RF Instrumentation

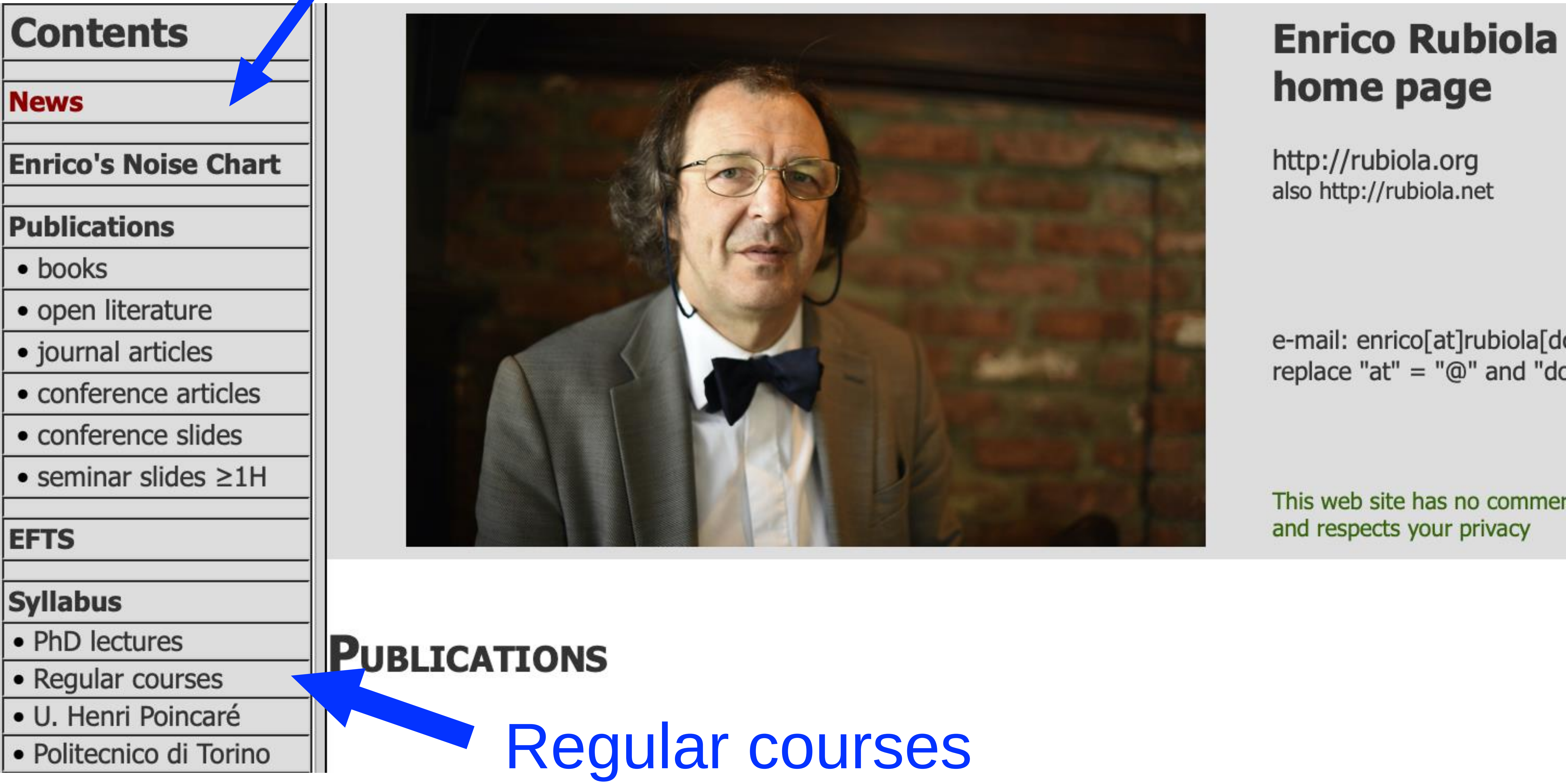
Enrico Rubiola

2020-2021

- Burden:
  21 H Lectures and work
  8 H Labs

Learning Material

Science and PhD courses only



Contents

News

Enrico's Noise Chart

Publications
- books
- open literature
- journal articles
- conference articles
- conference slides
- seminar slides ≥1H

EFTS

Syllabus
- PhD lectures
- Regular courses
- U. Henri Poincaré
- Politecnico di Torino

Enrico Rubiola home page

http://rubiola.org
also http://rubiola.net

e-mail: enrico[at]rubiola[d
replace "at" = "@" and "d

This web site has no commer
and respects your privacy

PUBLICATIONS

Regular courses

Lectures, Labs and Exam

14 séances CM/TD 21H

2 TP 8H

Jan

last exam

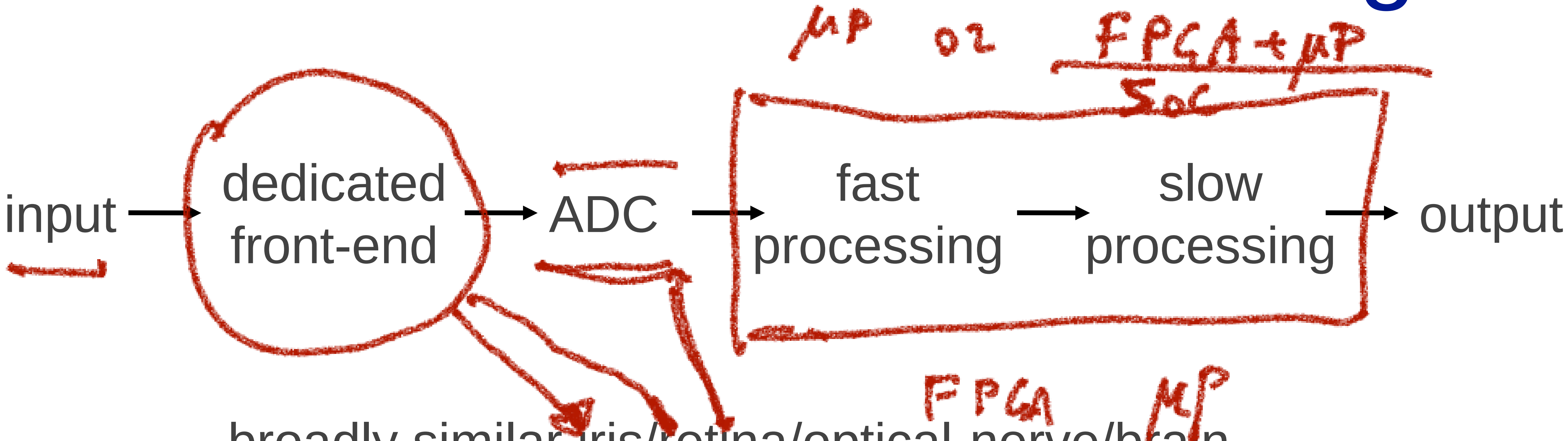
Digital Instruments

1.5 Hours

#1 Thursday, Sept 10, 2020

1.5 Hours

Where Are Instruments Going?



broadly similar iris/retina/optical-nerve/brain

FIXED POINT

Moore Law: describes exponentially growing technology

12-bit converters – from my memory
(years may not be accurate)

- 1985-1990 15-20 MS/s
- 2005 250 MS/s
- 2018 12 GS/s

System on Chip
FPGA + μP
(ADC/DAC + FPGA + μP)

Dedicated Front-End

RF

ADC a few GHz

- fast
- low impedance

30 - 300 Ω

50 - 100 Ω

$$\sqrt{\mu_0/\epsilon_0} = 377\,\Omega$$

$$Z \sim \sqrt{\frac{\mu_0}{\epsilon}} \leftarrow \mu \approx \mu_0$$

$$\epsilon > \epsilon_0$$

charge C

current I

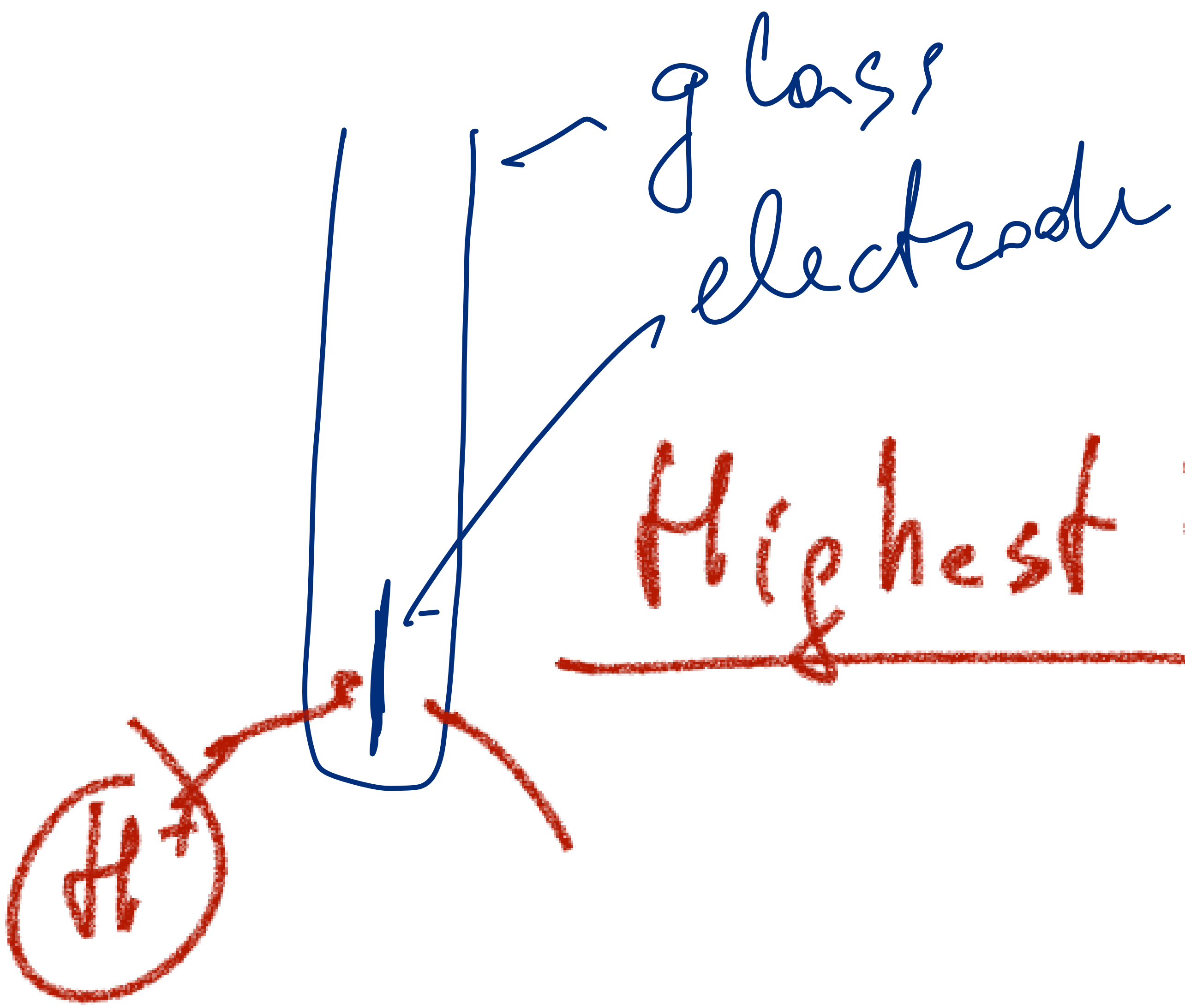
guided propagation

Dedicated Front. End

PH $\log_{10} \frac{\text{Protons}}{\text{mole}}$ ← Volume?

⊕

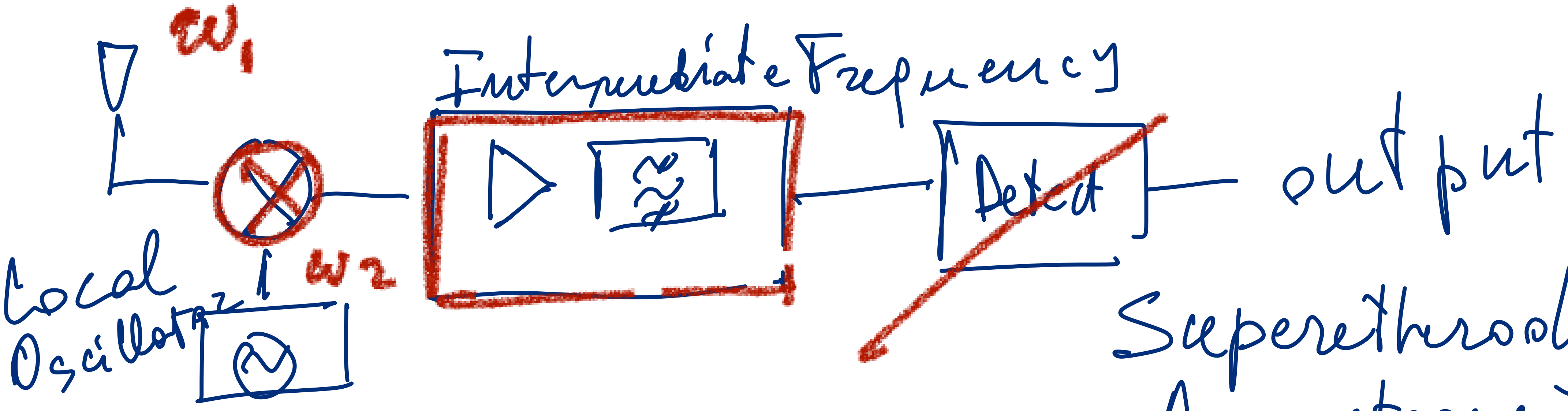
H_2O H^+



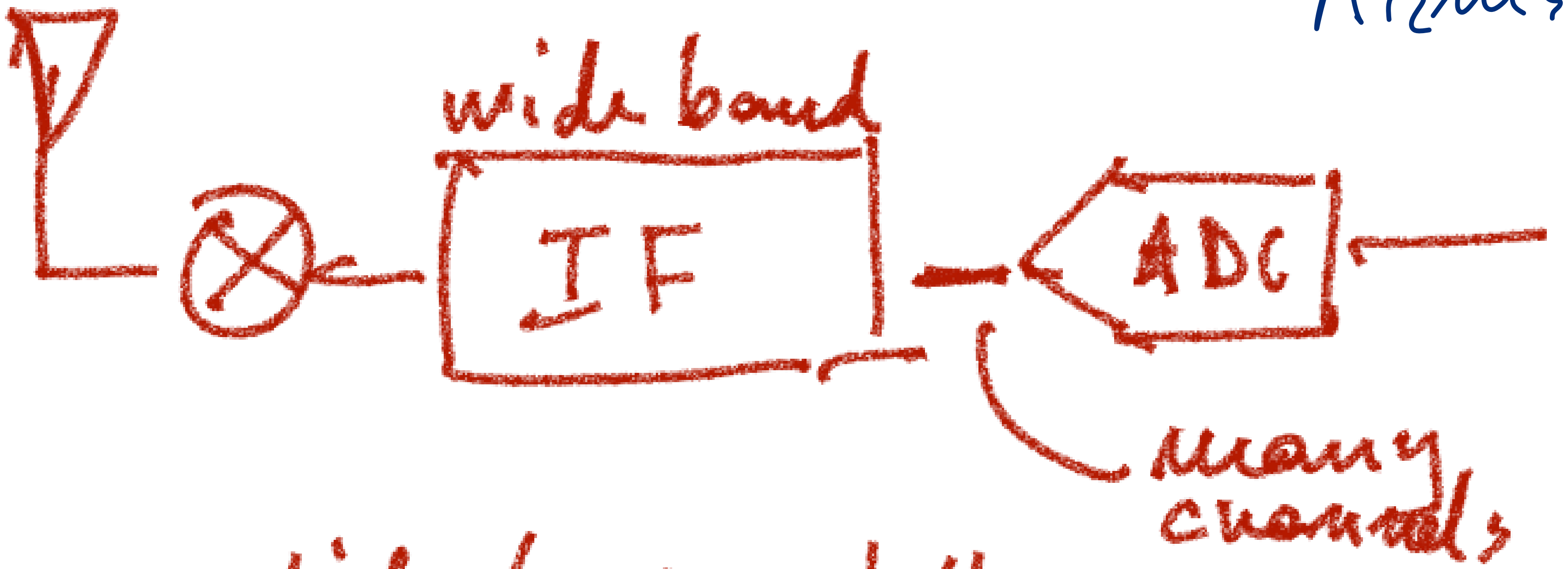
Highest Z

Where Are Telecom Going?

Need for down conversion progressively removed



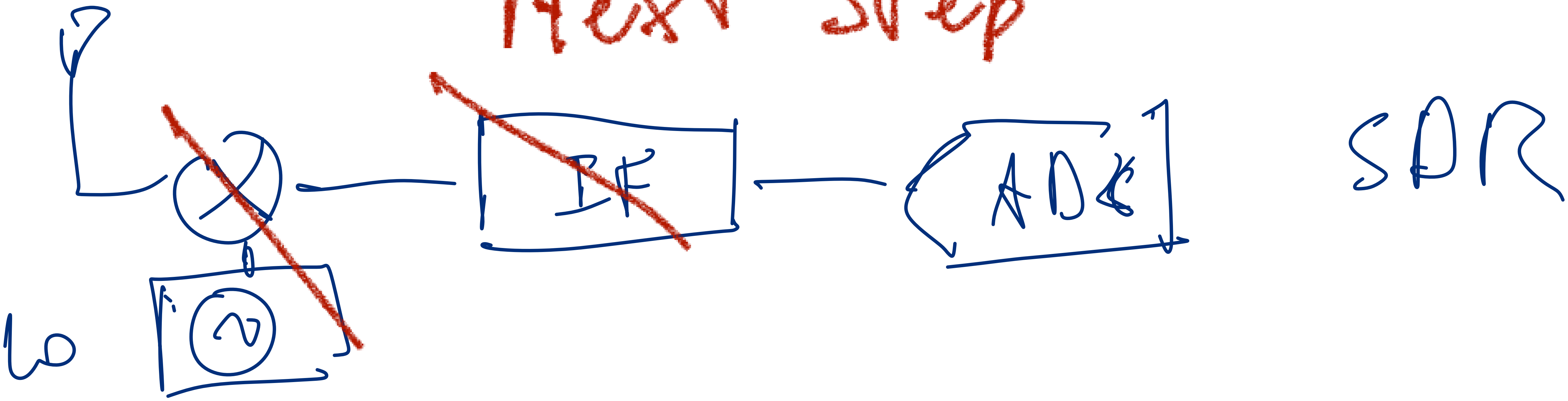
Superheterodyne
Armstrong 1930

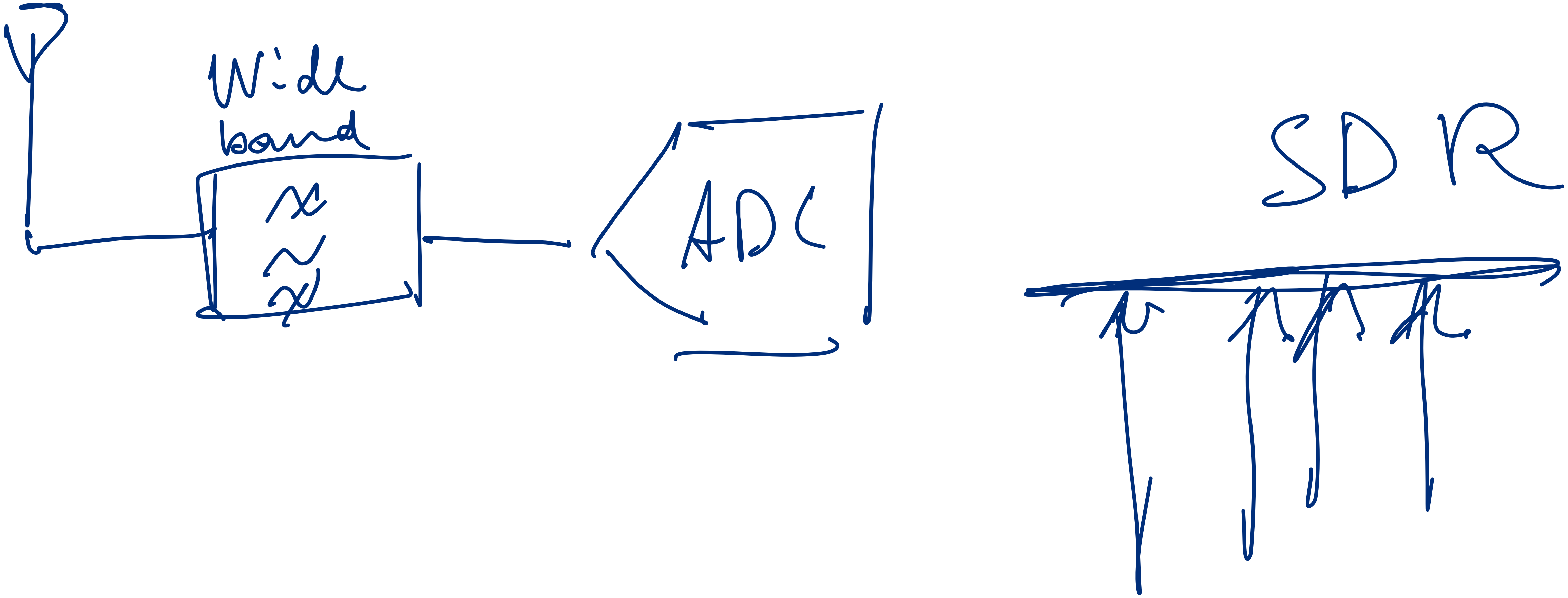


Identify/extract the channels in software

SDR = Software Defined Radio
Jean Michel Friedt

Next Step





most figures are from

ANALOG-DIGITAL CONVERSION

Walt Kester

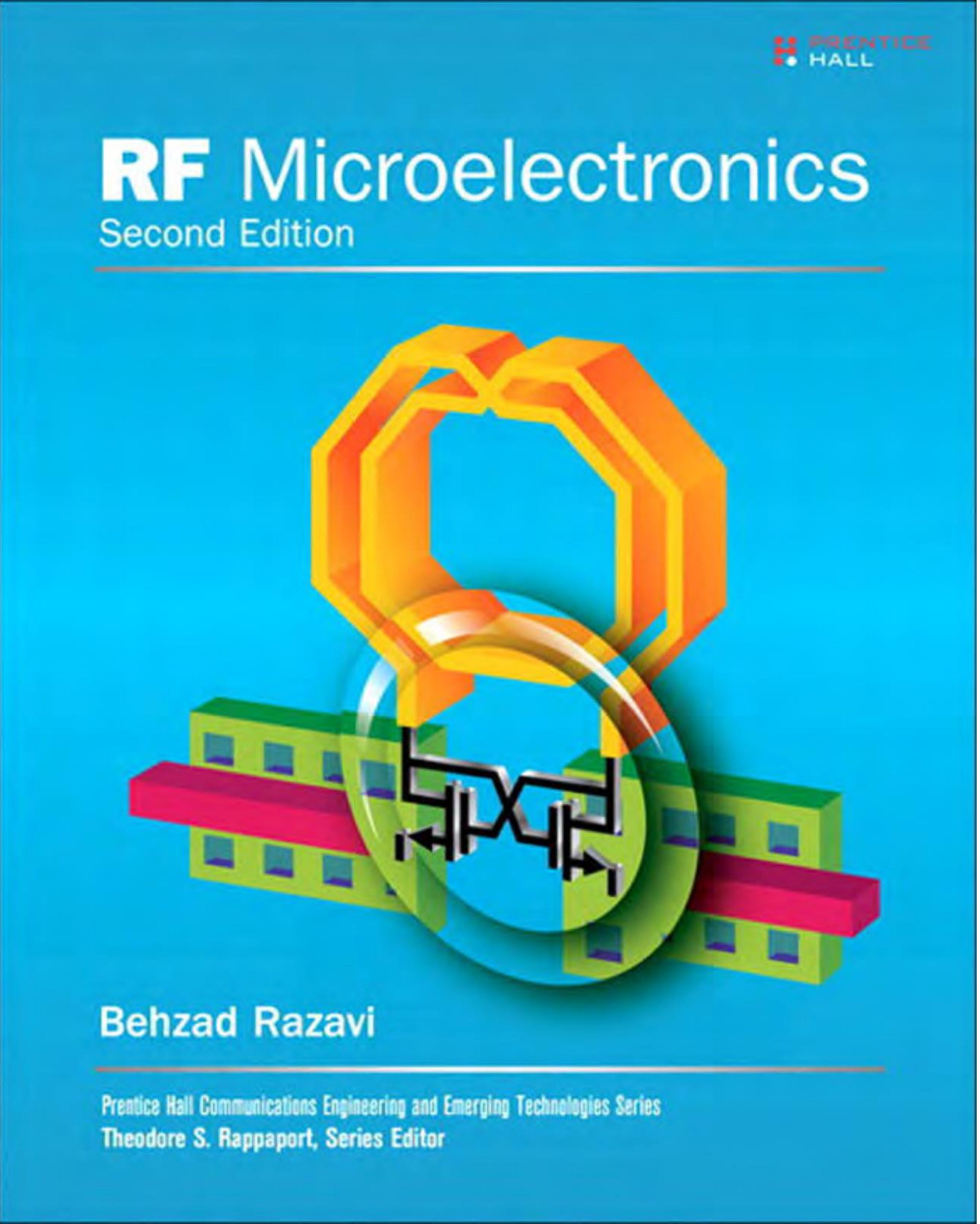
Editor

ANALOG
DEVICES

1138 pages
© AD, but free

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Useful textbook (950 pages)



PRENTICE HALL

RF Microelectronics
Second Edition

Behzad Razavi

Prentice Hall Communications Engineering and Emerging Technologies Series
Theodore S. Rappaport, Series Editor

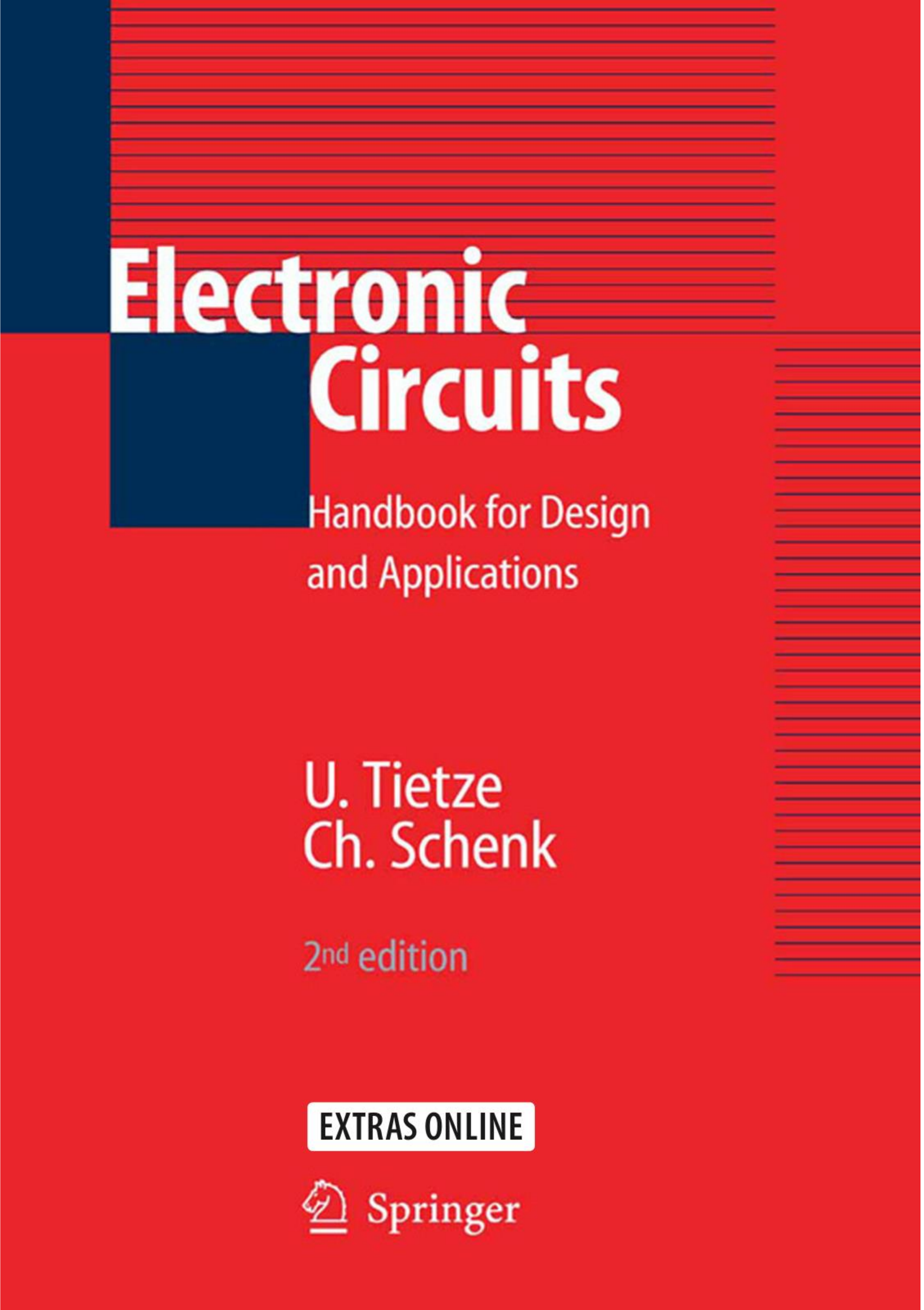
Genesis library

Useful textbook
recent (2015)
1225 pages

1 chapter
Analog meets Digital

for
experimentalists

Genesis library



Electronic Circuits

Handbook for Design and Applications

U. Tietze
Ch. Schenk

2nd edition

EXTRAS ONLINE

Springer

Useful textbook elderly (2002) but great 1544 pages

German soul

Genesis library

Conversion Basics

Analog to Digital Conversion

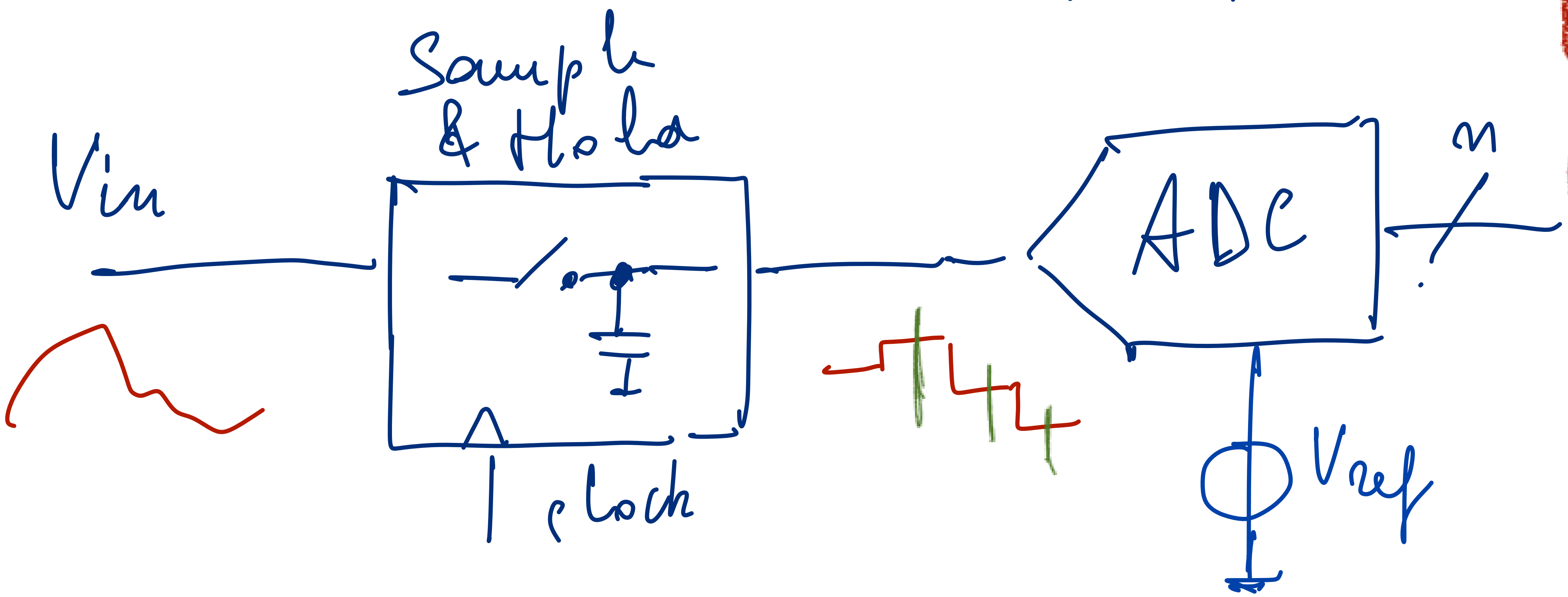
Basic scheme, with S/H, ADC, Href

$V_{in} \to N$

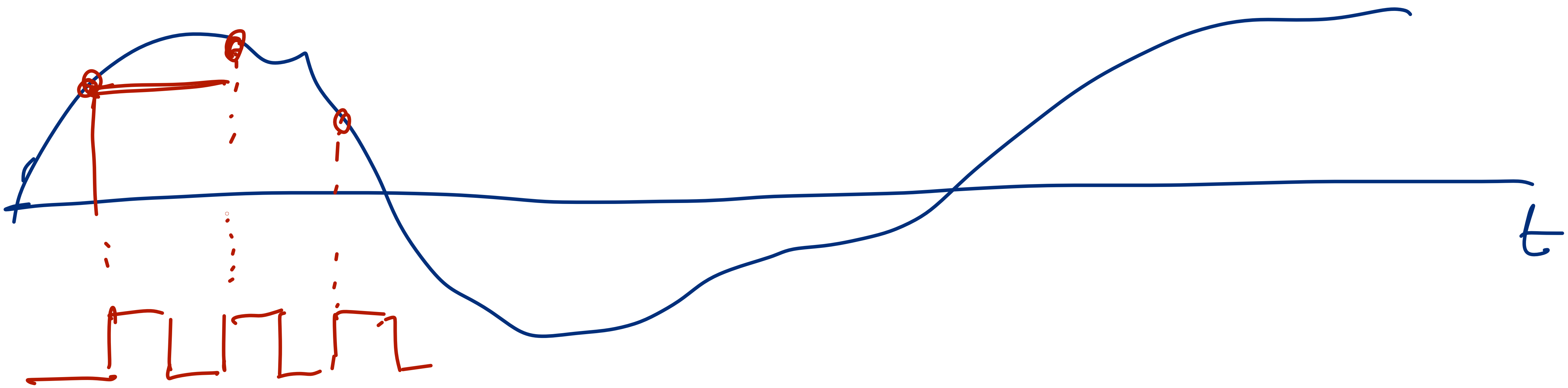
n bits

$$\boxed{N \approx K \frac{V_{in}}{V_{ref}}}$$

nint()



V_{LSB}



End of lecture 1

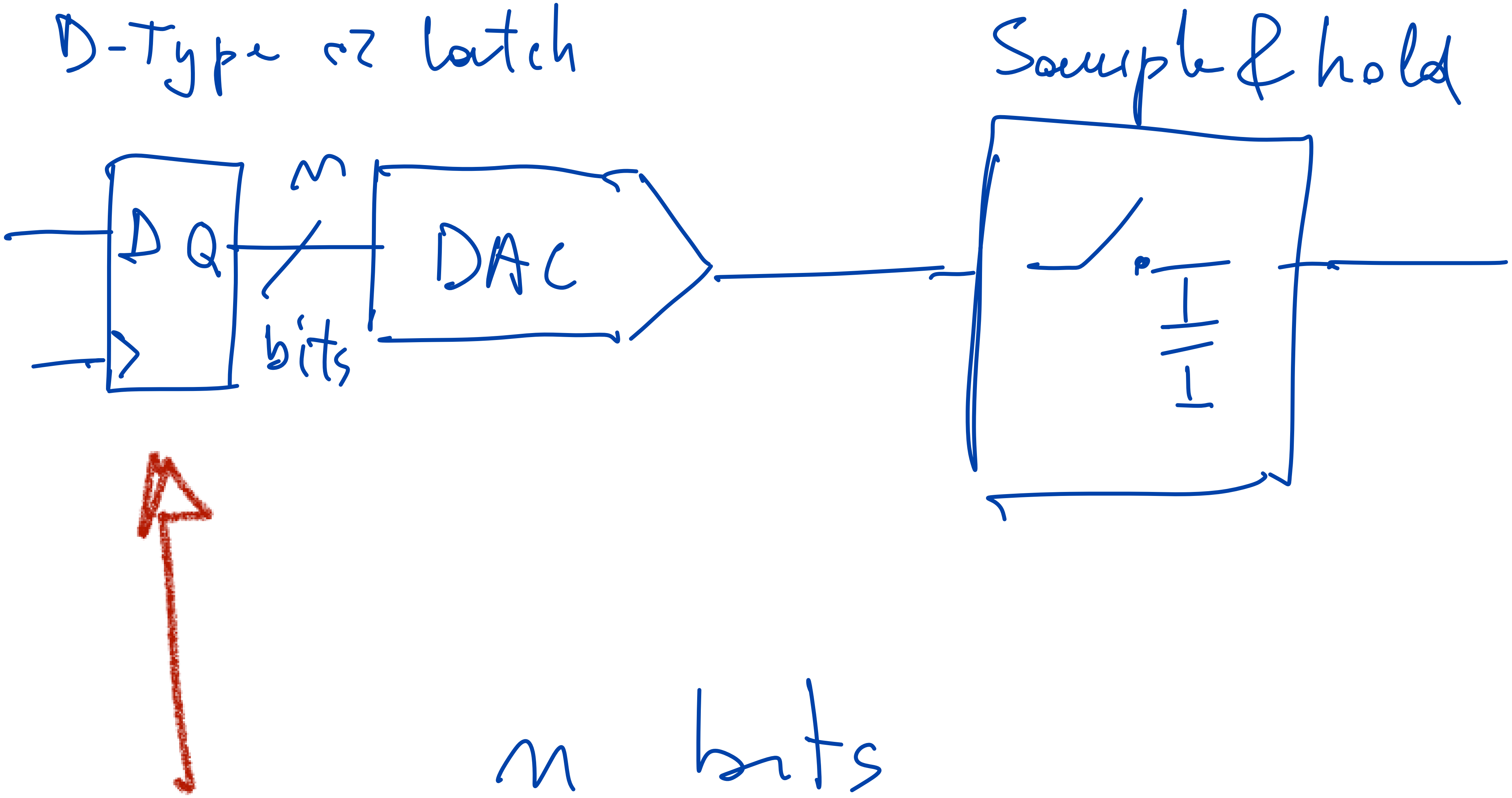
#2 Thursday, Sept 17, 2020

1.5 Hours

Digital to Analog Conversion

Basic scheme, with ADC, Vref, S/H,

both in/out must be stable

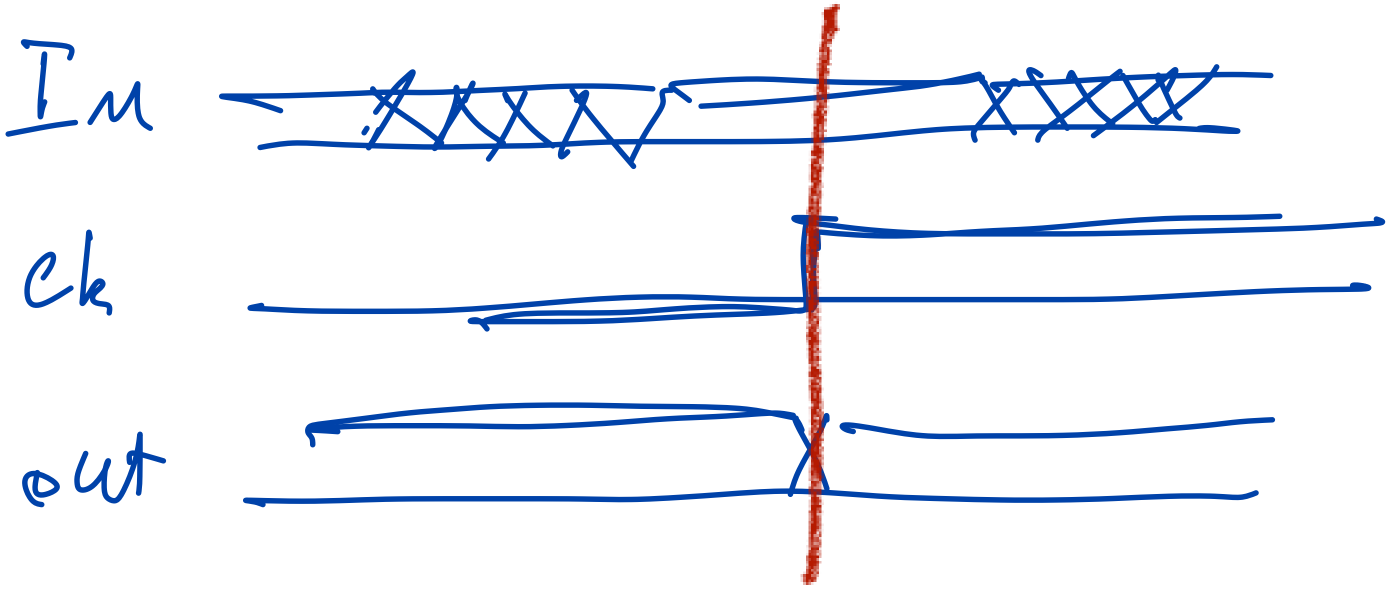


n bits

N values

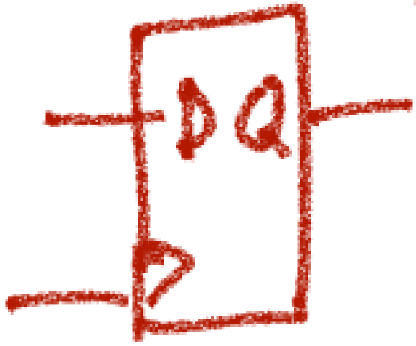
$N = 2^n$

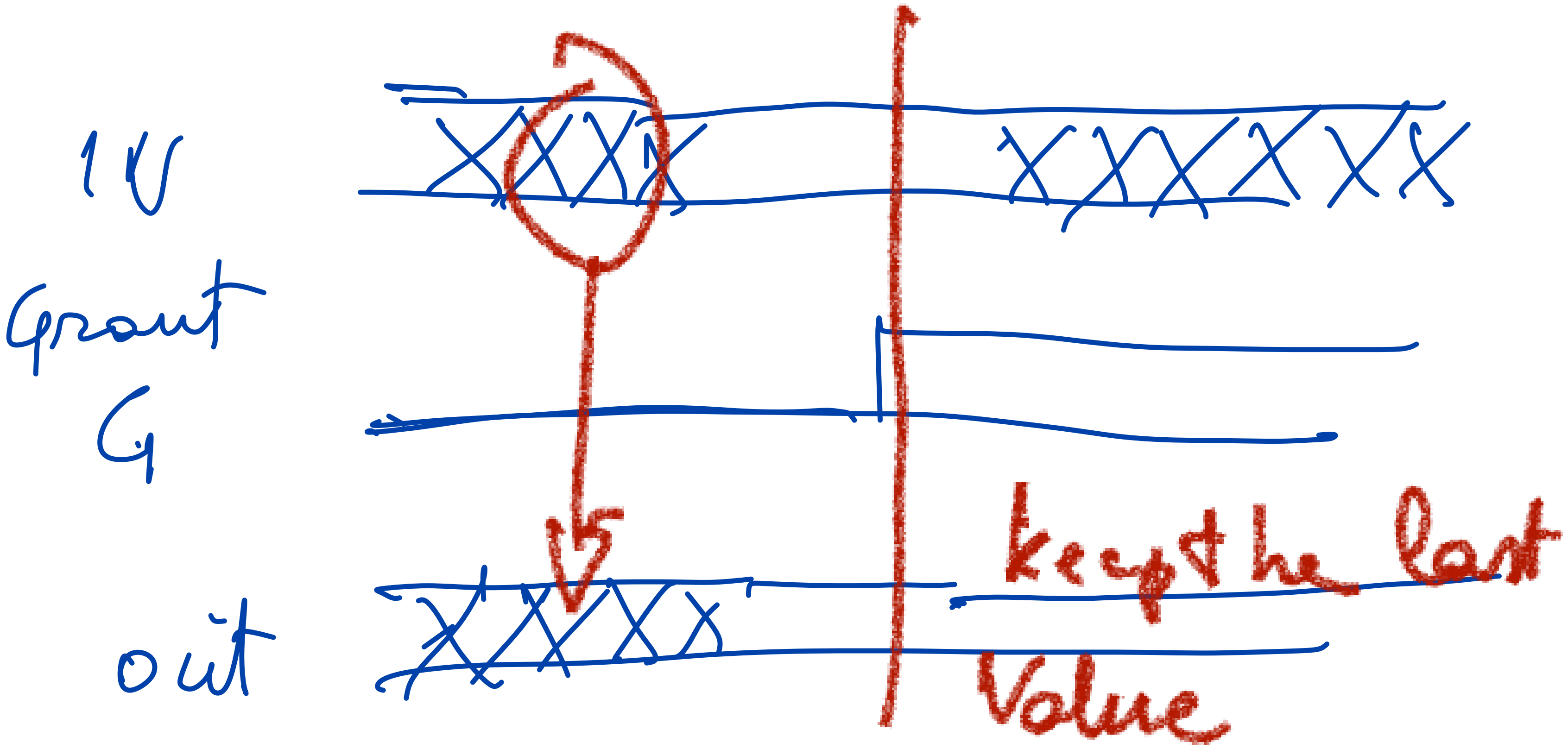
D.Type FF vs Latch



logic sampling

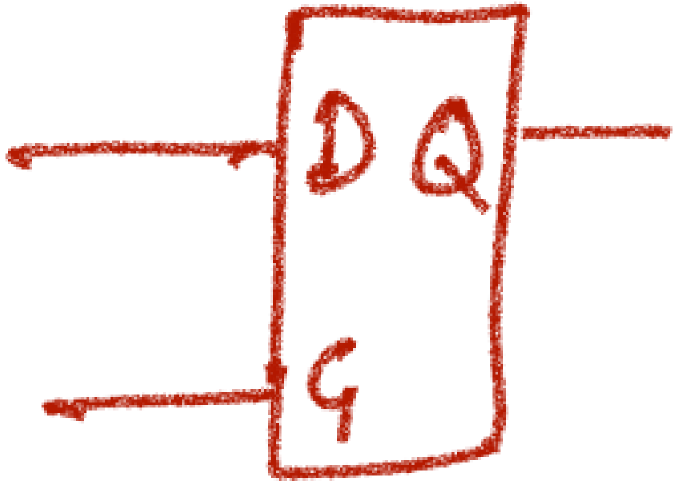
D.Type



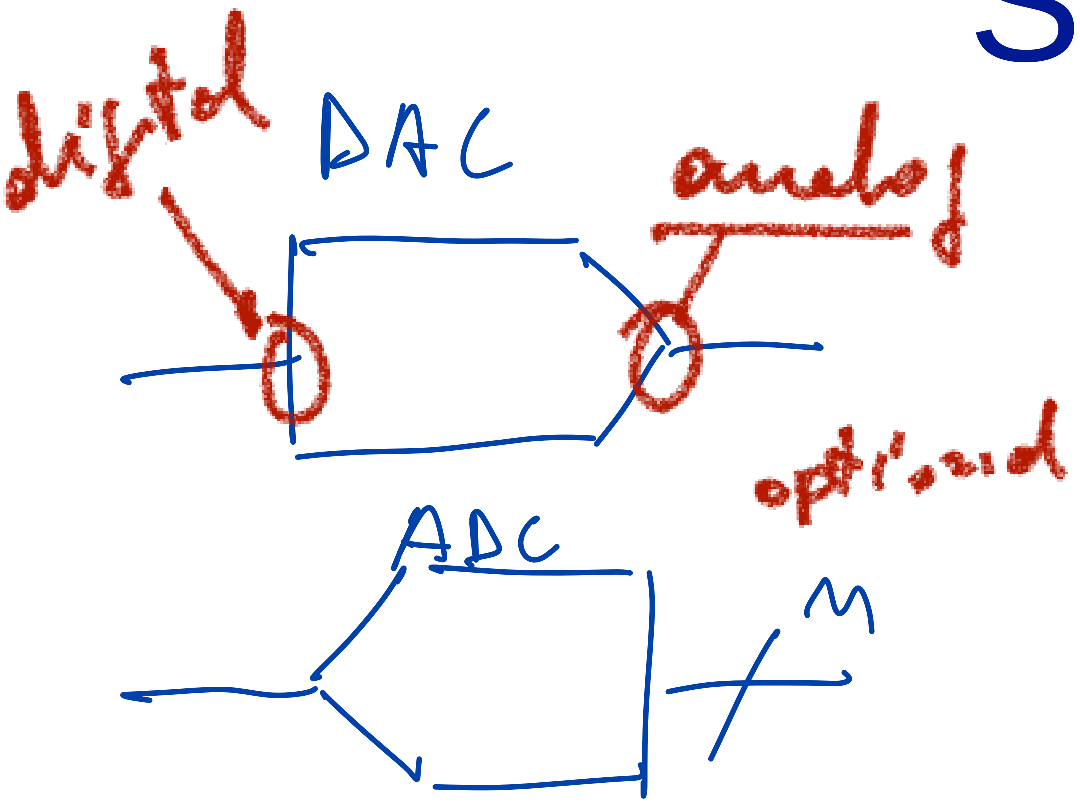


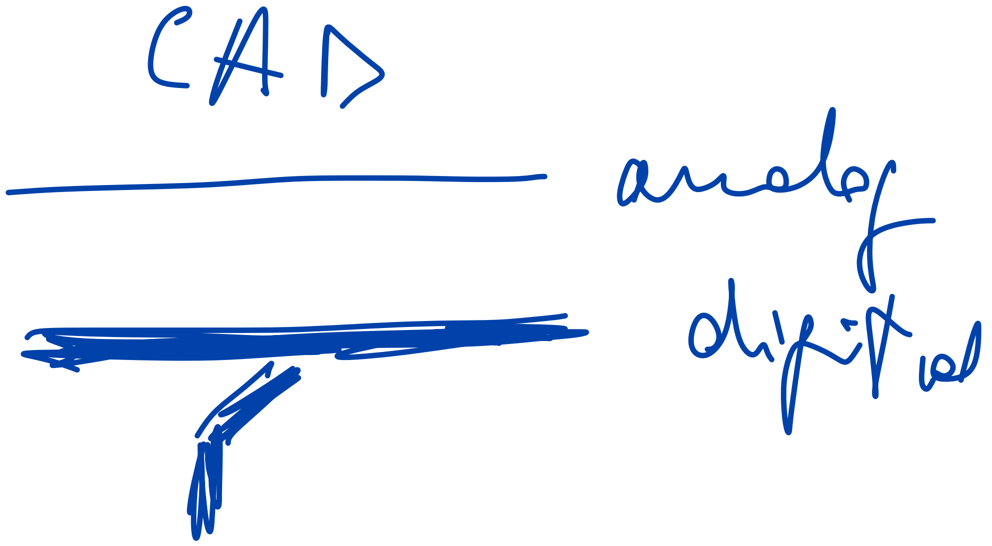
keep the last value

Latch



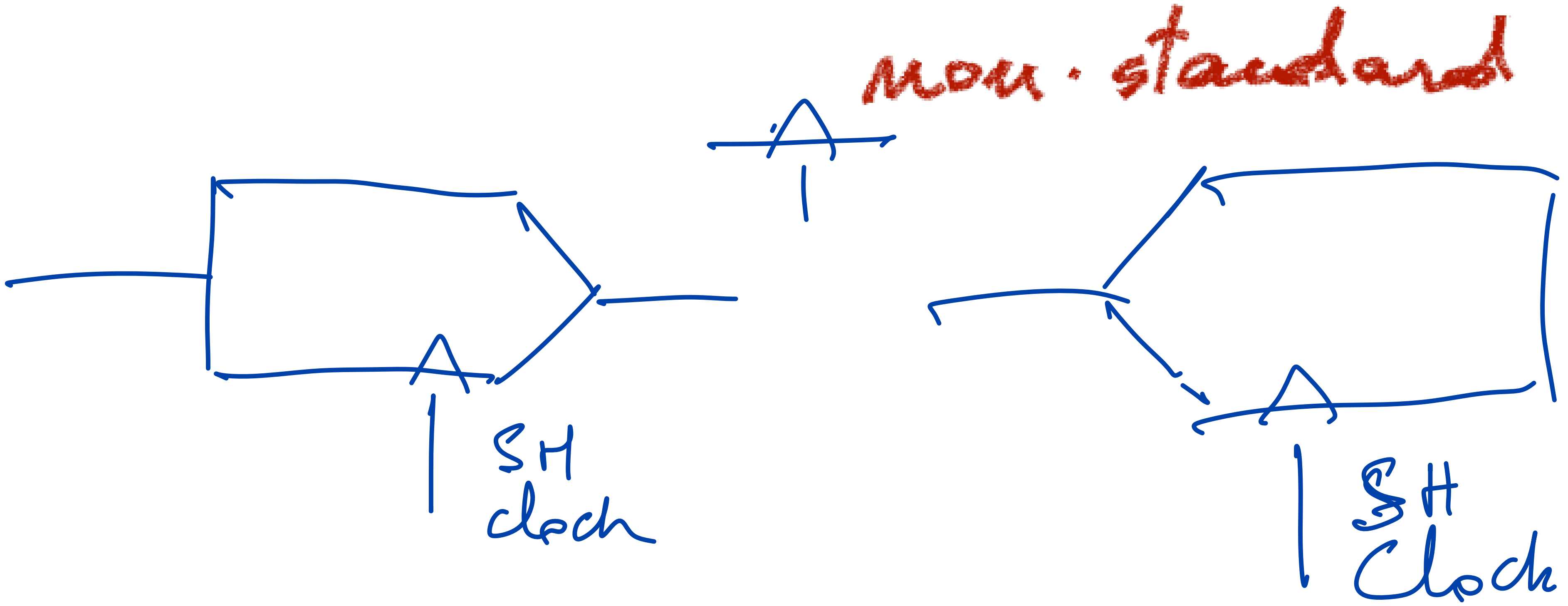
Symbols





Often latch and S/H inside

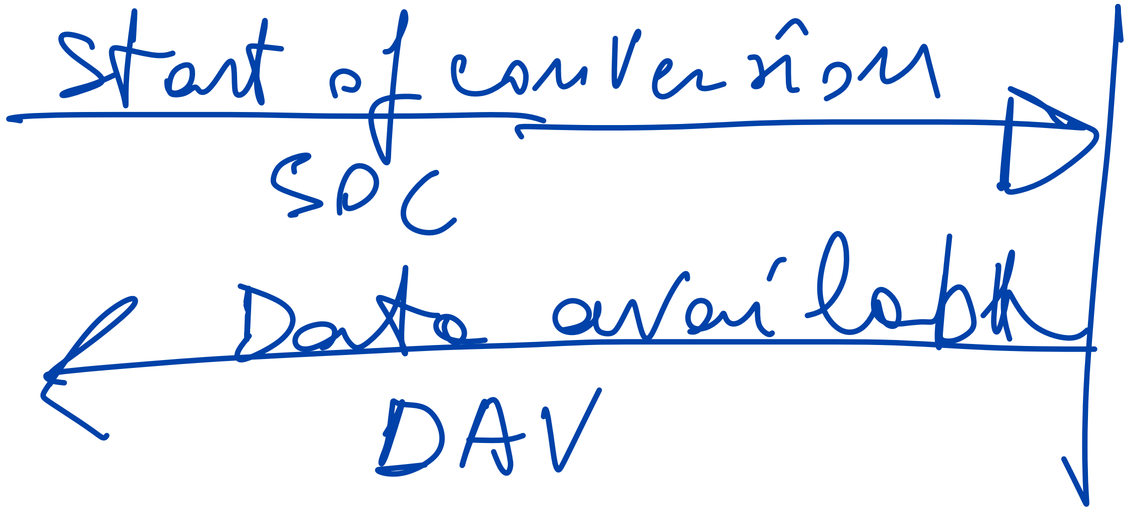
FF-D



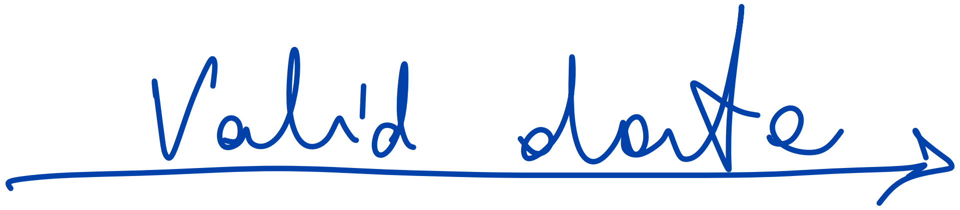
Timing and Protocol

Communication

ADC

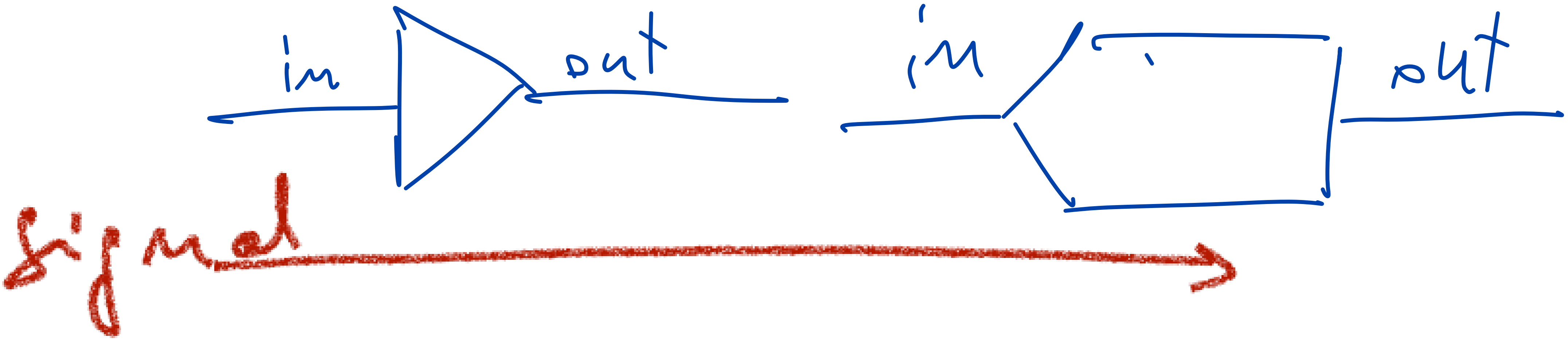


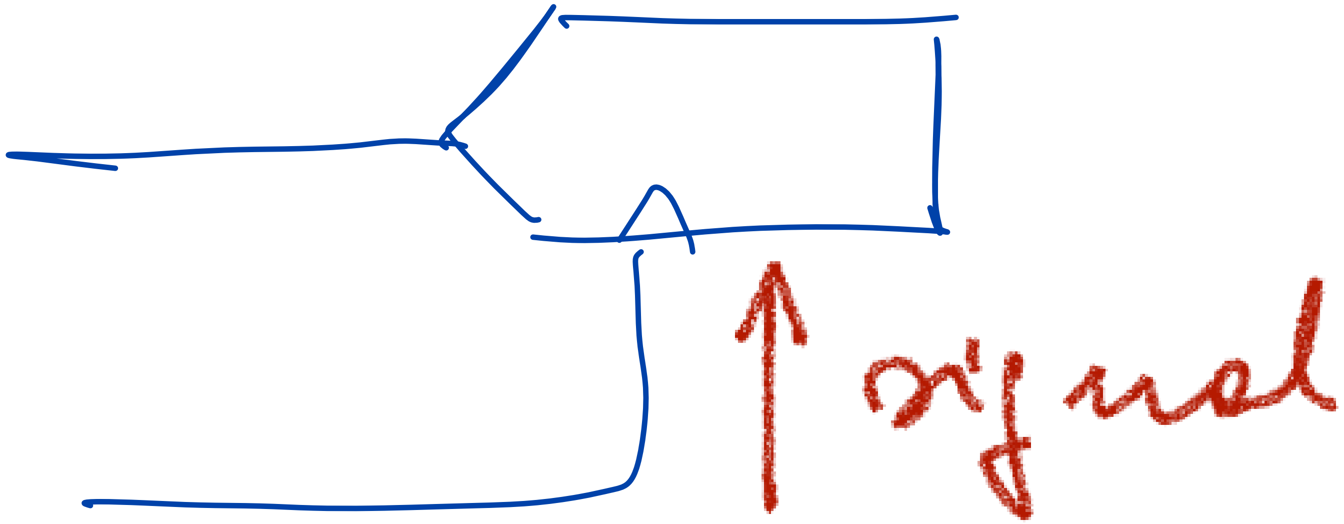
DAC



Drawing Rules

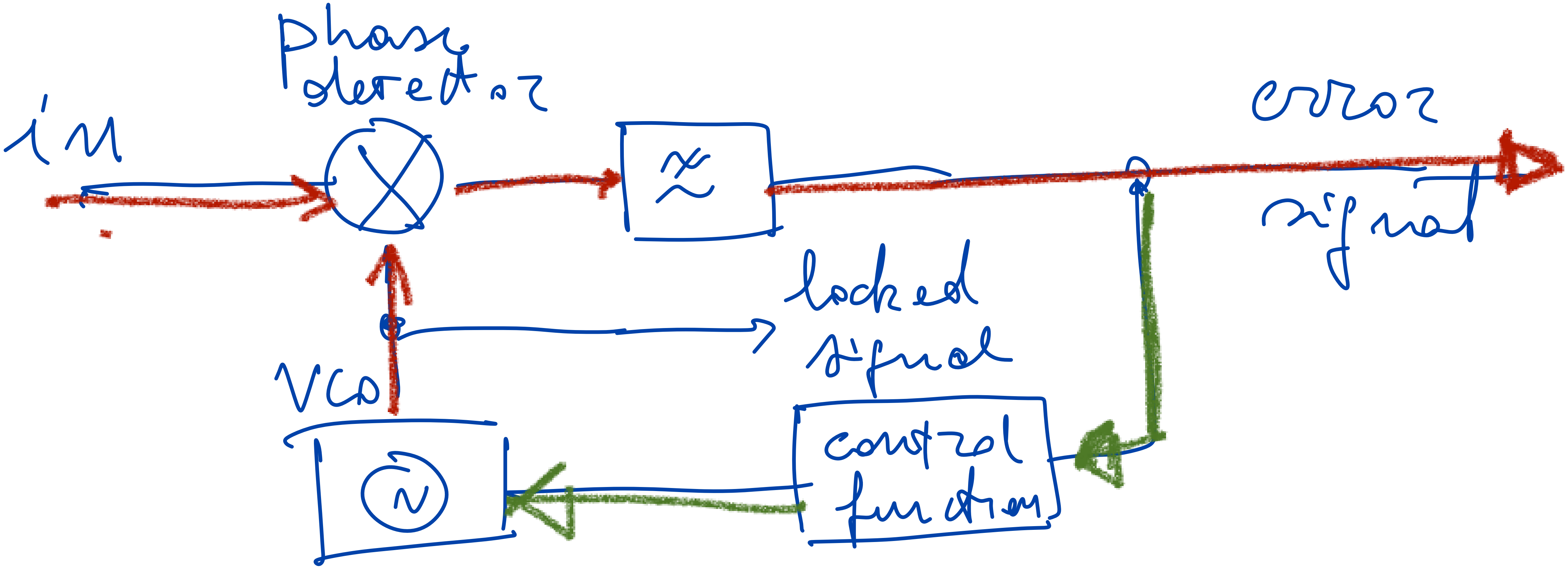
Make schematics clear





Feedback → change direction

PLL



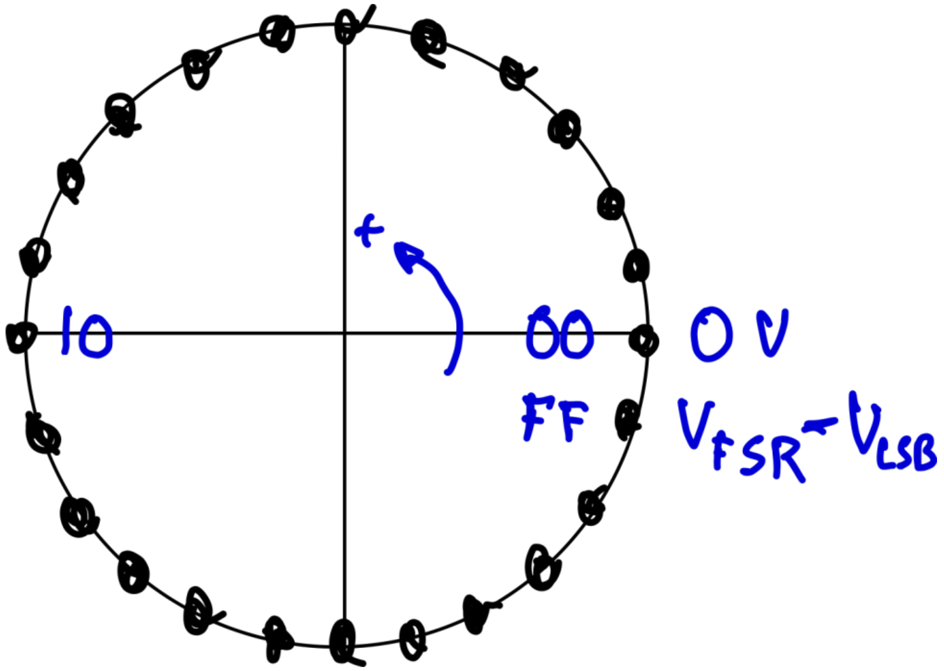
main path

feedback

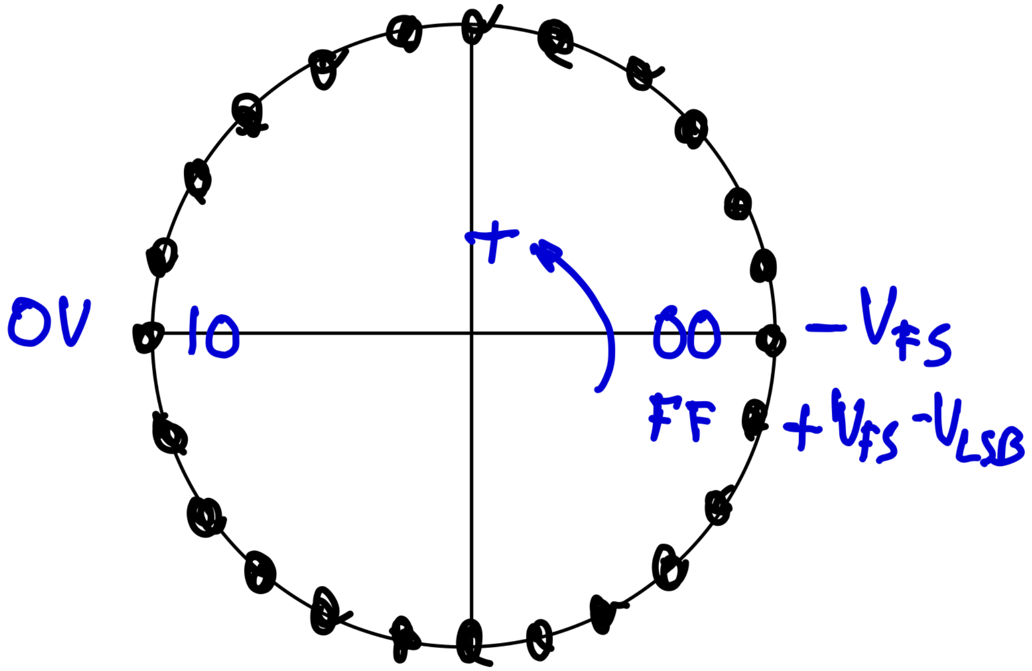
Horowitz
Hill

Binary Integer Numbers

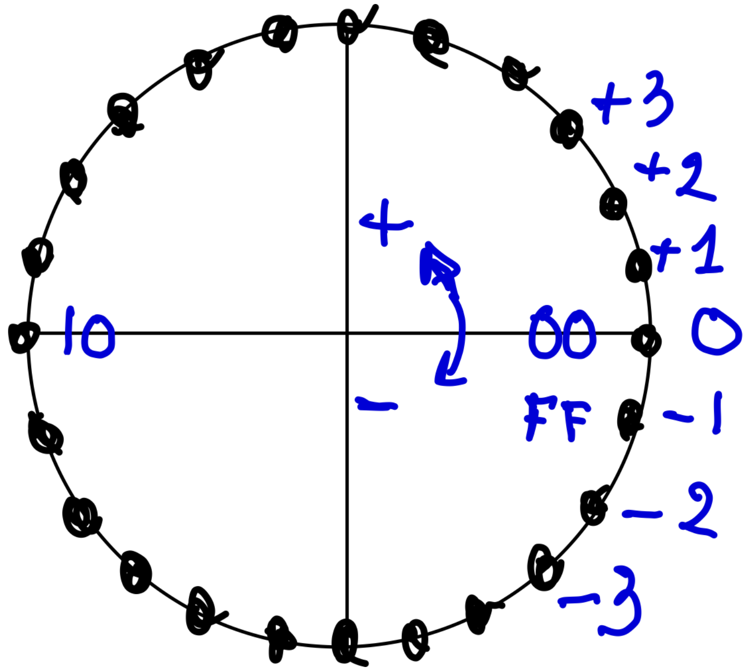
Integer
Computers & ADC/DAC



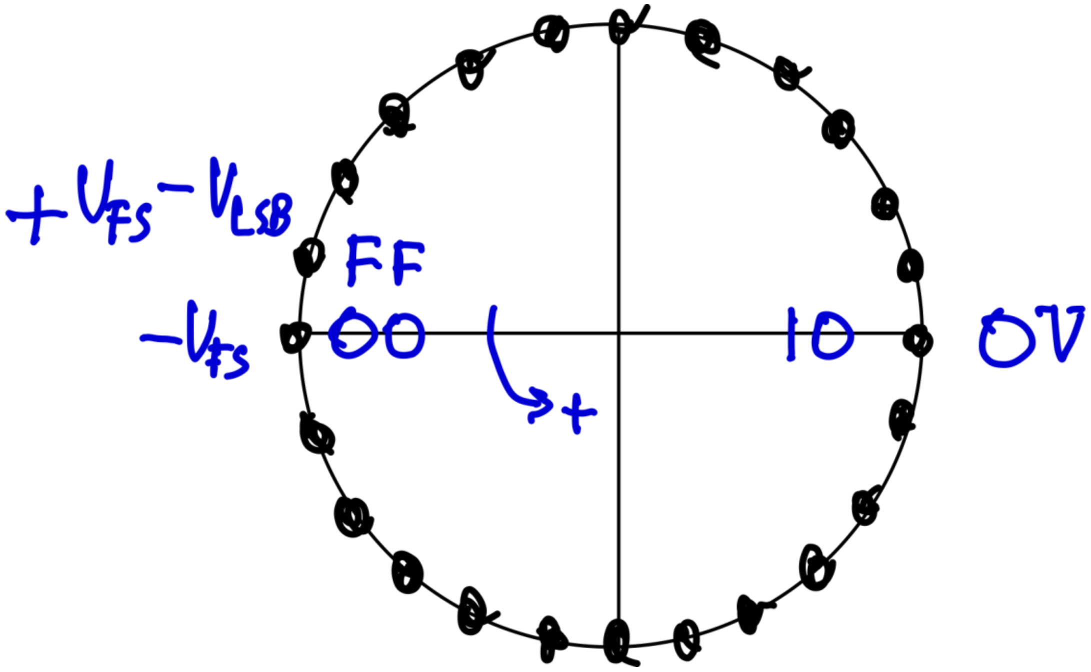
Offset binary
Computers & ADC/DAC



Two’s complement integer
computers only

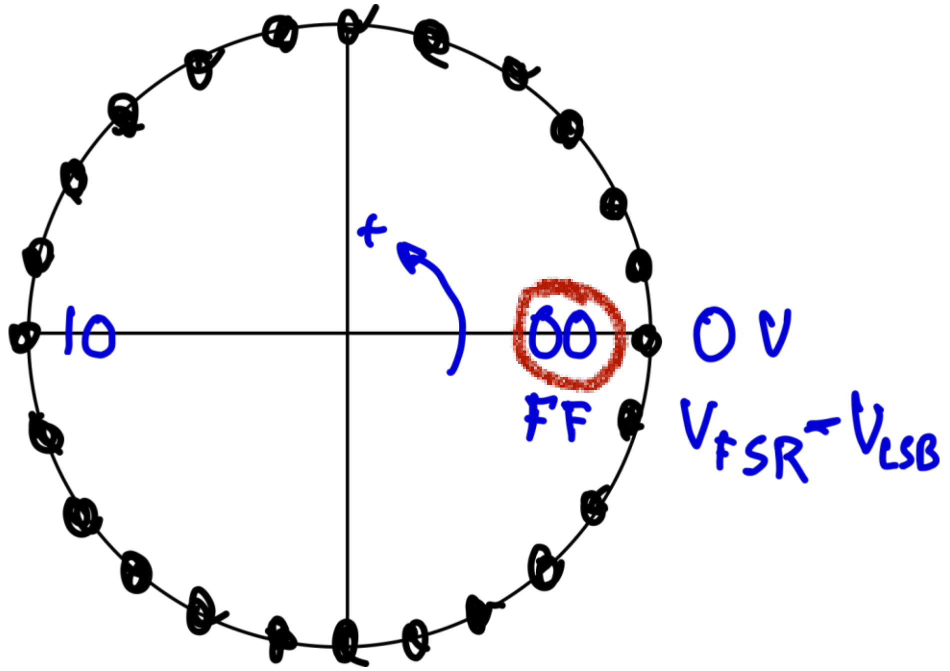


Same as above
The figure is rotated by π



Binary Integer Numbers

Integer
Computers & ADC/DAC



0 . . . .

8 bits 0 . . FF

10 bits 0 3FF

Modulo 2^n

$FF + 1 \bmod 2^8 = 0$

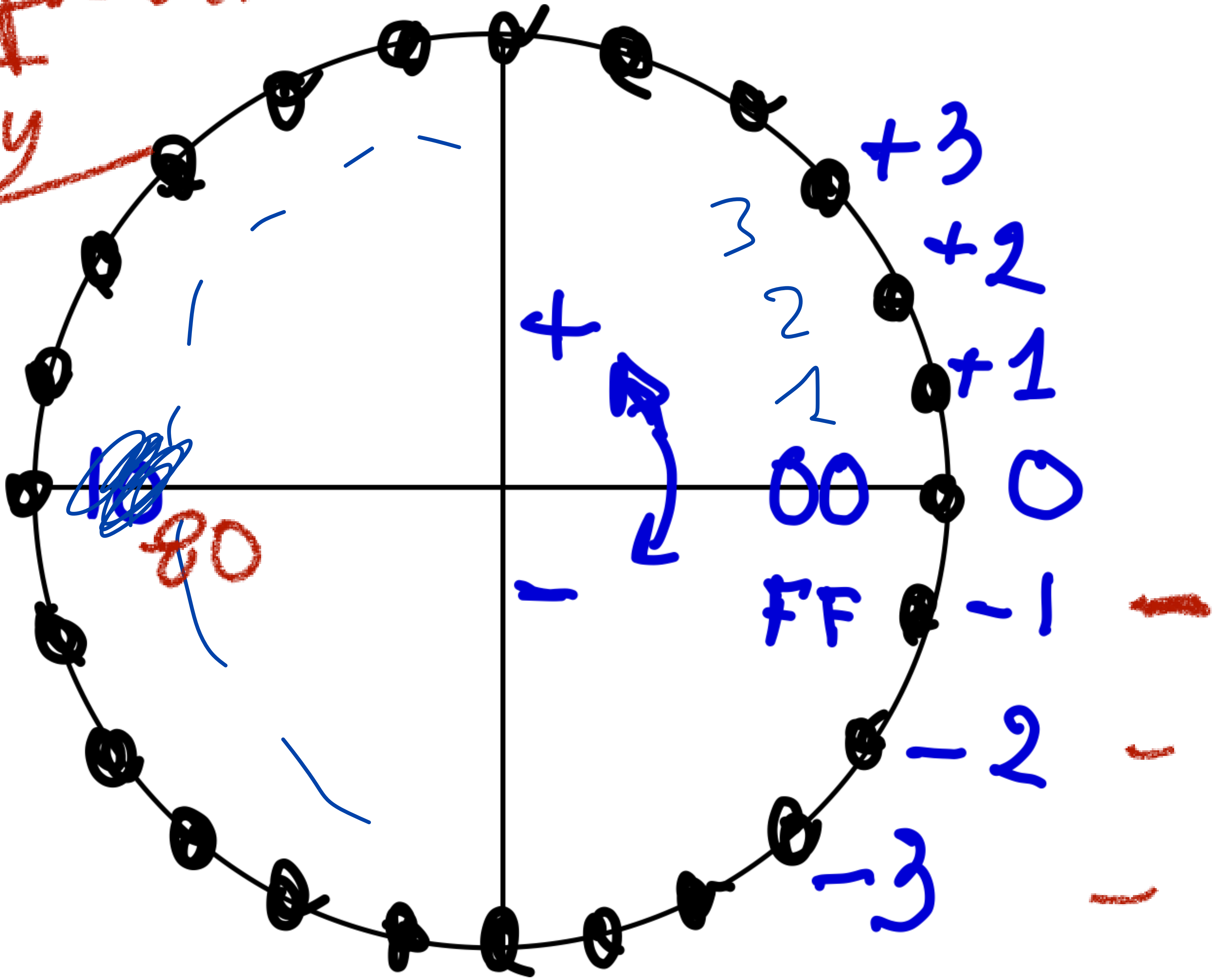
$FF + 1 \equiv 0 \mod 2^8$

Good for unipolar converters

Binary Integer Numbers

Finite fields

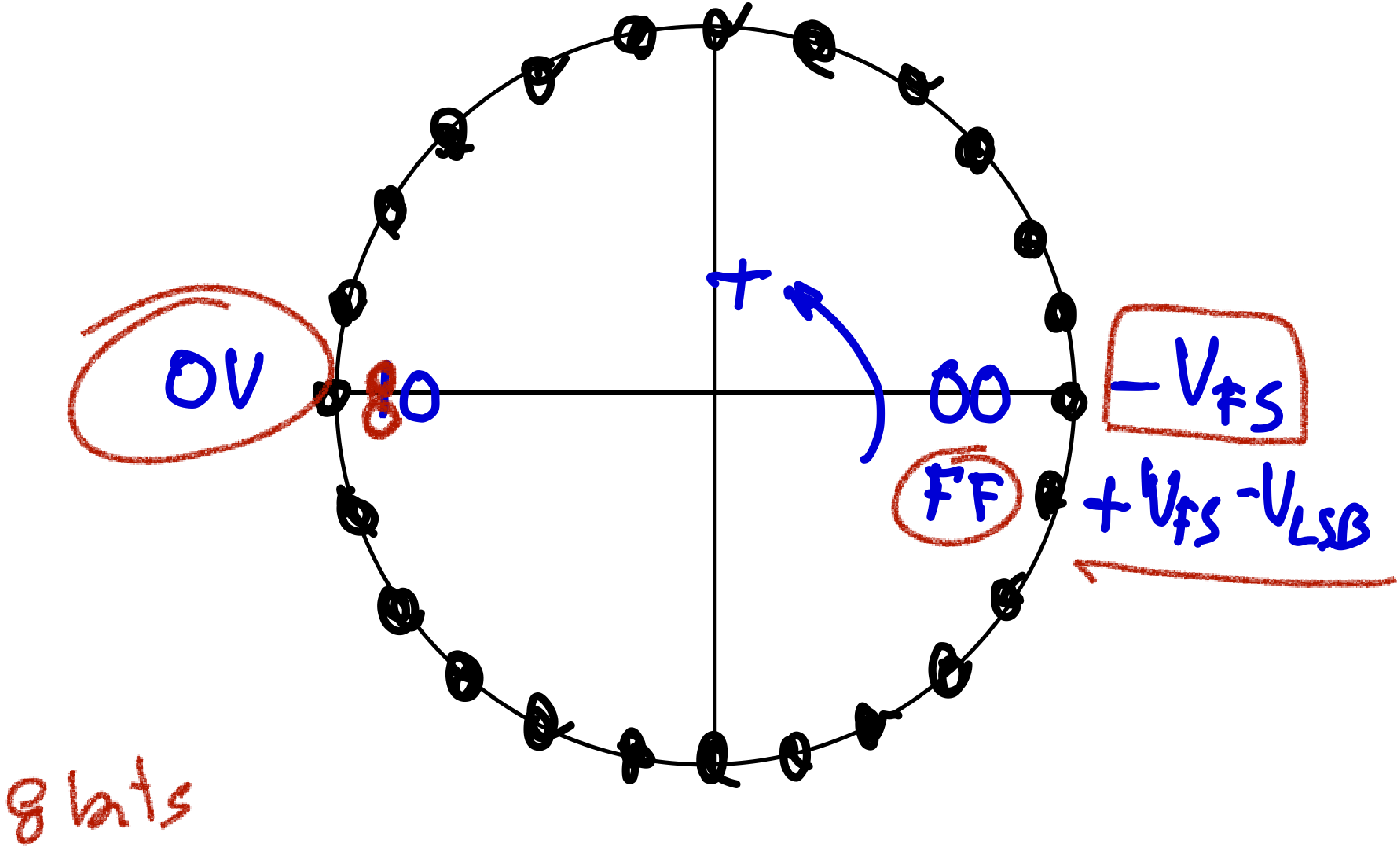
Cyclotomy



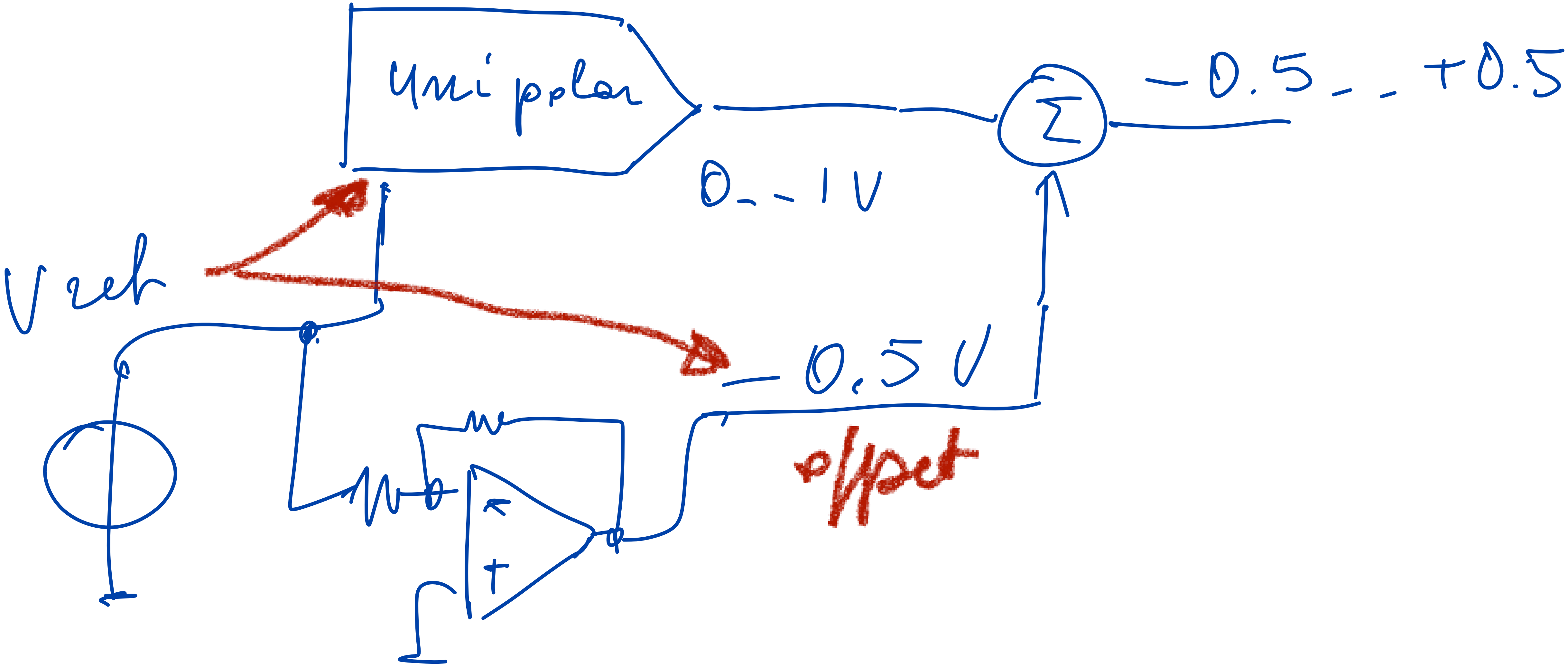
1 byte

Binary Integer Numbers

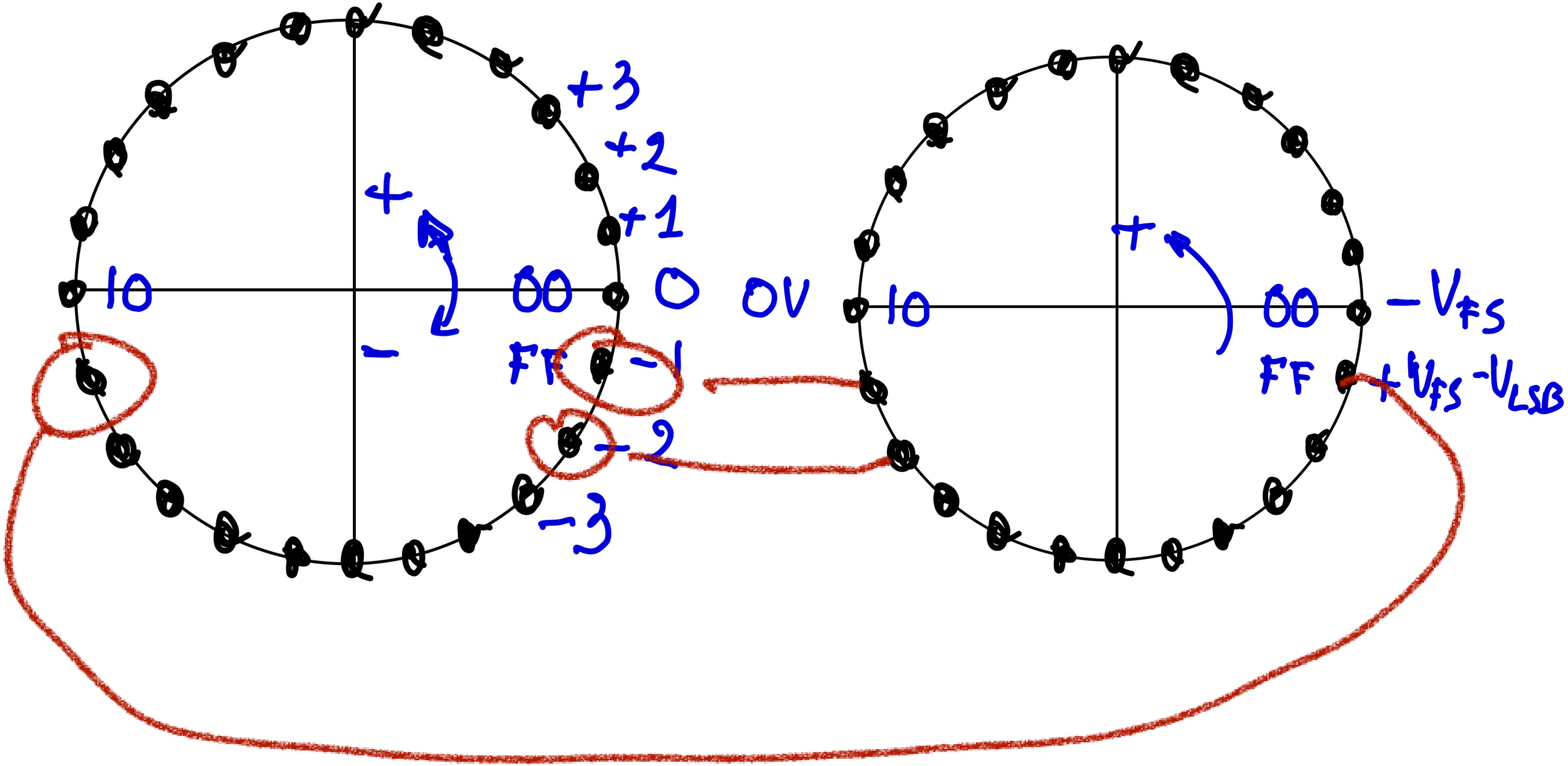
Offset binary
Computers & ADC/DAC



Vref errors or fluctuations impact on the range,
not on the Zero Value.



Binary Integer Numbers



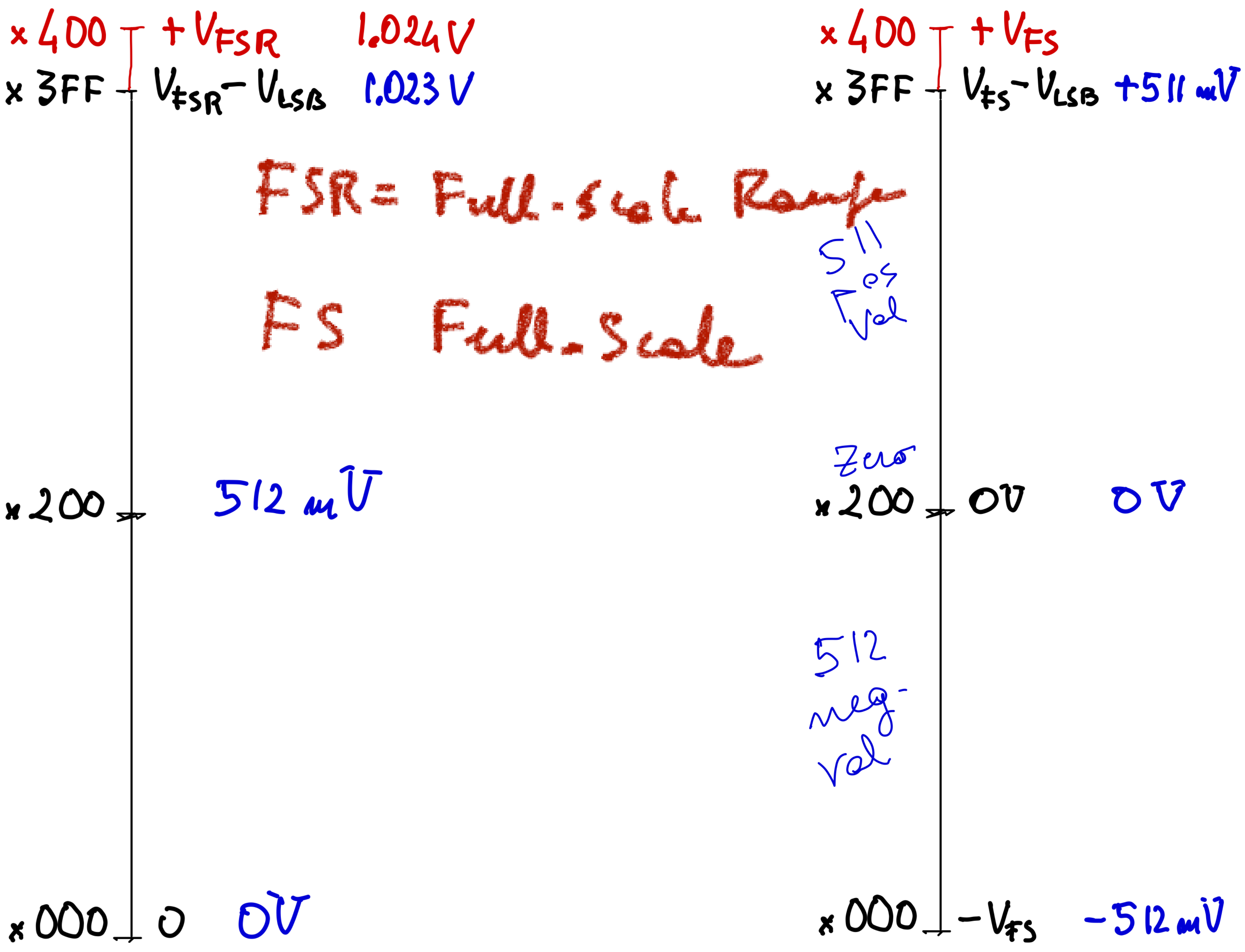
Two's complement integer
computers only

Offset binary
Computers & ADC/DAC

V_{FS}, V_{FSR}, and Actual Range

Example — 10 bit conversion

0 . . 1023



FSR = Full-scale Range

FS Full-Scale

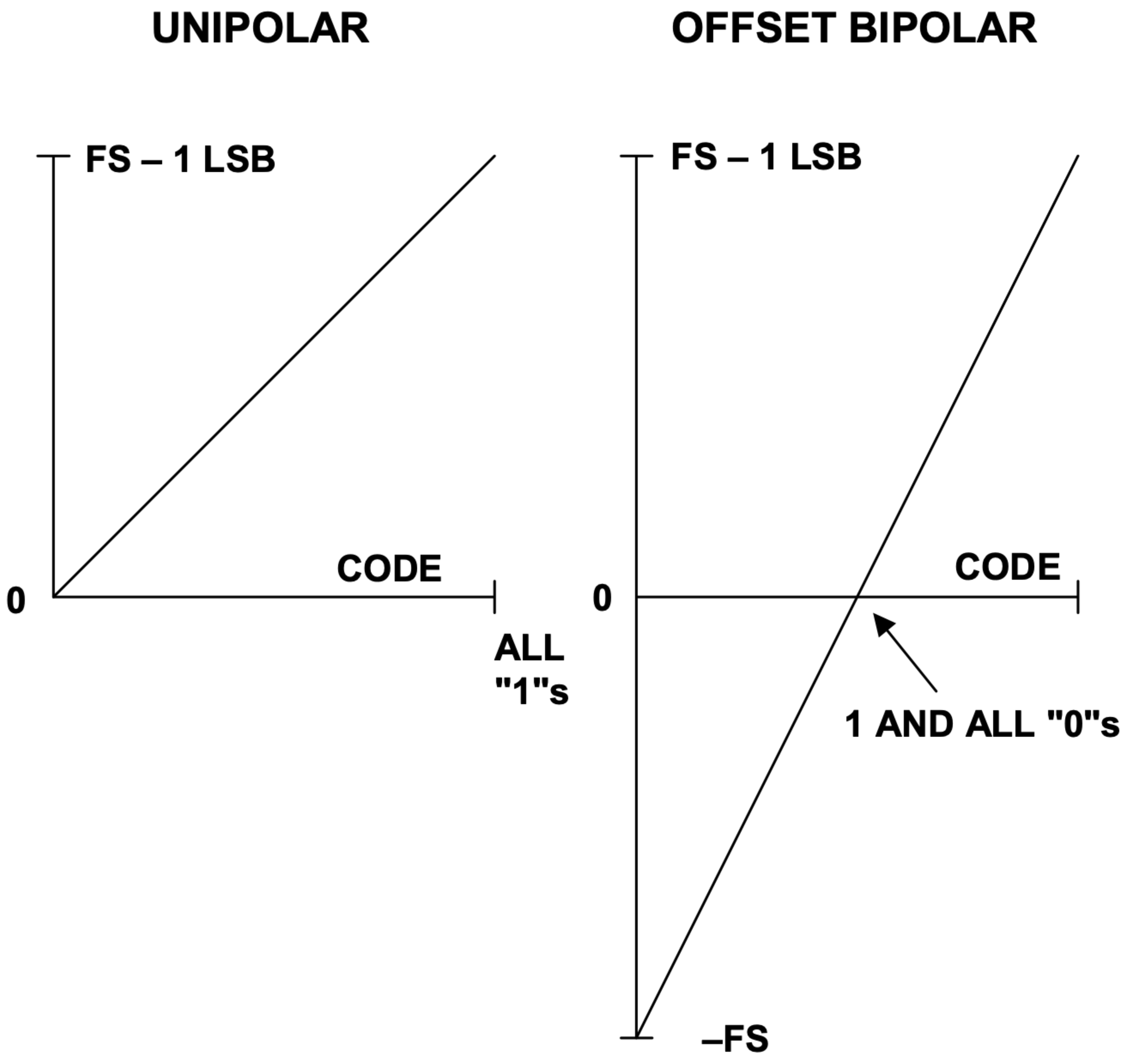
Example - hand-held digital voltmeter

20 V range → ±20V

V_{FS} = 20 V

V_{FSR} = 40 V

Conversion, Summary



W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

W. Kester, Fig 2.16 (excerpt)

Knuth Floor and Ceil Parentheses

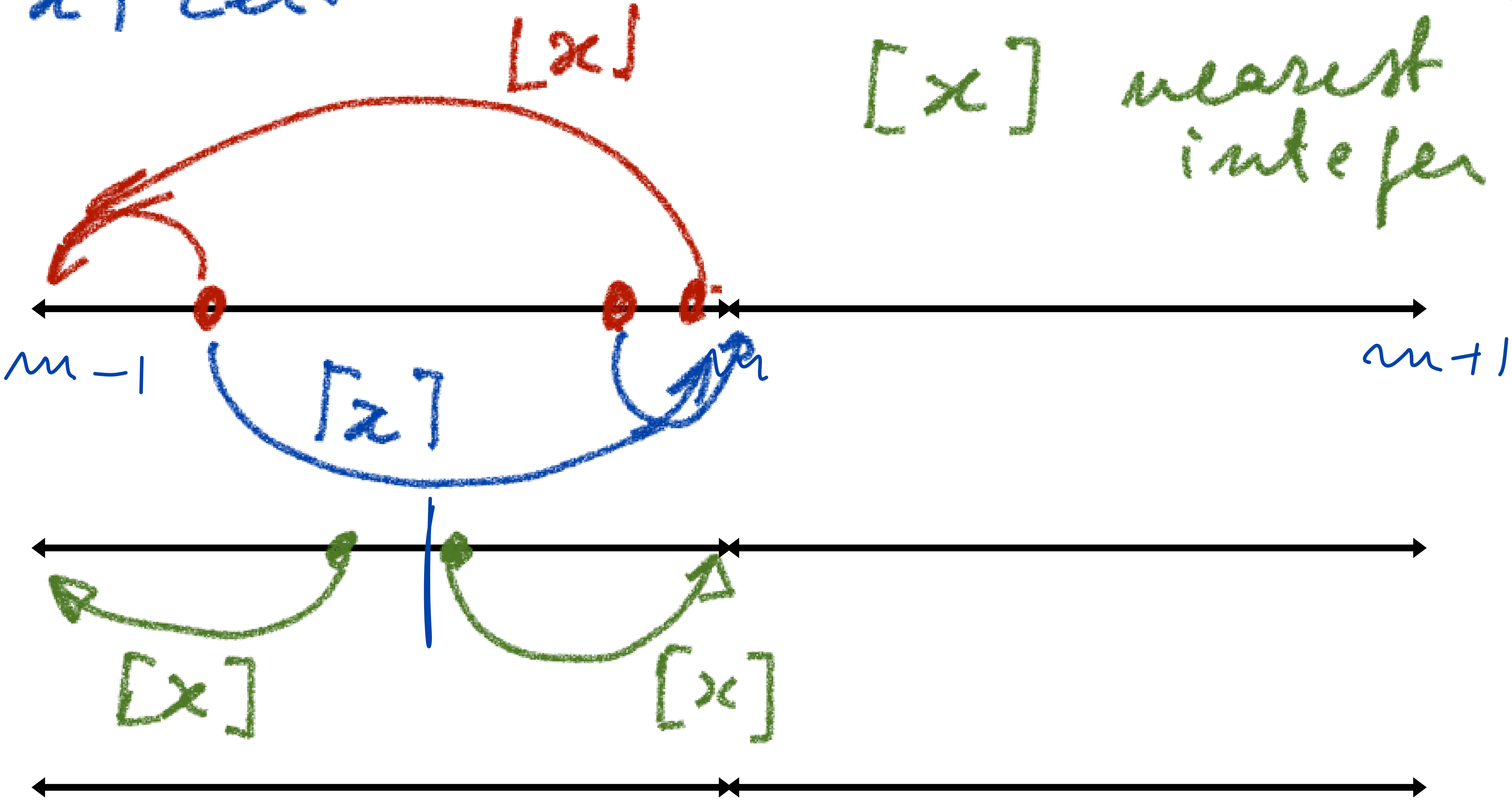
$\lfloor x \rfloor$ $\lceil x \rceil$ $[x]$

$\lfloor x \rfloor$ floor

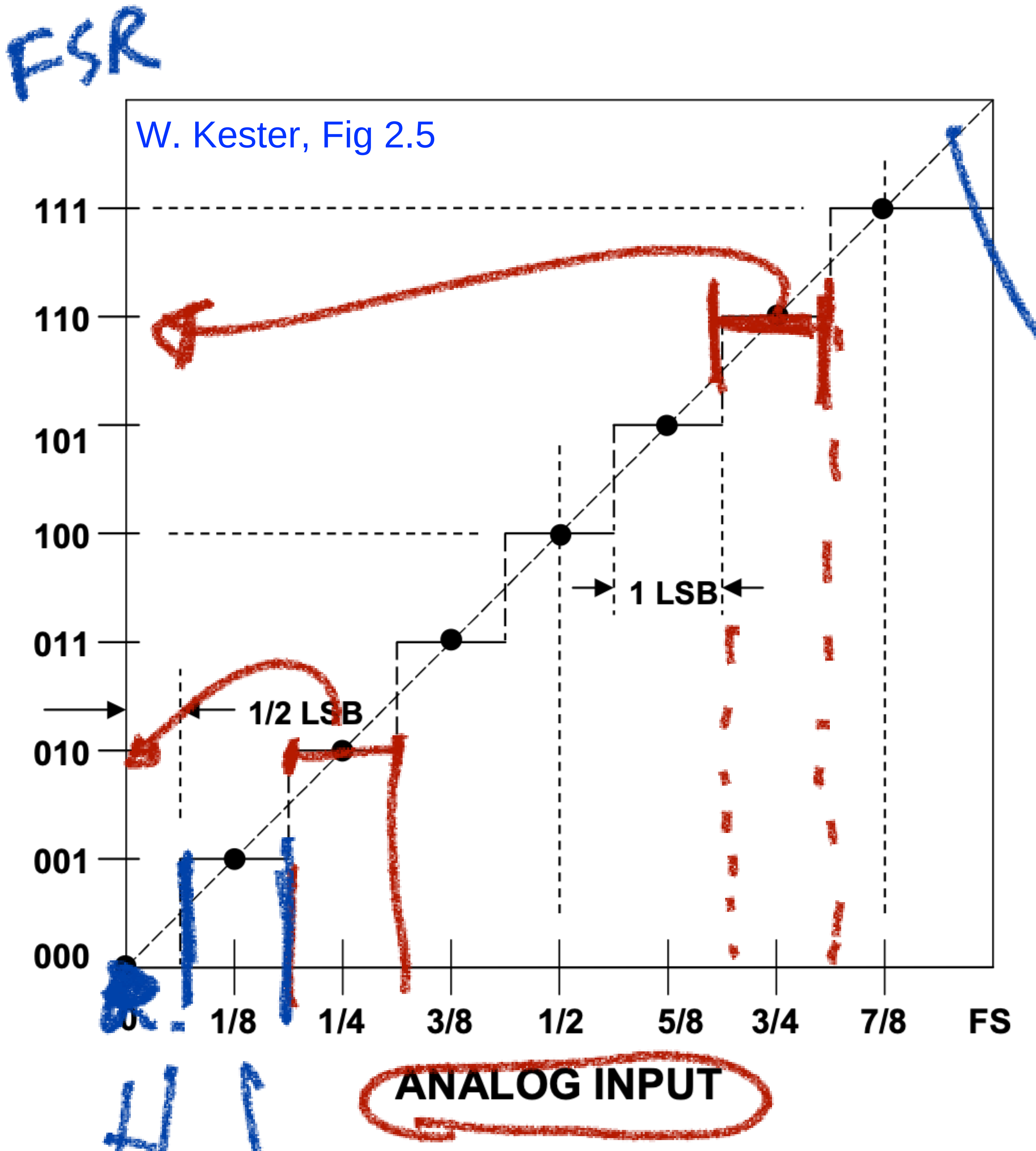
$\lceil x \rceil$ ceil

The Art of Programming (3 Vol. / Genius)

$[x]$ nearest integer



Example: 3-bit ADC



LSB
Least Significant Bit

$\frac{3}{2}$ LSB

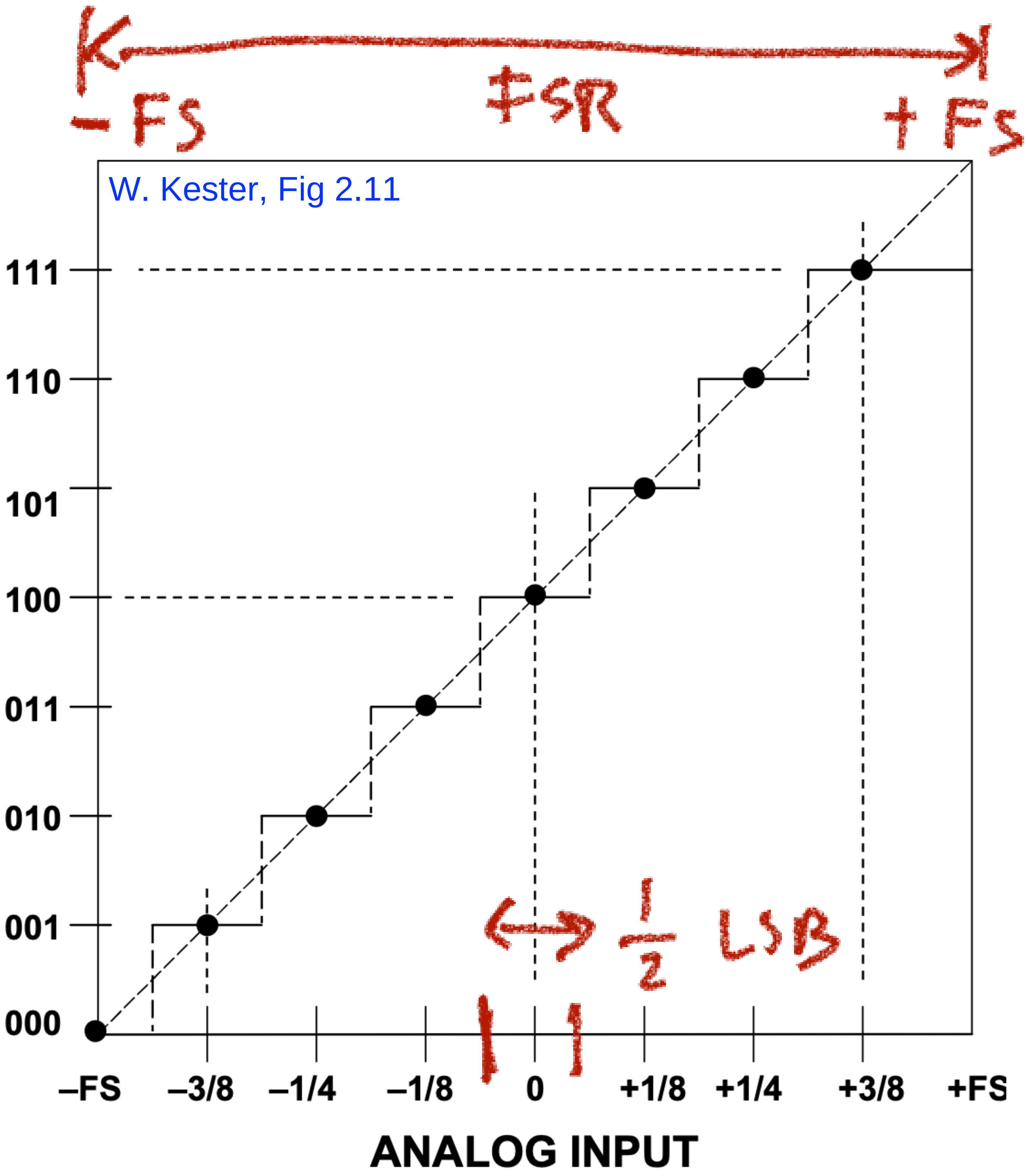
$\frac{1}{2}$ LSB

± $\frac{1}{2}$ LSB

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Bipolar Input – Example

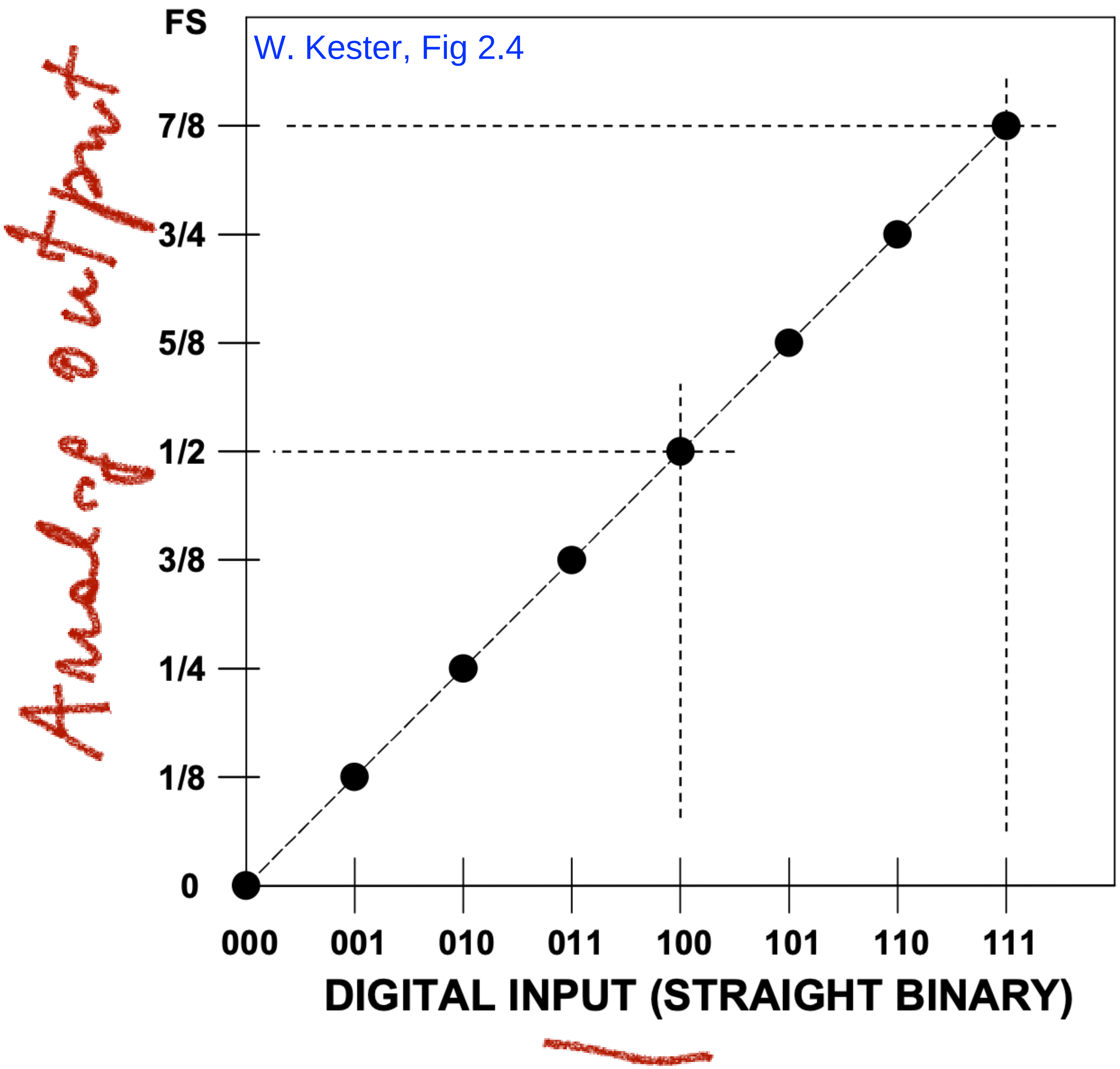
VFS vs VFSR



ADC

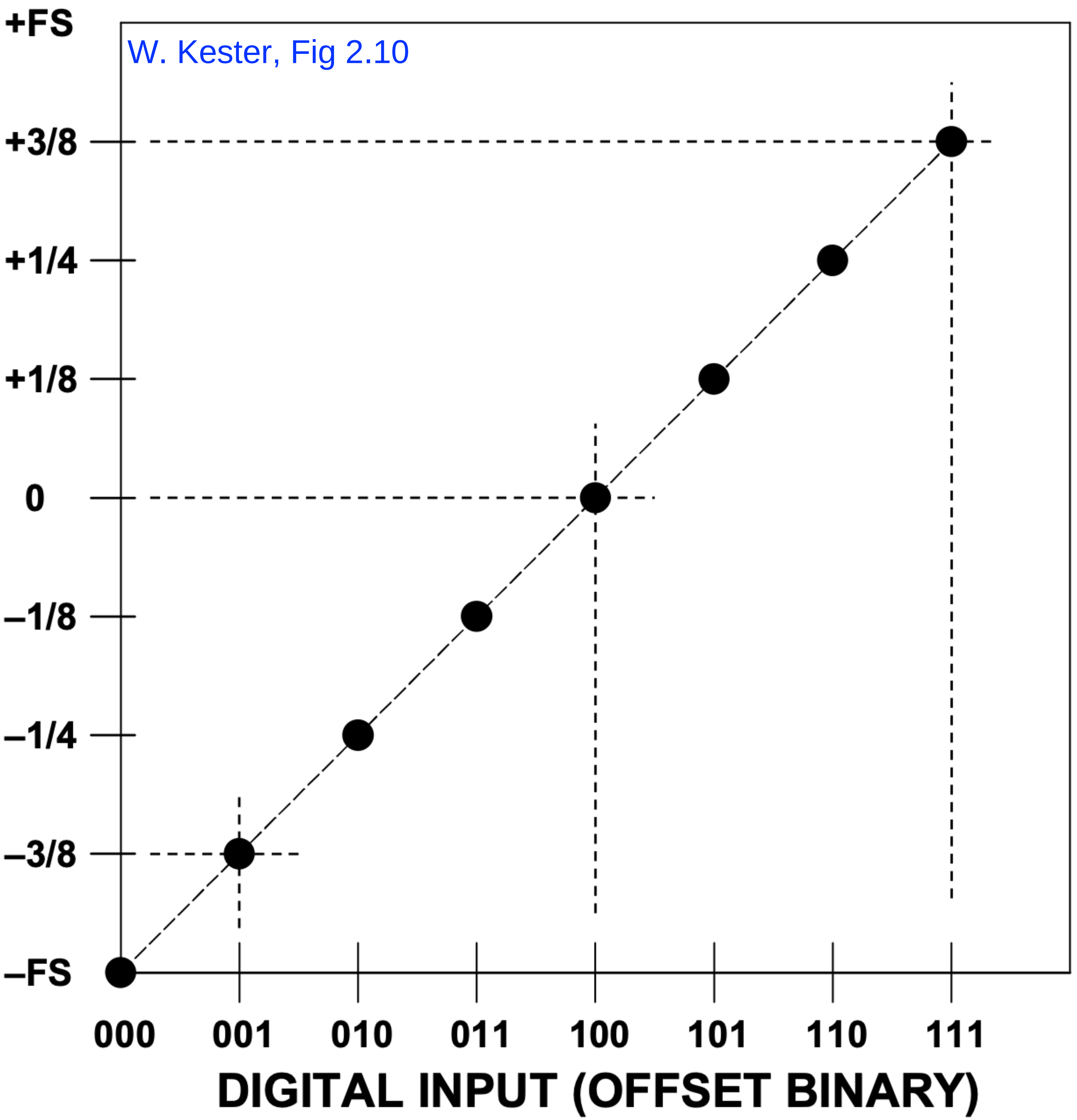
W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Example: 3-bit DAC



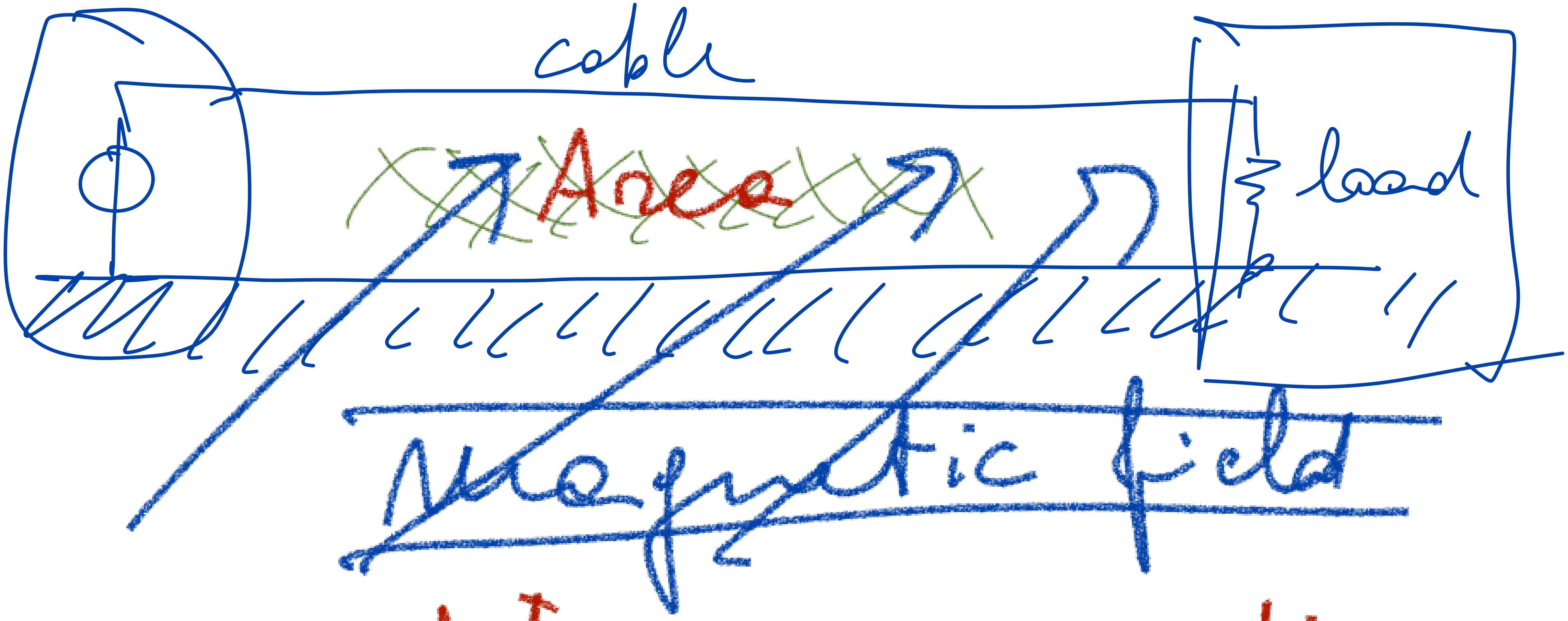
W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Bipolar Conversion



W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

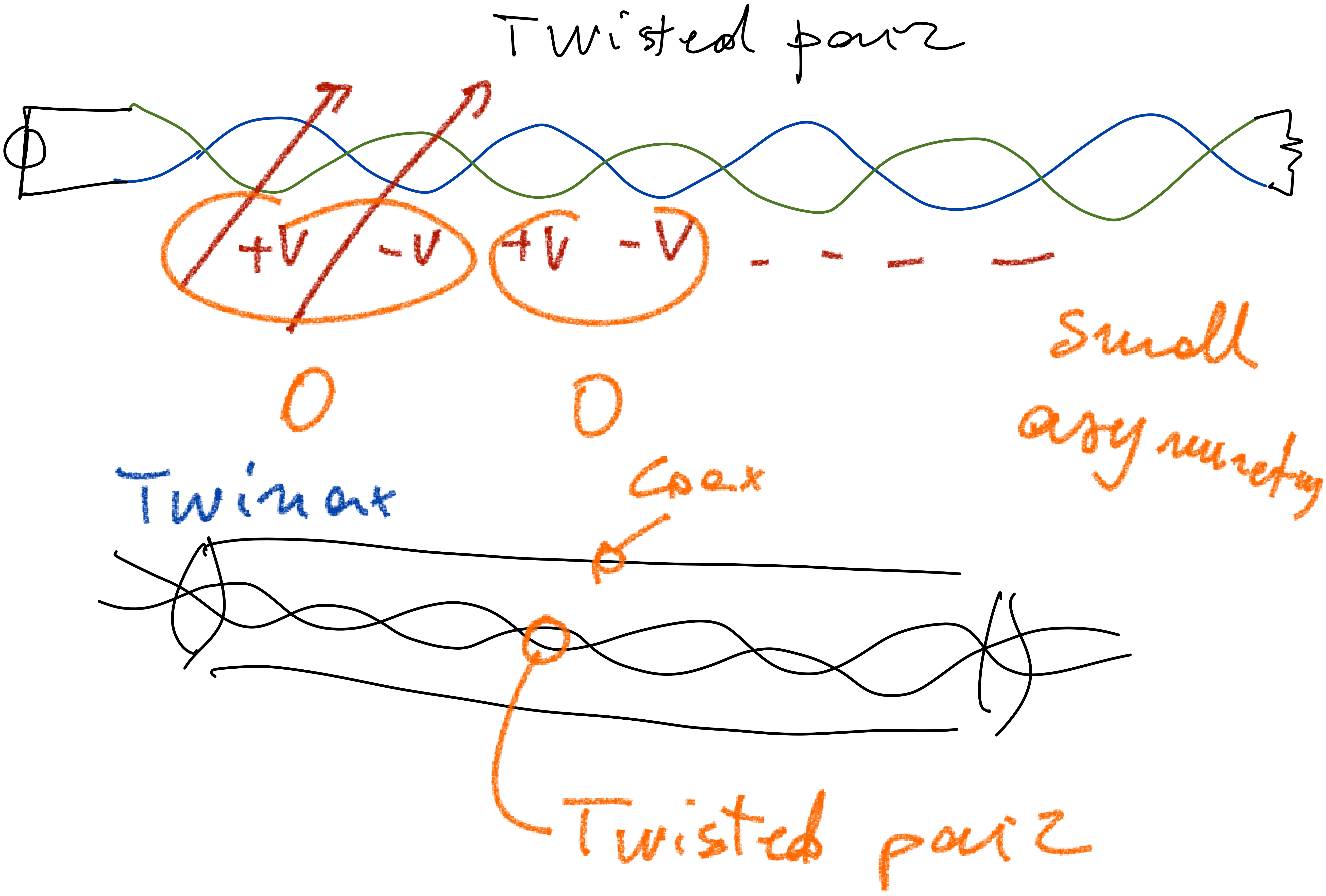
Unbalanced Signal

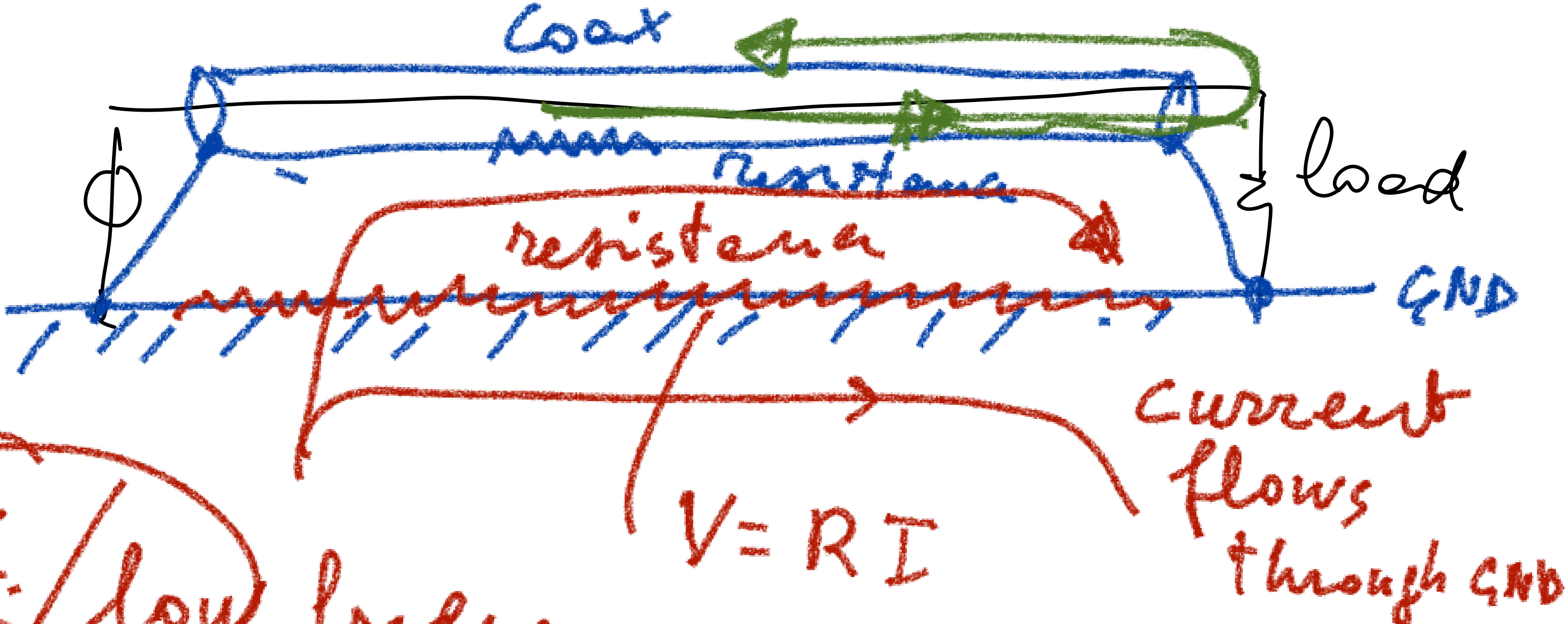


$$V = \frac{d\Phi}{dt}$$

magnetic electromotive force

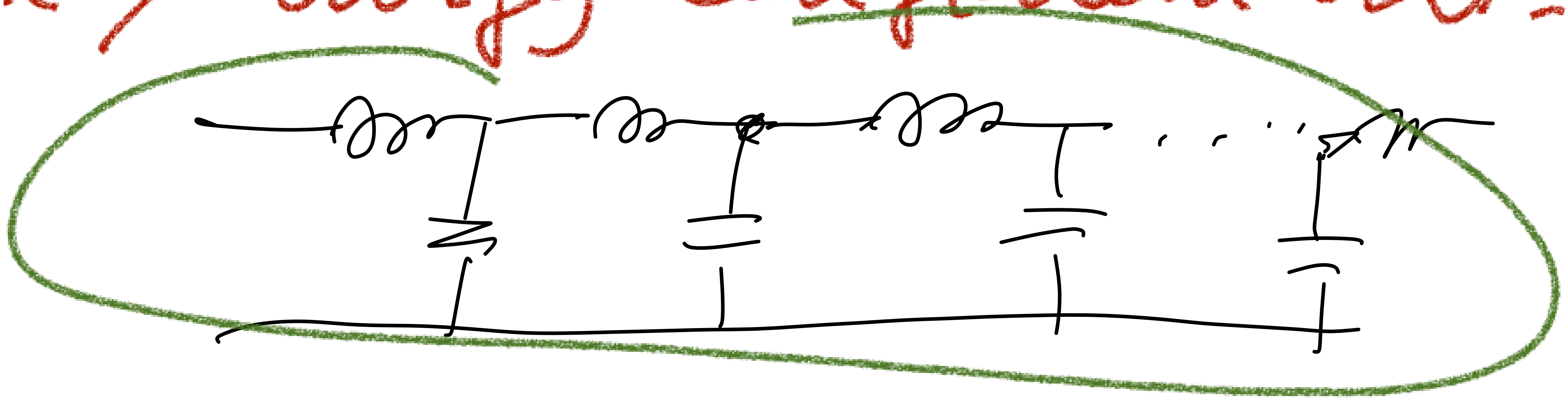
Balanced (Differential) Signal





DC / low frequency

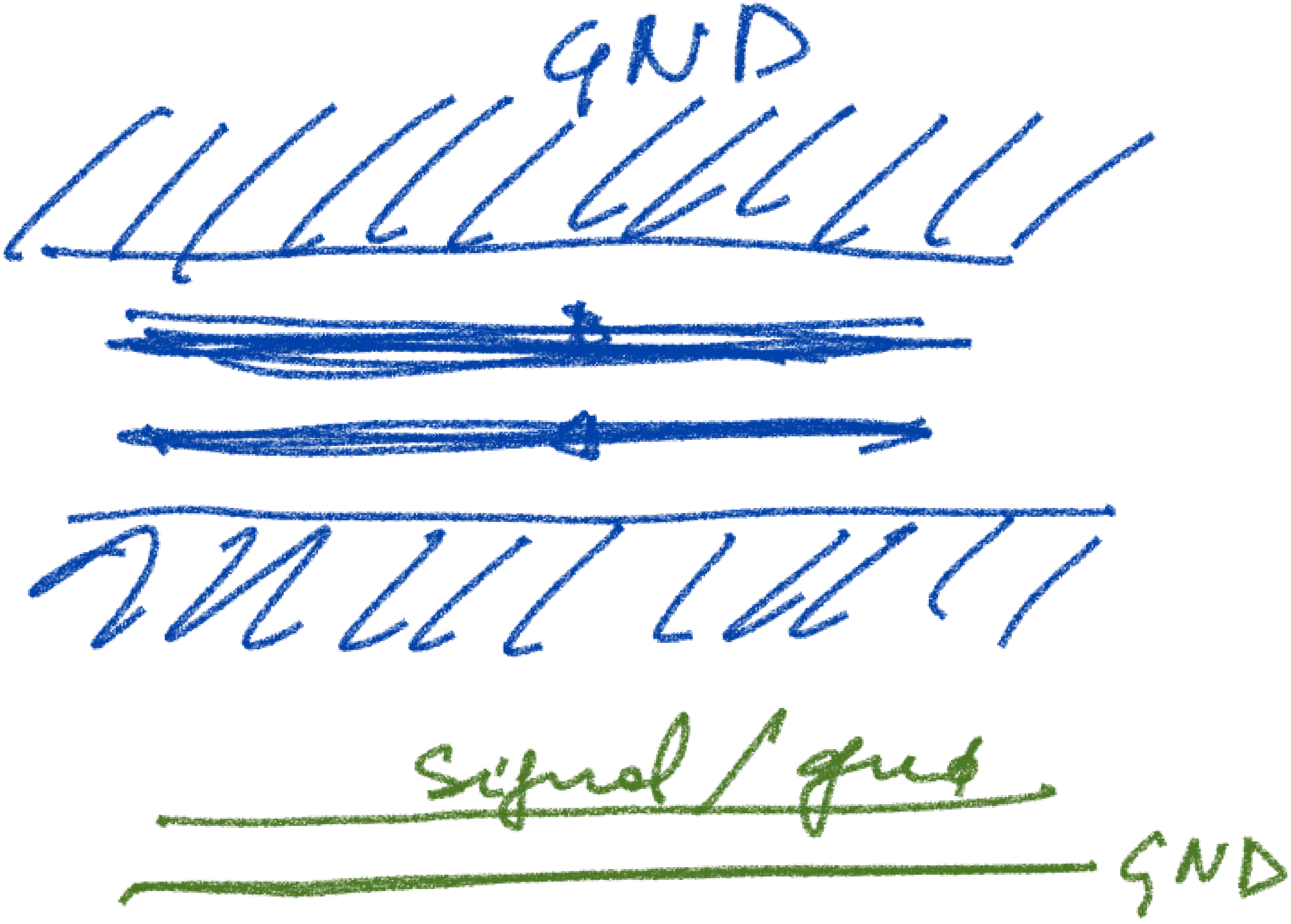
AC / energy confinement

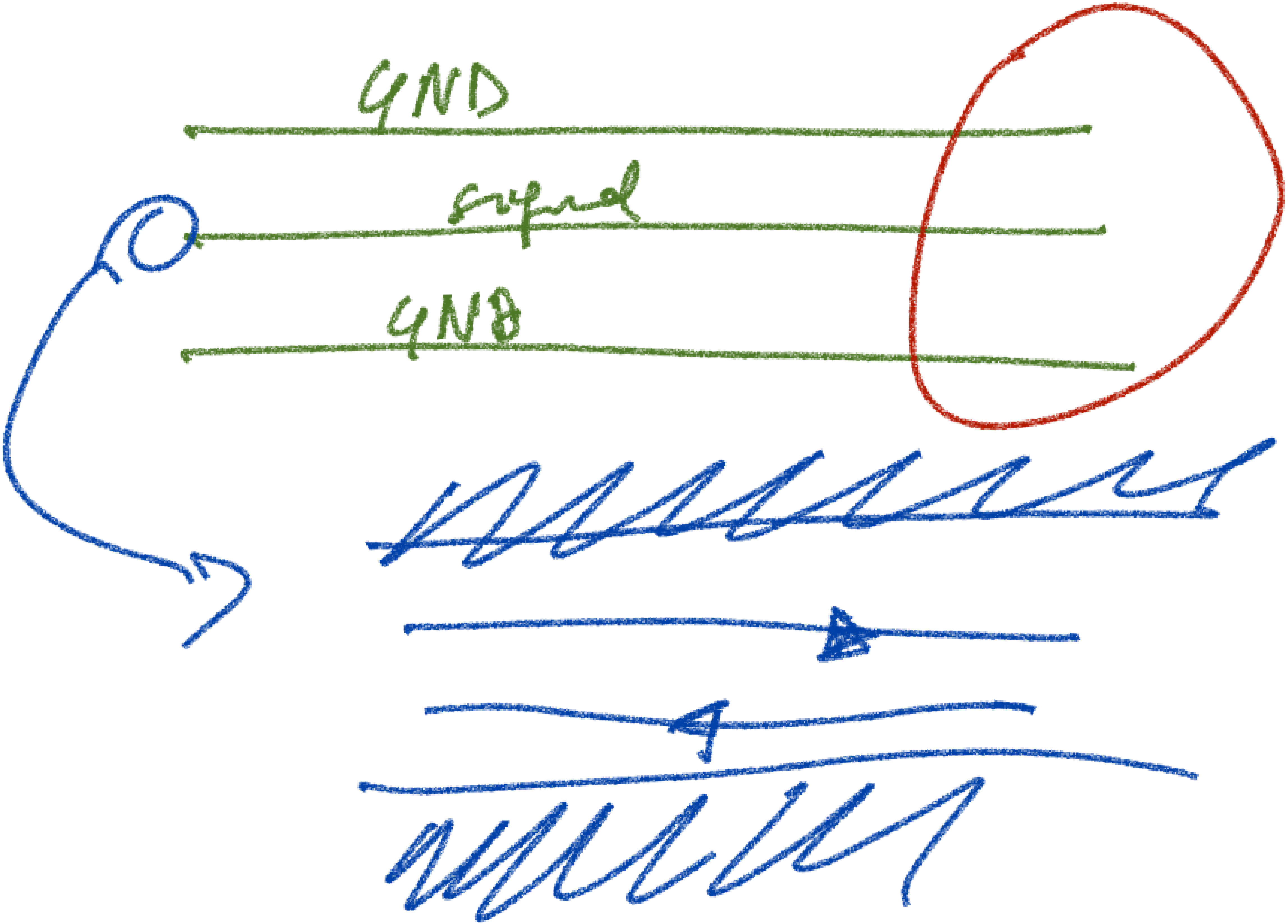


Cutoff frequency

RG 58	8-10 kHz
Twinax	2 kHz
Twisted pair	. - — -

printed circuitboard




GND
signal
GND

Electromagnetic Compatibility
Where to learn

- Jasper Goedbloed, Electromagnetic Compatibility, Prentice Hall 1990 (Philips)
- Henry W. Ott, Electromagnetic Compatibility, Wiley 2009
- Clayton R. Paul, Electromagnetic Compatibility, Wiley 2006
- Reinaldo Perez, ed., Handbook of Electromagnetic Compatibility, Academic Press 1995 (JPL)

Genesis library

End of lecture

#2

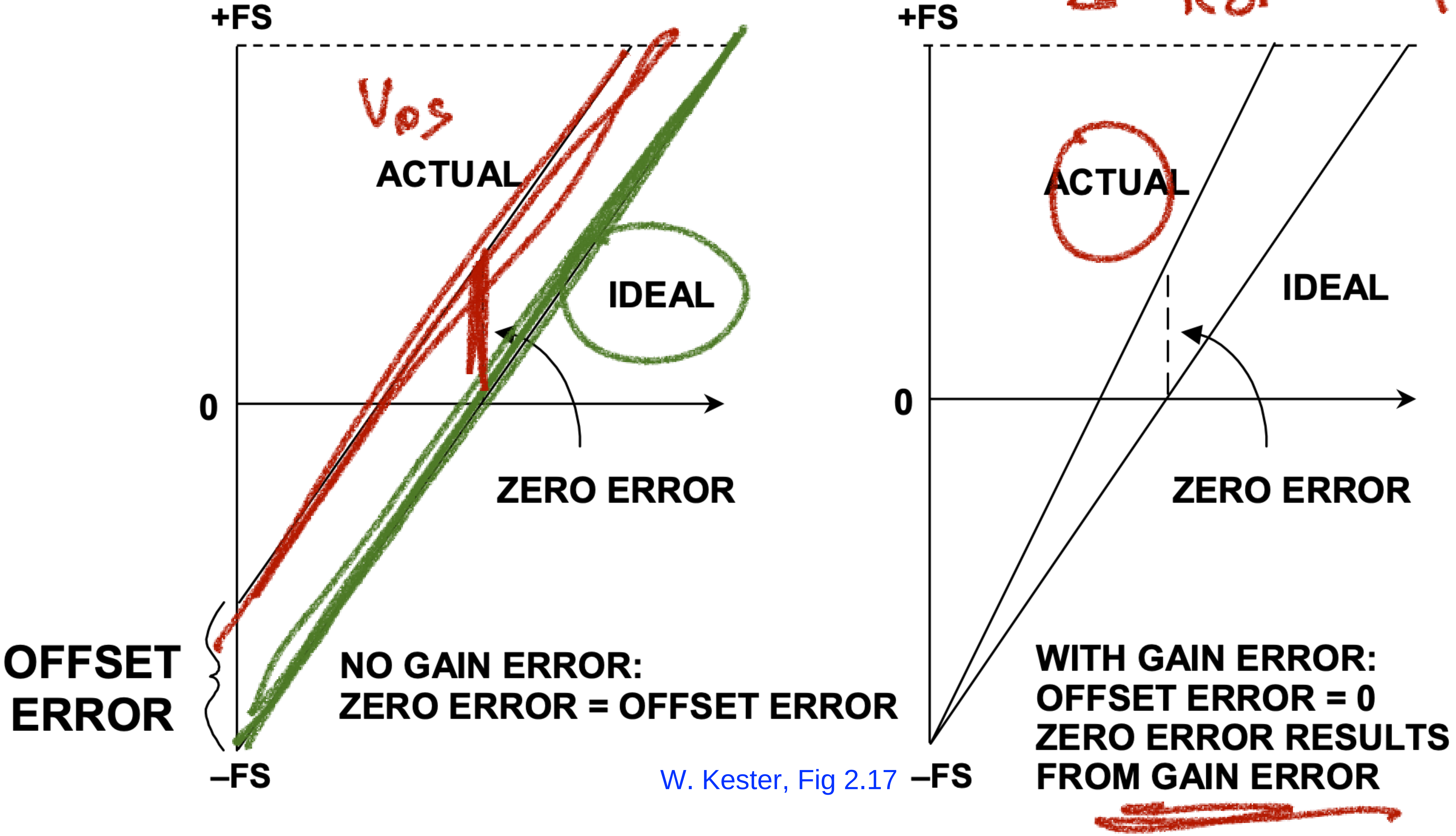
#3, Tuesday, Sept 22

1.5 Hours

Conversion Errors and Uncertainty

Offset and Gain

ΔV_{REF} or Gain



NO GAIN ERROR:
ZERO ERROR = OFFSET ERROR

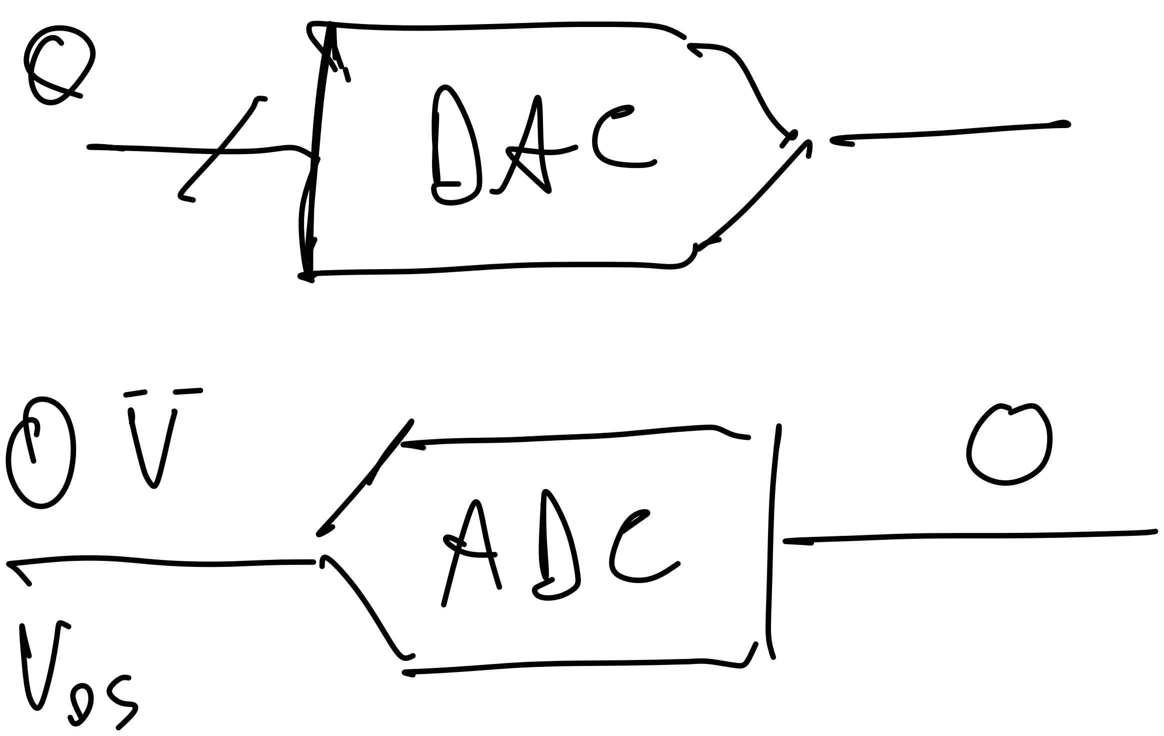
WITH GAIN ERROR:
OFFSET ERROR = 0
ZERO ERROR RESULTS FROM GAIN ERROR

W. Kester, Fig 2.17

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Gain Error and V_{REF}

Example: Flash DAC



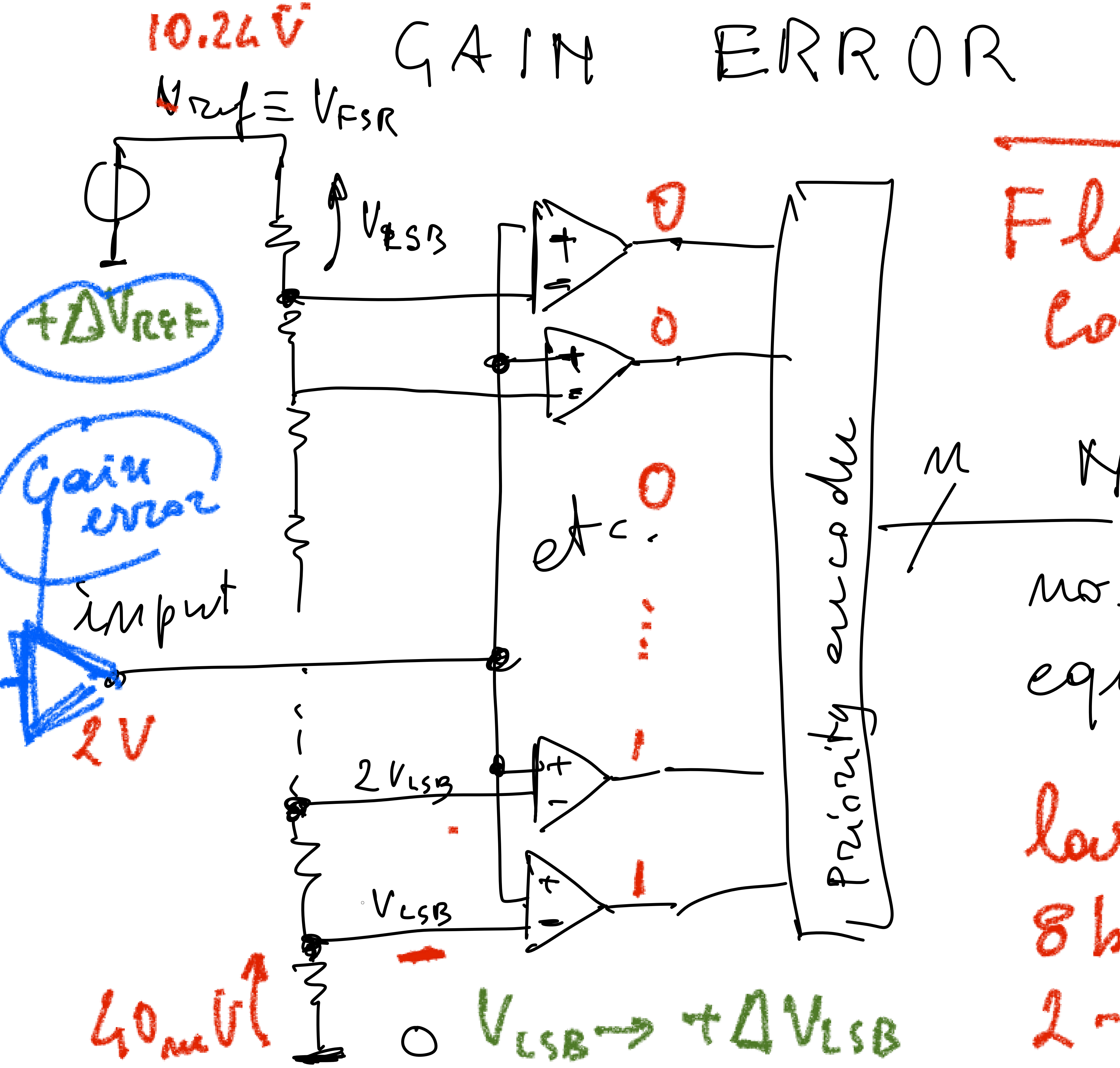
$0\,V \rightarrow +V_{os}$

$\xrightarrow[\text{Uncertainty}]{\text{Offset}}$

$V_{os} = 10\,mV$

$|V_{os,true}| \leq 10\,mV$

GAIN ERROR



8 bits

Flash Converter

N = no. of outputs equal to 1

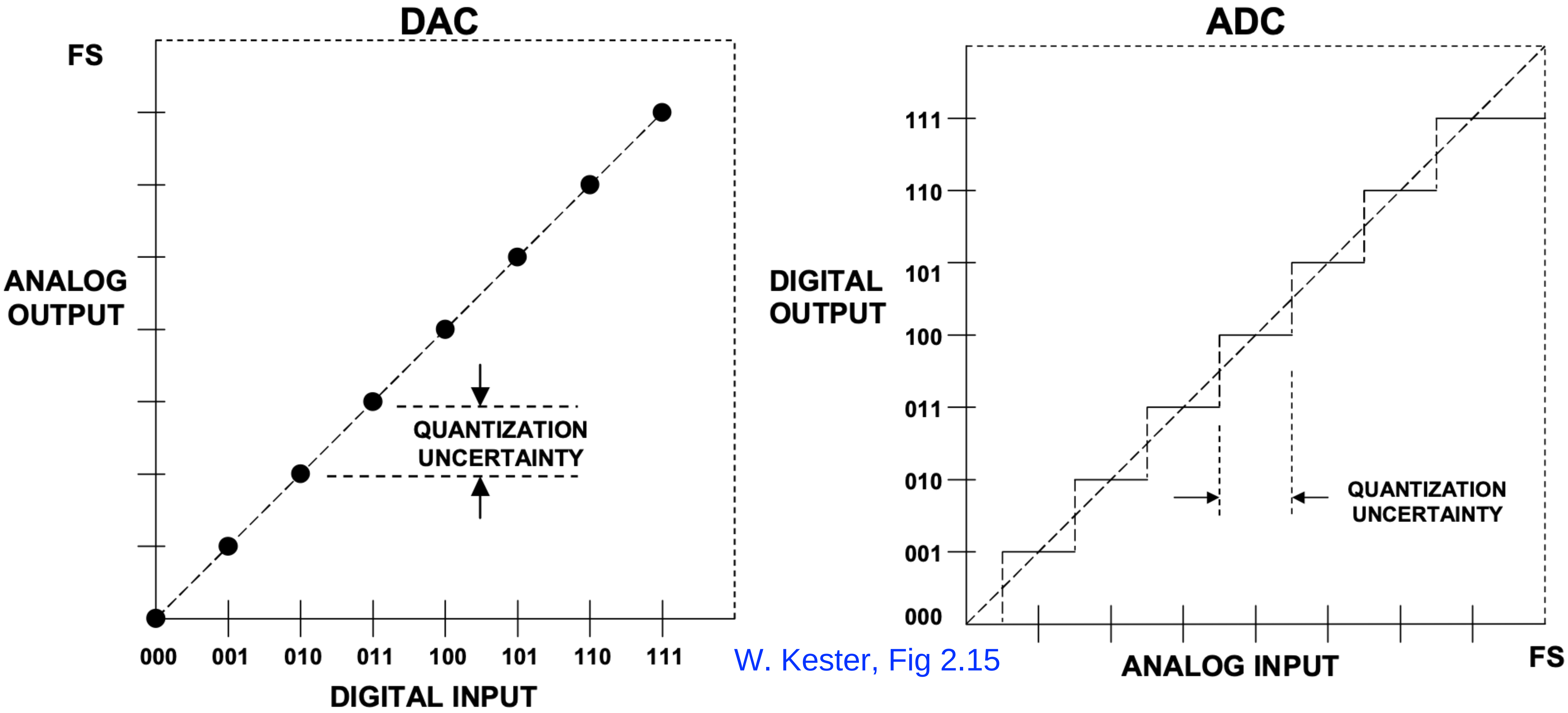
late 1980s
8 bits
2-3 μs

Gain Error and Offset

Example: Bipolar Flash DAC

Quantization Uncertainty

n bits —> 2^N levels



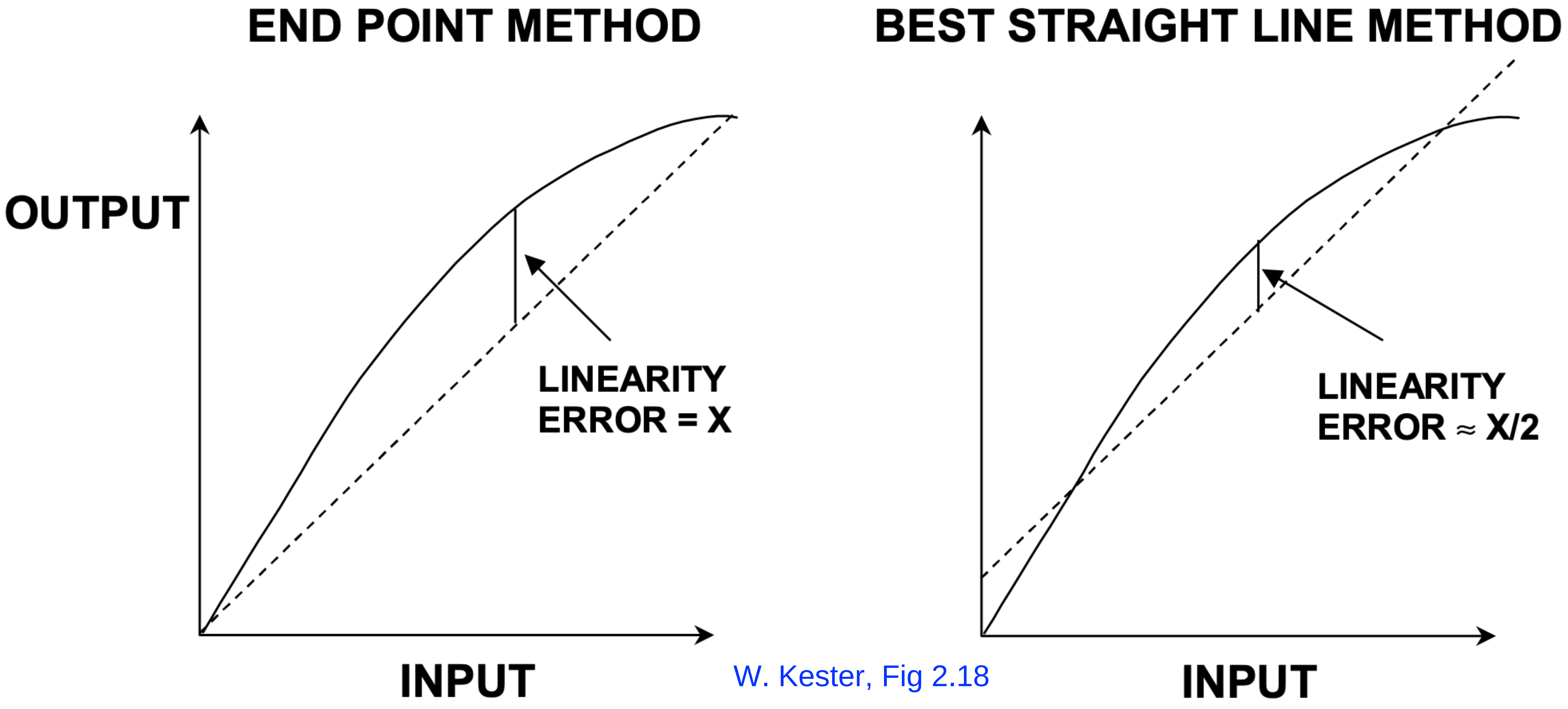
W. Kester, Fig 2.15

— Save for later —

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

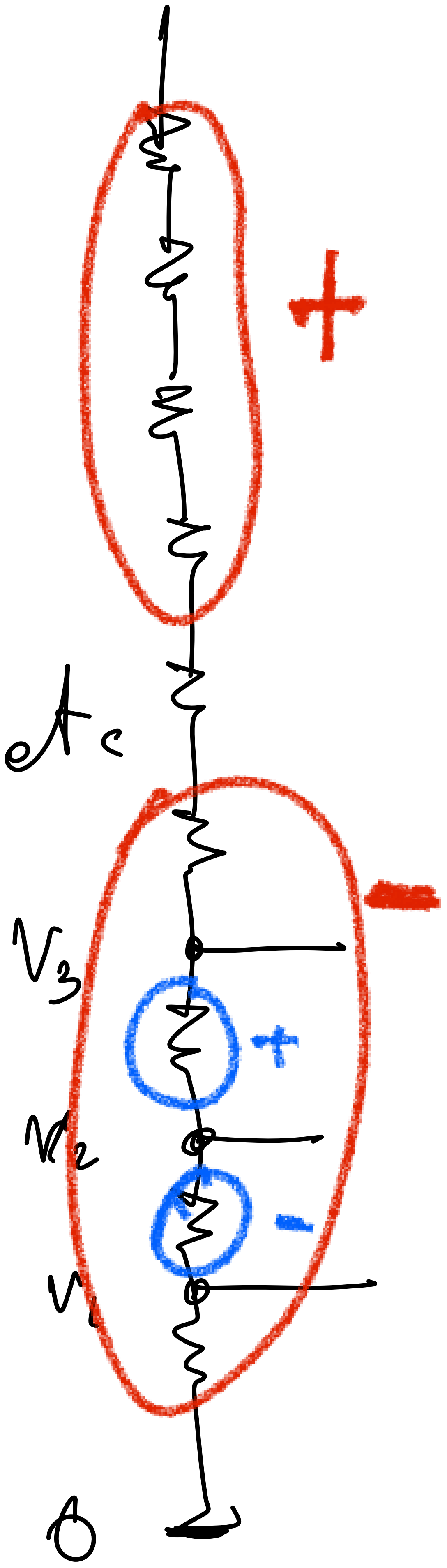
Integral Nonlinearity

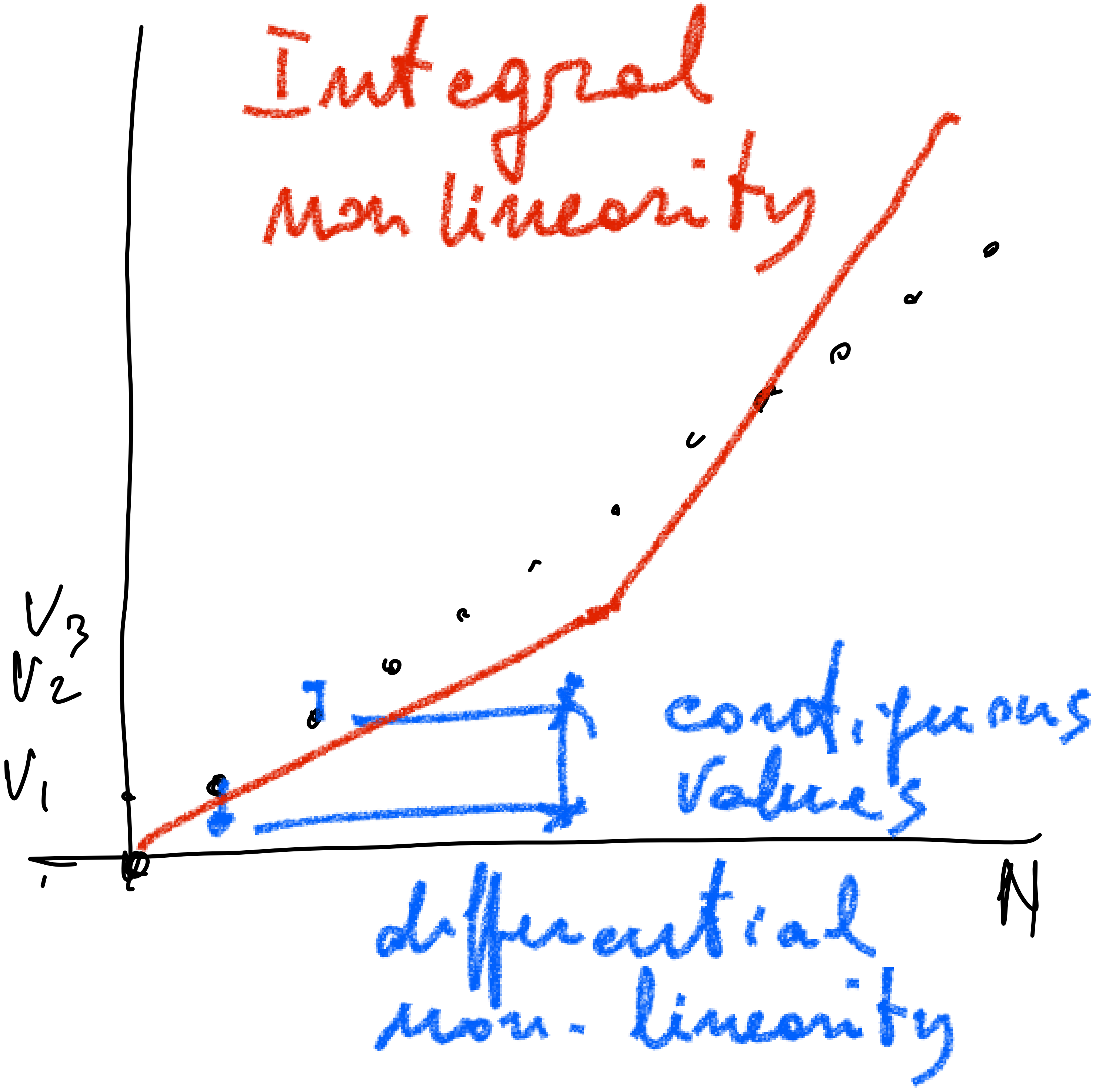
Example: Flash DAC



W. Kester, Fig 2.18

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3


+
etc
−
V3
+
V2
−
V1
0

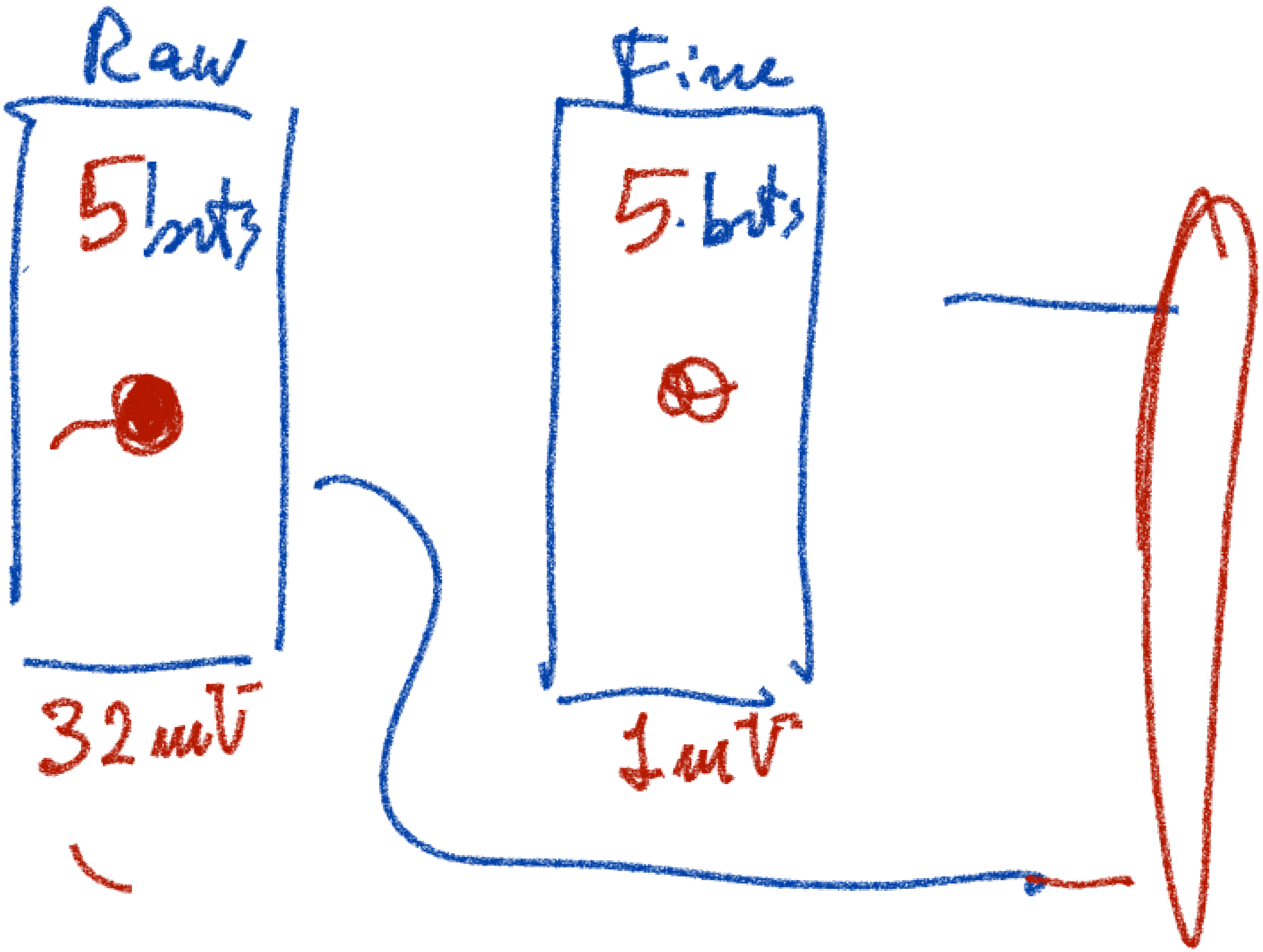

Integral non linearity
V3
V2
V1
continuous values
differential non-linearity
N

Two-stage Flash

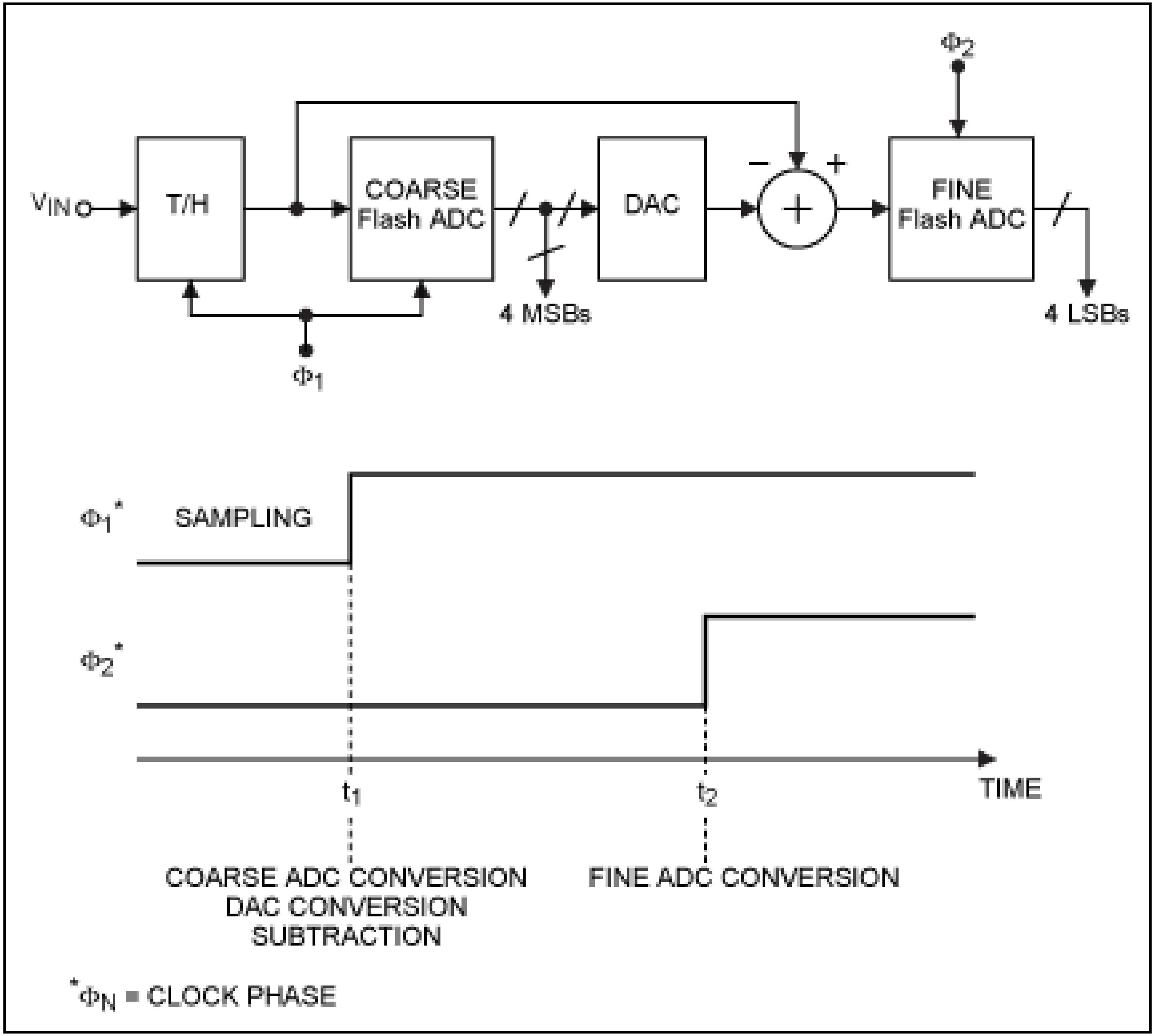
10 bits

1.024 V

$V_{LSB} = 1\,mV$

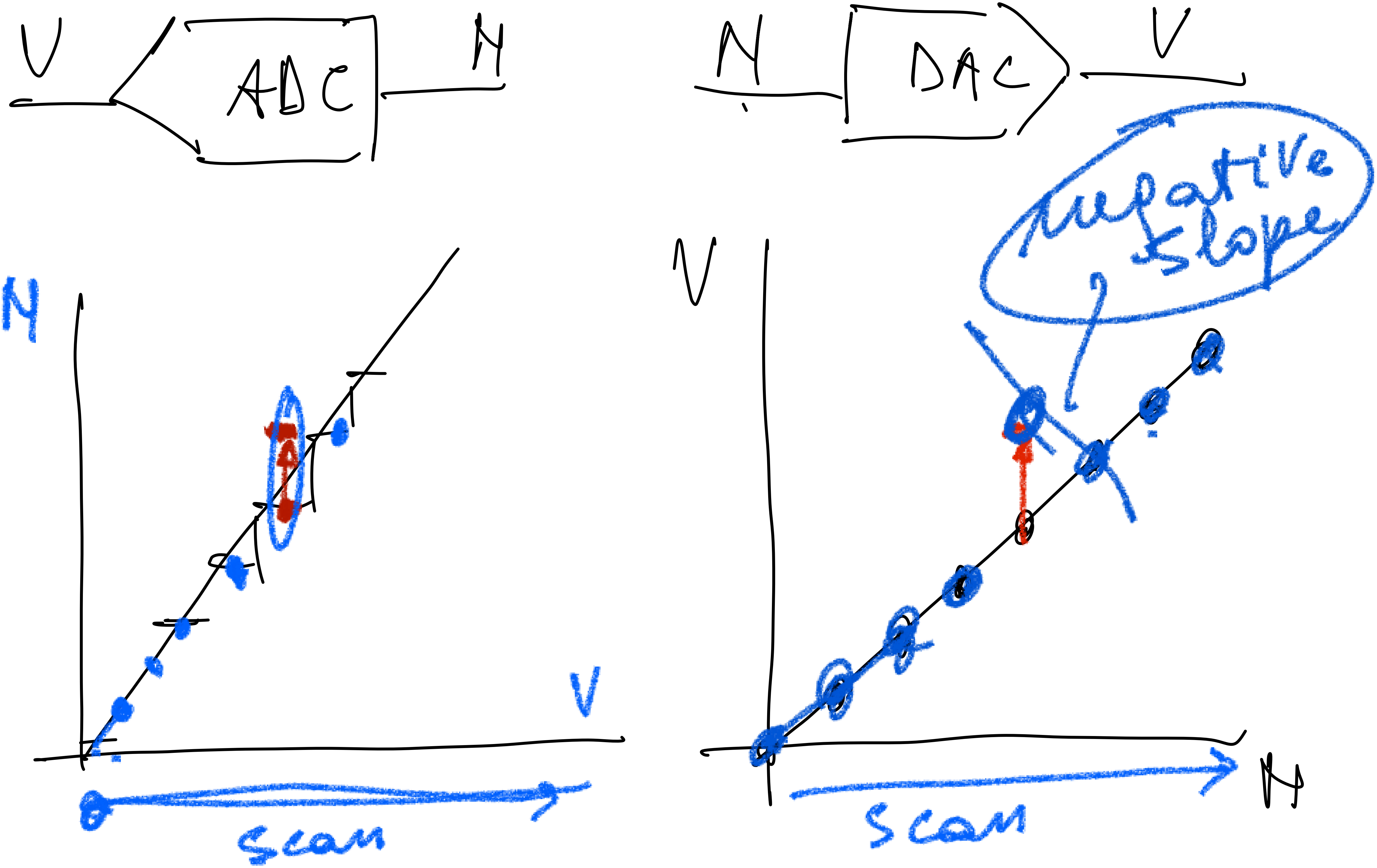


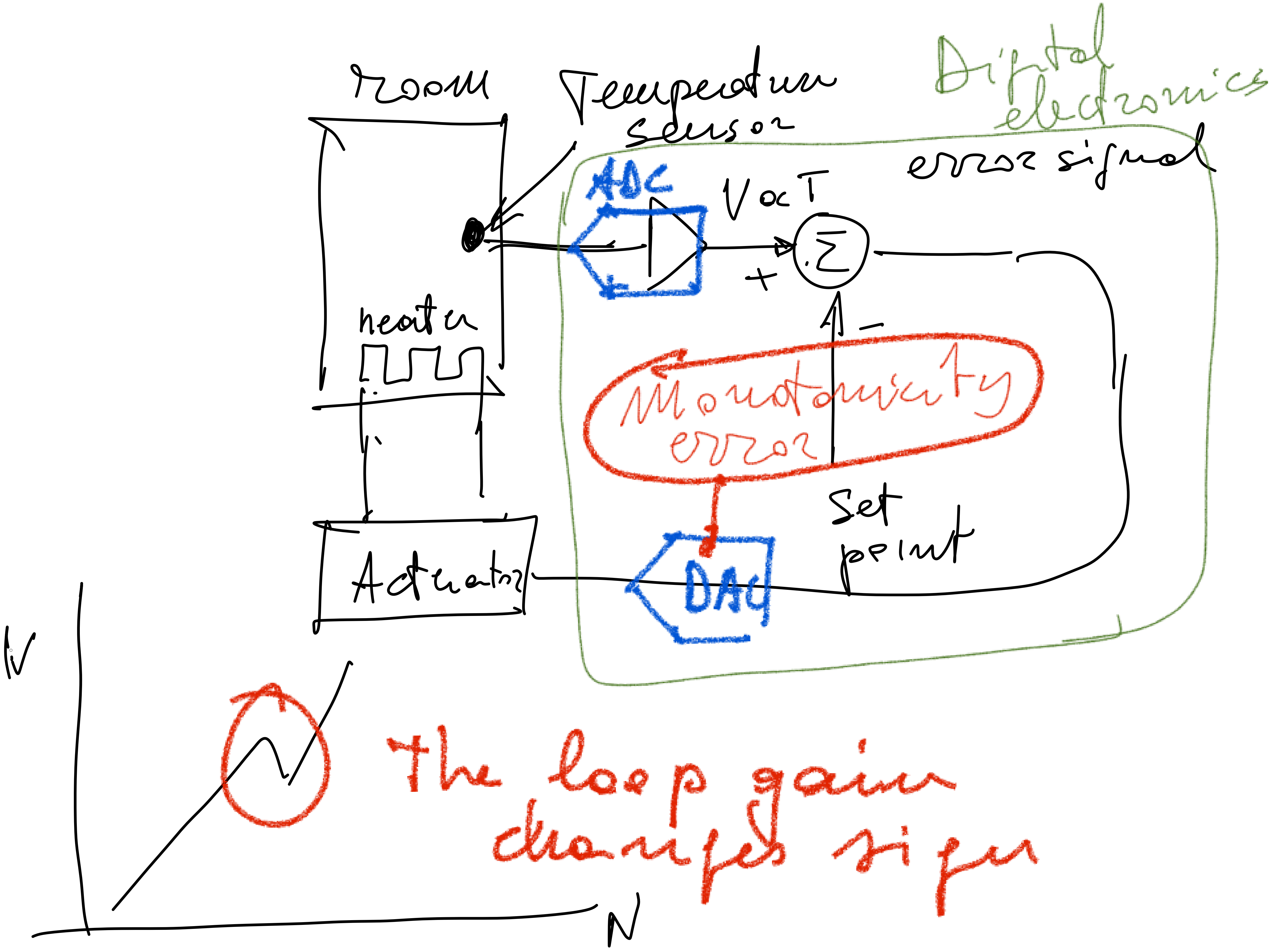
Two-Stage (Subranging) Flash ADC



Maxim Integrated, Tutorials 810, Fig.4 (Accessed 23 Sep 2020)

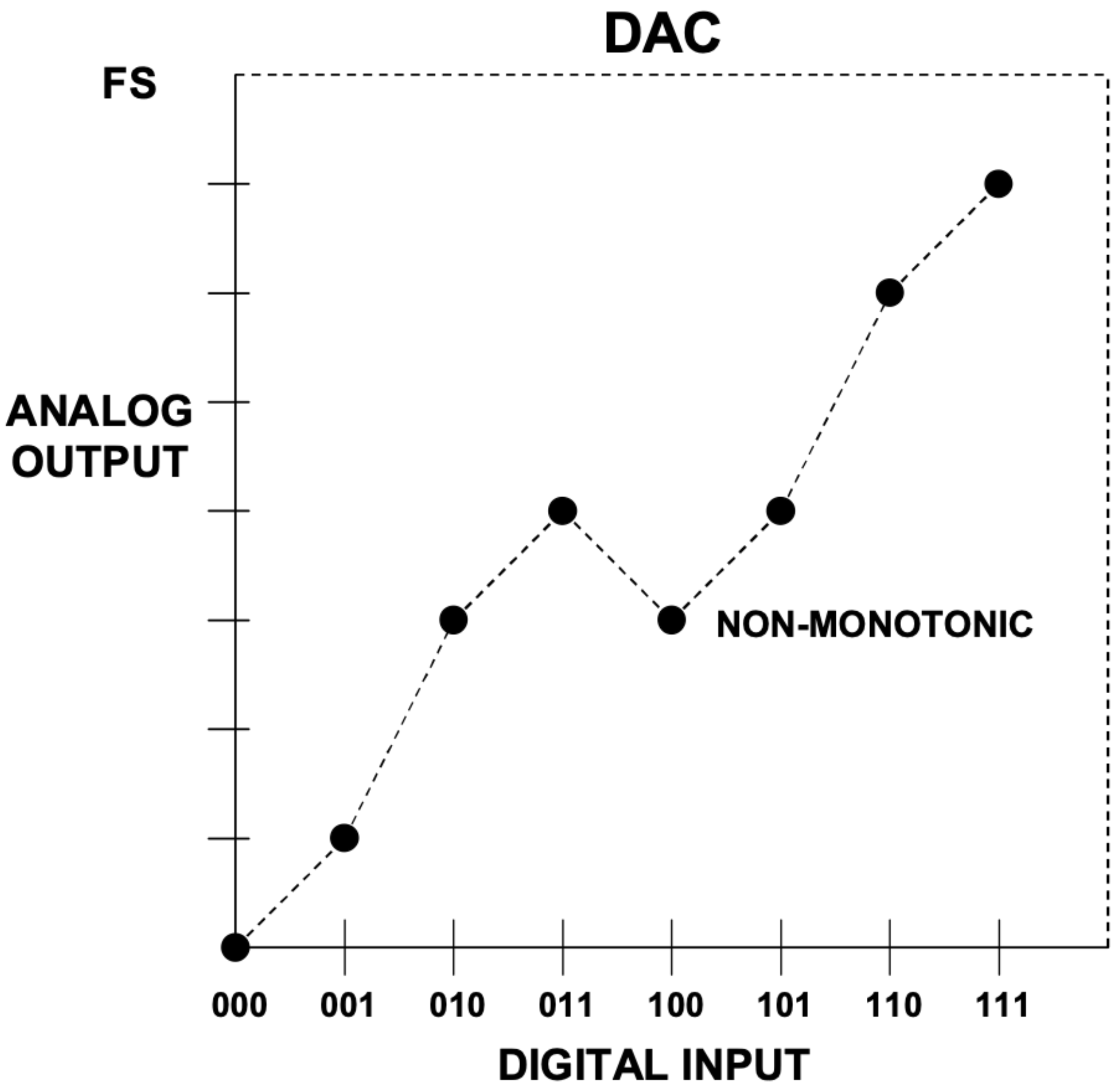
Missing Codes — Monotonicity error

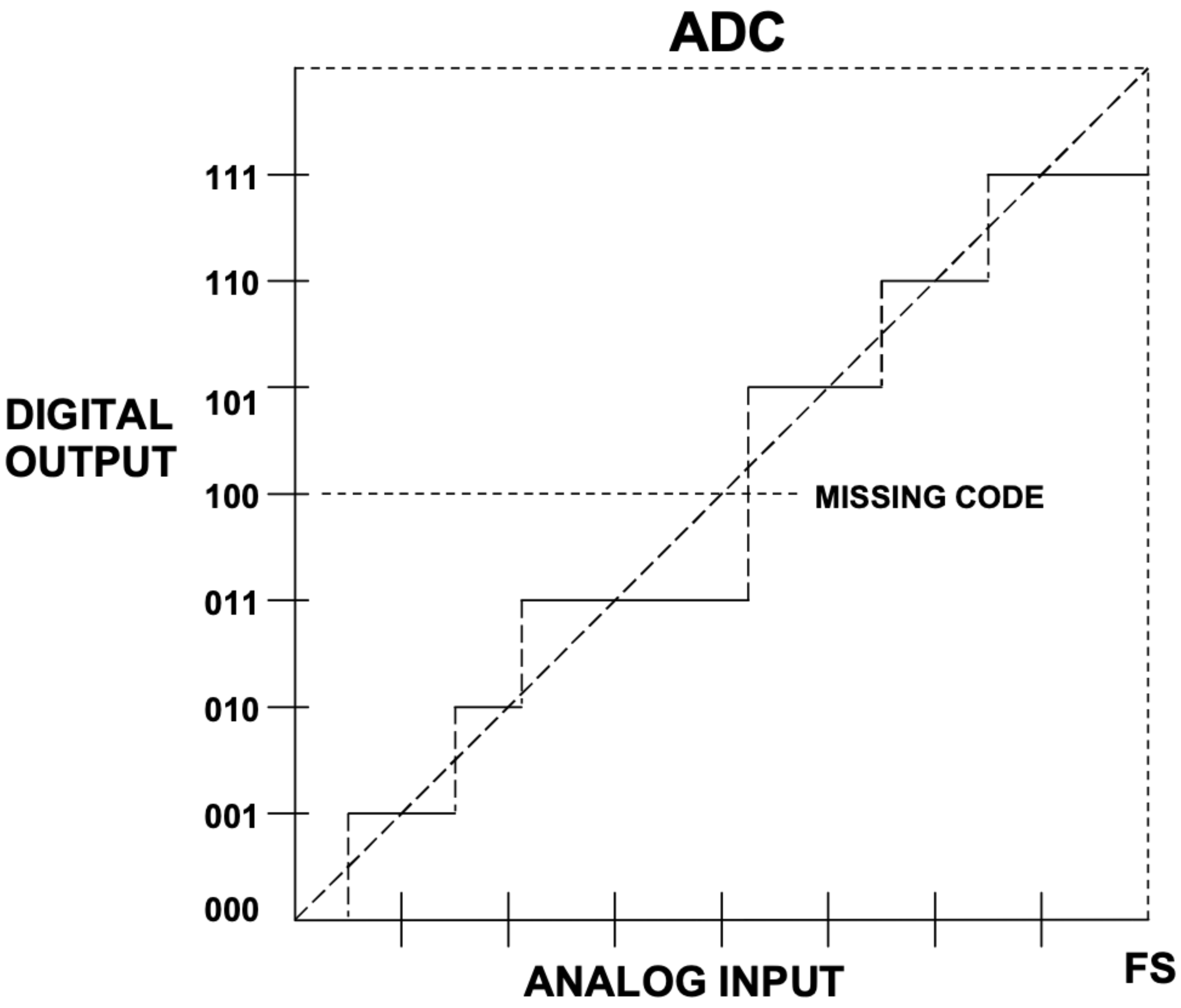




the loop gain changes sign

Differential Nonlinearity





W. Kester, Fig 2.19

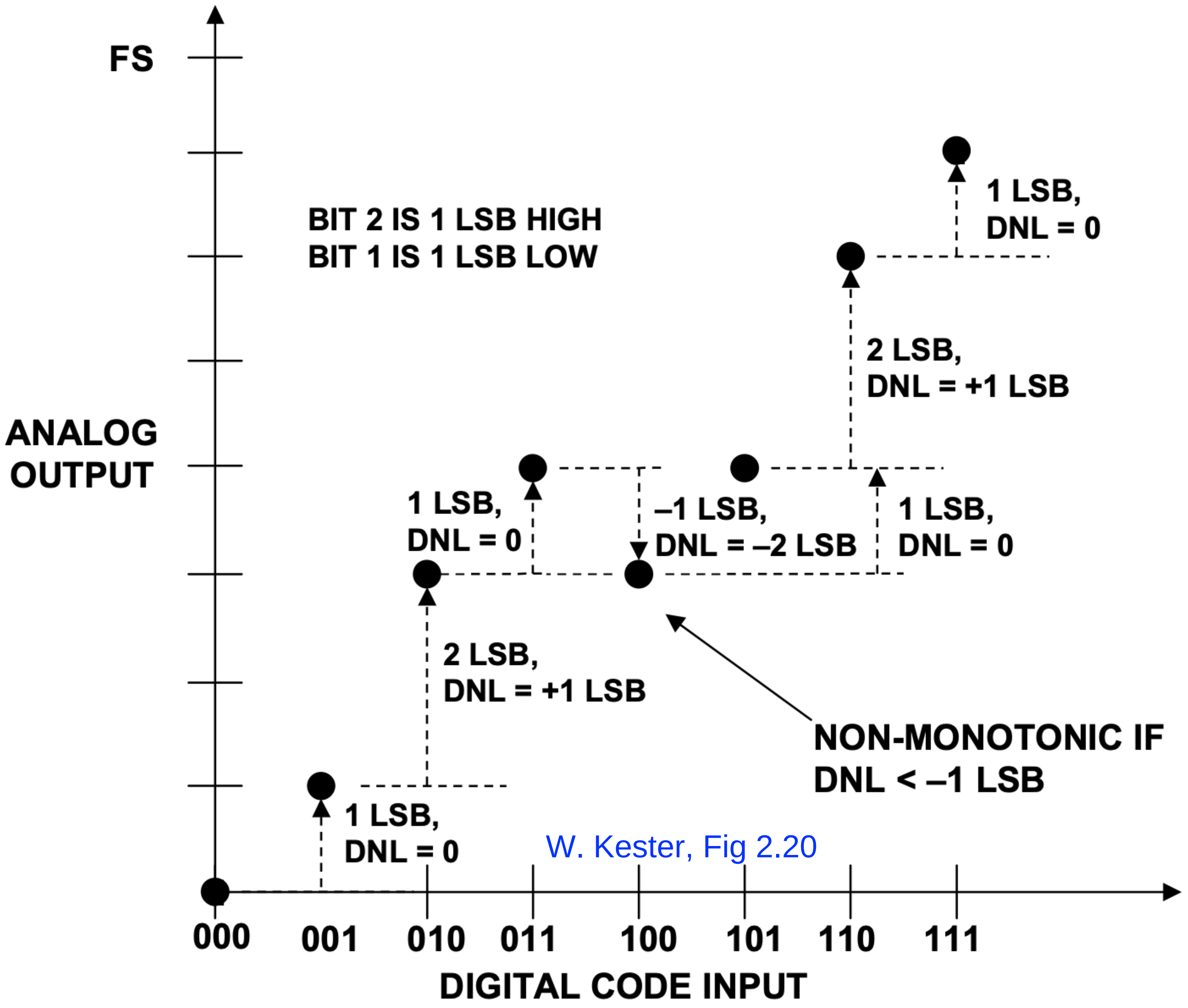
W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Monotonicity

Control folks are obsessed by monotonicity

Old-fashion controls

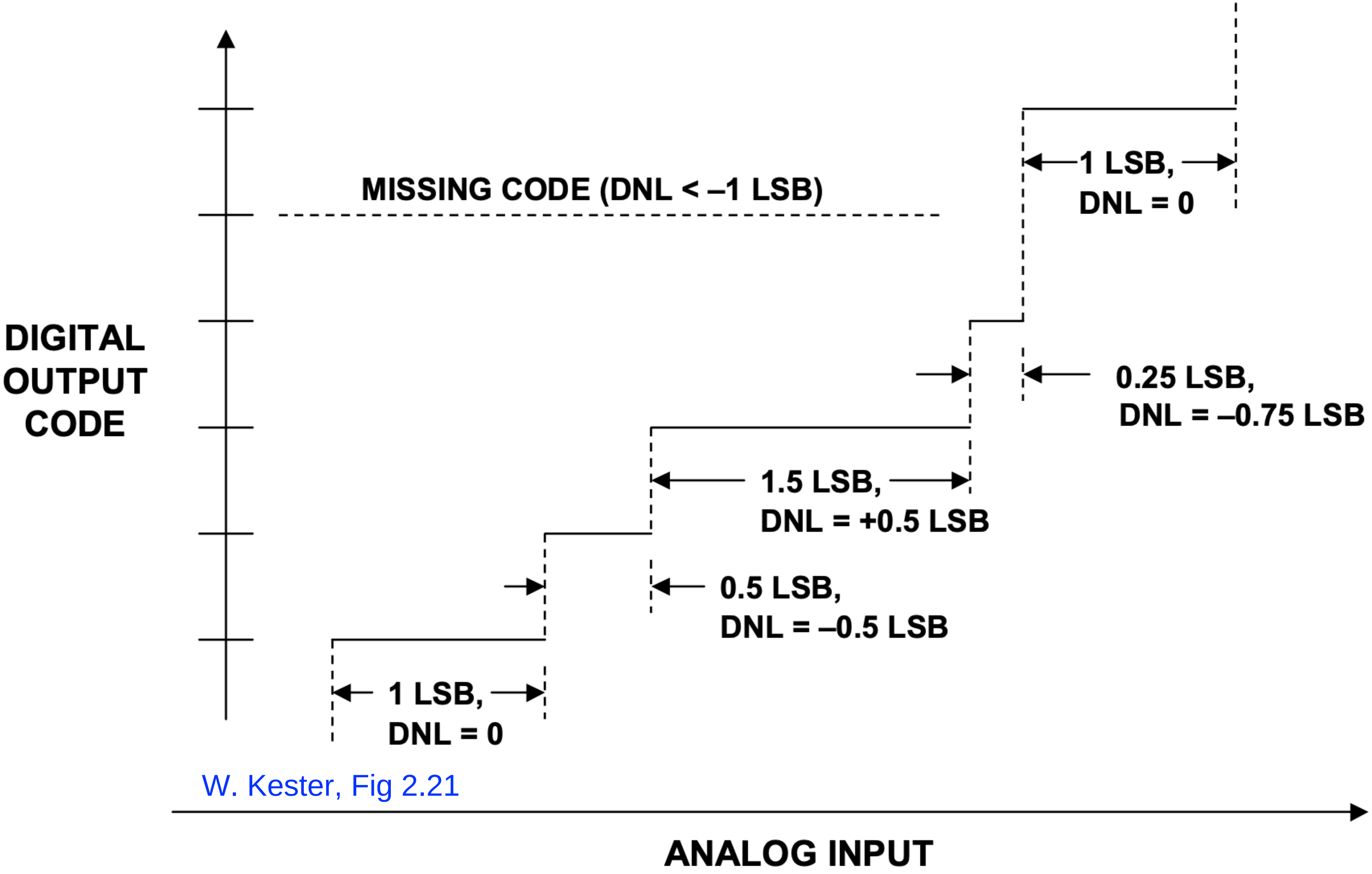
DAC Differential Nonlinearity



W. Kester, Fig 2.20

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

ADC Differential Nonlinearity

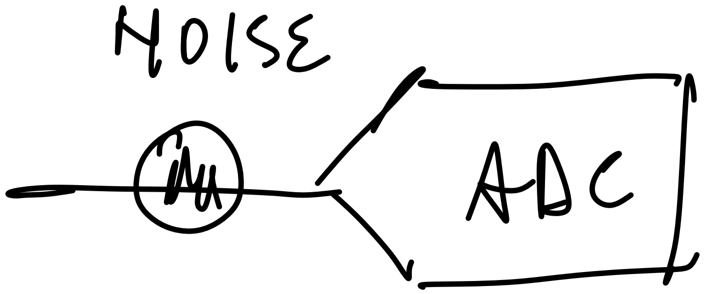


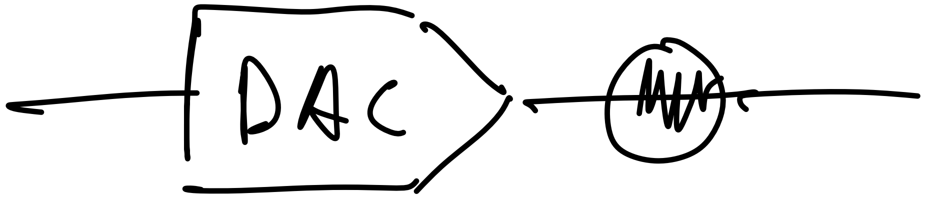
W. Kester, Fig 2.21

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Code Transition and Noise

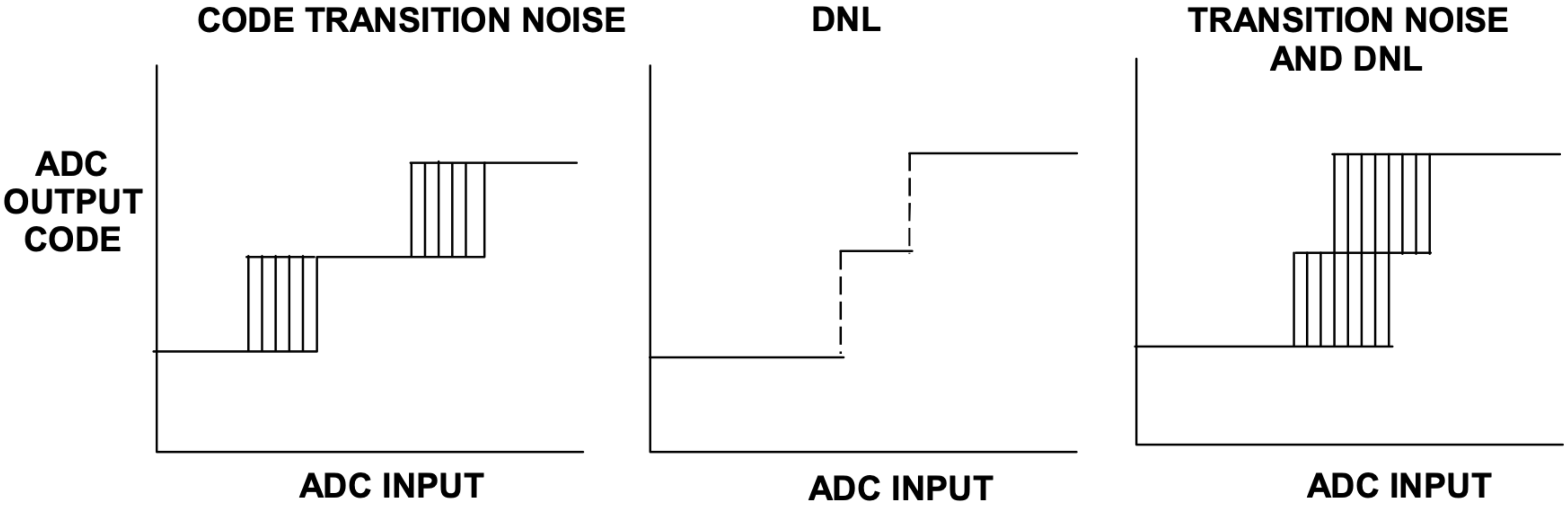
ENOB





Noise also affects Vref and clock.

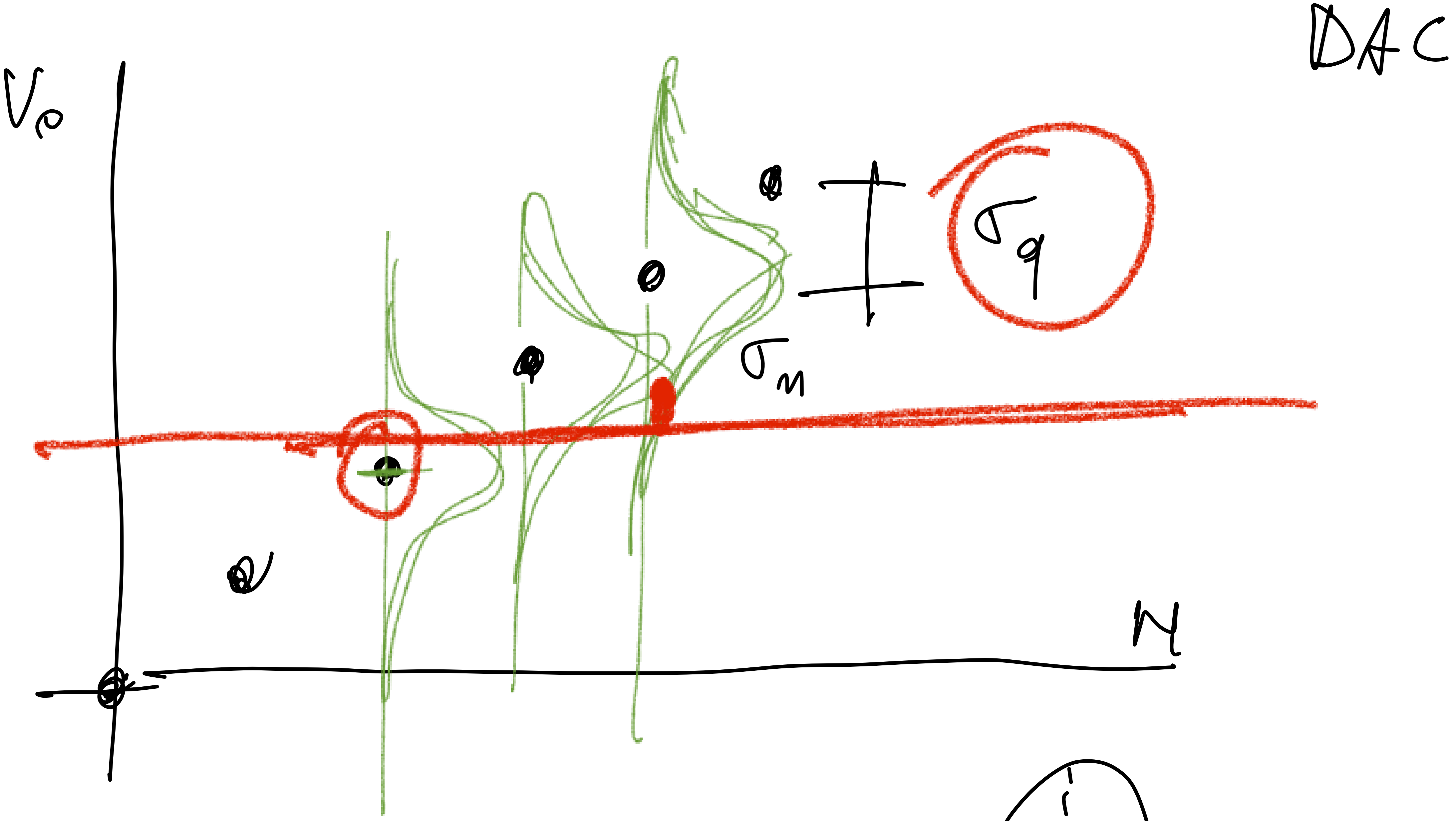
Noise can also affect Vref and clock.



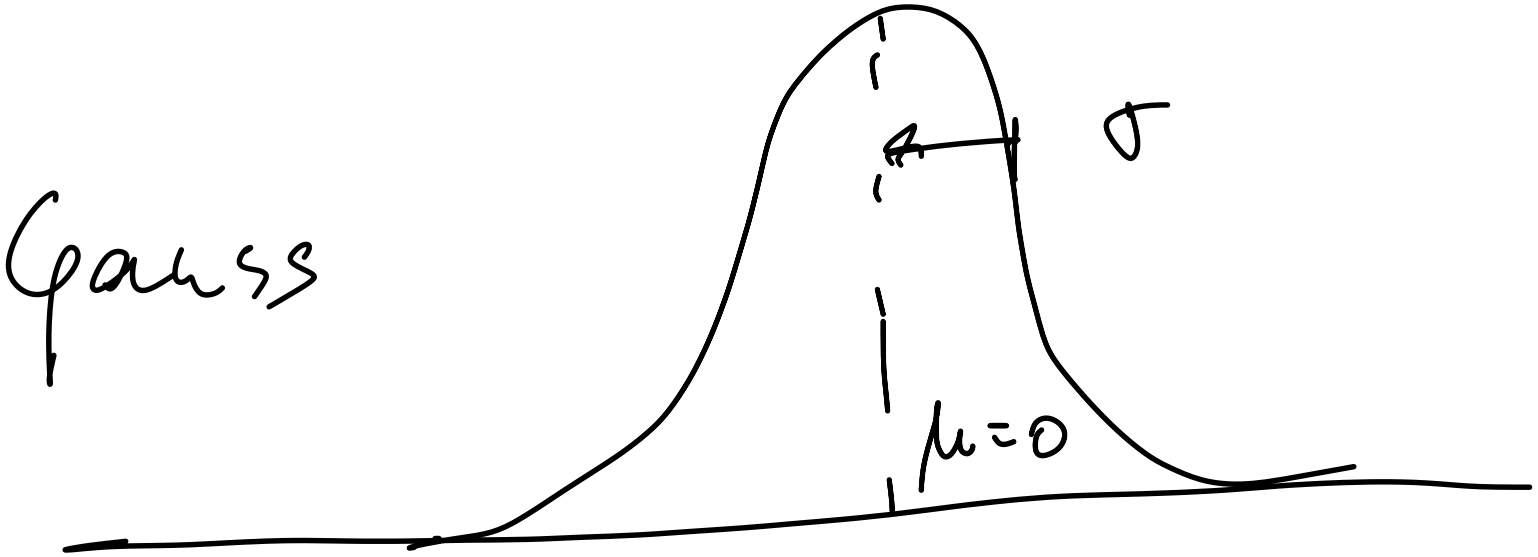
W. Kester, Fig 2.24

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

BUS Bits Vs Metrology



$$\sigma^2 = \sigma_q^2 + \sigma_M^2$$



Now, DNL Is No Longer Useful as It Used to be

We will explain why later

First, we need sampling and noise

DNL & Transition Noise

↓

Sampling

↓

spectral analysis

↓

Noise

↓

Transition Noise

End of lecture 3

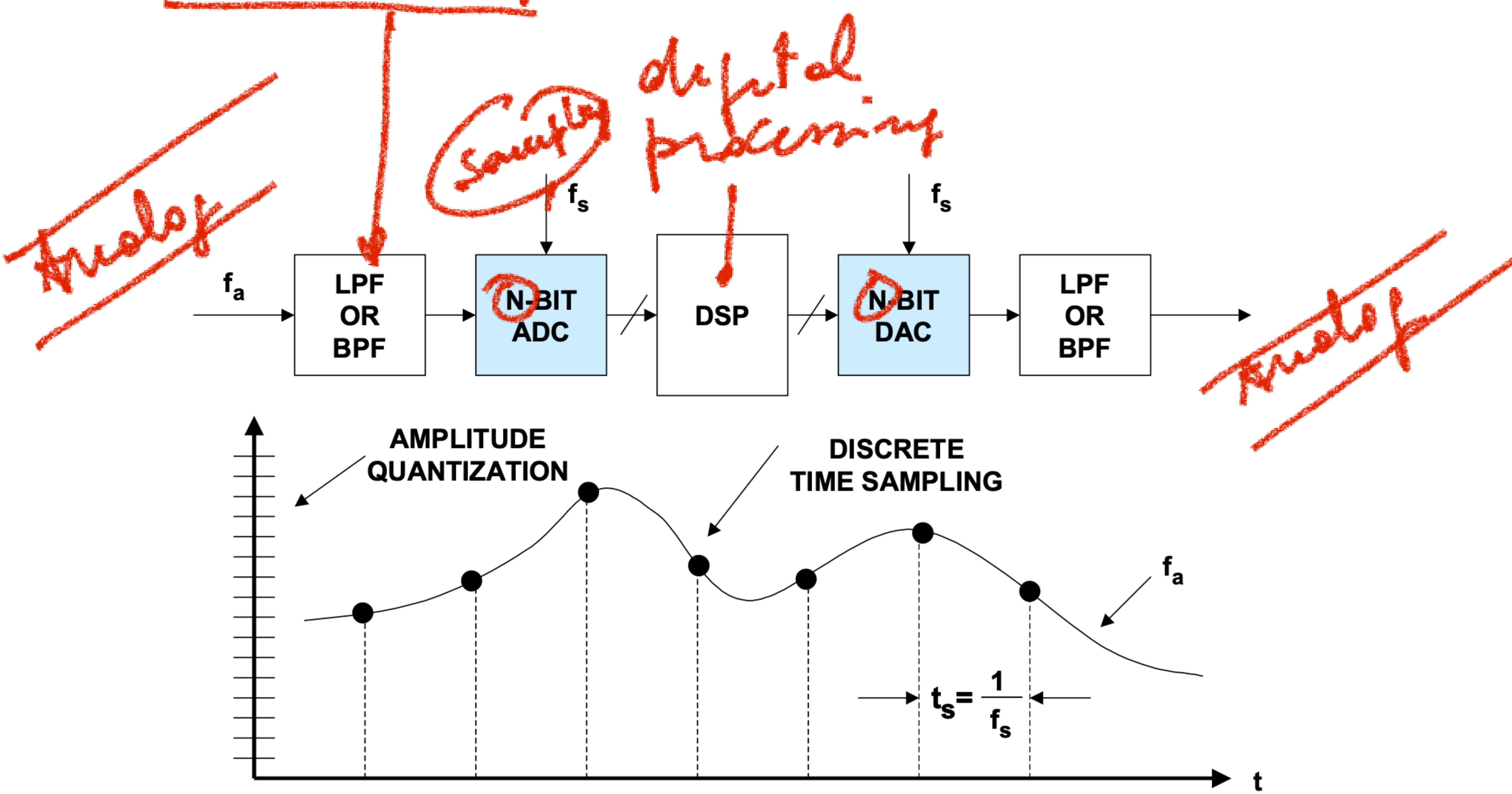
#4 Thursday, Sept 24, 2020

1.5 Hours

Sampling

Basic

Anti aliasing



W. Kester, Fig 2.25

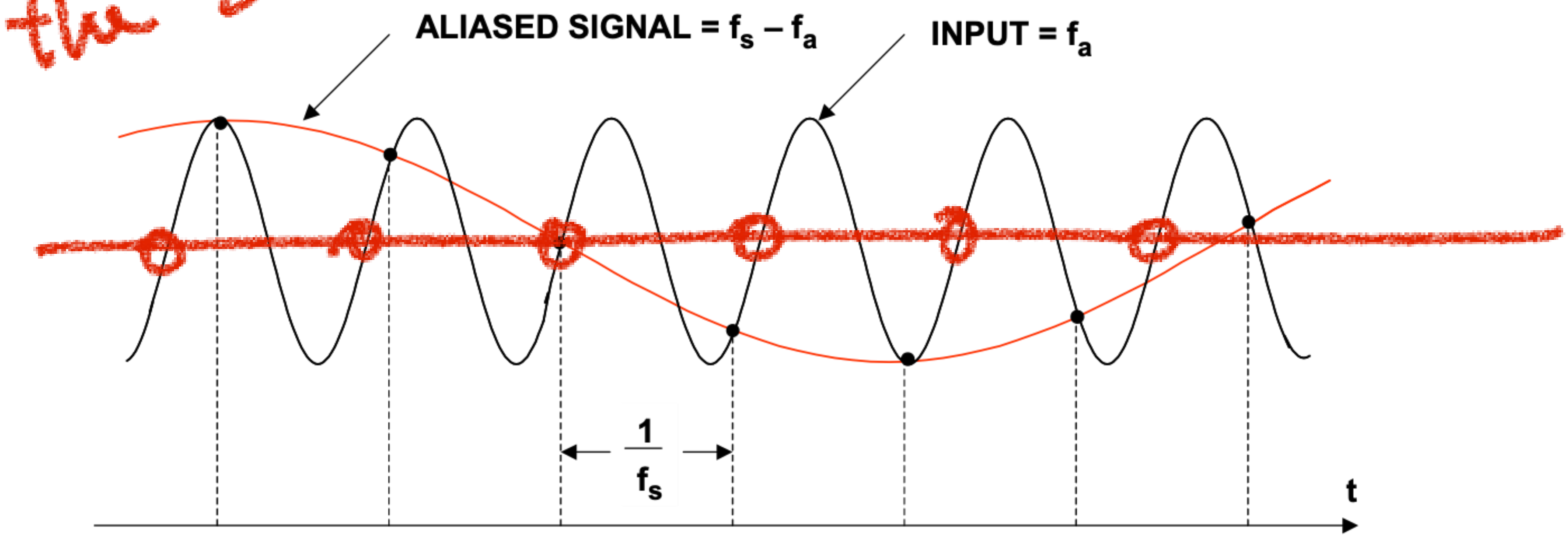
$f_N = \frac{1}{2} f_s$

Nyquist Sampling

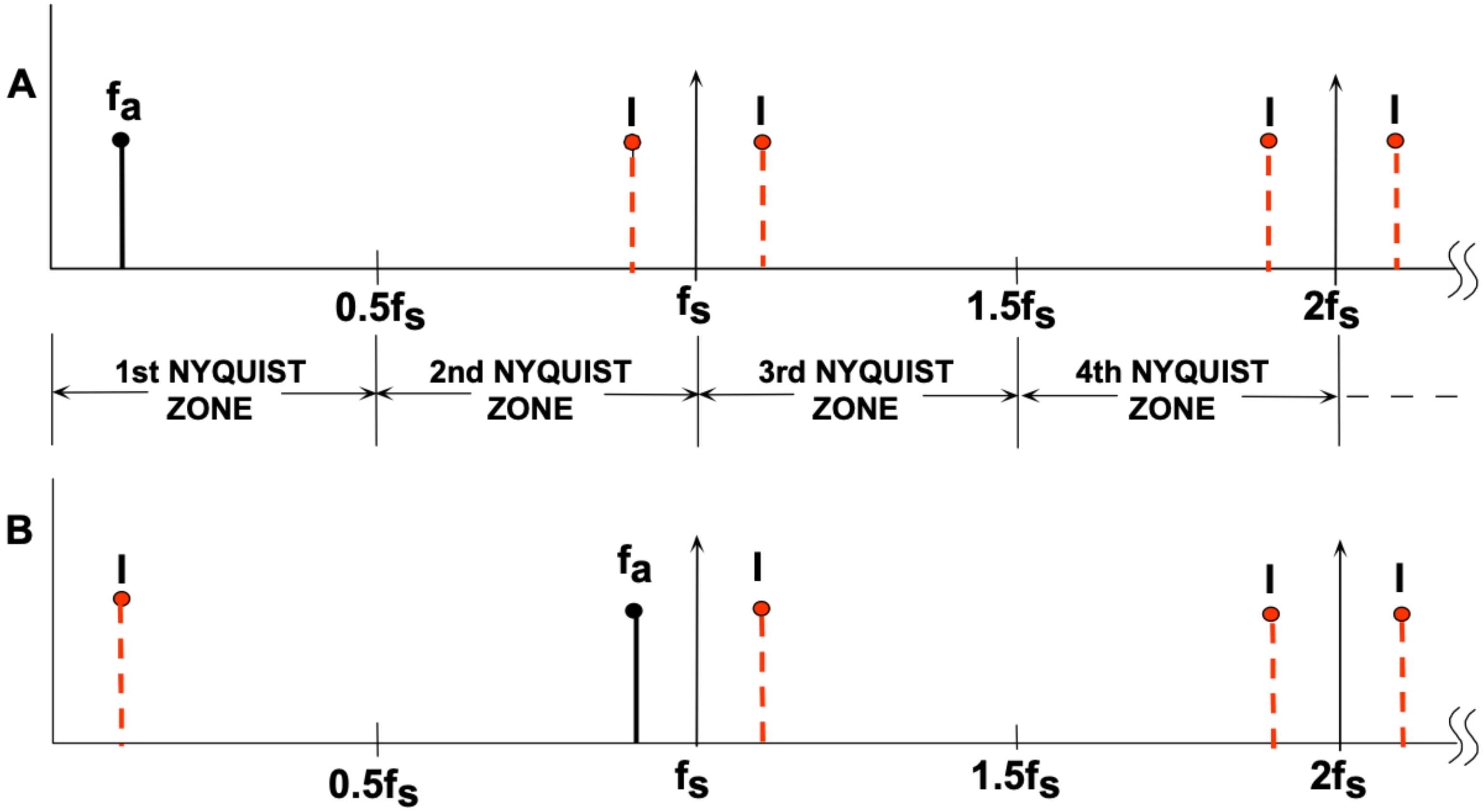
W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Aliasing

stroboscopic
sampling
freeze the rotation



W. Kester, Fig 2.30



W. Kester, Fig 2.31

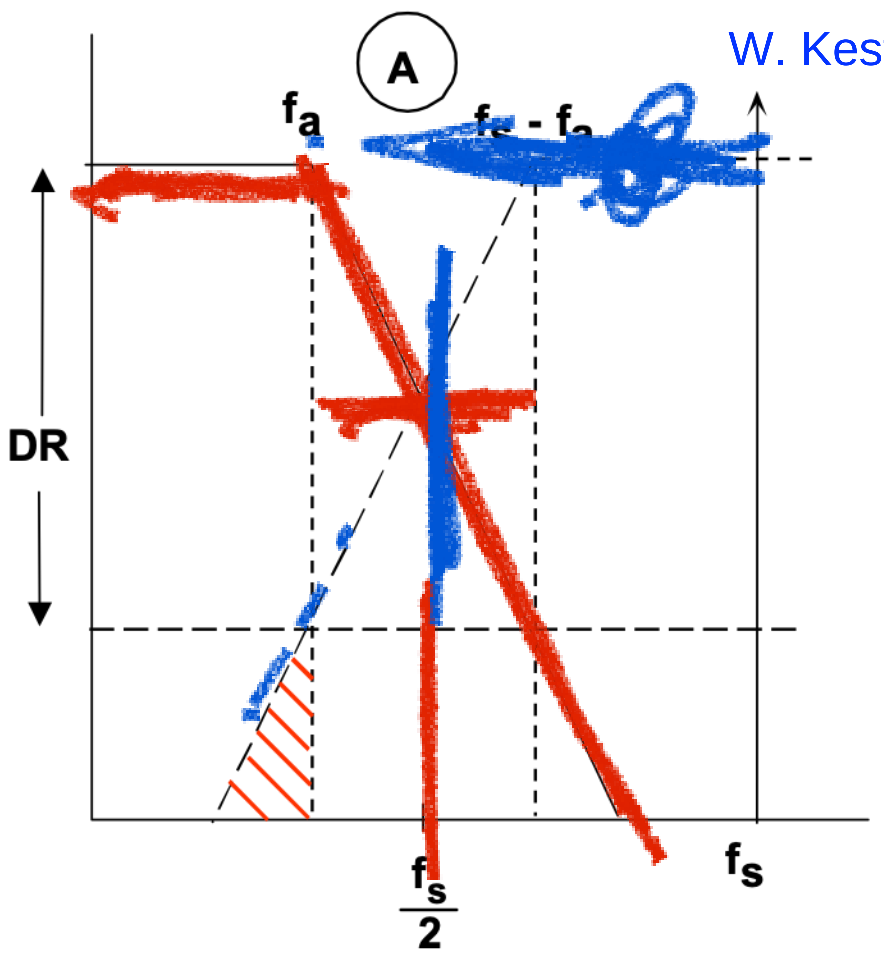
W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Aliasing

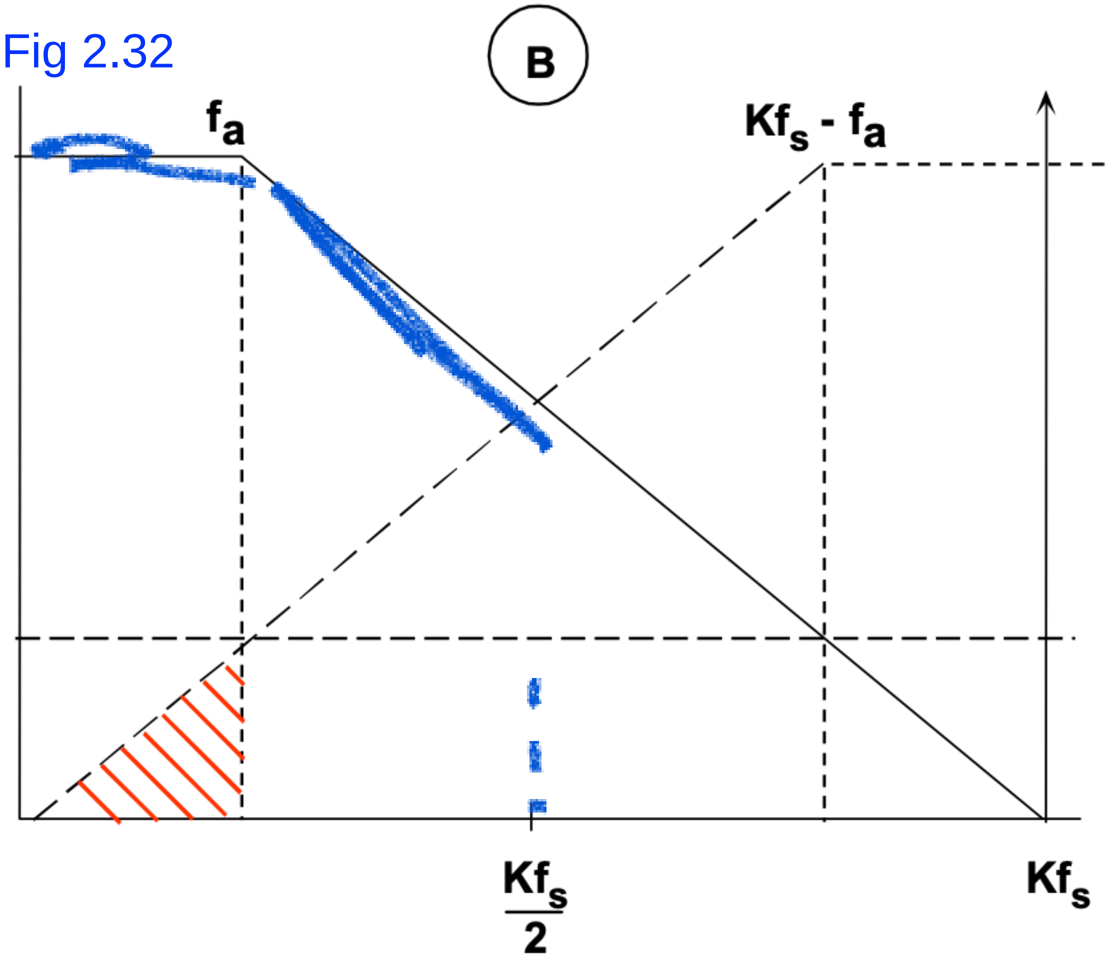

WELLS, FARGO & COMPANY.
U.S. MAIL
ILS OF THE ROAD
$2750 REWARD
$500 Each
REWARD
$250 Each

Antialiasing Filter & Oversampling

W. Kester, Fig 2.32

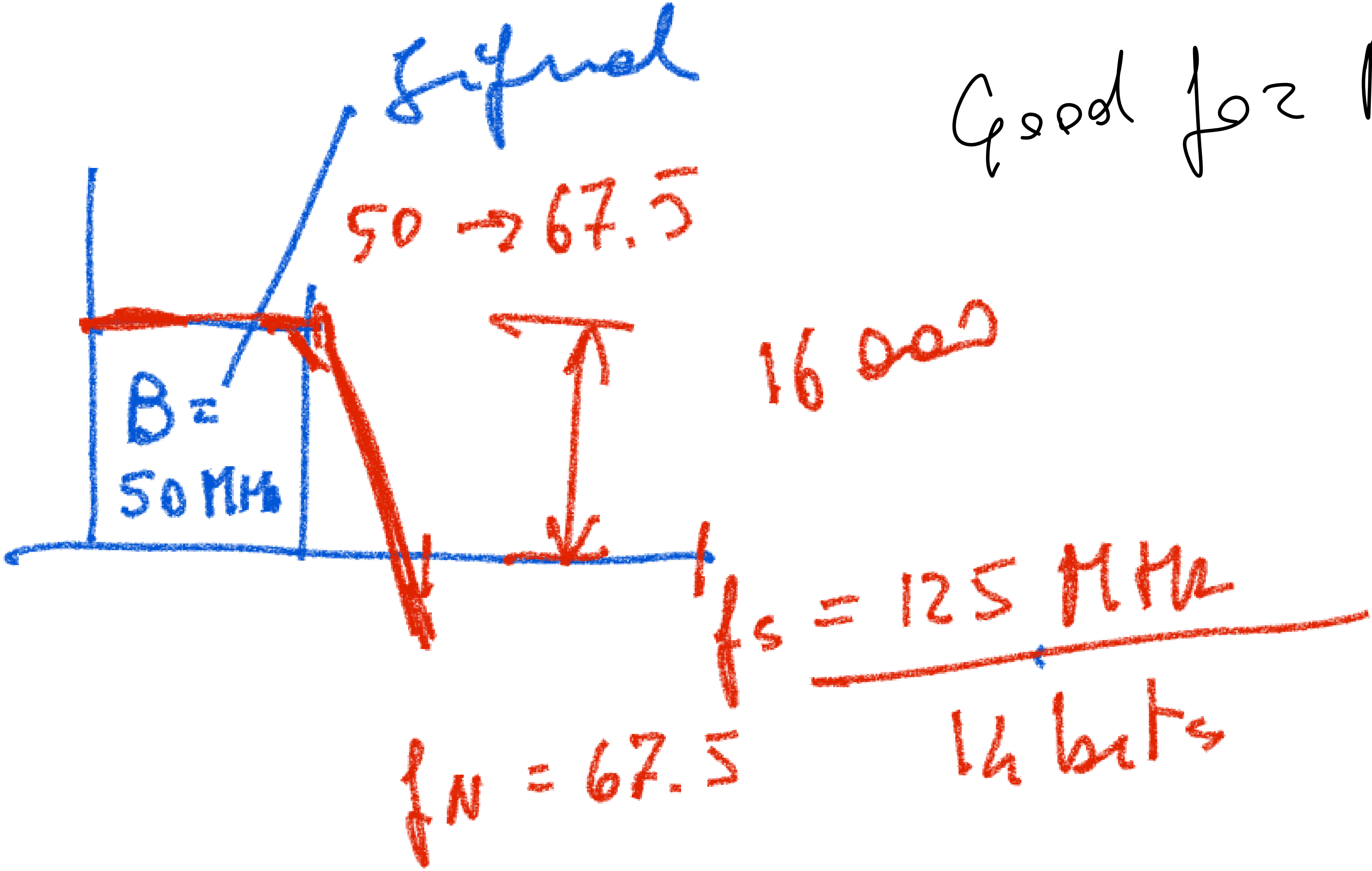


STOPBAND ATTENUATION = DR
TRANSITION BAND: f_a to $f_s - f_a$
CORNER FREQUENCY: f_a



STOPBAND ATTENUATION = DR
TRANSITION BAND: f_a to $Kf_s - f_a$
CORNER FREQUENCY: f_a

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3



Good for RF

---

low frequency filters

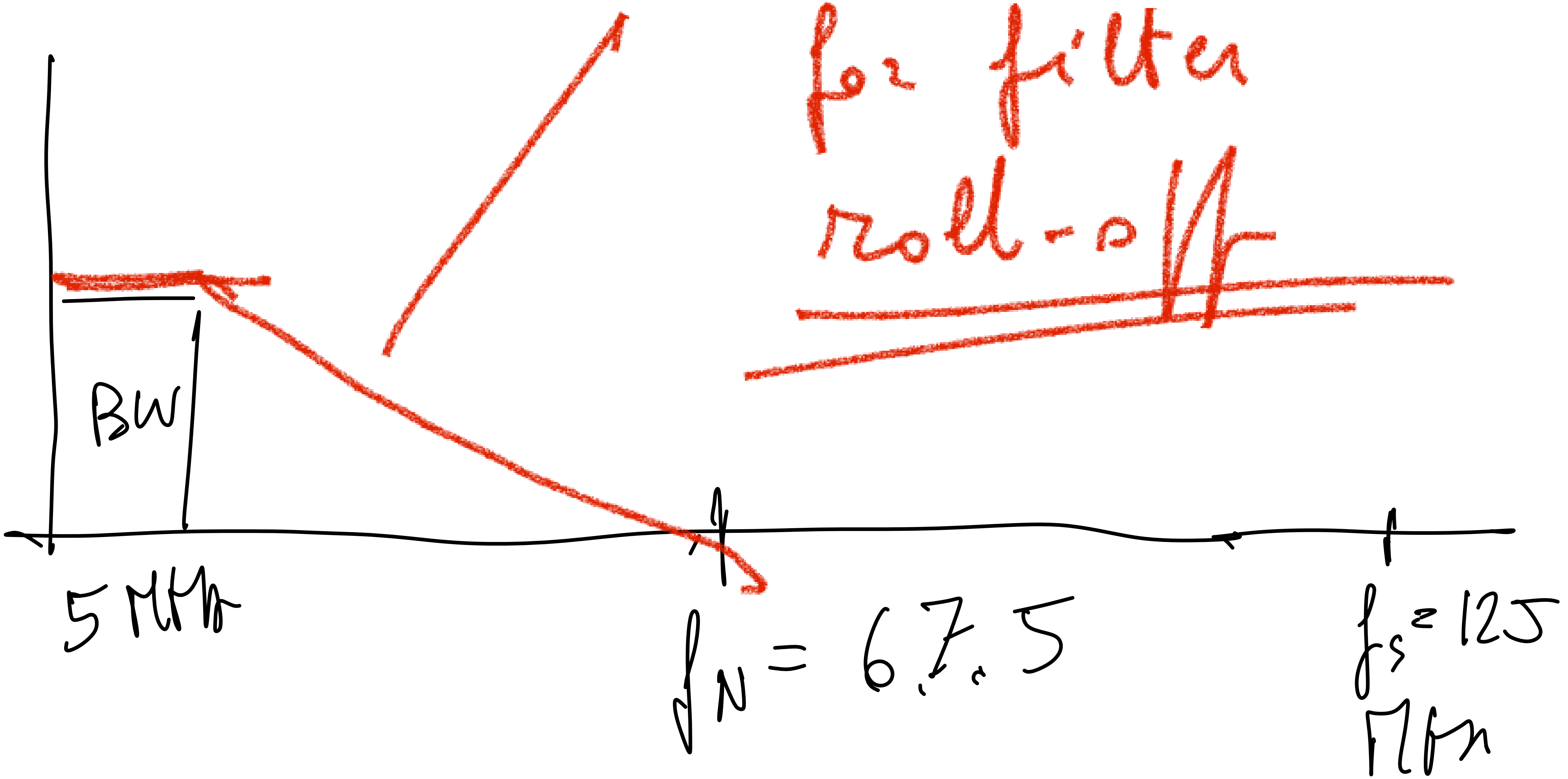
$$E = \frac{1}{2} L I^2$$

$$E = \frac{1}{2} C V^2$$

- Energy
- Physical size

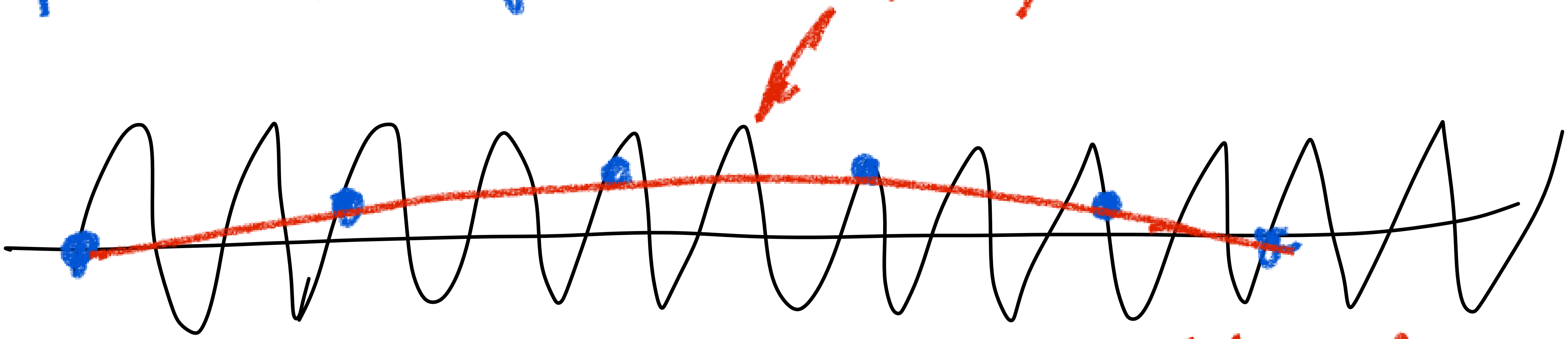
low-frequency < 1-10 MHz

> 1 decade for filter roll-off



New/recent ADCs intended for $f > f_N$

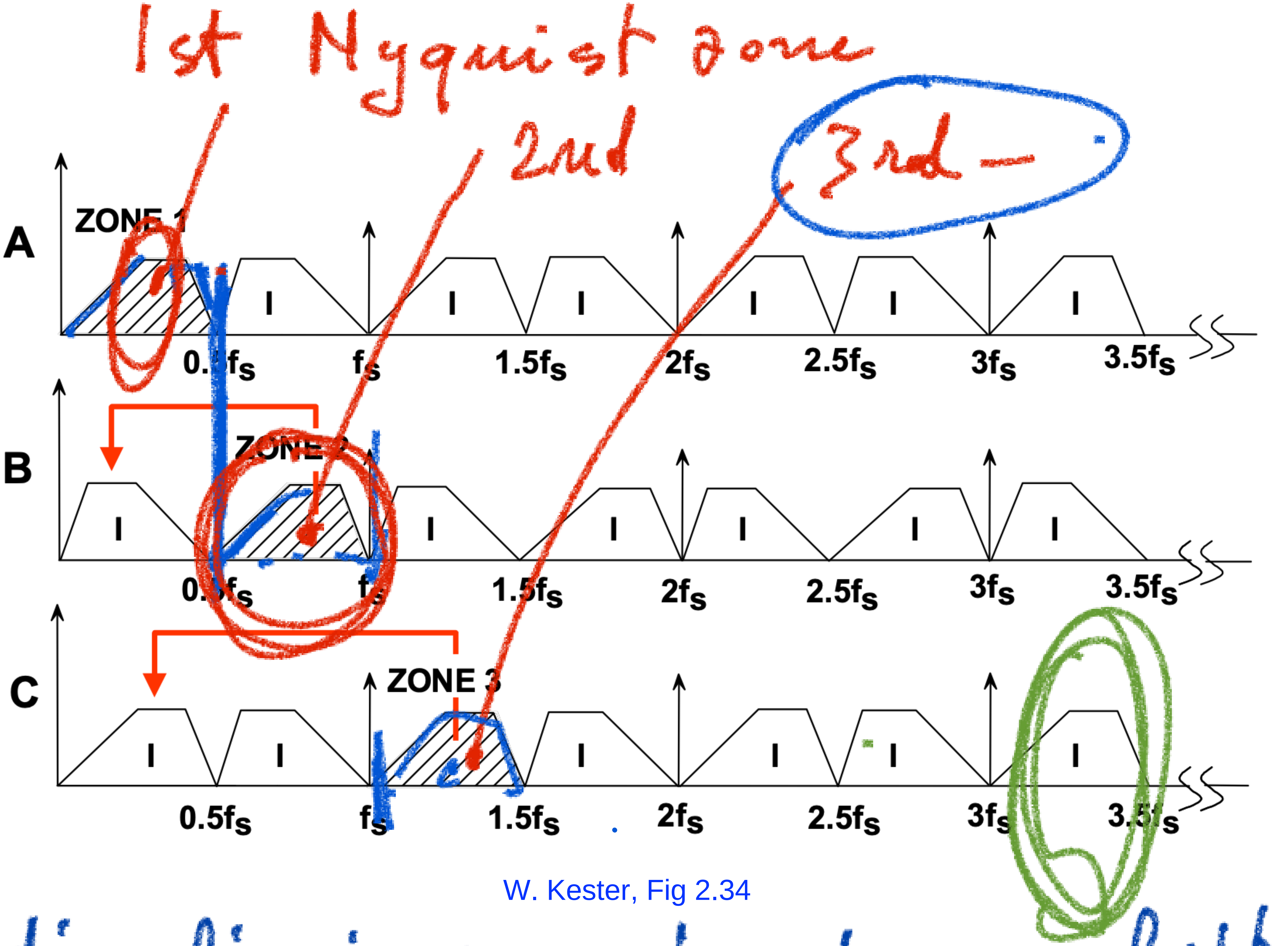
AM/PM



Amplitude Modulation

Phase ″

Intentional Undersampling

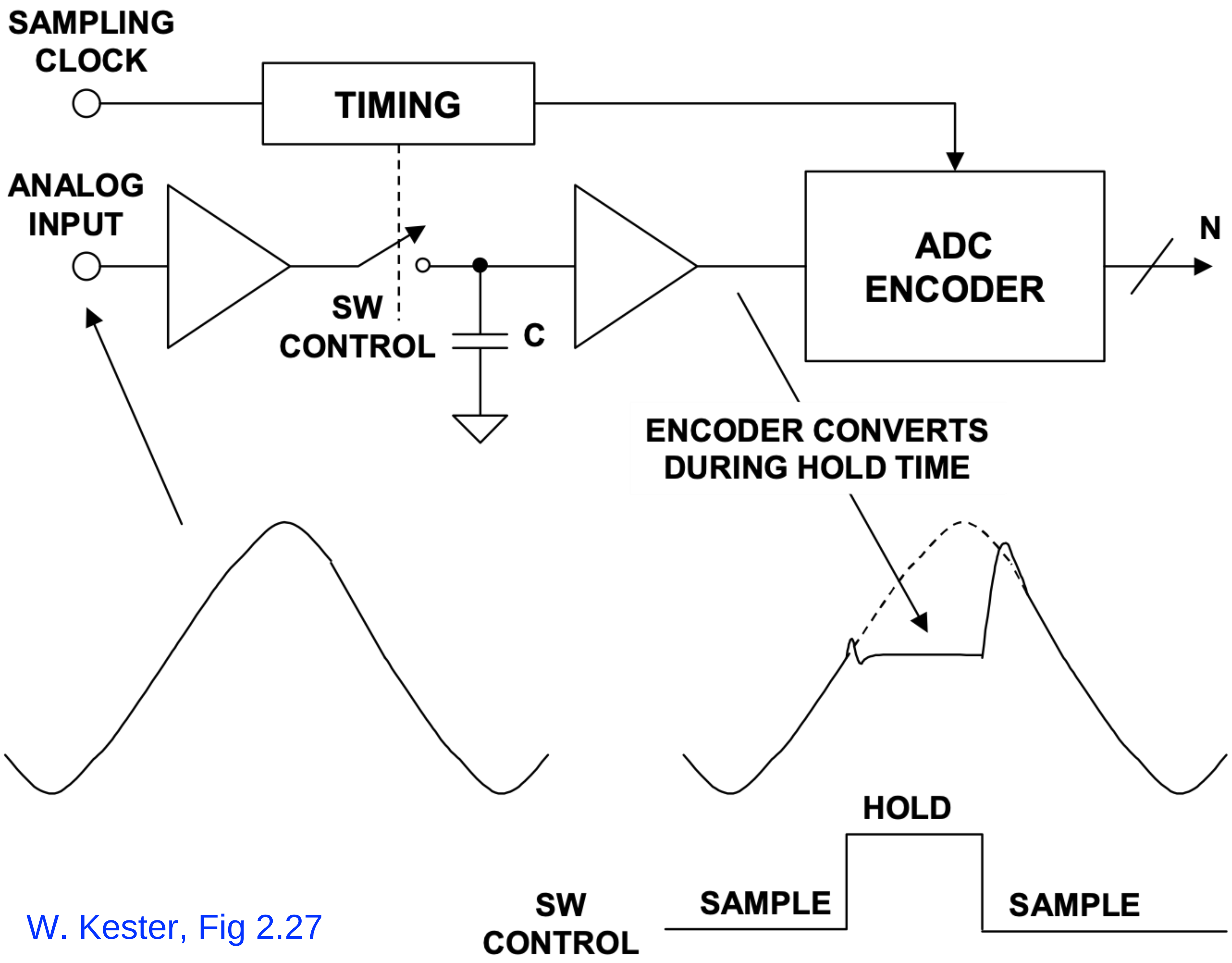


W. Kester, Fig 2.34

Anti-aliasing → band pass filter

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Sample and Hold



W. Kester, Fig 2.27

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Power, Spectra and Probability

Recall dB, dBm, dBV, dBm/Hz, dBV^2/Hz etc.

Power $(P_2/P_1)_{dB} = 10\ \mathrm{Log}_{10}(P_2/P_1)$

Power, dBm $P_{dBm} = 10\ \mathrm{Log}_{10}(P/P_{ref})$, $P_{ref} = 1$ mW

Voltage $(V_2/V_1)_{dB} = 20\ \mathrm{Log}_{10}(V_2/V_1)$

Current $(I_2/I_1)_{dB} = 20\ \mathrm{Log}_{10}(I_2/I_1)$

Voltage, dBV $V_{dBV} = 20\ \mathrm{Log}_{10}(V/V_{ref})$, $V_{ref} = 1$ V

Obvious extension, use
10 Log for power,
20 Log for voltage and current

$P = 105\ kW$

dBm

0 dBm = 1 mW

30 1 W

60 1 kW

80 → 100 kW

$\ln(10) = 2.305$

$\ln(1+x) = x$

small x

$$\log_{10} A = \frac{\ln(A)}{\ln(10)}$$

$$\left[\frac{P_2}{P_1}\right]_{dB} = 10\log_{10}\frac{P_2}{P_1}$$

$$\left[\frac{V_2}{V_1}\right]_{dB} = 20\log_{10}\frac{V_2}{V_1} \quad , \quad \left[\frac{I_2}{I_1}\right]_{dB} \qquad \frac{P_2 = V_2^2/R}{P_1 = V_1^2/R}$$

dBm $\quad 10\log_{10}\frac{P}{1\,mW}$

dBV $\quad 20\log_{10}\frac{V}{1\,V}$

Examples

- Car engine, 137 HP (102 kW) —> dBm

- Antenna signal, 300 µV —> dBV

- Antenna signal, 300 µV —> dBm, assume 50 Ω input

Variance (signal power)

dc

Time domain $$\sigma^2 = \frac{1}{T}\int_0^T |x(t) - \mu|^2 dt$$

signal power

remove dc

$P = \tilde{x}^2$

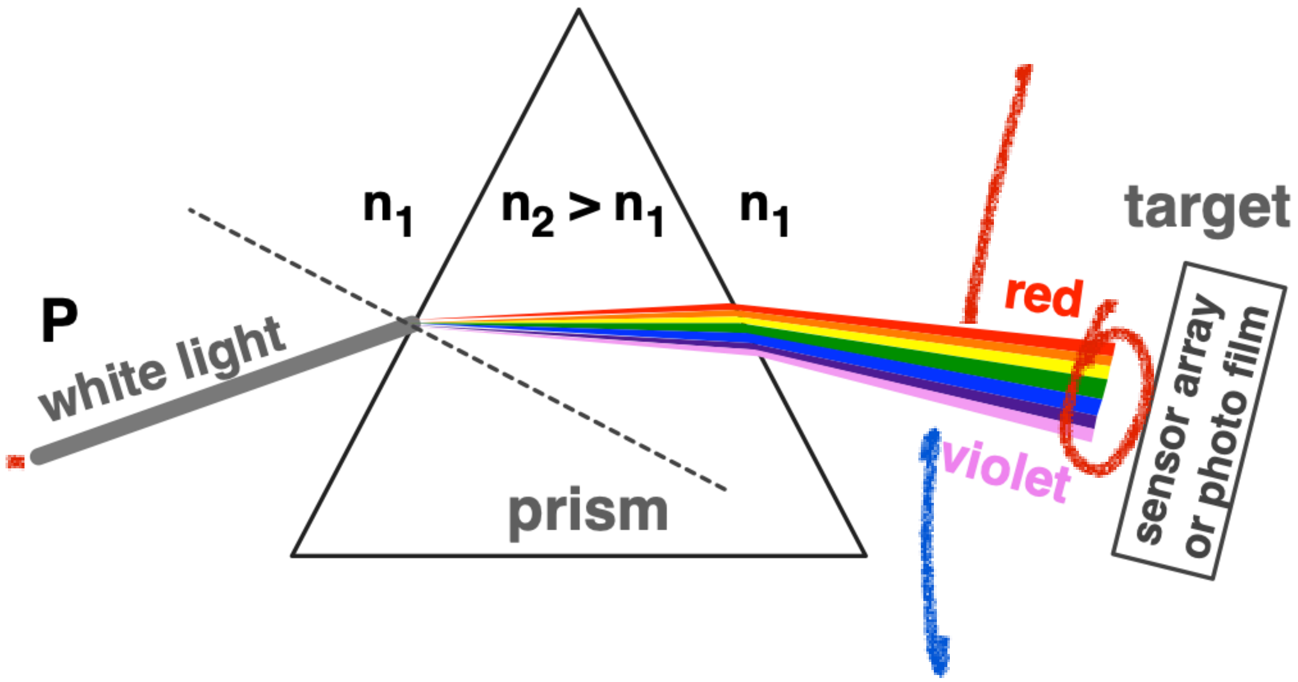
Average power

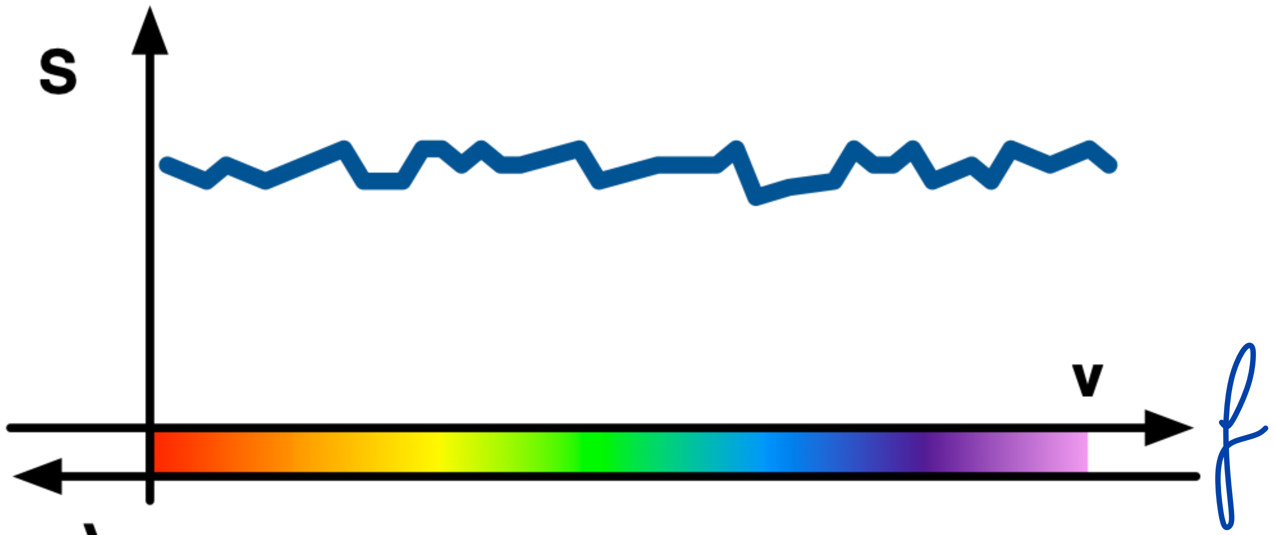
$\tilde{x}$

Power Spectral Density S(*f*)

PSD

B f_1, f_2 <—> pixel



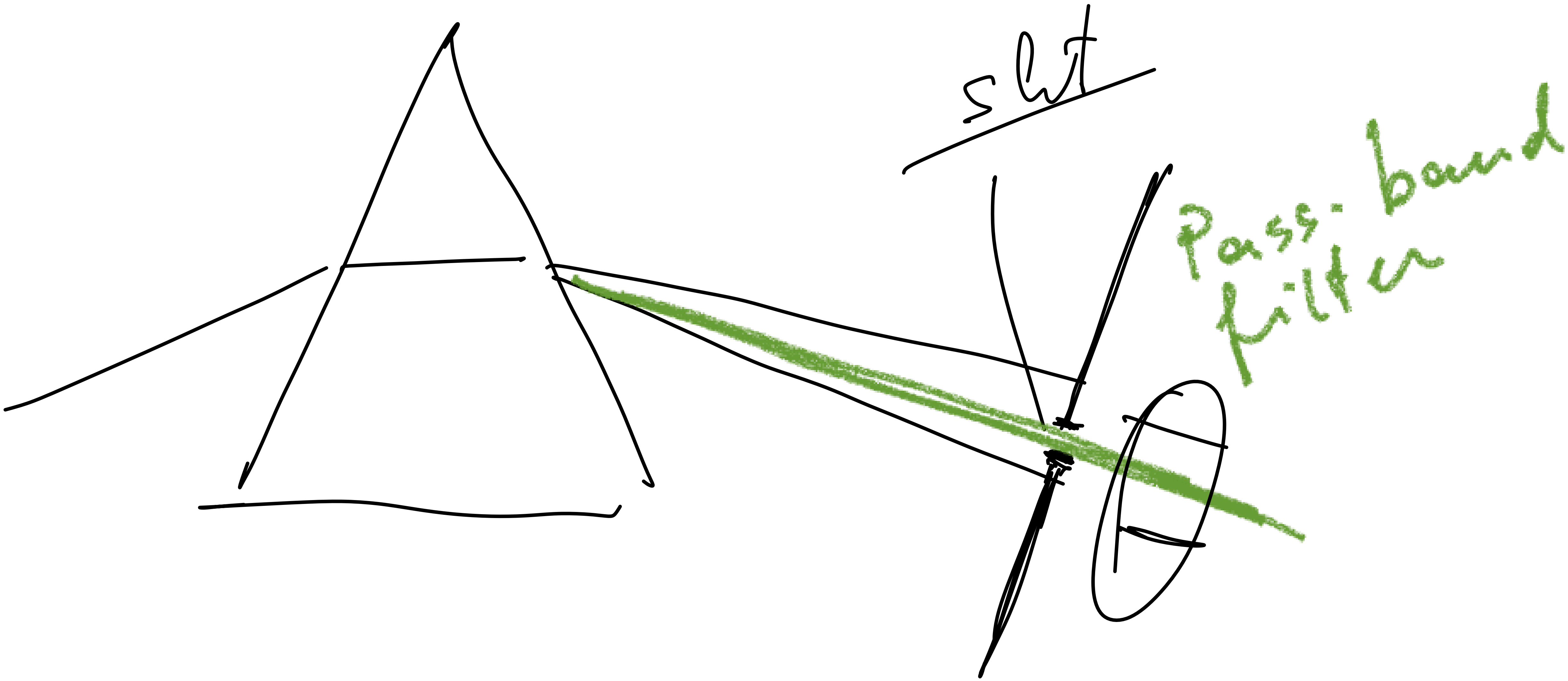


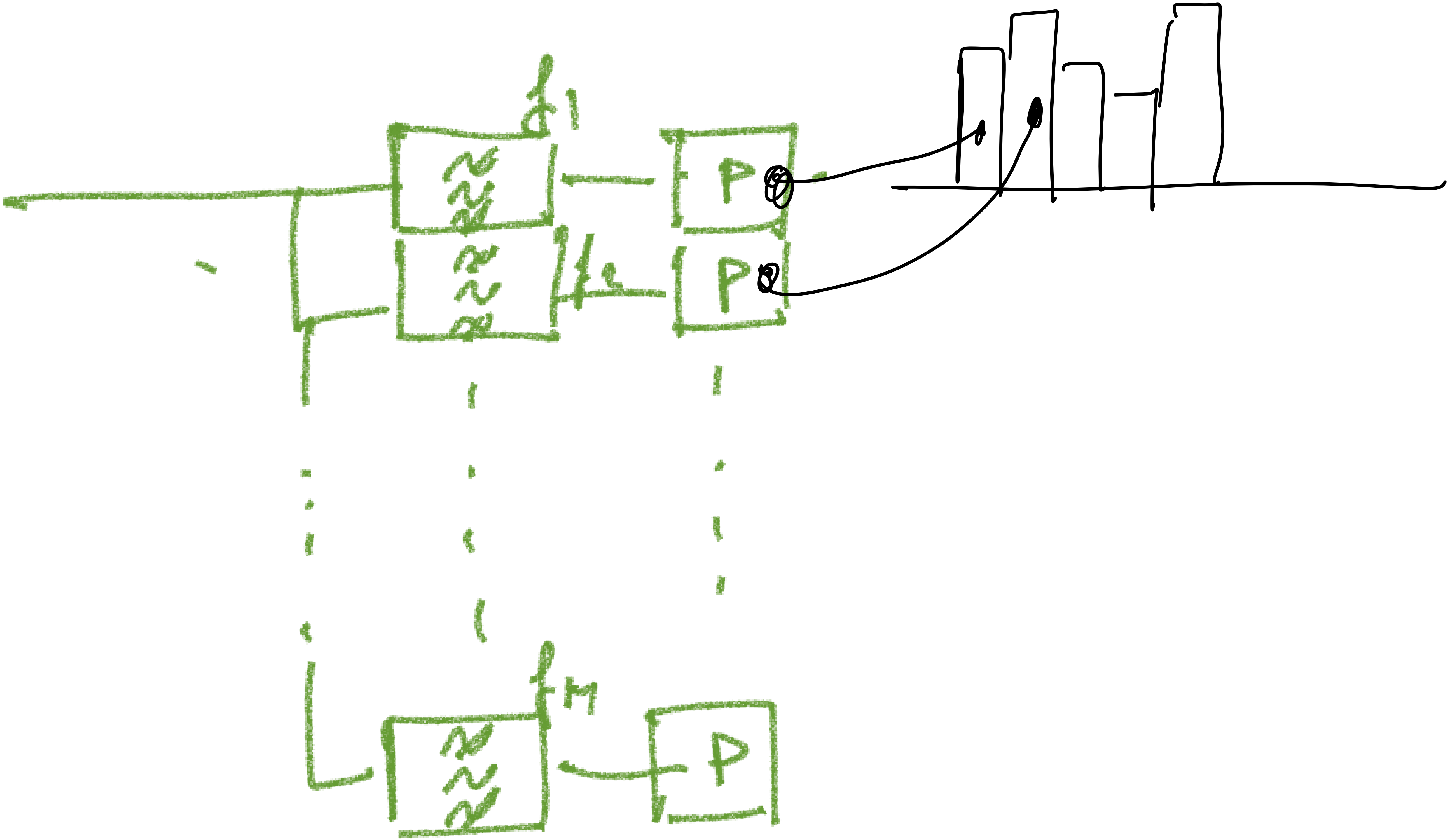
Continuous or discrete

- The PSD is the distribution of power vs. frequency (power in 1-Hz bandwidth)
- The PS is the distribution of energy vs. frequency (energy in 1-Hz bandwidth)
- Frequency can be continuous or discrete (histogram),

- In mathematics,
- the power is a square quantity
- the energy is power integrated in time

- Power (energy) in physics is a square (integrated) quantity
- PSD –> W/Hz (or V^2/Hz, A^2/Hz, etc.)
- PS –> J/Hz


slit
Pass-band filter


f1
P
f2
P
fM
P

Empiric Definition of the PSD

Experimental

$$\boxed{P = \int_0^\infty S(f)df}$$

Total power

$$P_{ab} = \int_a^b S(f)df$$

Power in a bandwidth

$$P_{ab} + P_{cd} = \int_a^b S(f)df + \int_c^d S(f)df$$

$$ab \cap cd$$

Power in separate bands adds up

Parseval Theorem

The power P (the variance σ^2) of a signal can be evaluated in time domain or in the frequency domain, and the result is the same

$$P = \int_0^{\infty} S(f)\, df$$

One-sided PSD

Frequency domain

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \; dt$$

avg. power

Time domain

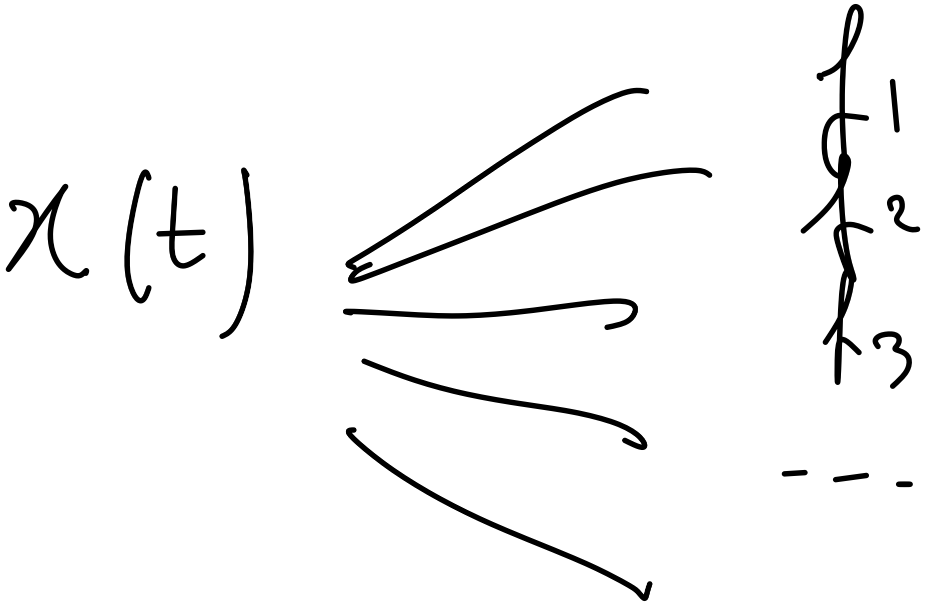
$\int_0^T$ for deterministic signals

Fourier Transform and PSD

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-i 2\pi f t}\, dt$$

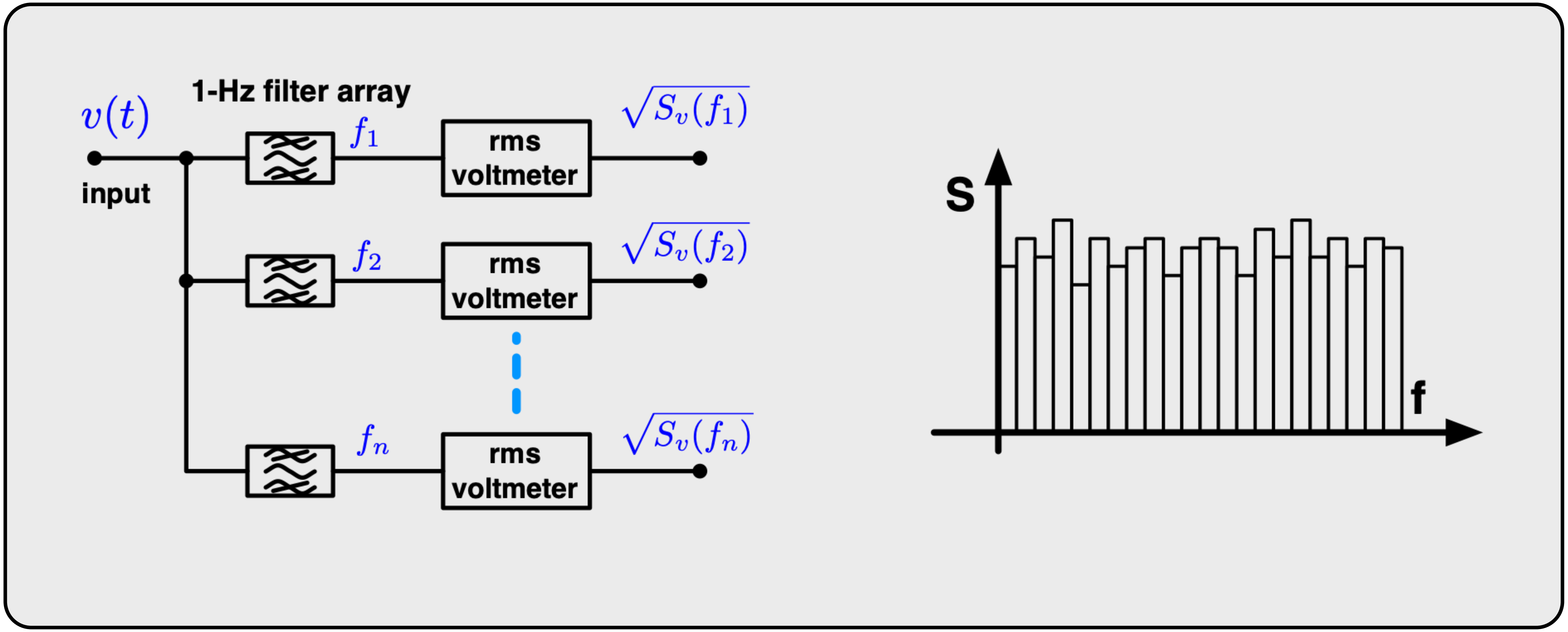
scalar product

$$(x, y) = x_1 y_1^* + x_2 y_2^* + x_3 y_3^* + \text{------}$$



$$X(n) = \sum x_k e^{-i 2\pi \frac{n}{N}}$$

Parallel Spectrum Analyzer



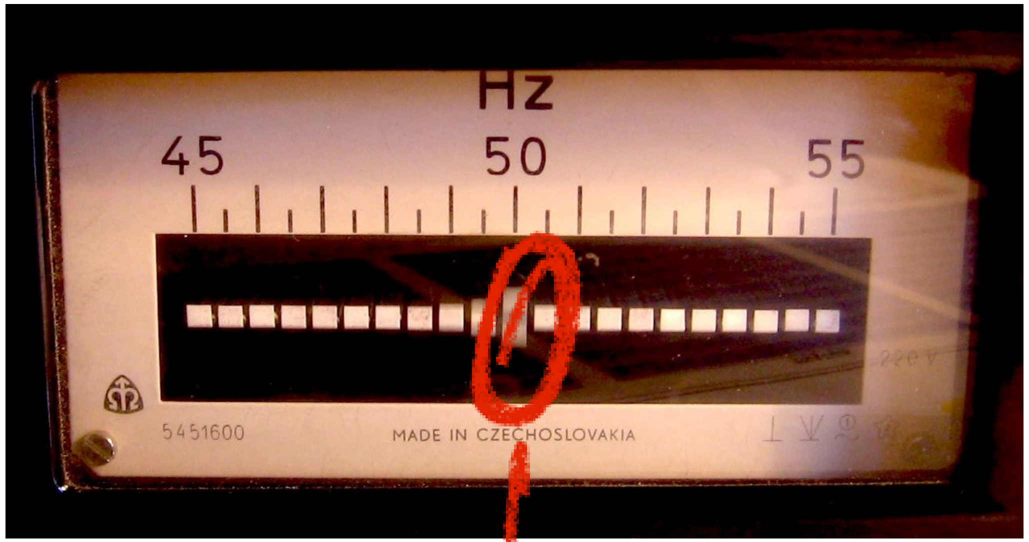
Rice representation —> Extension of the Fourier series

$$x(t) = \sum_{n=0}^{\infty} a_n(t)\cos(n\omega_0 t) - b_n(t)\sin(n\omega_0 t)$$

$$S_x(n\omega_0) = \left[a_n^2 + b_n^2\right]/\omega_0$$

ω_0 is the analysis bandwidth

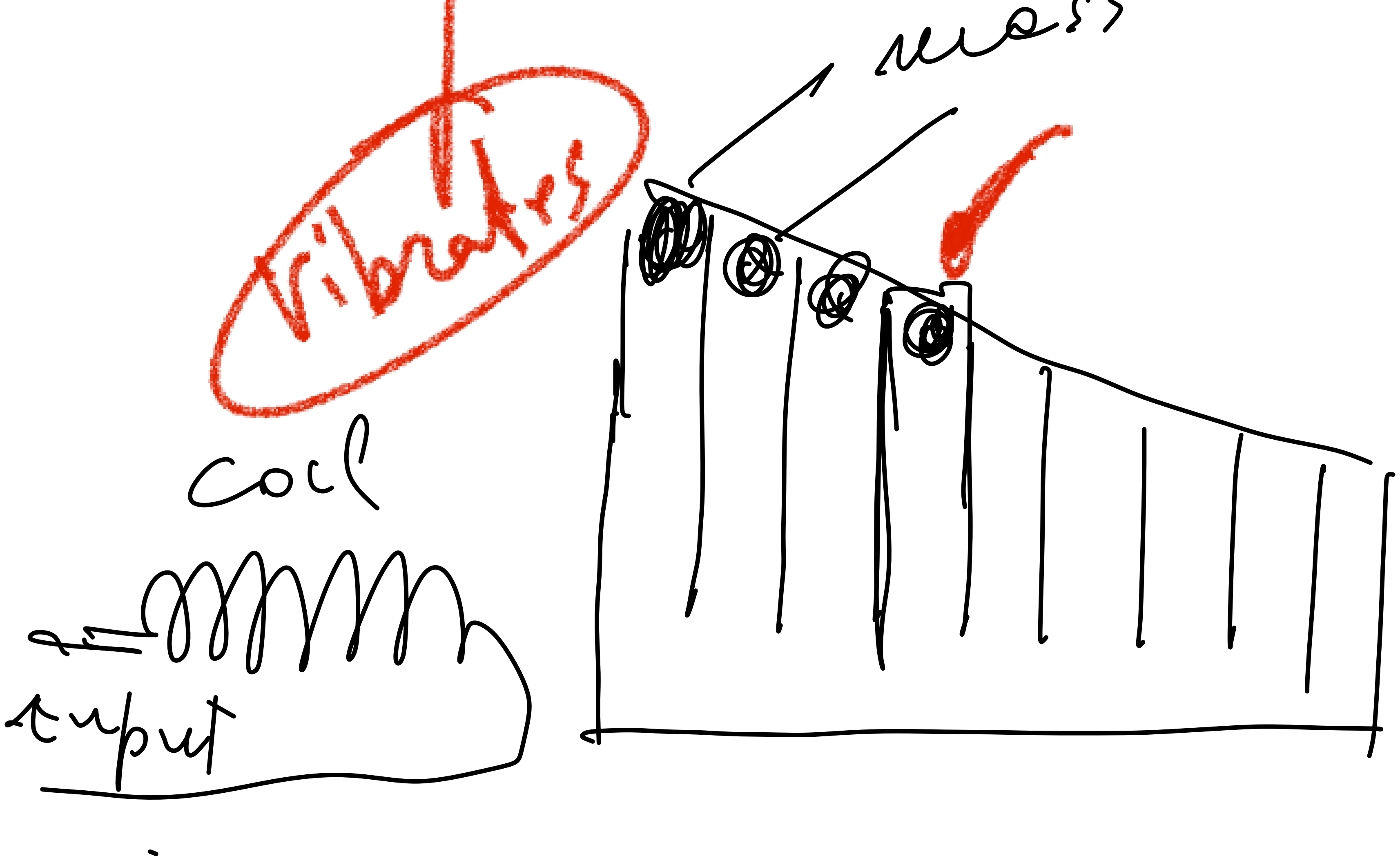
Vibrating-Reed Frequency Meter

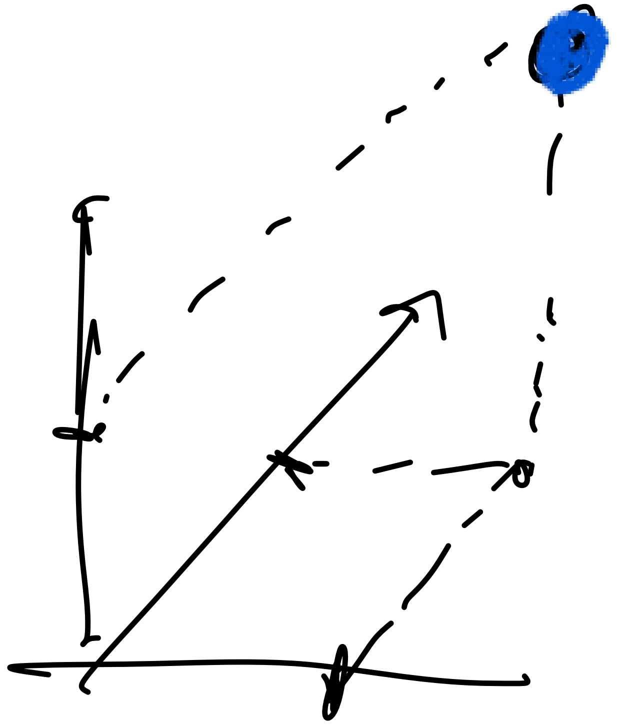


Julo, 24 mar 2006
(Wikimedia Commons)

Mass & Spring —> resonator

mass.Spring
resonator





IFT

$$x(t) = \int \underline{X(f)}\, \underbrace{e^{i2\pi ft}\, df}$$

components

base of the Hilbert space

$\hat{i}, \hat{j}, \hat{k}$

$$X(f,\Delta)\,X^*(f,\Delta f) \sim P(f,\Delta f)$$

T acquisition times / truncated signal

$$S_x(f) = \frac{1}{T} X_T(f)\, X_T^*(f)$$

acquisition time

single sided PSD

$$\frac{2}{T} X_T(f)\, X_T^*(f), \quad f>0$$

energy conservation

End of lecture

#4

#5 Thursday, October 1, 2020

1.5 Hours

DFT, FFT, FFTW, SFFT

The Discrete Fourier Transform (DFT) approximates the (continuous) FT

$$X\left(\frac{n}{NT_s}\right) = \sum_{k=0}^{N-1} x(kT_s)\, e^{-\imath 2\pi nk/N}$$

Basic DFT

T_s = sampling interval, $f_S = 1/T_s$

$n = 0 \ldots N-1$ integer frequency, $f = n/NT$

N

$\sim N^2$

$N \log_2 N$

- The direct computation of the DFT takes ≈ N^2 multiplications
- The FFT is an algorithm for Fast computation of the DFT that takes ≈ N log(N) multiplications
- The FFTW, “the Fastest Fourier Transform in the West,” is an even faster.
  N log(N) multiplications (M. Frigo, S.G. Johnson, MIT)
  See http://fftw.org/
- SFFT “faster-than-fast” Sparse (FFT, D.Katabi, P.Indyk, MIT)
  See http://groups.csail.mit.edu/netmit/sFFT/
- For the general user (does not implement FT algorithms)

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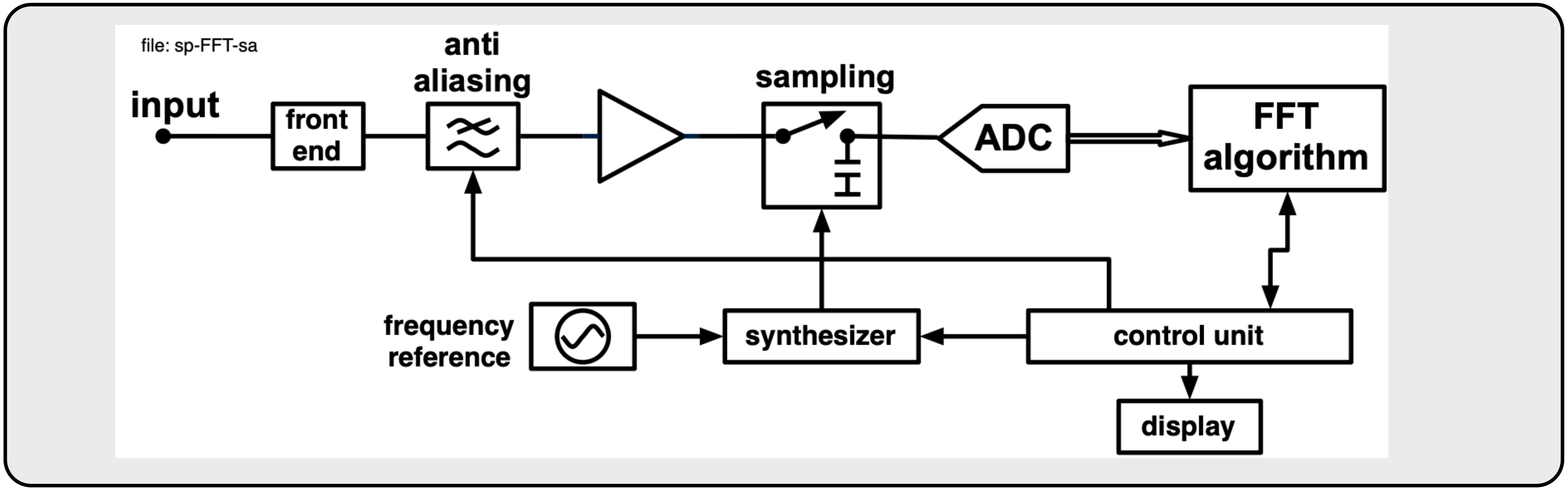
FFT Spectrum Analyzer



- Direct digitization of the input signal
- Fully digital process
- Limited to $f_{\text{max}} \approx 0.4 \times f_{\text{sampling}}$
- Tough tradeoff between resolution and max frequency

FT, DFT and PSD of Periodic Signals

FT contains Dirac delta(t) —> $|X(f)|^2$ does not exist

DFT $|X(f)|^2$ is always a valid quantity —> no problem

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-i 2\pi f t}\, dt$$

$$\int_{-\infty}^{\infty} \cos w_1 t \cos w_2 t \, dt = \underbrace{\ldots}_{\text{coef-}} \delta(t)$$

DFT No Dirac δ

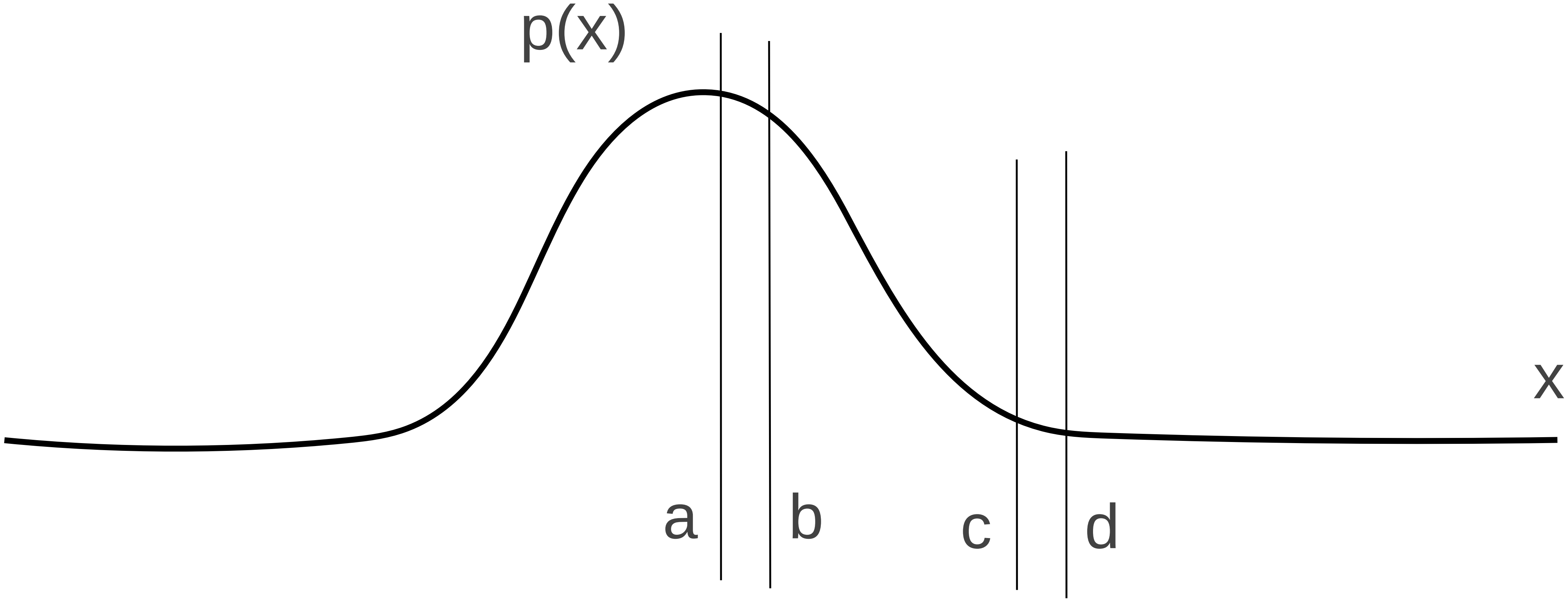
$$S_x(f) = \frac{2}{T} X(f) X^*(f)$$

FT ??

DFT $S_x = \frac{2}{T}$ --- (p. 97) Not problem

Power, Spectra and Probability

Probability Density Distribution



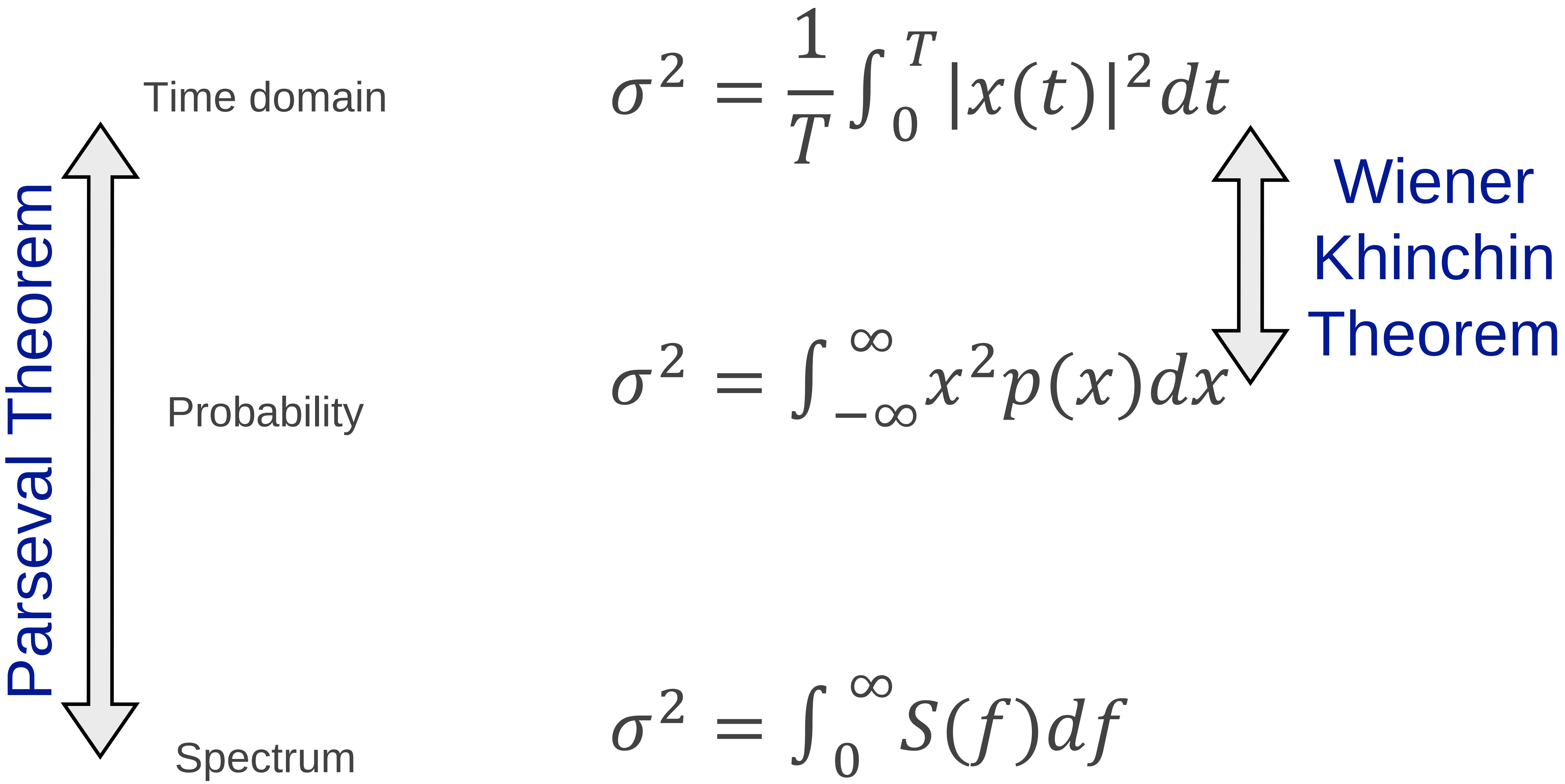
$$P_{ab} = \int_a^b p(x)dx$$

$$\int_{-\infty}^{\infty} p(x)dx = 1$$

$$P_{aborcd} = P_{ab} + P_{cd}$$

iff ab and cd are separated

Variance (signal power)



Parseval Theorem

Time domain

$$\sigma^2 = \frac{1}{T}\int_0^T |x(t)|^2 dt$$

Wiener Khinchin Theorem

Probability

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 p(x) dx$$

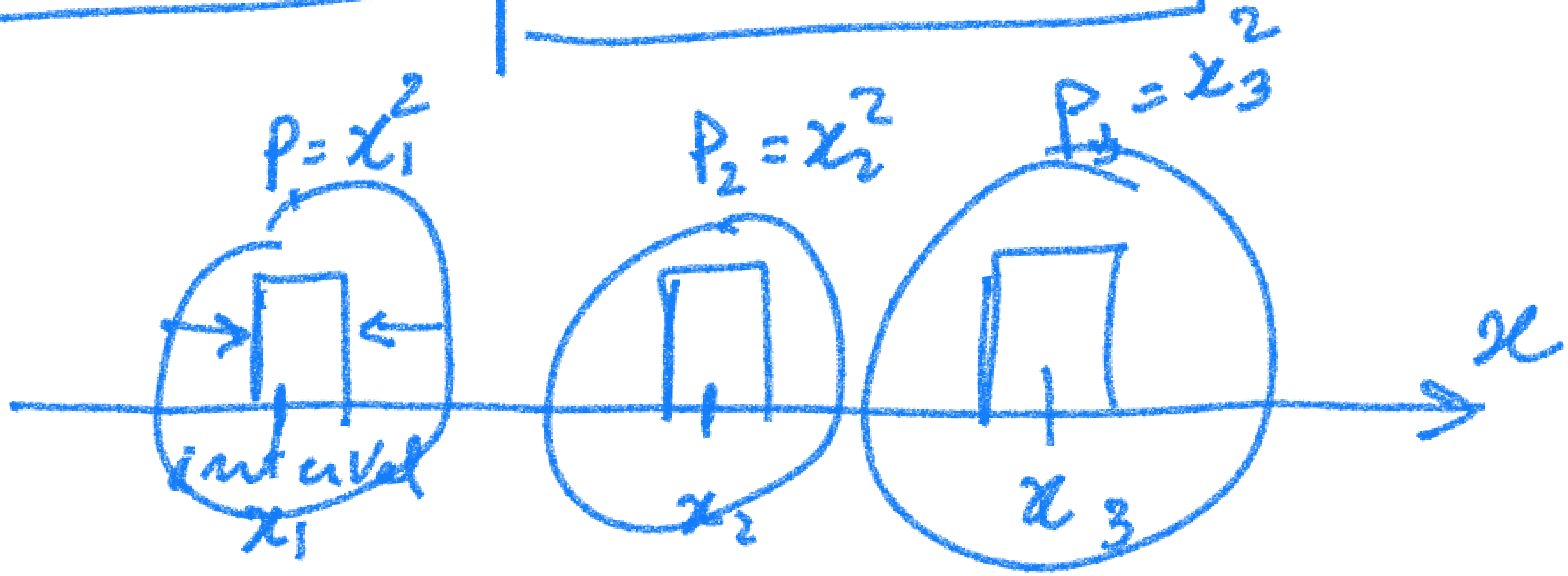
Spectrum

$$\sigma^2 = \int_0^{\infty} S(f) df$$

Why the Variance is Power

$\mathbb{P} \quad P \quad p$

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 p(x) dx$$

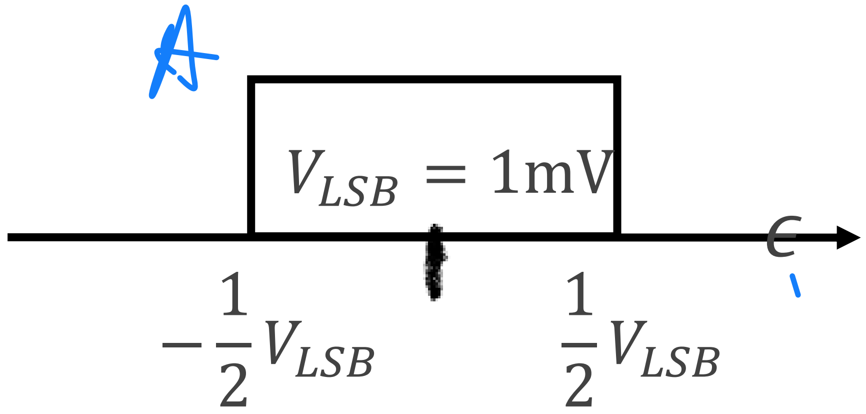


$\mathbb{P}(x \in x_1) = p(x_1) \rightarrow P_1$

$\mathbb{P}(x \in x_2) = p(x_2) \rightarrow P_2$

$\rightarrow P_3$

Example



Calculate

$$P\{320\mu V < \epsilon < 325\mu V\}$$

$$P = \int_a^b p(x)\,dx$$

b ← 325 μV

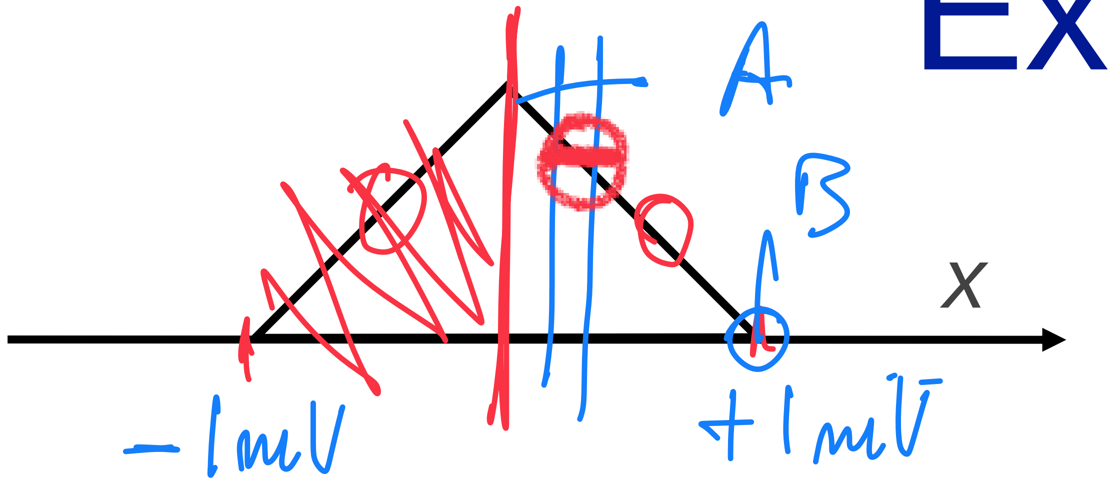
a = 320 μV

$$\boxed{= 0.005}$$

$$\int_{-\infty}^{\infty} p(x)\,dx = 1 \rightarrow \int_{-500\mu V}^{500\mu V} A = 1$$

$$A \times 1mV = 1 \qquad A = \frac{1000}{V}$$

Example



Calculate

$P\{320\mu V < x < 325\mu V\}$

$S = \frac{1}{2} AB = \frac{1}{2}$ $AB = 1$ $A = \frac{1}{1mV}.$ $A = \frac{1000}{V}$

$\boxed{y = A + Cx}$ $y = 0 \leftarrow x = B$

$A + CB = 0$ $CB = -A$ $C = \frac{-A}{B}$

$C = -\frac{1}{1mV} \cdot \frac{1}{1mV} = -\frac{10^6}{V}$

Gaussian (Normal) Distribution

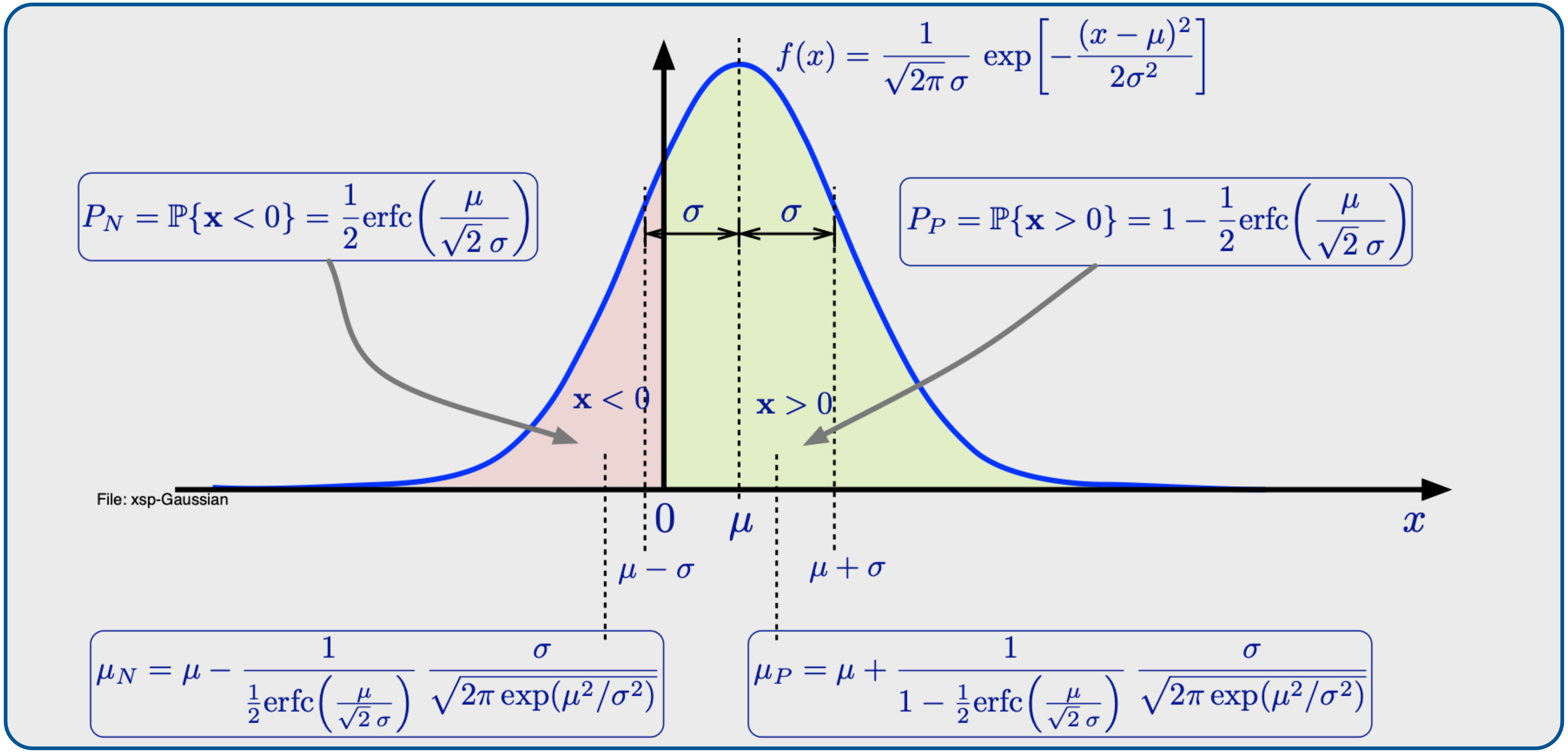
x is normal distributed with
zero mean μ and variance σ^2

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

$$\mathbb{E}\{f(x)\} = \mu$$

$$\mathbb{E}\{f^2(x)\} = \mu^2 + \sigma^2$$

$$\mathbb{E}\{|f(x) - \mathbb{E}\{f(x)\}|^2\} = \sigma^2$$



$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

$$P_N = \mathbb{P}\{\mathbf{x} < 0\} = \frac{1}{2}\mathrm{erfc}\left(\frac{\mu}{\sqrt{2}\,\sigma}\right)$$

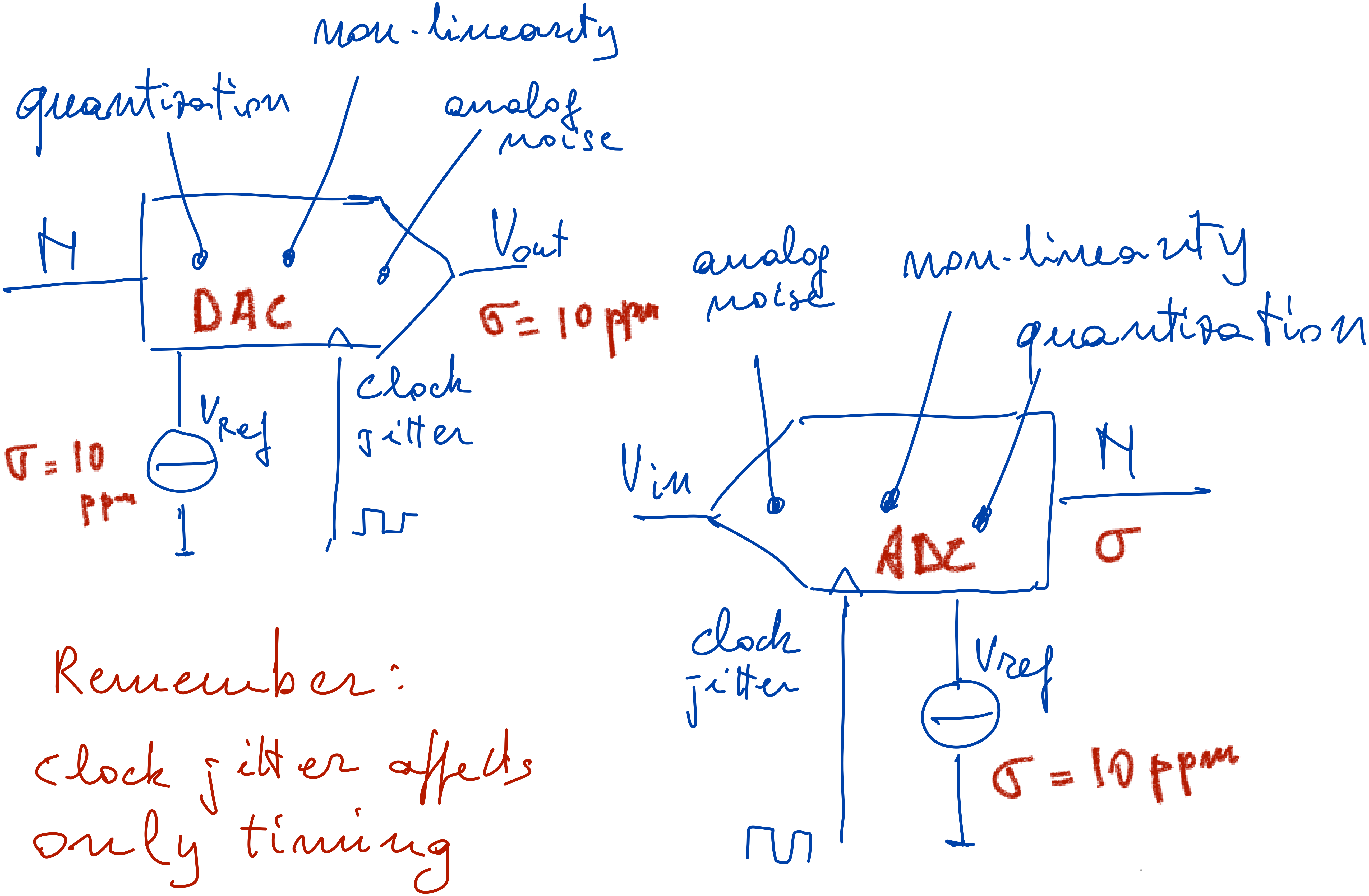
$$P_P = \mathbb{P}\{\mathbf{x} > 0\} = 1 - \frac{1}{2}\mathrm{erfc}\left(\frac{\mu}{\sqrt{2}\,\sigma}\right)$$

$$\mu_N = \mu - \frac{1}{\frac{1}{2}\mathrm{erfc}\left(\frac{\mu}{\sqrt{2}\,\sigma}\right)}\,\frac{\sigma}{\sqrt{2\pi\exp(\mu^2/\sigma^2)}}$$

$$\mu_P = \mu + \frac{1}{1 - \frac{1}{2}\mathrm{erfc}\left(\frac{\mu}{\sqrt{2}\,\sigma}\right)}\,\frac{\sigma}{\sqrt{2\pi\exp(\mu^2/\sigma^2)}}$$

Noise

Accuracy and Noise — The Big Picture



Remember:

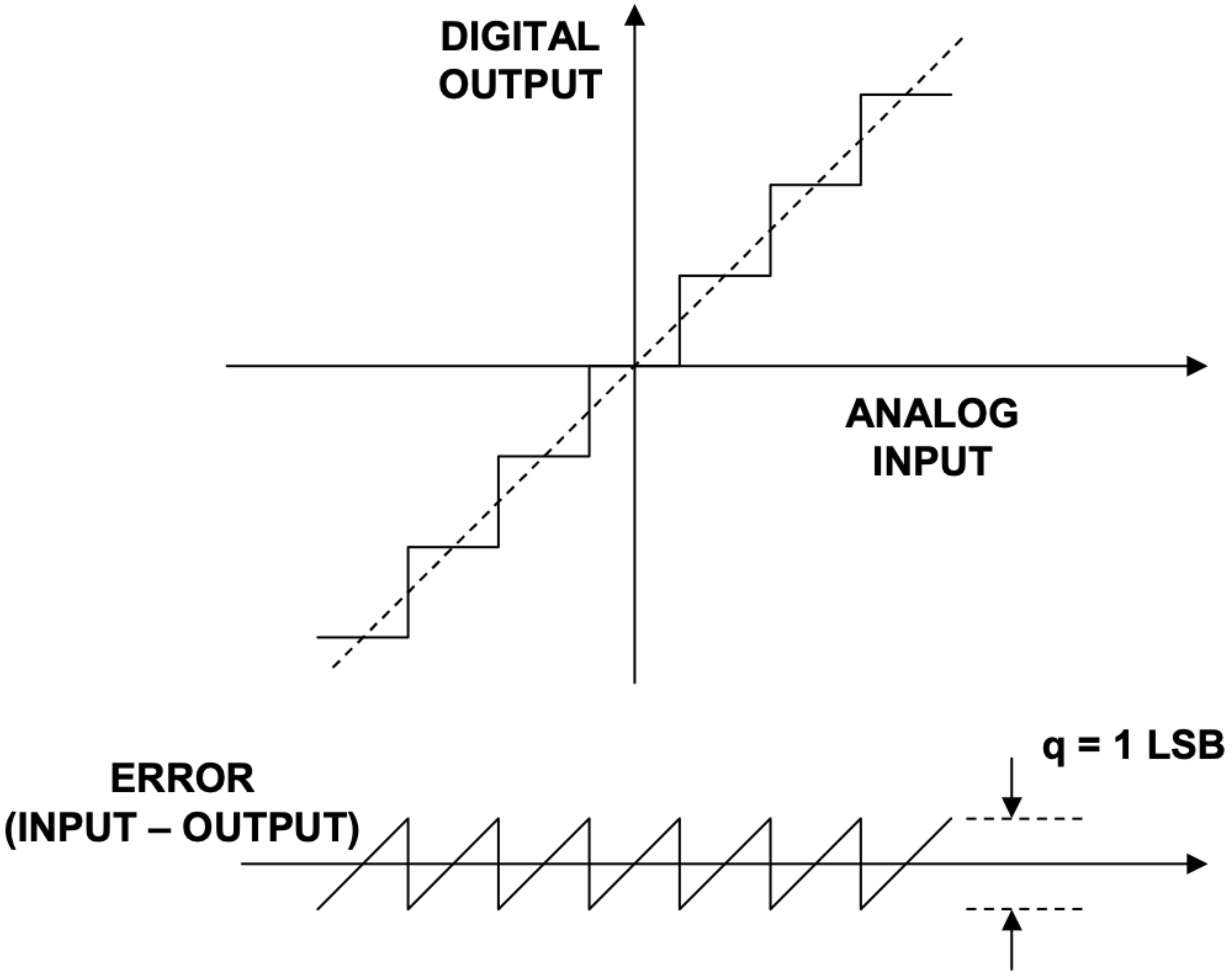
clock jitter affects only timing

End of Lecture #5

#6 Tuesday, Sept 26, 2019

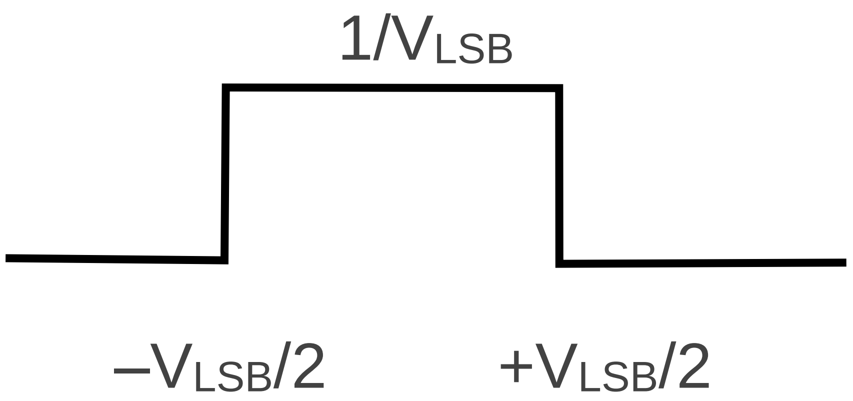
1.5 Hours

Quantization



W. Kester, Fig 2.37

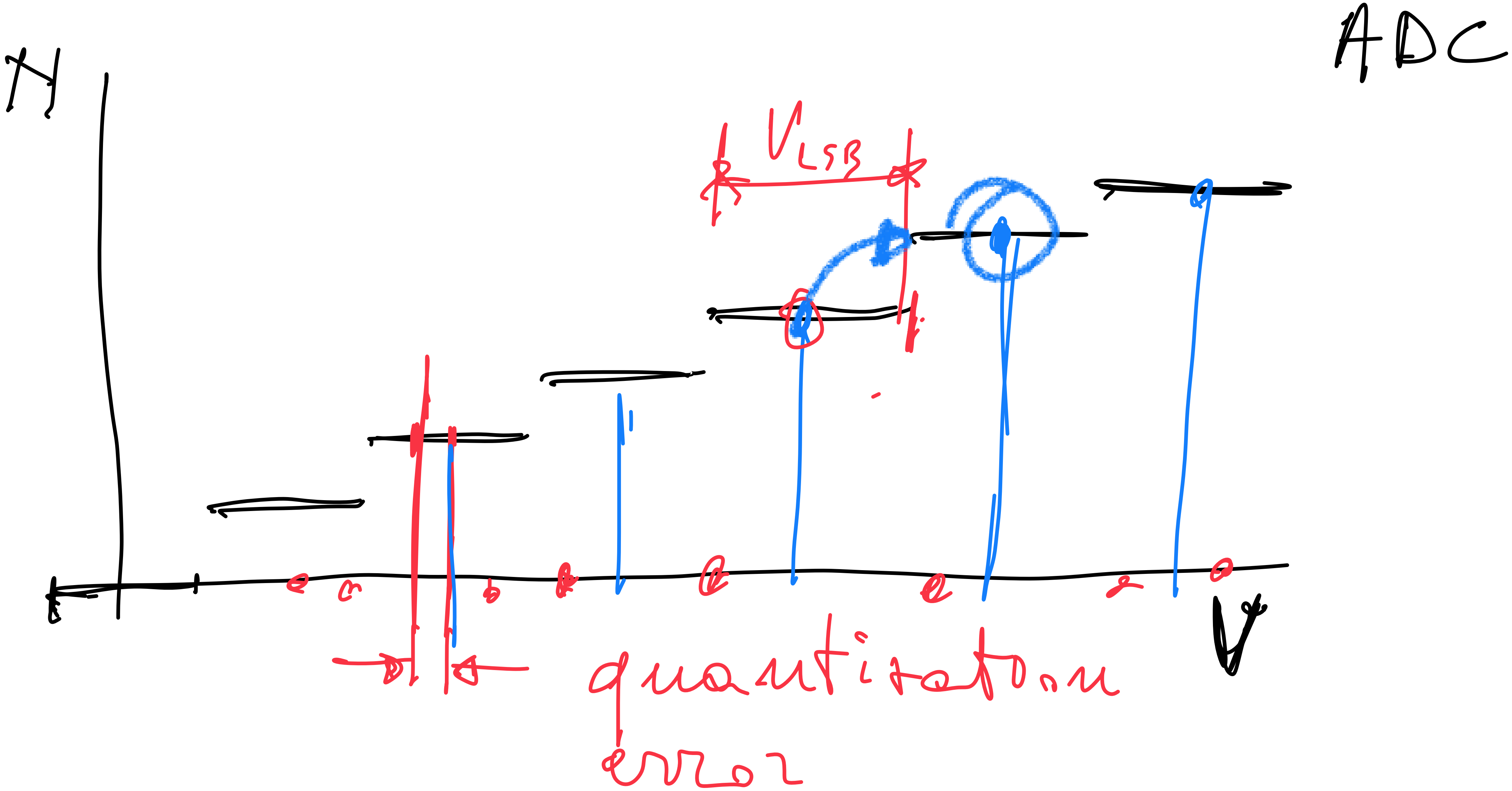
Probability density



$$\sigma^2 = \frac{1}{12} V_{LSB}^2$$

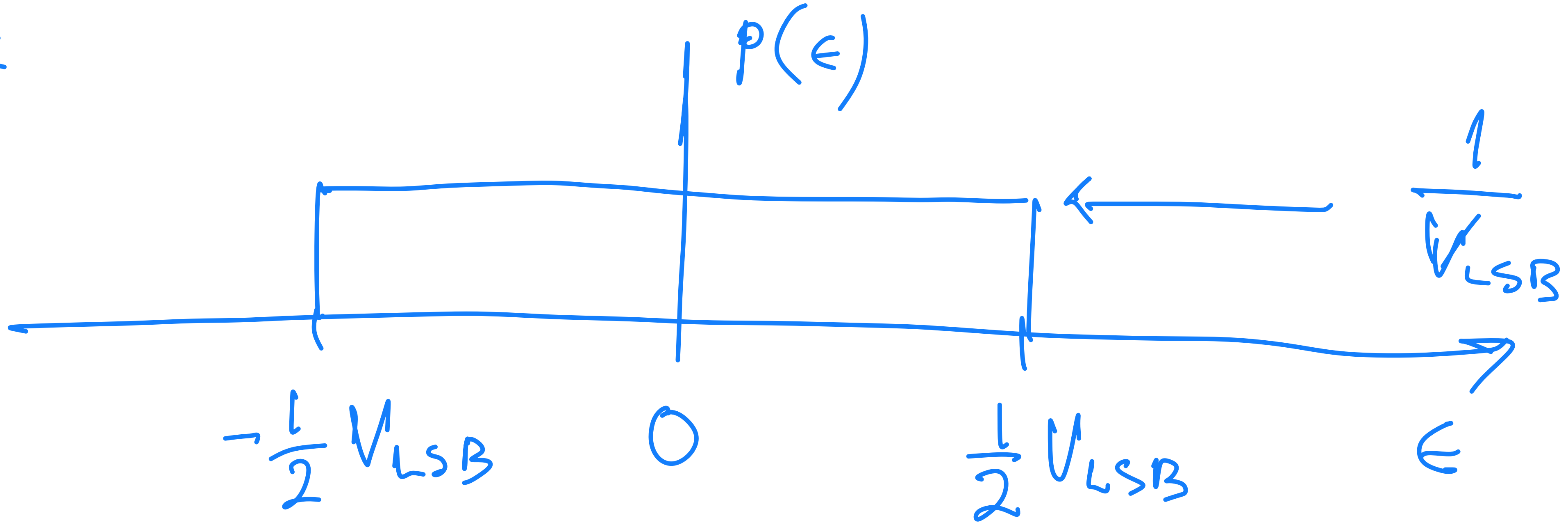
W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Quantization Noise, PDF



$$-\frac{1}{2}V_{LSB} < \epsilon < \frac{1}{2}V_{LSB}$$

PDF



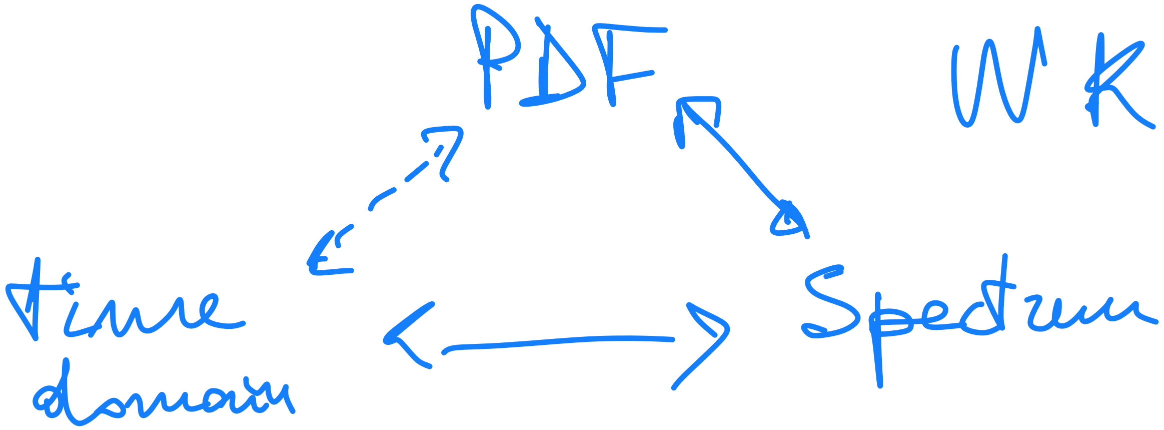
Remember

$$\int_{-\infty}^{\infty} p(x)\, dx = 1$$

Power

σ^2

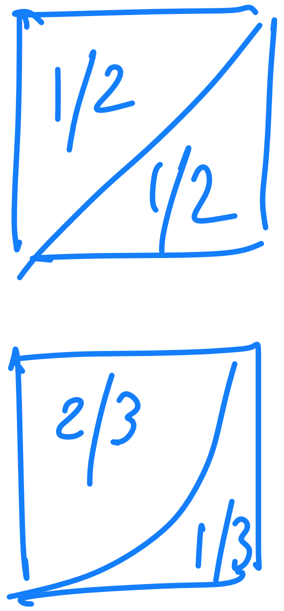
WK



Quantization Noise <---> Power

$$P = \sigma_q^2 = \int_{-\infty}^{\infty} x^2 \, p(x) \, dx$$

quantization



$$\sigma_q^2 = \int_{\cancel{-V_{LSB}/2}\;0}^{+V_{LSB}/2} 2\times \, x^2 \, \frac{1}{V_{LSB}} \, dx$$

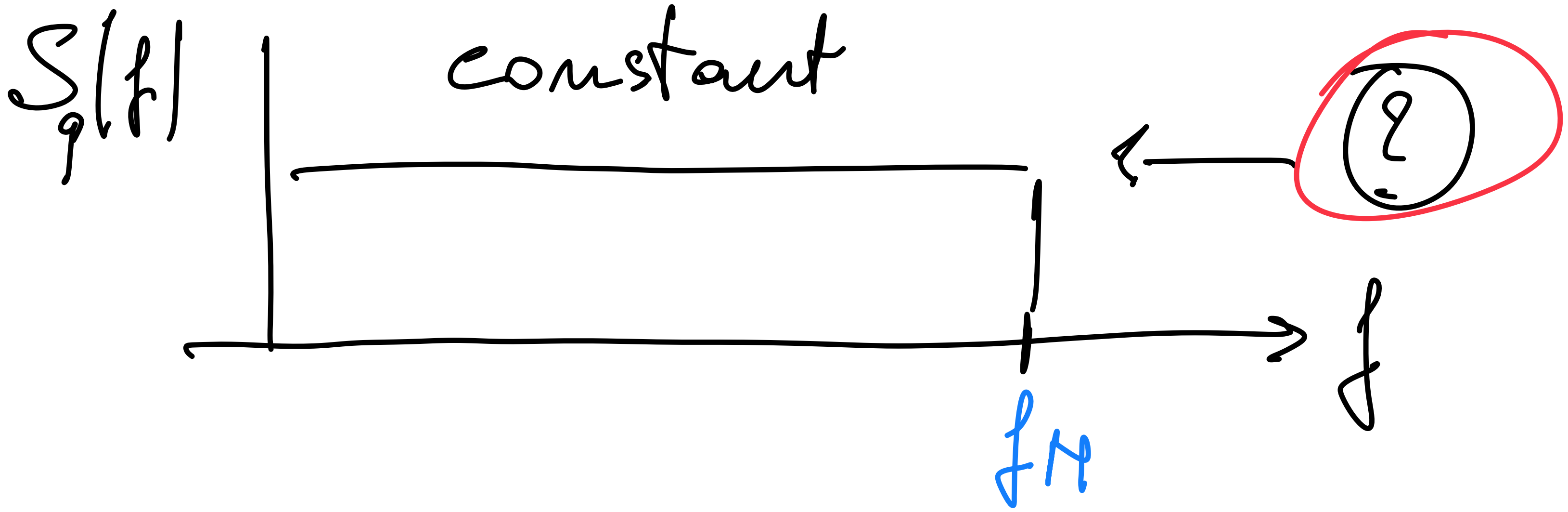
$$\frac{2}{V_{LSB}} \quad \frac{1}{3} x^3 \Big|_0^{V_{LSB}/2}$$

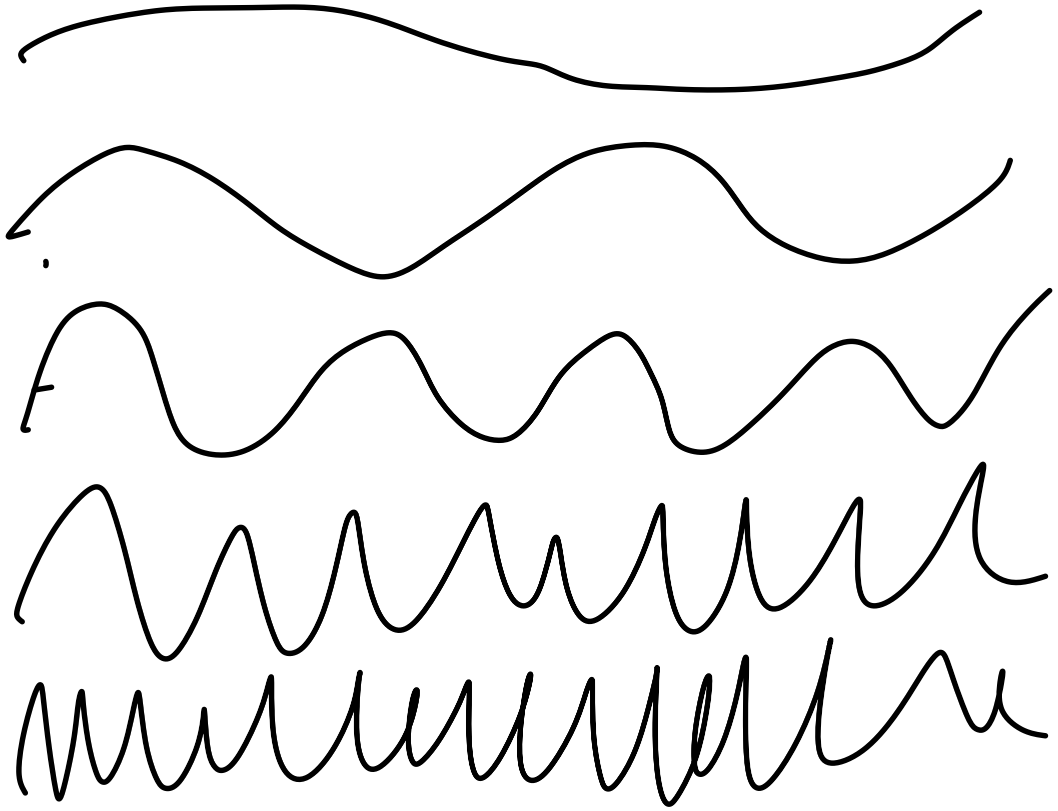
$$\boxed{\sigma_q^2 = \frac{1}{12} V_{LSB}^2}$$

$$V_{LSB} = \frac{V_{FSR}}{2^n}$$

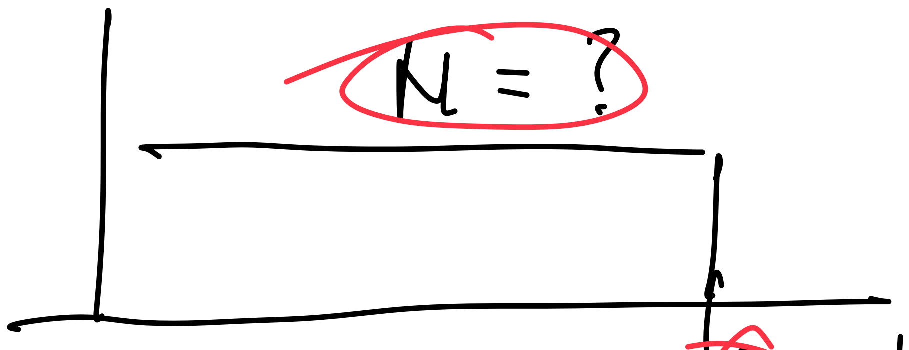
$$\boxed{\sigma_q^2 = \frac{1}{12} \; \frac{V_{FSR}^2}{2^{2n}}}$$

Quantization Noise, Spectrum





$$f_M = \frac{1}{2} f_s$$



$f_N = \frac{1}{2} f_s$

$$N \cdot f_N = \sigma_q^2 = \int_0^{\infty} S_q(f)\,df \quad \text{Parseval}$$

$$\frac{V_{LSB}^2}{12} = \sigma_q^2 = \int_{-\infty}^{\infty} x^2 p(x)\,dx \quad \text{WK}$$

$$N f_N = \frac{V_{LSB}^2}{12}$$

$$N = \frac{V_{LSB}^2}{12} \cdot \frac{1}{f_N} = \frac{V_{LSB}^2}{12} \frac{2}{f_s}$$

$$V_{LSB} = \frac{V_{FSR}}{2^M}$$

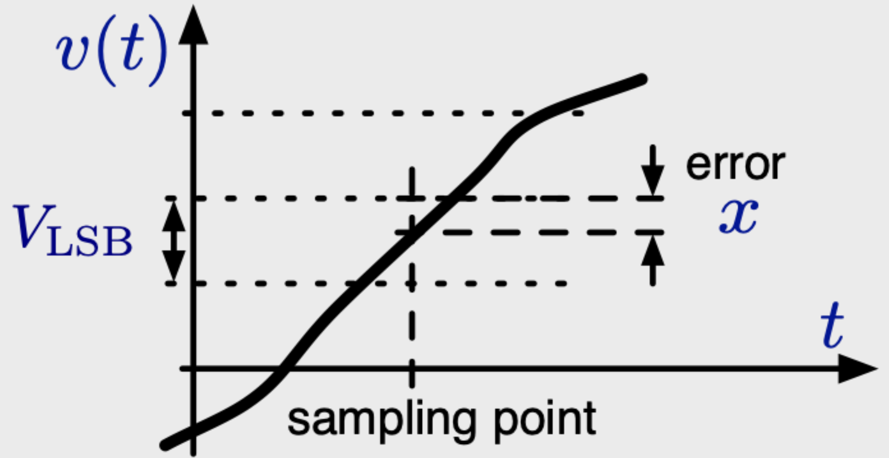
M = no of bits

$$N = \frac{1}{12} \frac{V_{FSR}^2}{2^{2M}} \cdot \frac{2}{f_s}$$

$$\boxed{\text{White Noise} \quad N = \frac{1}{6 f_s} \frac{V_{FSR}^2}{2^{2M}}}$$

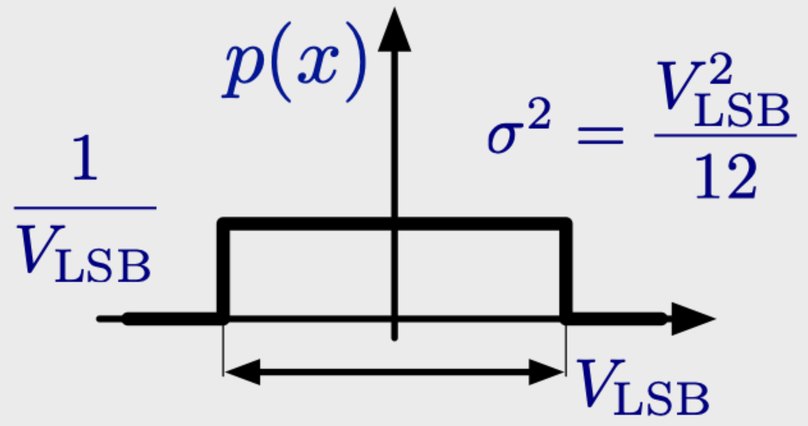
Quantization Noise

W. R. Bennett, Spectra of quantized signals, Bell System Tech J. 27(4), July 1948



Analog-to-digital conversion introduces a quantization error x [–$V_{LSB}/2 \leq x \leq +V_{LSB}/2$]

n-bit conversion: $V_{\mathrm{LSB}} = \dfrac{V_{\mathrm{FSR}}}{2^n}$



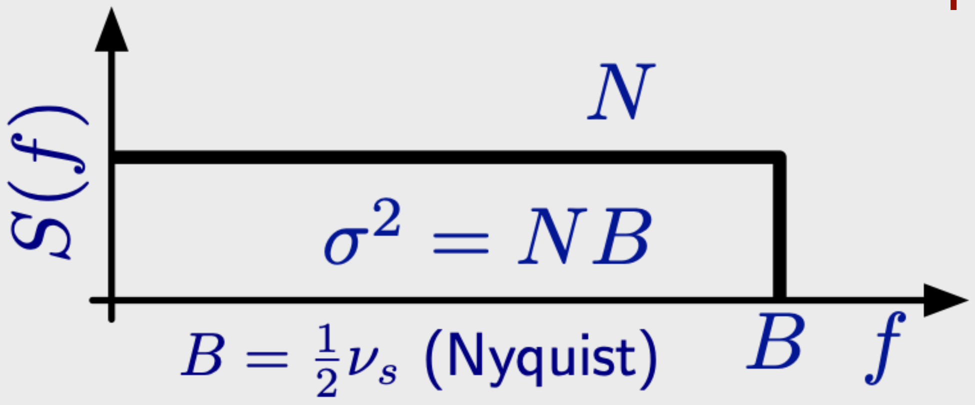
Wiener–Khintchine theorem: in ergodic systems, interchange time & ensemble

The noise can be calculated with statistics

$$\sigma^2 = \frac{V_{\mathrm{FSR}}^2}{12 \times 2^{2n}} \quad \mathrm{V}^2$$

1/12 –> –10.8 dB

2^{2n} –> 6 dB/bit



Parseval theorem: Energy (power) calculated in time and in frequency is the same

$$N = \frac{V_{\mathrm{FSR}}^2}{6 \times 2^{2n}\,\nu_s} \quad \mathrm{V}^2/\mathrm{Hz}$$

f_s

The Formula SNR = 6.02 n + 1.76 dB

~~Signal to Noise Ratio~~ ~~SNR~~

Signal to Quantization Ratio SQR

$$\hookrightarrow \sigma_q^2 = \frac{1}{12}\frac{V_{FSR}^2}{2^{2n}}$$

$$v(t) = \frac{V_{FSR}}{2}\cos(\omega t)$$

$$P(t) = \frac{V_{FSR}^2}{4R}\cos^2(\omega t)$$

signal

$$\sigma^2 = \frac{V_{FSR}^2}{8}$$

$$SQR = \frac{\cancel{V_{FSR}^2}/8}{\frac{1}{12}\frac{\cancel{V_{FSR}^2}}{2^{2n}}}$$

$$SQR = \frac{3}{2}2^{2n}$$

$$10\log_{10}\left(\frac{3}{2}\right) =$$

$$10\log_{10}\left(2^{2n}\right) =$$

The Formula SNR = 6.02 n + 1.76 dB

$$\mathrm{SNR} = \frac{P}{\sigma^2}$$

$$P = V_{\mathrm{pp}}^2/8R$$

$$\sigma^2 = V_{\mathrm{LSB}}^2/12R$$

Better called SQR
Signal to Quantization Ratio

$$\mathrm{SNR} = \frac{3}{2}\frac{V_{\mathrm{pp}}^2}{V_{\mathrm{LSB}}^2}$$

$$V_{\mathrm{pp}} \leq V_{\mathrm{FSR}}$$

$$V_{\mathrm{LSB}} = V_{\mathrm{FSR}}/2^n$$

Simplified

$$\mathrm{SNR} = \frac{3}{2}2^{2n}$$

$$10\log_{10}(3/2) = 1.76$$

$$10\log_{10}(2^{2n}) = 6.02n$$

$$6.02n + 1.76\mathrm{dB}$$

Full formula

$$\mathrm{SNR} = \frac{3}{2}2^{2n}\frac{V_{\mathrm{pp}}^2}{V_{\mathrm{FSR}}^2}$$

$$6.02n + 1.76 + 20\log_{10}\frac{V_{\mathrm{pp}}}{V_{\mathrm{FSR}}}\mathrm{dB}$$

End of lecture

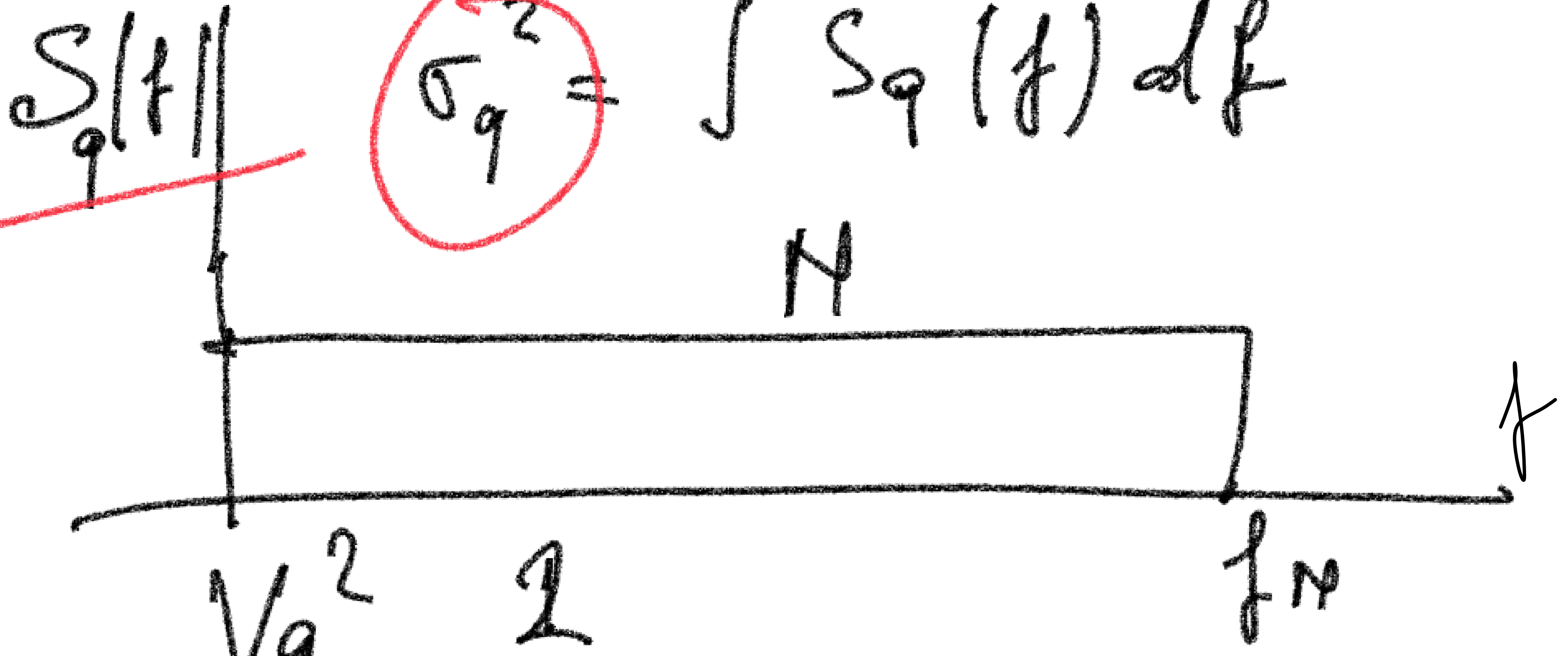
6

#7 Thursday, October 8, 2020

1.5 Hours

How To Get Lower f_s (Wrong)

$$\sigma_q^2 = \frac{V_q^2}{12}$$



$$\sigma_q^2 = \int S_q(f)\,df$$

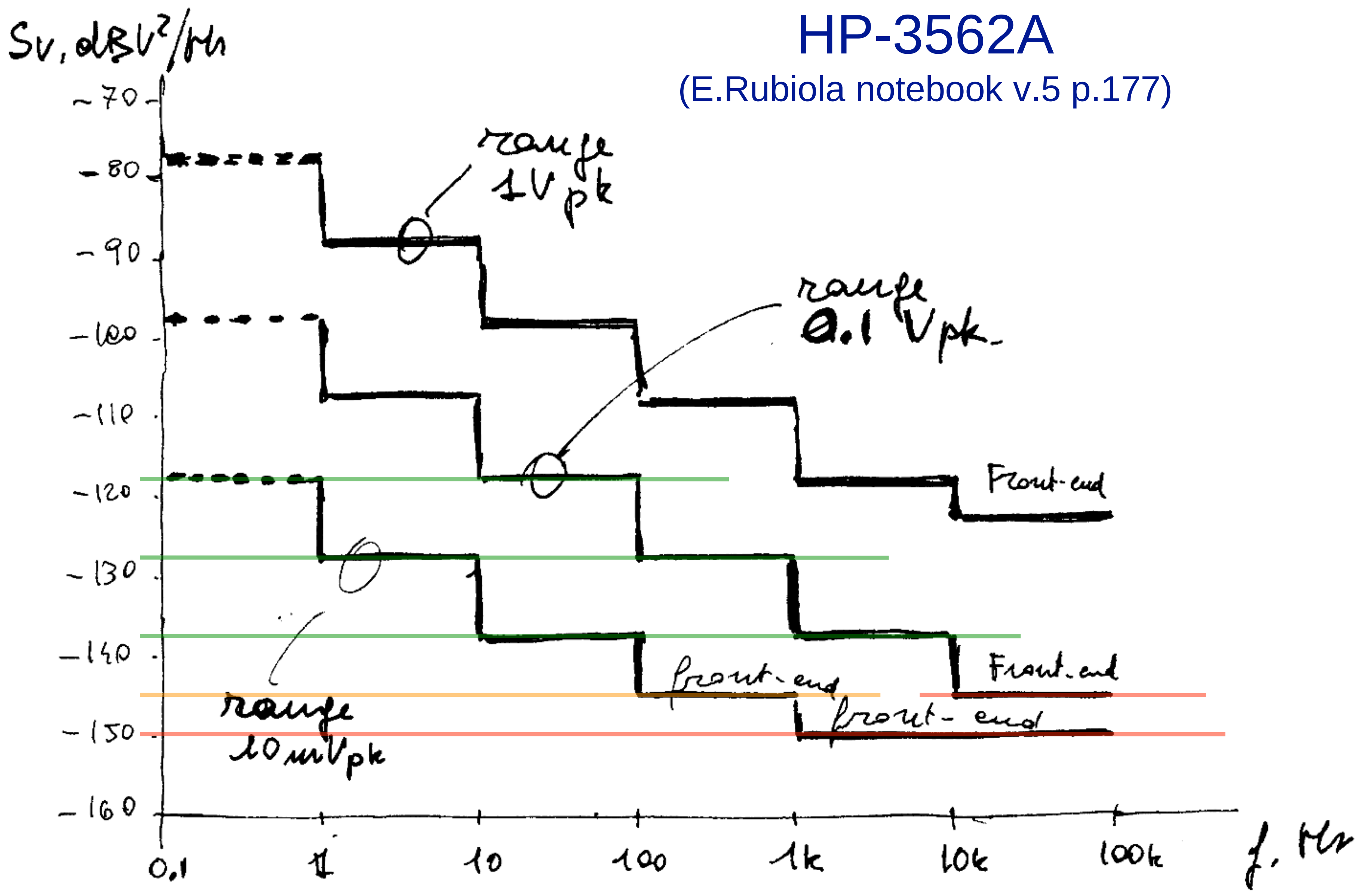
$$N f_N = \frac{V_q^2}{12}$$

$$N = \frac{V_q^2}{12}\,\frac{1}{f_s}$$ sampling

FFT

Example of FFT Analyzer Noise

Experimental observation


HP-3562A
(E.Rubiola notebook v.5 p.177)

Theoretical evaluation

DAC 12 bit resolution, including sign

range 10 $\mathrm{mV_{peak}}$
V_{fsr} = 20 mV (±10 mV)

resolution
$V_q = V_{fsr} / 2^{12}$
$= 4.88\ \mu\mathrm{V}$

total noise
$\sigma^2 = (4.88\ \mu\mathrm{V})^2 / 12$
$= 2{\times}10^{-12}\ \mathrm{V}^2$ (–117 dB)

quantization noise PSD
$S_v = \sigma^2 / B$
= –117 $\mathrm{dBV^2/Hz}$ with B = 1 Hz (etc.)

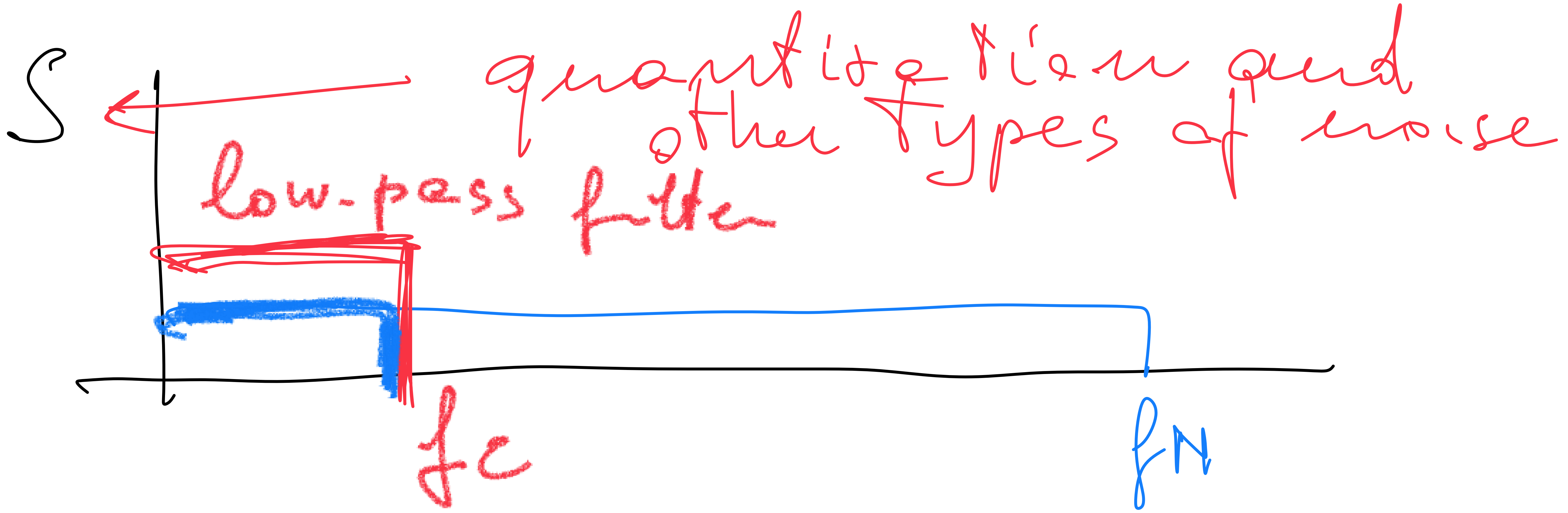
Front-end noise, evaluated from the plot

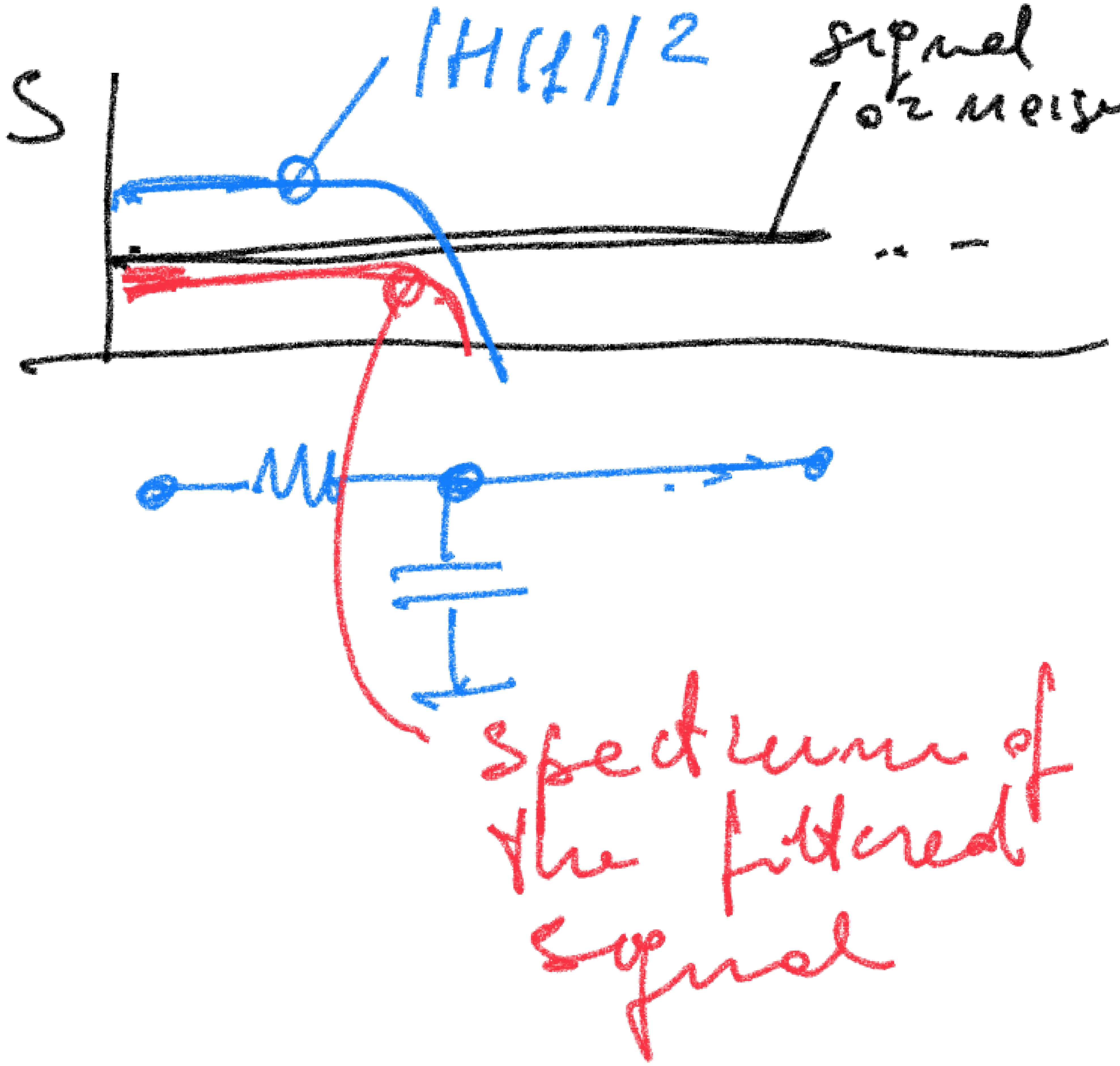
$S_v = 2{\times}10^{-15}\ \mathrm{V}^2$ (–150 dB), at 10–100 kHz
or 45 $\mathrm{nV/Hz^{1/2}}$

use Sv = 4kTR
R = 125 kΩ
or R = 100 kΩ and F = 1 dB (noise figure)

How To Get Lower f_s (Right)

FILTER



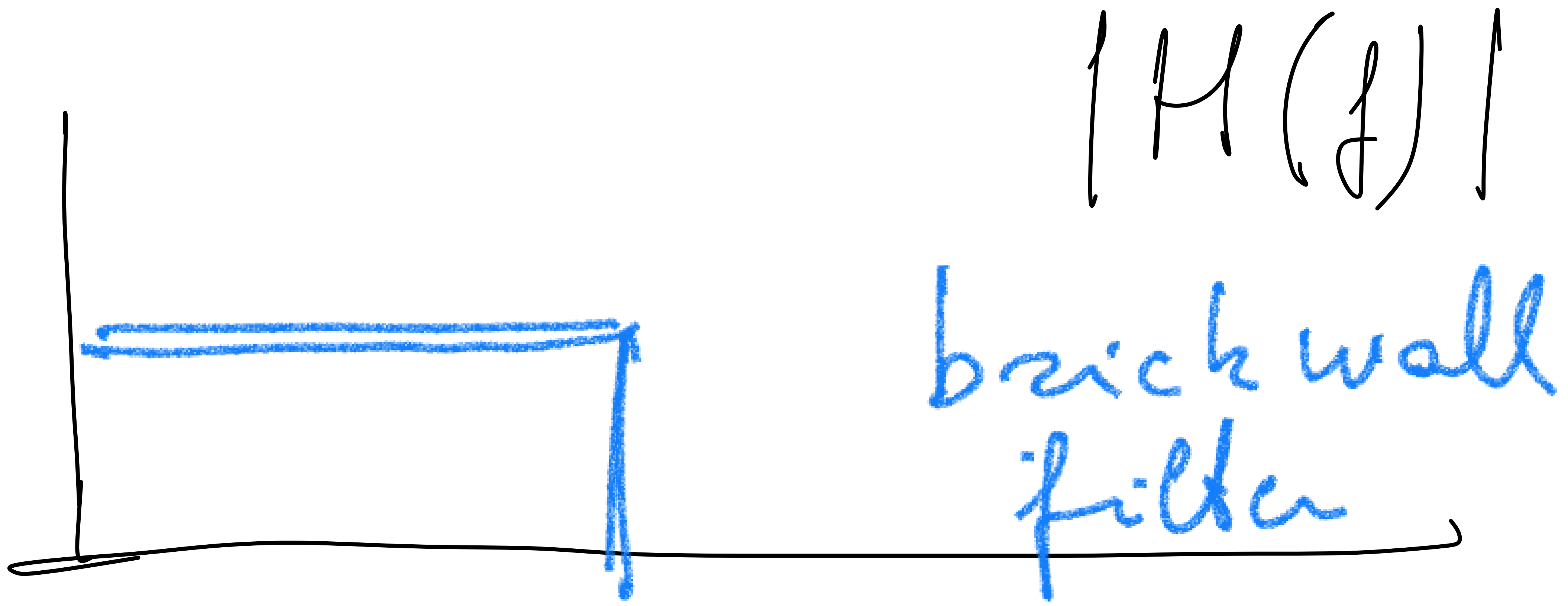


Frequency domain

$$S_{out}(f) = |H(f)|^2 S_{in}(f)$$

Time domain

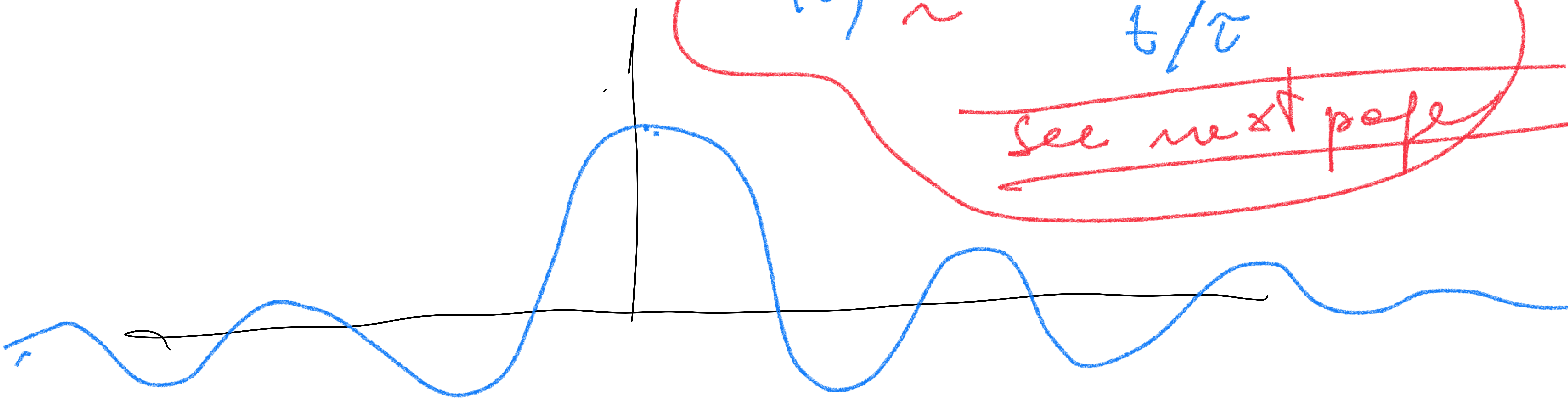
$$y(t) = h(t) * x(t)$$



check on the coefficient

$$h(t) \approx \frac{\sin(t/\tau)}{t/\tau}$$

see next page

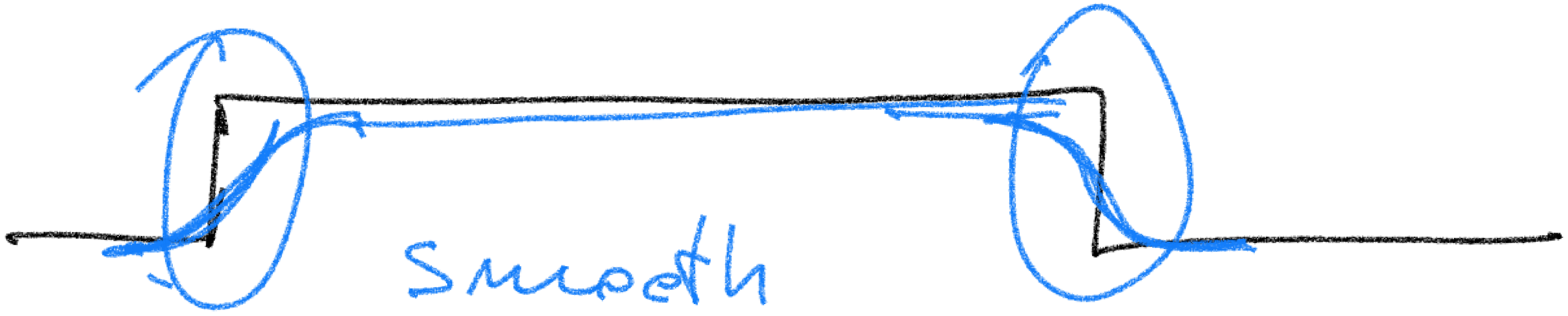


Convolution with appropriate $h(t)$

Digital $\int \dots \rightarrow \sum$

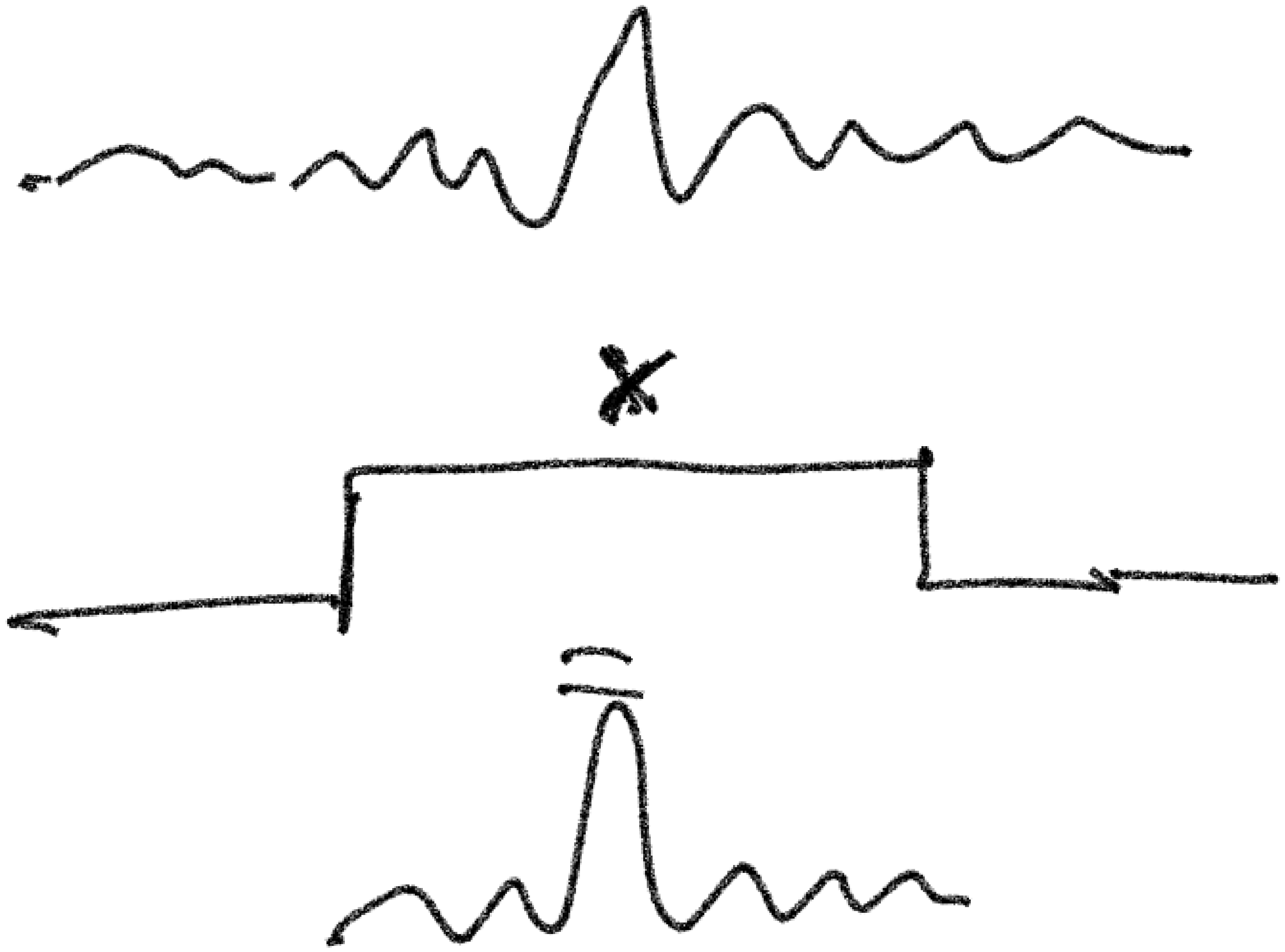
sinc → truncated sinc

truncation → multiply by 

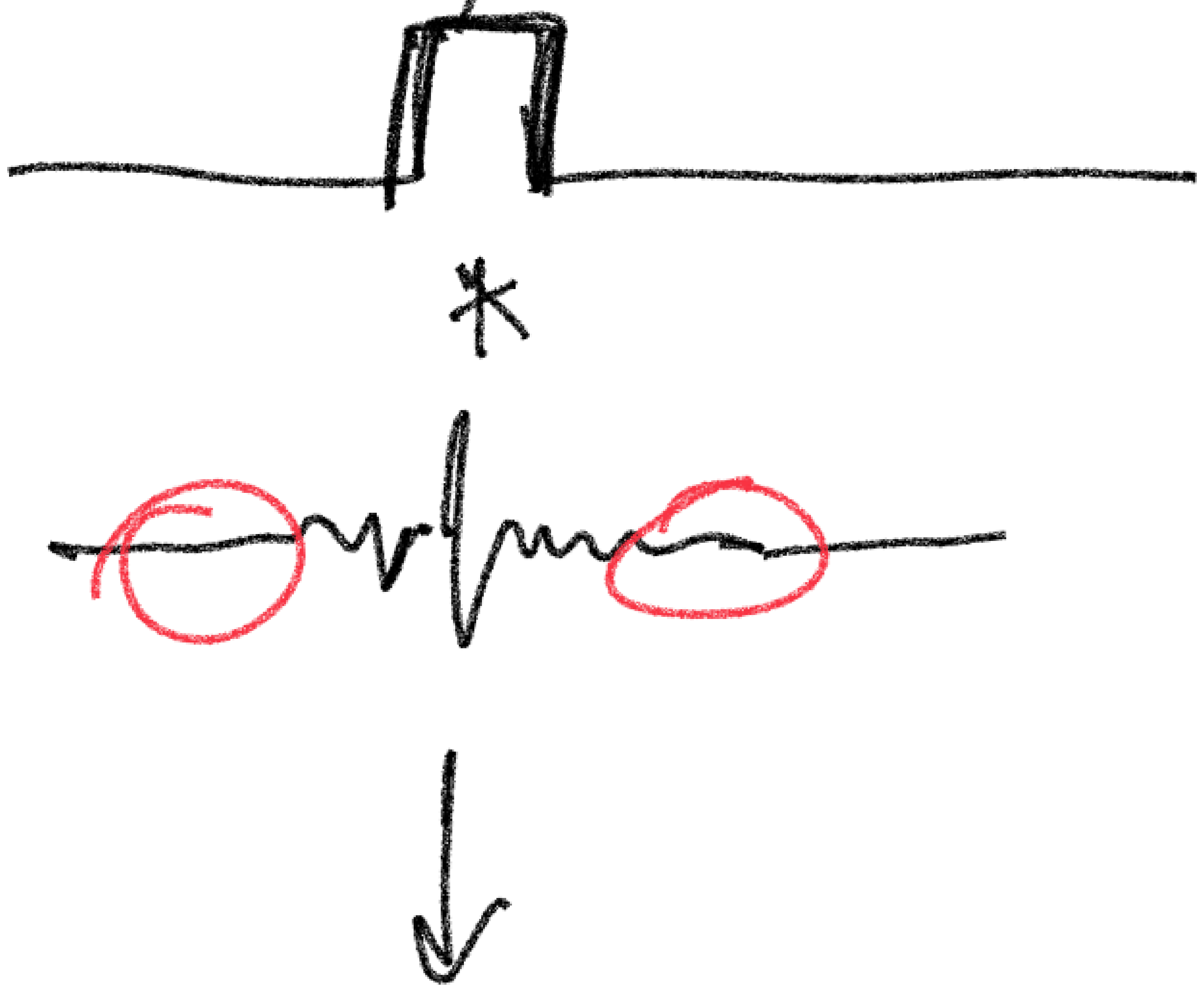


smooth edges

time



Spectra / freq



aliasing

Smooth edges

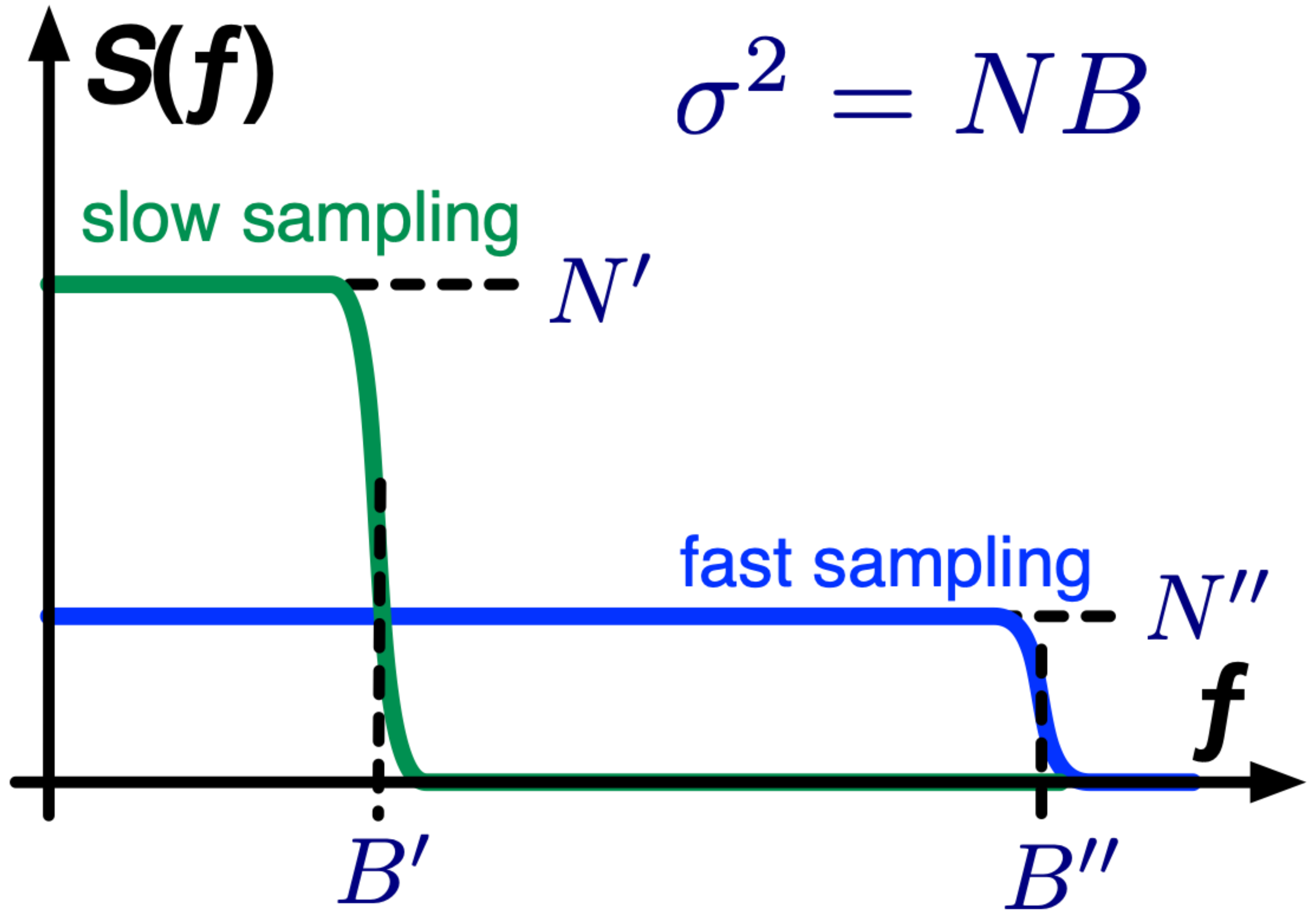
⇒ smaller energy in the side lobes

little aliasing

Digital Filter and Decimation

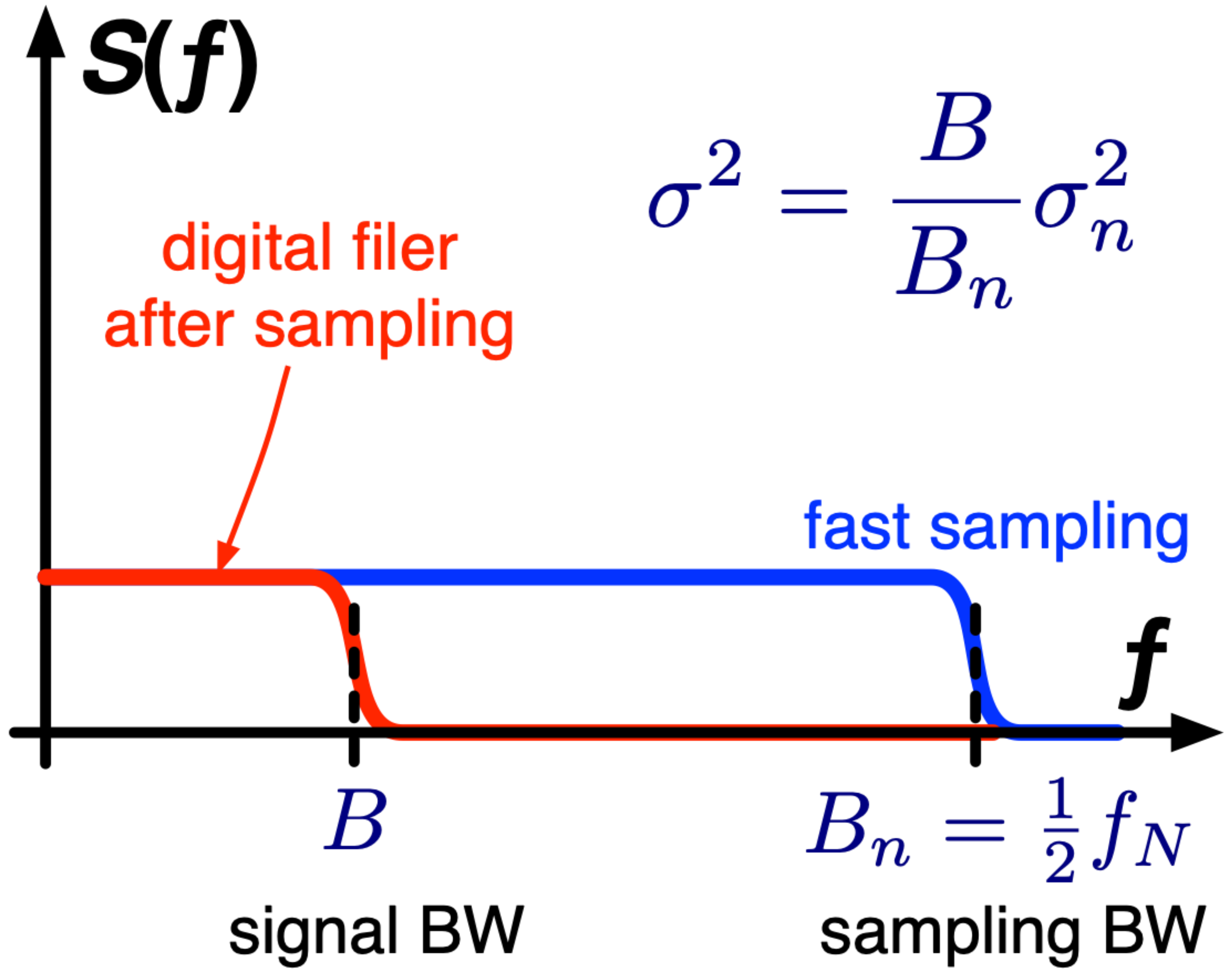
Noise, Sampling, and the Parseval theorem

$$\sigma^2 = \frac{V_{\mathrm{FSR}}^2}{12\times 2^{2m}}$$



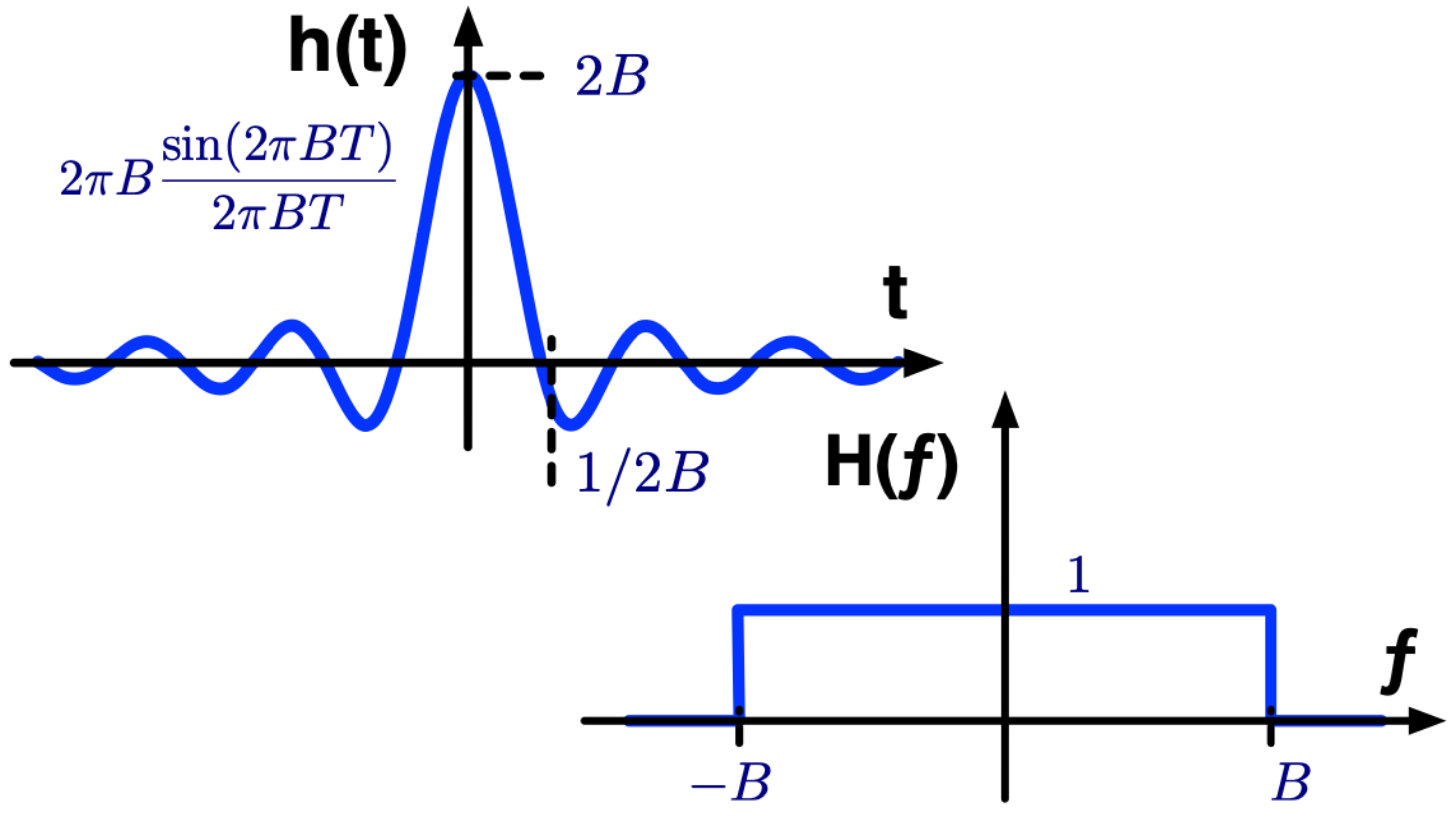
$$\sigma^2 = NB$$

Solution: Fast sampling, and filter

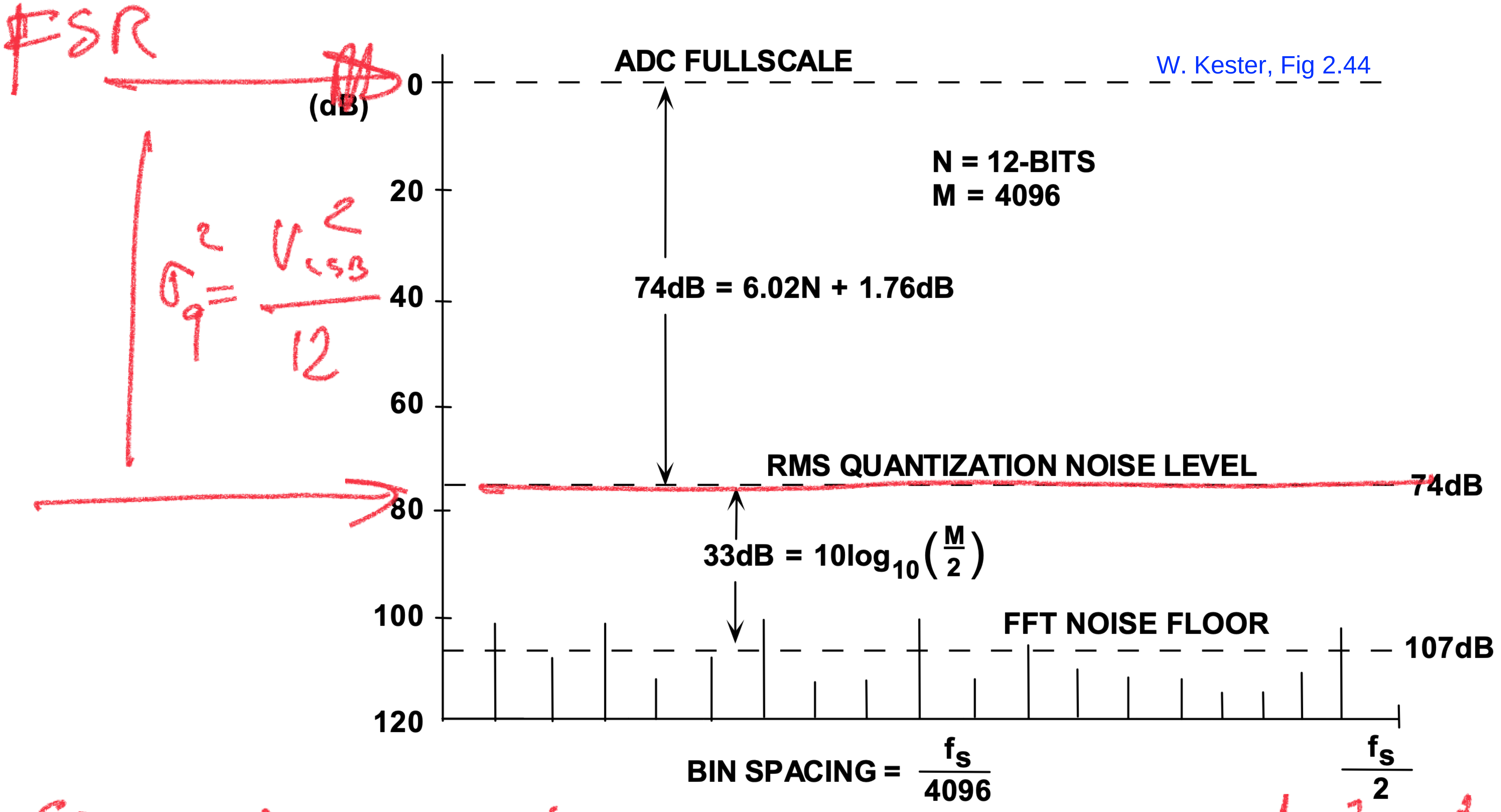


$$\sigma^2 = \frac{B}{B_n}\sigma_n^2$$

- Convolution with low-pass h(t)
- 127 coeff. Blackman-Harris kernel provides 70 dB stop-band attenuation
- Future: we will use >>127 coefficients
- Need more bits



Quantization



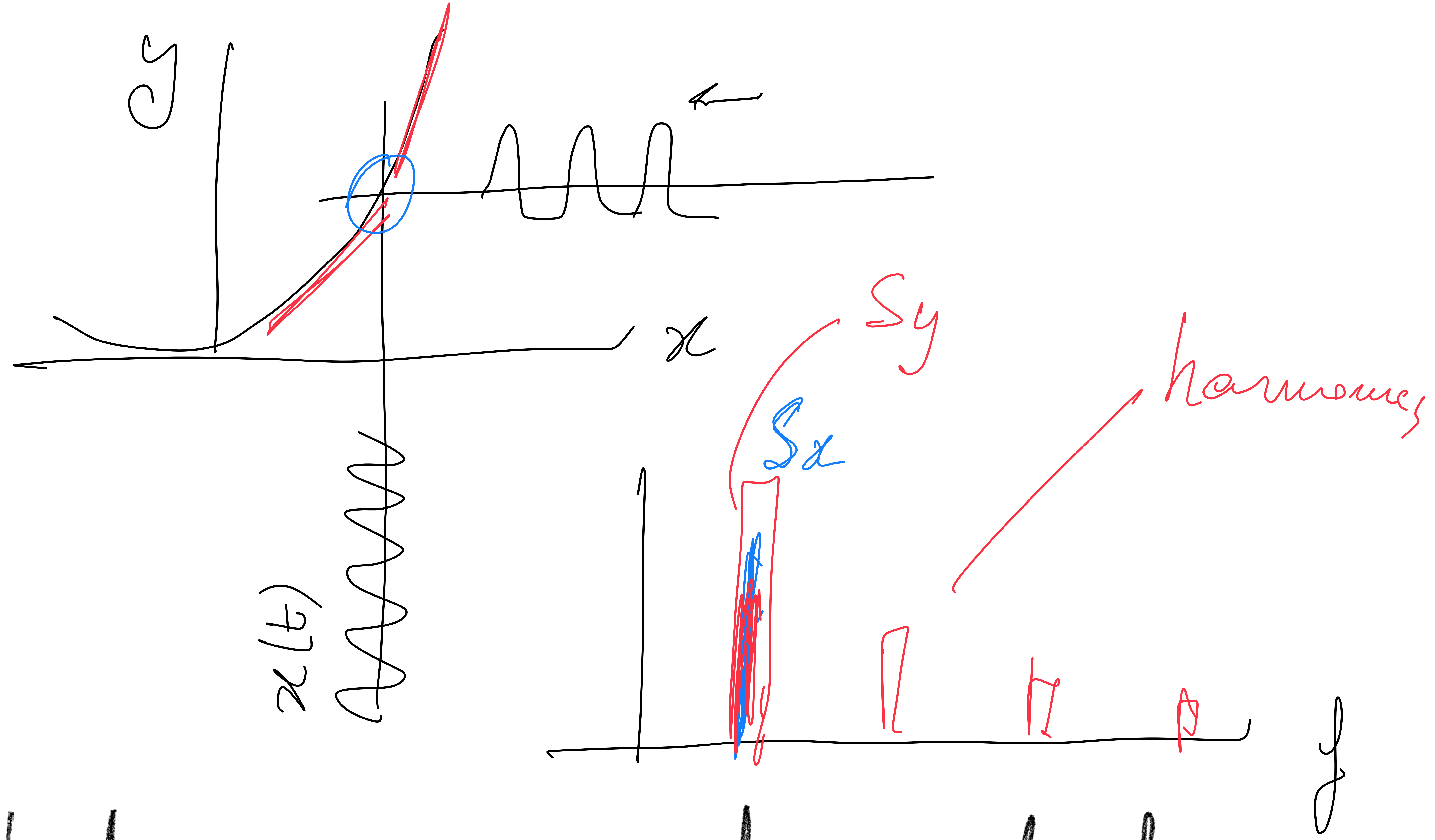
$$\sigma_q^2 = \frac{V_{LSB}^2}{12}$$

W. Kester, Fig 2.44

Signal processing → use more bits than the converter output

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

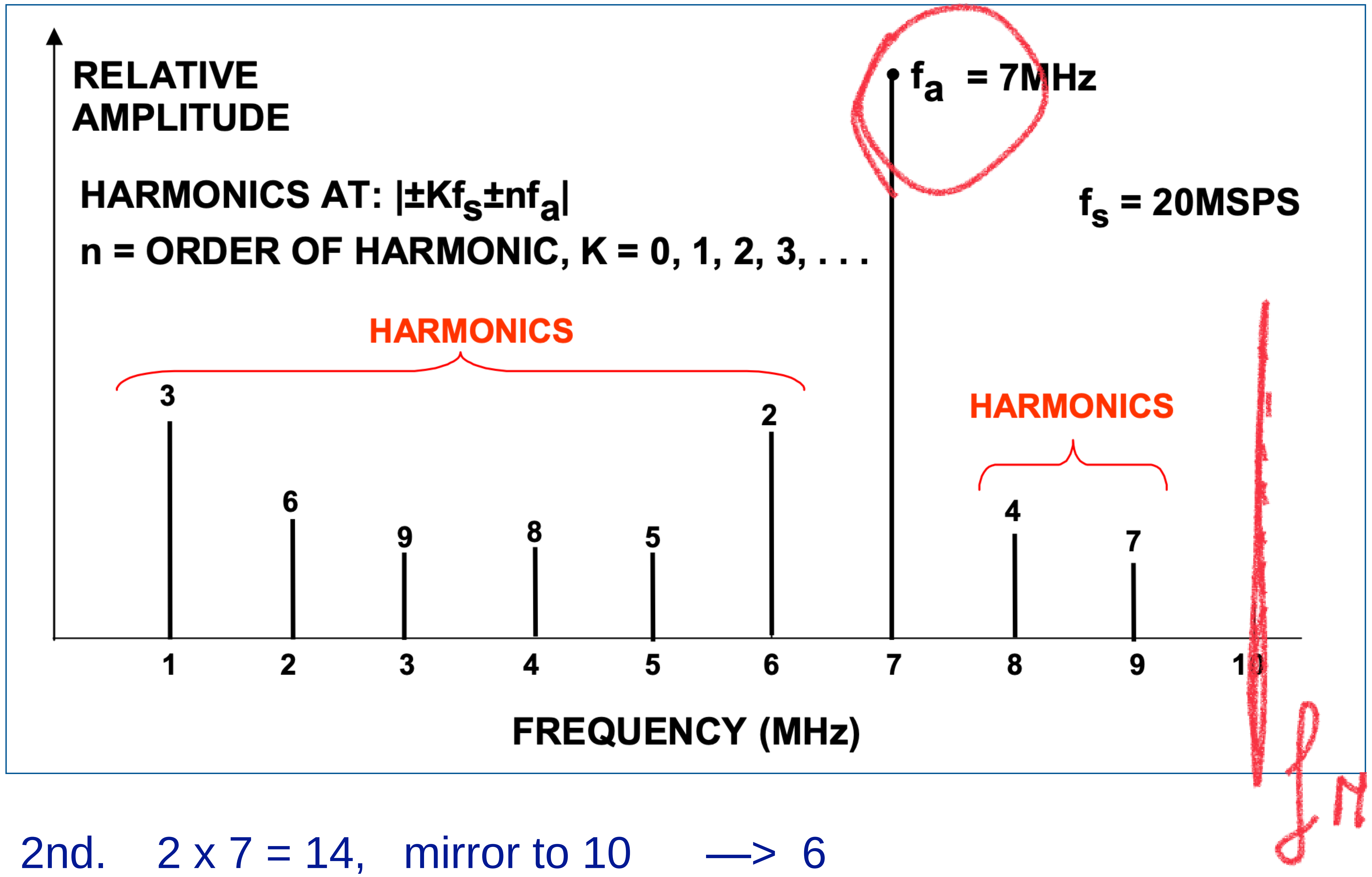
Distortion



What if the harmonics go beyond fM

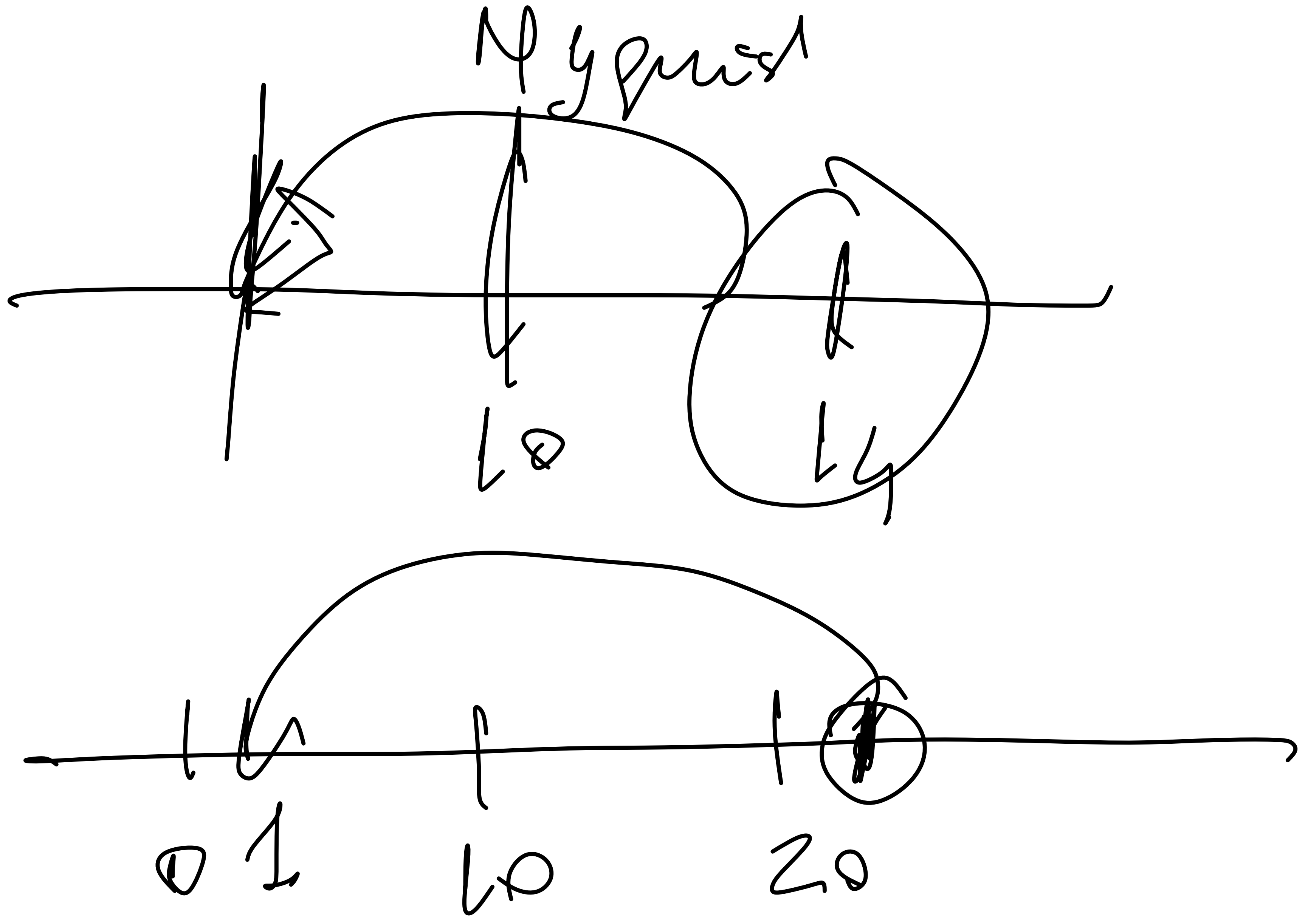
Distortion and Aliasing

W. Kester, Fig 2.44



Main signal
7 MHz sinusoid

Sampling f_s = 20 MHz
Nyquist f_N = 10 MHz

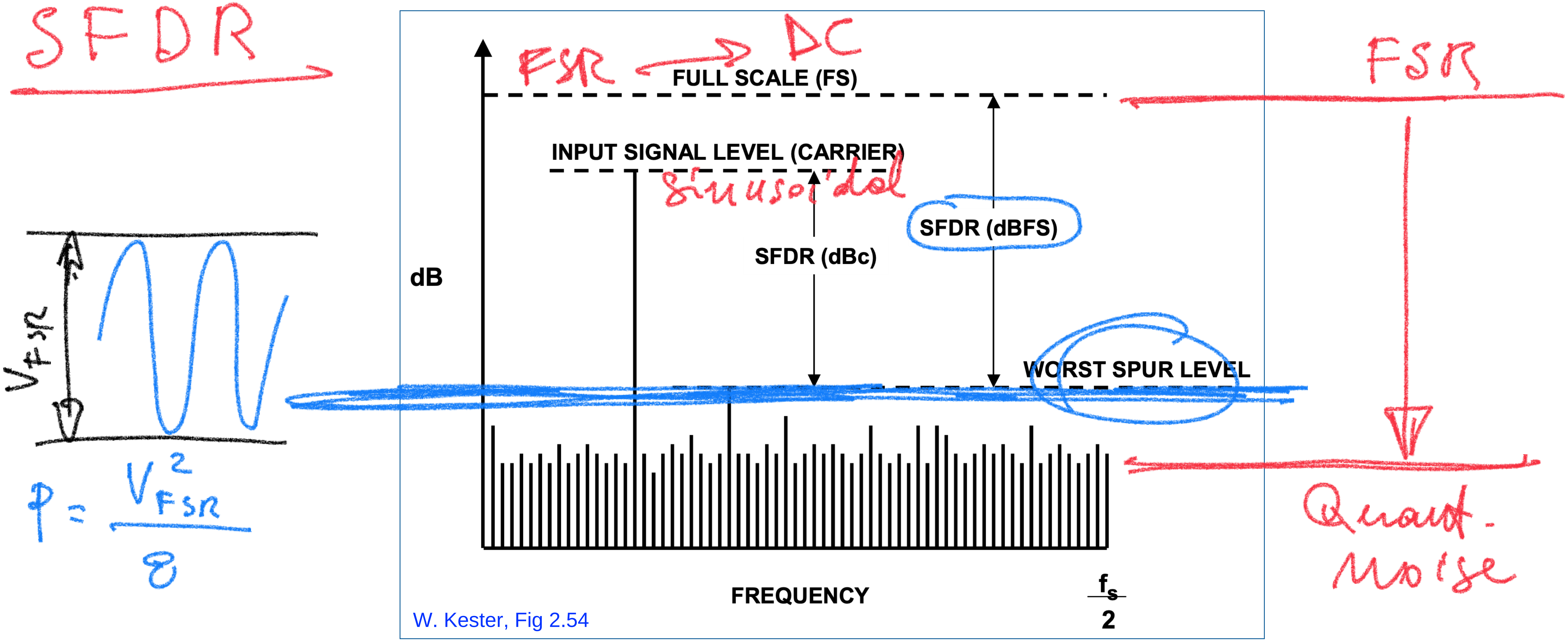


2nd. 2 x 7 = 14, mirror to 10 —> 6
3rd. 3 x 7 = 21, take away 20 —> 1
4th. 4 x 7 = 28, take away 20 —> 8
5th. 5 x 7 = 35, take away 20 —> 15, mirror to 10 —> 5
6th. 6 x 7 = 42, take away 40 —> 2
7th. 7 x 7 = 49, take away 40 —> 9
8th. 8 x 7 = 56, take away 40 —> 16, mirror to 10 —> 4

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Spurious-Free Dynamic Range

SFDR



W. Kester, Fig 2.54

$$P = \frac{V_{FSR}^2}{8}$$

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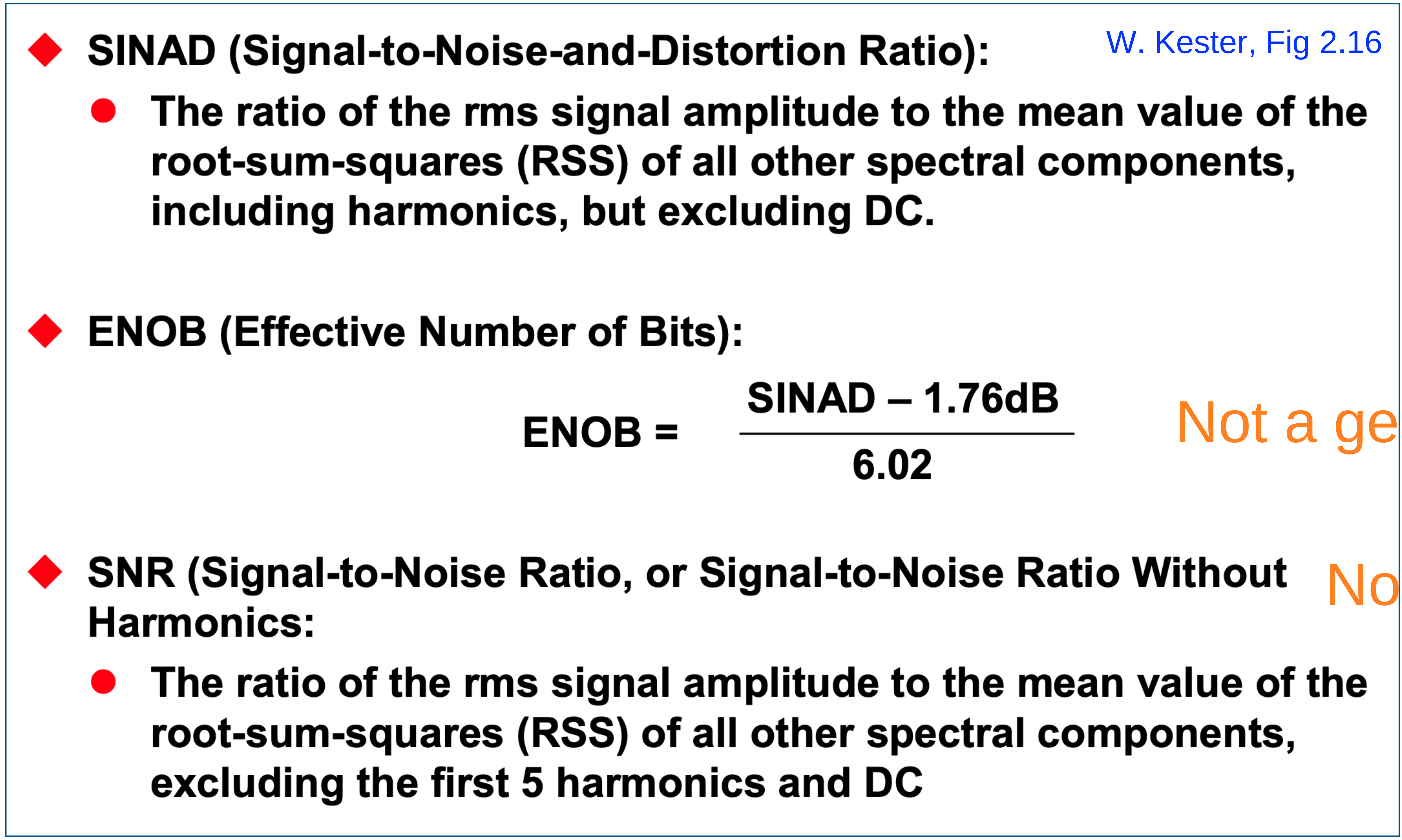
SINAD, ENoB, SNR



W. Kester, Fig 2.16

- **SINAD (Signal-to-Noise-and-Distortion Ratio):**
  - **The ratio of the rms signal amplitude to the mean value of the root-sum-squares (RSS) of all other spectral components, including harmonics, but excluding DC.**
- **ENOB (Effective Number of Bits):**

$$\text{ENOB} = \frac{\text{SINAD} - 1.76\text{dB}}{6.02}$$

Not a general definition!!!

- **SNR (Signal-to-Noise Ratio, or Signal-to-Noise Ratio Without Harmonics:**
  - **The ratio of the rms signal amplitude to the mean value of the root-sum-squares (RSS) of all other spectral components, excluding the first 5 harmonics and DC**

Not a general definition!!!

$$\text{SINAD} = \frac{V_{\text{rms}}}{\sqrt{\Sigma V_{\text{harmon}}^2} + \sqrt{\Sigma V_{\text{spurs}}^2} + \sqrt{\Sigma \ldots}}$$

Also SQR = Signal to Quantization Ratio

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Transition Noise

High-Speed Converters

- Analog noise is higher than quantization noise
- Given a voltage V –> random distribution of output N
- This correct –> $V^2 = V^2_{\text{analog}} + V^2_{\text{quant}}$
  (don’t spoil the resolution with insufficient no of bits)

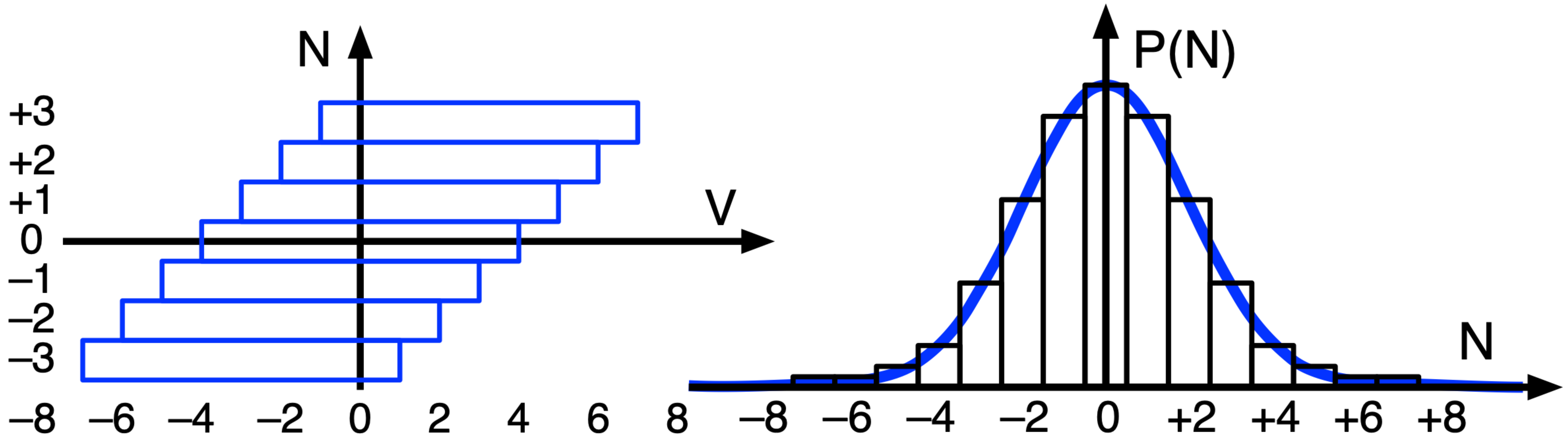
Information (bits)

$$I = \sum_i -p_i \log_2(p_i)$$

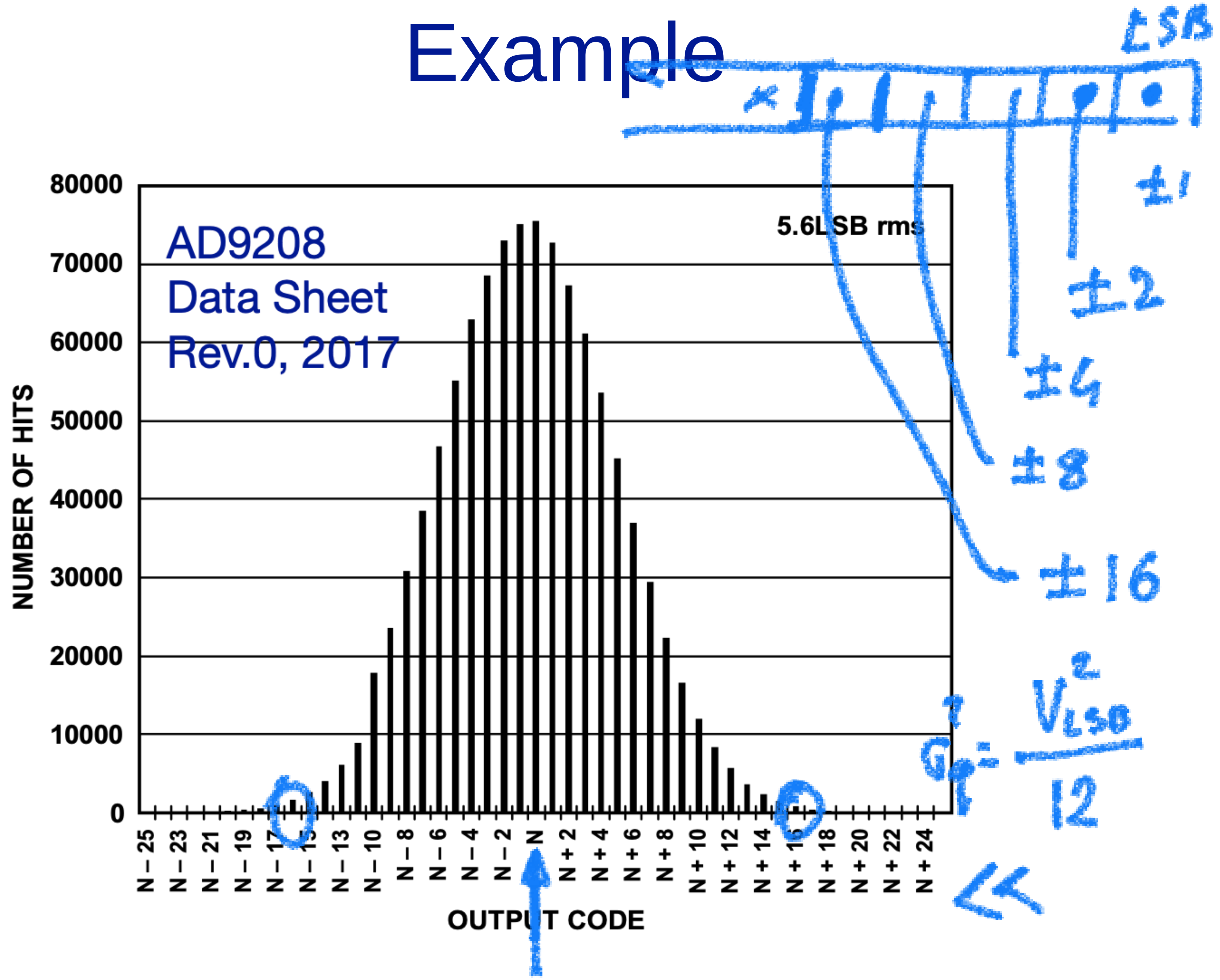
Equivalent No of Bits

$$\text{ENoB} = \log_2 \left[1 + \frac{V_{\text{FSR}}}{\sqrt{12}\,\sigma_V} \right]$$

Transition Noise



Example



$\sigma_q^2 = \frac{V_{LSB}^2}{12}$

- Actual noise includes Quantization, Analog noise, and Distortion
- $\boxed{\sigma^2_v} = \cancel{\sigma^2_q} + \sigma^2_a + \sigma^2_d$
- Random distribution of output N
- Metrology suggests to make σ^2_q negligible because BUS bits are cheap

Equivalent Number of Bits (ENoB)

1. Quantization
2. Analog noise
3. Distortion
4. Spurs

$$\sigma_q^2 = \frac{V_{\mathrm{LSB}}}{12} = \boxed{\frac{V_{\mathrm{FSR}}}{122^{2n}}}$$

← quantization

$$\sigma_a^2$$

$$\boxed{\sigma_q^2 \rightarrow \sigma_{\mathrm{Total}}^2} \Rightarrow \boxed{n \rightarrow \mathrm{ENoB}}$$

$$\sigma^2 = \frac{V_{\mathrm{FSR}}}{122^{\mathrm{ENoB}}}$$

Example AD9652

- Specs: V_{FSR} = 2 V, ENOB = 12 bits, f_s = 250 MHz
- Calculate σ_q , noise PSD, SQR.
- What happens if f_s is lowered to 100 MHz?

Information

- Bit
- Hartley (ban, dit)
- Natural logarithm

$\log_{10}$

Information (bits)

$$I = \sum_i -p_i \log_2(p_i)$$

event $i \rightarrow p_i$

1 bit (sign only)

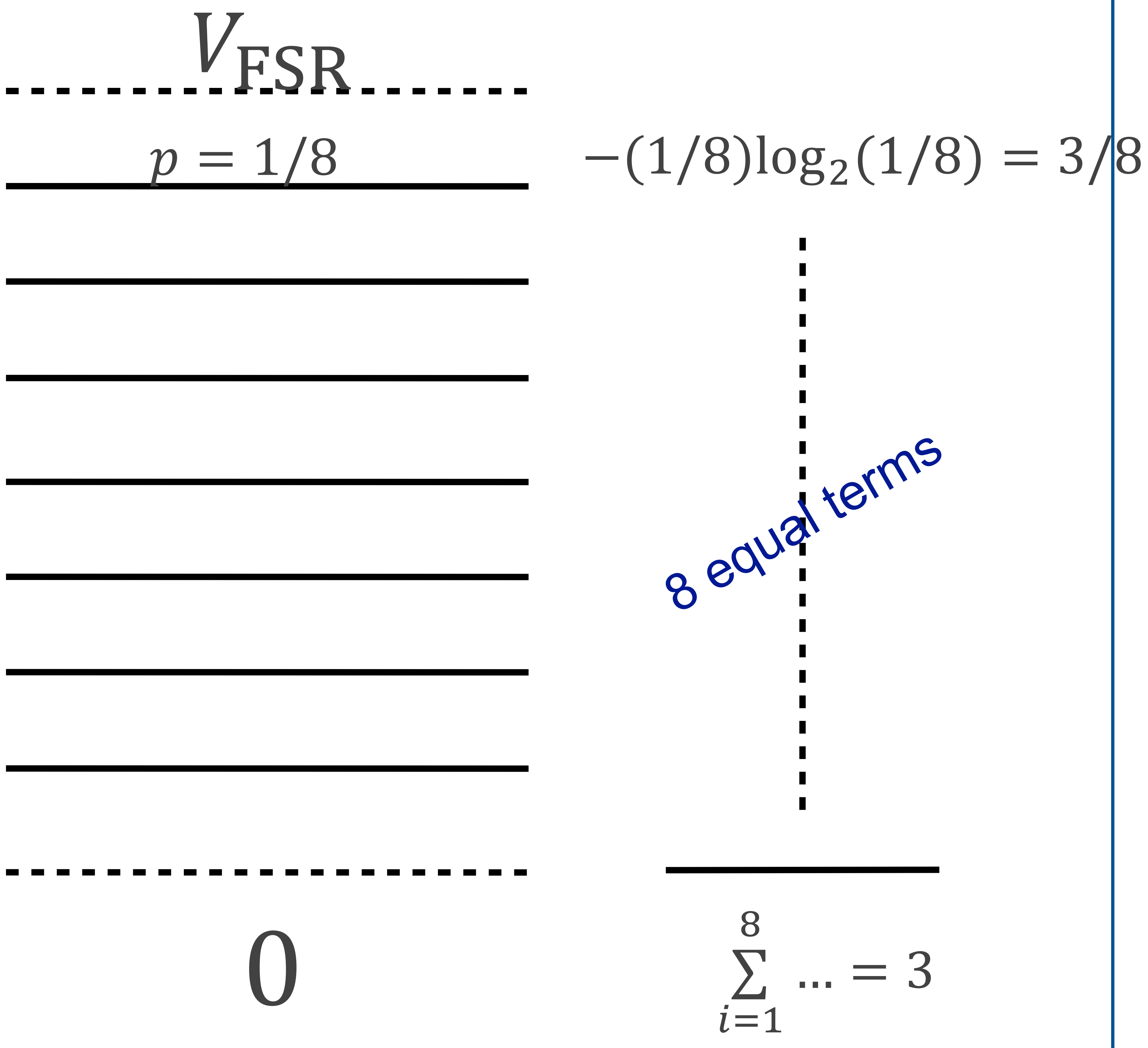
$50\% V > 0$

$50\% V < 0$

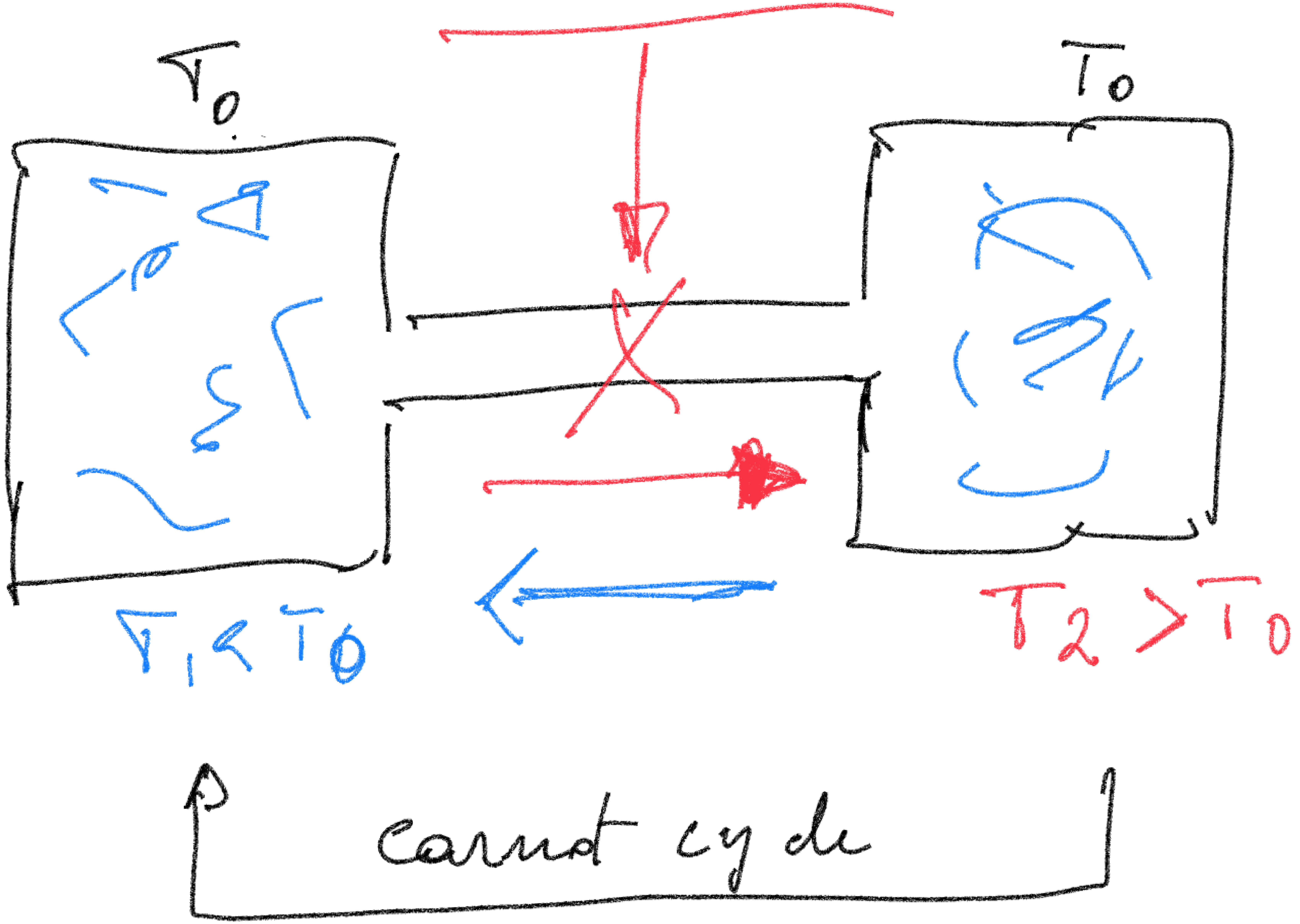
$-0.5\log_2(0.5) = 0.5$

$\sum = 1$

3 bits



Maxwell's Demon



End of Lecture #7

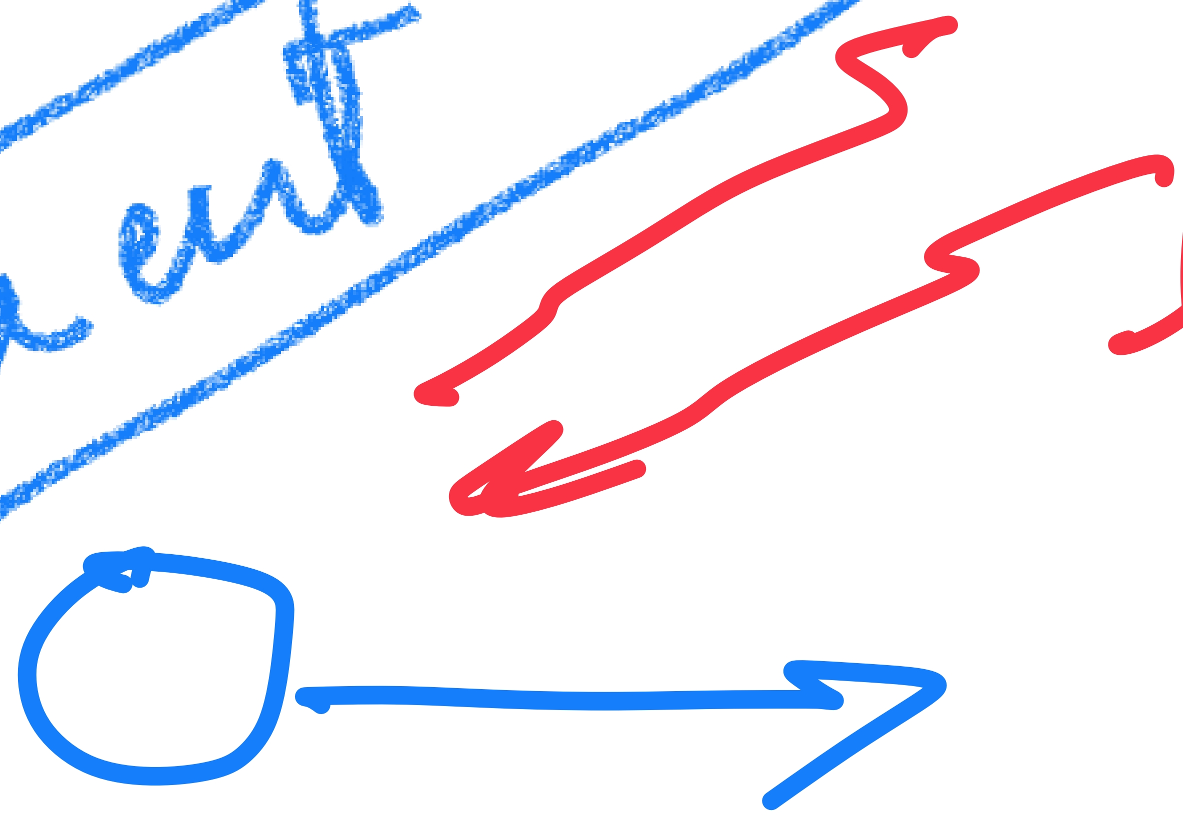
#8 Tuesday, October 15, 2019

1.5 Hours

Maxwell's Demon

- Photon energy?
- Constants, k vs h
- Something deeper

Gedankenexperiment

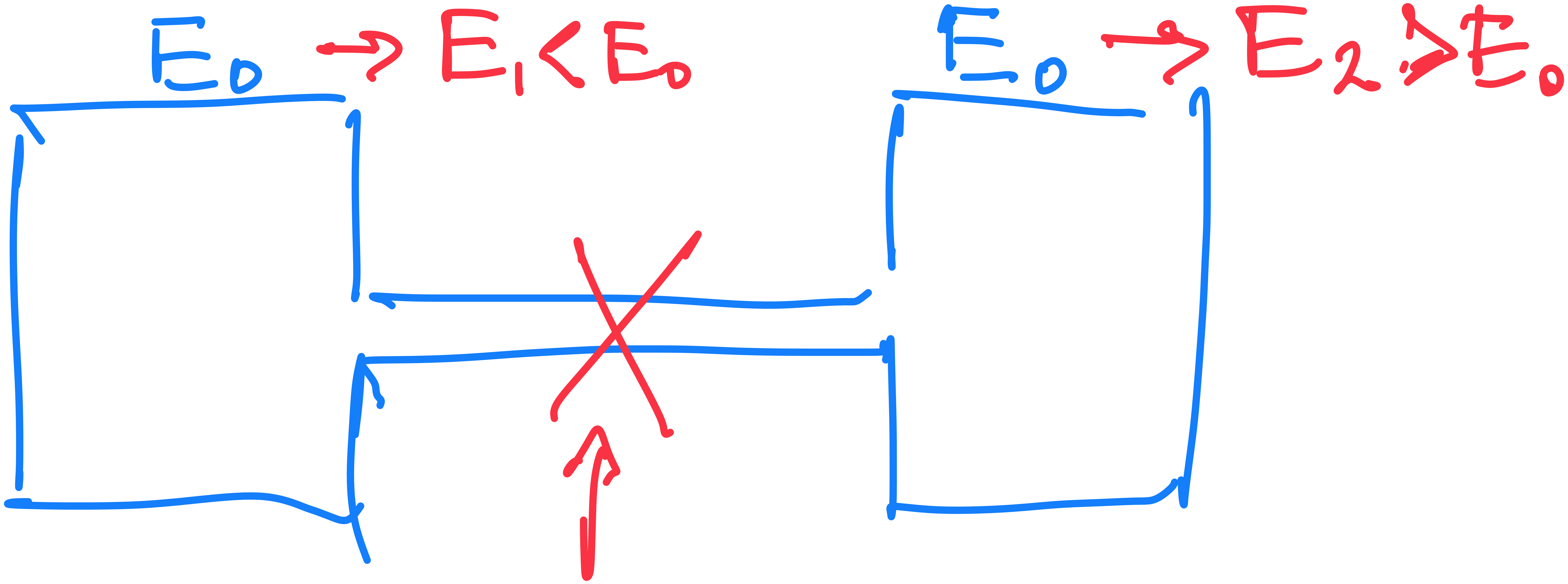


$\langle E \rangle = kT$

$\boxed{E = hf}$

Boltzmann

Planck



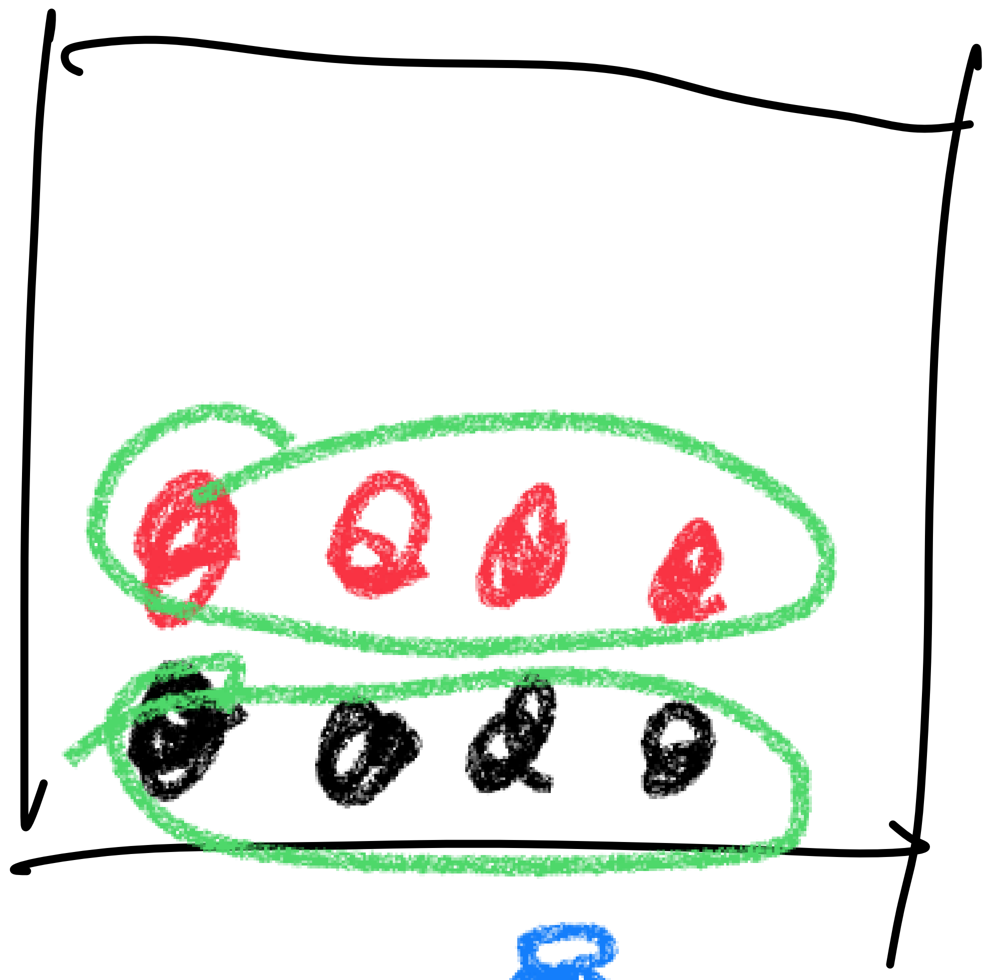
Information and energy are $(\approx)$ the same thing

Information – Black/Red(Green) Balls

+
- B/R, 50% each
- B/R, 25/75%
- Roulette

$$I = \sum -p_i \log p_i$$

probability of the i-th event

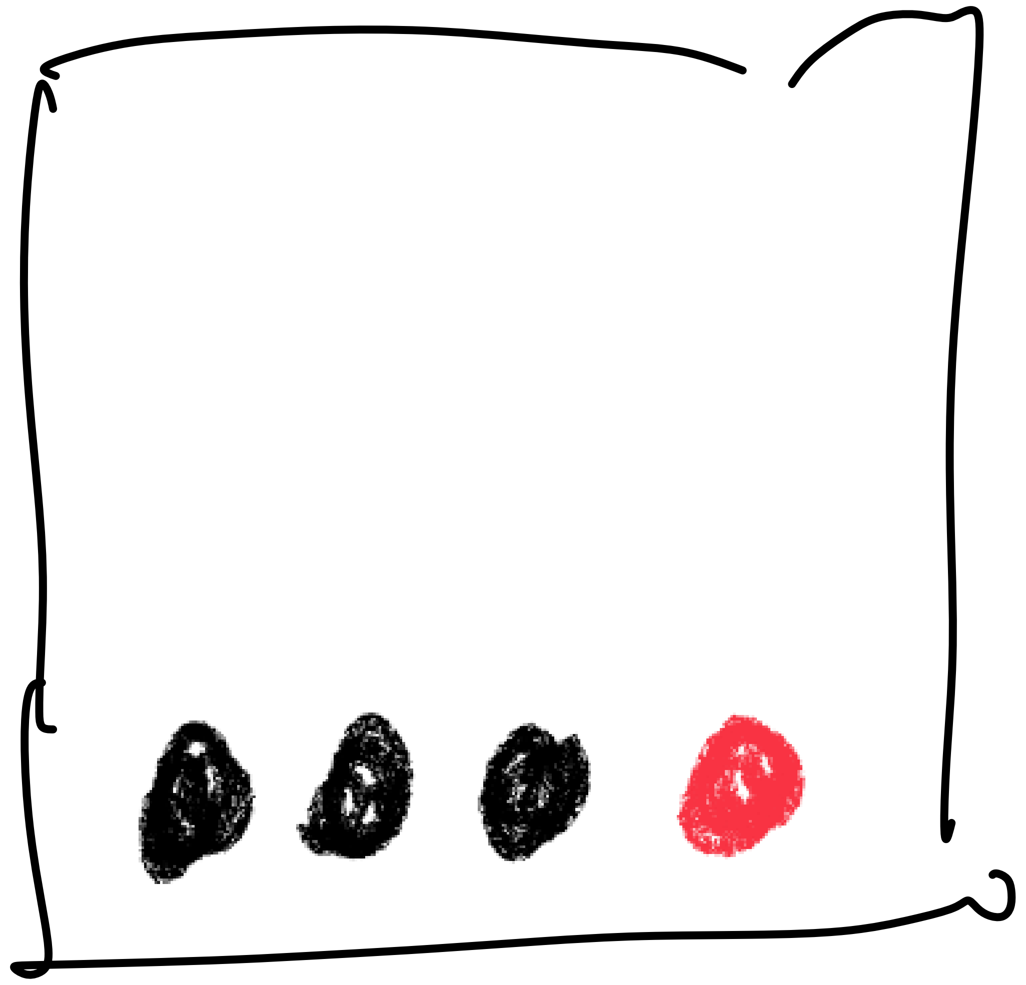


$$p_i = \frac{1}{8} \quad \forall i$$

✓ all different

$$I = \sum_{i=1}^{8} -\frac{1}{8} \log_2 \frac{1}{8}$$

$$I = \sum_{i=1}^{2} -\frac{1}{2} \log_2 \frac{1}{2} = 1 \text{bit}$$



$$I = \sum_i -p_i \log_2 p_i$$

$$-\frac{3}{4}\log_2\left(\frac{3}{4}\right) - \underbrace{\frac{1}{4}\log_2\left(\frac{1}{4}\right)}_{1/2}$$

0.311

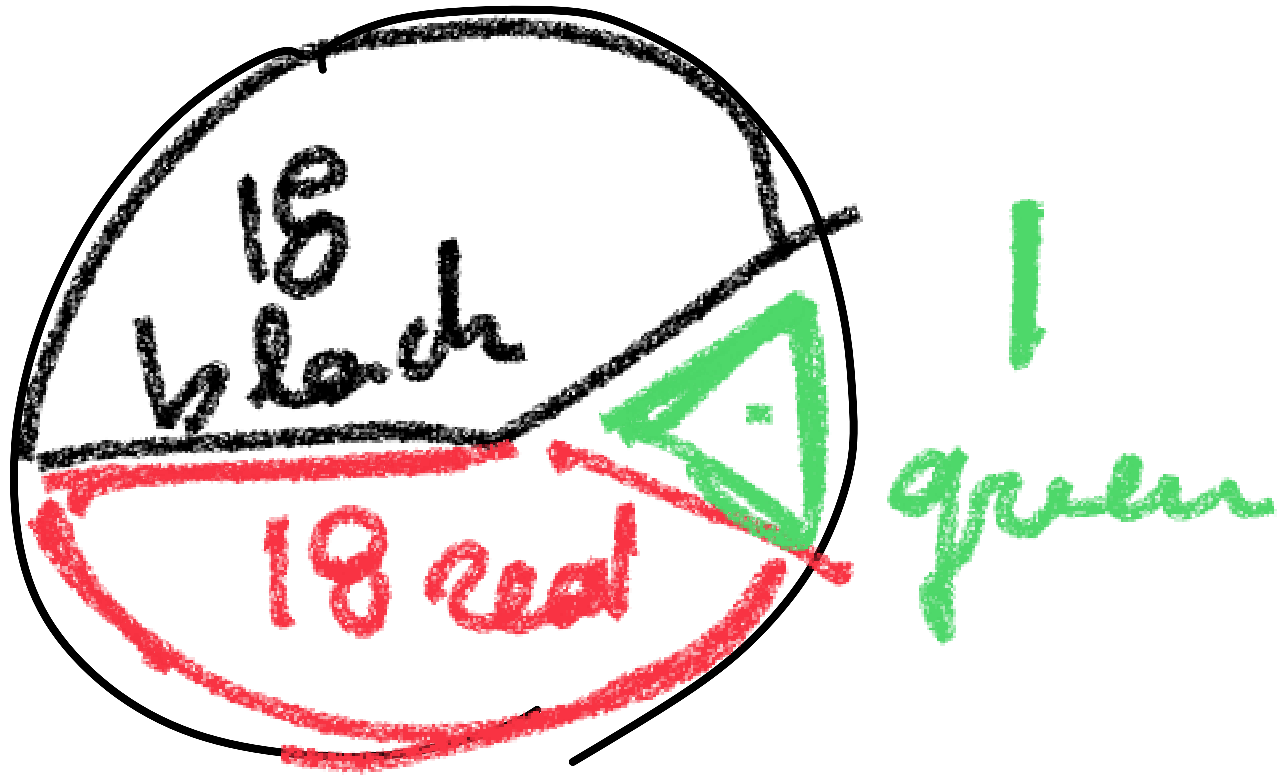
$$I = 0.81 \text{ bit}$$

$$\log_2(x) = \frac{\ln(x)}{\ln(2)}$$

$\ln(2)$

ROULETTE

38



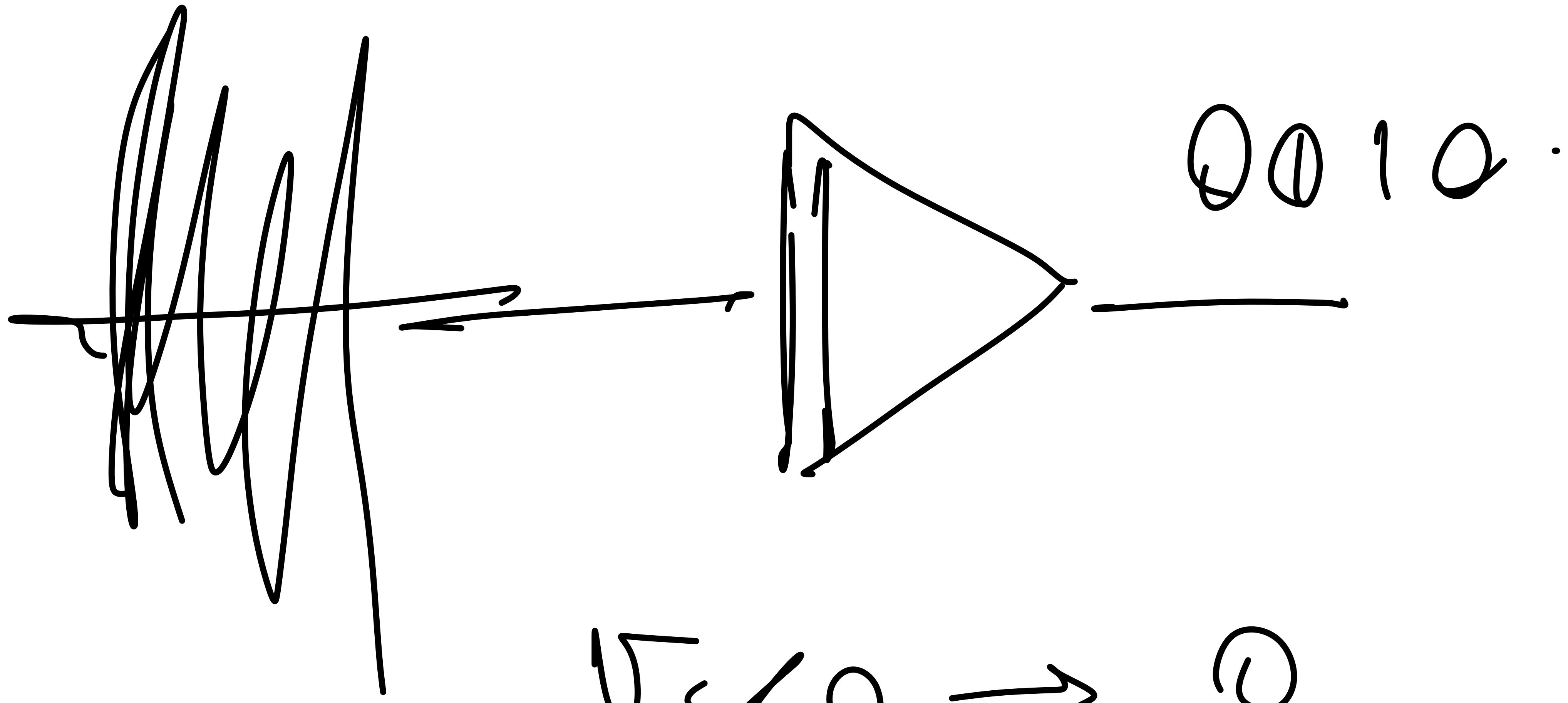
$$I = -\underbrace{\cancel{\frac{18}{37}}\,\overset{\approx -1}{\log_2}\left(\overset{\approx \frac{1}{2}}{\frac{18}{37}}\right)\times \cancel{2}}_{\substack{\text{black/red} \\ 1.01}} + \underbrace{\frac{1}{37}\log_2\left(\frac{1}{37}\right)}_{0.14}$$

$\log_2$ → bit

$\log_{10}$ → Hartley

$\ln$ → Thermodynamic Entropy

Information – Comparator



$v_i < 0 \rightarrow 0$

$v_i > 0 \rightarrow 1$

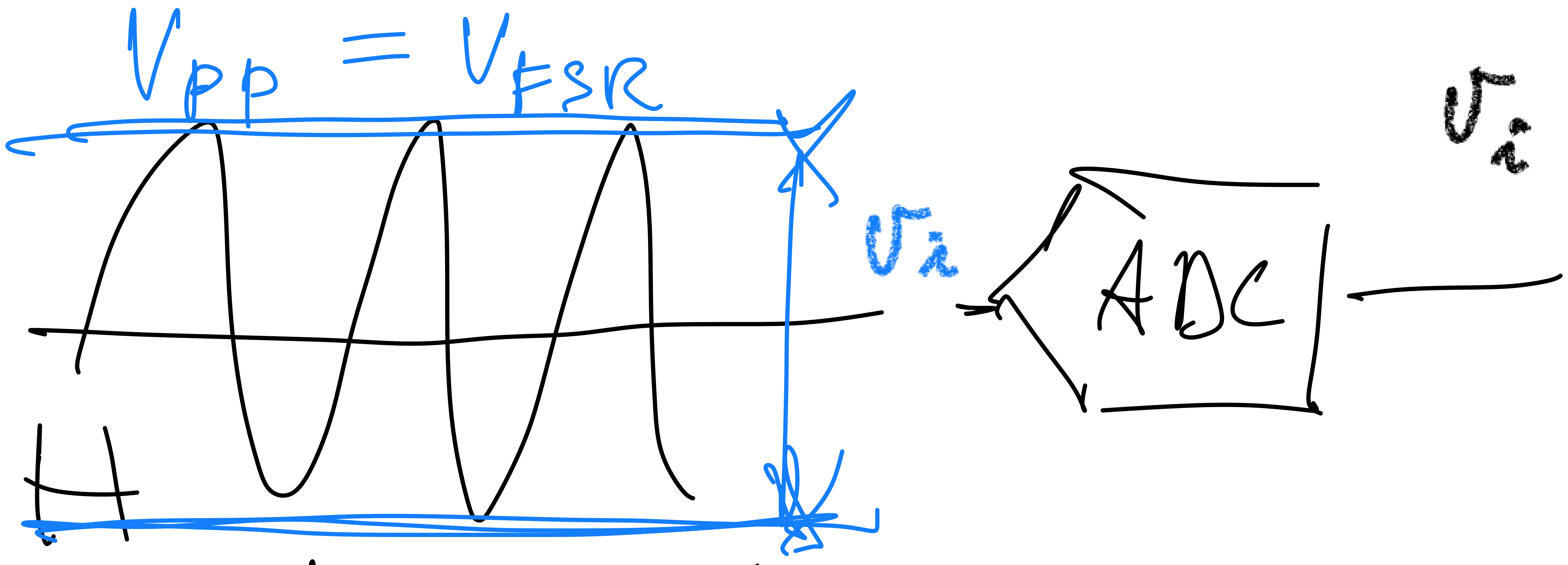
$P_+ = P_- = \frac{1}{2}$

$I = 1$ bit

ΣΔ DACs

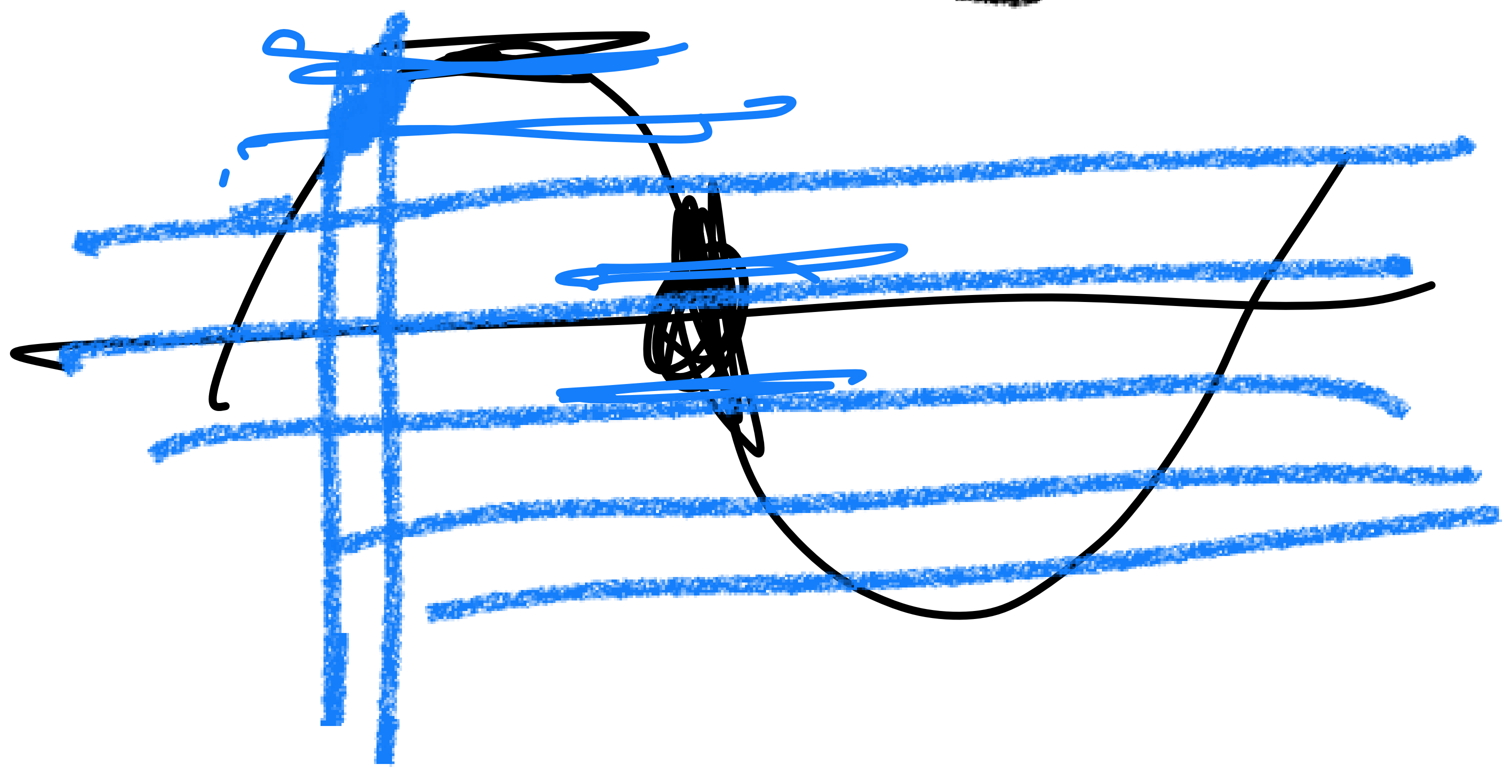
ADC

Information – 3 bit ADC & Sinusoid

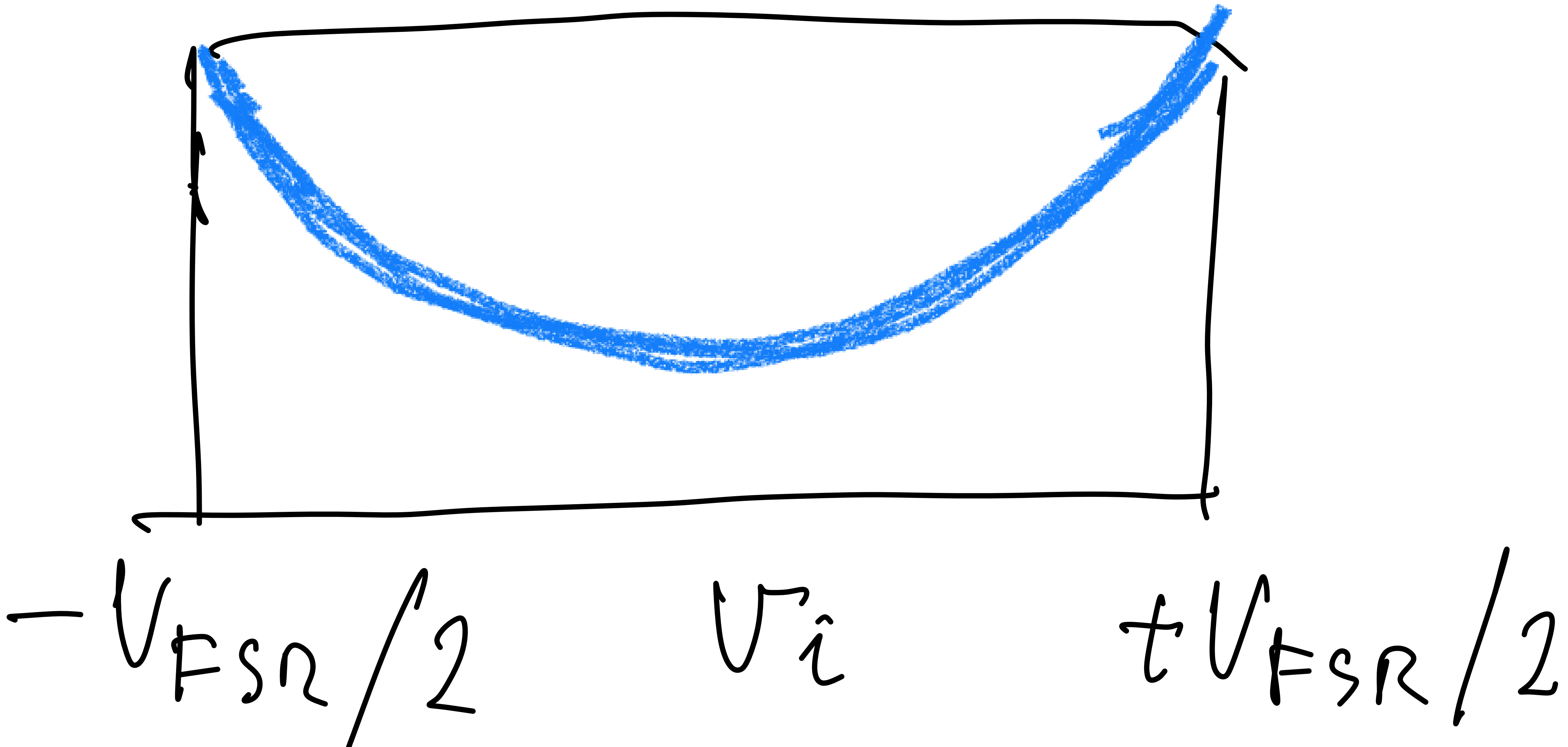


$v_i = V_p \cos(\omega t)$

$\hookrightarrow V_p = \frac{1}{2} V_{pp}$

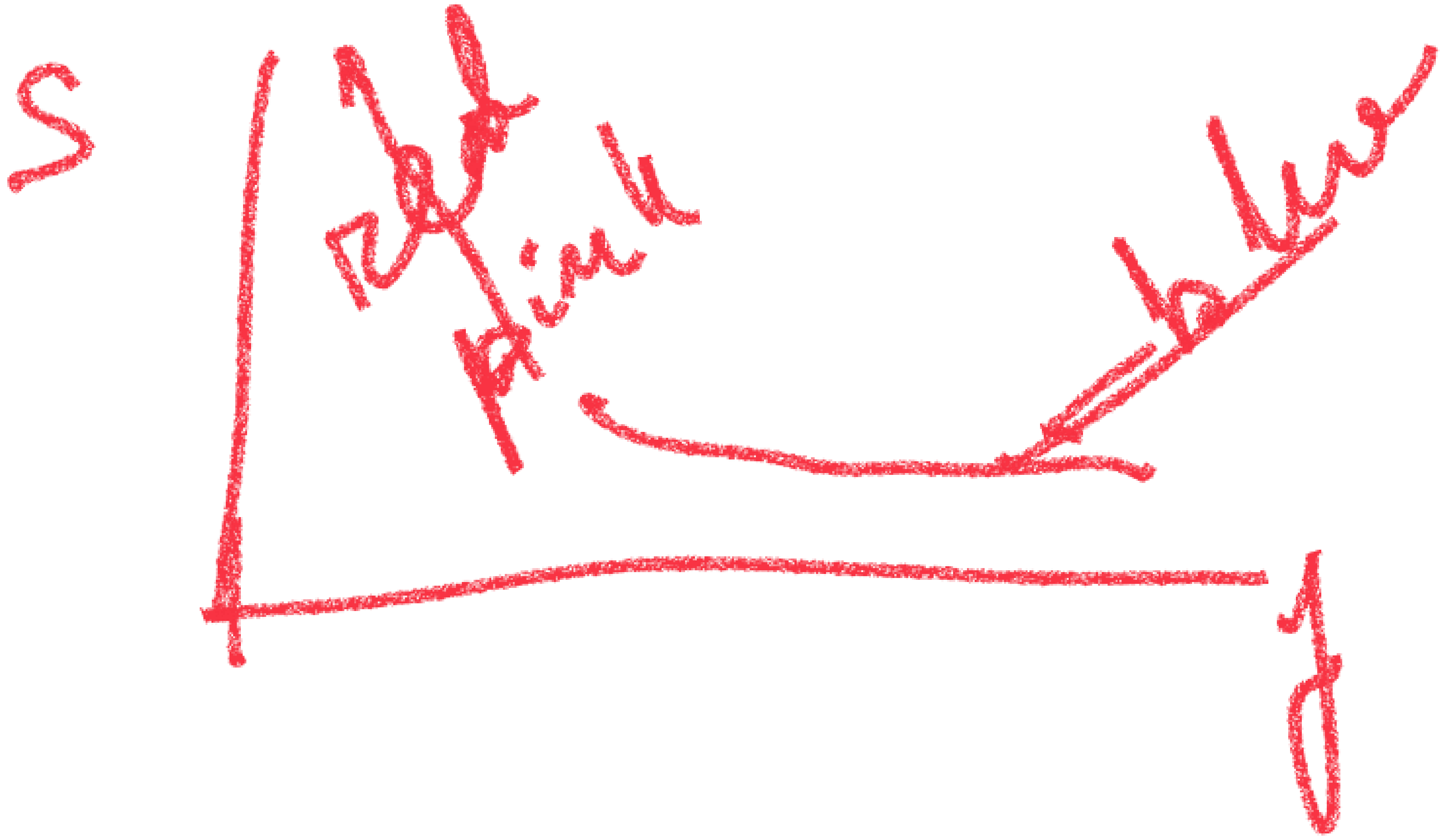


random origin of time



Analog Noise

- Thermal noise
- Shot noise
- Noise temperature
- Noise figure / Noise factor
- Flicker noise
- Colored noise



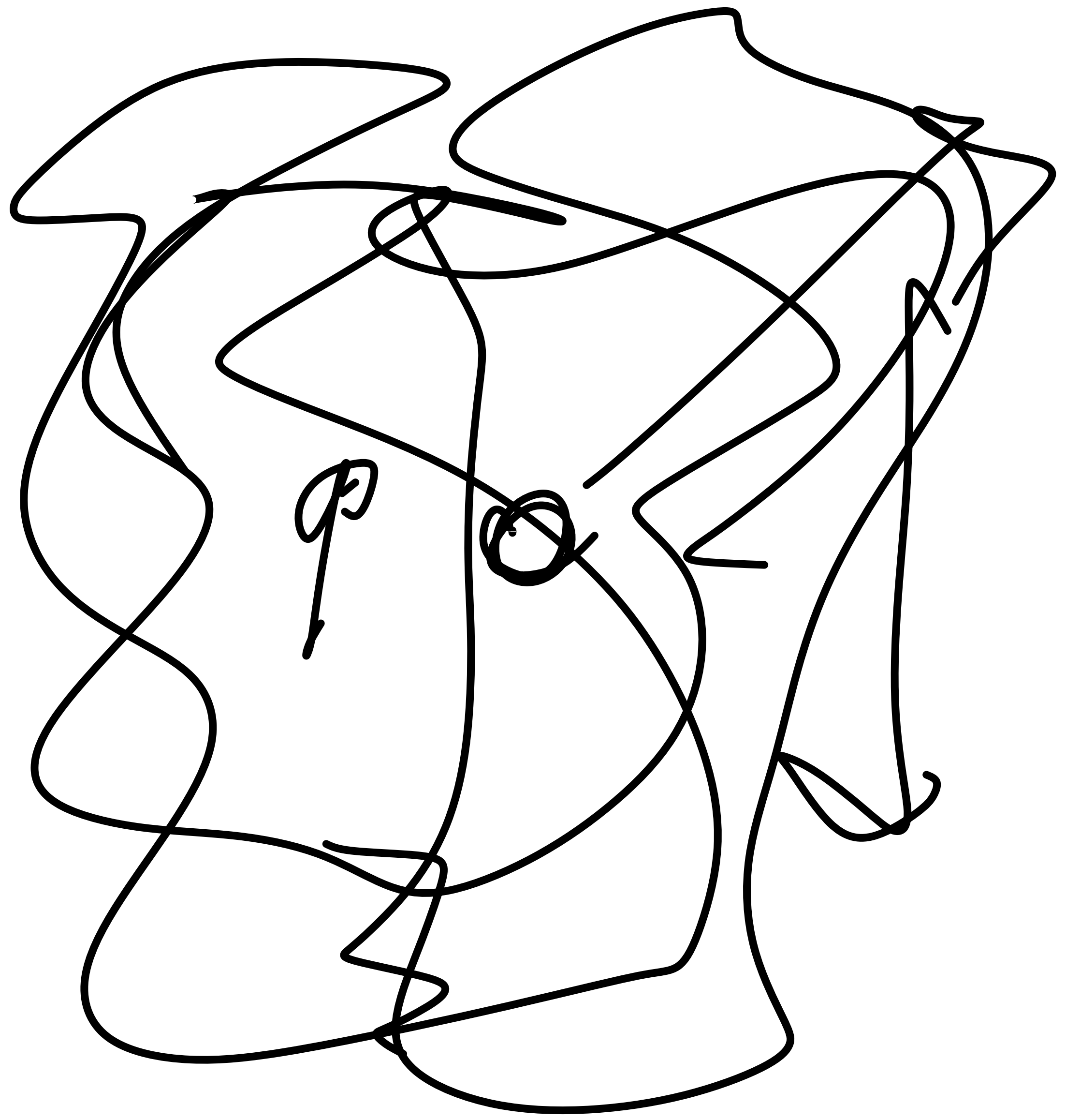
Accelerated Electrical Particle

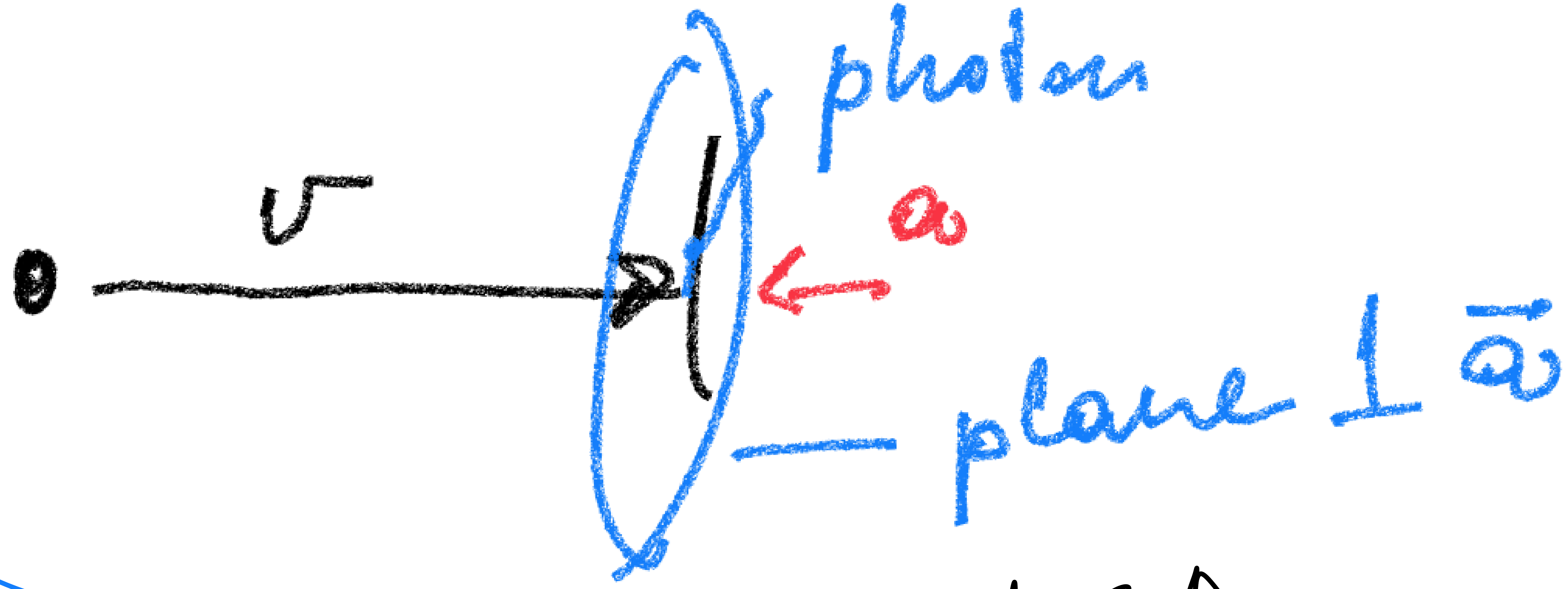
- Antenna
- Synchrotron radiation
- LED
- Blackbody radiation, gas-like electrons
- $E = kT/2$ per DOF
- Electrons, 2 DOF
- Neutron matter?

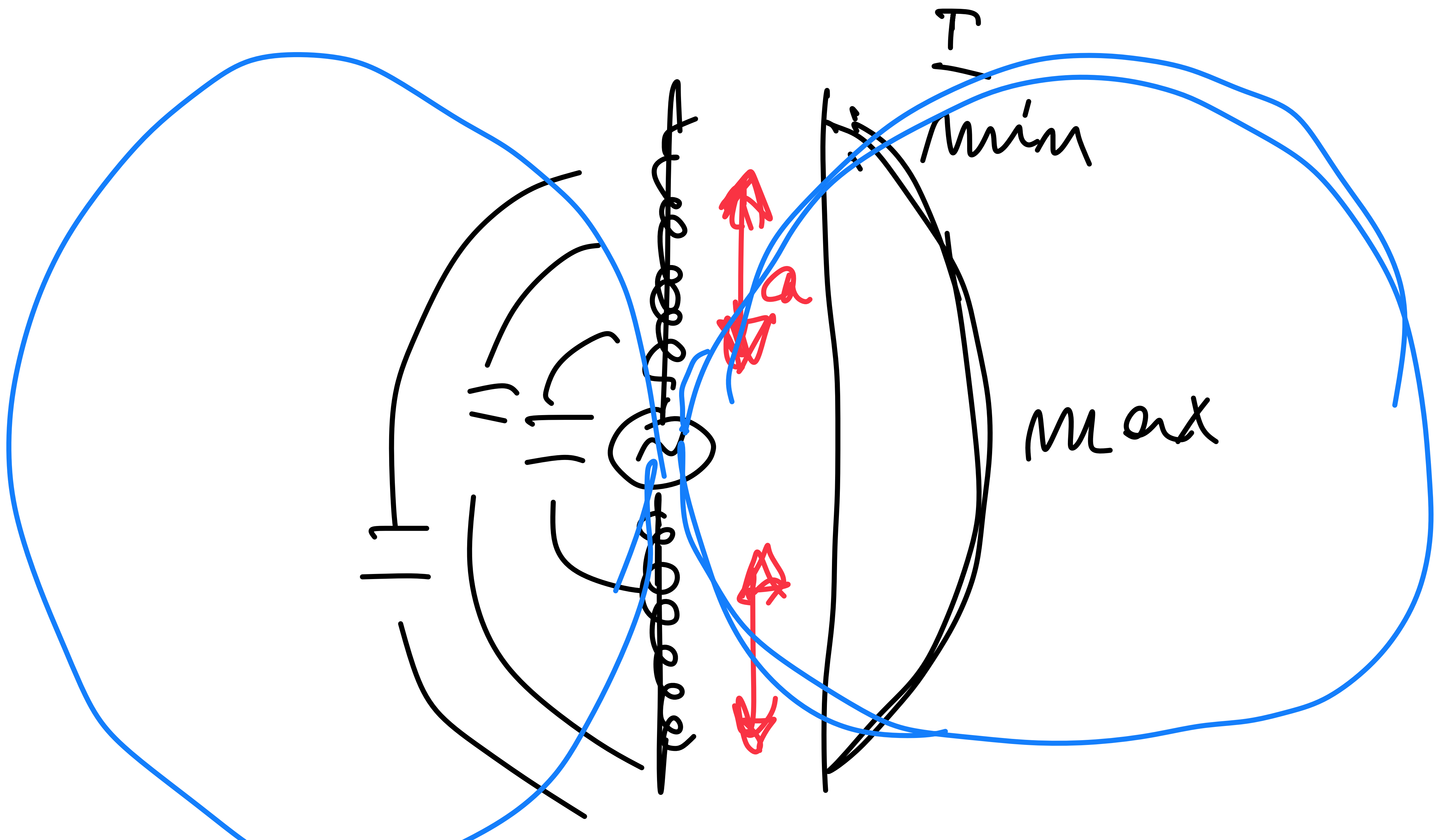
Keep on hand
$h = 6.62607004 \times 10^{-34}$ m^2 kg / s (J/Hz)
$k = 1.38064852 \times 10^{-23}$ m^2 kg s^{-2} K^{-1} (J/K), or 2 cal/mol

Larmor Theorem

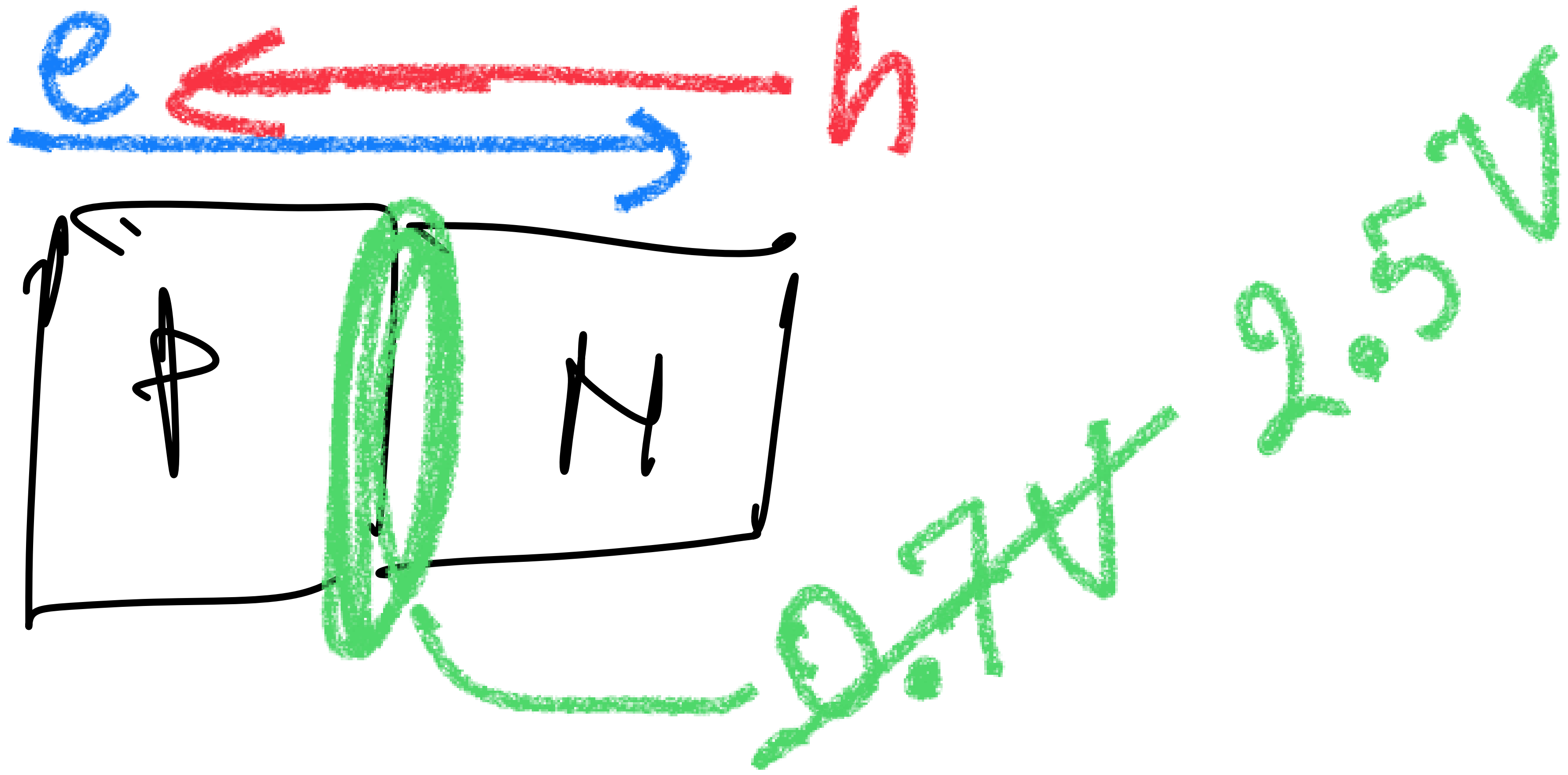
(Lucasian Chair)







LED



$E = hf \longrightarrow f = \frac{c}{\lambda}$ — $3 \cdot 10^{8}$

$0.53\ \mu m$

h: $6.6 \times 10^{-34}\ J/Hz$

$e = 1.602 \times 10^{-19}\ C$

$\rightarrow eV$

$\frac{3 \times 10^{8}}{5.3 \times 10^{-7}} = 5.7 \times 10^{14}$

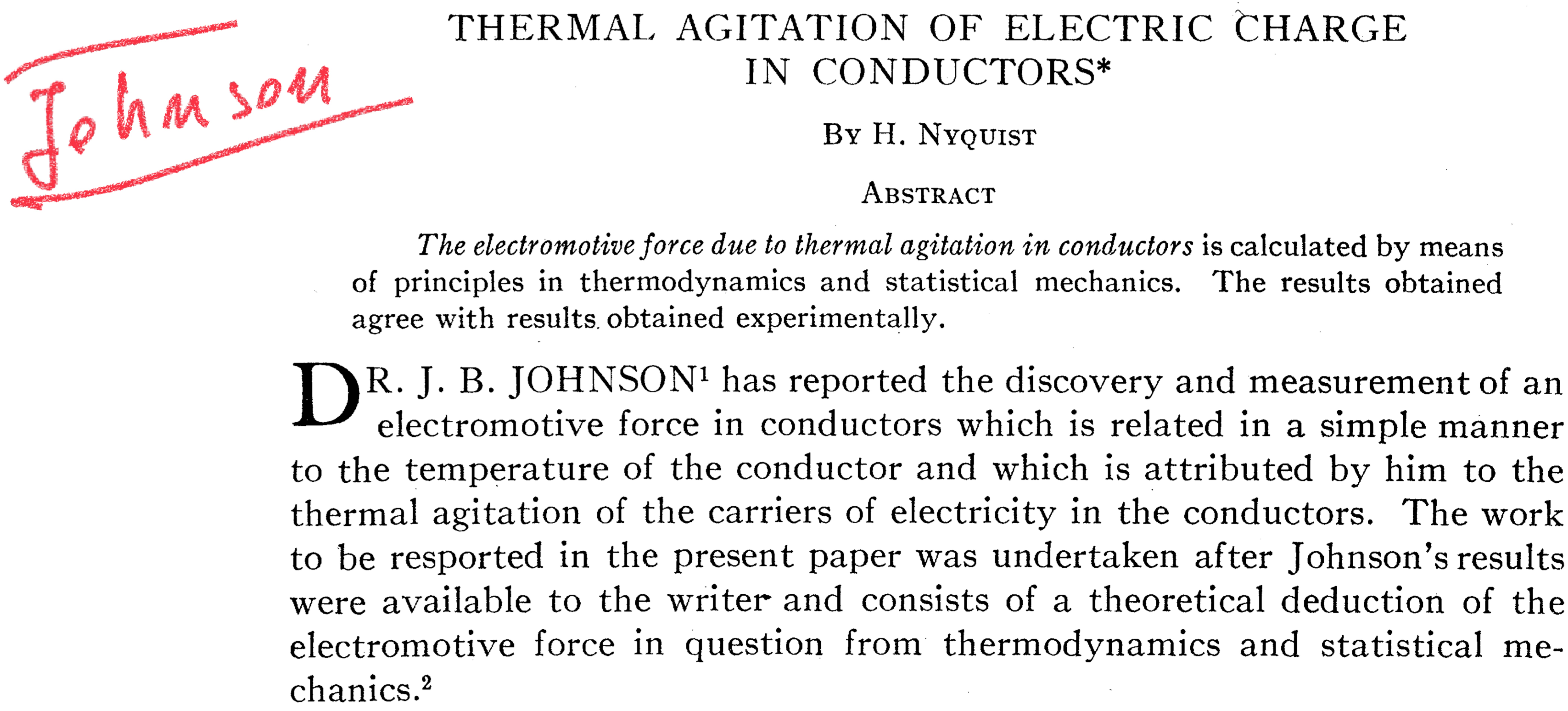
$3.7 \times 10^{-19}\ J$

$\downarrow \div e$

$2.33\ V$

Thermal Noise

Historical Article, Highly Educational



Johnson

THERMAL AGITATION OF ELECTRIC CHARGE IN CONDUCTORS*

By H. Nyquist

Abstract

The electromotive force due to thermal agitation in conductors is calculated by means of principles in thermodynamics and statistical mechanics. The results obtained agree with results obtained experimentally.

Dr. J. B. JOHNSON[1] has reported the discovery and measurement of an electromotive force in conductors which is related in a simple manner to the temperature of the conductor and which is attributed by him to the thermal agitation of the carriers of electricity in the conductors. The work to be resported in the present paper was undertaken after Johnson's results were available to the writer and consists of a theoretical deduction of the electromotive force in question from thermodynamics and statistical mechanics.[2]

Nyquist H - Thermal agitation of electric charges in conductors - Phys Rev 32(1) p110-113, July 1928

End of Lecture #8

#9 Tuesday, October 20, 2020

1.5 Hours

The Nyquist Article

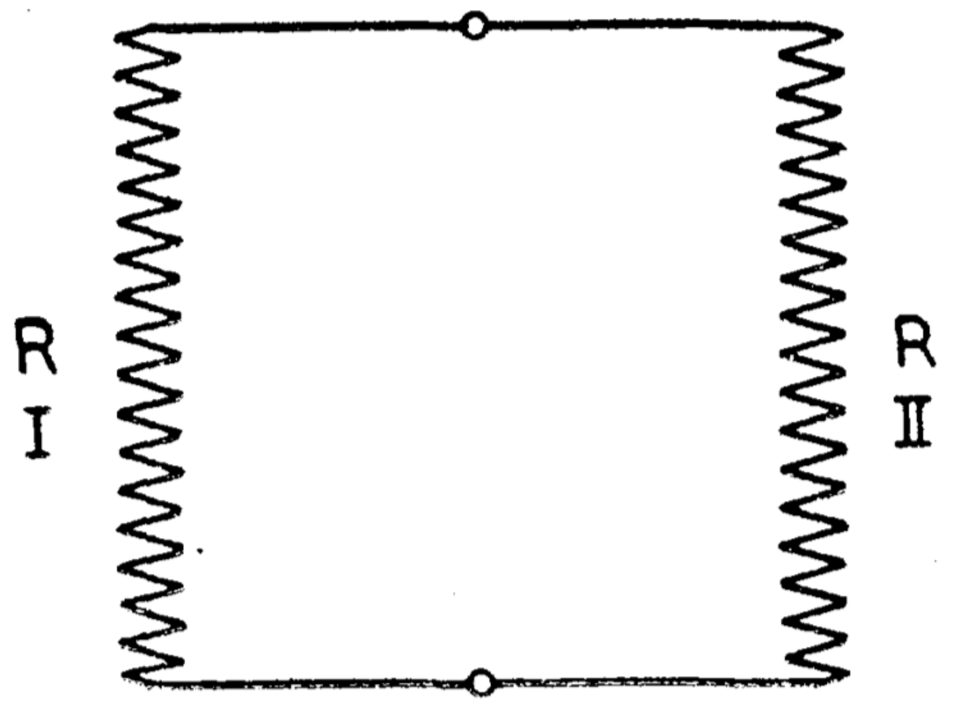


Fig. 1.

Circuits laws and thermodynamics
E = E(*f*, R, T)

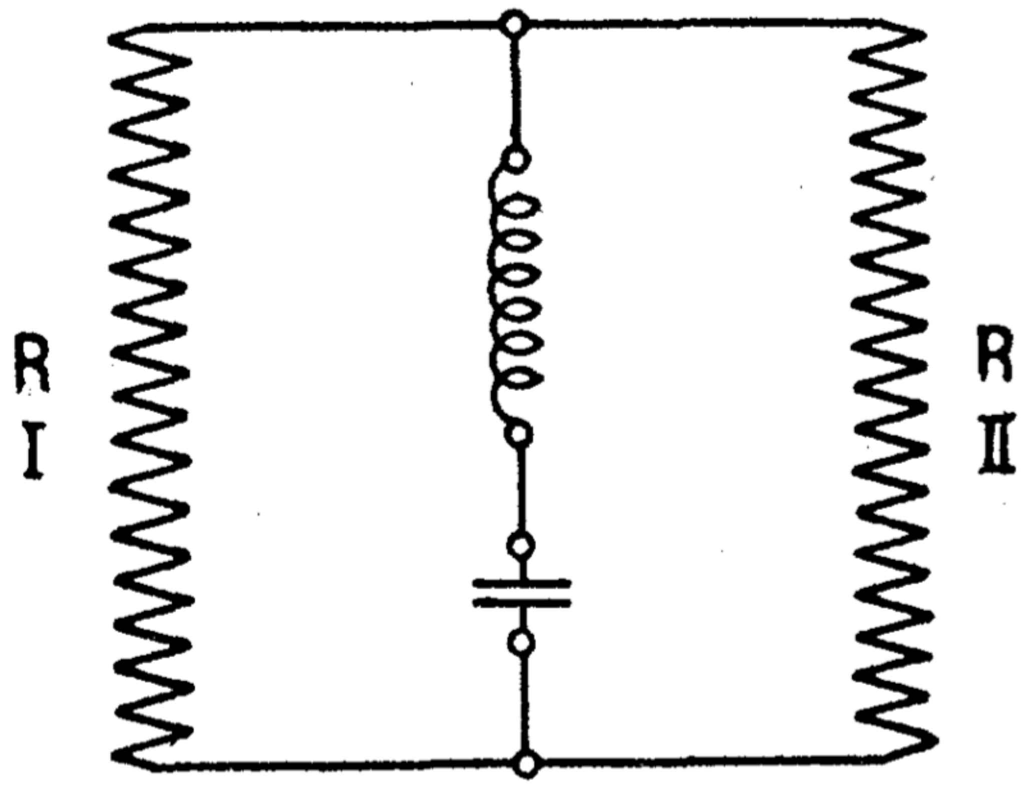


Fig. 2.

White
E = E(R, T)

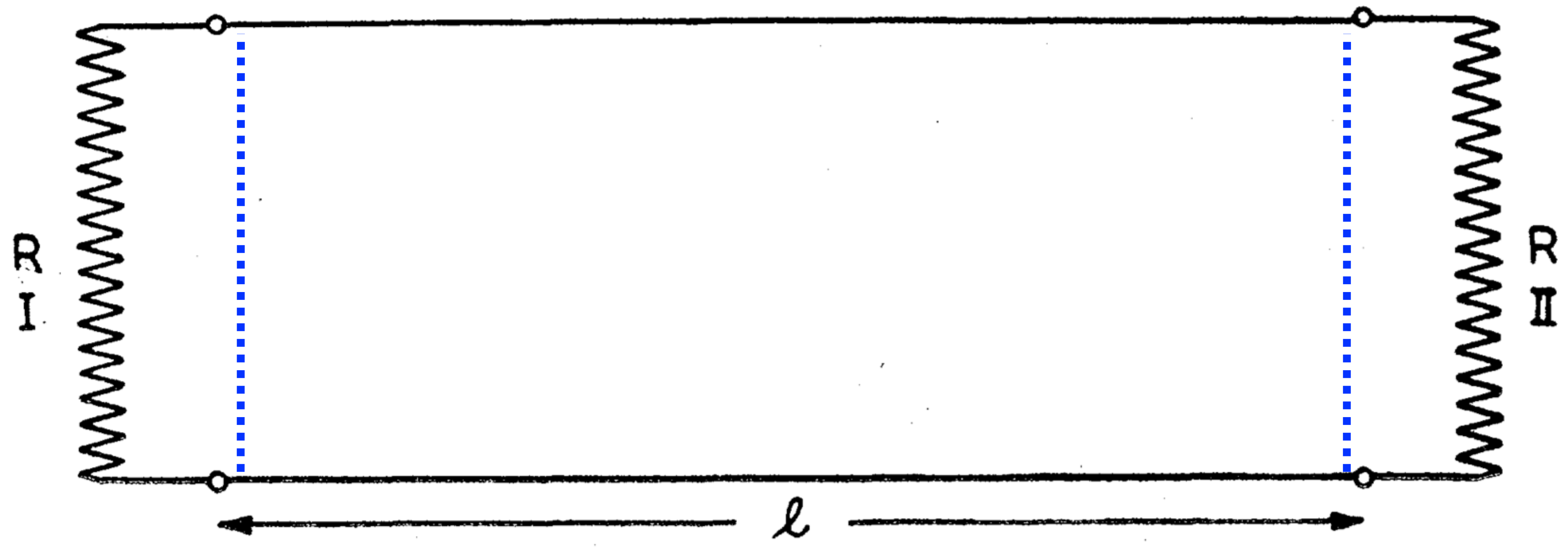


Fig. 3.

Line —> modes spaced by $\Delta\nu = v/2l$

$E = kT\mathrm{permode}$

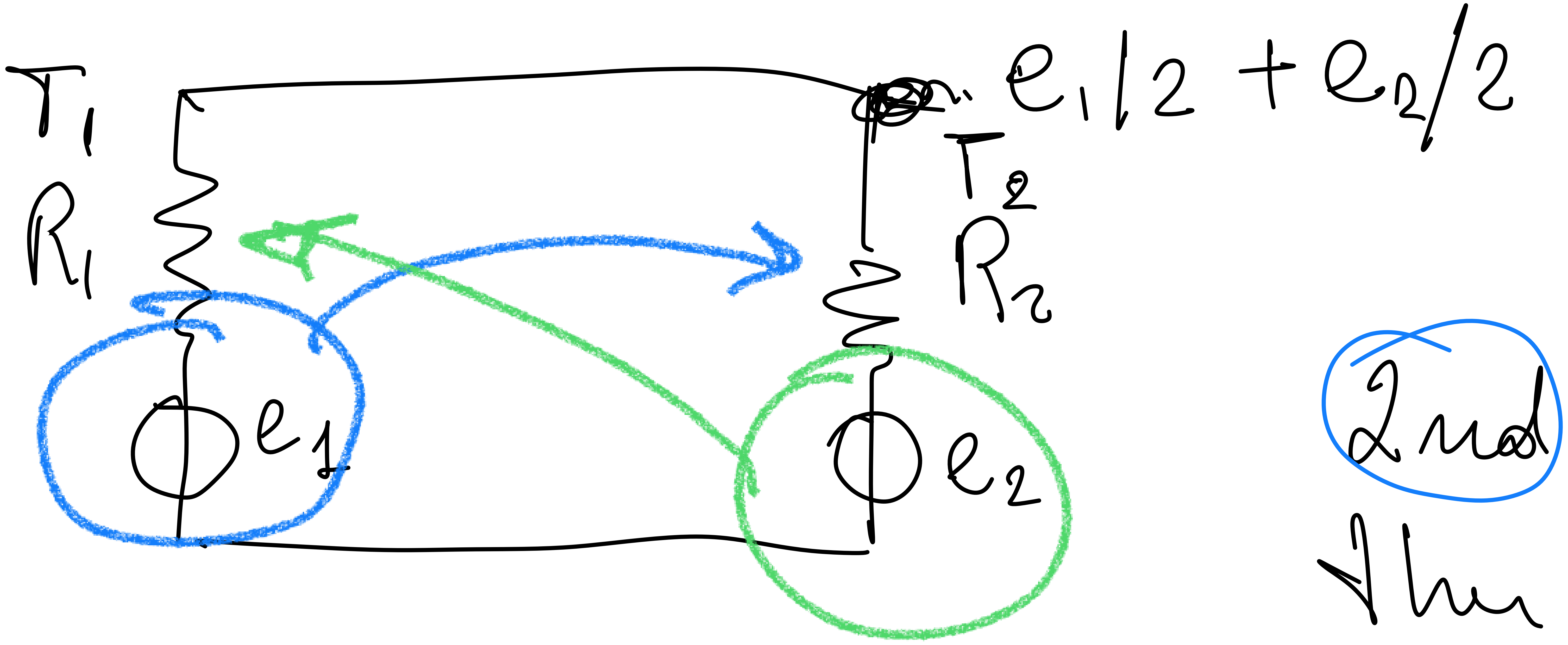
P transferred in d\nu and in time l/v is kT d\nu

Spectrum

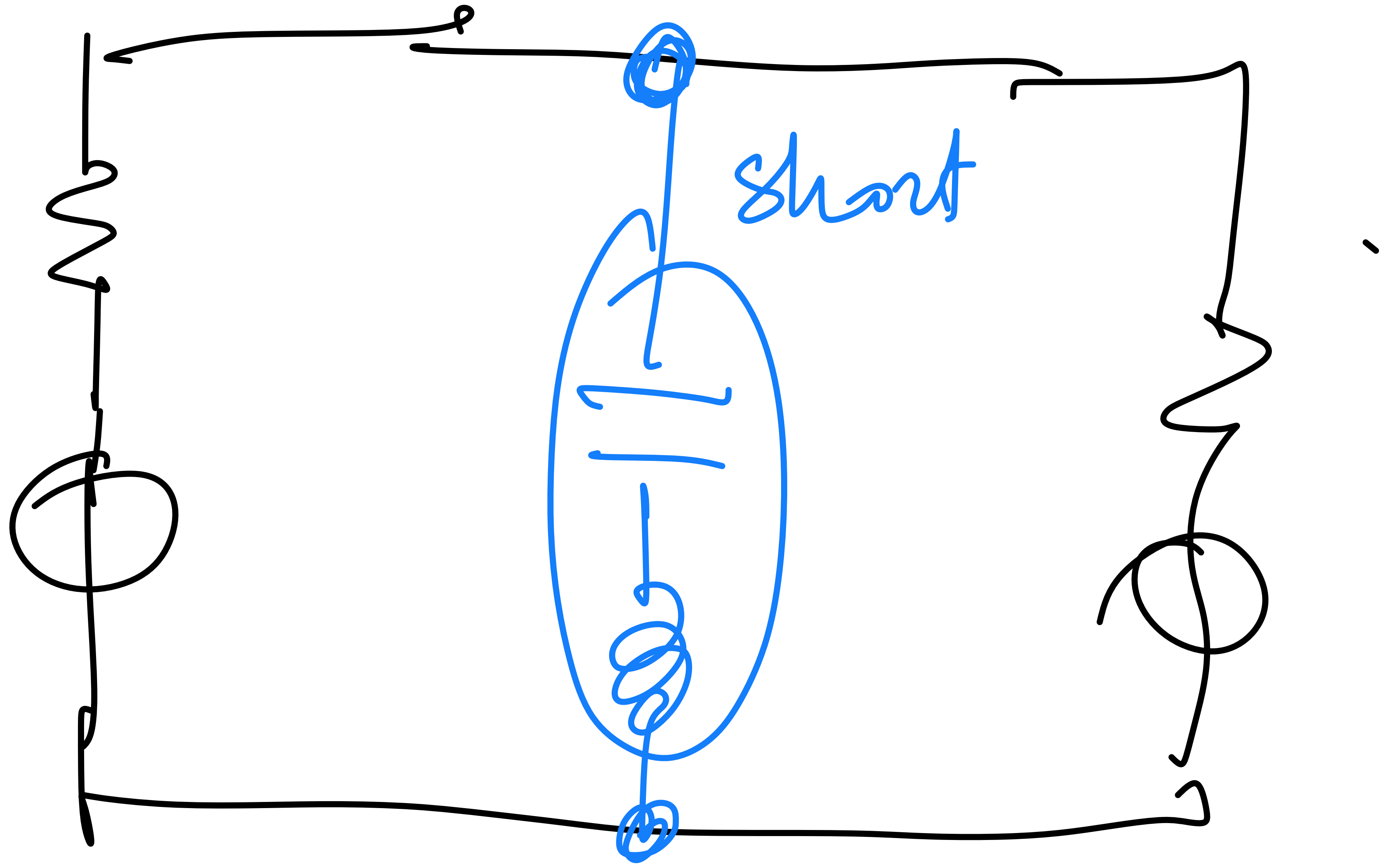
$$E^2 d\nu = 4RkTd\nu \tag{1}$$

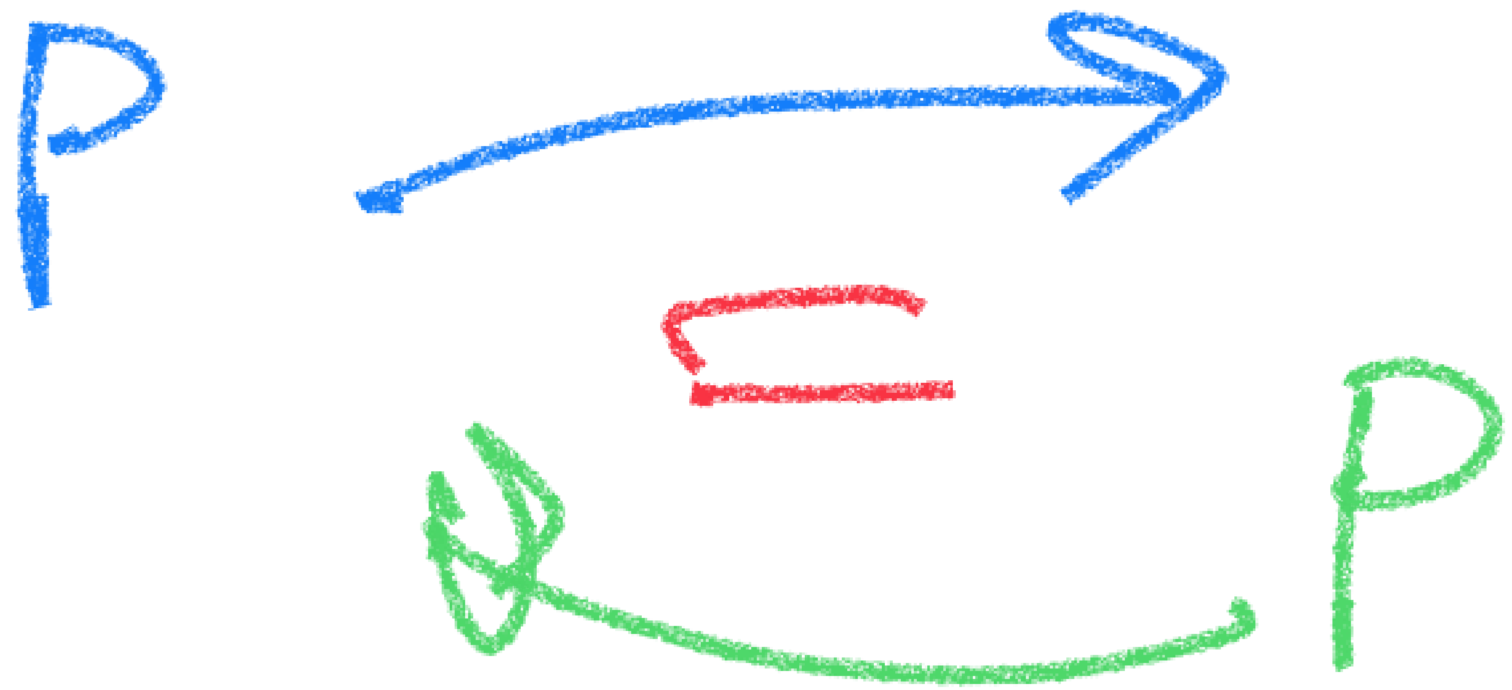
E^2 is the square voltage

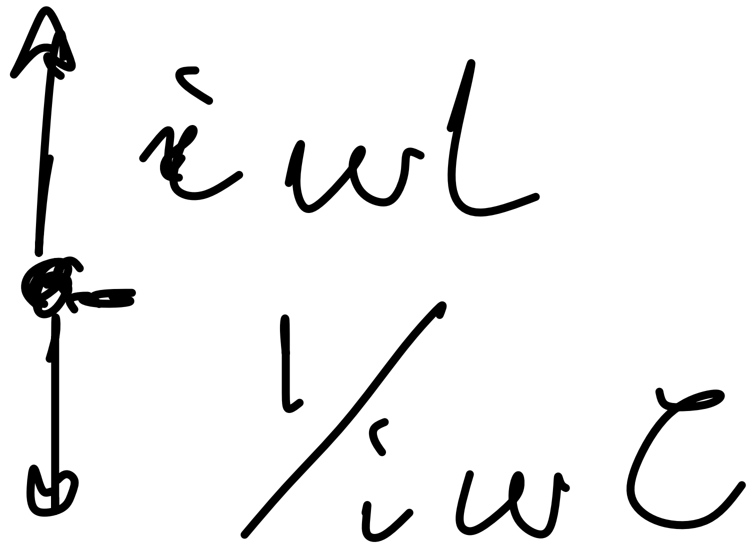
Nyquist H - Thermal agitation of electric charges in conductors - Phys Rev 32(1) p110-113, July 1928



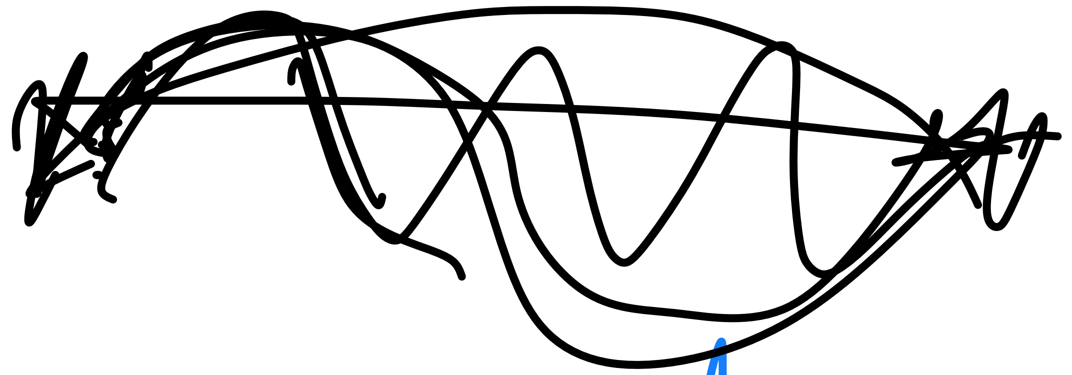
2nd principle of thermodynamics

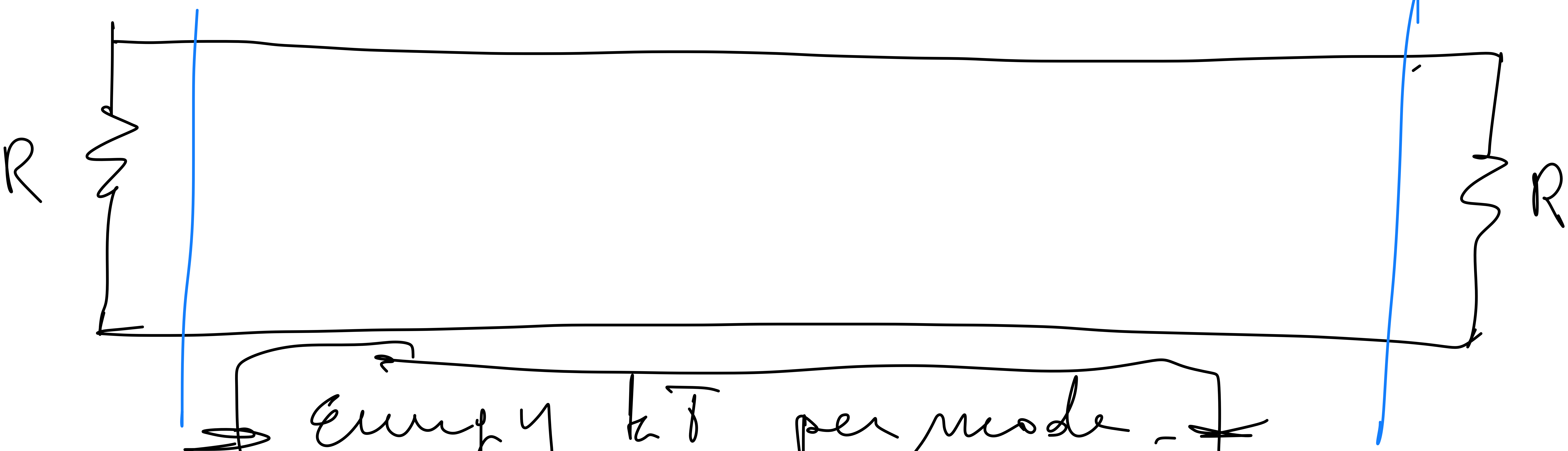






line $Z_0 = R$



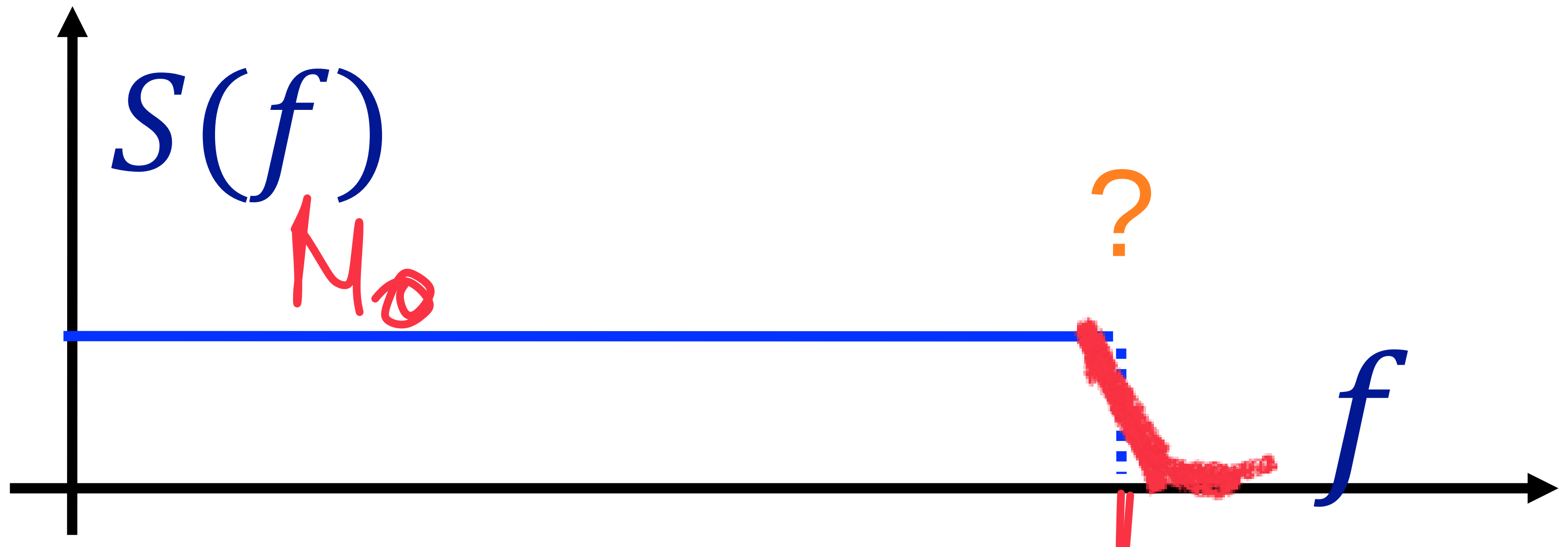


Energy kT per mode

$\langle v^2 \rangle$ on 1 Hz bandwidth

$$S_v = 4kTR$$

Cutoff Frequency



$P = \int_a^b S(f)df$

$P < \infty$

$a \to 0$
$b \to \infty$

$\boxed{hf = kT}$

accelerated charge

$E = \frac{1}{2} kT$ per degree of freedom

$E = hf$ Planck

accelerated electron → photon

$E_{photon} < E_{electron}$

$f_c = \frac{kT}{h}$

UV catastrophe, Sir James Jeans & John William Strutt, 3rd Baron Rayleigh —> Max Planck

$$f_c = \frac{kT}{h}$$

$$\frac{k}{h} = 2.29 \times 10^{10}$$

$$\boxed{22.9\ \mathrm{GHz/K}}$$

c
ν_{Cs}
N_A
K_{cd}

$$\boxed{\begin{aligned} k &= 1.380649 \times 10^{-23}\ \mathrm{J/K} \\ h &= 6.02607015 \times 10^{-34}\ \mathrm{W/Hz} \end{aligned}}$$

[J·s]

300 K

≈ 7 THz

≈ 10 μm

CO_2

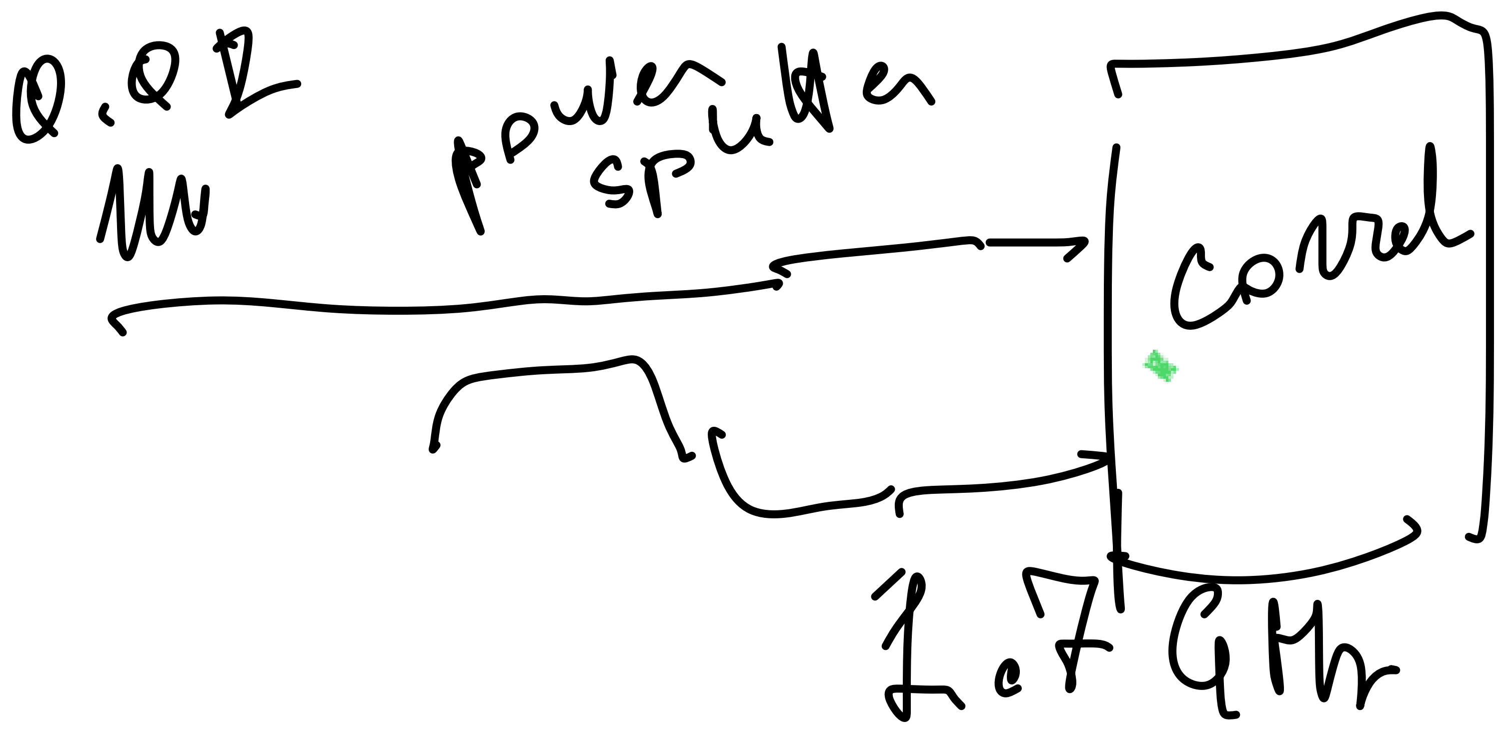
T = 4.2 K liquid He

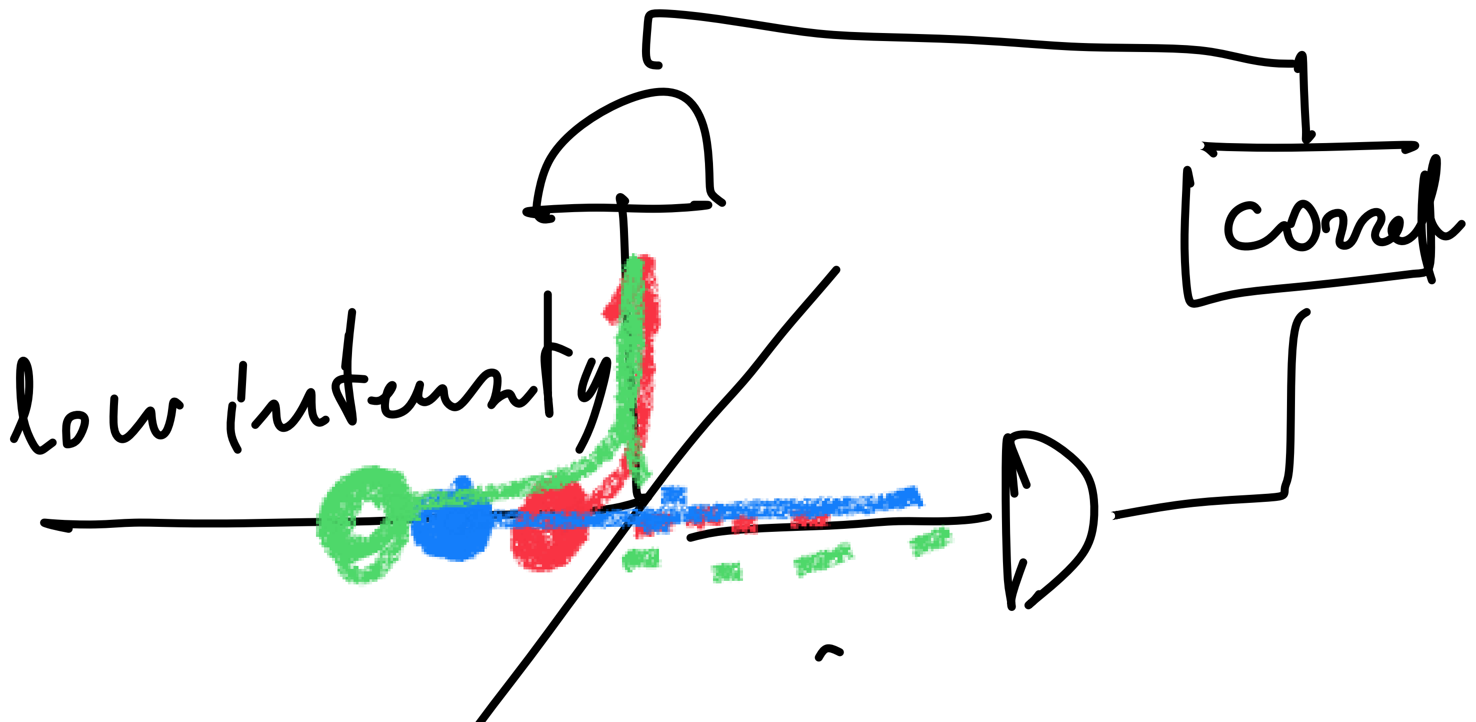
Dilution 0.01 K $^3He/^4He$ → $\boxed{200\ \mathrm{MHz}}$

electronics

< 200 GHz

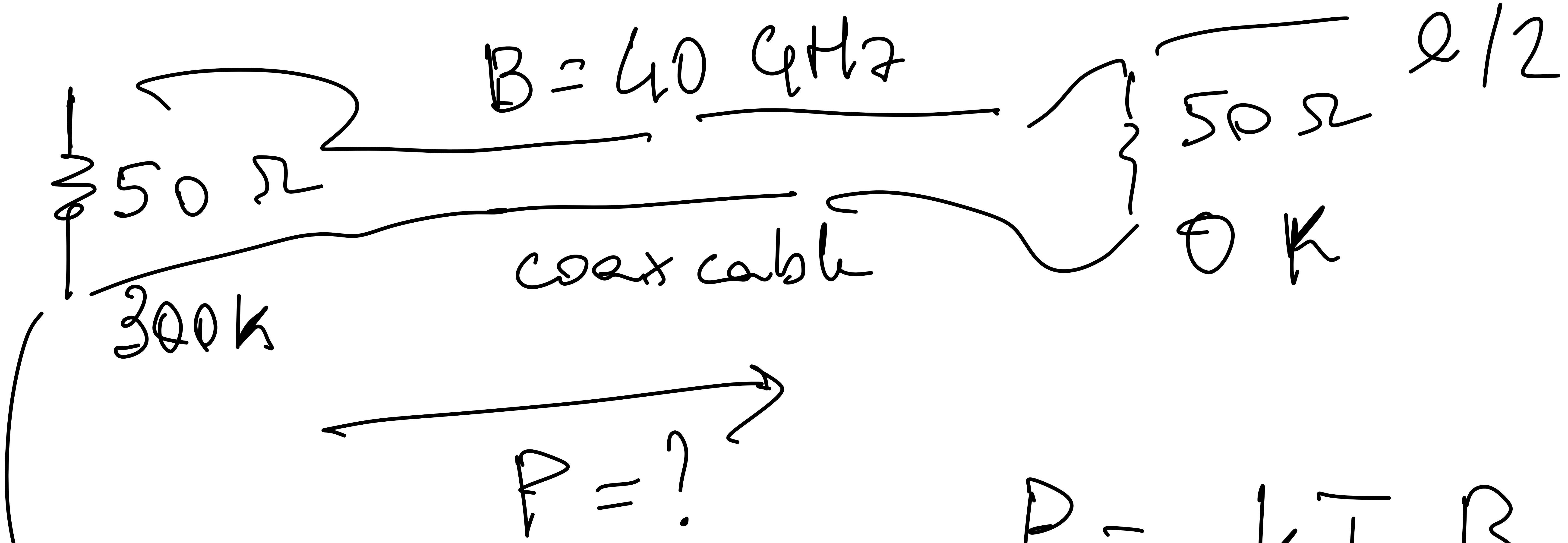
Christian Glattli





Hanbury · Brown · Twiss

Calculate the total power transferred from a resistor at T2 = 290 K to a cold (T1 = 0 K)



$$\boxed{e = \sqrt{4kTR}}$$

available = $\sqrt{kTR}$
the voltage in
max - transferred - power
conditions

$$P = kTB$$

$$4\times10^{-21} \times 4\times10^{10}$$

$$= 1.6\times10^{-10} \quad W$$

160 pW

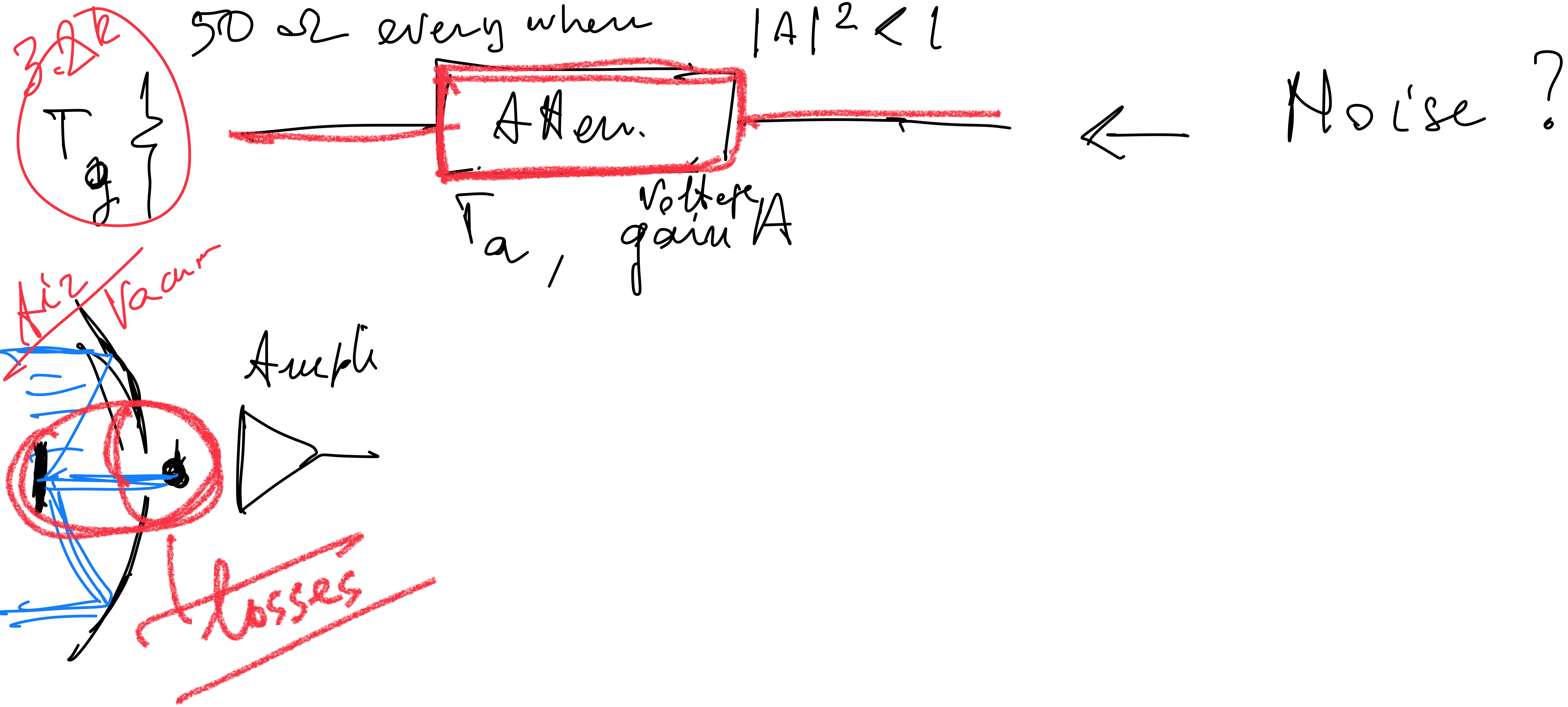
$$P_{1-2} = kT_1 B$$

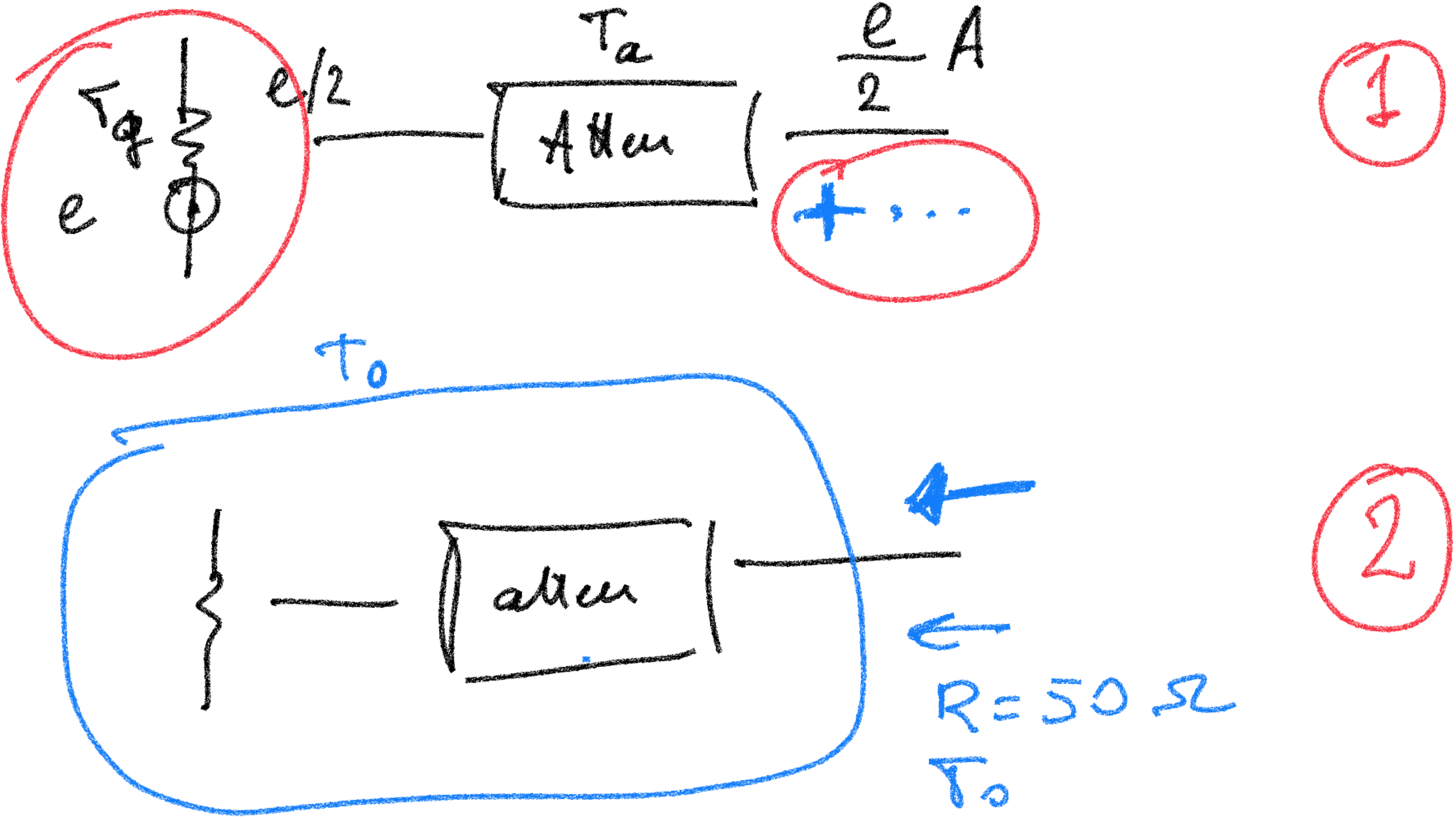
$$P_{2-1} = kT_2 B$$

$$P = k(T_1 - T_2) B$$

$$\boxed{1 \to 2}$$

300 - 77


3.2K
T_g
50 Ω everywhere
$|A|^2 < 1$
Atten.
T_a , voltage gain A
Noise ?
~~Air~~
Vacum
Ampli
Losses



T_g
e
e/2
T_a
Atten
e/2 A
+ , . .
1
T_0
atten
R=50 Ω
T_0
2

End of Lecture #9

#10 Thursday, October 22, 2020

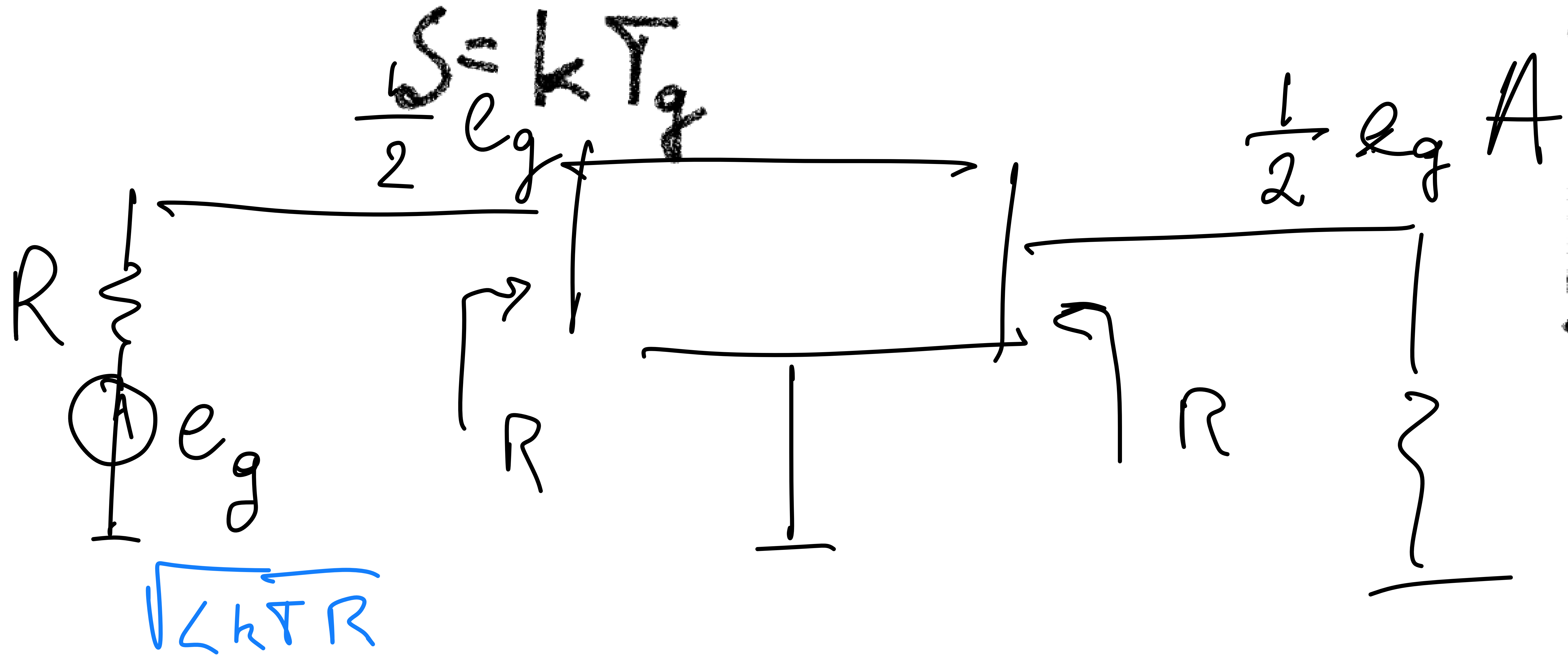
1.5 Hours

Meaning / Usefulness

$A^2 < 1$

$A = \frac{1}{2}$

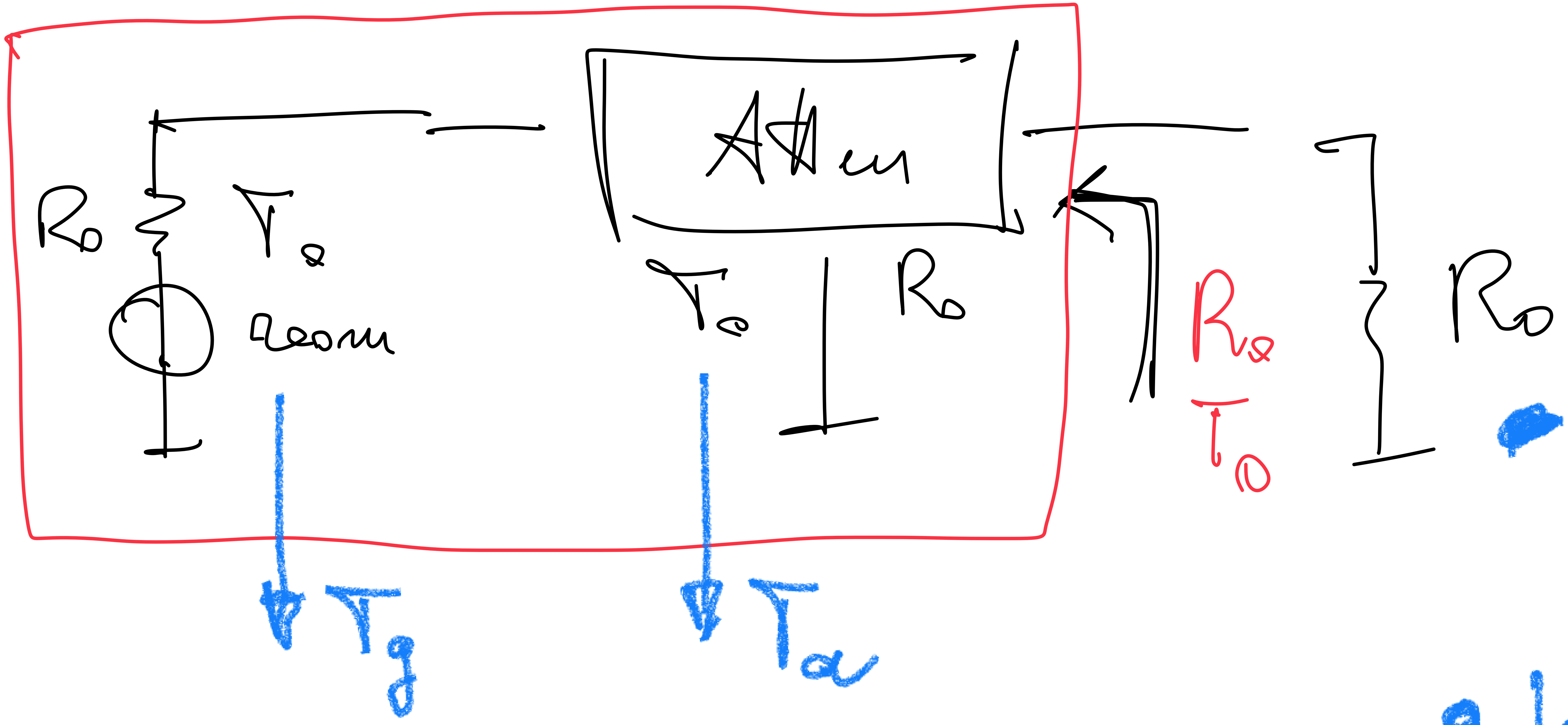
$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega$



$$\boxed{S = kT_g A^2}$$

The attenuator does not know the nature of e_g.

e_g is attenuated as any signal



kT_0

• $kT_0 A^2 + ?$

$\downarrow$

• $kT_0 (1-A^2)$

Output noise PSD

$$kT_g A^2 + kT_a (1-A^2)$$

$A^2 \rightarrow 1$ low loss

$T_{out} \simeq T_g$

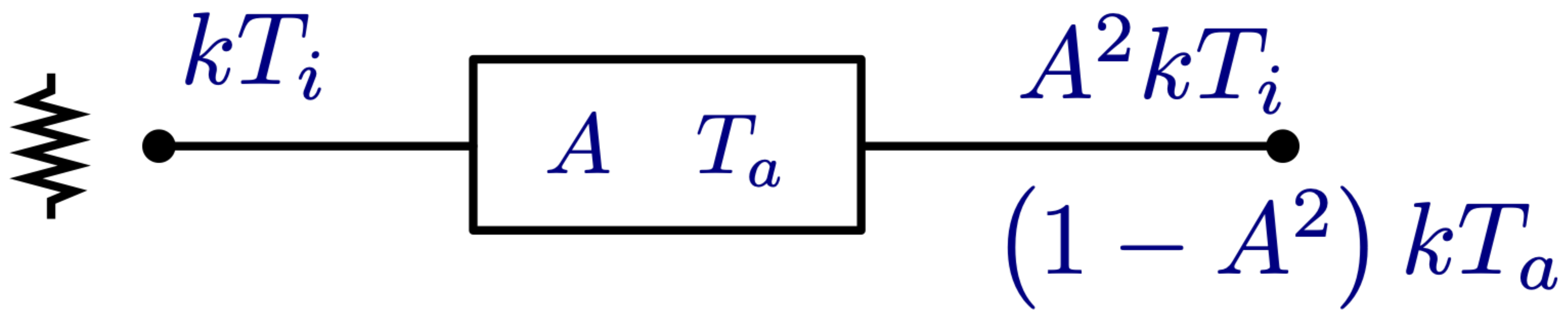
$A^2 \rightarrow 0$ high loss

$T_{out} \simeq T_a$

Noise under test → Signal

Noise of the instrument → Noise
(atten., amplifier, ADC)

Thermal Noise of a Dissipative Device



$$S(f) = A^2 kT_i + \left(1 - A^2\right) k\boxed{t_a}$$

upper case

$\longleftarrow T_a$

- Noise contribution of the input resistor
  - The attenuator makes no difference between “noise” and “signal”
  - The input signal is “amplified” by a factor $A^2 < 1$
- Noise contribution of the attenuator
  - At uniform temperature T the sum of the contributions must be kT
  - The input contributes A^2kT
  - The attenuator contributes the complement $(1–A^2)kT$
- The factors A^2kT and $(1–A^2)kT$ do not depend on temperature

Shot Noise

Matter
energy
charge
←
quantized

- Poisson process: emission at random time
  - No space correlation, no time correlation
  - Exponential PDF, $p = \exp(-at)$
  - Variance equals average, $\sigma^2 = \mu$Stream of electrons
- Applies to any kind of events: radioactive decay, photons, atoms, chemical reaction, whatever
- Photon stream, $P = \Phi E,\ E = hf = kc/\lambda$
- Current
  - Average $\langle I \rangle = q\Phi$
  - PSD $S_I(f) = 2q\langle I \rangle$
- White noise, cutoff frequency
- Example: photocurrent

Poisson Process

ensemble of random sources

- radioactivity
- chemical reaction
- emission of thermal photons
- electrons in vacuum lamp/ diode, transistor/

emission
- random time
- no memory

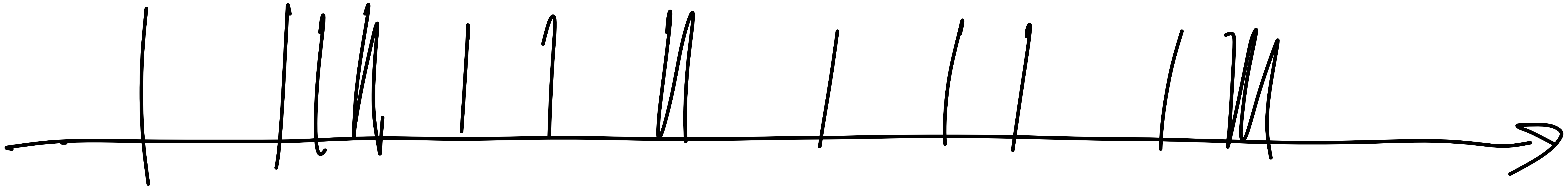
PDF e^{-at}

---

a avg no of particle in a unit of time

n cells

$\Phi = a n$



F. Vermotte

$$\boxed{\mu = \sigma^2}$$

Maximum Variance for a given μ.

electrons q

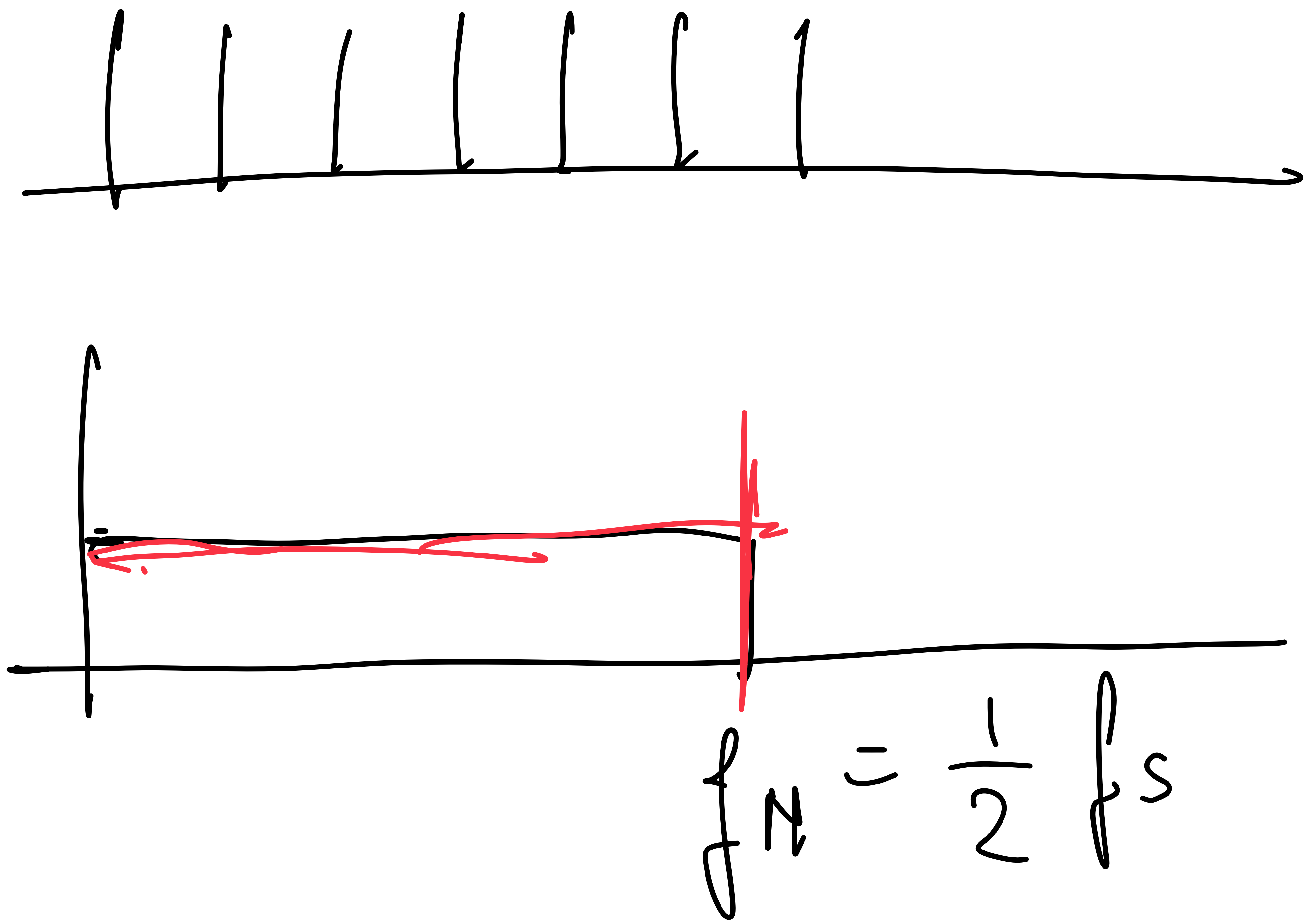
$2q^2\Phi A$

$$S_I = 2q\langle I\rangle \quad A^2/Hz$$

White noise

Φ no of particle per second

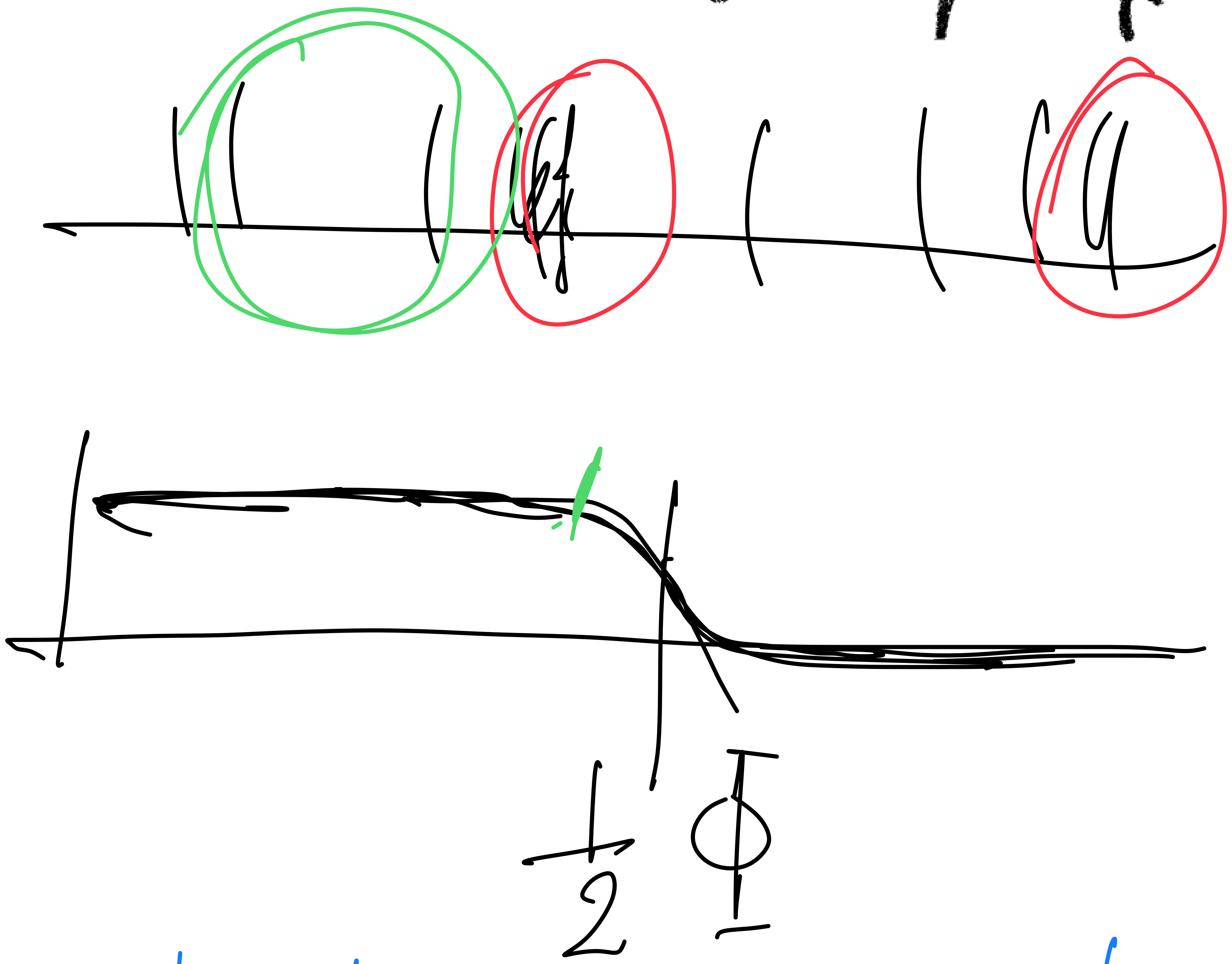
Coherent sampling



$f_N = \frac{1}{2} f_S$

Several Nyquist zones

Random sampling

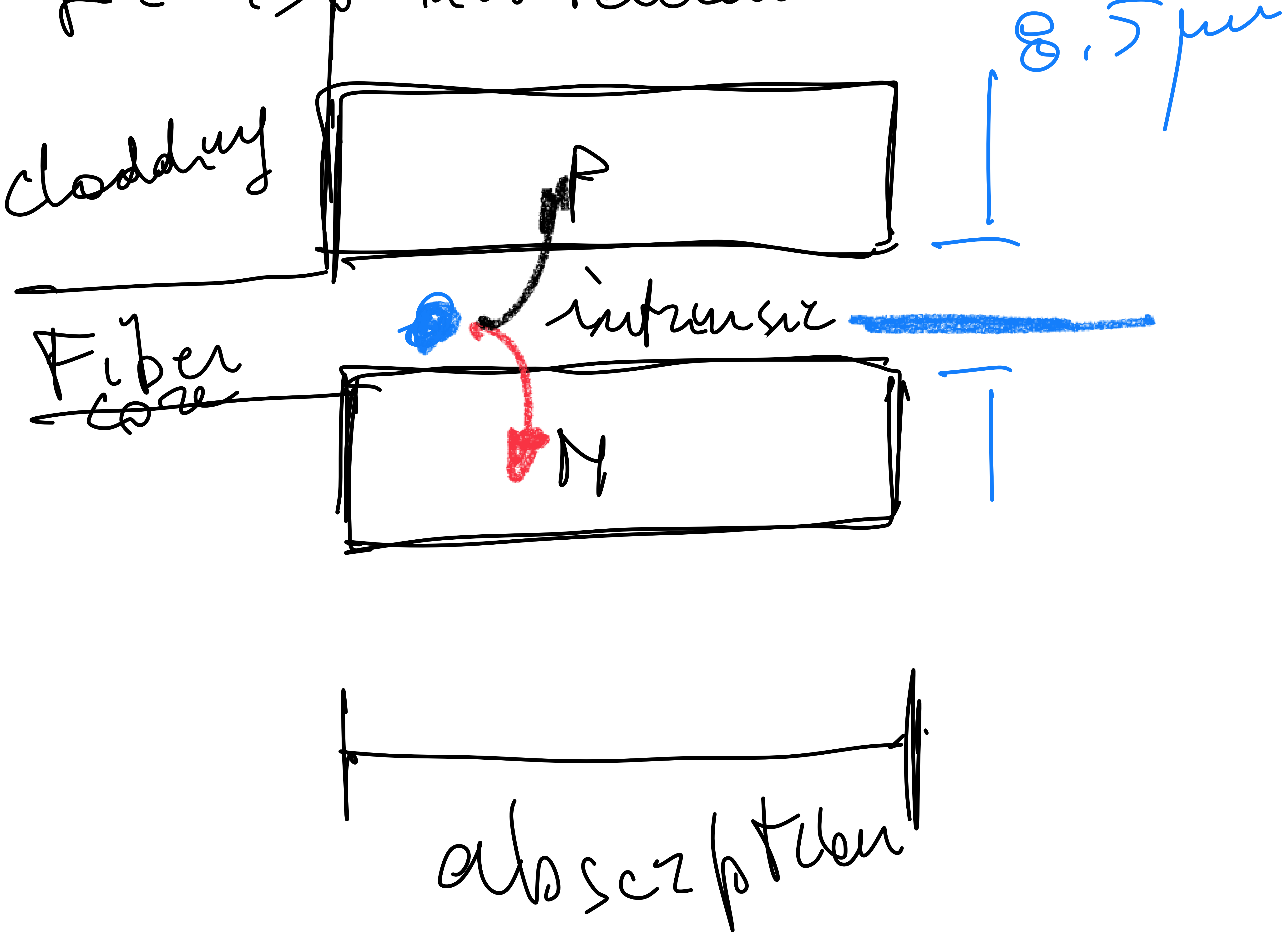


$\frac{1}{2}\Phi$

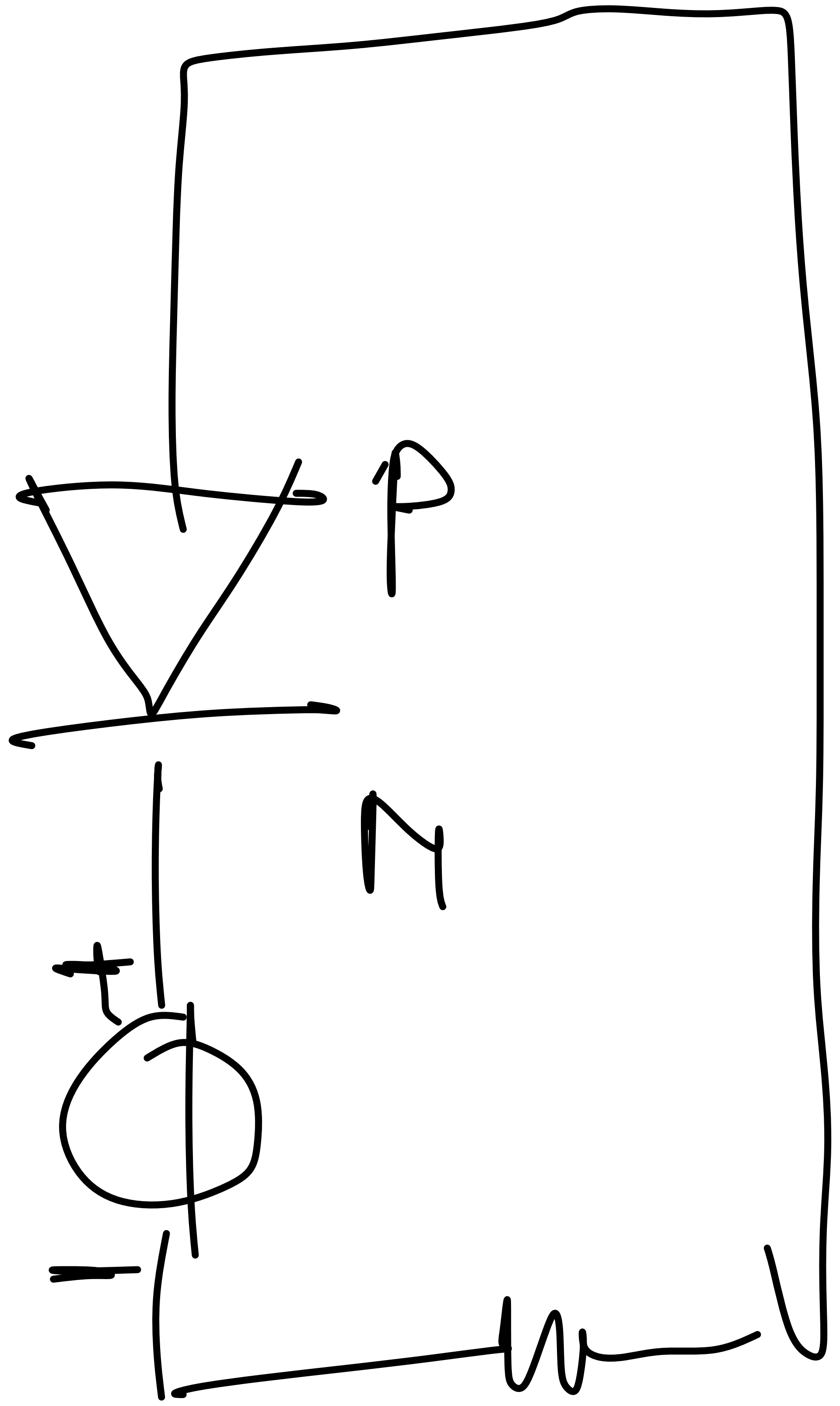
The term "Nyquist zone" has no meaning

Photodiode Noise

Fast diode
for 1550 nm Telecom



photon $\rightarrow$ e^- h^+



equiv. to

q

Ramo Theorem

Quantum efficiency

$$I = \eta \Phi q$$

$P = 1\,\mu W$

$\lambda = 1550\,nm$

$(f = 194\,THz)$

$I = ?$

S_I

$\eta = 0.75$

$E = hf$

$$\Phi = \frac{P}{E} = \frac{P}{hf} = \frac{P}{hc/\lambda}$$

$$\phi = \frac{10^{-6}}{6.6\times10^{-34}\cdot 2\times10^{14}} = 7.8\times10^{14}$$

$$\langle I\rangle = \eta\,\Phi\,q \approx 9.4\times10^{-5} \quad 94\,\mu A$$

$$S_I = 2q\langle I\rangle = 3\times10^{-23}\,A^2/Hz$$

$$(5.5\,pA/\sqrt{Hz})$$

$$\left.\begin{array}{l}\text{Signal} \propto P_{opt}^2 \\ \text{Noise} \propto P_{opt}\end{array}\right\}\ \text{threshold}$$

End of lecture

22-X-2020

#10

End of Lecture #10

#11 Thursday, November 5, 2020

1.5 Hours

Exam?

17 nov

16:30 – 18
(18:30)

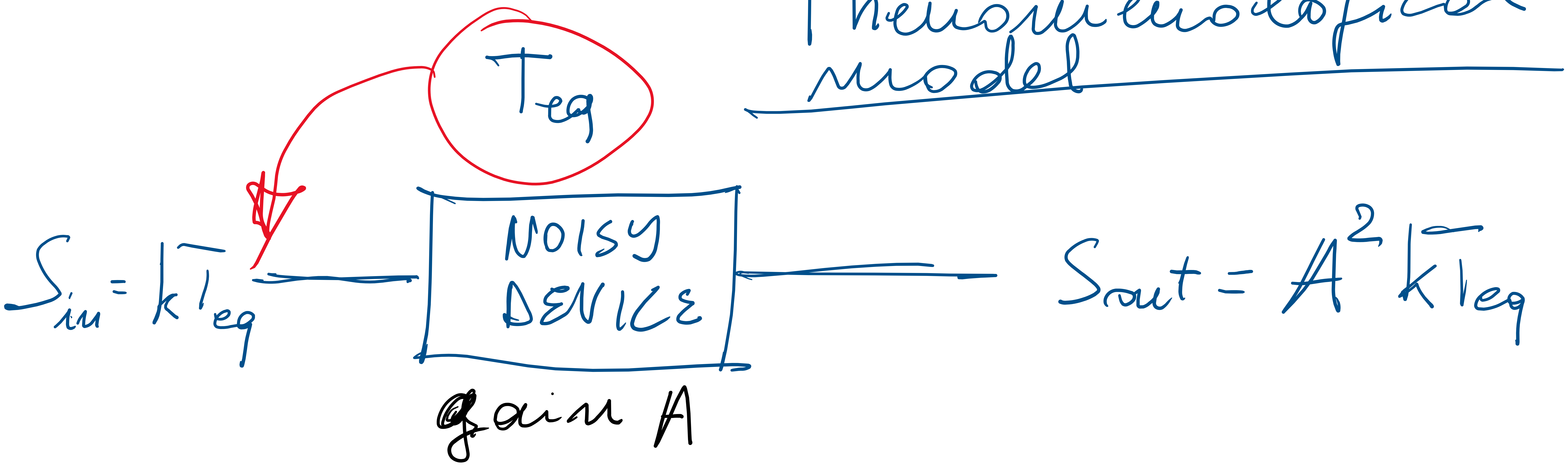
Presentiel ?

Pour moi, en ligne si
(et seulement si)
Vous avez une webcam

J'attend des instructions du doyen

Equivalent Noise Temperature

Phenomenological model



$S_{in} = kT_{eq}$

$S_{out} = A^2 kT_{eq}$

T_{eq} → the noise referred to the input is

$$S(f) = kT_{eq}$$

(thermal noise)

A : Voltage gain

A^2 power gain

Equivalent Noise Temperature

The case of our amplifier

Thermal noise

$$S(\nu) = \frac{h\nu}{e^{h\nu/kT}} \qquad S(\nu) = N_t \quad \text{constant for } h\nu \ll kT \qquad N_t = kT$$

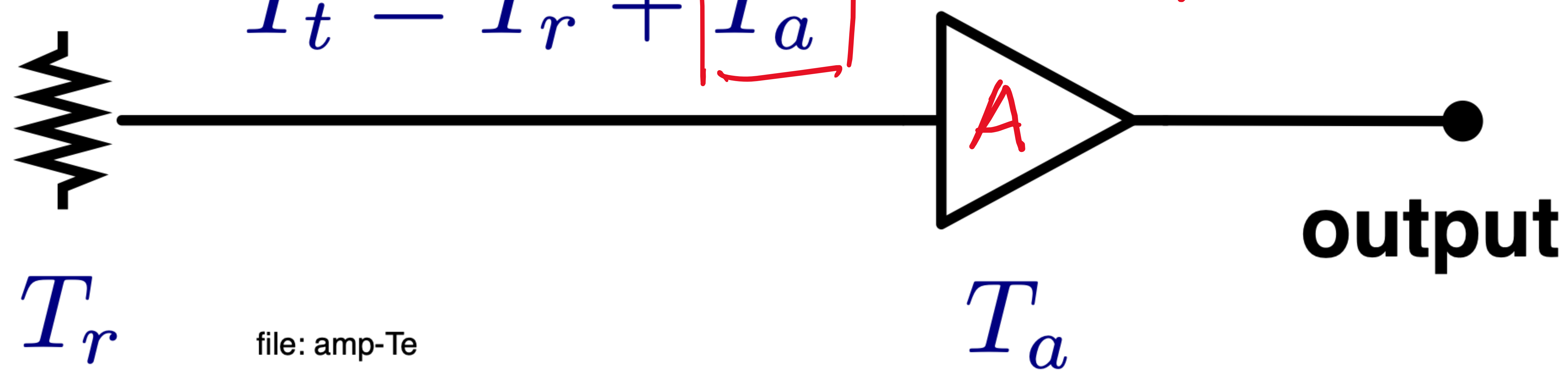
kT

Physical dimension

W/Hz ≡ J

T_t = total noise at the input

$$T_t = T_r + \boxed{T_a} \rightarrow T_{eq}$$



$k(T_a + T_r)$

- Warning: the noise temperature a radio-engineering concept
  - The physical nature of noise does not matter
  - Often misleading in optics: the shot noise contributes to the equivalent temperature

Equivalent temperature $\quad T_a$ defined by $N_t = k(T_a + T_r)$

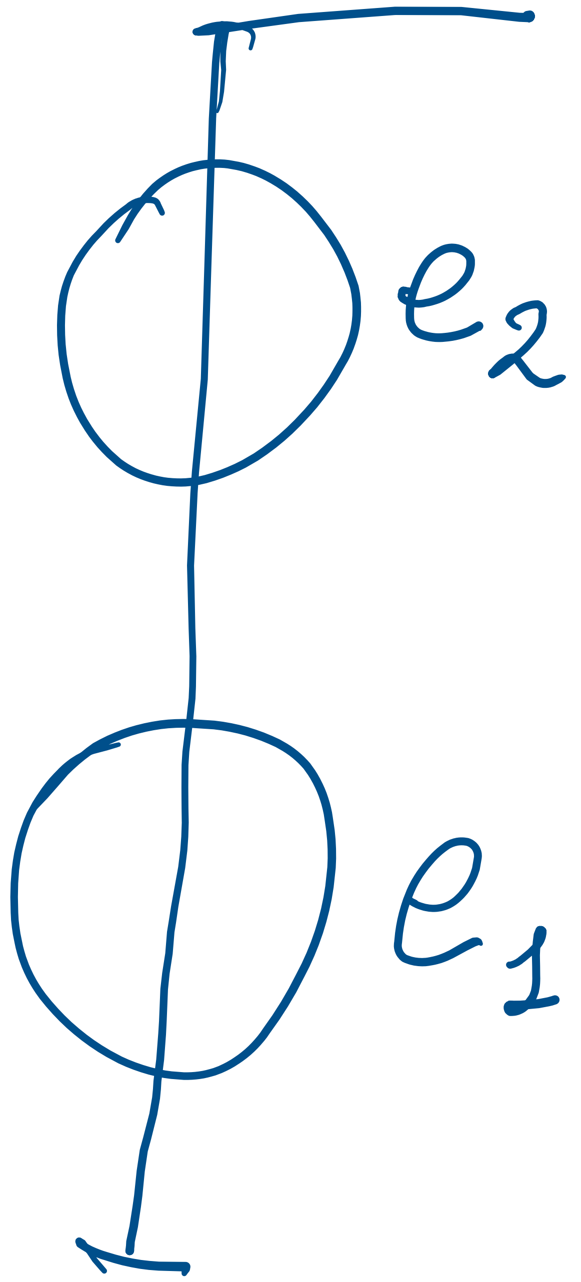
T_a is the equivalent noise temperature of the amplifier

defined in specified conditions (physical temperature and input resistance)

Adding Temperatures?

In general physics, we cannot add temperatures [intensive]

Electrical noise:
- Teq is not a temperature in a strict sense
- Teq is a way to express the variance of a random process.

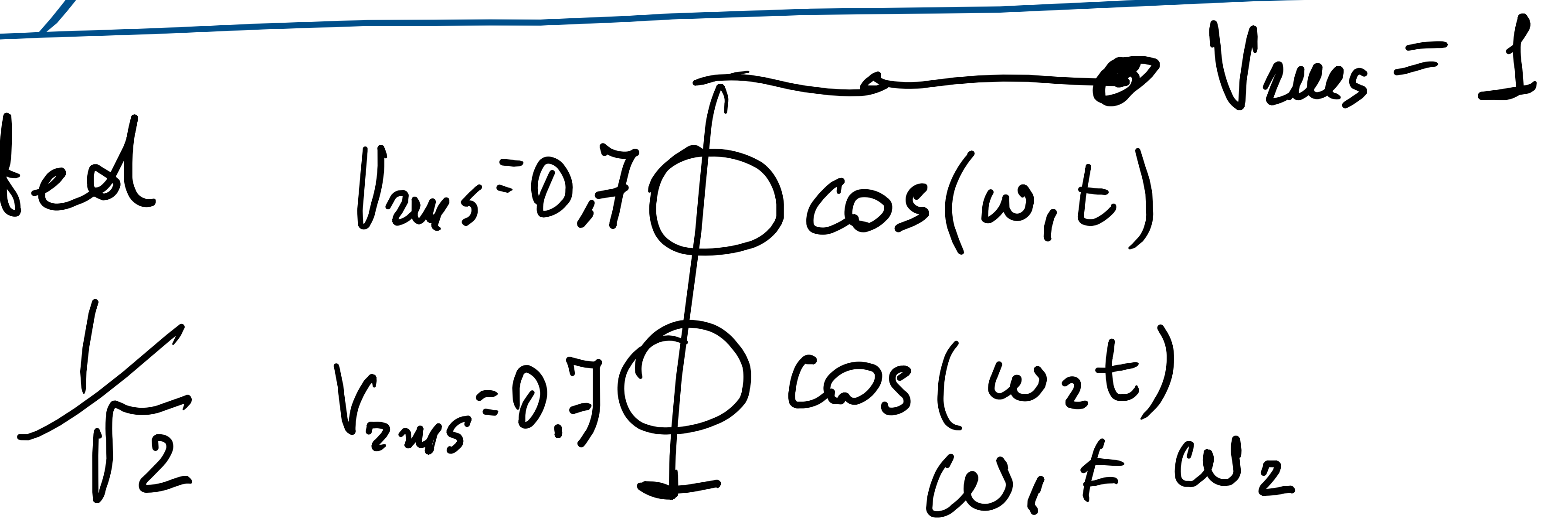


$$e = e_1 + e_2$$

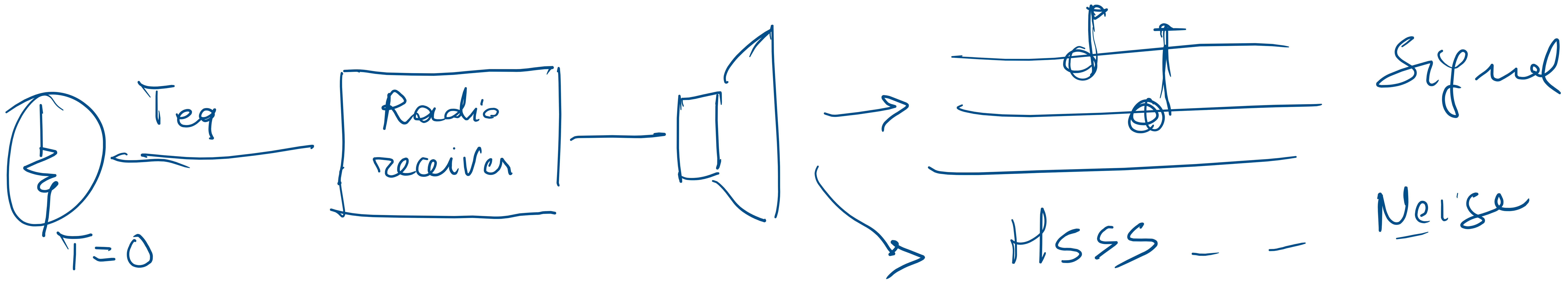
$$\sigma^2 = \sigma_1^2 + \sigma_2^2$$

(statistically independent) ←

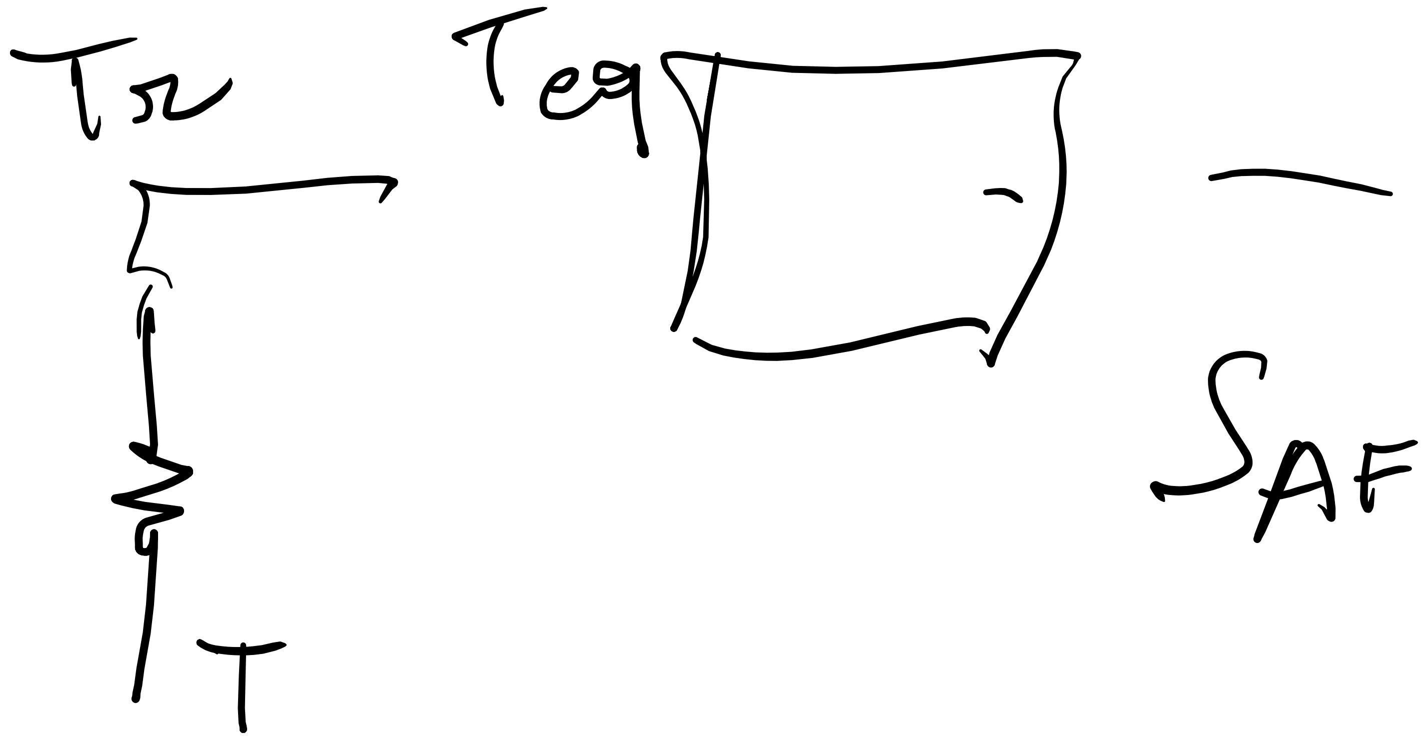
the necessary condition is weaker: uncorrelated signals



T_{eq} Is A Radio-Engineering Concept



$$S_{AF} = A^2 kT_{eq}$$



(close to room temperature

$$S_{AF} = A^2 k(T_s + T_{eq})$$

OPTICS

Shot noise > Thermal noise

Teq includes both Thermal & Shot

folks from optics are (sometimes) lost

A warning to folks from optics

Noise Factor

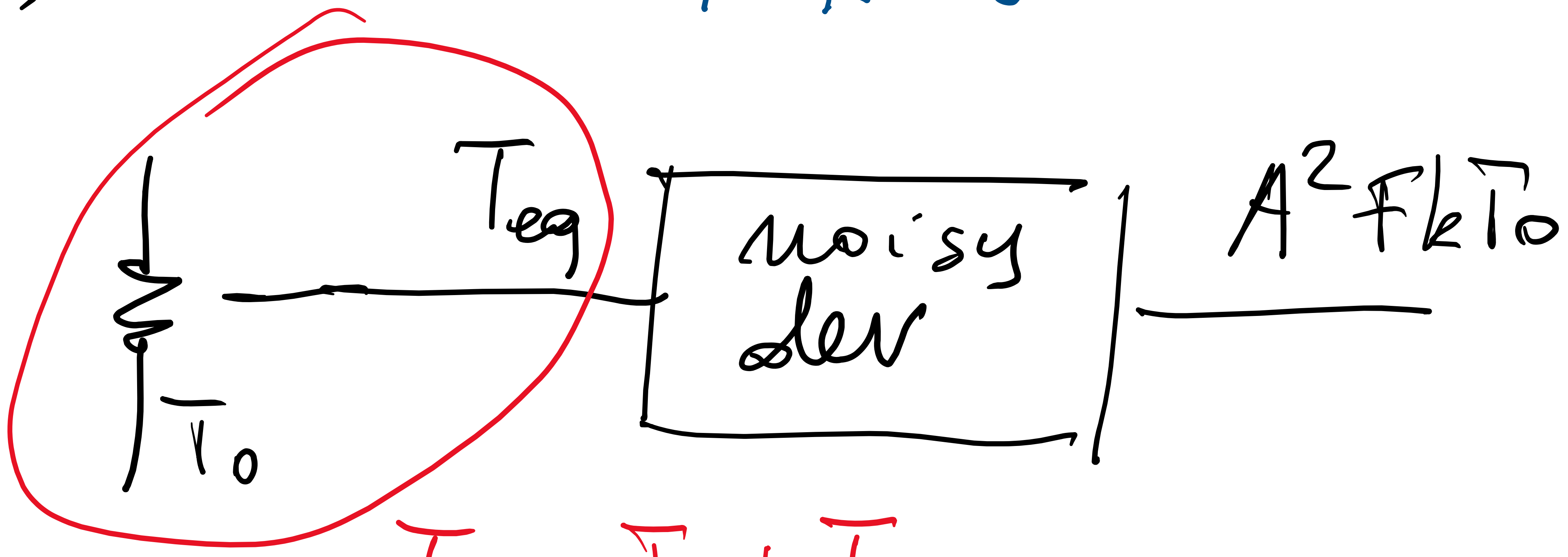
Most practical cases

$T \sim T_{room}$

(10–25 °C)

Total noise

$F k T_0$



$T_t = T_0 + T_{eq} \longrightarrow F T_0$

Reference Temperature

$T_0 = 290\ K$

17 °C

1 – reasonable value for T_{room}

2 – $kT_0 = 4 \times 10^{-21}$

Noise Figure

$$N = 10 \log_{10}(F) \quad dB$$

Noise Factor and Noise Figure

In all radio engineering (including digital RF systems) the noise is given as NF

A good amplifier
NF = 1 dB

Q: S = ... W/Hz referred at the amplifier input

$$NF = 10\log(F) \longrightarrow F = 10^{NF/10}$$

$$F = 10^{0.1} \simeq 1.25$$

$$S = \underset{1.25}{F}\ \underbrace{kT_o}_{}\ 4\times10^{-21}\ W/Hz$$

$$= 5\times10^{-21}\ W/Hz$$

$$T_t = 1.25T_o$$

T_o → input
$0{,}25T_o$ → ampli seul

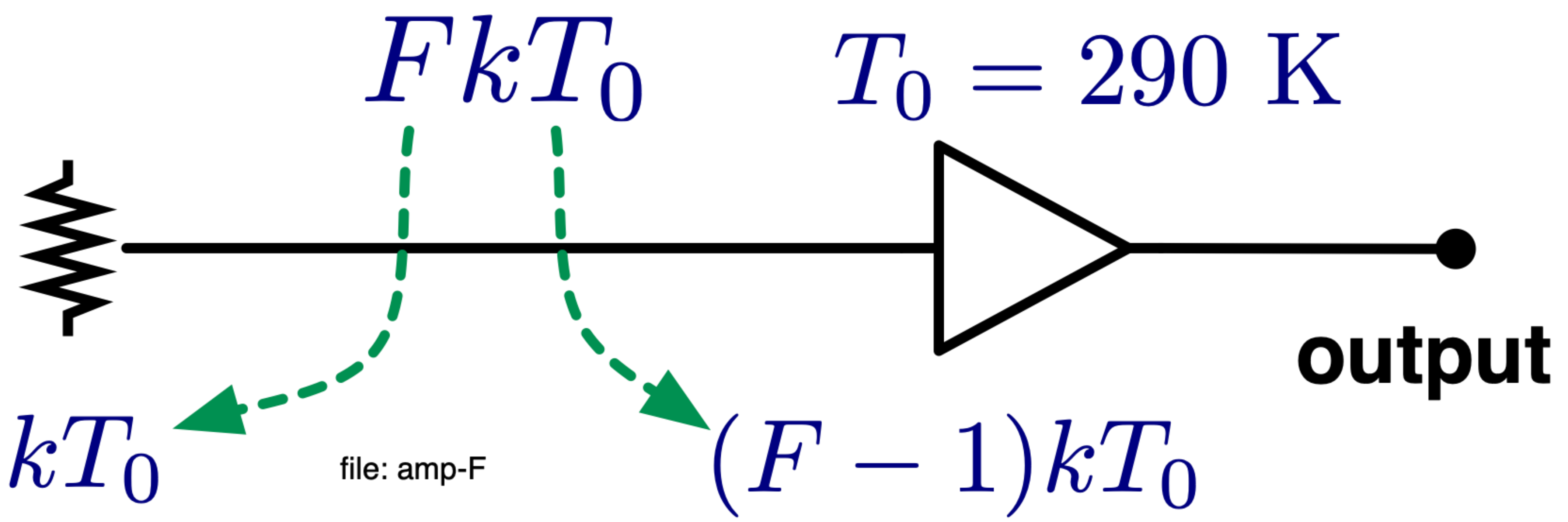
Noise Factor & Noise Figure

Noise Factor $$F = \frac{\mathrm{SNR(out)}}{\mathrm{SNR(in)}}$$ general definition

Int'l standard

this definition is equivalent To the definition given before

$$\mathrm{NF} = 10 \log_{10} F$$



Assume that the whole circuit is at the reference temperature T0 = 290 K (17 ºC)

The total noise referred to the amplifier input is FkT0

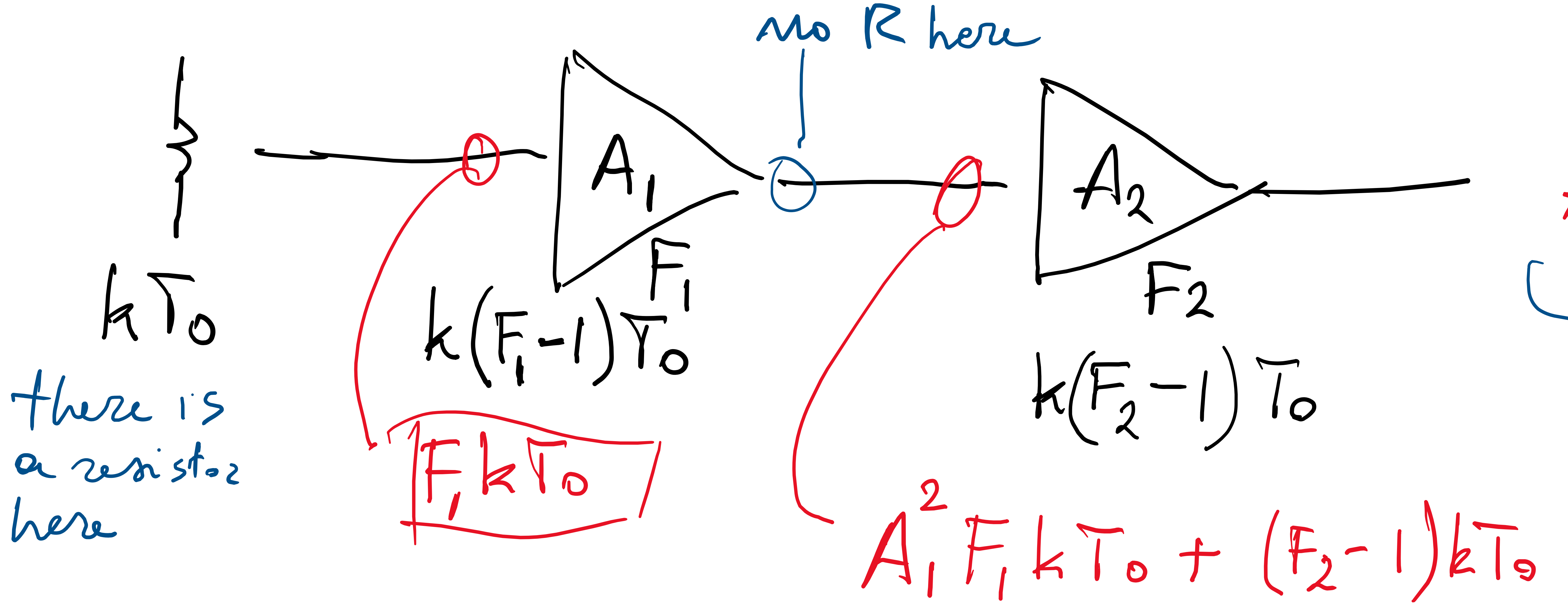
amplifiers and RF/µw devices

$$FkT_0 = kT_e = k(T_a + T_0) \qquad T_0 = 290\ \mathrm{K}$$

$$F = \frac{T_a + T_0}{T_0} \quad \text{and} \quad T_a = (F-1)T_0$$

Warning: the noise factor/figure is a radio-engineering concept, can be misleading in optics

Cascaded devices -- The Friis Formula



F

$$\underbrace{A_1^2 A_2^2 F_1 kT_0 + A_2^2 (F_2-1)kT_0}$$

$$\div A_1^2 A_2^2$$

Total noise referred to the input

$$S = F_1 kT_0 + \frac{F_2-1}{A_1^2} kT_0$$

$$= \left(F_1 + \frac{F_2-1}{A_1^2}\right) kT_0$$

Consequences

$$F = F_1 + \frac{F_2-1}{A_1^2}$$

practical implication

The total noise is determined by the 1st amplifier (take with care)

3 ampli (just an academic excuse)

$$F = F_1 + \frac{F_2-1}{A_1^2} + \frac{F_3-1}{A_1^2 A_2^2} \text{ etc.}$$

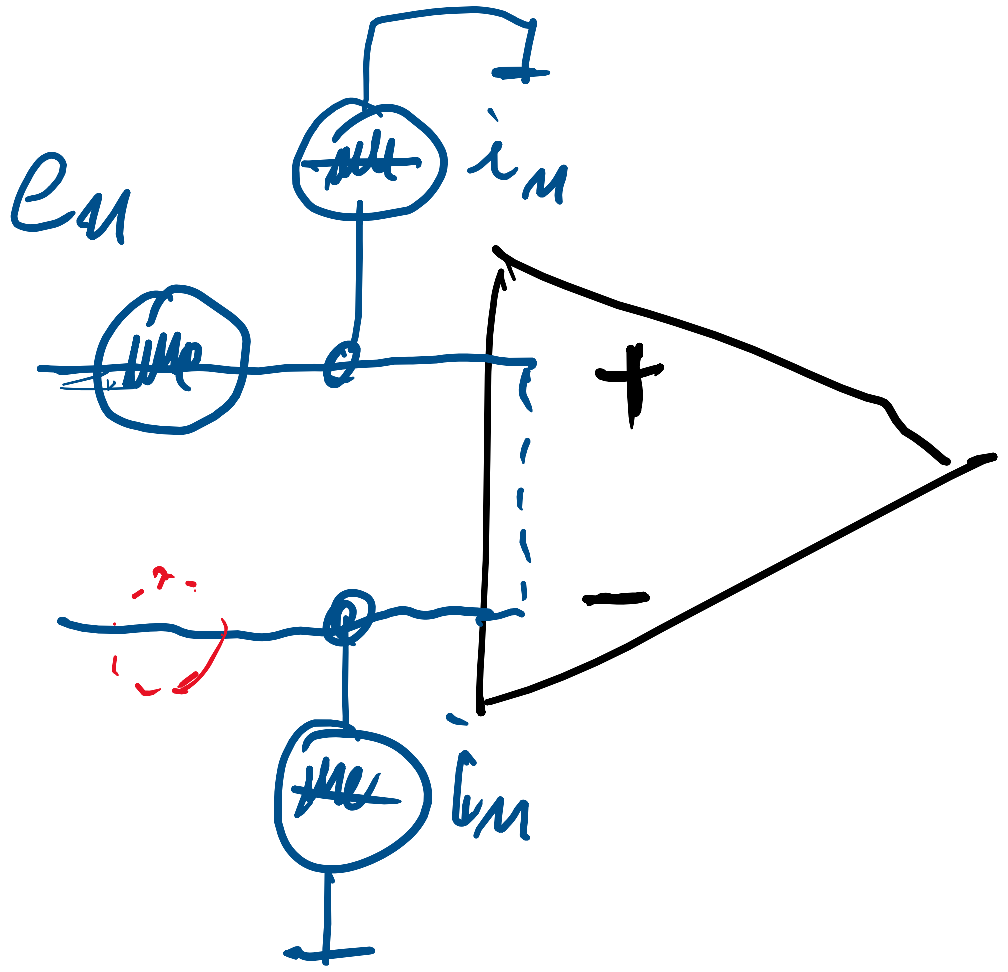
The Rothe Dahlke Model

$$\sqrt{S_v} \longrightarrow e_n \ \ \mathrm{nV}/\sqrt{\mathrm{Hz}}$$

$$\sqrt{S_I} \longrightarrow e_n \ \ \mathrm{pA}/\sqrt{\mathrm{Hz}}$$

→ noise in Operational amplifier

→ also valid in RF and with converters

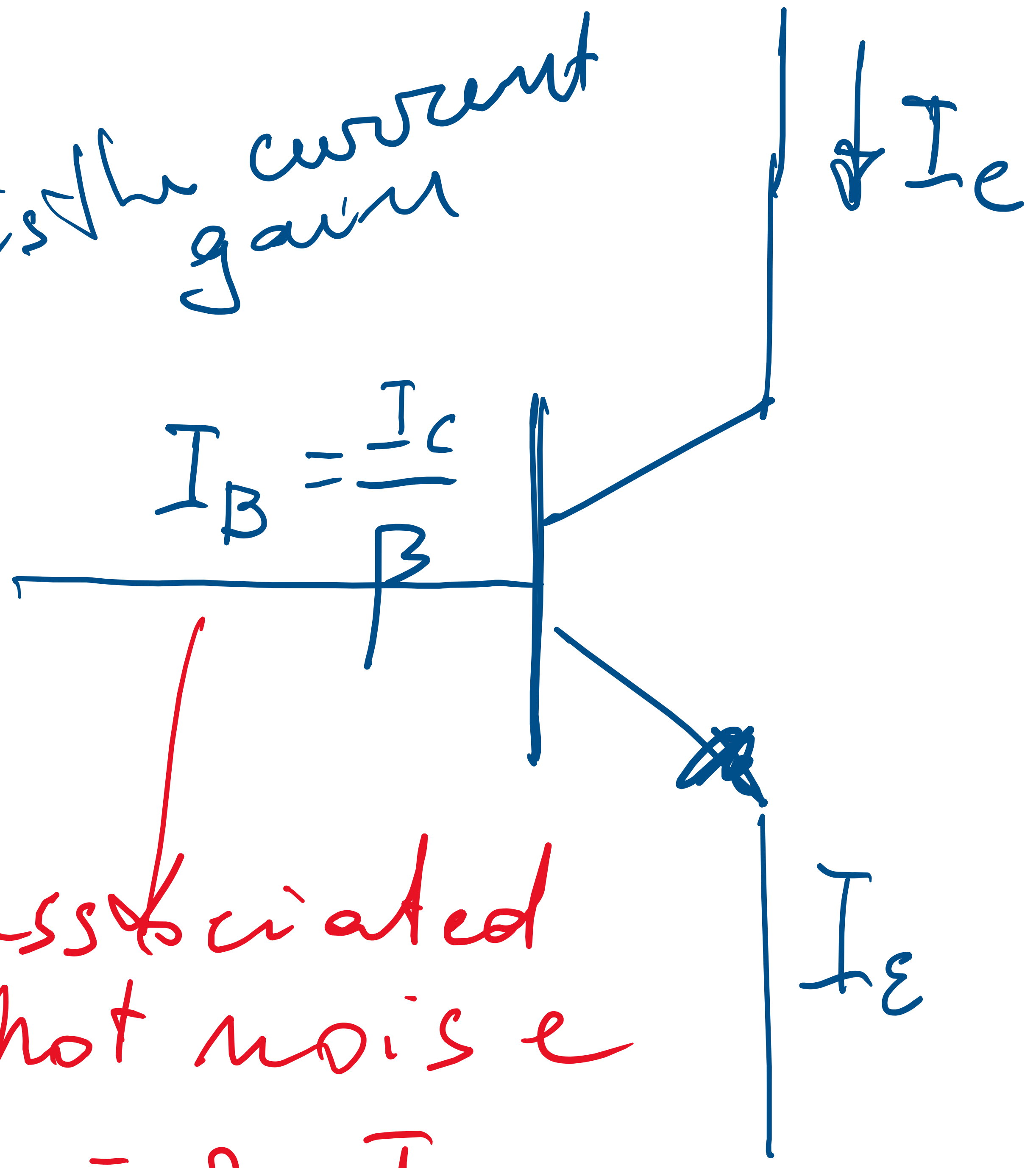


virtual short

$V_+ = V_-$

e_n is either at the +/- input

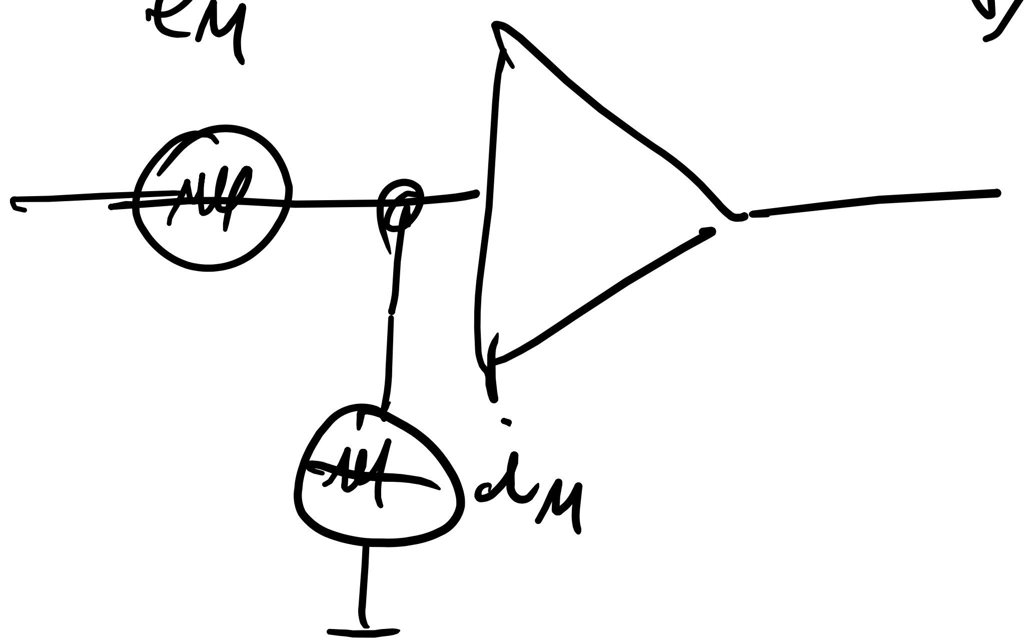
β is the current gain



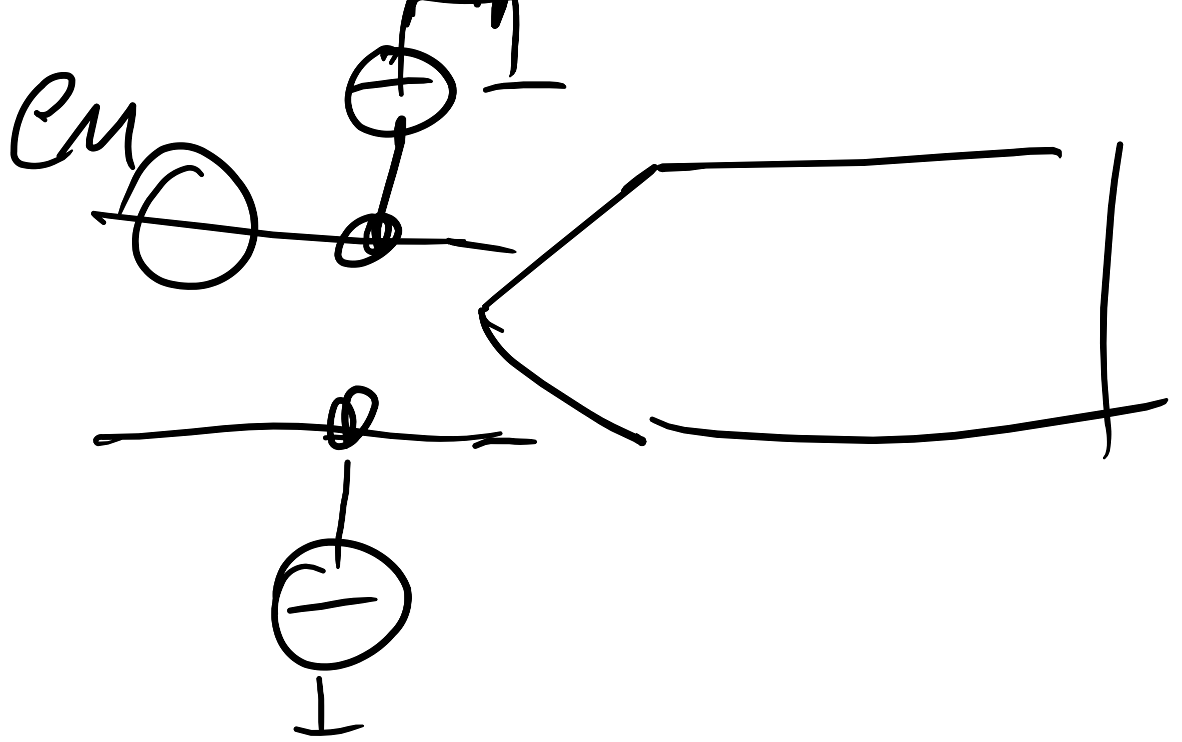
associated shot noise

$S_I = 2qI_B$

Microwave/RF ampli
Single-input ADCs

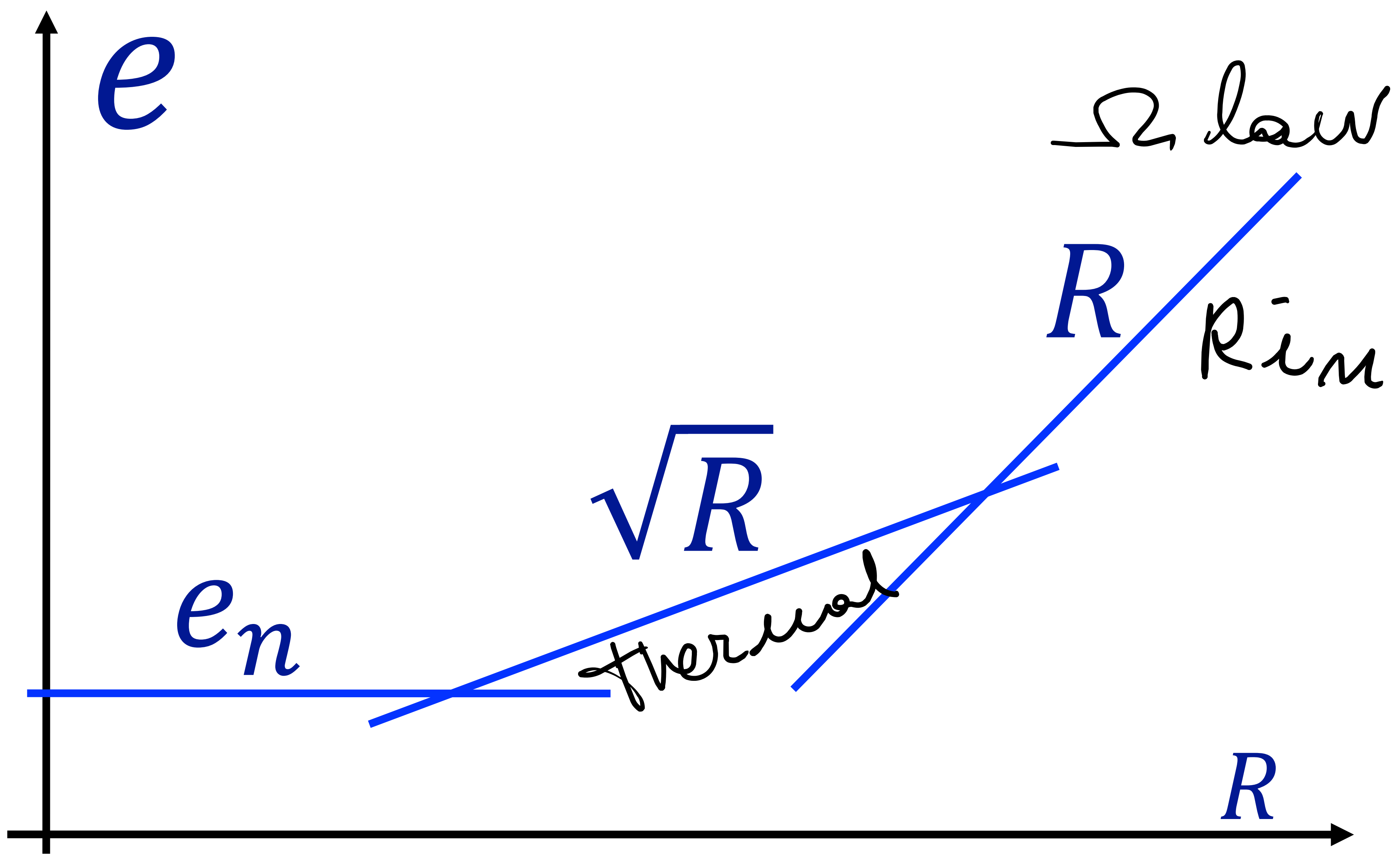


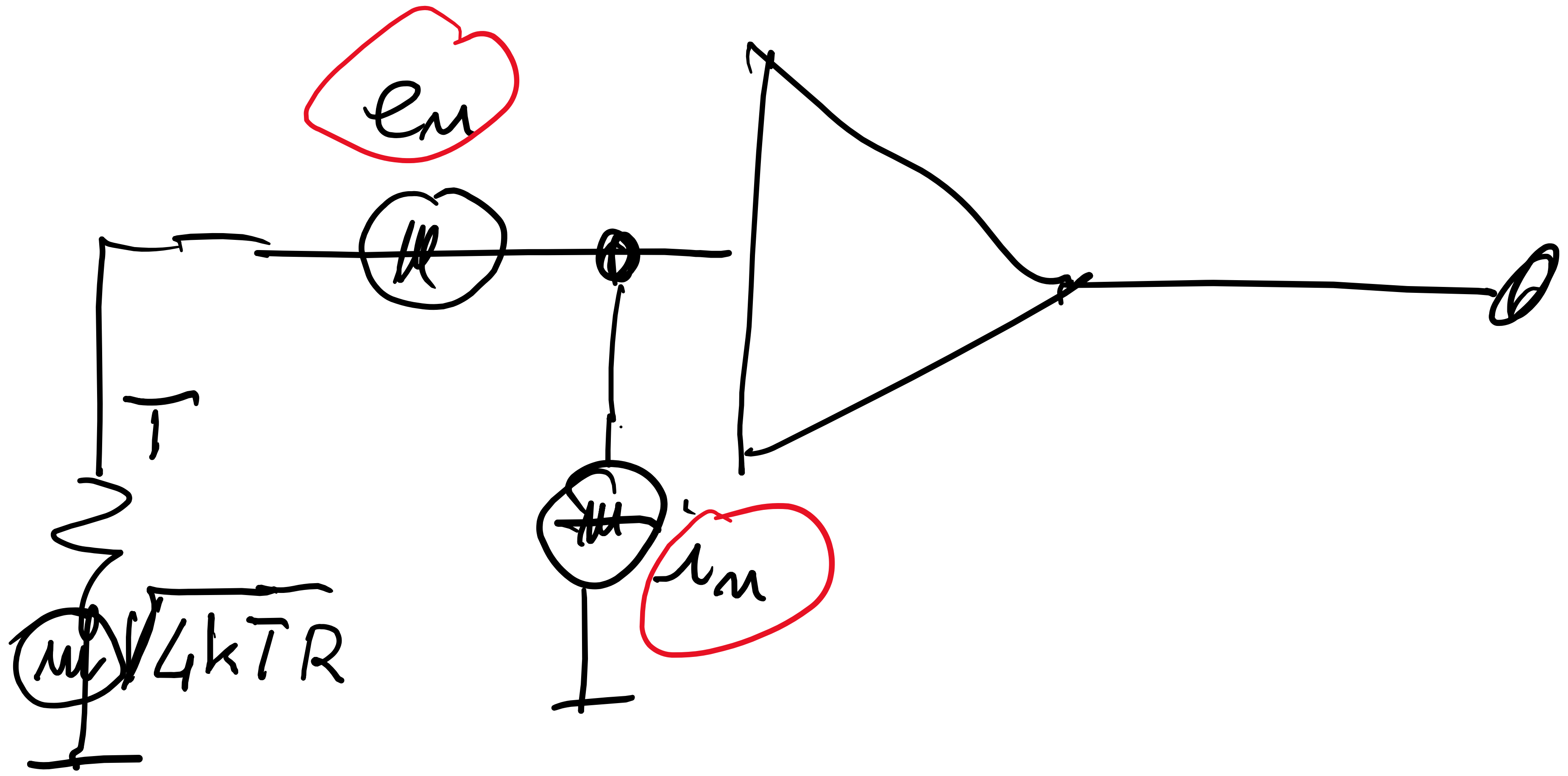
Differential input ADCs



Rothe H, Dahlke W - Theory of Noisy Fourpoles - Proc IRE 44(6) p.811-818, June 1956

Adding a Resistor to the RD Model





$$e^2 = e_n^2 + R^2 i_n^2 + 4kTR$$

$$e = \sqrt{\quad}$$

const.

∝ R

End of Lecture #11

Impedance-Matching Concepts

- Maximum power transfer
  - The impedance matching we are used to

$$Z_g = Z_L^*$$

- Lowest noise
  - Straightforward consequence of the Rothe Dahlke model

$$R_b = e_n / i_n$$

  - …but you loose signal
- Highest SNR
  - Choose the best compromise between the above

Flicker Noise

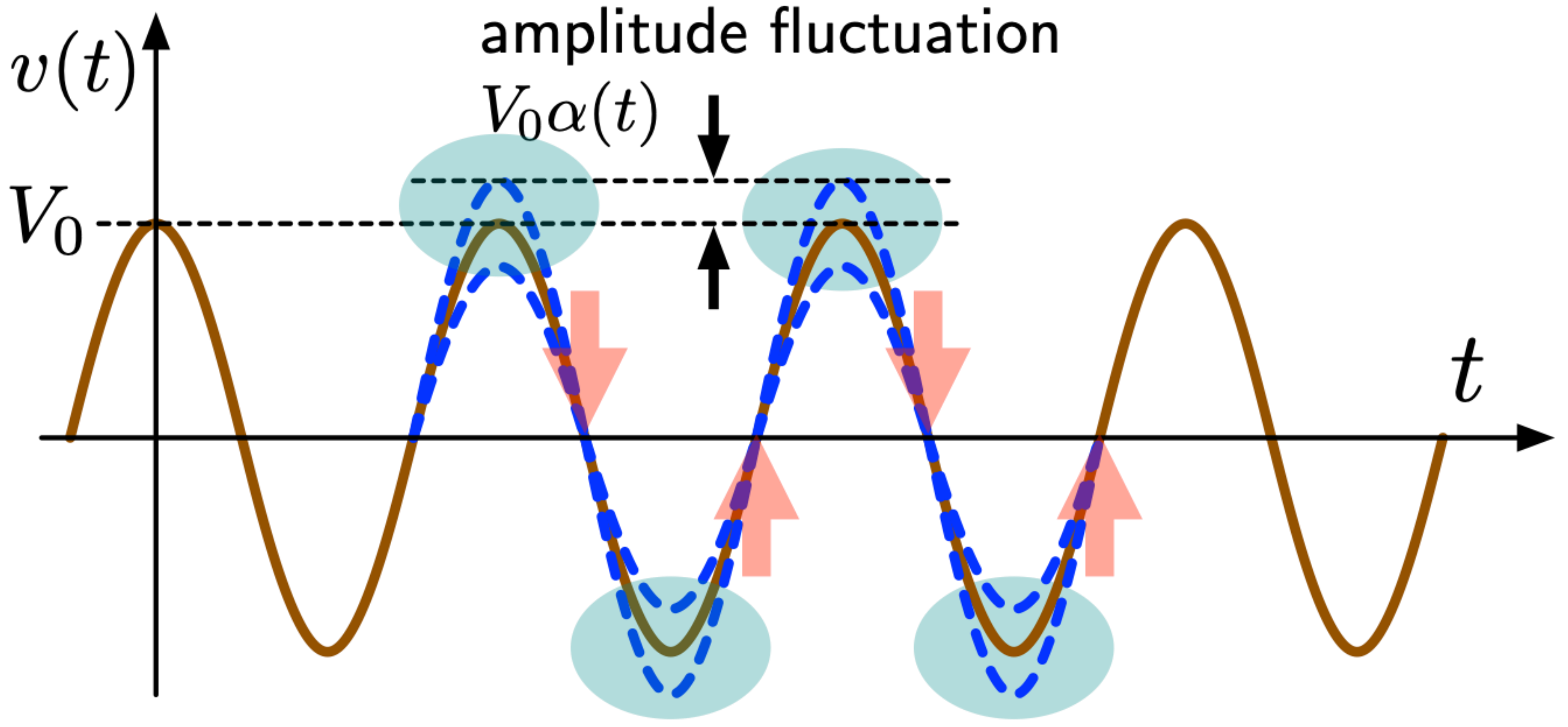
Flicker Noise

Flicker Noise

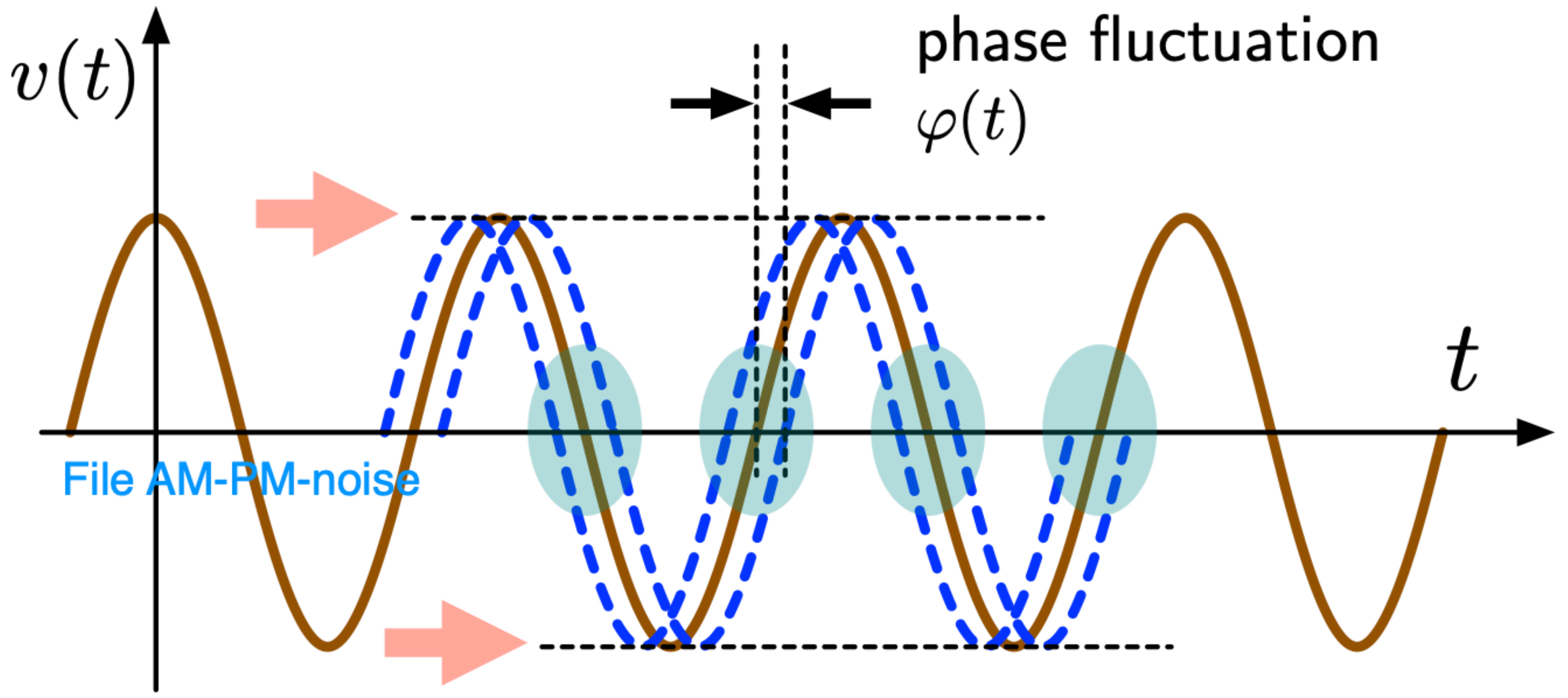
Phase Noise

Phase and Amplitude Noise

amplitude fluctuation



phase fluctuation



polar coordinates $$v(t) = V_0\left[1+\alpha(t)\right]\cos\left[\omega_0 t + \varphi(t)\right]$$

Cartesian coordinates $$v(t) = V_0\cos\omega_0 t + n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t$$

under low noise approximation

$$|n_c(t)| \ll V_0 \quad \text{and} \quad |n_s(t)| \ll V_0$$

It holds that

$$\alpha(t) = \frac{n_c(t)}{V_0} \quad \text{and} \quad \varphi(t) = \frac{n_s(t)}{V_0}$$

Noise in Digital Circuits

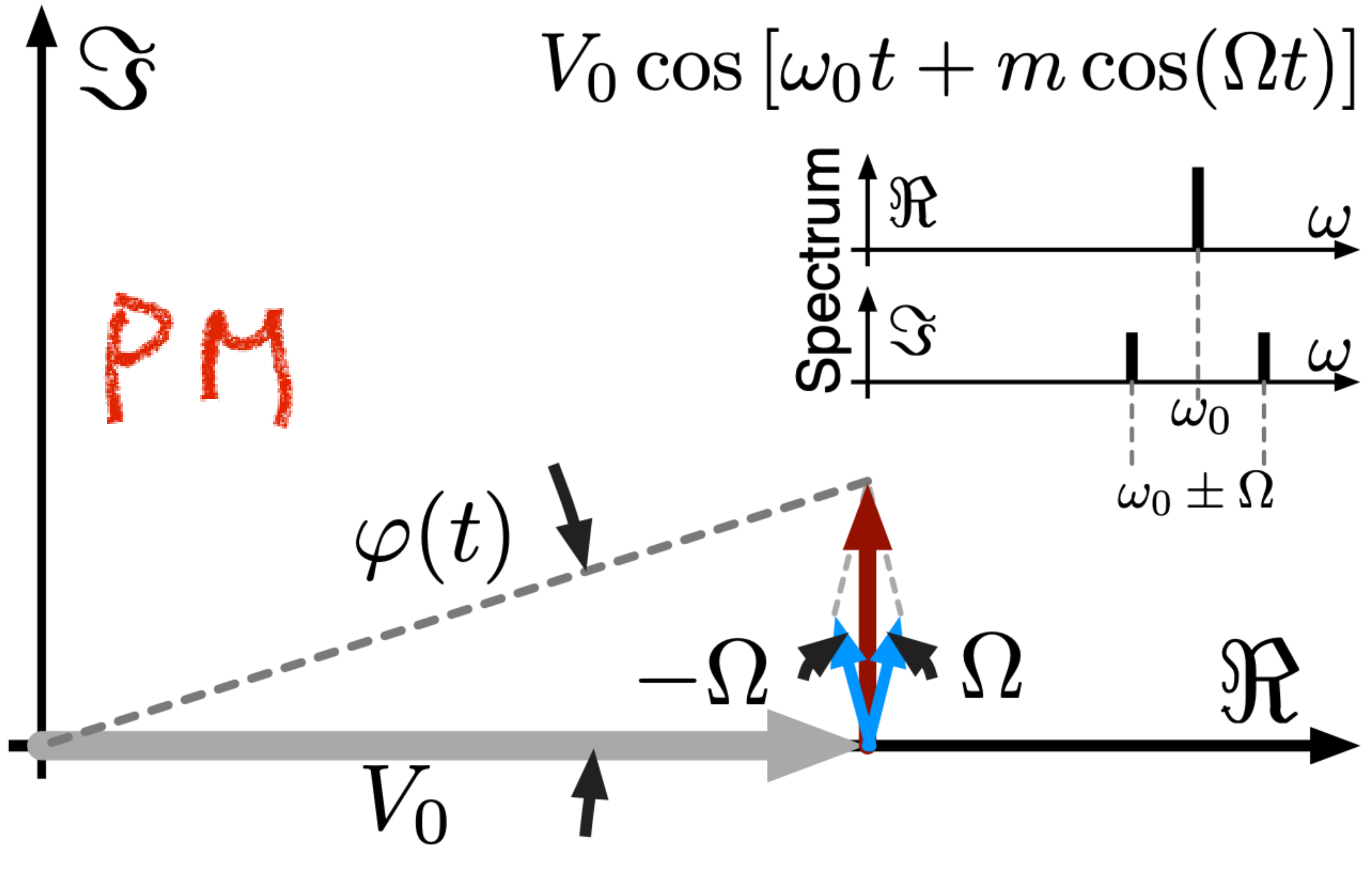
Noise in Analog/Digital Circuits

Modulation and Sidebands

m modulation index

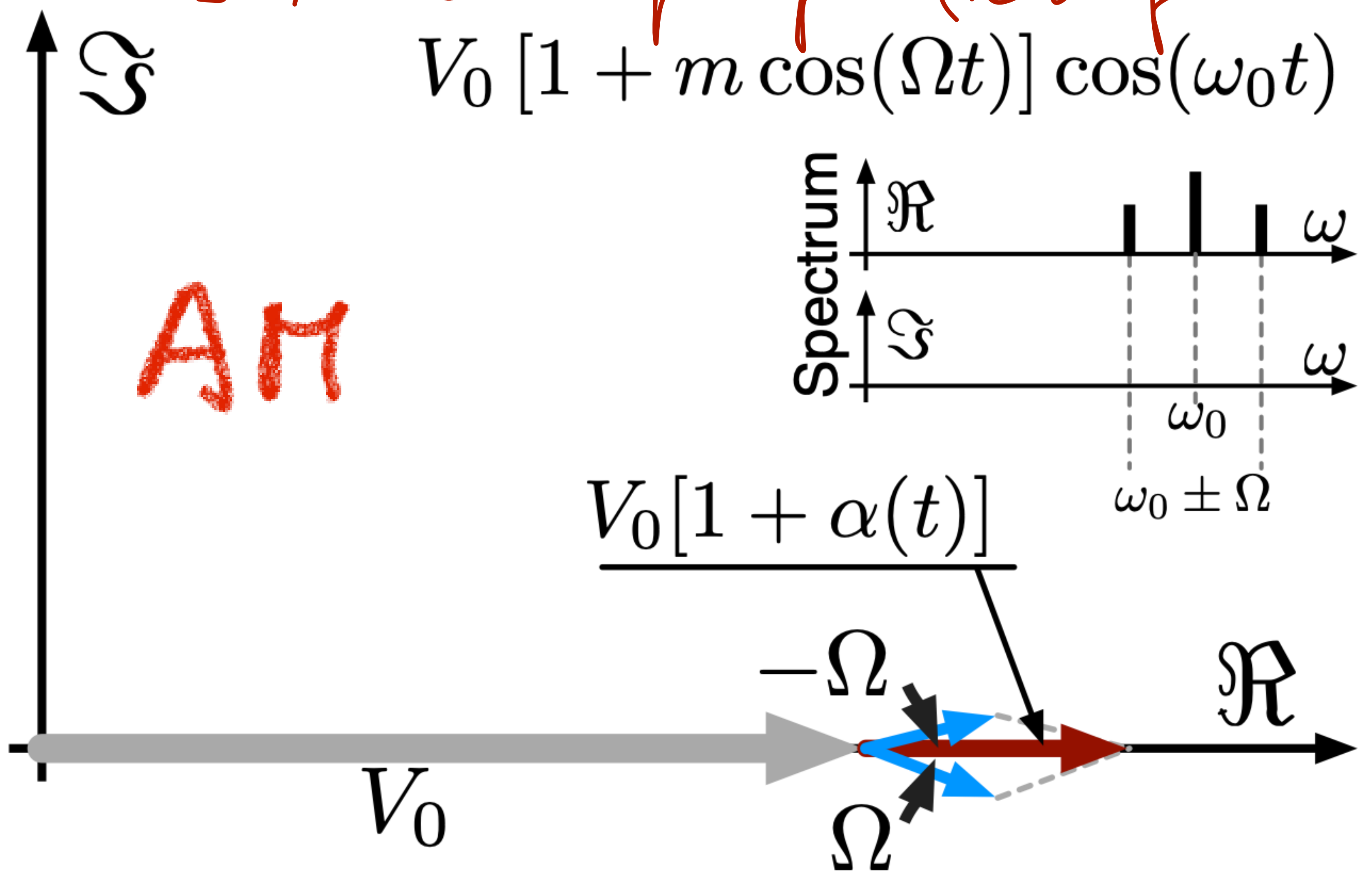
ω_0 carrier frequency

Ω low-freq. (information)



$V_0 \cos\left[\omega_0 t + m\cos(\Omega t)\right]$

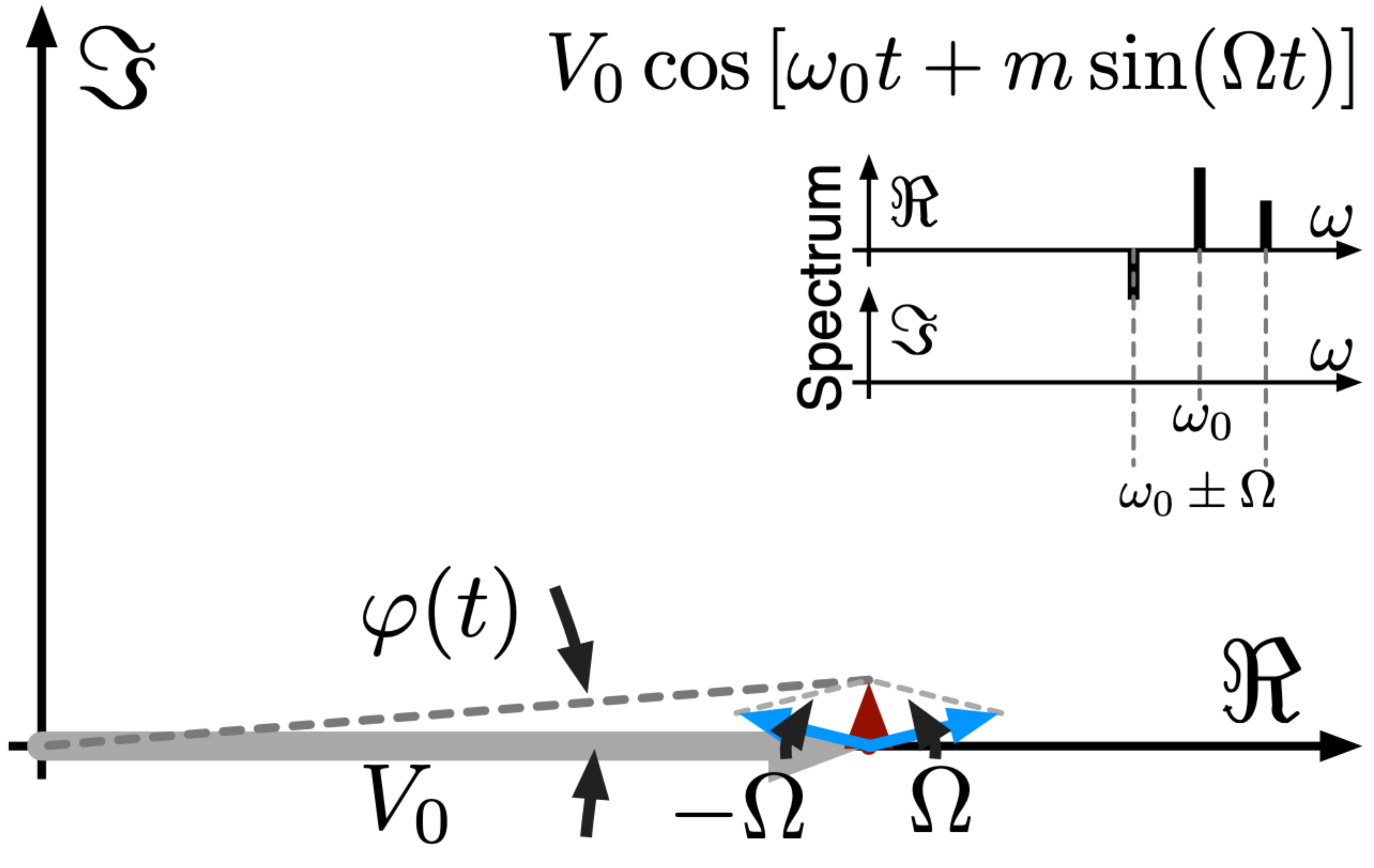
PM



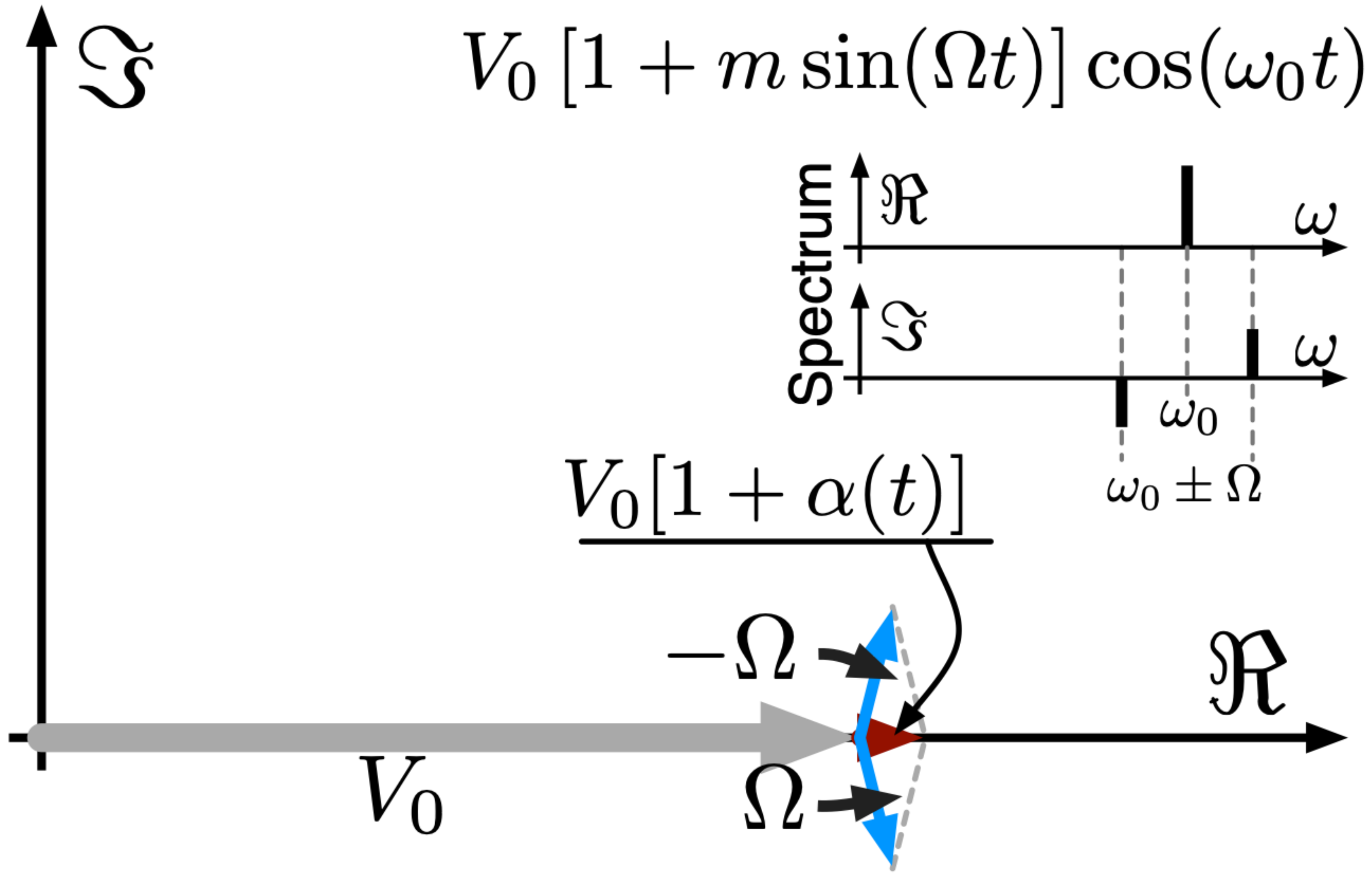
$V_0\left[1 + m\cos(\Omega t)\right]\cos(\omega_0 t)$

AM

File Modulation-summary



$V_0 \cos\left[\omega_0 t + m\sin(\Omega t)\right]$



$V_0\left[1 + m\sin(\Omega t)\right]\cos(\omega_0 t)$

Phase Modulation – Math

Phase modulated signal, with modulation index *m*

$$v(t) = e^{j(\omega_0 + m \sin \omega_m)t}$$

The full frequency domain representation contains an infinite number of sidebands ruled by the Jacobi–Anger expansion

$$e^{jm \sin\theta} = \sum_{n=-\infty}^{\infty} J_n(m)\, e^{jn\theta}$$

For small *m*, the expansion can be truncated to 3 terms, $n = -1 \ldots +1$

Use the asymptotic expansion $J_0(\mathrm{m}) \approx 1$, $J_{-1}(m) \approx -m/2$, $J_1(m) \approx m/2$,

$$v(t) = e^{j\omega_0 t} + \frac{m}{2} e^{j(\omega_0 + \omega_m)t} - \frac{m}{2} e^{j(\omega_0 - \omega_m)t}$$

Freeze ω_0 —> phase vector representation

$$V(t) = 1 + \frac{m}{2}\left[e^{j\omega_m t} - e^{-j\omega_m t}\right]$$

use

$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

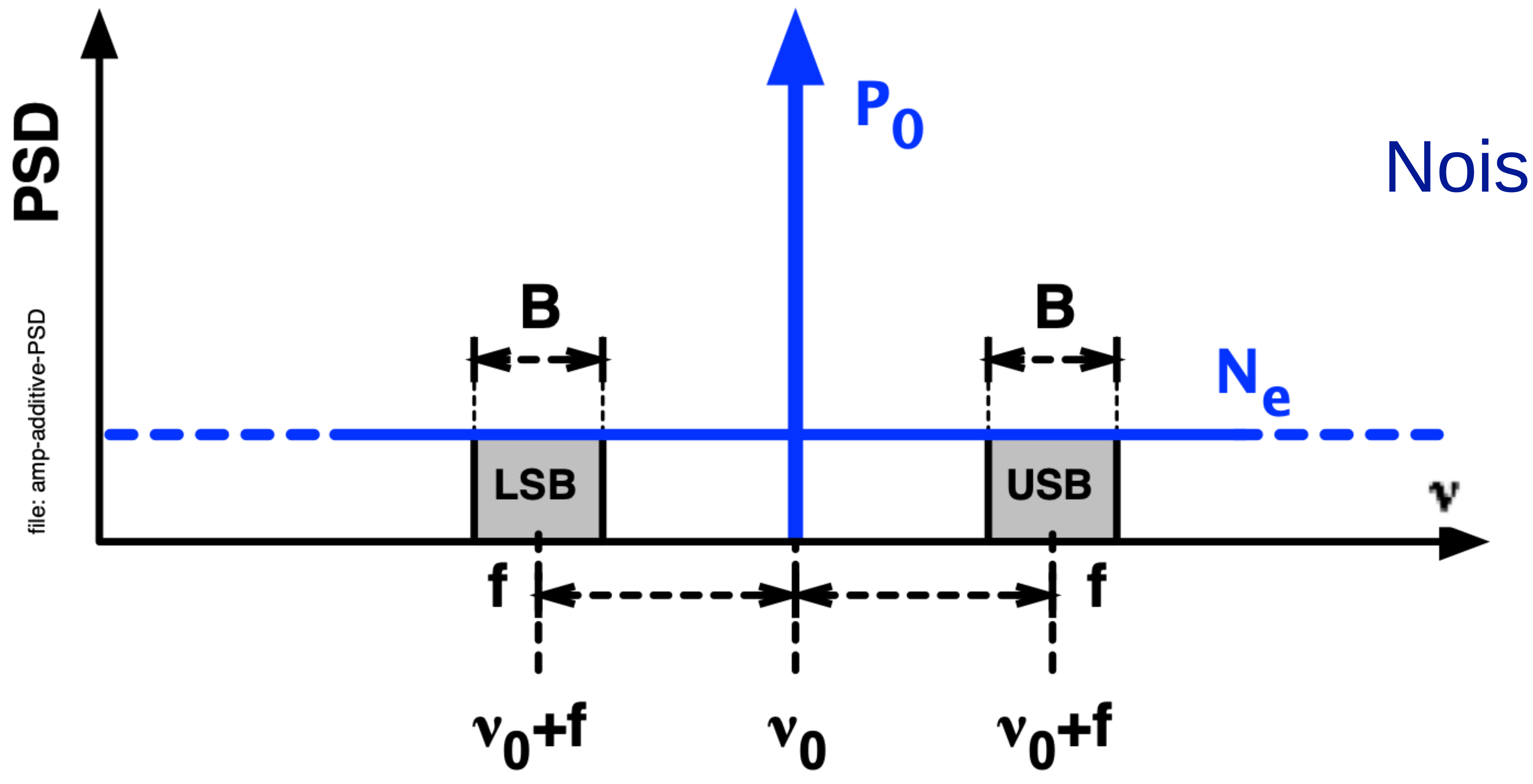
equivalent to

$$V(t) = 1 + jm \sin(\omega_m t)$$

A swinging phase θ is equivalent to a swinging frequency $\Delta f = (1/2\pi)\,(d\theta/dt)$

$$(\Delta f)(t) = m \frac{\omega_m}{2\pi} \cos(\omega_m t) = m f_m \cos(\omega_m t)$$

Phase and Amplitude Noise



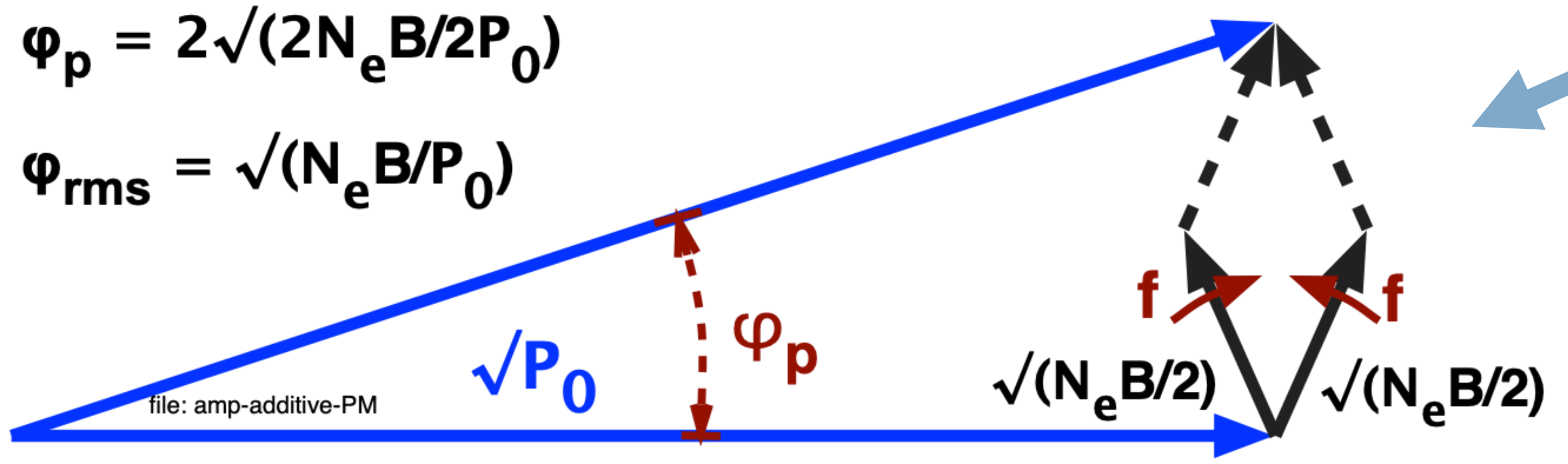
Noise is equally split between
AM and PM

PM (rms)

$$v_{\rm usb}(t) = i\sqrt{N_e B/2}\, e^{i2\pi f t}$$
$$v_{\rm lsb}(t) = -i\sqrt{N_e B/2}\, e^{-i2\pi f t}$$

$\varphi_p = 2\sqrt{(2N_eB/2P_0)}$

$\varphi_{rms} = \sqrt{(N_eB/P_0)}$

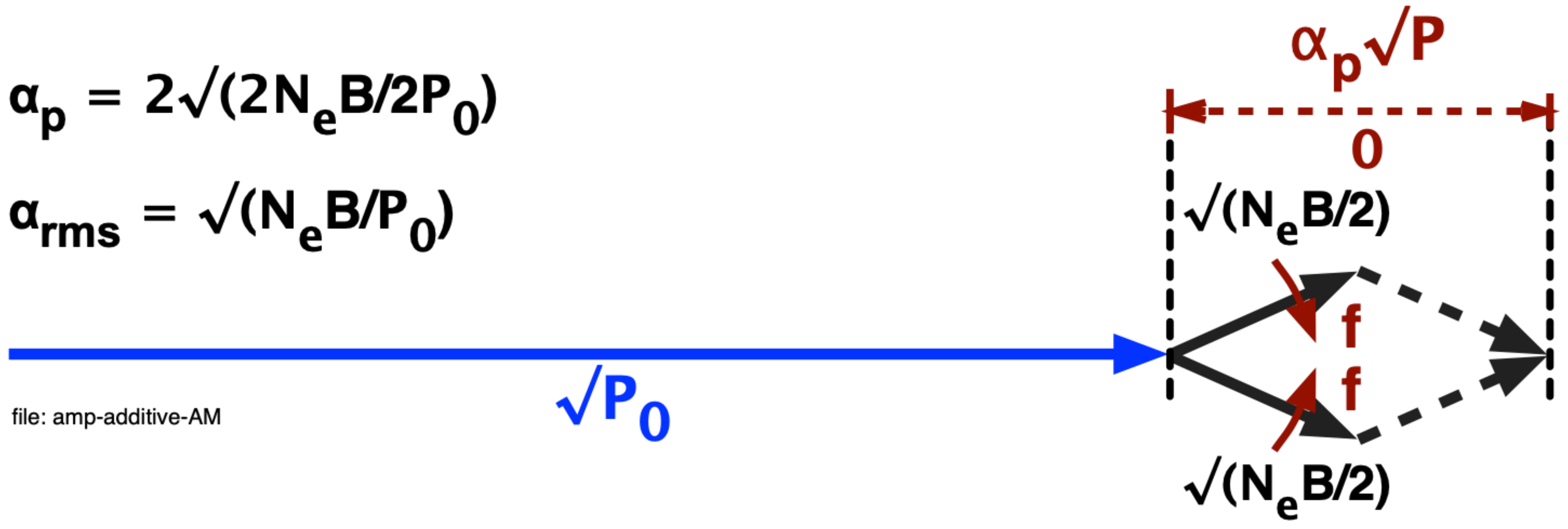


AM (rms)

$$v_{\rm usb}(t) = \sqrt{N_e B/2}\, e^{i2\pi f t}$$
$$v_{\rm lsb}(t) = \sqrt{N_e B/2}\, e^{-i2\pi f t}$$

$\alpha_p = 2\sqrt{(2N_eB/2P_0)}$

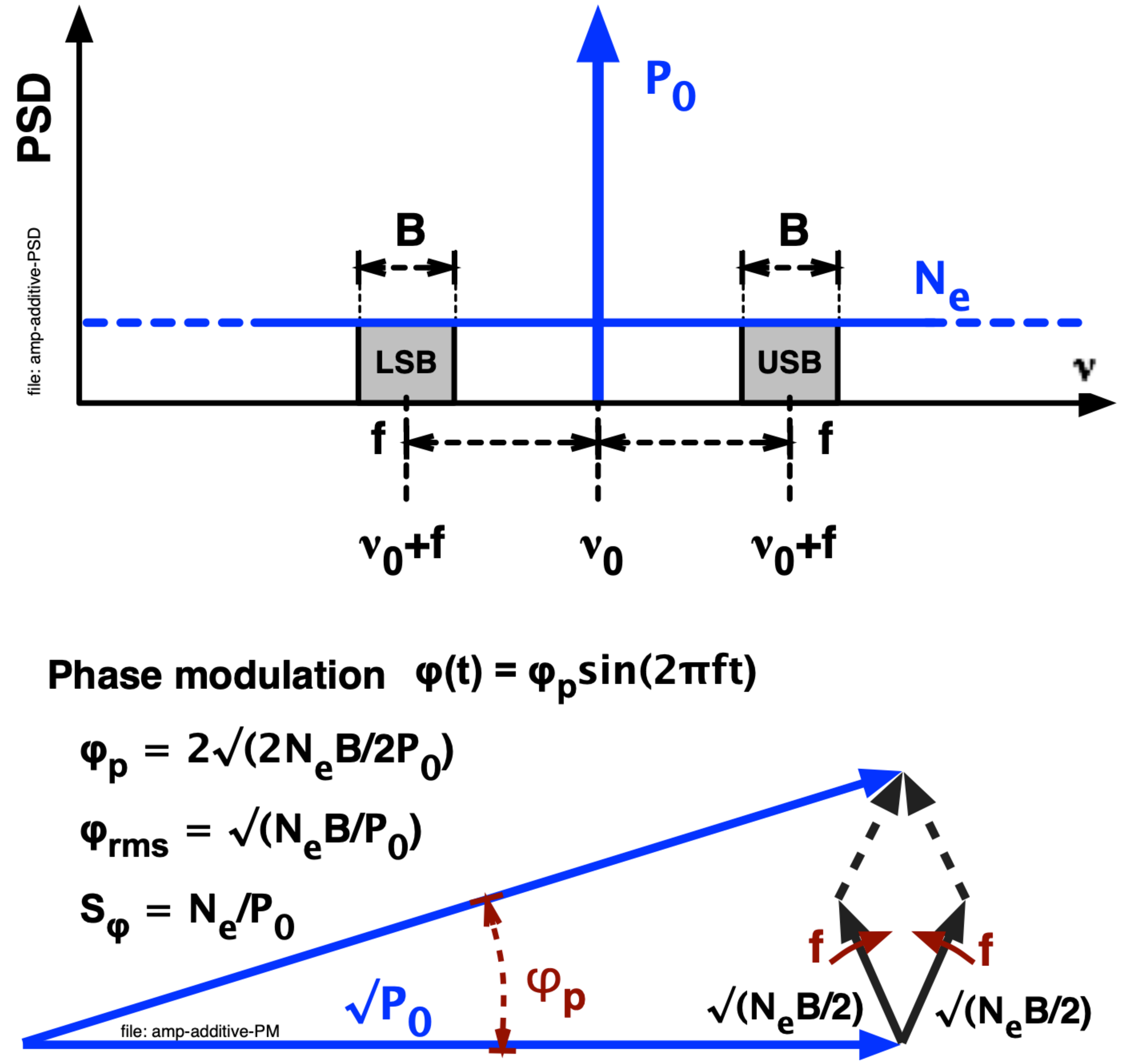
$\alpha_{rms} = \sqrt{(N_eB/P_0)}$



Normalize on B

$$S_\varphi(f) = \frac{N_e}{P_0}\,, \qquad S_\alpha(f) = \frac{N_e}{P_0}$$

Bandwidth



Phase modulation $\varphi(t) = \varphi_p \sin(2\pi f t)$

$\varphi_p = 2\sqrt{(2N_eB/2P_0)}$

$\varphi_{rms} = \sqrt{(N_eB/P_0)}$

$S_\varphi = N_e/P_0$

Phase modulation

Two symmetrical sidebands

Fourier analysis

hence

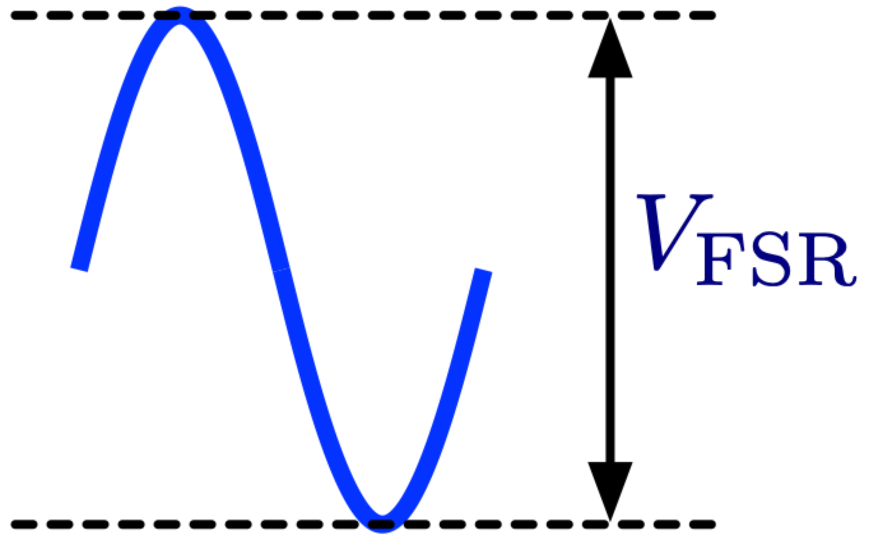
$f < \nu_0$

carrier

otherwise LSB goes to negative frequency

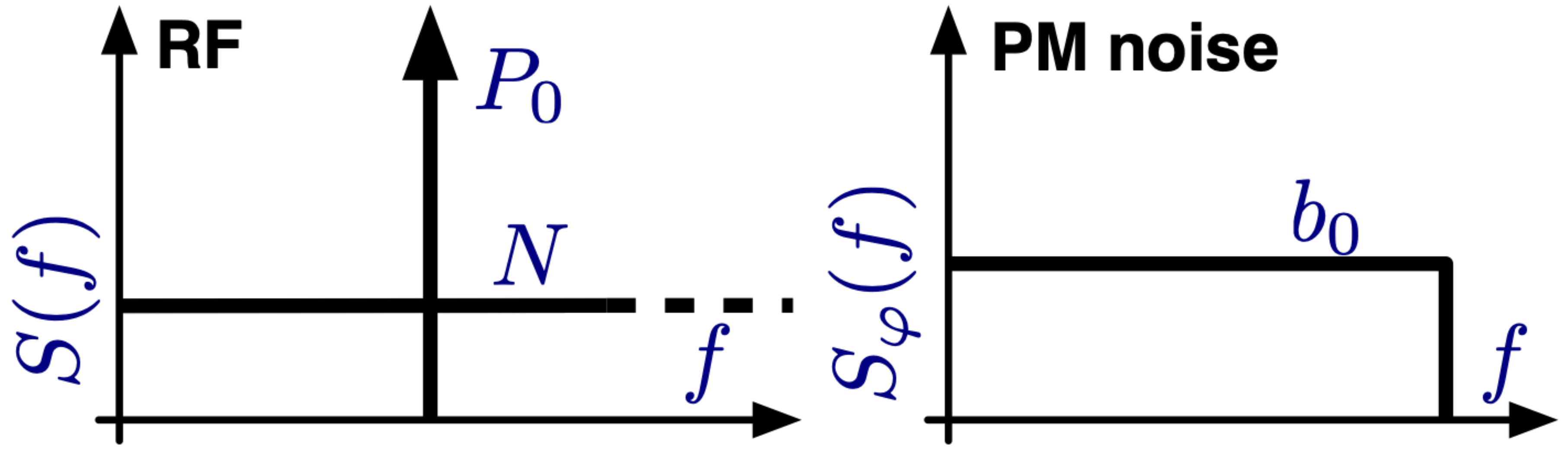
Warning: Frequency dividers also divide the bandwidth
Aliasing is around the corner

Quantization and PM Noise



The maximum power is

$$P_0 = \frac{1}{8} V_{\mathrm{FSR}}^2 \qquad \mathrm{V}^2$$



In the presence of white noise *N*,
the PM noise is

$$b_0 = \frac{N}{P_0} \qquad \mathrm{rad}^2/\mathrm{Hz}$$

white phase noise

noise floor

carrier

Recall the quantization noise

$$N = \frac{V_{\mathrm{FSR}}^2}{6 \times 2^{2n}\,\nu_s}$$

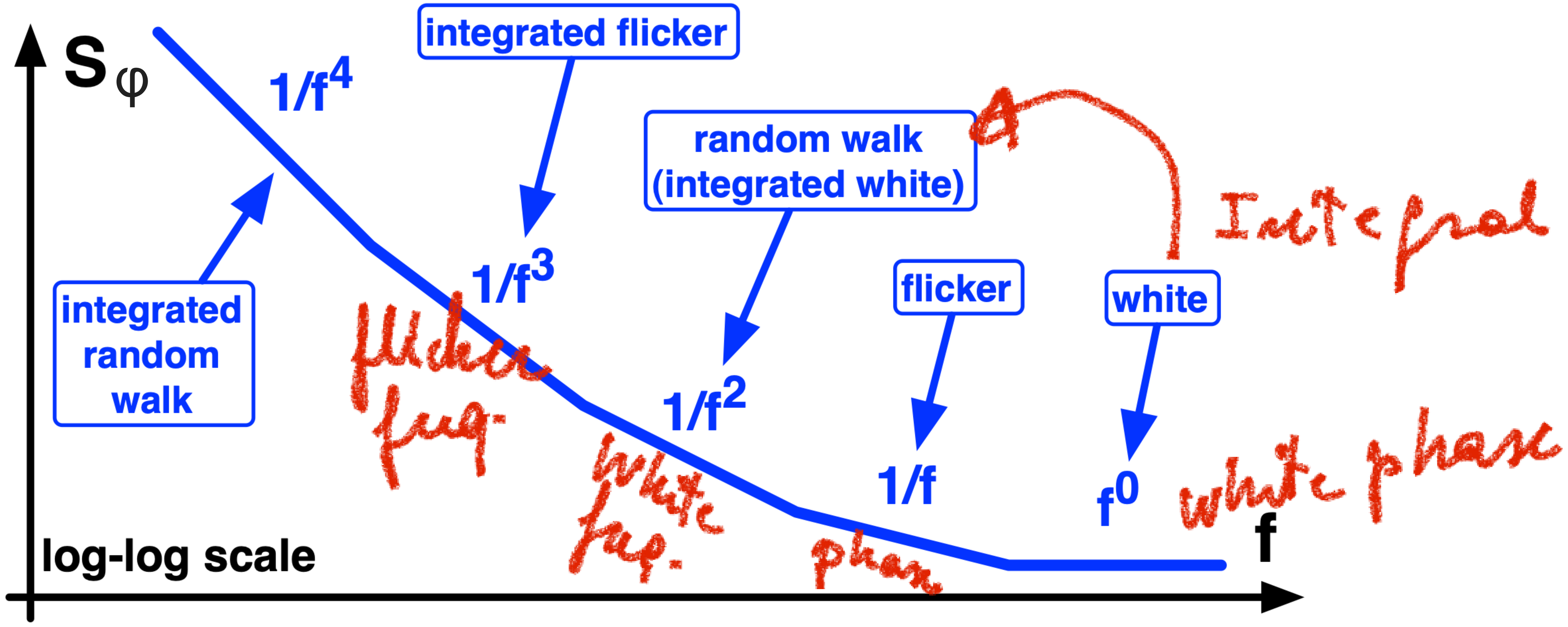
sampling freq

The white PM noise is

$$b_0 = \frac{4}{3}\,\frac{1}{2^{2n}\,\nu_s} \qquad \mathrm{rad}^2/\mathrm{Hz}$$

Example:
14 bit, 1 GHz –> –173 dB
14 bit , 400 MHz –> –169 dB
12 bit, 300 MHz –> –156 dB

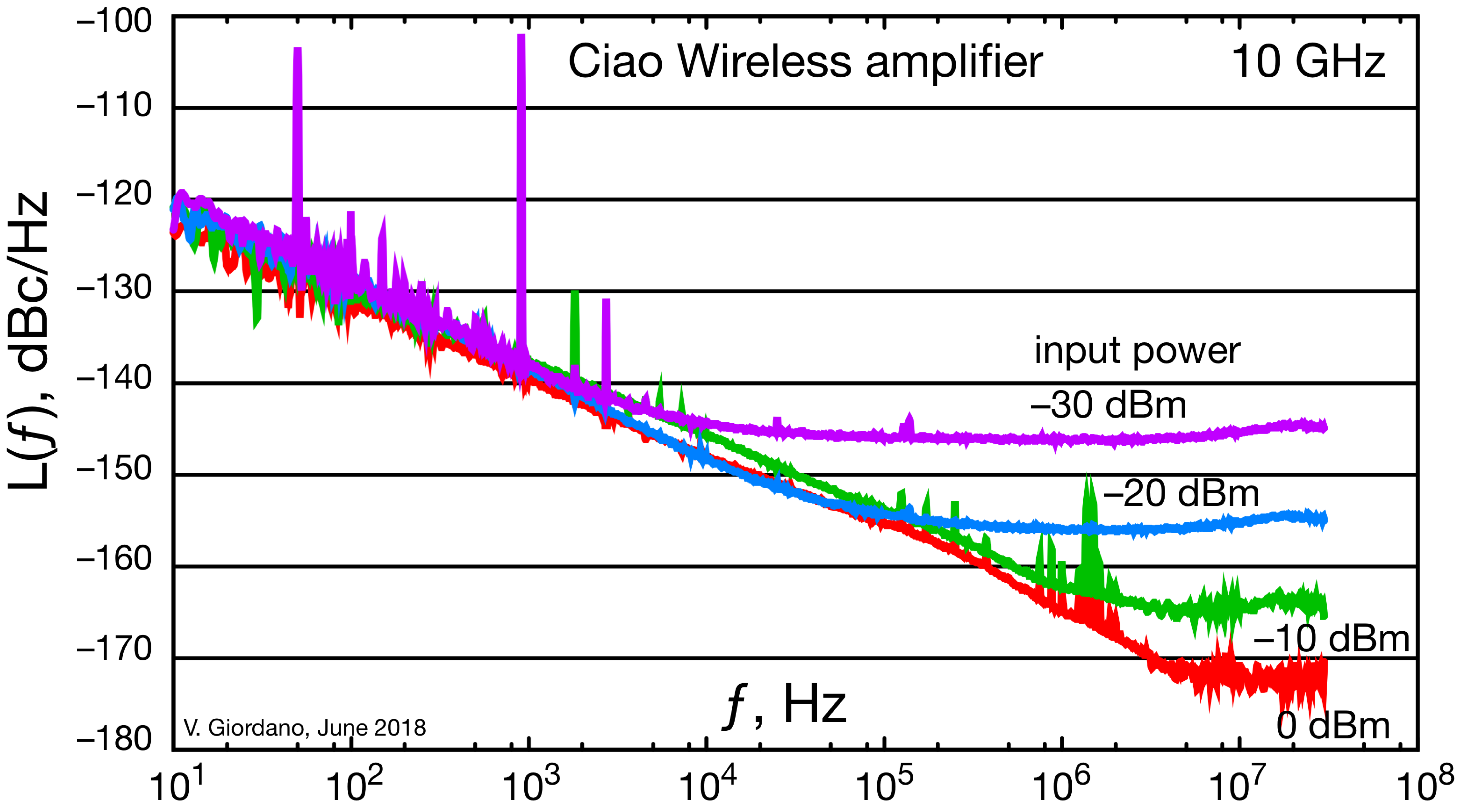
Polynomial Law



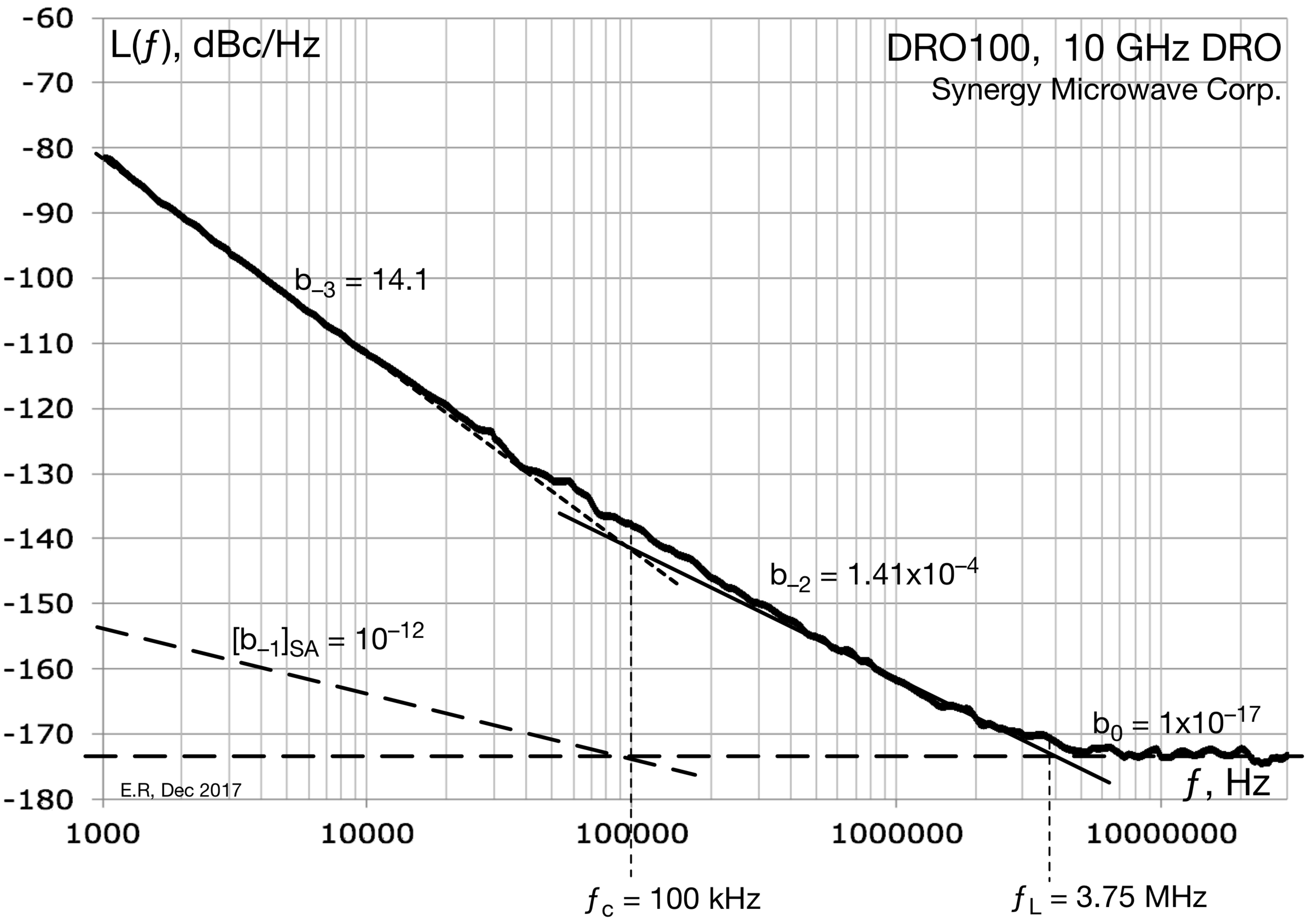
A model which is useful to describe the close-in noise

- Oscillator PM: $S_\varphi(f) = \ldots + b_{-4}/f^4 + b_{-3}/f^3 + b_{-2}/f^2 + b_{-1}/f + b_0$
- Oscillator FM: $S_y(f) = \ldots + h_{-2}/f^2 + h_{-1}/f + h_0 + h_1 f + h_2 f^2$
- Oscillator AM: $S_\alpha(f) = h_{-1}/f + h_0$ (chiefly, but not only)
- 2-port device PM: $S_\varphi(f) = b_{-1}/f + b_0$ (chiefly, but not only)
- 2-port device AM: $S_\alpha(f) = h_{-1}/f + h_0$ (chiefly, but not only)

Example – Microwave Amplifier



Example – Microwave Oscillator

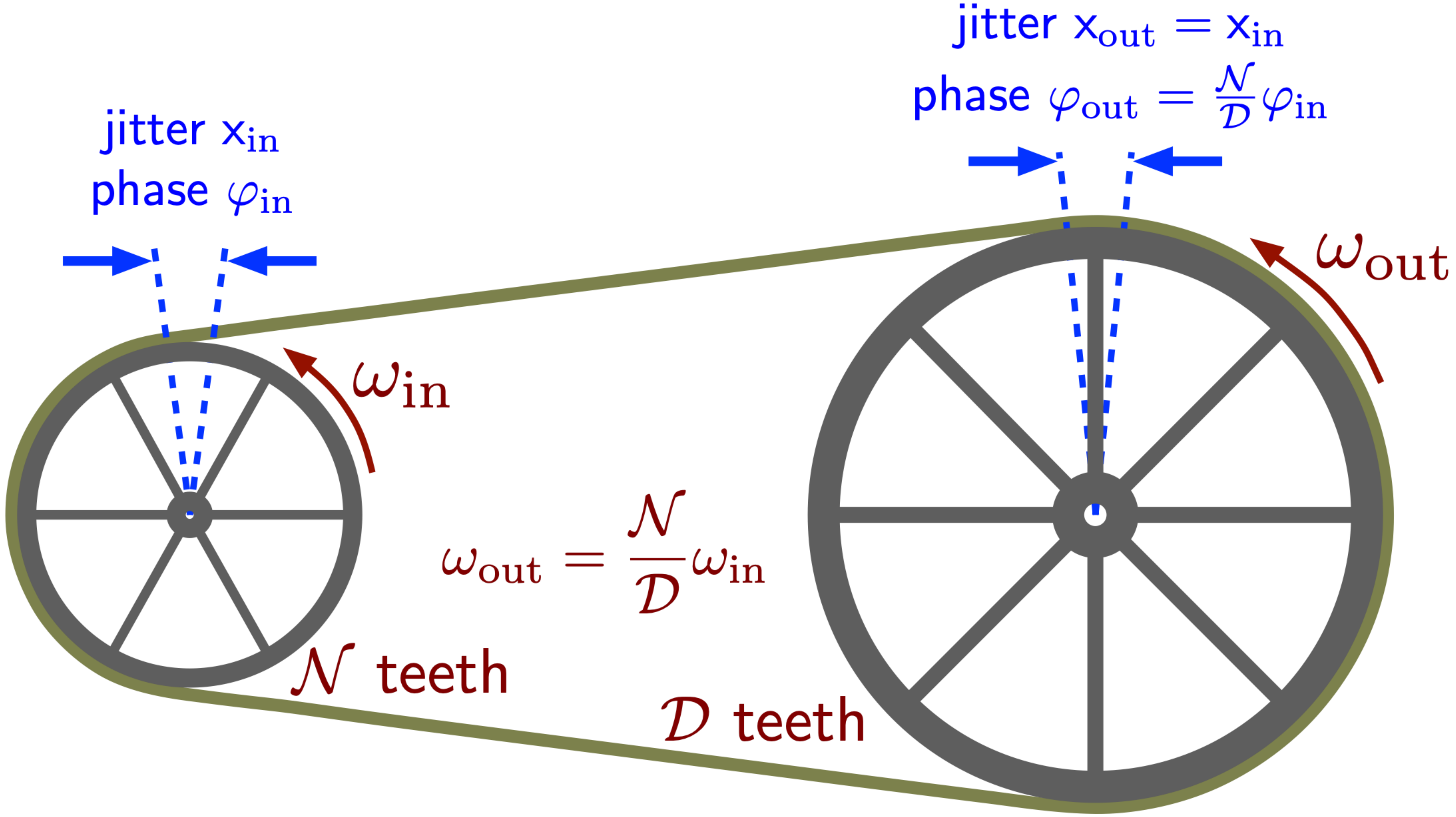


Phase-Type and Time-Type Phase Noise

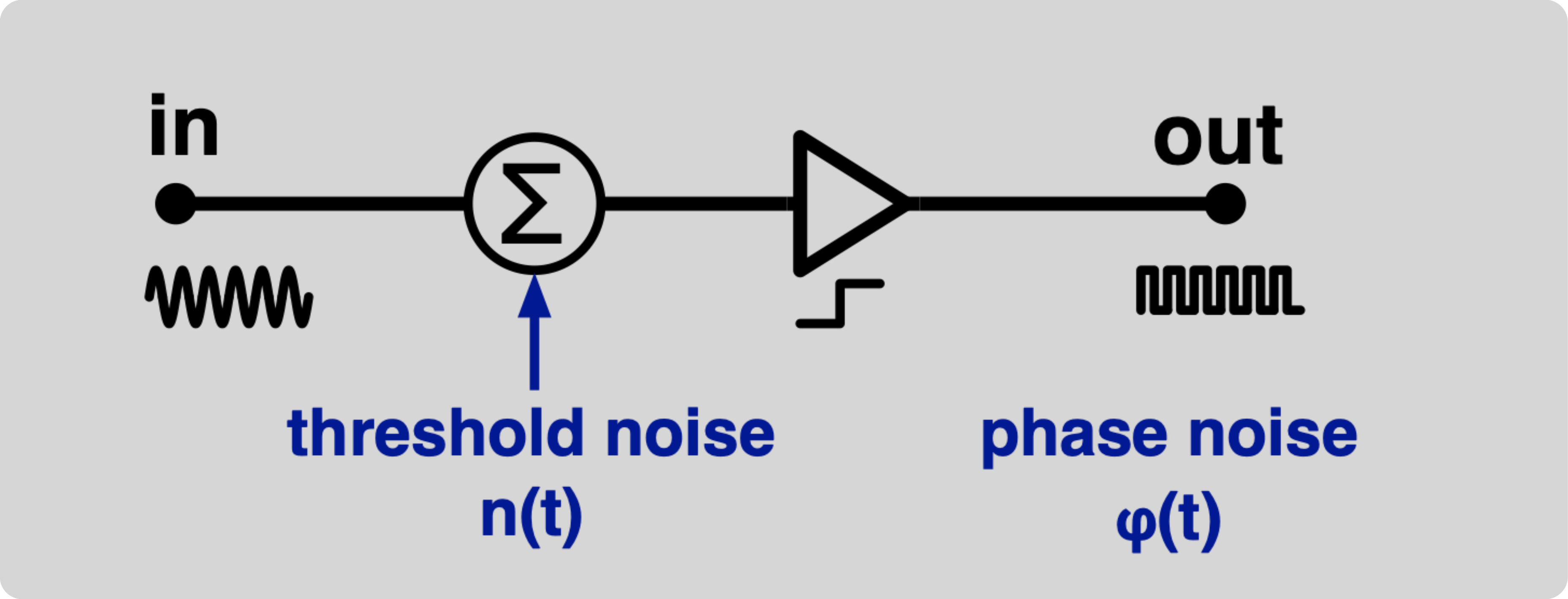
Phase Time Fluctuations

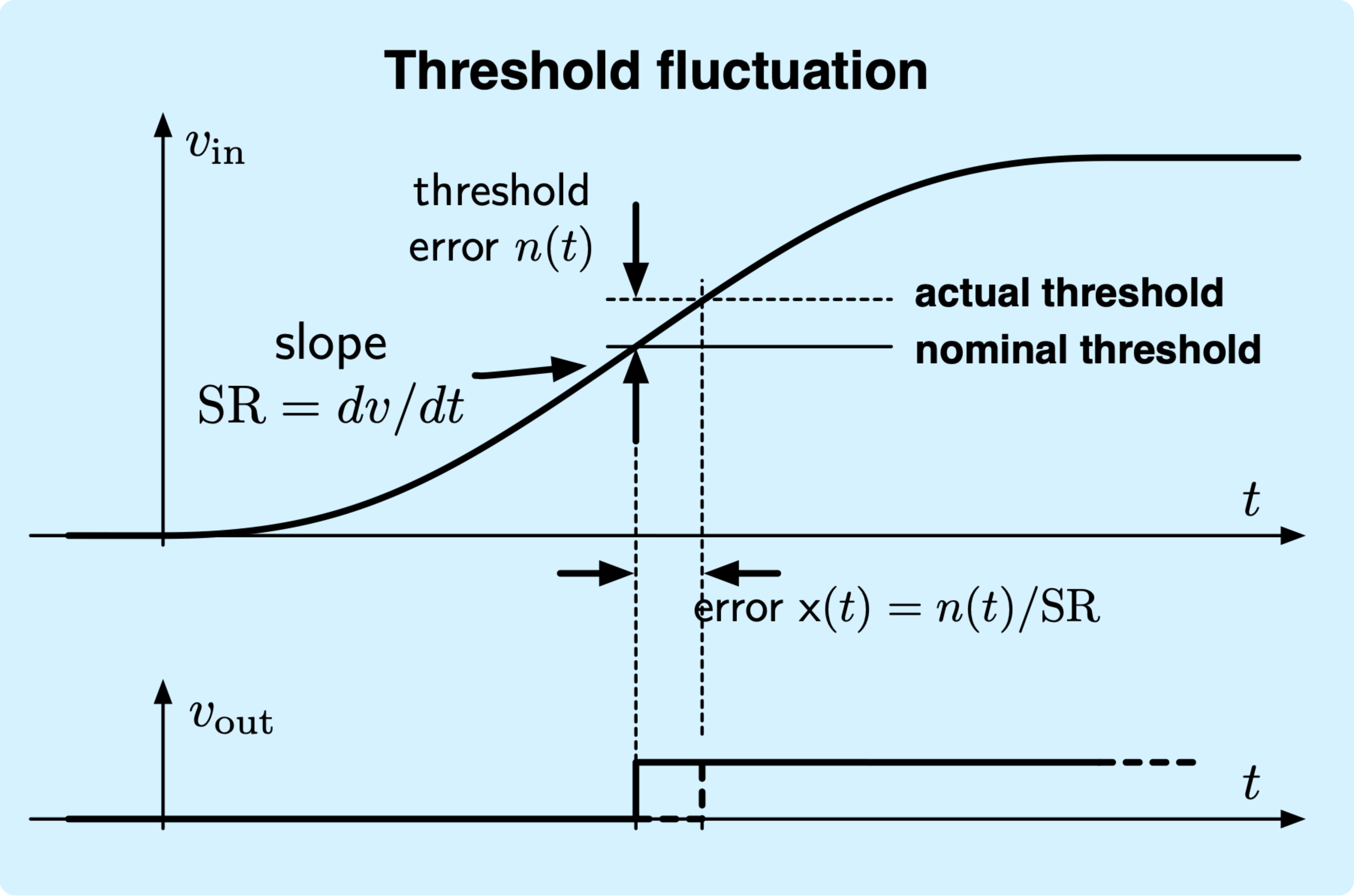
$$v(t) = V_0 \left[1 + \alpha(t)\right] \cos\left[2\pi\nu_0 t + \varphi(t)\right] \qquad \mathrm{x}(t) = \frac{\varphi(t)}{2\pi\nu_0}$$

- ITU defines jitter as the variations in the significant instants of a clock or data signal, vs a “perfect” clock
- Jitter —> Usually fast phase changes f > a few tens of Hz
- Wander —> Usually slower phase changes (due to temperature, voltage, etc)
- Designers first care about consistency of logic functions,
  - First, maximum timing error
  - Sometimes RMS value and probability distribution
- Time and Frequency community focuses on
  - PM noise spectra
  - Delay spectra
  - Two-sample variances (ADEV, TDEV, etc.)


jitter x_{in}
phase φ_{in}
ω_{in}
$\mathcal{N}$ teeth
jitter $x_{\mathrm{out}} = x_{\mathrm{in}}$
phase $\varphi_{\mathrm{out}} = \frac{\mathcal{N}}{\mathcal{D}}\varphi_{\mathrm{in}}$
ω_{out}
$\omega_{\mathrm{out}} = \frac{\mathcal{N}}{\mathcal{D}}\omega_{\mathrm{in}}$
$\mathcal{D}$ teeth

Phase Noise in the Input Stage





mechanism

$$\mathrm{x}(t) = \frac{n(t)}{(\mathrm{SR})(t)}$$

$$\varphi(t) = \frac{2\pi\nu_0\, n(t)}{(\mathrm{SR})(t)}$$

Phase Noise in the Input Stage

Sinusoidal signal

$$v(t) = V_0\,[1+\alpha(t)]\cos\left[2\pi\nu_0 t + \varphi(t)\right] \quad \Rightarrow \quad \mathrm{SR} = 2\pi\nu_0\,V_0$$

$$\mathsf{x}(t) = \frac{n(t)}{\mathrm{SR}} \quad \longrightarrow \quad \mathsf{x}(t) = \frac{1}{2\pi\nu_0}\,\frac{n(t)}{V_0}$$

$$\varphi(t) = \frac{2\pi\nu_0\,n(t)}{\mathrm{SR}} \quad \longrightarrow \quad \varphi(t) = \frac{n(t)}{V_0}$$

phase-type

(**φ**-type)

noise

$$S_\varphi(f) = \frac{S_n(f)}{V_0^2}$$

$\text{constant vs } \nu_0$

End of Lecture #12

End of Lecture #13

Examples and Suggested Exercises

Examples

1. White noise, –150 dBm/Hz
   Calculate the power in B = 10 MHz centered at f_0 = 100 MHz

2. Pure 100 MHz carrier, +10 dBm power, sketch the PSD
   Assume ideally narrow bandwidth, continuous PSD

3. Pure 100 MHz carrier, +10 dBm power, sketch the PSD
   Assume discrete PSD, RBW = 5 kHz —> Give the definition of RBW

4. Amplitude-modulated signal,
   Carrier f_0 = 100 MHz, P_0 = 10 dBm
   Sidebands f_m = 500 Hz, P_{SB} = 0 dBm
   - Assume ideally narrow bandwidth, continuous PSD
   - Assume discrete PSD, RBW = 5 kHz

Example 1

White noise, –150 dBm/Hz

Calculate the power in B = 10 MHz centered at f_0 = 100 MHz

Example 2

Pure 100 MHz carrier, +10 dBm power, sketch the PSD
Assume ideally narrow bandwidth.

Example 3

- Pure 100 MHz carrier, +10 dBm power, sketch the PSD
  Assume discrete PSD, RBW = 5 kHz

Example 4

Amplitude-modulated signal, sketch the PSD

Carrier f_0 = 100 MHz, P_0 = 10 dBm

Sidebands f_m = 500 Hz, P_{SB} = 0 dBm

- Assume ideally narrow bandwidth, continuous PSD
- Assume discrete PSD, RBW = 5 kHz

Examples & Exercises

1. Calculate the total power transferred from a resistor at T_2 = 290 K to a cold (T_1 = 0 K)
2. A resistor of temperature T_R = 4.2 K is connected to a 6 dB attenuator at the temperature T_A = 300 K. Which is the equivalent temperature T_E seen at the other end of the attenuator?
   Tip: Trust the 2nd principle of thermodynamics.
3. Same as (1), but the two resistors are connected by WR28 waveguide (21.1-42.2 GHz bandwidth). Assume the cable loss free.
4. Same as (3), but now T_1 = 77 K

$$\Phi = \frac{P}{E} \longrightarrow \frac{\eta P}{h\nu} \longrightarrow \frac{\eta P}{hc/\lambda} \qquad \frac{0.7 \cdot 10^{-9}}{6.6 \times 10^{-34} \cdot 3 \times 10^{8} / 1.55 \times 10^{-6}}$$

$$hc/\lambda \longrightarrow 1.3 \times 10^{-19}\ \text{J} \sim 0.8\ \text{eV}$$

$$\eta P = 0.7 \times 10^{-9} = 7 \times 10^{-10}$$

$$\Phi \quad 5.4 \times 10^{9}\ e/s$$

$$f_c = \frac{1}{2}\Phi = 2.7\ \text{GHz}$$

$$S_I = 2qI \qquad I = q\Phi = 1.6 \times 10^{-19} \times 5.4 \times 10^{9}$$

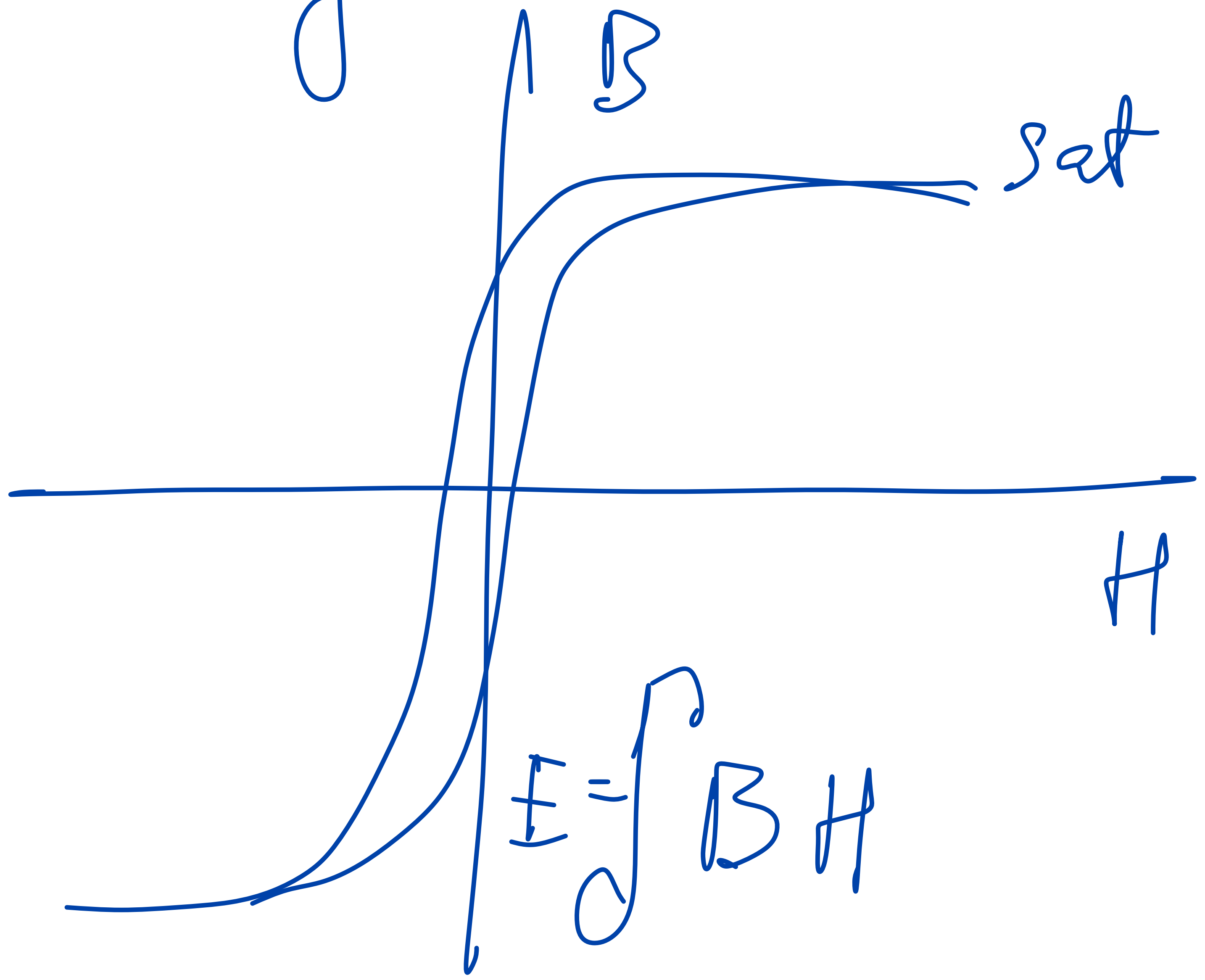
$$8.6 \times 10^{-10}\ \text{A}$$

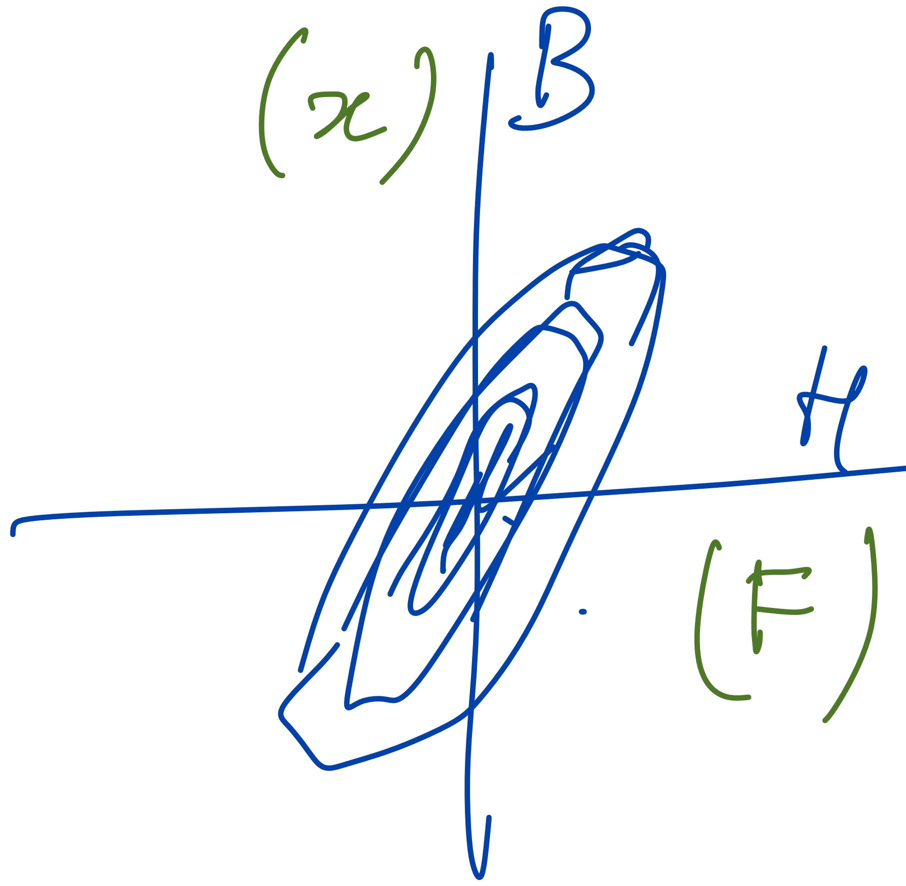
$$= 2.8 \times 10^{-28}\ \text{A}^2/\text{Hz}$$

$$\longrightarrow 1.7 \times 10^{-14}\ \text{A}/\sqrt{\text{Hz}} \qquad 17\ \text{fA}/\sqrt{\text{Hz}}$$

- Carbon microphones (this is where it was discovered)
- Ferromagnetic materials (Barkhausen effect)
- Semiconductors
- Mechanical stability
- Pulsars
- Nile flooding (!!!)
- … etc.

Magnetic materials





Small H

Aspect ratio

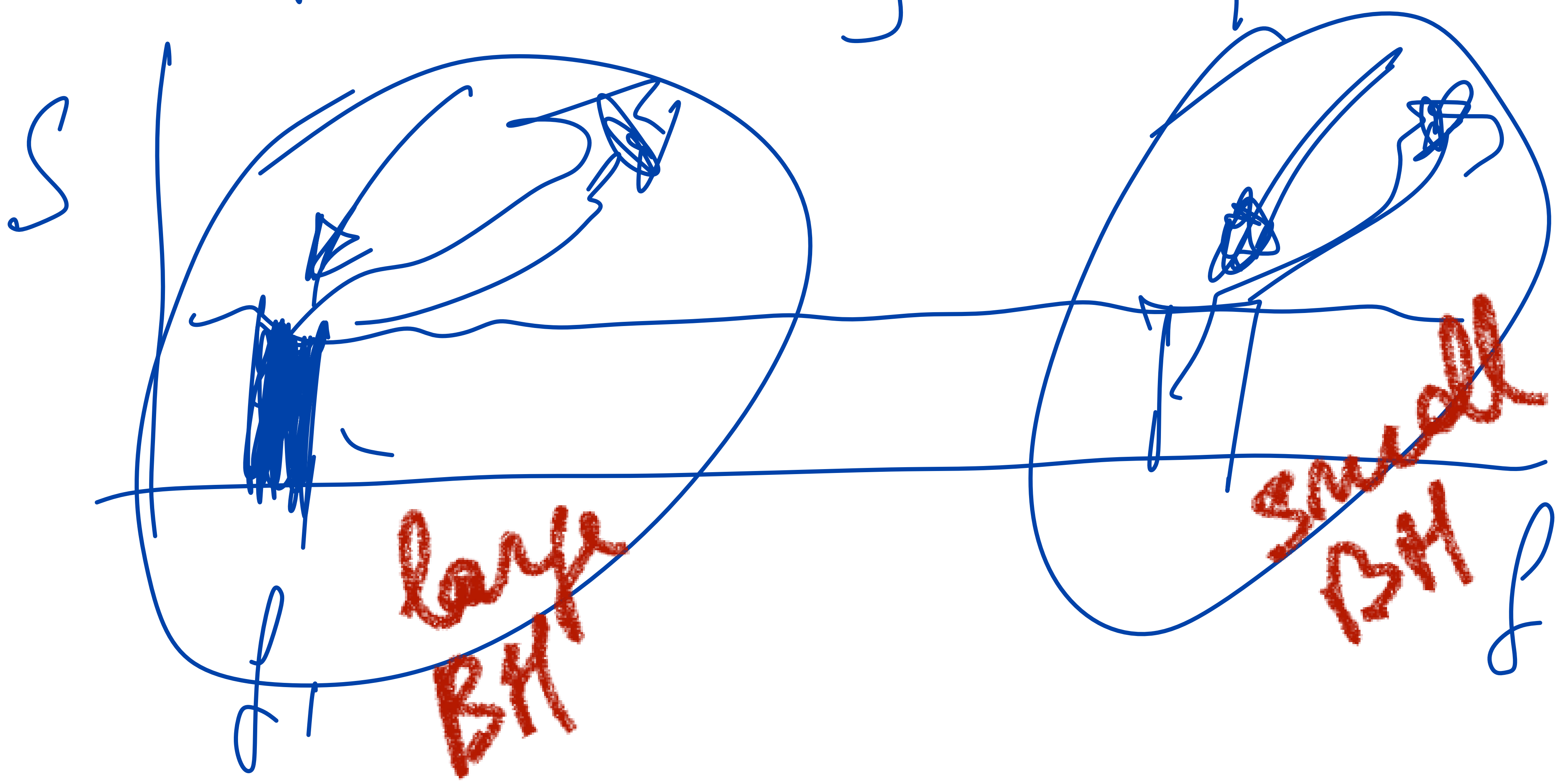
independent of size

$\sum H_0 \cos(\omega t)$

$E \sim H_0^2$

$P \sim H_0^2 \omega$

Thermodyn. equilibrium



1-2 T

Weiss

Langevin

x $\qquad$ $X = \int x(t)\, e^{i\omega t}\, dt$ $\qquad$ $\left[\frac{V}{Hz}\right]$

$[V]$ $\qquad$ $[s]$

$[Hz^{-1}]$

S $\qquad$ $[V^2/Hz]$

$|X|^2$ $\qquad$ $[V^2/Hz^2]$

$f>0$: energy conservation

$\frac{2}{T}|X|^2$ $\qquad$ $[V^2/Hz]$

acquisition time

The End