femto-st SCIENCES & TECHNOLOGIES

INRiM ISTITUTO NAZIONALE DI RICERCA METROLOGICA

The Origin and the Measurement of Phase Noise in Oscillators

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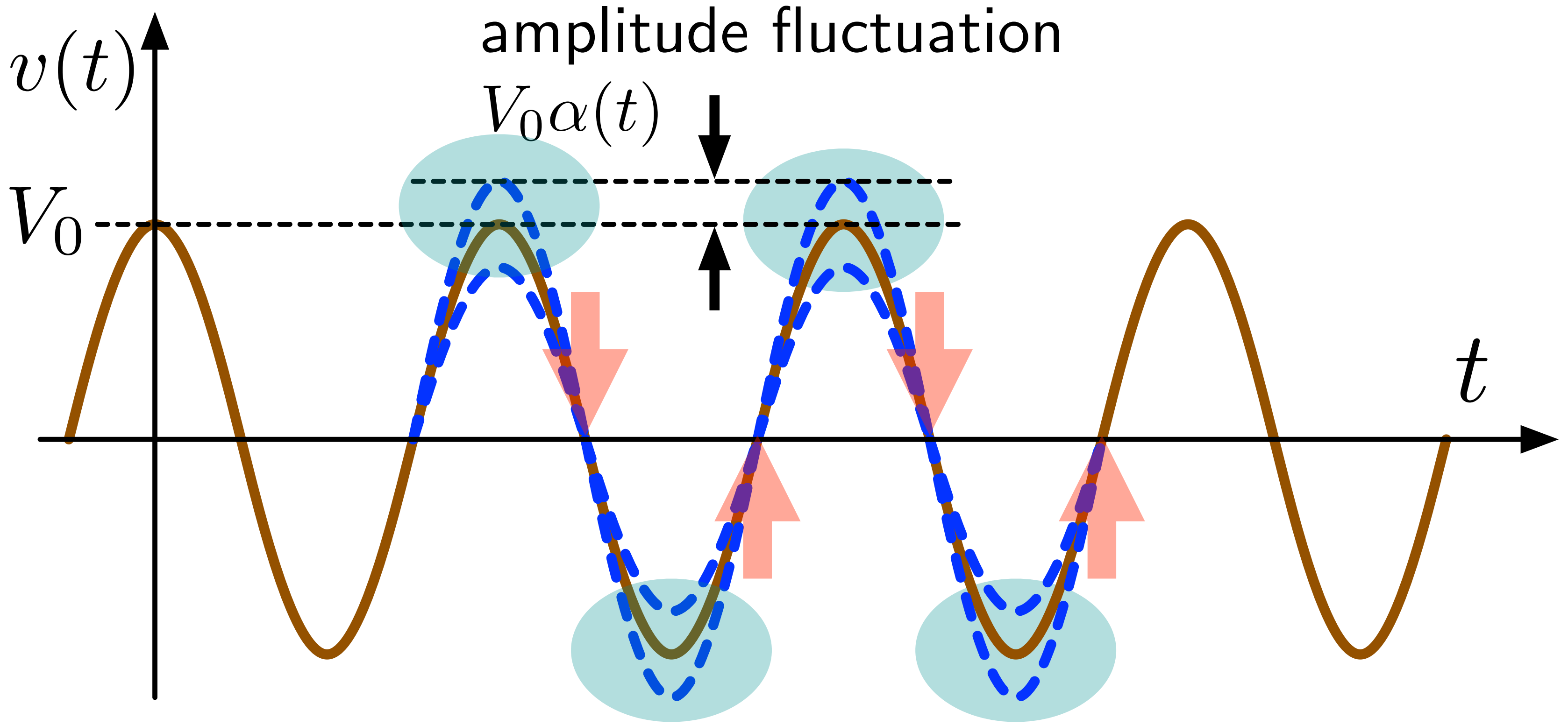
Outline

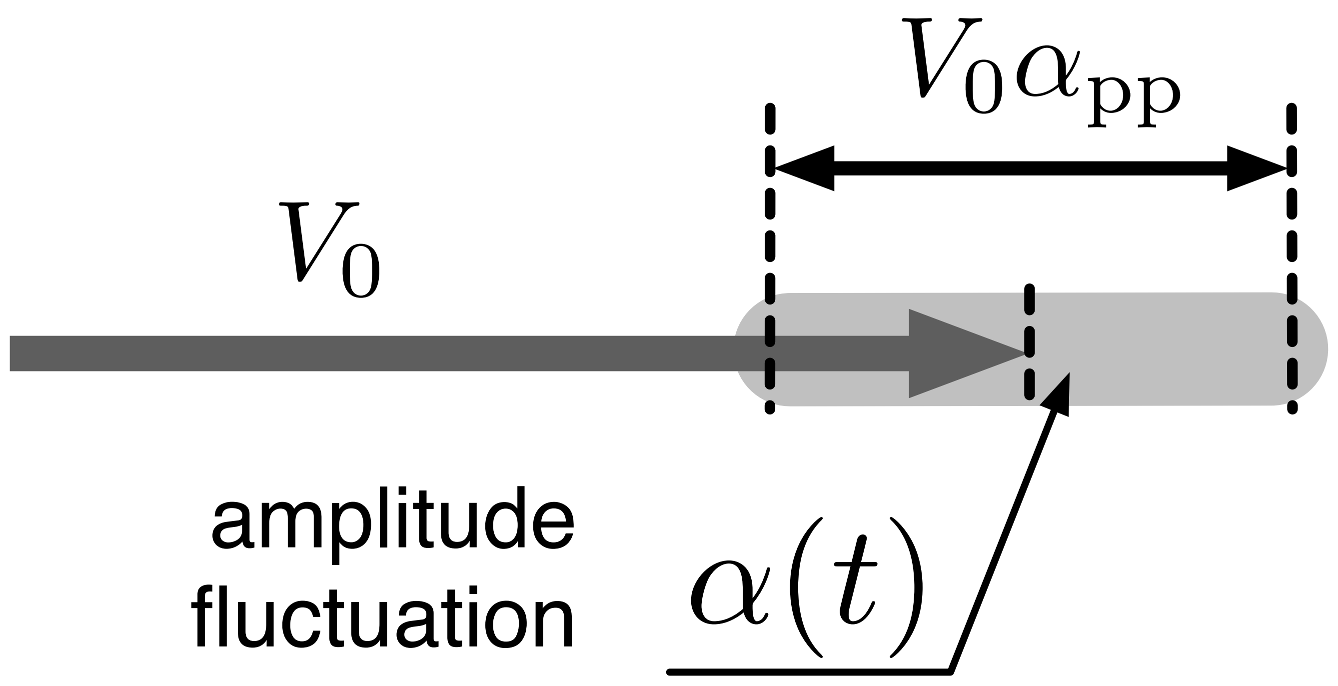
- Clock signal, phase noise, and friends
- The Leeson effect
- Noise in resonators
- Instruments, uncertainty, and loopholes
- …and something more

home page http://rubiola.org

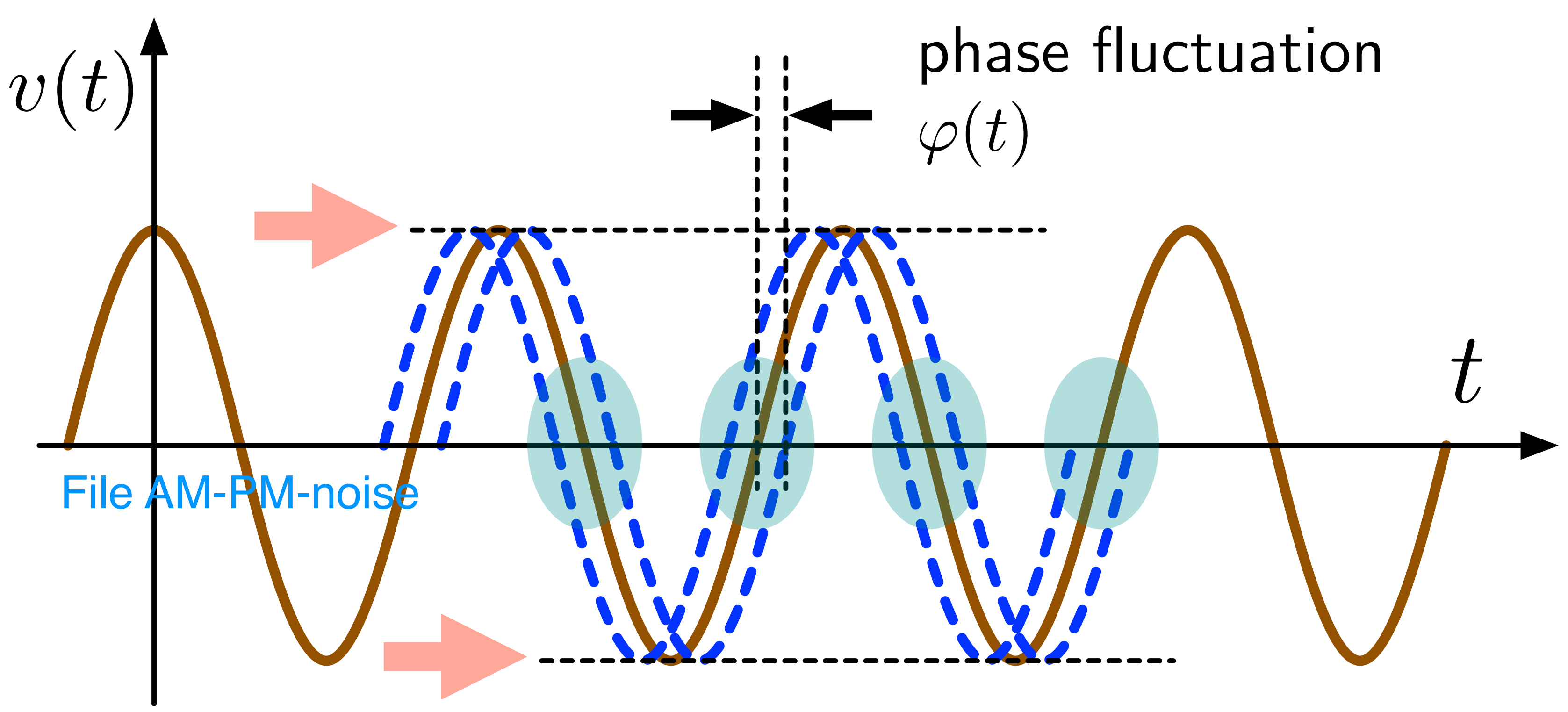
The Clock Signal

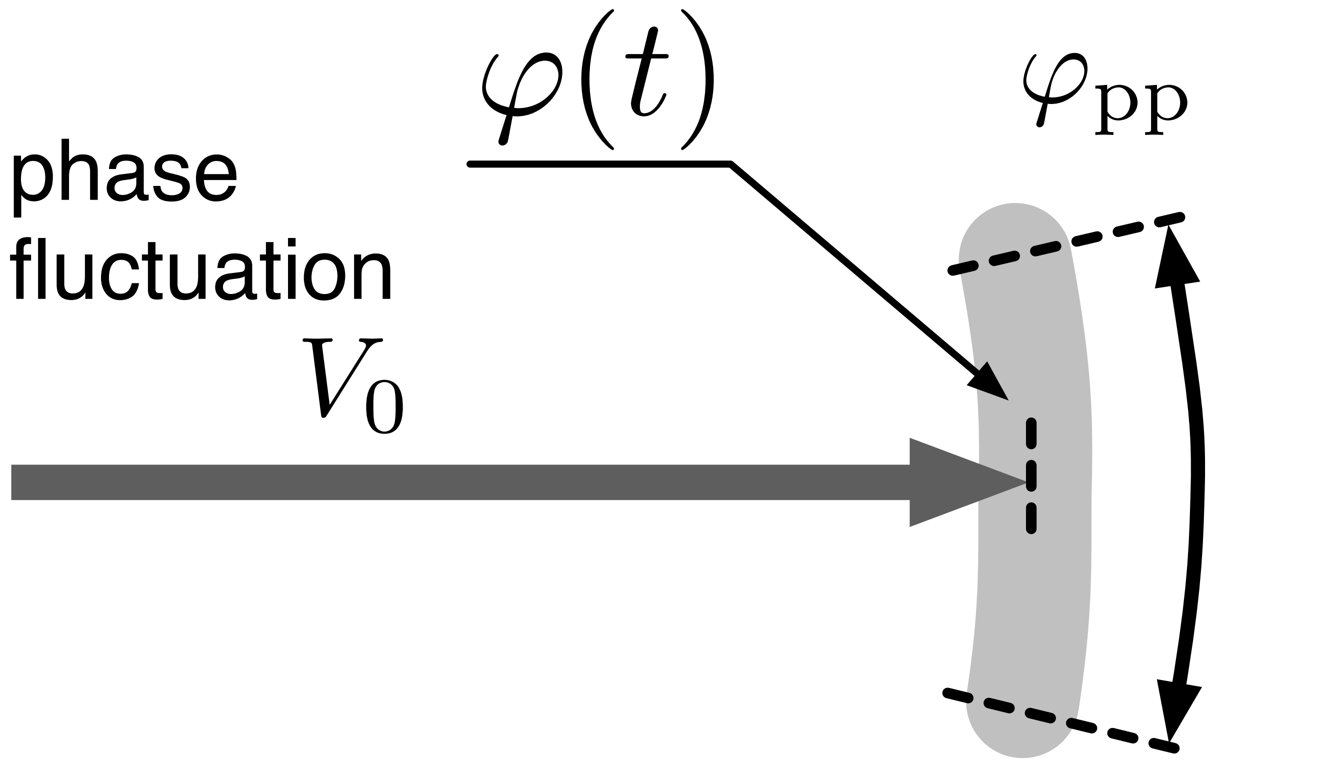
amplitude fluctuation





phase fluctuation





polar coordinates $$v(t) = V_0\,[1+\alpha(t)]\cos\left[\omega_0 t + \varphi(t)\right]$$

Cartesian coordinates $$v(t) = V_0 \cos\omega_0 t + n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t$$

under low noise approximation

$$|n_c(t)| \ll V_0 \quad \text{and} \quad |n_s(t)| \ll V_0$$

It holds that

$$\alpha(t) = \frac{n_c(t)}{V_0} \quad \text{and} \quad \varphi(t) = \frac{n_s(t)}{V_0}$$

$S_\varphi(f)$ and $\mathscr{L}(f)$

Phase noise PSD $S_\varphi(f)$

$$S_\varphi(f) = 2\,\mathcal{F}\left\{C_{\varphi\varphi}(\tau)\right\},\ \ f>0 \qquad \text{(Autocovariance)}$$

$$S_\varphi(f) = 2\,\mathbb{E}\left\{\Phi(f)\Phi^*(f)\right\},\ \ f>0 \qquad \text{(WK theorem)}$$

$$S_\varphi(f) \approx \frac{2}{T}\left\langle \Phi(f)\Phi^*(f)\right\rangle_m,\ \ f>0 \qquad \text{(measured)}$$

Units

Sφ –> [rad²/Hz]

10 Log_{10}(Sφ) –> [dBad²/Hz]

The IEEE Std 1139-1999 defines $\mathscr{L}(f)$ as

$$\mathscr{L}(f) = \frac{1}{2} S_\varphi(f)$$

$(\mathscr{L})_{dB} = (S\varphi)_{dB} - 3$ dB

Units

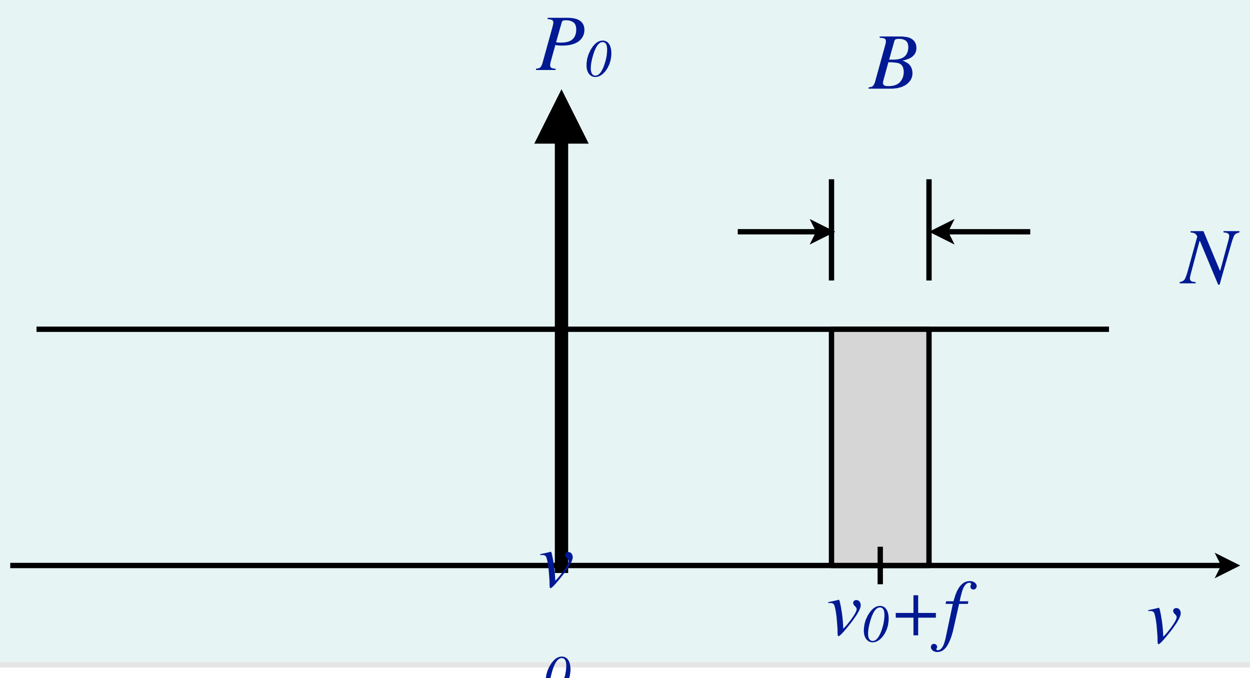
10 $\text{Log}_{10}(\mathscr{L})$ –> [dBc/Hz]

Unit of angle √2 rad ≈ 80°

The obsolete definition of $\mathscr{L}(f)$ is

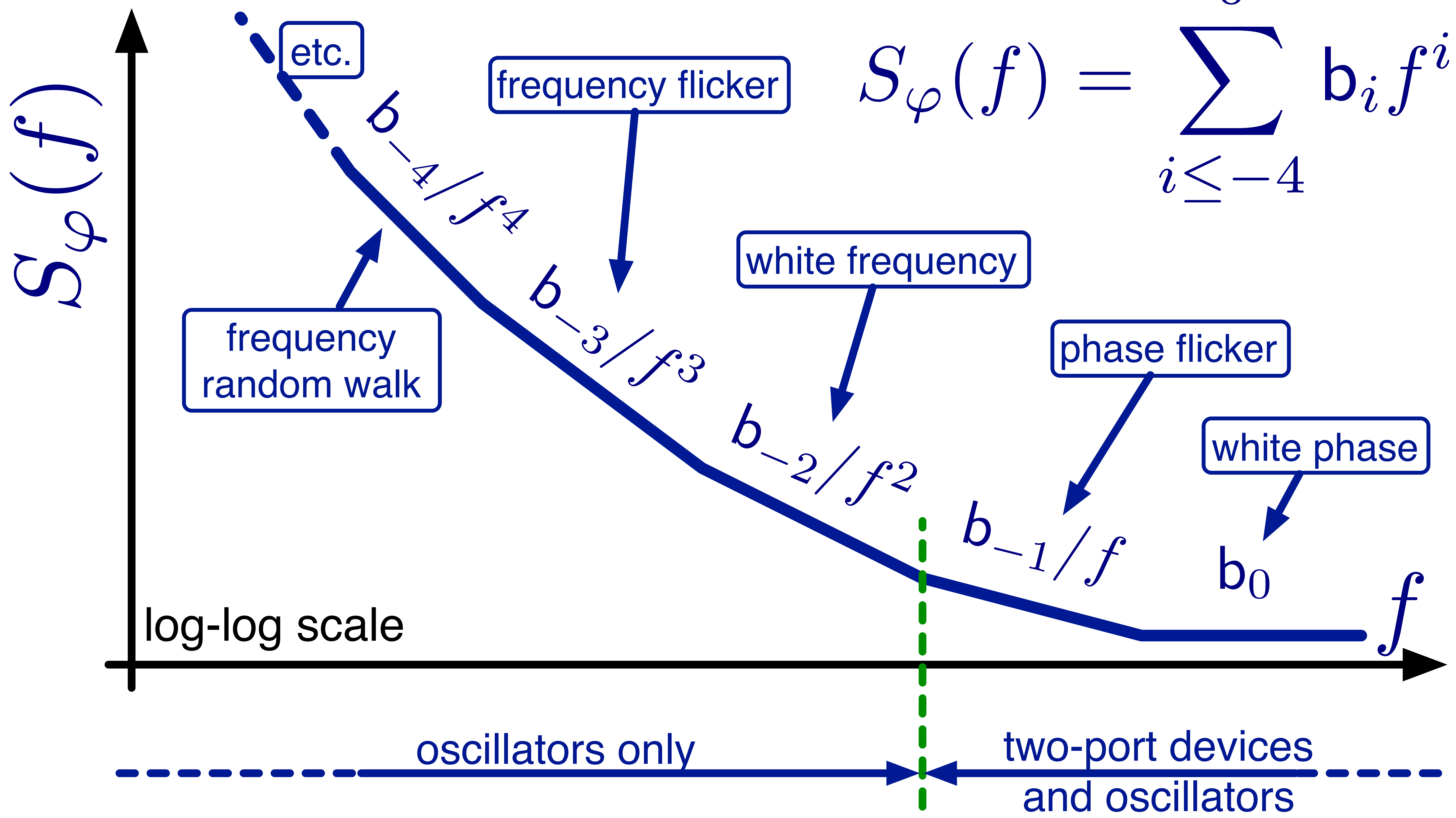
$$\mathscr{L}(f) = \frac{\text{SSB power in 1 Hz band}}{\text{carrier power}}$$

The problem with this definition is that it does not divide AM noise from PM noise, which yields to ambiguous results



Polynomial Law

$$S_\varphi(f) = \sum_{i \le -4}^{0} \mathsf{b}_i f^i$$



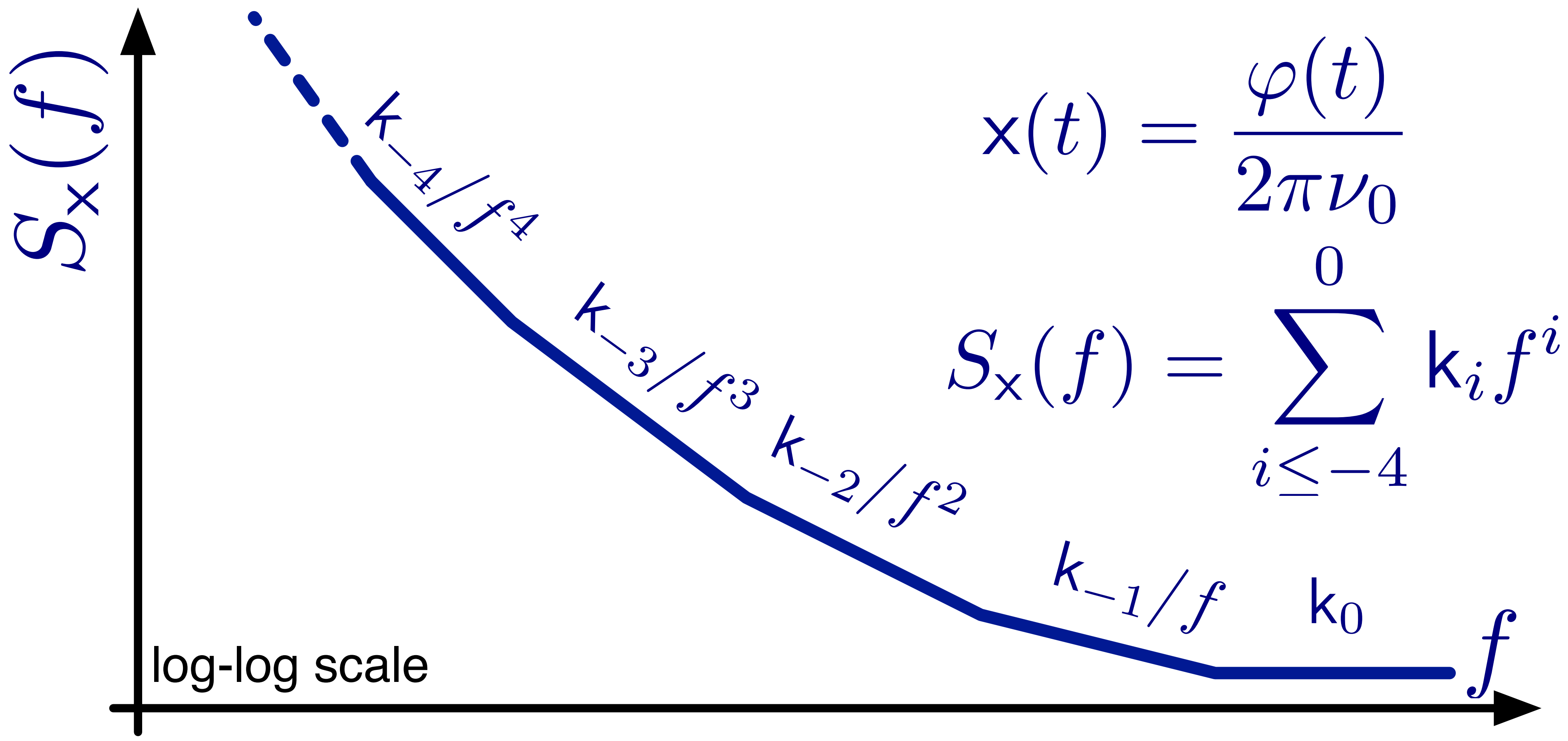
The integrated 1/*f* noise is amazingly small $\ln(A_U/\tau_P) \simeq 140$

Phase Noise —> Time & Frequency Fluctuations

Time fluctuation spectrum

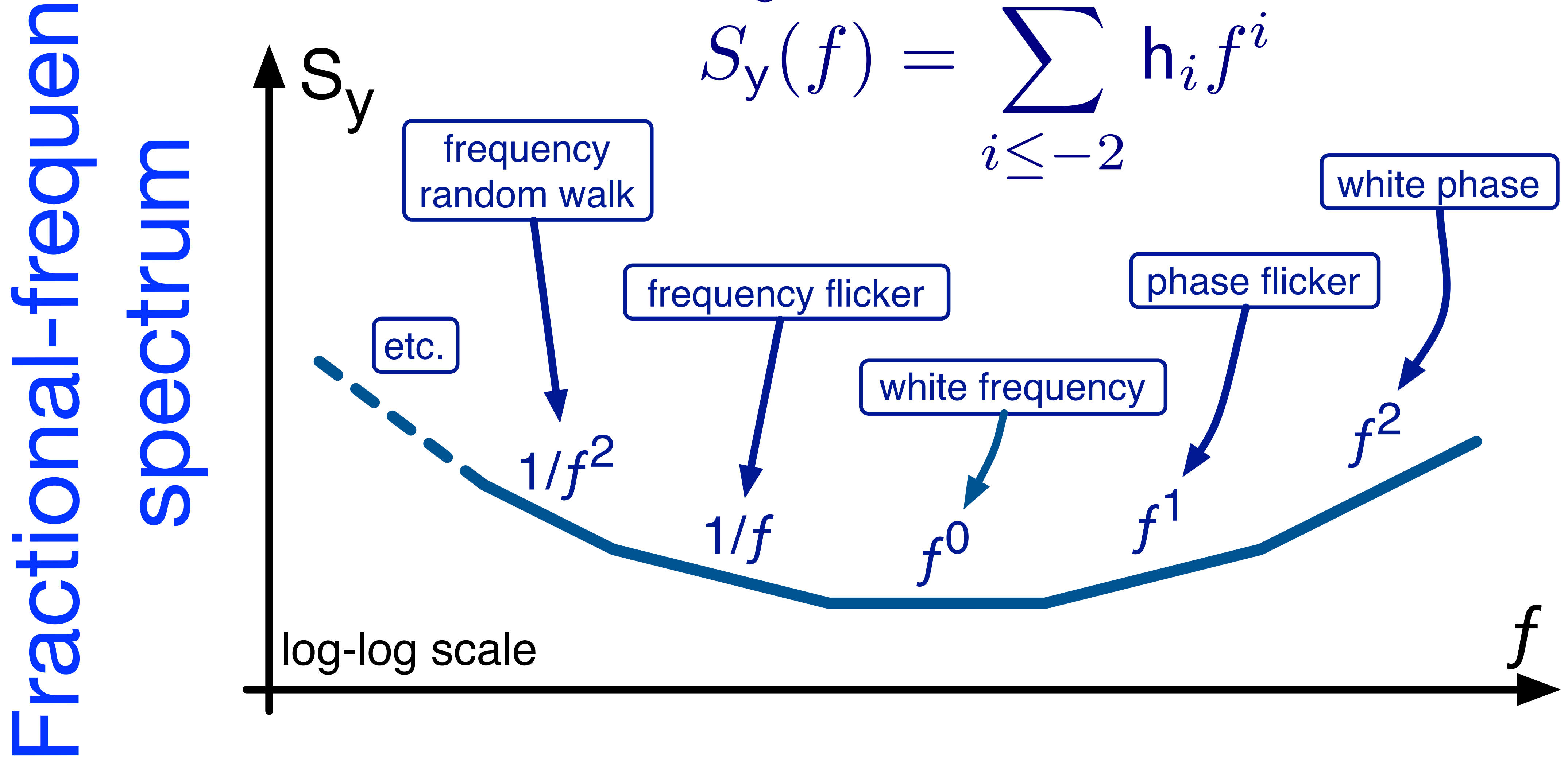
$$\mathrm{x}(t) = \frac{\varphi(t)}{2\pi\nu_0}$$

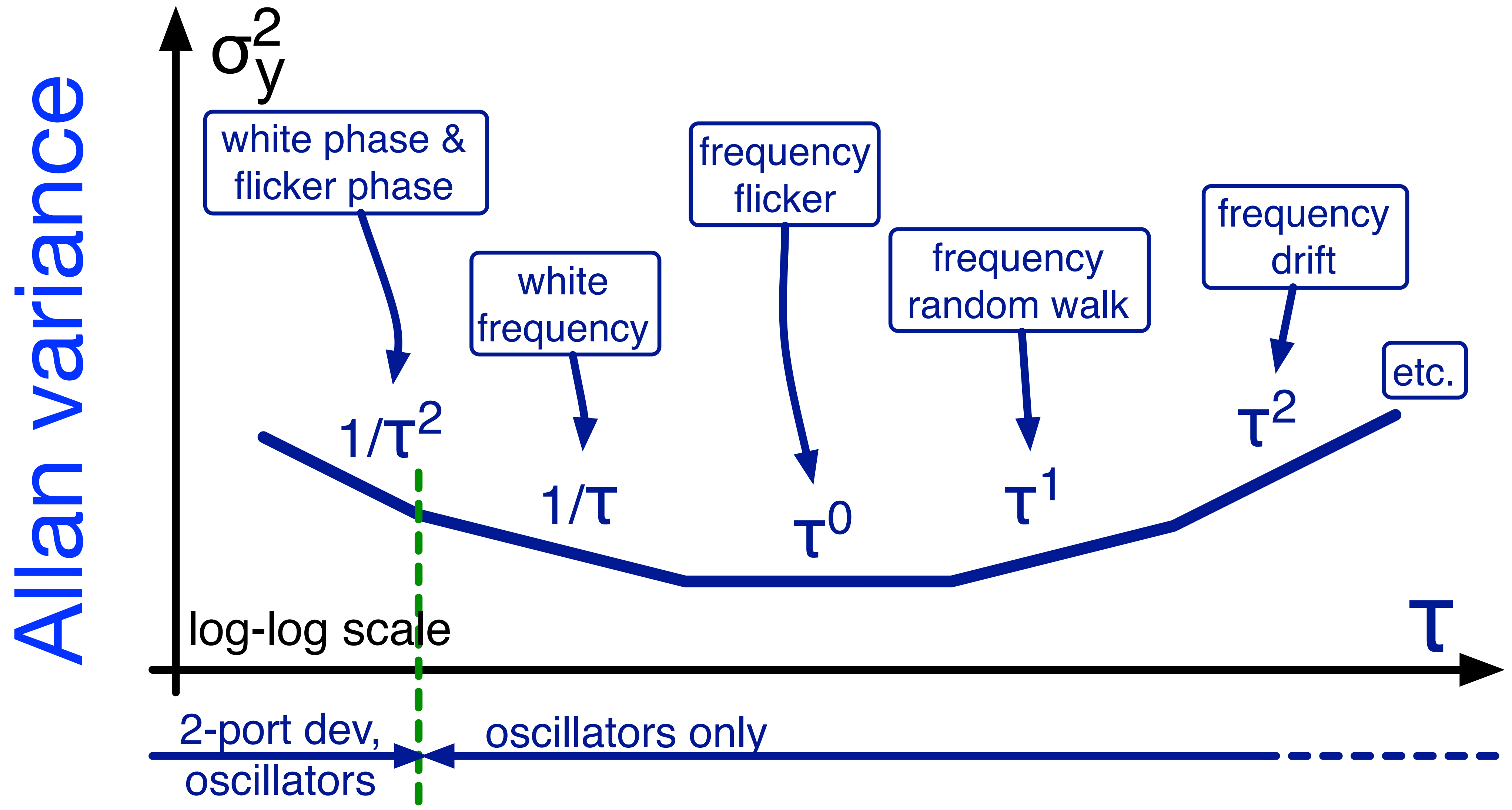
$$S_{\mathrm{x}}(f) = \sum_{i \le -4}^{0} \mathrm{k}_i f^i$$



$$\mathrm{y}(t) = \frac{\dot{\varphi}(t)}{2\pi\nu_0}$$

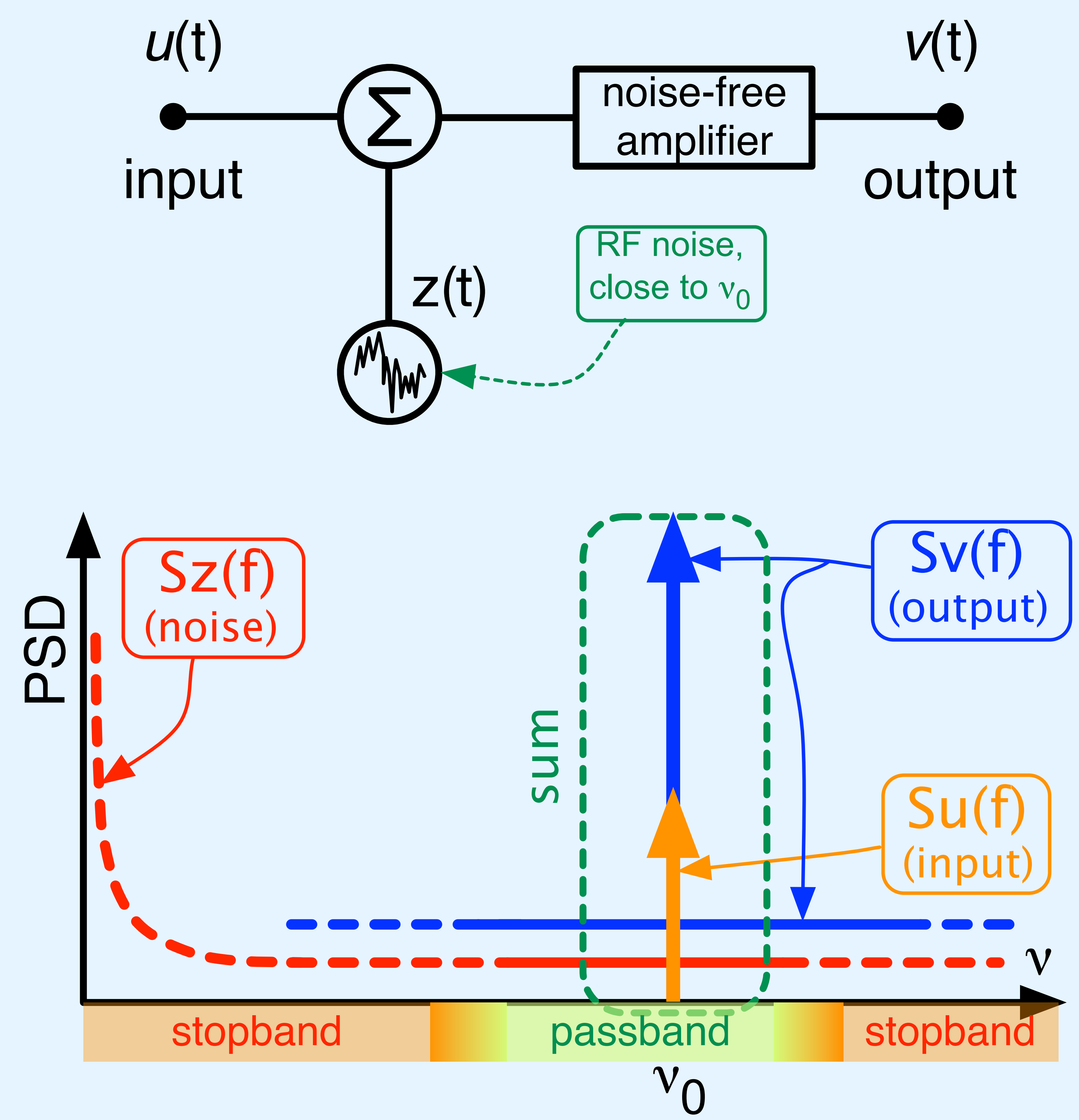
$$S_{\mathrm{y}}(f) = \sum_{i \le -2}^{2} \mathrm{h}_i f^i$$



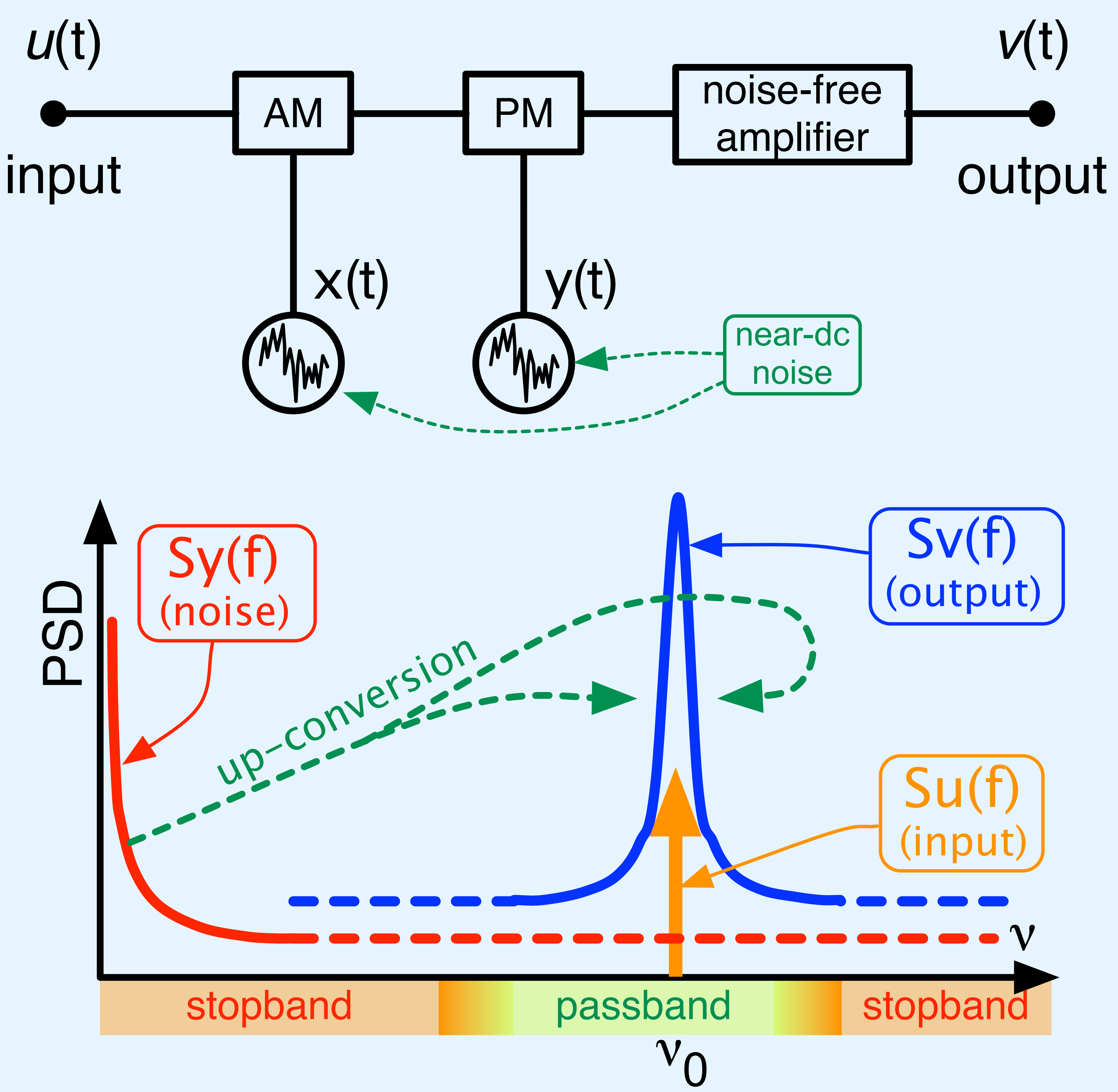


You may like the Enrico's Noise Chart on http://rubiola.org

Additive vs Parametric Noise



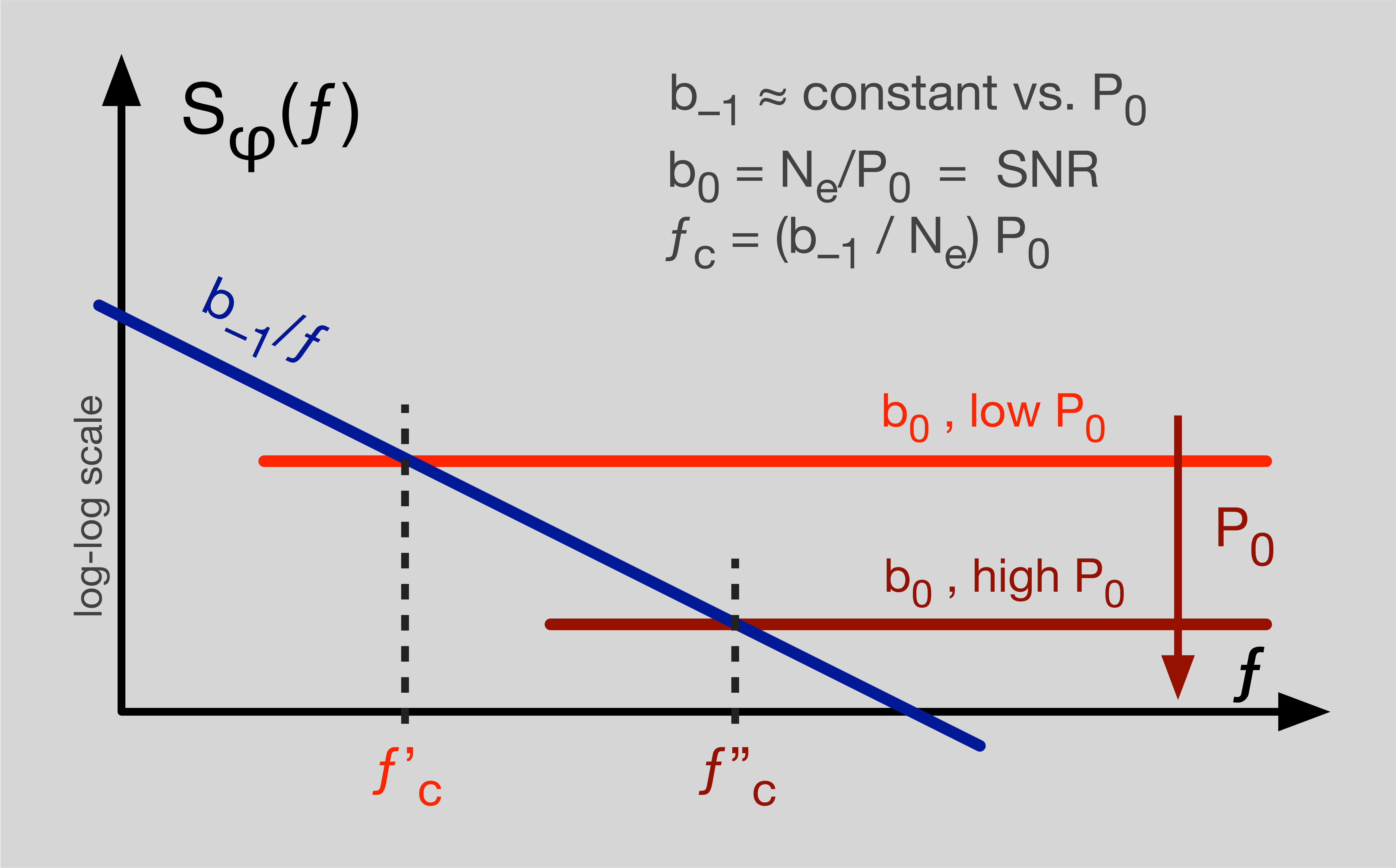
the noise sidebands are independent of the carrier



the noise sidebands are proportional to the carrier

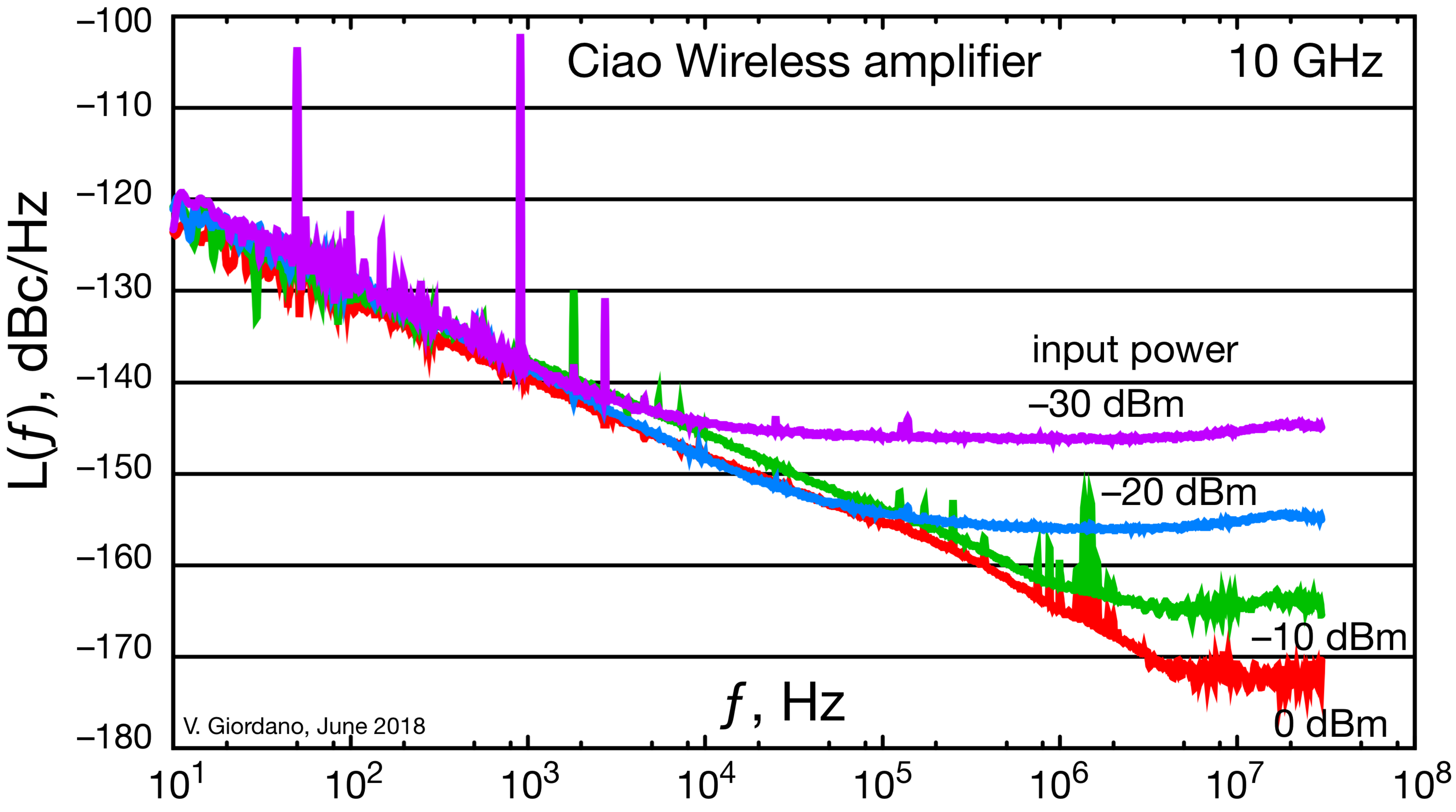
R. Boudot, E. Rubiola, Phase noise in RF and microwave amplifiers IEEER T UFFC 59(12) p.2613-2624, December 2012.

Combining White and Flicker Noise



The corner frequency f_c, sometimes specified in data sheets is a misleading parameter because it depends on P_0

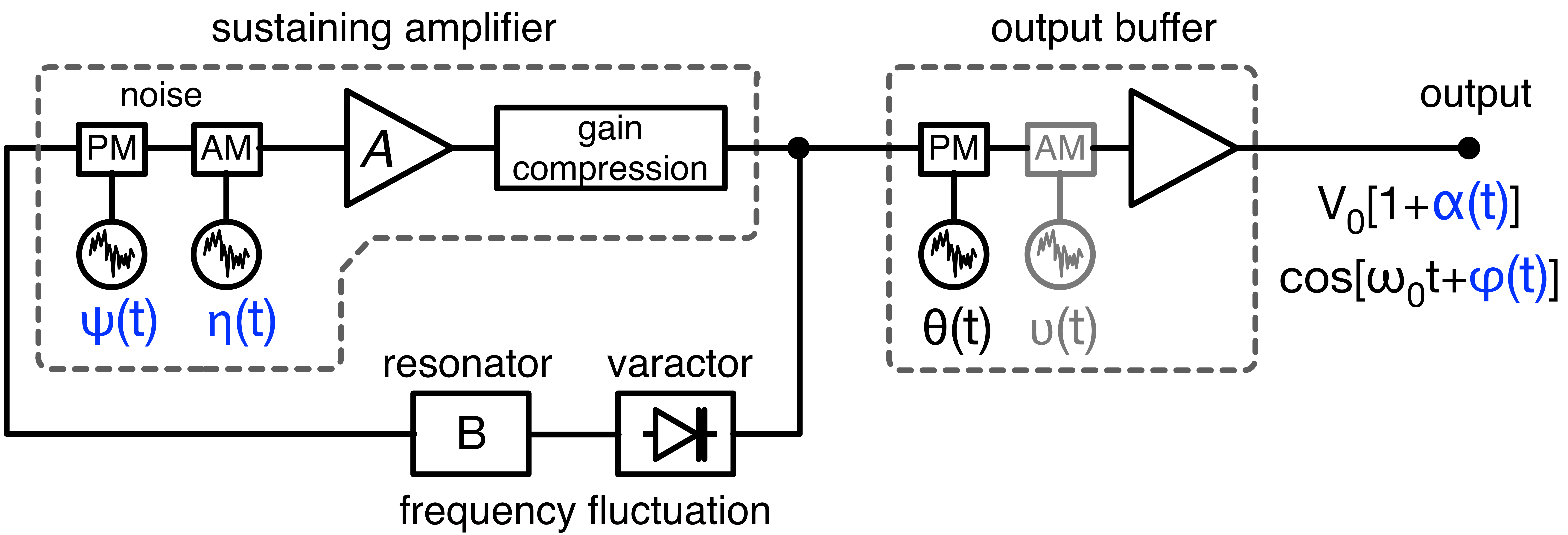
Example – Microwave Amplifier



The Leeson Effect

An Oscillator Model

With trivial changes — This describes oscillators from low RF to lasers



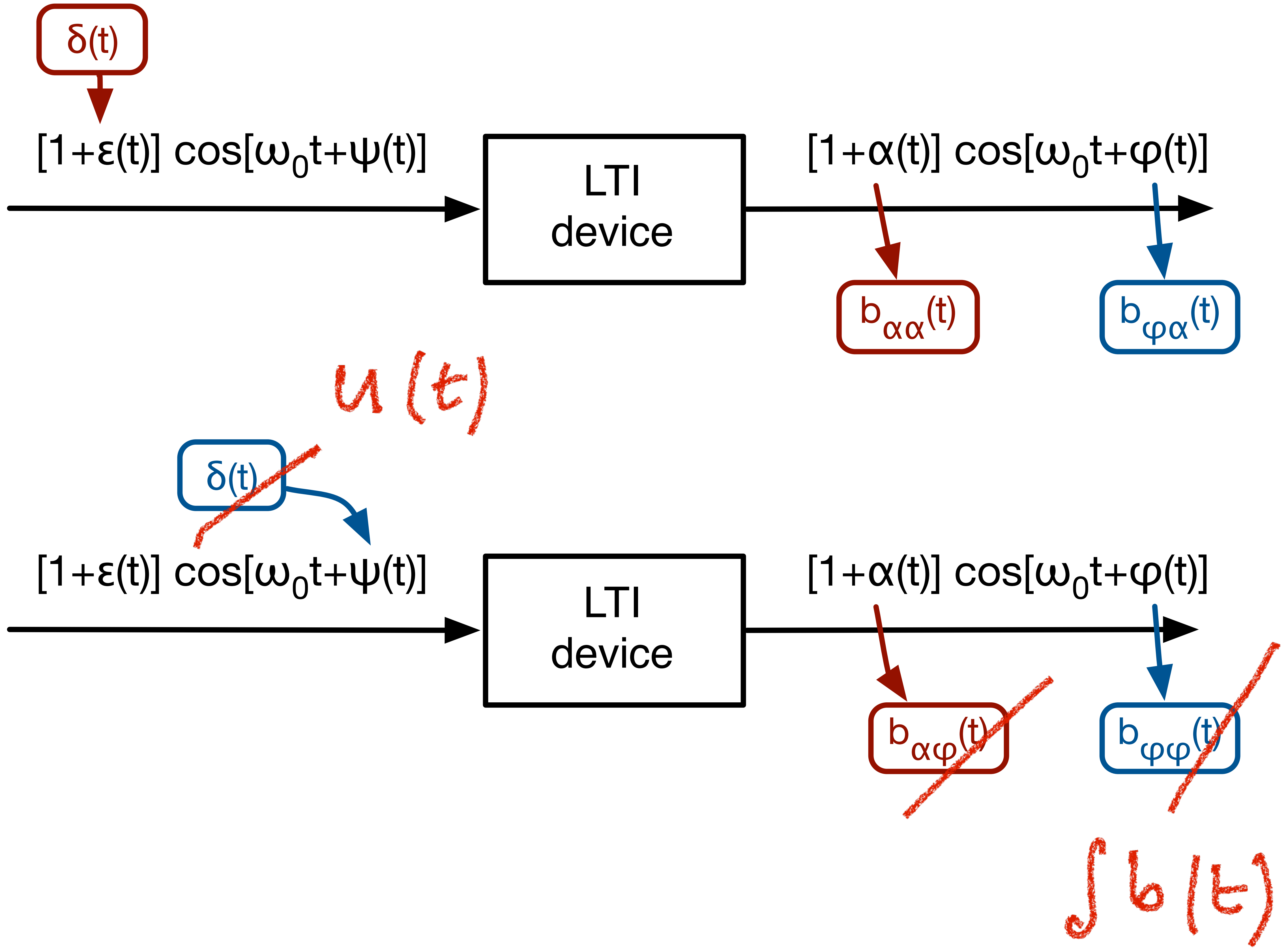
Steady oscillation

- Barkhausen condition $A(f)\,\beta(f) = 1$
- Gain compression sets $|A\beta| = 1$,
- Closed-loop condition sets $\arg(A\beta) = 0$

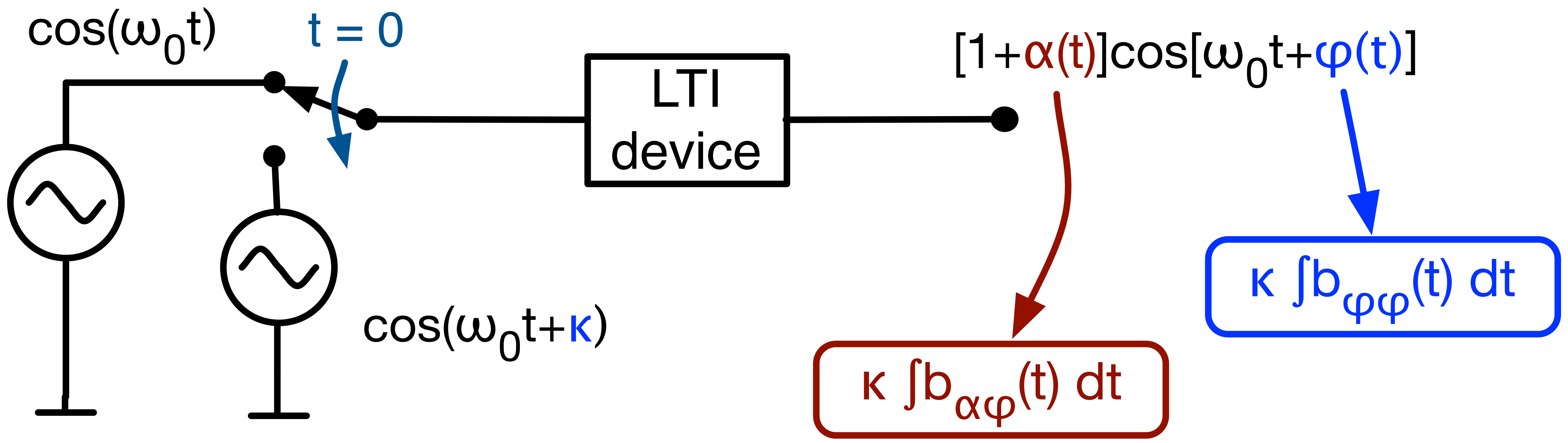
Phase noise & frequency stability

- Leeson effect (Q, ν_0, noise)
- Resonator stability
- Output buffer

A General Method to Solve PN Problems



set a small phase or amplitude step κ at $t=0$, and linearize for $\kappa \to 0$



- A Dirac $\delta(t)$ in a trig function?
- NOPE
- Replace with Heaviside $u(t) = \int \delta(t)\, dt$
- Calculate the response
- Differentiate

Convolution theorem

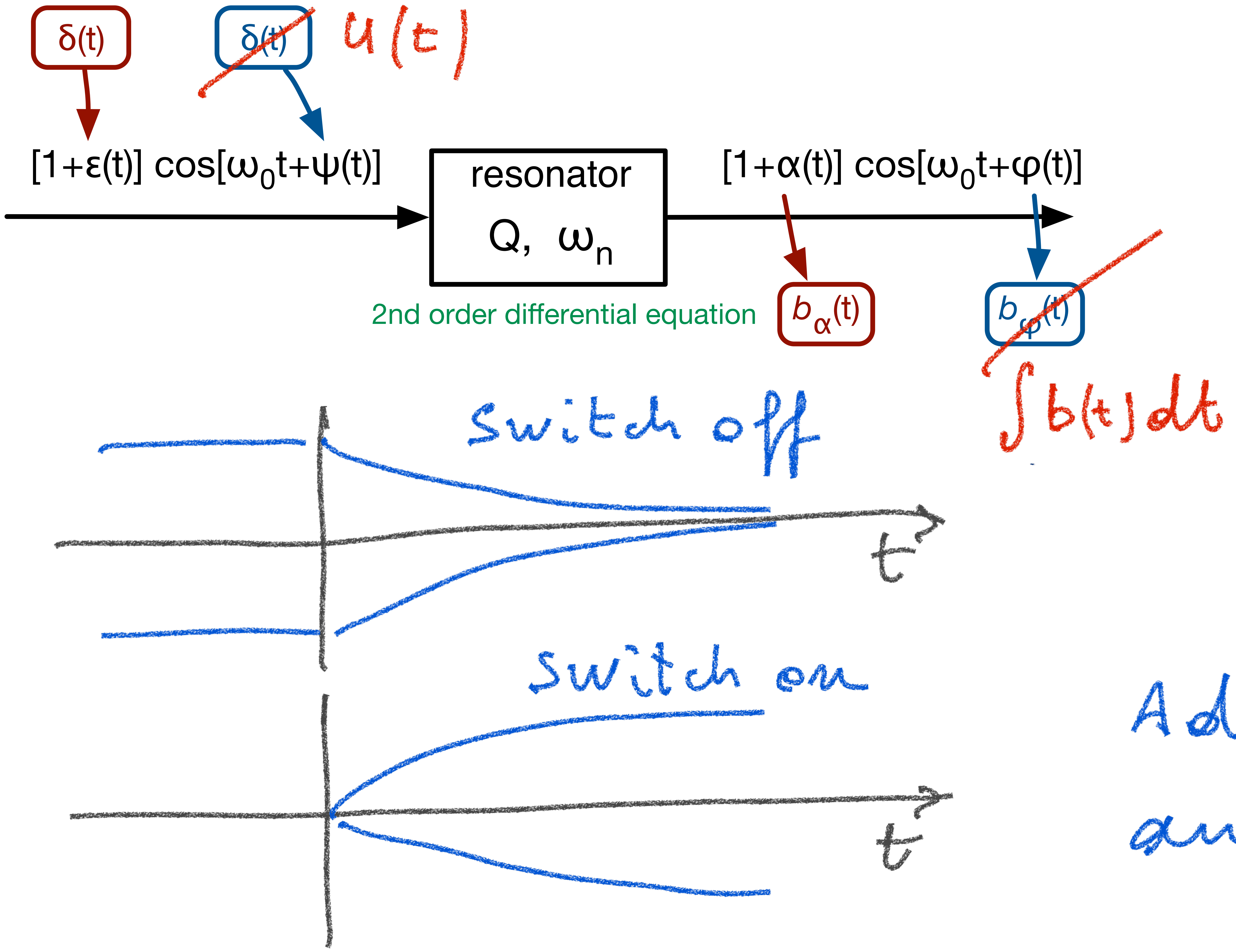
$$x(t) * y'(t) = x'(t) * y(t)$$

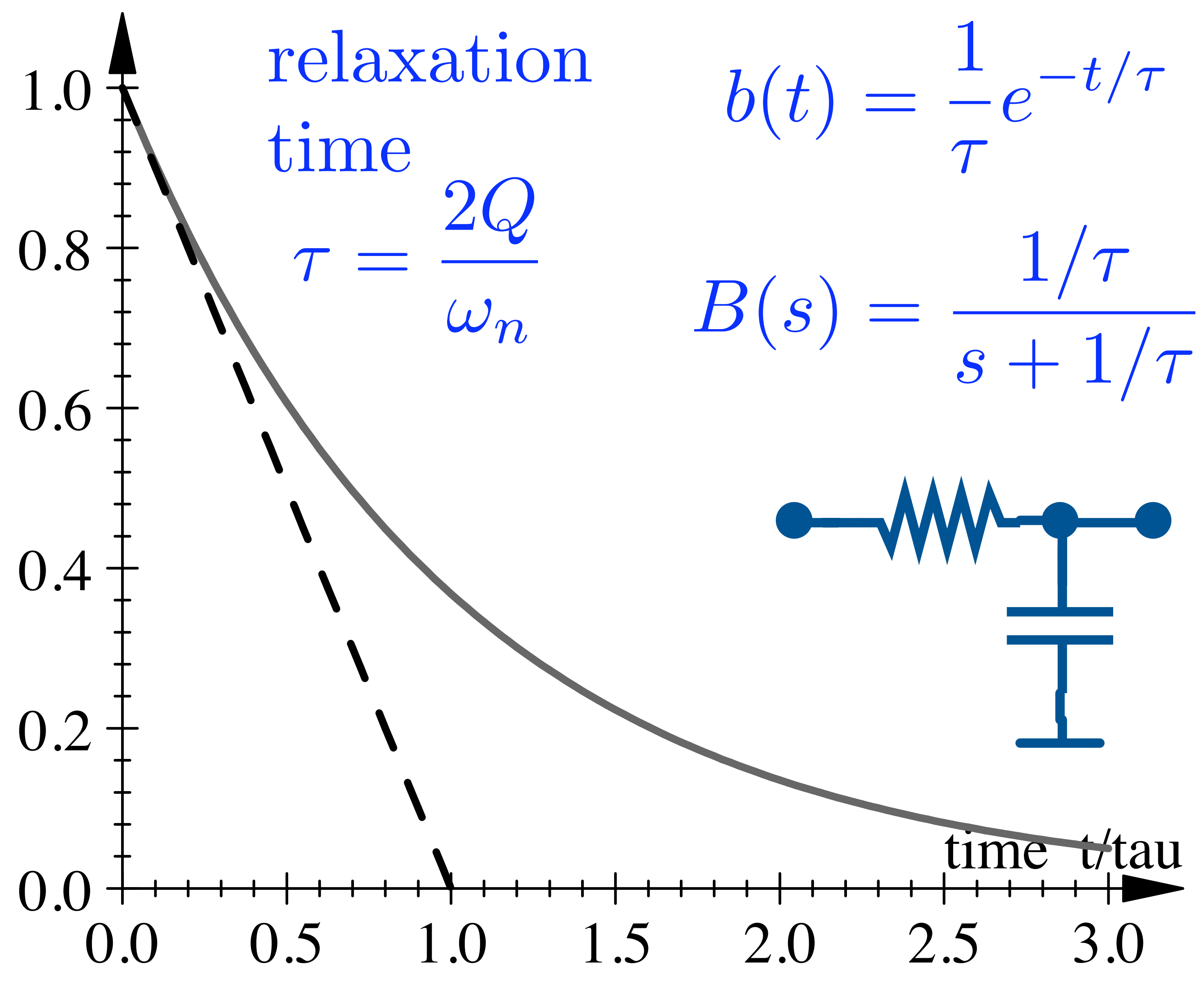
There is a catch

- We assume that the fluctuations are averaged over multiple limit cycles

(and goodbye Floquet vectors)

Resonator's Impulse Response



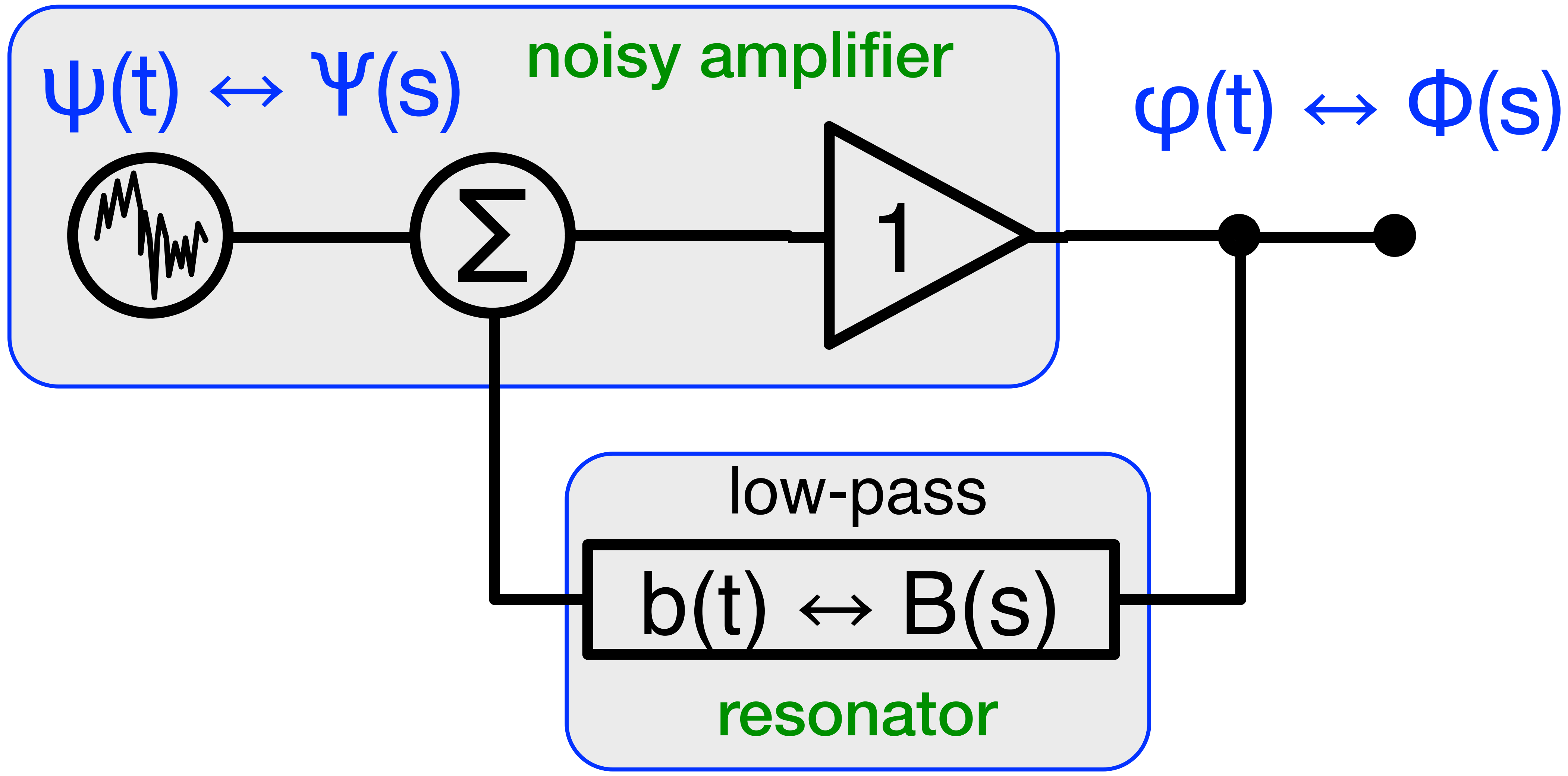


Add the transients, and differentiate

E.Rubiola, Phase noise and frequency stability in oscillators, Cambridge 2008,
E. Rubiola, R. Brendel, A generalization of the Leeson effect, arXiv:1004.5539 [physics.ins-det]

The Leeson Effect

Phase-noise model



Laplace transform inverse-transform pairs
φ(t) ↔ Φ(s)
ψ(t) ↔ Ψ(s)
b(t) ↔ B(s)

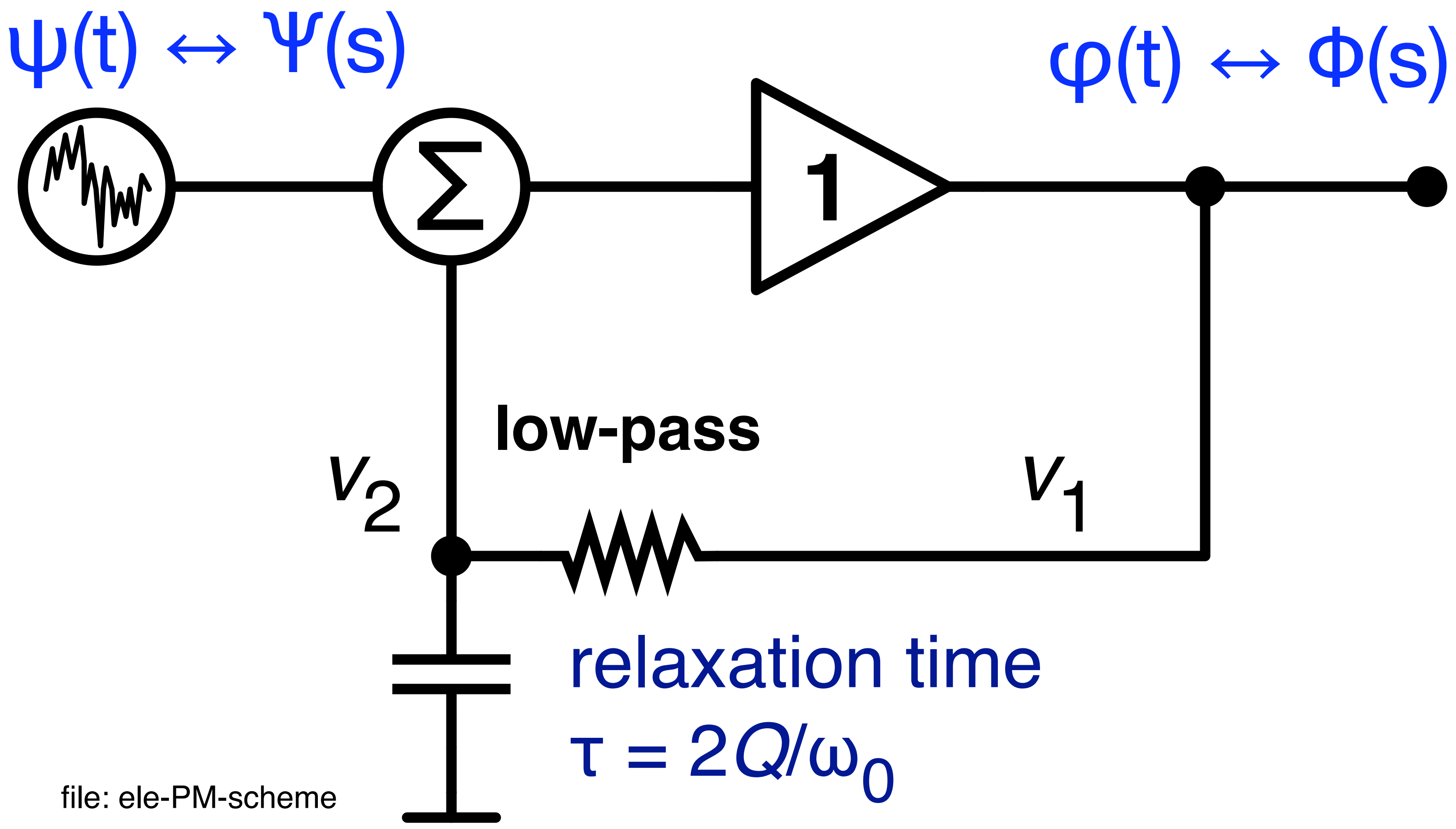
phase-noise transfer function

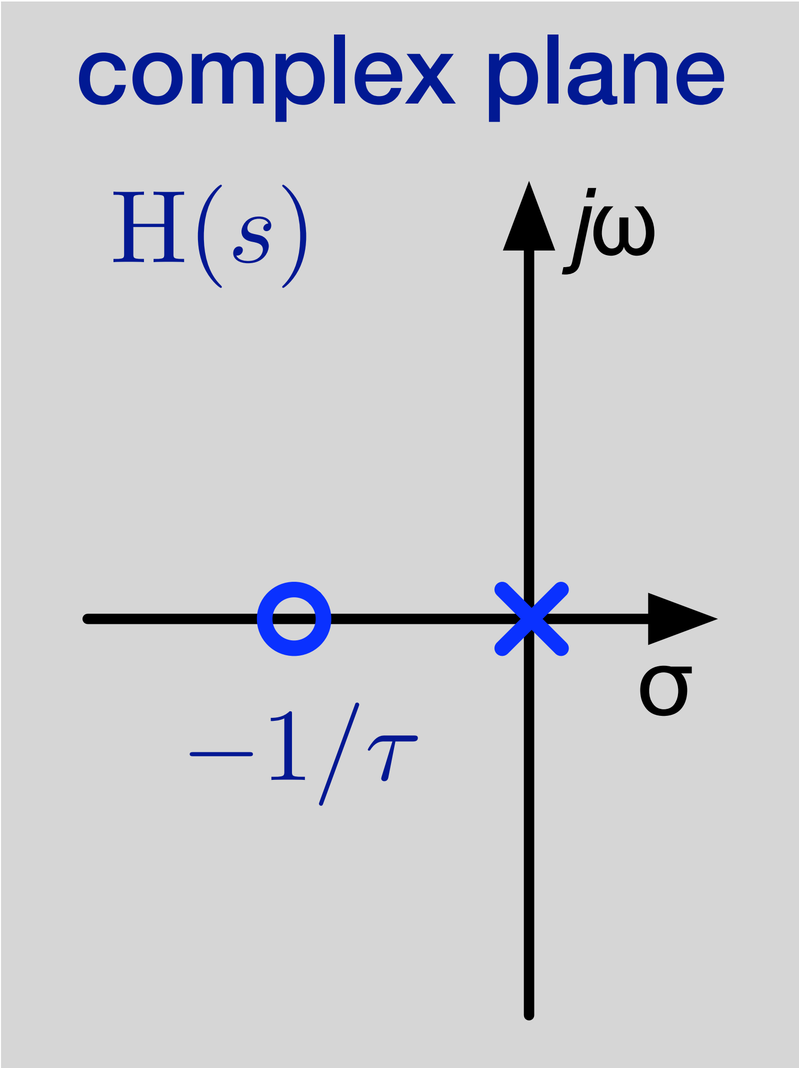
$$\mathrm{H}(s) = \frac{\Phi(s)}{\Psi(s)}$$ **definition**

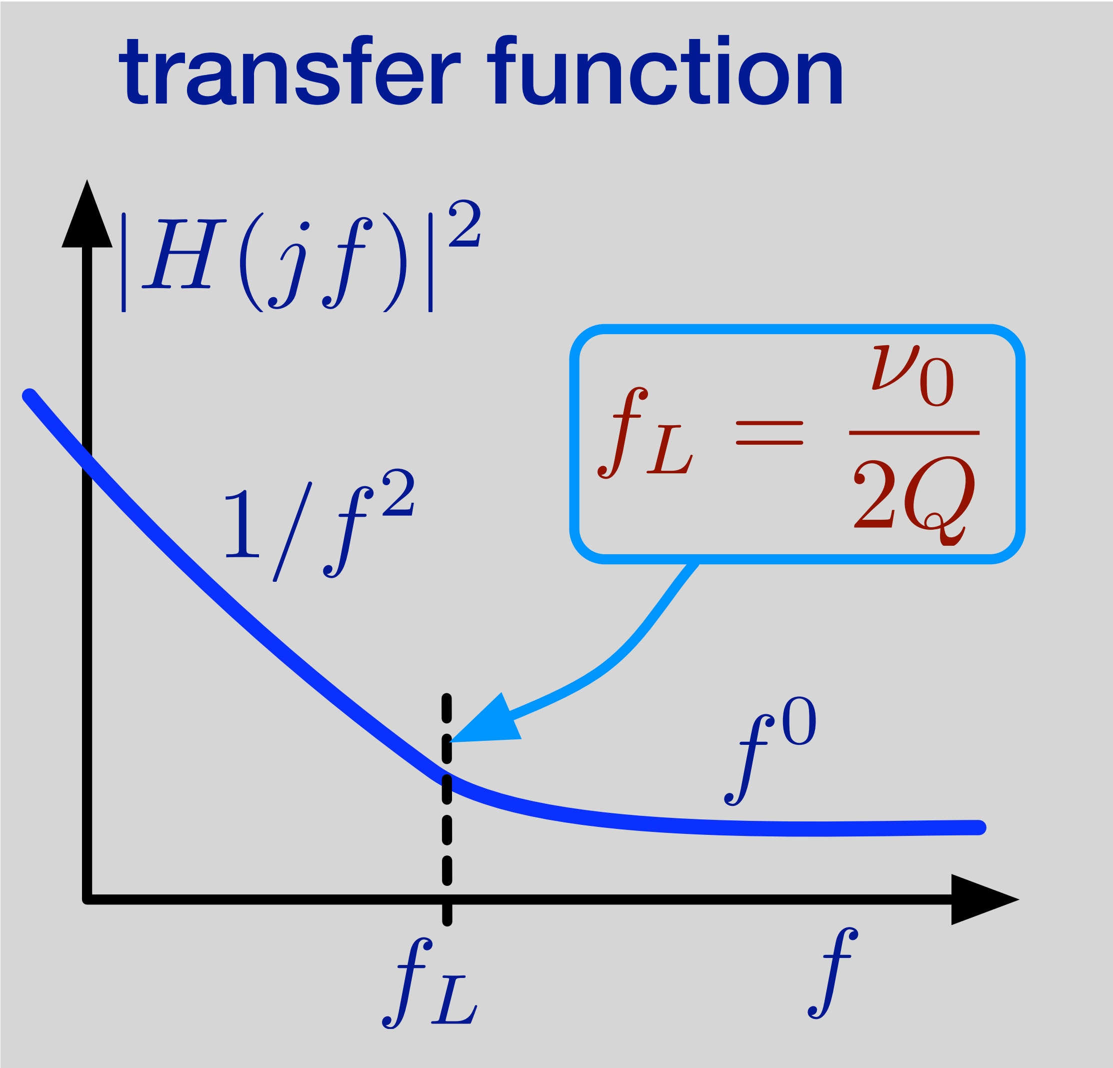
$$\mathrm{H}(s) = \frac{1}{1 - B(s)}$$ **feedback theory**

$$B(s) = \frac{1/\tau}{s + 1/\tau}$$

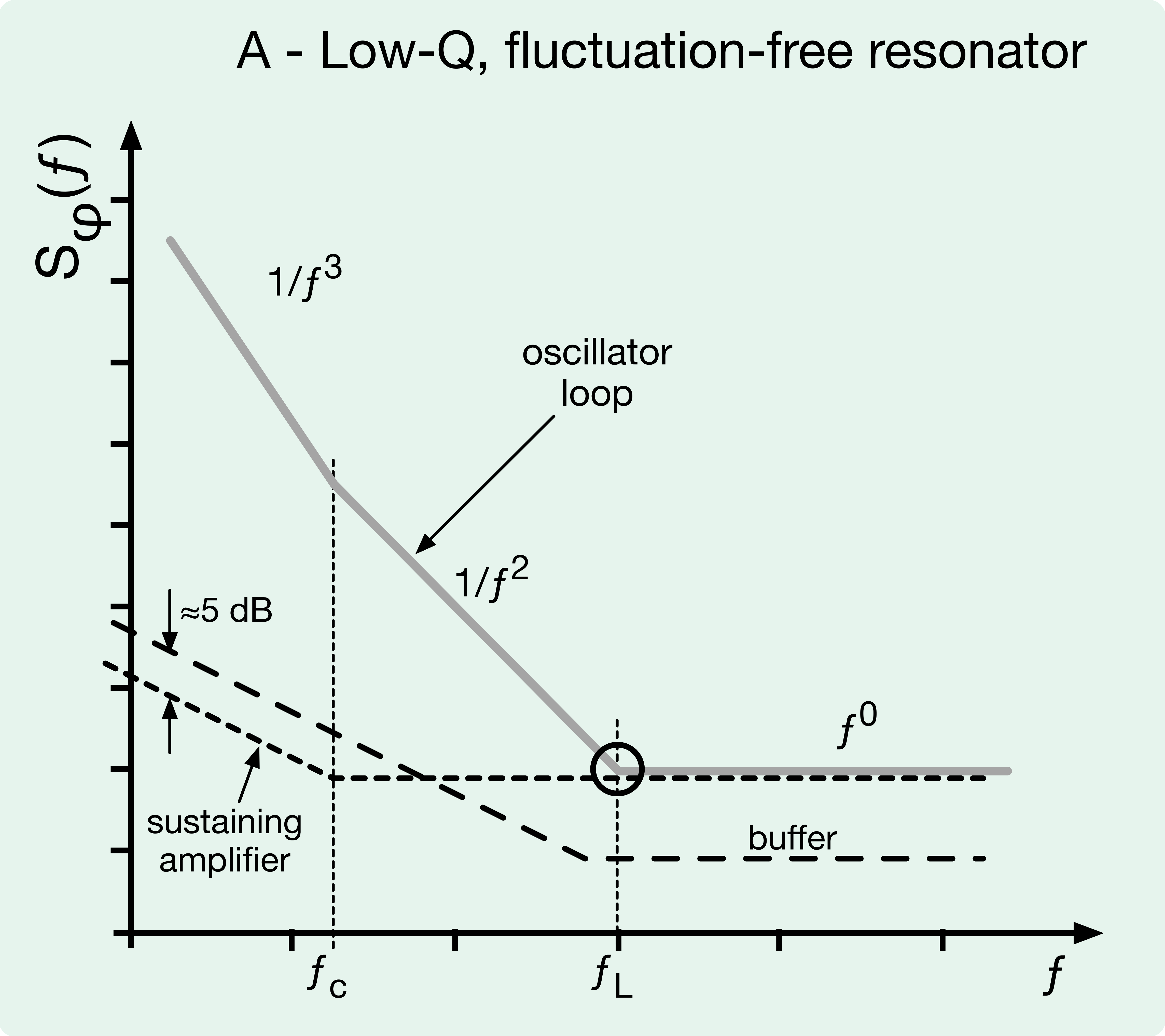
$$\mathrm{H}(s) = \frac{1 + s\tau}{s\tau}$$ **Leeson effect**

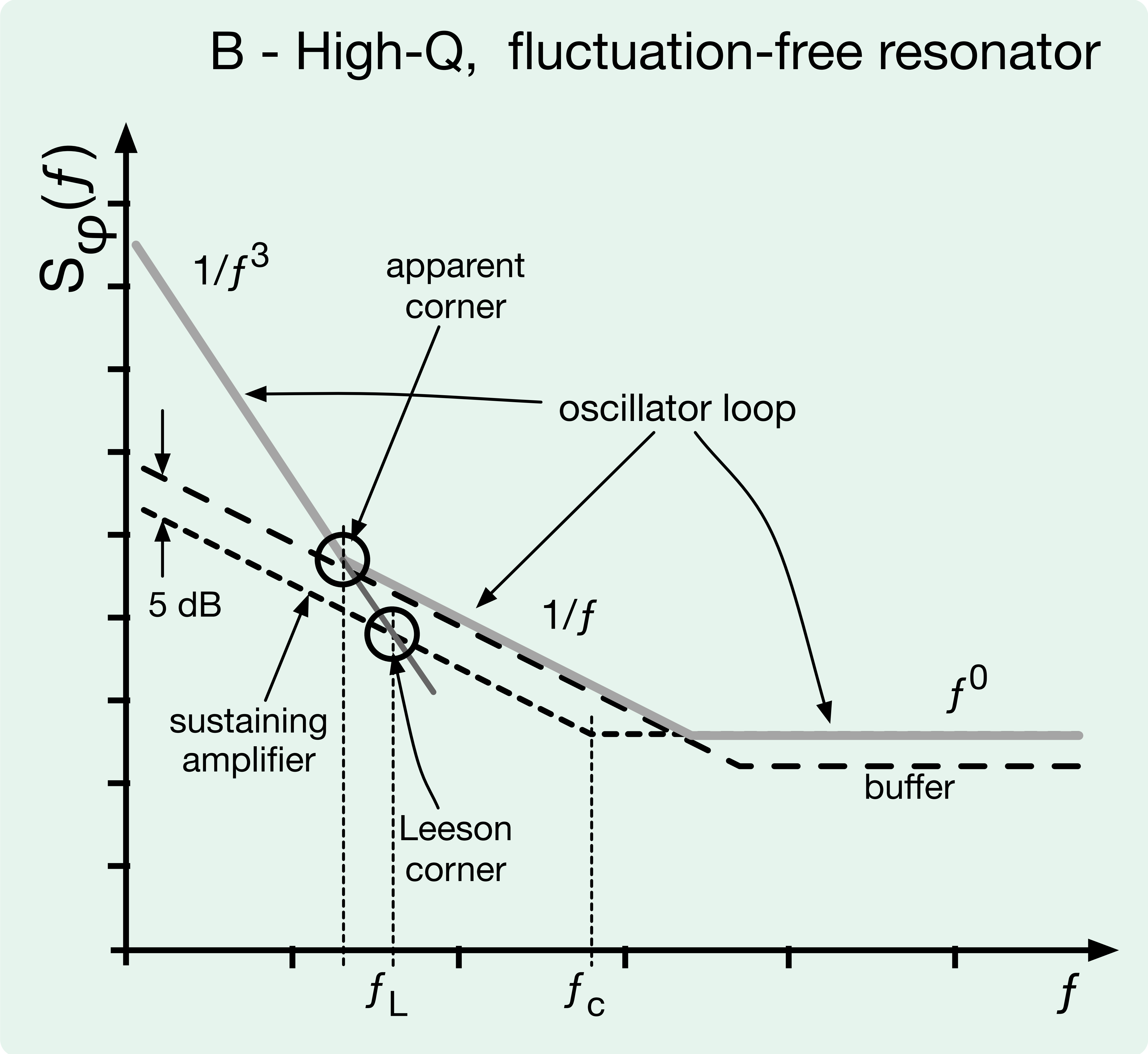






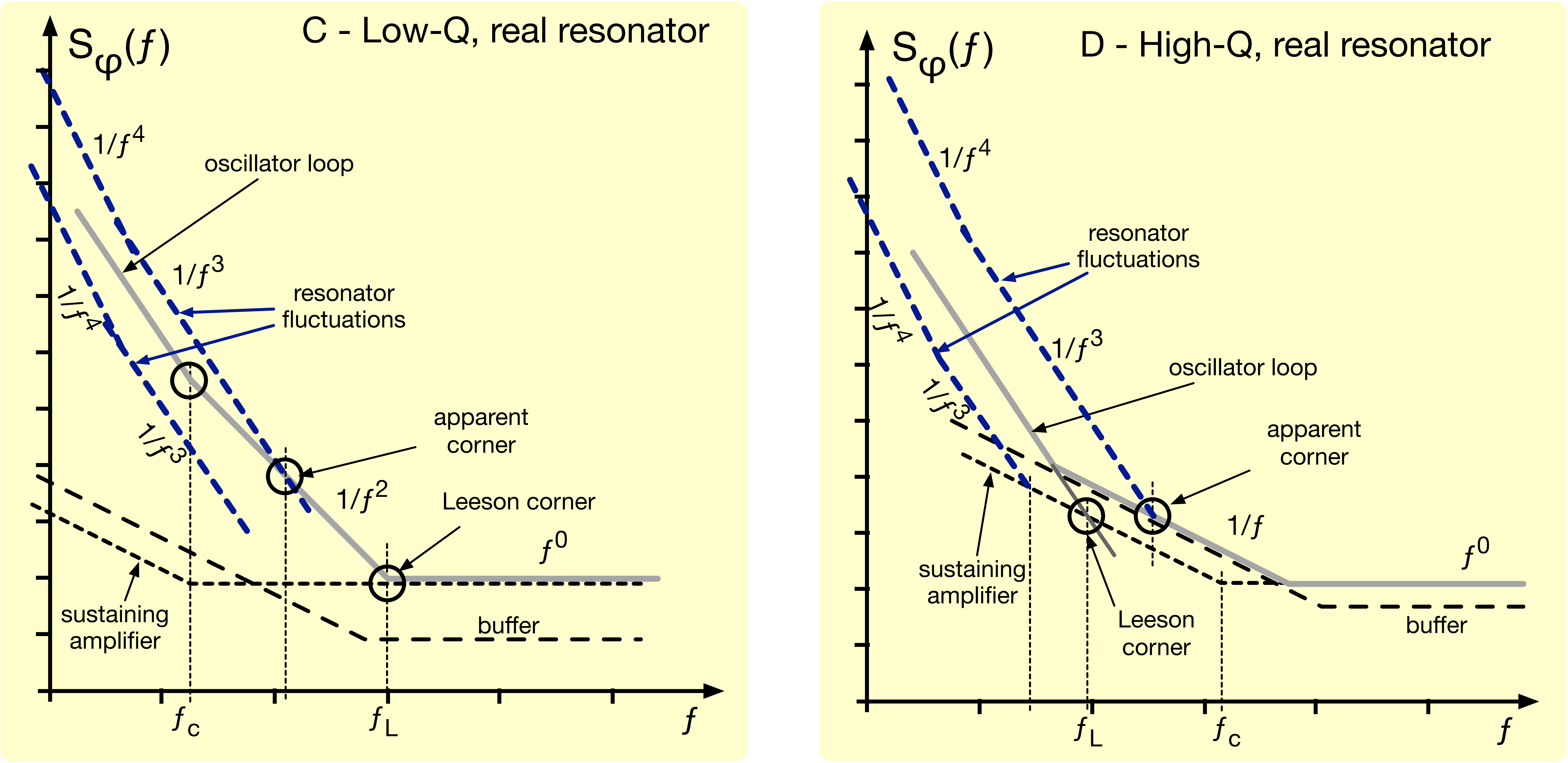
Oscillator Noise – Real Amplifier





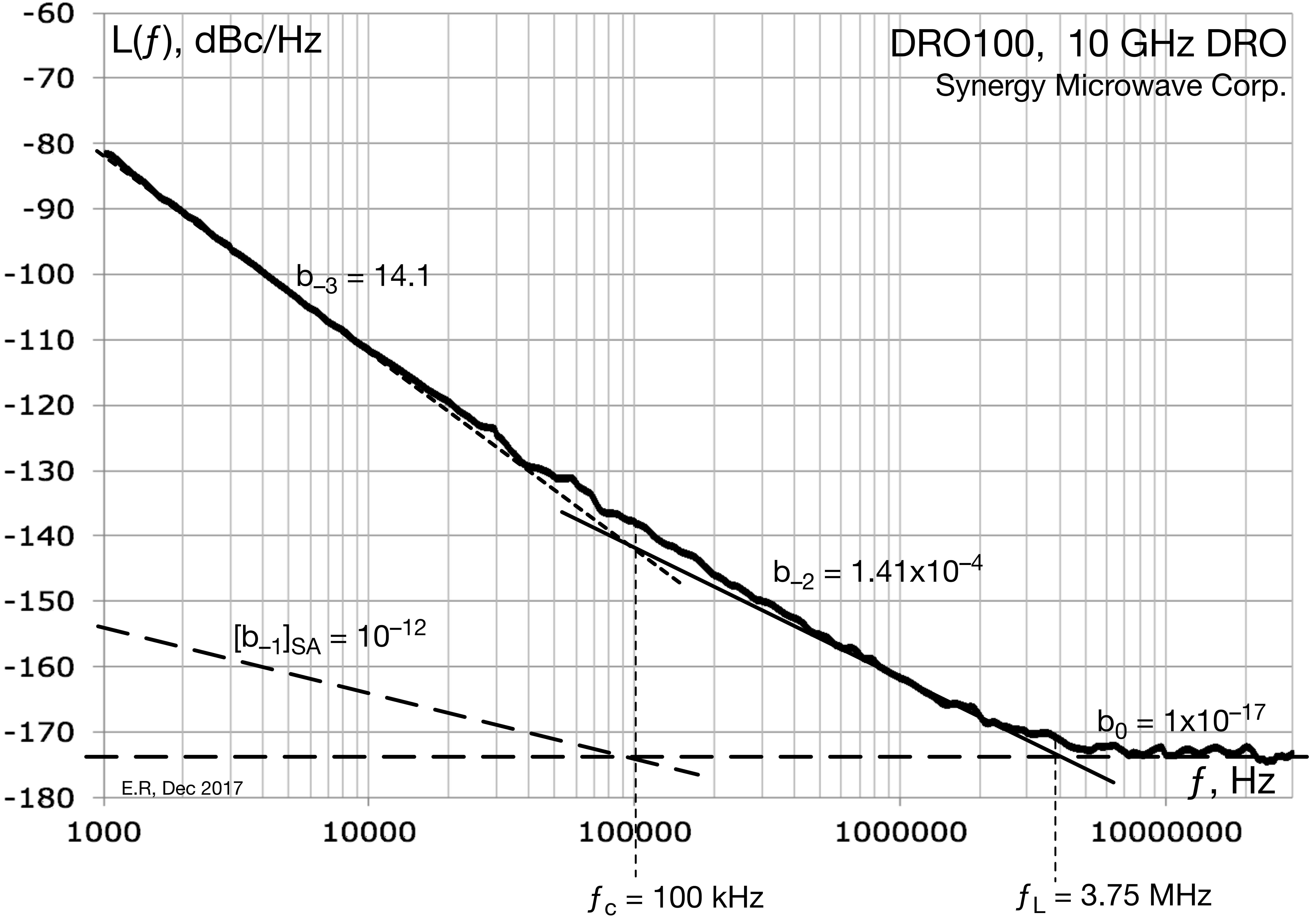
The sustaining-amplifier noise is $S_\varphi(f) = b_0 + b_{-1}/f$ (white and flicker)

The Effect of the Resonator Noise



The oscillator tracks the resonator's natural frequency, and its fluctuations

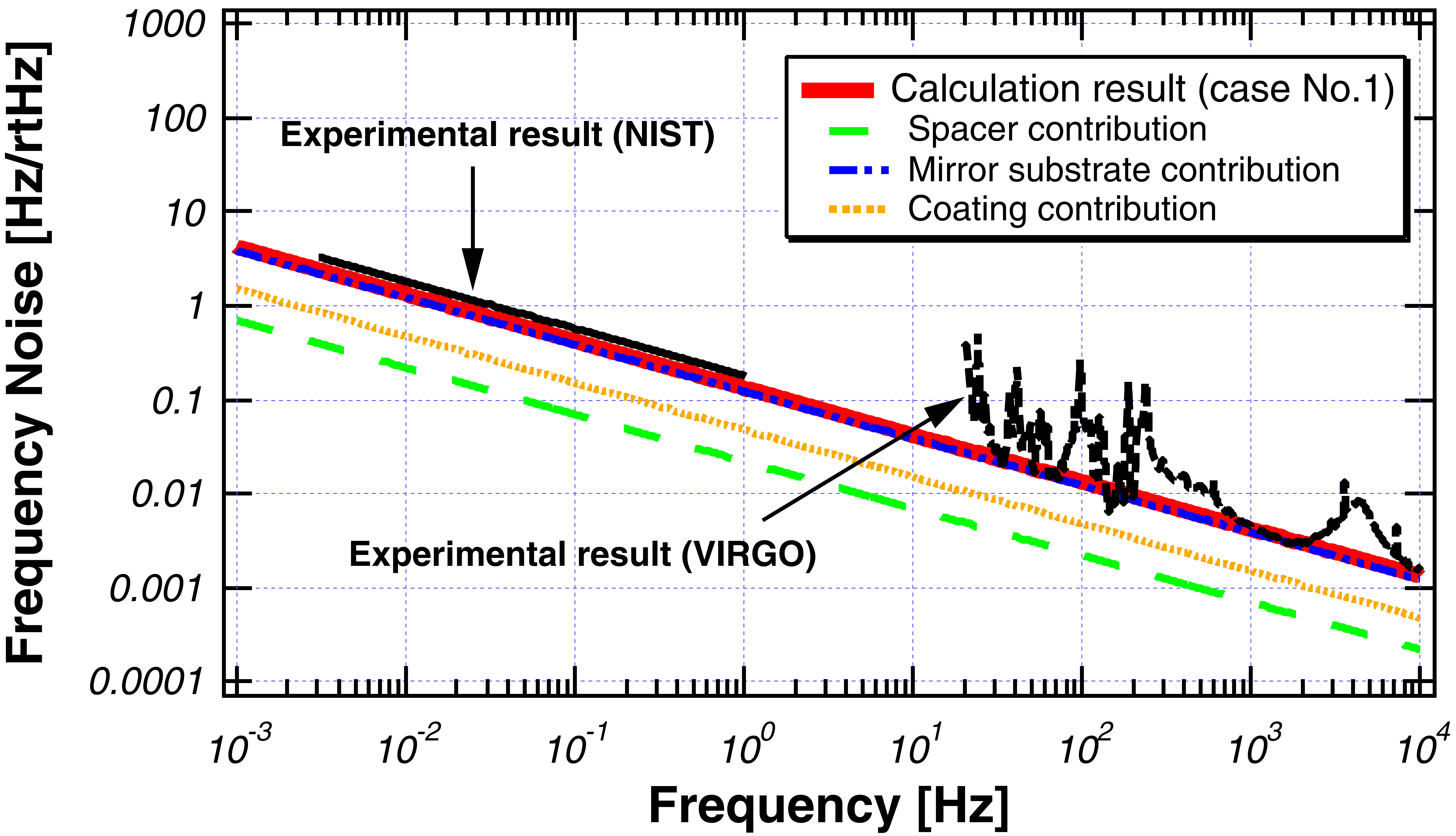
Example from the Real World



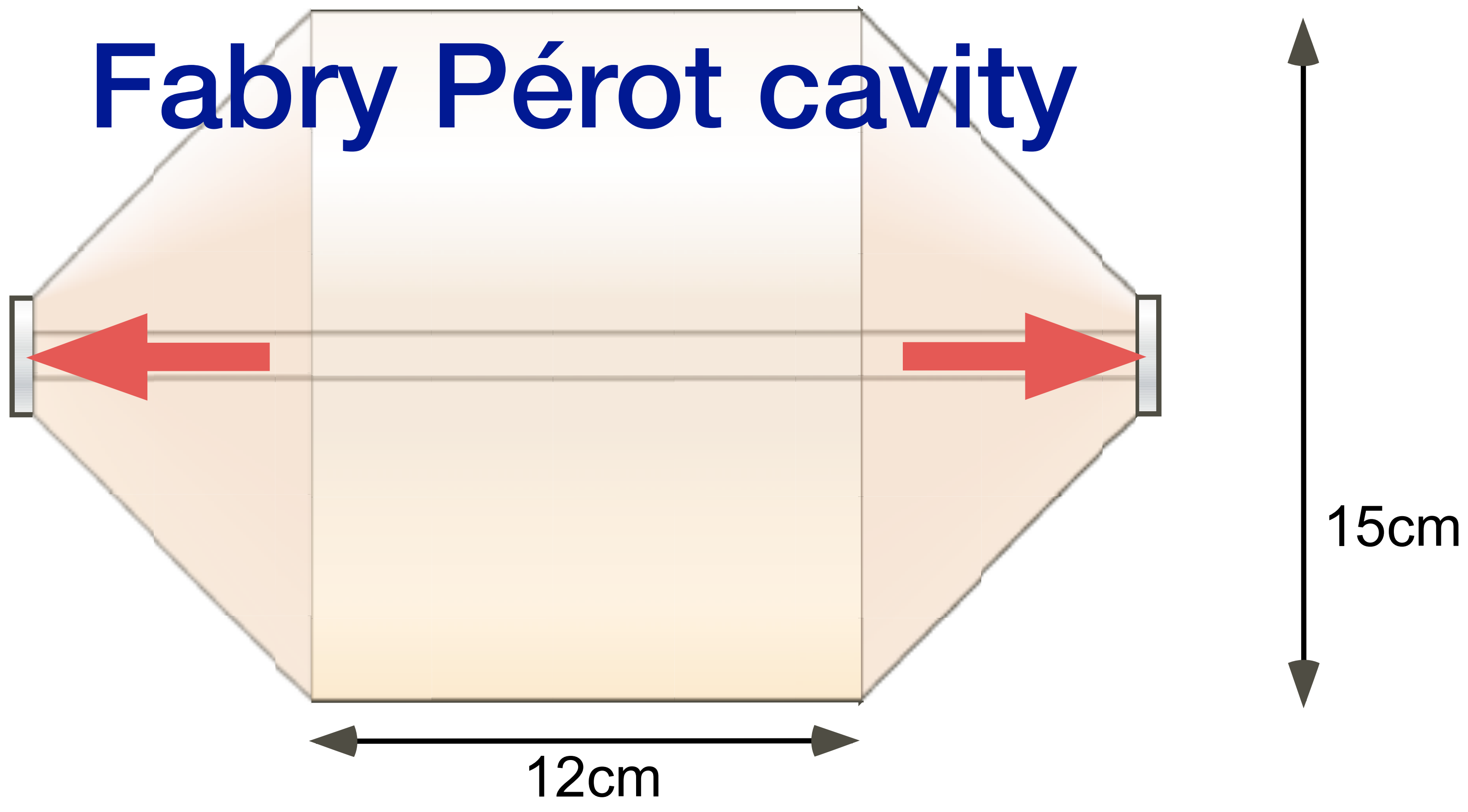
Resonator Noise

In some fortunate cases, the origin of $1/f$ frequency noise is known

Figures from: Numata K, Kemery A, Camp J, Thermal-noise limit in the frequency stabilization of lasers with rigid cavities, PRL 93(25) 250602, Dec 2004



Fabry Pérot cavity



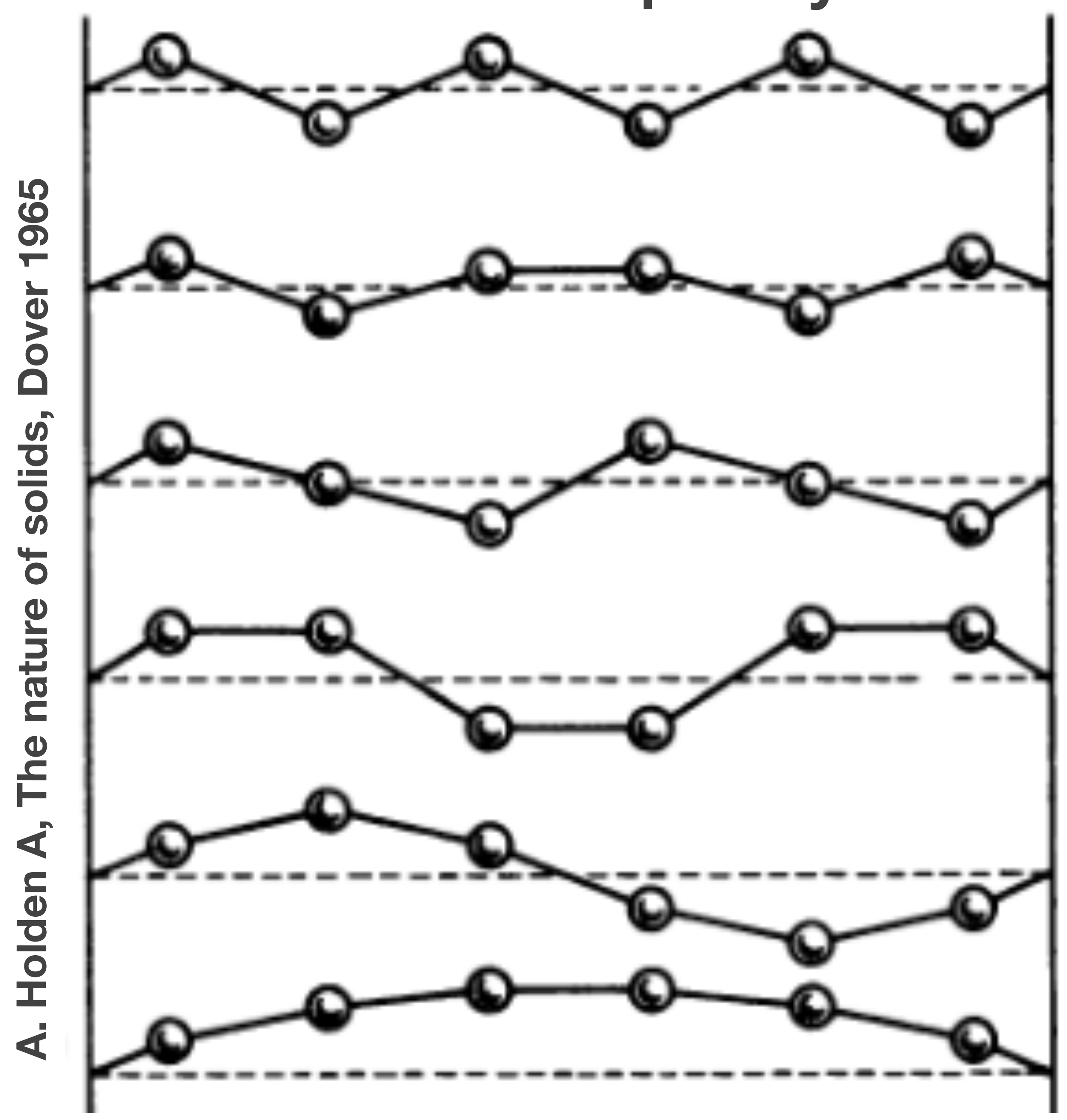
Numata provides fairly accurate prediction of $1/f$ noise

See also T. Kessler, T. Legero, U. Sterr, Thermal noise in optical cavities revisited, JOSA-B 29(1), 2012

1/ƒ Noise and FD Theorem

Flicker (1/ƒ) dimensional fluctuation is powered by thermal energy

Debye-Einstein theory
for heath capacity

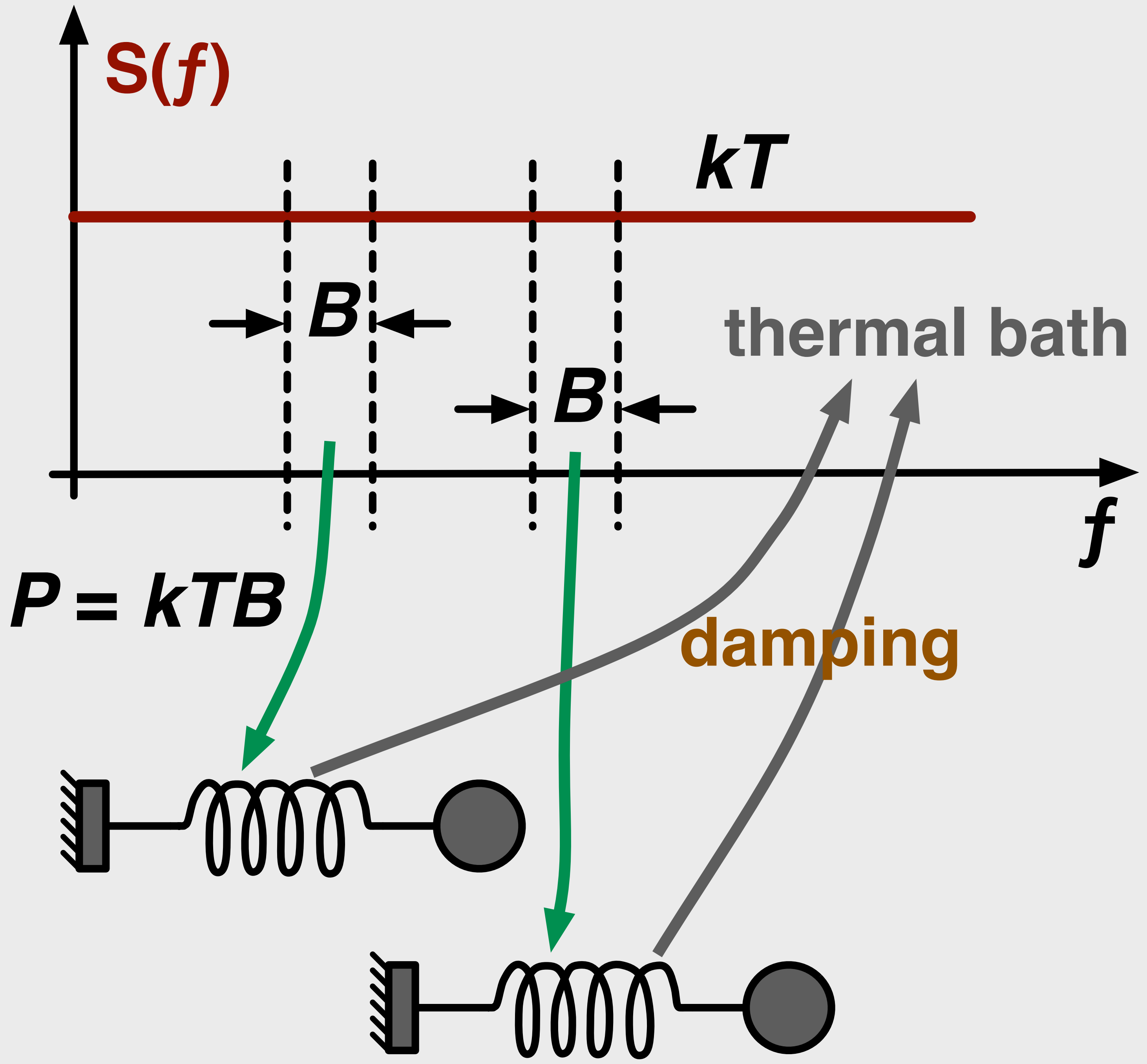


A single theory explains

- Heath capacity
- Elasticity
- Thermal expansion

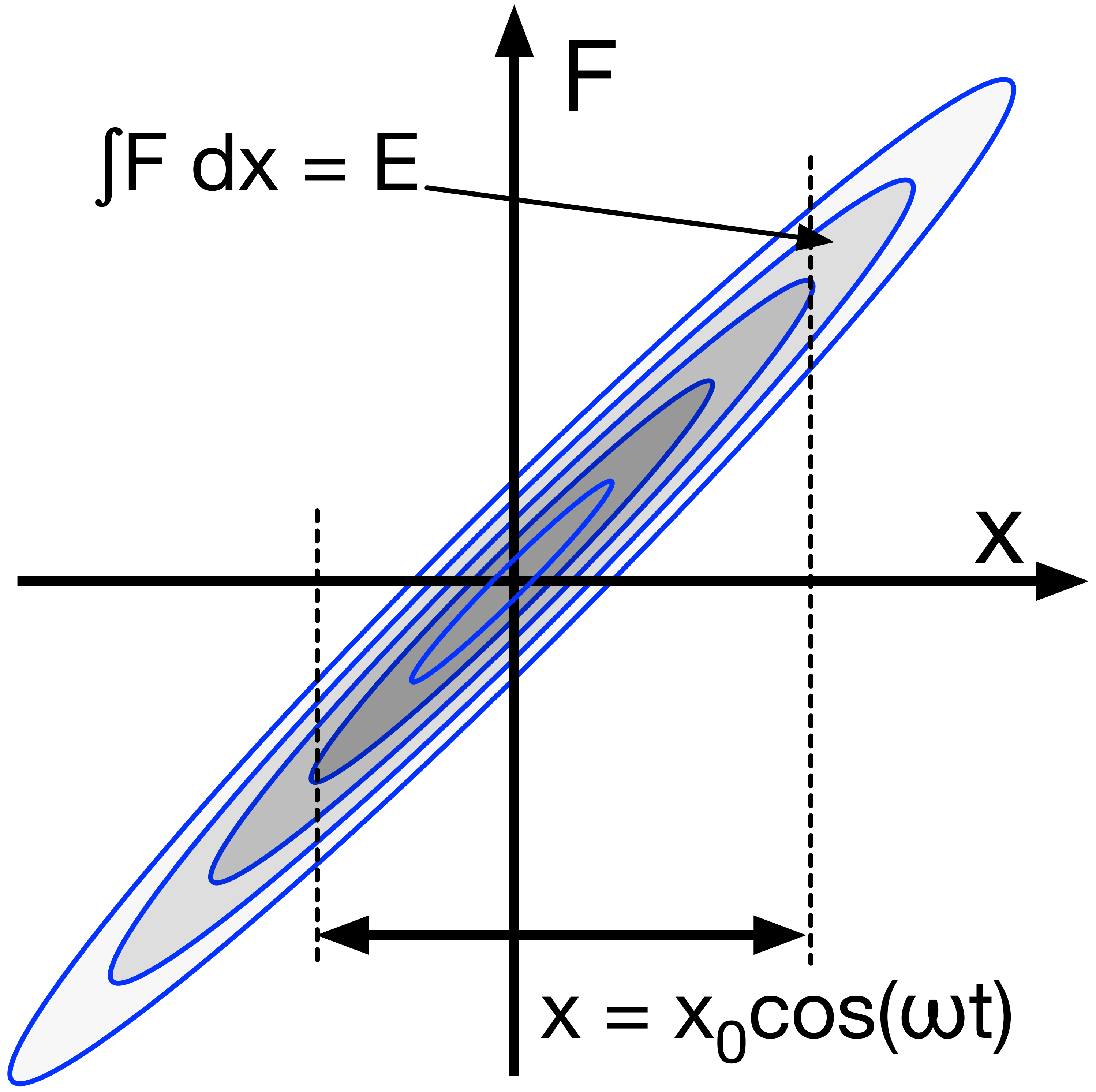
… and fluctuations

Fluctuation Dissipation



Thermal equilibrium applies to all parts of spectrum

Thermal $1/f$ from Structural Dissipation



Dissipation in solids is structural (hysteresis)

There is no viscous dissipation

Structural dissipation

nanoscale, instantaneous

Dissipated energy $E = \int F\,dx$

Small vibrations

The hysteresis cycle keeps the aspect ratio

$E \propto x_0^2$ lost energy in a cycle

Thermal equilibrium

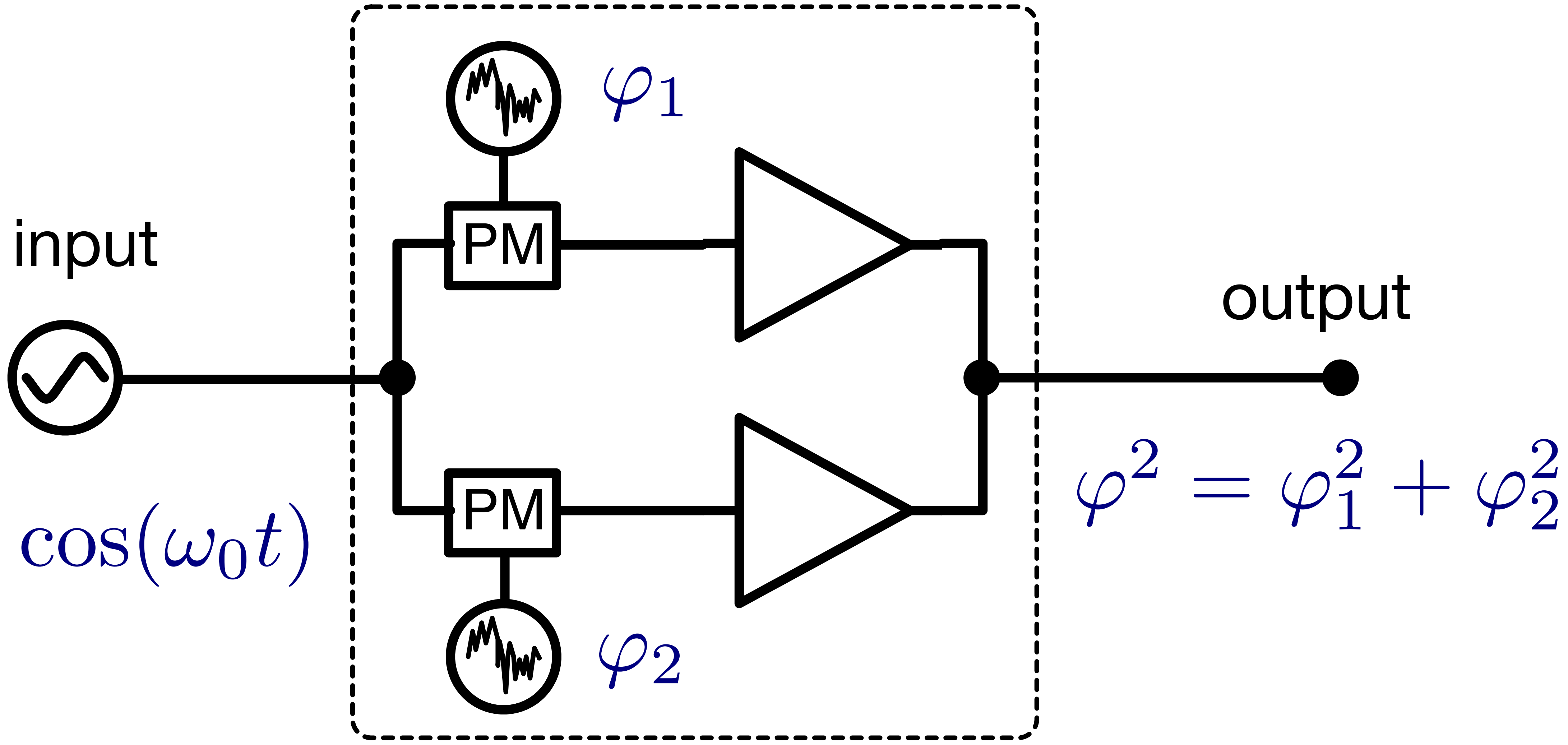
$P = kT$ in 1 Hz BW

$P \propto kTx_0^2$

$x_0^2 \propto 1/f$ —> flicker

The Volume Law

Experiment



- The $1/f$ coefficient b_{-1} is independent of power
- The flicker of a branch does not increase at P/2
- At the output,
  - the carrier adds up coherently
  - the phase noise adds up statistically
- With m branches, the $1/f$ PM noise is reduced by 1/m
- White noise cannot be reduced in this way

Gedankenexperiment

- Flicker is of microscopic origin because it has Gaussian PDF
- Join the m branches into a compound
- $1/f$ noise is proportional to 1/V, the volume of the active region

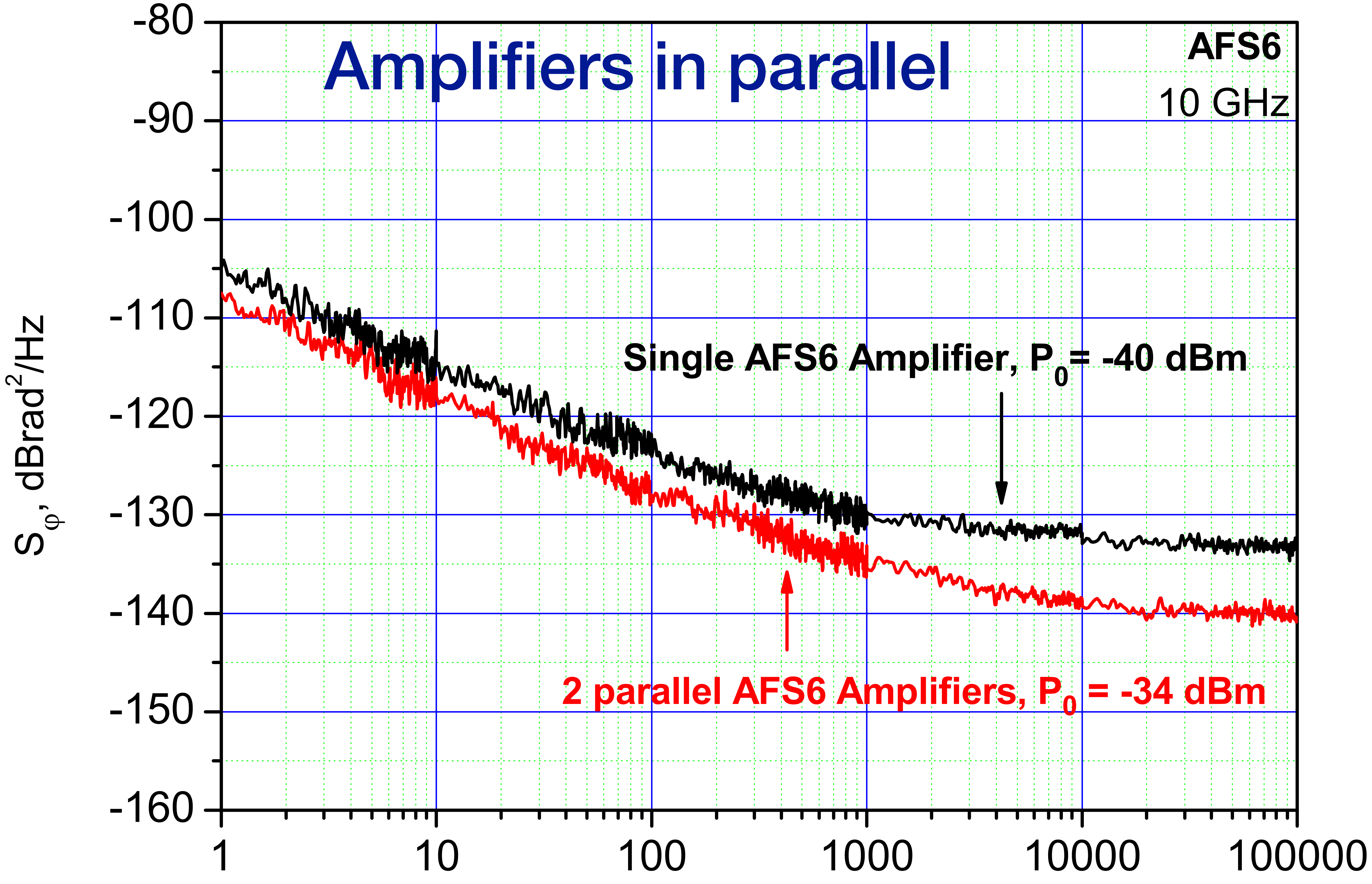
Volume Law

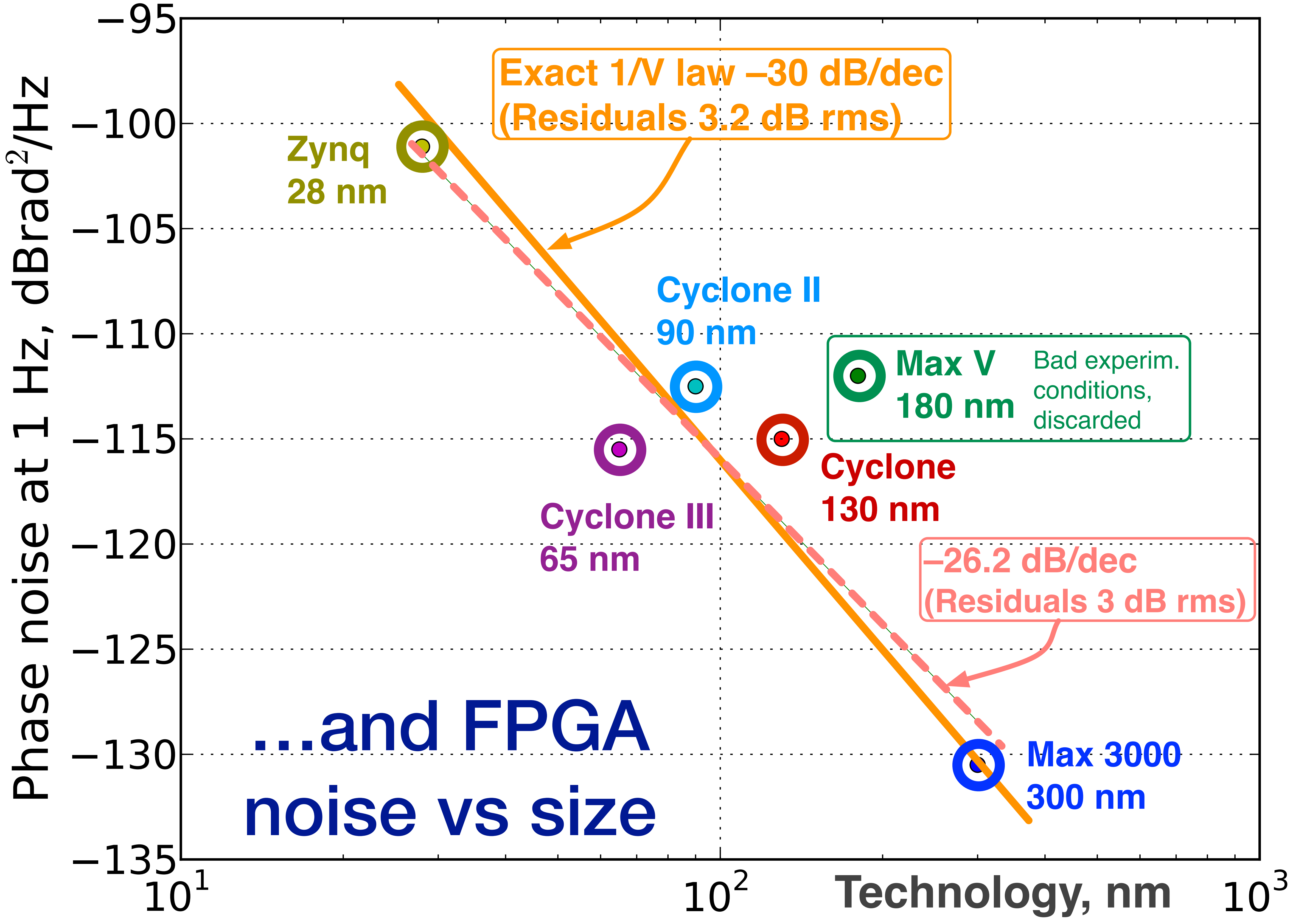
optical resonator (10^{-7}?)
($50\ \mu m^2$) × (π × 5.5 mm)
≈ 1×10^{-12} m^3

optical fiber (10^{-12})
($50\ \mu m^2$) × (2 km)
≈ 1×10^{-7} m^3

5 MHz quartz (10^{-13})
$0.3 \times [\pi \times (2/2\ \text{cm})^2] \times (0.1\ \text{mm})$
≈ 1×10^{-8} m^3

sapphire resonator (<10^{-16})
$0.1 \times [\pi \times (5/2\ \text{cm})^2] \times (2.5\ \text{cm}) \approx 5\times10^{-6}\ \text{m}^3$

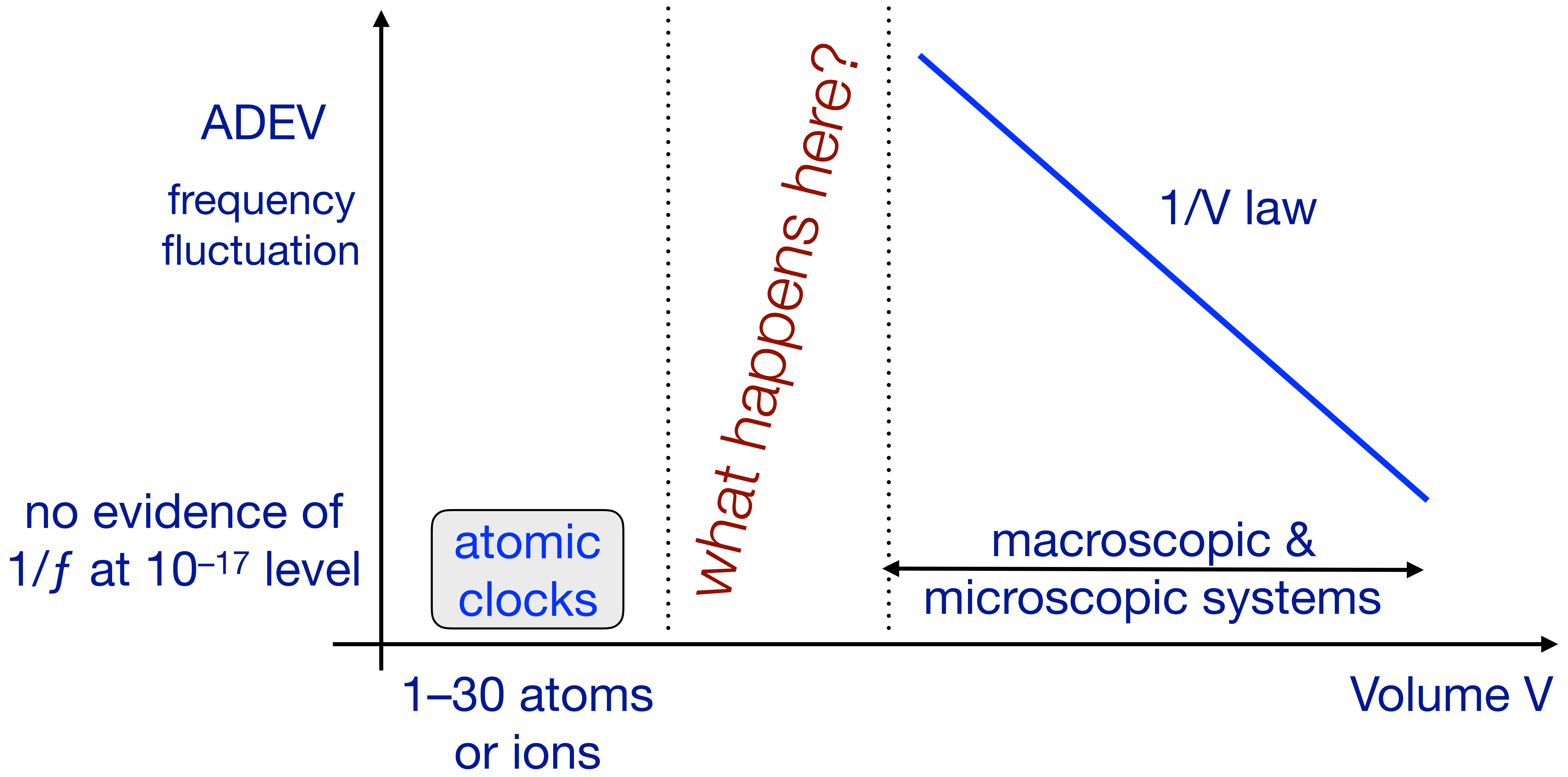




Challenges / Questions for This Community

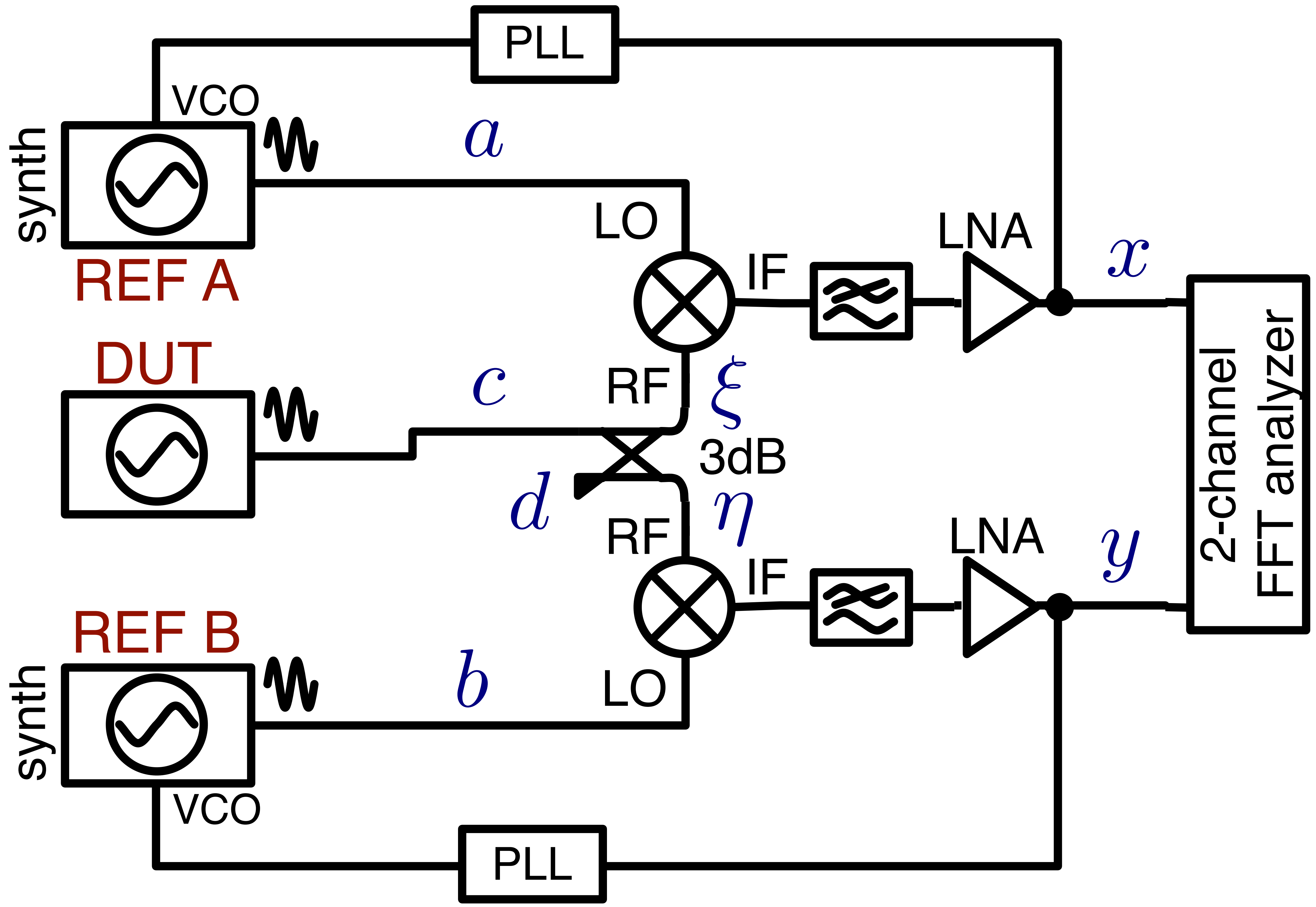
(1) Which resonators can be described by hysteresis ?

(2) and …



Phase Noise Measurement Sets

The NIST Scheme



Average cross spectrum <S_{yx}>

- rejects a and b
- ideally, converges to S_c

However

- dark-port noise
  <S_{yx}> converges to $S_c - S_d$
- DUT AM noise
  - correlated DC fluctuation
  - positive or negative correlation
- …and other flaws

Consequence

- the error can be negative

S_{yx} with Correlated Term κ≠0

All the DUT signal goes in Re{S_{yx}}, Im{S_{yx}} contains only noise

Real

$$\Re\left\{\langle S_{yx}\rangle_m\right\} = \frac{1}{T}\left\{\langle B'A' + B''A''\rangle_m + \langle B'C' + B''C''\rangle_m + \langle A'C' + A''C''\rangle_m + \langle (C')^2 + (C'')^2\rangle_m\right\}$$

var=1/2 (B′, A′); var=1/2, var= $\kappa^2/2$ (B′, C′); var=1/2, var= $\kappa^2/2$ (A′, C′); var= $\kappa^2/2$ (C′)

Bessel K_0, avg=0, var=1/4 (B′A′, B″A″)

Bessel K_0, avg=0, var=$\kappa^2/4$ (B′C′, B″C″, A′C′, A″C″)

white, χ^2, 2 DF, avg = κ^2, var = κ^4 ($(C')^2$, $(C'')^2$)

- $\langle B'A' + B''A''\rangle_m$: Gaussian, avg = 0, var = 1/2m
- $\langle B'C' + B''C''\rangle_m$: Gaussian, avg = 0, var = κ^2/2m
- $\langle A'C' + A''C''\rangle_m$: Gaussian, avg = 0, var = κ^2/2m
- $\langle (C')^2 + (C'')^2\rangle_m$: white, χ^2, 2m deg. of freedom, avg = κ^2, var = κ^4/m

Gaussian, avg = 0, var = (1+2κ^2)/2m

Note: DF < 2m
See vol.XVI p.56

Imaginary

$$\Im\left\{\langle S_{yx}\rangle_m\right\} = \frac{1}{T}\left\{\langle B''A' + B'A''\rangle_m + \langle B''C' - B'C''\rangle_m + \langle A'C'' - A''C'\rangle_m\right\}$$

Bessel K_0, avg = 0, var = 1/4 (B″A′, B′A″)

Bessel K_0, avg = 0, var = κ^2/4 (B″C′, B′C″, A′C″, A″C′)

- $\langle B''A' + B'A''\rangle_m$: Gaussian, avg = 0, var = 1/2m
- $\langle B''C' - B'C''\rangle_m$: Gaussian, avg = 0, var = κ^2/2m
- $\langle A'C'' - A''C'\rangle_m$: Gaussian, avg = 0, var = κ^2/2m

Gaussian, avg = 0, var = (1+2κ^2)/2m

Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ^2
var{A'} = var{A"} = var{B'} = var{B"} = 1/2, and var{C'} = var{C"} = κ^2/2

A, B, C are independent Gaussian noises
Re{ } and Im{ } are independent Gaussian noises

Problems with Digital Electronics

- Large averaging power for cheap encourages thoughtless trust in noise rejection
- Manufacturers use |Syx| instead of Re{Syx}

…and something has to go wrong

Type-A, Type B, and Null Uncertainty

2.28
Type A evaluation of measurement uncertainty
Type A evaluation

evaluation of a component of **measurement uncertainty** by a statistical analysis of **measured quantity values** obtained under defined measurement conditions

A —> Data-series statistics

Averaging multiple FFTs results in reduced uncertainty

2.29
Type B evaluation of measurement uncertainty
Type B evaluation

evaluation of a component of **measurement uncertainty** determined by means other than a **Type A evaluation of measurement uncertainty**

B —> Ensemble statistics

Averaging multiple FFTs results in reduced uncertainty

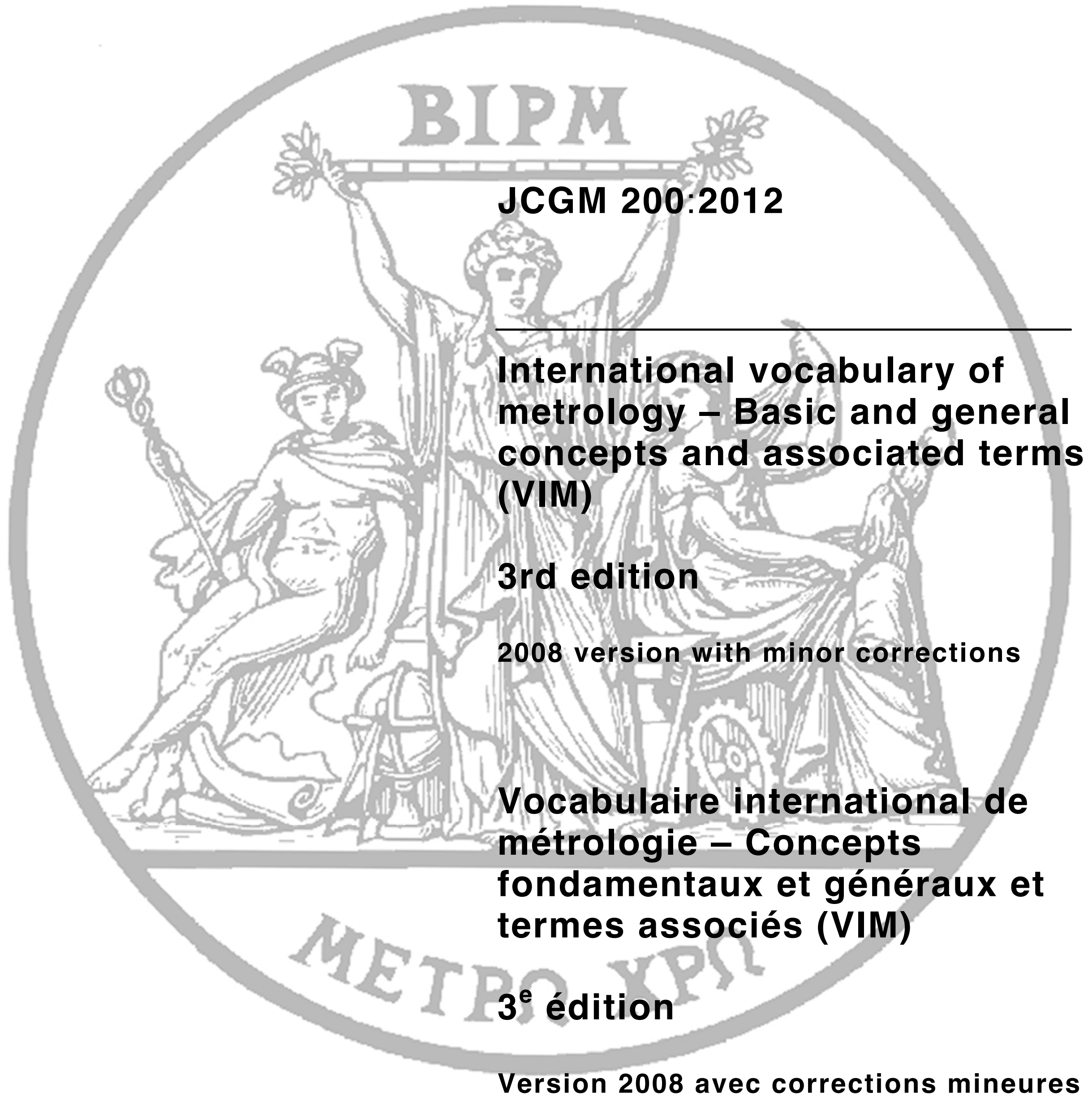
4.29
null measurement uncertainty

measurement uncertainty where the specified **measured quantity value** is zero

NOTE 1 Null measurement uncertainty is associated with a null or near zero **indication** and covers an interval where one does not know whether the **measurand** is too small to be detected or the indication of the **measuring instrument** is due only to noise.

Null —> Detection threshold

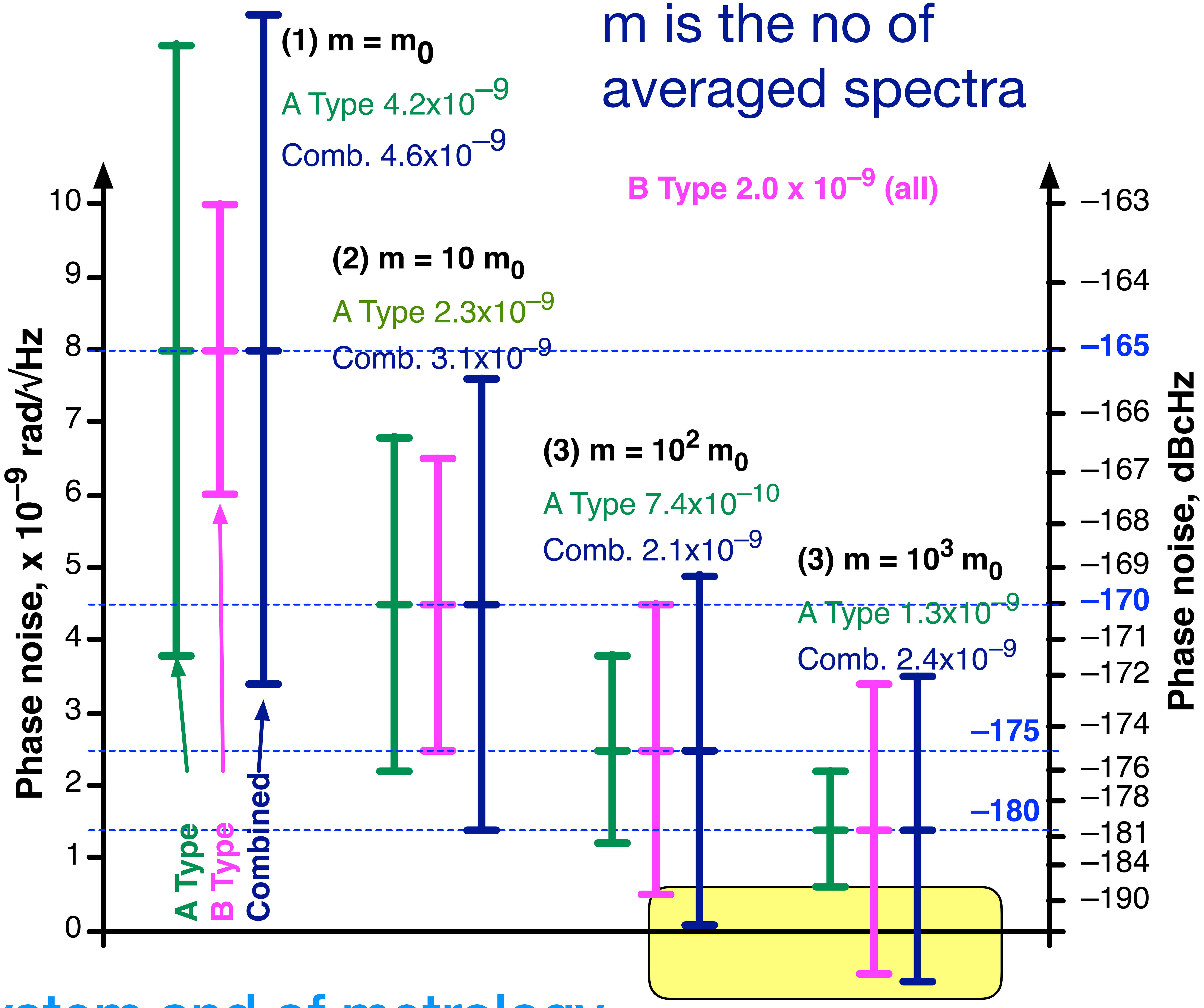
When the error bar Q±U hits 0,
the outcome is Q=0 with null uncertainty U



Type-A, Type B, and Null Uncertainty

- A-type (noise-like) uncertainty
- B-type (system) uncertainty
- Combined $U^2 = A^2+B^2$
- "Regular" case $S \rightarrow S_0 \pm U$
- Zero uncertainty, applies to $S > 0$

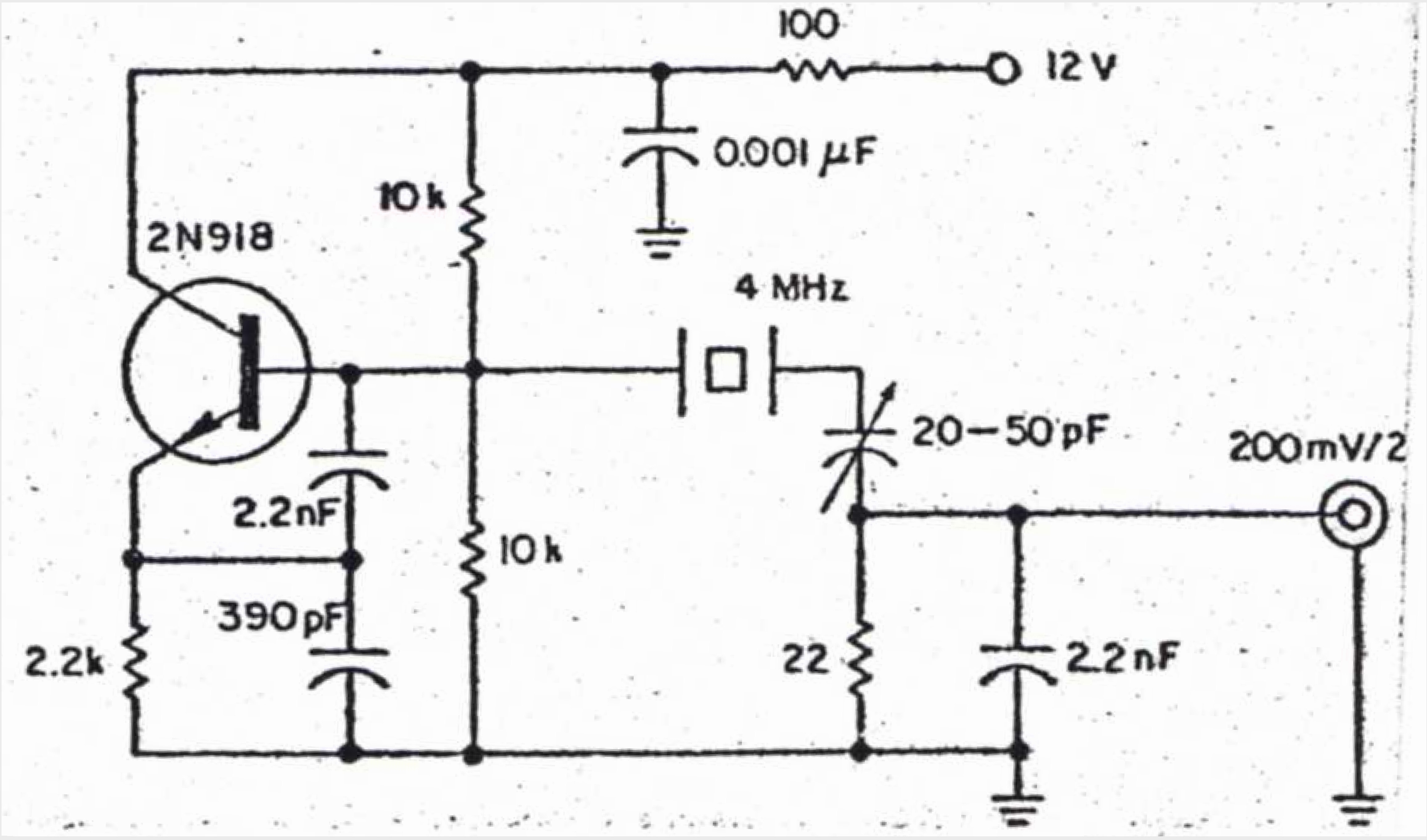
  When $S_0 \pm U$ hits 0,
  the outcome is 0
  with zero-uncertainty of U

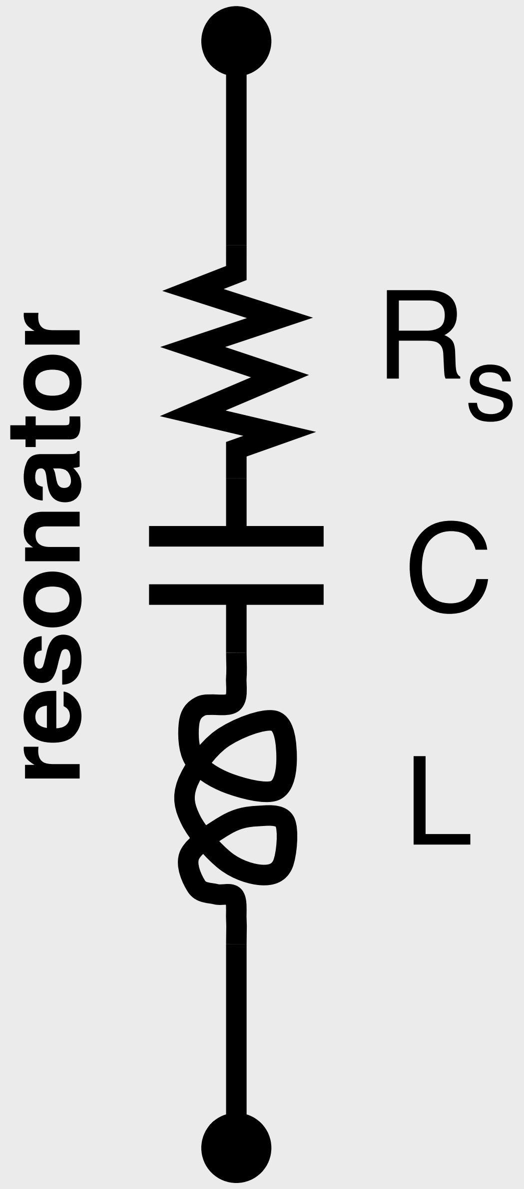


Pushing the instruments to the limit
takes deep understanding of the system and of metrology

The Rohde-Colpitts Oscillator

U. L. Rohde, Electronic Design Oct 11, 1975 p.11, 14

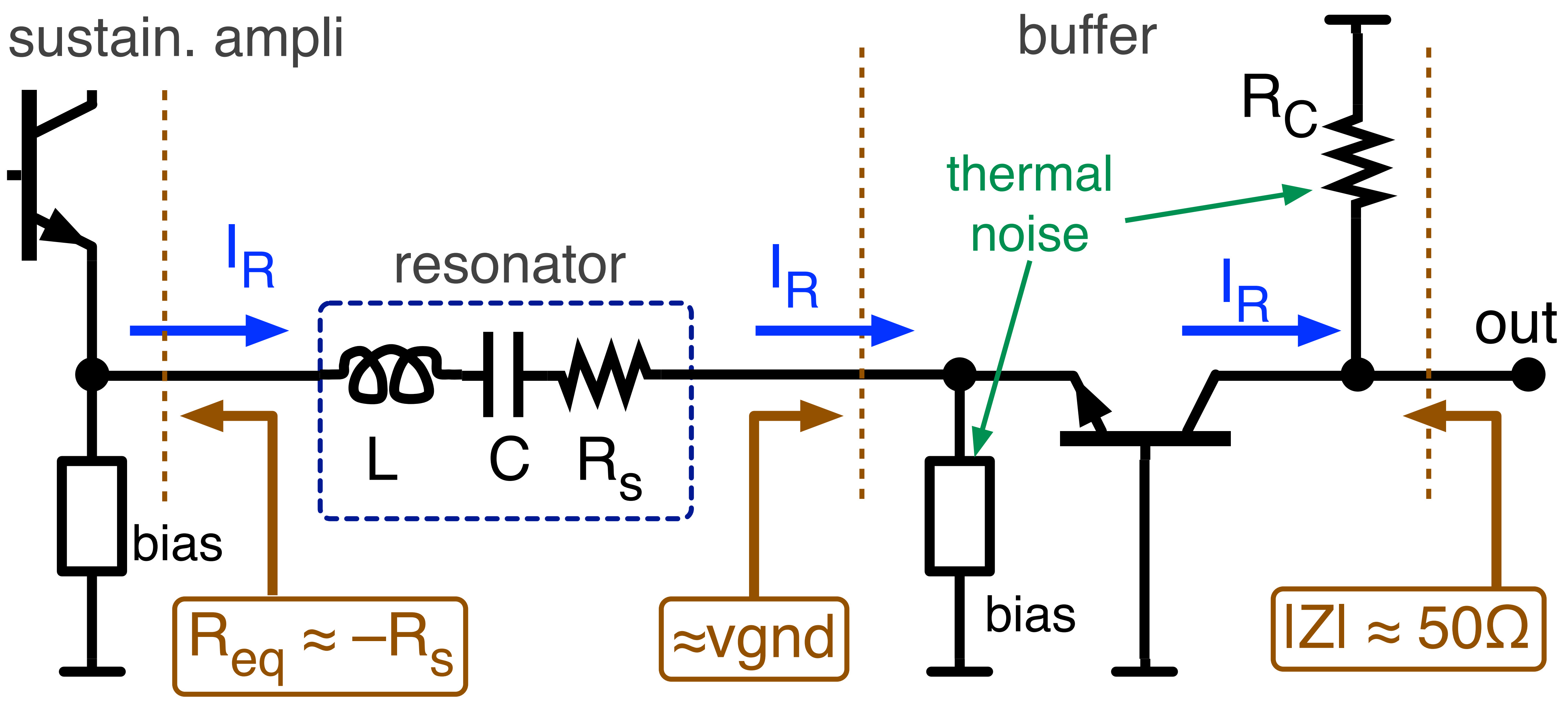




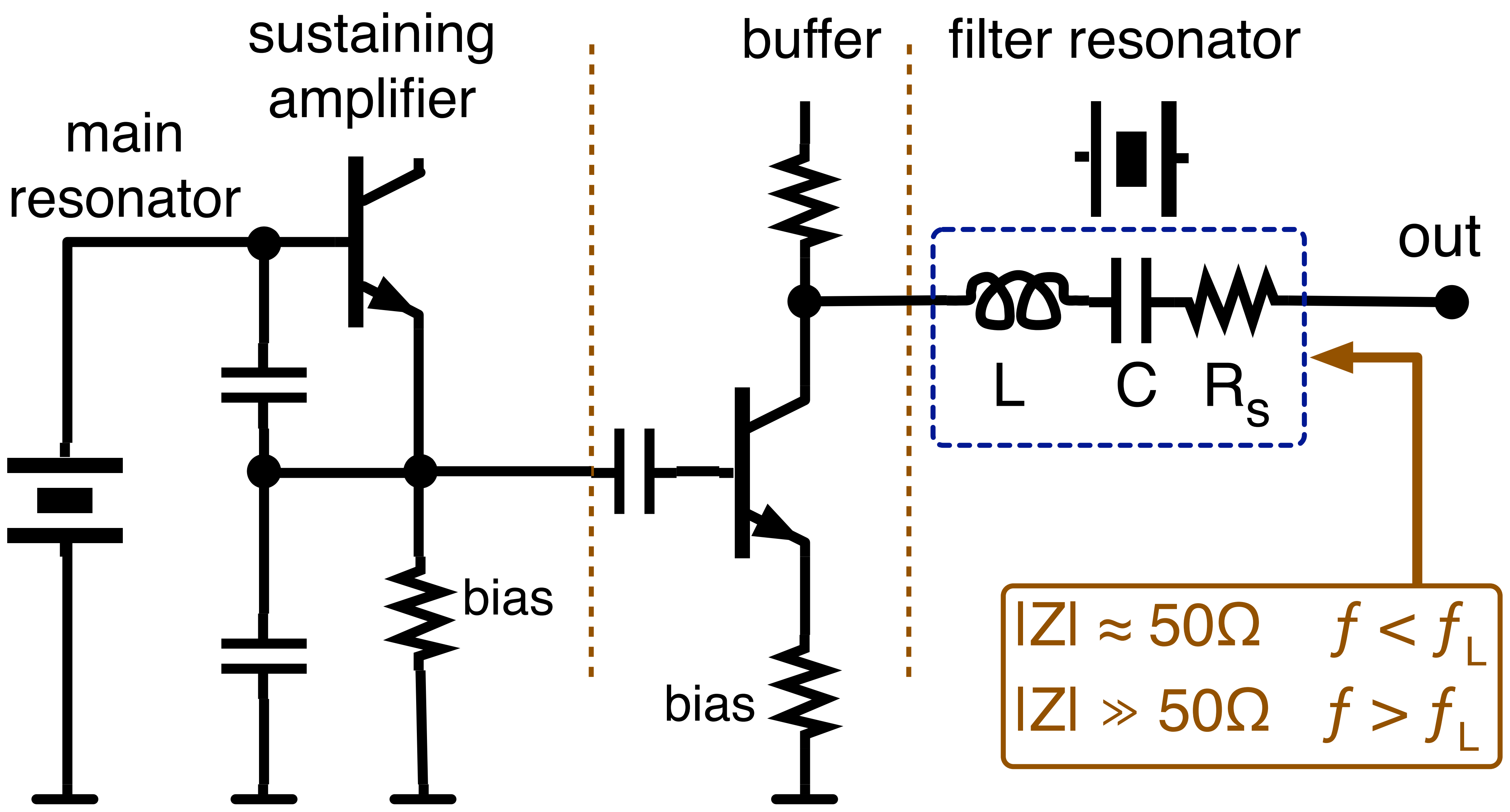
- Off resonance, either $X_L >> R_S$ or $X_C >> R_S$
- The motional resistance R_S is not coupled to the output
  - No thermal noise from R_S —> output
- The quartz also filters out harmonics and spurs

Practical Ultralow-Noise Oscillators

Thermal-limited oscillator



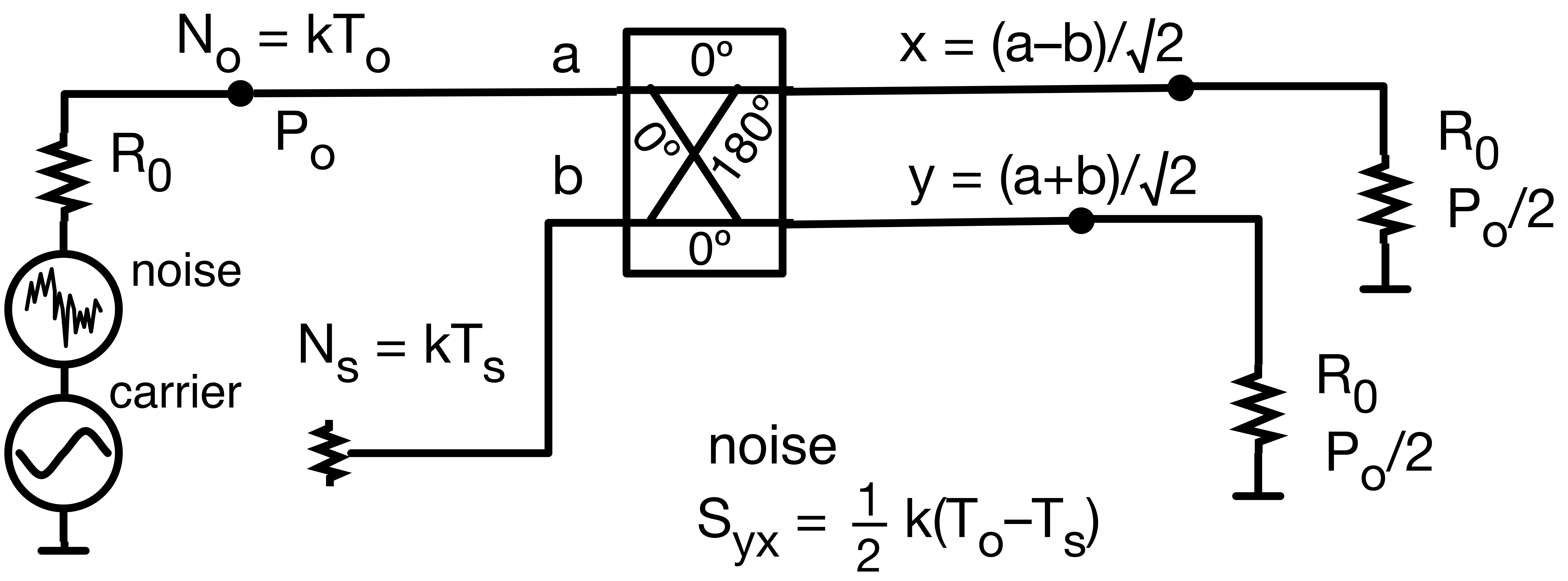
Sub-thermal oscillator



Something weird must happen

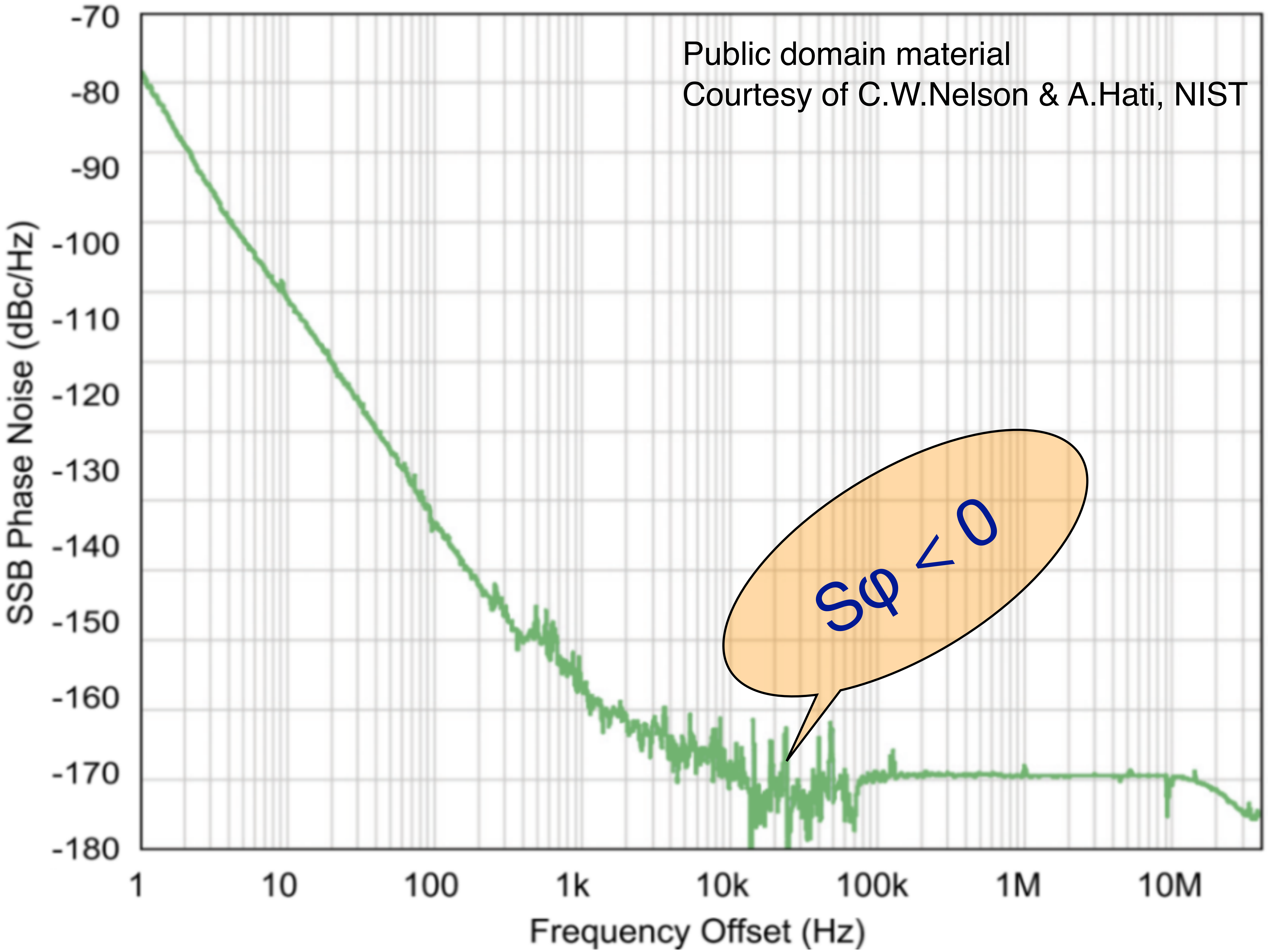
What Happens at the Instrument Input

The correlation thermometer has been in use since ≈1960



Also internal crosstalk, and AM to DC-fluctuation in the mixers

…But Instruments Display $|S_{yx}(f)|$

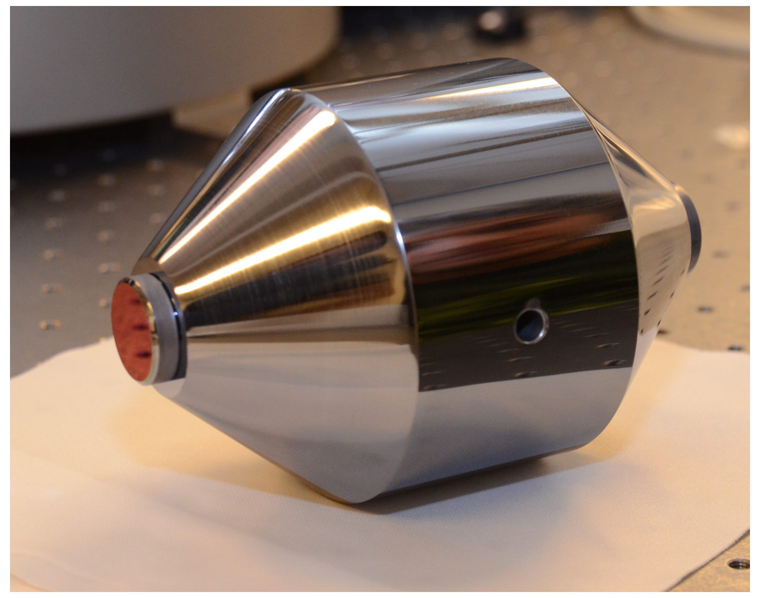


A forthcoming paper by Y.Gruson, U.L.Rohde, A.Roth, A.Rus, E.Rubiola

…And Something More

Oscillator Instability Measurement Platform

Microwave photonic oscillators

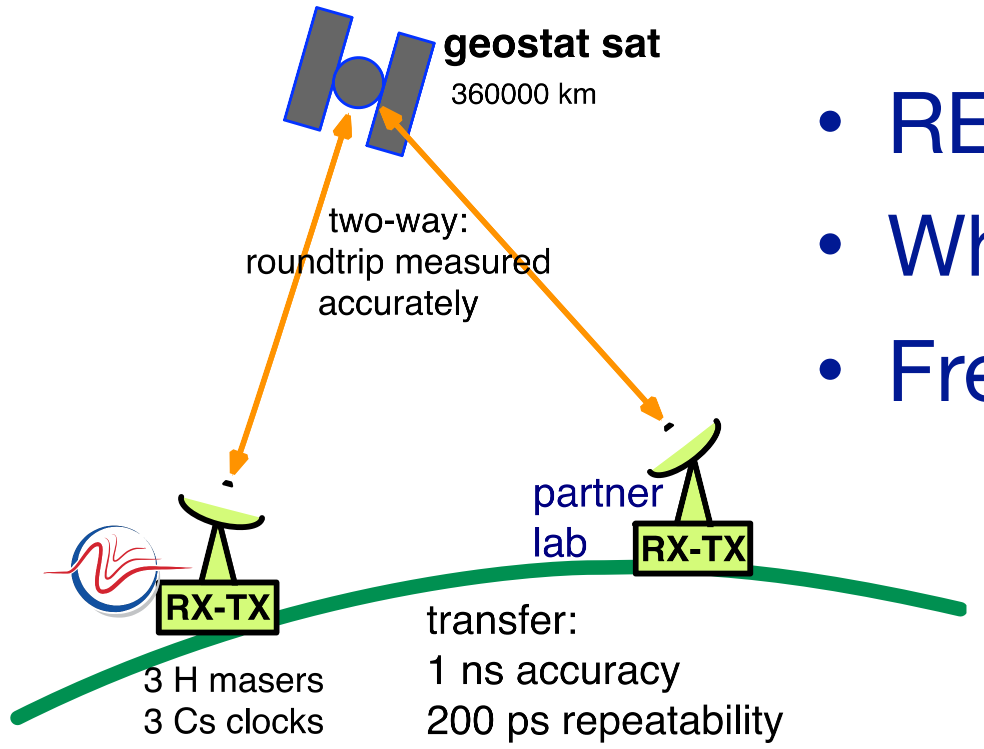


- Si Monocrystal FP, Bragg mirrors
- 17 K natural turning point
- Projected stability 3E–17
- First tests

Also

- Spherical FP cavity, 1E–15 stability
- Compact FP cavity, A3 size breadboard

- BIPM CMC
- COFRAC, top level
- 2 FS combs
- Shielded chamber
- 10 mK dilution cryo
- Prop-Integr Temp & hygro contr. rooms
- ULISS traveler



- REFIMEVE+ in progress
- White Rabbit
- Frequency distribution on fibers

Time System

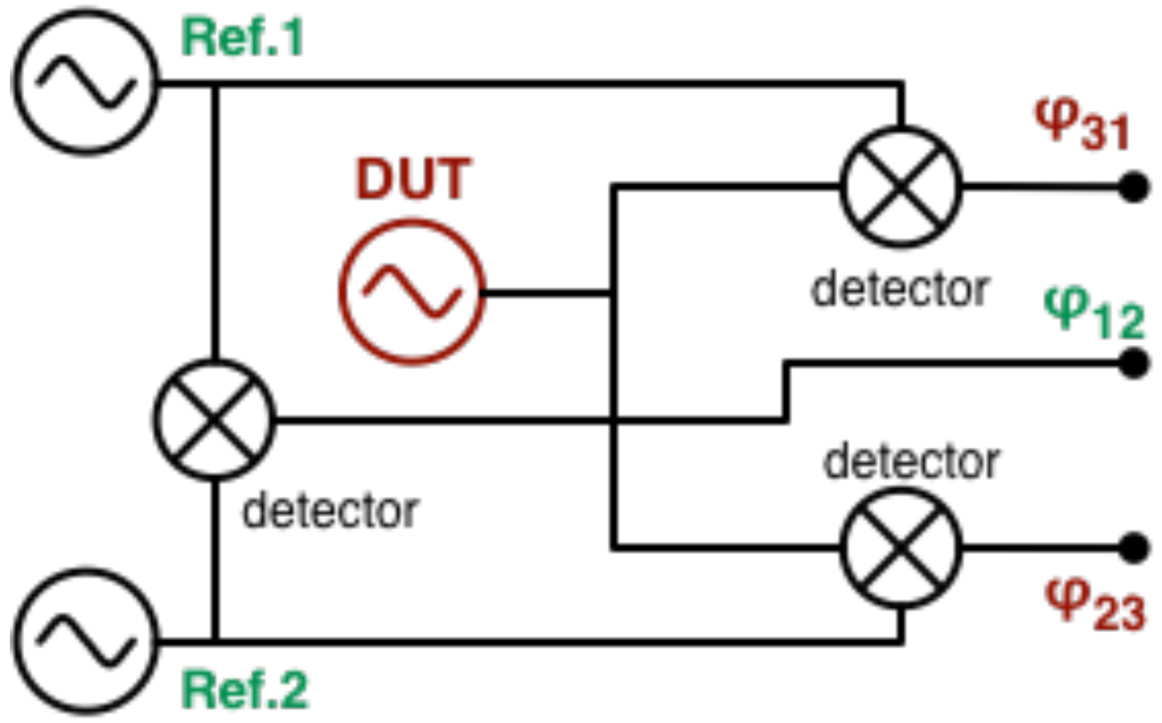
3rd Time site in France

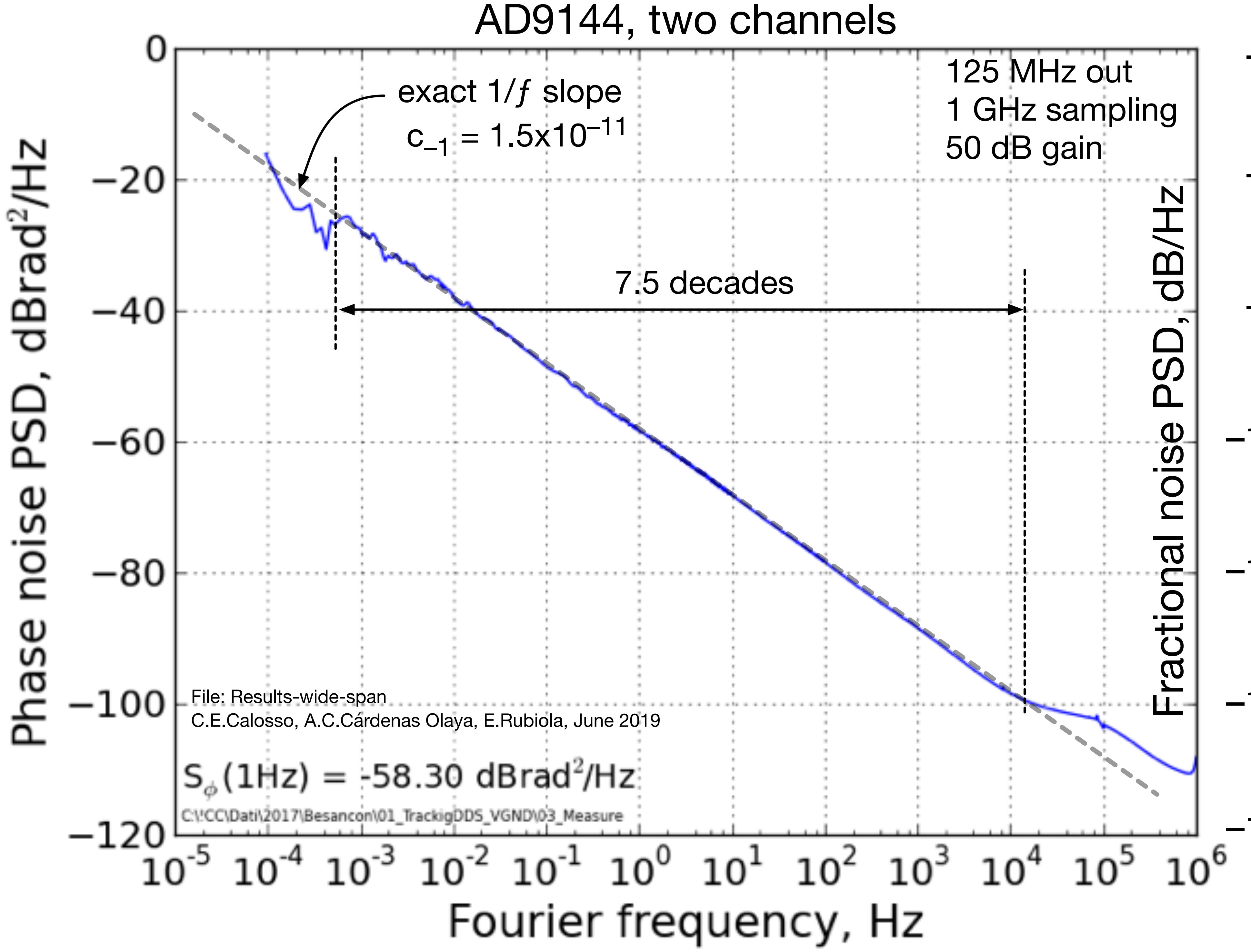
- 3 H masers & 3 CS
- TWSTFT
- Common view GPS

Digital Electronics & Metrology

Three-cornered hat noise measurements

Ω, Λ Π counters are commercial products

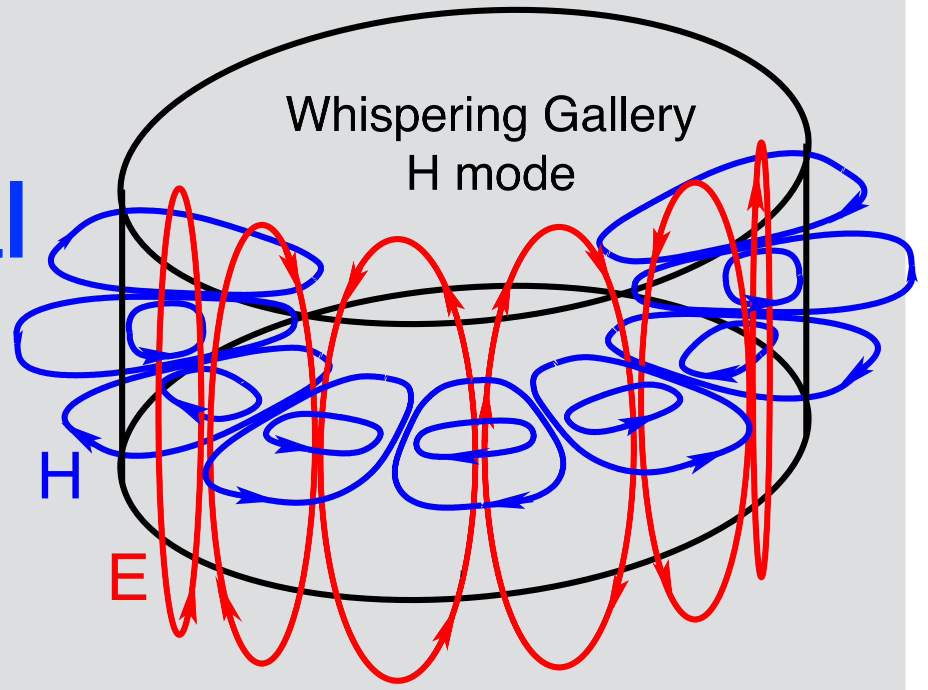




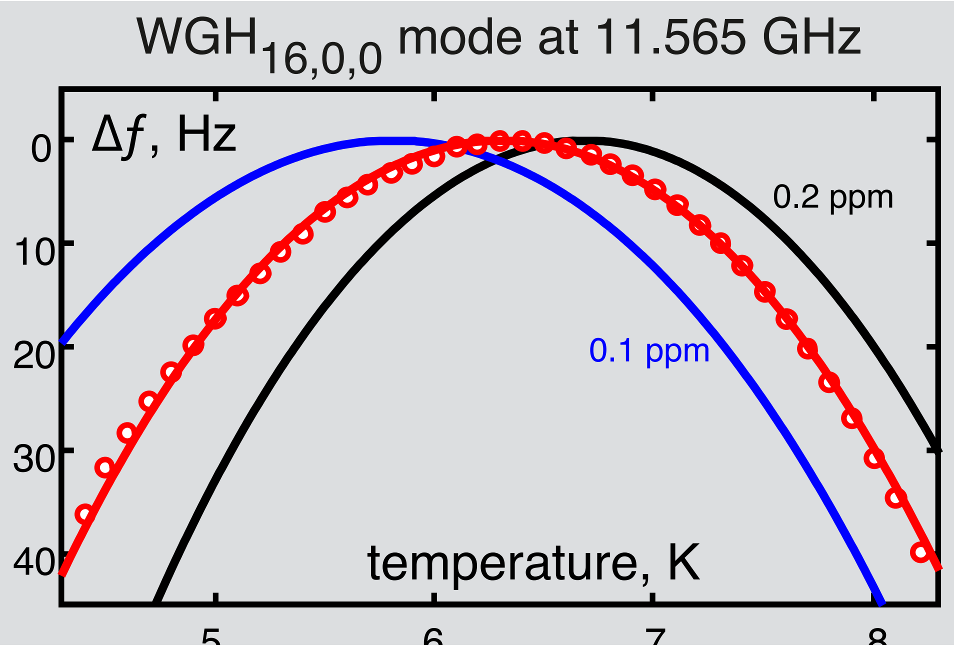
Liquid-He Sapphire Oscillator

Cr^{3+} Fe^{3+} doped
Al_2O_3 mono crystal
$\phi \approx 5$ cm, $H \approx 3$ cm

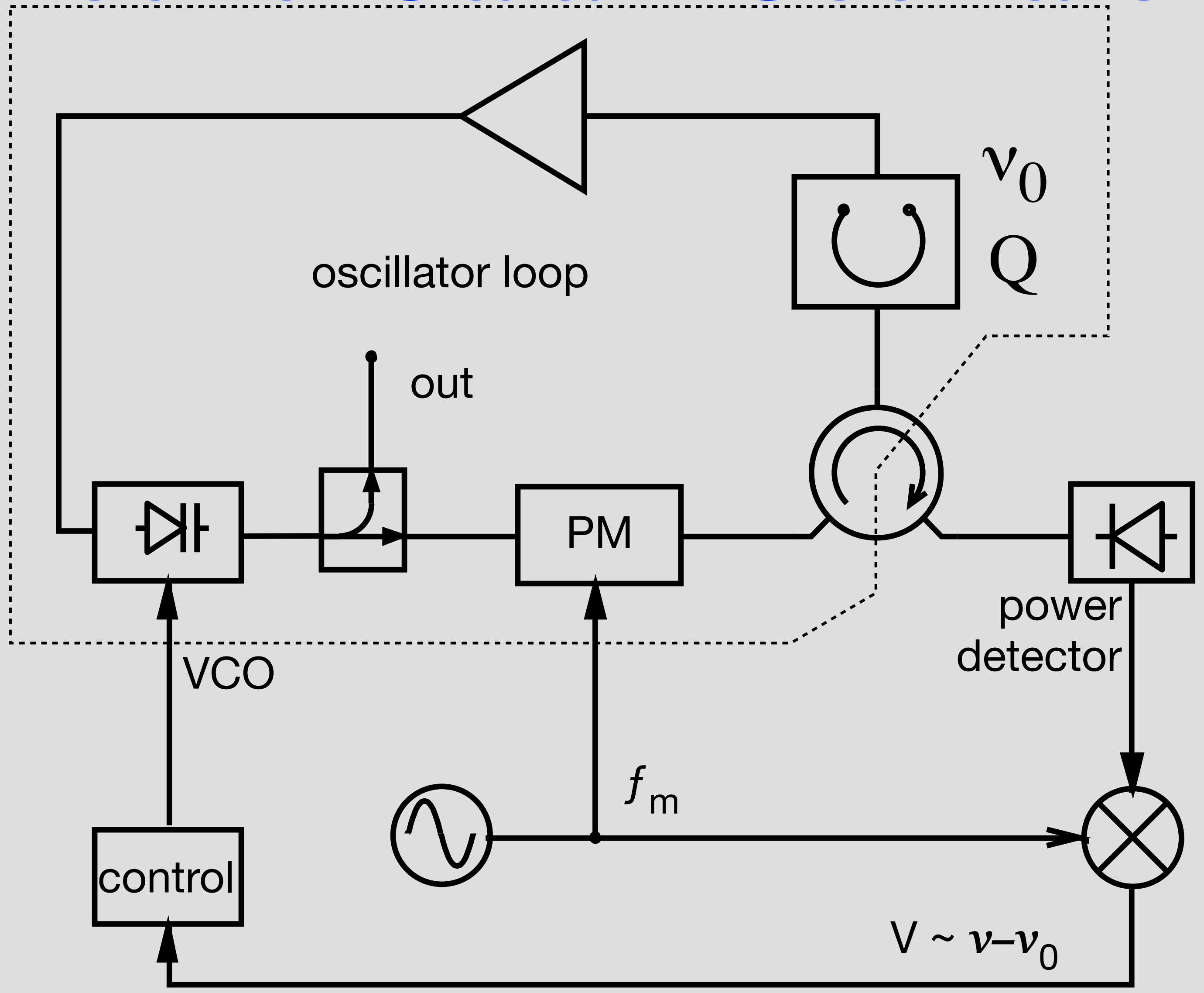
10 GHz resonance
$Q \approx 2\times10^9$ at 5–7 K



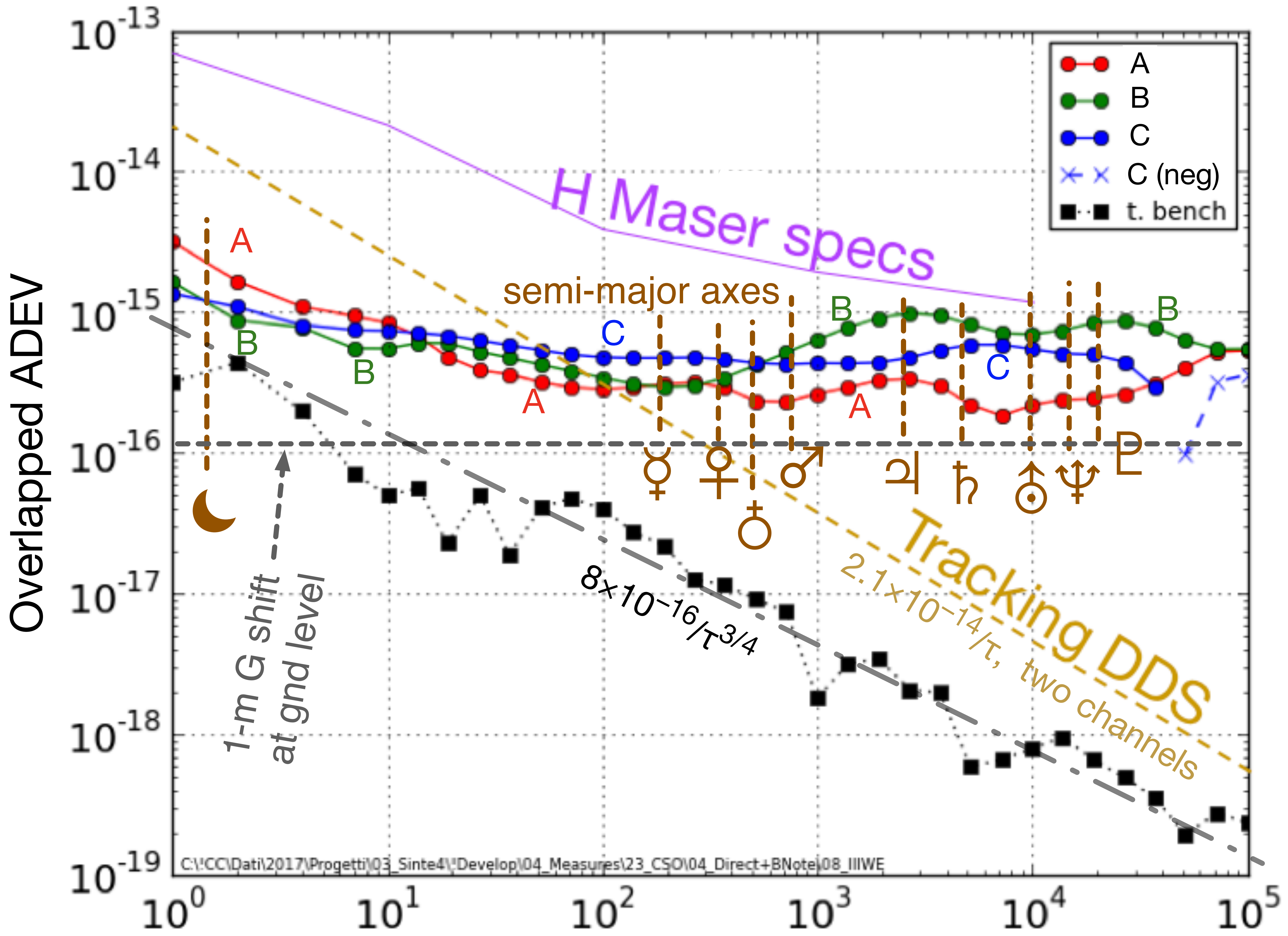
Paramagnetic temperature compensation



Pound-Galani Oscillator

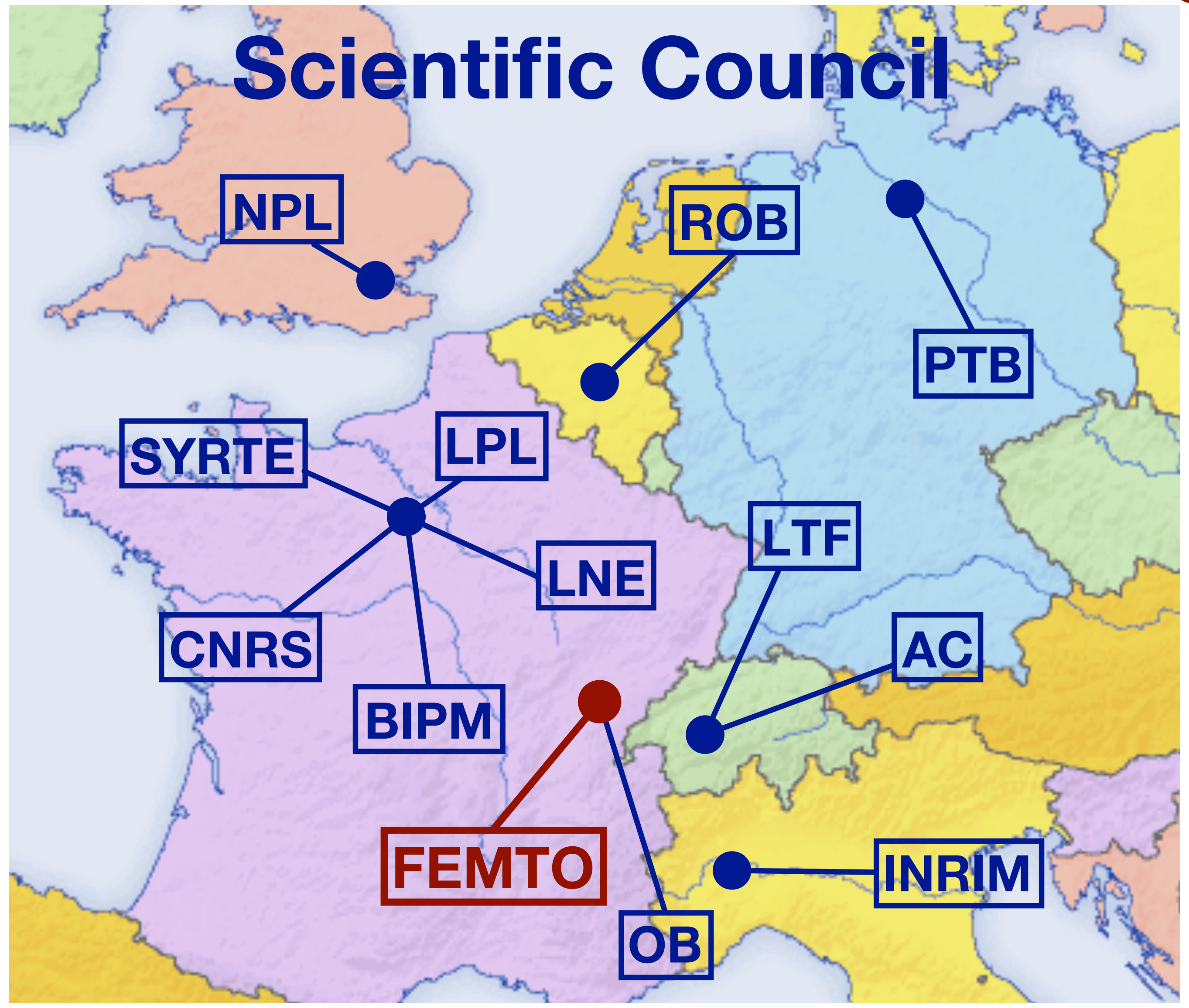


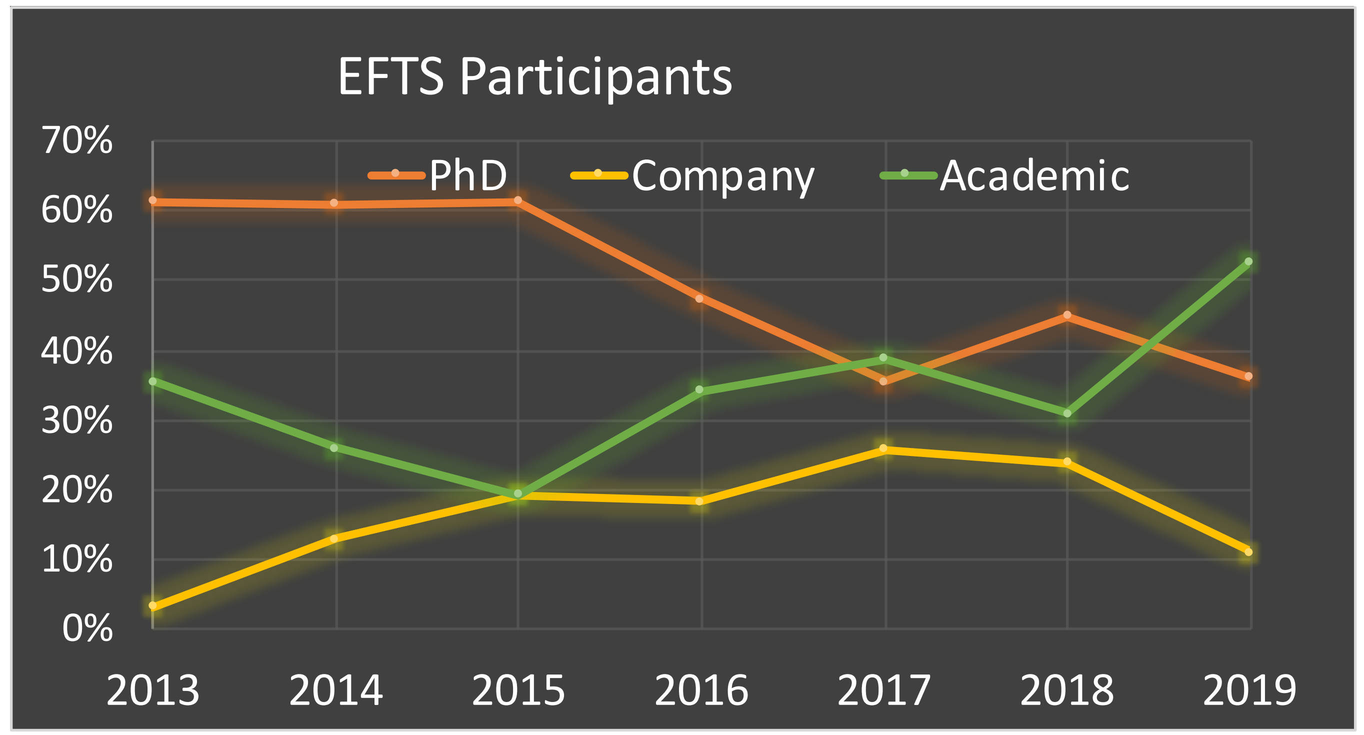
- Pound frequency lock to the cavity
- The same cavity is used in the VCO

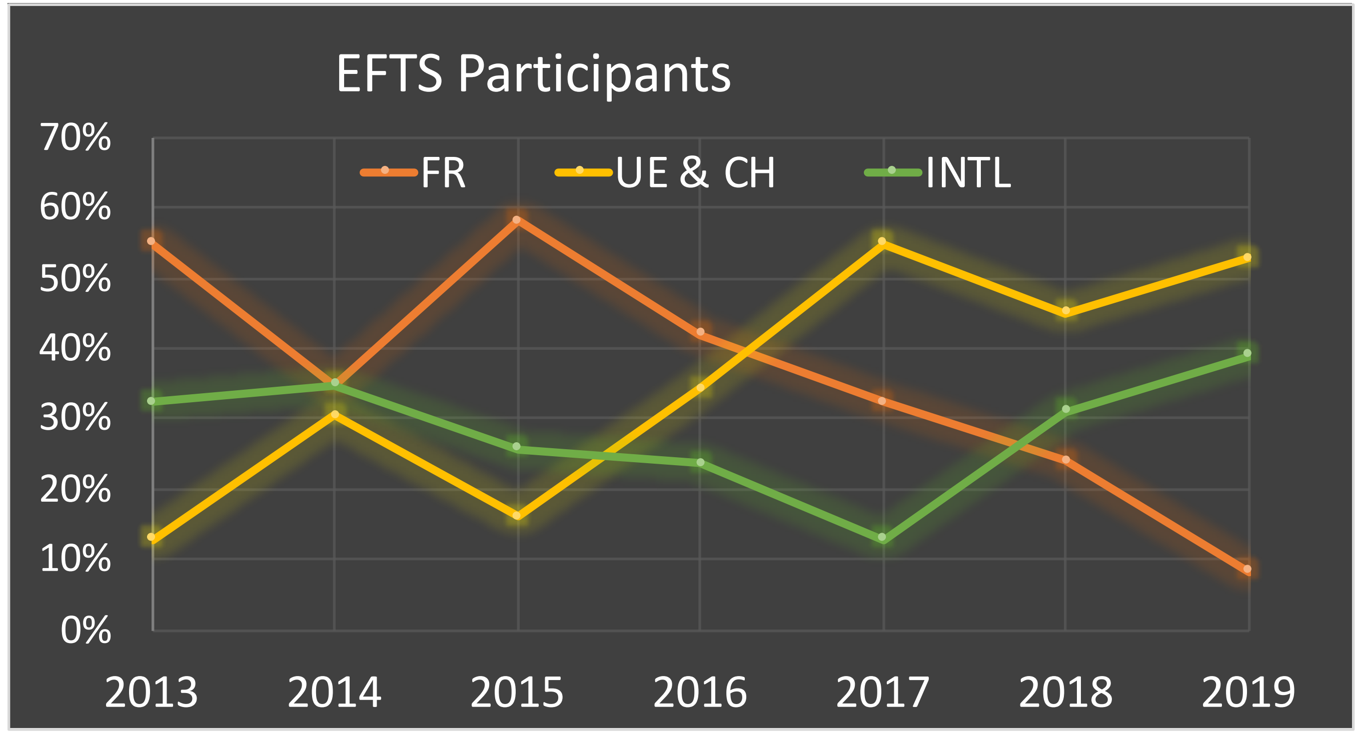


European Frequency and Time Seminar

- Crash course on T&F for newcomers
- Oscillators, measurement, atomic standards, time scales, and general topics
- Broad target audience: PhD/PostDoc Students, Academics, Private Company Engineers
- Balance between academic and applied issues
- Instructors from leading European institution
- Plenary **lectures 23 H**, **labs 12 H** in small groups
- Capped no of participants, set by the labs







Every year in Besançon, end June / beginning July

http://efts.eu