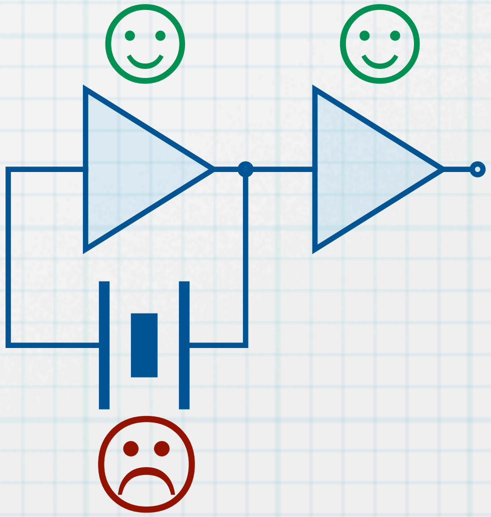
On the 1/f noise in ultra-stable quartz oscillators

Enrico Rubiola and Vincent Giordano
FEMTO-ST Institute, Besançon, France
(CNRS and Université de Franche Comté)

Outline

* Amplifier noise
* Leeson effect
* Interpretation of $S_\varphi(f)$
* Examples



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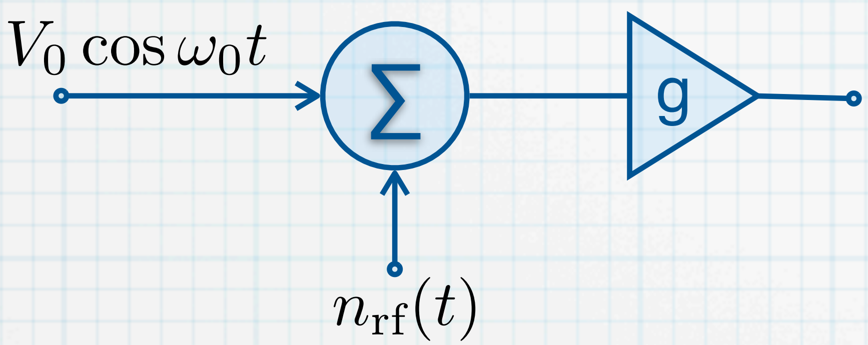
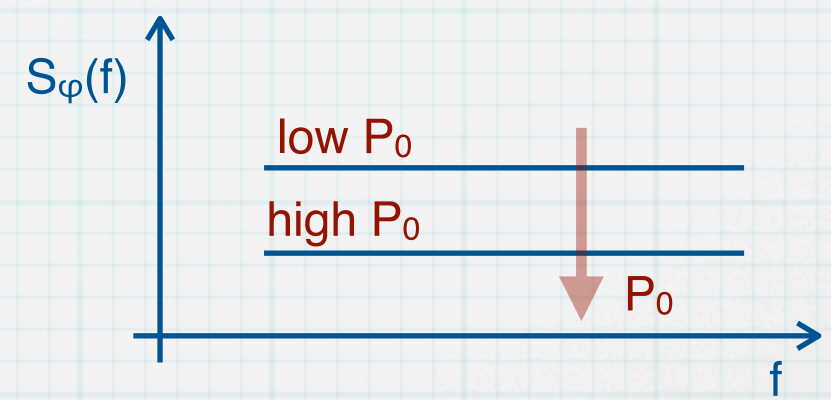
Amplifier white noise

Noise figure F
Input power P_0



power law $$S_\varphi = \sum_{i=-4}^{0} b_i f^i$$

white phase noise $$b_0 = \frac{F k T_0}{P_0}$$



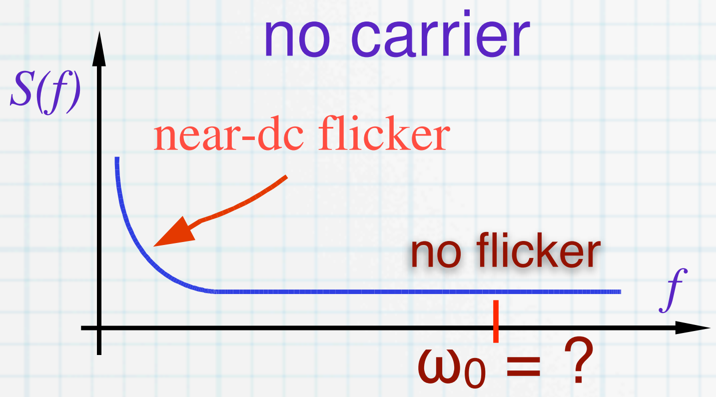
Cascaded amplifiers (Friis formula)

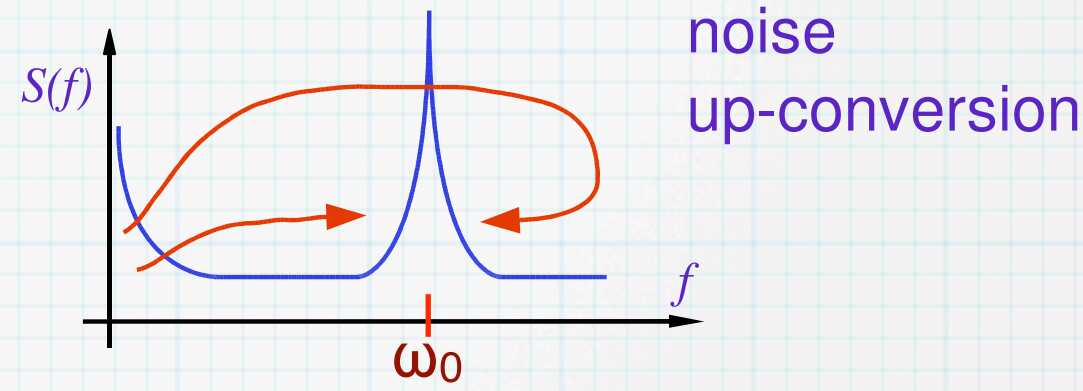
$$N = F_1 k T_0 + \frac{(F_2 - 1) k T_0}{g_1^2} + \ldots$$

As a consequence, (phase) noise is chiefly that of the 1st stage

Amplifier flicker noise

parametric up-conversion of the near-dc noise





carrier + near-dc noise

$$v_i(t) = V_i\, e^{j\omega_0 t} + n'(t) + jn''(t)$$

the parametric nature of 1/f noise is hidden in n' and n''

substitute
(careful, this hides the down-conversion)

$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + \ldots$$

non-linear amplifier

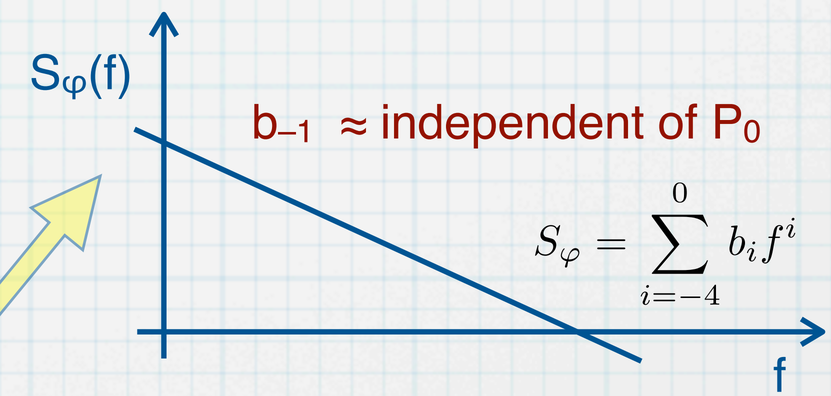
expand and select the ω_0 terms

$$v_o(t) = V_i \Big\{ a_1 + 2a_2 \big[n'(t) + jn''(t) \big] \Big\} e^{j\omega_0 t}$$

get AM and PM noise

$$\alpha(t) = 2\,\frac{a_2}{a_1}\, n'(t) \qquad \varphi(t) = 2\,\frac{a_2}{a_1}\, n''(t)$$

independent of V_i (!)



m cascaded amplifiers

$$(b_{-1})_{\text{cascade}} = \sum_{i=1}^{m} (b_{-1})_i$$

In practice, each stage contributes ≈ equally

Resonator in the phase space

1 – theory of linear systems

Dirac (impulse at t): $\delta(t) \rightarrow$ [$h(t)$] $\rightarrow h(t)$ ⟺ (Laplace transform) ⟺ $1 \rightarrow$ [$H(s)$] $\rightarrow H(s)$

Heaviside (step): $U(t) \rightarrow$ [$h(t)$] $\rightarrow \int h(t)\,dt$ ⟺ (Laplace transform) ⟺ $1/s \rightarrow$ [$H(s)$] $\rightarrow (1/s)\,H(s)$

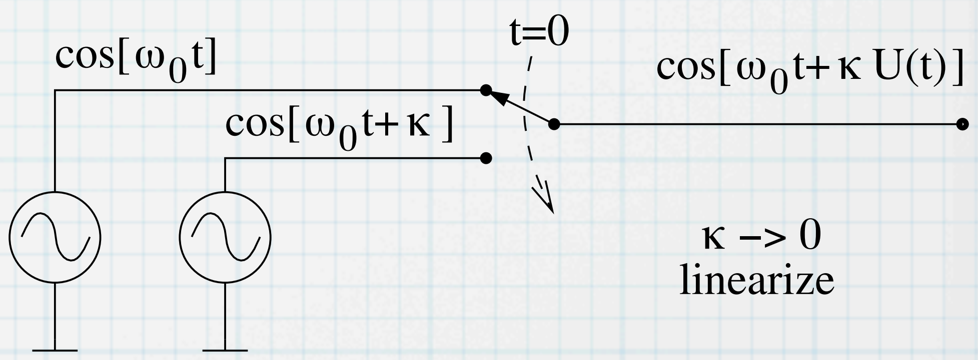
2 – resonator phase response

$\cos[\omega_0 t + \delta(t)] \rightarrow$ [resonator] $\rightarrow \cos[\omega_0 t + b(t)]$

☹☠☈≹∞?!

$\cos\left[\omega_0 t + U(t)\right] \rightarrow$ [resonator] $\rightarrow \cos\left[\omega_0 t + \int b(t) dt\right]$

this is easy –>



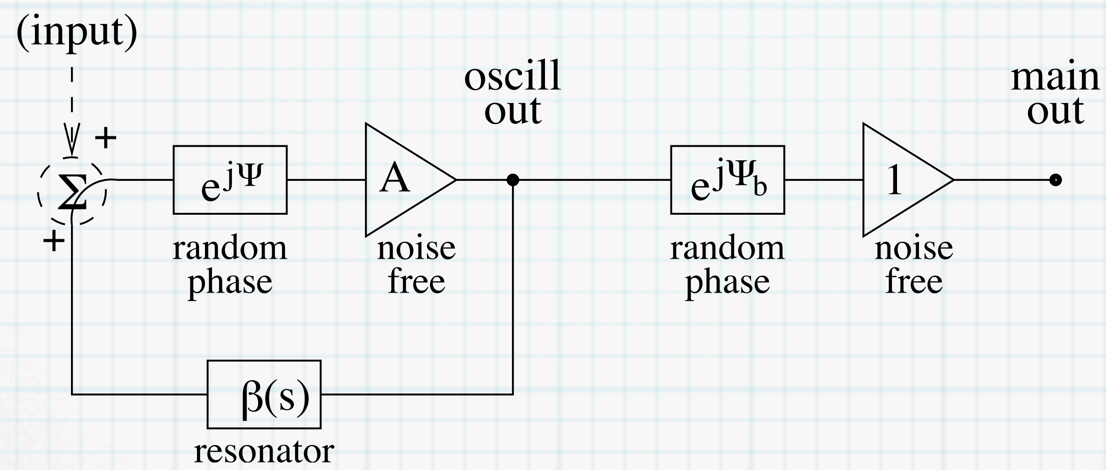
3 – the resonator phase response is a low-pass function

$U(t) \rightarrow$ [$b(t)$] $\rightarrow \int b(t)\,dt$ ⟺ (Laplace transform) ⟺ $1/s \rightarrow$ [$B(s)$] $\rightarrow (1/s)\,B(s)$

$$b(t) = \frac{1}{\tau}\,e^{-\frac{t}{\tau}} \qquad \tau = \frac{2Q}{\omega_0} \qquad B(s) = \frac{1}{1+s\tau}$$

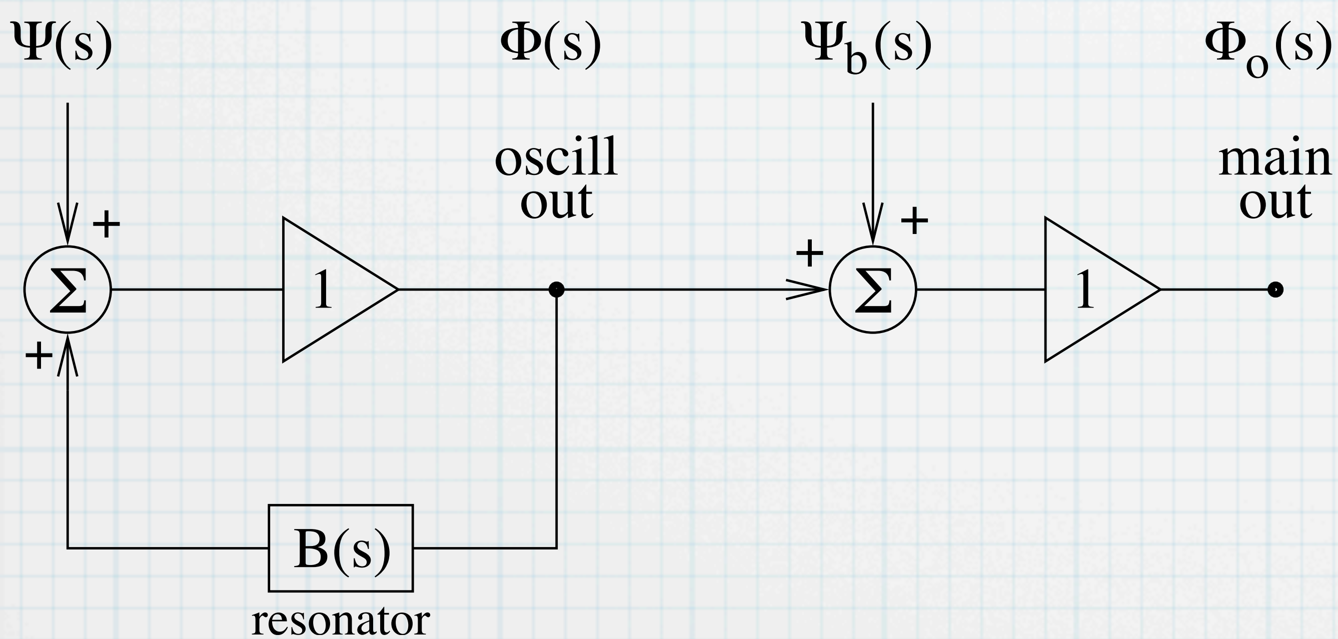
The Leeson effect

basic feedback theory



$$H(s) = \frac{A}{1 - A\beta(s)}$$

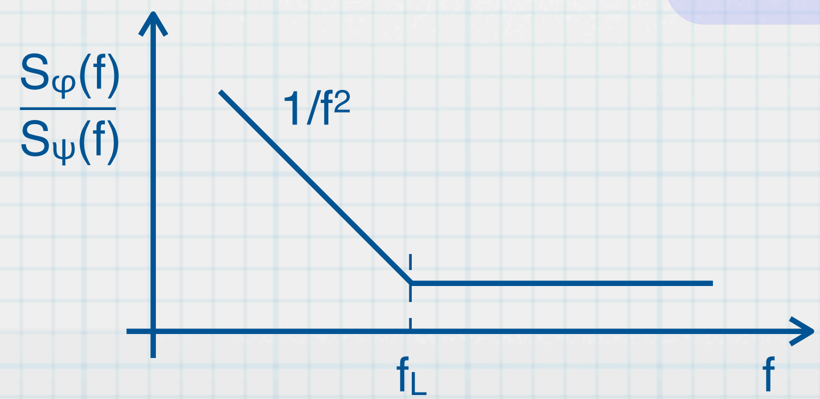
phase response – use the linear-feedback theory



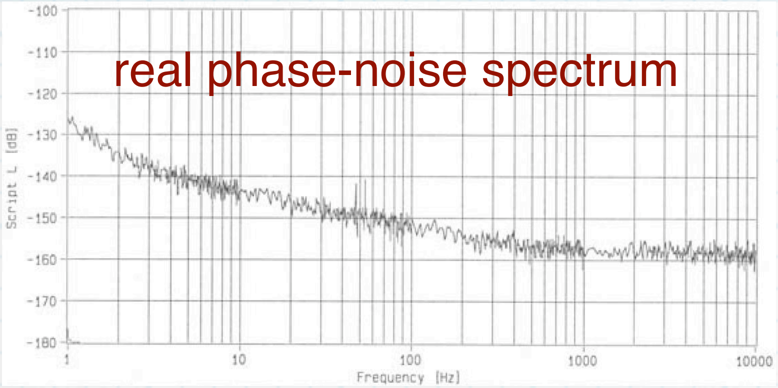
$$\mathcal{H}(s) = \frac{\Phi(s)}{\Psi(s)} = \frac{1}{1 - B(s)} = \frac{1 + s\tau}{s\tau}$$

$$|\mathcal{H}(j\omega)|^2 = \frac{1 + \omega^2\tau^2}{\omega^2\tau^2} \qquad \tau = \frac{2Q}{\omega_0}$$

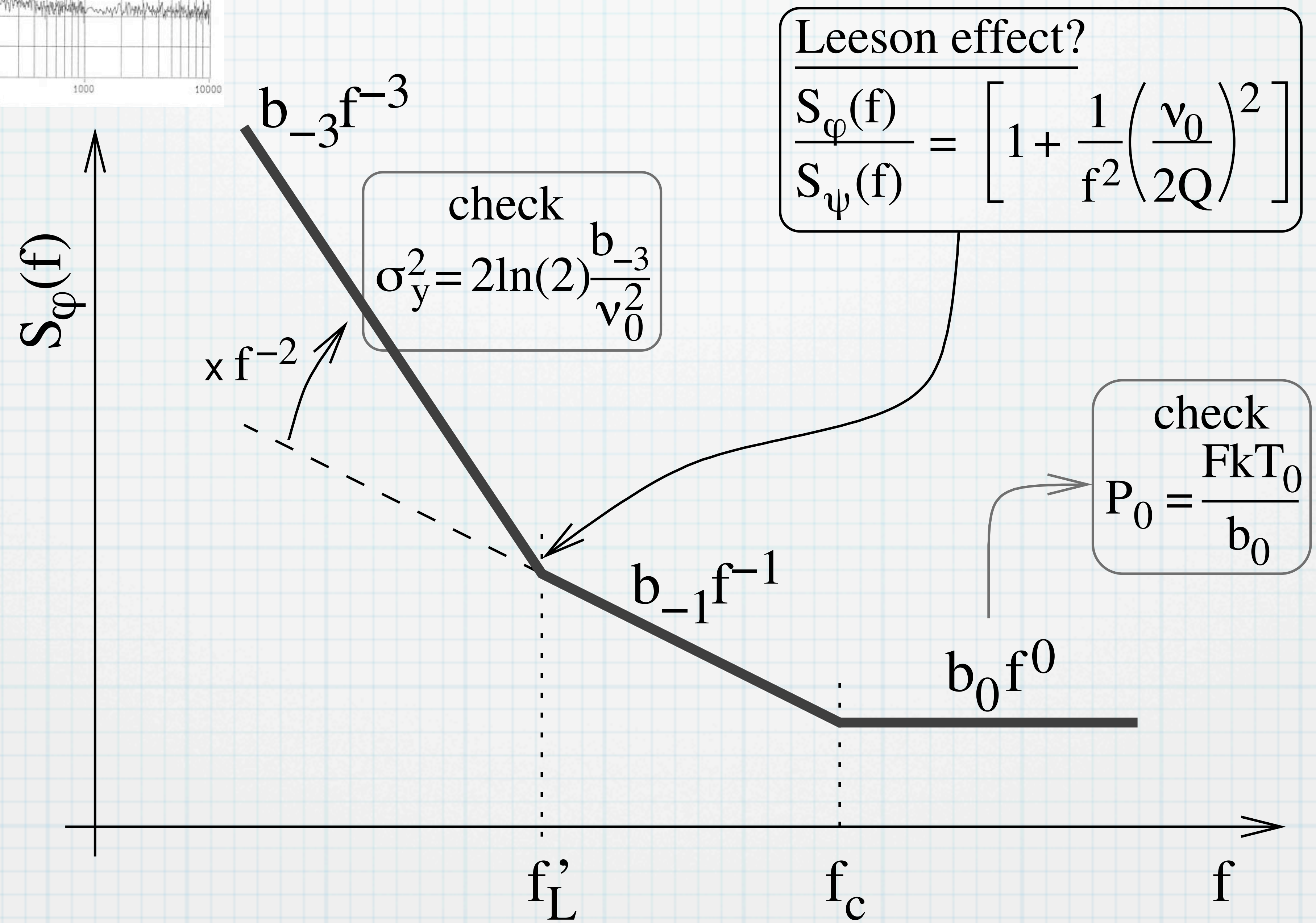
$$S_{\varphi\, o}(f) = \left[1 + \frac{1}{f^2}\frac{\nu_0^2}{4Q^2}\right] S_\psi(f) + S_{\psi\, b}(f)$$



Interpretation of $S_\varphi(f)$ [1]

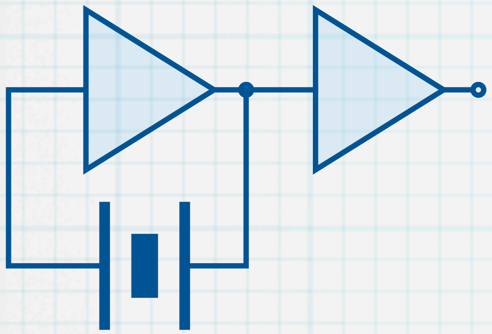


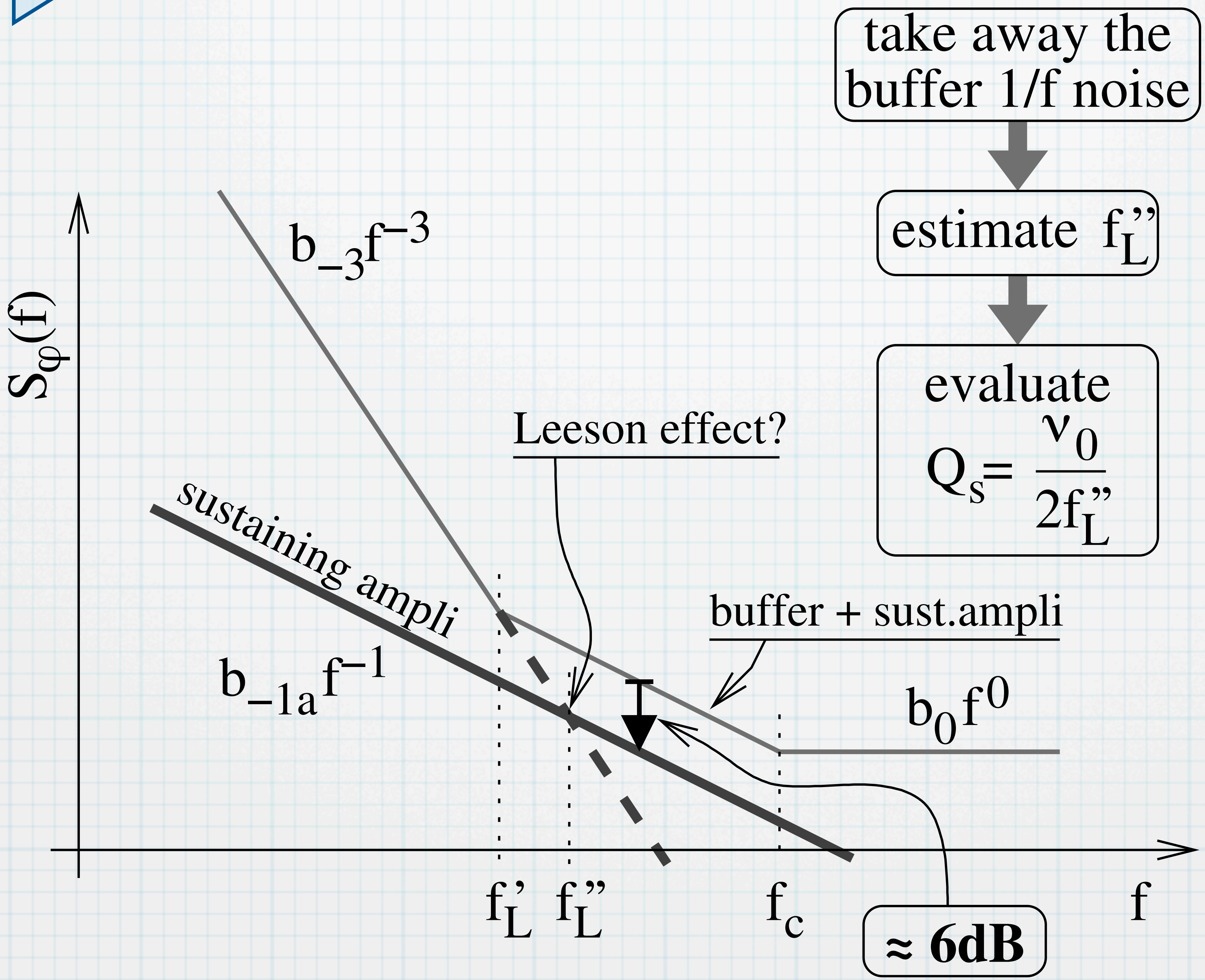
after parametric estimation



Sanity check:
– power P_0 at amplifier input
– Allan deviation σ_y (floor)

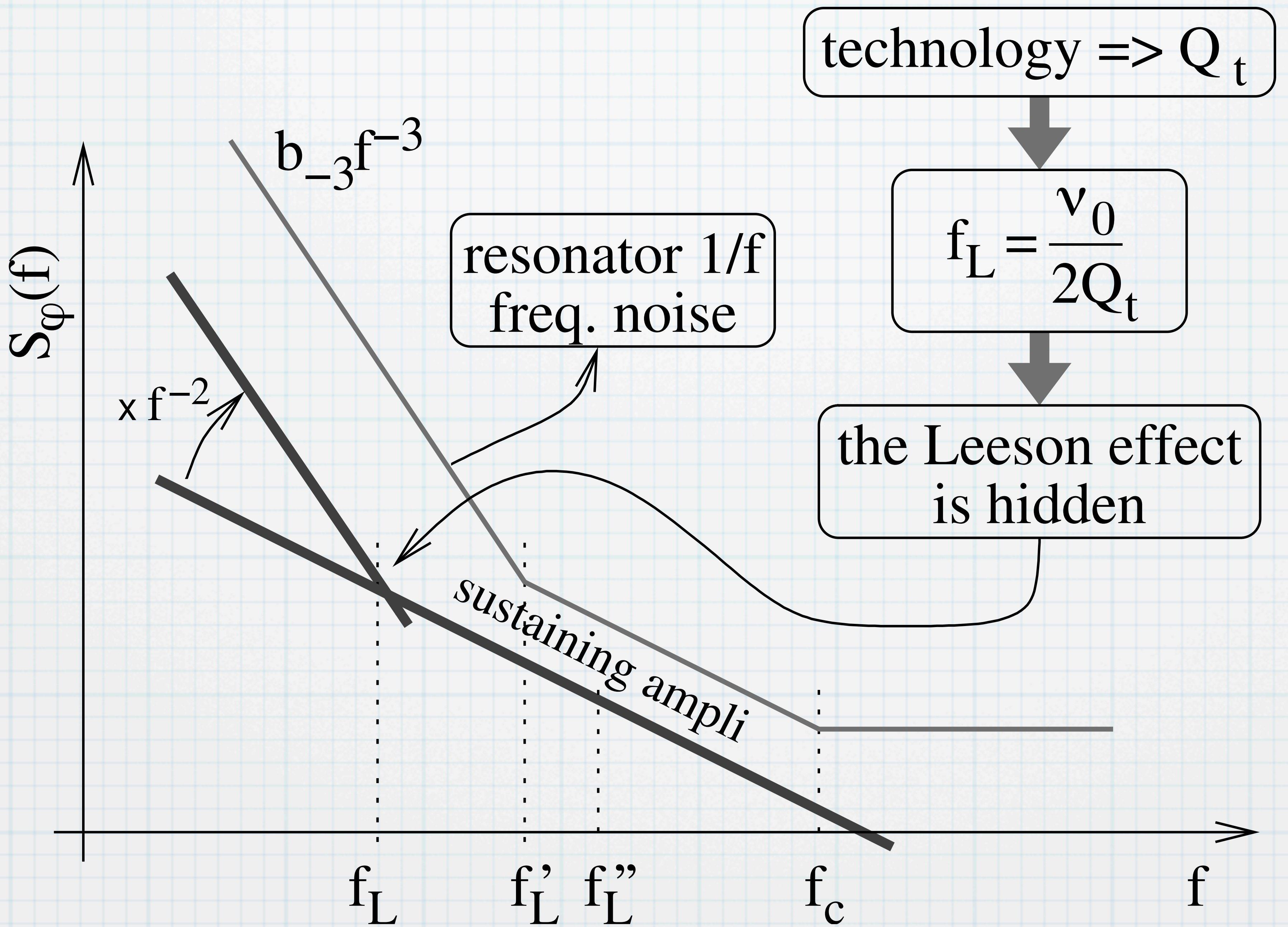
Interpretation of $S_\varphi(f)$ [2]





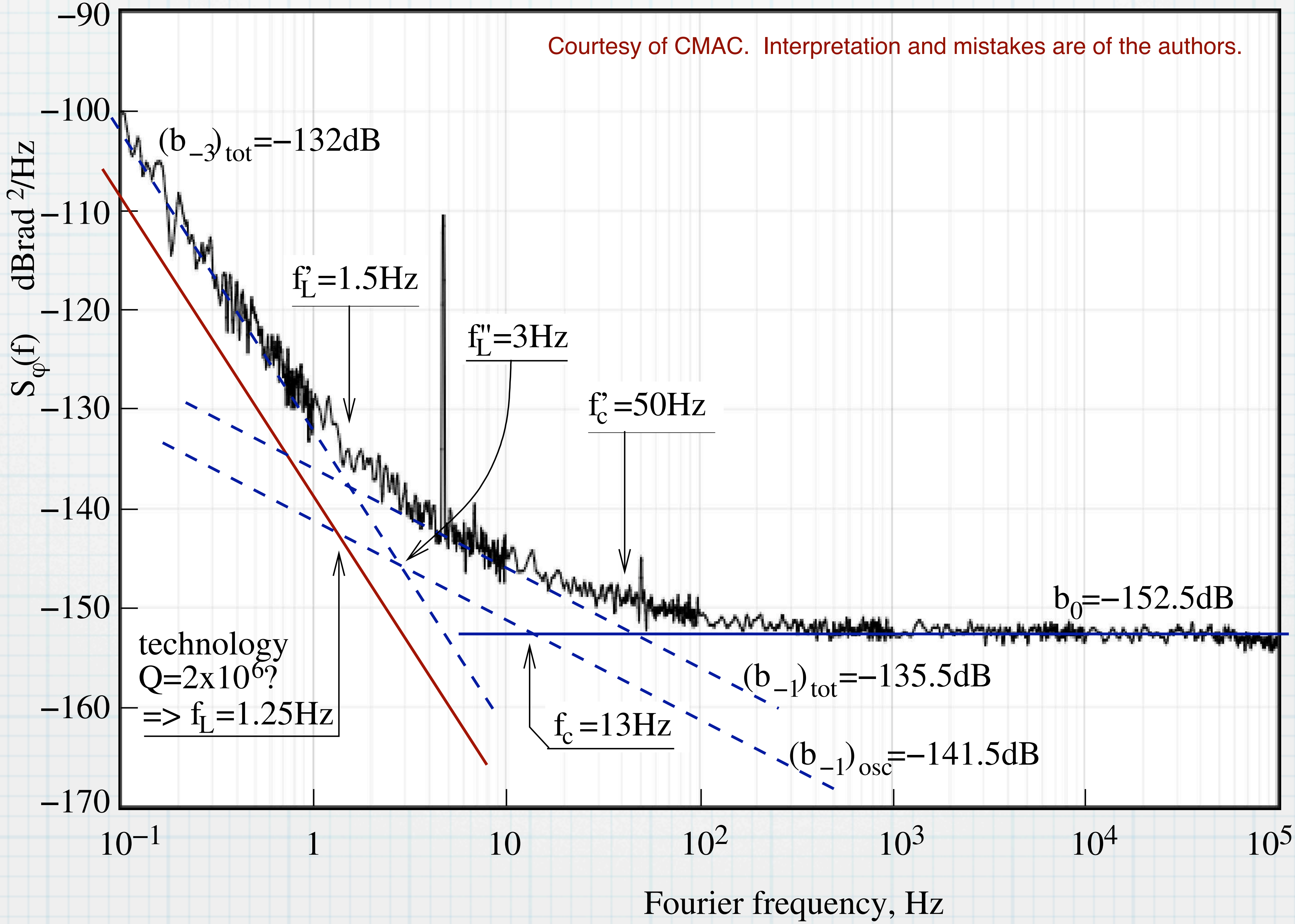
2–3 buffer stages => the sustaining amplifier contributes ≲ 25% of the total 1/f noise

Interpretation of $S_\varphi(f)$ [3]



Technology suggests a merit factor Q_t. In all xtal oscillators we find $Q_t \gg Q_s$

Example – CMAC Pharao



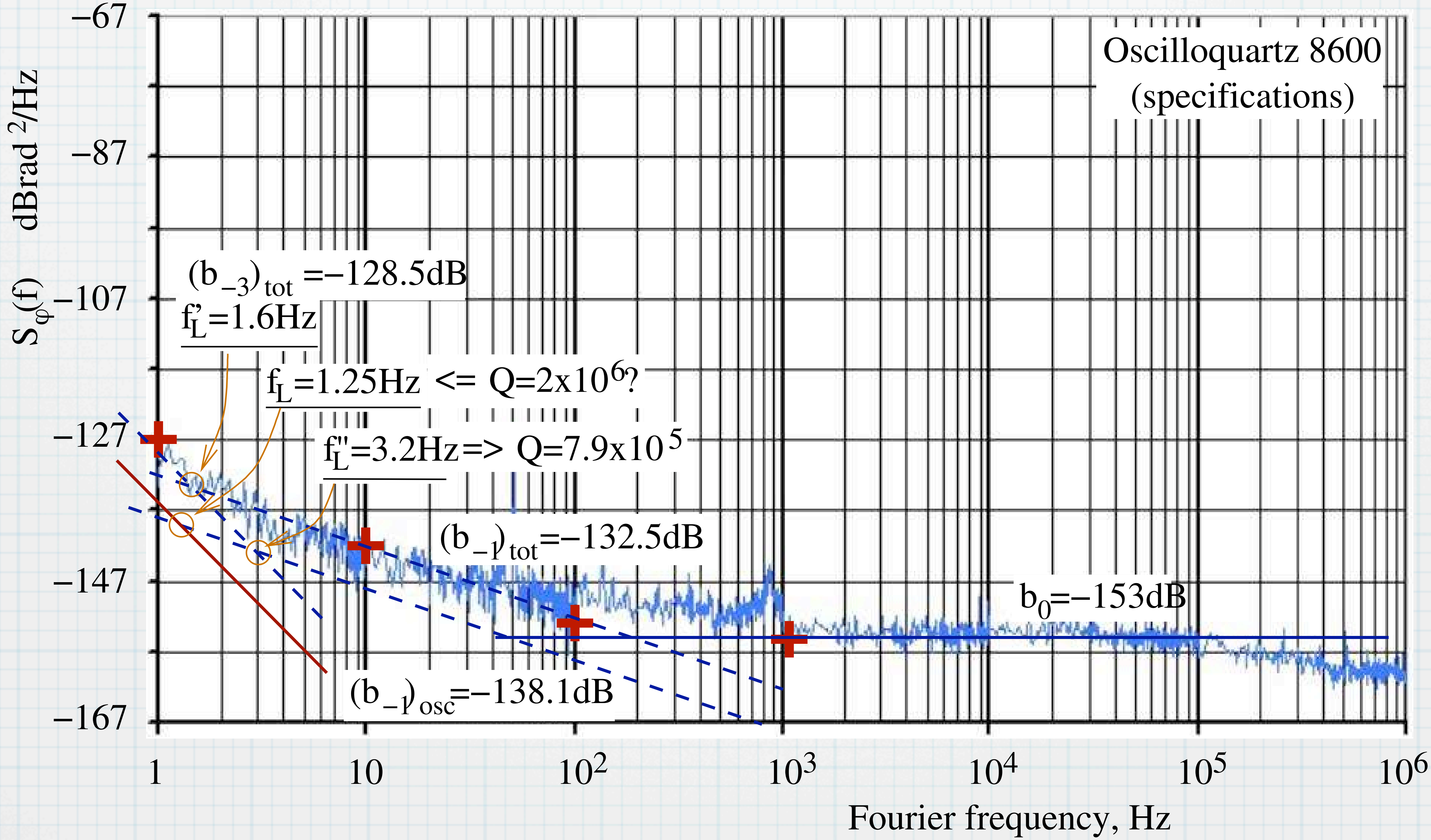
F=1dB b_0 => P_0=–20.5 dBm

$(b_{-3})_{osc}$ => σ_y=5.9x10^{-14}, Q=8.4x10^5 (too low)

Q $\stackrel{?}{=}$ 2x10^6 => σ_y=2.5x10^{-14} Leeson (too low)

Example – Oscilloquartz 8607

Courtesy of Oscilloquartz. Interpretation and mistakes are of the authors.



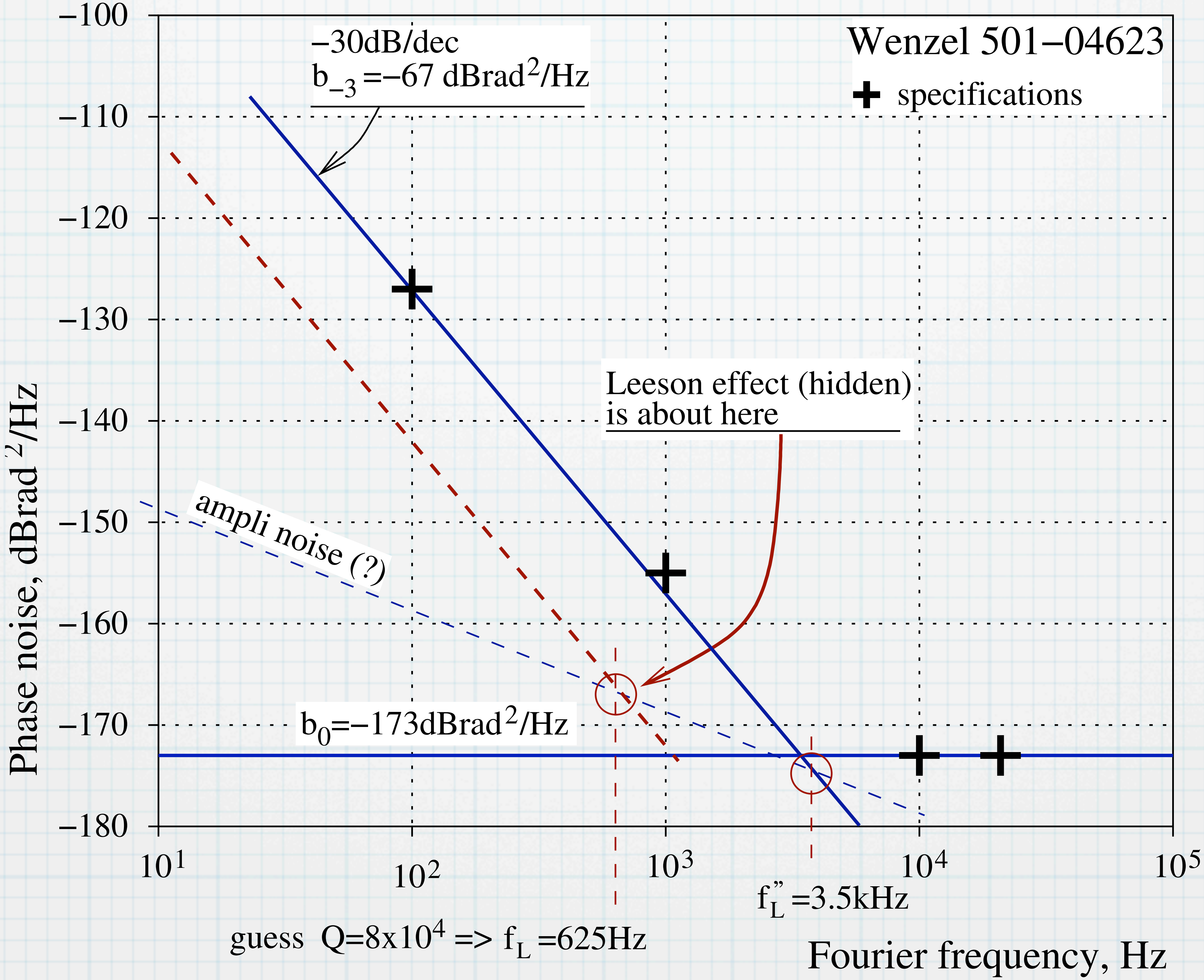
F=1dB b_0 => P_0=–20 dBm

$(b_{-3})_{osc}$ => σ_y=8.8x10^{-14}, Q=7.8x10^5 (too low)

Q $\stackrel{?}{=}$ 2x10^6 => σ_y=3.5x10^{-14} Leeson (too low)

Example – Wenzel 501-04623

Data are from the manufacturer web site. Interpretation and mistakes are of the authors.



Estimating $(b_{-1})_{ampli}$ is difficult because there is no visible 1/f region

F=1dB b_0 => P_0=0 dBm

$(b_{-3})_{osc}$ => $\sigma_y=5.3\times10^{-12}$ Q=1.4x10⁴

$Q \overset{?}{=} 8\times10^4$ => $\sigma_y=9.3\times10^{-13}$ (Leeson)

Other oscillators

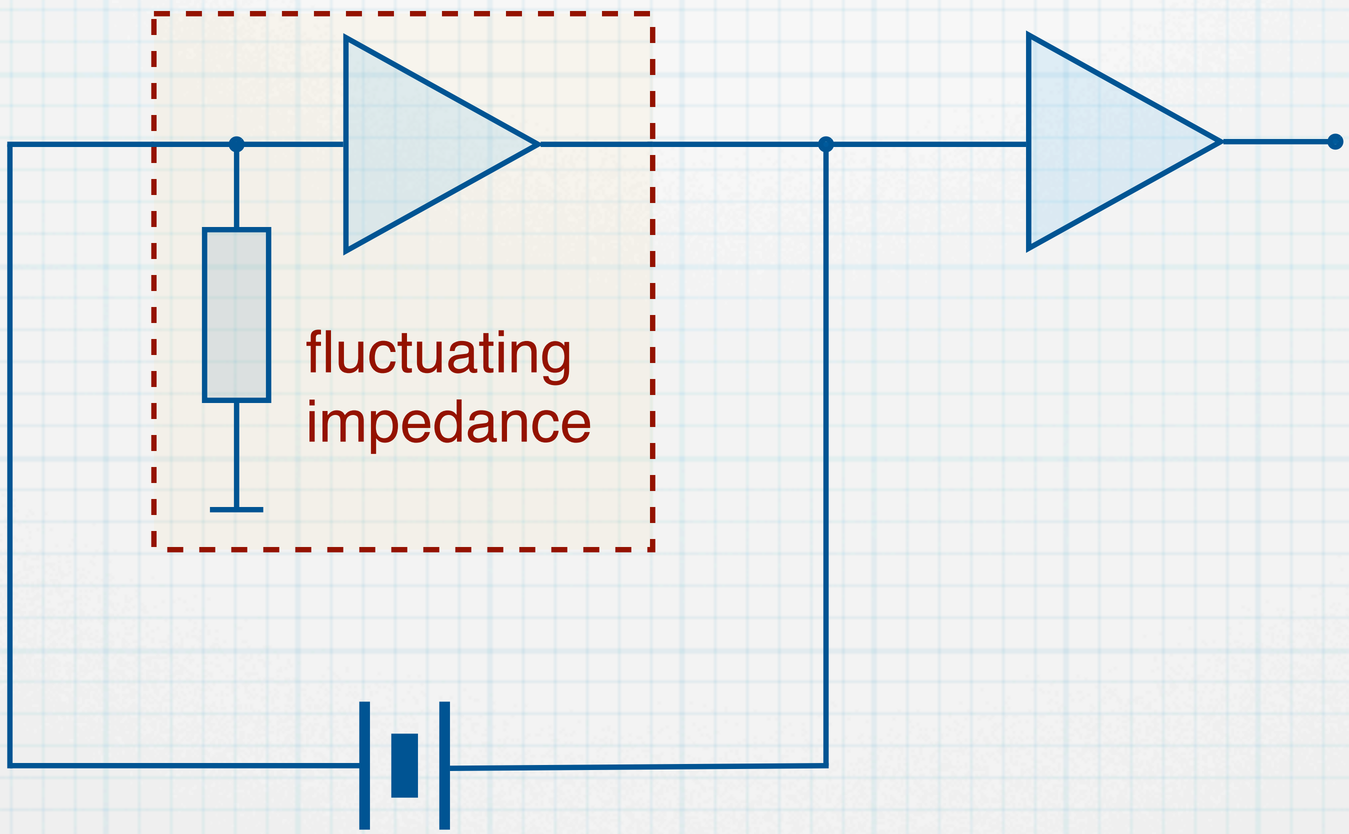
Oscillator	ν_0	$(b_{-3})_{\text{tot}}$	$(b_{-1})_{\text{tot}}$	$(b_{-1})_{\text{amp}}$	f'_L	f''_L	Q_s	Q_t	f_L	$(b_{-3})_{\text{L}}$	R	Note
Oscilloquartz 8600	5	-124.0	-131.0	-137.0	2.24	4.5	5.6×10^5	1.8×10^6	1.4	-134.1	10.1	(1)
Oscilloquartz 8607	5	-128.5	-132.5	-138.5	1.6	3.2	7.9×10^5	2×10^6	1.25	-136.5	8.1	(1)
CMAC Pharao	5	-132.0	-135.5	-141.1	1.5	3	8.4×10^5	2×10^6	1.25	-139.6	7.6	(2)
FEMTO-ST LD prot.	10	-116.6	-130.0	-136.0	4.7	9.3	5.4×10^5	1.15×10^6	4.3	-123.2	6.6	(3)
Agilent 10811	10	-103.0	-131.0	-137.0	25	50	1×10^5	7×10^5	7.1	-119.9	16.9	(4)
Agilent prototype	10	-102.0	-126.0	-132.0	16	32	1.6×10^5	7×10^5	7.1	-114.9	12.9	(5)
Wenzel 501-04623	100	-67.0	-132 ?	-138 ?	1800	3500	1.4×10^4	8×10^4	625	-79.1	15.1	(6)
unit	MHz	dB rad^2/Hz	dB rad^2/Hz	dB rad^2/Hz	Hz	Hz	(none)	(none)	Hz	dB rad^2/Hz	dB	

Notes
(1) Data are from specifications, full options about low noise and high stability.
(2) Measured by CMAC on a sample. CMAC confirmed that $2\times10^6 < Q < 2.2\times10^6$ in actual conditions.
(3) LD cut, built and measured in our laboratory, yet by a different team. Q_t is known.
(4) Measured by Hewlett Packard (now Agilent) on a sample.
(5) Implements a bridge scheme for the degeneration of the amplifier noise. Same resonator of the Agilent 10811.
(6) Data are from specifications.

$$R = \left.\frac{(\sigma_y)_{\text{oscill}}}{(\sigma_y)_{\text{Leeson}}}\right|_{\text{floor}} = \sqrt{\frac{(b_{-3})_{\text{tot}}}{(b_{-3})_L}} = \frac{Q_t}{Q_s} = \frac{f''_L}{f_L}$$

Warning: an effect not accounted for still remains

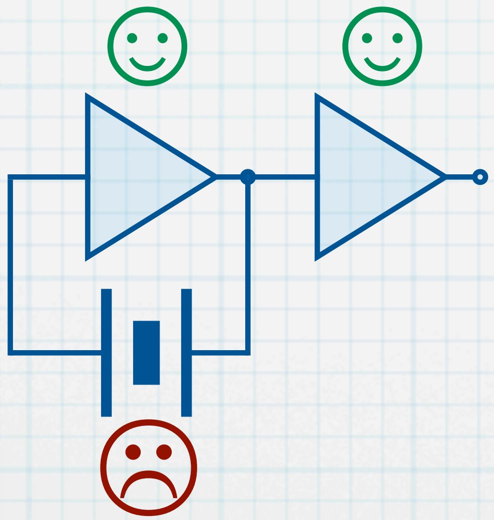
A fluctuating impedance that affects the input without participating to the gain



This does not fit general experience on amplifiers, yet it is to be reported

Conclusions

- The analysis of $S_\varphi(f)$ provides insight in the oscillator
- The oscillator $1/f^3$ phase noise (Allan variance floor) originates from:
  - amplifier 1/f noise, via the Leeson effect
  - resonator instability
- In actual oscillators, the **resonator** instability turns out to be the dominant effect



qed

Full text available on http://arxiv.org/abs/physics/0602110

Talk slides and full text (20 pages, pdf) available on http://rubiola.org

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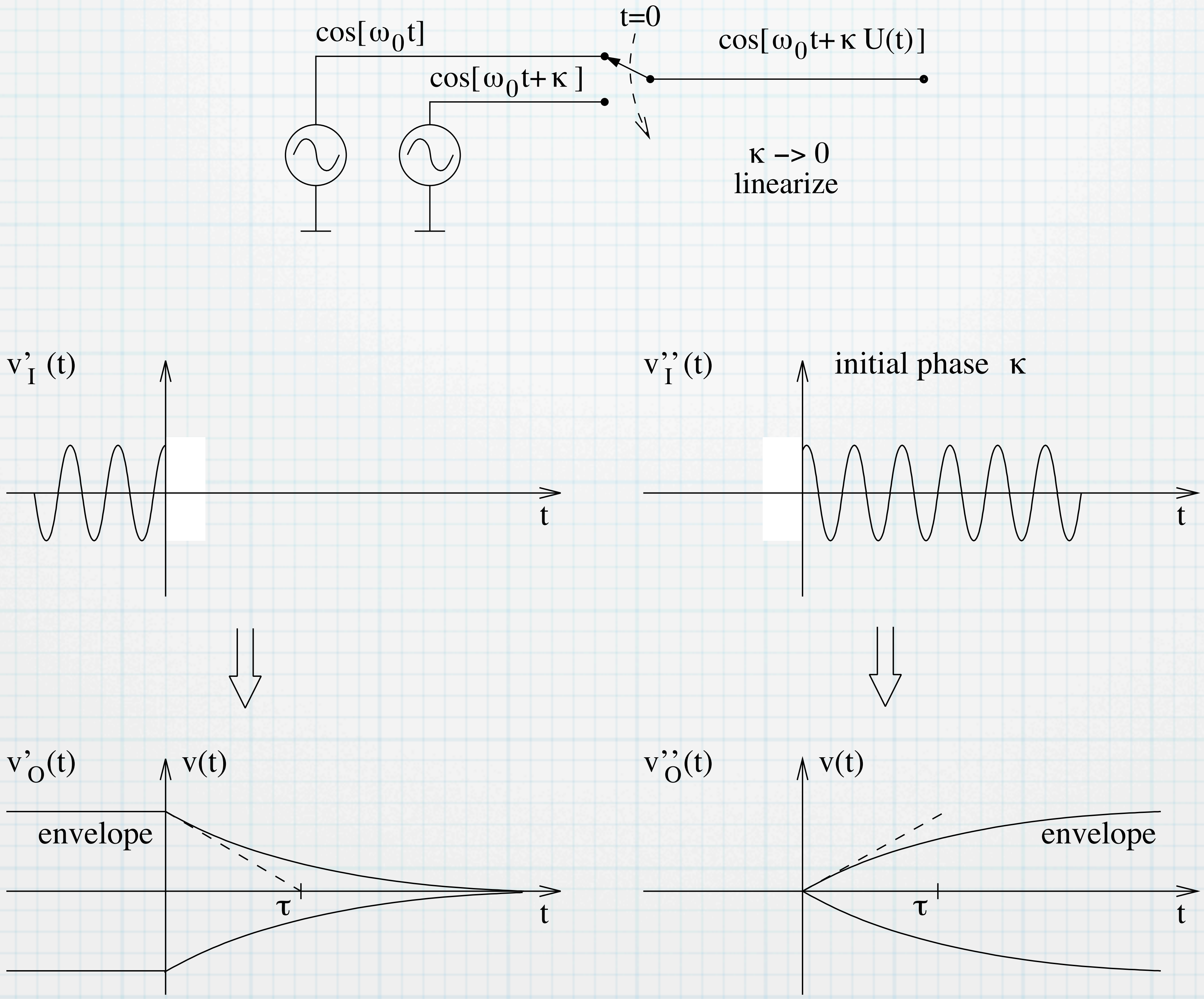
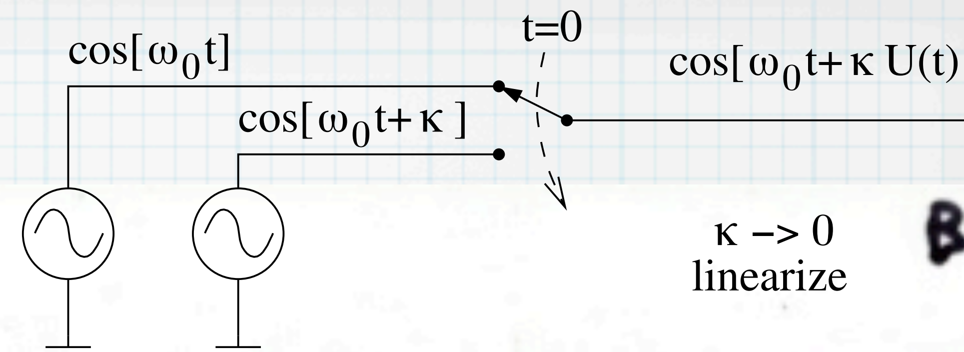
Details



$$\cos(\omega_0 t)\, e^{-t/\tau} \qquad \tau = \frac{2Q}{\omega_0} \qquad \cos(\omega_0 t + \kappa)\,\left[1 - e^{-t/\tau}\right]$$

Details [2]



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DETAILS

$v(t)$ resonator output voltage

$$v(t) = \cos(\omega_0 t)\, e^{-t/\tau} + \cos(\omega_0 t+\kappa)\left[1-e^{-t/\tau}\right]$$

decay growth

$$v(t) = \cos(\omega_0 t)\, e^{-t/\tau} + \left[\cos(\omega_0 t)\cos(\kappa) - \sin(\omega_0 t)\sin(\kappa)\right]\left[1-e^{-t/\tau}\right]$$

use $\kappa \ll 1$ $\cos\kappa = 1$ $\sin\kappa = \kappa$

$$v(t) = \cos(\omega_0 t)\, e^{-t/\tau} + \left[\cos\omega_0 t - \kappa\sin\omega_0 t\right]\left[1-e^{-t/\tau}\right]$$

$$= \cos(\omega_0 t)\, e^{-t/\tau} + \cos(\omega_0 t)\left[1-e^{-t/\tau}\right] - \kappa\sin(\omega t)\left[1-e^{-t/\tau}\right]$$

$$v(t) = \cos(\omega_0 t) - \kappa\sin(\omega_0 t)\left[1-e^{-t/\tau}\right]$$

Re Im Fresnel vector

FRESNEL VECTOR V

$\kappa\left[1 - e^{-t/\tau}\right]$

$$\arg(V) = \kappa\left[1 - e^{-t/\tau}\right]$$

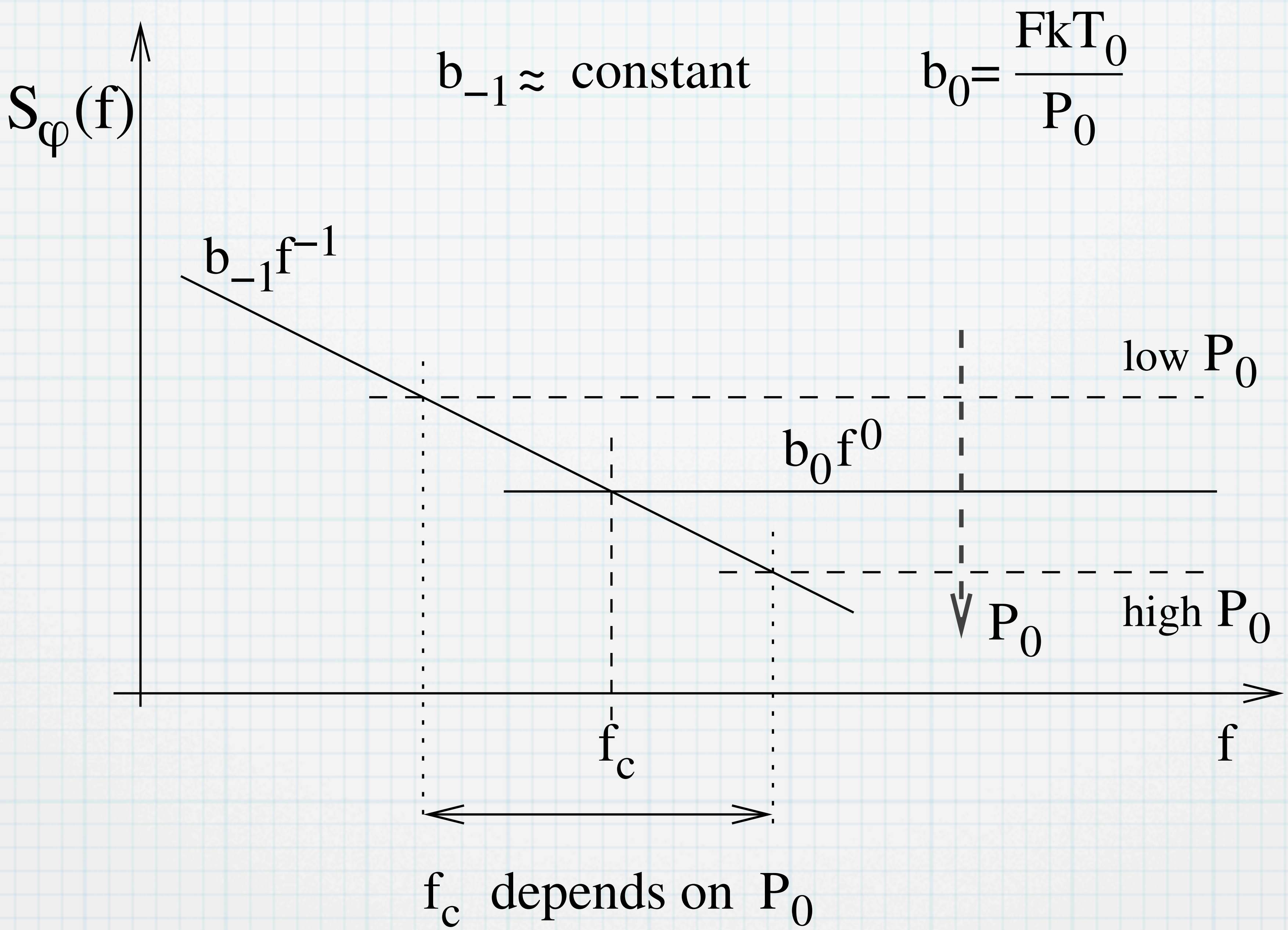
unity step →
- replace $\kappa u(t) \to u(t)$
- don't forget that all this holds for $\kappa \ll 1$

step response $\quad b_u(t) = 1 - e^{-t/\tau}$

impulse response $\quad b(t) = \frac{1}{\tau} e^{-t/\tau} \quad$ derivative

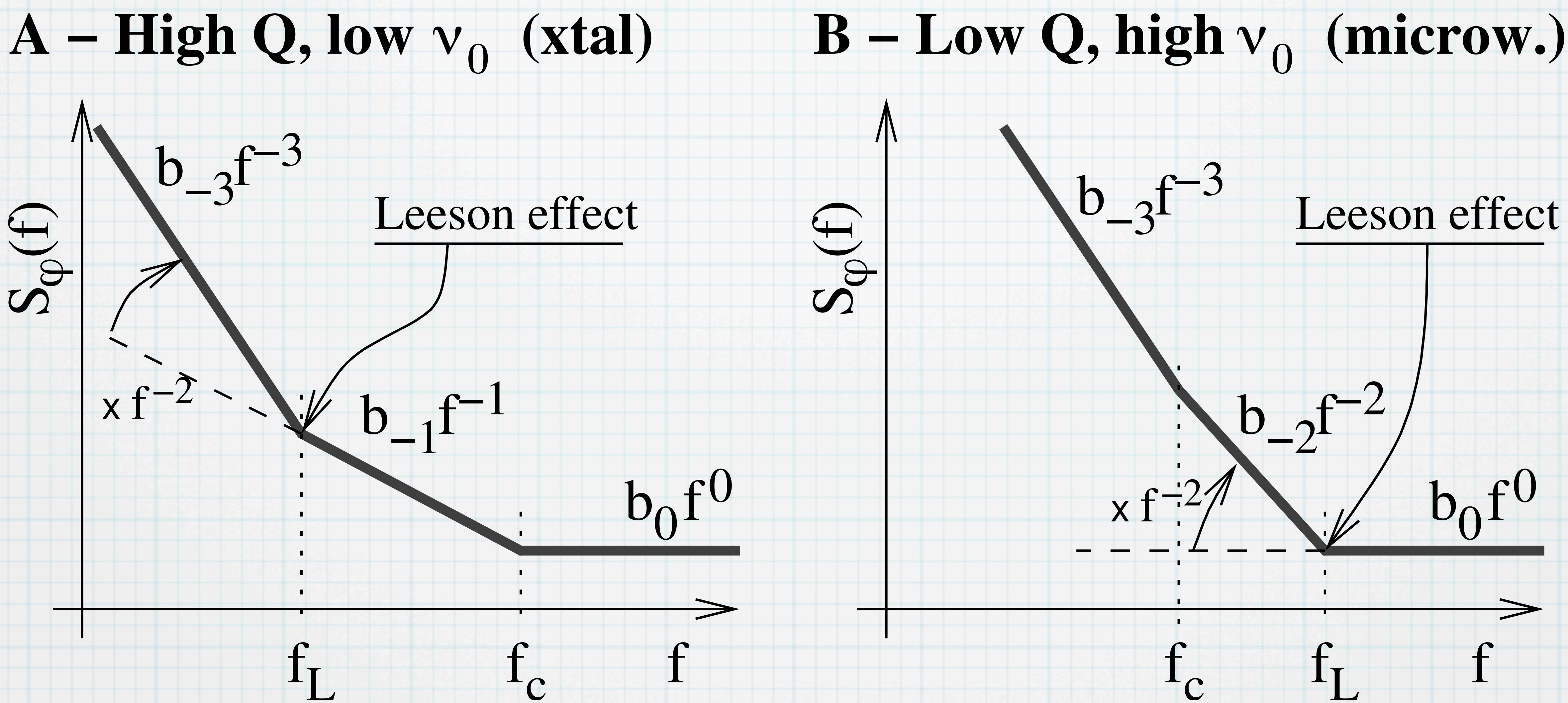
$$B(s) = \frac{1/\tau}{s + 1/\tau} = \frac{1}{s\tau + 1}$$

Summary of the amplifier phase noise



- White PM noise is inversely proportional to P_0
- Flicker PM noise is about independent P_0
- The corner frequency f_c follows

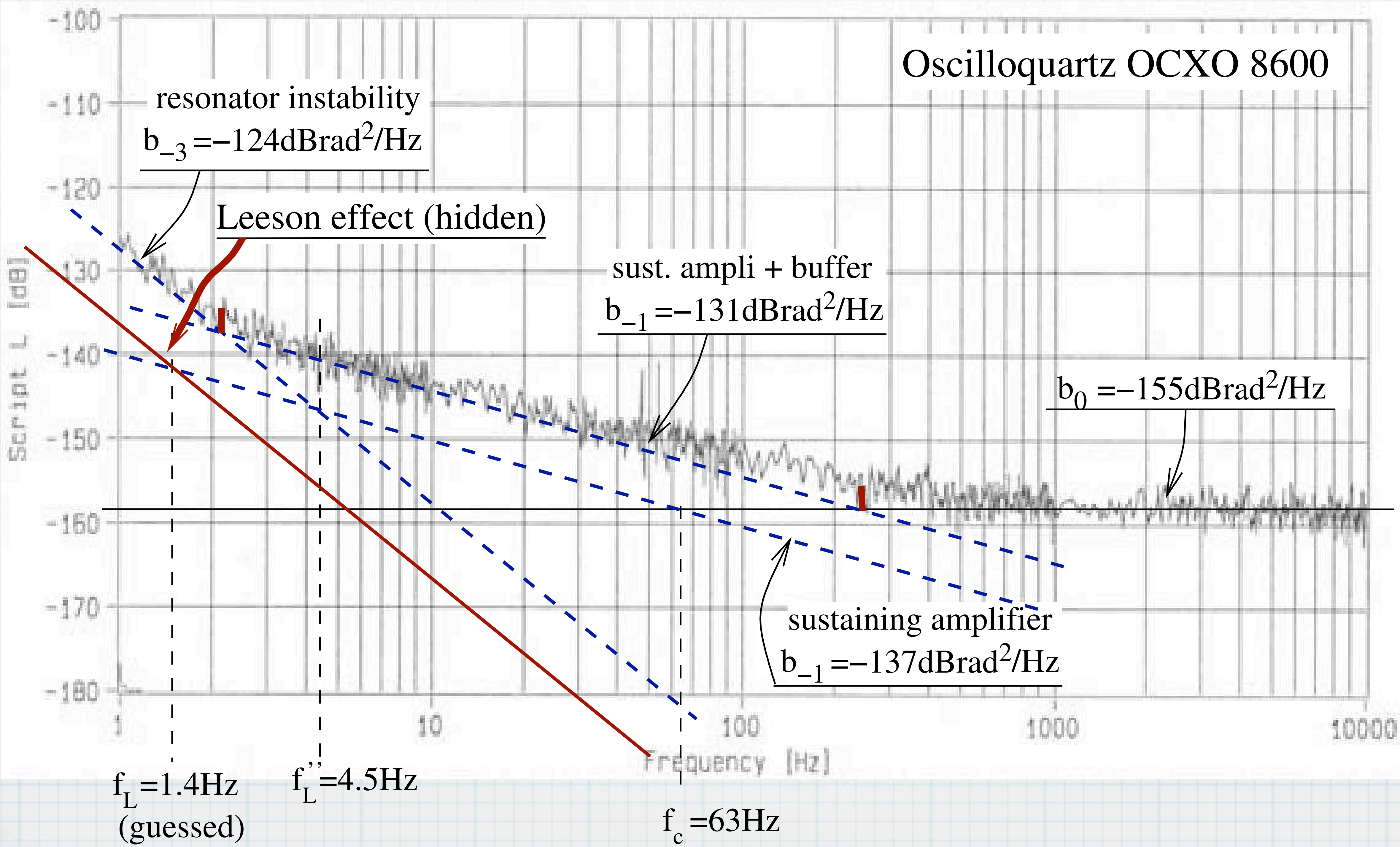
The Leeson effect



two typical patterns

Example – Oscilloquartz 8600

Courtesy of Oscilloquartz. Interpretation and mistakes are of the authors

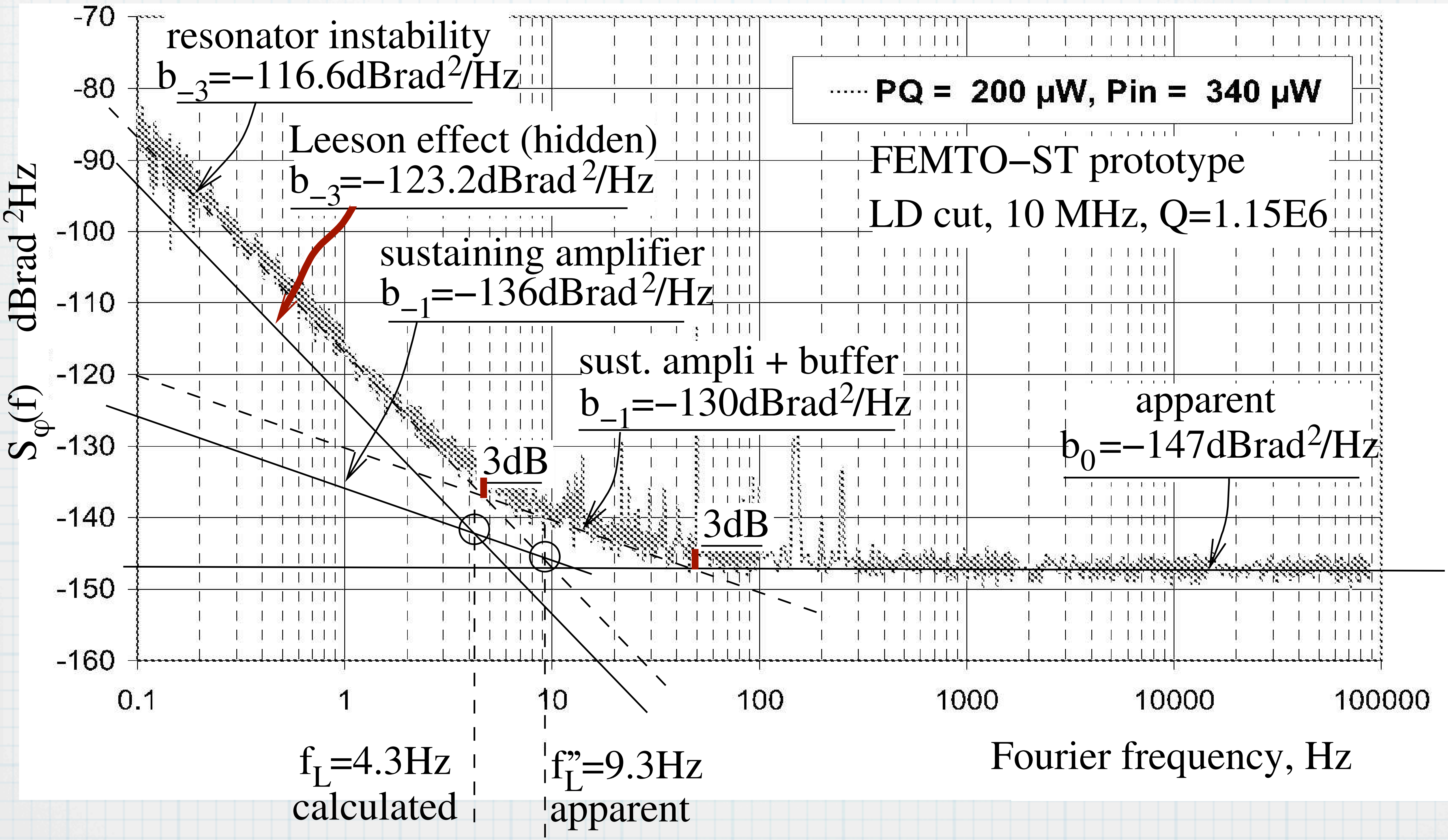


F=1dB b_0 => P_0=−18 dBm

$(b_{-3})_{osc}$ => $\sigma_y=1.5\times10^{-13}$, $Q=5.6\times10^5$ (too low)

$Q \stackrel{?}{=} 1.8\times10^6$ => $\sigma_y=4.6\times10^{-14}$ Leeson (too low)

Example – FEMTO-ST prototype



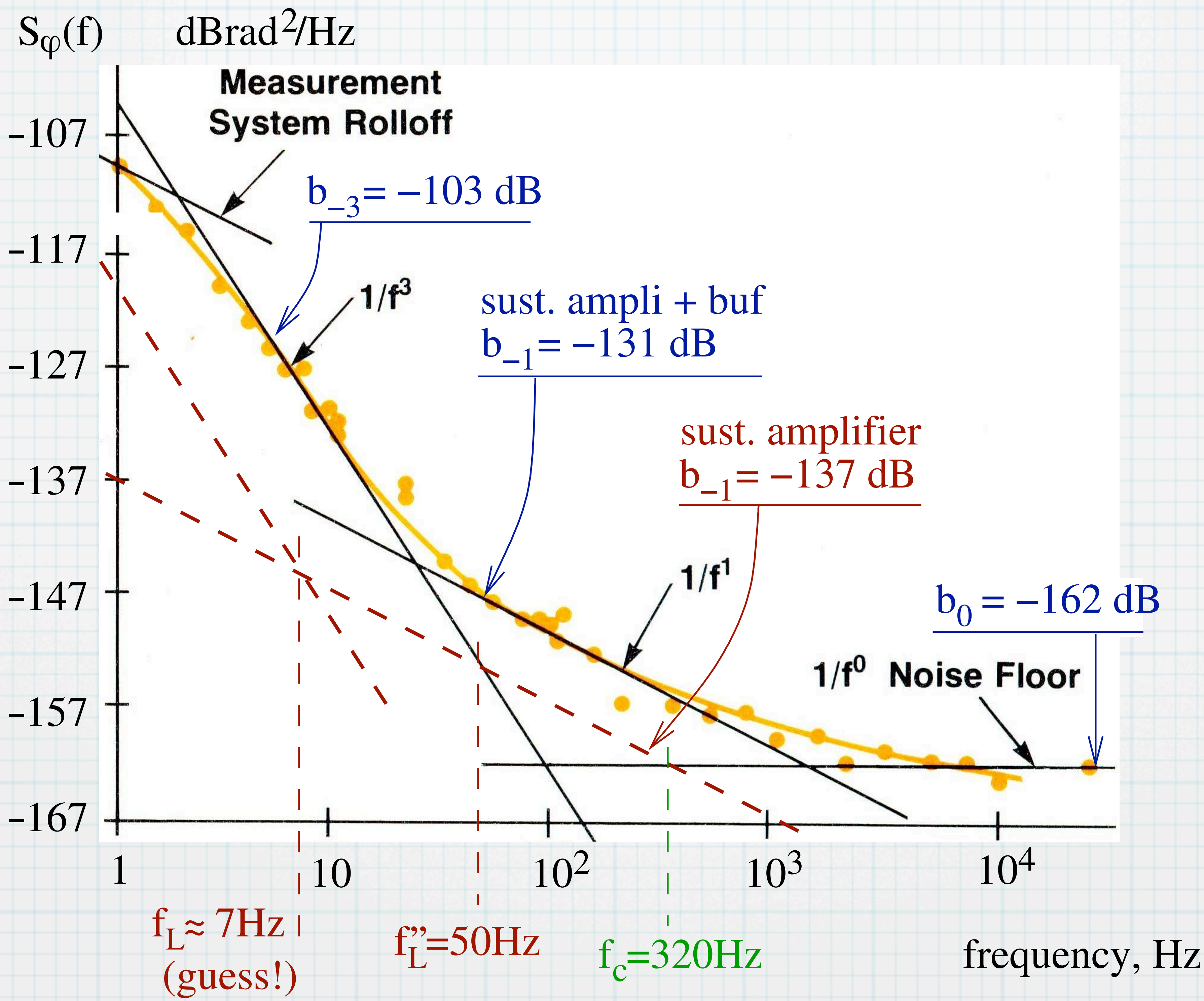
F=1dB b_0 => P_0=–26 dBm

(there is a problem)

$(b_{-3})_{osc}$ => $\sigma_y=1.7\times10^{-13}$, $Q=5.4\times10^{5}$ (too low)

$Q=1.15\times10^{6}$ => $\sigma_y=8.1\times10^{-14}$ Leeson (too low)

Example – Agilent 10811

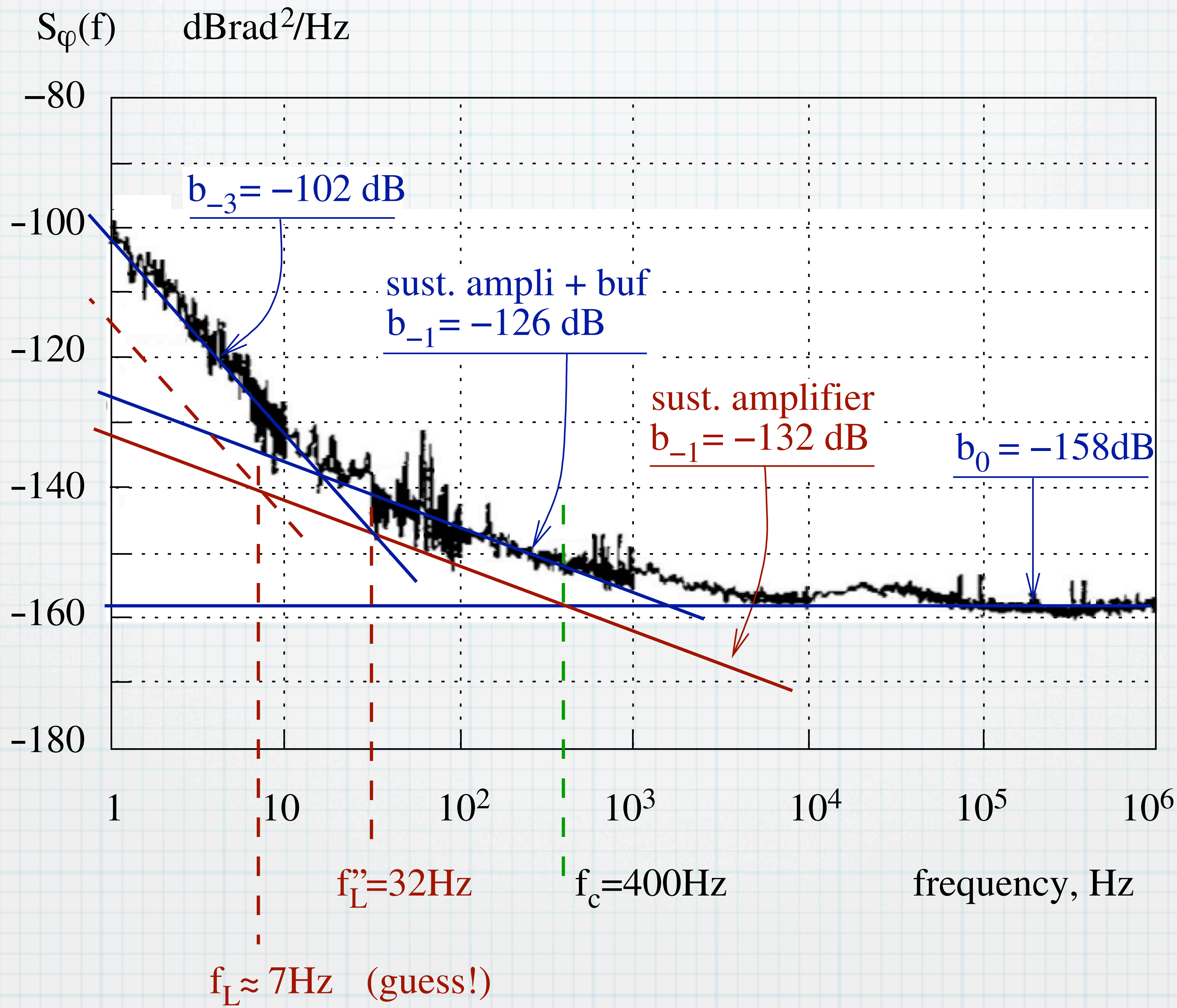


F=1dB b_0 => P_0=–11 dBm

$(b_{-3})_{osc}$ => $\sigma_y=8.3\times10^{-13}$, $Q=1\times10^5$ (too low)

$Q \overset{?}{=} 7\times10^5$ => $\sigma_y=1.2\times10^{-13}$ Leeson (too low)

Example – Agilent prototype



F=1dB b_0 => P_0=–12 dBm

$(b_{-3})_{osc}$ => $\sigma_y=9.3\times10^{-13}$ $Q=1.6\times10^5$

$Q \stackrel{?}{=} 7\times10^5$ => $\sigma_y=2.1\times10^{-13}$ (Leeson)