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UNIVERSITÉ DE FRANCHE-COMTÉ

Photonic microwave oscillators

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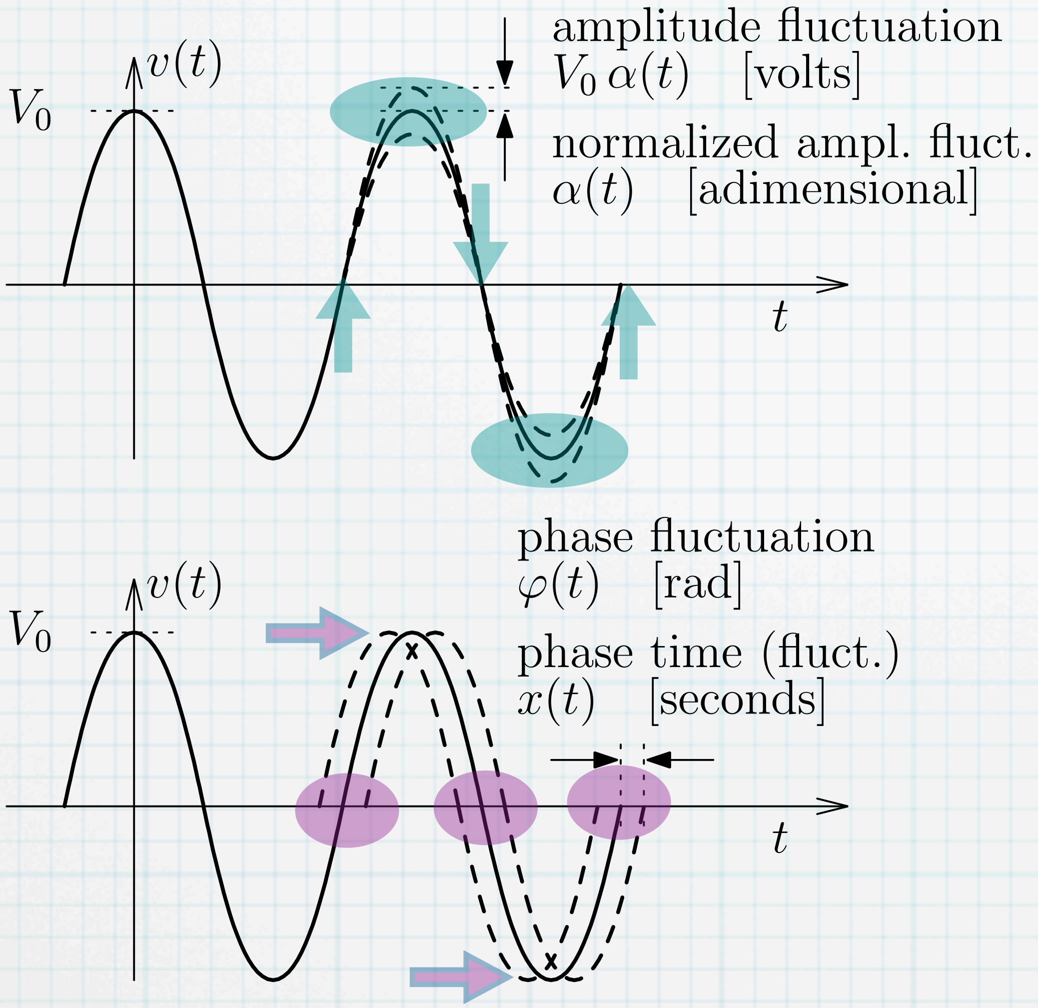
Outline

* Phase noise and frequency stability
* Delay-line instrument
* Correlation instrument
* Delay line oscillator
* Nonlinear AM oscillations
* Optical resonators

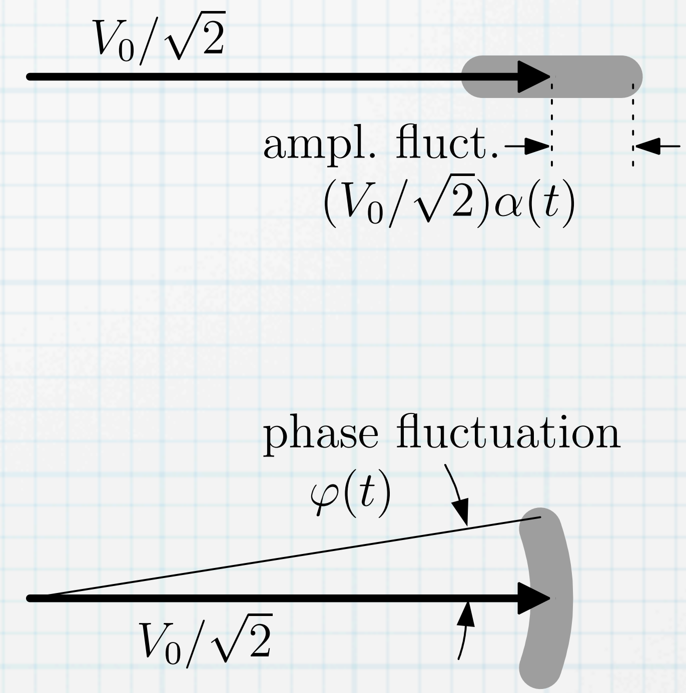
home page http://rubiola.org

Phase and amplitude noise noise

Time Domain



Phasor Representation



polar coordinates $$v(t) = V_0\,[1+\alpha(t)]\cos\,[\omega_0 t + \varphi(t)]$$

Cartesian coordinates $$v(t) = V_0\cos\omega_0 t + n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t$$

under low noise approximation

$$|n_c(t)| \ll V_0 \quad \text{and} \quad |n_s(t)| \ll V_0$$

It holds that

$$\alpha(t) = \frac{n_c(t)}{V_0} \quad \text{and} \quad \varphi(t) = \frac{n_s(t)}{V_0}$$

Phase noise & friends

random phase fluctuation

$$S_\varphi(f) = \text{PSD of } \varphi(t)$$

power spectral density

it is measured as

$$S_\varphi(f) = \mathbb{E}\left\{\Phi(f)\Phi^*(f)\right\}$$

$$S_\varphi(f) \approx \langle\Phi(f)\Phi^*(f)\rangle_m$$

$$\mathcal{L}(f) = \frac{1}{2} S_\varphi(f) \quad \text{dBc}$$

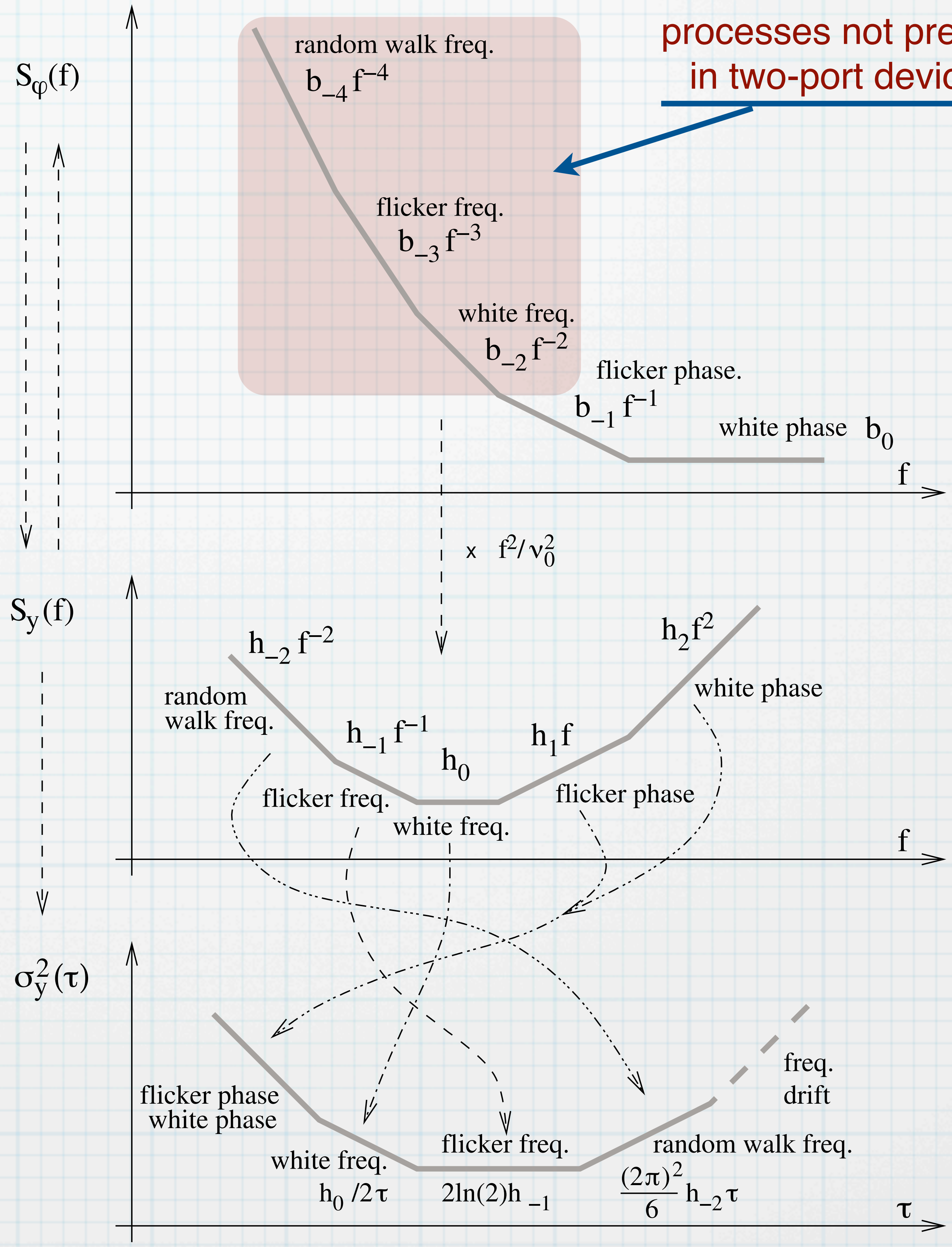
random fractional-frequency fluctuation

$$y(t) = \frac{\dot{\varphi}(t)}{2\pi\nu_0} \quad \Rightarrow \quad S_y = \frac{f^2}{\nu_0^2} S_\varphi(f)$$

Allan variance
(two-sample wavelet-like variance)

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{y}_{k+1} - \overline{y}_k\right]^2\right\}.$$

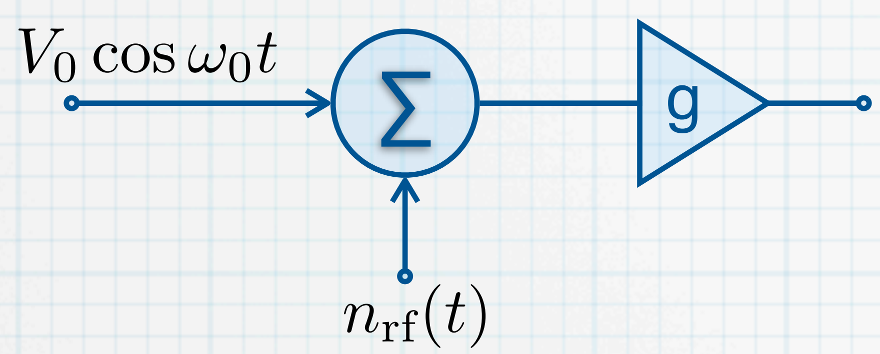
approaches a half-octave bandpass filter (for white),
hence it converges for processes steeper than 1/f

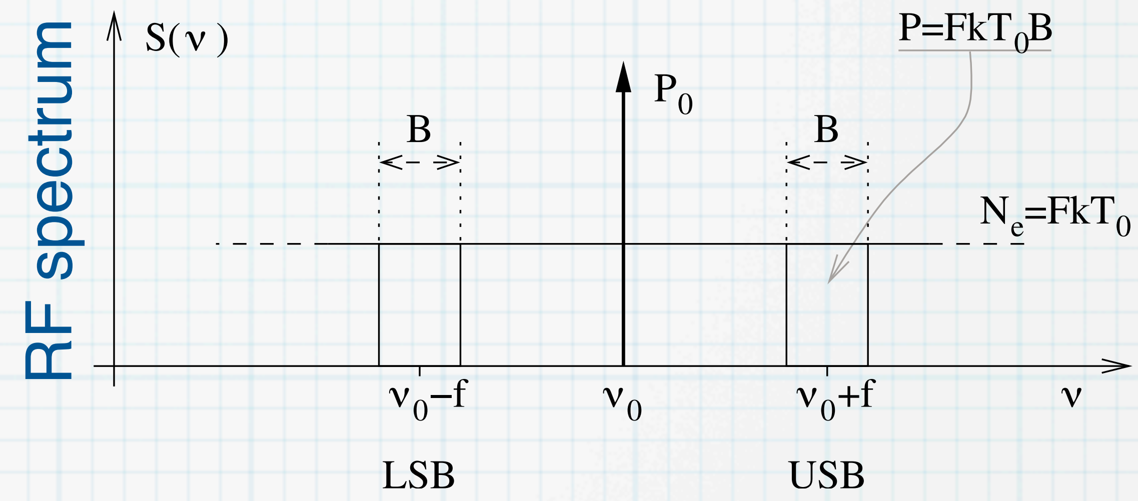


E. Rubiola, *Phase Noise and Frequency Stability in Oscillators*, Cambridge 2008

Amplifier white noise

Noise figure F, Input power P_0



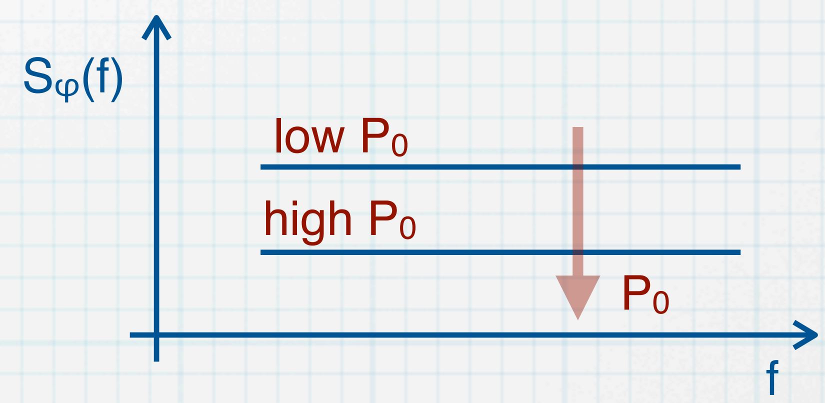


power law

$$S_\varphi = \sum_{i=-4}^{0} b_i f^i$$

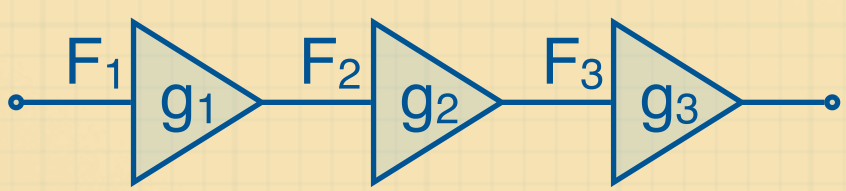
white phase noise

$$b_0 = \frac{FkT_0}{P_0}$$



Cascaded amplifiers (Friis formula)

The (phase) noise is chiefly that of the 1st stage



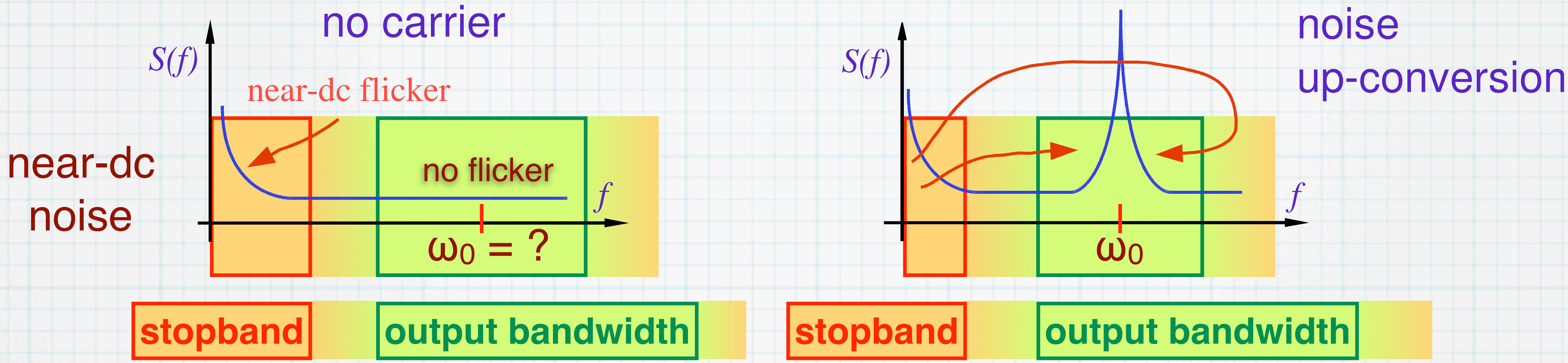
$$N = F_1 kT_0 + \frac{(F_2 - 1)kT_0}{g_1^2} + \ldots$$

H. T. Friis, Proc. IRE **32** p.419-422, jul 1944

The Friis formula applied to phase noise

$$b_0 = \frac{F_1 kT_0}{P_0} + \frac{(F_2 - 1)kT_0}{P_0 g_1^2} + \ldots$$

Amplifier flicker noise



carrier **near-dc noise**

$$v_i(t) = V_i\, e^{j\omega_0 t} + n'(t) + jn''(t)$$

the parametric nature of 1/f noise is hidden in n' and n''

substitute
(careful, this hides the down-conversion)

$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + \dots$$

non-linear (parametric)amplifier

expand and select the ω_0 terms

$$v_o(t) = V_i \Big\{ a_1 + 2a_2 \big[n'(t) + jn''(t) \big] \Big\} e^{j\omega_0 t}$$

The noise sidebands are proportional to the input carrier

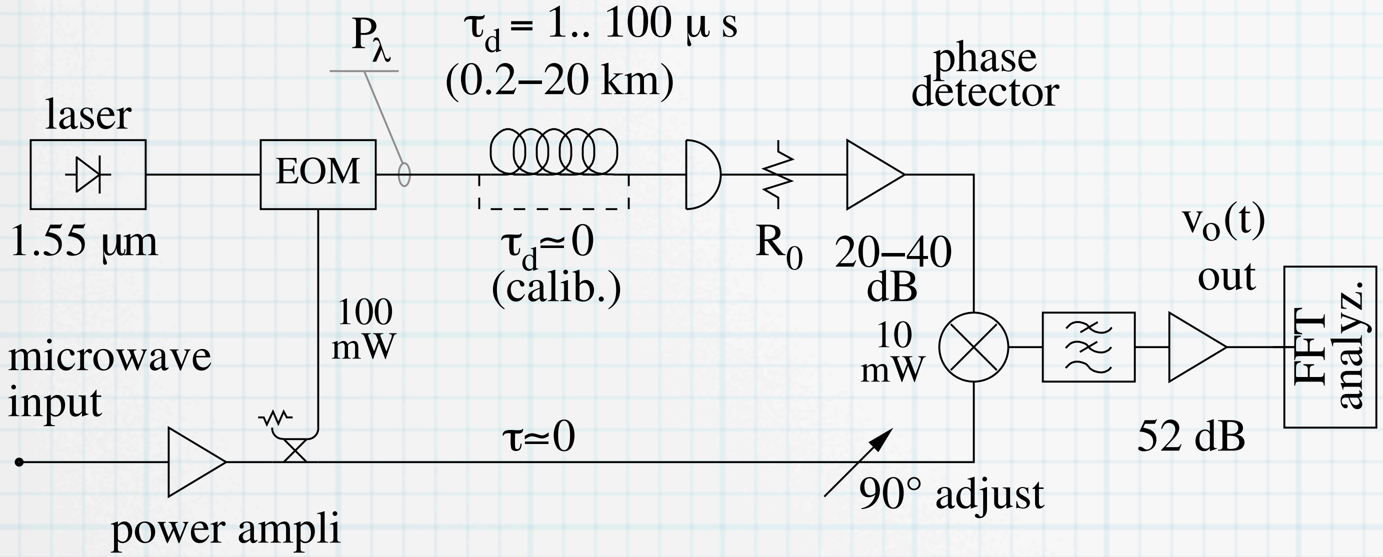
get AM and PM noise

$$\alpha(t) = 2\, \frac{a_2}{a_1}\, n'(t) \qquad \varphi(t) = 2\, \frac{a_2}{a_1}\, n''(t)$$

The AM and the PM noise are independent of V_i , thus of power

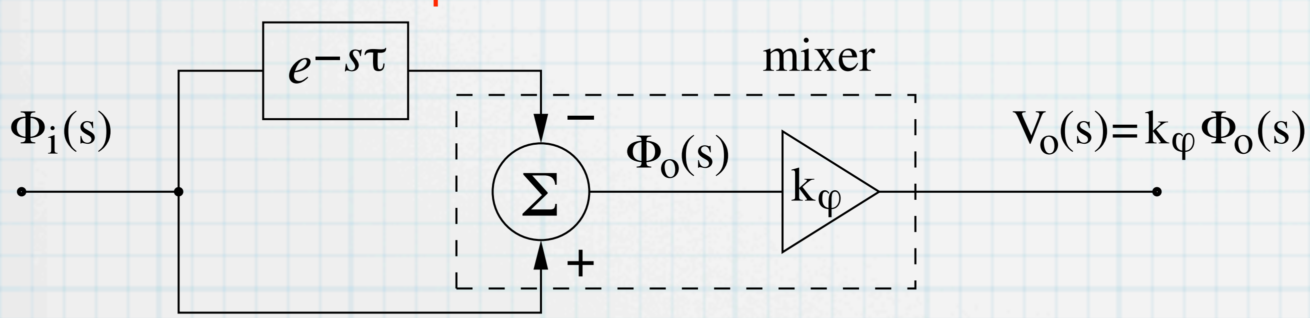
Delay line theory

Rubiola-Salik-Huang-Yu-Maleki, JOSA-B 22(5) p.987–997 (2005)



Note that here one arm is a microwave cable

Laplace transforms



$$\Phi_o(s) = (1-e^{-s\tau})\,\Phi_i(s)$$

- **delay –> frequency-to-phase conversion**
- **works at any frequency**
- **long delay (microseconds) is necessary for high sensitivity**
- **the delay line must be an optical fiber**
  fiber: attenuation 0.2 dB/km, thermal coeff. 6.8 10^{-6}/K
  cable: attenuation 0.8 dB/m, thermal coeff. ~ 10^{-3}/K

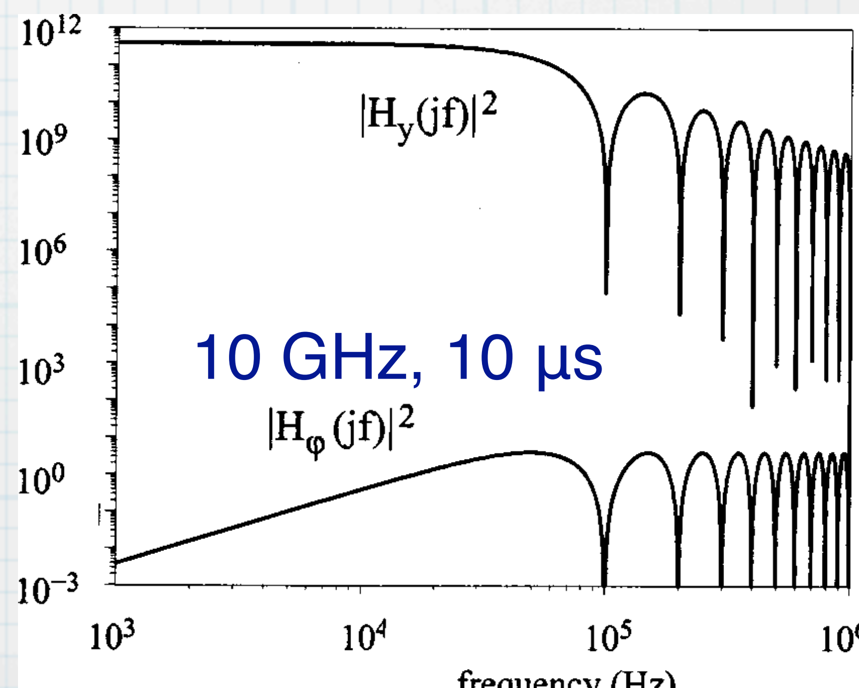
Laplace transforms

$$\Phi(s) = H_\varphi(s)\Phi_i(s)$$

$$|H_\varphi(f)|^2 = 4\sin^2(\pi f \tau)$$

$$S_y(f) = |H_y(f)|^2\, S_{\varphi\, i}(s)$$

$$|H_y(f)|^2 = \frac{4\nu_0^2}{f^2}\sin^2(\pi f \tau)$$



White noise

intensity modulation $$P(t) = \overline{P}(1 + m\cos\omega_\mu t)$$

photocurrent $$i(t) = \frac{q\eta}{h\nu}\,\overline{P}(1 + m\cos\omega_\mu t)$$

microwave power $$\overline{P}_\mu = \frac{1}{2}m^2 R_0 \left(\frac{q\eta}{h\nu}\right)^2 P^2$$

shot noise $$N_s = 2\frac{q^2\eta}{h\nu}\,\overline{P}R_0$$

thermal noise $$N_t = FkT_0$$

total white noise (one detector)

shot **thermal**

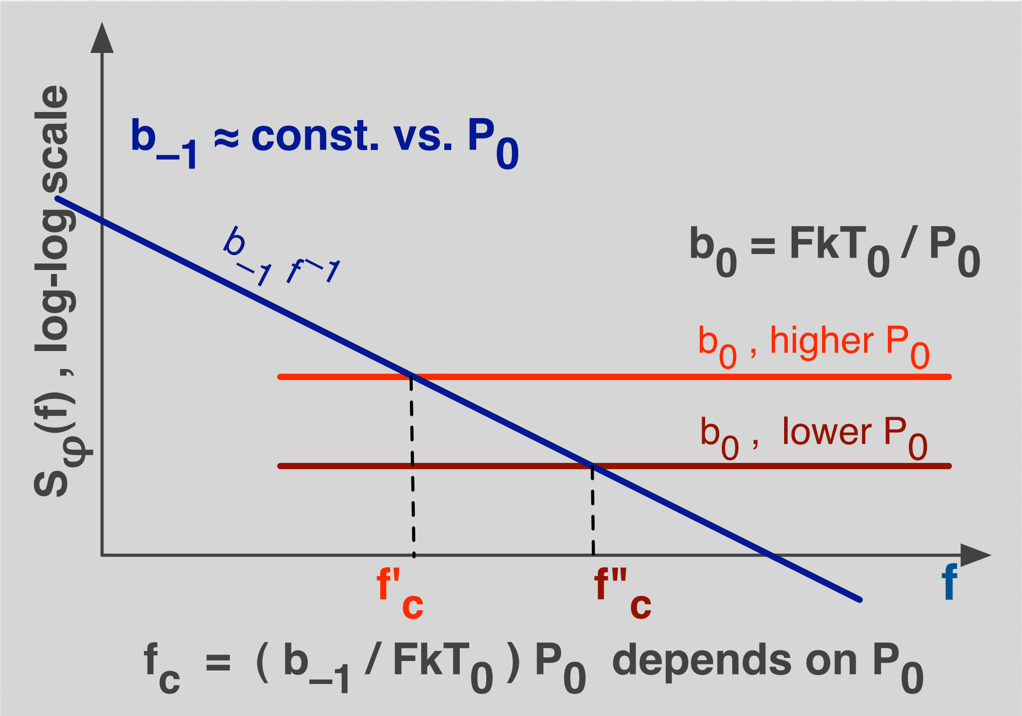
$$S_{\varphi 0} = \frac{2}{m^2}\left[2\frac{h\nu_\lambda}{\eta}\,\frac{1}{\overline{P}} + \frac{FkT_0}{R_0}\left(\frac{h\nu_\lambda}{q\eta}\right)^2\left(\frac{1}{\overline{P}}\right)^2\right]$$

total white noise (P/2 each detector)

$$S_{\varphi 0} = \frac{16}{m^2}\left[\frac{h\nu_\lambda}{\eta}\,\frac{1}{\overline{P}} + \frac{FkT_0}{R_0}\left(\frac{h\nu_\lambda}{q\eta}\right)^2\left(\frac{1}{\overline{P}}\right)^2\right]$$

Flicker (1/f) noise

- **experimentally determined (takes skill, time and patience)**
- **amplifier GaAs: $b_{-1} \approx -100$ to -110 dBrad²/Hz,**
  SiGe: $b_{-1} \approx -120$ dBrad²/Hz
- **photodetector $b_{-1} \approx -120$ dBrad²/Hz**
  Rubiola & al. IEEE Trans. MTT (& JLT) 54 (2) p.816–820 (2006)
- **mixer $b_{-1} \approx -120$ dBrad²/Hz**
- **contamination from AM noise (delay => de-correlation => no sweet point**
  (Rubiola-Boudot, IEEE Transact UFFC 54(5) p.926–932 (2007)
- **optical fiber**
- **The phase flicker coefficient b_{-1} is about independent of power**
- **in a cascade, $(b_{-1})_{tot}$ adds up, regardless of the device order**



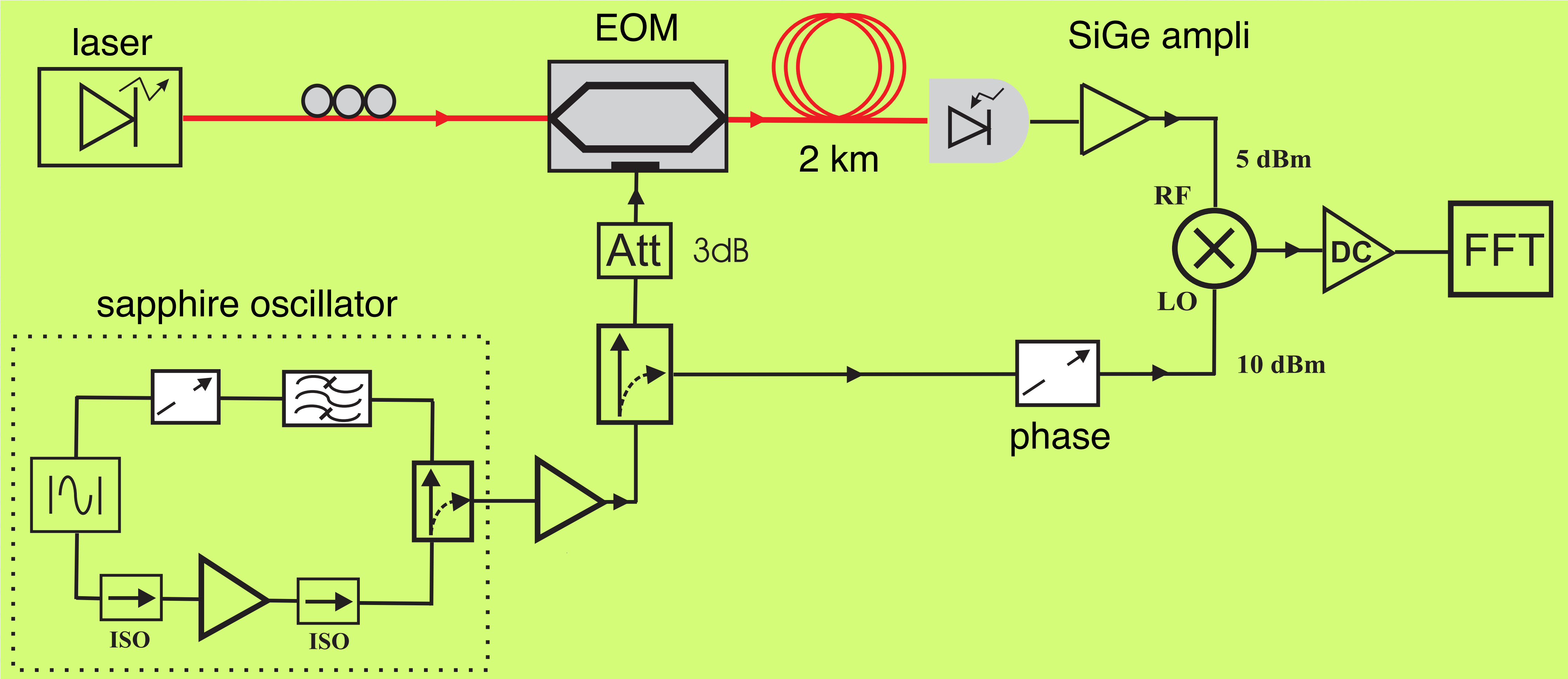
The Friis formula applies to white phase noise

$$b_0 = \frac{F_1 k T_0}{P_0} + \frac{(F_2 - 1) k T_0}{P_0 g_1^2} + \dots$$

In a cascade, the 1/f noise just adds up

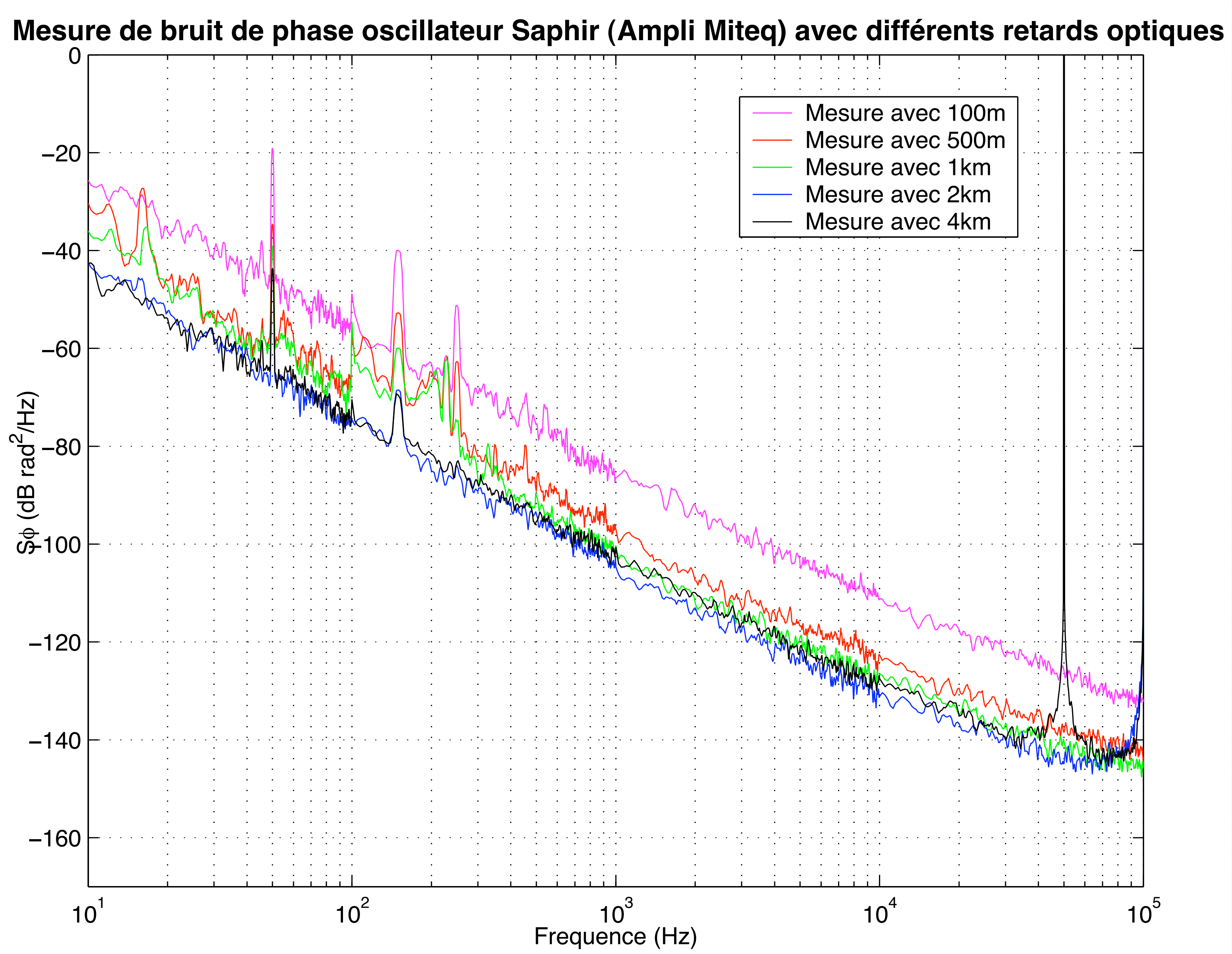
$$(b_{-1})_{\text{tot}} = \sum_{i=1}^{m} (b_{-1})_i$$

Single-channel instrument



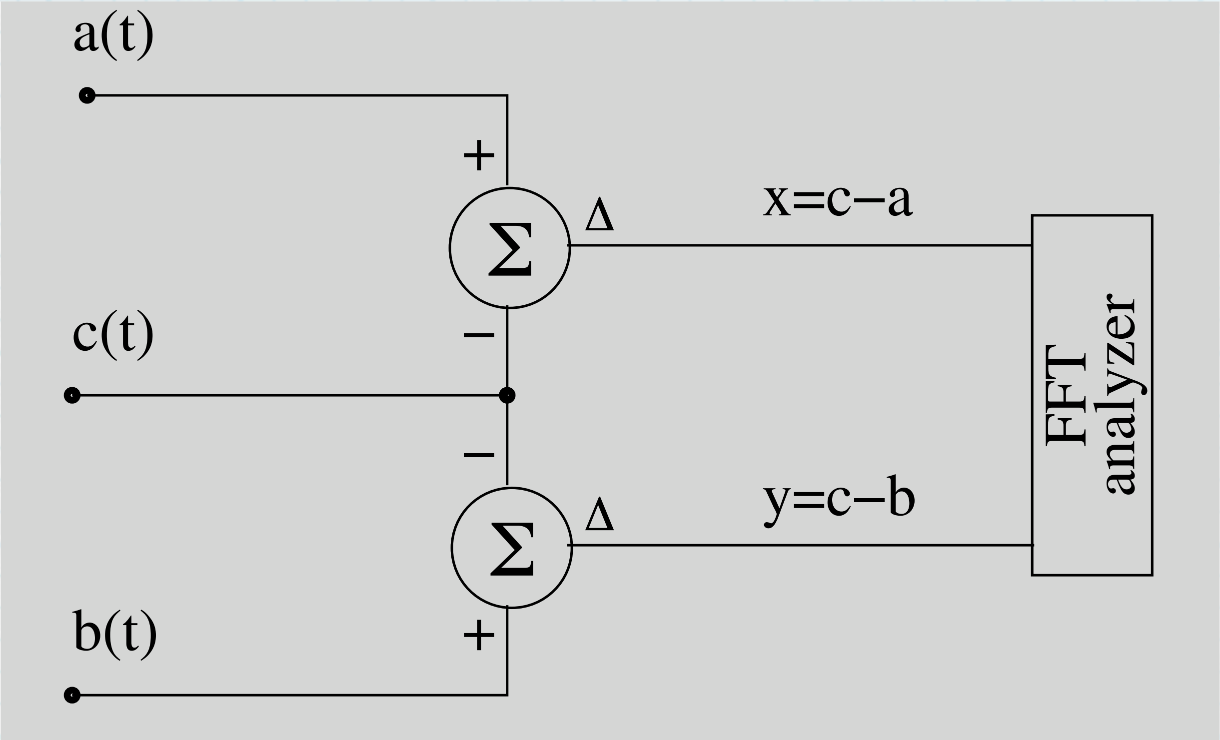
- The laser RIN can limit the instrument sensitivity
- In some cases, the AM noise of the oscillator under test turns into a serious problem (got in trouble with an Anritsu synthesizer)

Measurement of a sapphire oscillator



- **The instrument noise scales as 1/τ, yet the blue and black plots overlap**
  magenta, red, green => instrument noise
  blue, black => noise of the sapphire oscillator under test
- **We can measure the $1/f^3$ phase noise (frequency flicker) of a 10 GHz sapphire oscillator (the lowest-noise microwave oscillator)**
- **Low AM noise of the oscillator under test is necessary**

Basics of correlation spectrum measurements



phase noise measurements		
DUT noise, normal use	a, b c	instrument noise DUT noise
background, ideal case	a, b c = 0	instrument noise no DUT
background, with AM noise	a, b c ≠ 0	instrument noise AM-to-DC noise

$S_{yx} = \mathbb{E}\left\{YX^*\right\}$ W. K. theorem

$S_{yx} = \left\langle YX^*\right\rangle_m$ measured, m samples

a, b and c are incorrelated

expand $X = C - A$ and $Y = C - B$

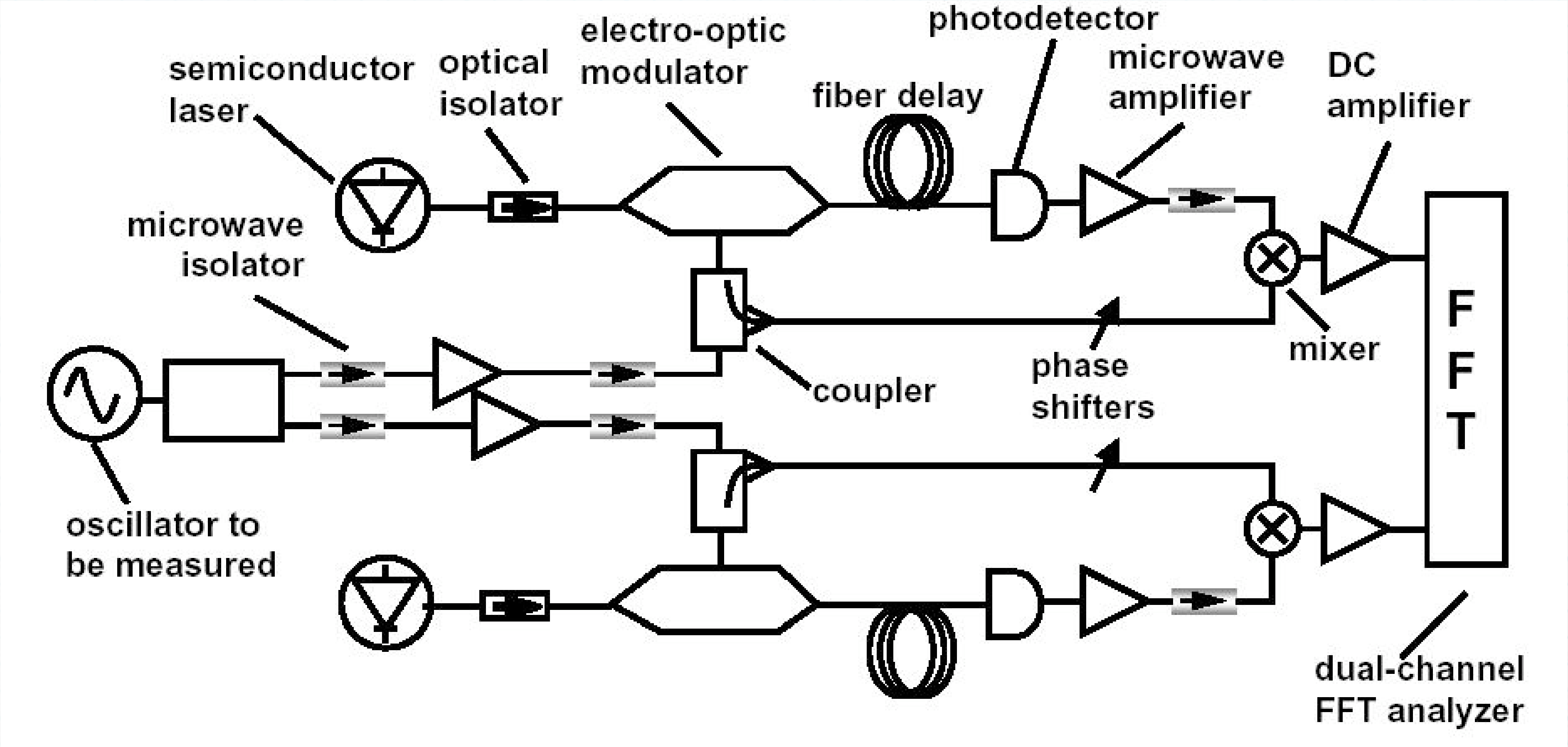
$S_{yx} = S_{cc}$ a, b, c independent

$S_{yx} = S_{cc} + O(\sqrt{1/m})$ measured, m samples

Averaging on a sufficiently large number *m* of spectra is necessary to reject the single-channel noise

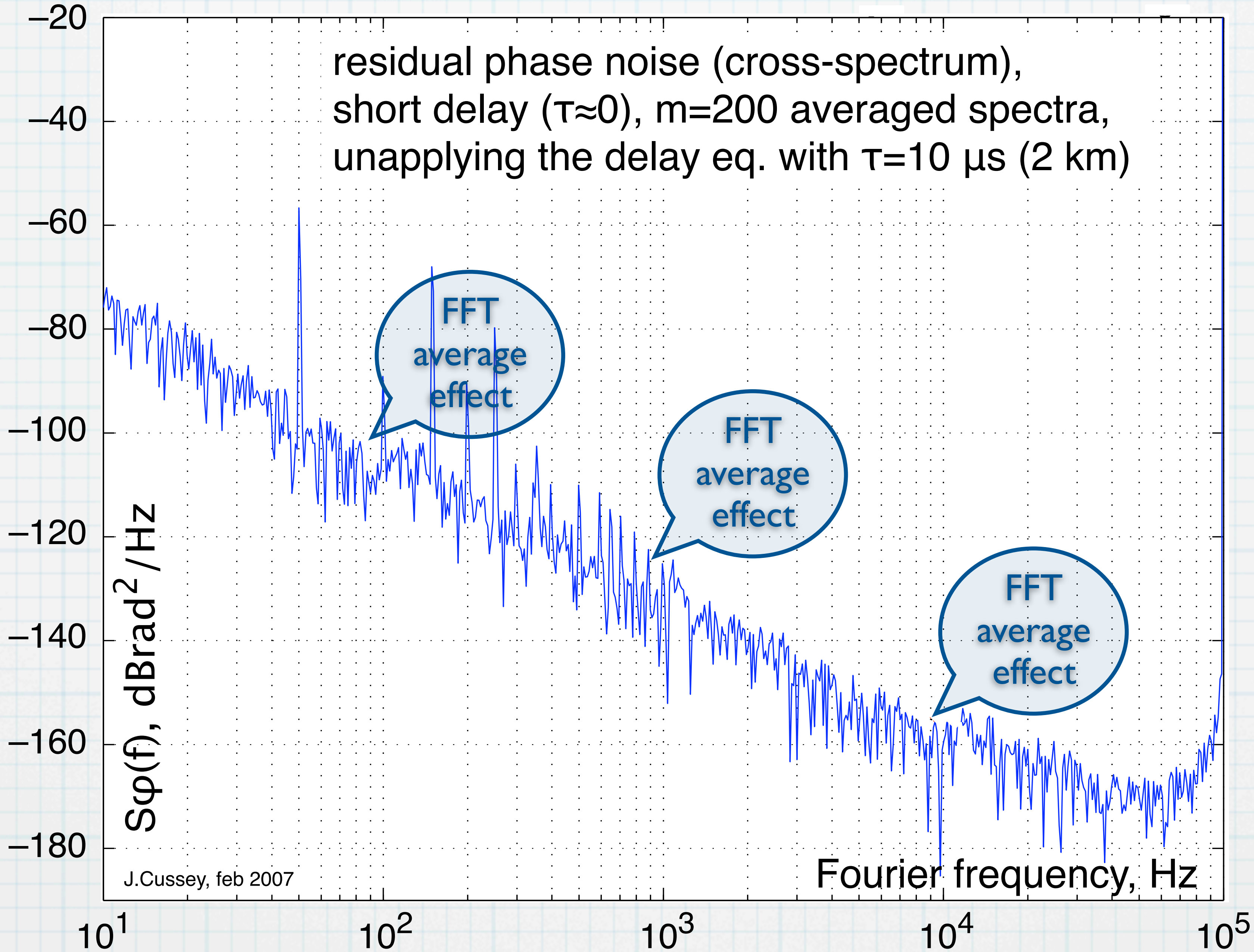
Dual-channel (correlation) instrument

Salik, Yu, Maleki, Rubiola, Proc. Ultrasonics-FCS Joint Conf., Montreal, Aug 2004 p.303-306



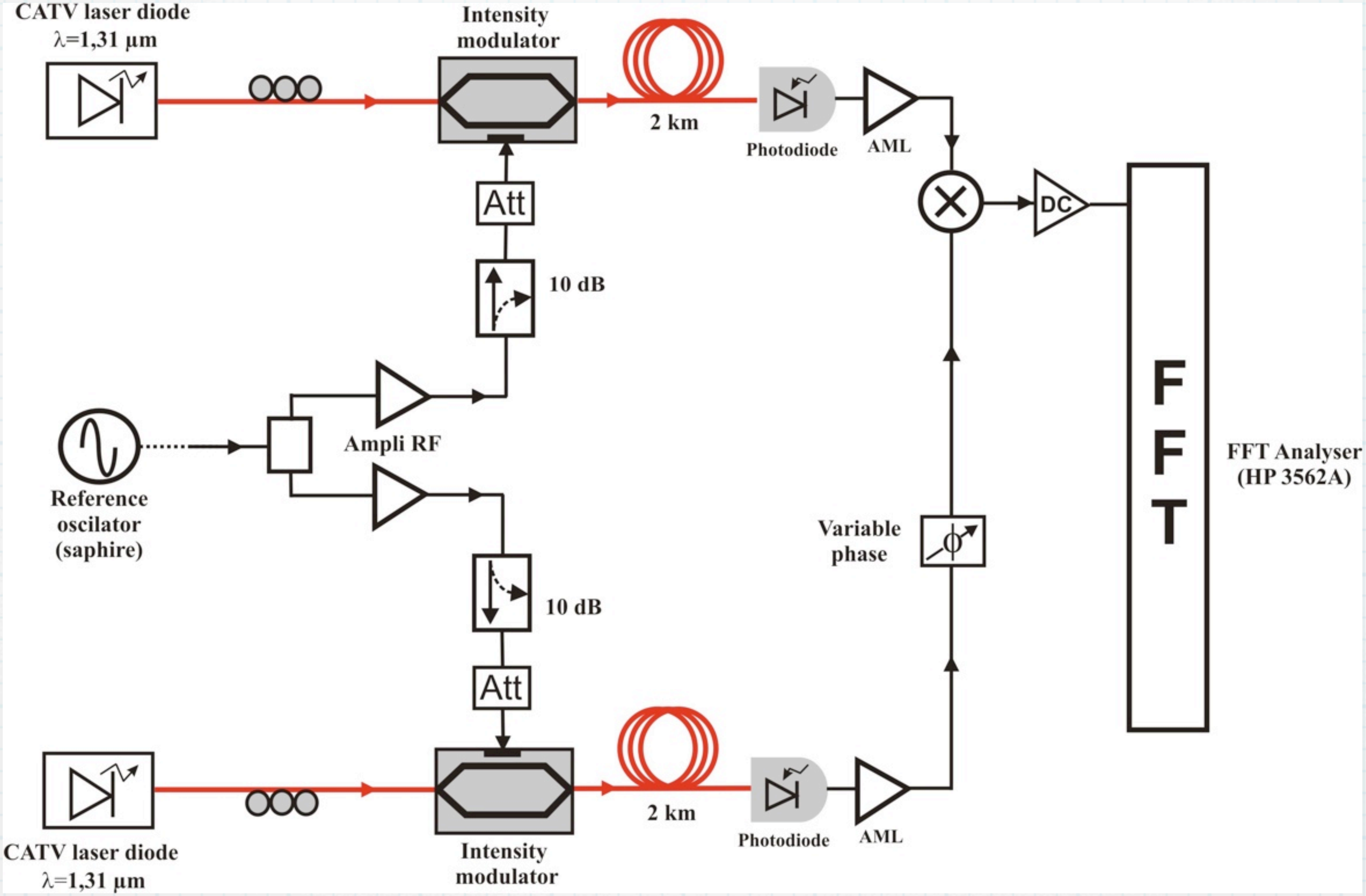
- uses cross spectrum to reduce the background noise
- requires two fully independent channels
- separate lasers for RIN rejection
- optical-input version is not useful because of the insufficient rejection of AM noise
- implemented at the FEMTO-ST Institute

Dual-channel (correlation) measurement



the residual noise is clearly limited by the number of averaged spectra, m=200

Measurement of the optical-fiber noise

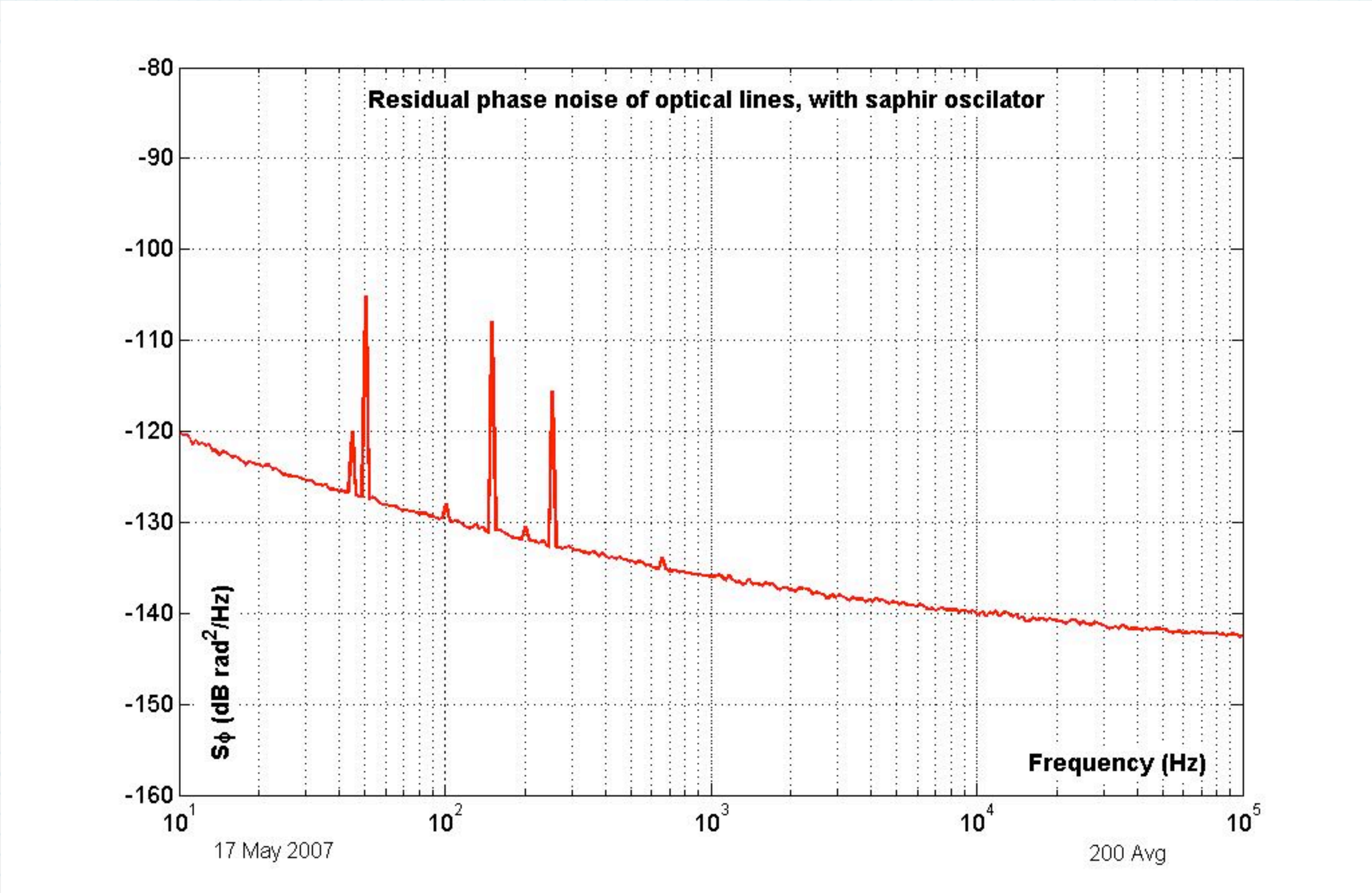


- matching the delays, the oscillator phase noise cancels
- this scheme gives the **total** noise

  2 × (ampli + fiber + photodiode + ampli) + mixer

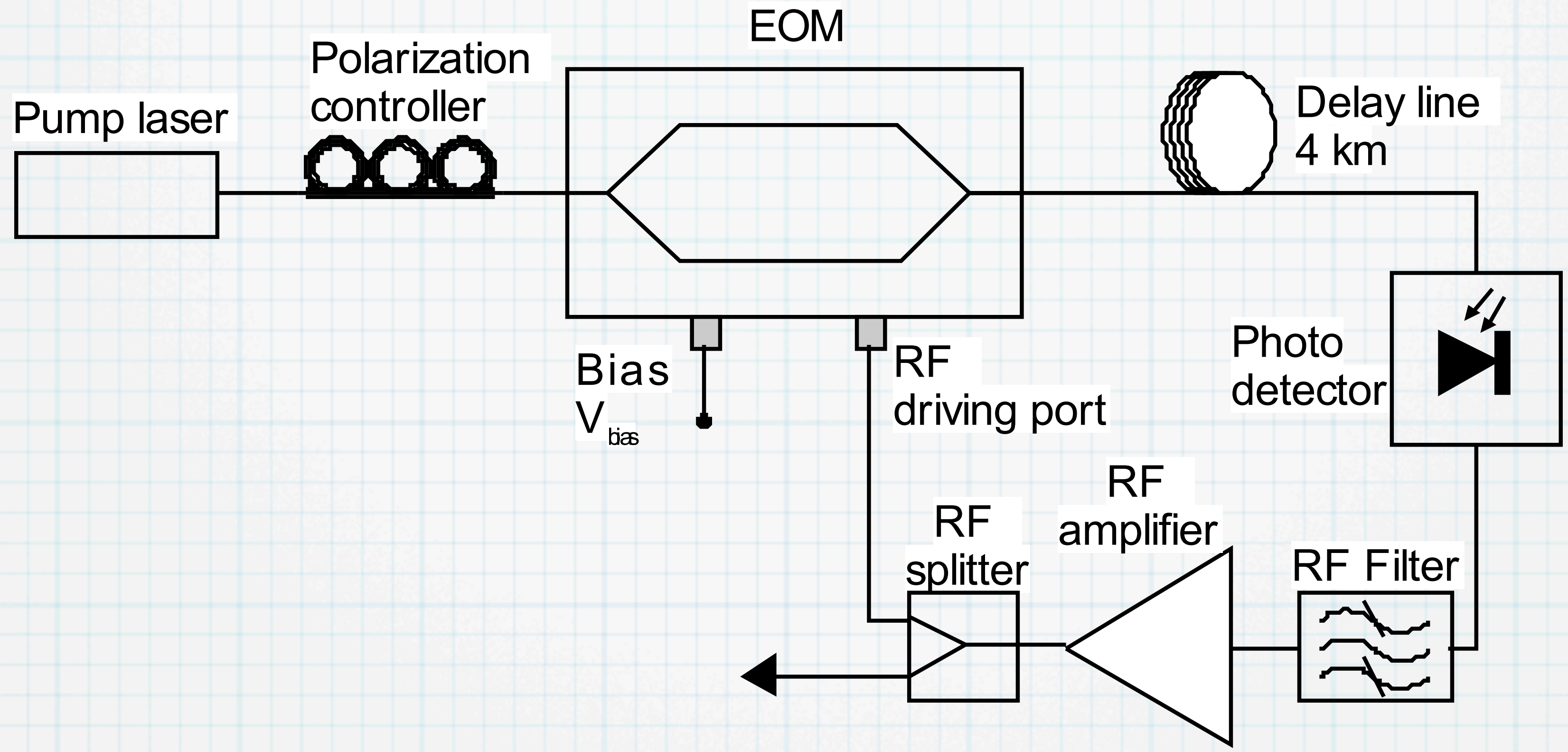
  thus it enables only to assess an upper bound of the fiber noise

Phase noise of the optical fiber



- **The method enables only to assess an upper bound of the fiber noise** **$b_{-1} \leq 5\times10^{-12}$ rad²/Hz for L = 2 km (–113 dBrad²/Hz)**
- **We believe that this residual noise is the signature of the two GaAs power amplifier that drives the MZ modulator**

Delay-line oscillator



$$Q_{\mathrm{eq}} = \pi \nu_0 \tau$$

$Q_{eq}=3\times10^5 \leftarrow L=4$km

$$f_L = \frac{\nu_0}{2Q}$$

$$f_L = \frac{1}{4\pi^2\tau^2}$$

$f_L=8$kHz

$$h_{-1} = b_{-3}/\nu_0^2$$ 6.3×10^{-24}

$$\sigma_y^2 = 2\ln(2)\, h_{-1}$$ 8.8×10^{-24}

10^{-11}

Leeson formula

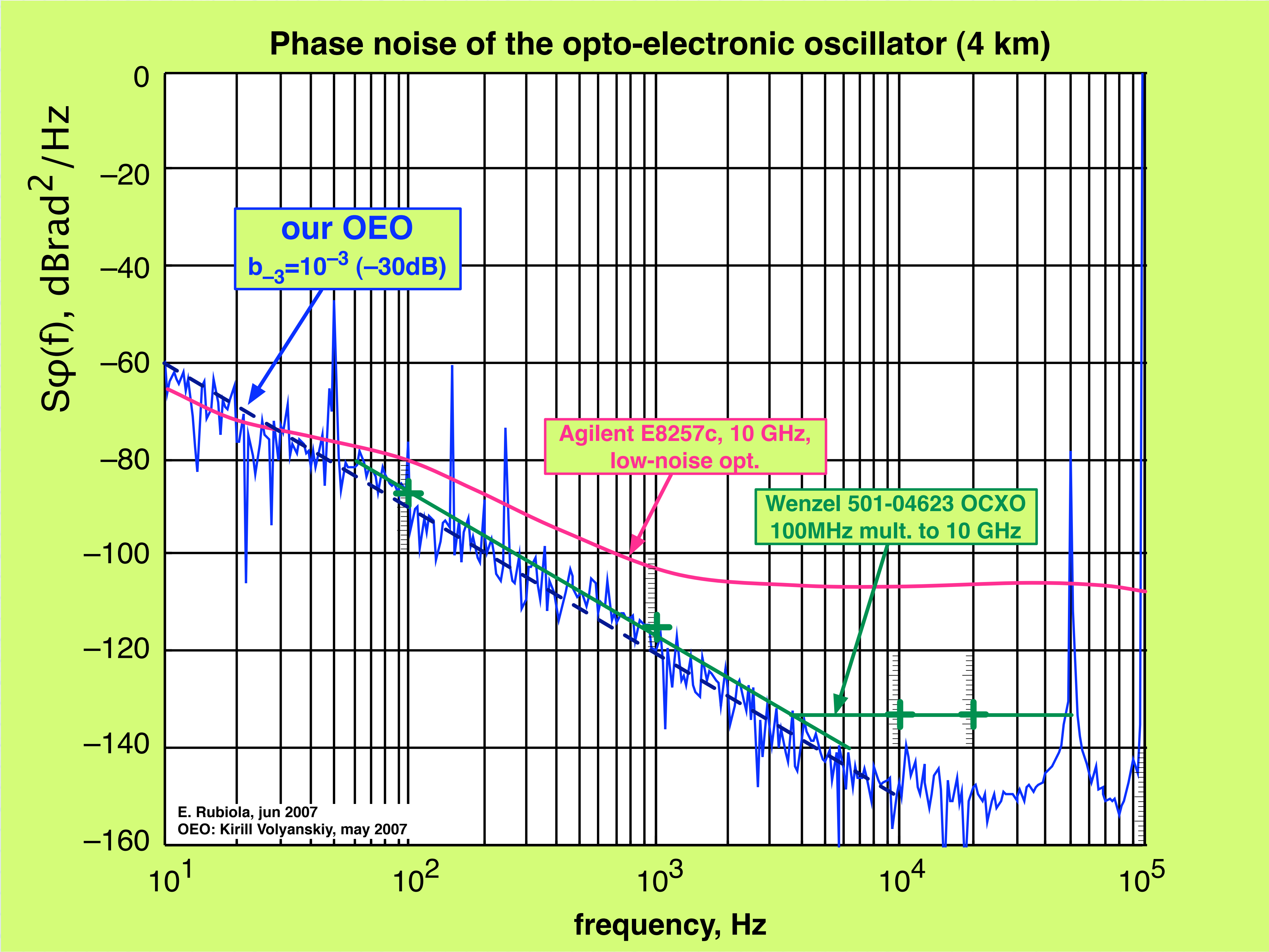
$$S_\varphi(f) \simeq \frac{f_L^2}{f^2}\, S_\psi(f) \quad \text{for } f \ll f_L$$

$b_{-3} = 6.3\times10^{-4}$ (–32 dB)

$$\sigma_y \simeq 2.9\times10^{-12}$$ **Allan deviation**

E. Rubiola, *Phase Noise and Frequency Stability in Oscillators*, Cambridge 2008

Delay-line oscillator



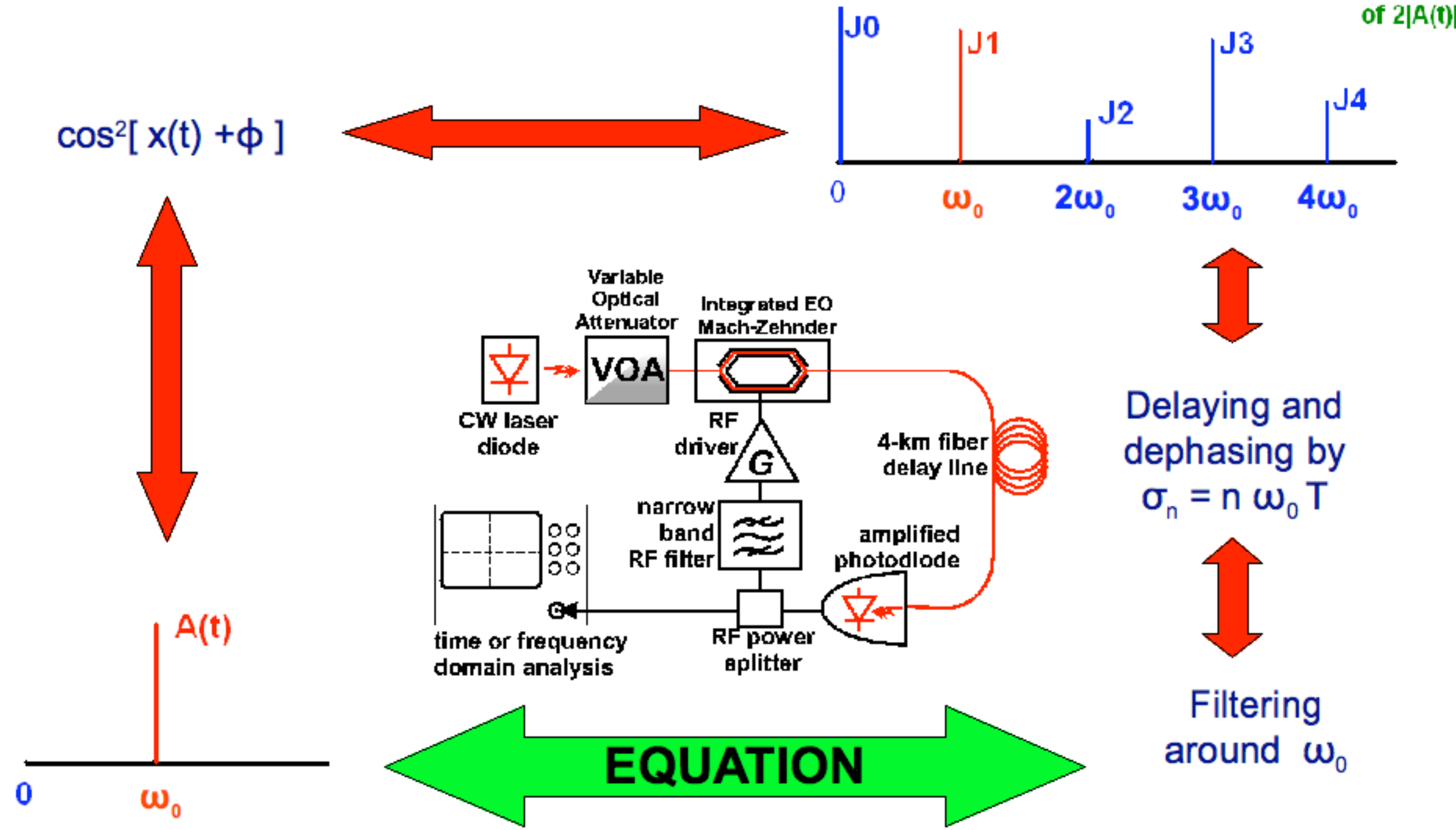
- 1.310 nm DFB CATV laser
- Photodetector DSC 402 (R = 371 V/W)
- RF filter ν_0 = 10 GHz, Q = 125
- RF amplifier AML812PNB1901 (gain +22dB)

expected phase noise
$b_{-3} \approx 6.3\times10^{-4}$ (–32 dB)

Nonlinear model

Anger-Jacobi expansion

$$e^{iz\cos\alpha} = \sum_{n=-\infty}^{+\infty} i^n J_n(z) e^{in\alpha}$$



A complex envelope equation

The complex envelope amplitude of the microwave obeys the equation

$$\dot{\mathcal{A}} = -\mu\mathcal{A} - 2\mu\gamma e^{-i\sigma} \cdot \mathrm{Jc}_1\left[2|\mathcal{A}_T|\right]\mathcal{A}_T$$

where $\mathrm{Jc}_1(x) = \mathrm{J}_1(x)/x$ is the Bessel-cardinal function

Looks like sinus cardinal, but the maximum is ½ instead of 1

$\mu = \Delta\omega/2$ = half-bandwith of the filter ($= 2\pi \times 10$ MHz)
$\gamma = \beta \sin 2\phi$ = effective normalized gain (can vary from -5 to 5)
$\sigma = \Omega_0 T$=microwave round-trip phase shift

The solutions of interest are:

- ***A(t)*** $\equiv$ ***0*** **(no oscillations)**
- ***A(t)*** $\equiv$ ***C*** ^{te} $\neq$ ***0*** **(pure monochromatic)**

These states are *fixed points* of the envelope equation.

We have to study the existence and the stability of the fixed point solutions, particularly for the solution $A(t)=C \neq 0$ which is of great technological interest.

Stability of the oscillating solution

It corresponds to the solution ***A(t)* ≡ *A*$_o$ ≠ *0*** with

$$\mathrm{Jc}_1[2|\mathcal{A}_o|] = -\frac{1}{2\gamma}e^{i\sigma}$$

Perturbation equation

$$\delta\dot{\mathcal{A}} = -\mu \cdot \delta\mathcal{A} - 2\mu\gamma\{\mathrm{Jc}_1[2|\mathcal{A}_o|] + 2|\mathcal{A}_o|\mathrm{Jc}_1'[2|\mathcal{A}_o|]\}\delta\mathcal{A}_T$$

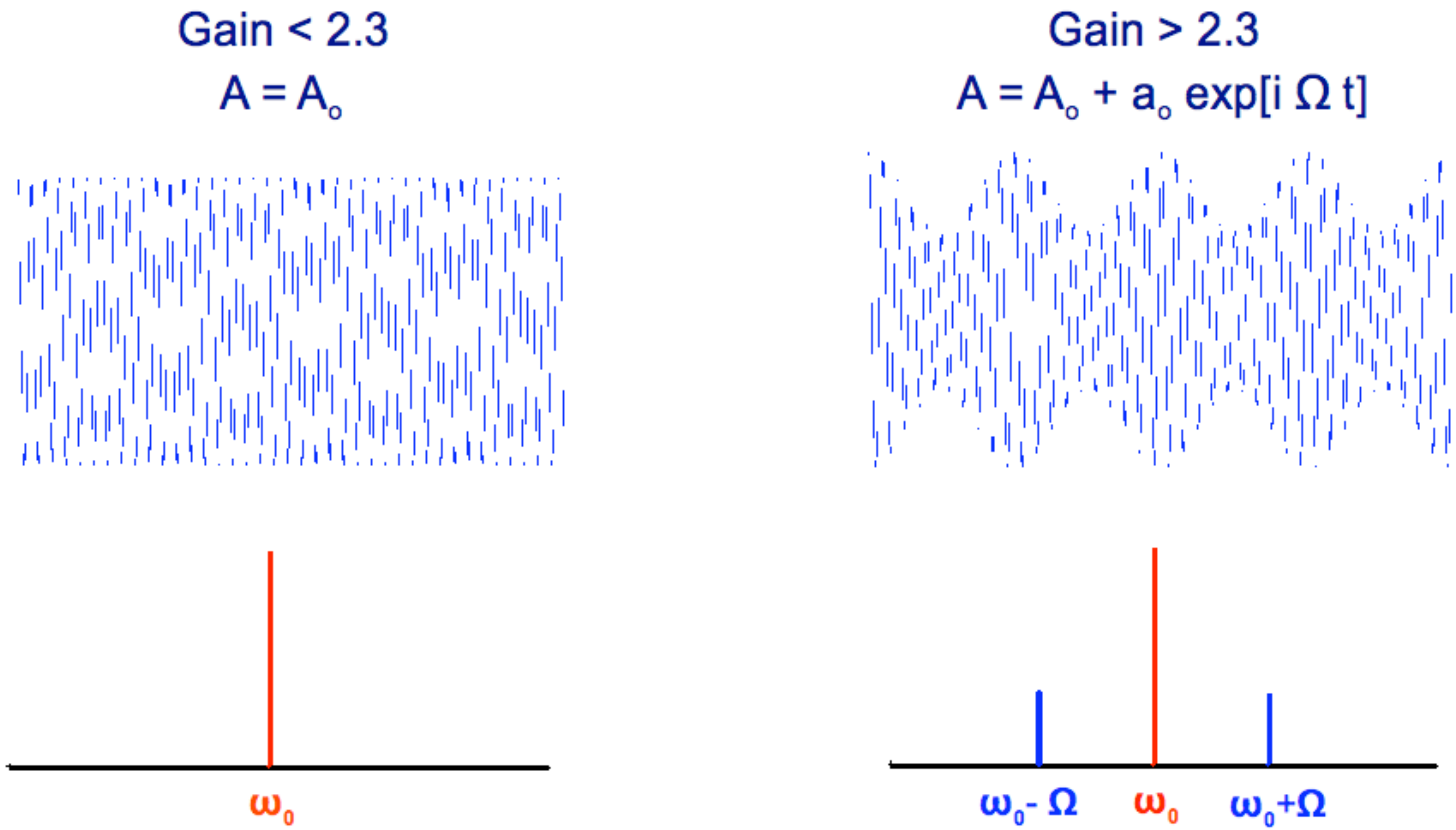
Stability condition

$$\left|\frac{1}{2} + \frac{|\mathcal{A}_o|\mathrm{Jc}_1'[2|\mathcal{A}_o|]}{\mathrm{Jc}_1[2|\mathcal{A}_o|]}\right| < \frac{1}{2}$$ fulfilled when $1 < \gamma < 2.3$, when $e^{-i\sigma} = -1$

What does occur beyond 2.3 ???

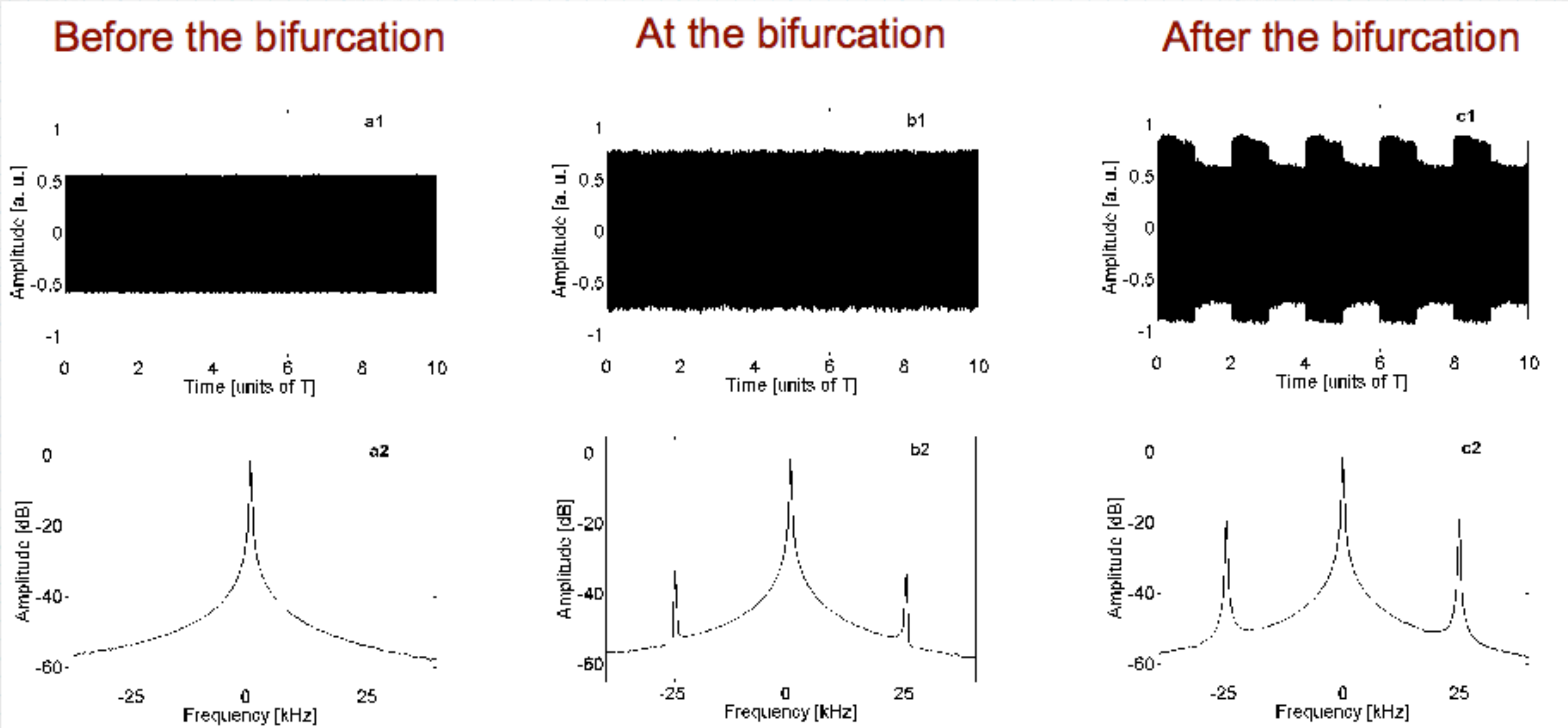
A Hopf bifurcation

X



The bifurcation at $\gamma=2.3$ should qualitatively modify the Fourier spectrum of OEOs

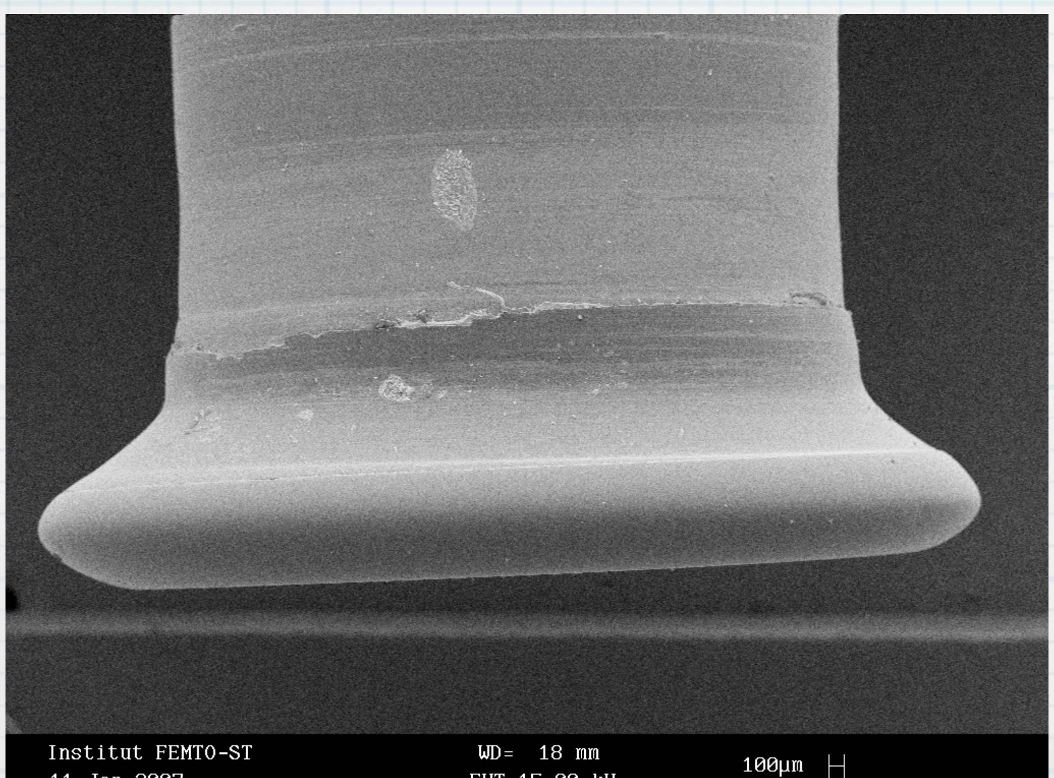
Hopf bifurcation, observed

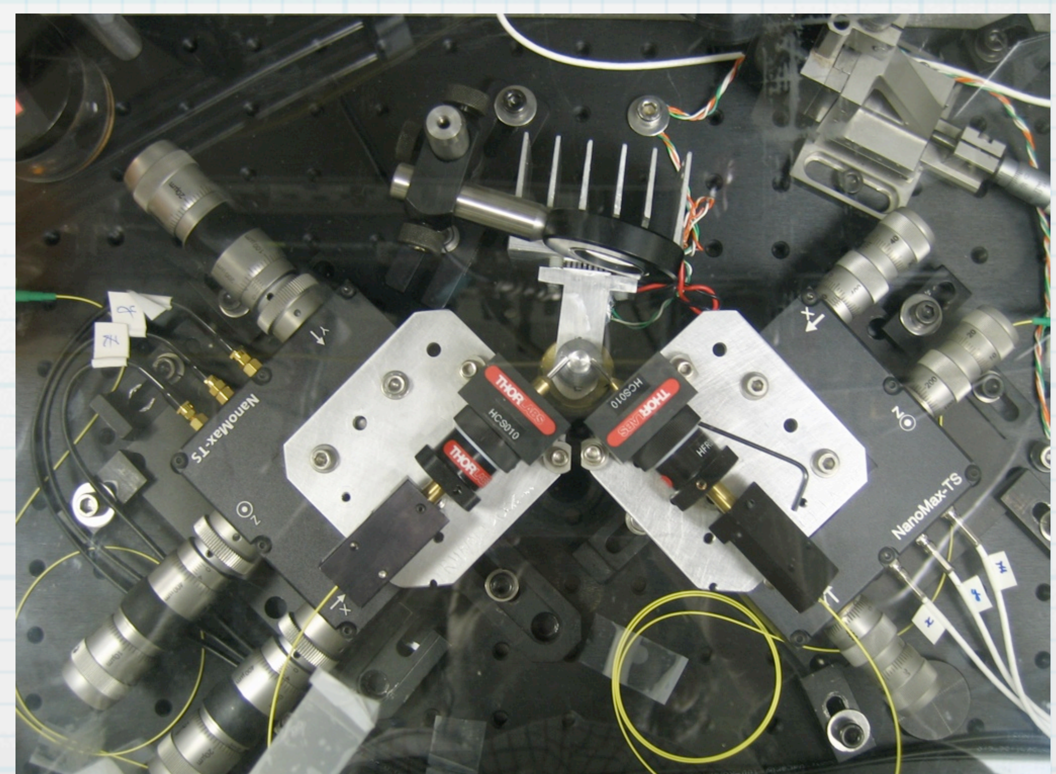


The Hopf bifurcation leads to the emergence of robust modulation side-peaks in the Fourier spectrum, which may drastically affect the phase noise performance of OEOs

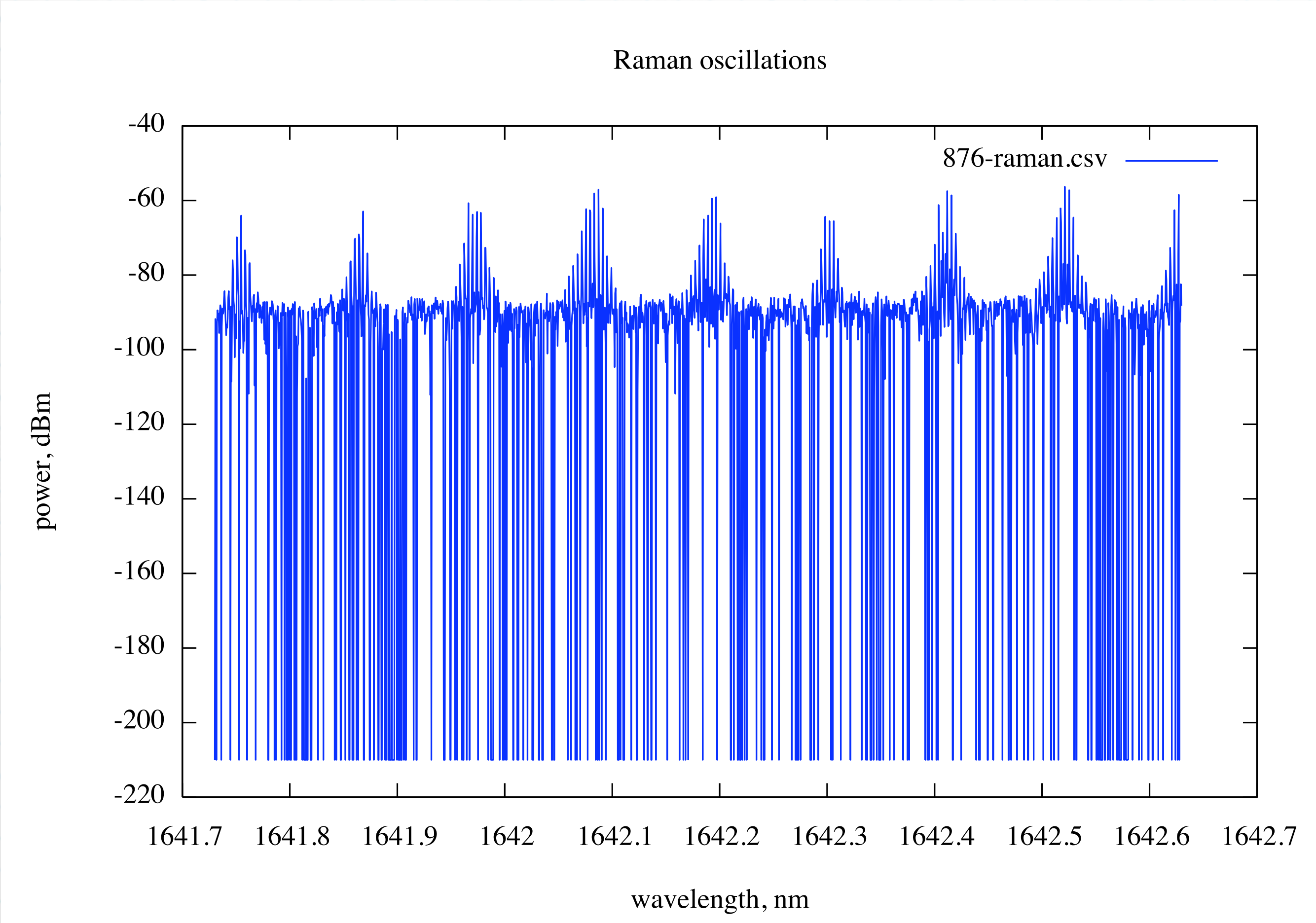
Small resonators

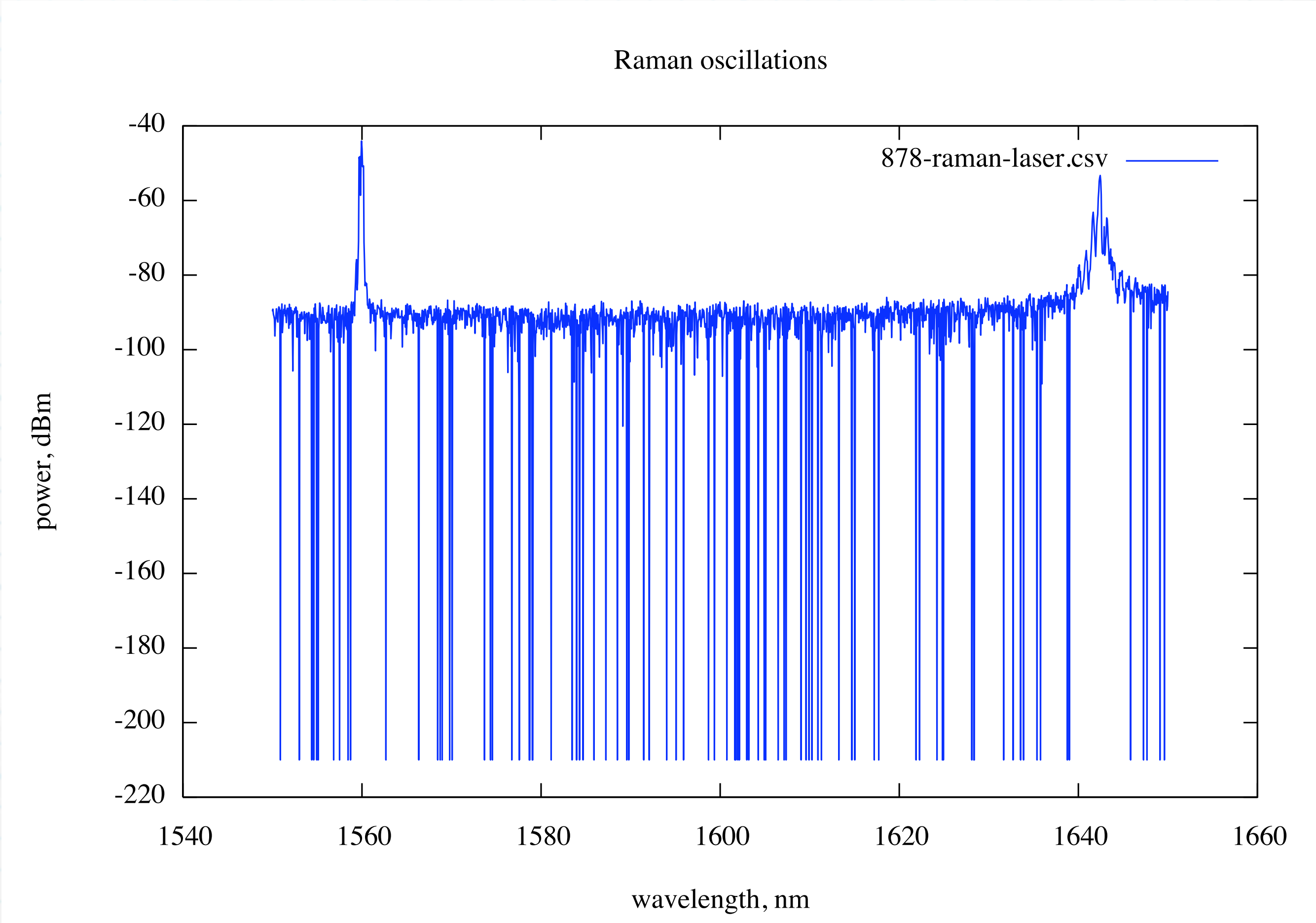
* Technology development in progress (quartz CaF_2 , MgF_2)
* A bunch of technical problems (and Ryad Bendoula left)
* Taper coupling still problematic
* some interesting phenomena observed





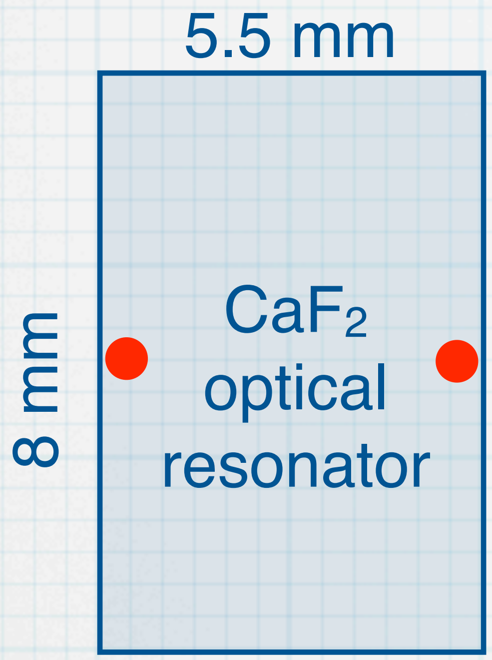
Raman oscillations





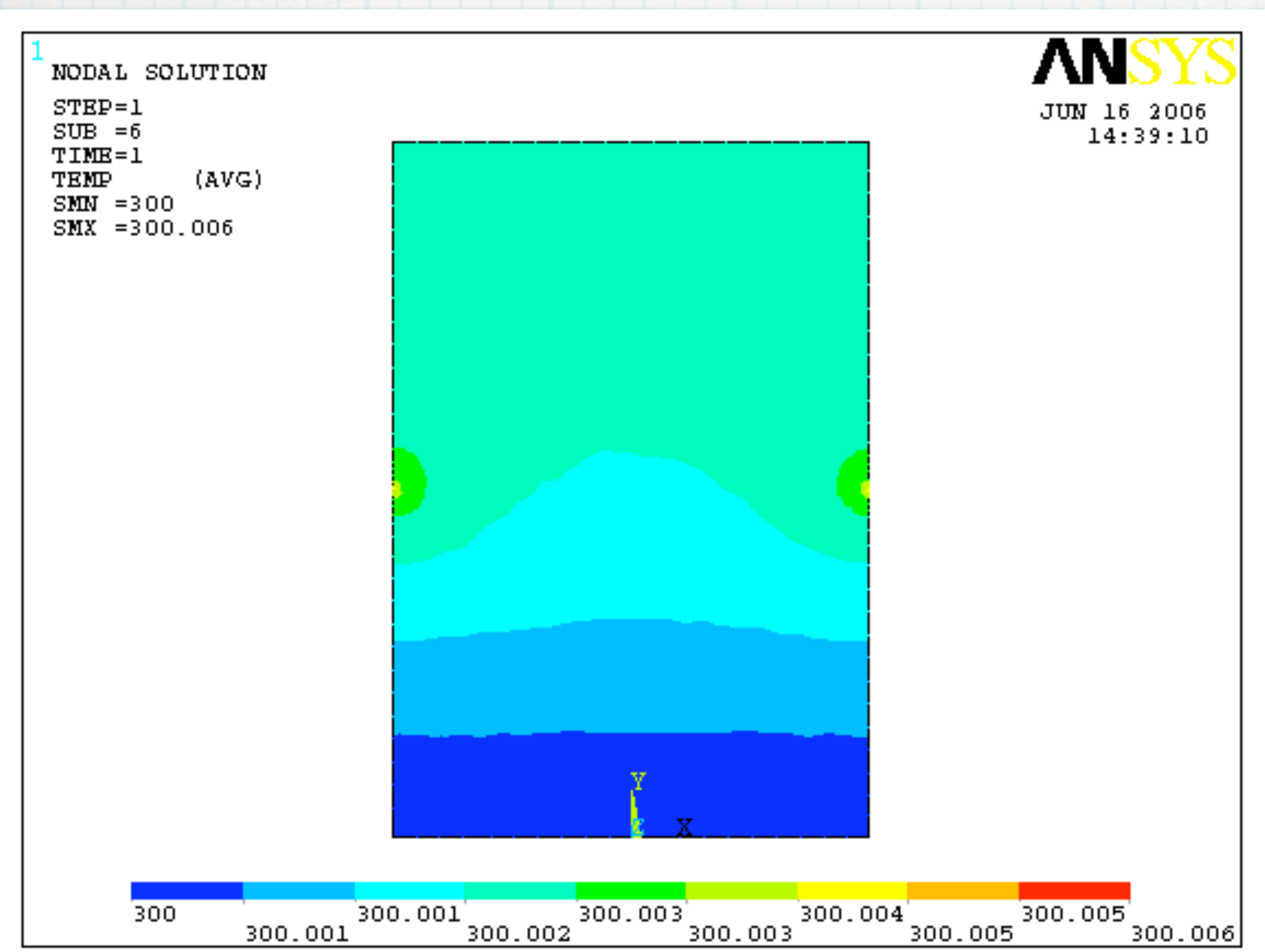
- **The Raman amplification is a quantum phenomenon of nonlinear origin that involves optical phonons.**
- **An amplifier inserted in a high-Q cavity turns into an oscillator, like masers and lasers.**
- **Oscillation threshold ~ $1/Q^2$**
- **In CaF2 pumped at 1.56 µm, Raman oscillation occurs at 1.64 µm**
- **Due to the large linewidth, the Raman oscillation appears as a bunch of (noisy) spectral lines spaced by the FSR (12 GHz, or 100 pm in our case)**
- **Raman phonons modulate the optical properties of the crystal, which induces noise at the pump frequency (1.56 µm)**

High temperature gradient

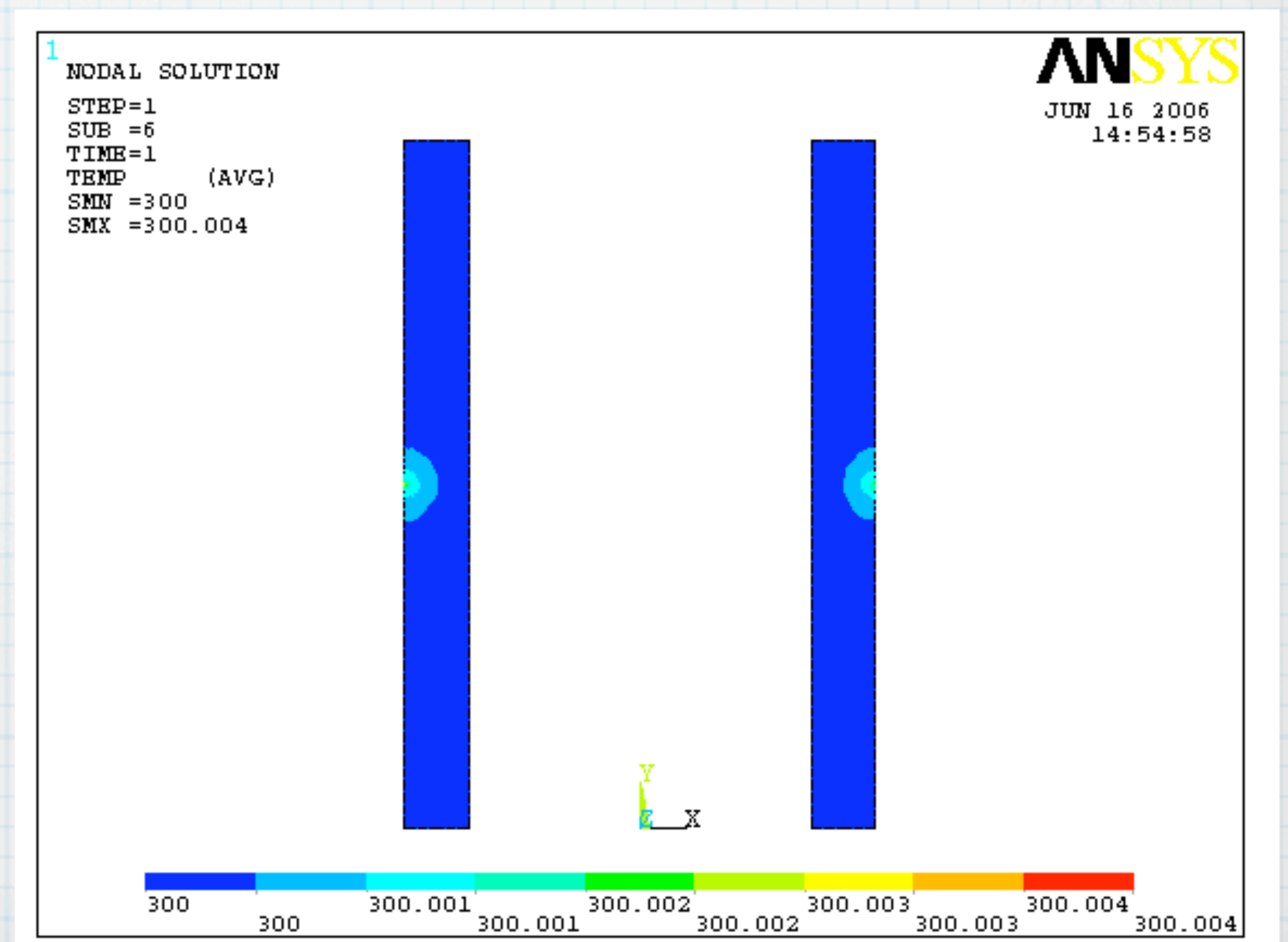


- cross section of the field region 1 μm^2
- CaF_2 thermal conductivity 9.5 W/mK
- dissipated power 300 µW
- wavelength 1.56 µm
- air temperature 300 K
- still air thermal conductivity 10 W/m^2K
- simplification: the heat flow from the mode region is uniform

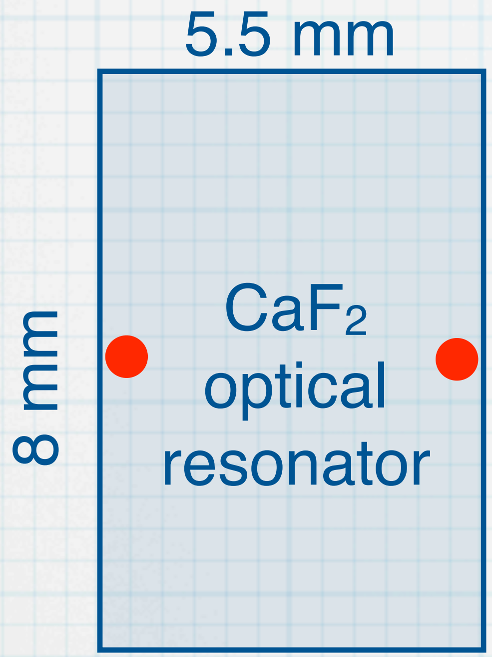
bottom plane at a reference temperature



inner bore at a reference temperature

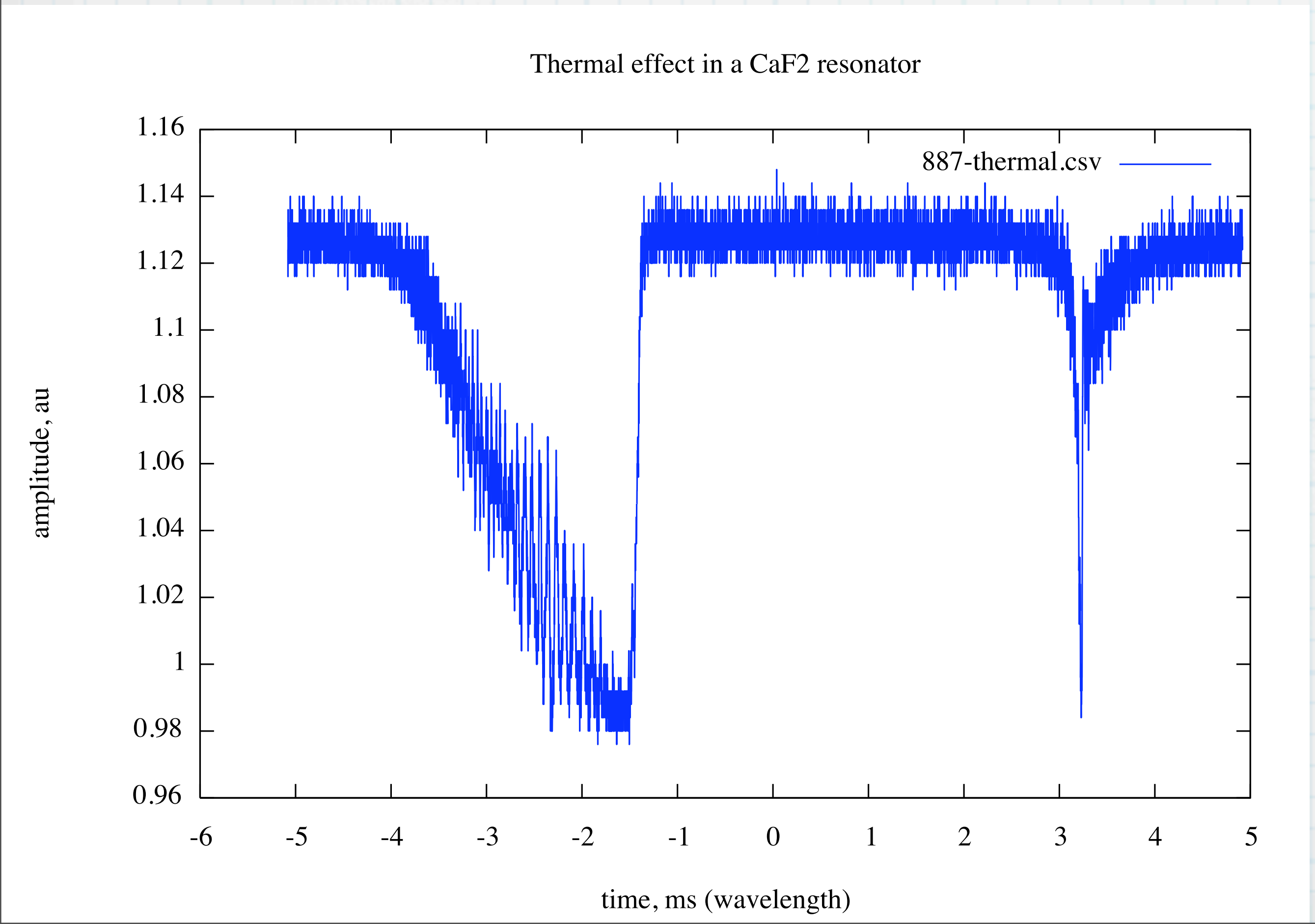


Thermal effect on frequency

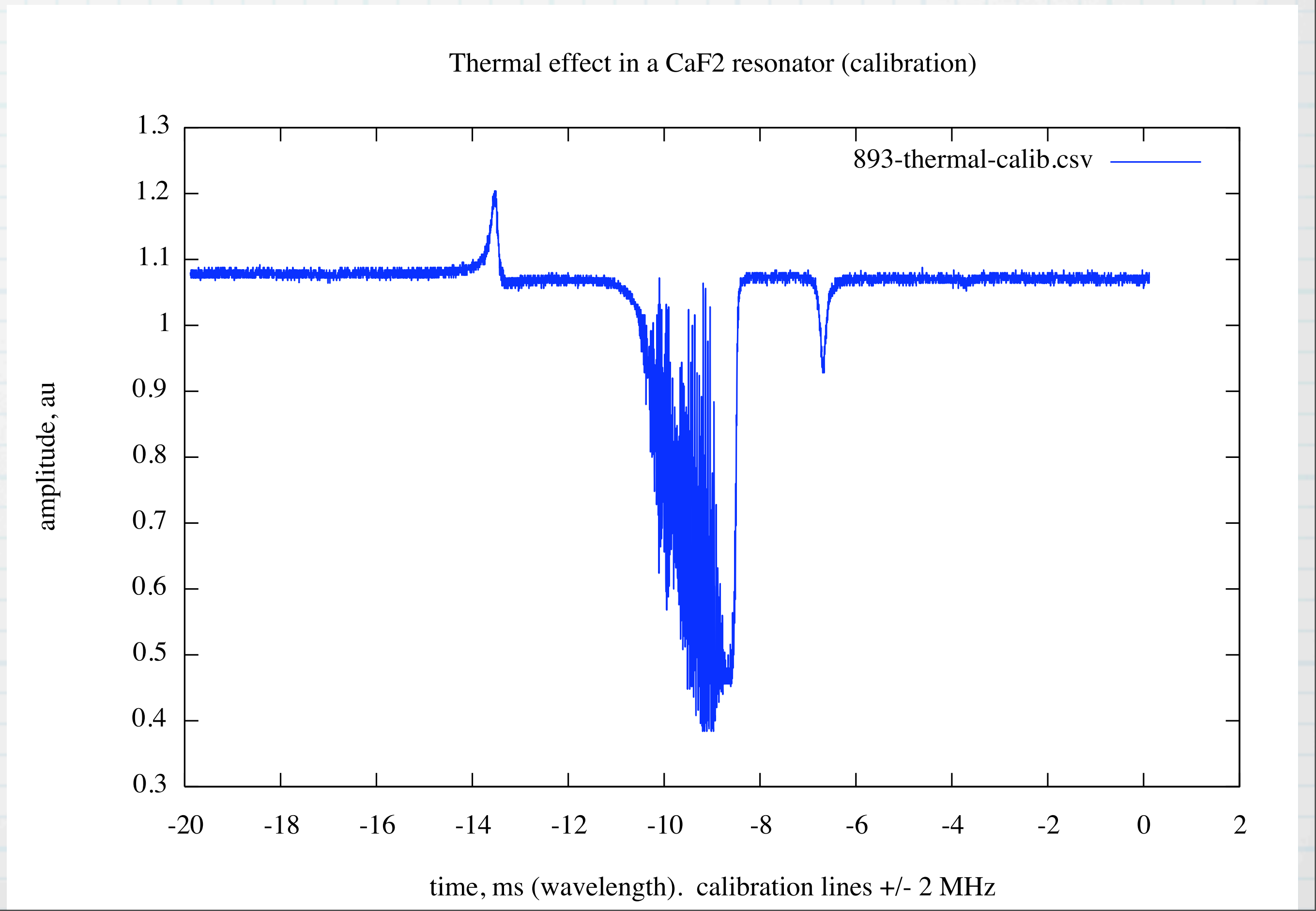


- wavelength 1.56 µm (ν_0=192 THz)
- Q=5x10^9 –> BW=40 kHz
- a dissipated power of 300 µW shifts the resonant frequency by 1.2 MHz (6x10^{-9}), i.e., 37.5 x BW
- time scale about 60 µs
- Q>10^{11} is possible with CaF_2 and other crystals!!

laser scan



calibration (2 MHz phase modulation)



Low-power oscillator operation

Assume: $\lambda = 1560$ nm, $\rho = 0.8$ A/W, $R = 50$ Ohm, $(P_\lambda)_{peak} = 2\times10^{-5}$ W (20 µW)

Shot noise (m=1)

$$I_{RMS} = \frac{1}{\sqrt{2}}\,\rho\overline{P}_\lambda$$

$$S_I = 2q\overline{I} = 2q\,\rho\overline{P}_\lambda$$

$$SNR = \frac{1}{4}\,\frac{\rho\overline{P}_\lambda}{q}$$

In practice, –131 dBrad²/Hz

Thermal noise (m=1)

$$I_{RMS} = \frac{1}{\sqrt{2}}\,\rho\overline{P}_\lambda$$

$$S_I = \frac{4kT}{R} \quad \text{or} \quad \frac{4FkT}{R}$$

$$SNR = \frac{1}{8}\,\frac{\rho^2\overline{P}_\lambda^2 R}{kT}$$

In practice, –110 dBrad²/Hz with F=0 dB (!!!)

- Thermal noise is dominant: below threshold, SNR ~ $1/P_\lambda^2$
- Thermal noise can be reduced (10 dB or more?) using VGND amplifiers
- What about flicker of photodetectors with integrated VGND amplifier?
- Dramatic impact on the (phase) noise floor