Artifacts and Errors in Cross-Spectrum Phase Noise Measurements

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Outline

- Experiments and results
- Theoretical background
  (attenuator, xSpectrum, power splitter, oscillators)
- Interpretation of the results
- Uncertainty concepts

Full text

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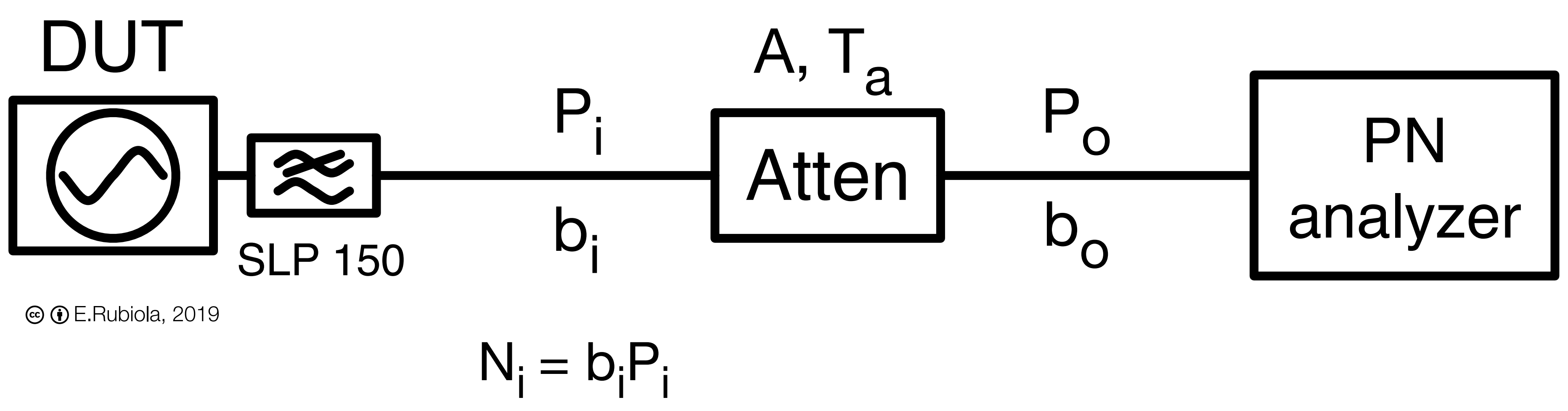
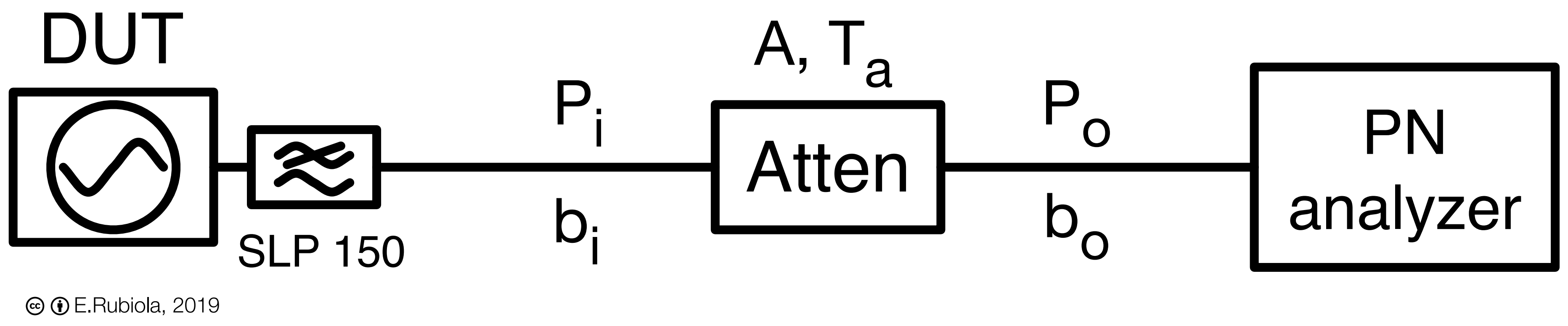
Experiments



The Story Starts with

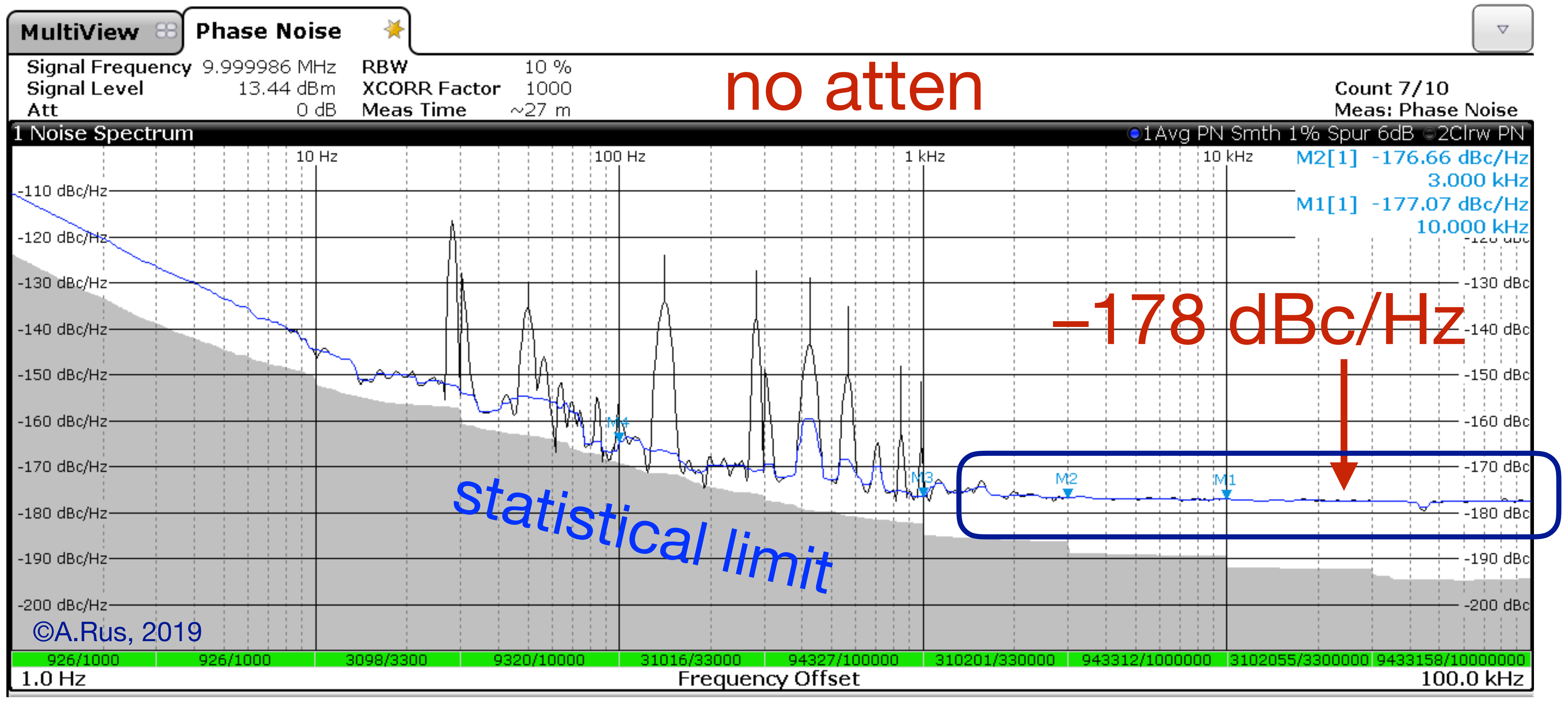
Adrian inserted an attenuator between OCXO and a FSWP-8 PN analyzer

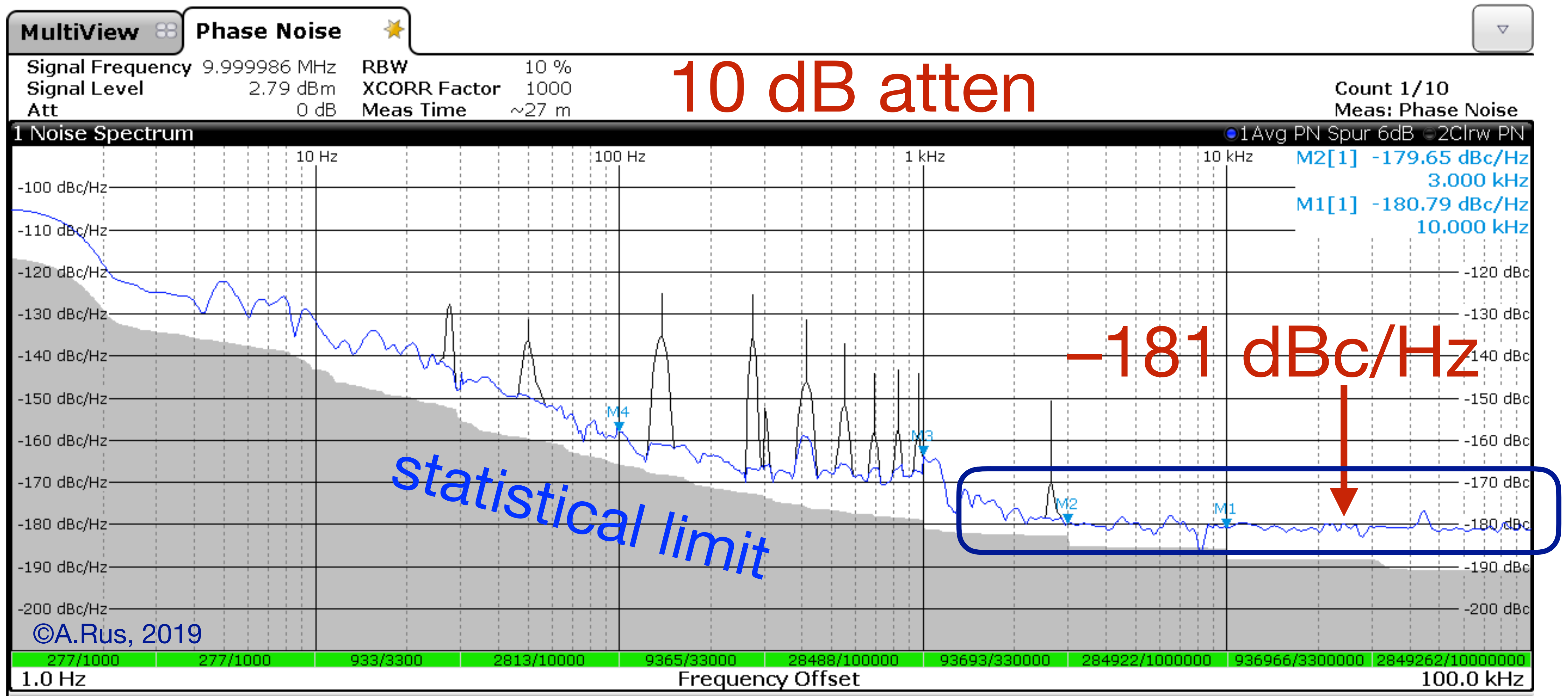


$N_i = b_i P_i$

Surprise, the white noise floor got lower

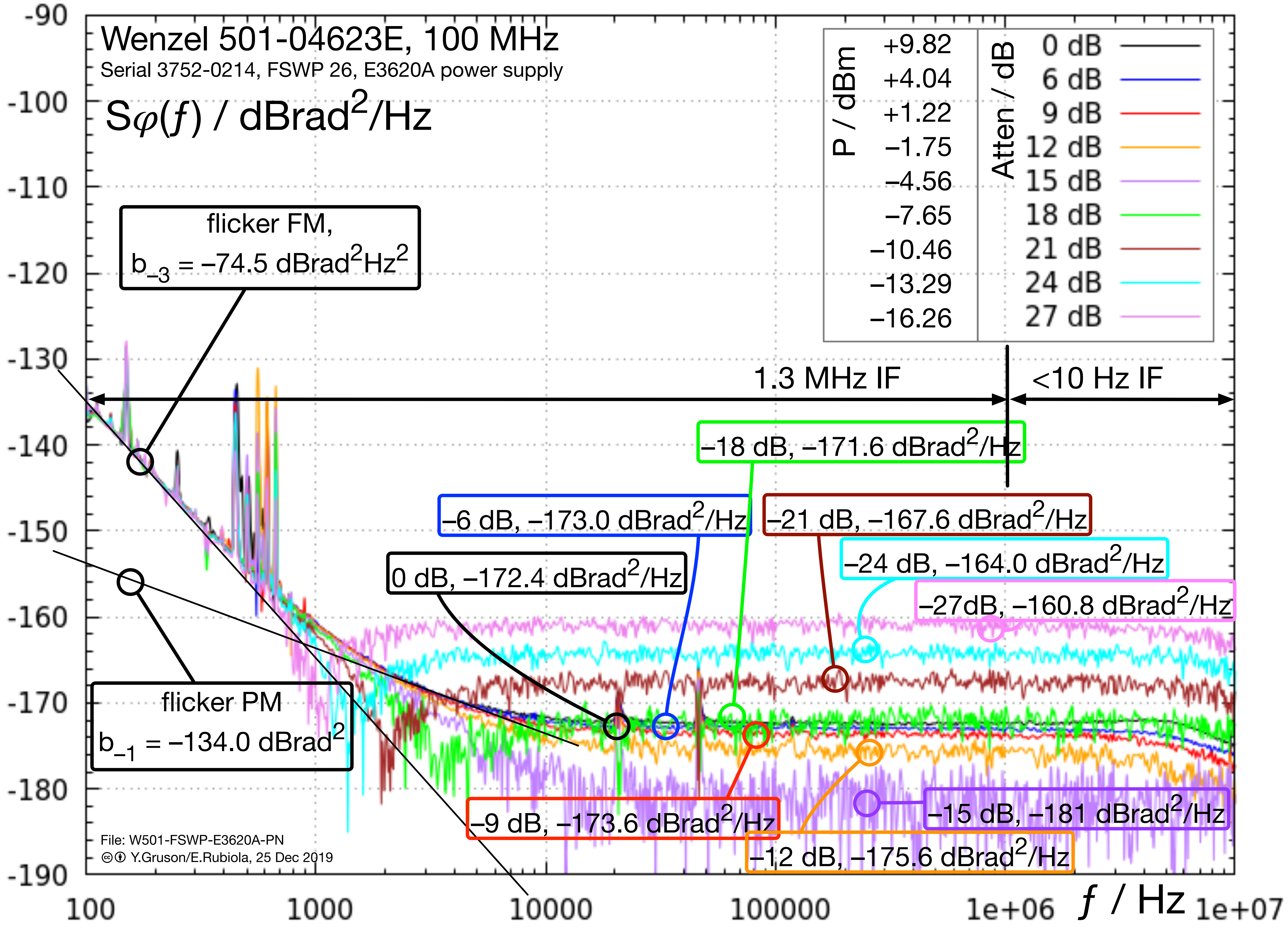
Wenzel 501-04538, 10 MHz OCXO



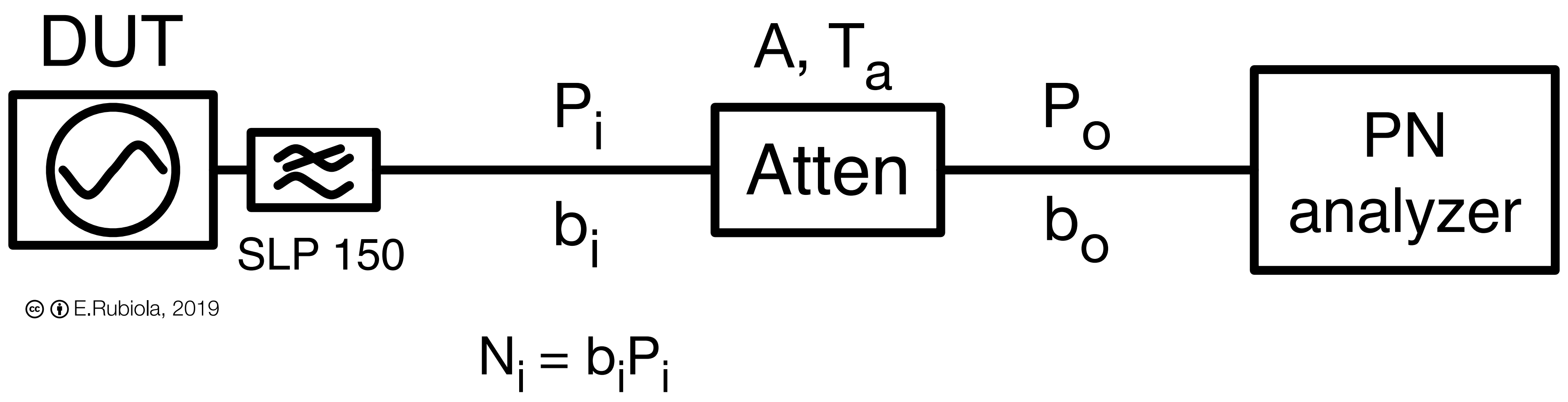


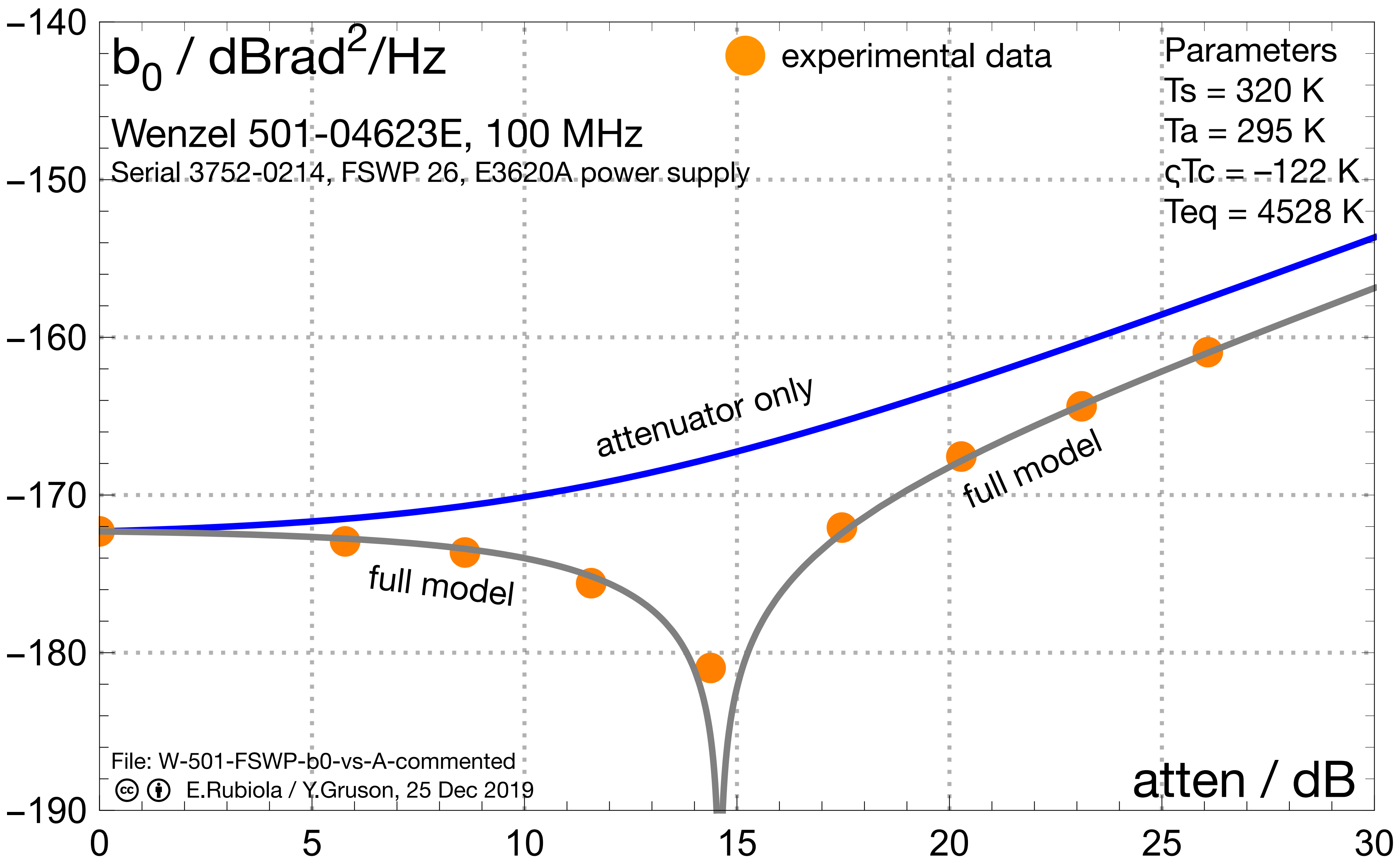
Experiment (an Old Low-Noise OCXO)

Wenzel 501-04623E & Rohde Schwarz FSWP



R&S FSWP-26 PN Analyzer





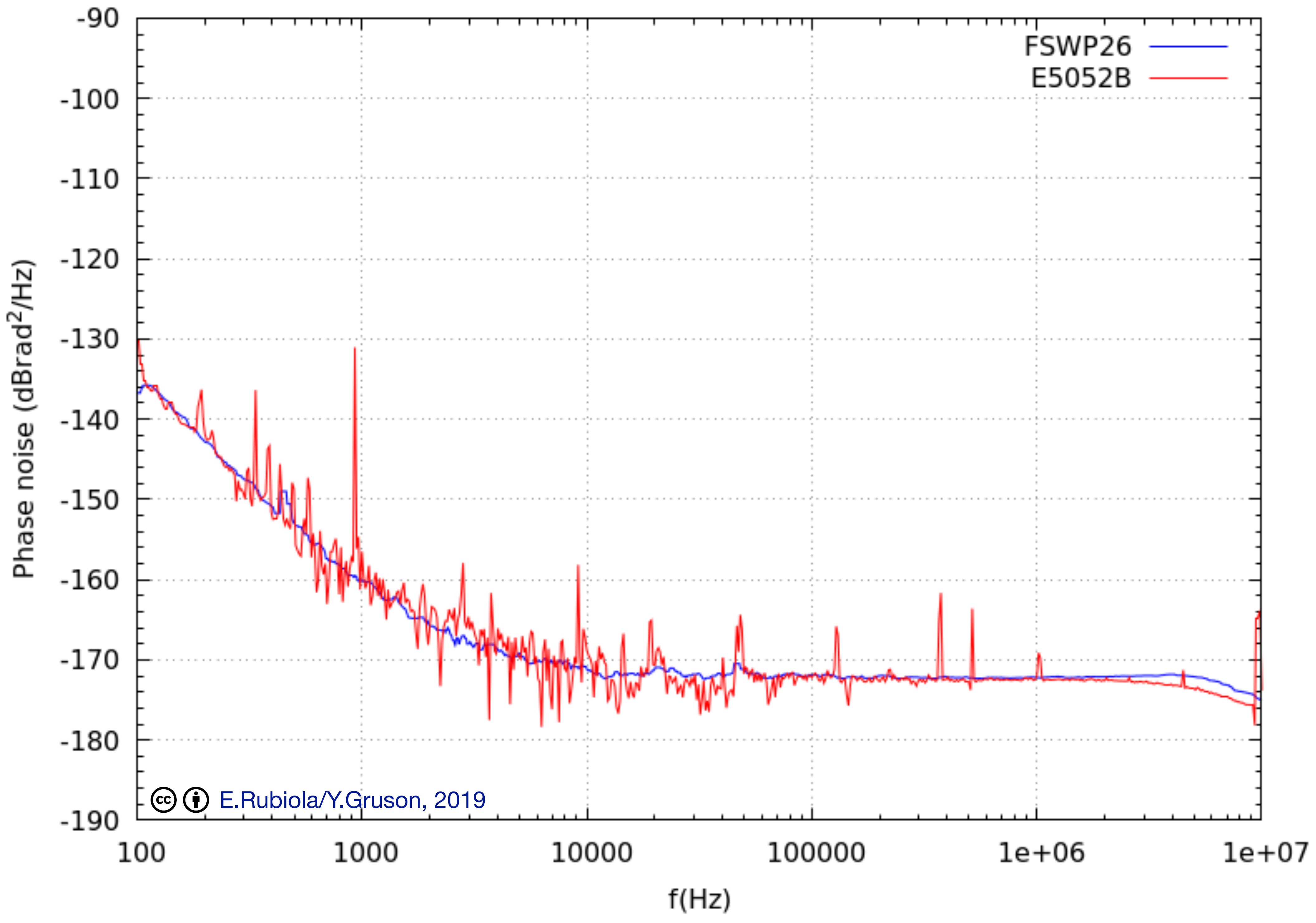
Attenuation —> erratic effect

Calibration Consistency

Wenzel 501-04623E – Compare Rohde Schwarz FSWP to Keysight E5052B

0 dB attenuation

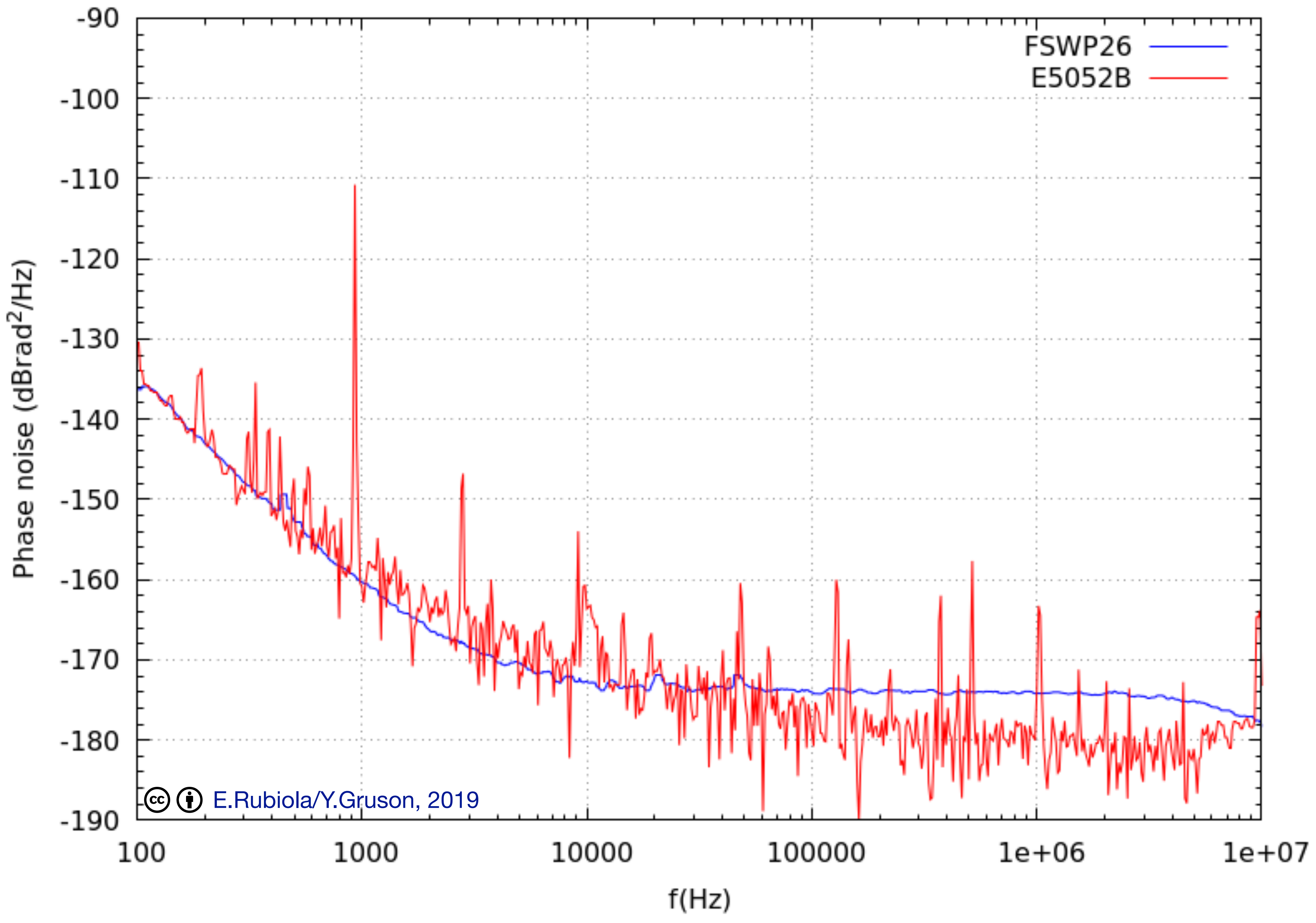
exact overlap —> calibration is OK



9 dB attenuation

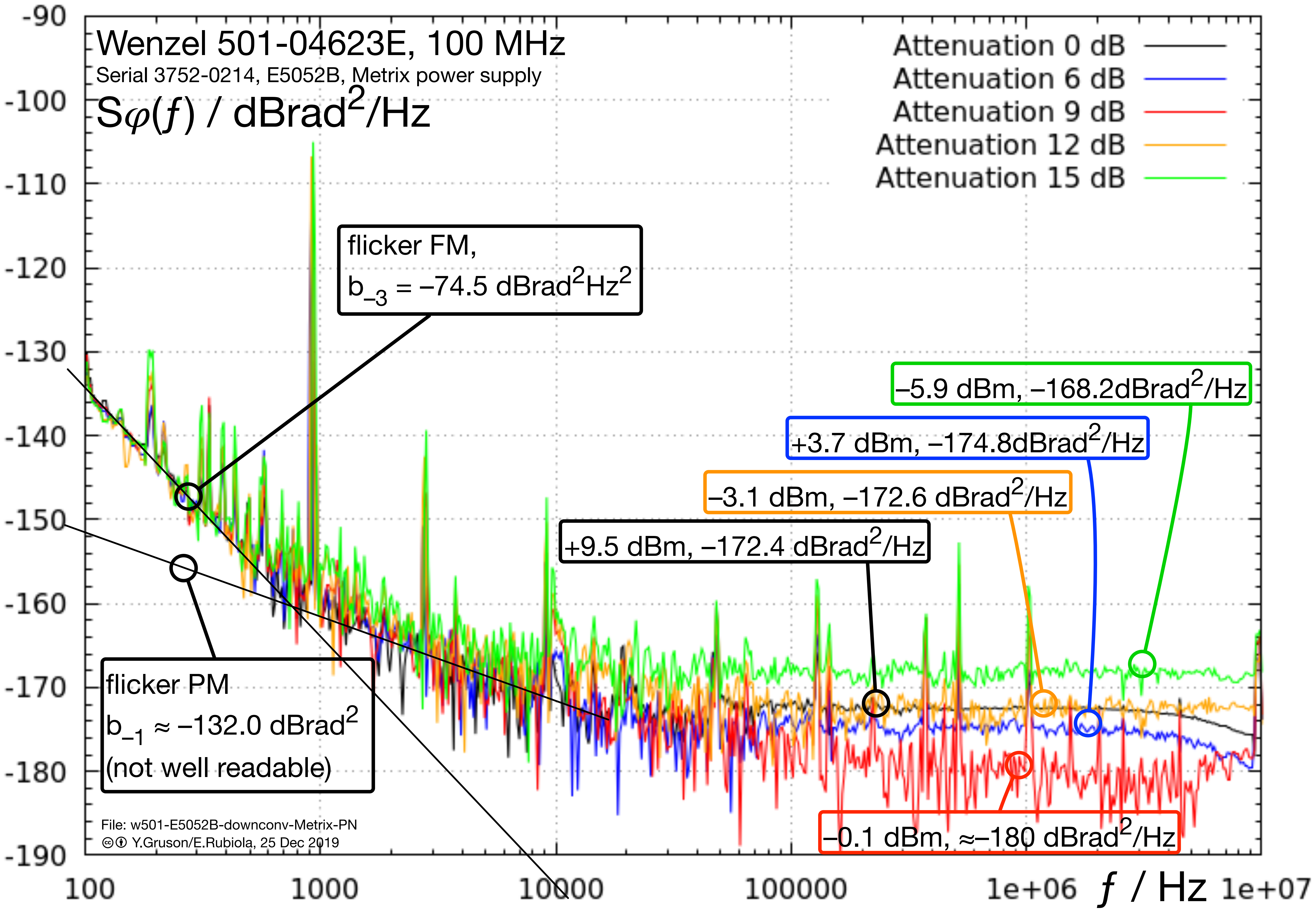
$1/f^3$ overlap, as expected

white PM —> different effects

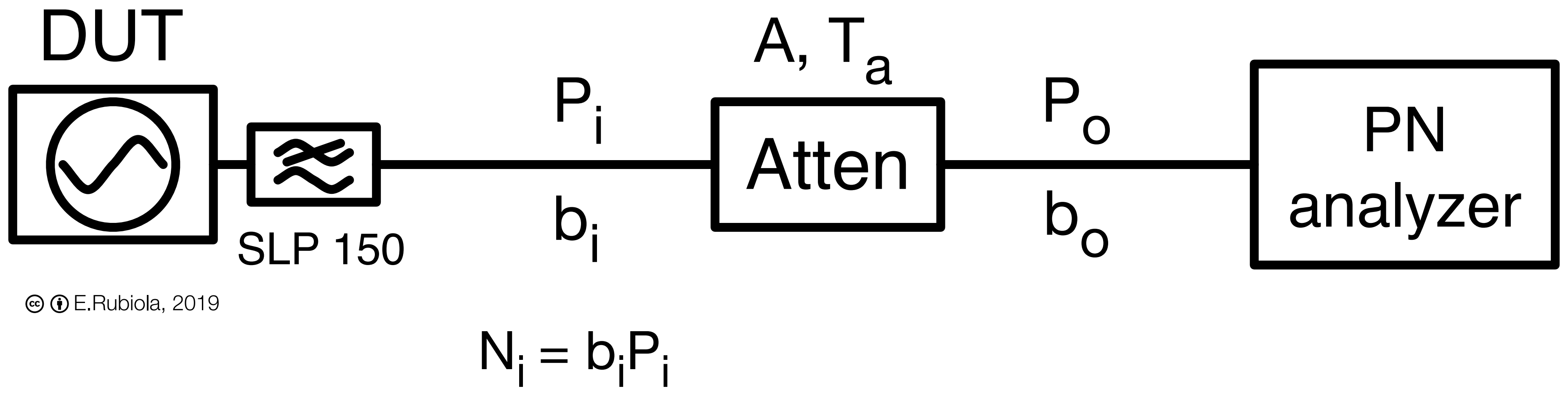


Reproducibility (Different PN Analyzer)

Wenzel 501-04623E & Keysight E5052B



Keysight E5052 PN Analyzer



No attenuation (A=1)

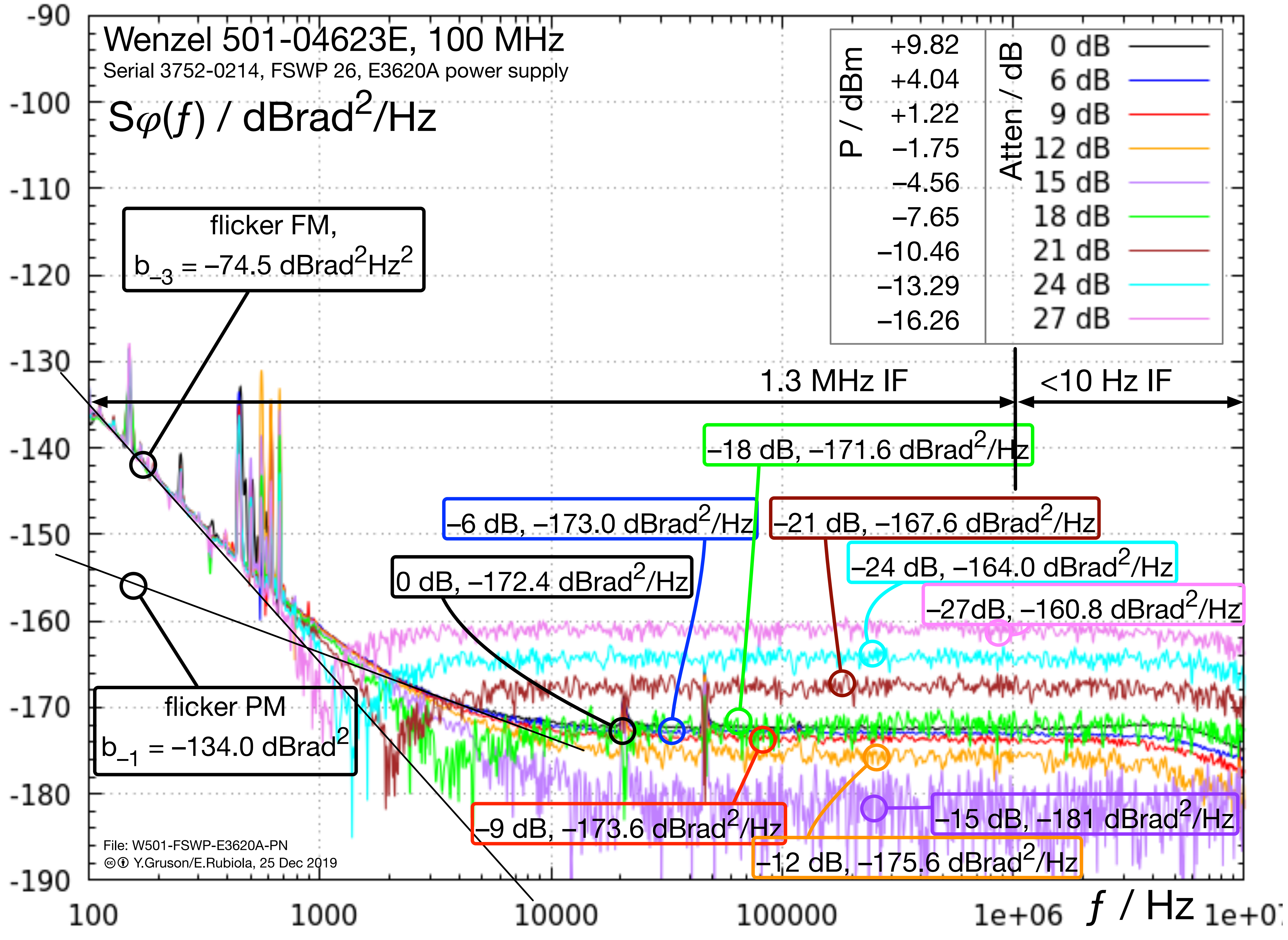
- Same b_{-3} = –74.5 dBrad²Hz²
- Negligible b_{-2}
- (unreadable b_{-1})
- Same b_0 = –172.4 dBrad²/Hz

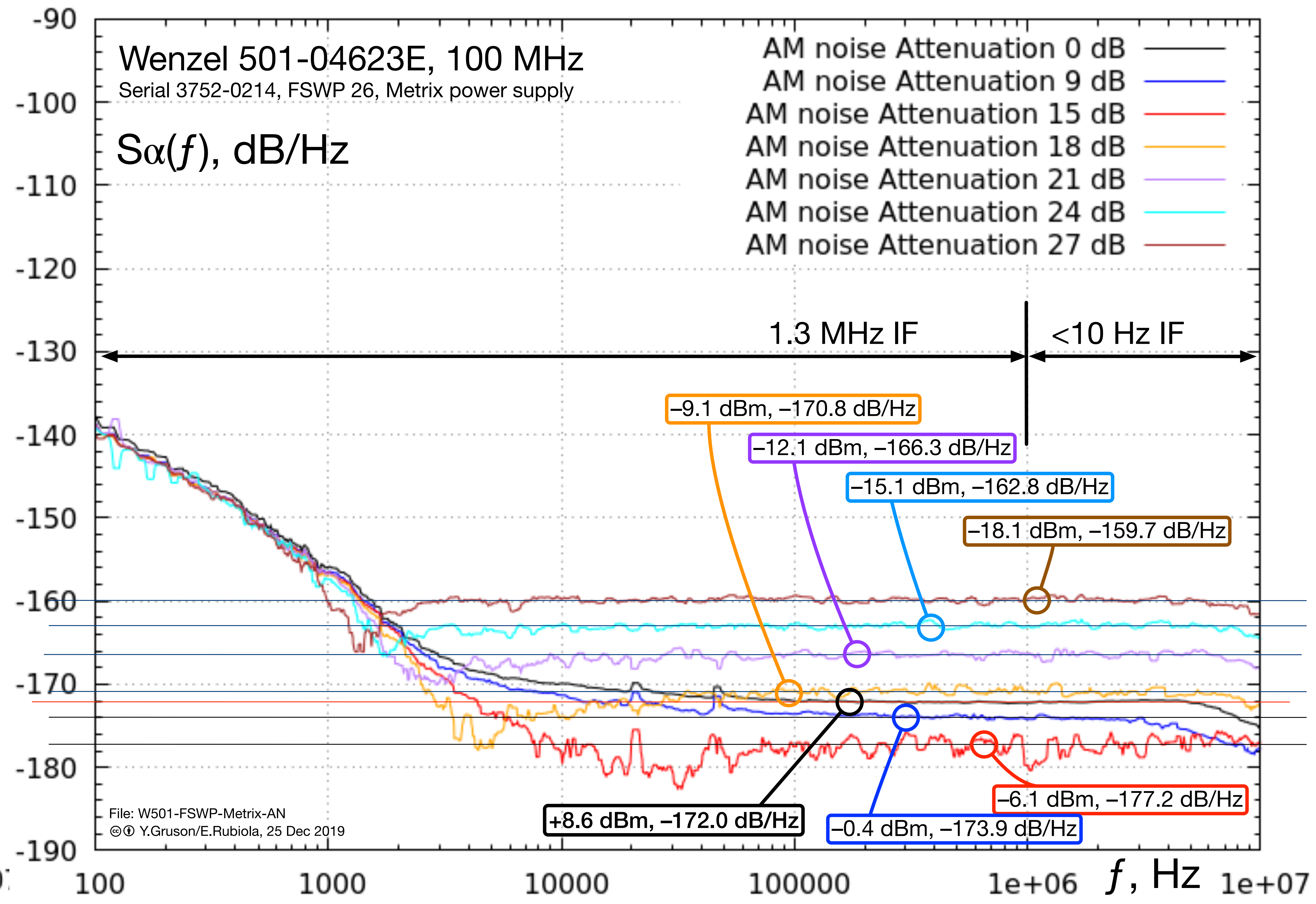
With attenuation (A<1)

- Minimum PM noise at 9 dB
- *P* range —> 15 dB max atten

PM vs AM Noise

Wenzel 501-04623E & Rohde Schwarz FSWP



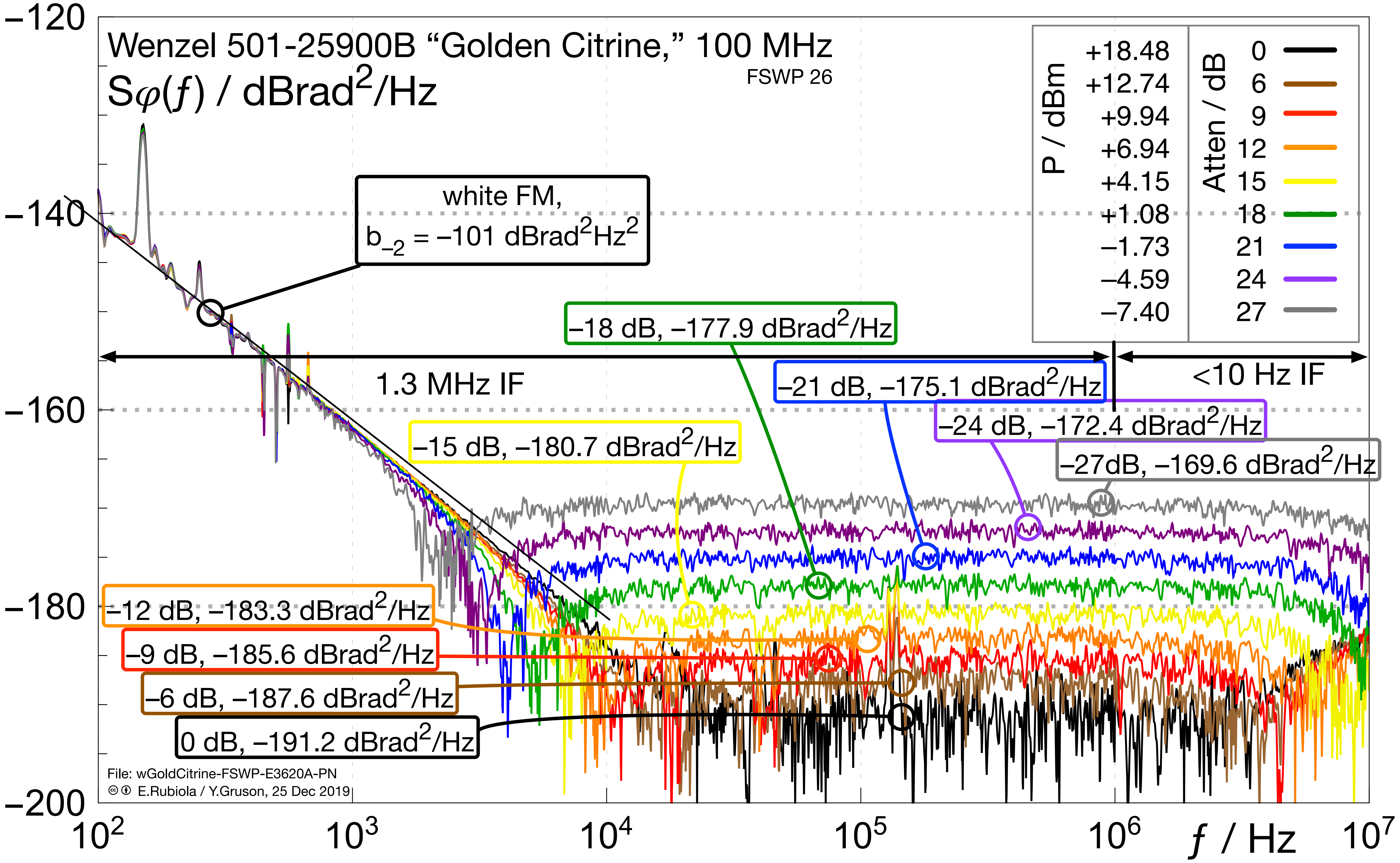


A similar phenomenon is observed

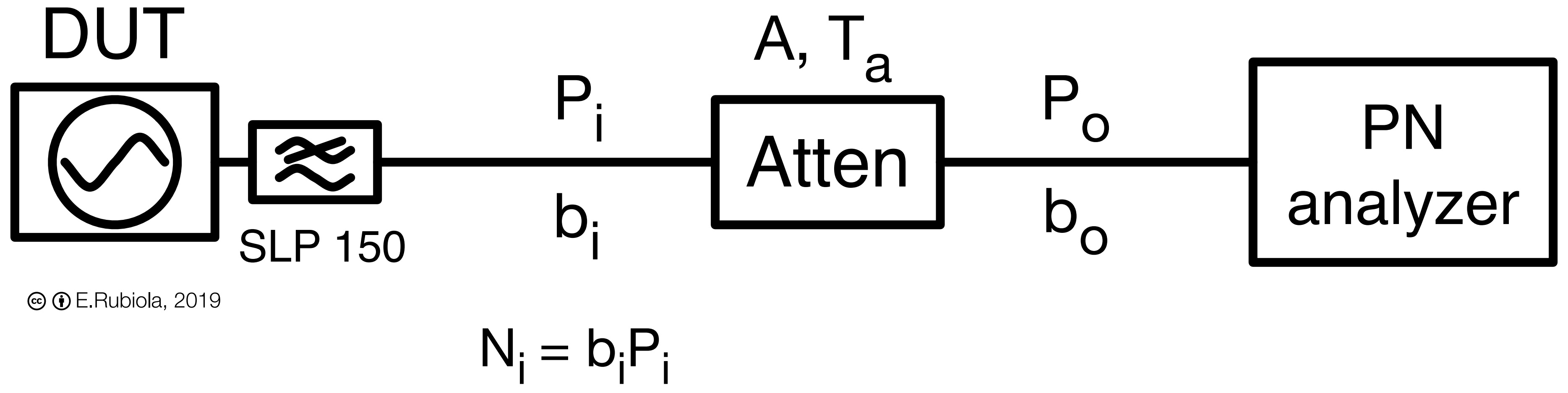
(Measuring AM, we spent less care in details)

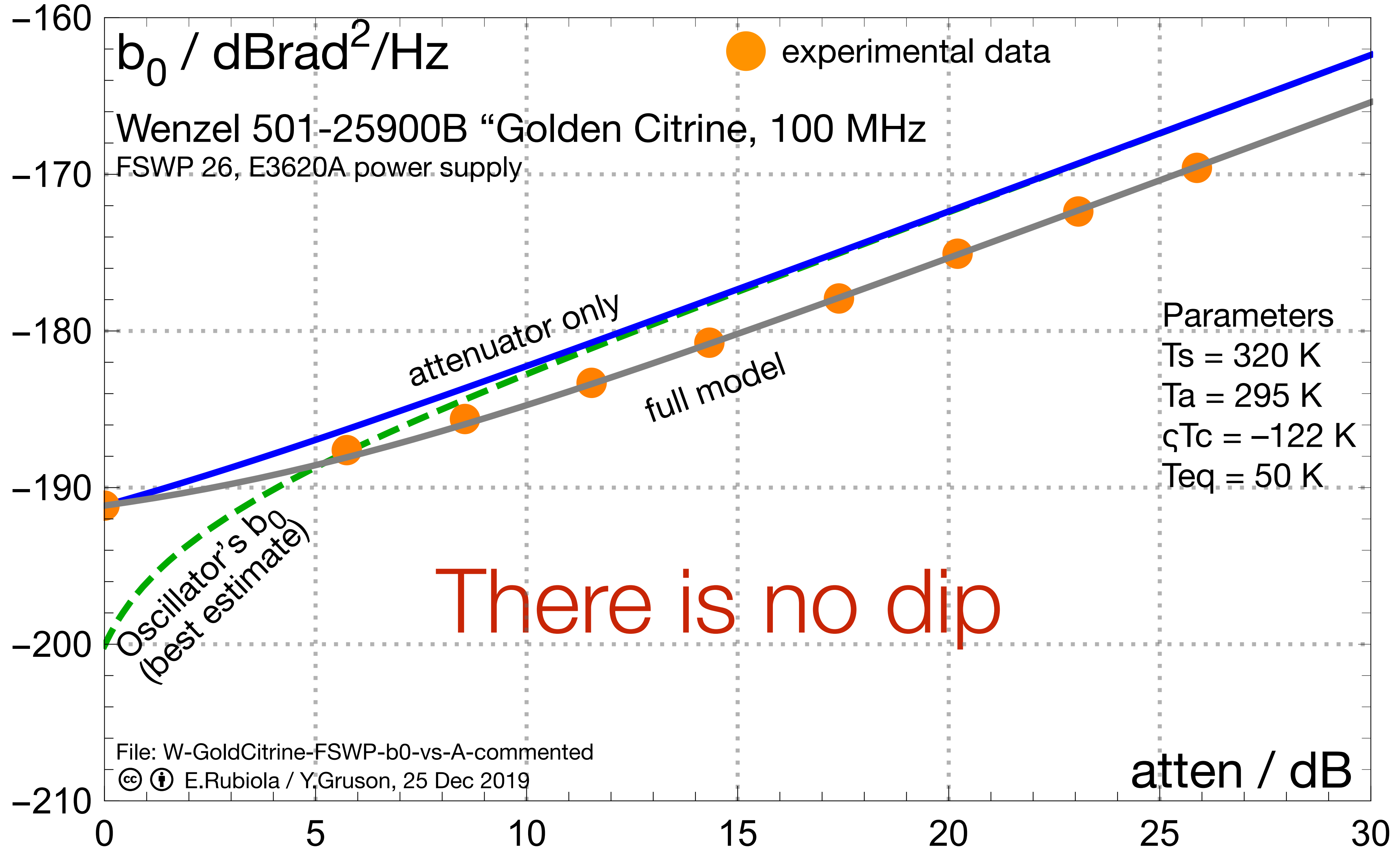
Experiment (Newest Low-Noise OCXO)

Wenzel 501-25900B “Golden Citrine” & Rohde Schwarz FSWP



R&S FSWP-26 PN Analyzer





Theoretical Background

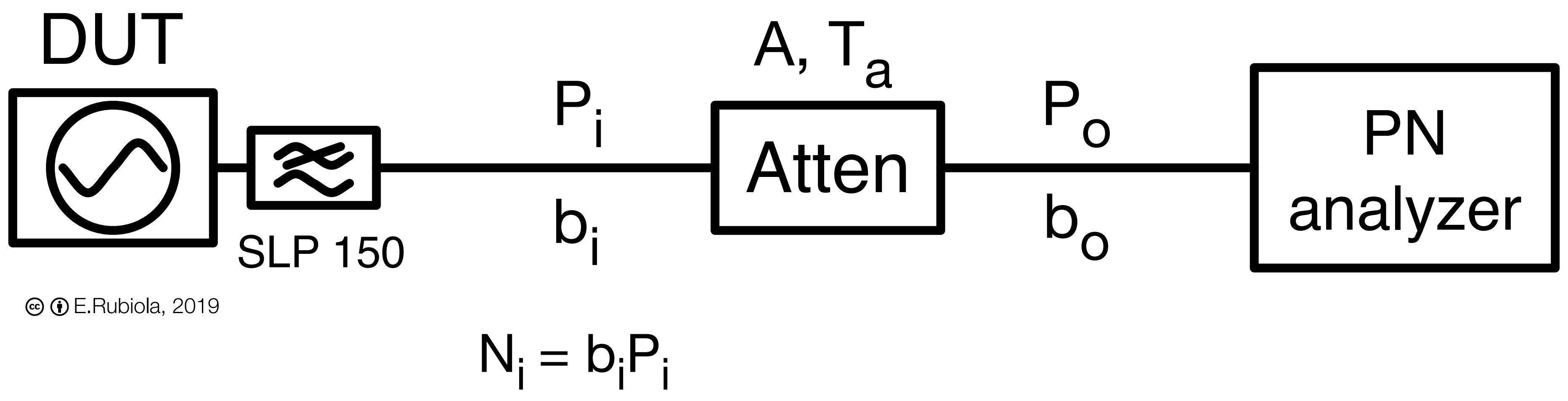
- Attenuator
- Cross Spectrum method
- Power splitters
- Quartz oscillators

We must understand the full chain

The Physics of the Attenuator

White RF Noise

radio astronomy and radiometry



$$N_i = kT_i$$ input

$$N_o = kT_iA^2 + kT_a\left(1 - A^2\right)$$ output

- Input kT_i —> out kT_iA^2, as any signal
- Term $kT_a(1–A^2)$ needed for the total noise be kT when everything is at T

White PM Noise

same concept, just ported

$$S_\varphi = \mathrm{b}_0$$ polynomial law

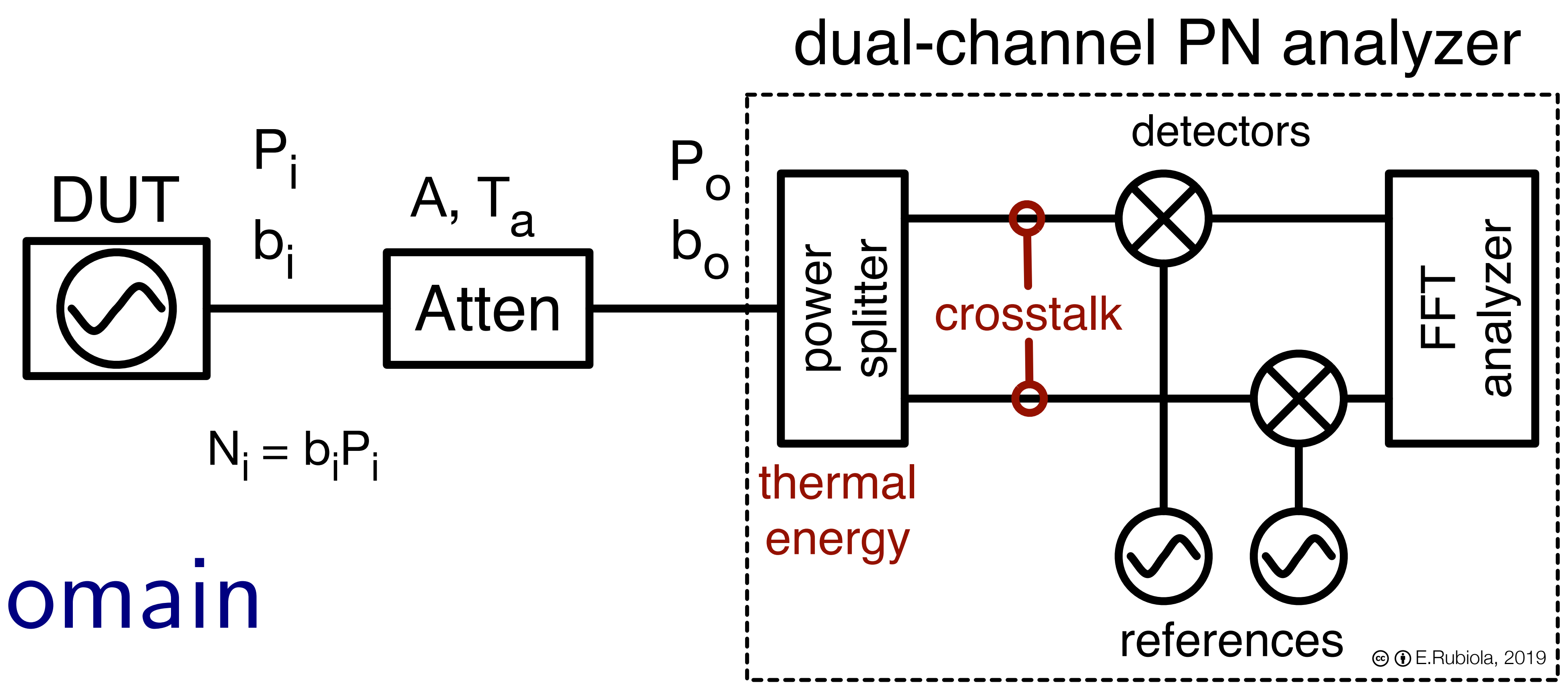
$$\mathrm{b}_i = \frac{kT_i}{P_i}$$ input "b$_0$"

$$P_o = A^2 P_i$$ output power

output "b$_0$"

$$\mathrm{b}_o = \frac{kT_i}{P_i} + \frac{kT_a\left(1 - A^2\right)}{A^2 P_i}$$

The Cross Spectrum Method



Time domain

$$x = a + c + \varsigma_x d$$

$$y = b + c + \varsigma_y d$$

Fourier transforms

±1

$$X = A + C + \varsigma_x D$$

$$Y = B + C + \varsigma_y D$$

DUT signal

disturbing signal, or crosstalk

single channel noise

One-sided cross spectrum

$$S_{yx}(f) = \frac{2}{T}\left[Y(f)\,X^*(f)\right]$$

Expand

$$S_{yx} = \frac{2}{T}\left[B + C + \underset{\pm D}{\varsigma D}\right]\left[A + C + \underset{\pm D}{\varsigma D}\right]^*$$

Uncorrelated A, B, C, D

$$S_{yx} = S_{cc} + \varsigma S_d$$

result

DUT spectrum

±1

bias

S_{yx} with a Correlated Term

A, *B* = instrument background
C = DUT noise
channel 1 $X = A + C$
channel 2 $Y = B + C$
A, *B*, *C* are independent Gaussian noises
Re{ } and Im{ } are independent Gaussian noises

Normalization: in 1 Hz bandwidth var{*A*} = var{*B*} = 1, var{*C*}=κ^2
var{*A*'} = var{*A*"} = var{*B*'} = var{*B*"} = 1/2, and var{*C*'} = var{*C*"} = $\kappa^2/2$

Cross-Spectrum

$$\langle S_{yx}\rangle_m = \frac{2}{T}\langle Y X^*\rangle_m = \frac{2}{T}\langle (Y' + iY'') \times (X' - iX'')\rangle_m$$

Expand using

$$X = (A' + iA'') + (C' + iC'') \qquad \text{and} \qquad Y = (B' + iB'') + (C' + iC'')$$

Split S_{yx} into three sets

$$\langle S_{yx}\rangle_m = \Big[\langle S_{yx}\rangle_m\Big]_{\text{instr}} + \Big[\langle S_{yx}\rangle_m\Big]_{\text{mixed}} + \Big[\langle S_{yx}\rangle_m\Big]_{\text{DUT}}$$

background only | background and DUT noise | DUT noise only

... and work it out !!!

S_{yx} with Correlated Term $\kappa \neq 0$

All the DUT signal goes in Re{S_{yx}}, Im{S_{yx}} contains only noise

Real

$\sigma^2 = 1/2$ | $\sigma^2 = 1/2$ | $\sigma^2 = \kappa^2/2$ | $\sigma^2 = 1/2$ | $\sigma^2 = \kappa^2/2$ | $\sigma^2 = \kappa^2/2$

$$\Re\left\{\langle S_{yx}\rangle_m\right\} = \frac{2}{T}\left\{\langle B'A' + B''A''\rangle_m + \langle B'C' + B''C''\rangle_m + \langle A'C' + A''C''\rangle_m + \langle (C')^2 + (C'')^2\rangle_m\right\}$$

Bessel K_0, $\mu = 0$, $\sigma^2 = 1/4$

Bessel K_0, $\mu = 0$, $\sigma^2 = \kappa^2/4$

χ^2, DF = 2, $\mu = \kappa^2$, $\sigma^2 = \kappa^4$

Gaussian, $\mu = 0$, $\sigma^2 = 1/2m$

Gaussian, $\mu = 0$, $\sigma^2 = \kappa^2/2m$

Gaussian, $\mu = 0$, $\sigma^2 = \kappa^2/2m$

χ^2, DF = 2m, $\mu = \kappa^2$, $\sigma^2 = \kappa^4/m$ (the signal)

Gaussian, $\mu = 0$, $\sigma^2 = (1+2\kappa^2)/2m$

Note: DF < 2m
See vol.XVI p.56

← noise → ← signal →

Imaginary

$$\Im\left\{\langle S_{yx}\rangle_m\right\} = \frac{2}{T}\left\{\langle B''A' + B'A''\rangle_m + \langle B''C' - B'C''\rangle_m + \langle A'C'' - A''C'\rangle_m\right\}$$

Bessel K_0, $\mu = 0$, $\sigma^2 = 1/4$

Bessel K_0, $\mu = 0$, $\sigma^2 = \kappa^2/4$

Gaussian, $\mu = 0$, $\sigma^2 = 1/2m$

Gaussian, $\mu = 0$, $\sigma^2 = \kappa^2/2m$

Gaussian, $\mu = 0$, $\sigma^2 = \kappa^2/2m$

Gaussian, $\mu = 0$, $\sigma^2 = (1+2\kappa^2)/2m$

(no signal)

Cross-Spectrum Estimators

Absolute-value estimator

$$\widehat{S_{yx}} = \left|\langle S_{yx}\rangle_m\right|$$

- Mostly used
- Biased
- Slow <— noise in Im{S_{yx}}
- Hides negative outcomes

$$\mathbb{E}\{\widehat{\mathsf{b}_o}\} = \left|\frac{kT_i}{P_i} + \frac{k(1-A^2)T_a}{A^2P_i} + \frac{k(\varsigma T_c - T_s)}{A^2P_i}\right|$$

Real-part estimator

$$\widehat{S_{yx}} = \Re\left\{\langle S_{yx}\rangle_m\right\}$$

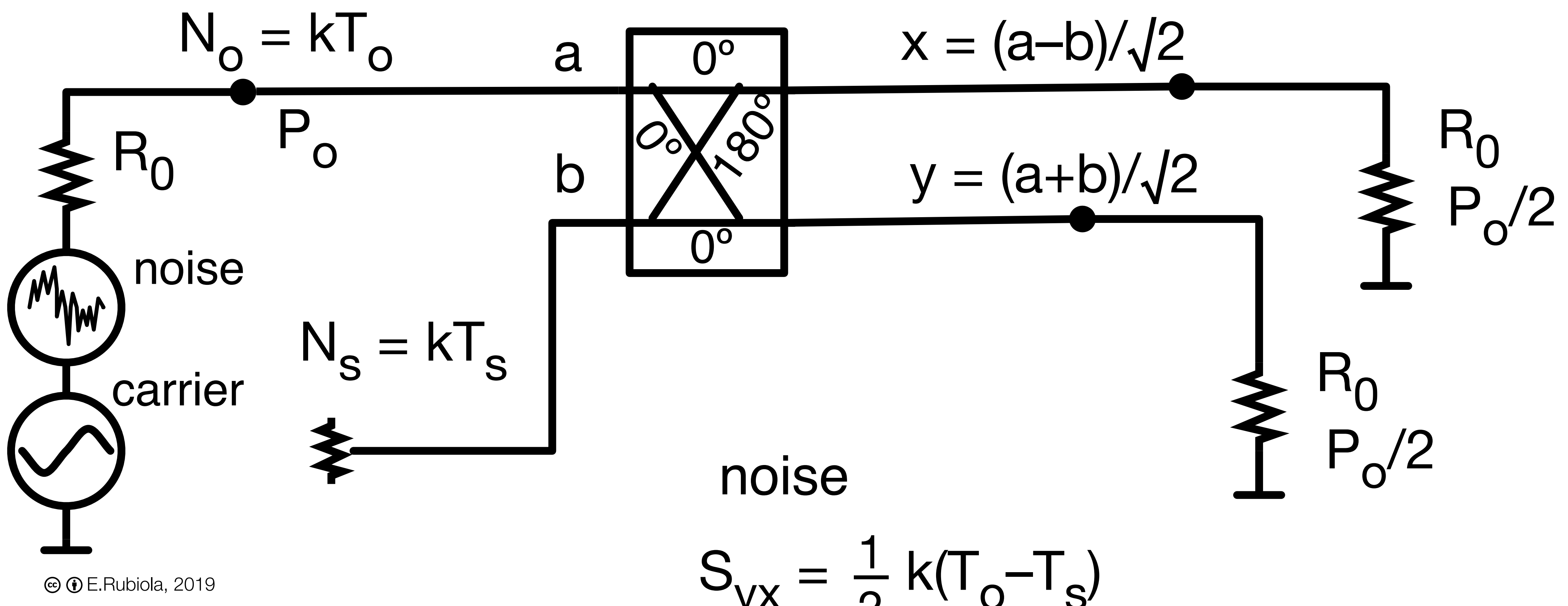
- Not used in PM noise analyzers (?)
- Unbiased
- Fastest
- Shows negative outcomes

$$\mathbb{E}\{\widehat{\mathsf{b}_o}\} = \frac{kT_i}{P_i} + \frac{k(1-A^2)T_a}{A^2P_i} + \frac{k(\varsigma T_c - T_s)}{A^2P_i}$$

Account for a crosstalk term ςT_c

Power Splitters

3 dB (loss-free) power splitter

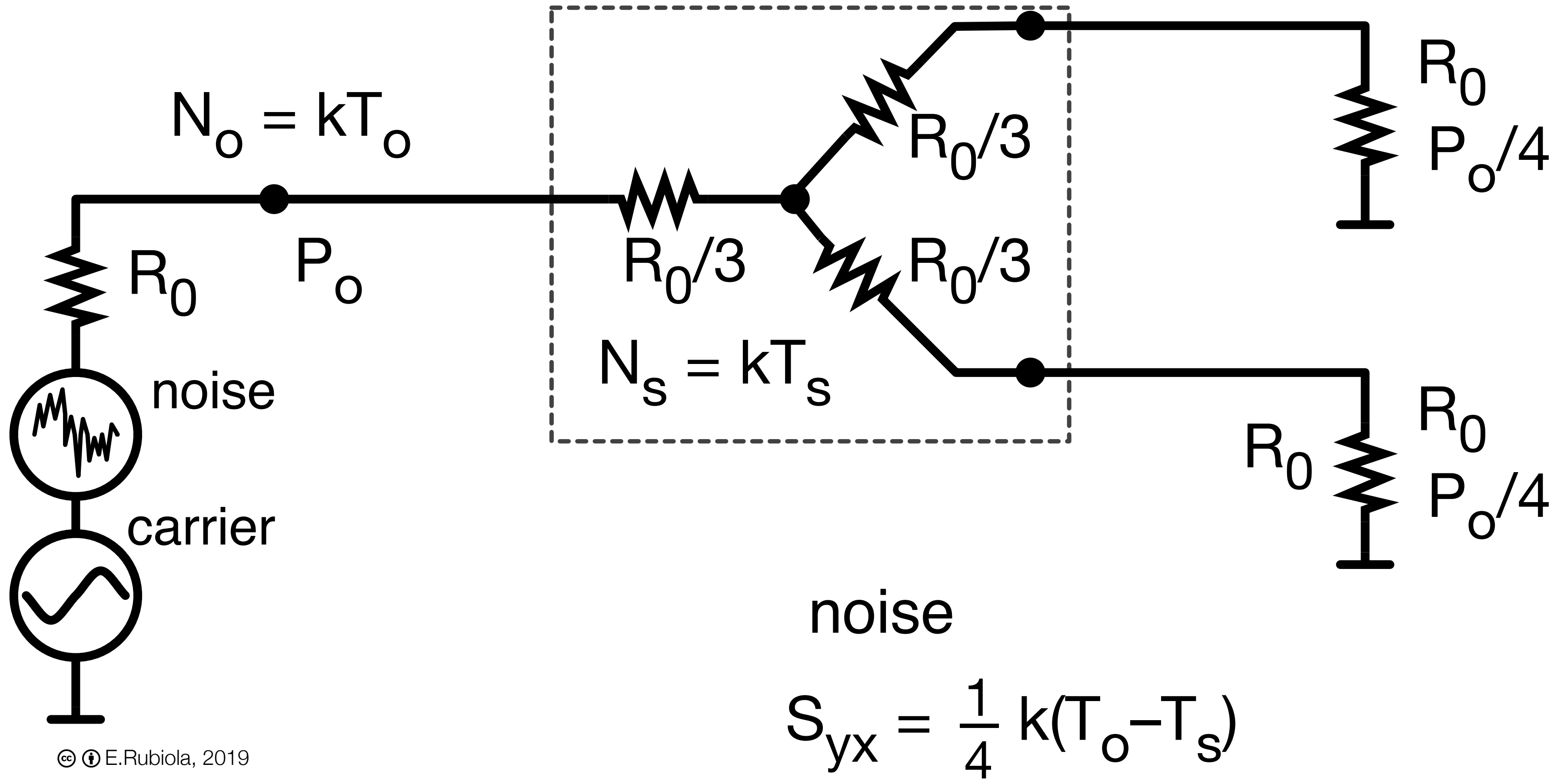


noise

$S_{yx} = \frac{1}{2} k(T_o - T_s)$

$S_{\varphi\varphi} = k(T_o - T_s) / P_o$

6 dB (resistive) power splitter



noise

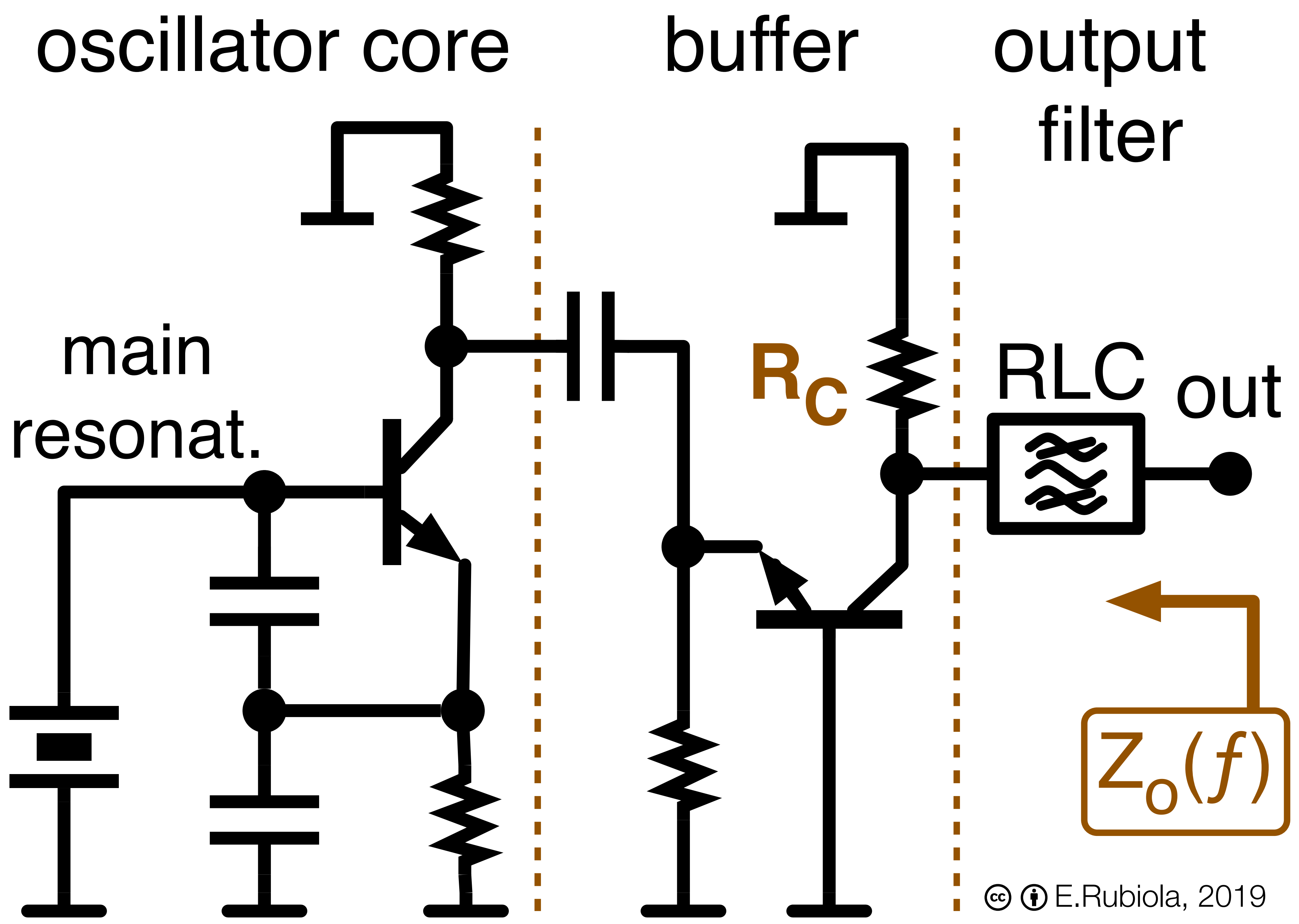
$S_{yx} = \frac{1}{4} k(T_o - T_s)$

$S_{\varphi\varphi} = k(T_o - T_s) / P_o$

Same PM noise

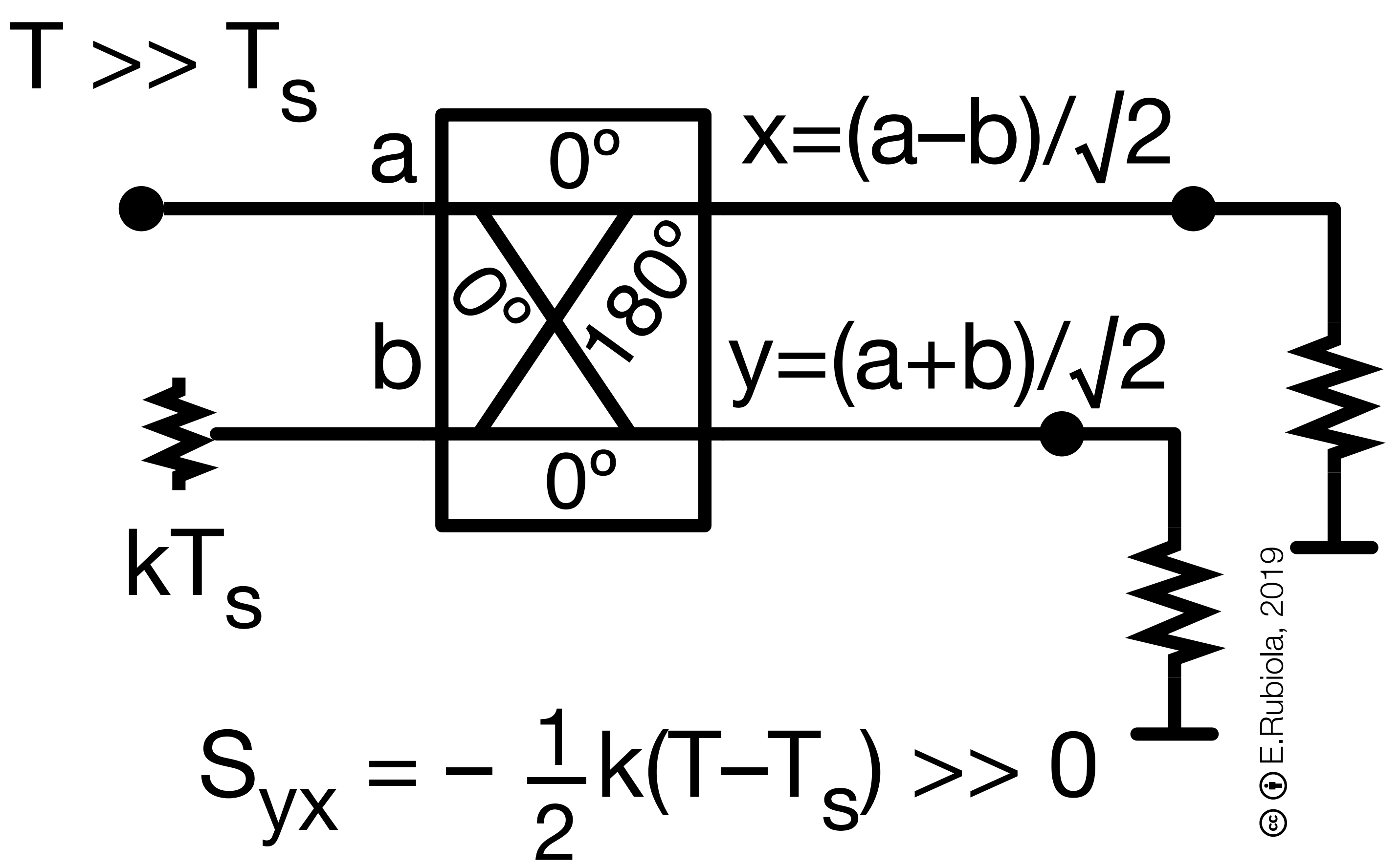
$$S_{\varphi\varphi} = \frac{k(T_o - T_s)}{P_o}$$

The Old-Style Quartz Oscillator

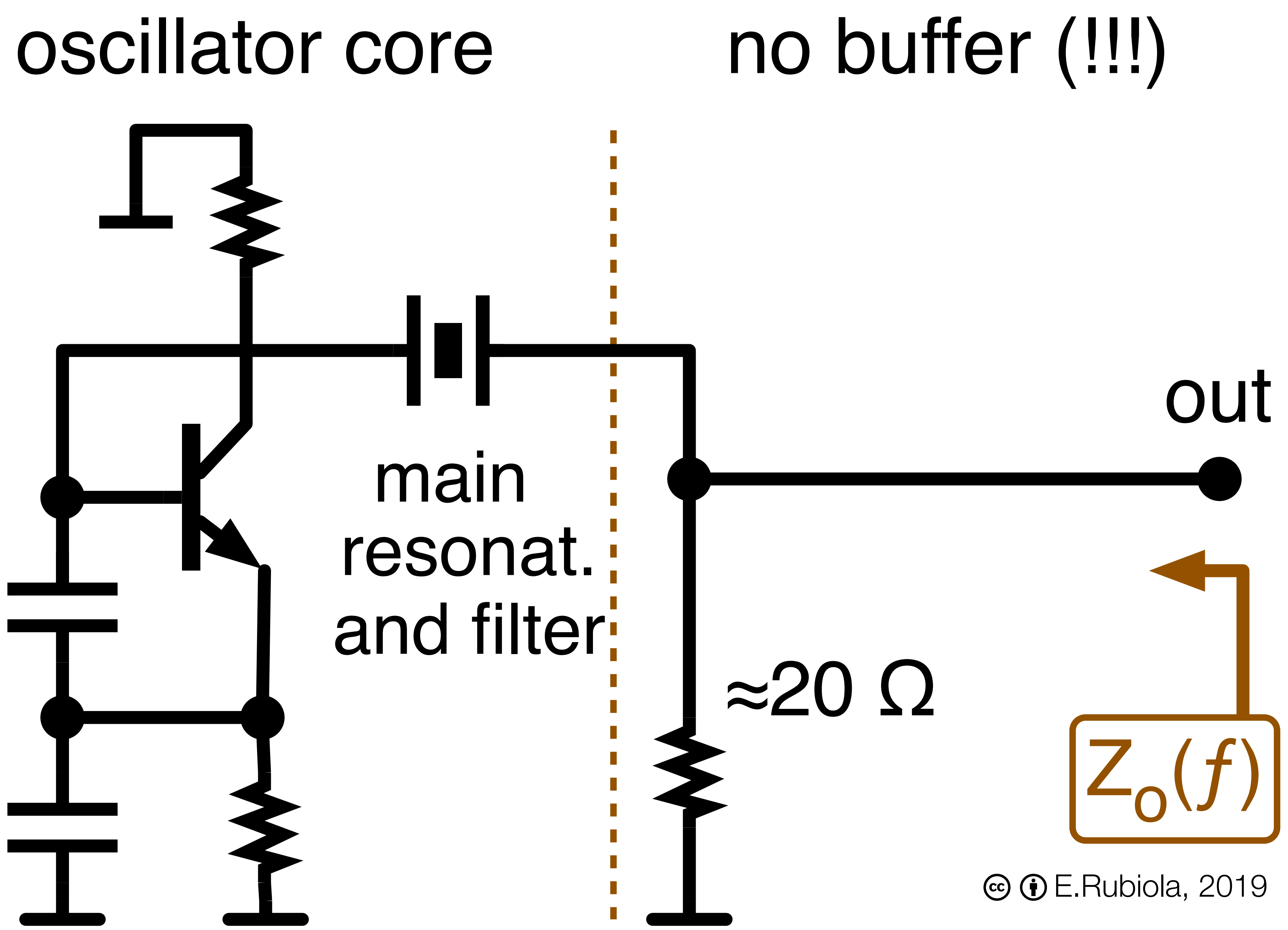


No problem with this

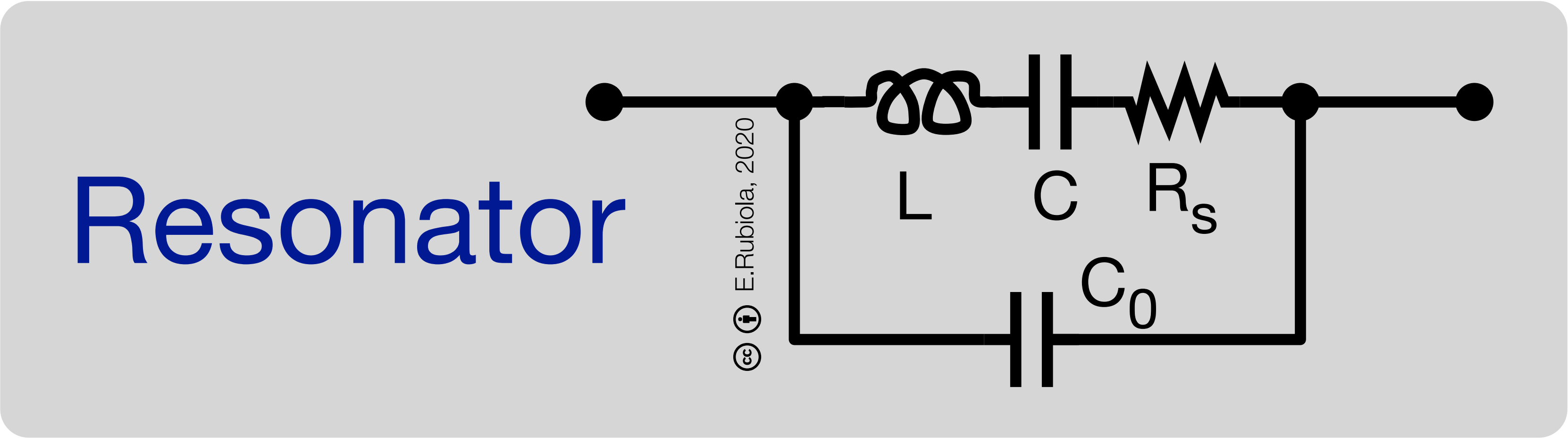
- The buffer amplifies the core noise
- T>>Ts
- Small under-estimation of the DUT noise



The Rohde Oscillator



U. L. Rohde, “Crystal oscillator provides low noise,” Electronic Design, vol. 21, p. 11 &14, Oct. 11, 1975



Out of the xtal bandwidth

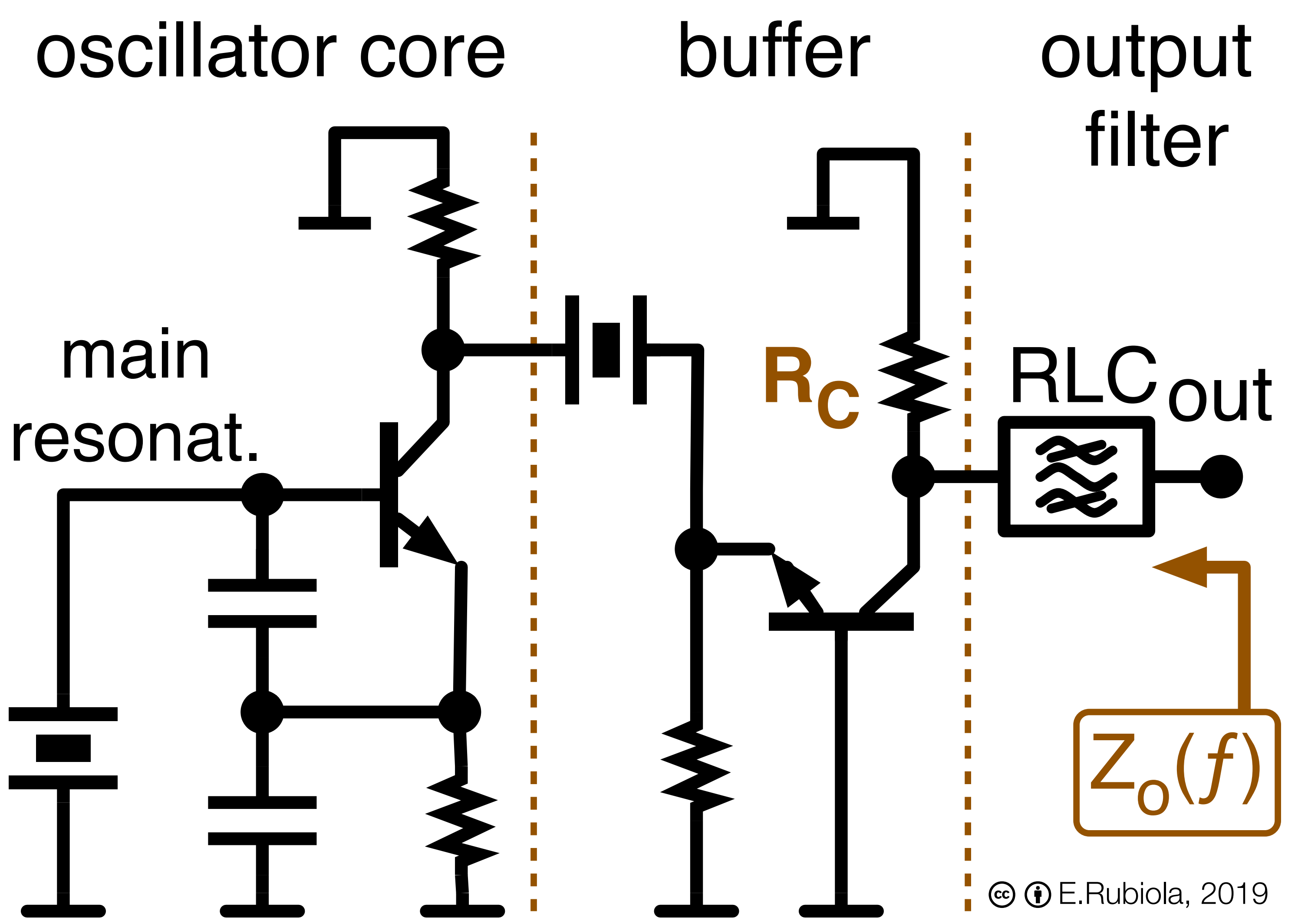
- Sustaining ampli and Rs are not coupled to the output
- The output noise is that of a small resistor

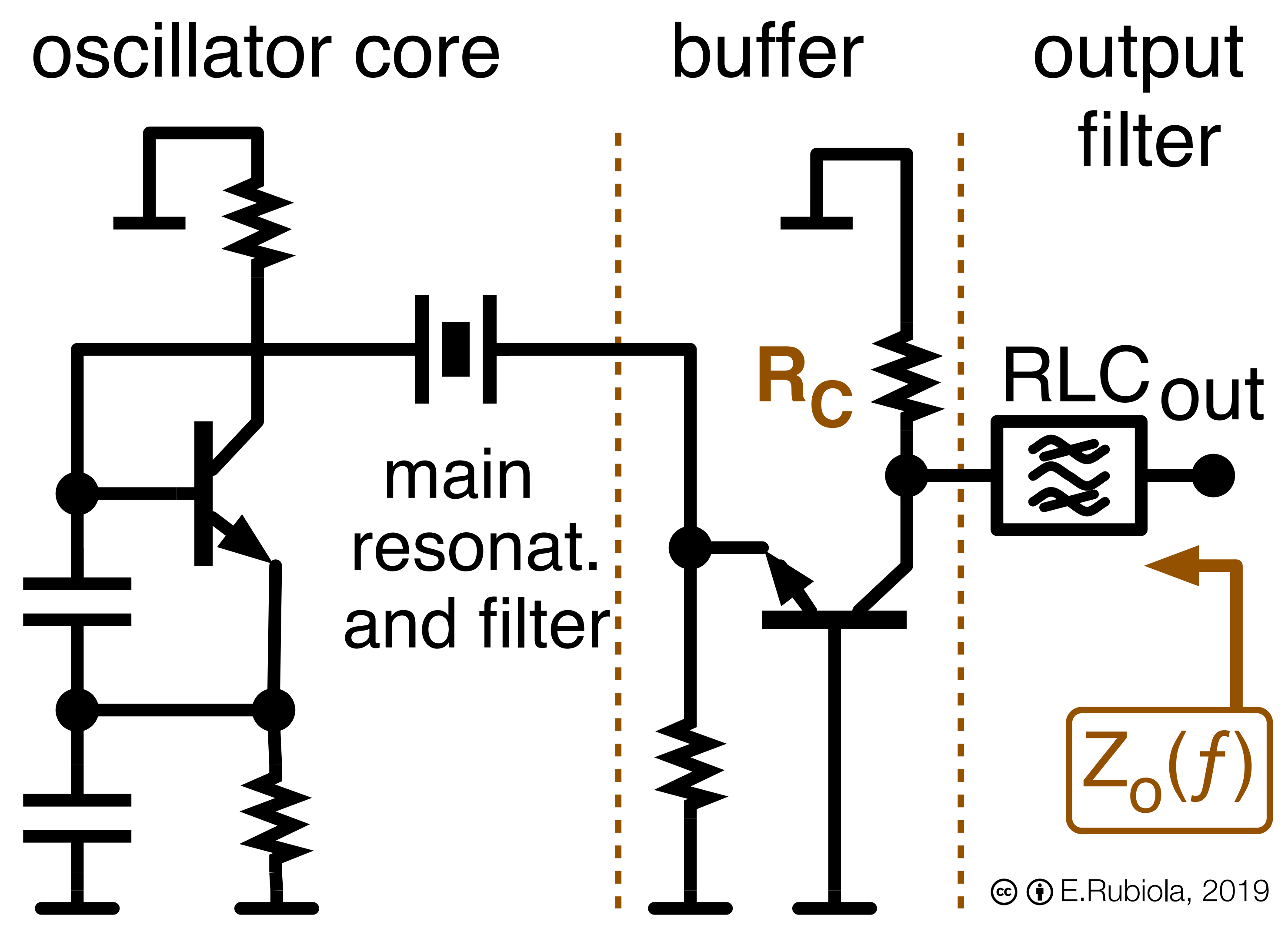
50 Ω => 0.9 nV/√Hz

20 Ω => 0.57 nV/√Hz

The cross spectrum was still not used

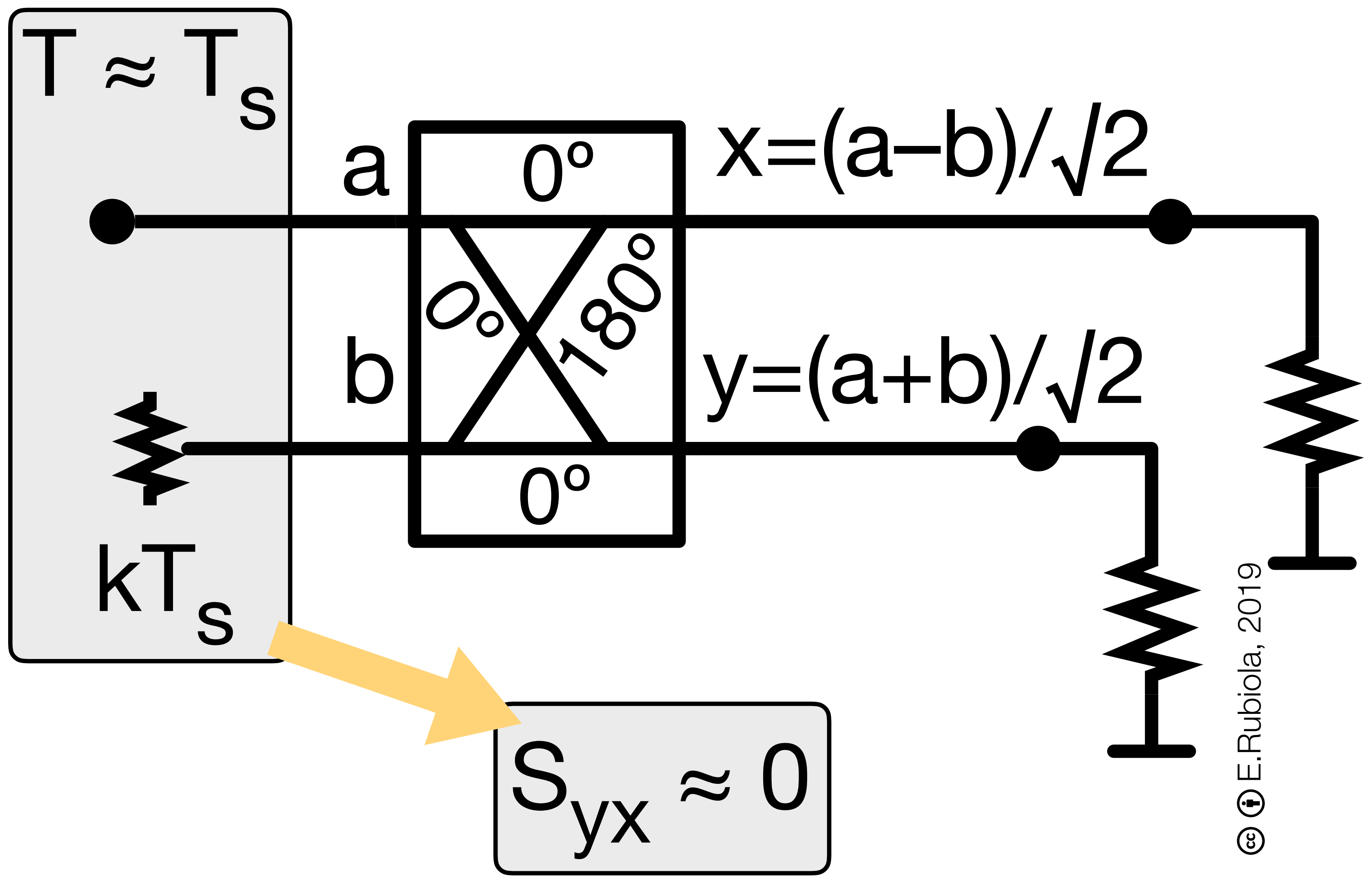
The Thermally-Limited Oscillator



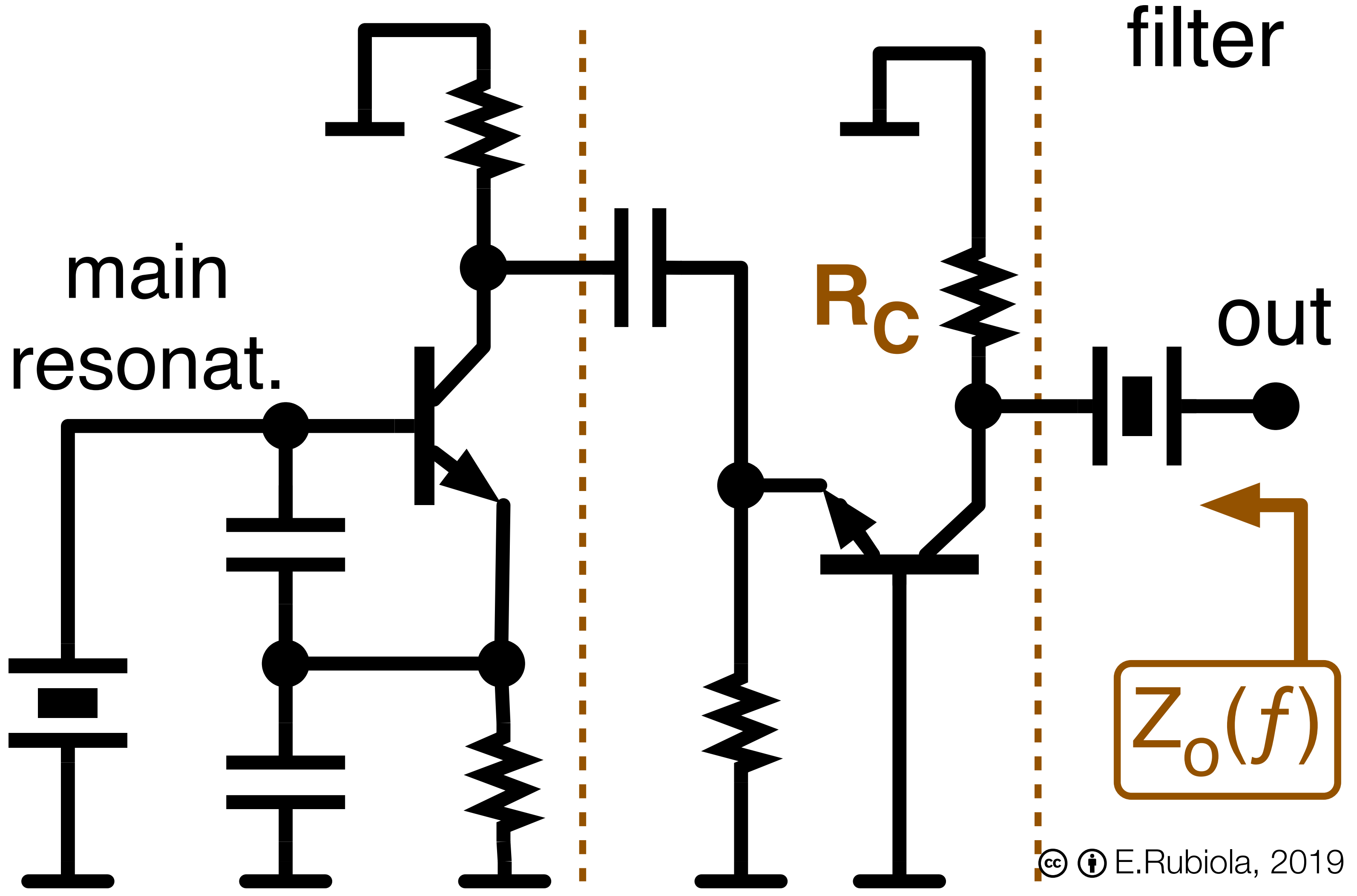


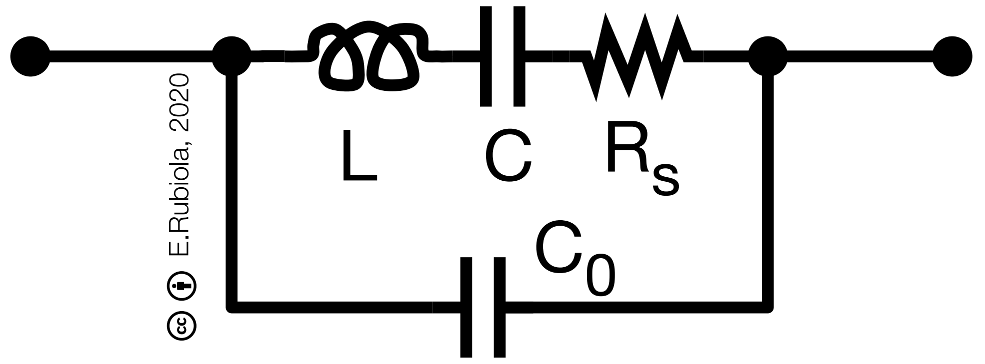
Out of the xtal bandwidth

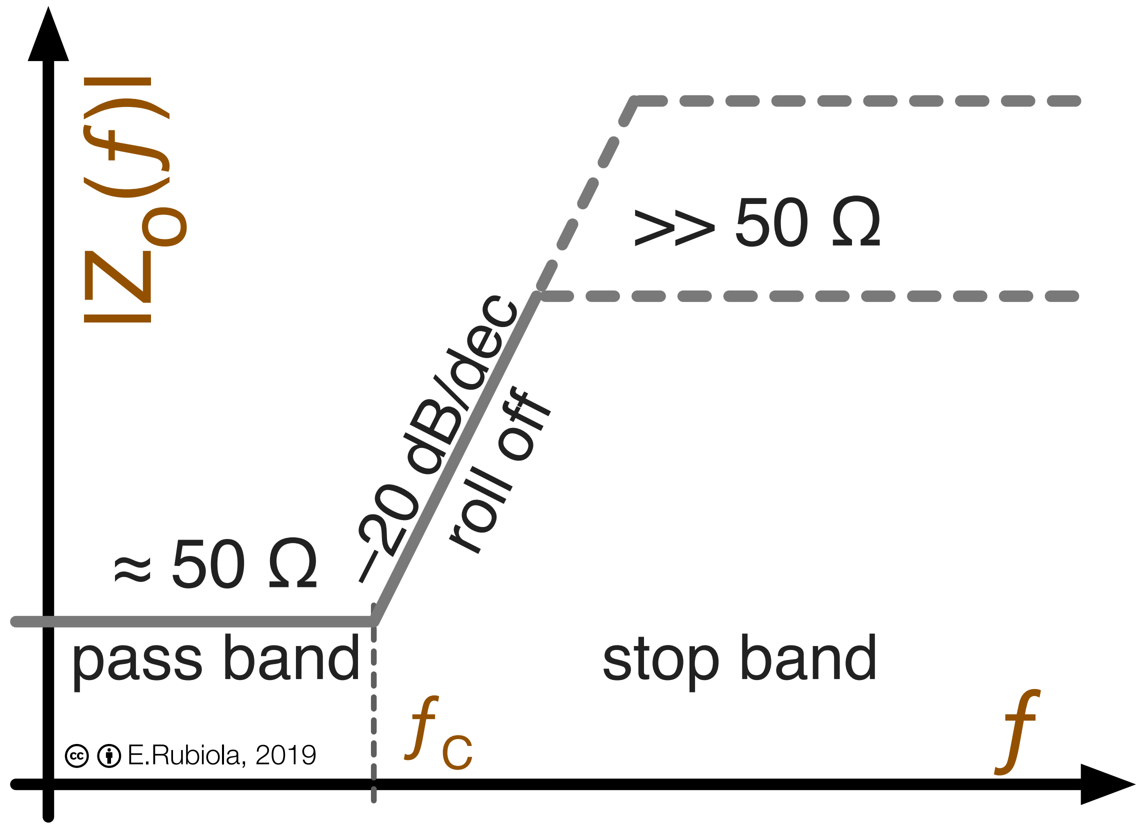
- Sustaining ampli and Rs are not coupled to the output to the CB amplifier
- The output noise is that of R_C
- The cross spectrum is $Syx \approx 0$



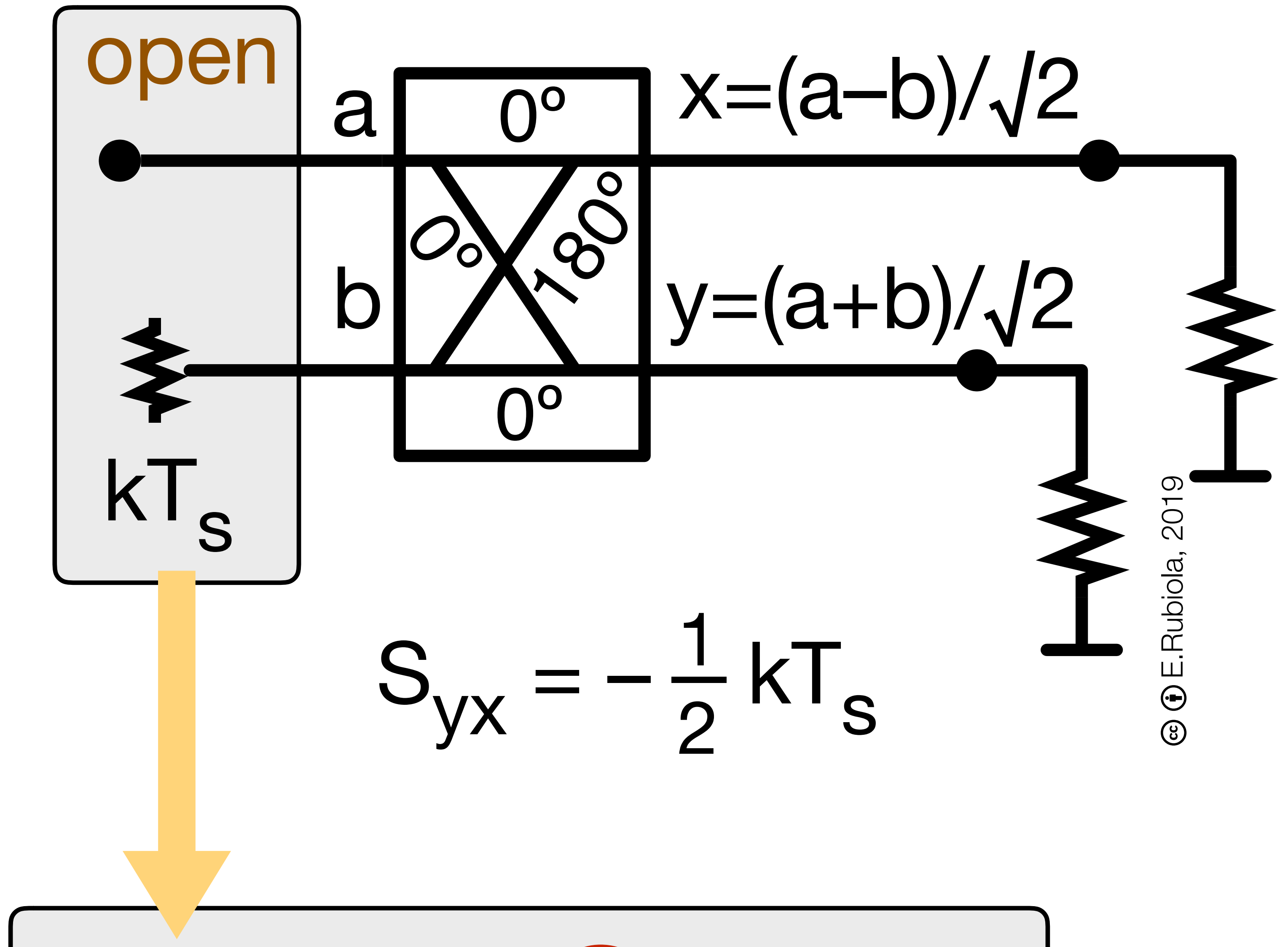
The Sub-Thermally-Limited Oscillator







- Beyond f_C,
  - $|Z_o| >> 50\ \Omega$
  - The power-splitter input is open circuit



… and $S_{yx} < 0$

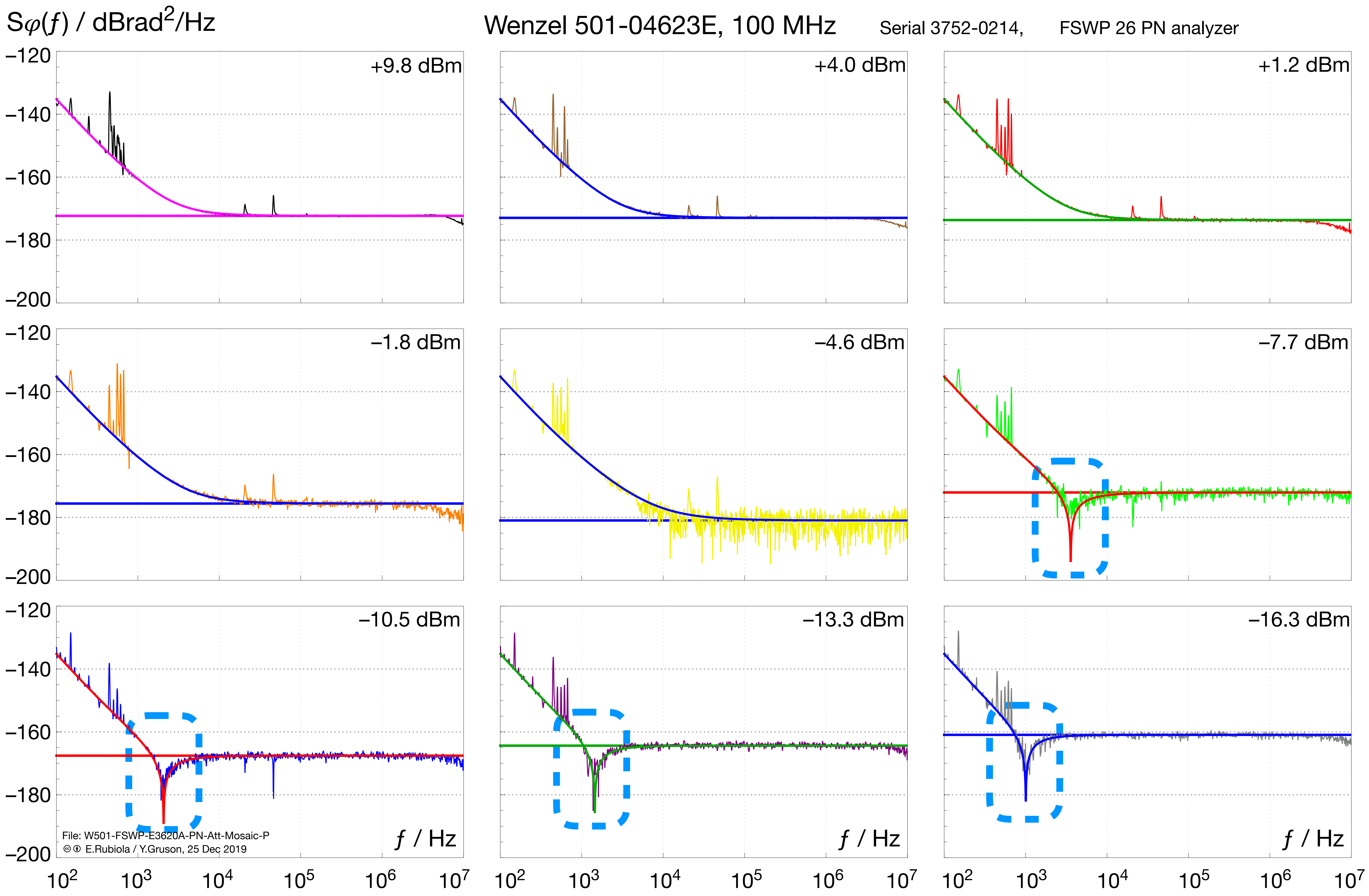
Interpretation

OCXO attenuator splitter & crosstalk

$$\mathbb{E}\{S_{yx}(f)\} = \frac{\mathrm{b}_{-3}}{f^3} + \frac{\mathrm{b}_{-2}}{f^2} + \frac{\mathrm{b}_{-1}}{f} + \frac{kT_i}{P_i} + \frac{k(1-A^2)T_a}{A^2 P_i} + \frac{k(\varsigma T_c - T_s)}{A^2 P_i}$$

Theory vs Experiment

$$\mathbb{E}\{\widehat{S_{yx}(f)}\} = \left| \frac{\mathrm{b}_{-3}}{f^3} + \frac{\mathrm{b}_{-2}}{f^2} + \frac{\mathrm{b}_{-1}}{f} + \frac{kT_i}{P_i} + \frac{k(1-A^2)T_a}{A^2P_i} + \frac{k(\varsigma T_c - T_s)}{A^2P_i} \right|$$



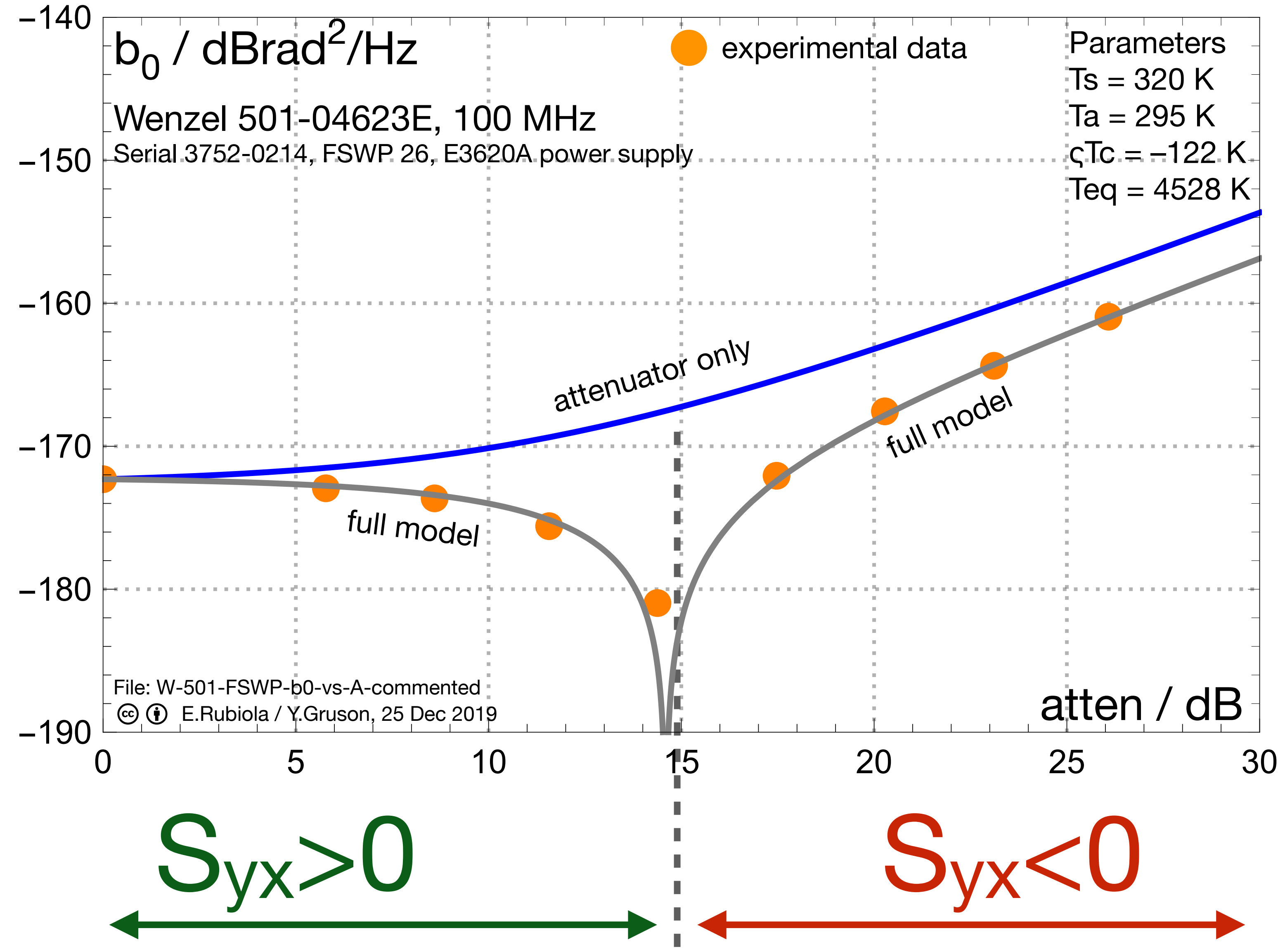
Parameters

T_s = 320 K

T_a = 295 K

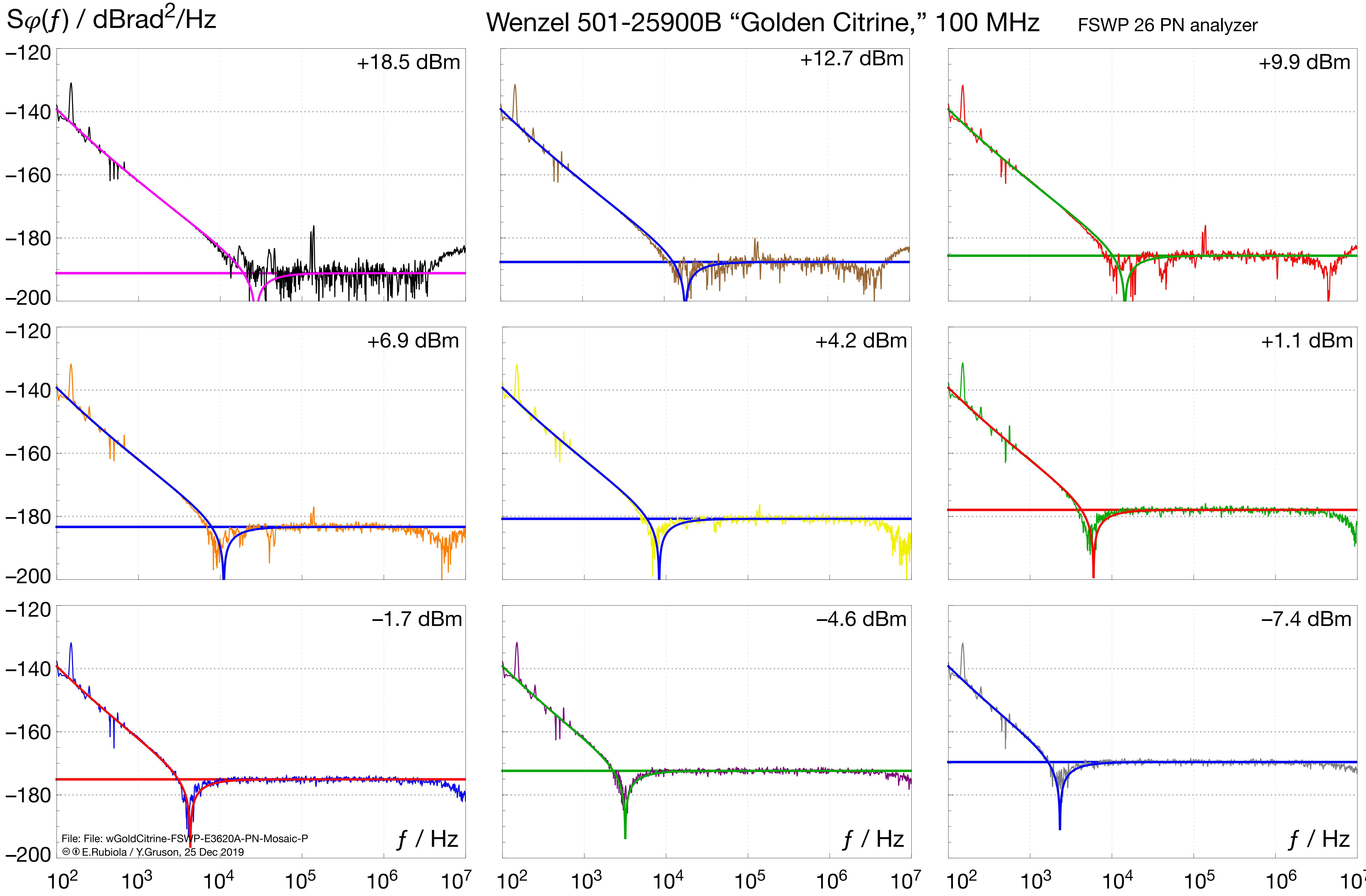
ςT_c = −122 K

T_{eq} = 4528 K



Theory vs Experiment

$$\mathbb{E}\{\widehat{S_{yx}(f)}\} = \left| \frac{\mathrm{b}_{-3}}{f^3} + \frac{\mathrm{b}_{-2}}{f^2} + \frac{\mathrm{b}_{-1}}{f} + \frac{kT_i}{P_i} + \frac{k(1-A^2)T_a}{A^2 P_i} + \frac{k(\varsigma T_c - T_s)}{A^2 P_i} \right|$$



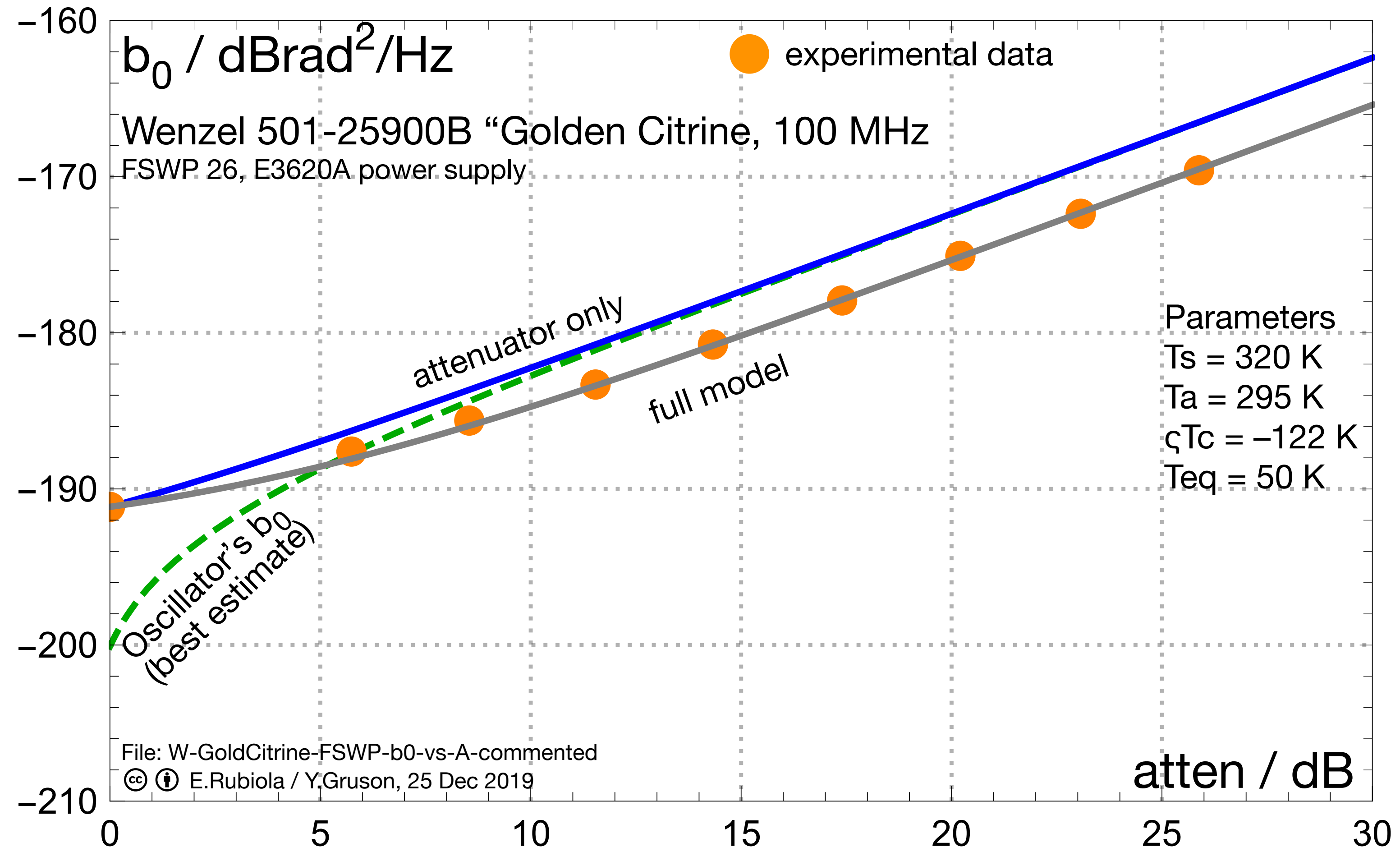
Parameters

T_s = 320 K

T_a = 295 K

ςT_c = –122 K

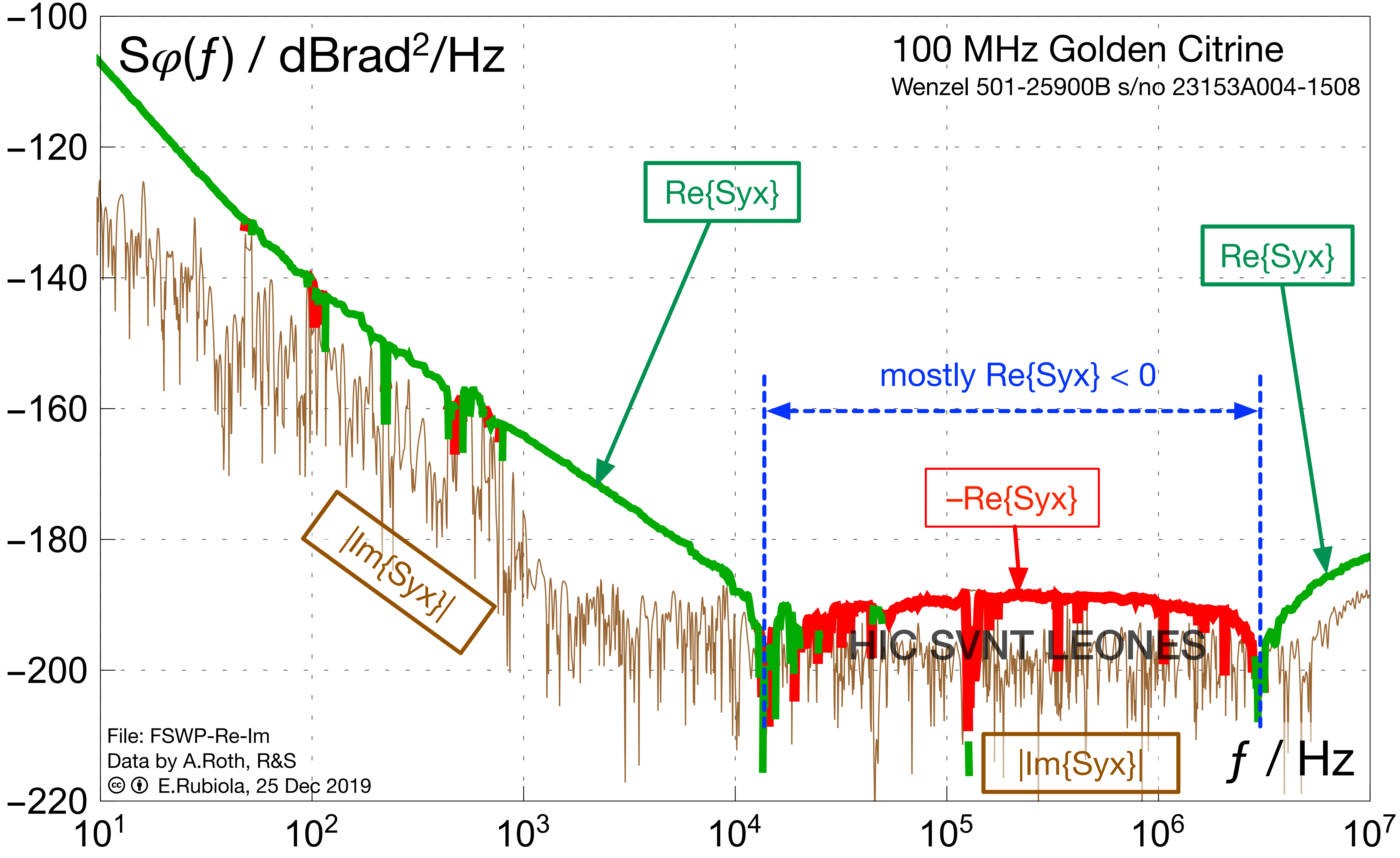
T_{eq} = 50 K



$S_{yx} < 0$

Experiment

Hacked FSWP-26 –> extract Re{S_{yx}} and Im{S_{yx}}



$1/f^2$ and $1/f^3$ regions, all right

- Re{S_{yx}} ≫ 0
- |Im{S_{yx}}| ≪ |Re{S_{yx}}|

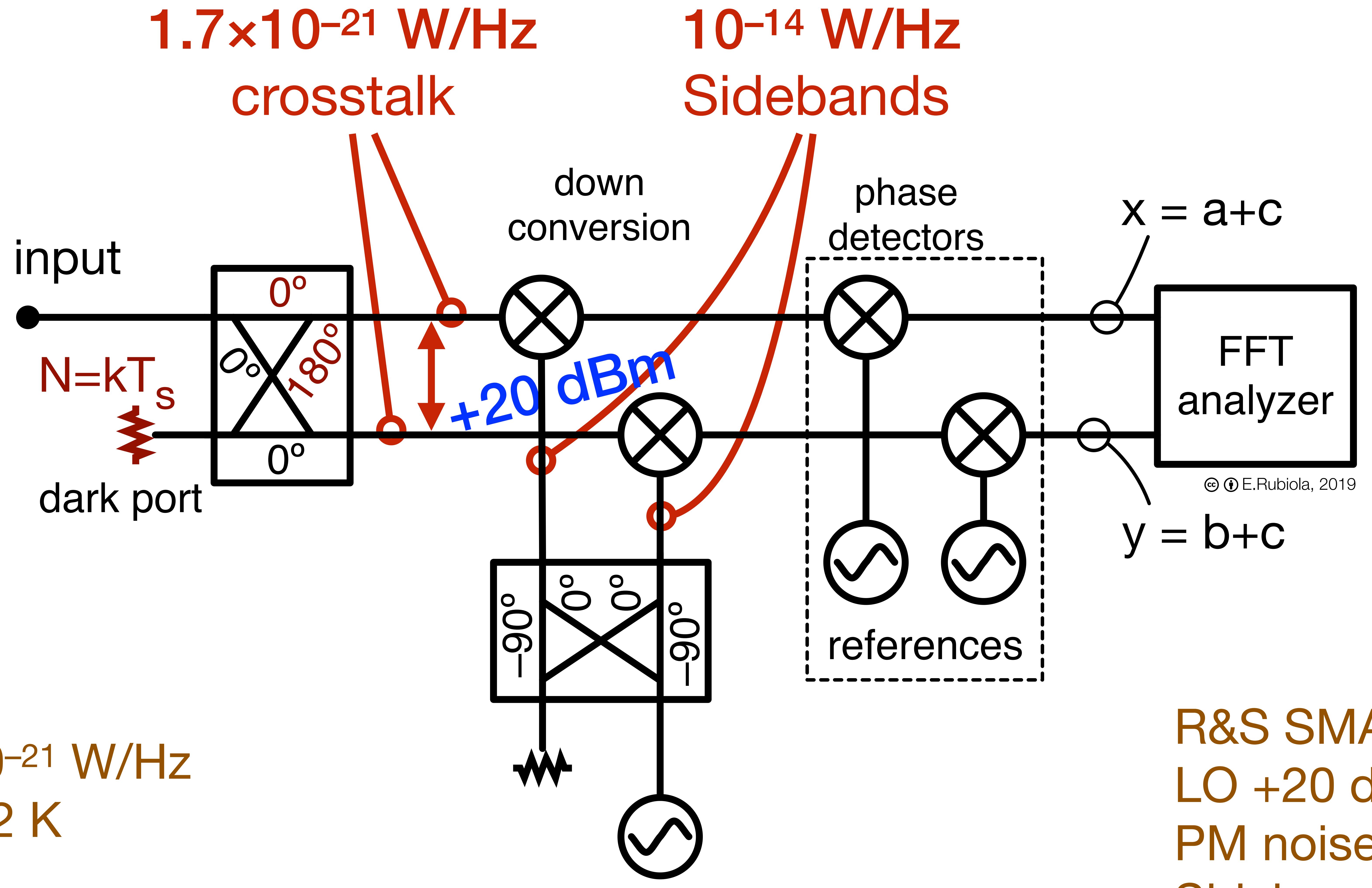
White region wrong

- Re{S_{yx}} < 0

HIC SVNT LEONES (here be dragons)
unexplored land, or to a land where humans are not permitted

Guess the Origin of Crosstalk

Locate where strongest and weakest signal are close together

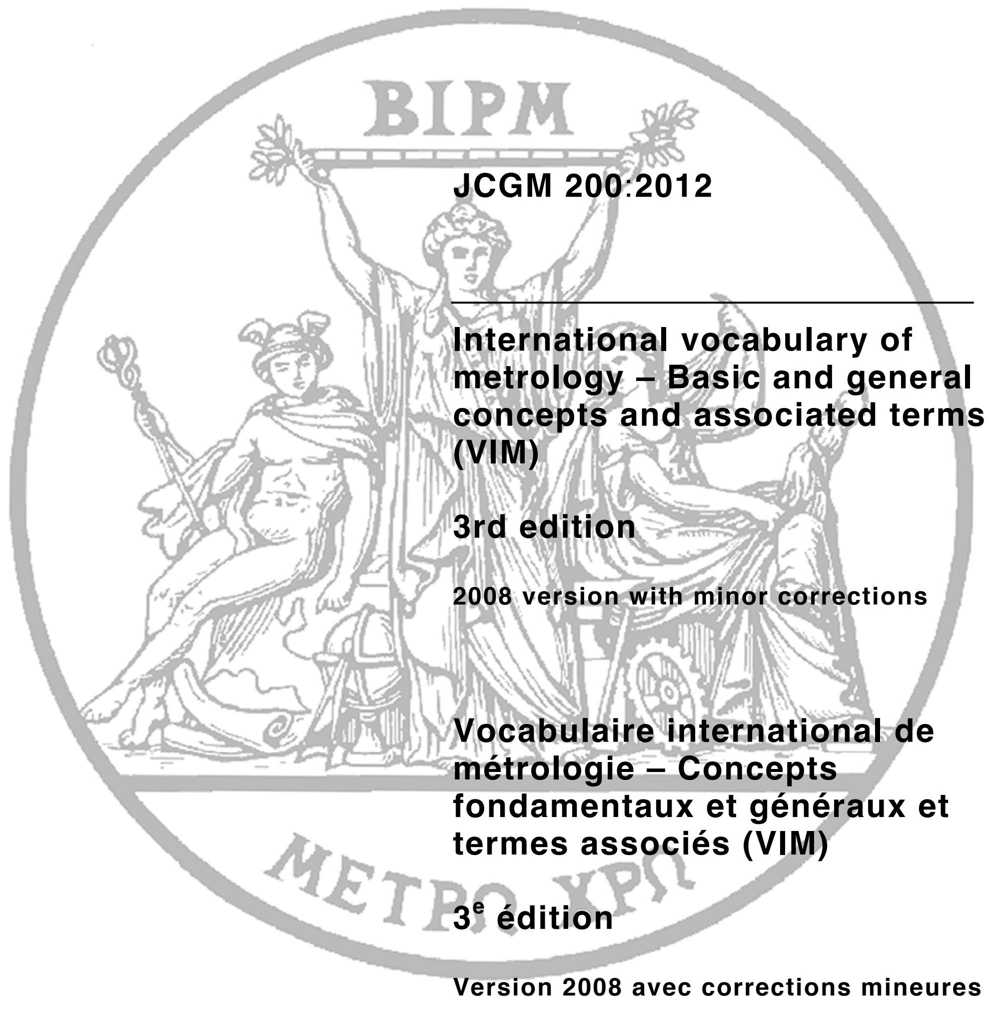


$kT_c = 1.7{\times}10^{-21}$ W/Hz
with $T_c = 122$ K

R&S SMA100A synthesizer
LO +20 dBm, ≈100 MHz
PM noise –160 dBrad²/Hz
Sidebands –140 dBm/Hz

Uncertainty

Learn form the Scriptures



Learn from the VIM

JCGM 200:2012, International Vocabulary of Metrology

Joint Committee for Guides in Metrology

Time series

2.28 – Type A evaluation of measurement uncertainty
Type A evaluation
evaluation of a component of measurement uncertainty by a statistical analysis of measured quantity values obtained under defined measurement conditions

System

2.29 – Type B evaluation of measurement uncertainty
Type B evaluation
evaluation of a component of measurement uncertainty determined by means other than a Type A evaluation of measurement uncertainty

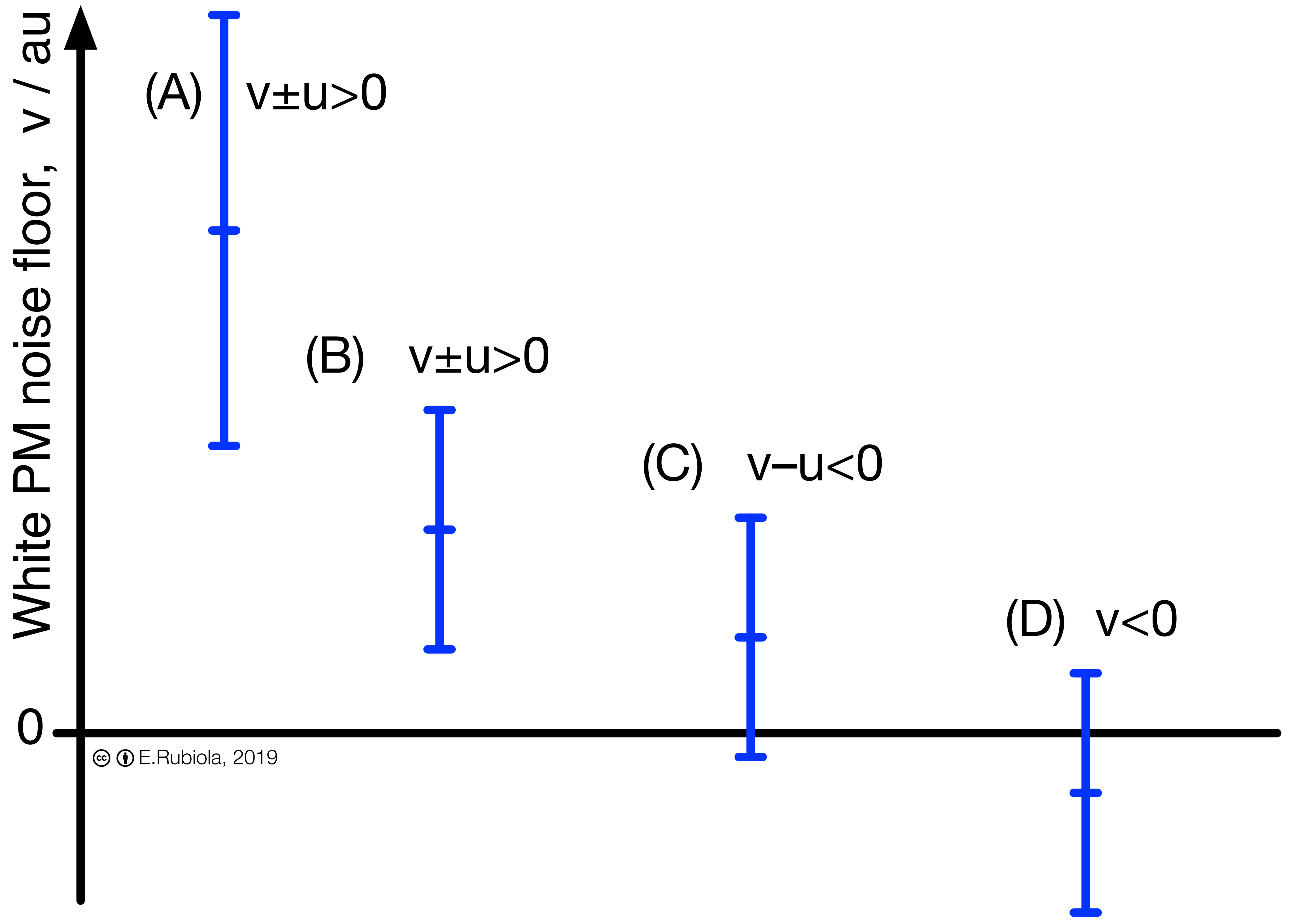
4.29 – null measurement uncertainty
measurement uncertainty where the specified measured quantity value is zero

2.52 – influence quantity
quantity that, in a direct measurement, does not affect the quantity that is actually measured, but affects the relation between the indication and the measurement result

2.27 – definitional uncertainty
component of measurement uncertainty resulting from the finite amount of detail in the definition of a measurand

Uncertainty

Example (one bin of $S_{yx}(f)$)



Define the measurement conditions

- Impedance in the full BW is critical

Define the measurement target

- The full high-resolution spectrum?
- Coefficients of the polynomial law?
- Any better idea?

Example of outcomes

(A) Correct

(B) Correct (likely smaller u_A)

(C) Null measurement uncertainty applies

(D) Should not be reported (worse than (C))

Conclusions

- Use averaging wisely
- Correct for the temperature of the power splitter (trivial if the instrument measures P_0)
- Be wary of crosstalk
- $\langle \mathrm{Re}\{S_{yx}\}\rangle_m$ is more reliable (and faster) than $|\langle S_{yx}\rangle_m|$
- Define the measurand and the operating conditions
- Understand Type-A, Type B and Null uncertainty
- Be wary of too-low-noise oscillators

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  AR-E is not related to A.Rus (nor to any of us)
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