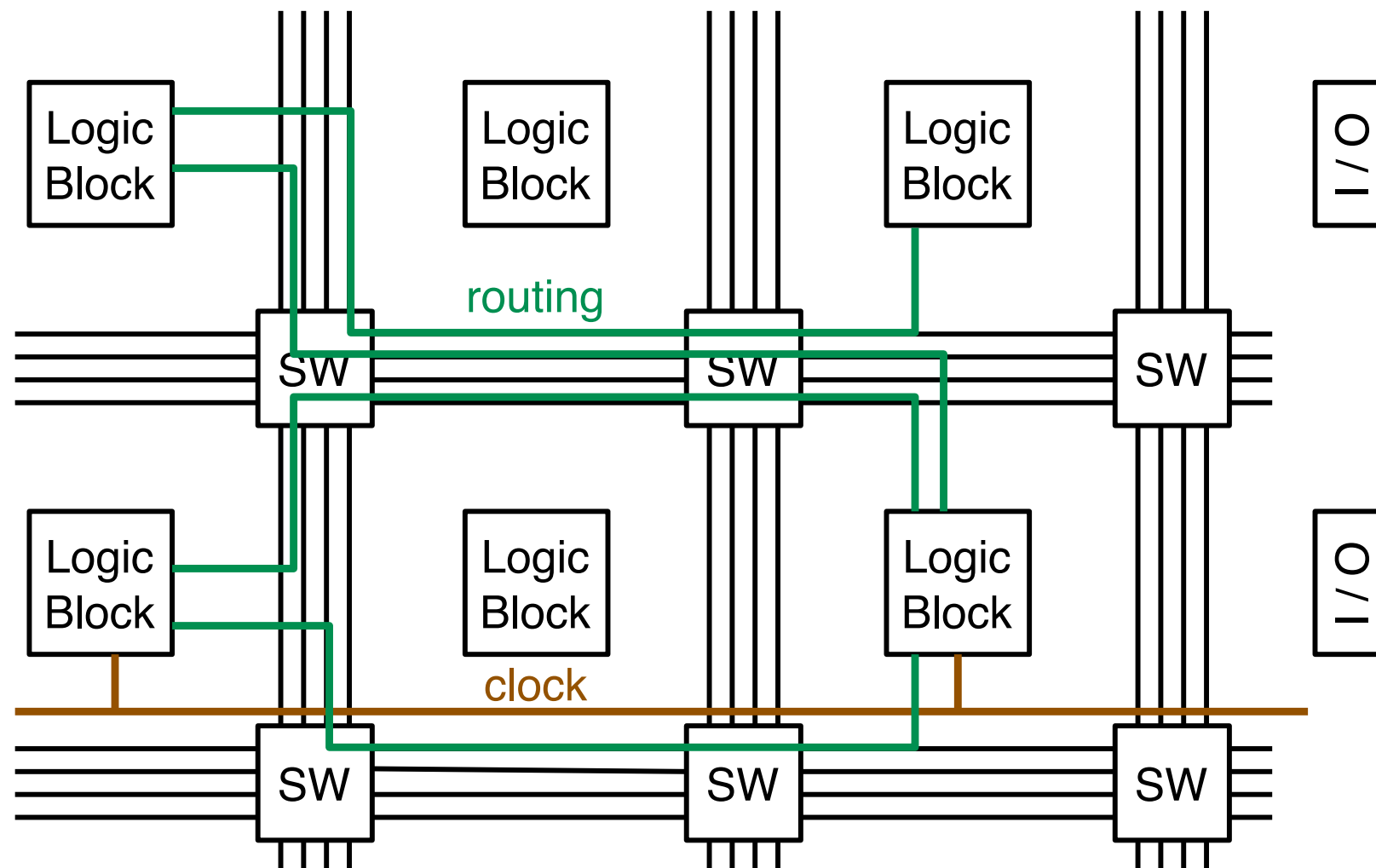
# 2 – FPGAs

- Noise Mechanisms
- Examples
- Additional Facts



# Noise Mechanisms

# FPGA Interconnection Structure



Device dependent blocks

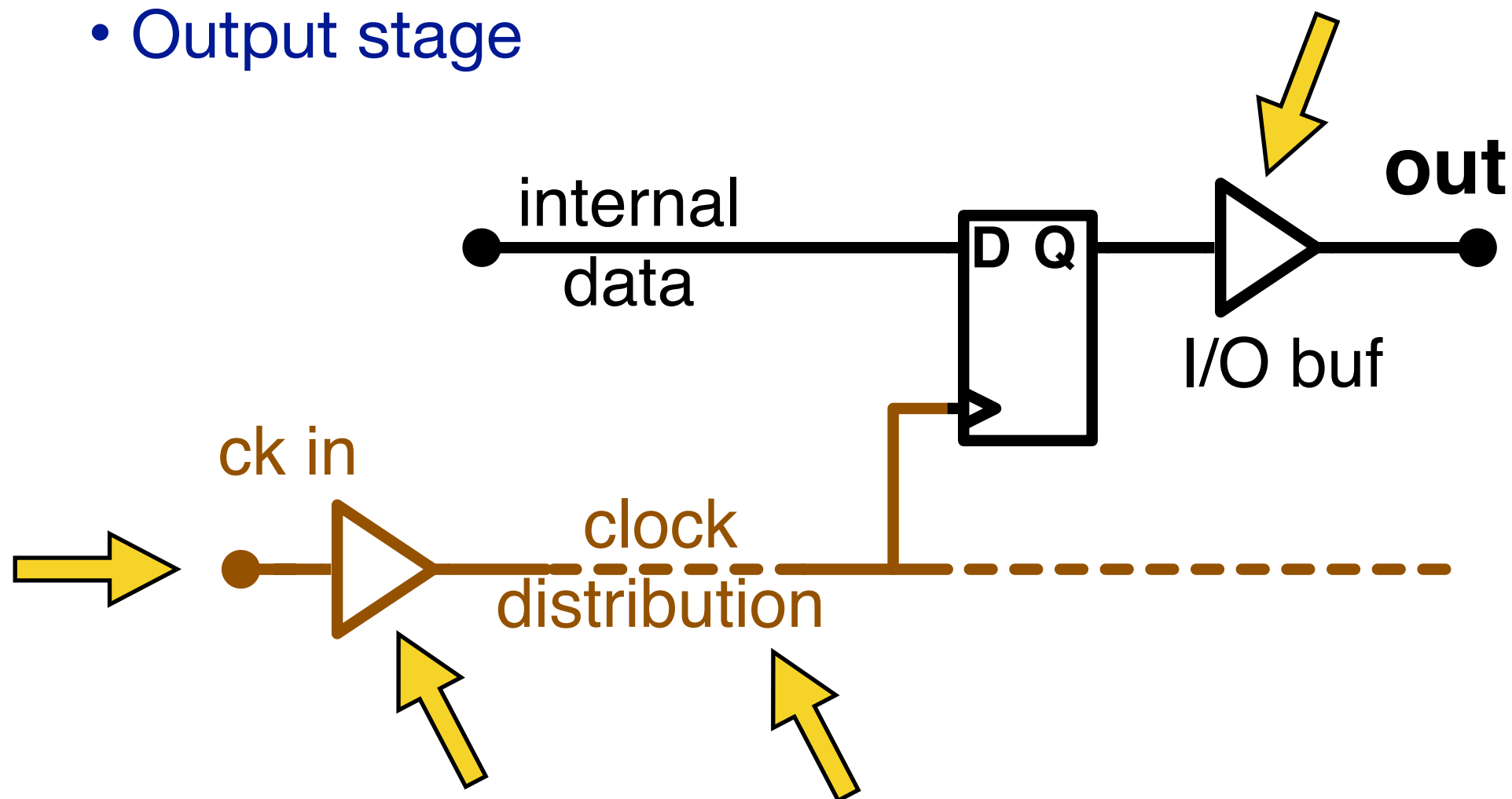
- Input/Output
- RAM
- PLL
- NCO
- ...etc.

## Delay & jitter

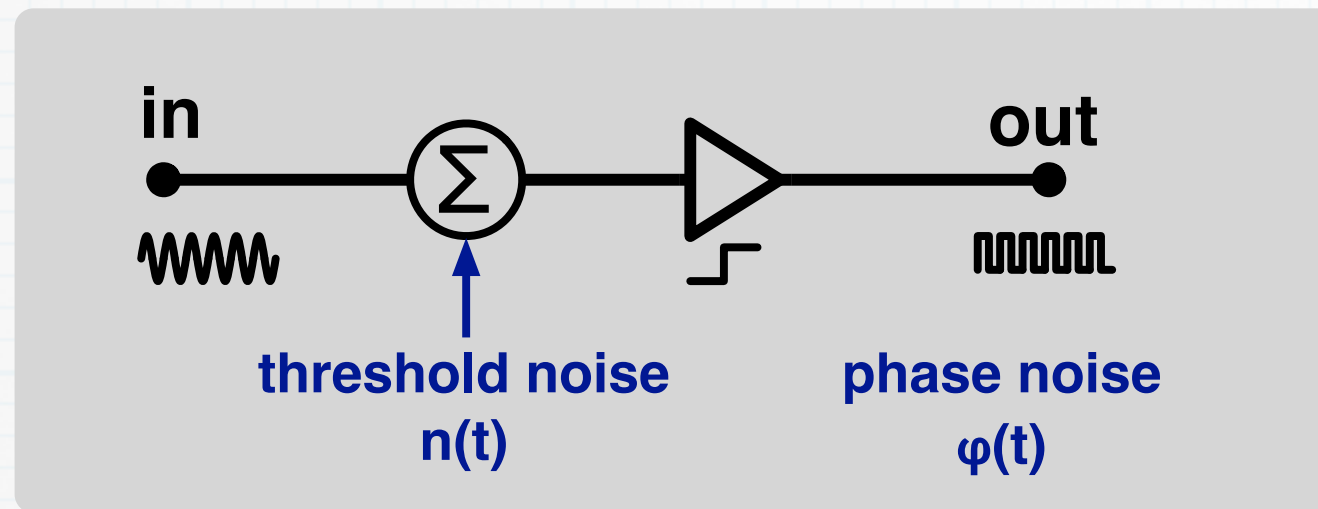
- General routing through switch points
  - Delay & jitter rather uniform in a block
  - Large spread over the interconnect matrix
- Dedicated clock lines managed separately
  - Low and predictable delay & jitter

# Output Jitter Limitations

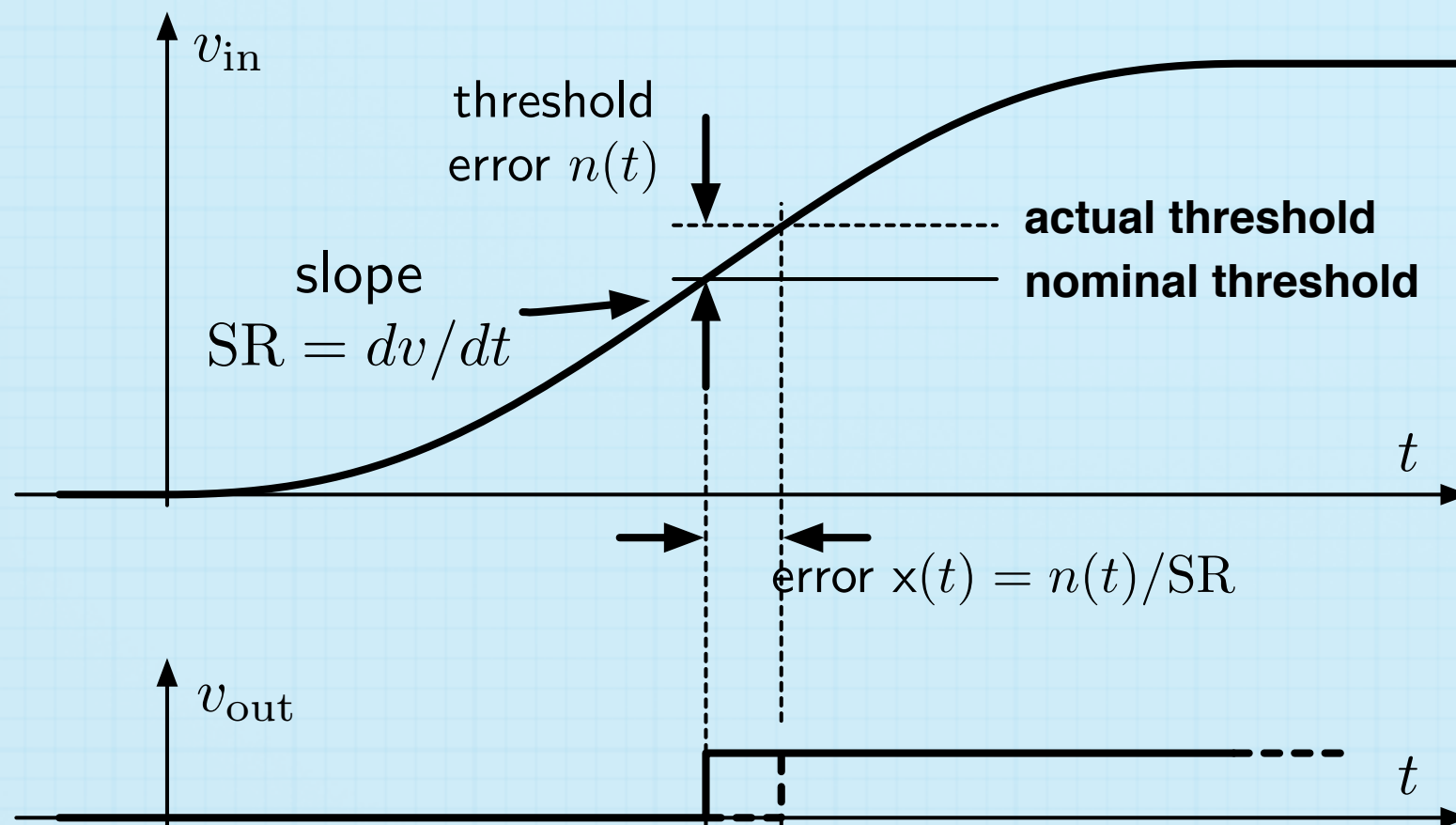
- Output can be synchronized to the clock
- Jitter cannot be smaller than
  - External clock signal
  - Clock input stage
  - Clock distribution
  - Output stage



# Phase Noise in the Input Stage



## Threshold fluctuation



## mechanism

$$x(t) = \frac{n(t)}{(SR)(t)}$$

$$\varphi(t) = \frac{2\pi\nu_0 n(t)}{(SR)(t)}$$

# Phase Noise in the Input Stage

**Sinusoidal signal**

$$v(t) = V_0 [1 + \alpha(t)] \cos [2\pi\nu_0 t + \varphi(t)] \implies \text{SR} = 2\pi\nu_0 V_0$$

$$x(t) = \frac{n(t)}{\text{SR}} \longrightarrow x(t) = \frac{1}{2\pi\nu_0} \frac{n(t)}{V_0}$$

$$\varphi(t) = \frac{2\pi\nu_0 n(t)}{\text{SR}} \longrightarrow \varphi(t) = \frac{n(t)}{V_0}$$

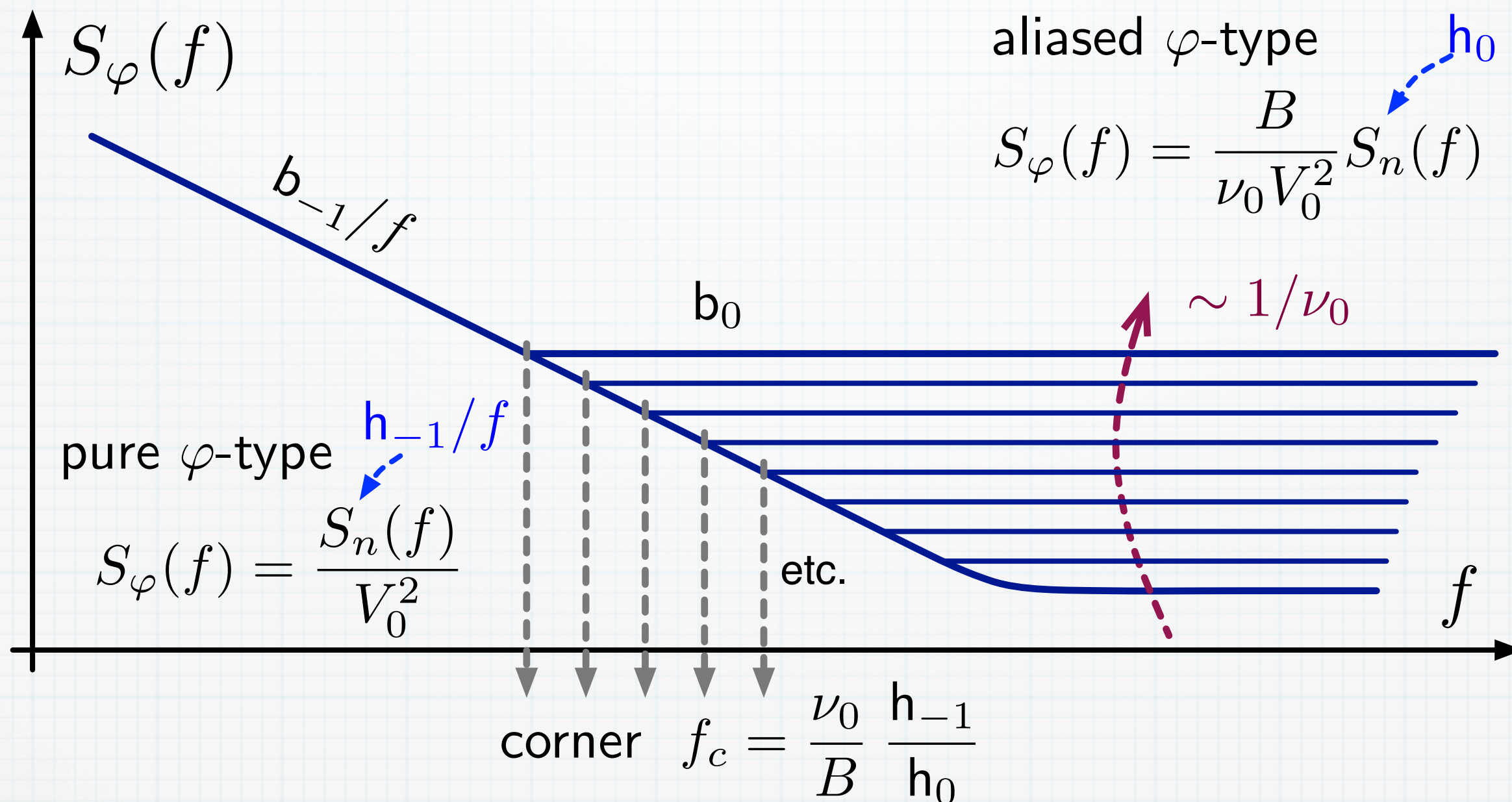
**$\varphi$ -type  
noise**

$$S_\varphi(f) = \frac{S_n(f)}{V_0^2}$$

constant vs  $\nu_0$

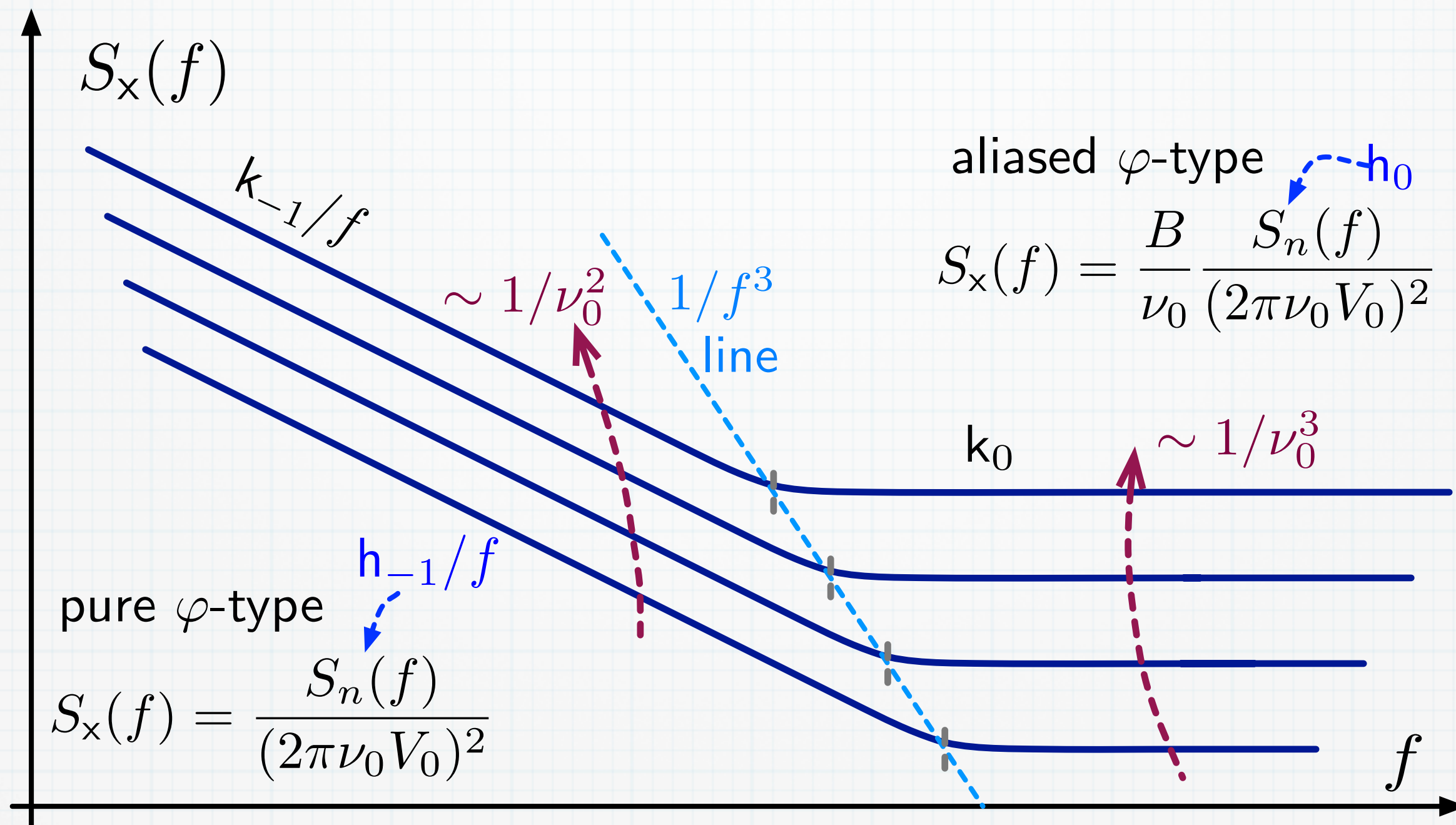
# $\varphi$ -type PM Noise

Remember that white noise is subject to aliasing, flicker is not



Power law  $S_n(f) = \sum h_i f^i$  [do not mistake with  $S_y(f)$ ]

# $\varphi$ -type Jitter

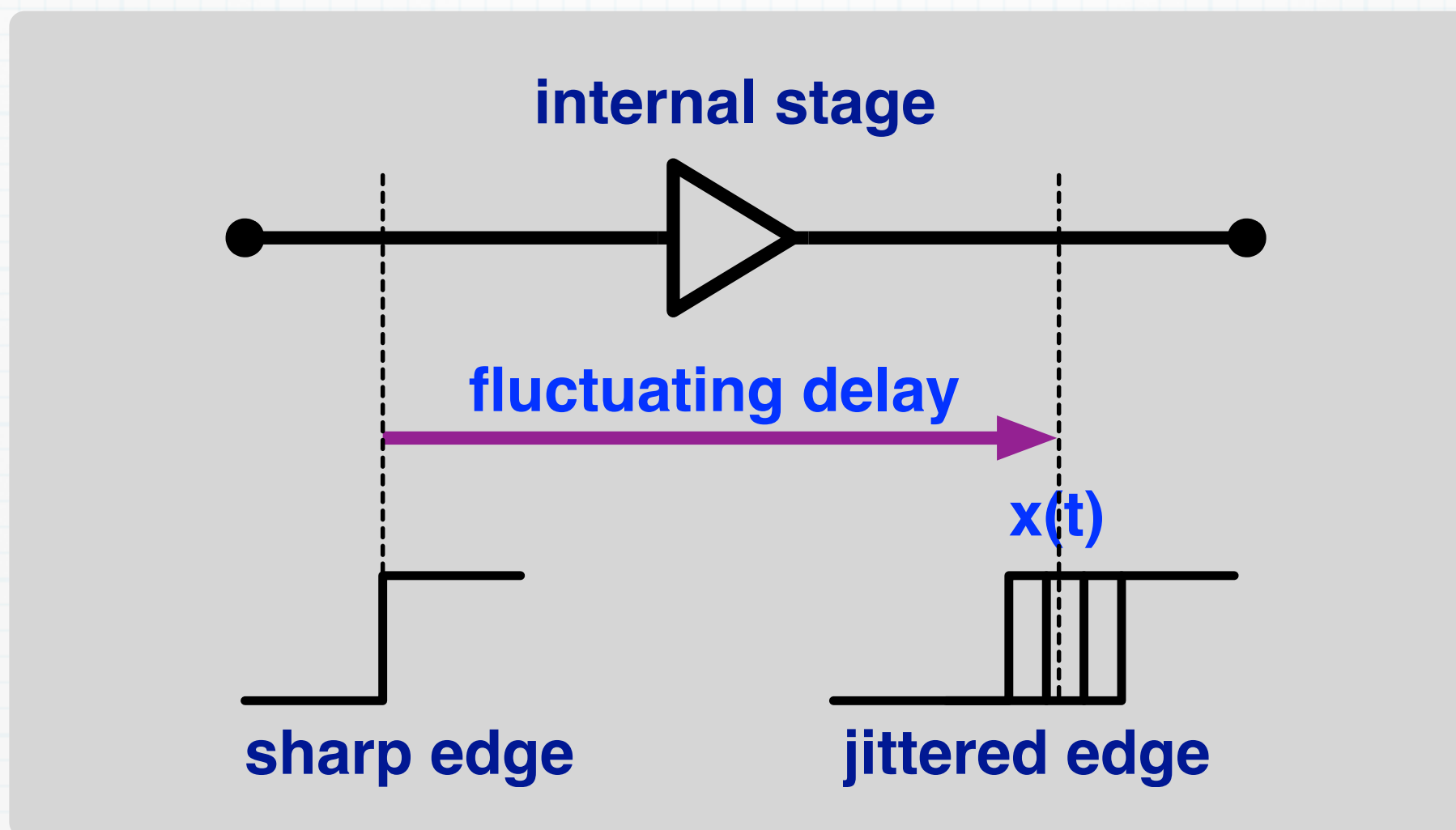


Power law  $S_x(f) = \sum k_i f^i$  and  $S_n(f) = \sum h_i f^i$



# Internal Delay Fluctuation

**x-type noise**

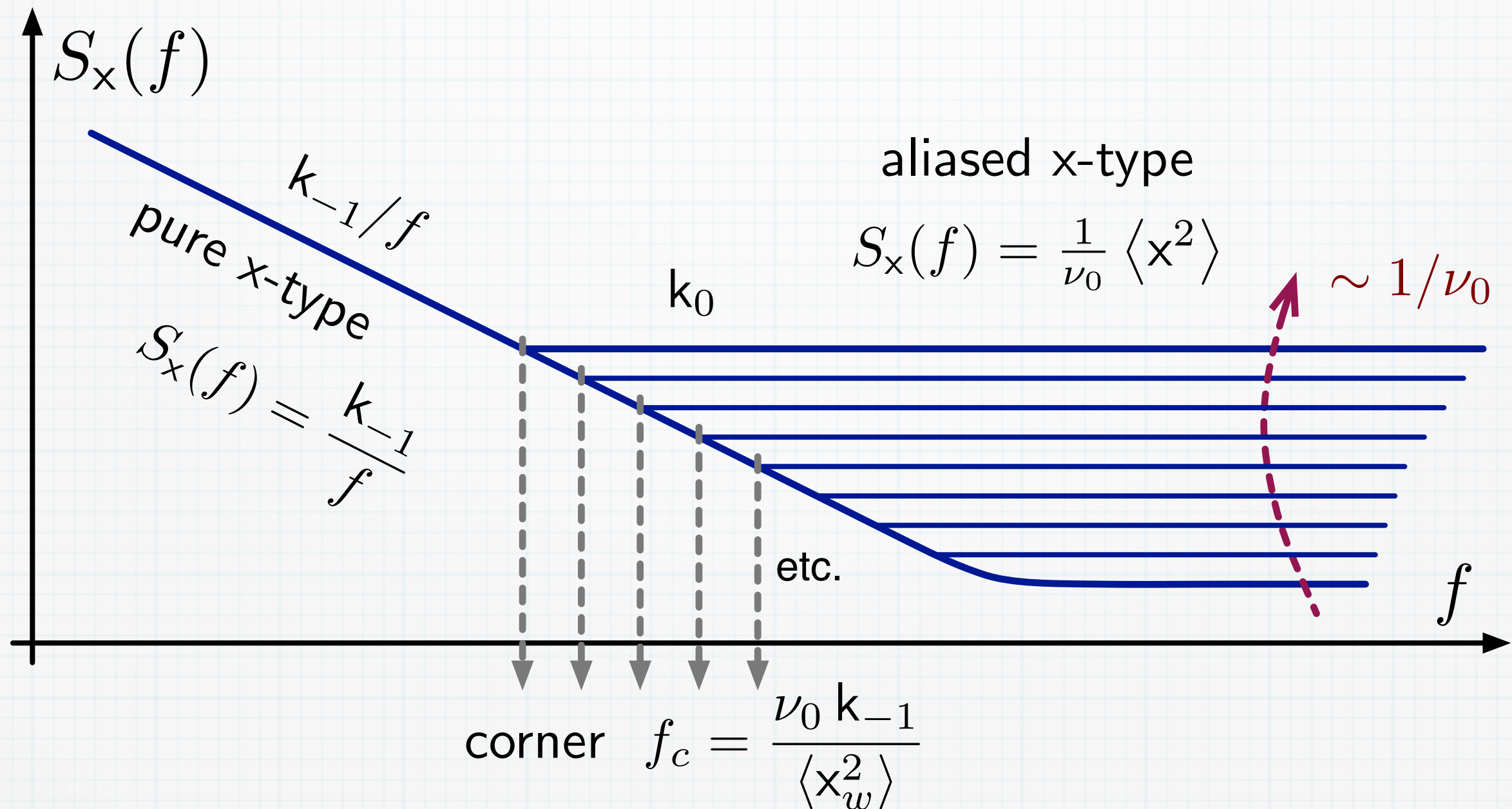


- The internal delay fluctuates by an amount  $x(t)$
- This has nothing to do with threshold and frequency



# x-type Jitter

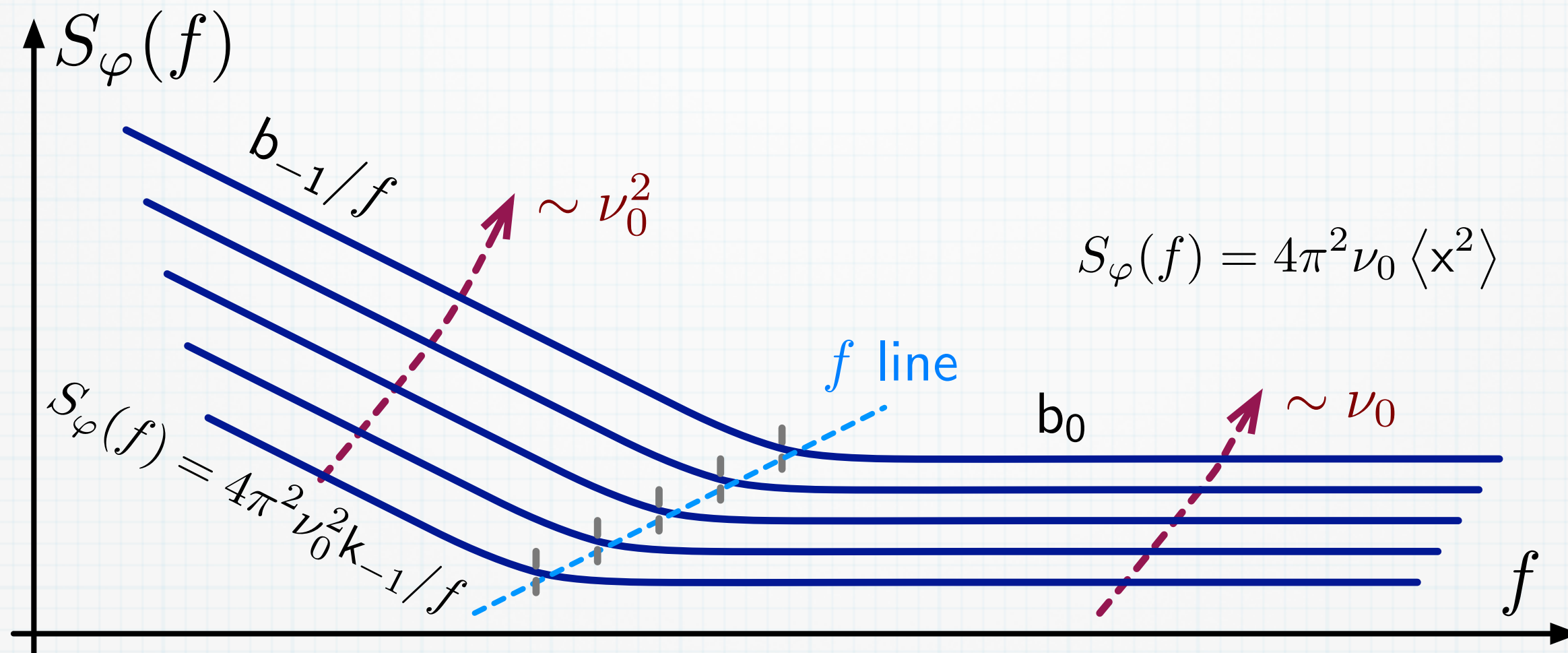
Remember that white noise is subject to aliasing, flicker is not



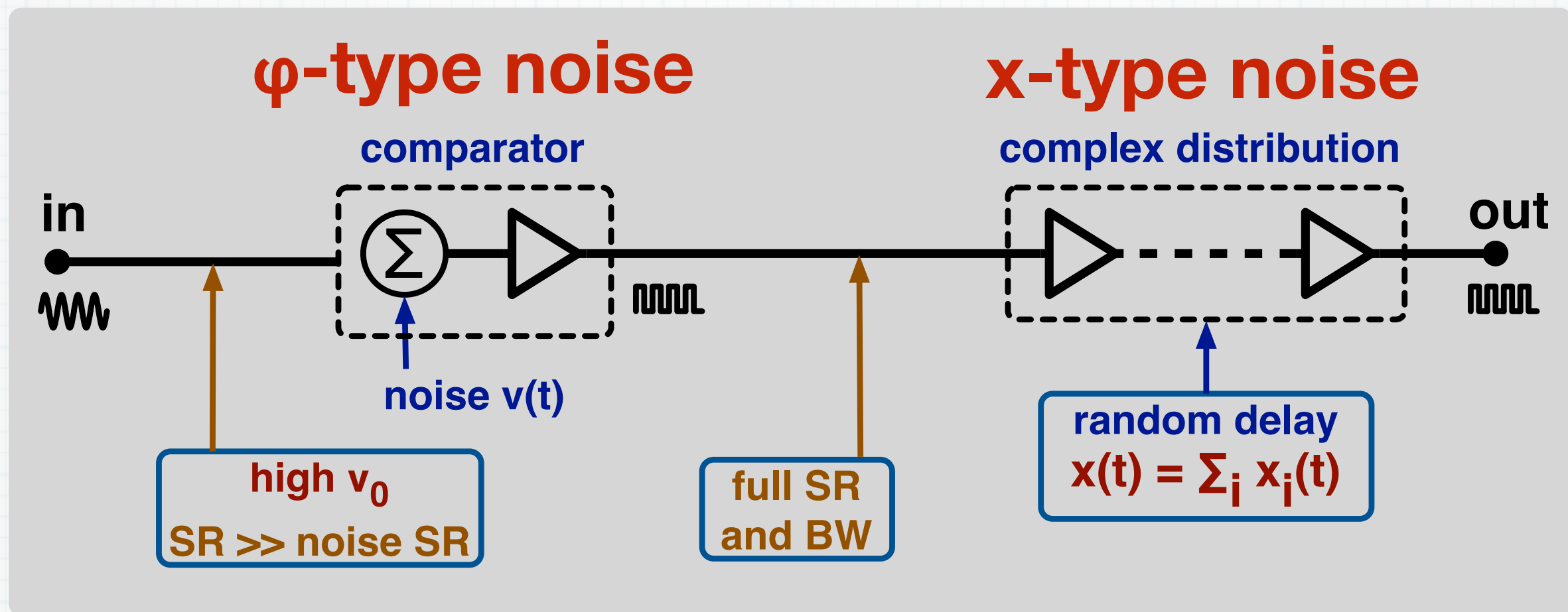
Power law  $S_x(f) = \sum k_i f^i$

# x-Type PM Noise

Remember that white noise is subject to aliasing, flicker is not



# Full Noise Mechanism

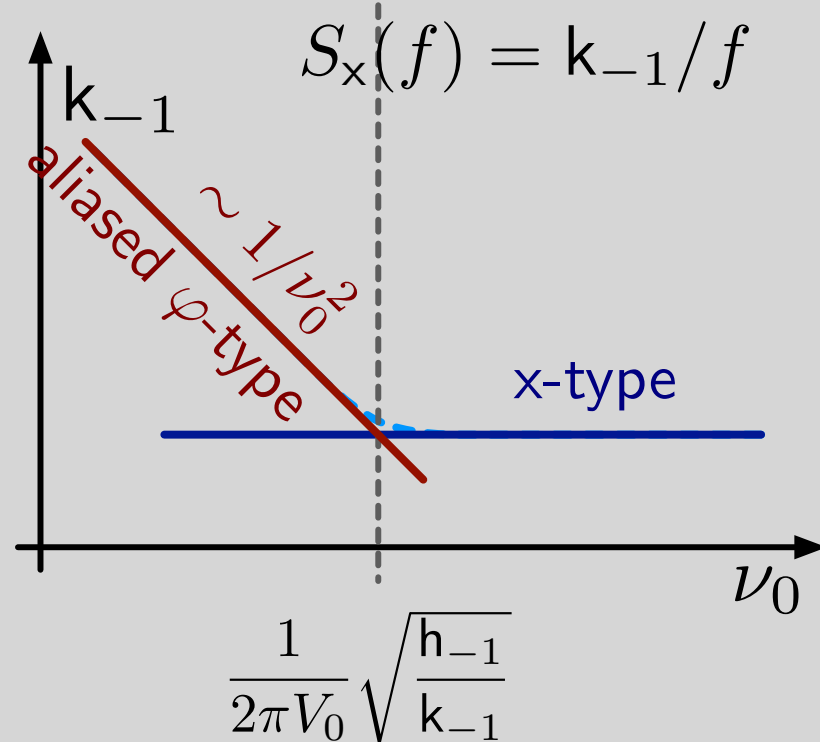
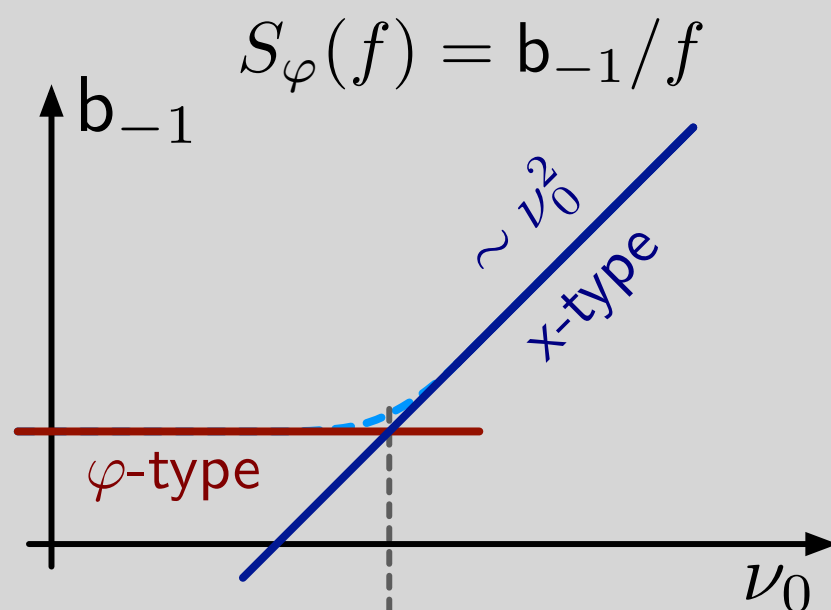


- The  $\varphi$ -type noise may show up or not, depending on input noise and SR
- At the comparator out, the edges attain full SR and bandwidth of the technology
- Complex distribution  $\rightarrow$  independent fluctuations add up

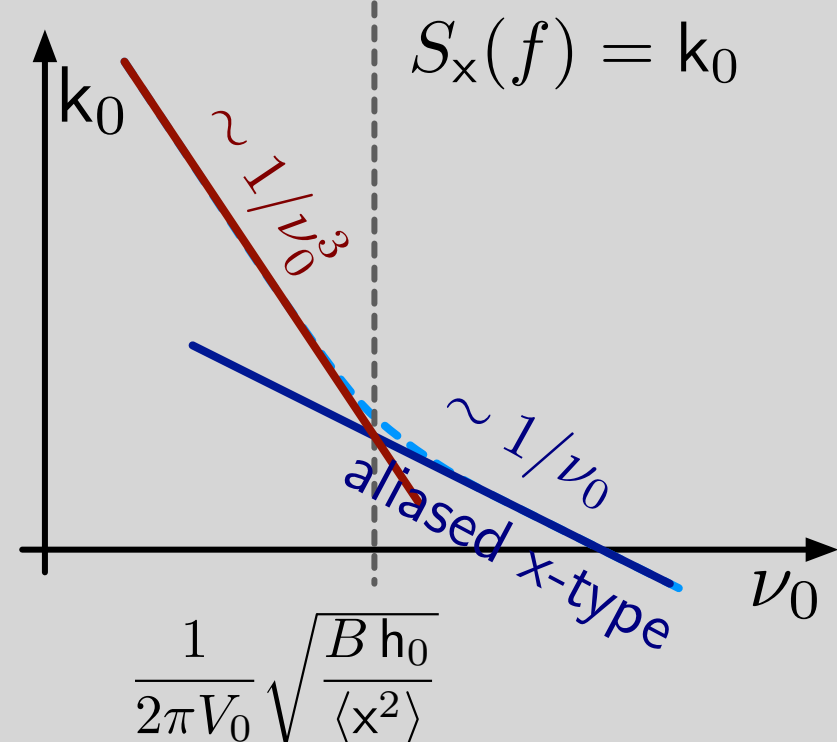
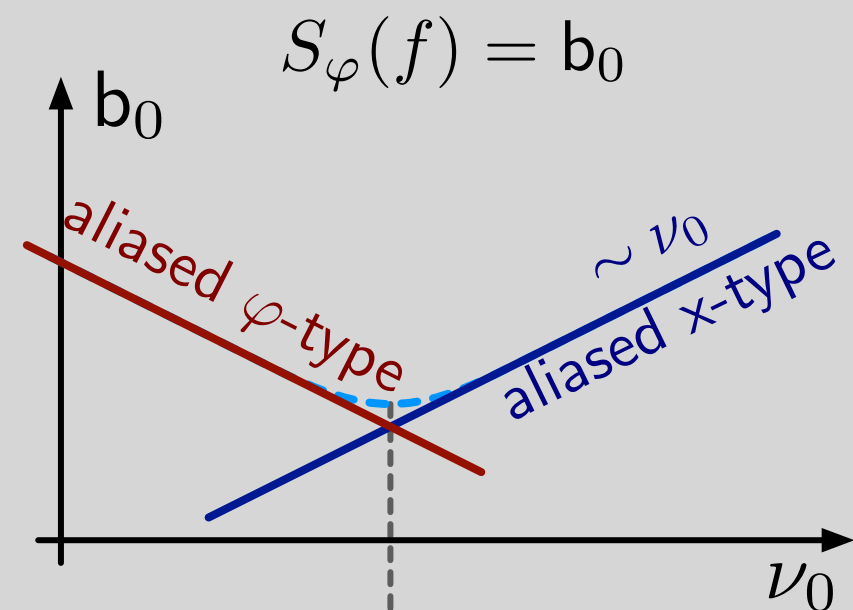
$$x(t) = \sum_i x_i(t) \quad \text{and} \quad \langle x^2(t) \rangle = \sum_i \langle x_i^2(t) \rangle$$

# Full Noise Mechanism

## Flicker (not aliased)



## White (aliased)



# Summary of the Noise Types

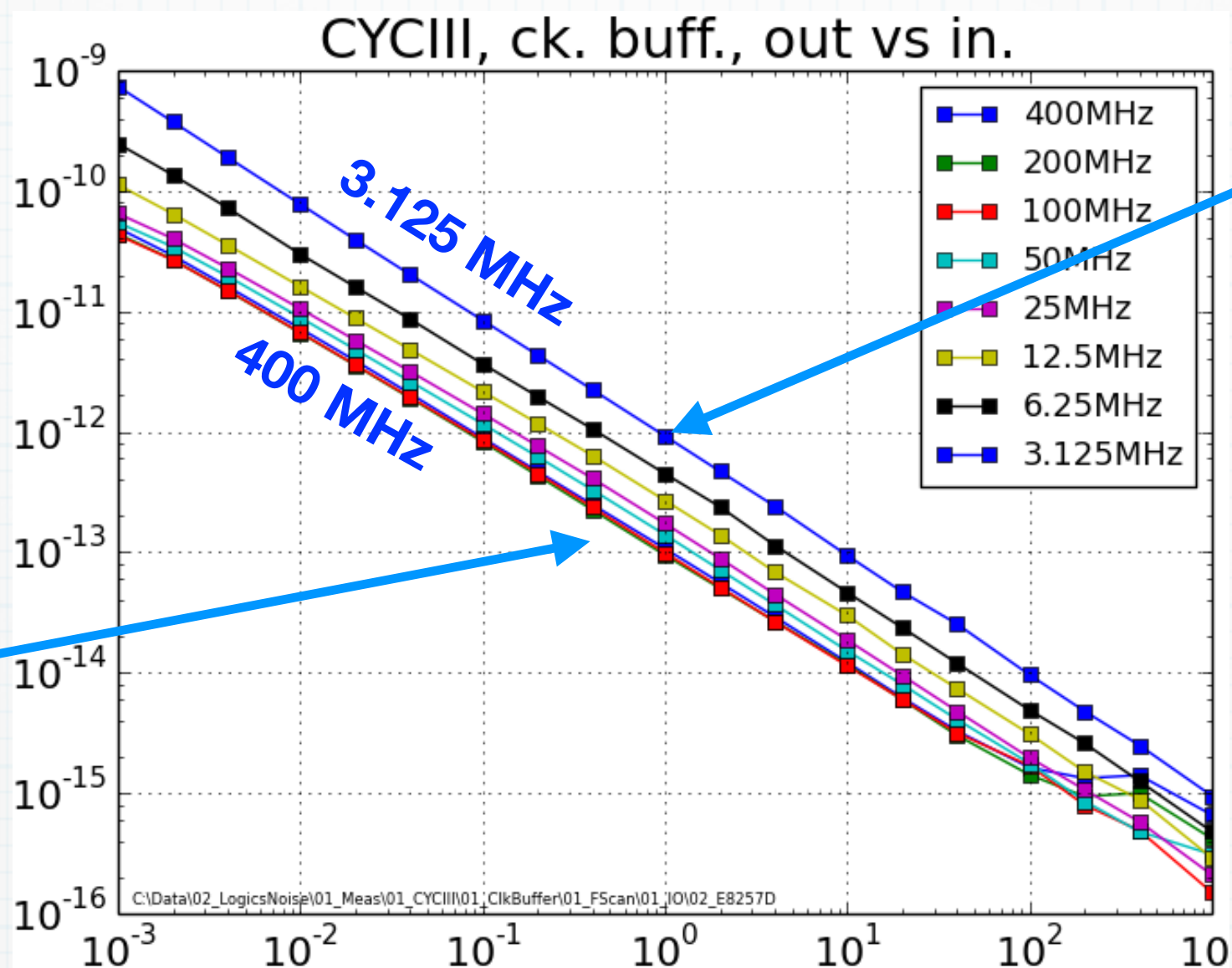
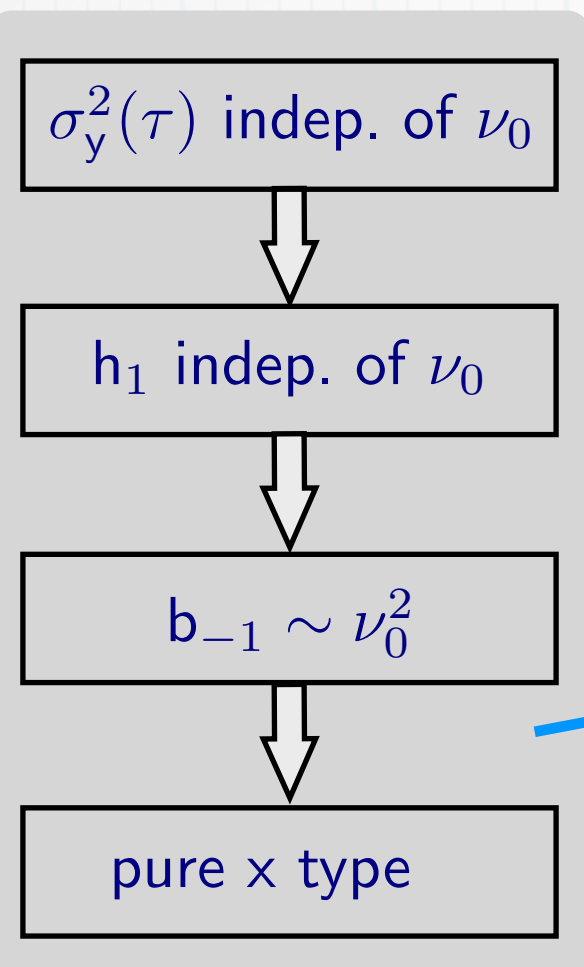
Noise class	Dependence on $\nu_0$	
	$S_\varphi(f)$	$S_x(f)$
Pure $\varphi$ -type	$C$ vs. $\nu_0$	$1/\nu_0^2$
Aliased $\varphi$ -type	$1/\nu_0$	$1/\nu_0^3$
Pure x-type	$\nu_0^2$	$C$ vs. $\nu_0$
Aliased x-type	$\nu_0$	$1/\nu_0$

- **Pure x-type.** High speed circuits, inside the device. Must be  $1/f$ , otherwise aliasing shows up
- **Aliased x-type.** High speed circuits, inside the device, at low switching frequency. The clock must be either high frequency  $\sin()$ , or sharp square wave, so that the threshold has no effect.

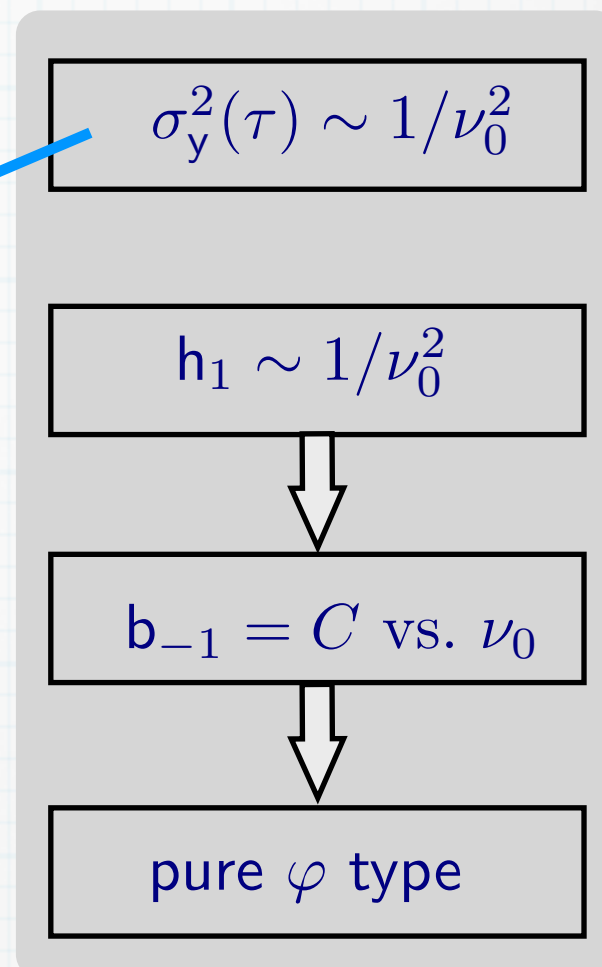


# Noise Types and AVAR

## High frequency

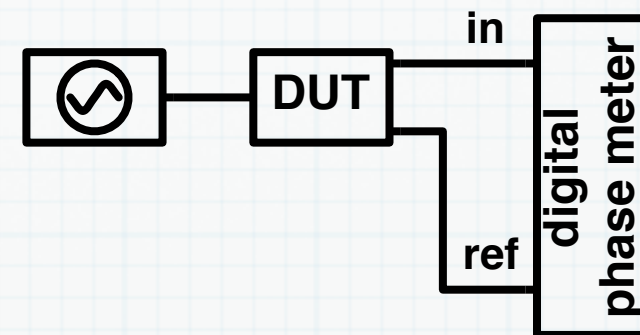
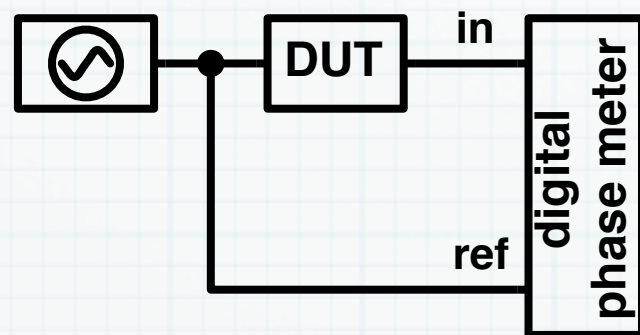


## Low frequency



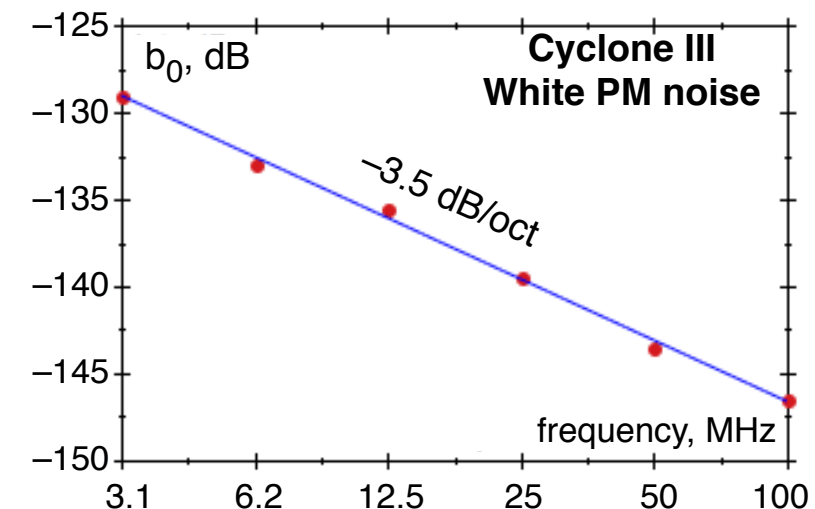
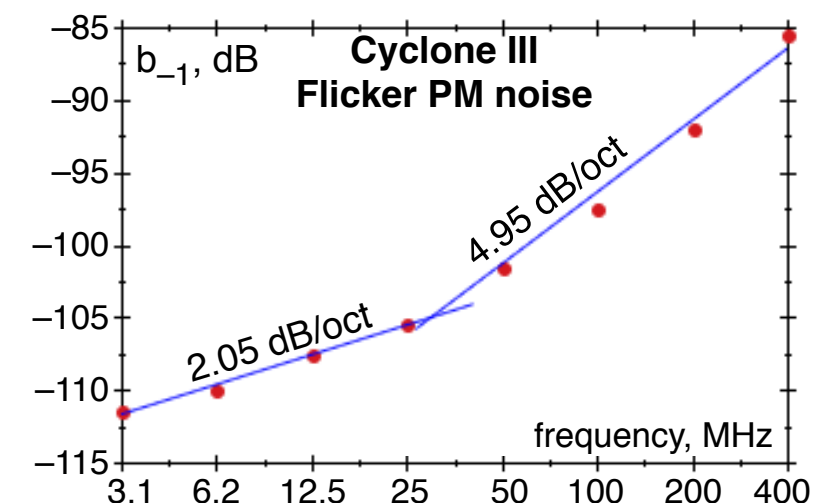
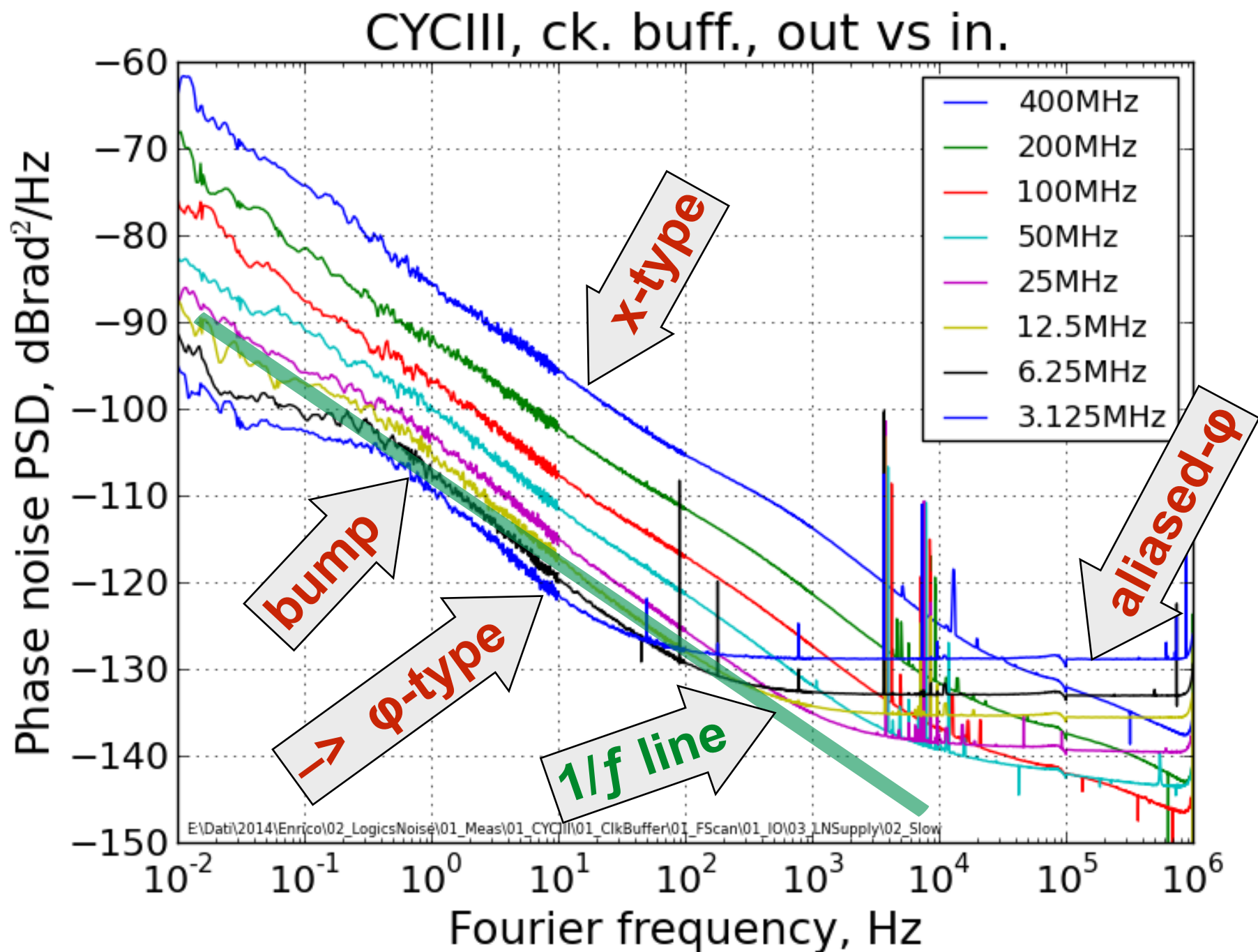
# Examples

**Measurements are performed with the Symmetricom (Microsemi) DS 5125 and DS 5120 dual-channel phase meter**



**Allow (input frequency)  $\neq$  (ref frequency)**

# Cyclone III Clock Buffer



## Flicker

- High  $v_0$   $\rightarrow$  scales as  $v_0$  (x-type)
- Low  $v_0$ ,  $\rightarrow$  to  $\phi$ -type (bumps 0.1–10 Hz)

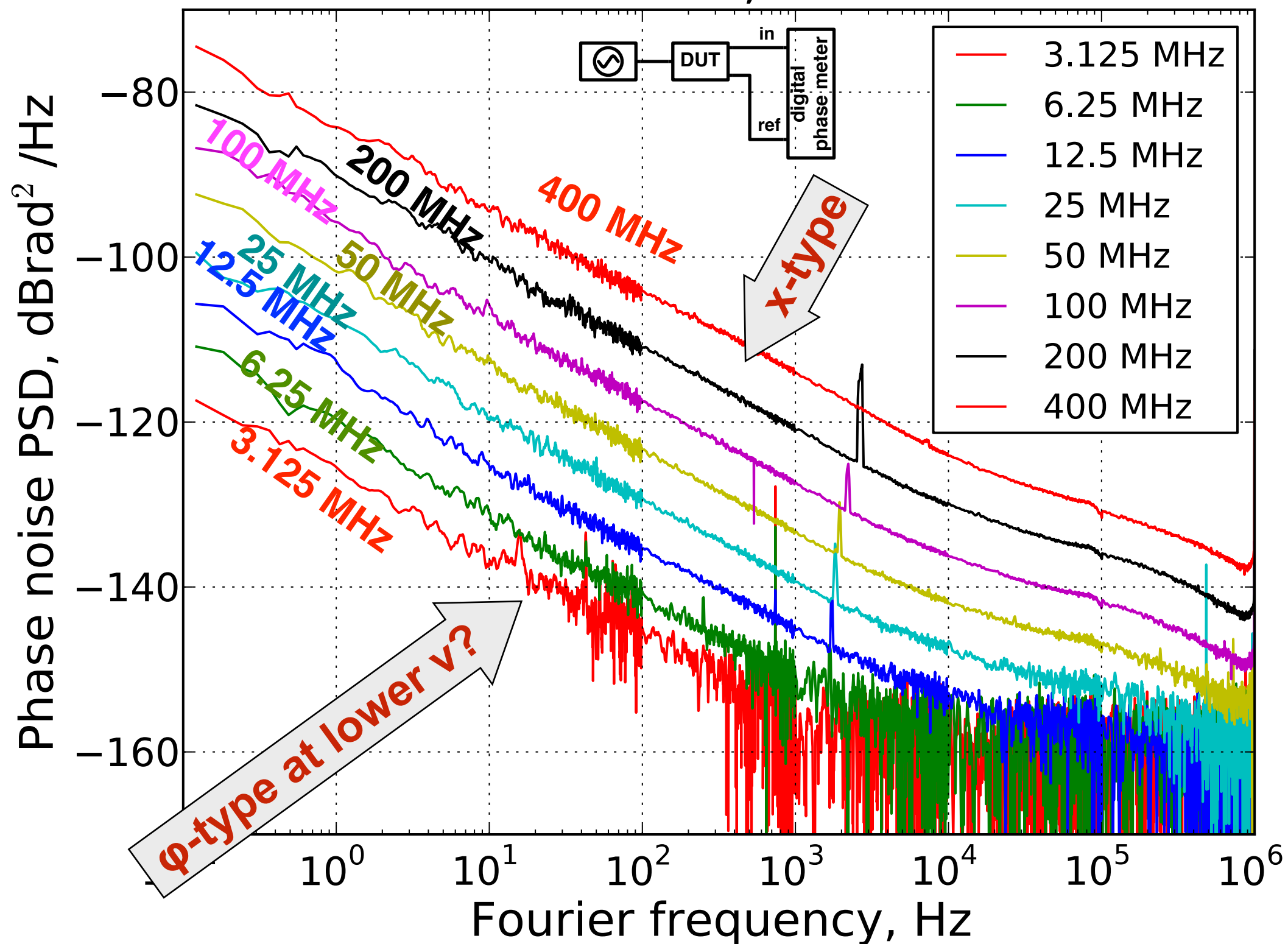
## White

- Aliasing shows up at low  $v_0$



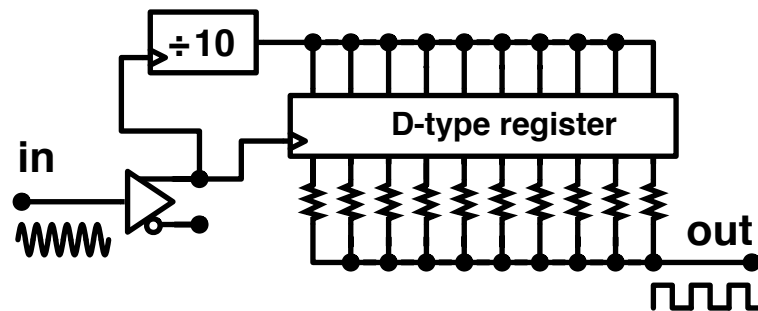
# Cyclone III Output Buffer

Double buffer, Out vs Out

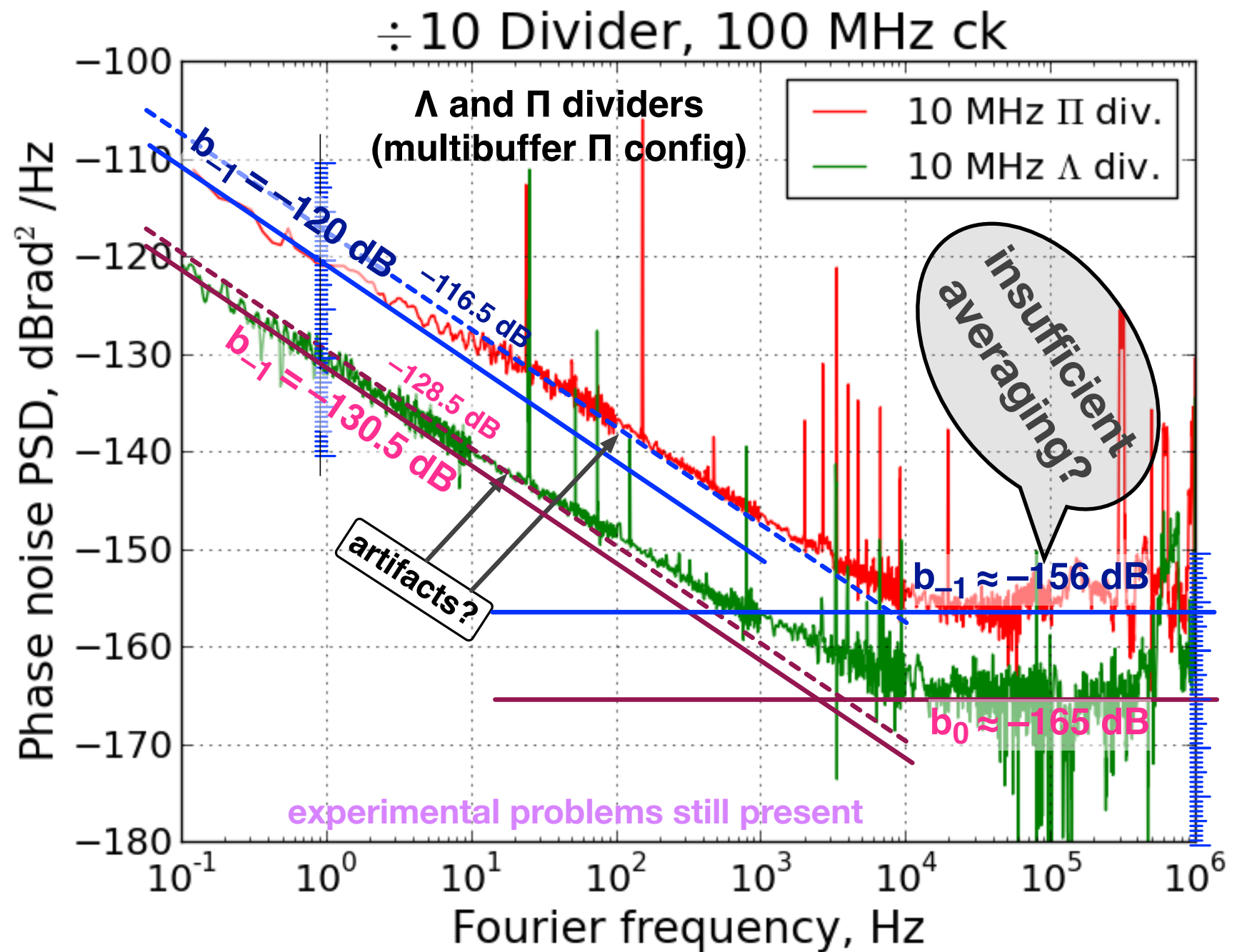
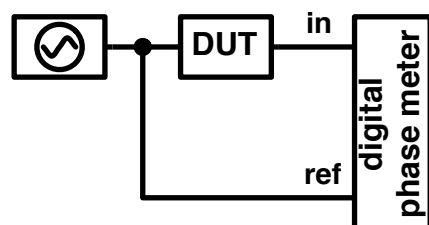
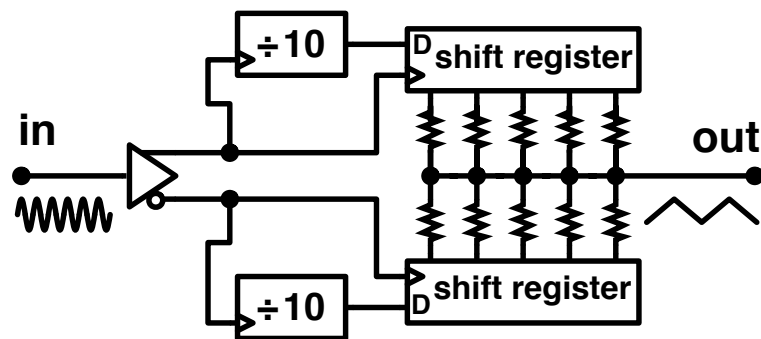


# MAX 3000 CPLD [300 nm] (1)

## $\Pi$ divider

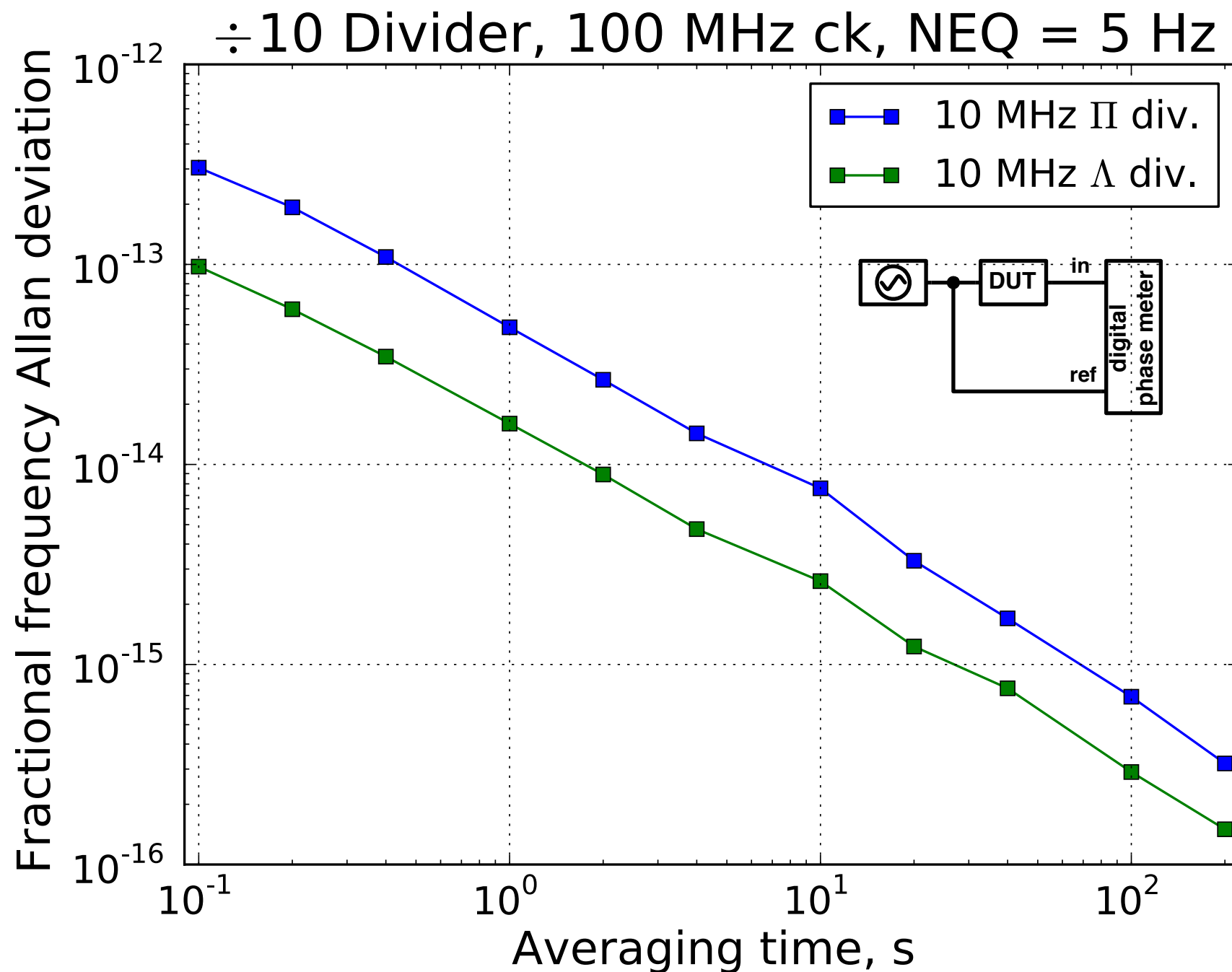


## $\Lambda$ divider



- **Flicker region → Negligible aliasing**
- The  $\Pi$  divider is still not well explained
- The  $\Lambda$  divider exhibits low  $1/f$  and low white noise

# MAX 3000 CPLD [300 nm] (2)

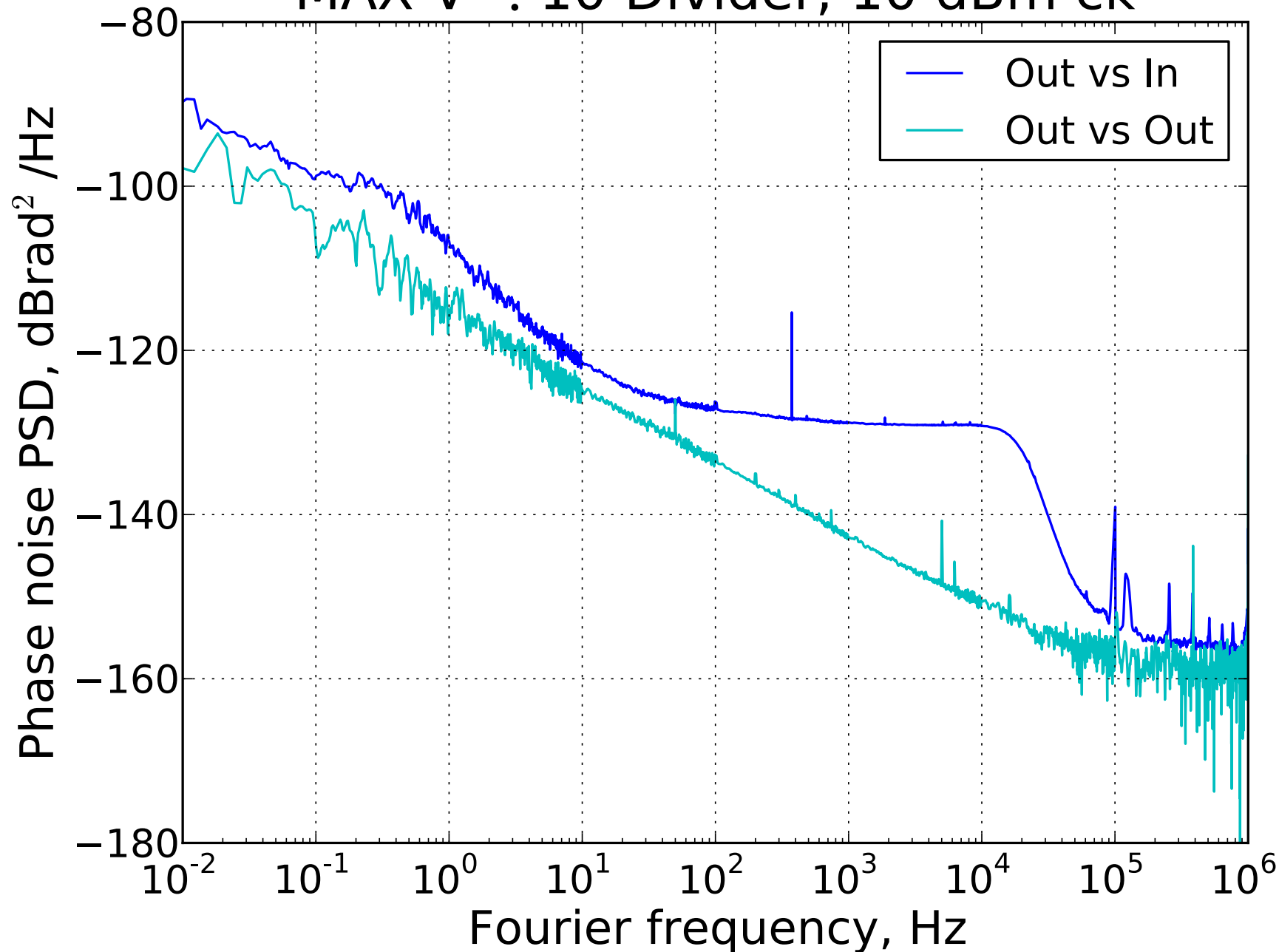


- Slope  $1/\tau$ , typical of white and flicker PM noise
- The  $\Lambda$  divider performs  $2 \times 10^{-14}$  at  $\tau = 1$  s, 10 MHz output

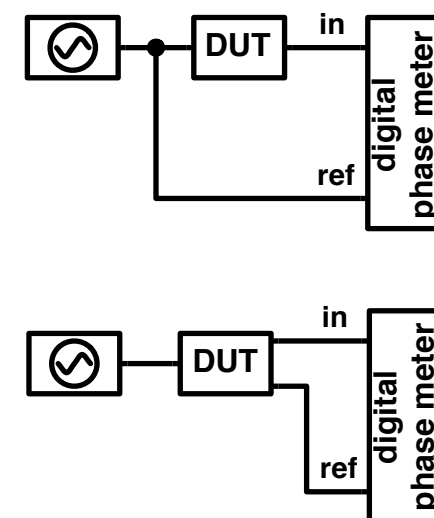
# Max V CPLD [180 nm]

**We do not trust this spectrum (bump  $\rightarrow$  supply voltage?)**

MAX V  $\div 10$  Divider, 10 dBm ck

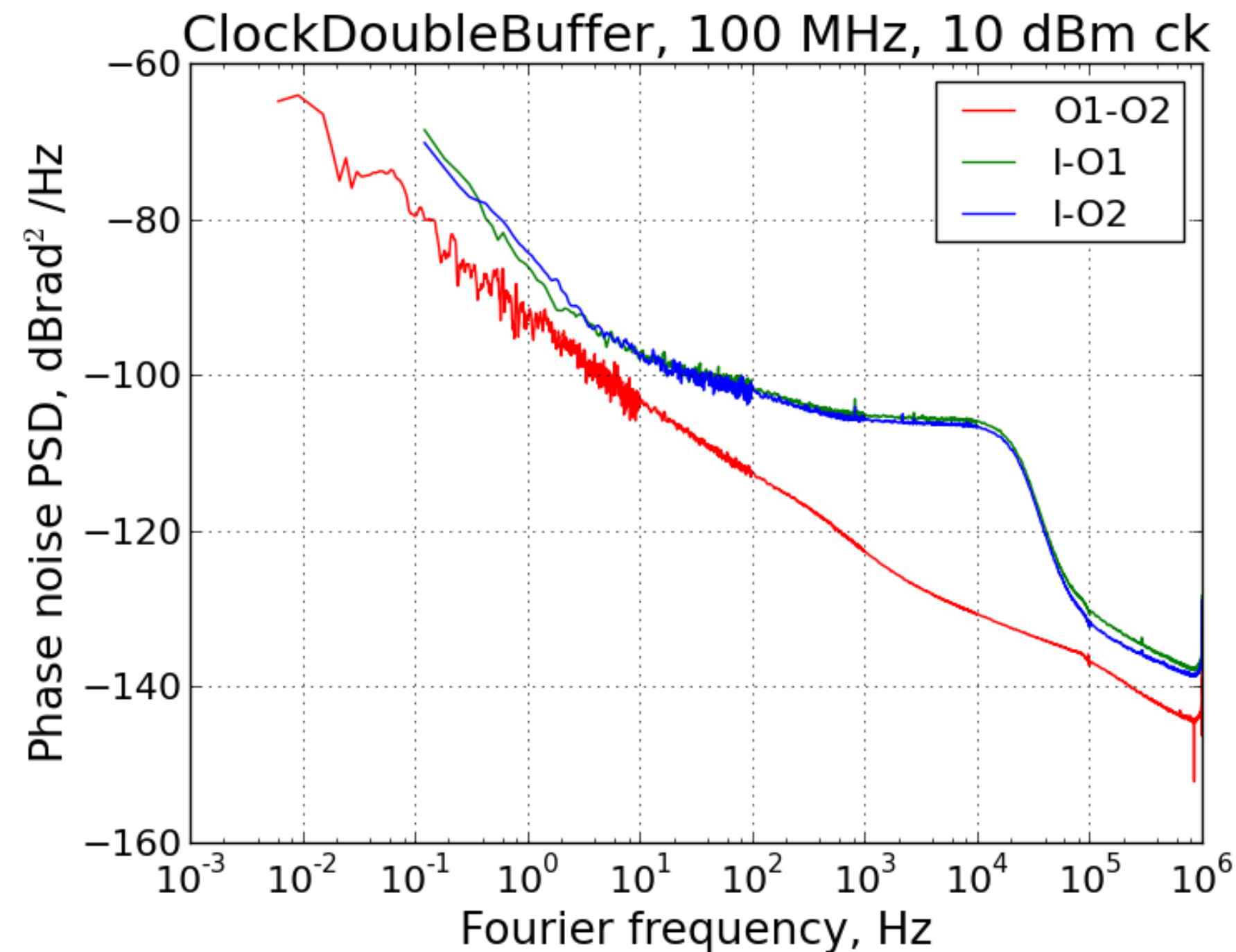


- **Two lambda dividers**
- **output-to-output and common clock,**
- **low  $f$ , emphasizes the  $1/f$  noise**
- **Same, only output-to-output and common clock**



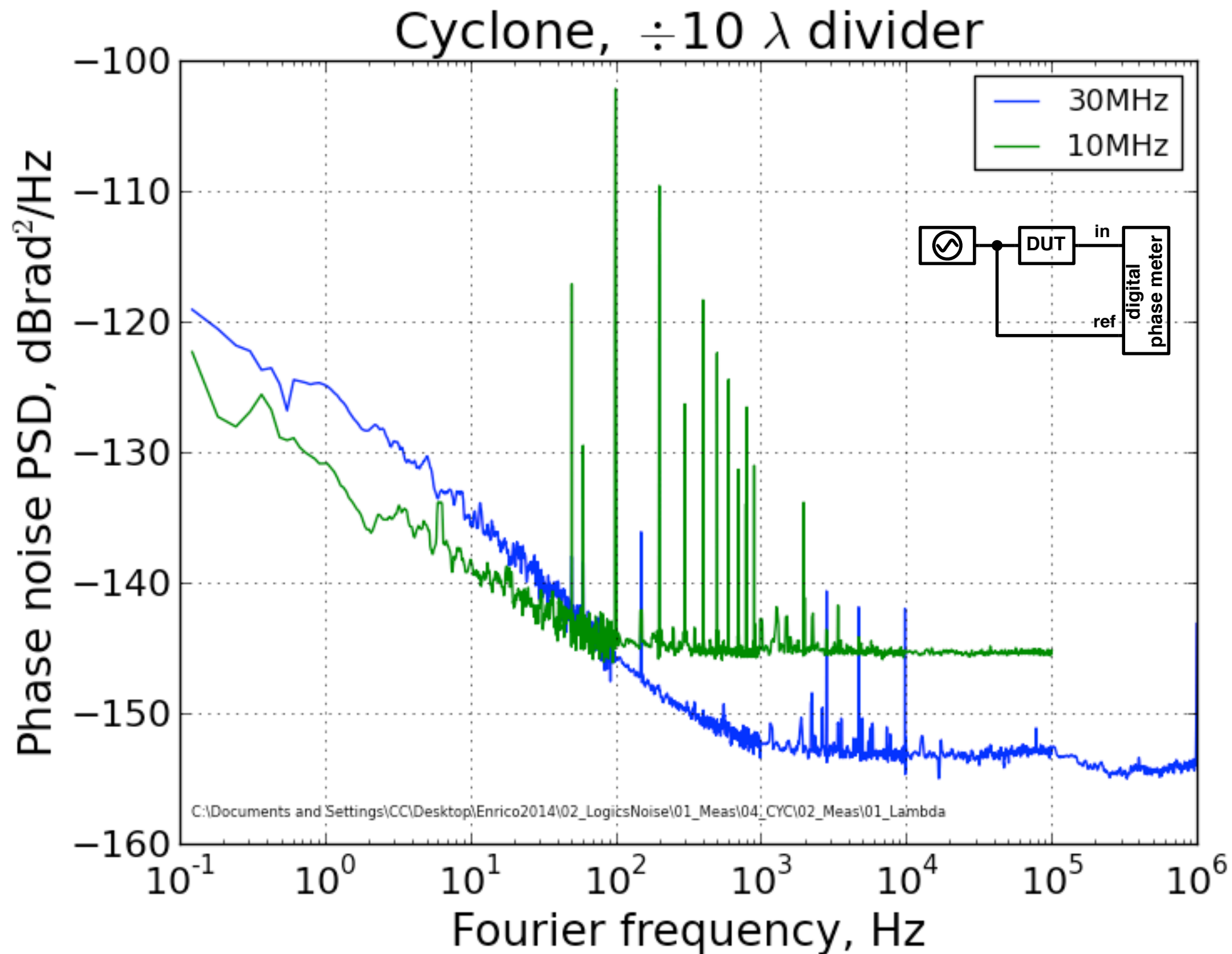
# Max V CPLD [180 nm]

**We do not trust this spectrum (bump  $\rightarrow$  supply voltage?)**



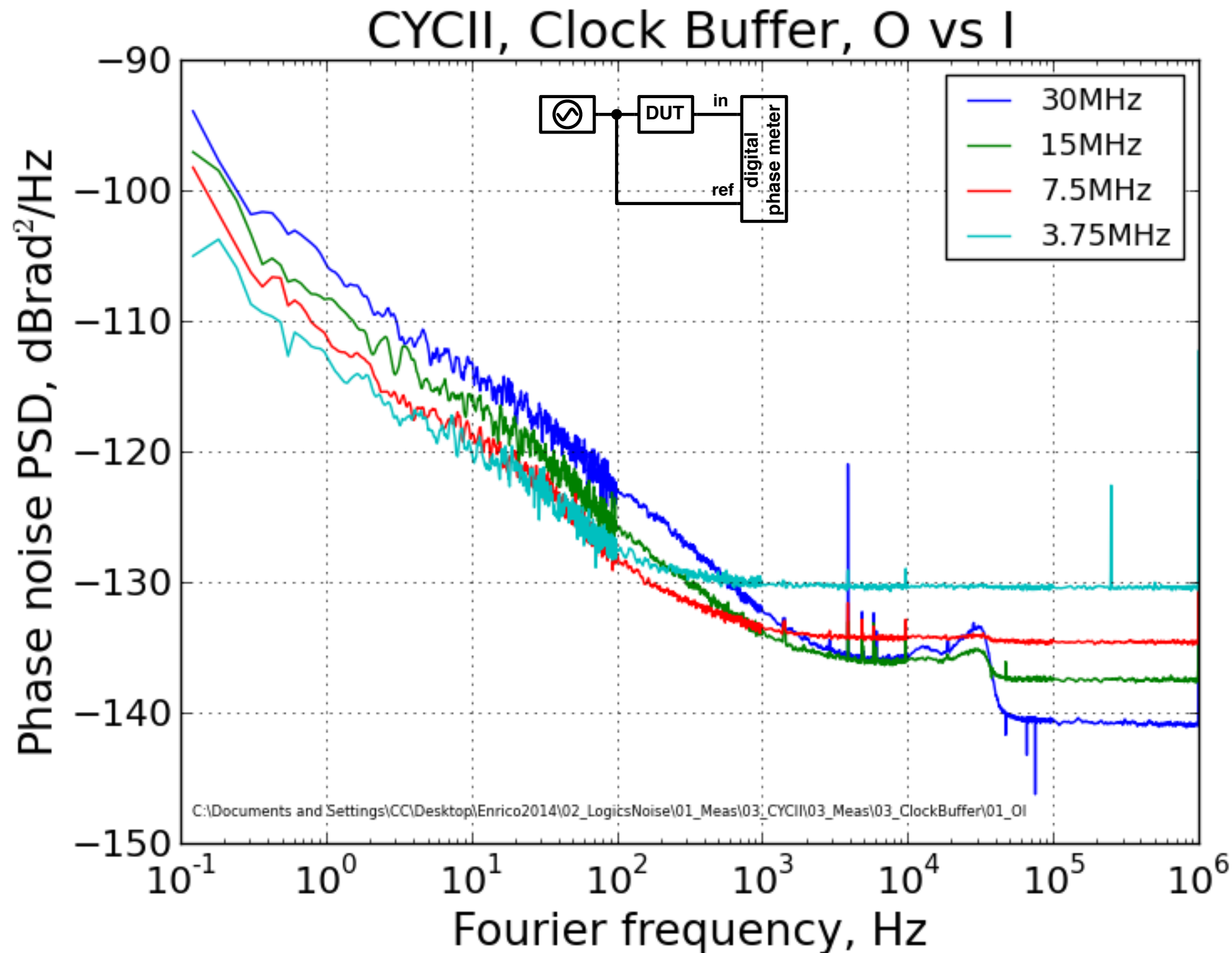
- Two lambda dividers
- output-to-output and common clock,
- low  $f$ , emphasizes the  $1/f$  noise
- Clock, difference between the two outputs

# Cyclone II $\Lambda$ Divider [90 nm]

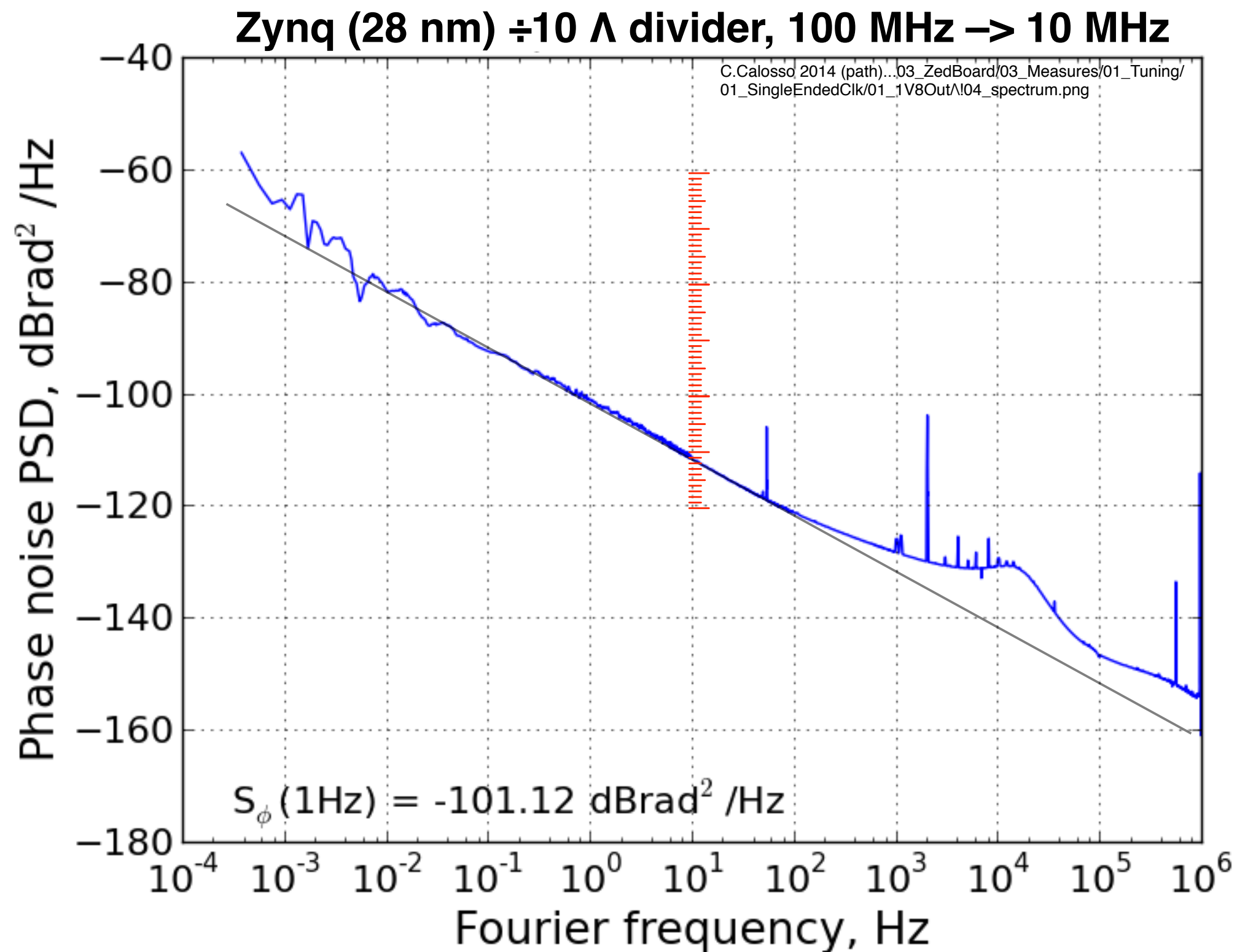




# Cyclone II Clock Buffer [90 nm]<sup>35</sup>

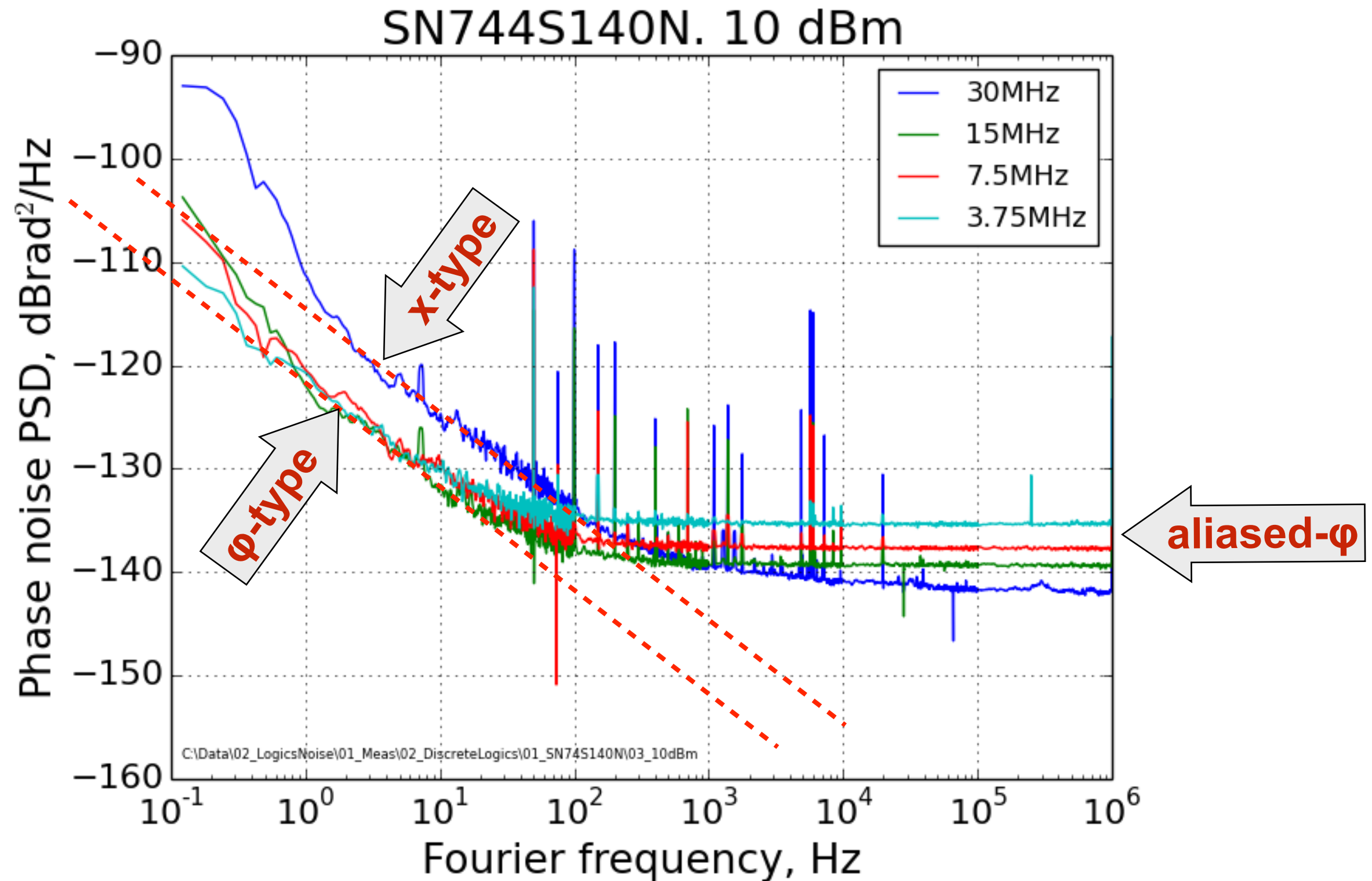


# Zynq (28 nm), $\Lambda$ Divider





# 74S140 – Old TTL 50 $\Omega$ Driver

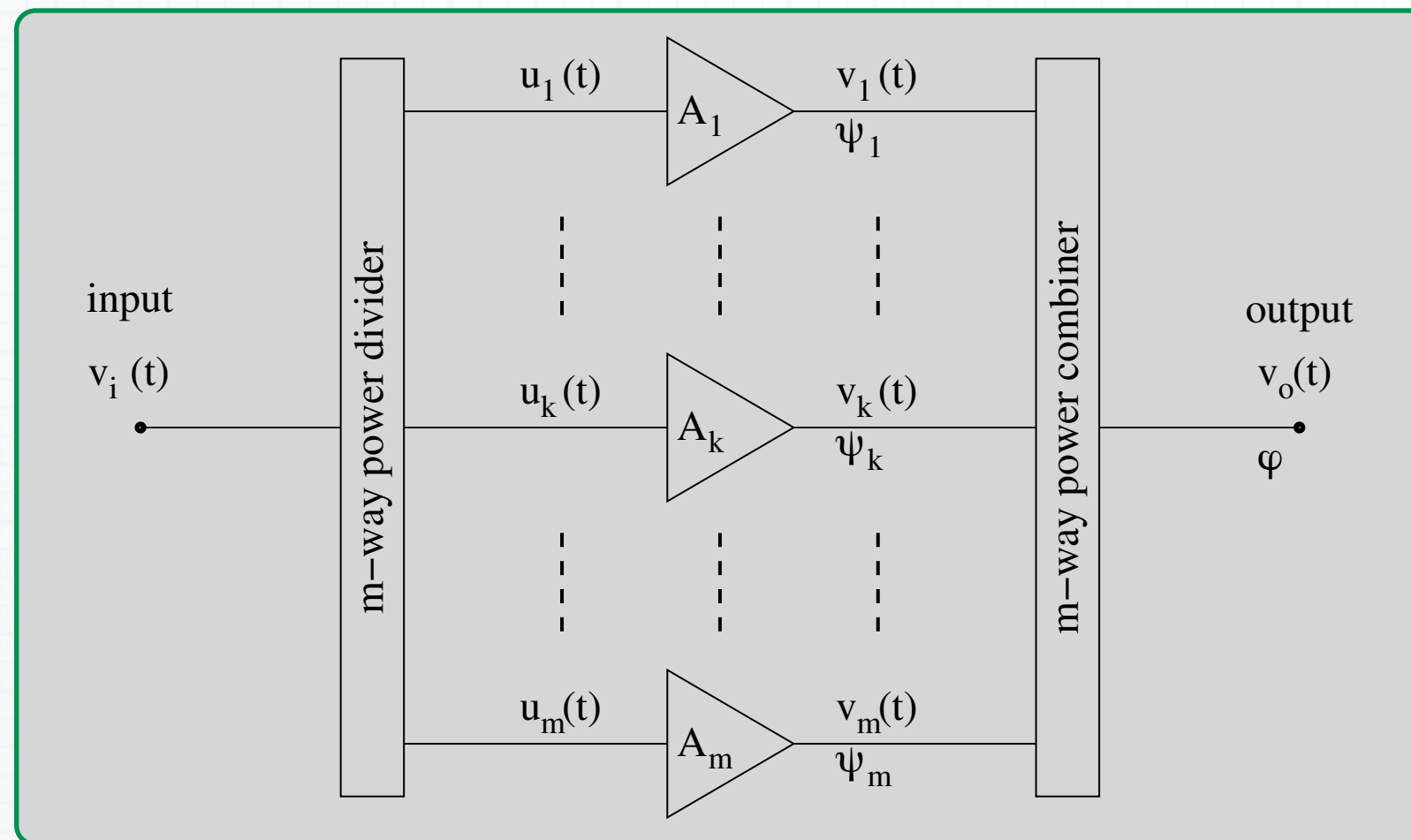


# Some Facts

## Related to Phase and Noise

- Volume Law
- Input Chatter
- Internal PLL
- Thermal Effects
- .....

# Rationale for the Volume Law



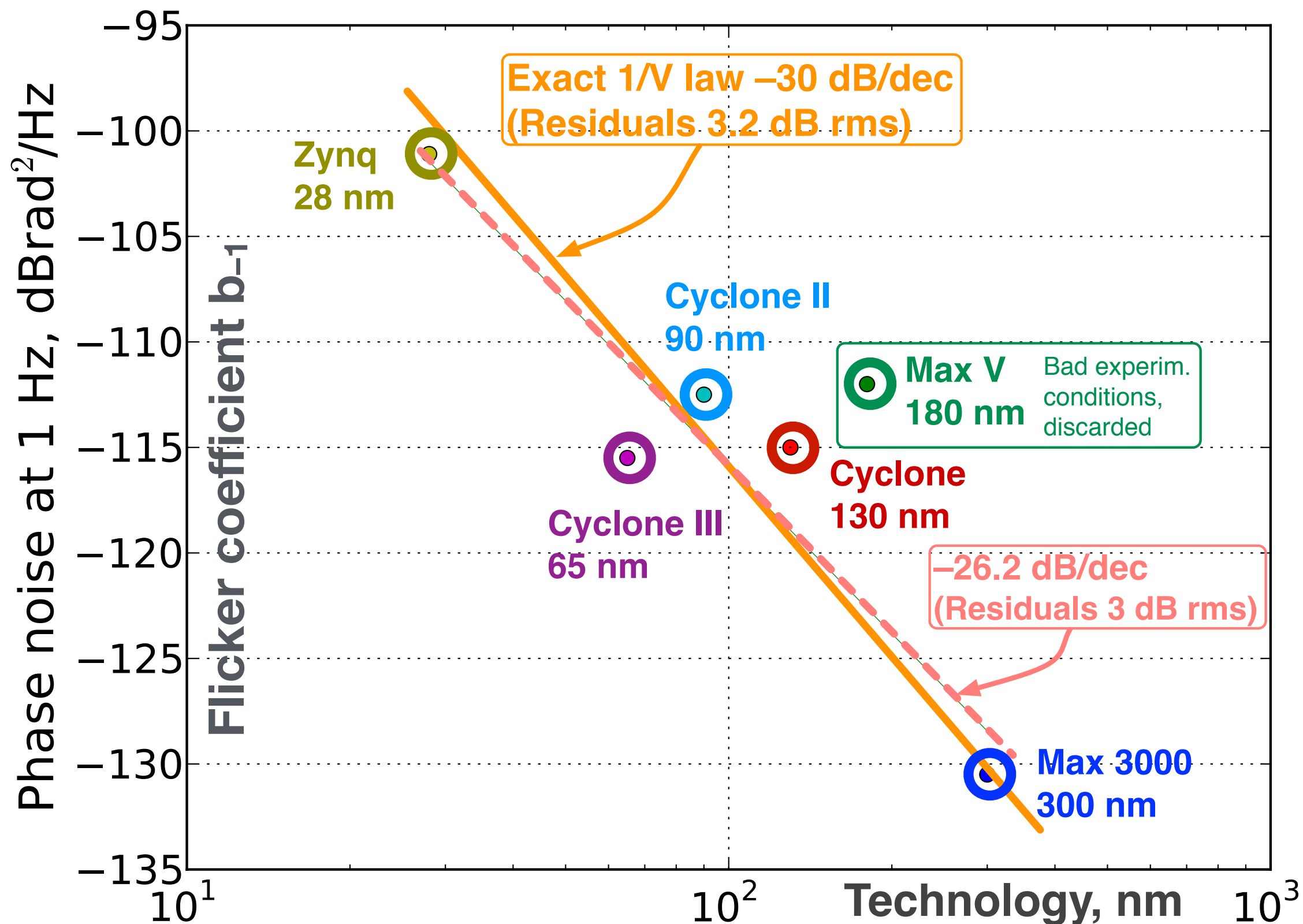
$$b_{-1} = \frac{1}{m} [b_{-1}]_{\text{cell}}$$

- Flicker coeff  $b_{-1}$  is  $\approx$  independent of power
- The flicker of a branch is not increased by splitting the input power
- The carrier adds up coherently, the phase noise adds up statistically
- Hence, the  $1/f$  phase noise is reduced by a factor  $m$

## Gedankenexperiment

- Flicker is of microscopic origin (Gaussian  $\rightarrow$  central limit theorem)
- Join the  $m$  branches of a parallel device forming a compound
- $1/f$  PM is proportional to the inverse size of the active region

# The Volume Law!



# Input Chatter (1/3)

Chatter occurs when the RMS Slew Rate of noise exceeds the slew rate of the pure signal

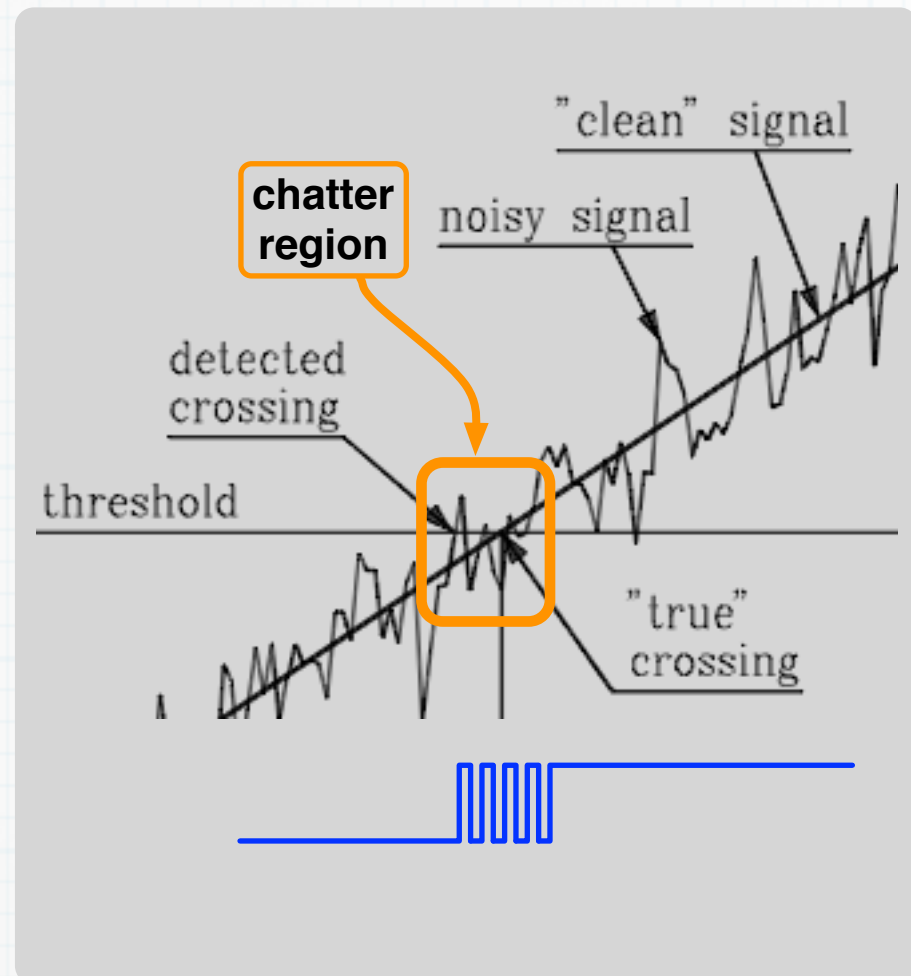
## Pure signal

$$v(t) = V_0 \cos(2\pi\nu_0 t)$$

$$SR = 2\pi\nu_0 V_0$$

## Wide band noise

$$\begin{aligned} \langle SR^2 \rangle &= 4\pi^2 \int_0^B f^2 S_V(f) df \\ &= \frac{4\pi^2}{3} \sigma_V^2 B^2 \quad (\text{rms}) \end{aligned}$$



## Chatter threshold

$$\nu_0^2 = \frac{1}{3} \frac{S_v B^3}{V_0^2}$$

## Example

- $V_0 = 100$  mV peak
- 10 nV/ $\sqrt{\text{Hz}}$  noise
- 650 MHz max  $\rightarrow$  2 GHz noise BW
- Chatter threshold  $\nu = 5.2$  MHz

**With high-speed devices,  
chatter can occur at  
unexpectedly high frequencies**







# Input Chatter – Example (3/3)

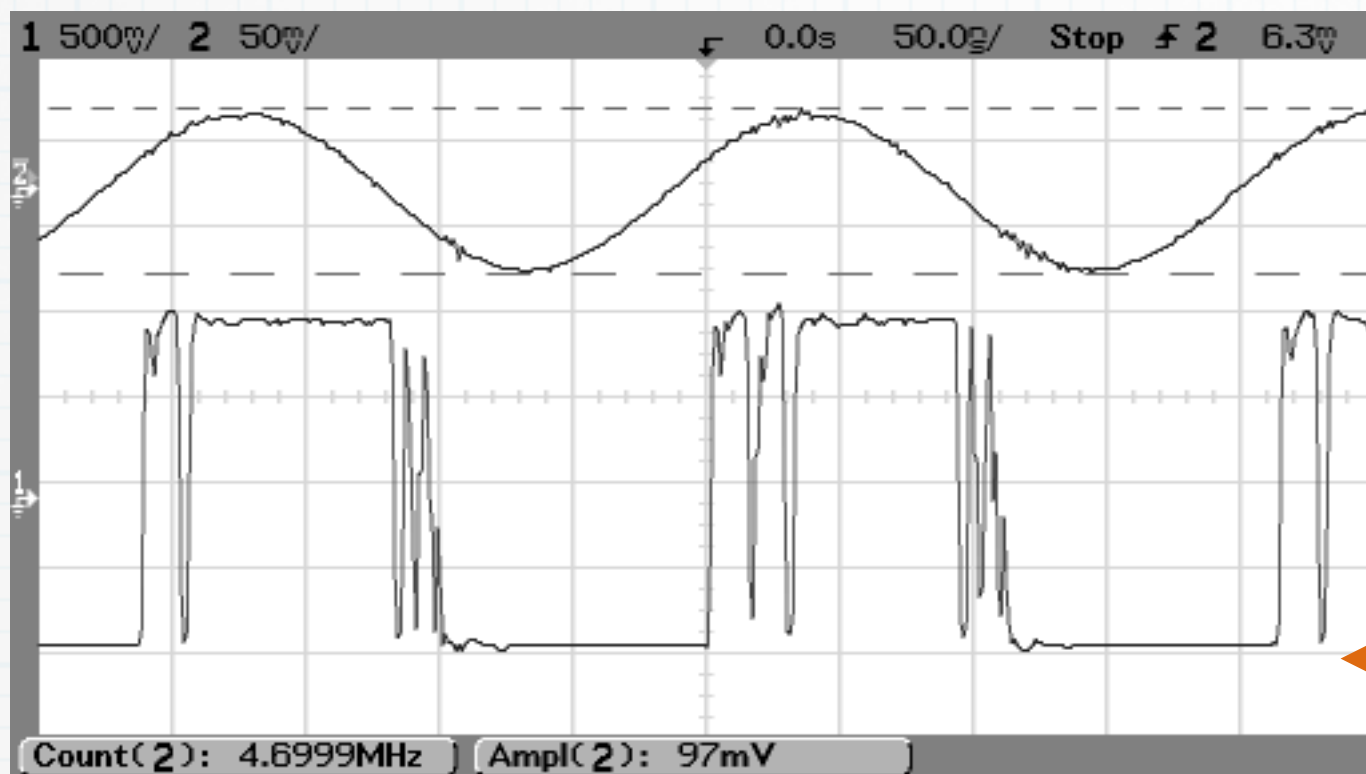
Good agreement with theory

## Experiment

- Cyclone III FPGA
- Estimated noise  $10 \text{ nV}/\sqrt{\text{Hz}}$
- Estimated BW 2 GHz

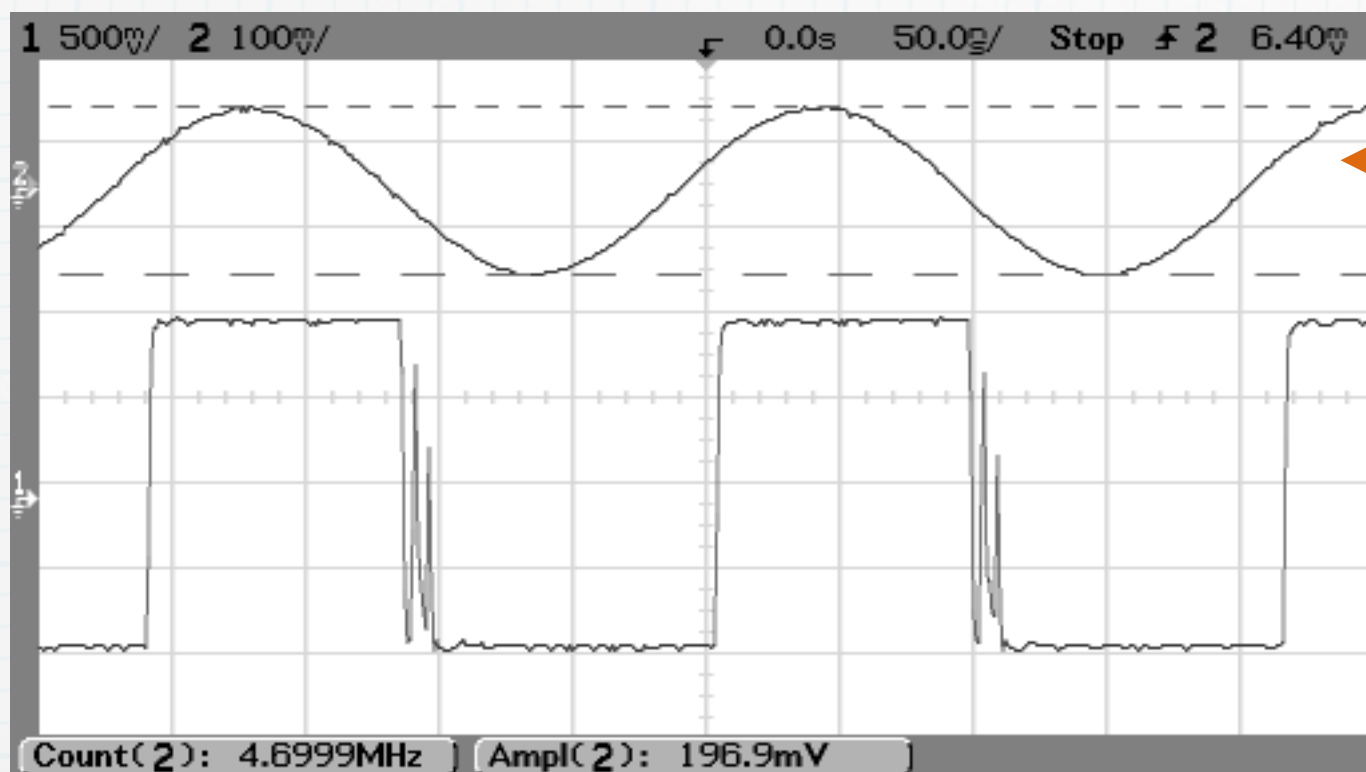
$$V_0 = 50 \text{ mV} (100 \text{ mV}_{\text{pp}})$$

$$v_0 = 4.7 \text{ MHz}$$



$$V_0 = 100 \text{ mV} (200 \text{ mV}_{\text{pp}})$$

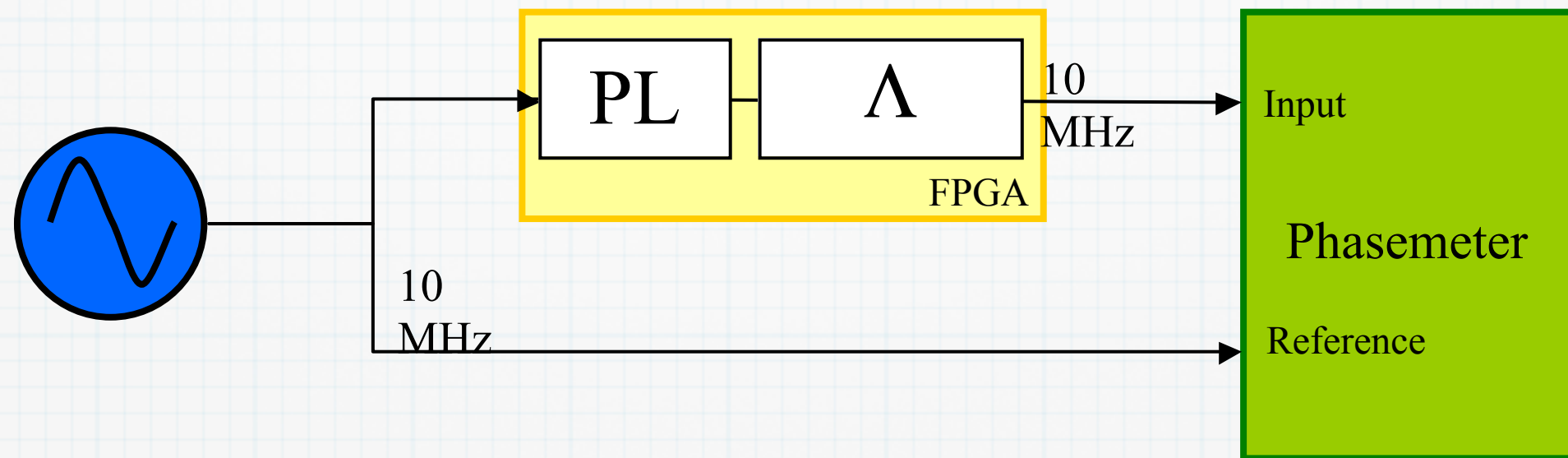
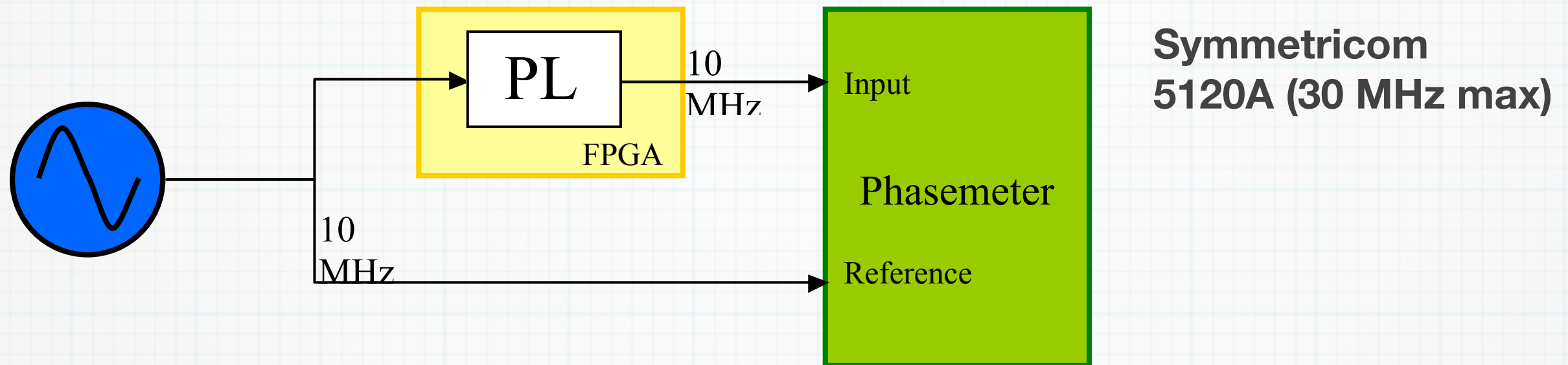
$$v_0 = 4.7 \text{ MHz}$$



Asymmetry shows up

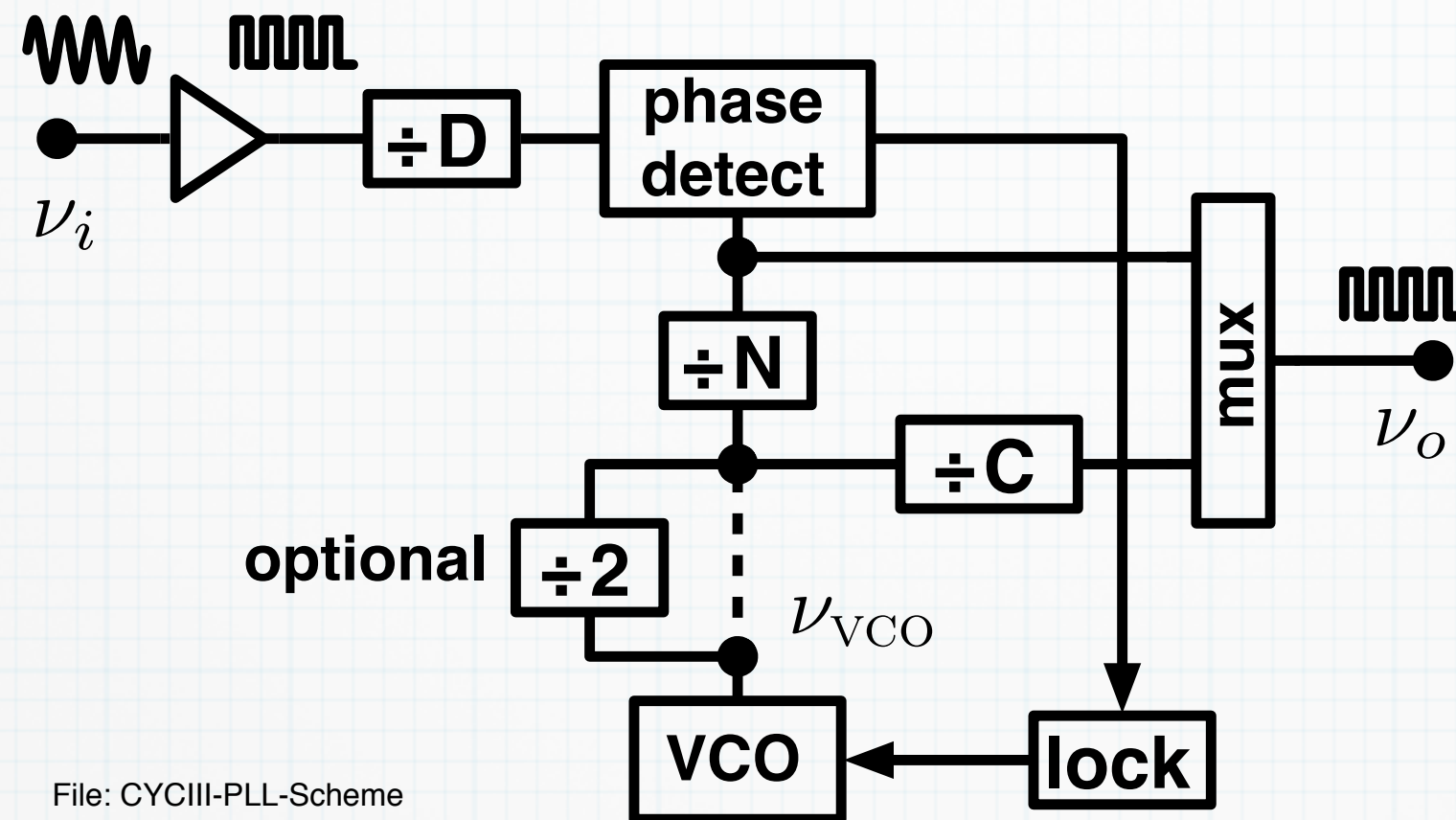
Explanation takes a detailed electrical model, which we have not

# Cyclone II Internal PLL (1/4)



- A  $\Lambda$  divider (inside the FPGA) enables the measurement
  - The divider noise is low enough
  - A trick to work at low frequency

# Cyclone II Internal PLL (2/4)

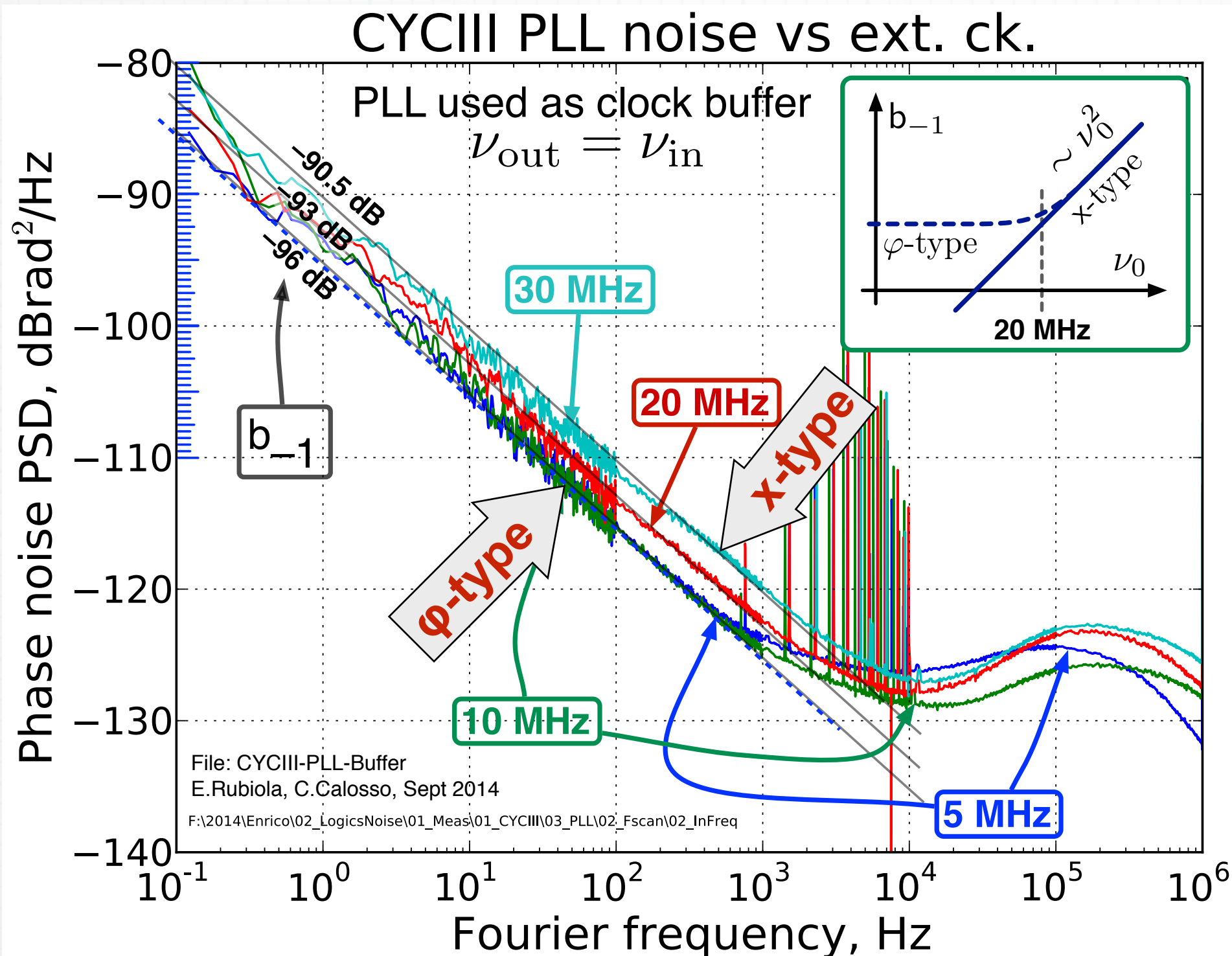


File: CYCIII-PLL-Scheme

- Low-Q LC oscillator ( $Q \approx 10$ ), 0.6–1.3 GHz
- Optional  $\div 2$  always present
- We set  $D = 1$  (for lowest noise)
- QUARTUS decides  $C$  and  $N$

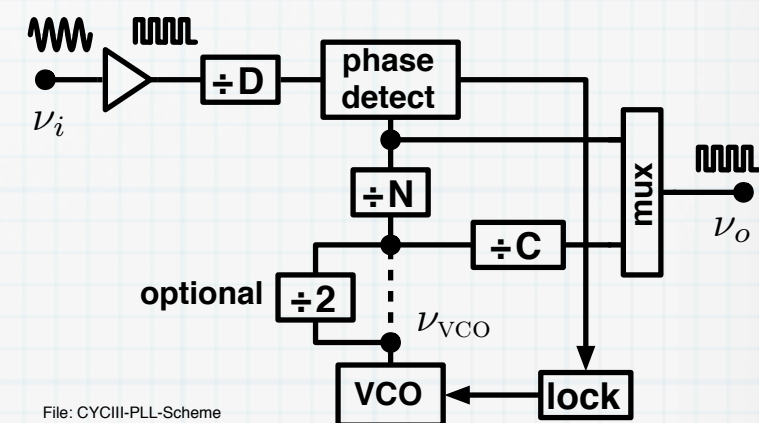
# Cyclone II Internal PLL (3/4)

## PLL used as a buffer



Crossover between  
phi-type and x-type  
at 20 MHz

- $\phi$ -type  
16  $\mu\text{rad}/\sqrt{\text{Hz}}$  @ 1 Hz
- x-type  
220 fs/ $\sqrt{\text{Hz}}$  @ 1 Hz

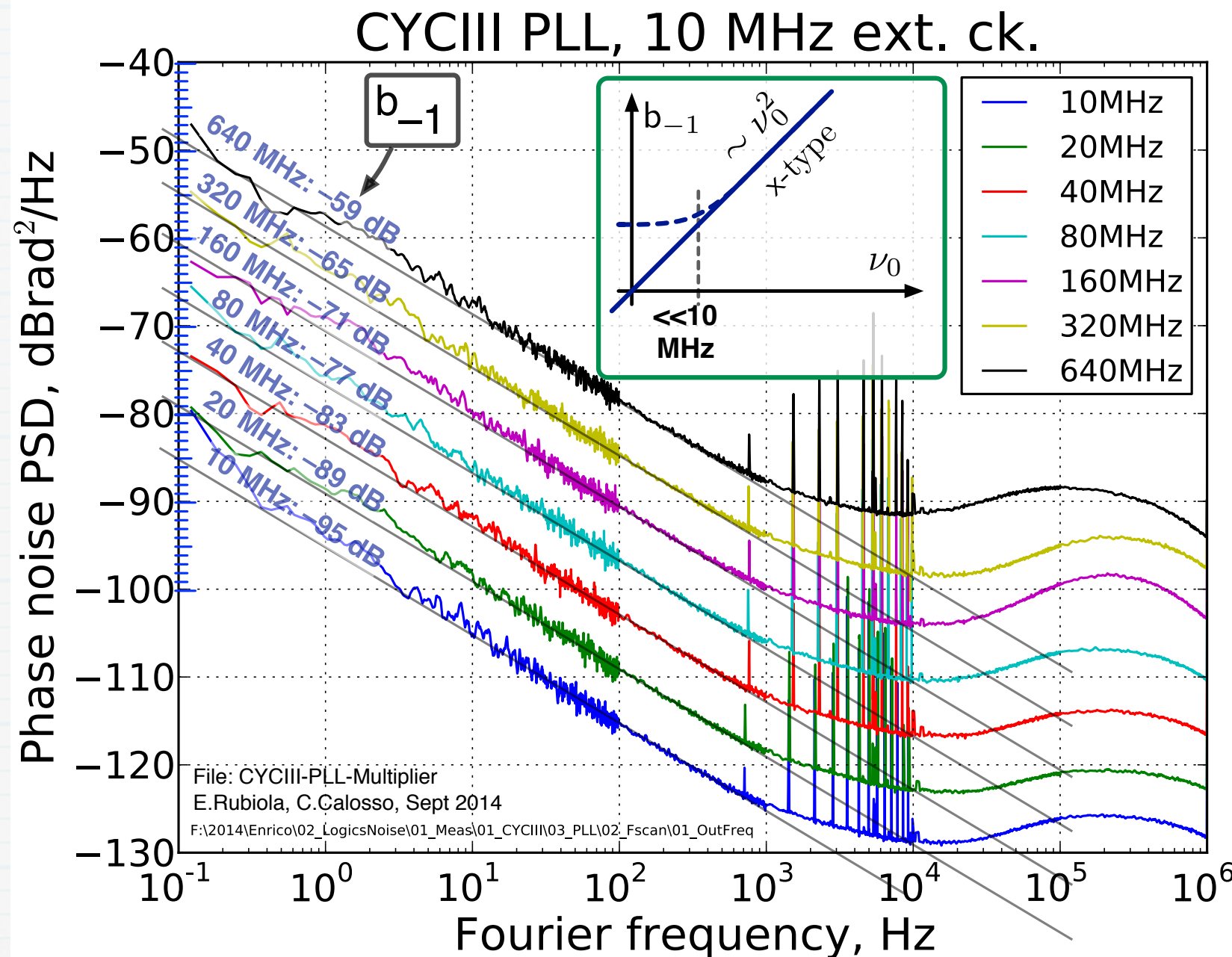


**x-type → analog noise in the phase detector**



# Cyclone II Internal PLL (4/4)

## PLL used as a frequency multiplier



10 MHz input

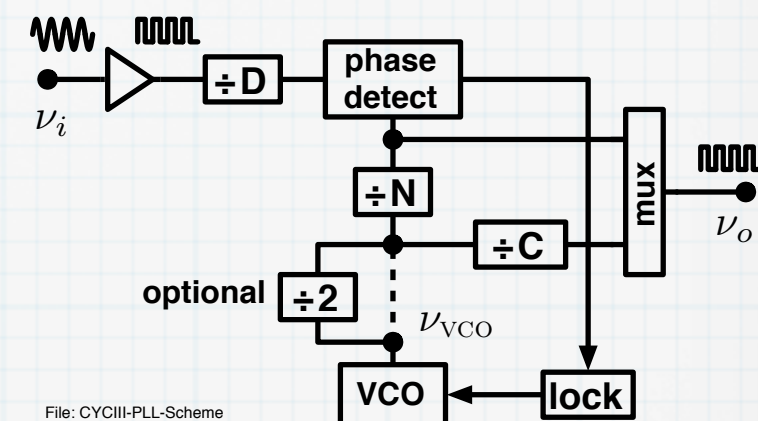
$N \times 10$  MHz out

Stability

$1.5 \times 10^{-12}$  @ 1 s

( $f_H = 500$  Hz)

$-115 \text{ dB} + 20 \log_{10}(v_0)$   
in MHz

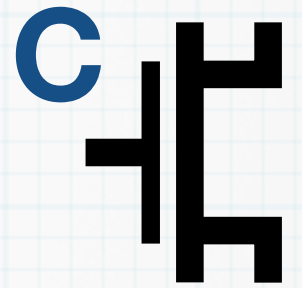


- $1/f$  phase noise is dominant
- Scales as  $N^2 \rightarrow$  analog noise in the phase detector

# Thermal Effects (1/3)

## Principle

- FPGA dissipation change  $\Delta P$  by acting on frequency
- Energy  $E = CV^2$  dissipated by the gate capacitor in a cycle

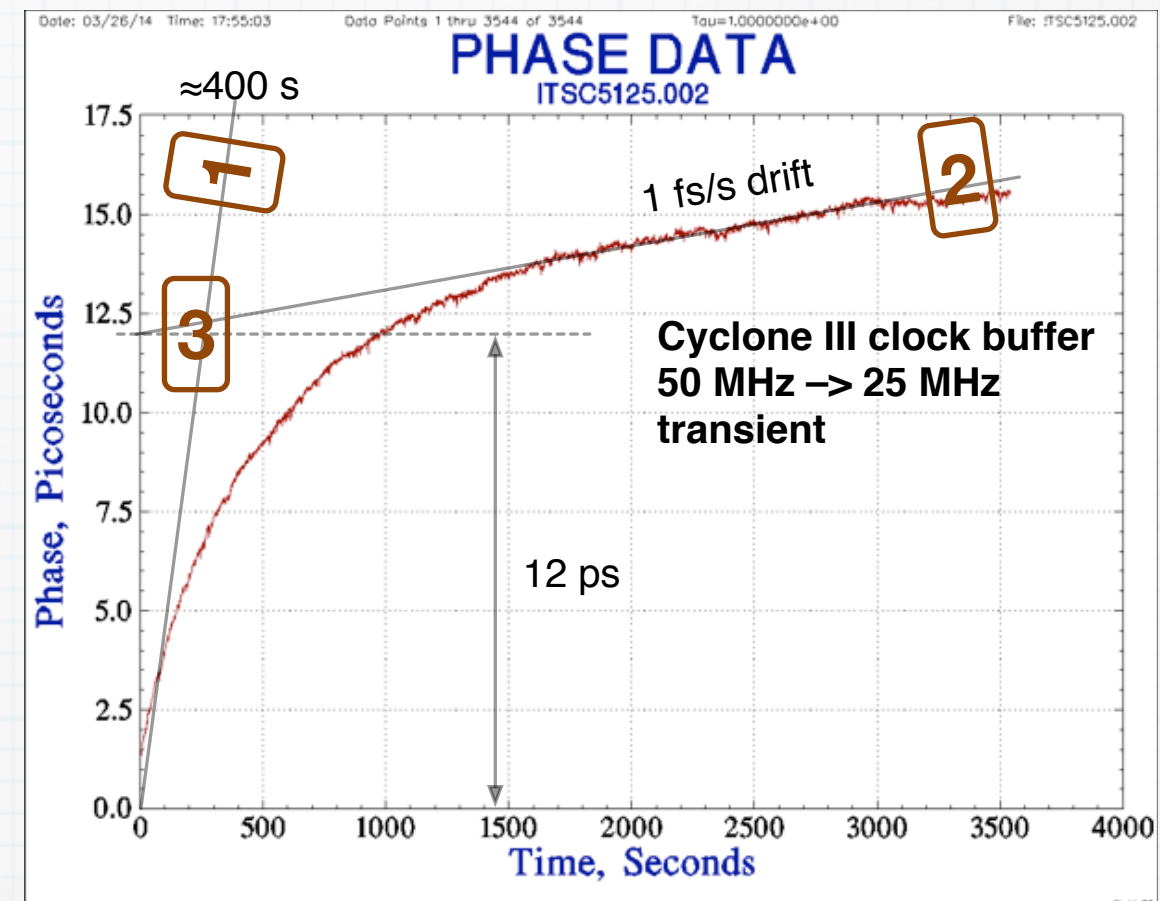


## Conditions

- Cyclone III used as a clock buffer
- Environment temperature fluctuations are filtered out with a small blanket (necessary)
- Two separate measurements (phase meter and counter) → trusted result

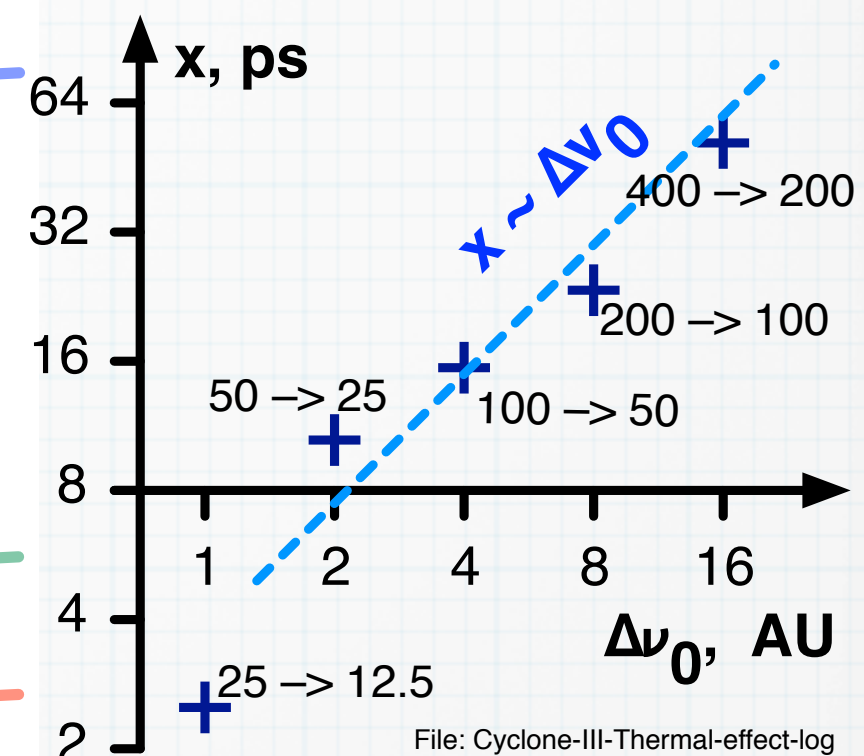
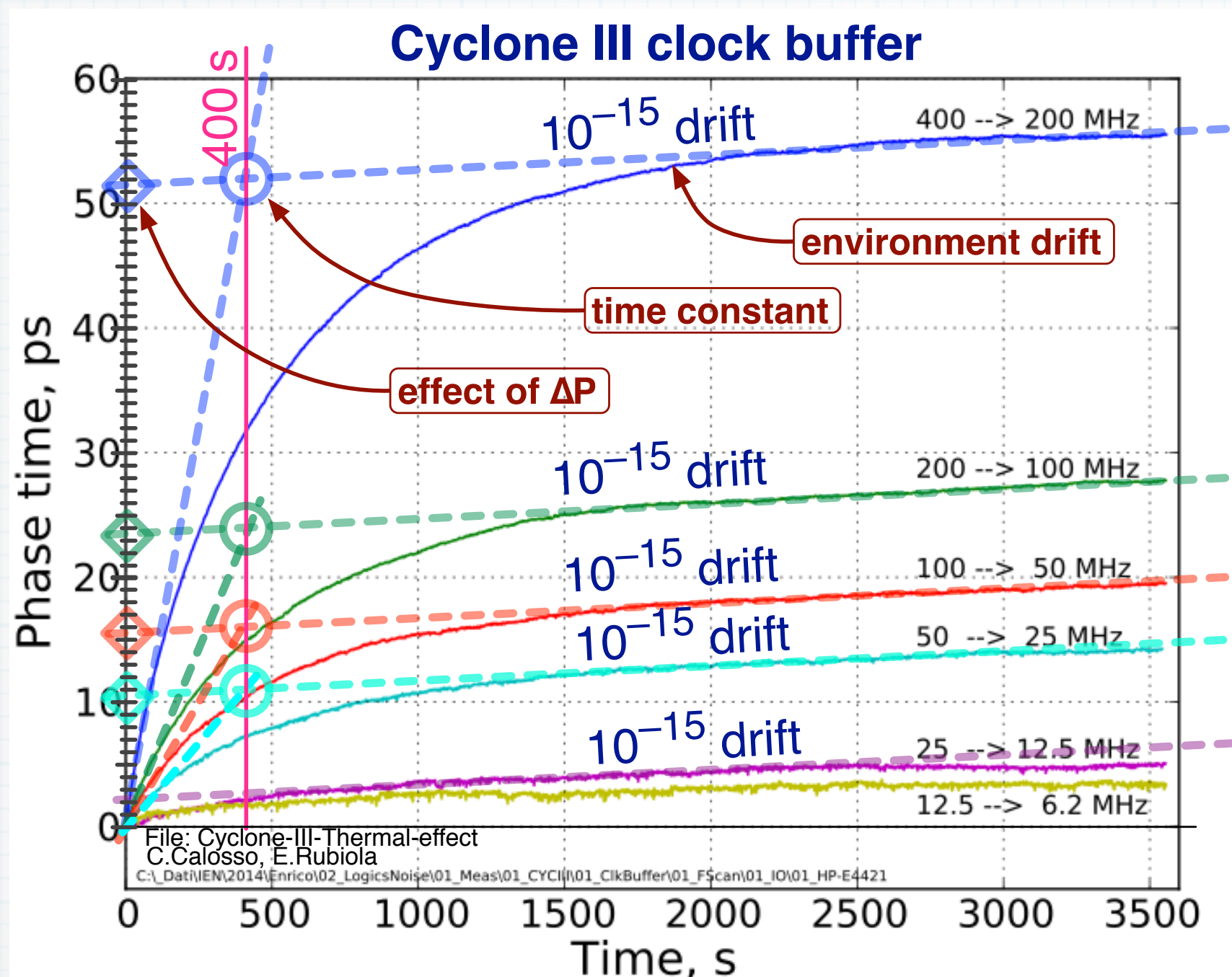
## Outcome

- (1) Thermal transient, due to the change of the FPGA dissipation
- (2) Slow thermal drift, due to the environment
- (3) Overall effect of  $\Delta P$





# Thermal Effects (2/3)

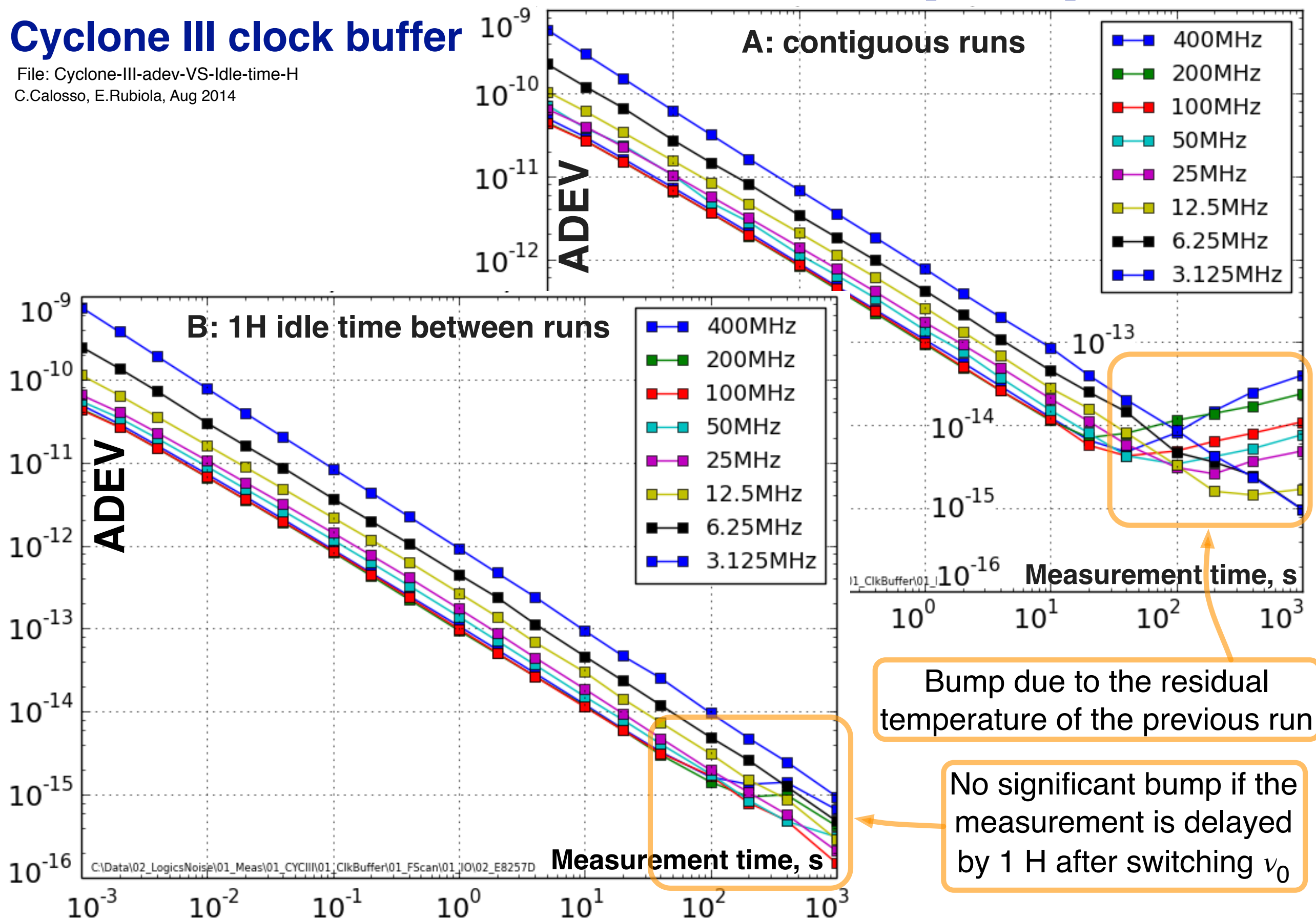


**Warning: In real applications, other parts of the same FPGA impact on the temperature, thus on phase – drift is possible**

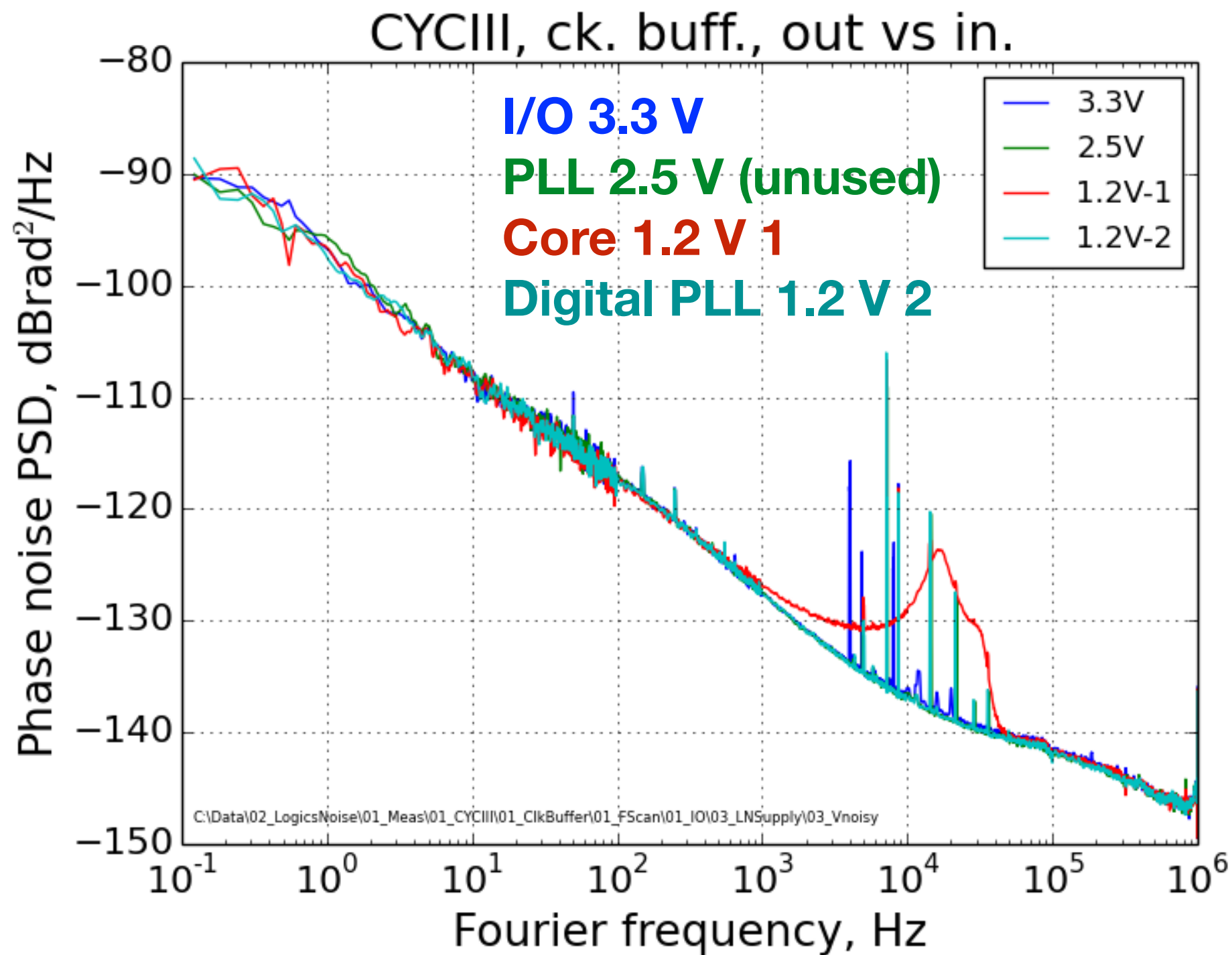
# Thermal Effects (3/3)

## Cyclone III clock buffer

File: Cyclone-III-adev-VS-Idle-time-H  
C.Calosso, E.Rubiola, Aug 2014

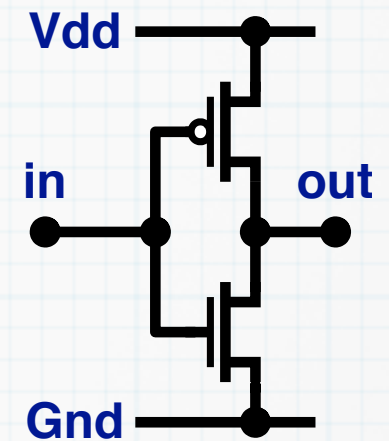
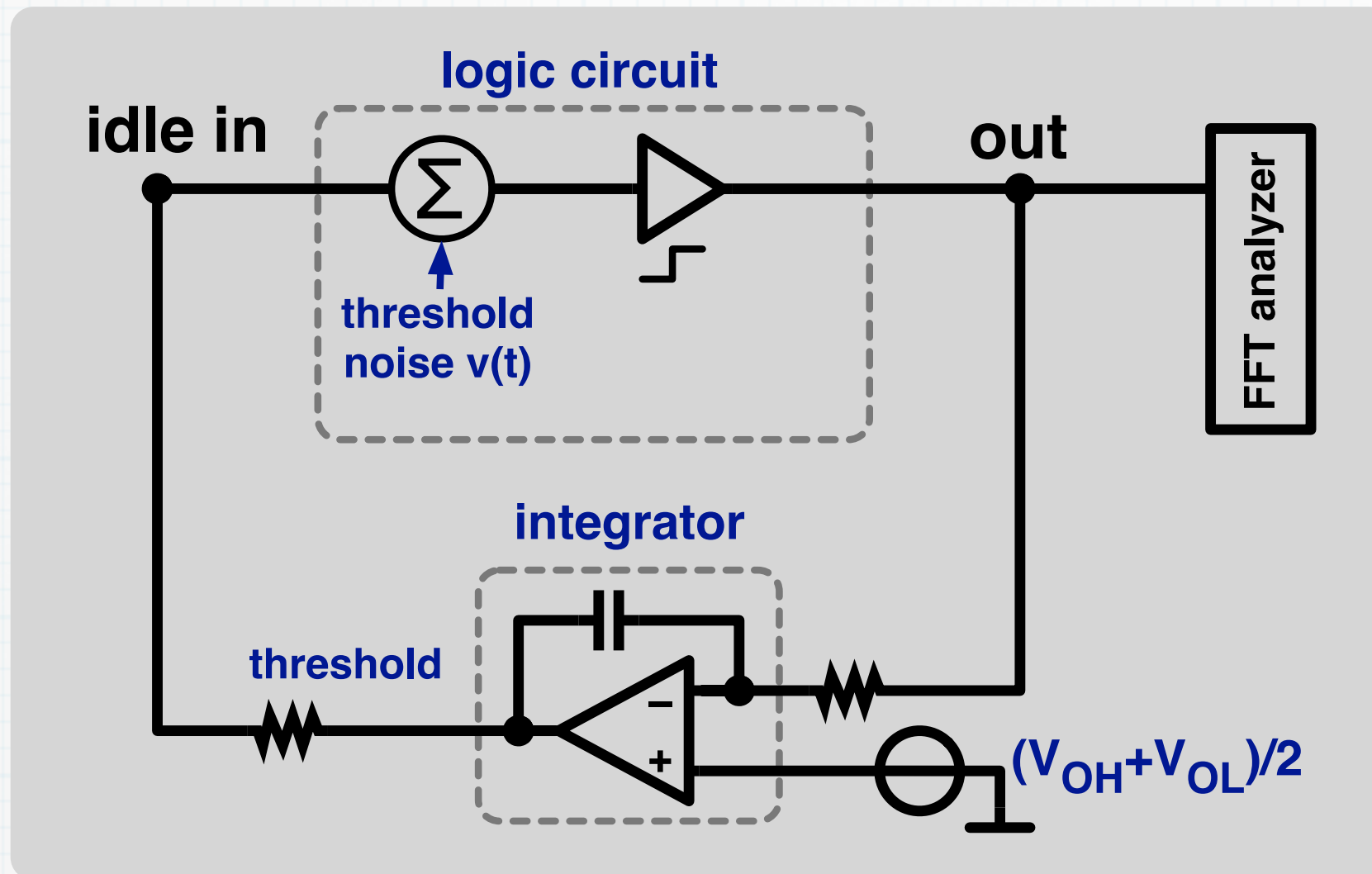


# Cyclone II, Voltage Supply



- All but one low-noise voltage supplies
- The noise is critical only in the core supply

# Threshold-Noise Measurement



- Keep the logic at the threshold, where it shows analog gain
- Measure the voltage (current) fluctuation needed to stabilise at the threshold
- Works only on simple (old) circuits
- Threshold-mismatched cascade → gain not accessible
- FPGAs complexity make the analog gain inaccessible

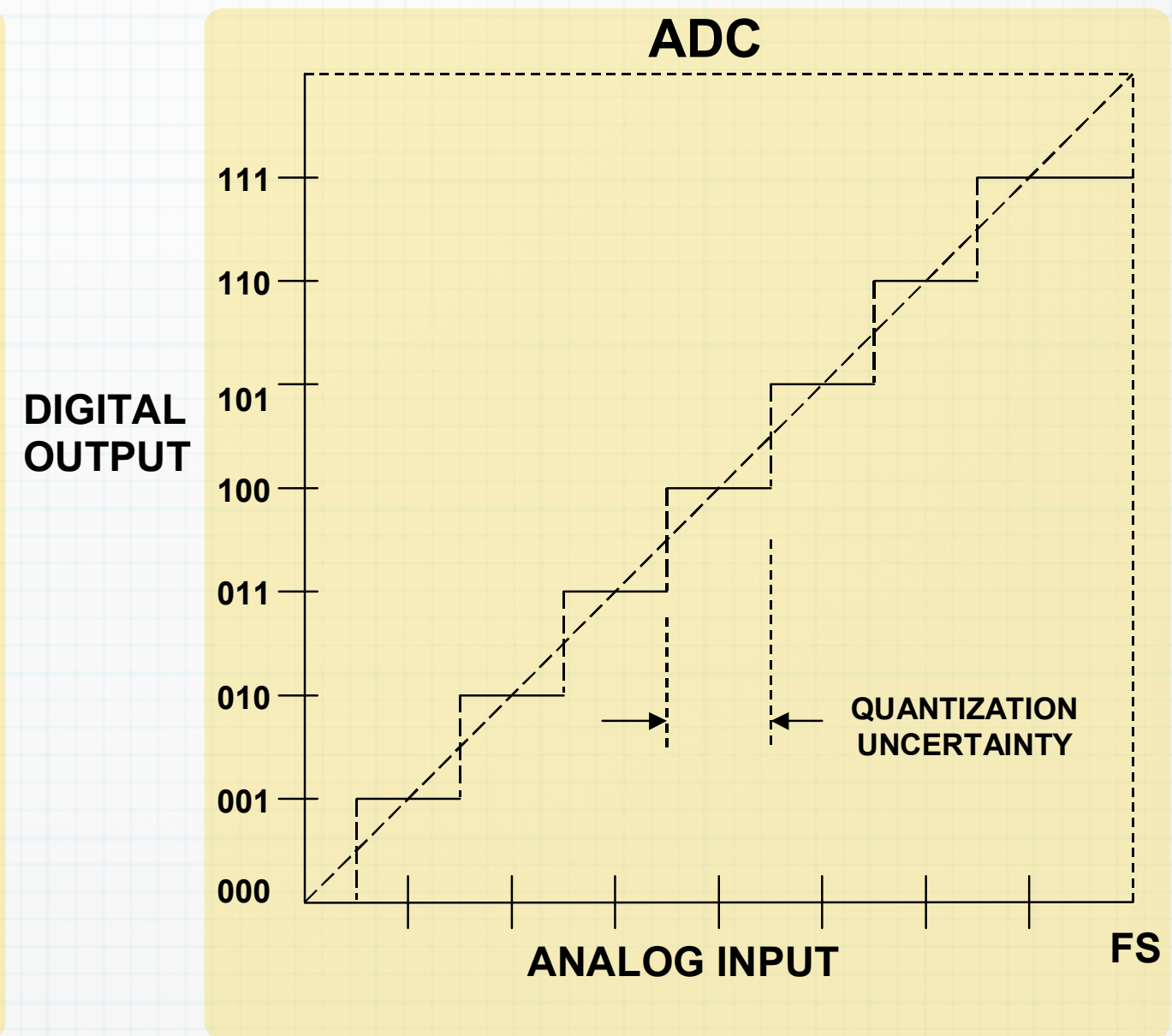
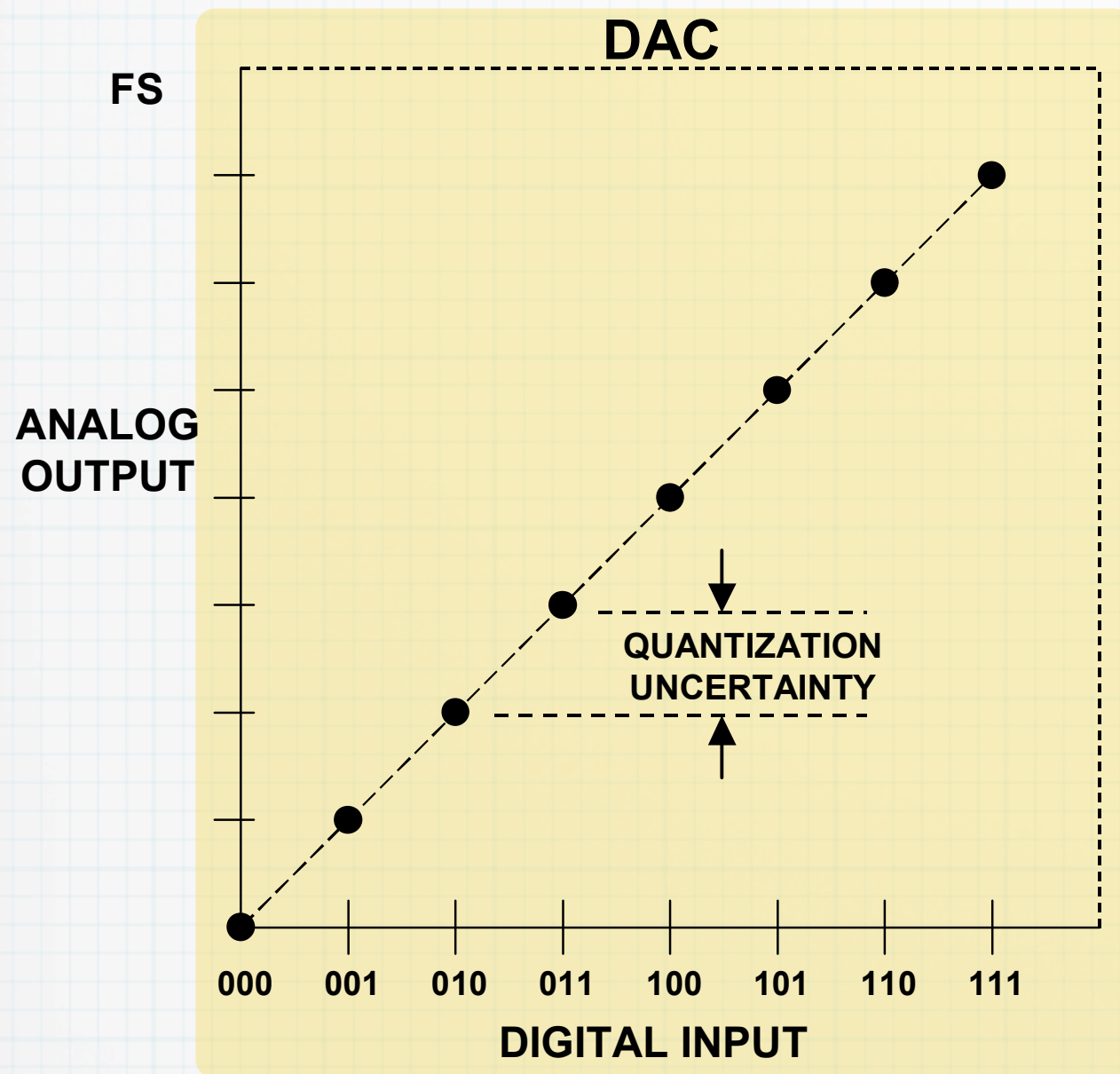


# 3 – ADCs

# Basics

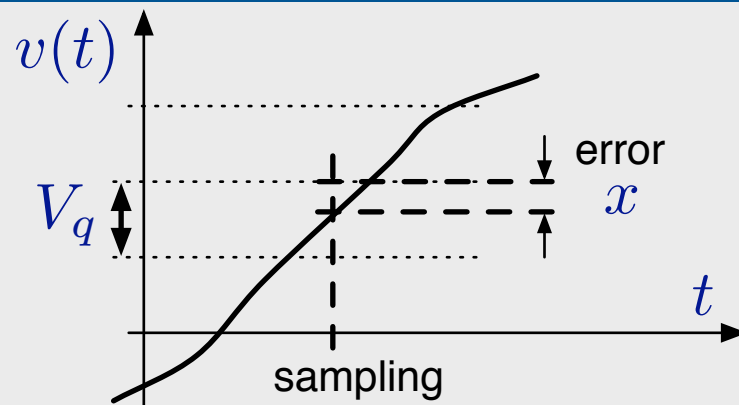


# Transfer Function & Quantization

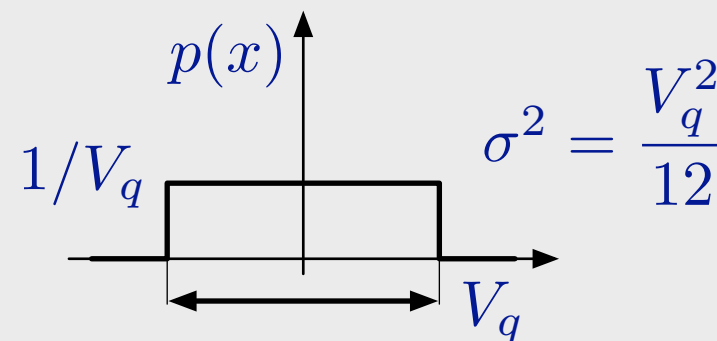


Kester W (ed), *Analog-Digital Conversion*, Fig.2.15, p.2.14,  
Analog Devices 2004, ISBN 0-916550-27-3

# Spectrum of the Quantization Noise

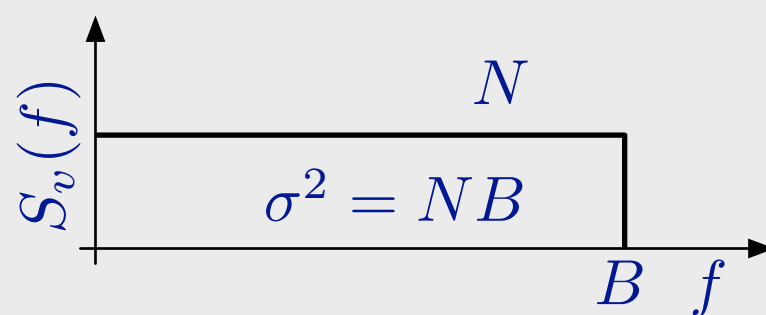


The analog-to-digital converter introduces a quantization error  $x$ ,  $-V_q/2 \leq x \leq +V_q/2$



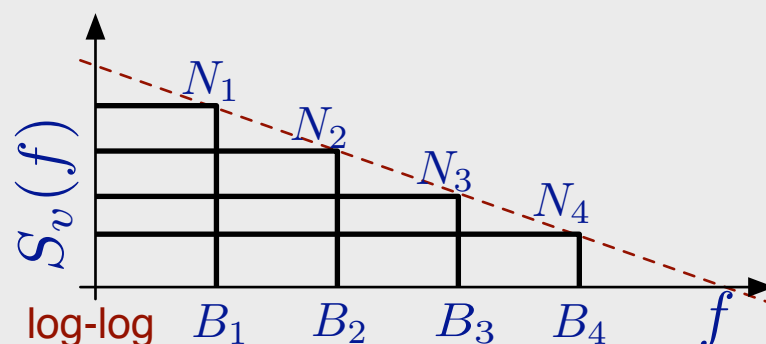
Ergodicity suggests that the quantization noise can be calculated statistically

$$\sigma^2 = \frac{V_q^2}{12}$$



The Parseval theorem states that energy and power can be evaluated by integrating the spectrum

$$NB = \frac{V_q^2}{12}$$



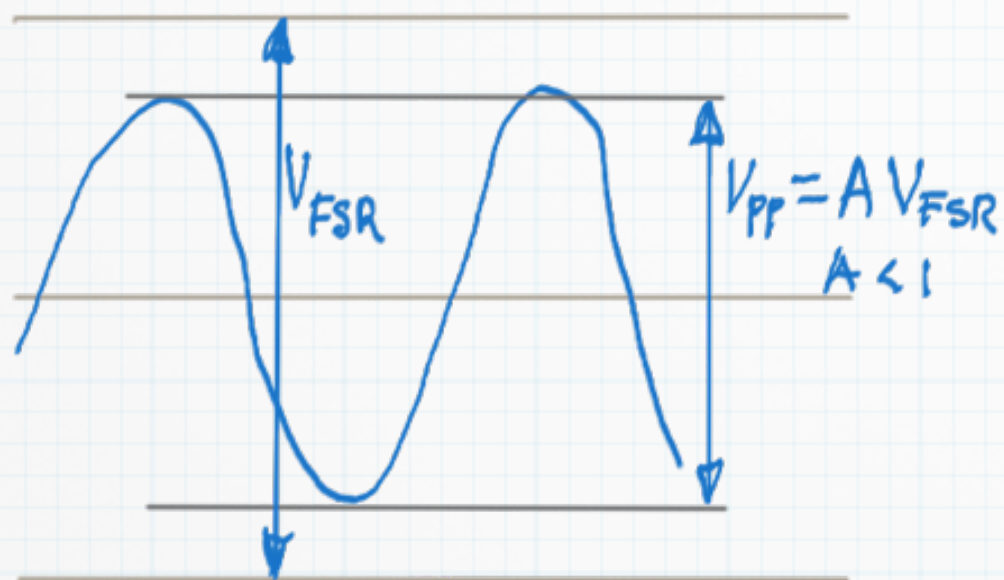
Changing  $B$  in geometric progression (decades) yields naturally  $1/B$  (flicker) noise

$$N = \frac{V_q^2}{12B}$$

**1/12 is  
-10.8 dB**

# Quantization & Sinusoidal Signals

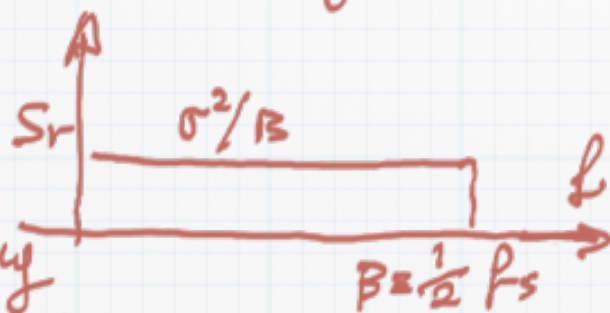
sampling frequency  $f_s$



Warning: We assume that the noise power is equally distributed in  $0 \dots B$ .

This is not true  $S_r$  in our case because sampling and carrier are highly coherent. See Widrow-Kollar Appendix G for details.

Anyway, we temporarily accept the uniform distribution, hoping that the reality is not too far.



Signal power

$$P_0 = \frac{V_{PP}^2}{8} = \frac{A^2 V_{FSR}^2}{8}$$

Noise power

$$\sigma^2 = \frac{V_{LSB}^2}{12}$$

$$\text{SNR} = (3/2) 2^{2m}$$

$$6.02 \text{ m} + 1.76 \text{ dB}$$

Parseval theorem

$$S_v = \frac{\sigma^2}{B} \Rightarrow S_v = \frac{V_{LSB}^2}{6 f_s}$$

Phase noise  $S_{\phi} = b_0$  (white)

$$b_0 = \frac{S_v}{P_0} = \frac{V_{LSB}^2}{V_{FSR}^2} \cdot \frac{4}{3 A^2 f_s}$$

$$b_0 = \frac{1}{(2^m)^2} \cdot \frac{4}{3 A^2 f_s}$$

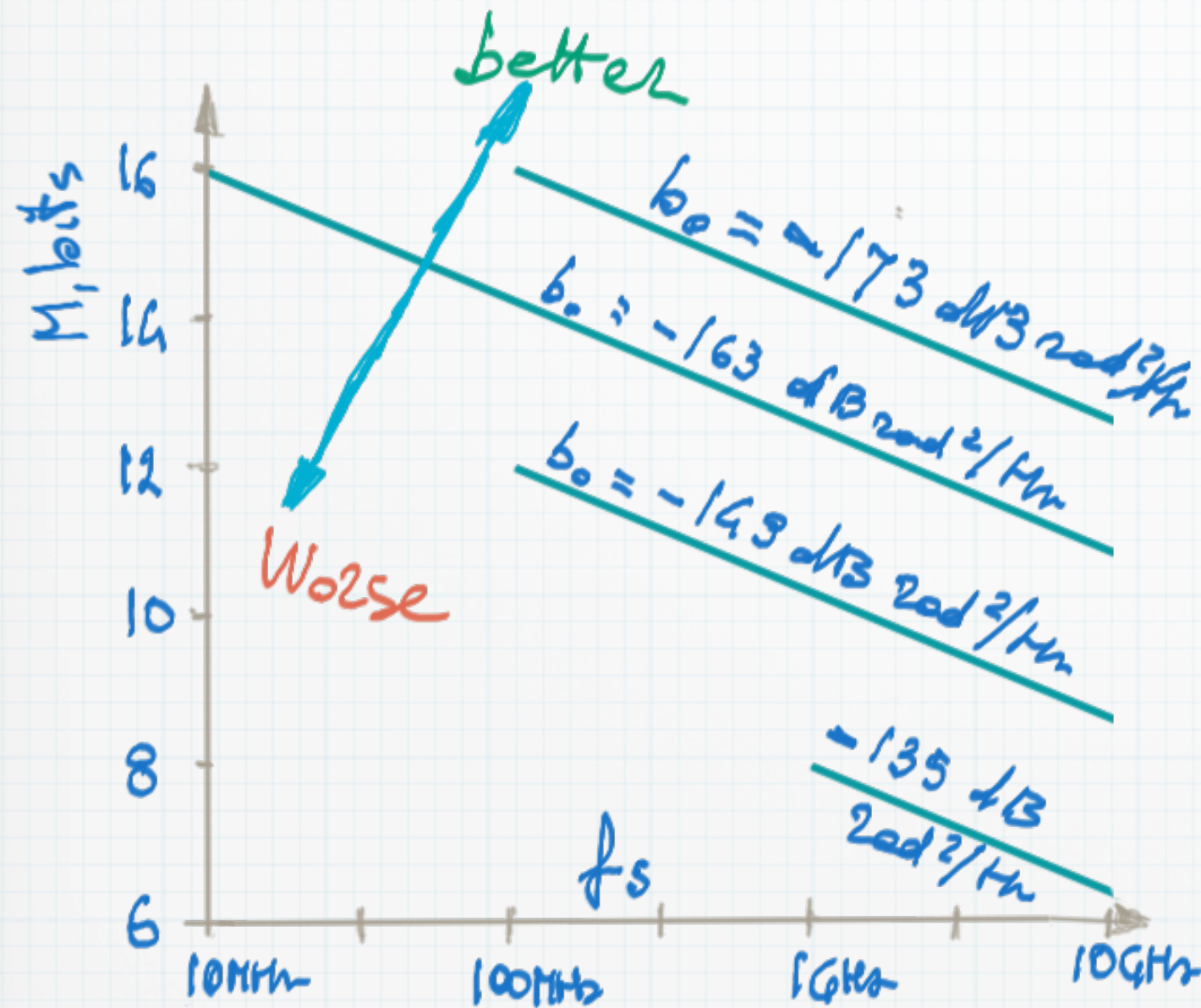
$$6.02 \text{ m} + 1.25$$

$$-10 \log_{10} f_s \text{ dB}$$

Approximation (fairly large  $V_p$ )  
 $A^2 = 2/3$  (-1.8 dB)  $\Rightarrow b_0 \approx \frac{1}{(2^m)^2} \cdot \frac{2}{f_s}$



# Phase Noise



$$b_0 = \frac{1}{(2^M)^2} \frac{4}{3A^2 f_s}$$

$$b_0 \approx \frac{1}{(2^M)^2} \frac{2}{f_s}$$

Cost of 6 dB improvement

- 1 bit
- factor-of-4  $f_s$

Obvious conclusion:  
practical ADCs feature  
lower  $b_0$  at low  $f_s$  because  
of the higher no. of bits

# Selection of dual-channel converters

Go Digital

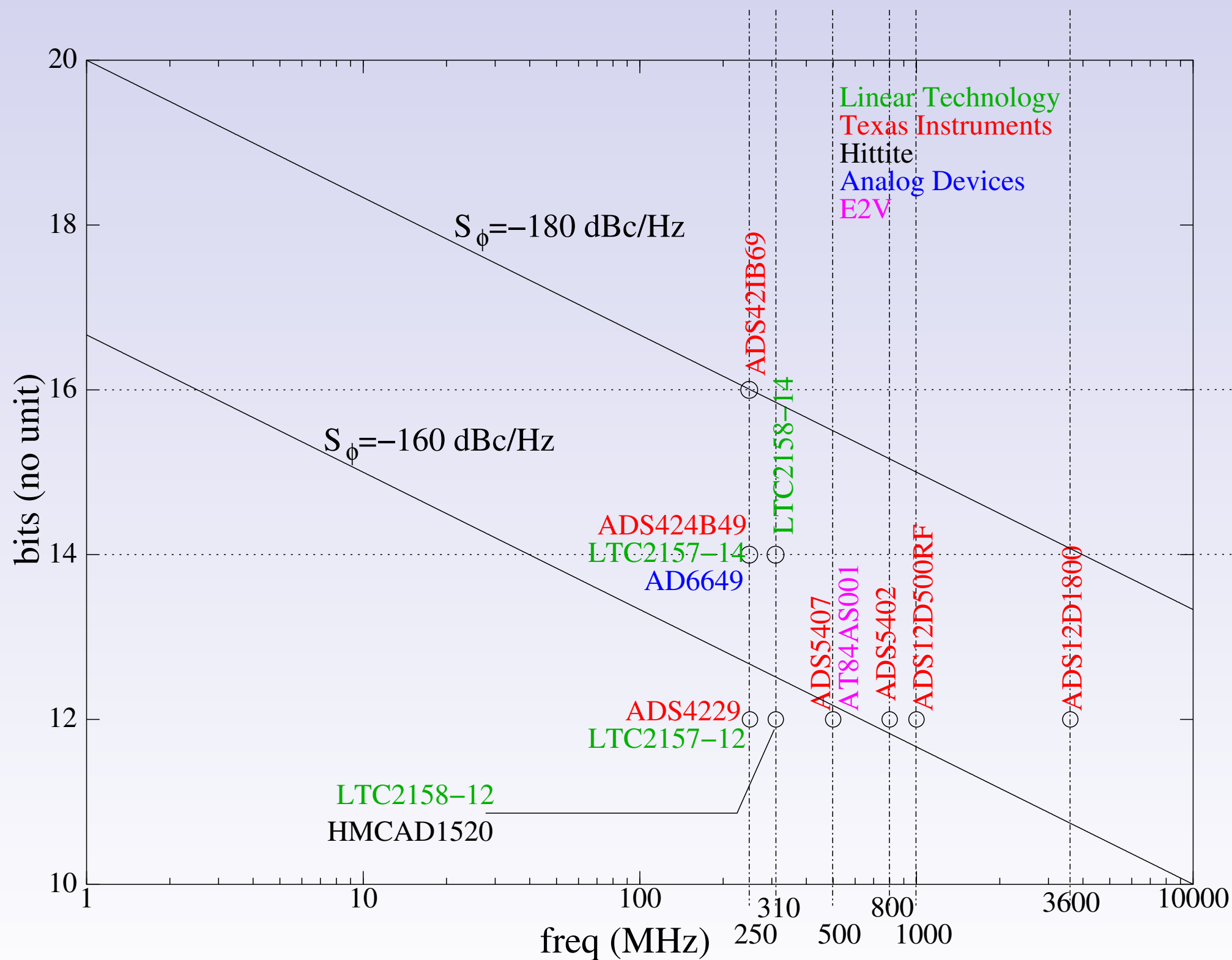
G.  
GOAVEC-  
MEROU

In phase measurements, a dual-channel converter is preferred because the two track-and-hold get the same clock, so the with minimum differential jitter

ADC

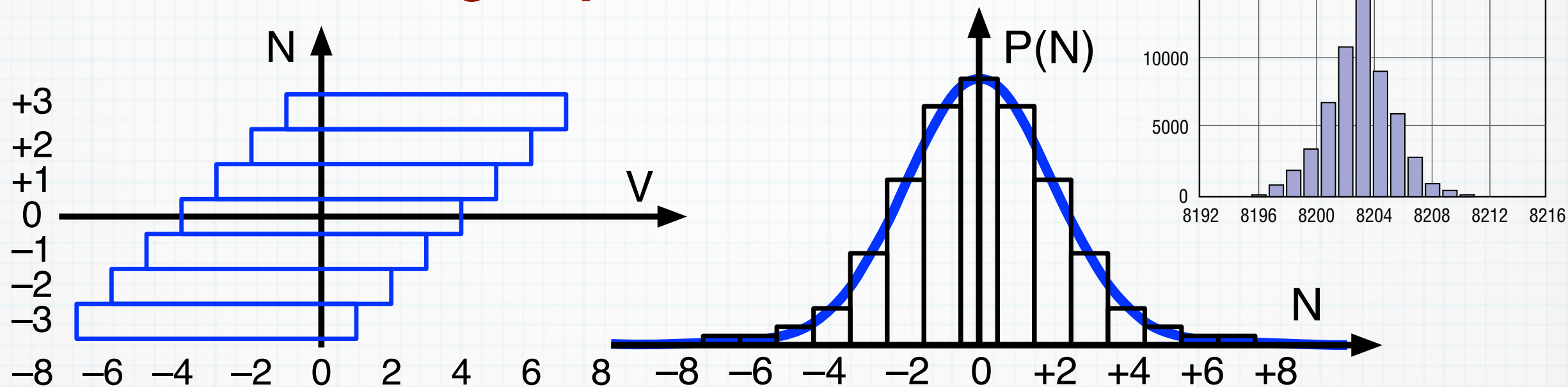
ADC  
classification

In progress



# Transition Noise

## High-Speed Converters



- Analog noise is higher than quantization noise
- Given a voltage  $V \rightarrow$  random distribution of output  $N$
- **This correct**  $\rightarrow V^2 = V_{\text{analog}}^2 + V_{\text{quant}}^2$   
(don't spoil the resolution with insufficient no of bits)

### Information (bits)

$$I = \sum_i -p_i \log_2(p_i)$$

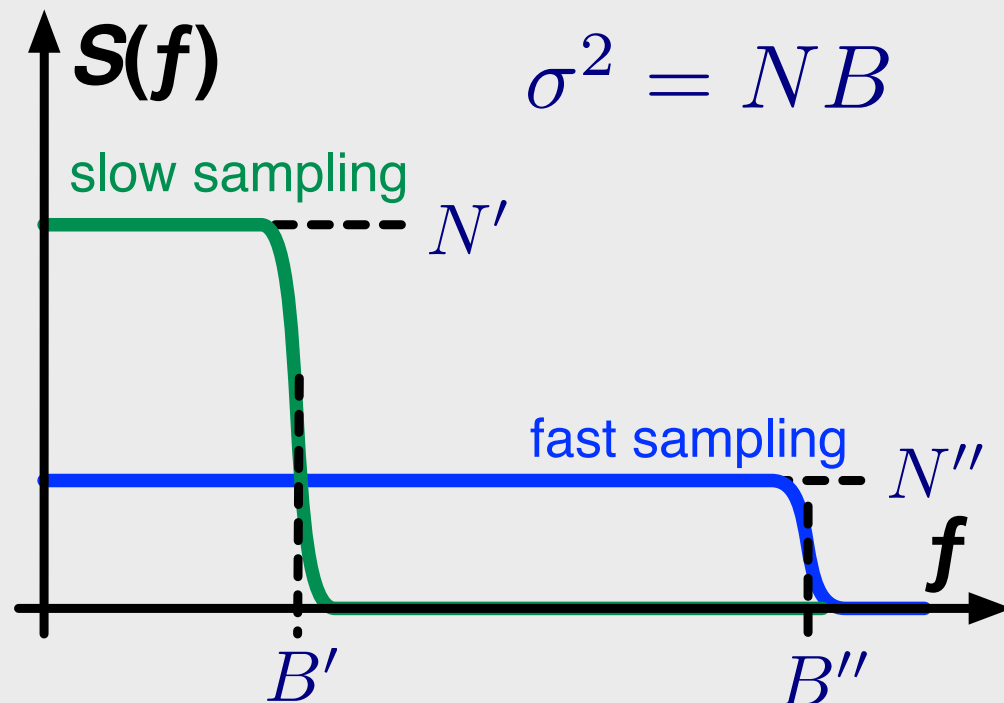
### Equivalent No of Bits

$$\text{ENoB} = \log_2 \left[ 1 + \frac{V_{\text{FSR}}}{\sqrt{12} f_N \sigma_V} \right]$$

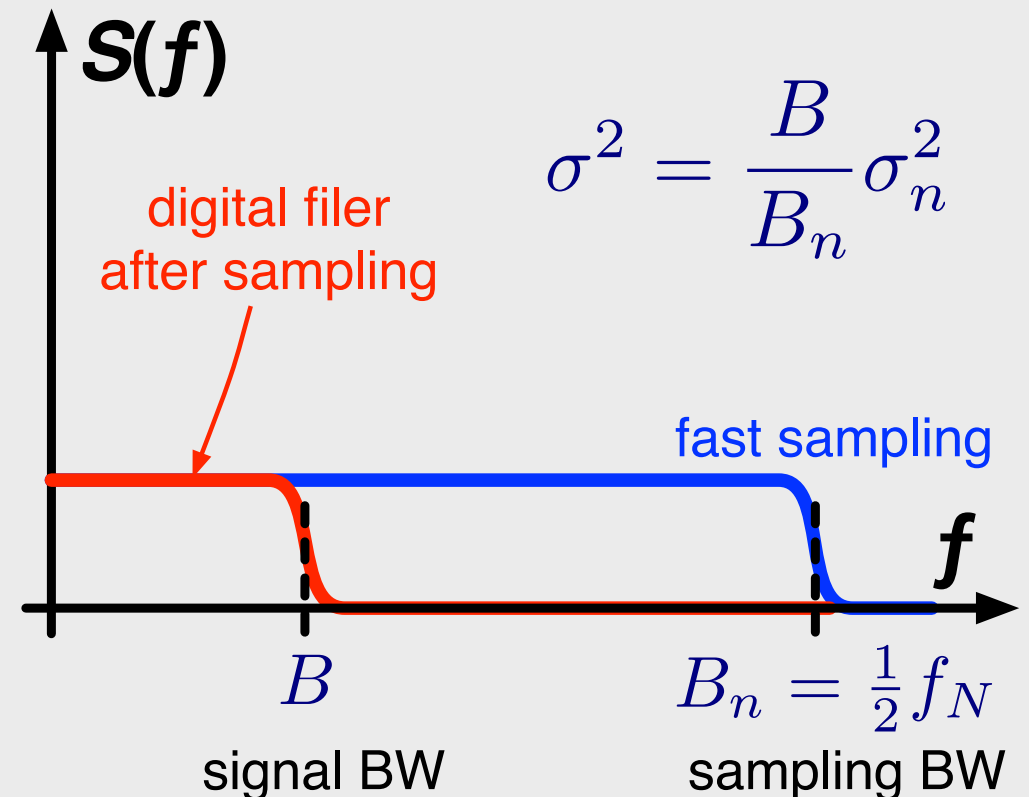


# Digital Filter and Decimation

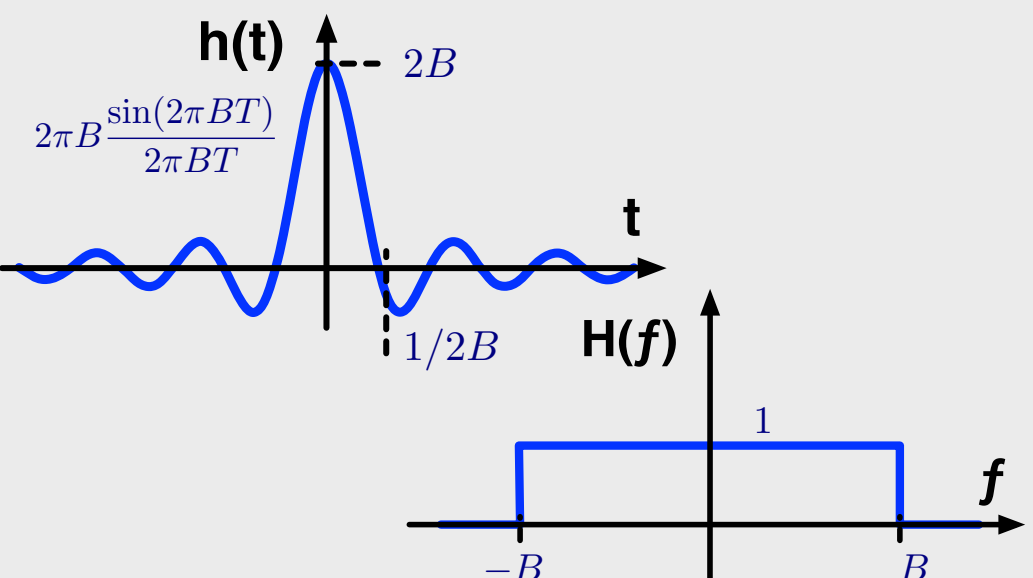
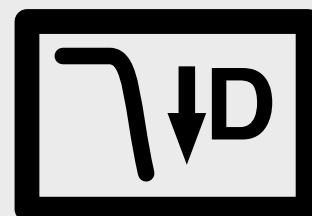
Noise, Sampling, and the Parseval theorem  $\sigma^2 = \frac{V_{\text{FSR}}^2}{12 \times 2^{2m}}$



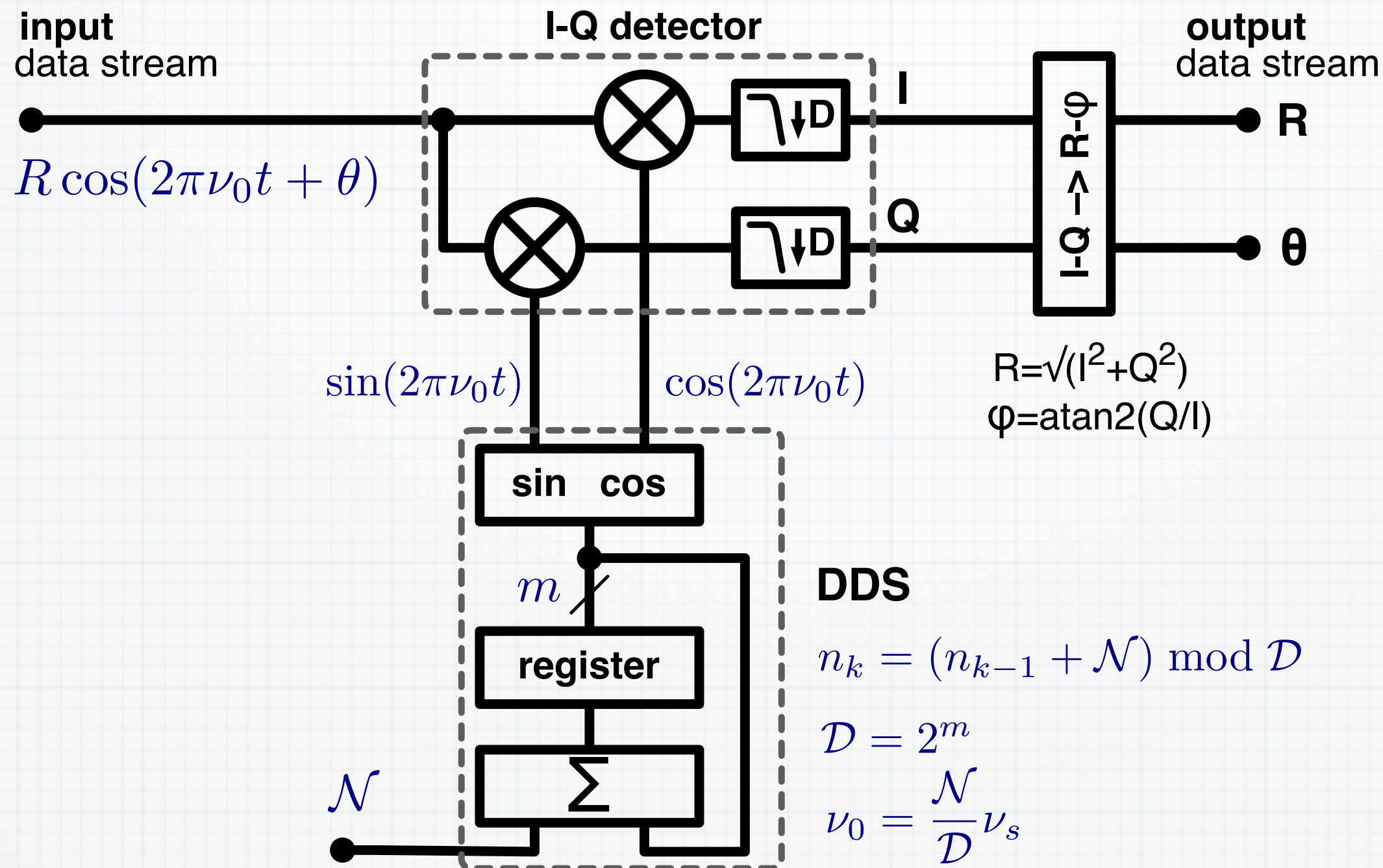
**Solution:**  
Fast sampling, and filter



- Convolution with low-pass  $h(t)$
- 127 coeff. Blackman-Harris kernel provides 70 dB stop-band attenuation
- Future: we will use  $\gg 127$  coefficients



# Digital Down Conversion



# Examples

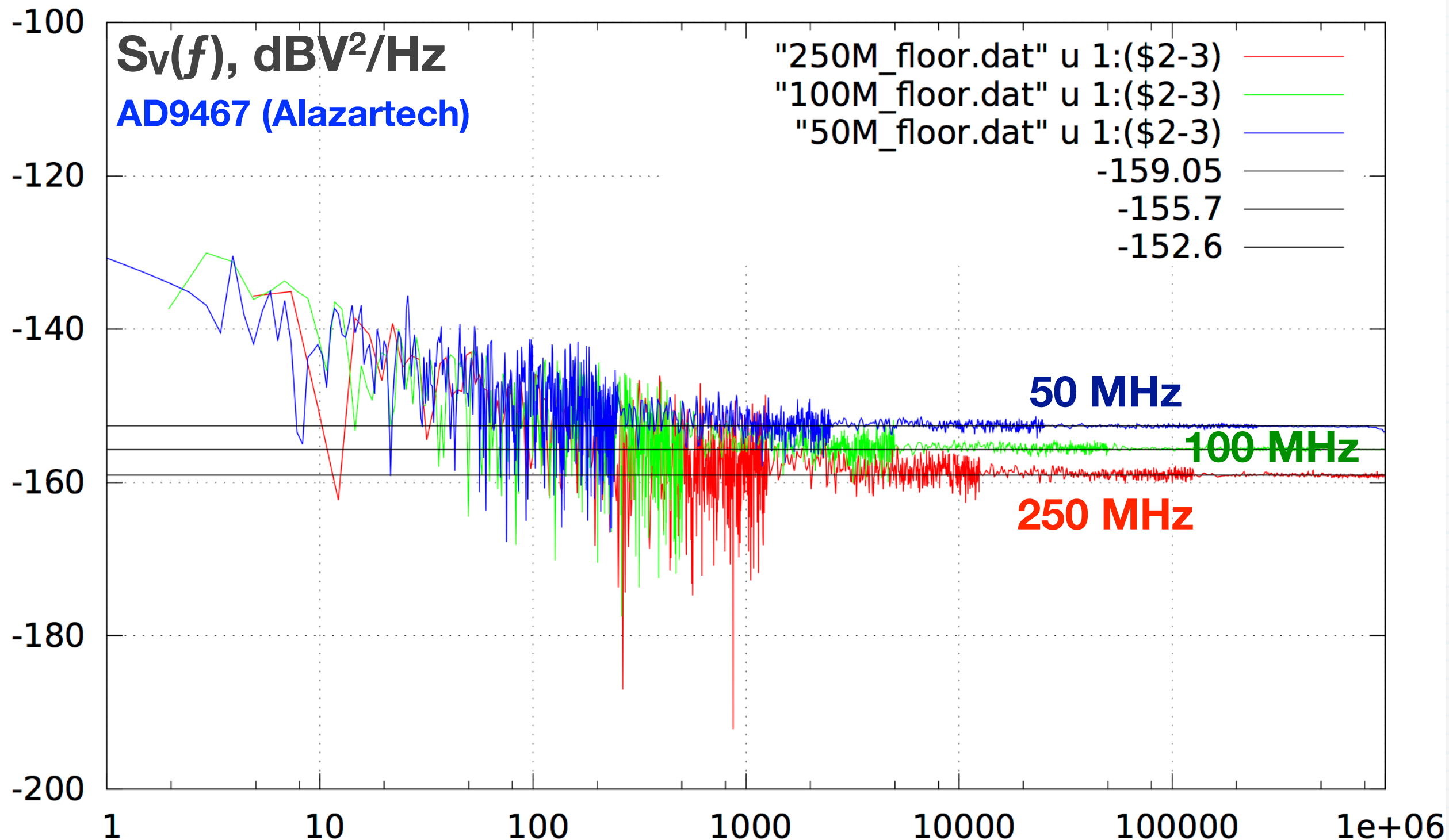
# Hardware

ADC type	AD9467 / Single Alazartech	LTC2145 / Dual Red Pitaya	LTC2158 / Dual Eval board
Platform	Computer	Zynq (onboard)	Zynq (separated)
Sampling $f$ Input BW	250 MHz 900 MHz	125 MHz 750 MHz	310 MHz 1250 MHz
Bits / ENoB	16 / 12	14 / 12	14 / 12
Exp.noise ( $2V_{fsr}$ )	-158 dBV <sup>2</sup> /Hz	-155 dBV <sup>2</sup> /Hz	-159 dBV <sup>2</sup> /Hz
Delay / Jitter	1.2 ns / 60 fs	0? / 100 fs diff 0? / 80 fs single	1 ns / 150 fs
Power supply	1.8 V & 3.3 V 1.33 W	1.8 V 190 mW	1.8 V 725 mW

Dissipation is relevant to thermal stability

For reference, 100 fs jitter is equivalent to			
carrier $f$	$\phi$ rms	$S\phi(f) = b_0$	$10 \log_{10}[L(f)]$
10 MHz	6.3 $\mu$ rad	$4 \times 10^{-18}$ rad <sup>2</sup> /Hz	-177 dBc/Hz
100 MHz	63 $\mu$ rad	$4 \times 10^{-17}$ rad <sup>2</sup> /Hz	-167 dBc/Hz

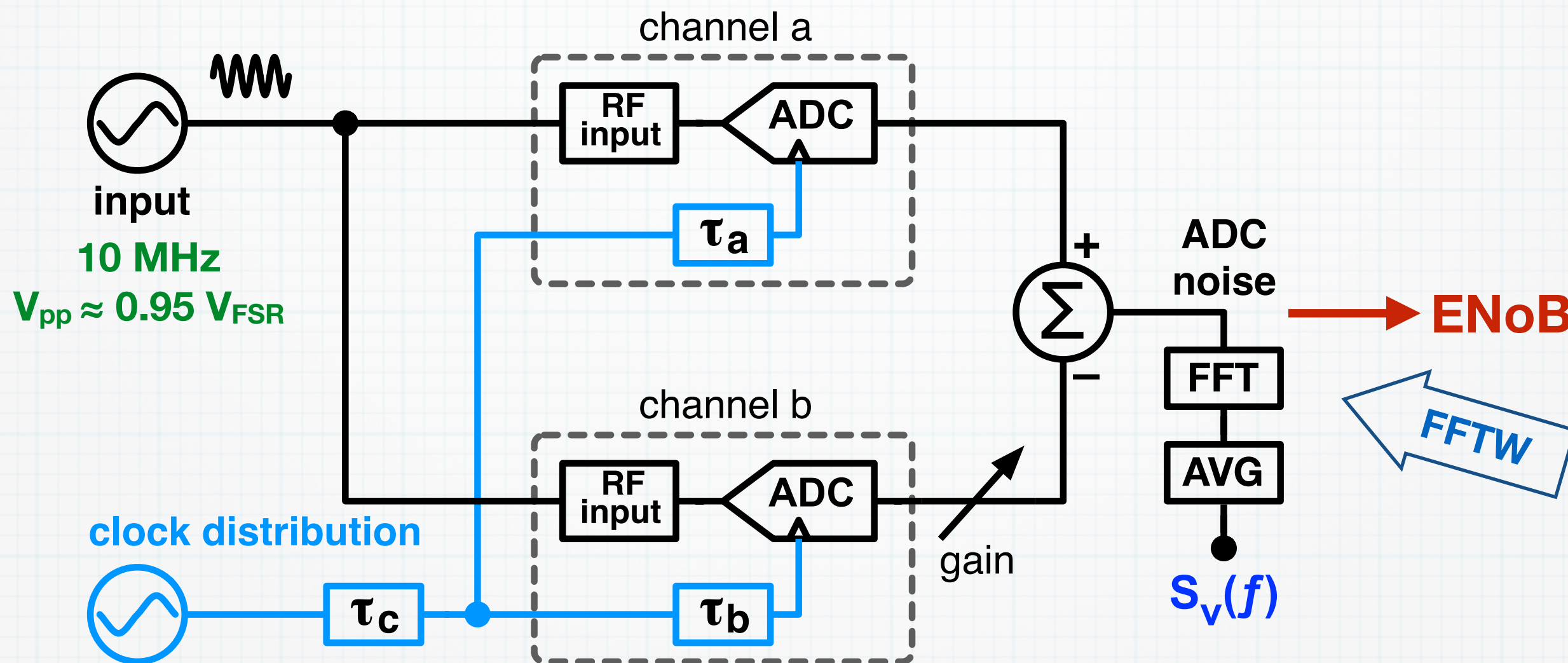
# Sampling Frequency



**The observed floor fits the theory**  
**We always use the highest sampling frequency**



# Transition Noise Measurement

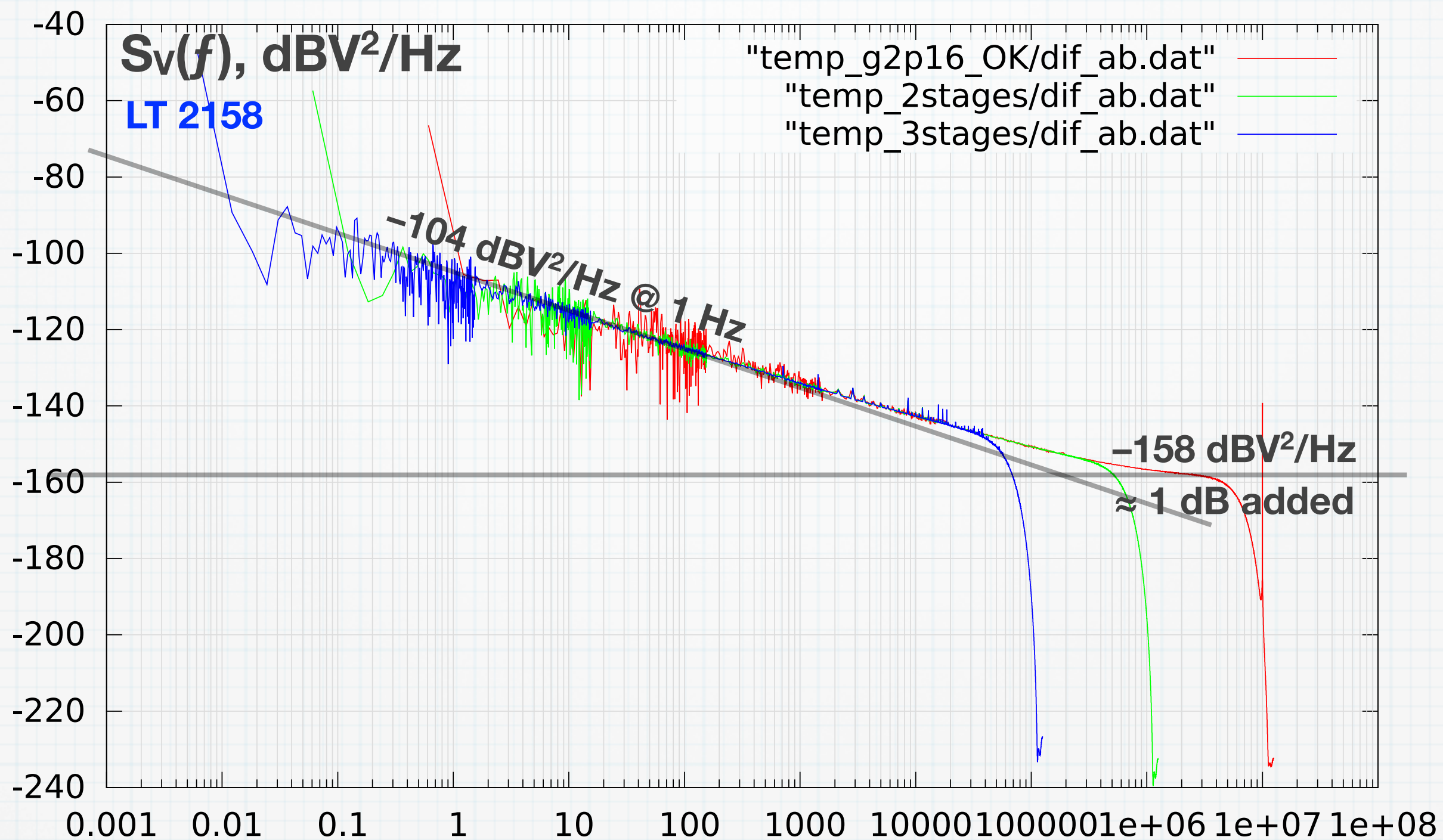


The differential clock jitter introduces additional noise due to the asymmetry between AM and PM

At 10 MHz input,  $\approx 100$  fs the effect of jitter does not show up

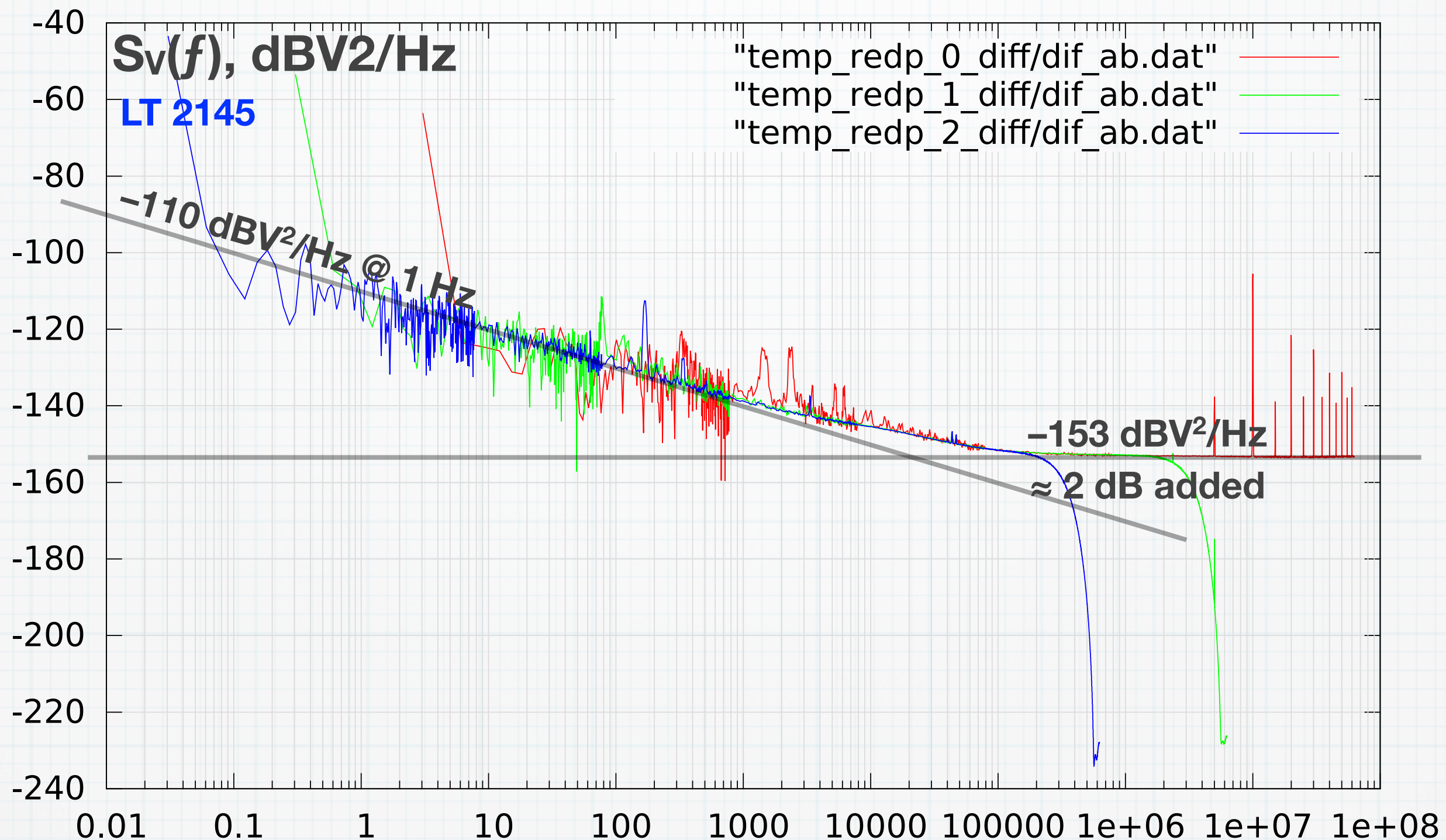


# LT 2158 Noise



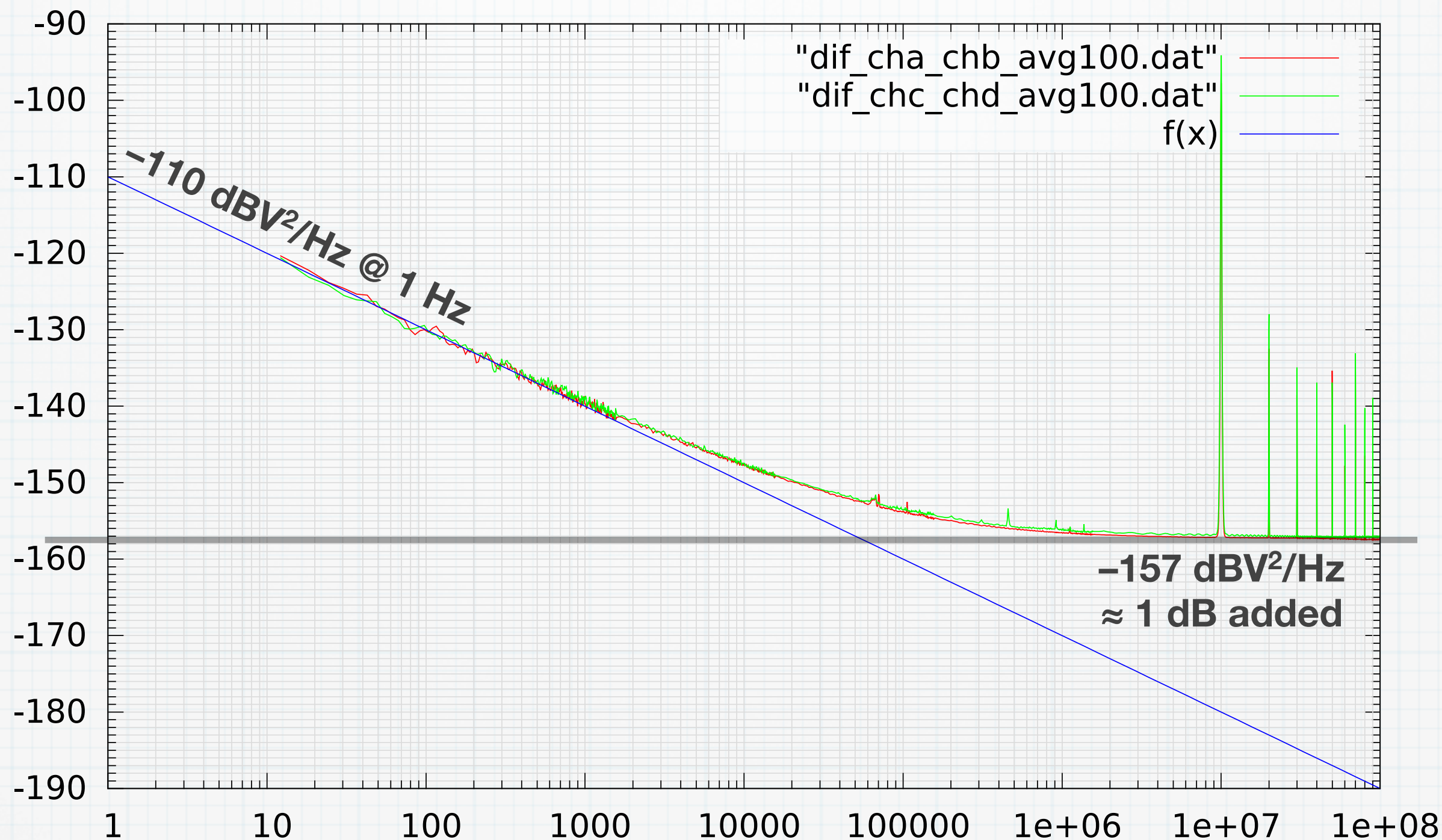
10 MHz,  $V_{pp} \approx 0.95 V_{FSR}$

# LT2145 (Red Pitaya) Noise



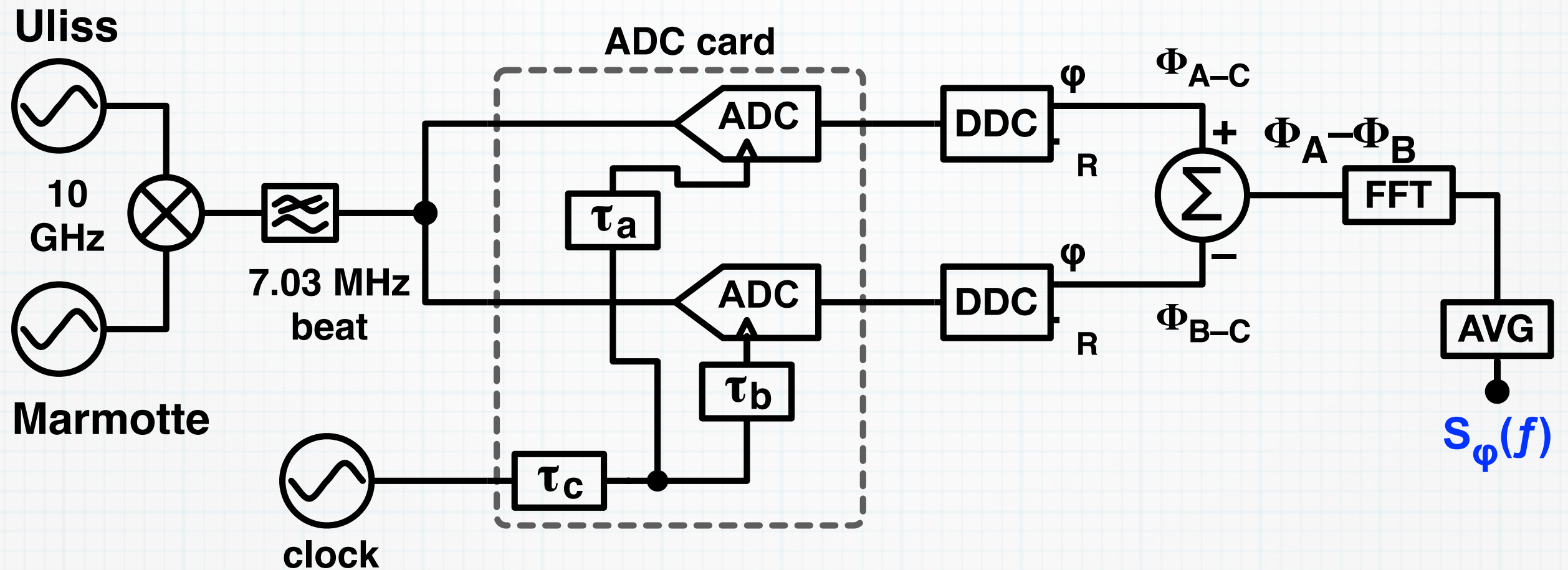
**10 MHz,  $V_{pp} \approx 0.95 V_{FSR}$**

# AD9467 (Alazartech) Noise



**10 MHz,  $V_{pp} \approx 0.95 V_{FSR}$**

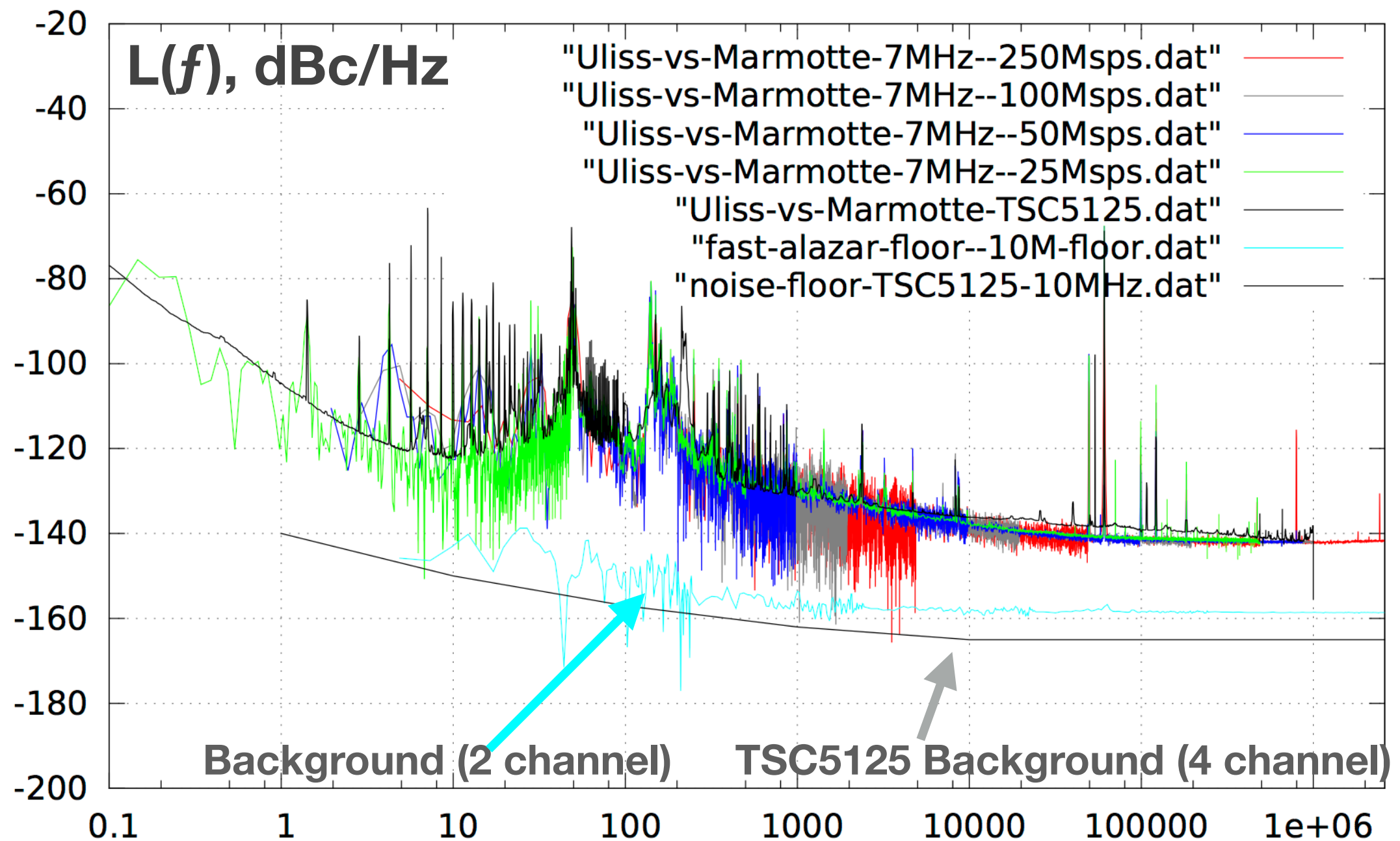
# Application to 10 GHz Cryogenic Oscillators



- Rejects the common-path jitter
- Takes in the differential jitter

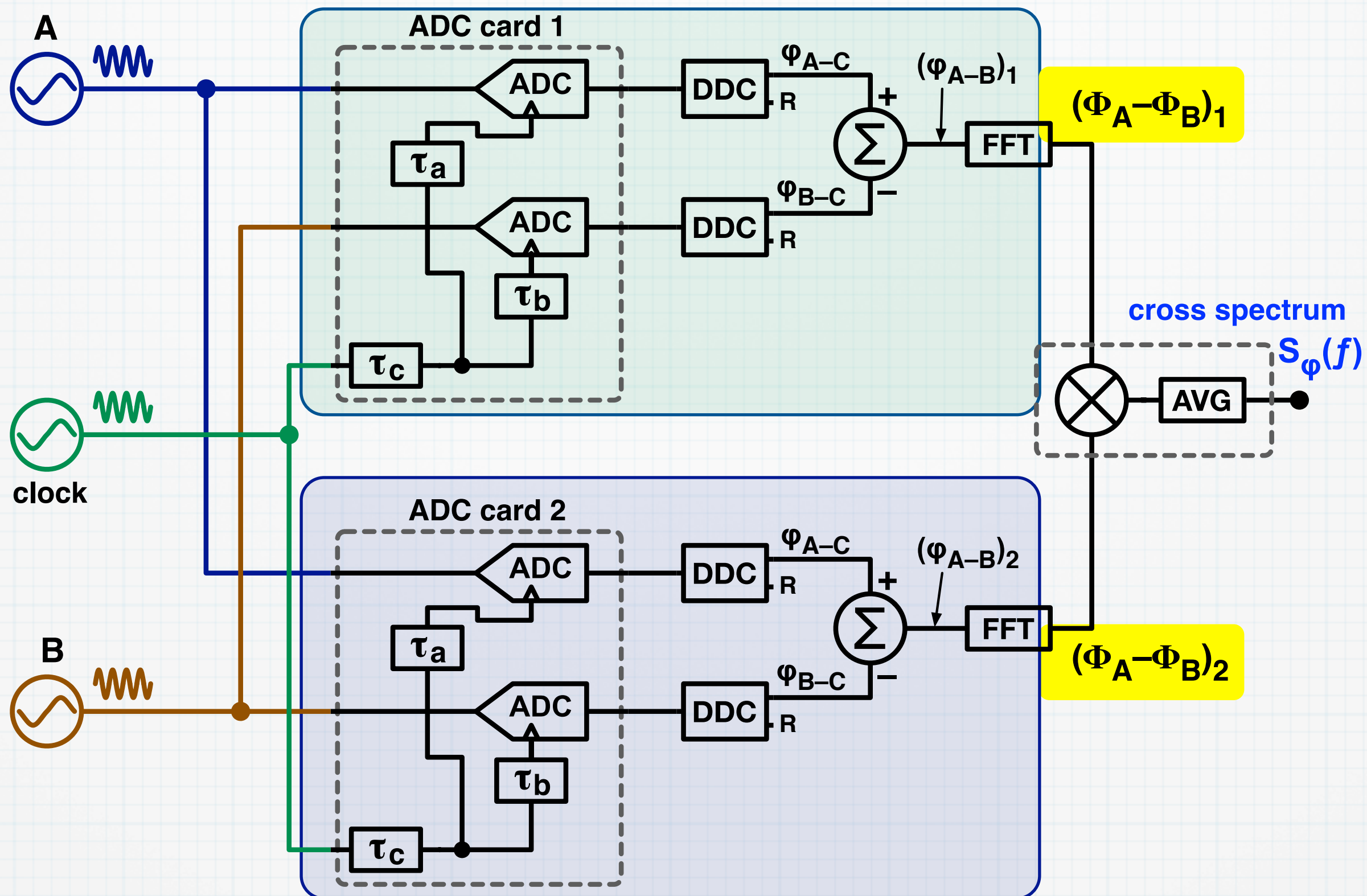


# Results



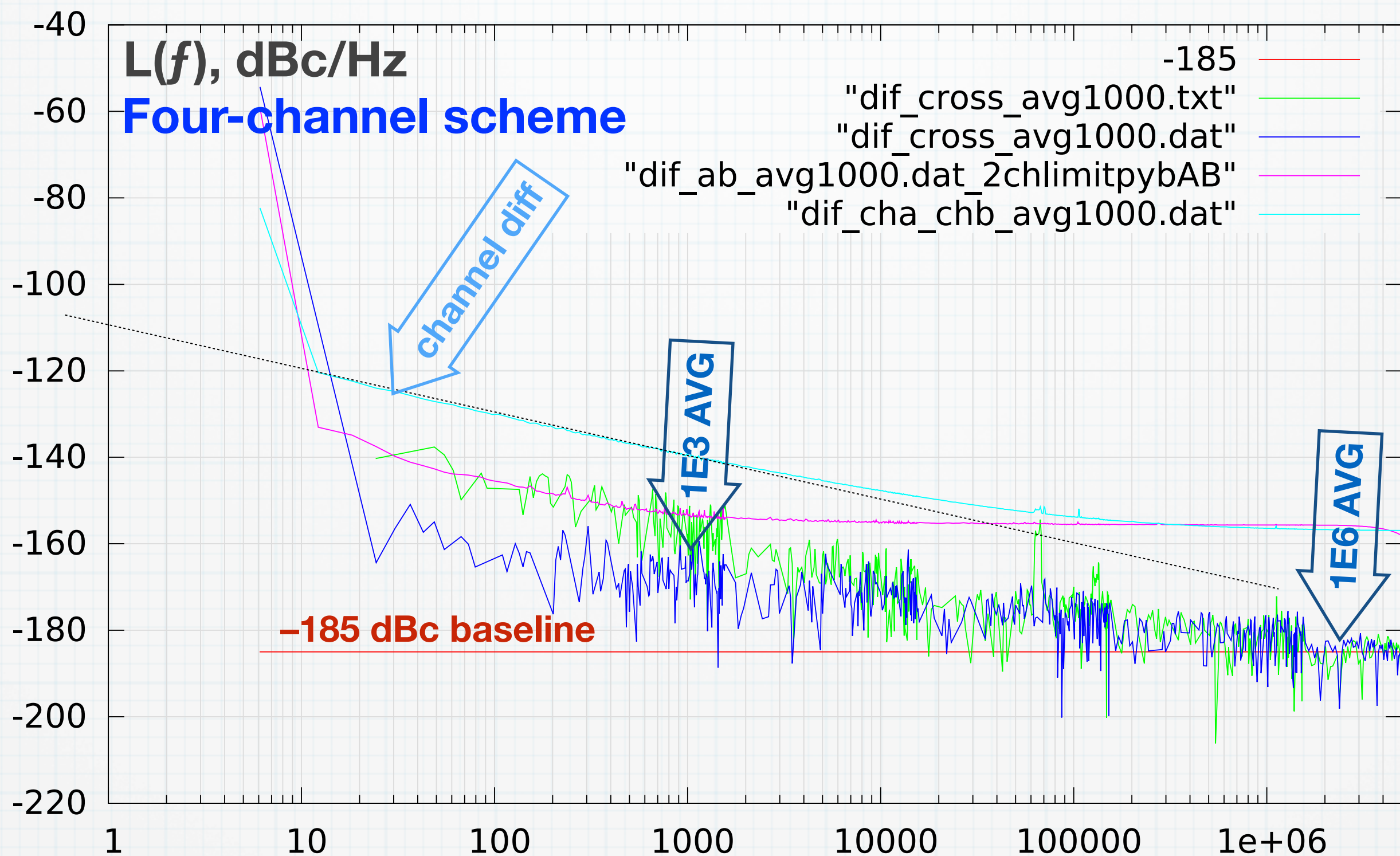
- Background noise 5–6 dB higher than that of the TSC5125
- We use 2 channel cross spectrum
- TSC5125 uses 4 channel cross spectrum

# The Four-Channel Scheme



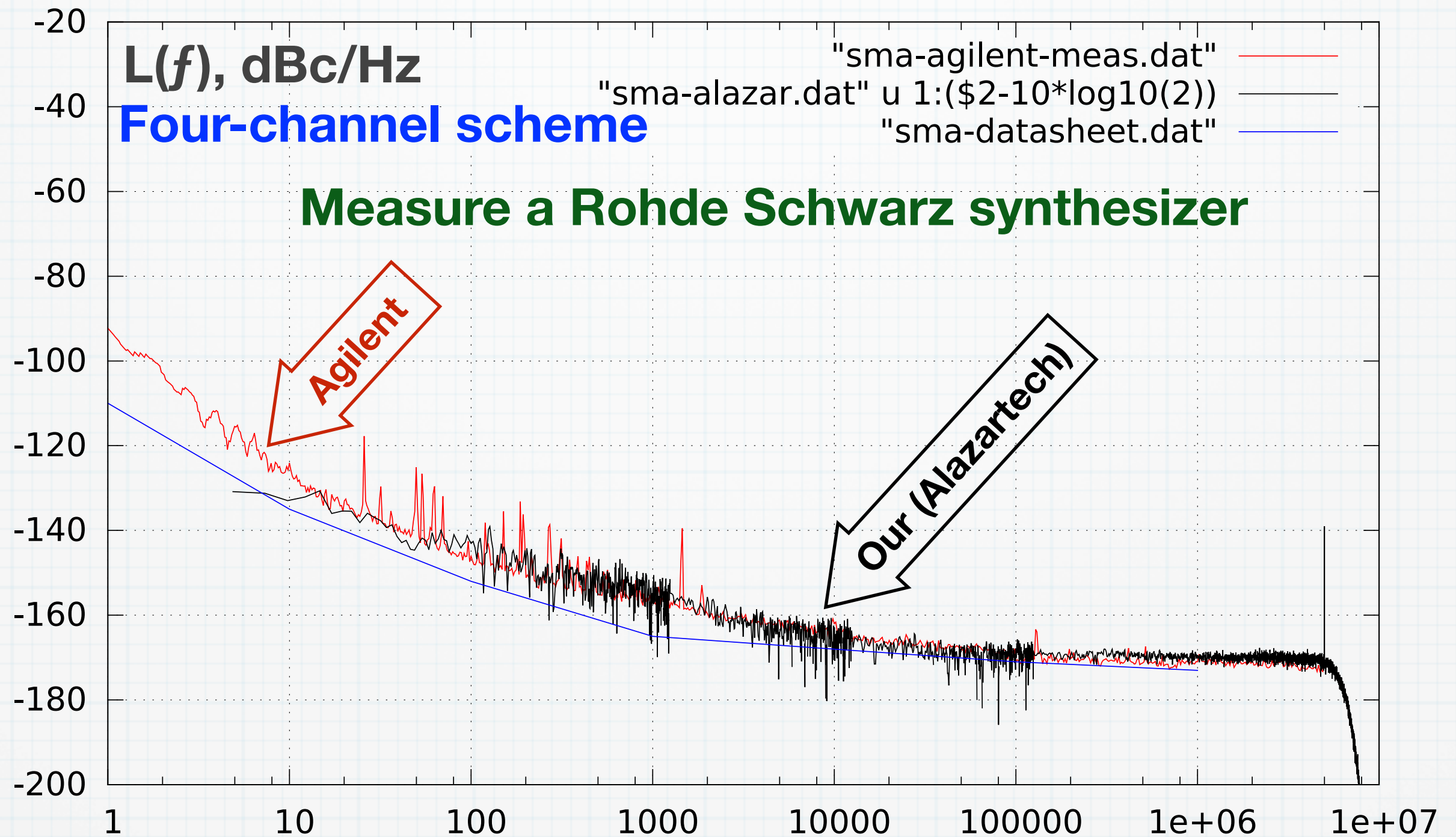


# Background Noise



# Compared to a Commercial Instrument

– this is done only to make sure that there is no calibration mistake –



# Conclusions

- **White noise**
  - **Depends on  $F_s$  and ENoB**
  - **Fits well the expectation**
- **Flicker  $-110 \text{ dBV}^2/\text{Hz}$  best found**
- **First phase noise measurements, (direct & beat)**
- **Background  $-185 \text{ dBc}$  with 4-channel scheme**
- **Modeling common-mode and differential jitter in progress**
- **Unwanted correlated effects still unknown**

# 4 – DDSs

- Basics
- Advanced
- Experiments

# Basics

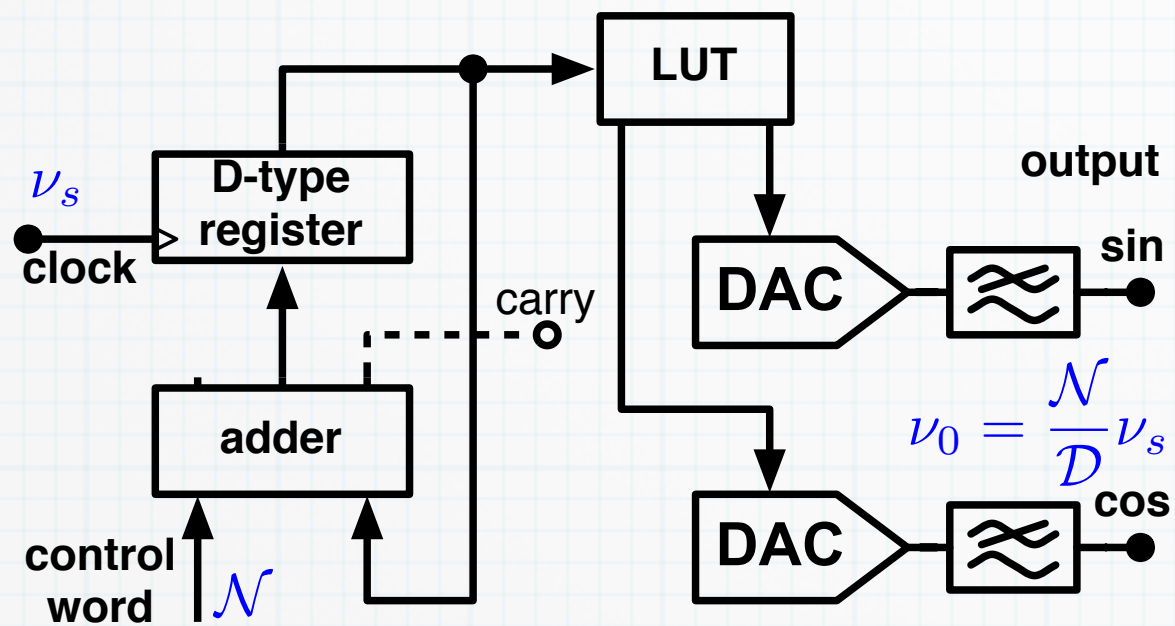


# Basic DDS scheme

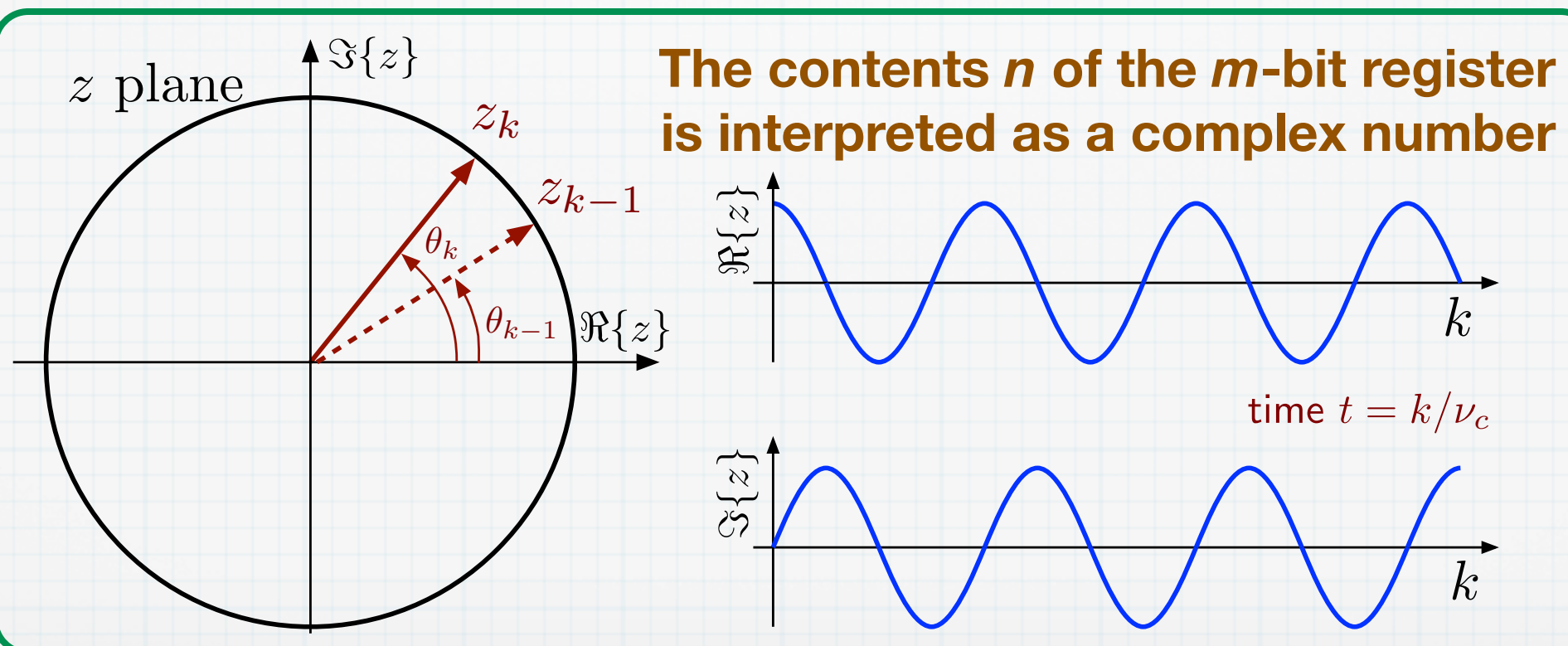
integer:  $n_k = (n_{k-1} + \mathcal{N}) \bmod \mathcal{D}$

complex:  $z_k = z_{k-1} \exp(j\eta)$

phase:  $\theta_k = (\theta_{k-1} + \eta) \bmod 2\pi$



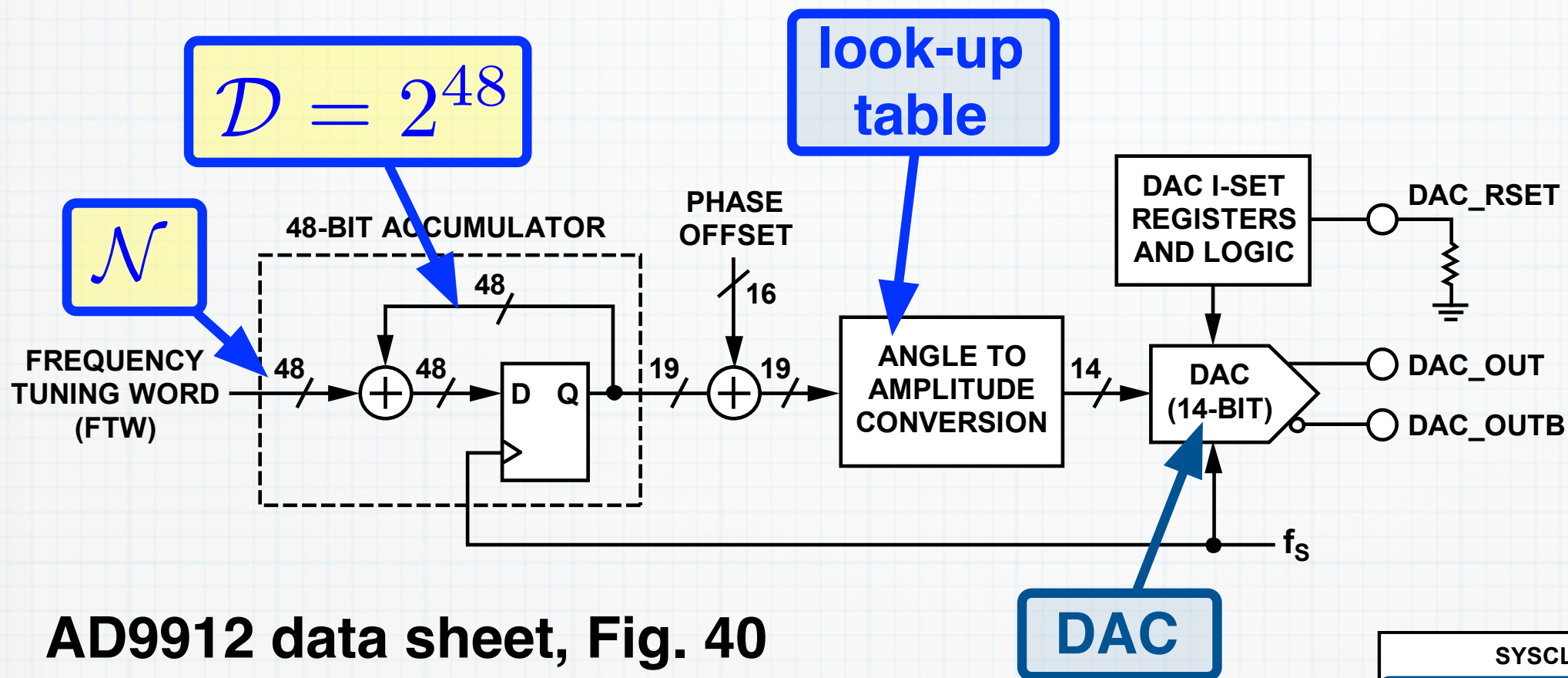
quantity	digital	analog
state variable	$n$	$\theta = 2\pi \frac{n}{\mathcal{D}}$
assoc. complex		$z = e^{j\theta}$
modulo	$\mathcal{D} = 2^m$	$2\pi$
increment	$\mathcal{N}$	$\eta = 2\pi \frac{\mathcal{N}}{\mathcal{D}}$
time	$k, 0, 1, 2, \dots$	$t = k/\nu_s$
clock freq. $\nu_s$	output freq. $\nu_0 = \frac{\mathcal{N}}{\mathcal{D}} \nu_s$	



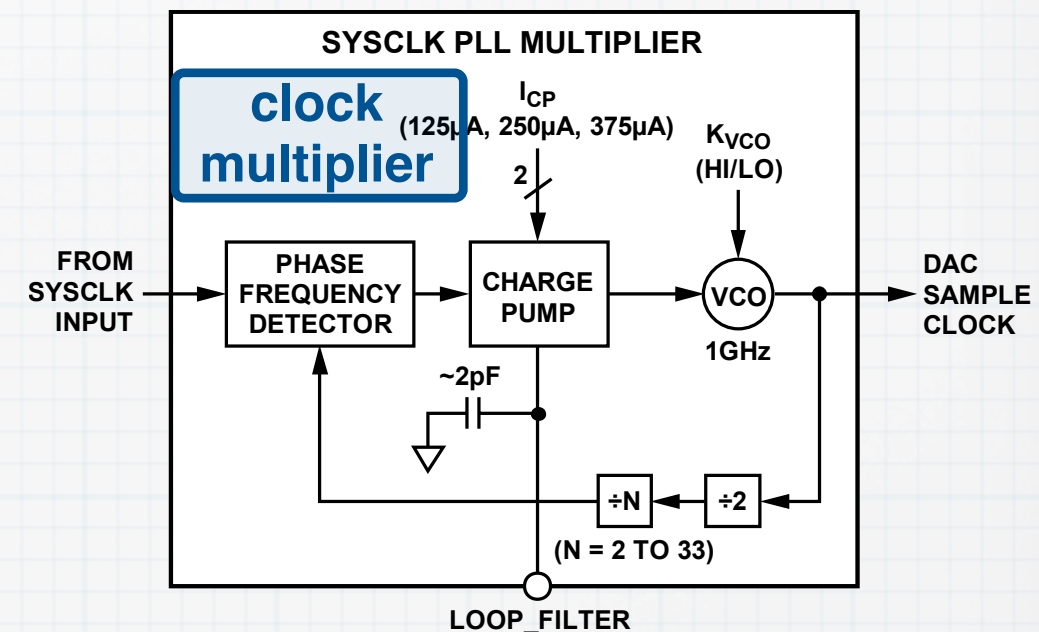


# AD9912, a popular fast DDS

48 bit accumulator, 14 bit DAC, 1 GHz clock



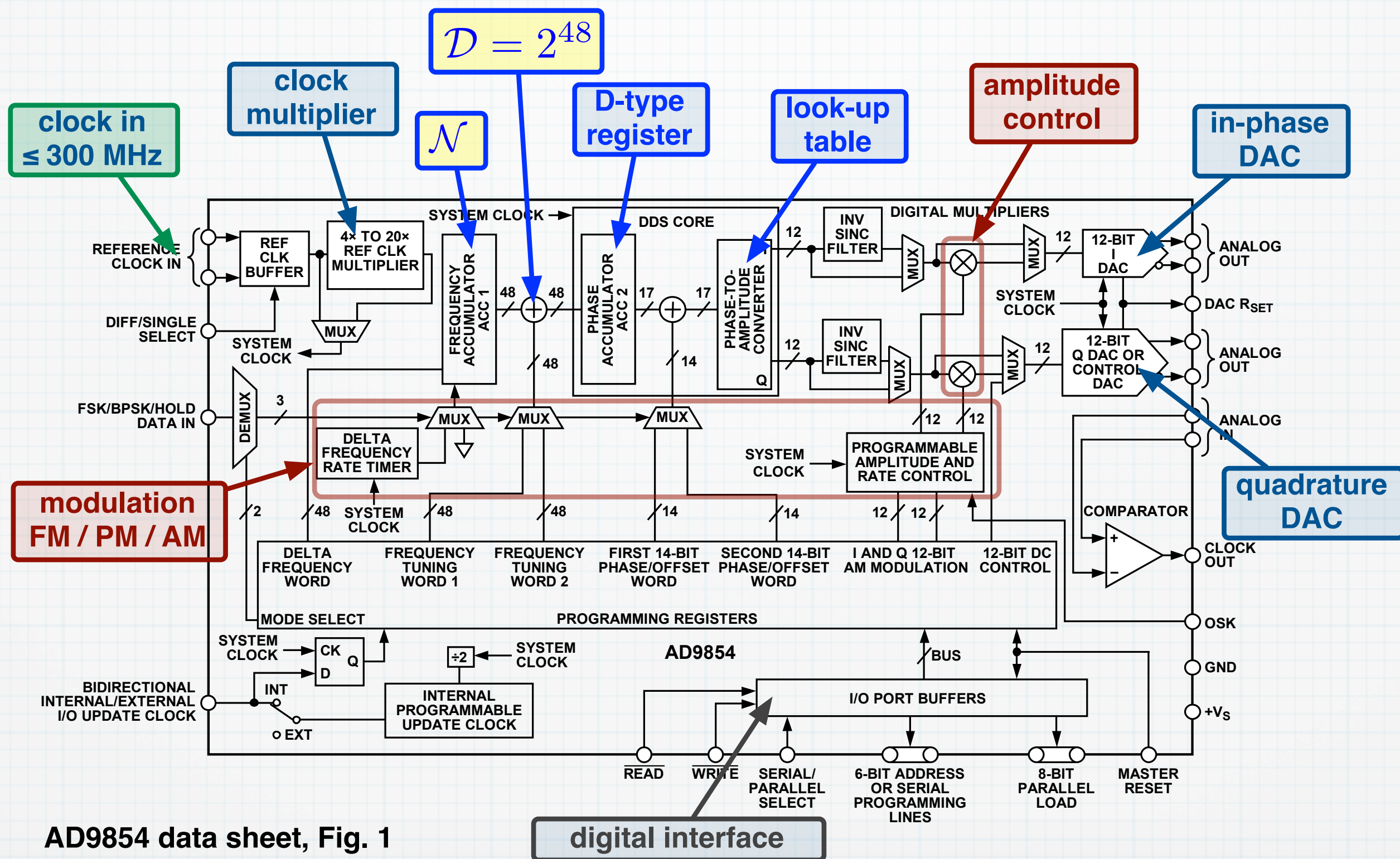
AD9912 data sheet, Fig. 40



AD9912 data sheet, Fig. 45

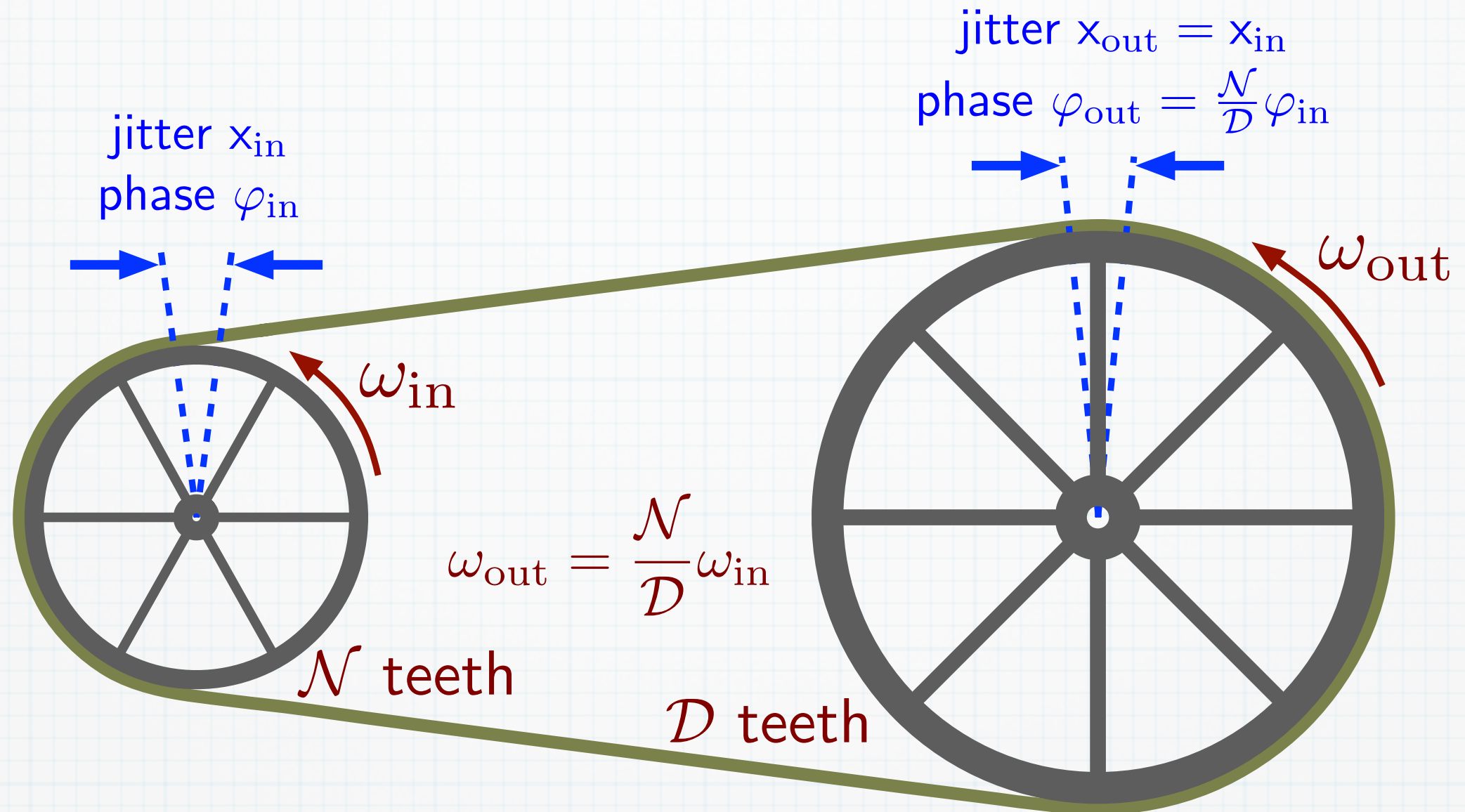
# AD9854, a popular DDS

48 bit accumulator, 300 MHz clock,  
12 bit DAC, I-Q output, AM/PM/FM capability



AD9854 data sheet, Fig. 1

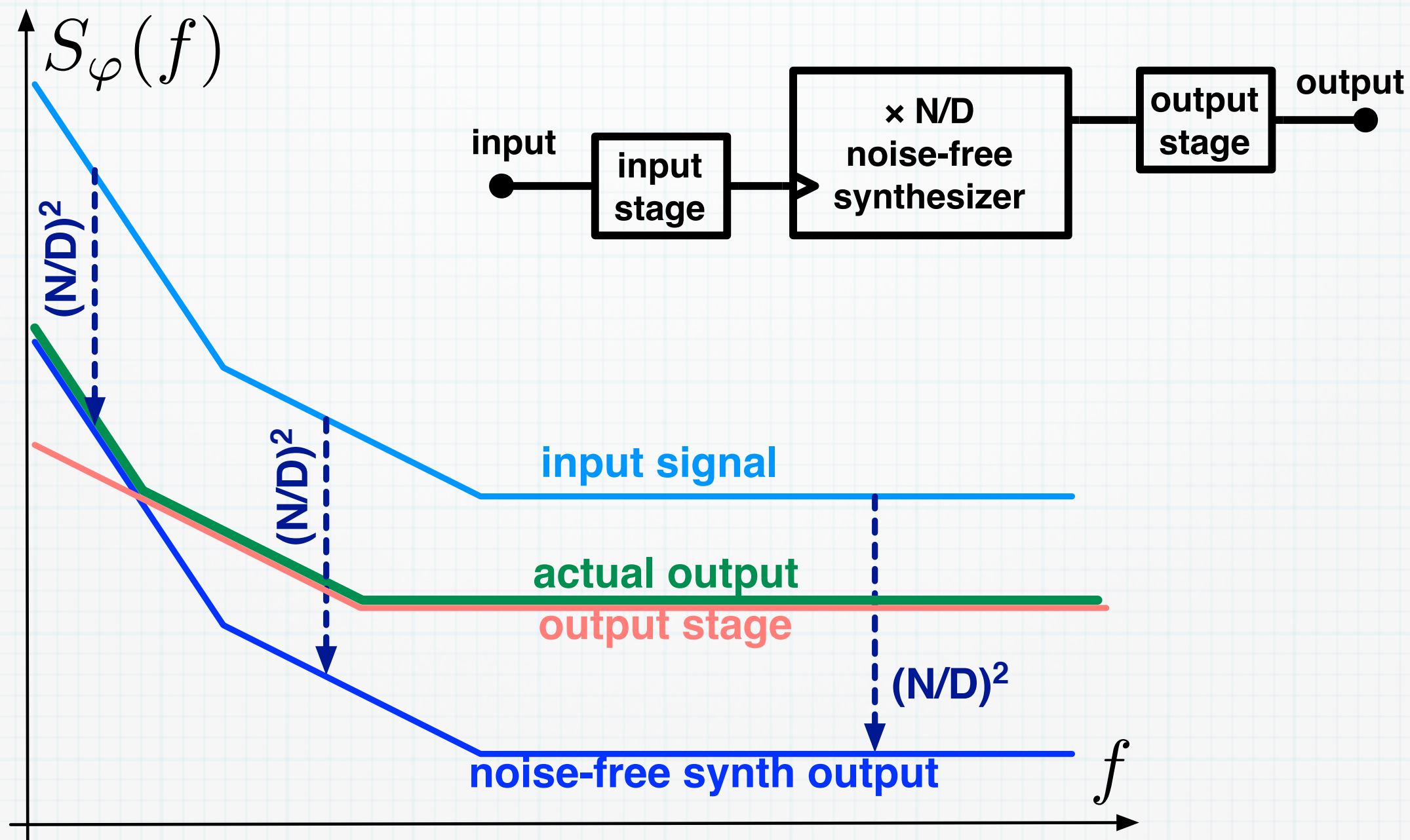
# The noise-free synthesizer



- The noise-free synthesizer propagates the jitter  $x$  (phase time)
- So, it scales the phase  $\varphi$  as  $N/D$ ,
- and the phase spectrum  $S_\varphi$  as  $(N/D)^2$
- Notice the absence of sampling

# The Egan model

for phase noise in frequency dividers

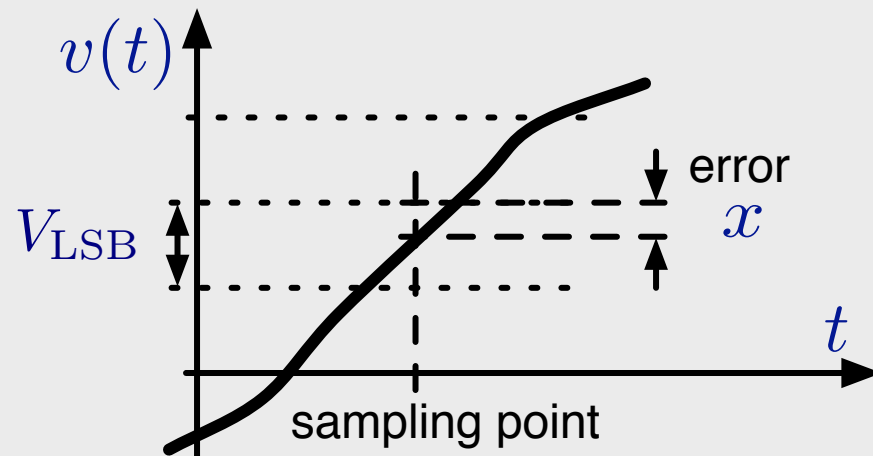


For  $N/D \ll 1$ , the scaled-down noise hits the output-stage limit



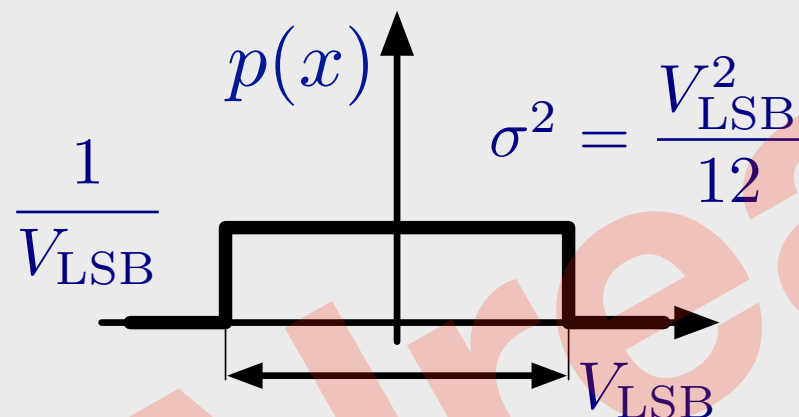
# Quantization noise

W. R. Bennett, Spectra of quantized signals, Bell System Tech J. 27(4), July 1948



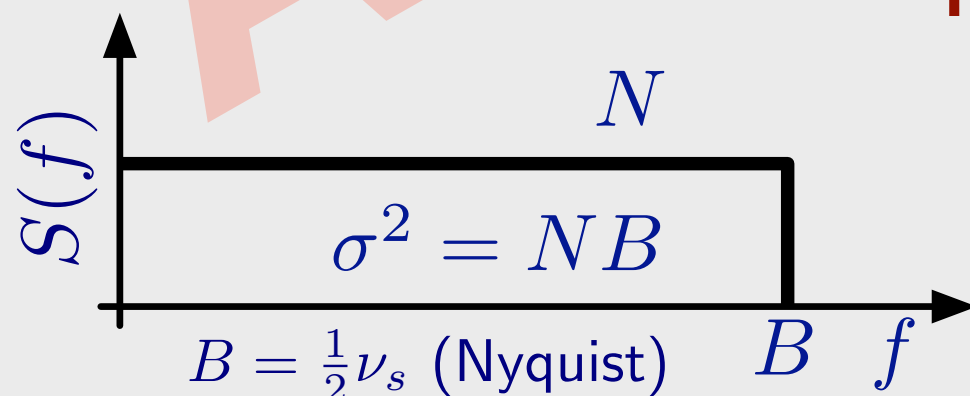
Analog-to-digital conversion introduces a **quantization error**  $x$   $[-V_{\text{LSB}}/2 \leq x \leq +V_{\text{LSB}}/2]$

$$n\text{-bit conversion: } V_{\text{LSB}} = \frac{V_{\text{FSR}}}{2^n}$$



**Wiener-Khintchine theorem:** in ergodic systems, interchange time / ensemble  
The noise can be calculated with statistics

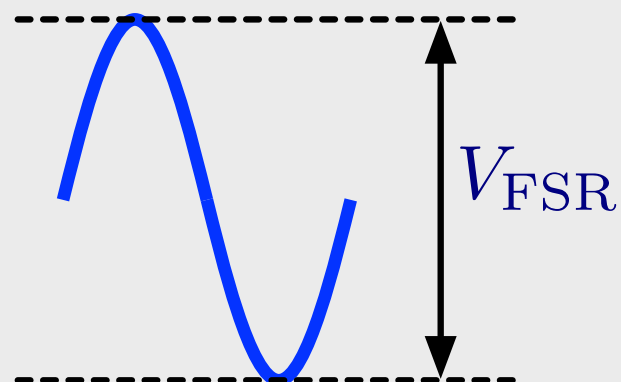
$$\sigma^2 = \frac{V_{\text{FSR}}^2}{12 \times 2^{2n}} \quad V^2 \quad \begin{array}{l} 1/12 \rightarrow -10.8 \text{ dB} \\ 2^{2n} \rightarrow 6 \text{ dB/bit} \end{array}$$



**Parseval theorem:** Energy (power) calculated in time and in frequency is the same

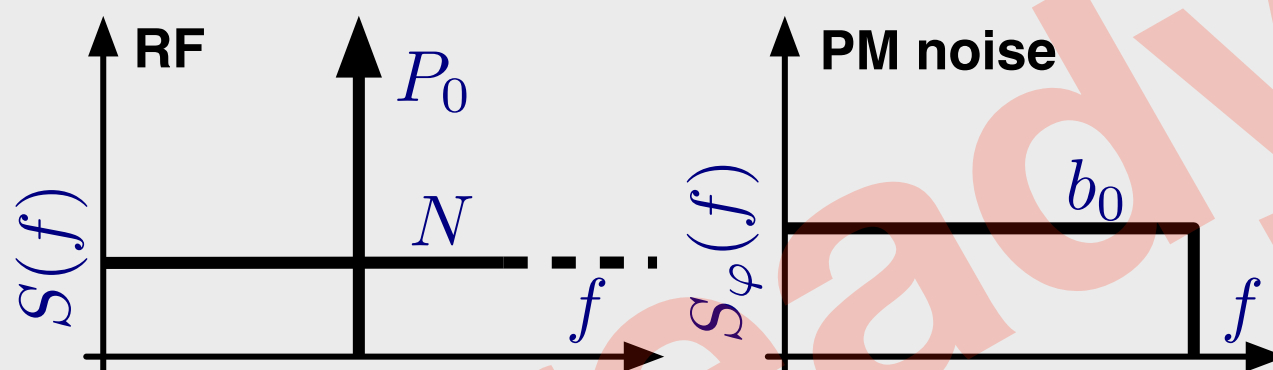
$$N = \frac{V_{\text{FSR}}^2}{6 \times 2^{2n} \nu_s} \quad V^2/\text{Hz}$$

# Quantization and PM noise



The **maximum power** is

$$P_0 = \frac{1}{8} V_{\text{FSR}}^2$$



In the presence of white noise  $N$ ,  
the **PM noise** is

$$b_0 = \frac{N}{P_0} \quad \text{rad}^2/\text{Hz}$$

Recall the  
quantization noise

$$N = \frac{V_{\text{FSR}}^2}{6 \times 2^{2n} \nu_s}$$

**The white PM noise is**

$$b_0 = \frac{4}{3} \frac{1}{2^{2n} \nu_s} \quad \text{rad}^2/\text{Hz}$$

**Example:**

14 bit, 1 GHz  $\rightarrow$  -173 dB

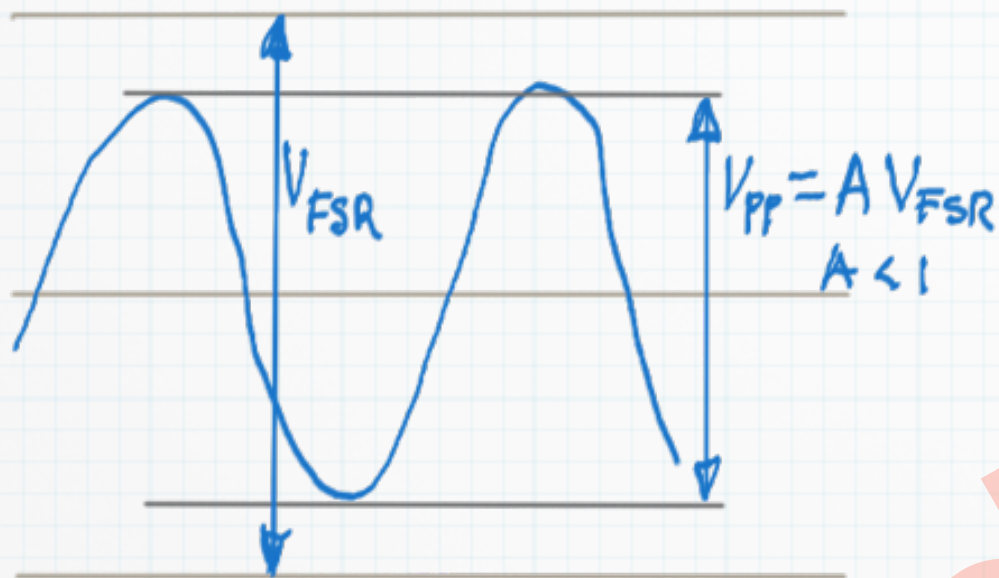
14 bit, 400 MHz  $\rightarrow$  -169 dB

12 bit, 300 MHz  $\rightarrow$  -156 dB



# Background noise

Sampling frequency  $f_s$



Signal power

$$P_0 = \frac{V_{PP}^2}{8} = \frac{A^2 V_{FSR}^2}{8}$$

Noise power

$$\sigma^2 = \frac{V_{LSB}^2}{12}$$

Parseval theorem

$$S_v = \frac{\sigma^2}{B} \Rightarrow S_v = \frac{V_{LSB}^2}{6 f_s}$$

Phase noise  $S_\phi = b_0$  (white)

$$b_0 = \frac{S_v}{P_0} = \frac{V_{LSB}^2}{V_{FSR}^2} \cdot \frac{4}{3 A^2 f_s}$$

$$b_0 = \frac{1}{(2^M)^2} \cdot \frac{4}{3 A^2 f_s}$$

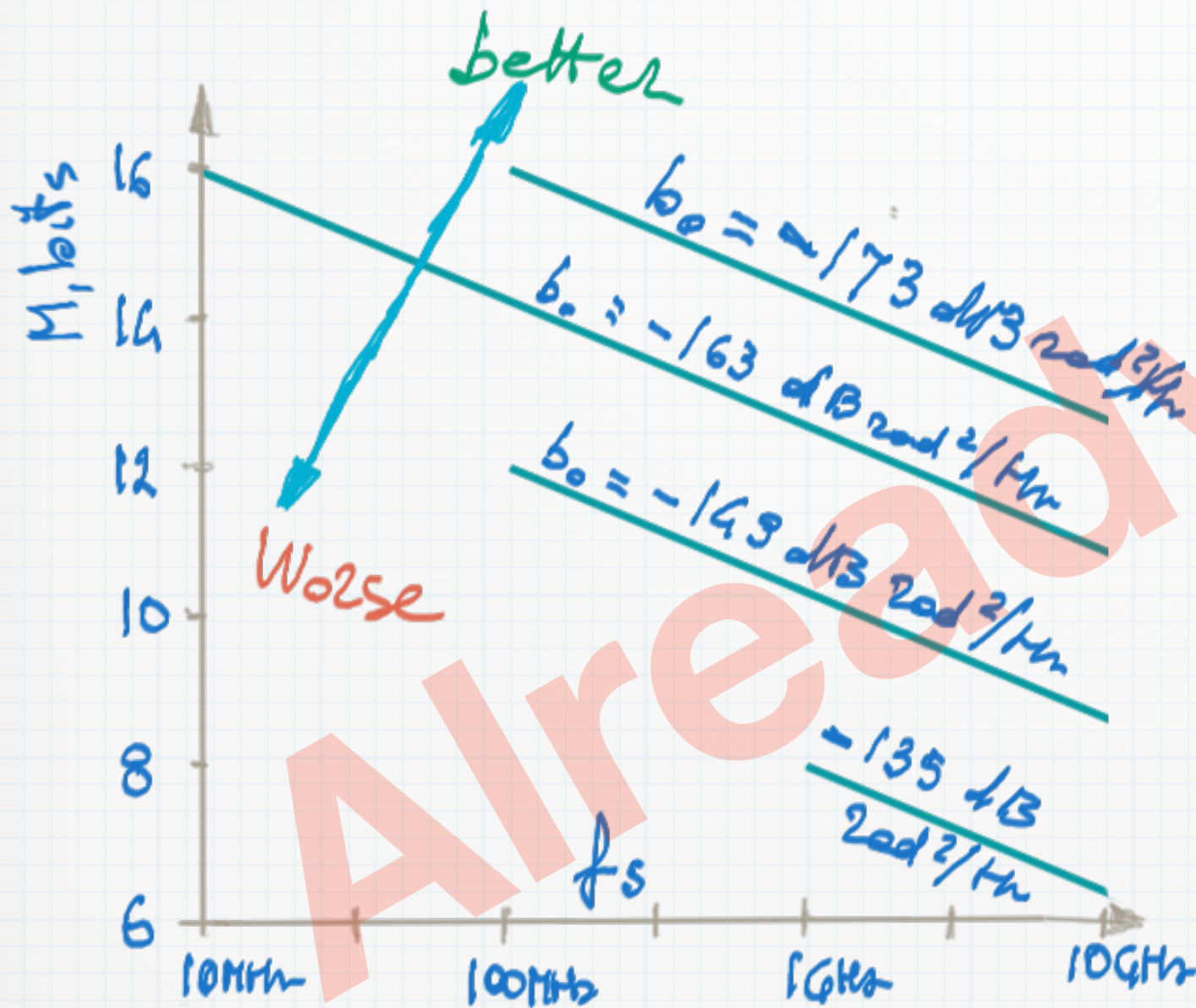
Approximation (fairly large  $V_P$ )  
 $A^2 = 2/3$  (-1.8dB)  $\Rightarrow b_0 \approx \frac{1}{(2^M)^2} \cdot \frac{2}{f_s}$

Warning: We assume that the noise power is equally distributed in  $0 \dots B$ .

This is not true in our case because sampling and carrier are highly coherent. See Widrow-Kollar Appendix G for details.

Anyway, we temporarily accept the uniform distribution, hoping that the reality is not too far.

# Background noise



$$b_0 = \frac{1}{(2^M)^2} \frac{4}{3A^2 f_s}$$

$$b_0 \approx \frac{1}{(2^M)^2} \frac{2}{f_s}$$

Cost of 6 dB improvement

- 1 bit
- factor-of-4  $f_s$

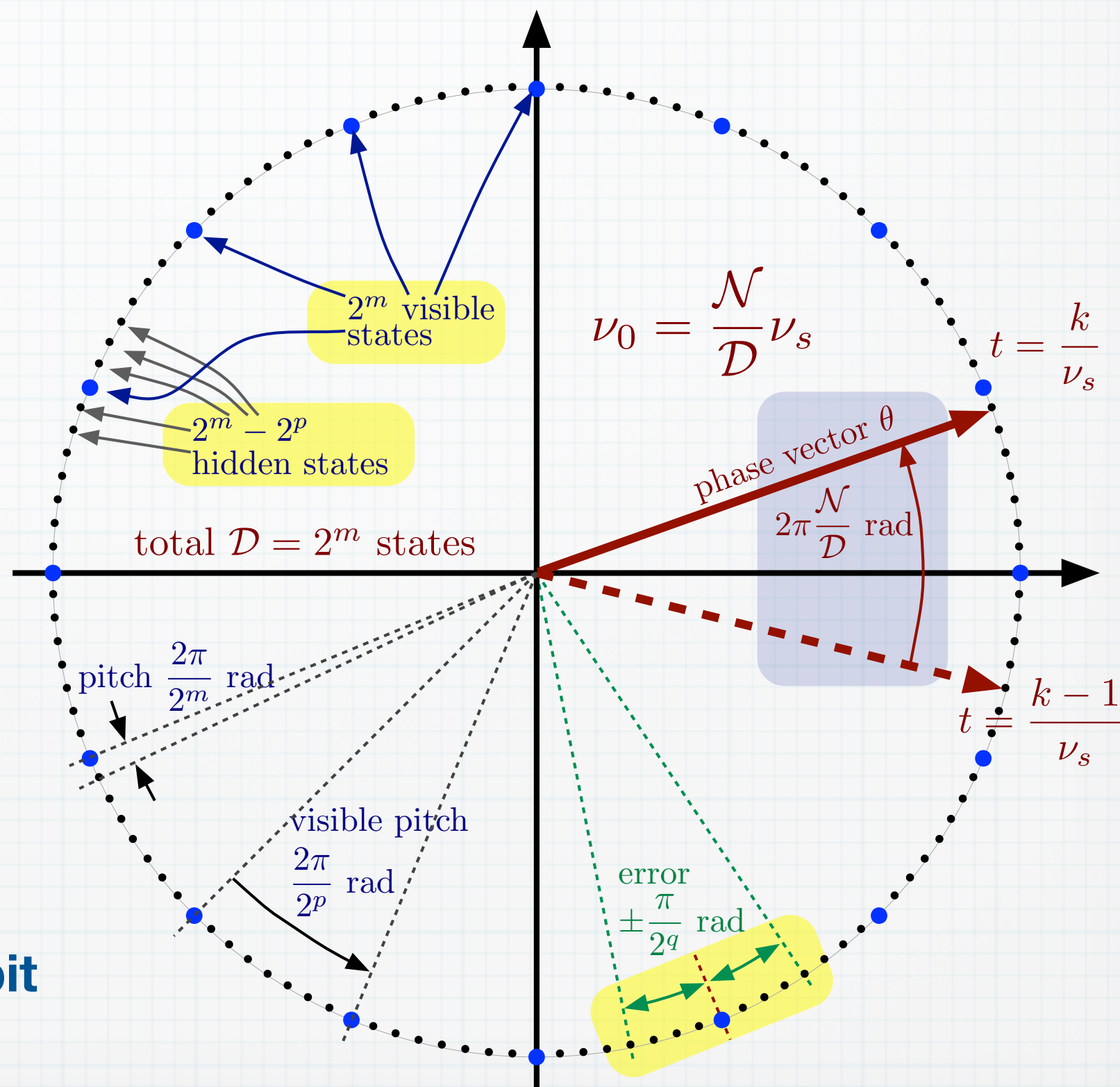
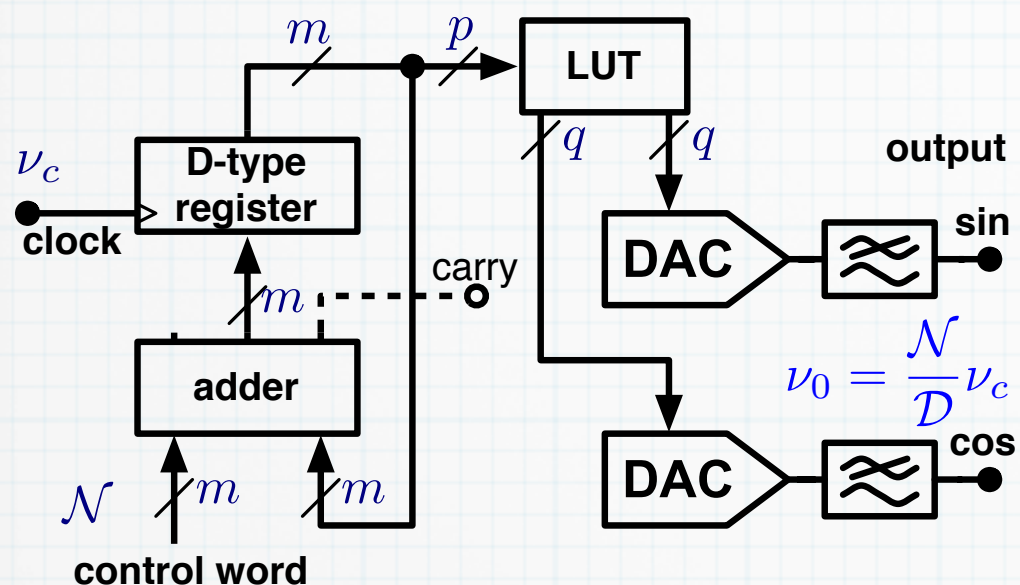
Obvious conclusion:  
practical ADCs feature  
lower  $b_0$  at low  $f_s$  because  
of the higher no. of bits

# Advanced



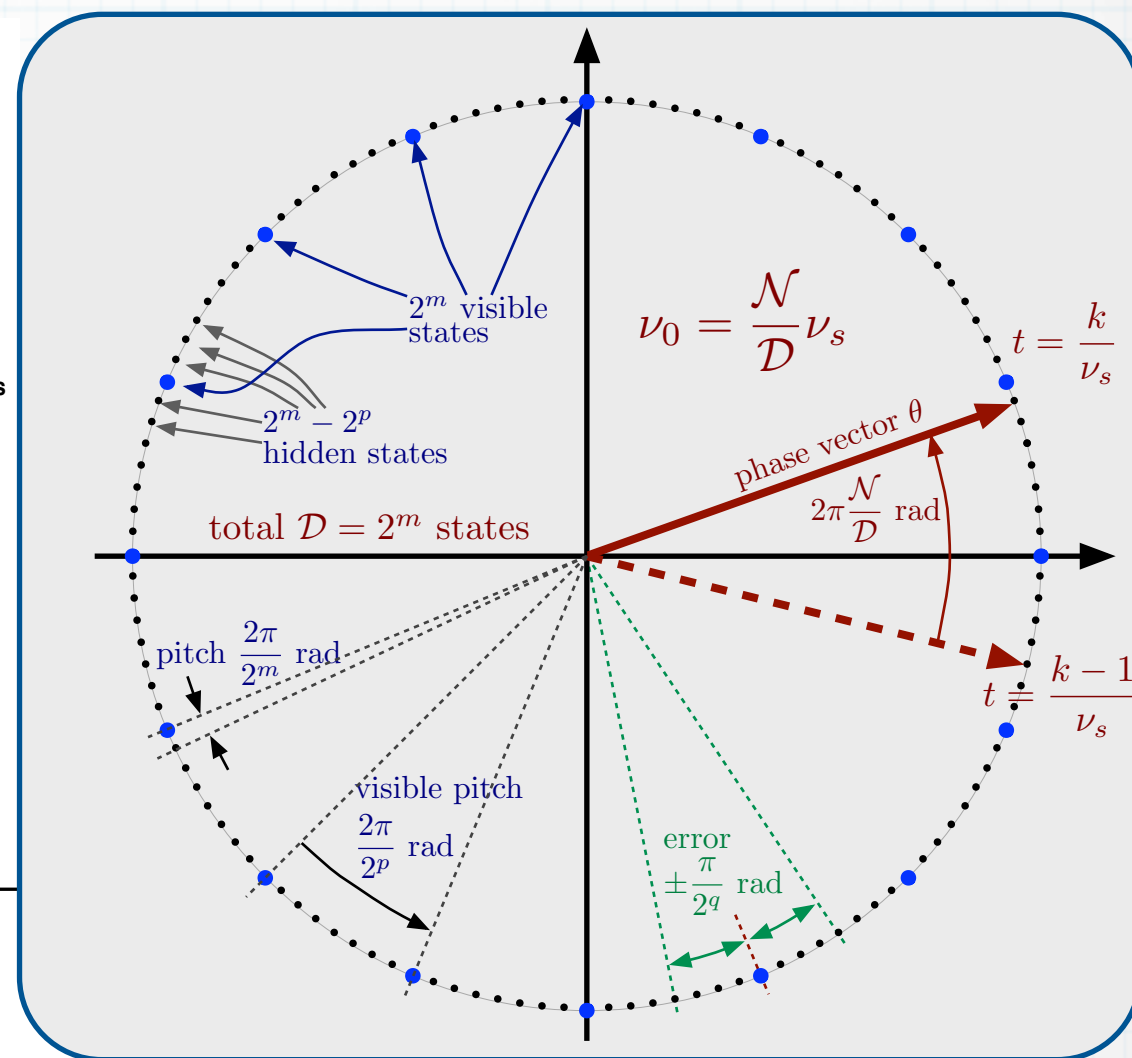
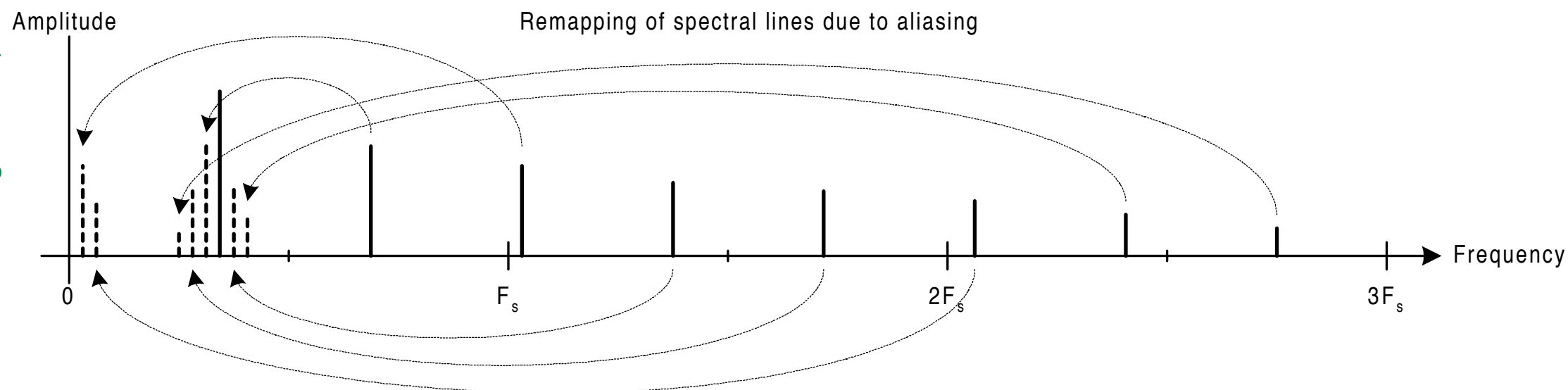
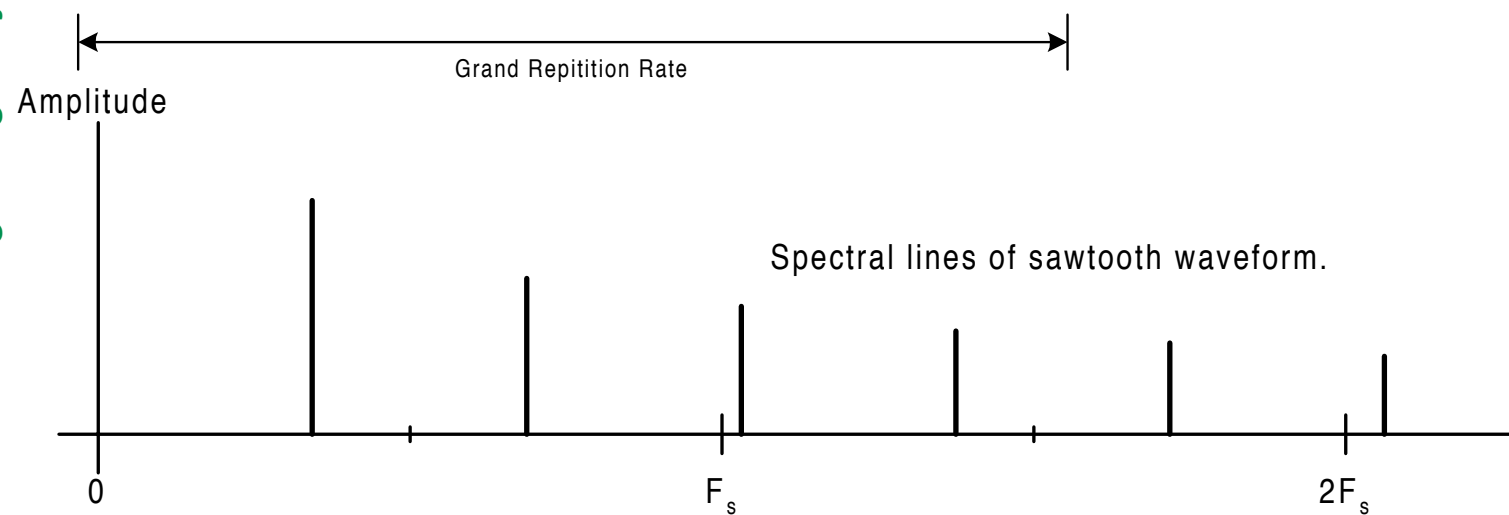
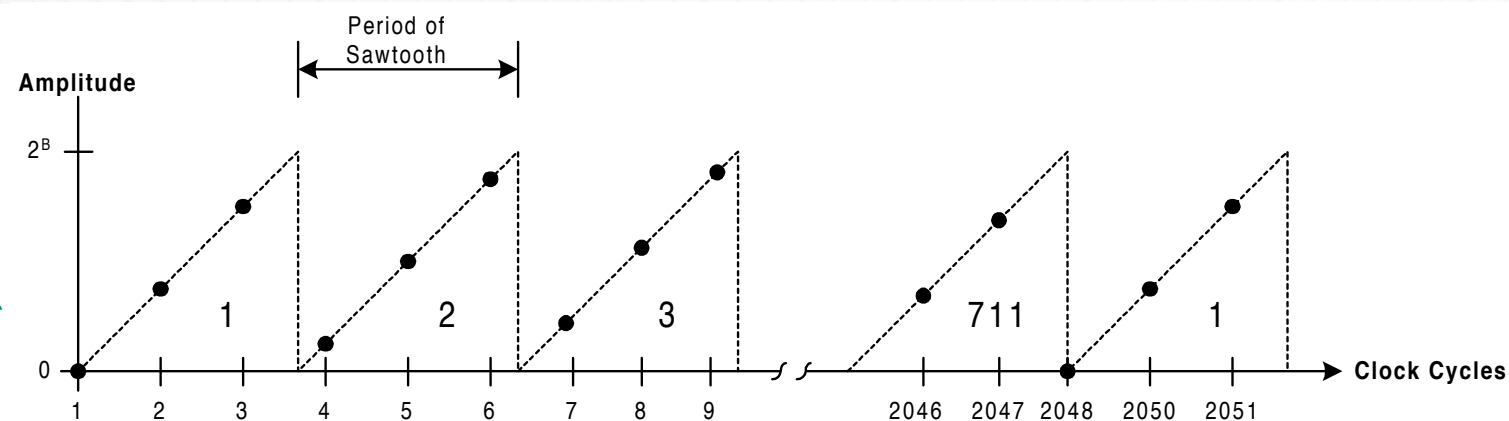
# State-variable truncation

$$n_k = (n_{k-1} + \mathcal{N}) \bmod \mathcal{D}, \quad \mathcal{D} = 2^m$$



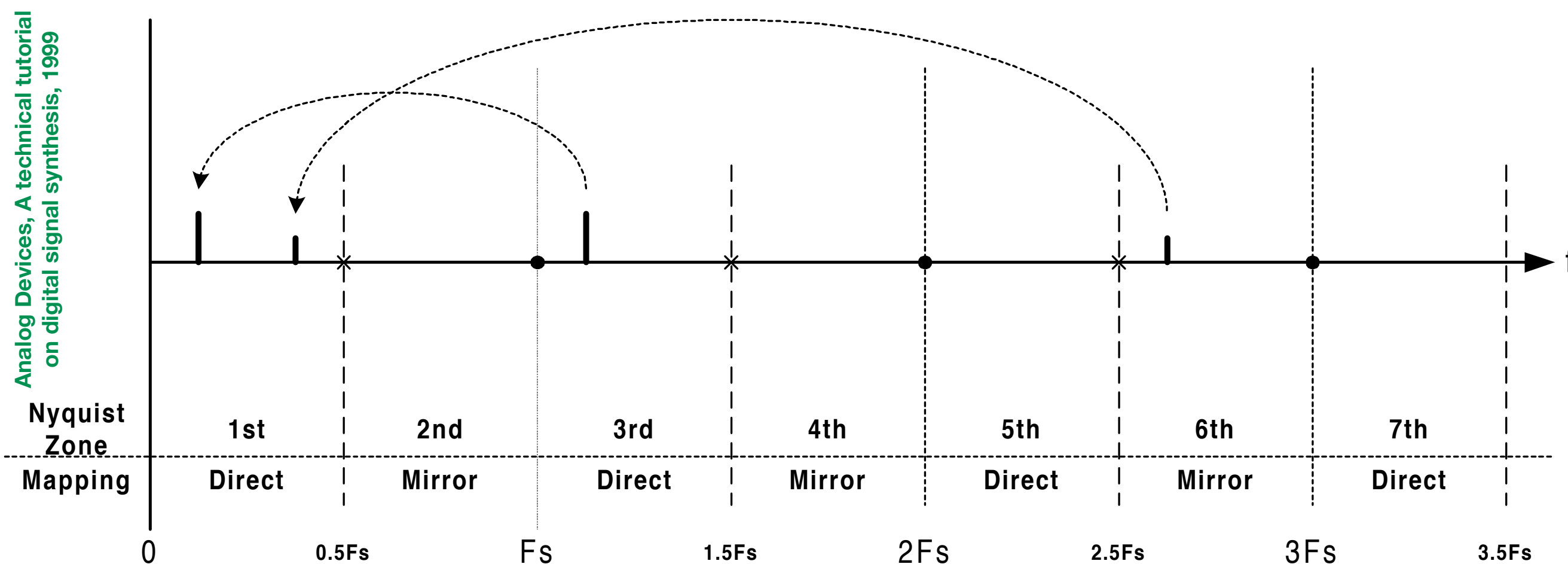
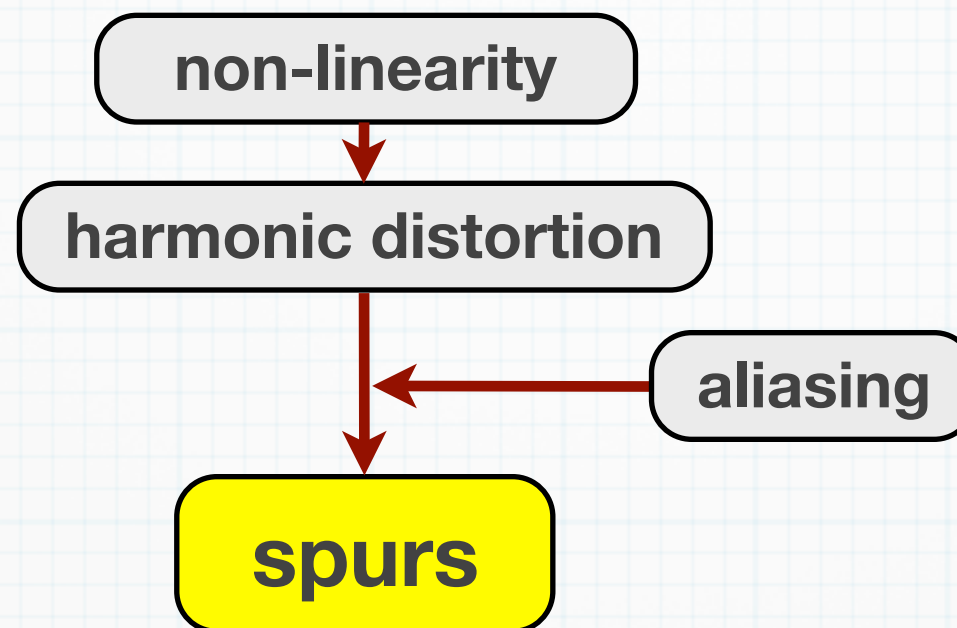
- Only quantization shows up with full m-bit conversion
- Technology  $\rightarrow$  q max
- Why  $p > q$
- Slow pseudorandom beat, 3d 6h 11m 15s @ 1 GHz, 48 bit
- Spurs  $\rightarrow$  next

# Truncation generates spurs



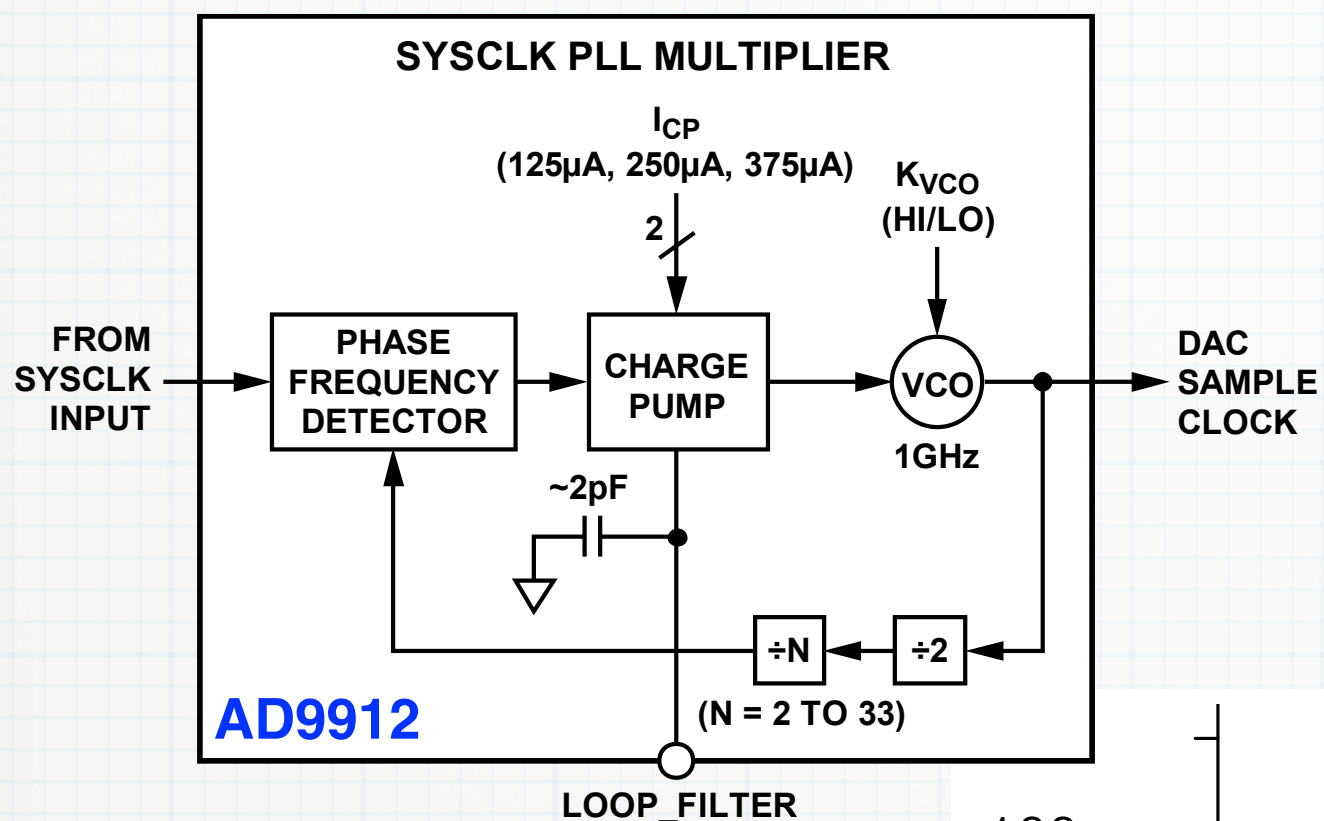
The power of spurs comes at expenses of white noise – yet not as one-to-one

# Nonlinearity generates spurs



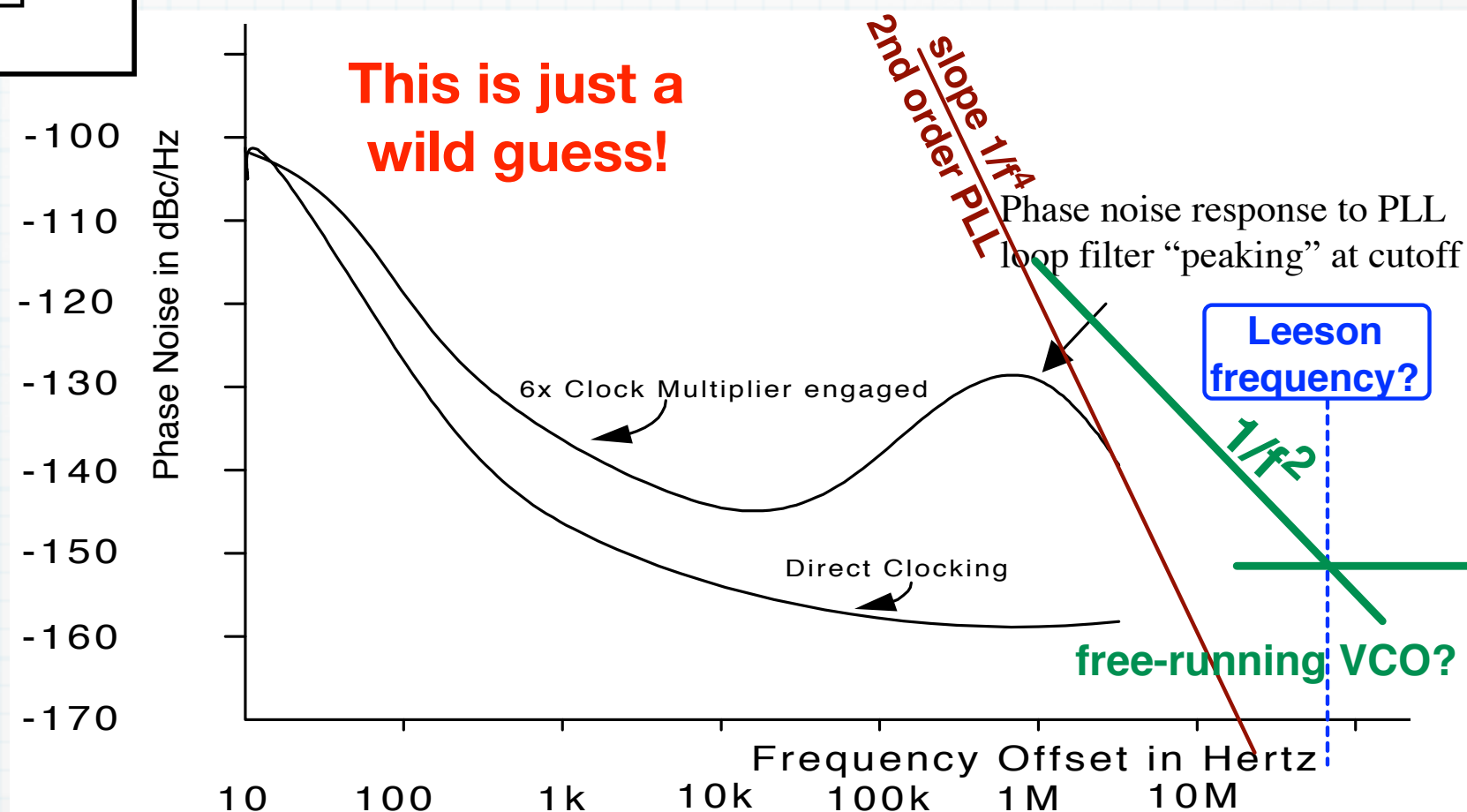


# PLL clock multiplier



- On-Silicon LC oscillator
- (also used in other AD devices)
- Literature suggests  $Q \approx 5...10$
- Leeson  $f_L = 50-100$  MHz
- Tight PLL is needed
- Divider noise
- No data from the manufacturer

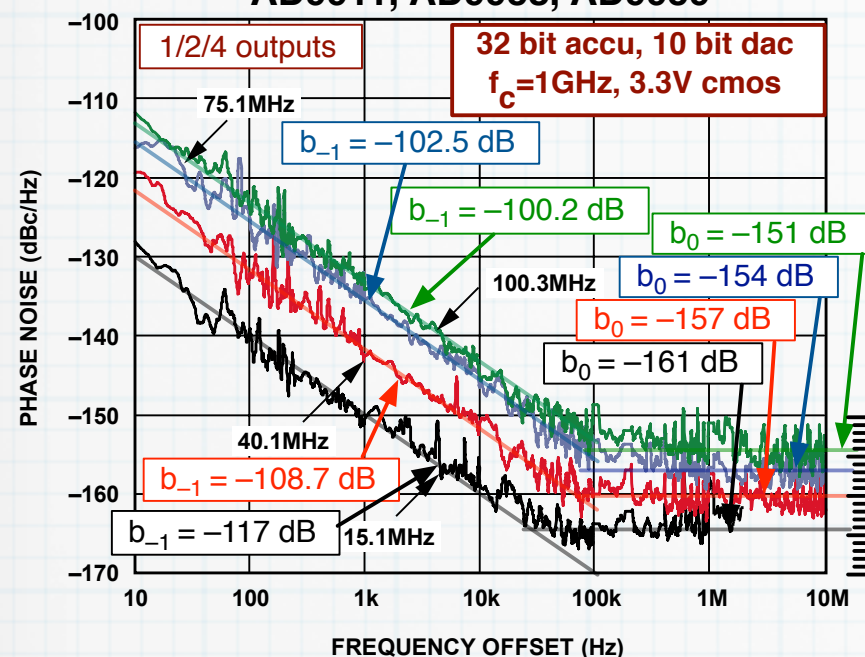
The AD 9854 is likely similar, yet the VCO frequency is 300 MHz



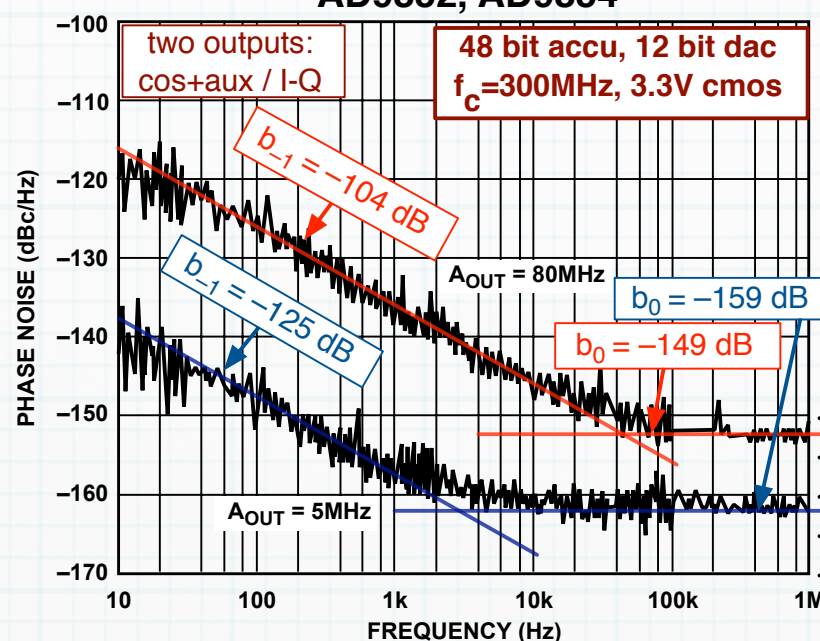
# 3.3 V: lower PM noise than 1.8 V

Probably related to the cell size and to the dynamic range

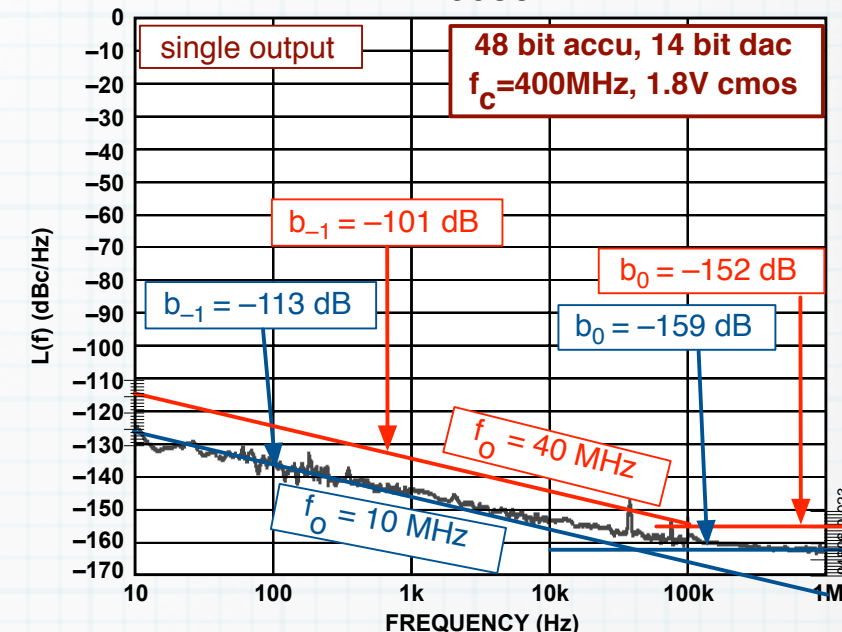
AD9911, AD9958, AD9959



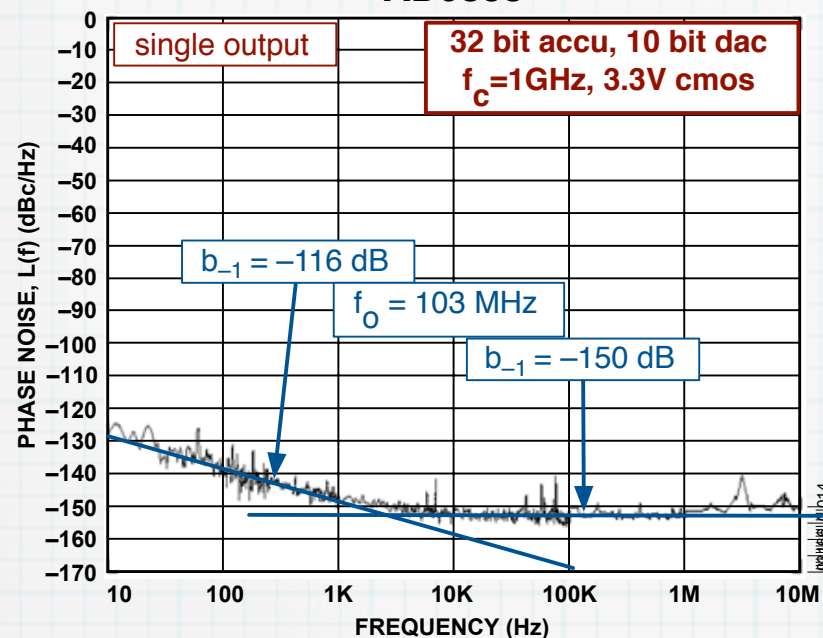
AD9852, AD9854



AD9956



AD9858



AD9912

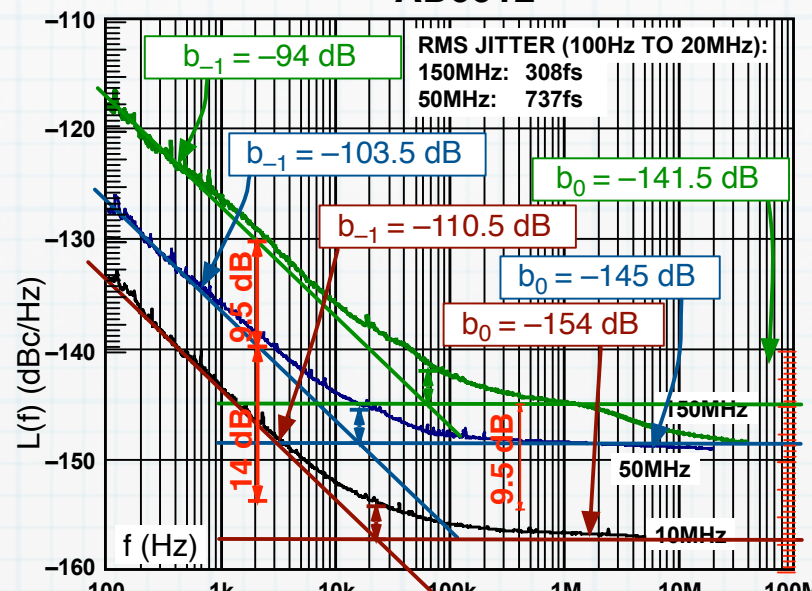
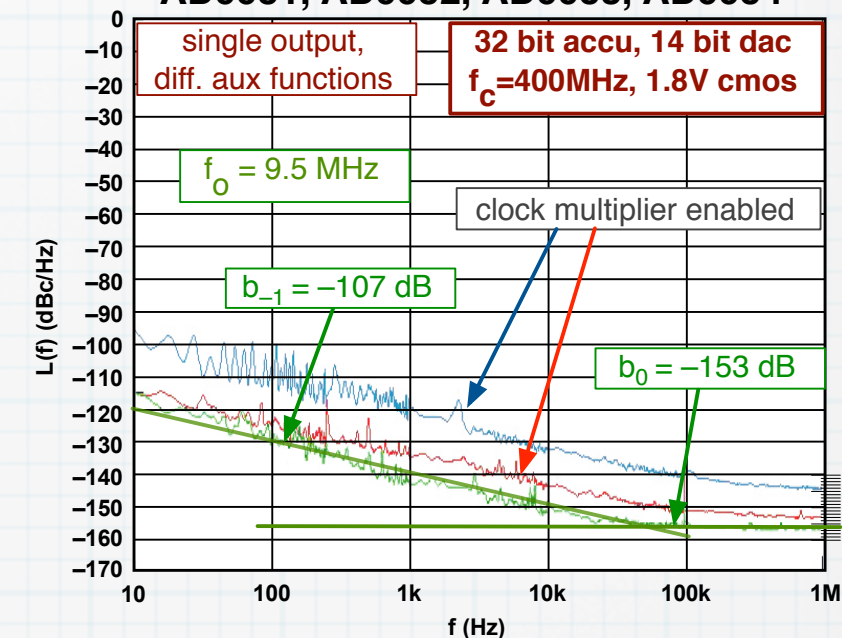


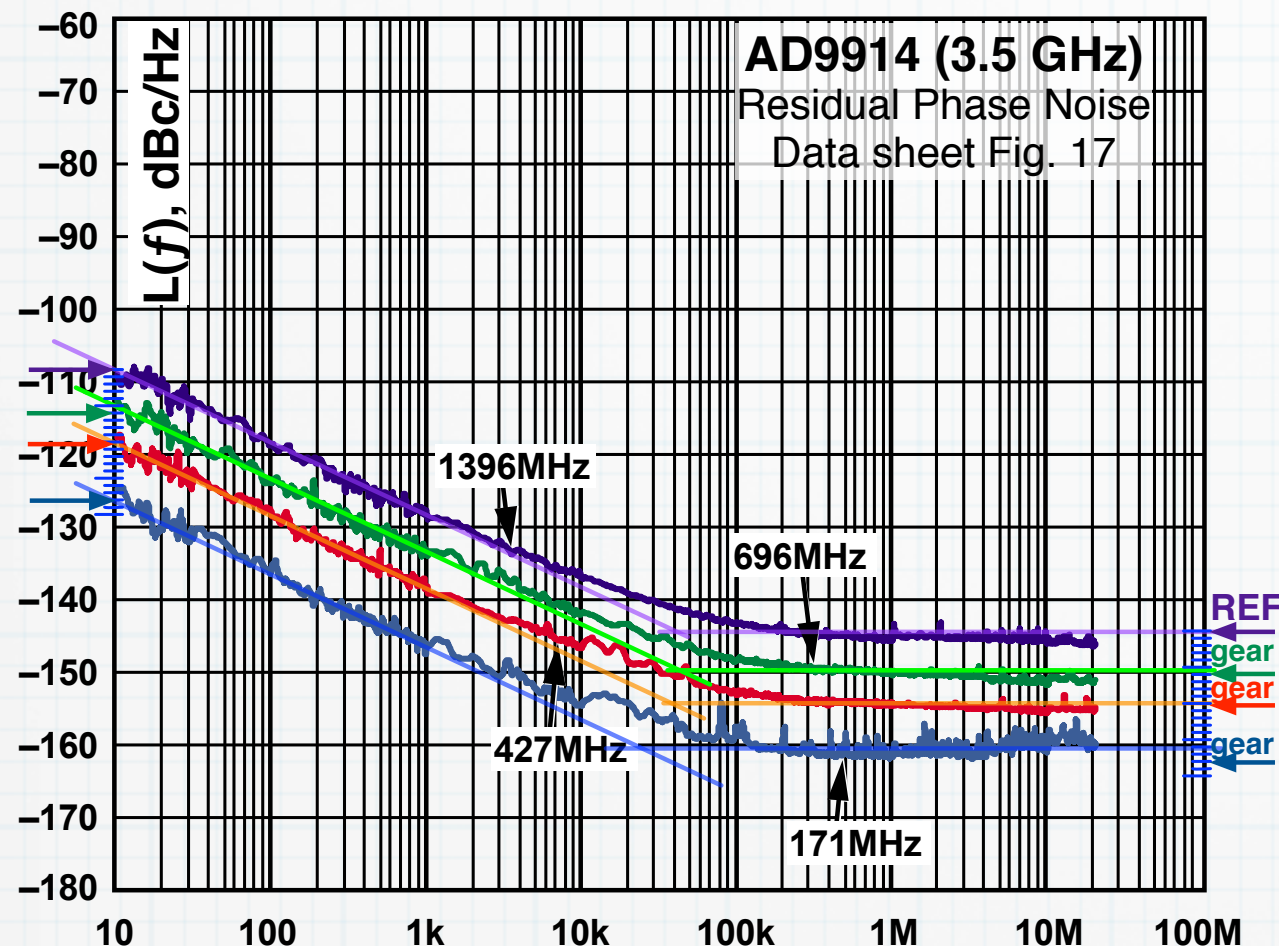
Figure 16. Absolute Phase Noise Using CMOS Driver at 3.3 V, SYSCLK = 1 GHz Wenzel Oscillator (SYSCLK PLL Bypassed) DDS Run at 200 MSPS for 10 MHz

AD9951, AD9952, AD9953, AD9954

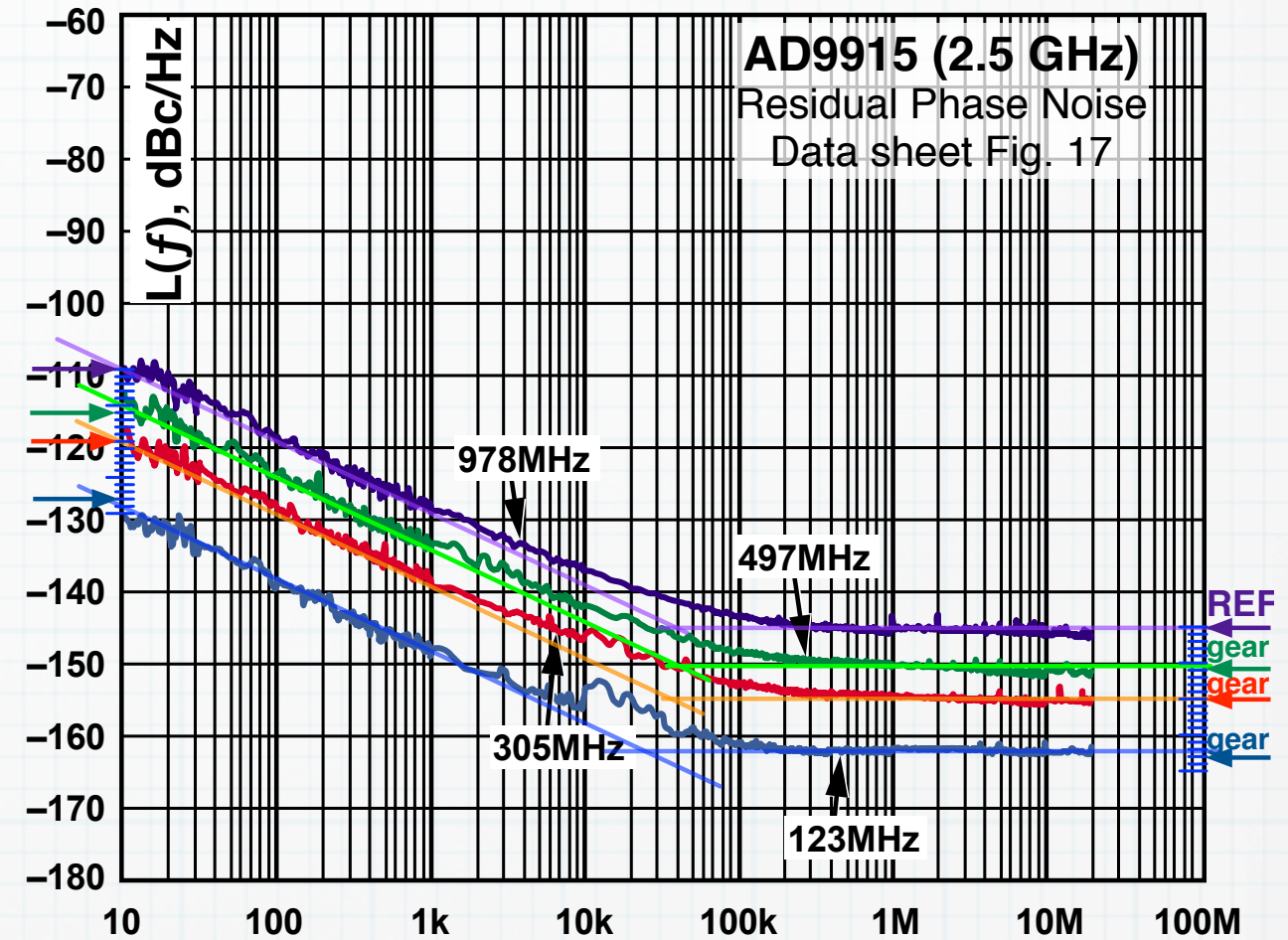


# High-Frequency DDSs

**AD9914 12 bit, 3.5 GHz**  
**64 bit accumulator (190 pHz res)**



**AD9915 12 bit, 2.5 GHz**  
**64 bit accumulator (135 pHz res)**



**Residual noise is close to that of the gear-box model**

**Plots are from the manufacturer data sheet**

**Whether spurs are removed or not, is not said**

# Experiments

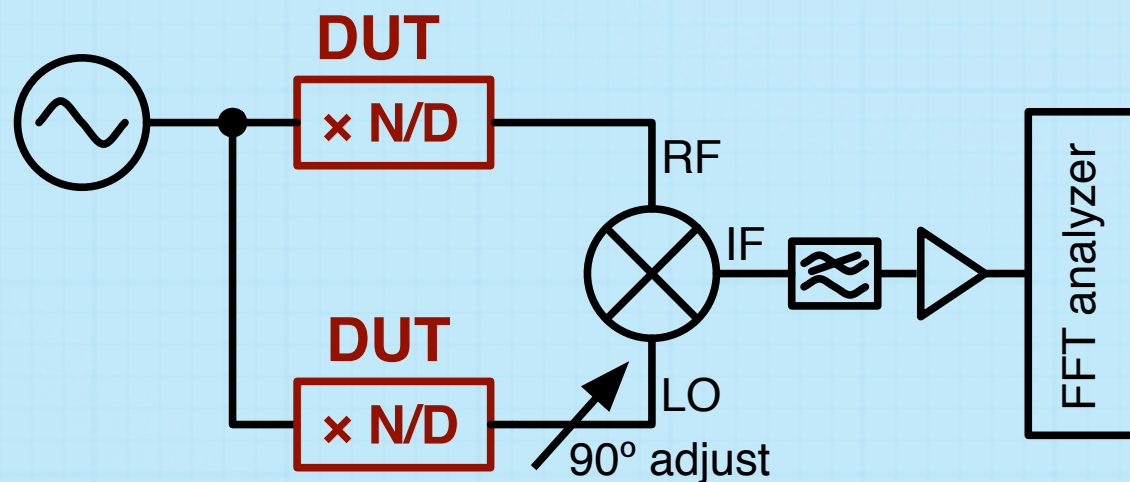
- **AD9912 demo board**
- **AD9854 (9914) demo board**
- **Claudio's AD9854 board**
  - **V1 – Current feedback OPA output stage**
    - **25 $\Omega$  input impedance, 8 nV/ $\sqrt{\text{Hz}}$  noise, kHz coupled**
  - **V2 – Balun and MAV-11 RF output amplifier**
    - **F = 3.6 dB, AC coupled ( $\geq 1$ –2) MHz**
      - **Specified above 50 MHz, yet works well below**



# Experimental method (PM noise)<sup>95</sup>

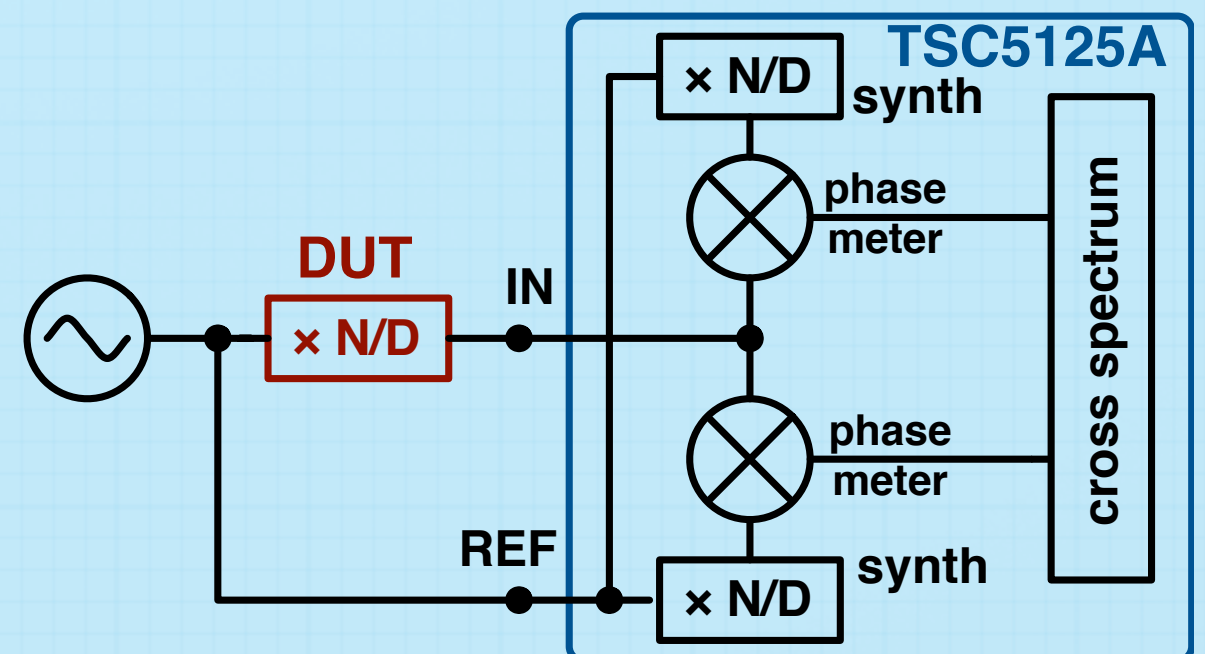
- Pseudorandom noise, slow beat (days)
- The probability that two accumulators are in phase is  $\approx 0$
- Two separate DDS driven by the same clock have a random and constant delay
- The delay de-correlates the two realizations, which makes the phase measurement possible

## Single channel



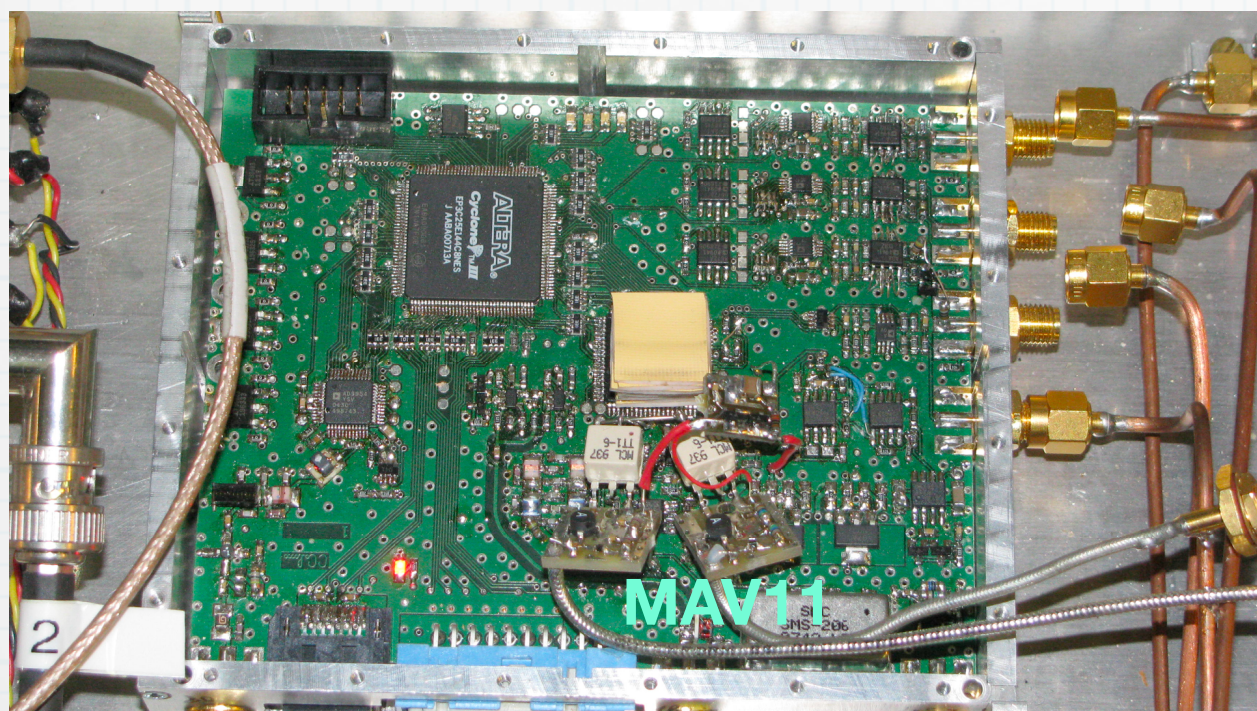
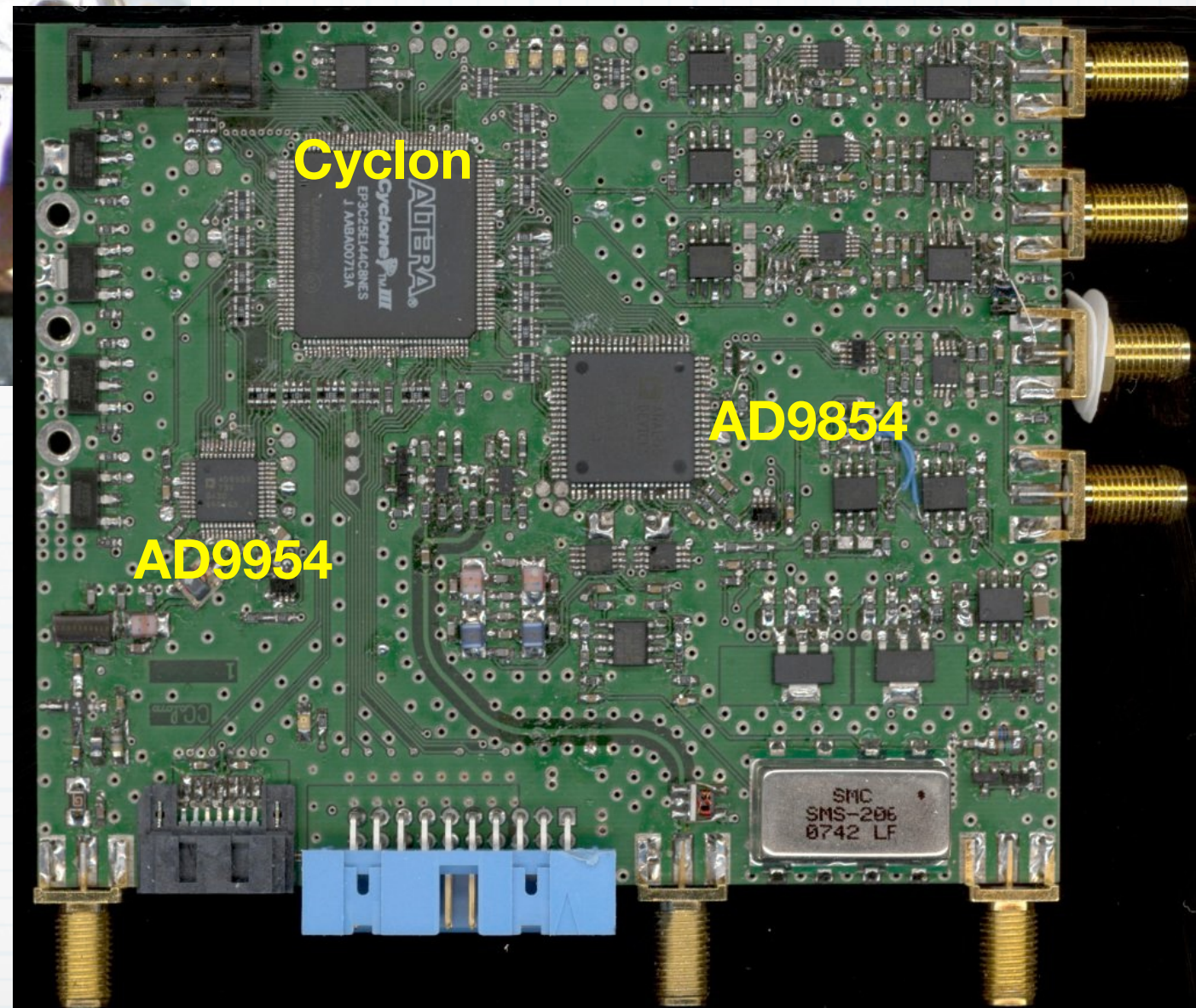
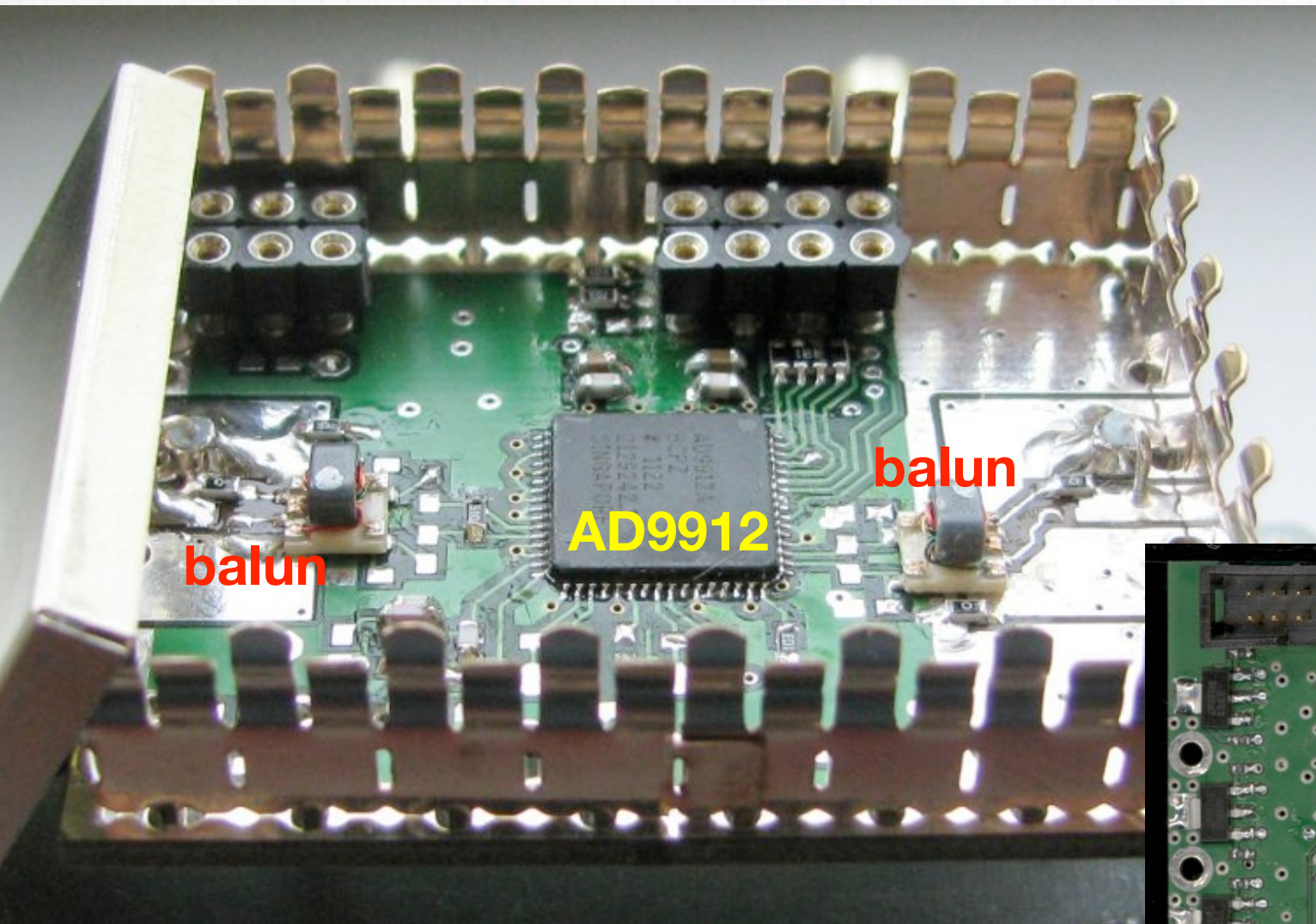
## Dual channel

kind of virtual mixer, after (sub)sampling & direct ADC



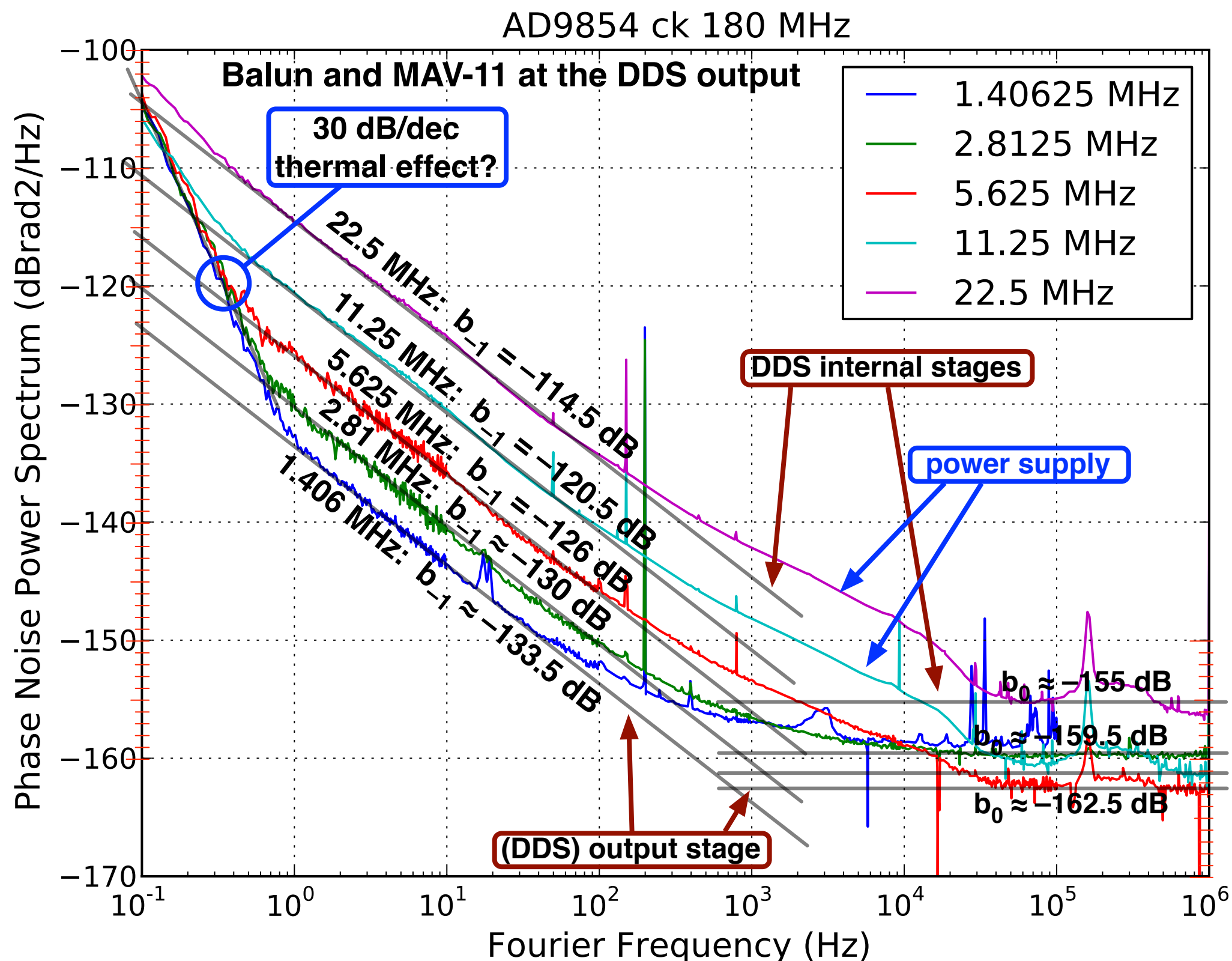


# Claudio's prototypes



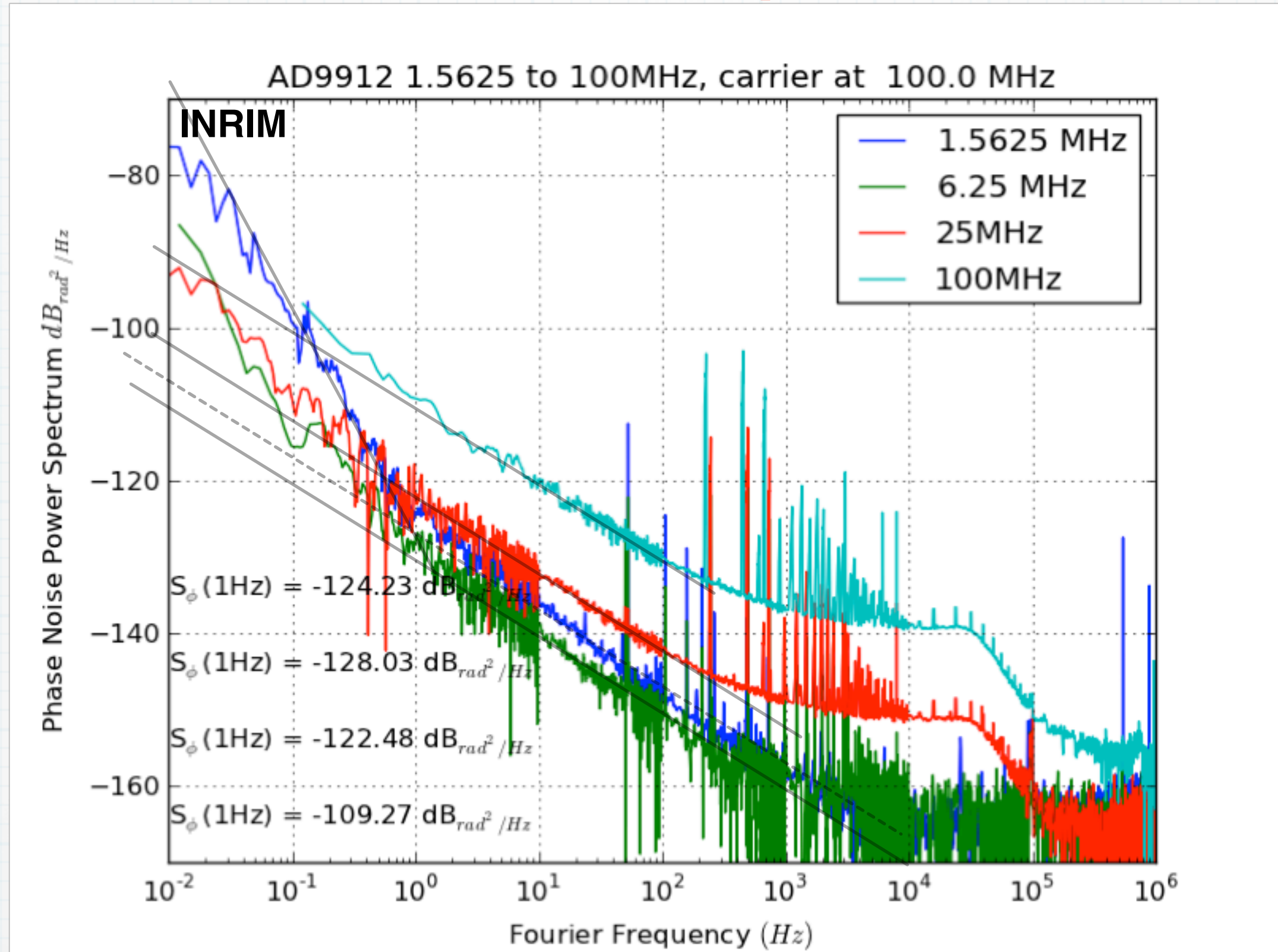


# PM noise vs. output frequency

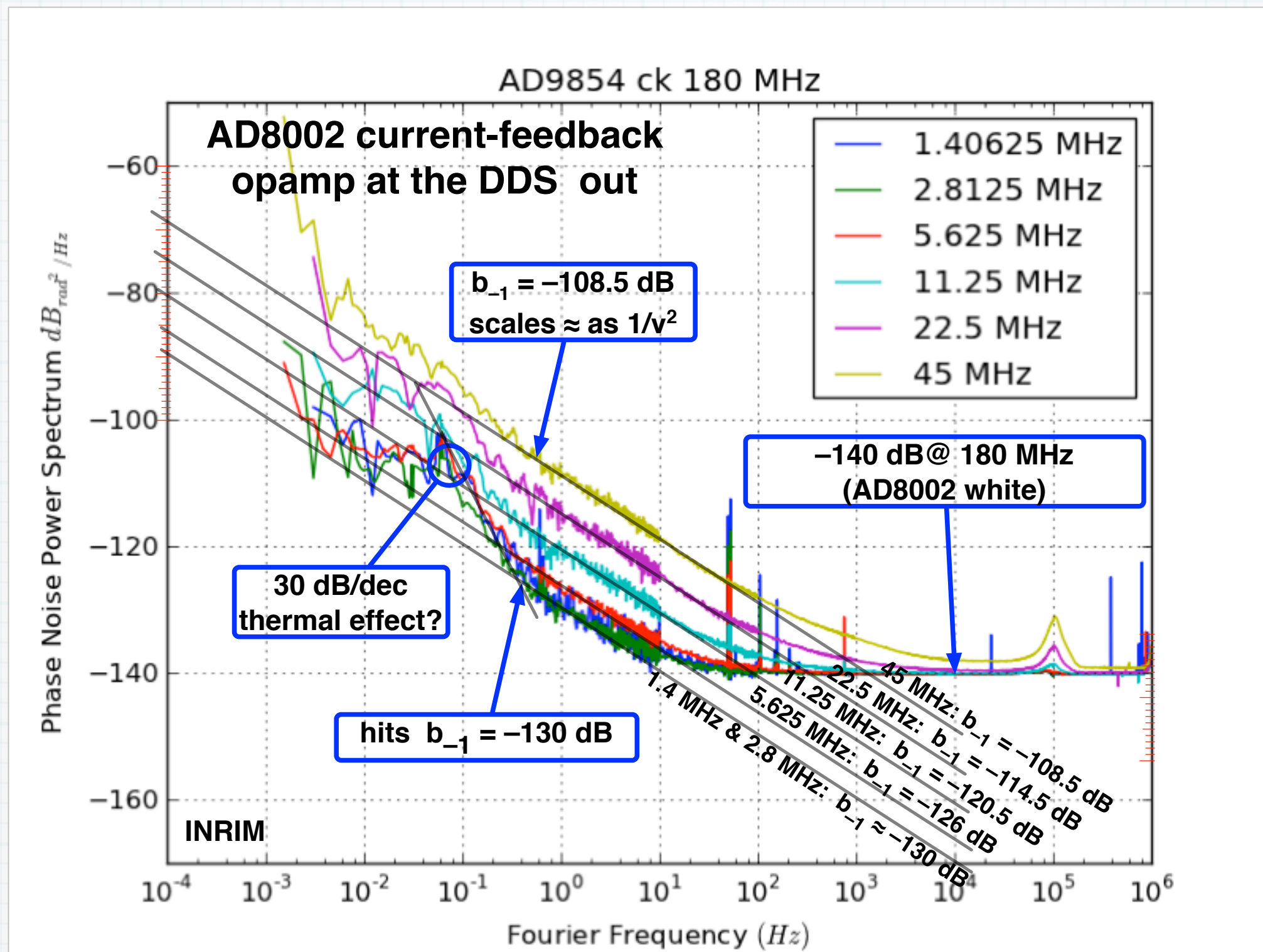


# AD9912 noise vs. out frequency

– low Fourier frequencies –



# PM noise vs. output frequency

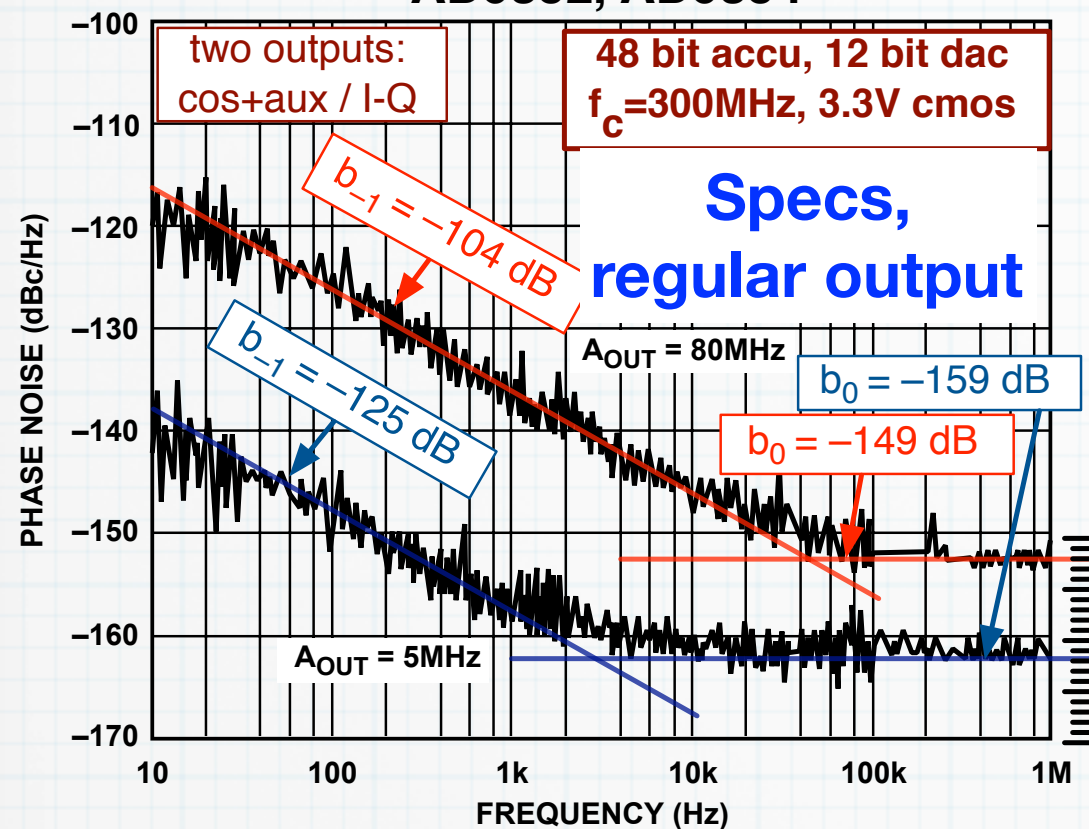


- The -140 dB floor is due to AD8002 at the DDS output
- The flicker is unchanged (comes from the DDS)



# AD9854 noise

AD9852, AD9854



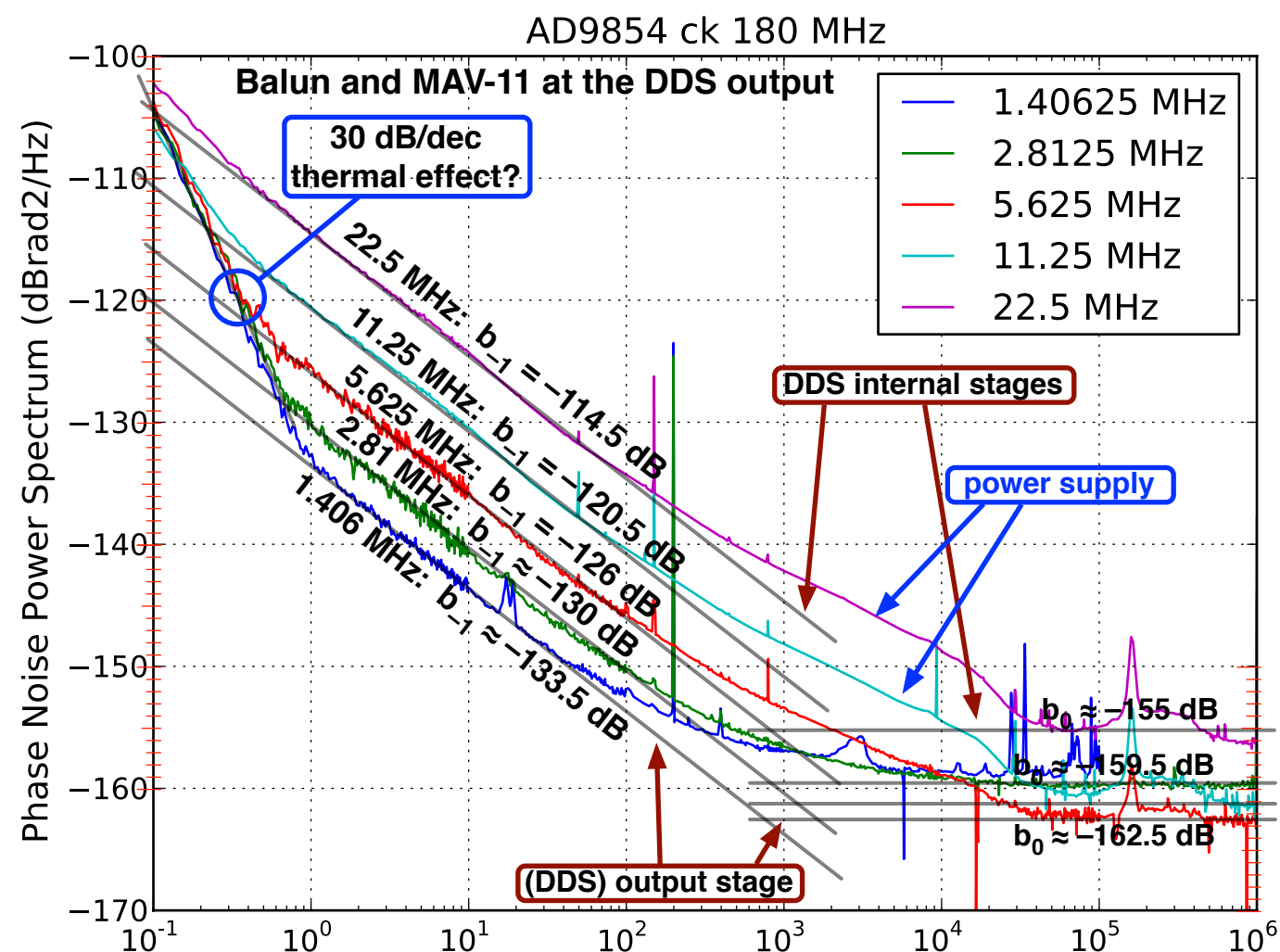
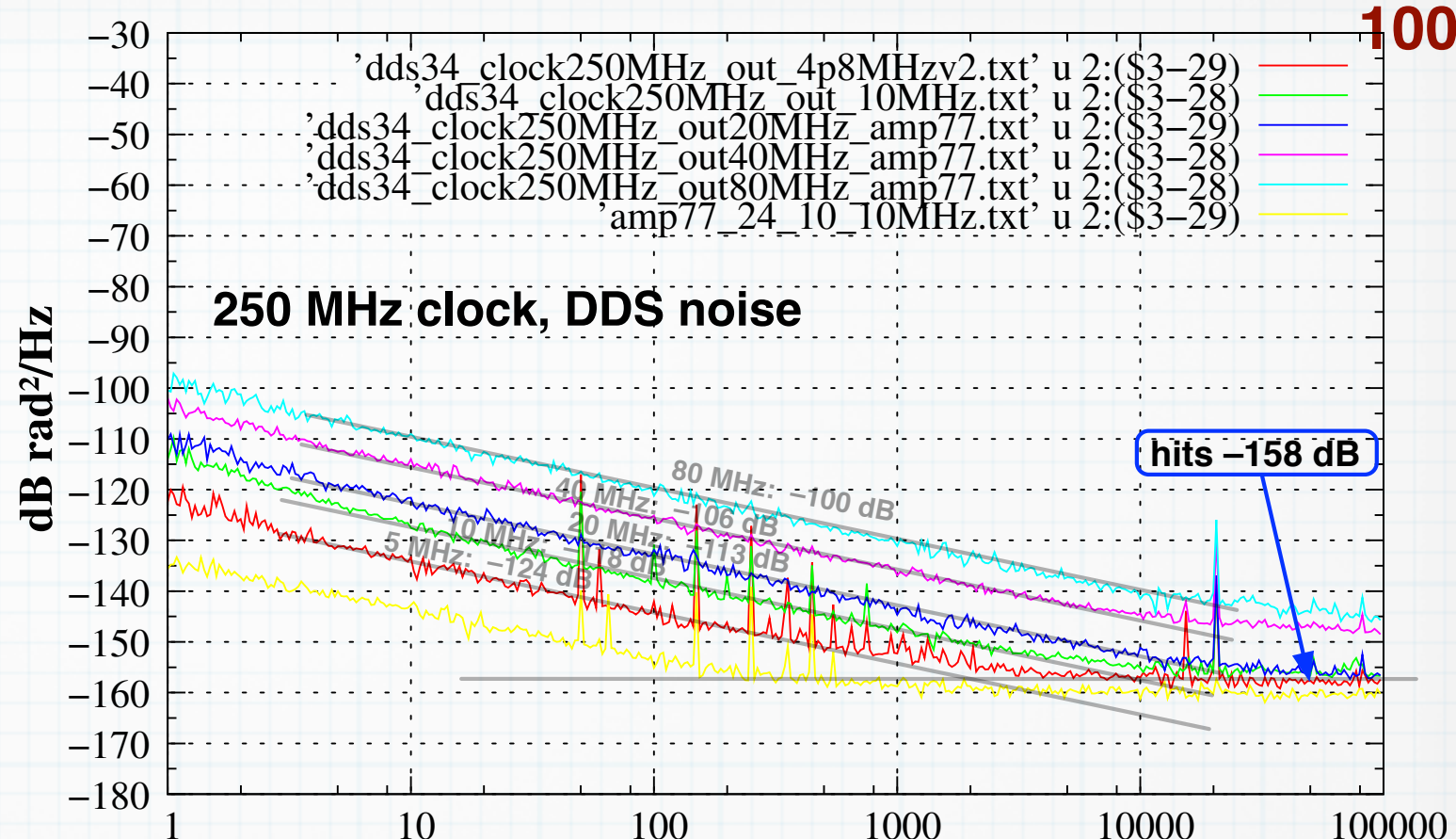
Flicker is in fair agreement

White is made low by spurs

Basic formula for white noise

$$b_0 = \frac{4}{3} \frac{1}{2^{2n} \nu_s} \text{ rad}^2/\text{Hz}$$

who	meas, dB	math, dB	clock, MHz
specs	-159	-155.8	300
YG	-158	-155.0	250
CC	-162.5	-153.6	180

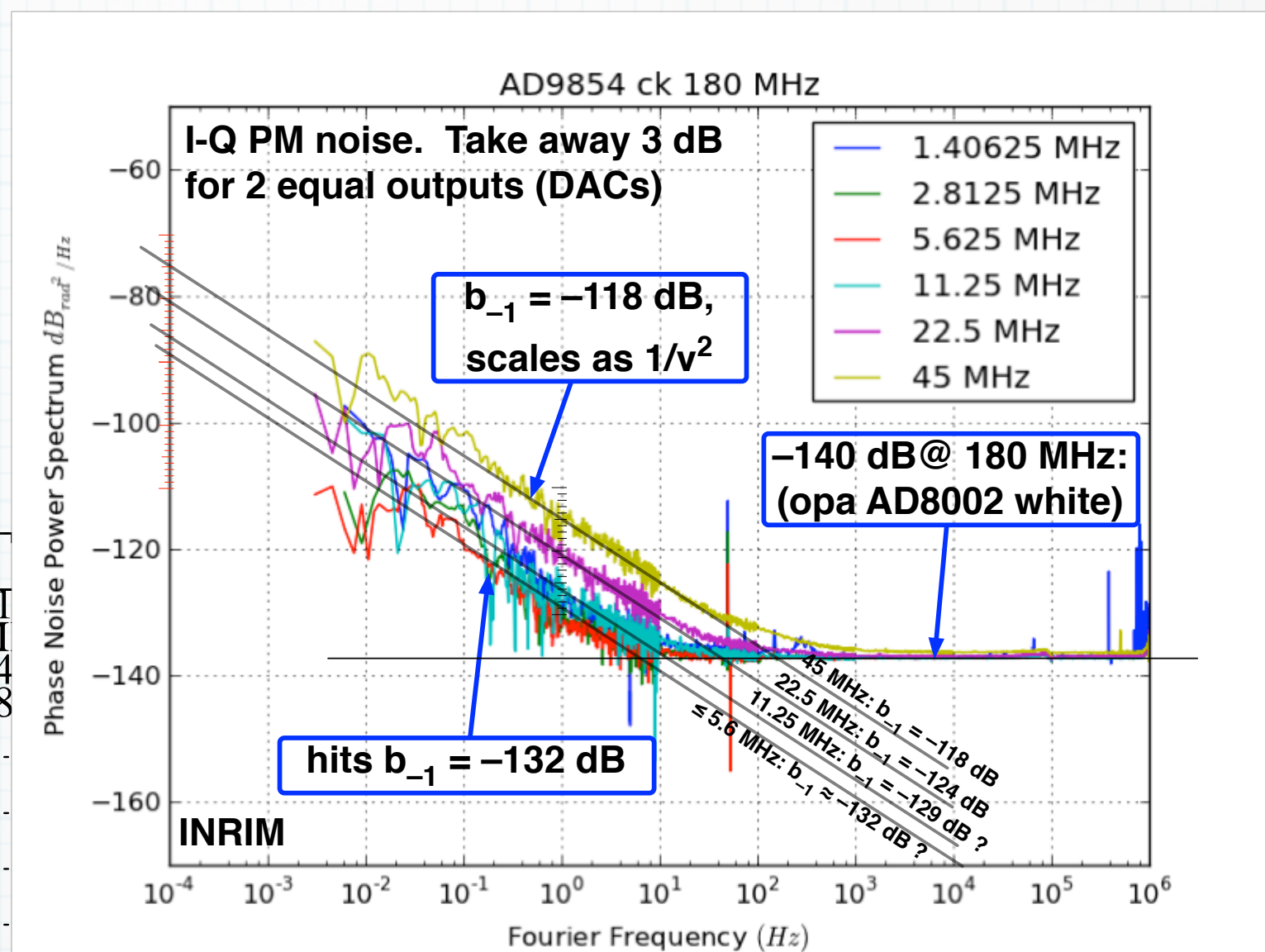
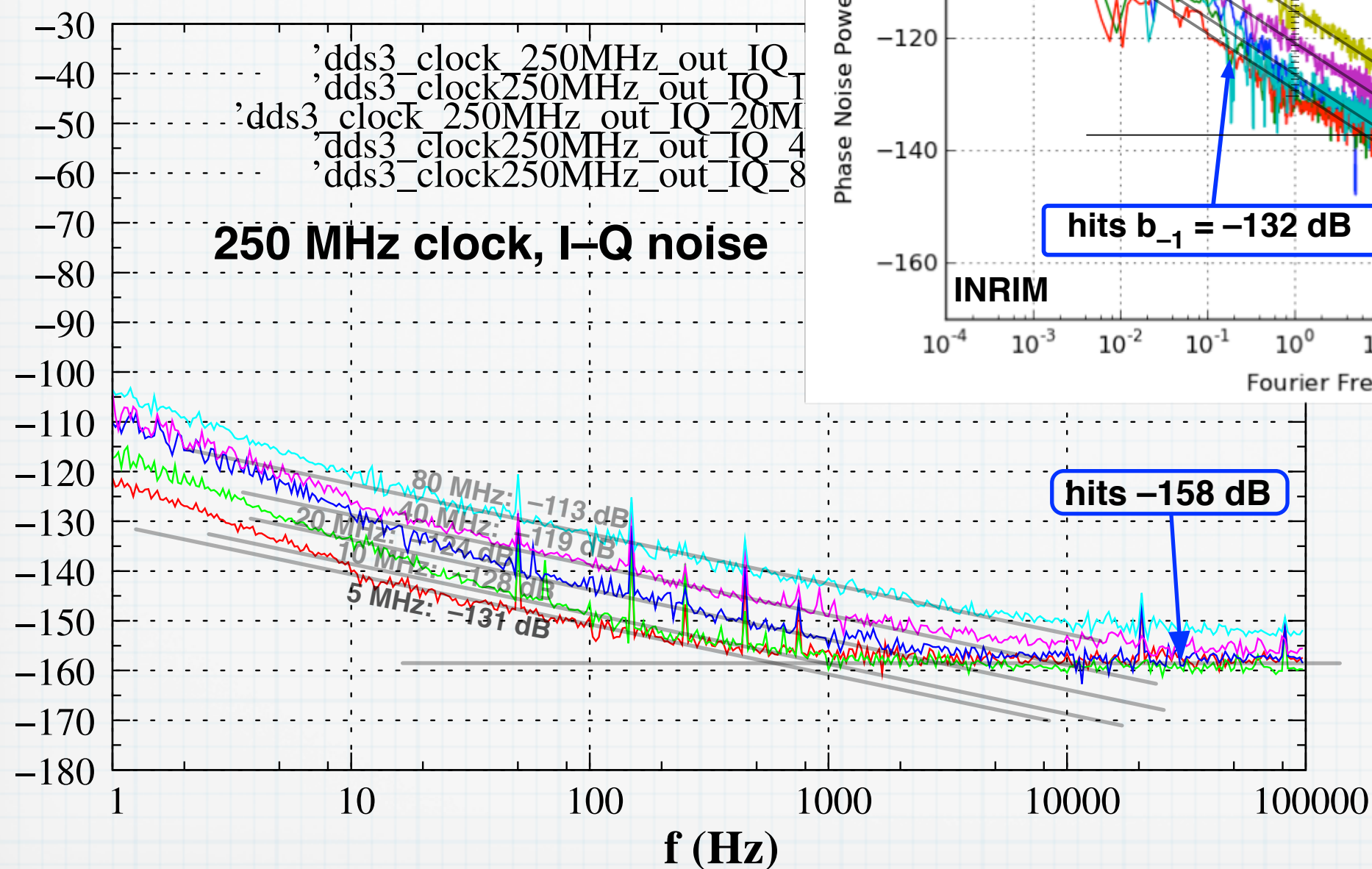




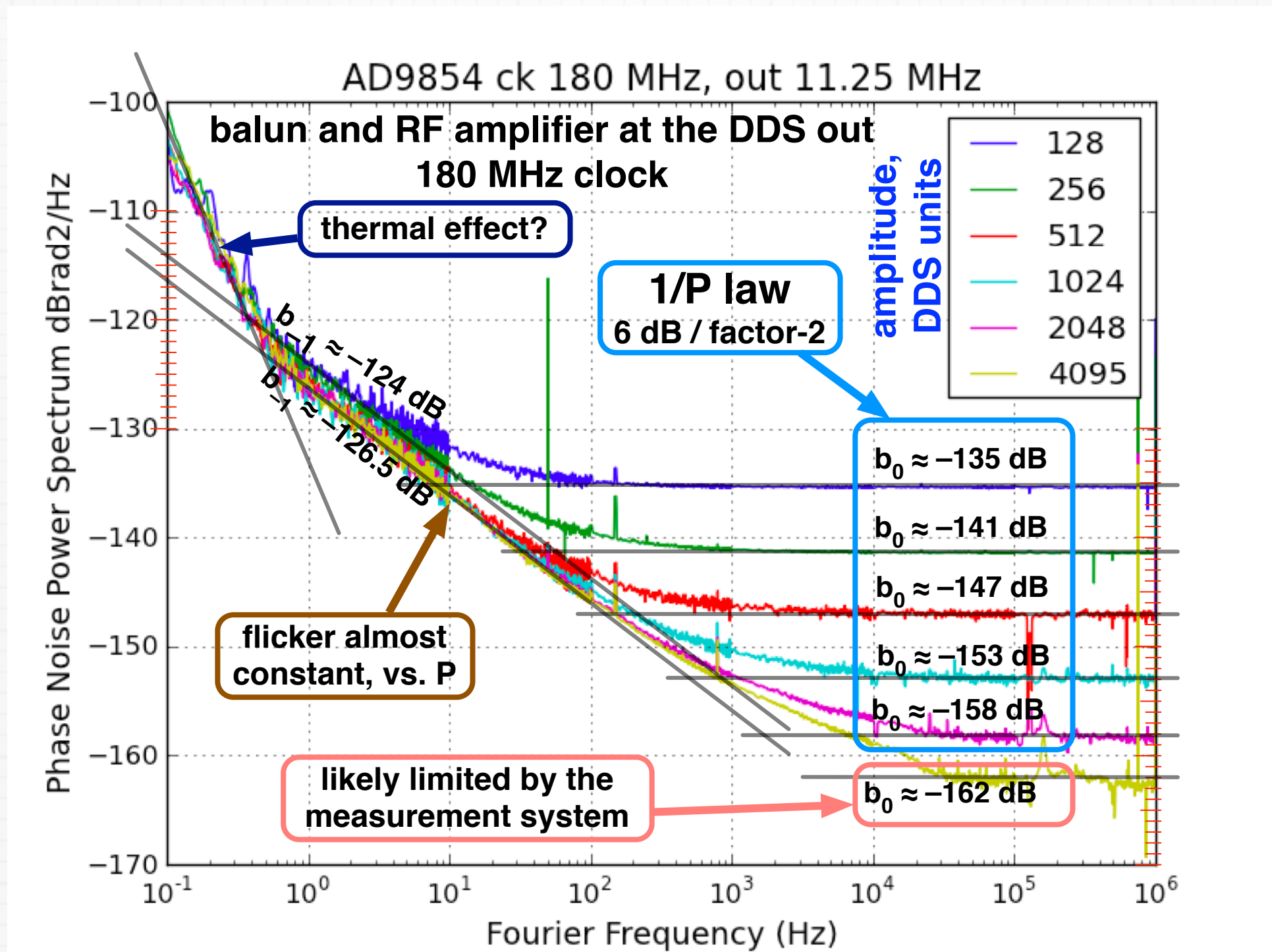
# AD9854 I-Q noise

Flicker is in quite a good agreement between YG and CC

I-Q spectra cannot be compared to specs



# PM noise vs. output amplitude

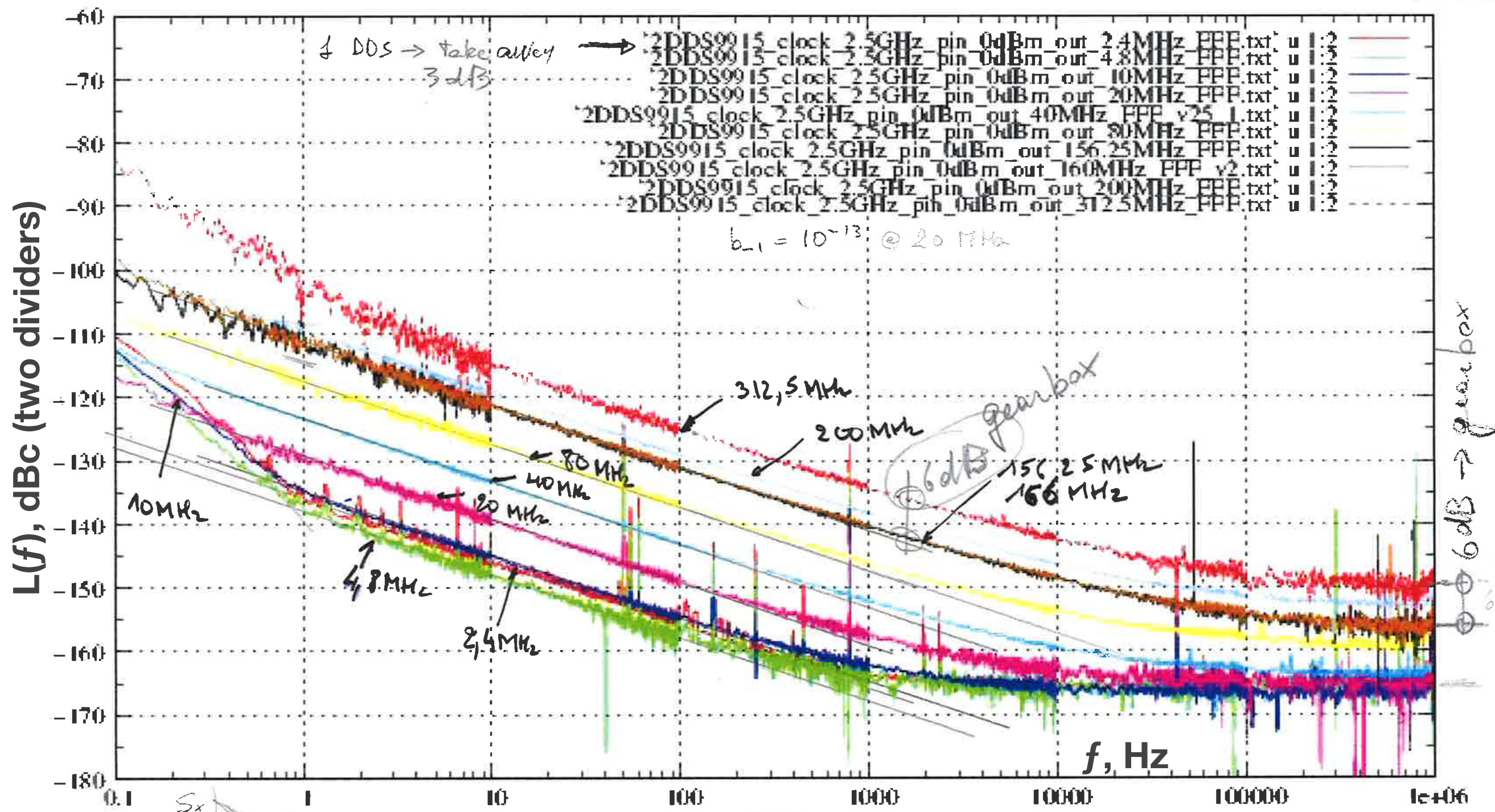


- PM noise scales 6 dB per factor-of-two output amplitude
- Signature of digital multiplication: lower amplitude is obtained by reducing the integer number at the DAC input

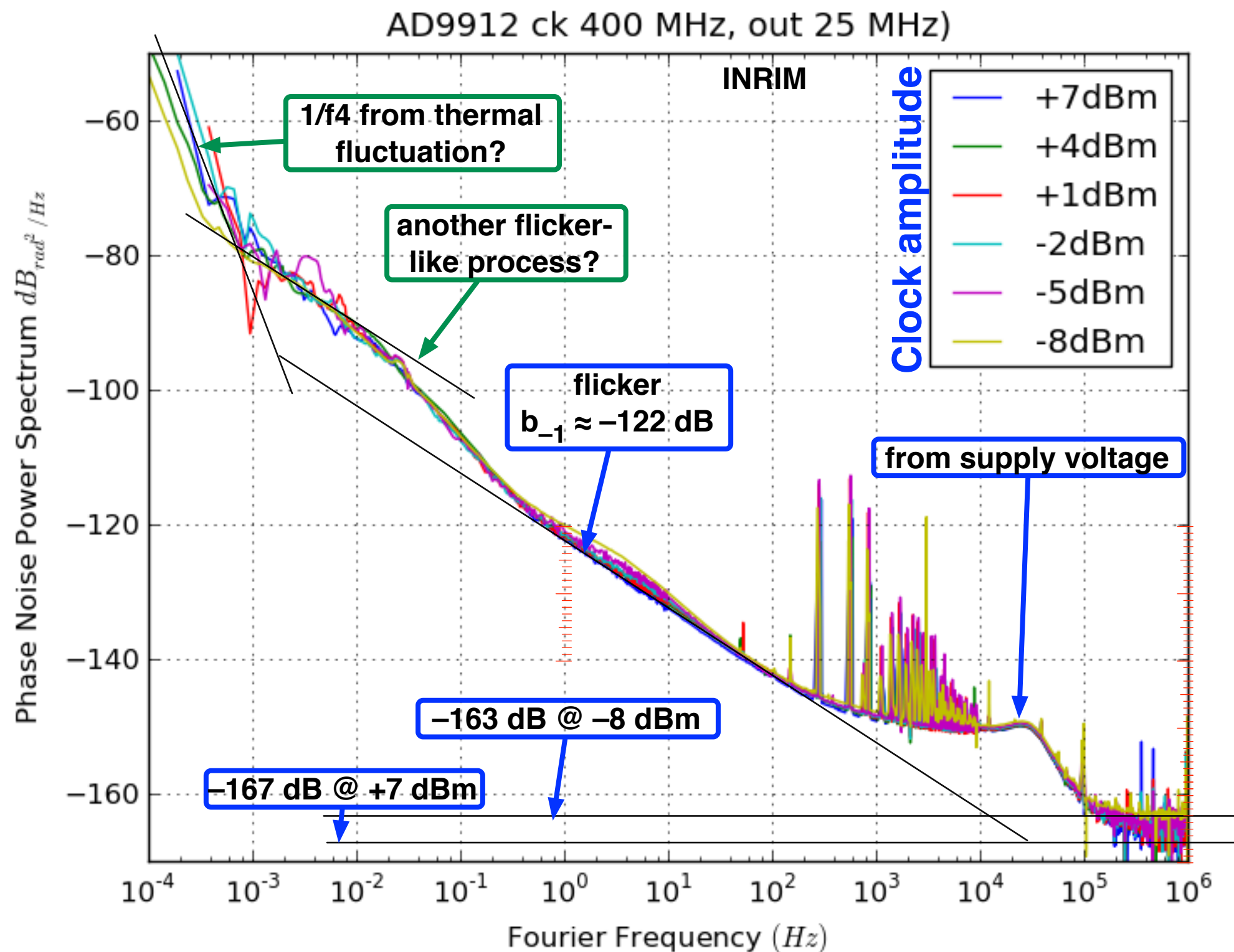


# High-frequency DDSs

AD9915 12 bit, 2.5 GHz  
64 bit accumulator (135 pHz res)

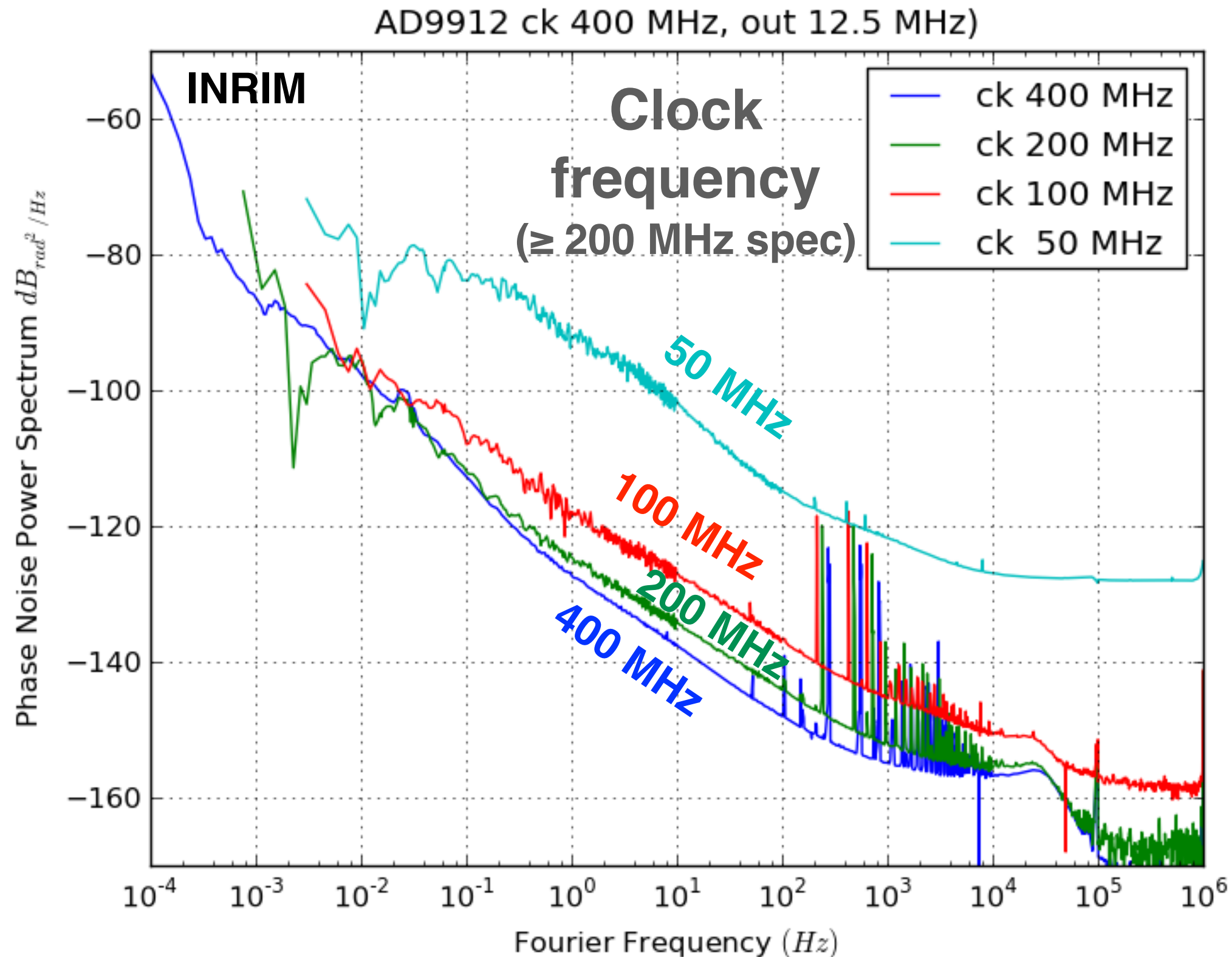


# PM noise vs. clock amplitude



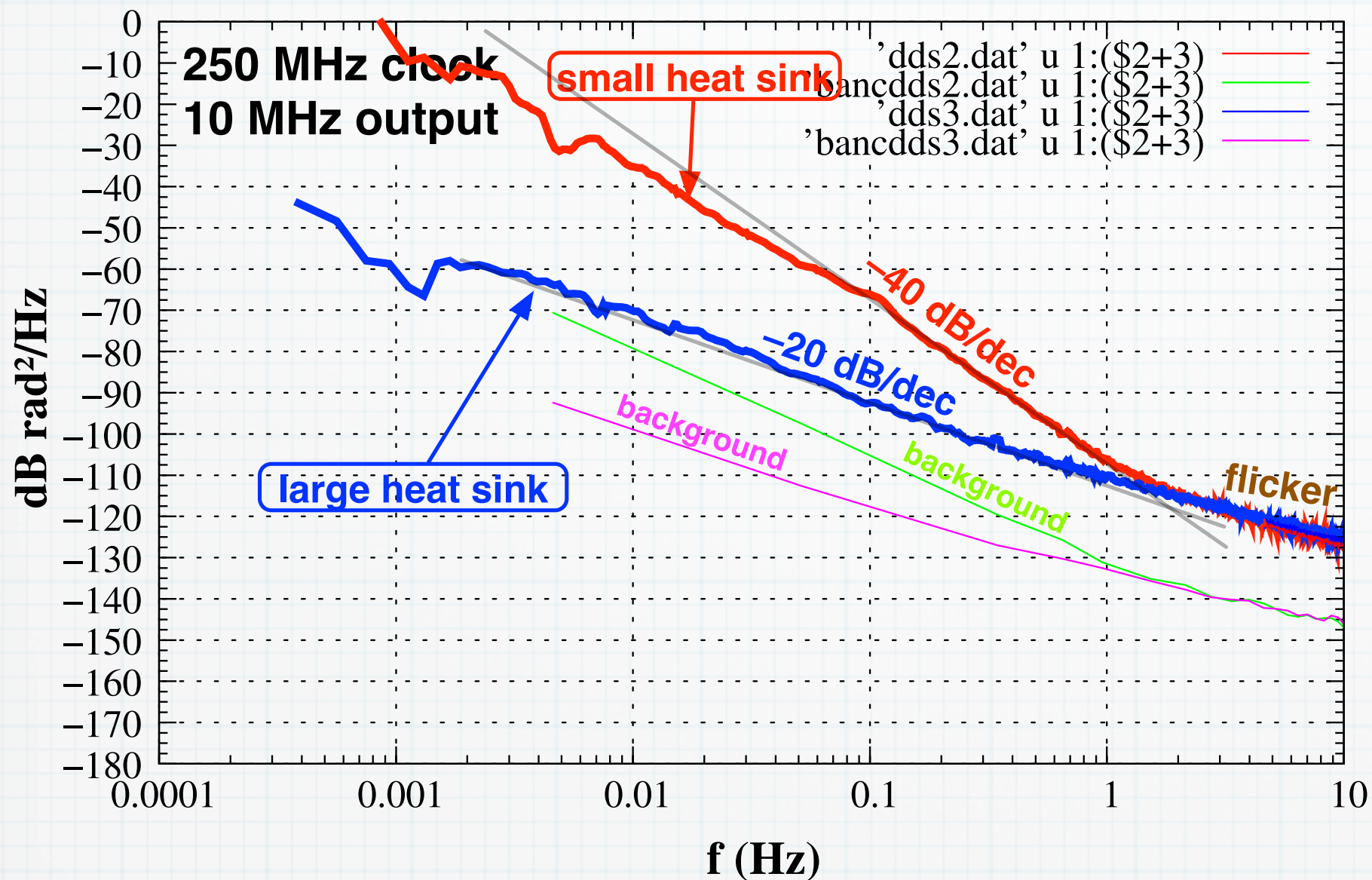


# The effect of the clock frequency

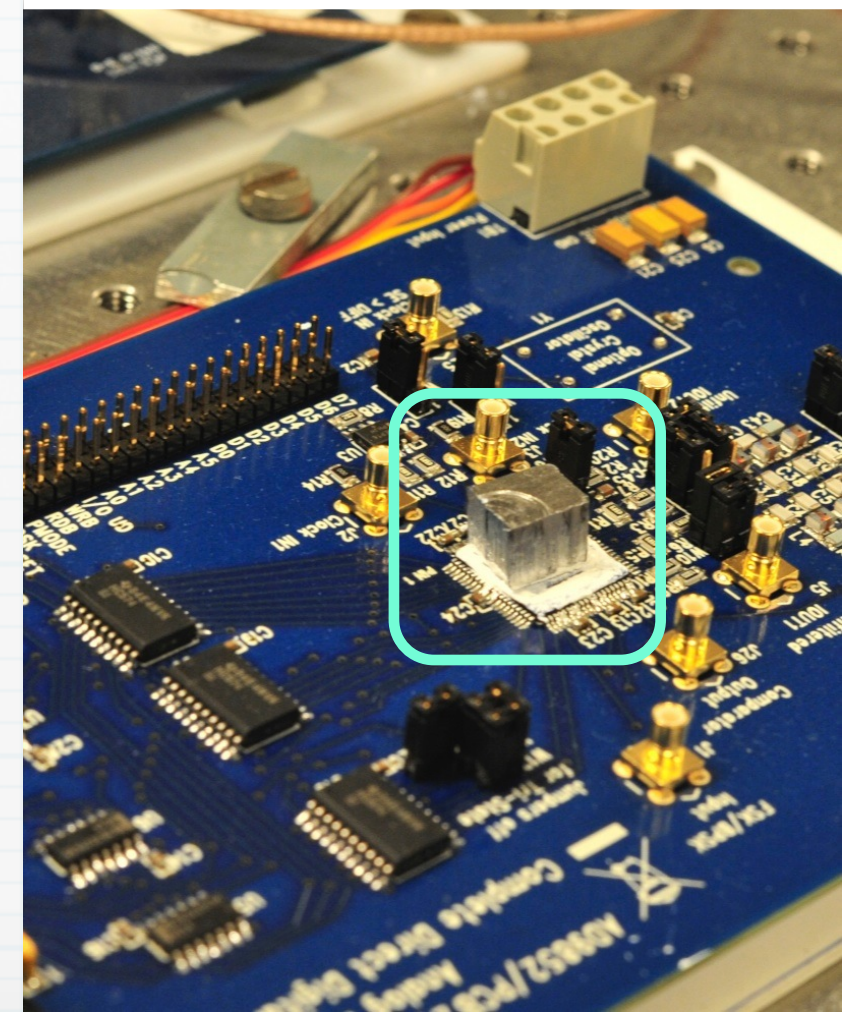
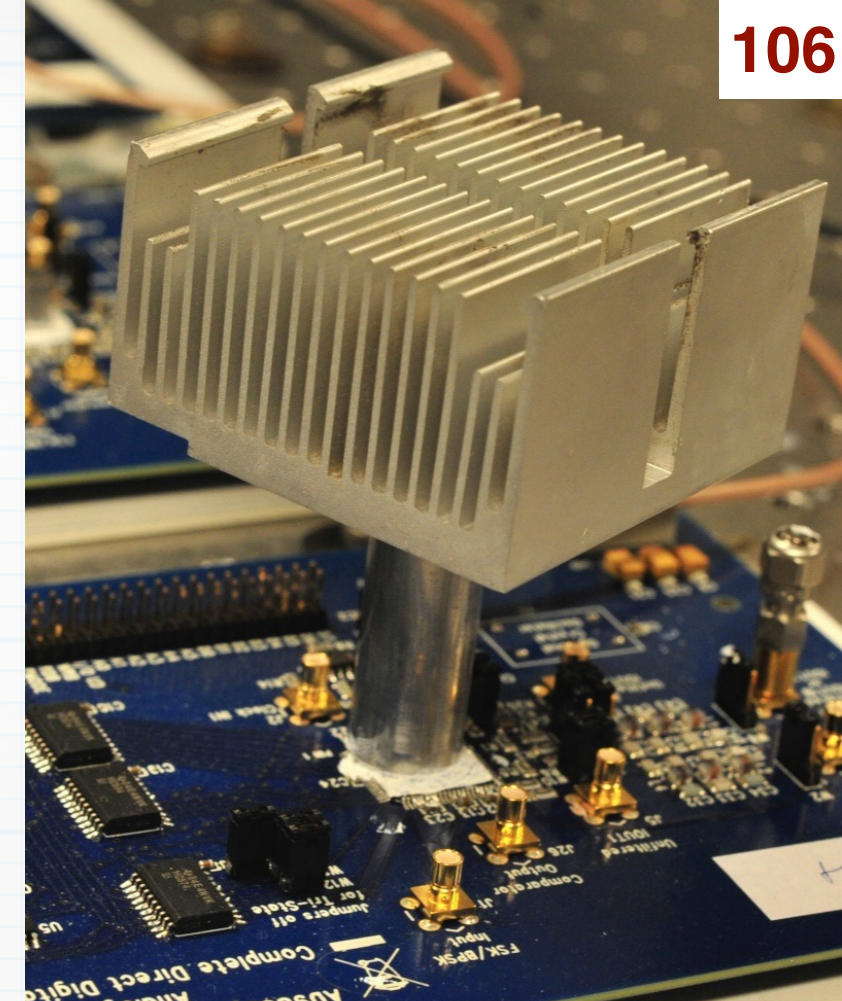




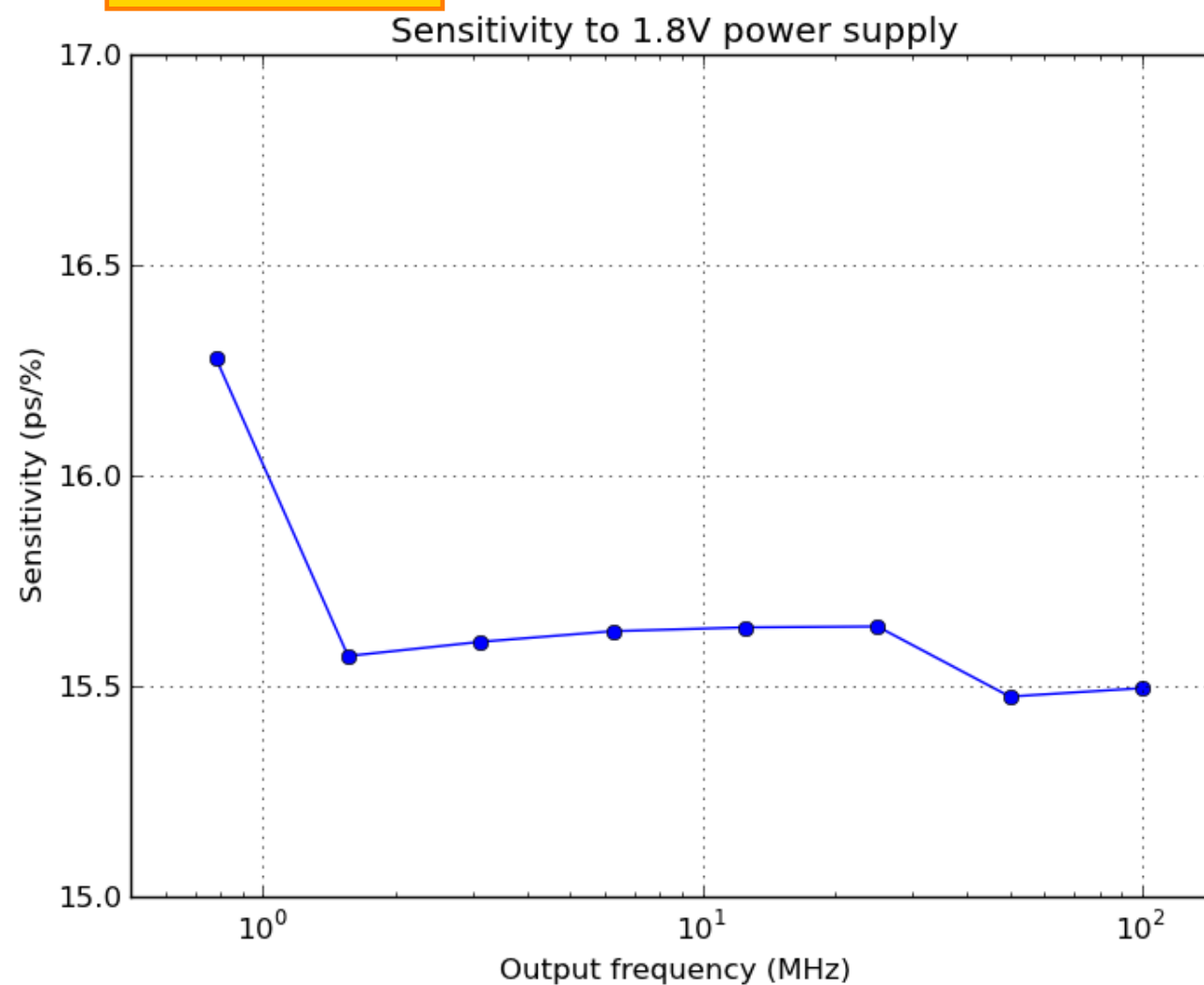
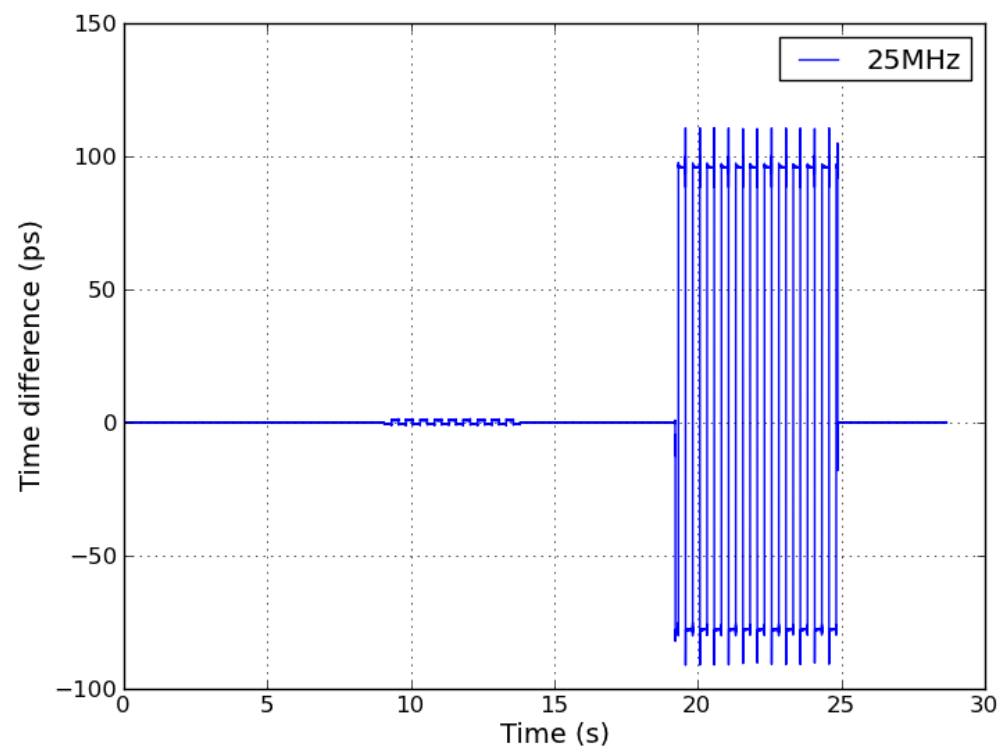
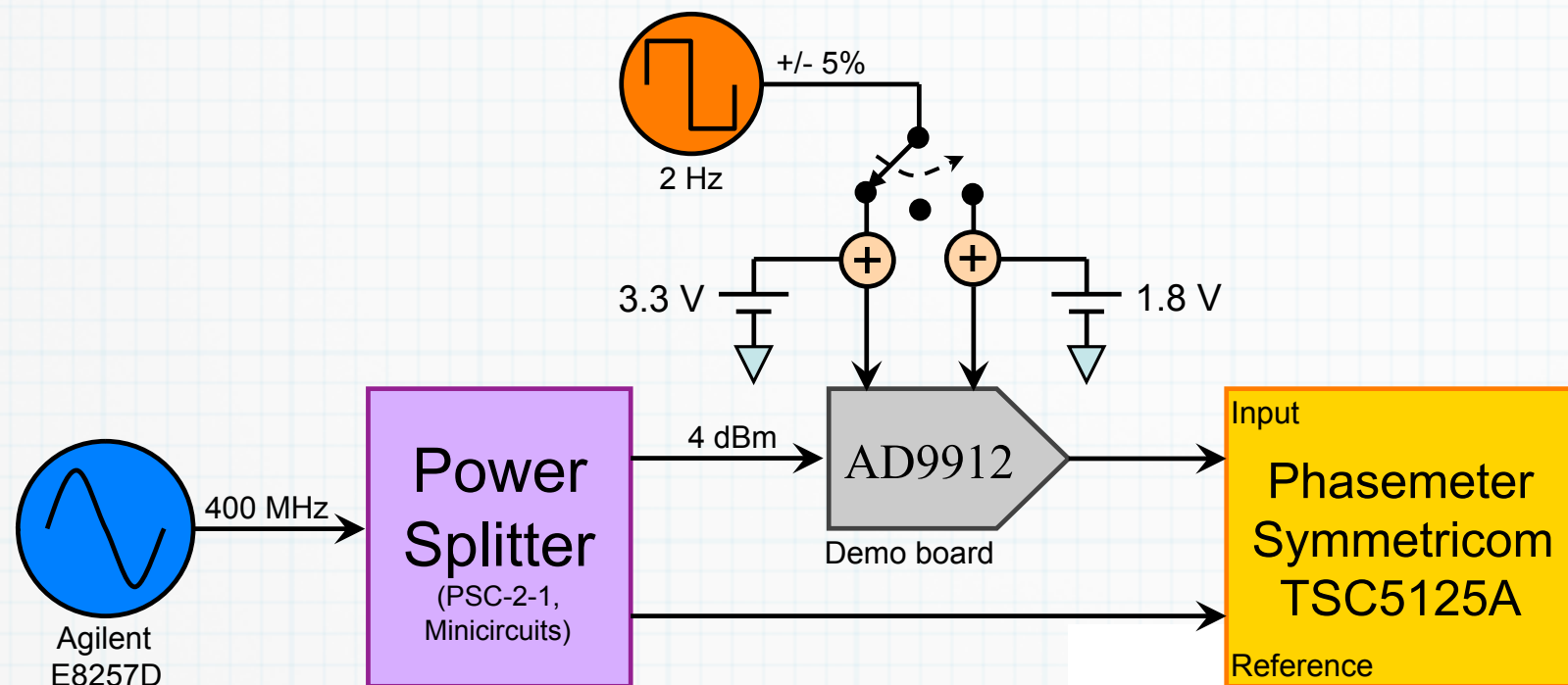
# Thermal effects



- Low-frequency temperature fluctuations induce phase noise
- A large thermal mass helps

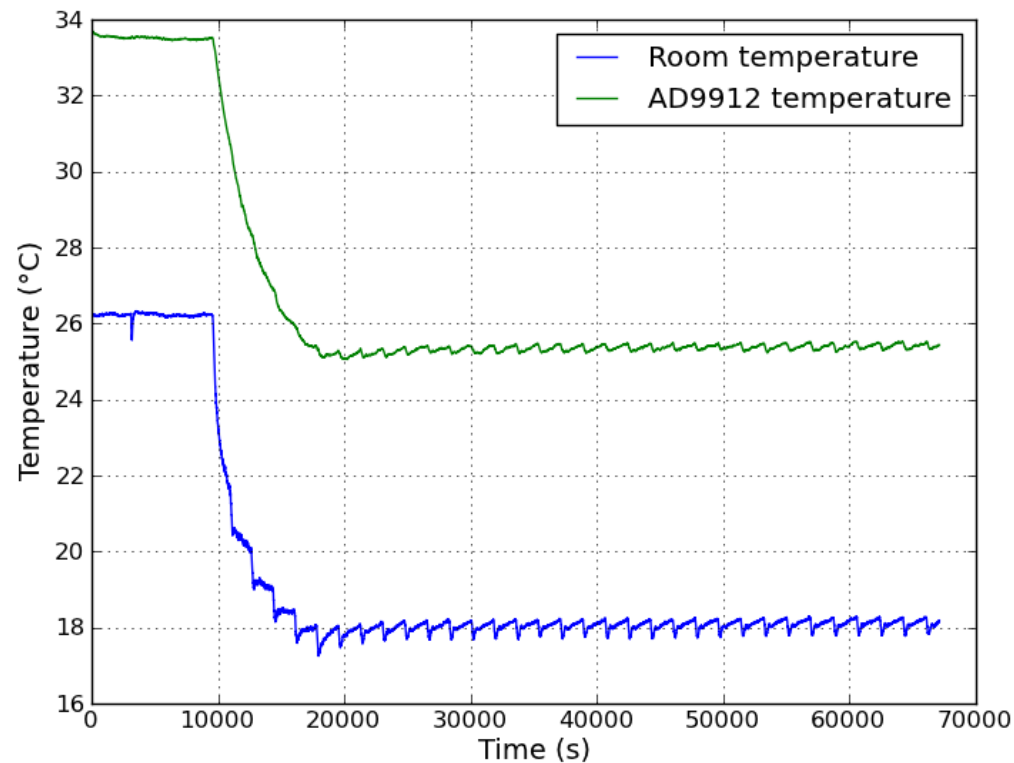


# AD9912 Voltage sensitivity



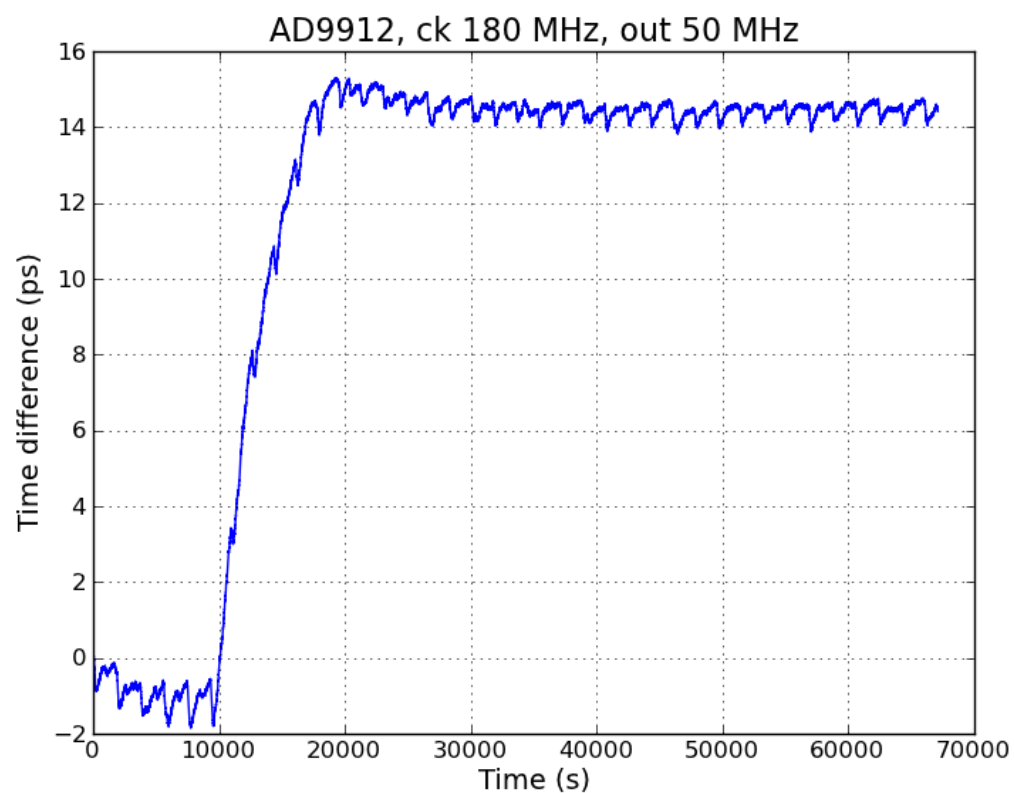


# AD9912 temperature sensitivity

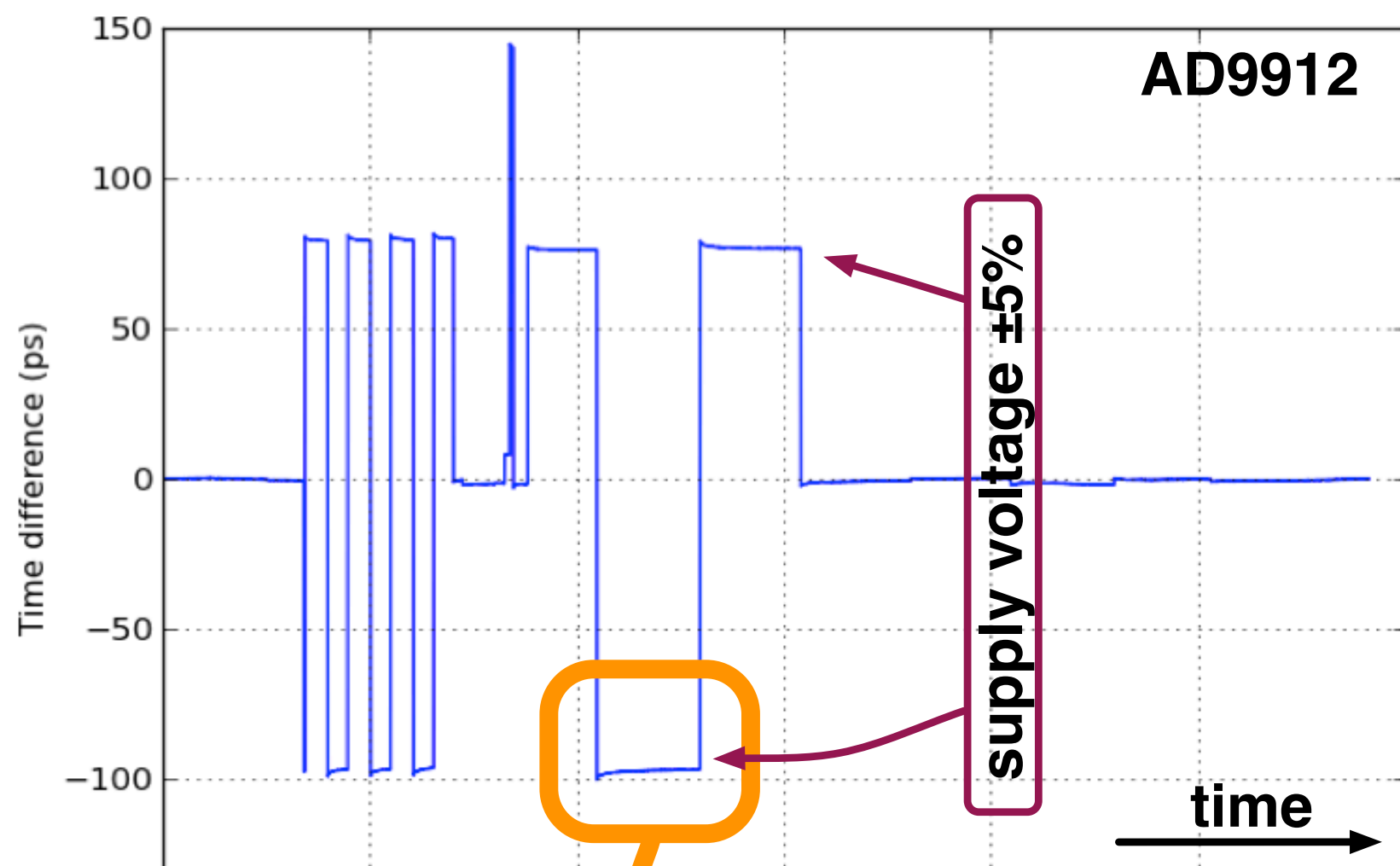


- Temperature control (chamber)

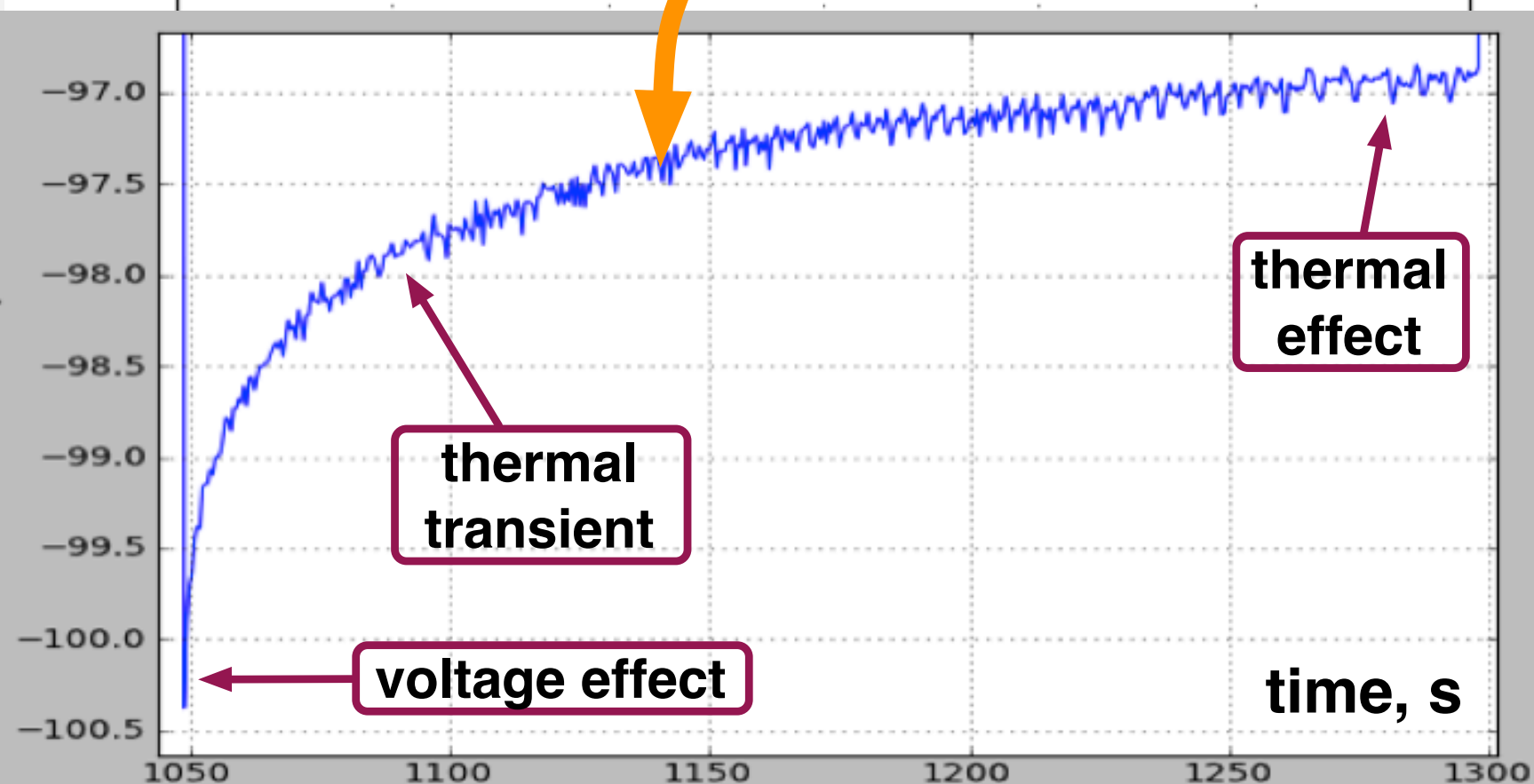
- Measured:  $-2$  ps/K



- Includes cables, baluns etc

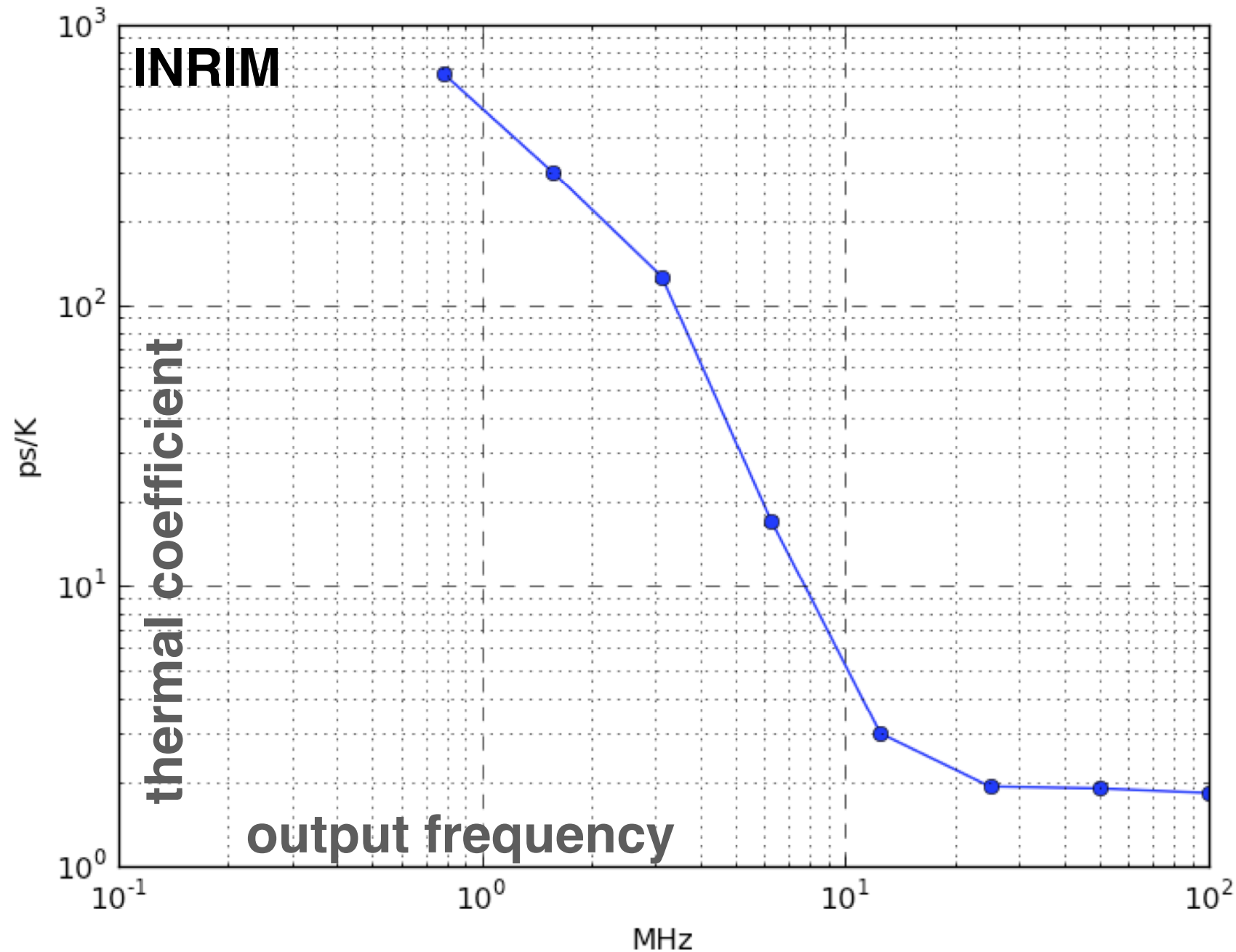


**AD9912**  
**sensitivity to**  
**temperature**  
**(alternate)**



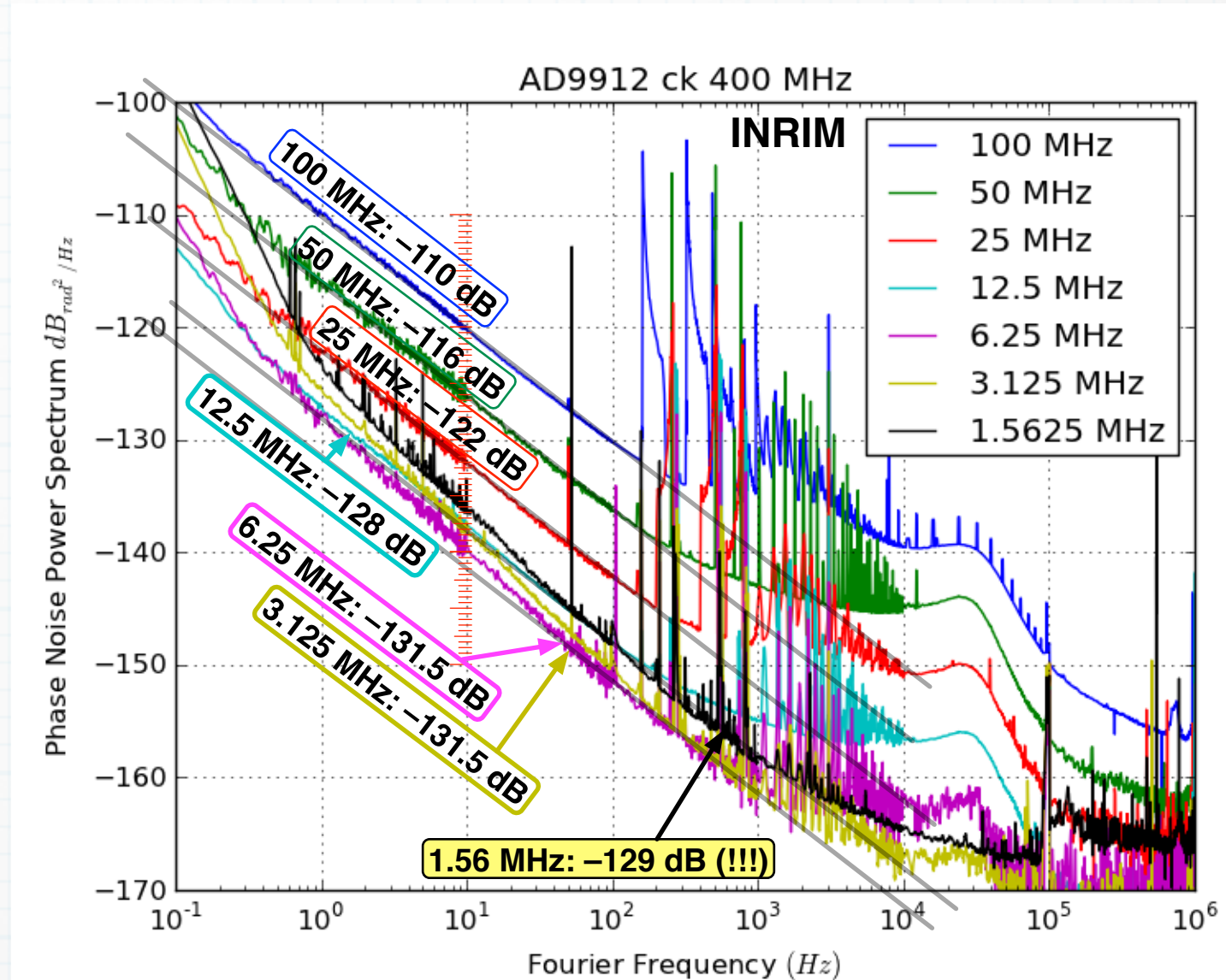
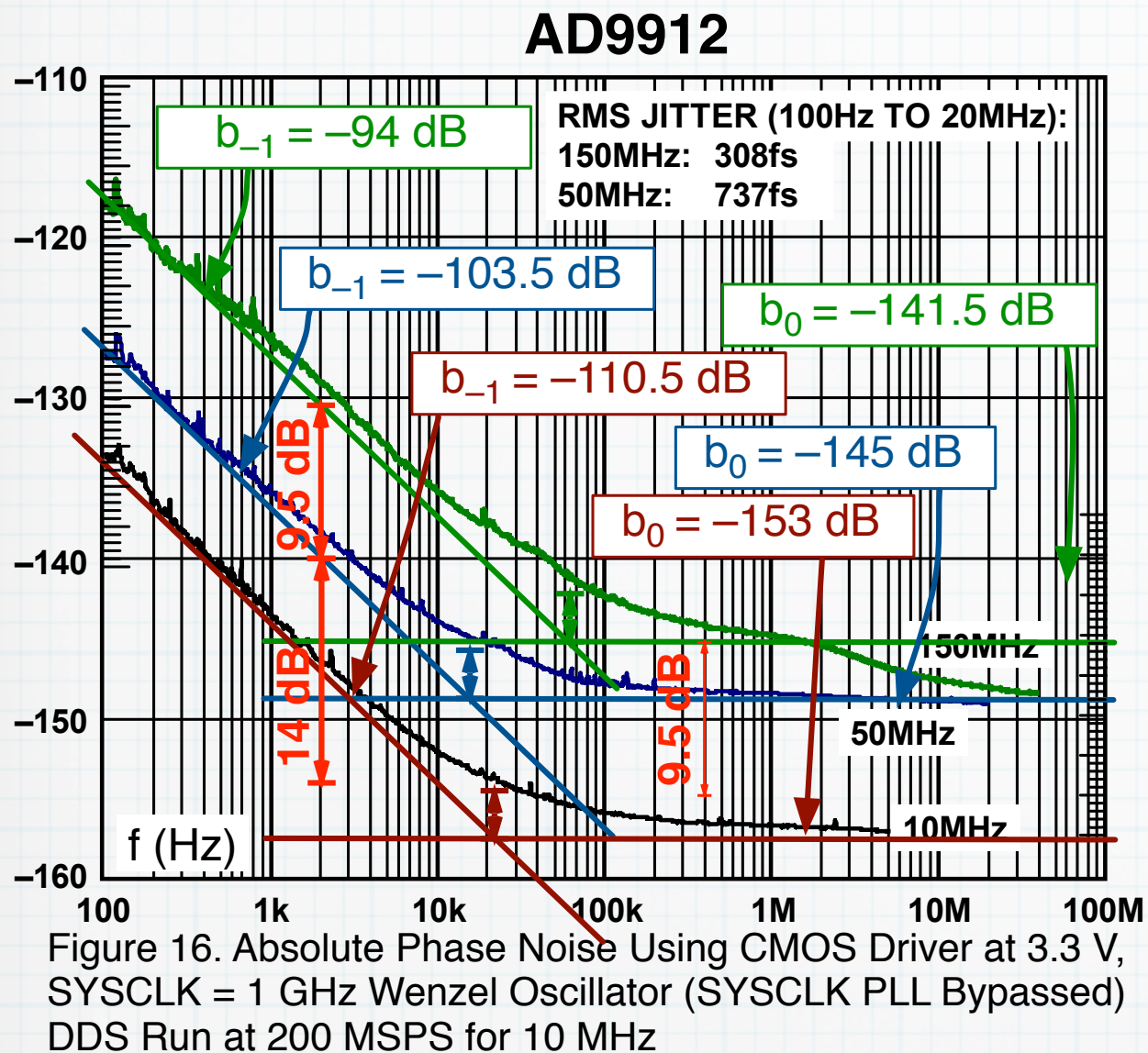


# AD9912 temperature sensitivity



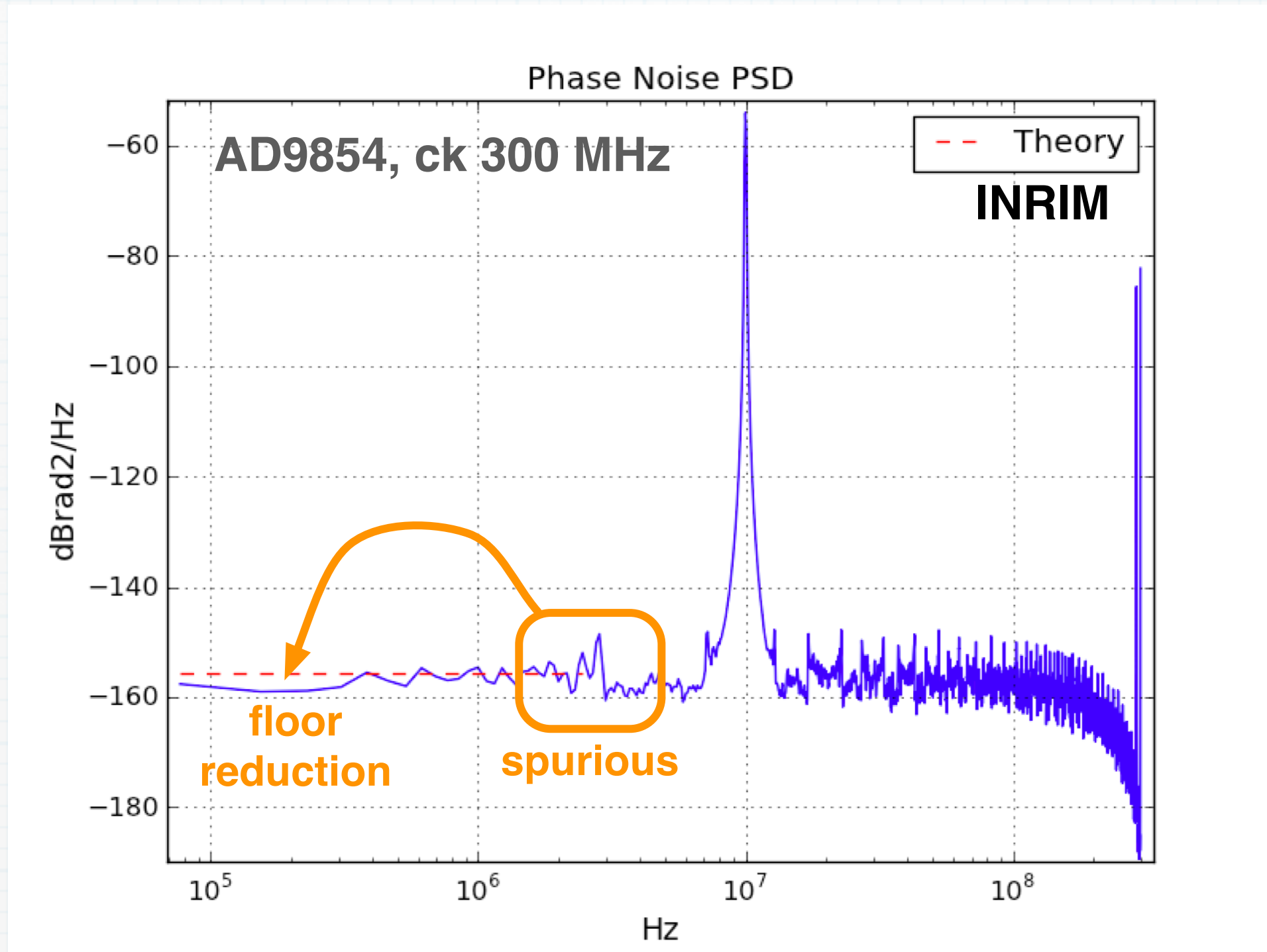
- High frequency:  $-2$  ps/K, constant
- Low frequency:  $1/v^3$  law

# PM noise of the AD 9912

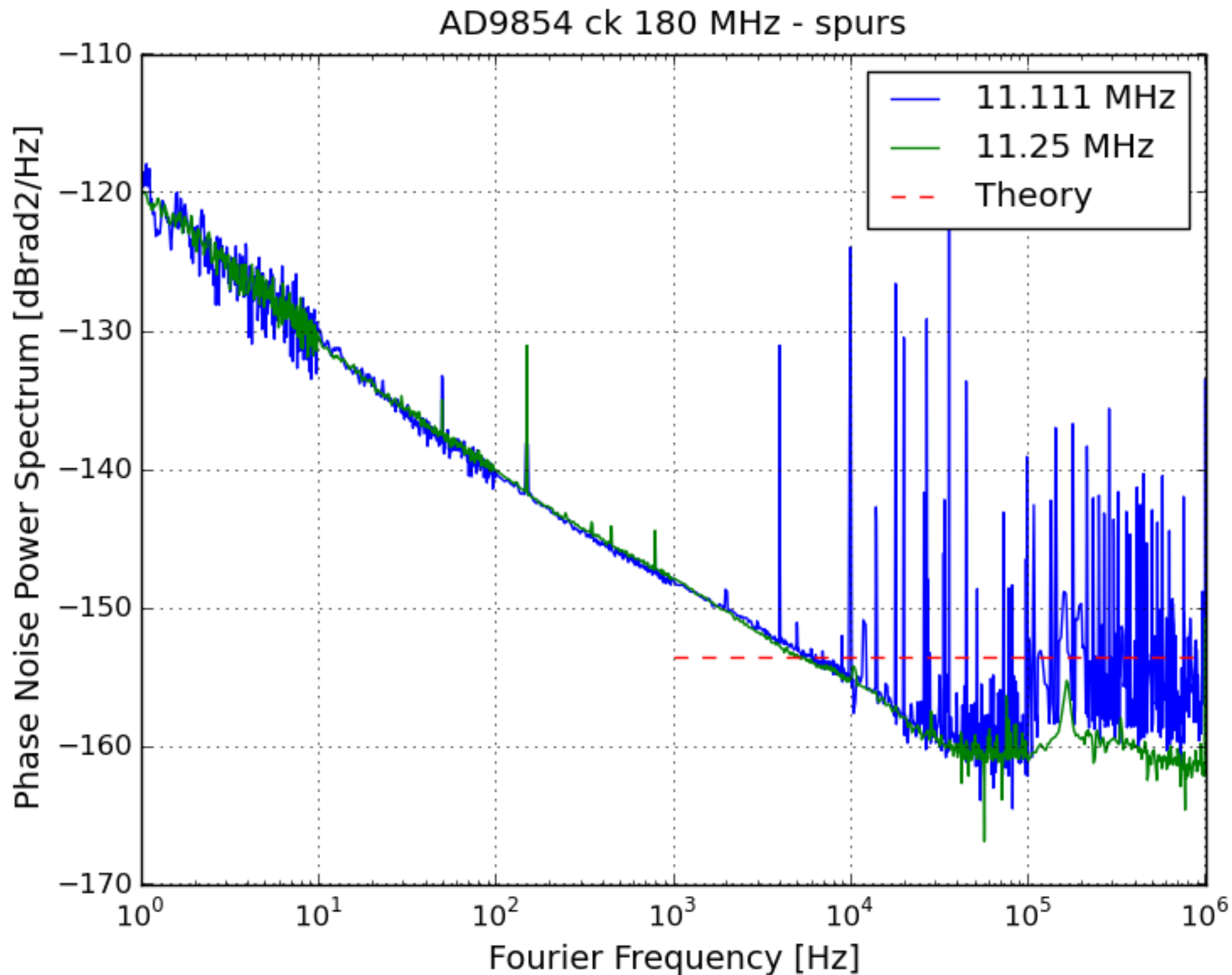


- At 50 MHz and 10/12.5 MHz we get  $\approx 15$  dB lower flicker than the data-sheet spectrum
- Experimental conditions unclear in the data sheets

# Spurs reduce the white noise



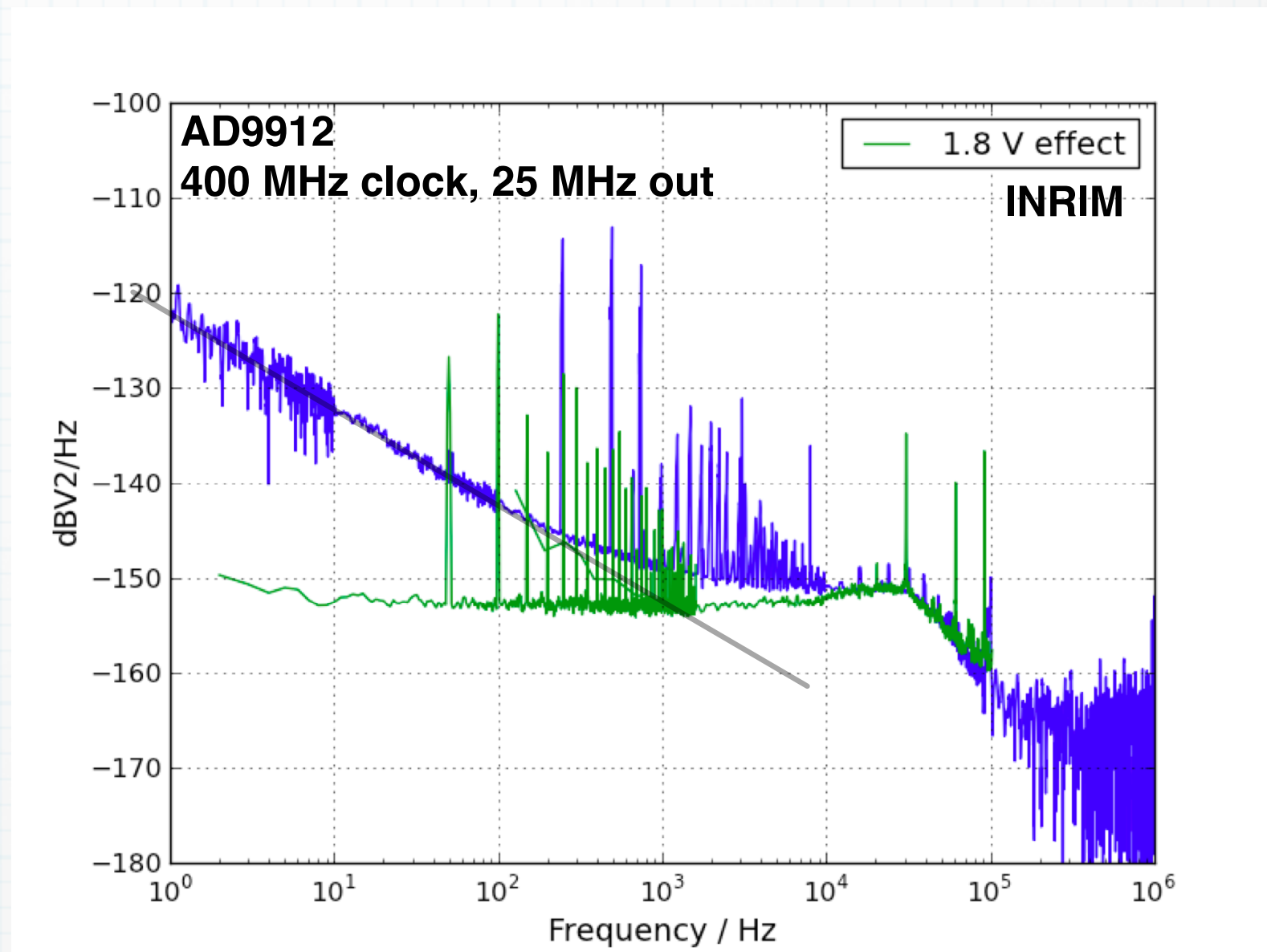
# Spurs can be amazing





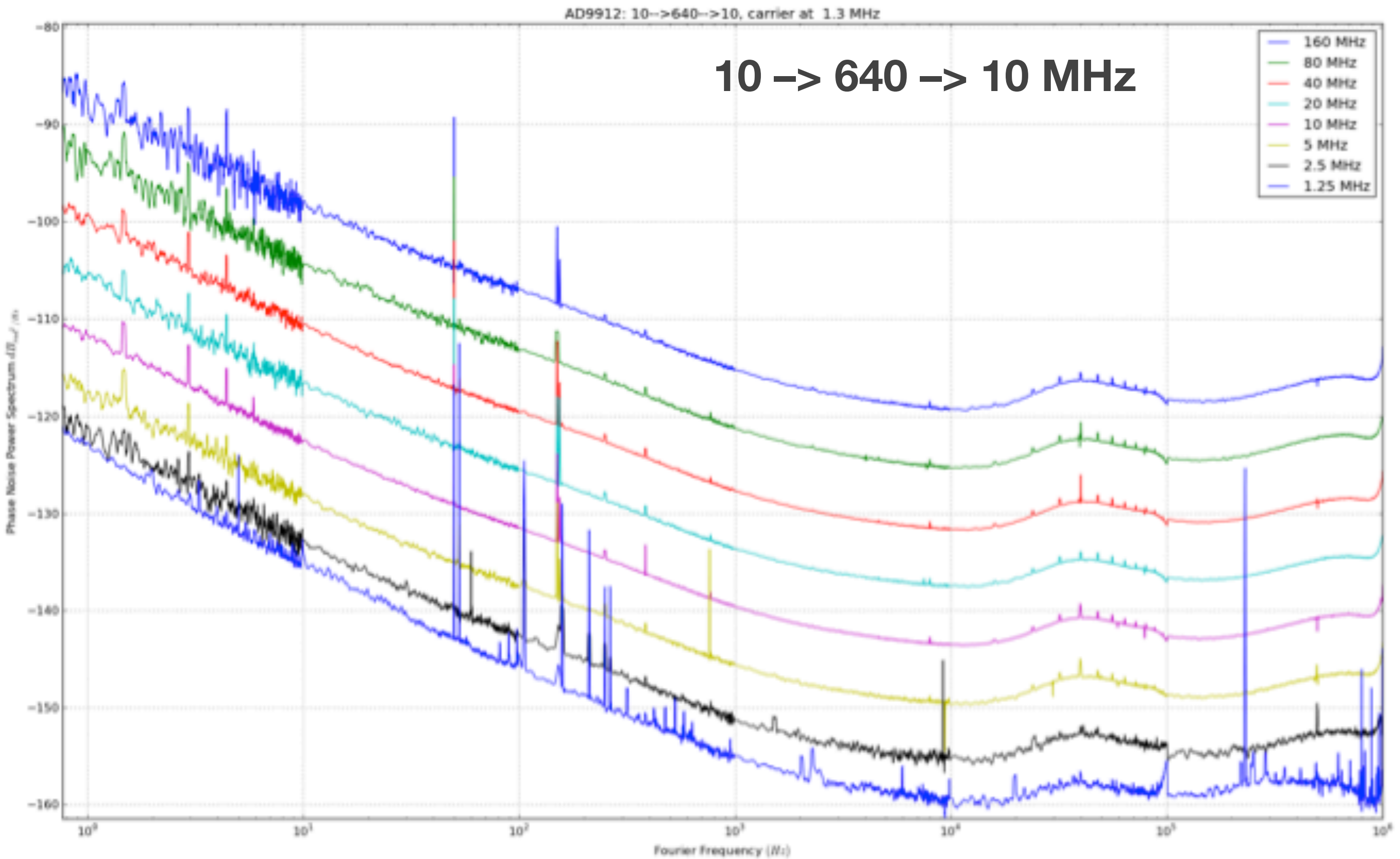
# More about a PM-noise bump

- Low PSRR (power-supply rejection ratio) of PM noise
- For instance The AD9912 at 25 MHz out has 15 ps/% supply-voltage sensitivity
- No bump at  $10^3$ – $10^5$  Hz is seen in the data-sheet spectra
- DC regulator may show a similar bump, alone or or with the output capacitor

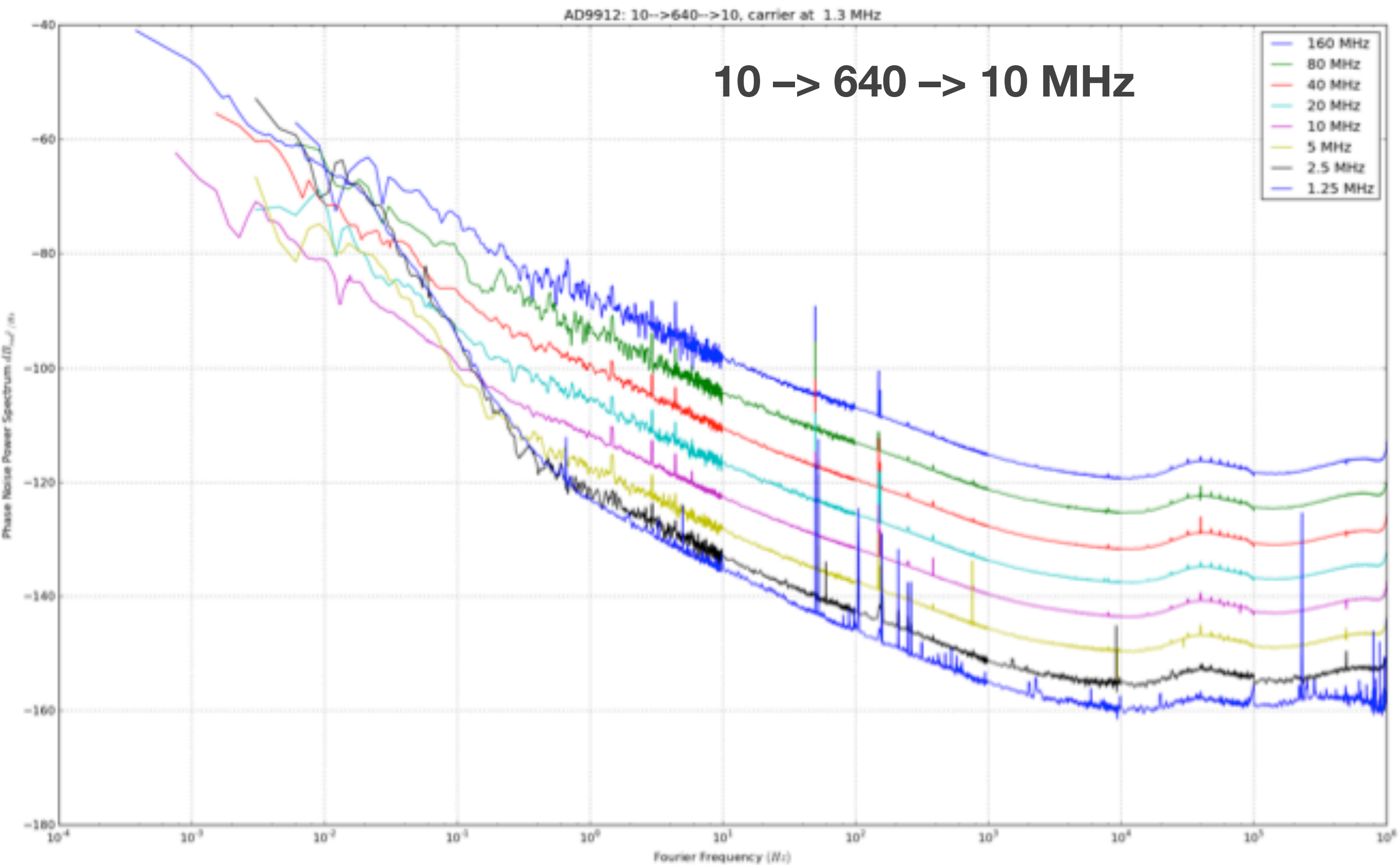


X7R SMD capacitor shows low ESR ( $\leq 5$  m $\Omega$ )

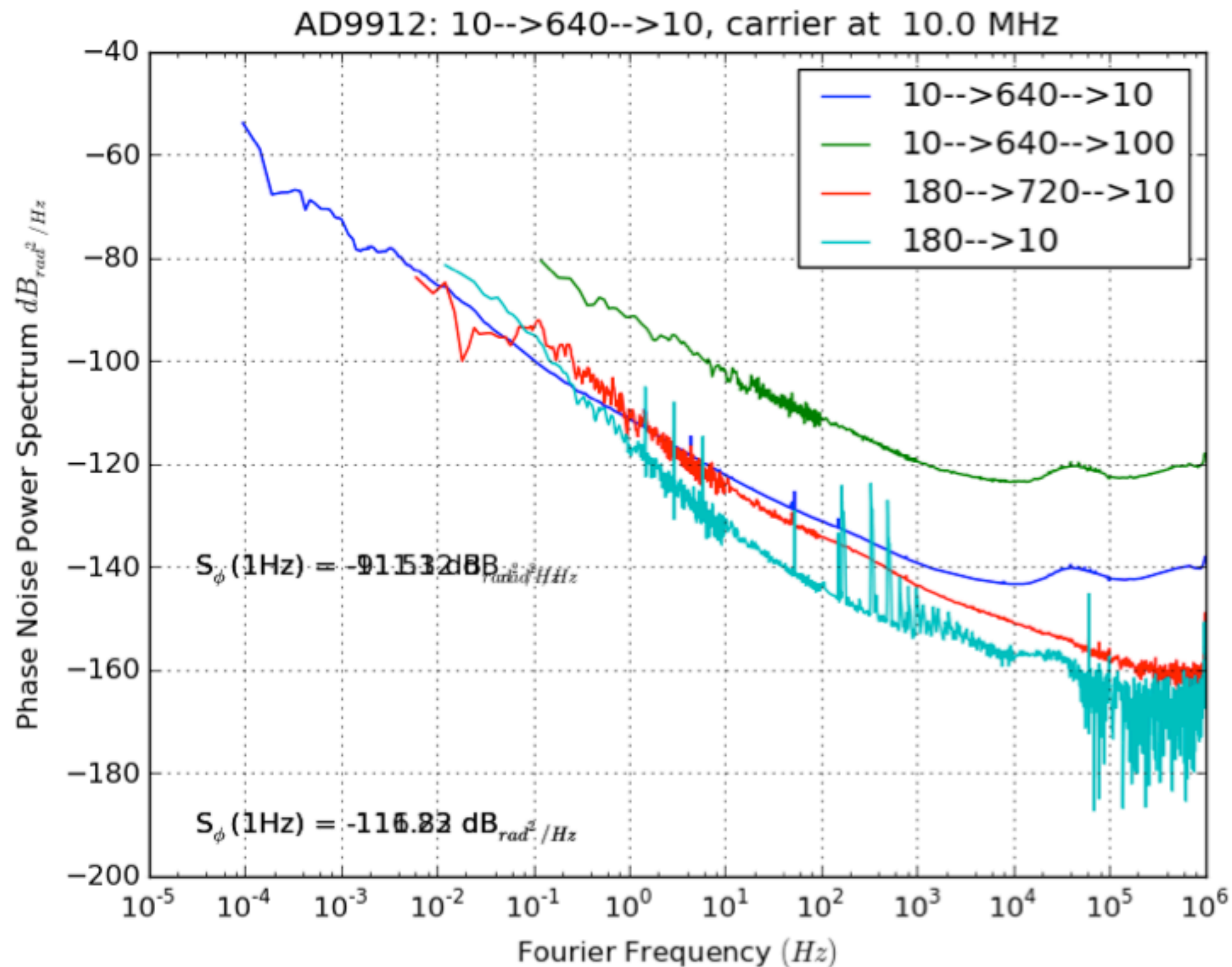
# PLL clock multiplier



# PLL clock multiplier

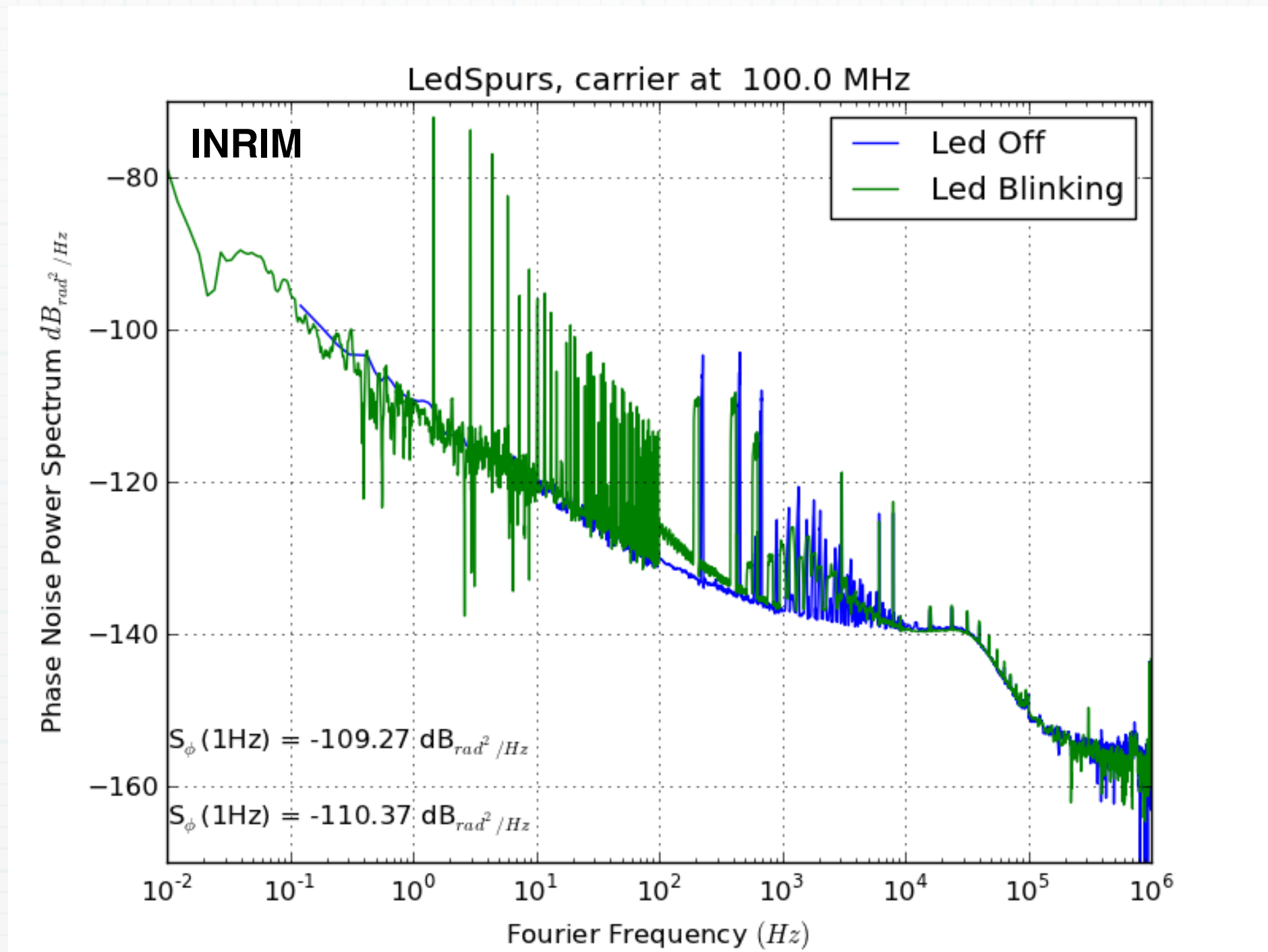


# PLL clock multiplier



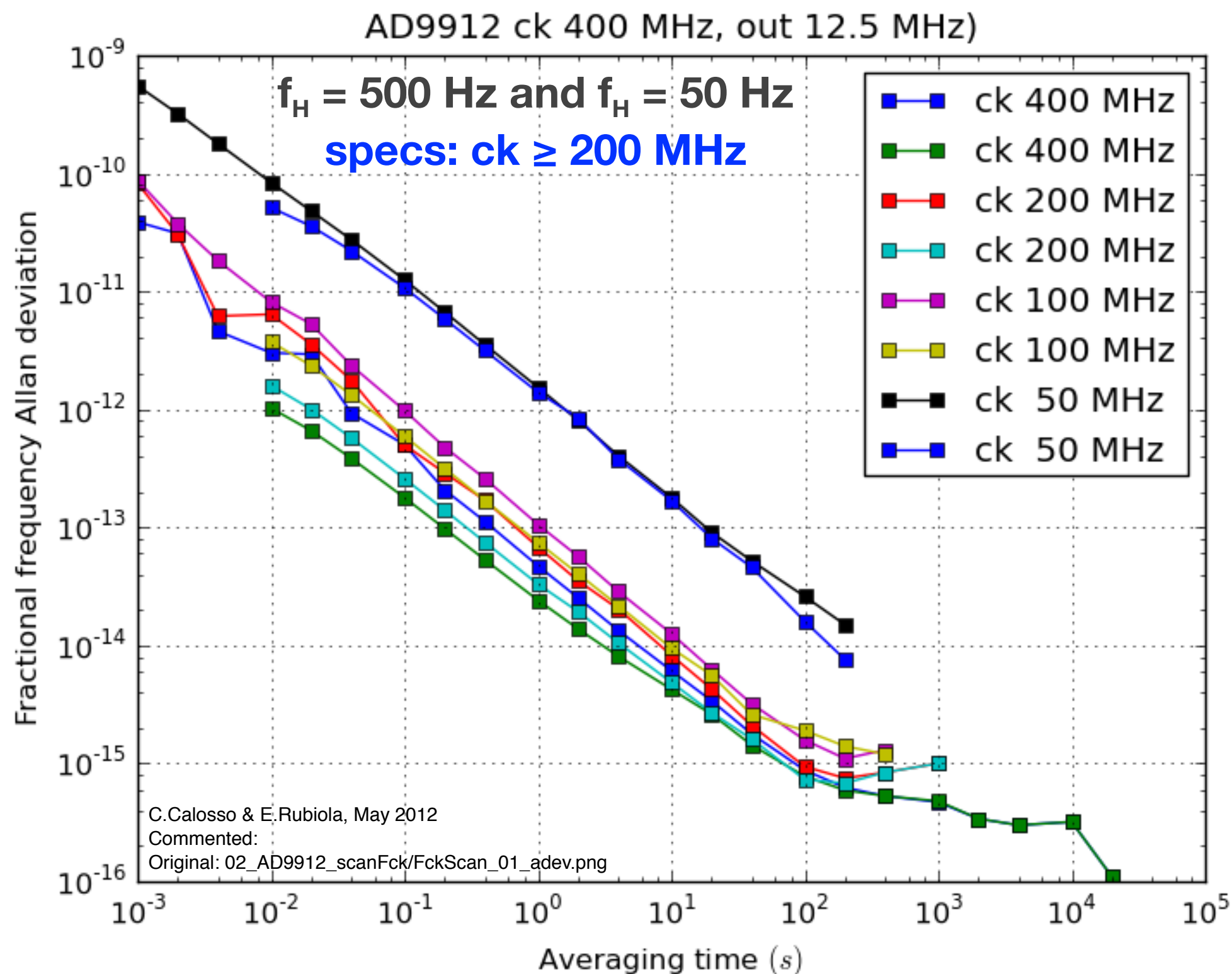


# Effect of other parts on the PCB

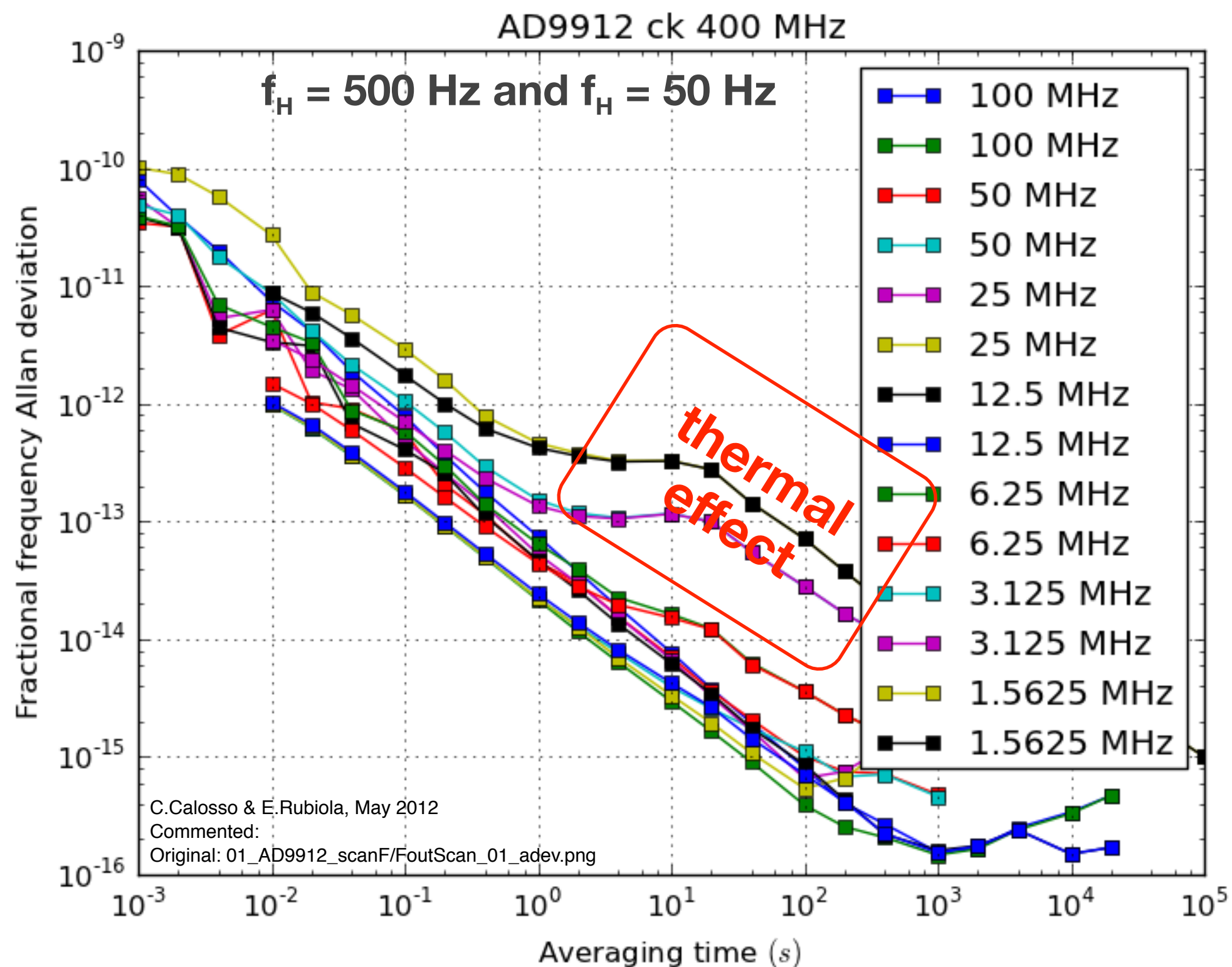


A blinking LED somewhere on the PCB spoils the output spectrum

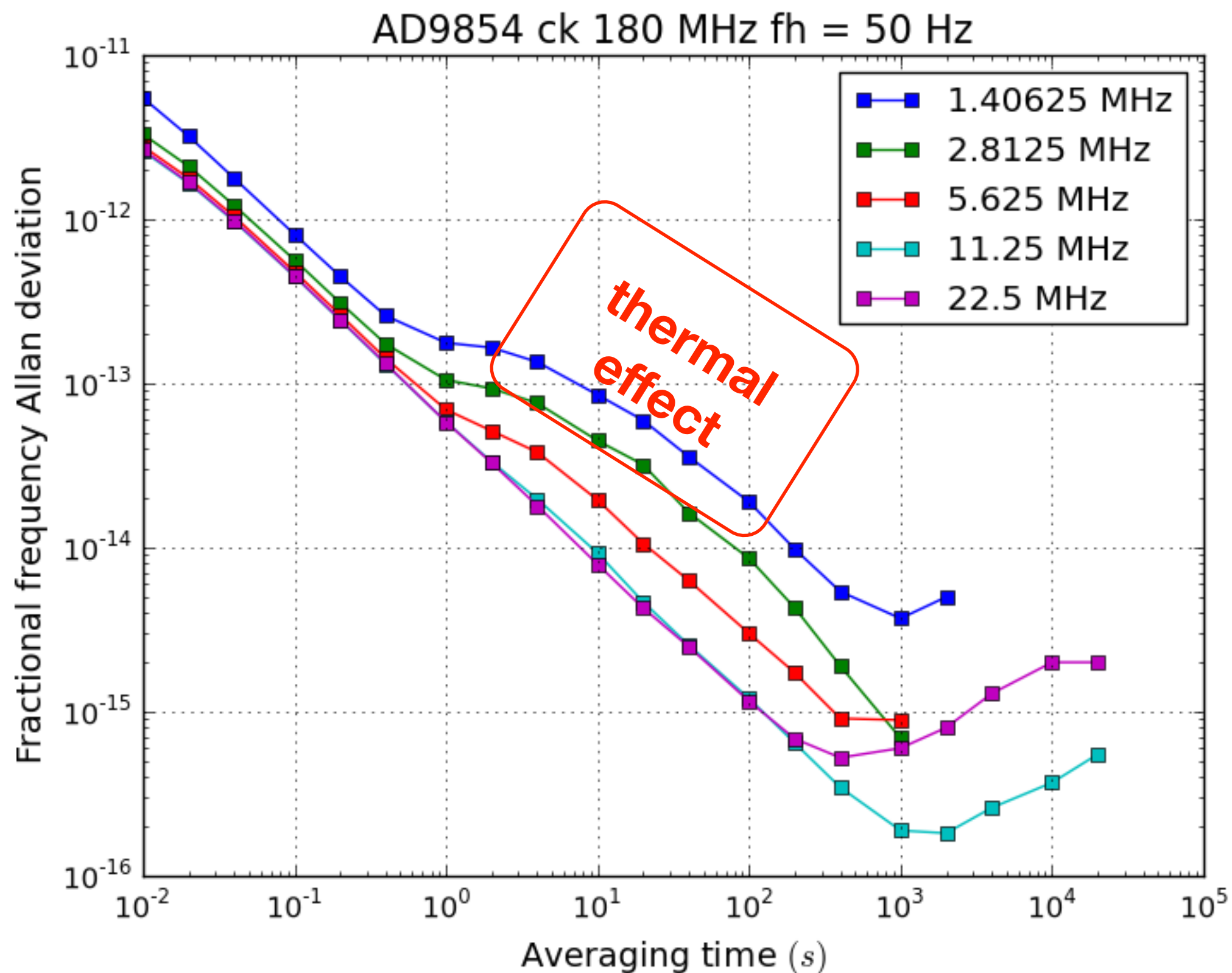
# ADEV vs. clock frequency



# ADEV vs. output frequency



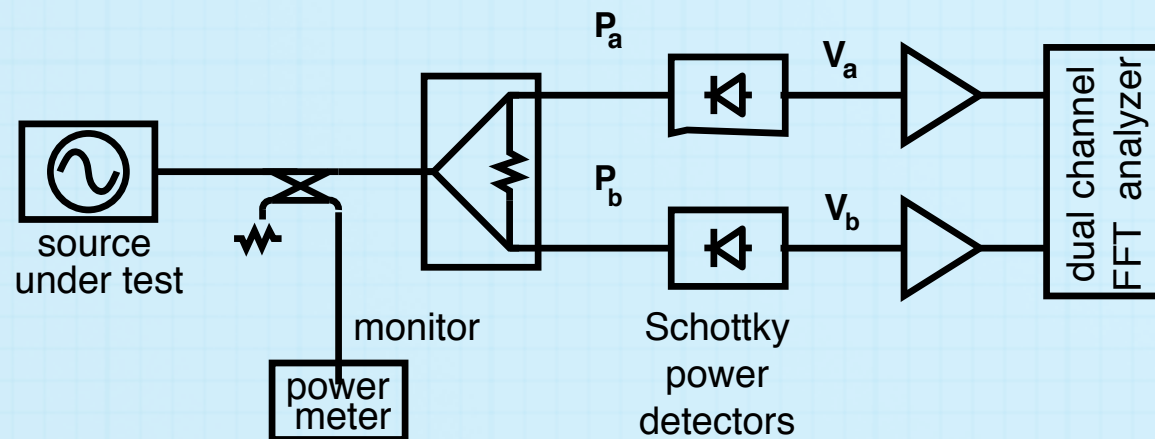
# ADEV vs. output frequency





# Experimental method (AM noise)

## Cross-spectrum



$$v_a(t) = 2k_a P_a \alpha(t) + \text{noise}$$

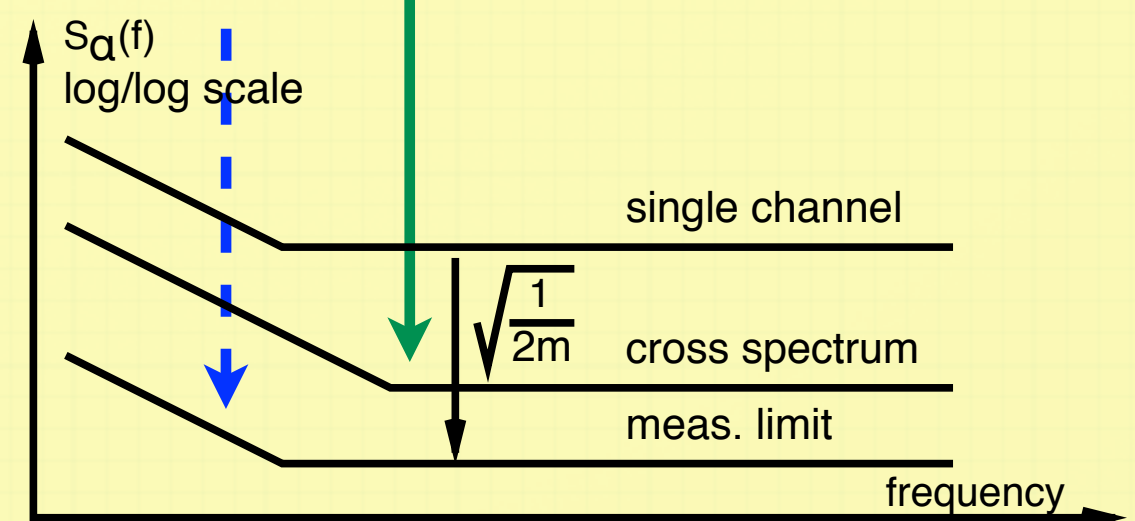
$$v_b(t) = 2k_b P_b \alpha(t) + \text{noise}$$

The cross spectrum  $S_{ba}(f)$  rejects the single-channel noise because the two channels are independent.

$$S_{ba}(f) = \frac{1}{4k_a k_b P_a P_b} S_{\alpha}(f)$$

- Averaging on  $m$  spectra, the single-channel noise is rejected by  $\sqrt{1/2m}$

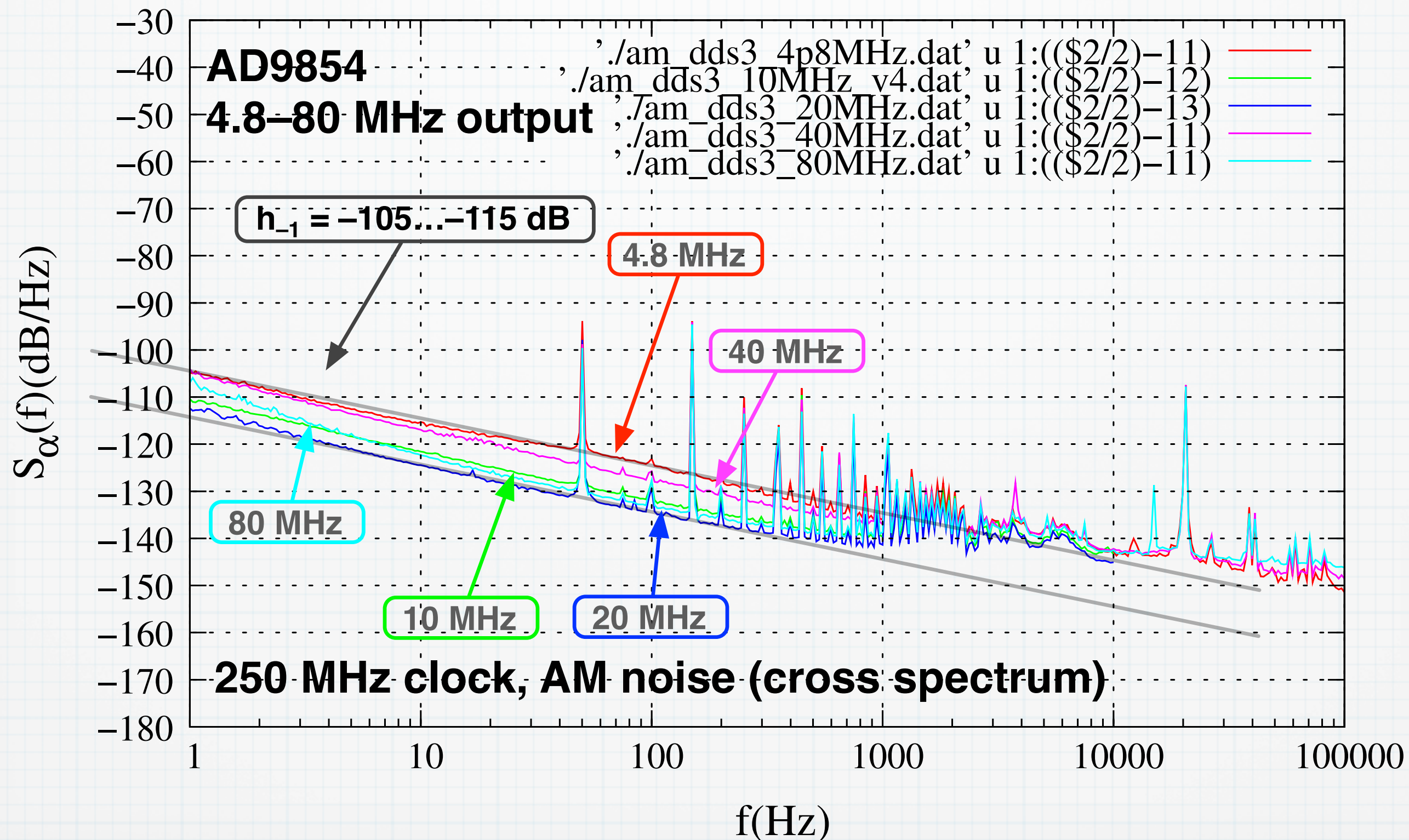
- A cross-spectrum higher than the averaging limit validates the measure



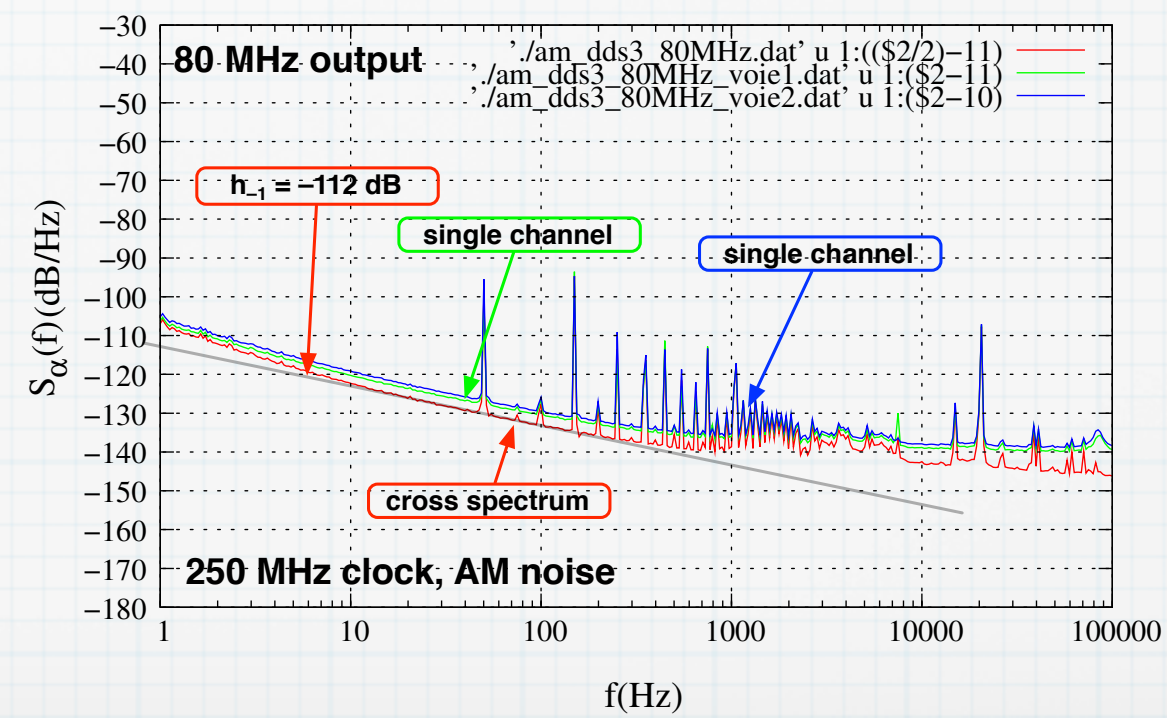
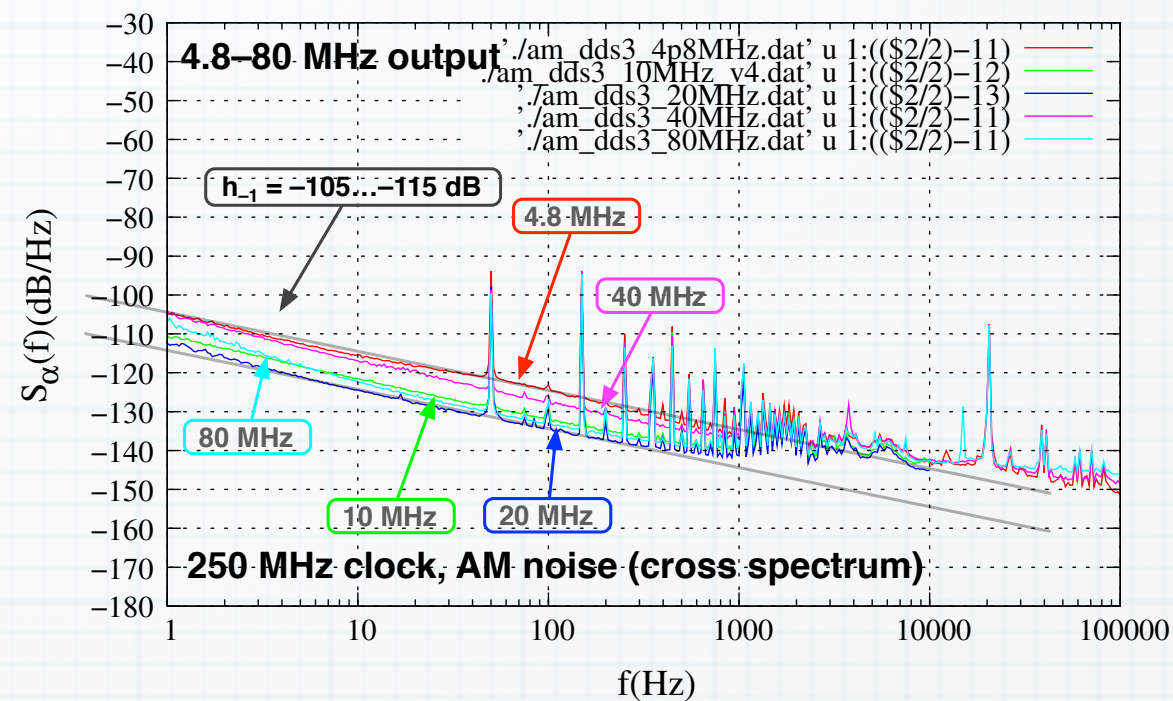
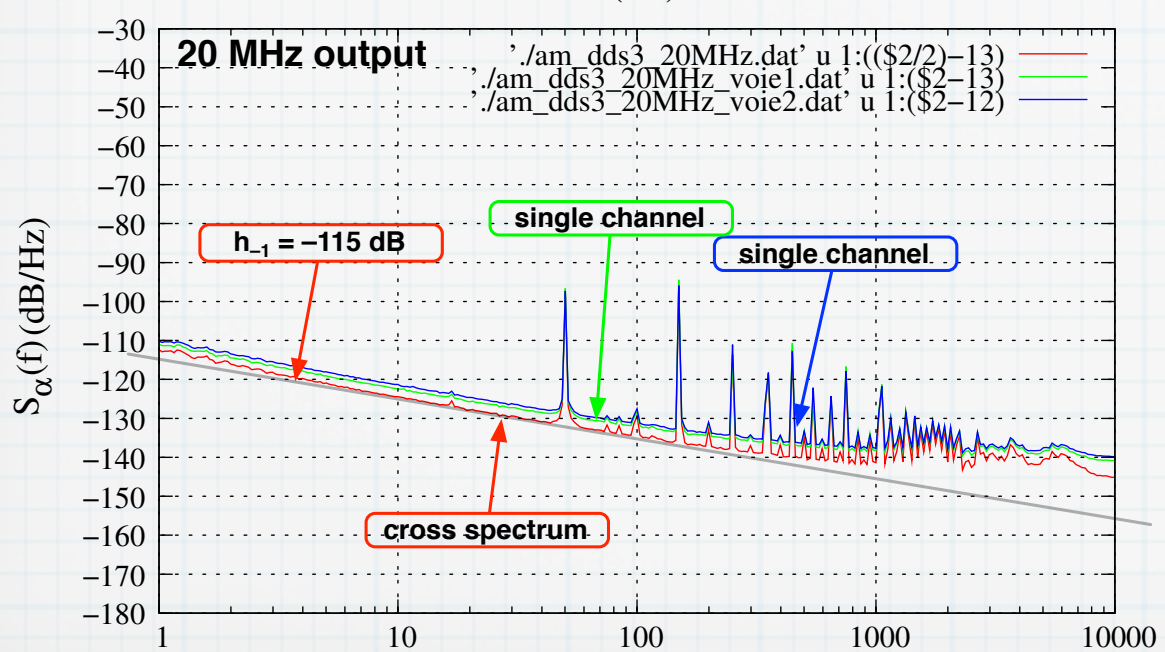
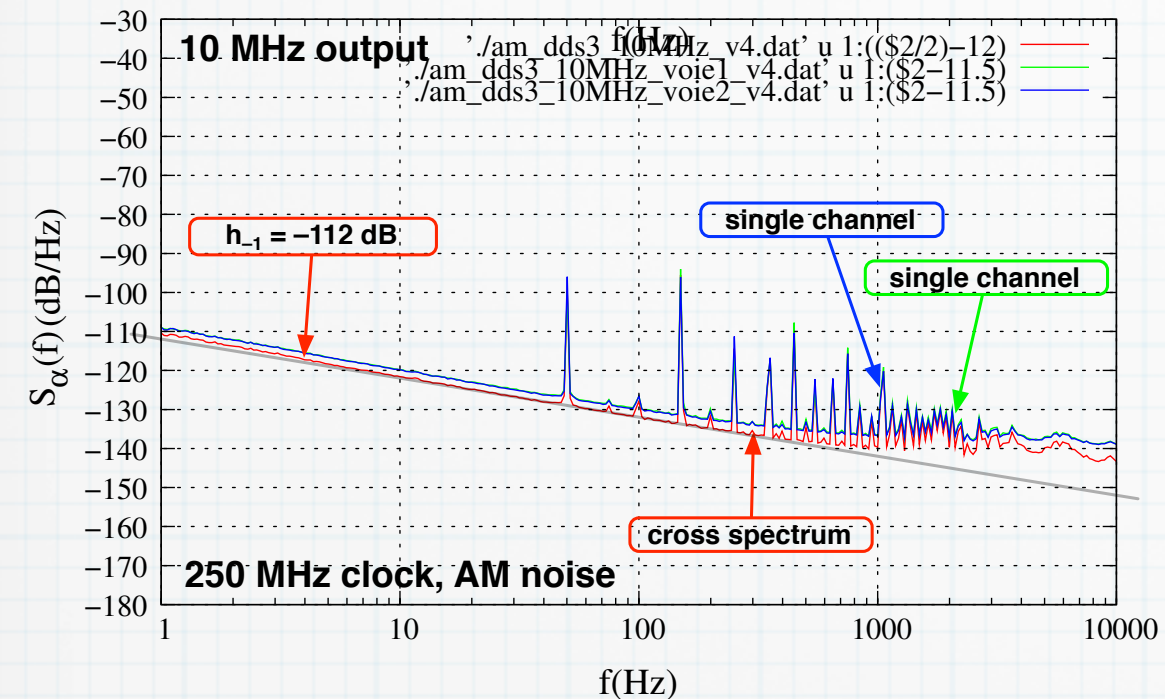
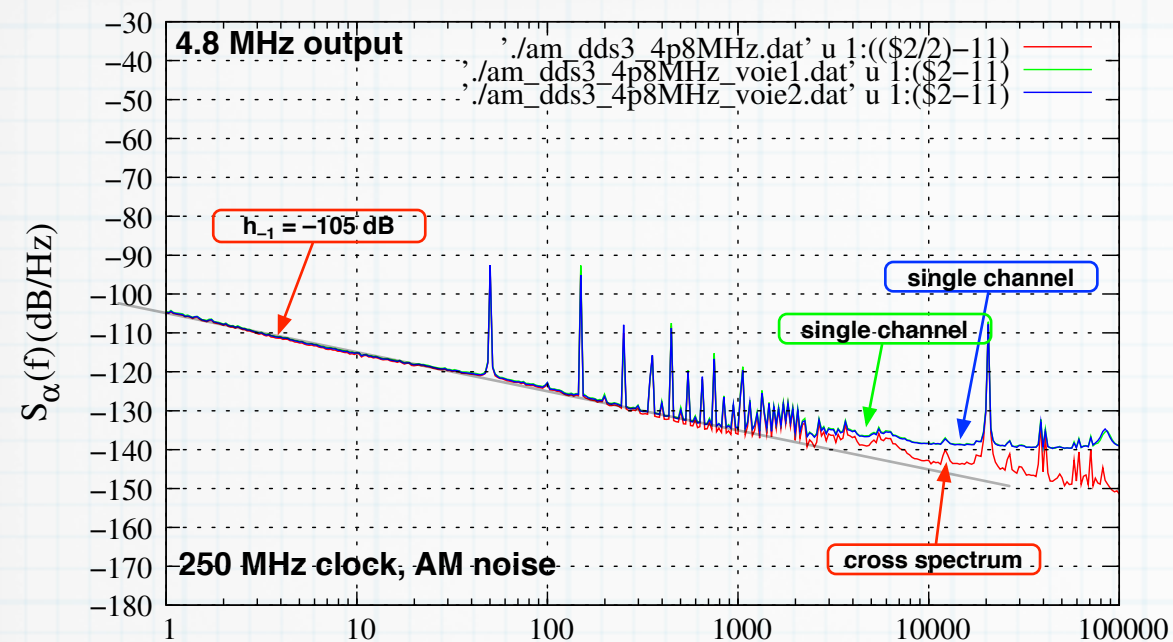
E. Rubiola, The measurement of AM noise of oscillators, arXiv:physics/0512082, Dec. 2005

E. Rubiola, F. Vernotte, The cross-spectrum experimental method, arXiv:1003.0113v1 [physics.ins-det], Feb. 2010

# AM noise (1)



# AM noise (2)





# Conclusions

- **Noise theory and model for the DDS**
- **A lot of still-not-published experimental data**
  - **Phase noise**
  - **Allan deviation**
  - **Amplitude noise**
- **Experiments done at INRIM and at FEMTO-ST**
- **Model and experimental data are in fair agreement**

**<http://rubiola.org>**



# 5 — Dividers

- $\Pi$  and  $\Lambda$  Dividers
- Microwave Dividers

# $\Pi$ and $\Lambda$ Dividers

# Motivations

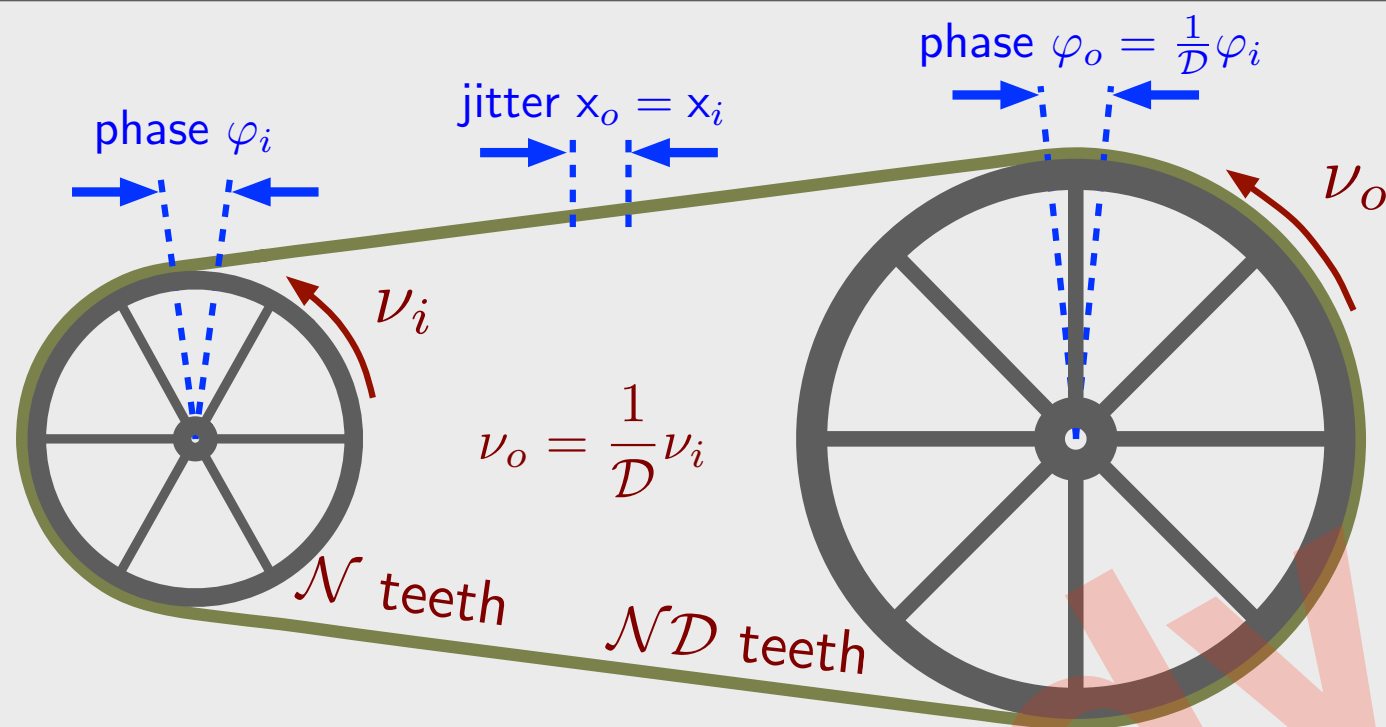
- **Seminal article by W. F. Egan (1990)**
  - Milestone in the domain, never forget it
  - However, TTL and ECL logic families are now obsolete
- **Microwave (photonics) → highest spectral purity**
- **Transfer the spectral purity to HF/VHF**
  - **Dividers are more comfortable than multipliers**
    - NIST now uses analog dividers
- **Nowadays digital electronics is fantastic**
  - **CPLD & FPGA → Easy to duplicate**
  - **High number of gates for cheap**
  - **High toggling frequency (1.5 GHz)**

W. F. Egan Egan WF, Modeling phase noise in frequency dividers, IEEE T UFFC 37(4), July 1990

E. Rubiola & al, Phase noise in the regenerative frequency dividers, IEEE T IM 41(3), June 1992

A. Hati & al, Ultra-low-noise regenerative frequency divider..., Proc IEEE IFCS, May 2012

# The Gear Work Model

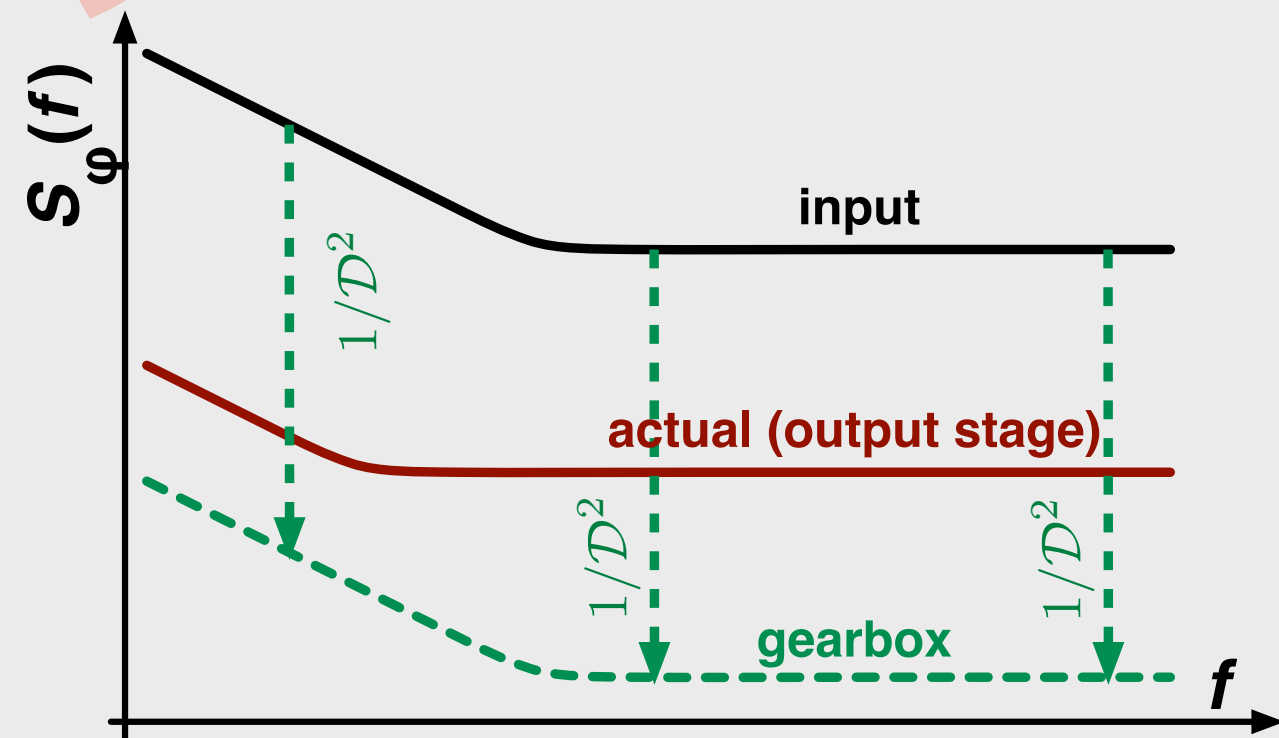
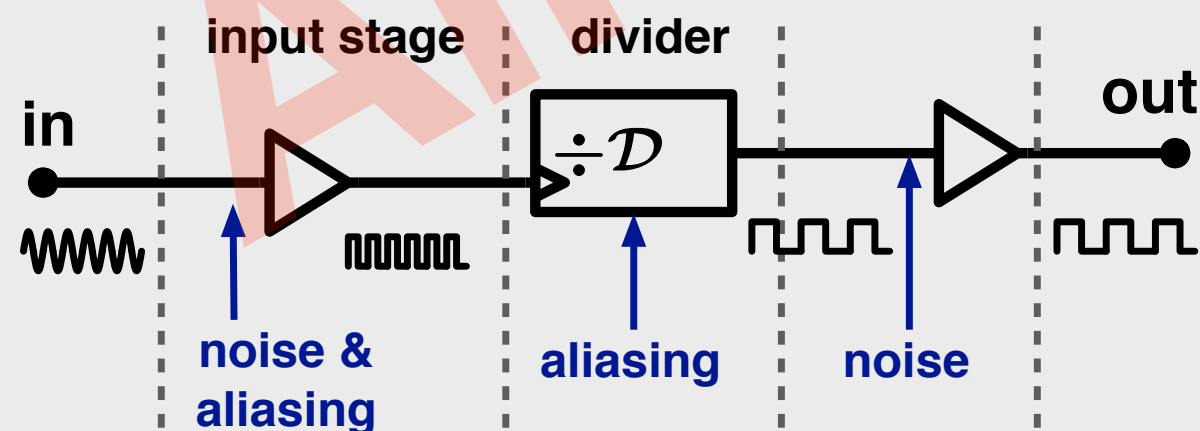


## • The noise-free divider

- Keeps the input jitter  $x(t)$  (phase-time fluctuation)
- Scales down
  - $\varphi$  by  $1/D$  [rad]
  - $S_\varphi$  by  $1/D^2$  [rad<sup>2</sup>/Hz]

## In the real divider

- $S_\varphi$  of the output often dominates
- Aliasing is present





# Parseval Theorem

The energy calculated in the **time domain** is equal to the energy calculated in the **frequency domain**

time domain

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

frequency domain

Average power, truncated signal

$$\sigma^2 = \frac{1}{T} \int_0^T |x(t)|^2 dt$$

One-sided PSD, truncated signal

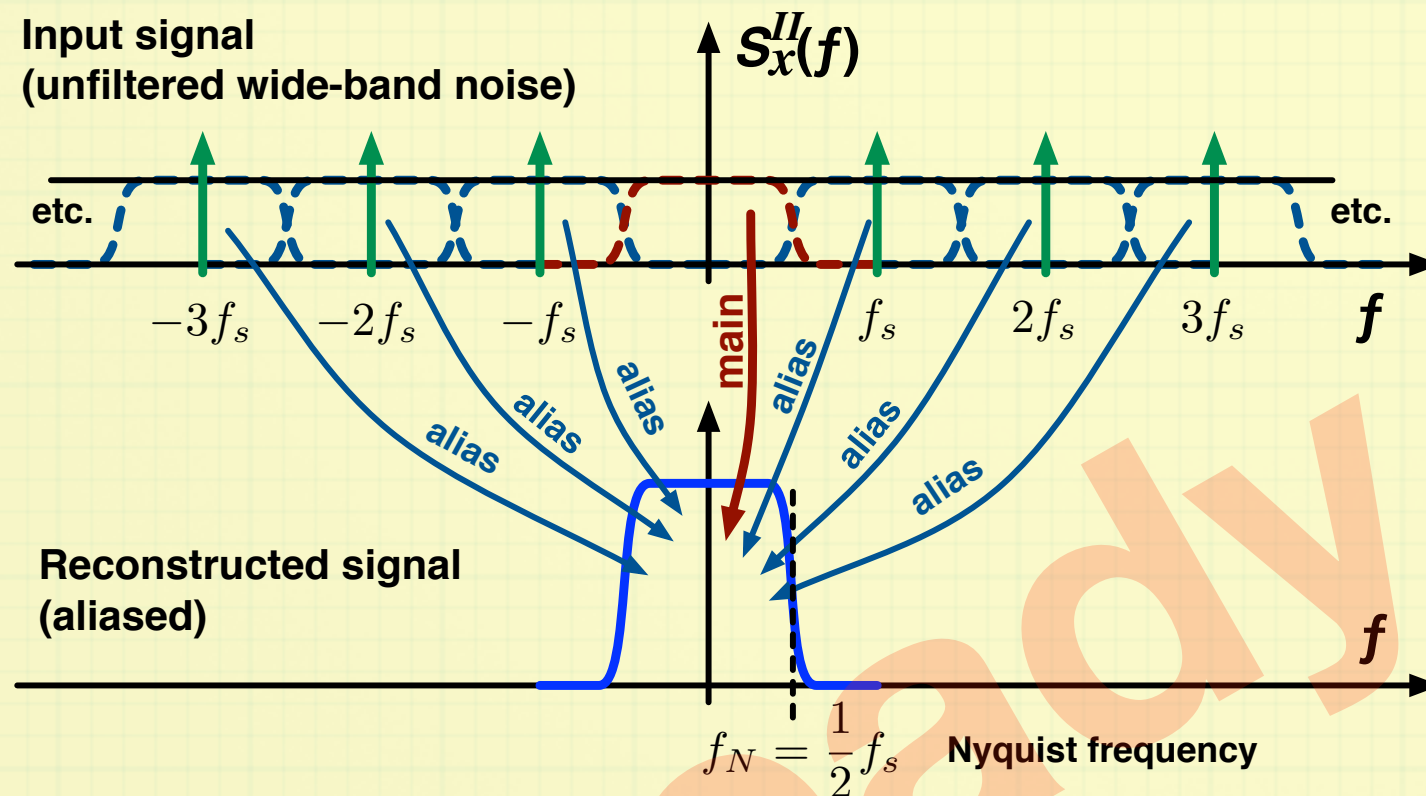
$$S_x(f) = \frac{2}{T} |X_T(f)|^2$$

$$\sigma^2 = \int_0^{\infty} S_x(f) df$$

For **ergodic** signals, the **time average** is equal to the **ensemble average**

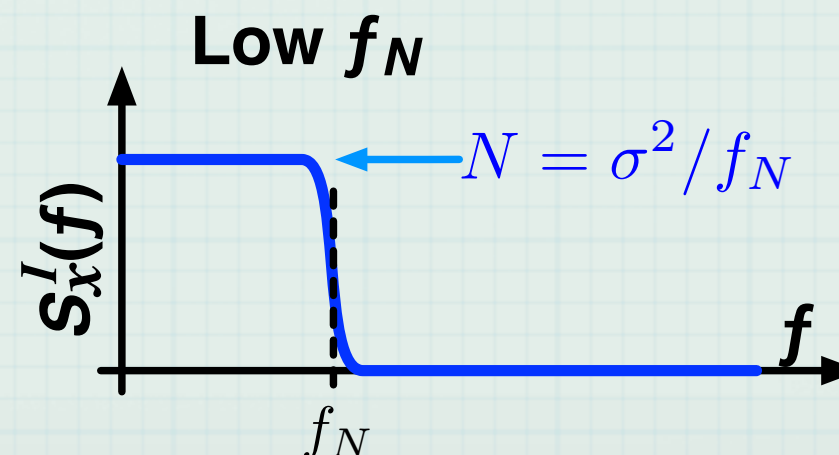
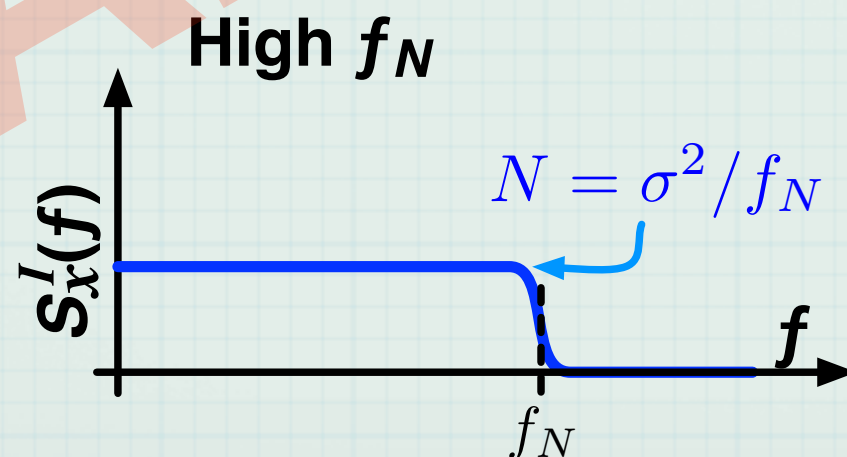
# Sampling and Aliasing

— Energy conservation applies to the unfiltered signal —

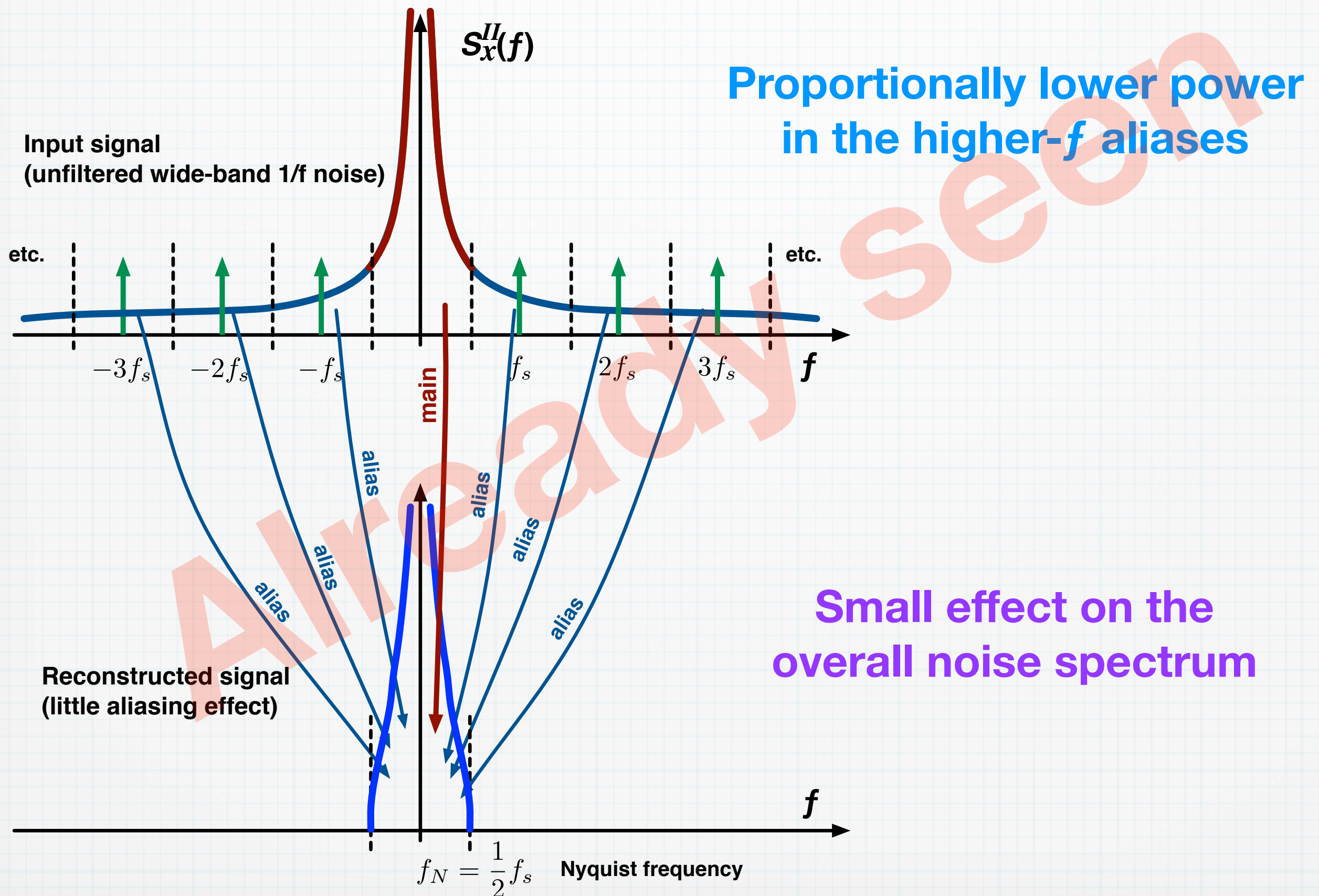


- Multiple aliases overlap to the main part of the spectrum
- With white noise, the PSD increases by  $B/f_N$  (Bandwidth / Nyquist  $f$ )

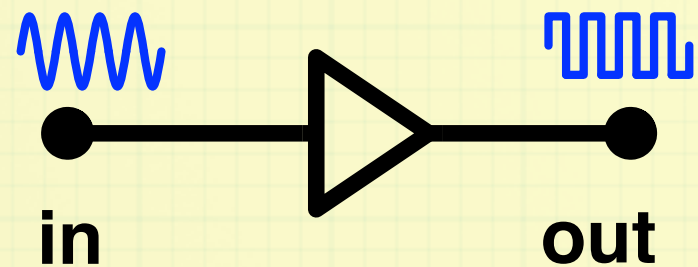
Downsampling increases the (PM) noise spectrum



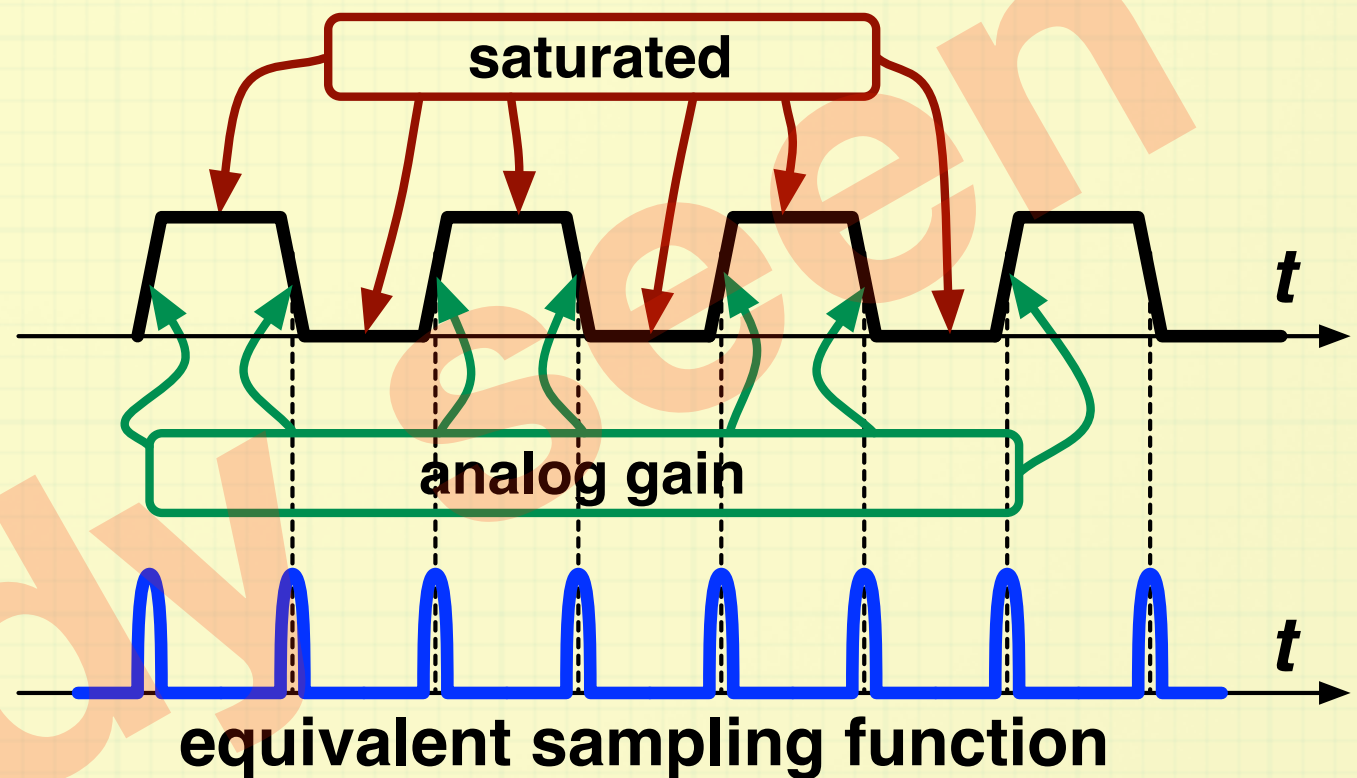
# Aliasing and $1/f$ Noise



# PM-Noise Aliasing in the Input Stage



Convert the input sinusoid into a square wave, as appropriate



- Edge-sampling at  $2v_i$  inherent in the sin-to-square conversion
- Full-bandwidth ( $B$ ) noise is taken in
- The phase-noise Nyquist frequency is  $v_i$
- The sampling process increases the noise by  $B/v_i$

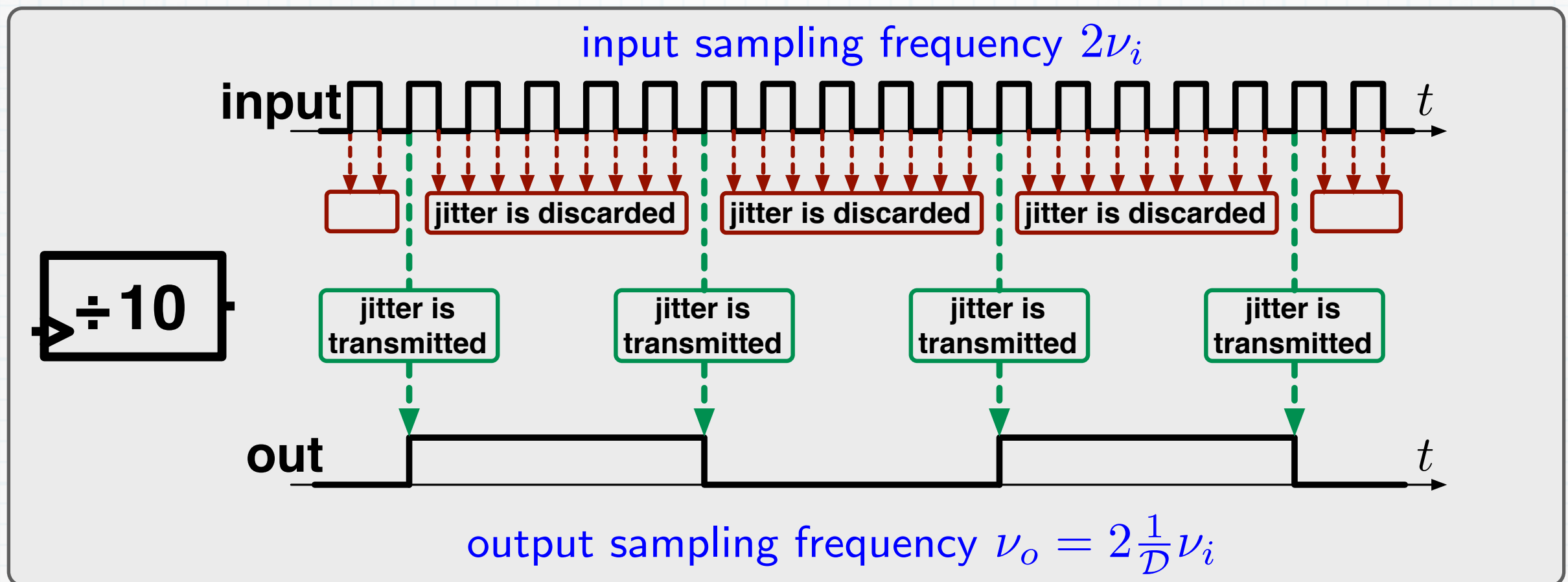
Eventually, clipping removes the AM noise [Pfaff 1974]



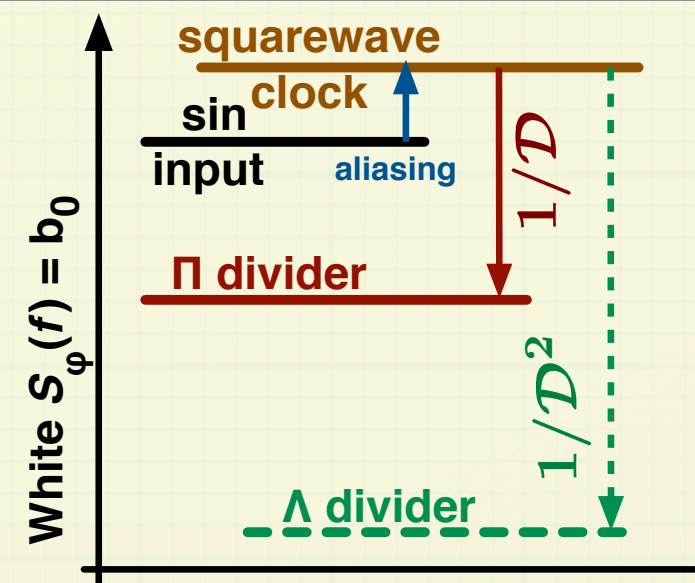
# Aliasing in $\Pi$ Divider

Regular synchronous divider

The Greek letter  $\Pi$  recalls the square wave  $\Pi \Pi \Pi \Pi$



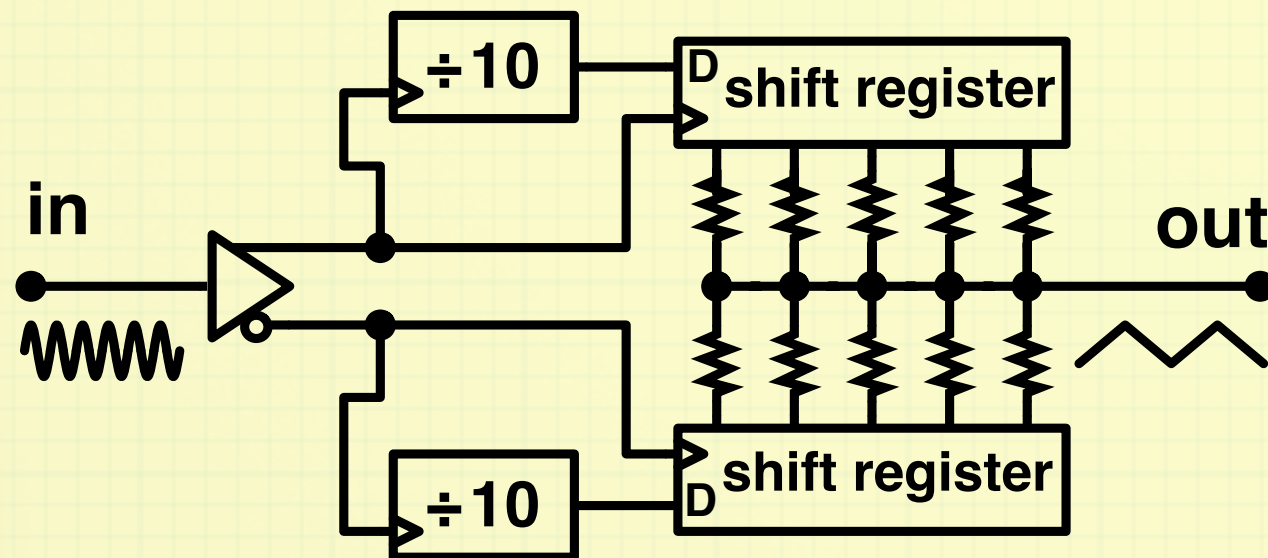
- The gearbox scales  $S_\phi$  down by  $1/D^2$
- The divider takes 1 edge out of  $D$ 
  - Raw decimation without low-pass filter
  - Aliasing increases  $S_\phi$  by  $D$
- Overall,  $S_\phi$  scales down by  $1/D$



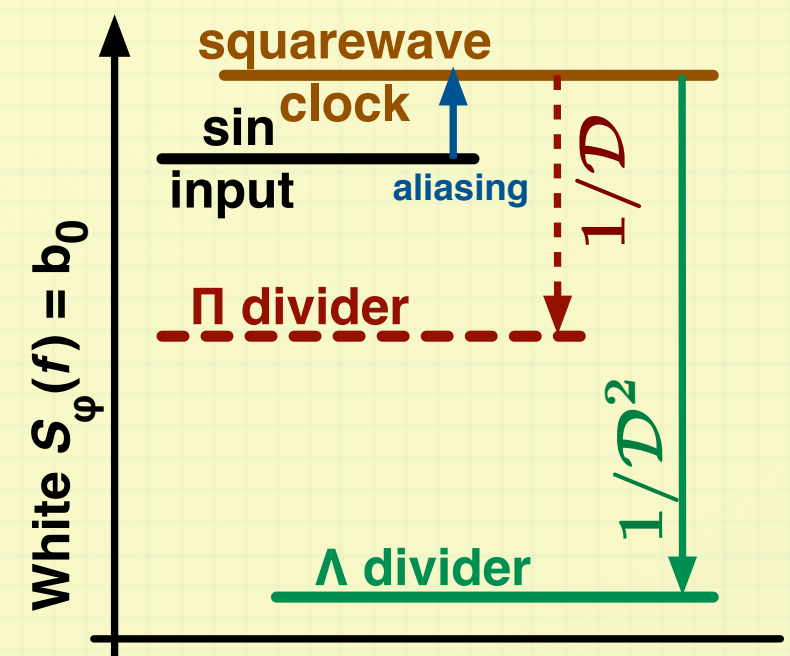
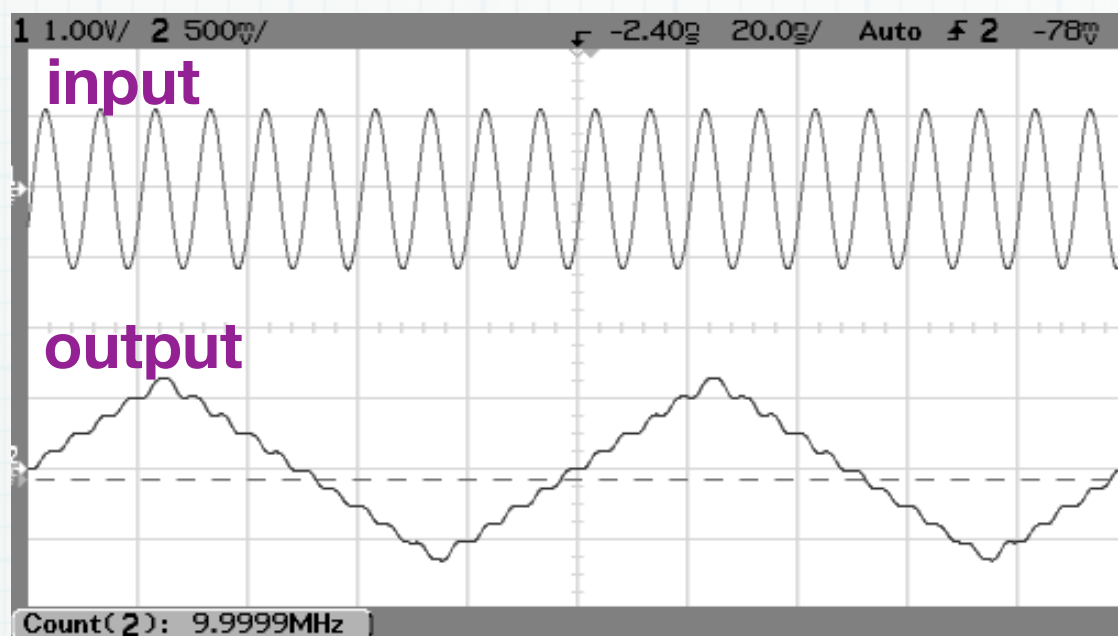
# The $\Lambda$ Divider – Little/no Aliasing

## New divider architecture

Series of Greek letters  $\Lambda\Lambda\Lambda\Lambda$  recalls the triangular wave



- Gearbox and aliasing  $\rightarrow 1/D$  law
- Add  $D$  independent realizations shifted by  $1/2$  input clock,
- reduce the phase noise by  $1/D$ ,
- ... and get back the  $1/D^2$  law



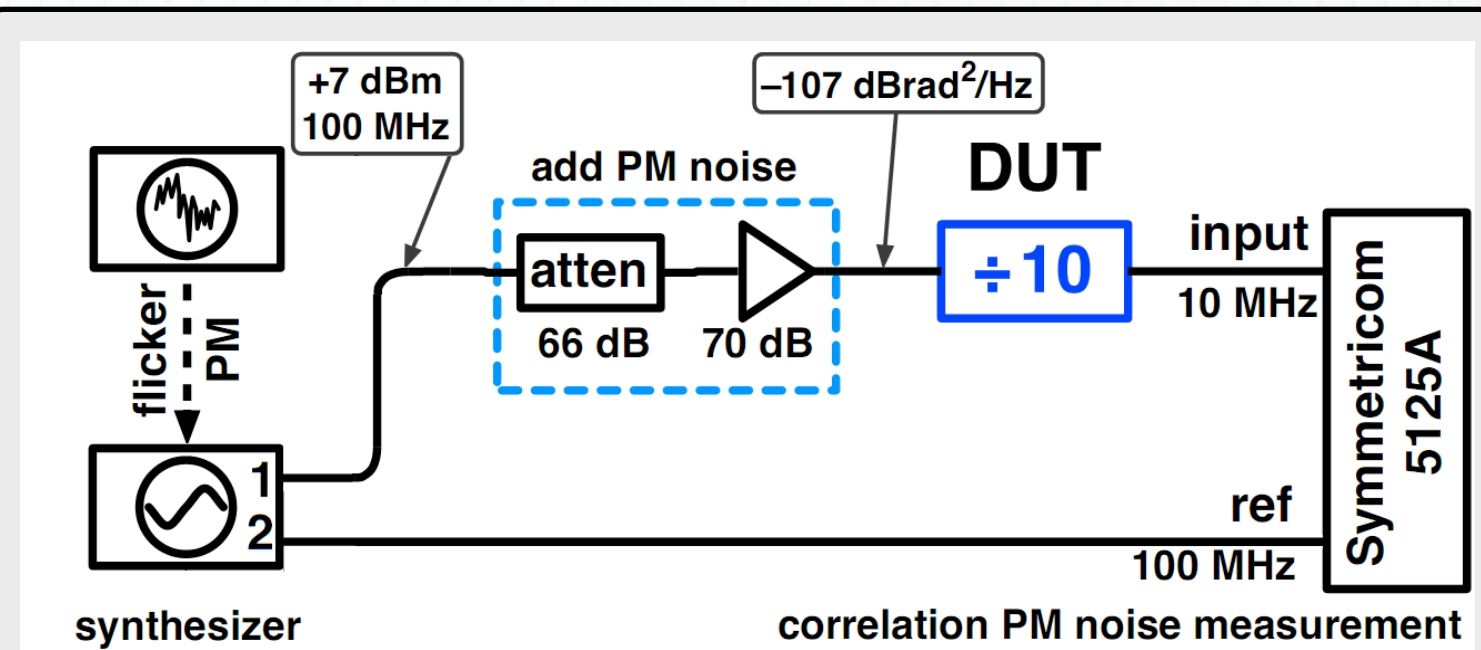
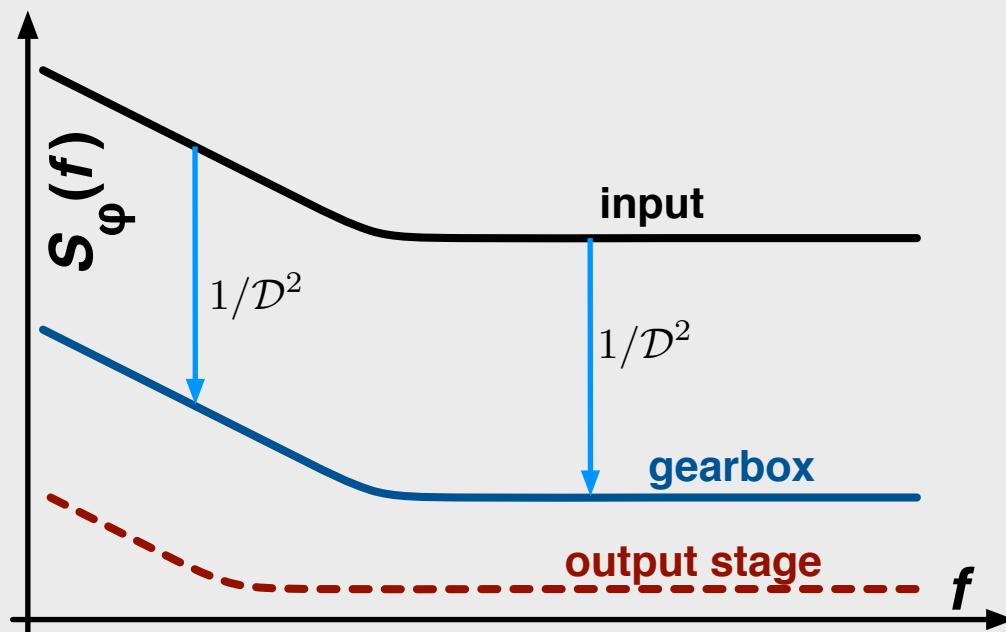
The names  $\Pi$  and  $\Lambda$  derive from the shape of the weight functions in our article on frequency counters

E. Rubiola, On the measurement of frequency ... with high-resolution counters, RSI 76 054703, 2005

# Experimental Method

Large input PM noise is used to emphasize the effect of aliasing

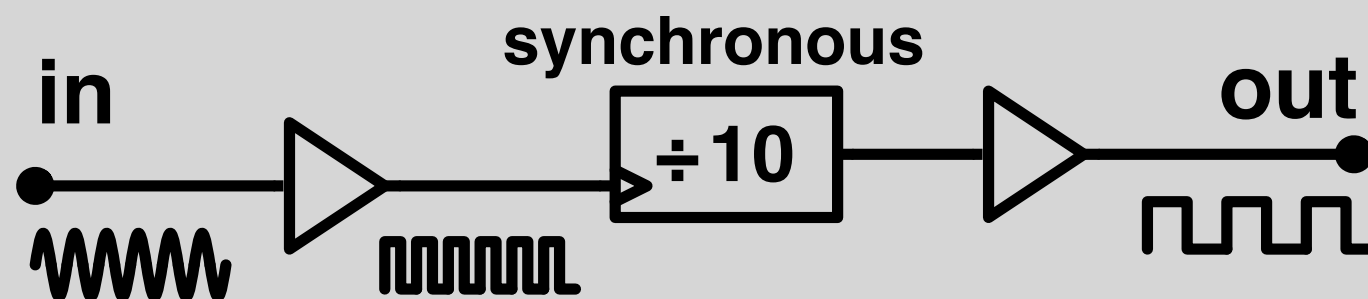
- Intentionally high PM noise at the input
- The scaled-down input noise is higher than the output-stage noise



- Large attenuation/ampli → noise
- Digital instruments for phase-noise measurement can handle  
 $f_{\text{input}} \neq f_{\text{reference}}$
- Correlation reduces the background

# Dividers Under Test

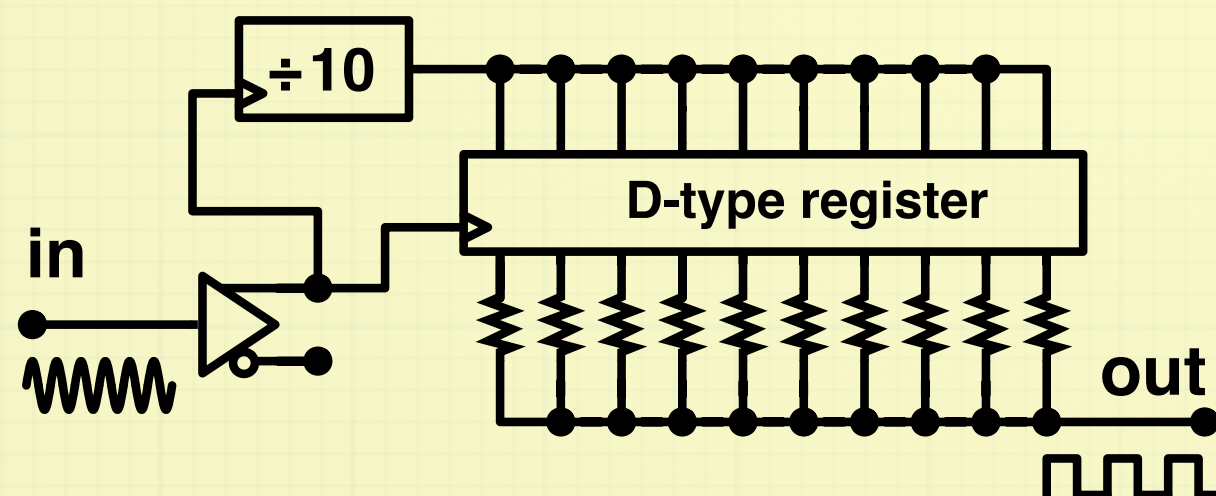
EPM3064A CPLD (Altera MAX 3000 Series, 64 macro-cells, speed grade 7 ns)



**Π divider**

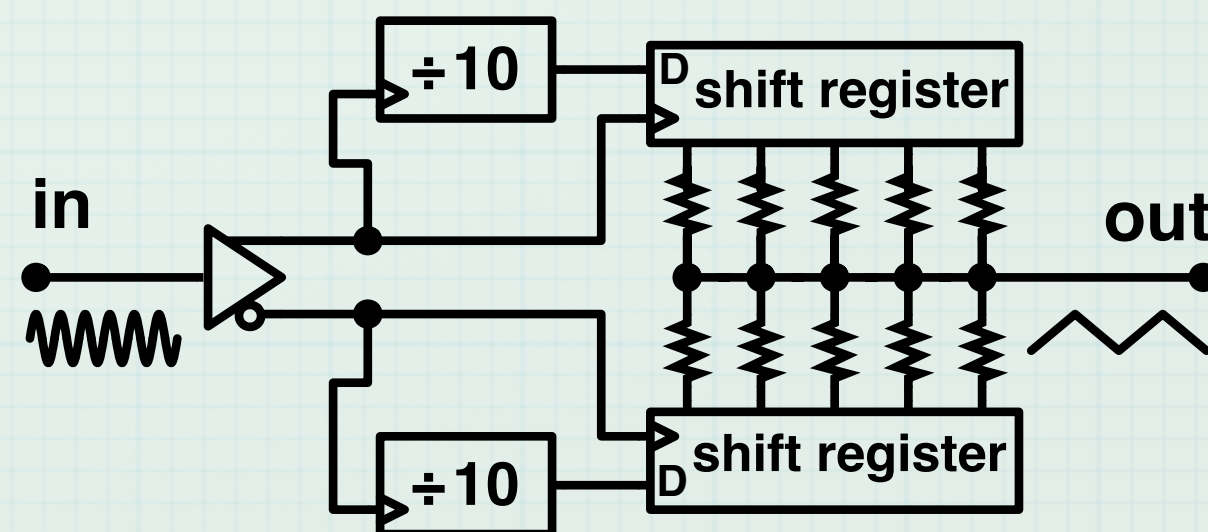
– the one everybody knows –

## Multi-buffer Π divider



The outputs are arguably independent  
Try to reduce the output-stage noise

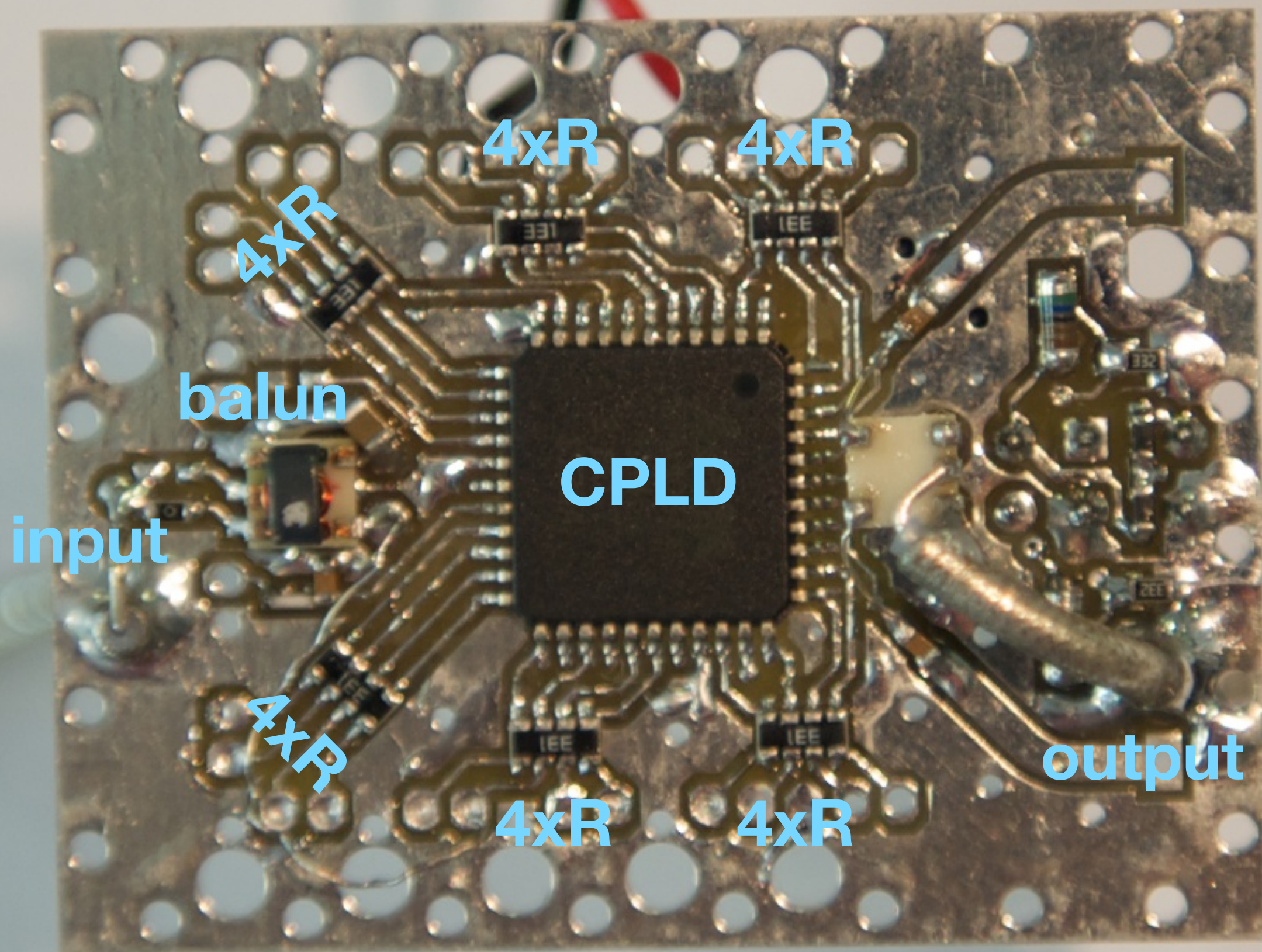
## Λ divider



White noise:  
The clock edges are independent  
Correct for aliasing

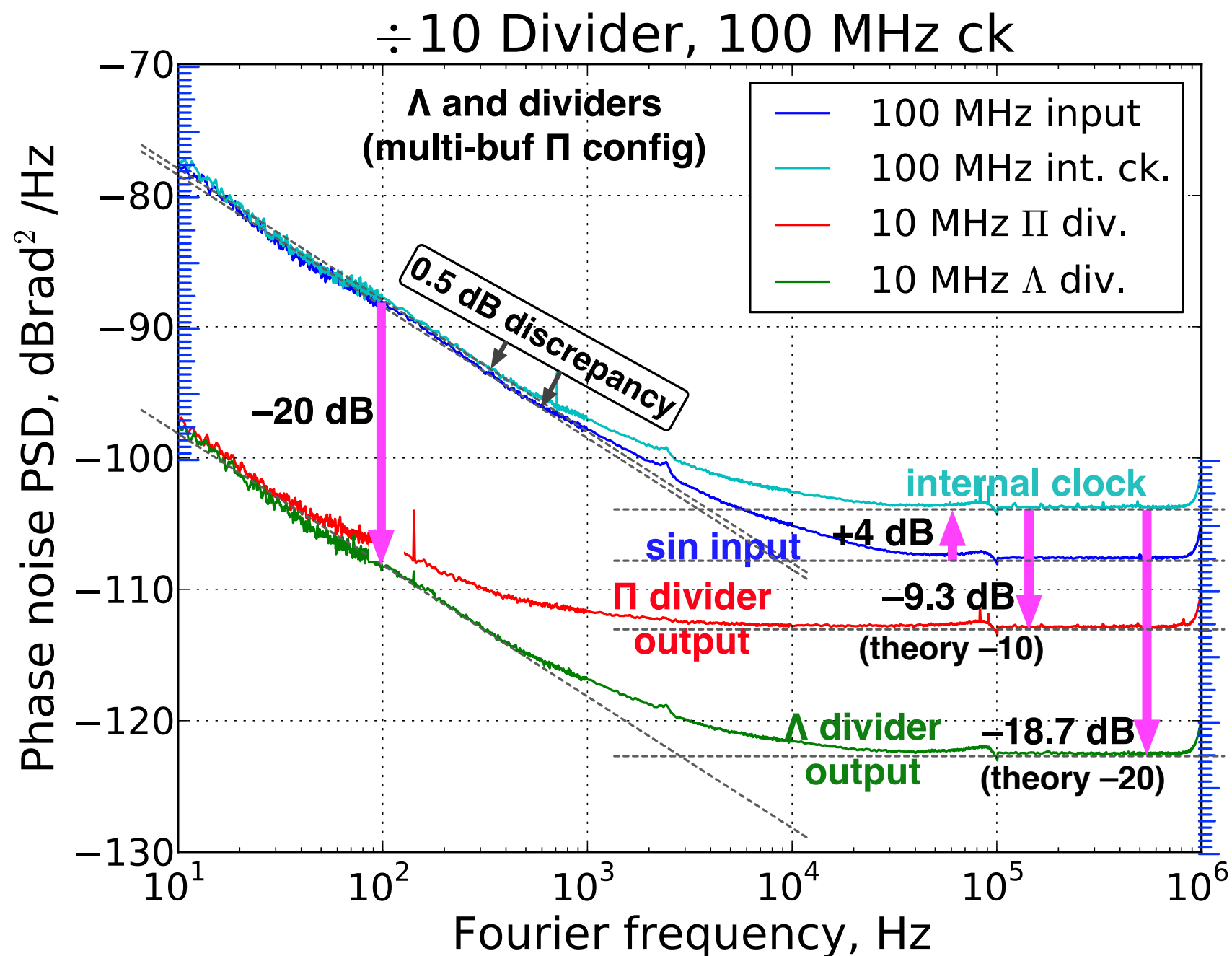


# As Simple as That...





# Results – Test on Aliasing



- **Flicker region**

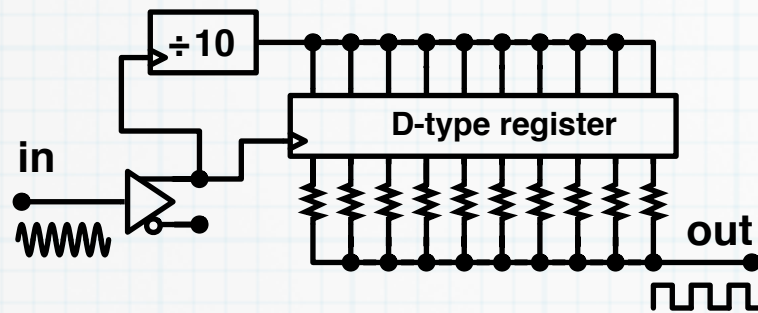
- Negligible aliasing
- $1/D^2$  law (-20 dB)

- **White region**

- Aliasing in the front-end  $\rightarrow$  +4 dB
- $1/D$  law and  $1/D^2$  law

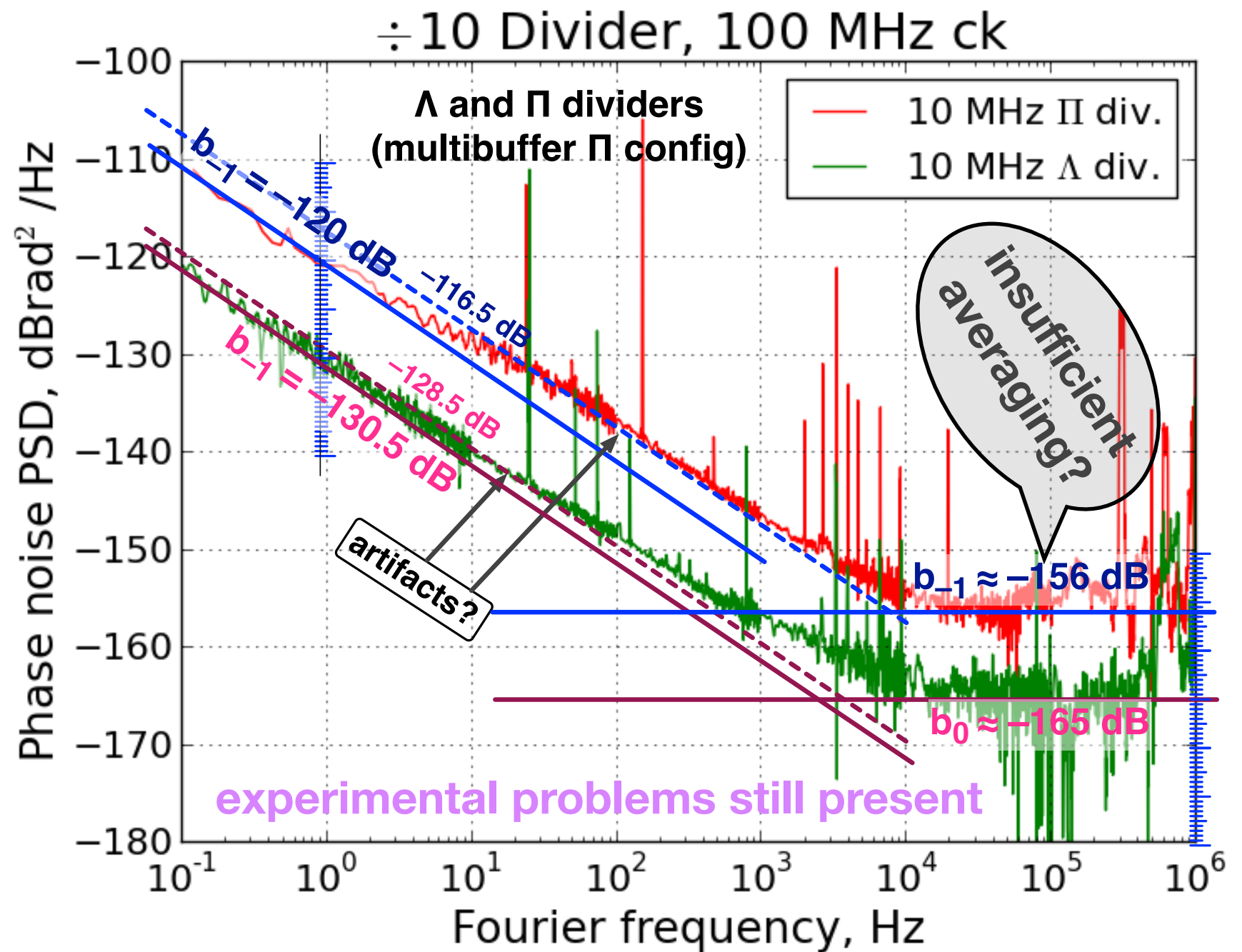
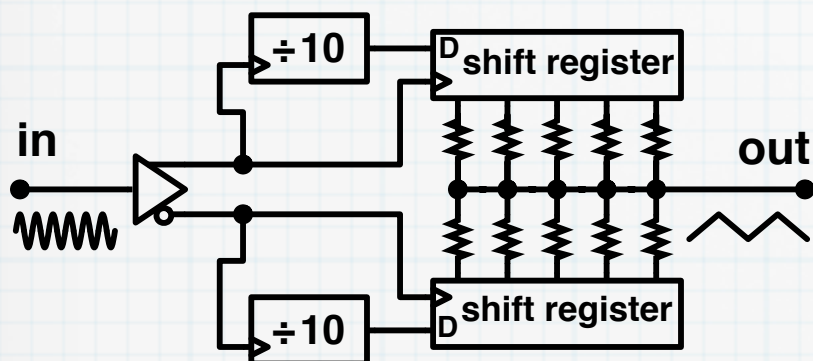
# Phase Noise of Real Dividers

## Multibuffer $\Pi$ divider



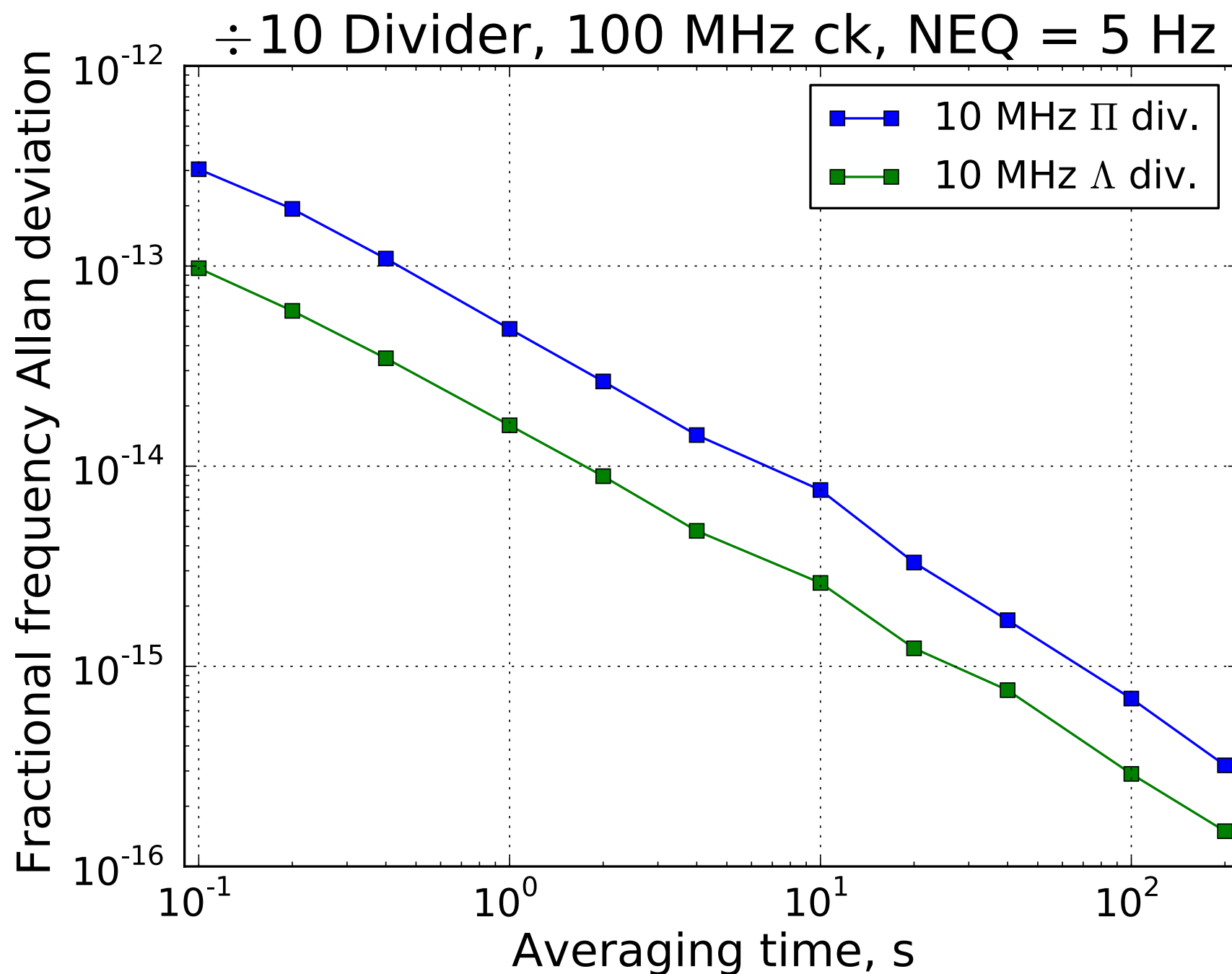
multiple outputs are  
expected to reduce the  
output-stage noise  
– not happened, why? –

## $\Lambda$ divider



- Flicker region  $\rightarrow$  Negligible aliasing
- The multibuffer  $\Pi$  divider is still not well explained
- The  $\Lambda$  divider exhibits low  $1/f$  and low white noise

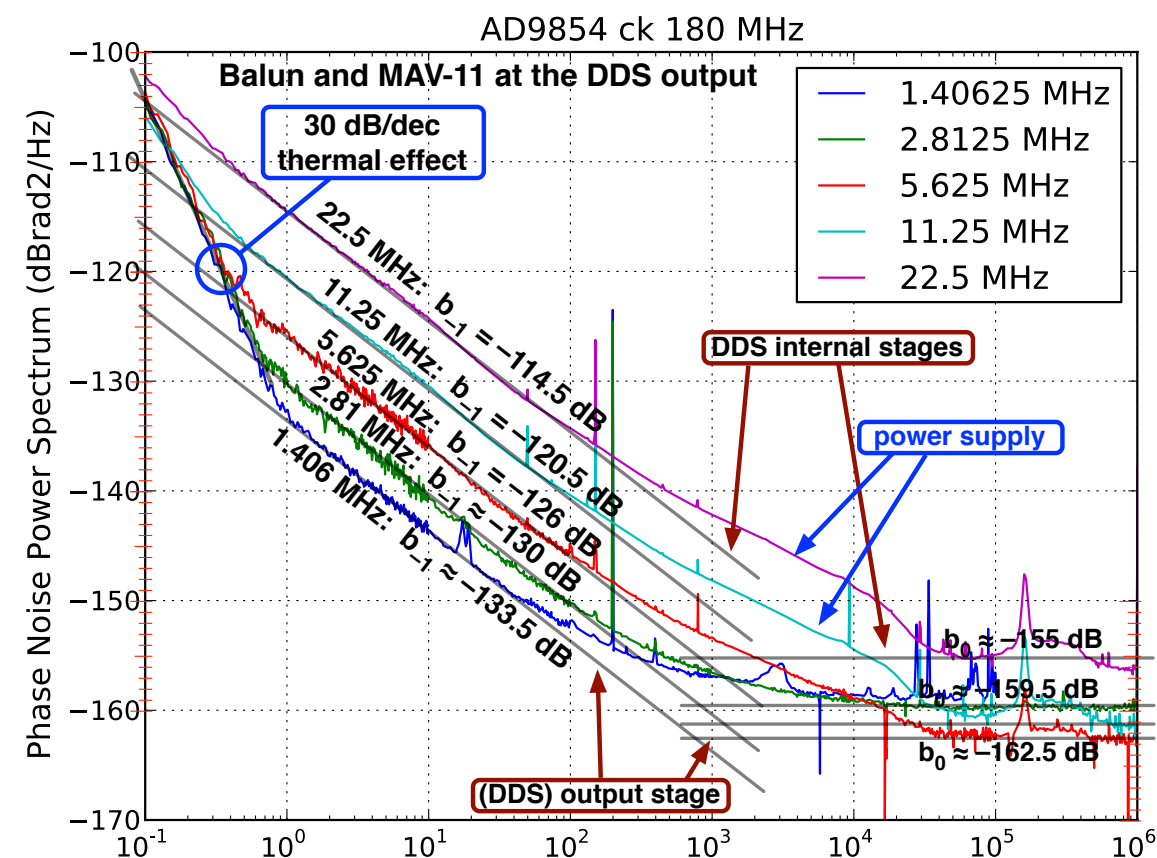
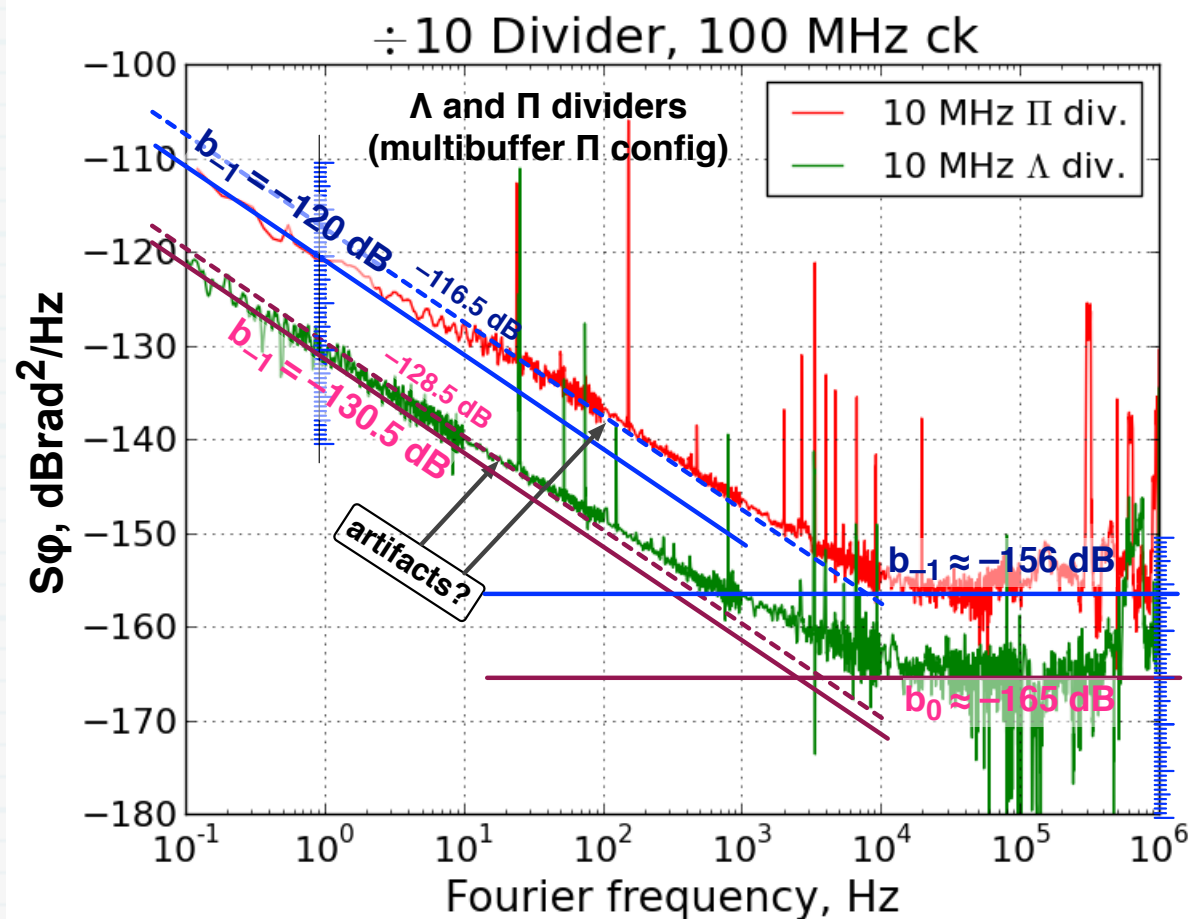
# Allan Deviation in Real Dividers



- Slope  $1/\tau$ , typical of white and flicker PM noise
- The  $\Lambda$  divider performs  $2 \times 10^{-14}$  at  $\tau = 1$  s, 10 MHz output

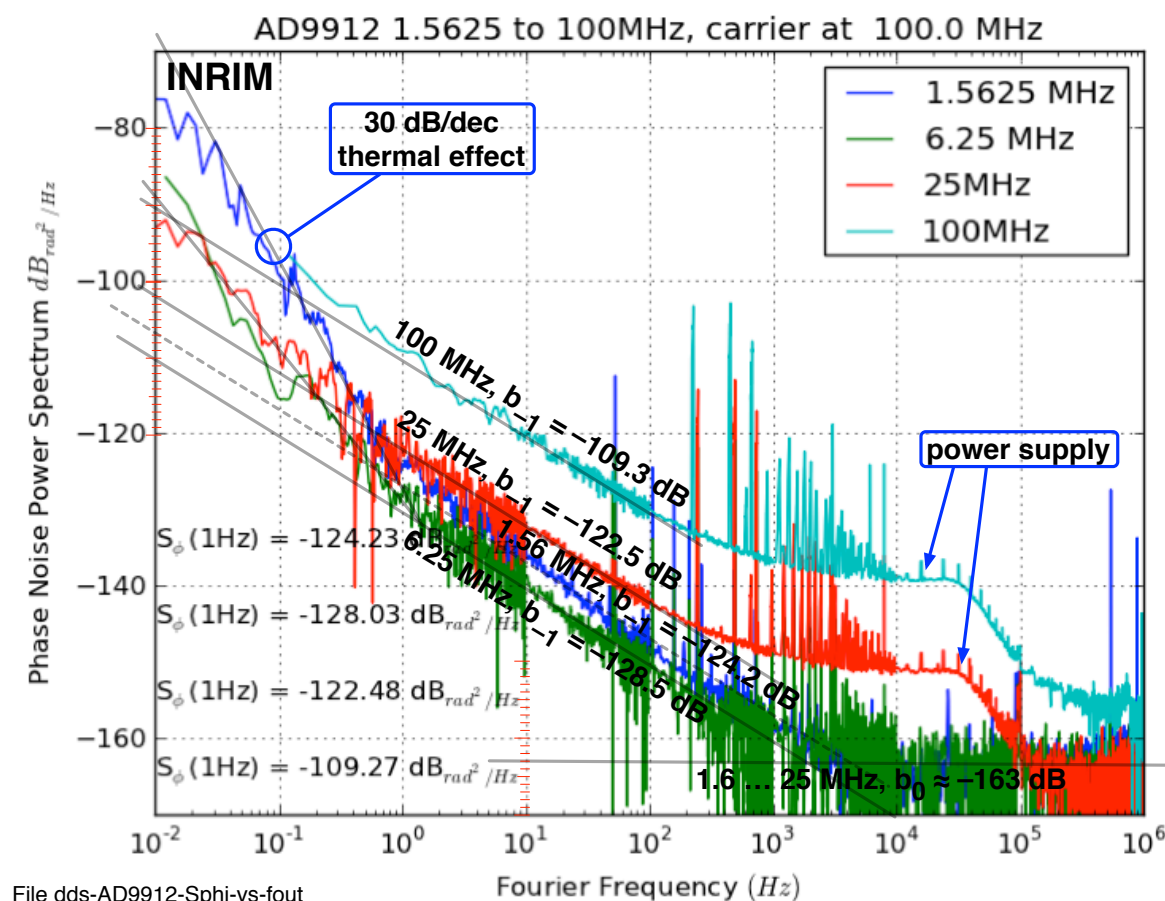


# The $\Lambda$ Divider Versus the DDS

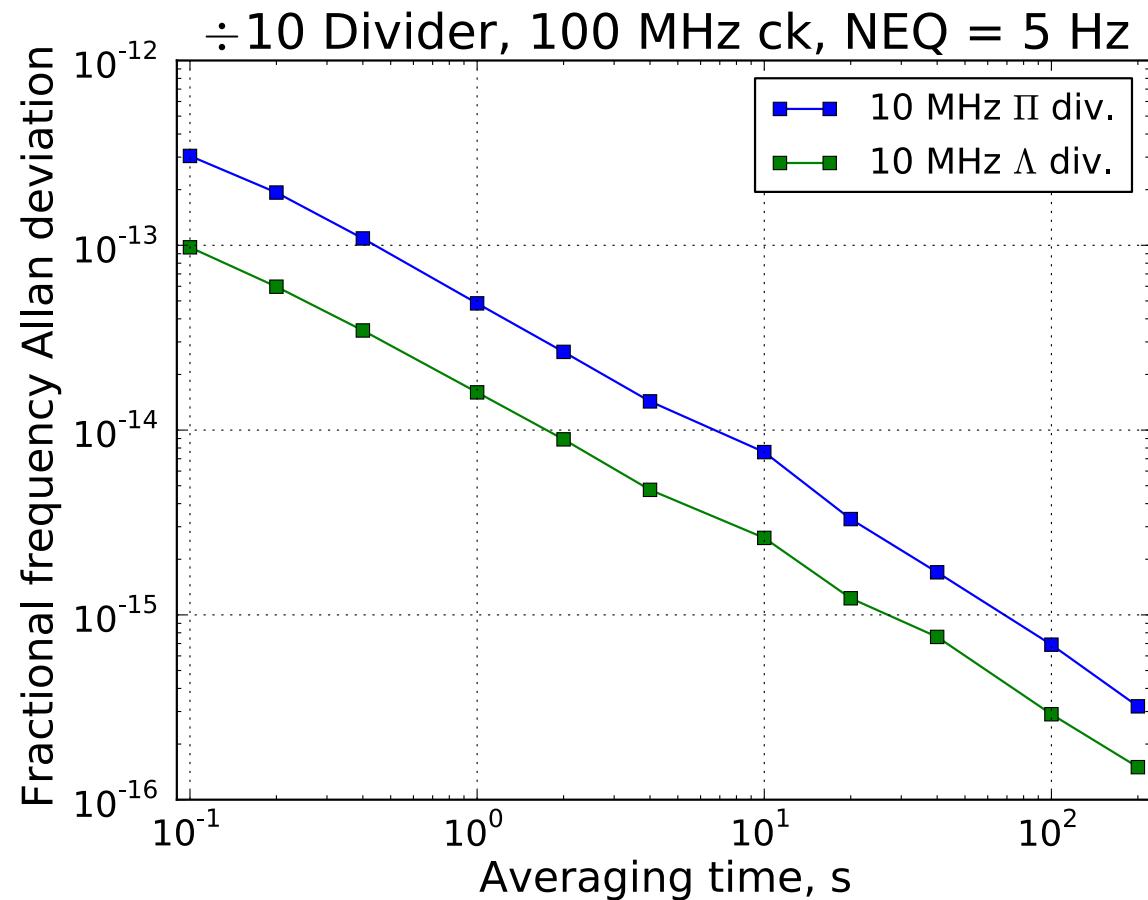


## Noise of the $\Lambda$ divider and two DDSs

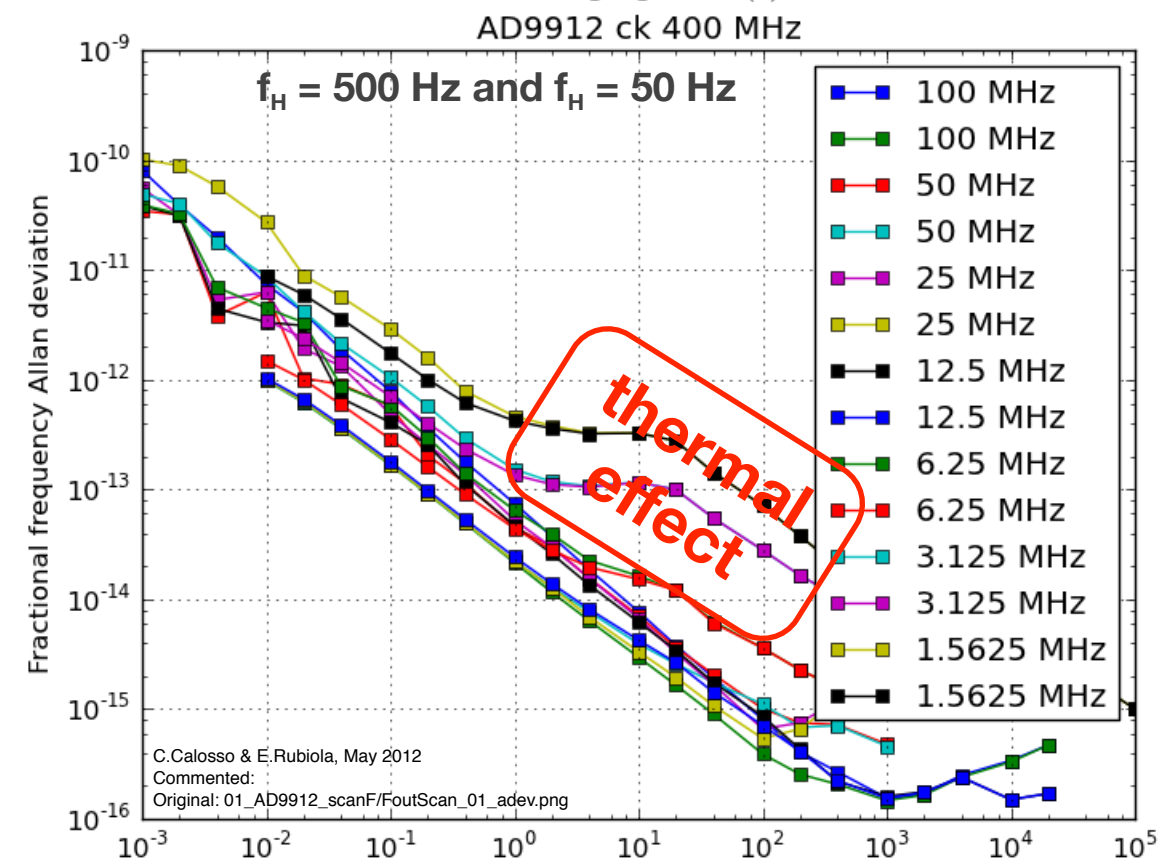
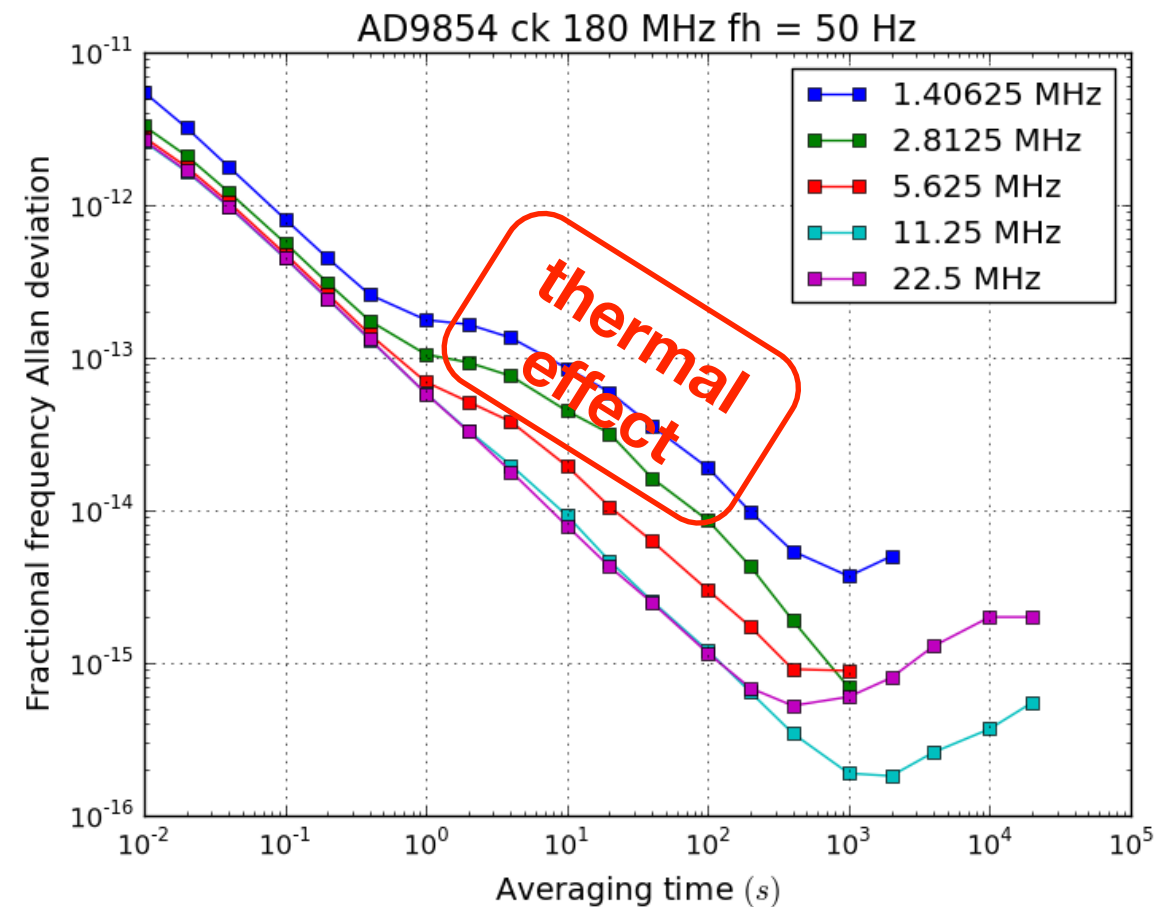
noise	$\Lambda$ div.	AD9854	AD9912
$b_0$	-165	-160	$\approx -163$
$b_{-1}$	-130.5	-121.5	-129 inferred
$b_{-2}$	—	—	-132 plot not shown
$b_{-3}$	—	-134	(seen at lower $\nu_o$ )



# The $\Lambda$ Divider Versus the DDS



- The  $\Lambda$  divider performs  $2 \times 10^{-14}$  at  $\tau = 1$  s, 10 MHz output
- Thermal effects make the DDS worse at  $\tau > 1$  s
- The  $\Lambda$  divider is free from thermal effects – at the scale shown



# The Bottom Line

- Aliasing in traditional dividers
  - Increases white noise
  - Has little effect on flicker
- Flicker in multi-buffer  $\Pi$  divider not understood yet
- The new  $\Lambda$  divider
  - Is little/no affected by aliasing
  - Exhibits the lowest PM noise
    - flicker:  $b_{-1} \approx -130$  dB
    - white:  $b_0 \approx -165$  dB
  - Features  $2 \times 10^{-14}$  at  $\tau = 1$  s, 10 MHz output
  - Is free from the thermal effects seen in DDSs at  $\tau > 1$  s

**home page <http://rubiola.org>**

Thanks – J. Groslambert, V. Giordano, M. Siccaldi, J.-M. Friedt

Grants from ANR (Oscillator IMP and First-TF network), and Region Franche Comte

# Microwave Dividers

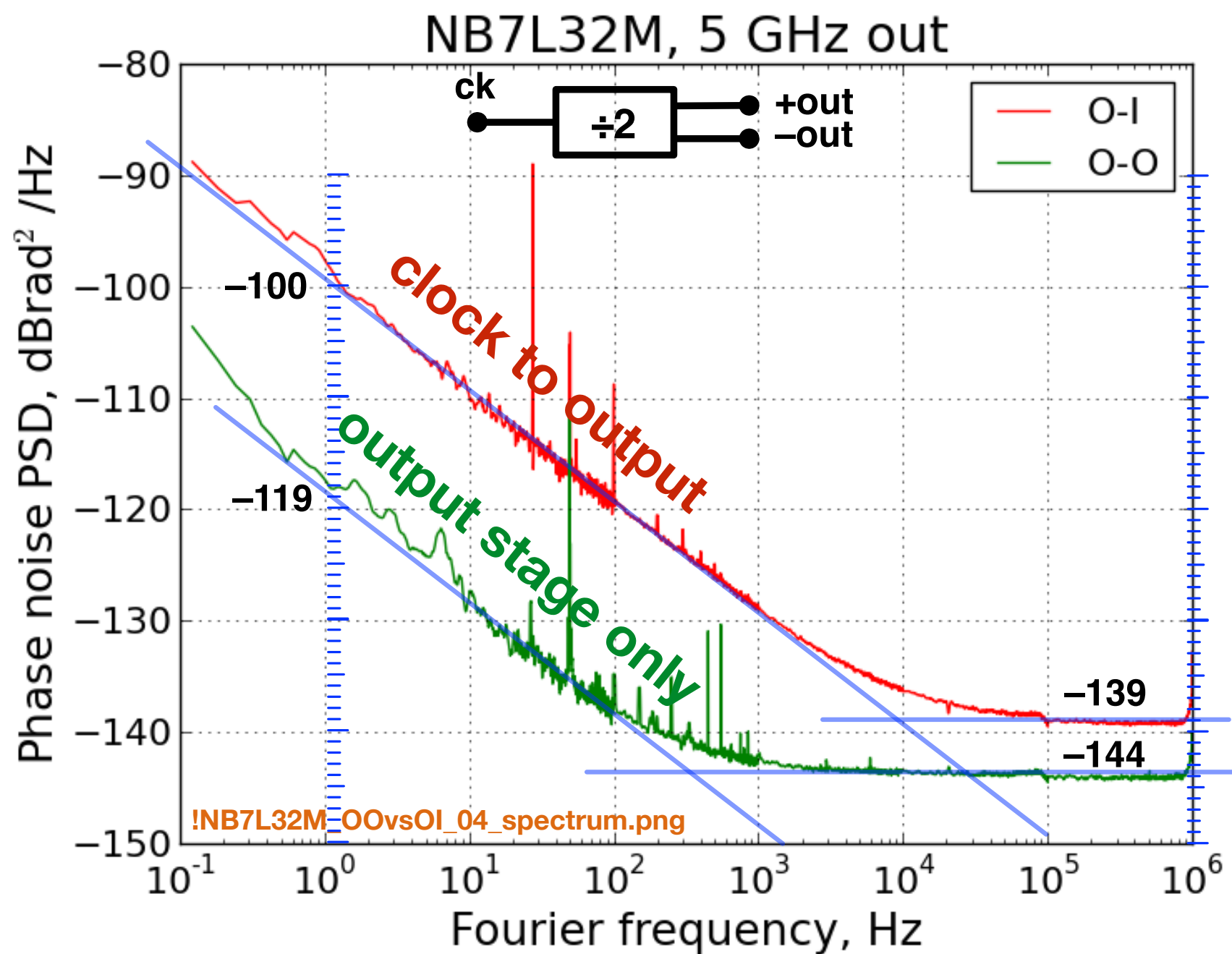


# NB7L32M $\div 2$ $\mu$ Wave Divider

$$10 \text{ GHz} \div 2 = 5 \text{ GHz}$$

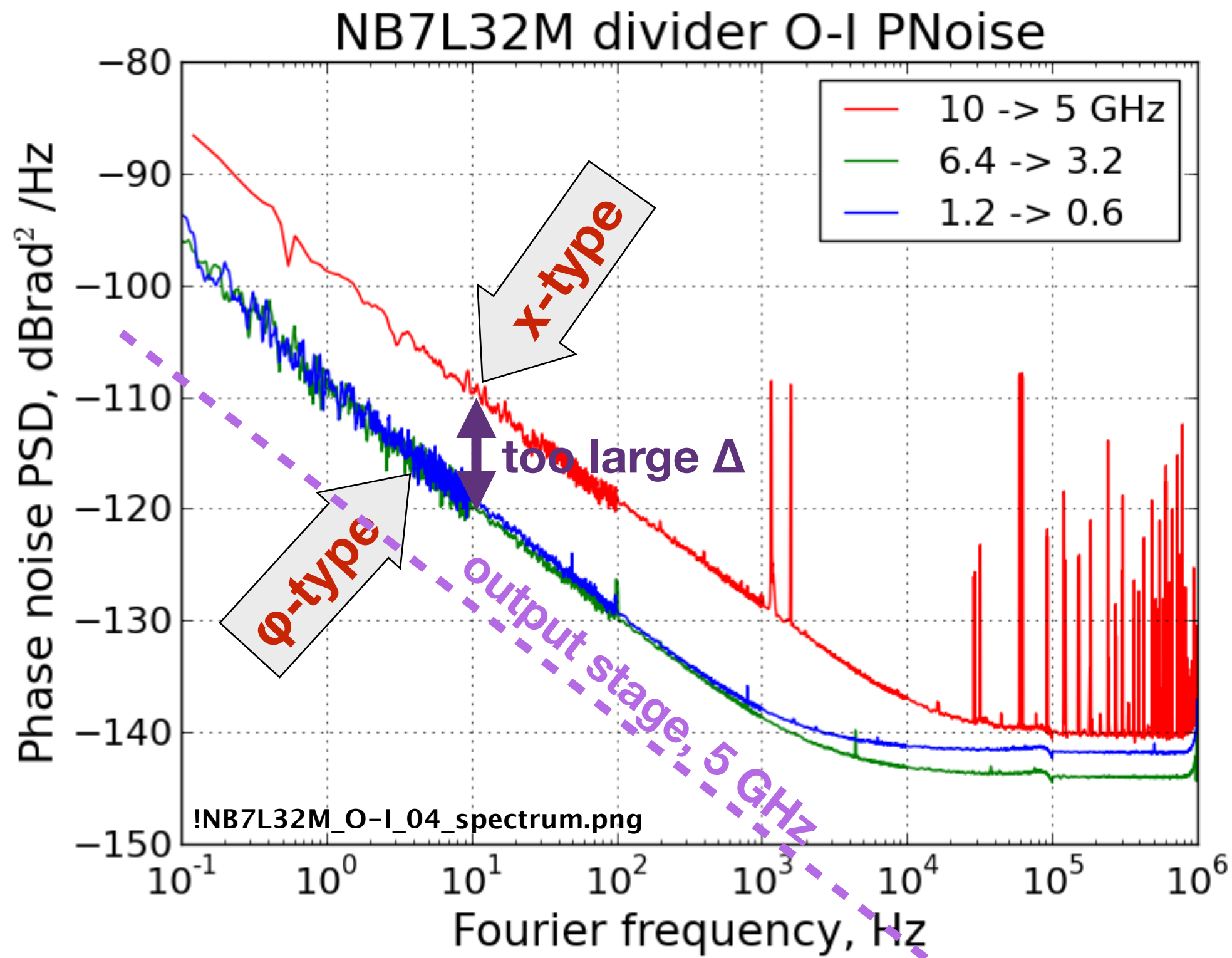
## Method

- Compare two dividers
- Use 5.01 GHz as a common oscillator, and beat
- Digital PM noise measurement at 10 MHz
- **O-I: Two equal dividers**
- **O-O: Two outputs of the same divider**
- Shown: spectrum of one divider



# NB7L32M $\div 2$ $\mu$ Wave Divider

Phase noise vs input frequency

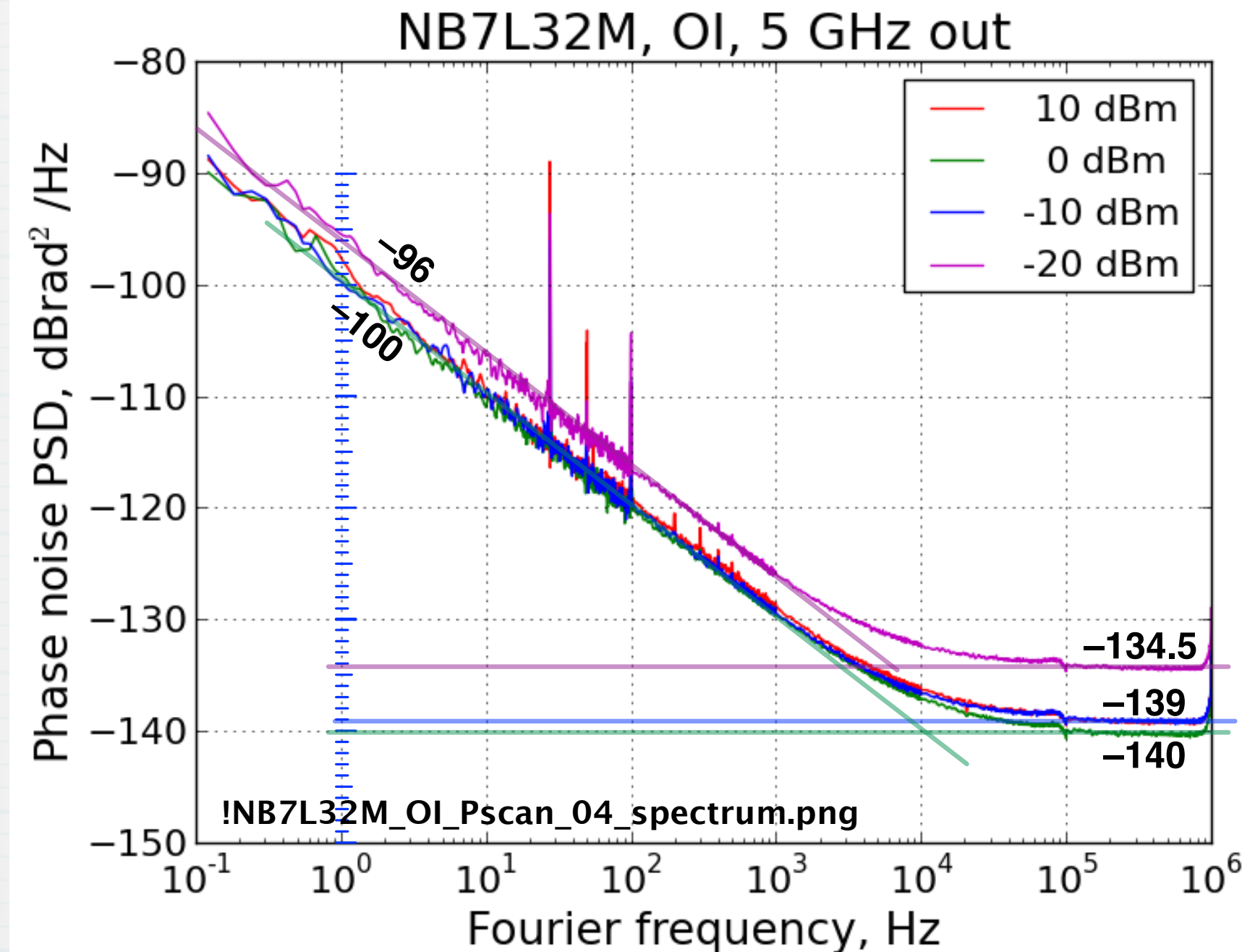


# NB7L32M $\div 2$ $\mu$ Wave Divider

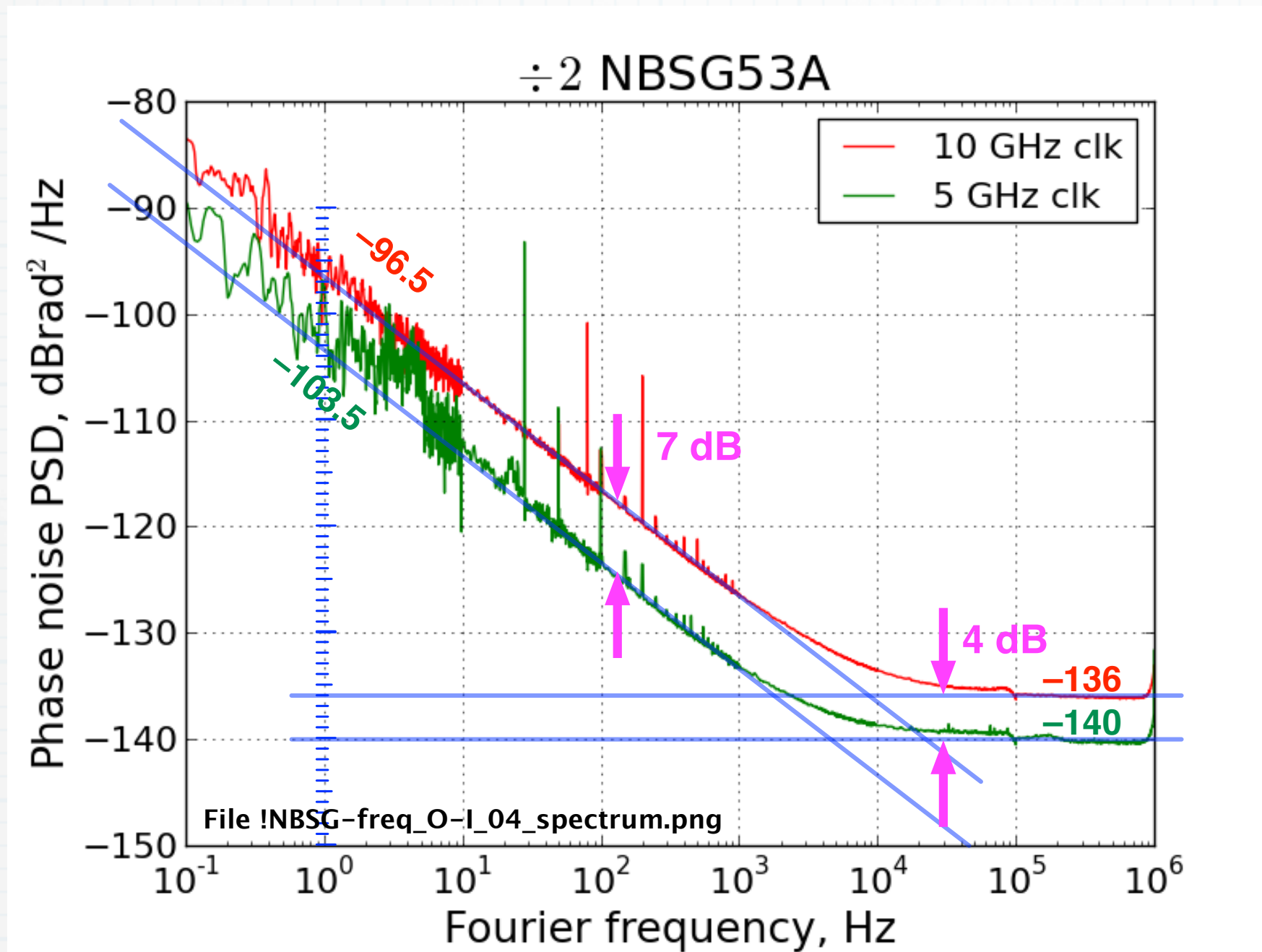
Works fairly well even at low input power (useful)

## Notes

- At  $-16$  dBm the white noise increases by 3 dB
- The critical power where  $(b_0)_\varphi = (b_0)_x$  is  $-16$  dBm
- Hence  $(b_0)_{x\text{-type}} \approx -140$  dB



# NBSG53A SiGe $\div 2$ $\mu$ Wave Divider



## Method

- Use **5.01** (**2.51**) GHz as a pivot oscillator, and mixers
- Digital PM noise measurement at 10 MHz
- One divider shown

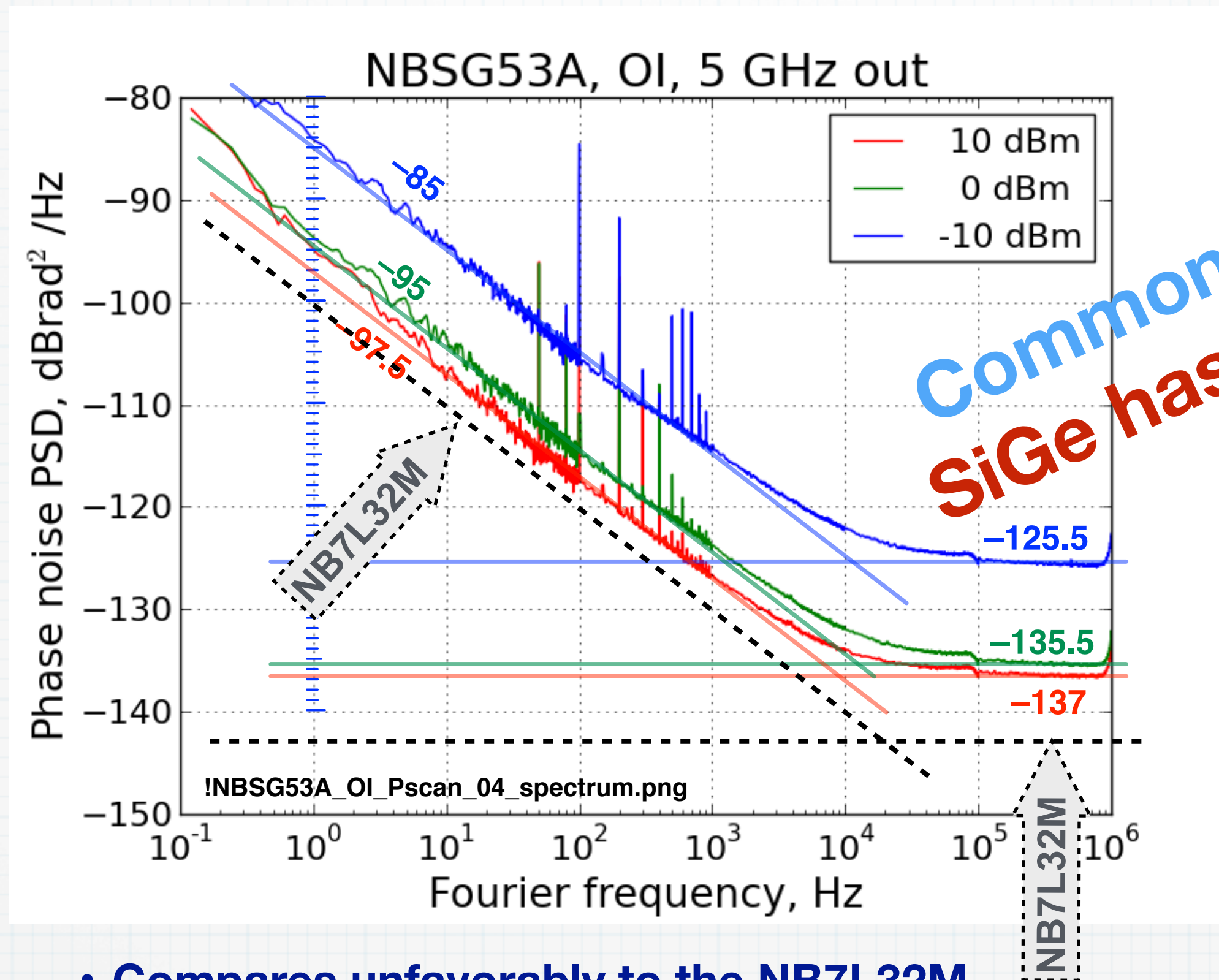
- $1/f$   $\rightarrow$  pure gearbox model (as expected)
- White  $\rightarrow$  aliased gearbox model (as expected)

Debugged:

- **-97.5 and -137** Likely, 1 dB discrepancy in w and  $1/f$  (mixer response)



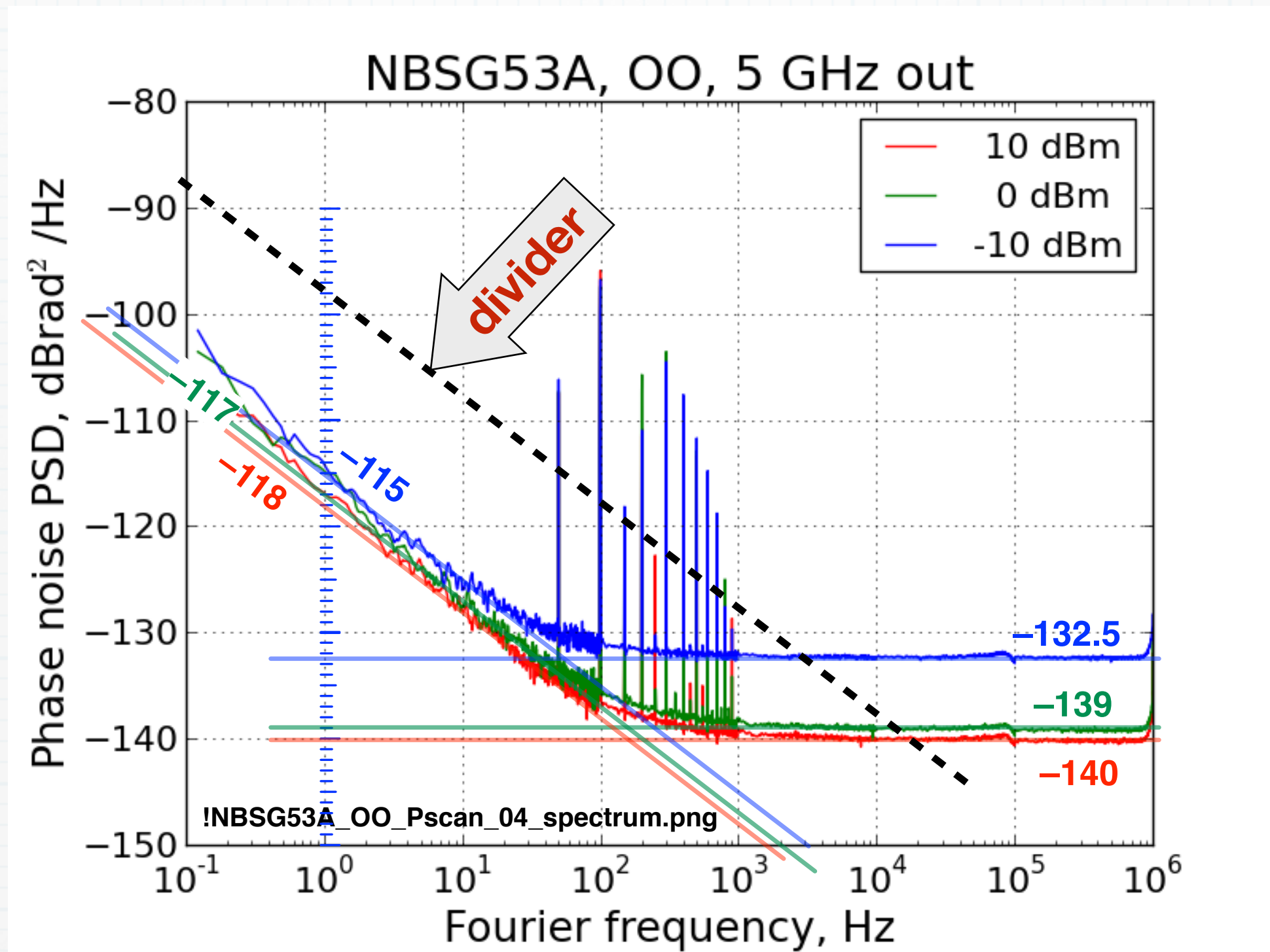
# NBSG53A SiGe $\div 2$ $\mu$ Wave Divider



- Compares unfavorably to the NB7L32M
- More noise and less tolerance to low power

# NBSG53A SiGe $\div 2$ $\mu$ Wave Divider

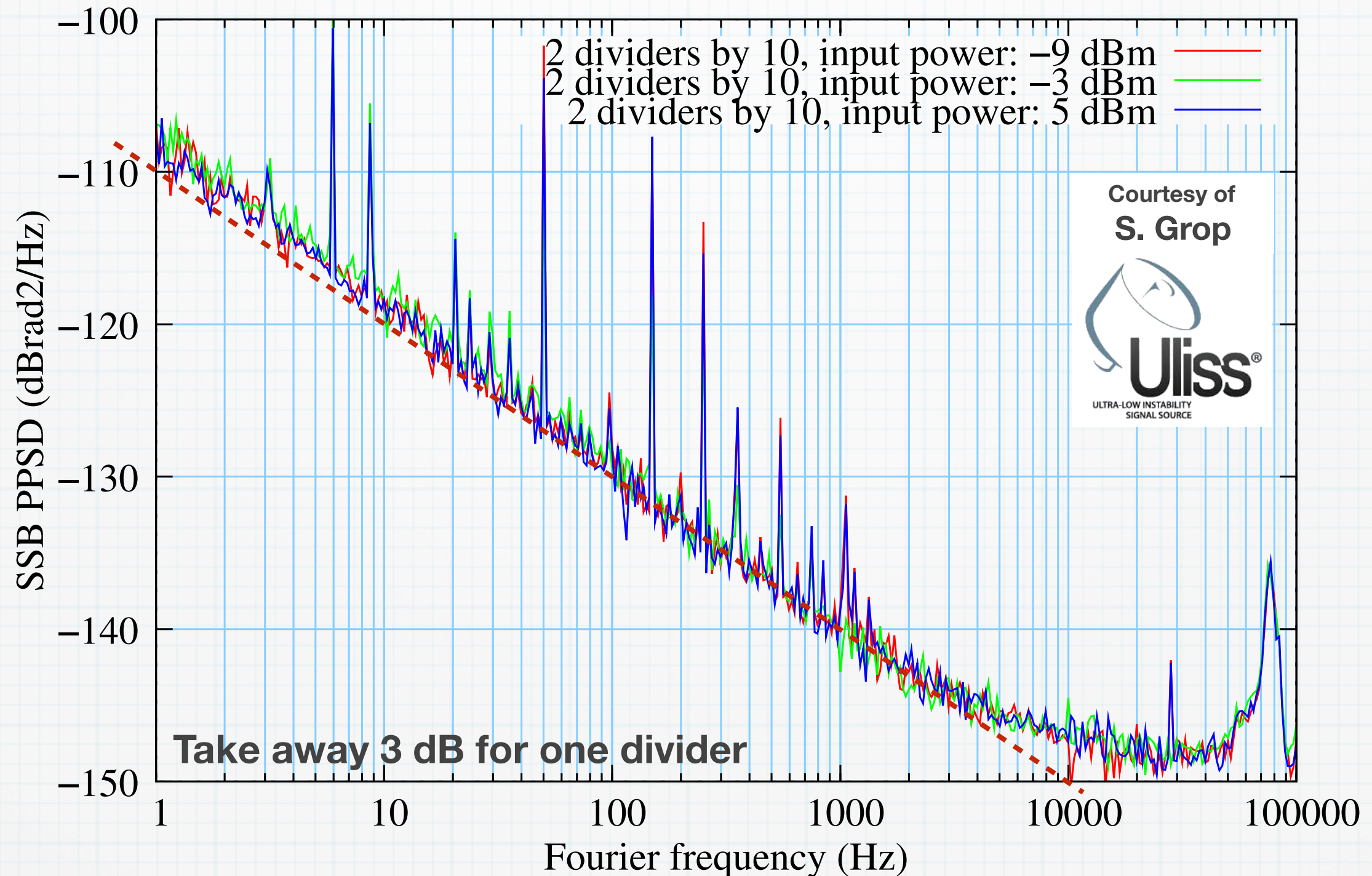
Output vs Output gives info about the output stage



The problem seems in the gear box, rather in the output stage

# Hittite HMC-C040, $\div 10$ Divider

2 Hittite divider by 10 HMC-C040,  $F_{in}=10\text{GHz}$



# Microwave Dividers Compared

	10 GHz ÷2	5 GHz ÷2	2.5 GHz ÷2	lower	
<b>NB7L32M</b>	<b>-100</b>	<b>-109</b>	<b>-109</b>	<b>-109</b>	<b>600 MHz</b> <b>1.2 GHz ÷2</b>
<b>NBSG53A</b>	<b>97.5</b>	<b>-103.5</b>		<b>-104</b>	<b>800 MHz</b> <b>1.6 GHz ÷2</b>
<b>HMC-C040</b>				<b>-113</b>	<b>1 GHz</b> <b>10 GHz ÷10</b>
<b>HMC705LP4</b>				<b>-121</b>	<b>0.5 GHz</b> <b>2.5 GHz ÷5</b>



# Suggested Readings

# Suggested Reading

Bernard Widrow,  
Istvan Kollar

*Quantization Noise*

Cambridge 2008

ISBN 978-0-511-40990-5

- Chapter 15: **Roundoff noise** in **FIR** digital filters and in **FFT** calculations
- Appendix G: **Quantization of a sinusoidal input**

## Quantization Noise

Roundoff Error in Digital Computation,  
Signal Processing, Control, and  
Communications

Bernard Widrow  
István Kollár

# Suggested Reading

**Walt Kester (editor)**

***Analog-Digital  
Conversion***

**Analog Devices 2004  
ISBN 0-916550-27-3**

**ANALOG-DIGITAL CONVERSION**

**Walt Kester**

**Editor**



# Suggested Reading

Marcel J.M. Pelgrom

## Analog-to-Digital Conversion



 Springer

**Marcel J. M. Pelgrom**

***Analog-to-Digital Conversion***

**Springer 2010**

**ISBN 978-90-481-8888-8**