The Magic of Correlation Measurements

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home page http://rubiola.org
Correlation measurements

Two separate instruments measure the same DUT. Only the DUT noise is common.

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\[ S_{\phi}(f) = \frac{1}{\sqrt{m}} \]

![Diagram showing correlation measurements](image)
Statistics

Boring but necessary exercises
A random process $x(t)$ is defined through a random experiment $e$ that associates a function $x_e(t)$ with each outcome $e$.  
- The set of all the possible $x_e(t)$ is called ensemble.
- The function $x_e(t)$ is called realization or sample function.
- The ensemble average is called mathematical expectation $E\{ \}$.

A random process is said stationary if its statistical properties are independent of time.
- Often we restrict the attention to some statistical properties.
- In physics, this is the concept of repeatability.

A random process $x(t)$ said ergodic if a realization observed in time has the statistical properties of the ensemble.
- Ergodicity makes sense only for stationary processes.
- Often we restrict the attention to some statistical properties.
- In physics, this is the concept of reproducibility.

Example: thermal noise of a resistor of value $R$
- The experiment $e$ is the random choice of a resistor $e$
- The realization $x_e(t)$ is the noise waveform measured across the resistor $e$
- We always measure $<x^2>=4kTRB$, so the process is stationary.
- After measuring many resistors, we conclude that $<x^2>=4kTRB$ always holds. The process is ergodic.
A relevant property of random noise

A theorem states that

there is no a-priori relation
between PDF\(^1\) and spectral measure

For example, white noise can originate from

• Poisson process (emission of a particle at random time)
• Random telegraph (random switch between two level)
• Thermal noise (Gaussian)

(1) PDF = Probability Density Function
Why Gaussian White Noise?

- Whenever randomness occurs at microscopic level, noise tends to be Gaussian (central-limit theorem)
- Most environmental effects are not “noise” in strict sense (often, they are more *disturbing* than noise)
- Colored noise types \((1/f, 1/f^2, \text{etc})\) can be whitened, analyzed, and un-whitened
- Of course, GW noise is easy to understand
Properties of Gaussian White noise with zero mean

\[ x(t) \leftrightarrow X(\text{if}) = X'(\text{if}) + iX''(\text{if}) \]

1. \( x(t) \leftrightarrow X(\text{if}) \) are Gaussian
2. \( X(\text{if}_1) \) and \( X(\text{if}_2) \), \( f_1 \neq f_2 
   1. are statistically independent,
   2. \( \text{var}\{X(\text{if}_1)\} = \text{var}\{X(\text{if}_2)\} \)
3. real and imaginary part:
   1. \( X' \) and \( X'' \) are statistically independent
   2. \( \text{var}\{X'\} = \text{var}\{X''\} = \text{var}\{X\}/2 \)
4. \( Y = X_1 + X_2 
   1. \( Y \) is Gaussian
   2. \( \text{var}\{Y\} = \text{var}\{X_1\} + \text{var}\{X_2\} \)
5. \( Y = X_1 \times X_2 
   1. \( Y \) is Gaussian
   2. \( \text{var}\{Y\} = \text{var}\{X_1\} \text{var}\{X_2\} \)
Properties of parametric noise

\[ x(t) \leftrightarrow X(\text{if}) = X'(\text{if}) + iX''(\text{if}) \]

1. **Pair** \( x(t) \leftrightarrow X(\text{if}) 
   
   1. there is no a-priori relation between the distribution of \( x(t) \) and \( X(\text{if}) \) (theorem)
   2. Central limit theorem: \( x(t) \) and \( X(\text{if}) \) end up to be Gaussian

2. \( X(\text{if}_1) \) and \( X(\text{if}_2) 
   
   1. generally, statistically independent
   2. \( \text{var}\{X(\text{if}_1)\} \neq \text{var}\{X(\text{if}_2)\} \) in general

3. Real and imaginary part, same frequency
   
   1. \( X' \) and \( X'' \) can be correlated
   2. \( \text{var}\{X'\} \neq \text{var}\{X''\} \neq \text{var}\{X\}/2 \)

4. \( Y = X_1 + X_2 \), zero-mean independent Gaussian r.v.
   \( \text{var}\{Y\} = \text{var}\{X_1\} + \text{var}\{X_2\} \)

5. If \( X_1 \) and \( X_2 \) are zero-mean independent Gaussian r.v.
   
   1. \( Y = X_1 \times X_2 \) is zero-mean Gaussian
   2. \( \text{var}\{Y\} = \text{var}\{X_1\} \text{var}\{X_2\} \)

The process has \( N \ldots 2N \) degrees of freedom, depending on correlation between \( X' \) and \( X'' \)
Children of the Gaussian distribution

Chi-square
\[ x^2 = \sum_i x_i^2 \]

Bessel K\(_0\)
\[ x = x_1 x_2 \]

Rayleigh
\[ x = \sqrt{(x_1^2 + x_2^2)} \]
Spectral measure\textsuperscript{1} and estimation

(1) Engineers call it Power Spectral Density (PSD)
The Spectral Measure
for stationary random process \( x(t) \)

**Autocovariance**
Improperly referred to as the correlation and denoted with \( R_{xx}(\tau) \)

\[
C(\tau) = \mathbb{E}\{[x(t) - \mu][x(t - \tau) - \mu]^*\}
\]

\[\mu = \mathbb{E}\{x\}\]

\[
S(\omega) = \mathcal{F}\{C(\tau)\} = \int_{-\infty}^{\infty} C(\tau) e^{-i\omega \tau} d\tau
\]

Spectral measure (two-sided)

For ergodic process, interchange ensemble and time average process \( x(t) \rightarrow \) realization \( x(t) \)

\[
C(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} [x(t) - \mu][x(t - \tau) - \mu]^* dt
\]

Wiener Khinchin theorem
for stationary ergodic processes

\[
S(\omega) = \lim_{T \to \infty} \frac{1}{T} X_T(\omega) X_T^*(\omega) = \lim_{T \to \infty} \frac{1}{T} |X_T(\omega)|^2
\]

\[
S^I(f) = 2S^{II}(\omega/2\pi), \quad f > 0
\]

In experiments we use the single-sided PSD

autocorrelation function

\[
R_{xx}(\tau) = \frac{1}{\sigma^2} \mathbb{E}\{[x(t) - \mu][x(t - \tau) - \mu]\}
\]

Fourier transform

\[
\mathcal{F}\{\xi\} = \int_{-\infty}^{\infty} \xi(t) e^{-i\omega t} dt
\]
Sum of random variables

1. The sum of Gaussian distributed random variables has Gaussian PDF

2. The central limit theorem states that
   For large m, the PDF of the sum of m statistically independent processes tends to a Gaussian distribution
   Let $X = X_1 + X_2 + \ldots + X_m$ be the sum of m processes of mean $\mu_1, \mu_2, \ldots, \mu_m$ and variance $\sigma_1^2, \sigma_2^2, \ldots, \sigma_m^2$. The process X has Gaussian PDF
   expectation $E\{X\} = \mu_1 + \mu_2 + \ldots + \mu_m$, and variance $\sigma^2 = \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_m^2$

3. Similarly, the average $<X>_m = (X_1 + X_2 + \ldots + X_m)/m$ has
   Gaussian PDF, $E\{X\} = (\mu_1 + \mu_2 + \ldots + \mu_m)/m$, and $\sigma^2 = (\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_m^2)/m$

4. Since white noise and flicker noise arise from the sum of a large number of small-scale phenomena, they are Gaussian distributed

PDF = Probability Density Function
Product of independent zero-mean Gaussian-distributed random variables

\[ x_1 \text{ and } x_2 \text{ are normal distributed with zero mean and variance } \sigma_1^2, \sigma_2^2 \]

\[ x = x_1 x_2 \]

\( x \) has Bessel \( K_0 \) distribution with variance \( \sigma = \sigma_1^2 \sigma_2^2 \)

\[ f(x) = \frac{1}{\pi \sigma} K_0 \left( -\frac{|x|}{\sigma} \right) \]

\[ \mathbb{E}\{f(x)\} = 0 \]

\[ \mathbb{E}\{|f(x) - \mathbb{E}\{f(x)\}|^2\} = \sigma^2 \]

Thanks to the central limit theorem, the average \( <X>_m = (X_1 + X_2 + \ldots + X_m)/m \) of \( m \) products has

- Gaussian PDF,
- average \( \mathbb{E}\{X\} = 0 \)
- variance \( \mathbb{V}\{X\} = \sigma^2 \)
Spectral Measure $S_{xx}(f)$
(Power Spectral Density)

X is white Gaussian noise
Take one frequency, $S(f) \rightarrow S$. Same applies to all frequencies

\[
\langle S_{xx} \rangle_m = \frac{1}{T} \langle XX^* \rangle_m
\]
\[
= \frac{1}{T} \langle (X' + iX'') \times (X' - iX'') \rangle_m
\]
\[
= \frac{1}{T} \langle (X')^2 + (X'')^2 \rangle_m
\]

white, Gaussian, avg = 0, var = 1/2

white, $\chi^2$, with 2m degrees of freedom
avg = 1, var = 1/m

\[
\text{dev} \quad \text{avg} = \sqrt{\frac{1}{m}}
\]

the $S_{xx}$ track on the FFT-SA shrinks as $1/m^{1/2}$

Normalization: in 1 Hz bandwidth
$\var{X} = 1$, and $\var{X'} = \var{X''} = 1/2$
Estimation of $|S_{xx}(f)|$

Running the measurement, $m$ increases and $S_{xx}$ shrinks => better confidence level.
Cross Spectrum Theory

Getting close to the real game

$$x = a + c$$

$$y = b + c$$

Diagram:

- DUT input signal $c(t)$
- Instrument A
  - Input signal $a(t)$ with instr. noise
  - Output $x = a + c$
- Instrument B
  - Input signal $b(t)$ with instr. noise
  - Output $y = b + c$
- Dual-channel FFT analyzer
**S_{yx} with correlated term (1)**

A, B = instrument background  
C = DUT noise  

channel 1  \( X = A + C \)  
channel 2  \( Y = B + C \)  

A, B, C are independent Gaussian noises  
\( \text{Re}\{\} \) and \( \text{Im}\{\} \) are independent Gaussian noises  

**Normalization:** in 1 Hz bandwidth \( \text{var}\{A\} = \text{var}\{B\} = 1, \) \( \text{var}\{C\} = \kappa^2 \)  
\( \text{var}\{A'\} = \text{var}\{A''\} = \text{var}\{B'\} = \text{var}\{B''\} = 1/2, \) \( \) and \( \text{var}\{C'\} = \text{var}\{C''\} = \kappa^2/2 \)

**Cross-spectrum**  
\[
\langle S_{yx} \rangle_m = \frac{1}{T} \langle Y X^* \rangle_m = \frac{1}{T} \langle (Y' + iY'') \times (X' - iX'') \rangle_m
\]

**Expand using**  
\[
X = (A' + iA'') + (C' + iC'') \quad \text{and} \quad Y = (B' + iB'') + (C' + iC'')
\]

**Split \( S_{yx} \) into three sets**  
\[
\langle S_{yx} \rangle_m = \langle S_{yx} \rangle_m \big|_{\text{instr}} + \langle S_{yx} \rangle_m \big|_{\text{mixed}} + \langle S_{yx} \rangle_m \big|_{\text{DUT}}
\]

background only  
background and DUT noise  
DUT noise only  

... and work it out !!!
**S_{yx} with correlated term \( \kappa \neq 0 \) (2)**

All the DUT signal goes in \( \text{Re}\{S_{yx}\} \), \( \text{Im}\{S_{yx}\} \) contains only noise

**Real**

\[
\text{Re}\left\{ \langle S_{yx} \rangle_m \right\} = \frac{1}{T} \left\{ \langle B' A' \rangle_m + \langle B'' A'' \rangle_m + \langle B' C' \rangle_m + \langle B'' C'' \rangle_m + \langle A' C' \rangle_m + \langle A'' C'' \rangle_m + \left( \langle C' \rangle^2 + \langle C'' \rangle^2 \right)_m \right\}
\]

Gaussian, \( \text{avg} = 0 \), \( \text{var} = \frac{1}{2}m \)

Bessel \( K_0 \), \( \text{avg} = 0 \), \( \text{var} = \frac{1}{4} \)

\[ \text{Set } A \]

Gaussian, \( \text{avg} = 0 \), \( \text{var} = \frac{1}{2} \) \( \kappa^2 \) \( 2m \) deg. of freedom

\[ \text{avg} = \kappa^2 \), \( \text{var} = \kappa^4 \) \( m \)

\[ \text{Set } C \]

**Imaginary**

\[
\text{Im}\left\{ \langle S_{yx} \rangle_m \right\} = \frac{1}{T} \left\{ \langle B'' A' \rangle_m + \langle B'' A'' \rangle_m + \langle B'' C' \rangle_m + \langle B'' C'' \rangle_m + \langle A' C'' \rangle_m + \langle A'' C' \rangle_m \right\}
\]

Gaussian, \( \text{avg} = 0 \), \( \text{var} = \frac{1}{4} \)

Bessel \( K_0 \), \( \text{avg} = 0 \), \( \text{var} = \frac{1}{2} \) \( \kappa^2 \)

\[ \text{Set } B \]

Gaussian, \( \text{avg} = 0 \), \( \text{var} = \frac{1}{2} \) \( \kappa^2 \) \( 2m \) deg. of freedom

\[ \text{avg} = \frac{1}{2}(1+2\kappa^2) \] \( 2m \)

**Normalization:** in 1 Hz bandwidth \( \text{var}\{A\} = \text{var}\{B\} = 1 \), \( \text{var}\{C\} = \kappa^2 \)
\( \text{var}\{A'\} = \text{var}\{A''\} = \text{var}\{B'\} = \text{var}\{B''\} = 1/2 \), and \( \text{var}\{C'\} = \text{var}\{C''\} = \kappa^2/2 \)

A, B, C are independent Gaussian noises

Re\{ \} and Im\{ \} are independent Gaussian noises

Note: DF < 2m
See vol.XVI p.56
Estimator \( \hat{S} = |\langle S_{yx} \rangle_m| \)

The instrument default

\[
|\langle S_{yx} \rangle_m| = \frac{1}{T} \sqrt{[\Re \{ \langle YX^* \rangle_m \}]^2 + [\Im \{ \langle YX^* \rangle_m \}]^2}
= \frac{1}{T} \sqrt{[\langle \mathcal{A} \rangle_m + \langle \mathcal{C} \rangle_m]^2 + [\langle \mathcal{B} \rangle_m]^2}.
\]

\( \kappa \to 0 \) Rayleigh distribution

\[
\langle \mathcal{E} \rangle_m = \sqrt{[\langle \mathcal{A} \rangle_m]^2 + [\langle \mathcal{B} \rangle_m]^2}.
\]

\[
\mathbb{E}\{\langle \mathcal{E} \rangle_m\} = \sqrt{\frac{\pi}{4m}} = \frac{0.886}{\sqrt{m}}
\]

\[
\mathbb{V}\{\langle \mathcal{E} \rangle_m\} = \frac{1}{m} \left( 1 - \frac{\pi}{4} \right) = \frac{0.215}{m}
\]

\[
\text{dev}\{|\langle S_{yx} \rangle_m|\} = \sqrt{\frac{4}{\pi} - 1} = 0.523
\]

Normalization: in 1 Hz bandwidth \( \text{var}(A) = \text{var}(B) = 1, \text{var}(C) = \kappa^2 \)

\( \text{var}(A') = \text{var}(A'') = \text{var}(B') = \text{var}(B'') = 1/2, \) and \( \text{var}(C') = \text{var}(C'') = \kappa^2/2 \)
Estimator $\hat{S} = \text{Re}\{<S_{yx}>_m\}$

**Best (unbiased) estimator**

\[
\begin{align*}
\langle \mathcal{Y} \rangle_m &= \langle \mathcal{A} \rangle_m + \langle \mathcal{C} \rangle_m \\
\mathbb{E} \{\langle \mathcal{Y} \rangle_m \} &= \kappa^2 \\
\mathbb{V} \{\langle \mathcal{Y} \rangle_m \} &= \frac{1 + 2\kappa^2 + 2\kappa^4}{2m} \\
\text{dev} \{\langle \mathcal{Y} \rangle_m \} &= \sqrt{\frac{1 + 2\kappa^2 + 2\kappa^4}{2m}} \approx \frac{1 + \kappa^2}{\sqrt{2m}} \\
\frac{\text{dev} \{\langle \mathcal{Y} \rangle_m \}}{\mathbb{E} \{\langle \mathcal{Y} \rangle_m \}} &= \sqrt{\frac{1 + 2\kappa^2 + 2\kappa^4}{\kappa^2 \sqrt{2m}}} \approx \frac{1 + \kappa^2}{\kappa^2 \sqrt{2m}}
\end{align*}
\]

0 dB SNR requires that $m=1/2\kappa^4$.

Example $\kappa=0.1$ (DUT noise 20 dB lower than single-channel background) averaging on $5\times10^3$ spectra is necessary to get SNR = 0 dB.

**Normalization:** in 1 Hz bandwidth $\text{var}(A) = \text{var}(B) = 1$, $\text{var}(C)=\kappa^2$

$\text{var}(A') = \text{var}(A'') = \text{var}(B') = \text{var}(B'') = 1/2$, and $\text{var}(C') = \text{var}(C'') = \kappa^2/2$
Ergodicity allows to interchange time statistics and ensemble statistics, thus the running index $i$ of the sequence and the frequency $f$. The average and the deviation calculated on the frequency axis are the same as the average and the deviation of the sequence of spectra.
Example:
Measurement of |Syx|
Measurement (C≠0), |Syx|

Running the measurement, m increases
Sxx shrinks => better confidence level
Syx decreases => higher single-channel noise rejection
Applications

The real fun starts here
Applications

- Radio-astronomy (Hanbury-Brown, 1952)
- Early implementations
- Radiometry (Allred, 1962)
- Noise calibration (Spietz, 2003)
- Frequency noise (Vessot 1964)
- Phase noise (Walls 1976)
- Dual delay line system (Lance, 1982)
- Phase noise (Rubiola 2000 & 2002)
- Effect of amplitude noise (Rubiola, 2007)
- Frequency stability of a resonator (Rubiola)
- Dual-mixer time-domain instrument (Allan 1975, Stein 1983)
- Amplitude noise & laser RIN (Rubiola 2006)
- Noise of a power detector (Grop & Rubiola, in progress)
- Noise in chemical batteries (Walls 195)
- Semiconductors (Sampietro RSI 1999)
- Electromigration in thin films (Stoll 1989)
- Fundamental definition of temperature
Radio-astronomy

Measurement of the apparent angular size of stellar radio sources

Jodrell Bank, Manchester

- The radio link breaks the hypothesis of symmetry of the two channels, introducing a phase $\theta$
- The cross spectrum is complex
- The antenna directivity results from the phase relationships
- The phase of the cross spectrum indicates the direction of the radio source

Cassiopeia A (or Cygnus A) radio source

500 m²

$500 \text{ m}^2$

$2\text{ km}$

$X(\text{if})$

$Y(\text{if}) e^{i\theta}$

$X(\text{if})$

$R_1$

Square law detector

Band-pass filter

Correlator

Radio Link

$X(\text{if})$

$Y(\text{if}) e^{i\theta}$

$X(\text{if})$

$R_2$

Square law detector

Band-pass filter

$R. \text{ Hanbury Brown} \ & \text{al., Nature 170(4338) p.1061-1063, 20 Dec 1952}$

R. Hanbury Brown, R. Q. Twiss, Phyl. Mag. ser.7 no.366 p.663-682
Thermal noise compensation

Correlation-and-averaging rejects the thermal noise

hybrid output

\[ y_1(t) = \frac{1}{\sqrt{2}} x_2(t) + \frac{1}{\sqrt{2}} x_1(t) \]
\[ y_2(t) = \frac{1}{\sqrt{2}} x_2(t) - \frac{1}{\sqrt{2}} x_1(t) \]

correlation

\[ \mathcal{R}_{y_1y_2}(\tau) = \lim_{\theta \to \infty} \frac{1}{\theta} \int_0^\theta y_1(t) y_2^*(t - \tau) \, dt \]
\[ = \frac{1}{2} \mathcal{R}_{x_2x_2}(\tau) - \frac{1}{2} \mathcal{R}_{x_1x_1}(\tau) \]

Fourier transform and thermal noise

\[ S_{y_1y_2}(f) = \frac{1}{2} S_{x_2}(f) - \frac{1}{2} S_{x_1}(f) \]
\[ S_{y_1y_2}(f) = \frac{k_B(T_2 - T_1)}{2} \]
Radiometry & Johnson thermometry

correlation and anti-correlation

\[ X = A + B \]

\[ Y = A - B \]

\[ S_{yx} = k \left( T_2 - T_1 \right) / 2 \]

noise comparator

Re-definition of the Kelvin?

- Thermal noise: $S = kT$
- Shot noise: $S = 2qI_{\text{avg}}R$
- High accuracy of $I_{\text{avg}}$ with a dc instrument

Poisson process

- Property of the Poisson process: $\mu = \sigma^2$

Thermal noise

- Boltzmann constant $N = kT$

Josephson effect

- $V_{\text{DC}} = \frac{h\nu}{2e}$

Planck constant

- Electron charge
- Second (Cesium)
Noise calibration

thermal noise \[ S = kT \]
shot noise \[ S = 2qI_{\text{avg}}R \]

high accuracy of \( I_{\text{avg}} \) with a dc instrument

Compare shot and thermal noise with a noise bridge

This idea could turn into a re-definition of the temperature

Fig. 1. Theoretical plot of current spectral density of a tunnel junction (Eq. 3) as a function of dc bias voltage. The diagonal dashed lines indicate the shot noise limit, and the horizontal dashed line indicates the Johnson noise limit. The voltage span of the intersection of these limits is \( 4k_B T/e \) and is indicated by vertical dashed lines. The bottom inset depicts the occupancies of the states in the electrodes in the equilibrium case, and the top inset depicts the out-of-equilibrium case where \( eV \gg k_B T \).

In a tunnel junction, theory predicts the amount of shot and thermal noise

Early implementations

1940-1950 technology

Spectral analysis at the single frequency $f_0$, in the bandwidth $B$

Need a filter pair for each Fourier frequency

Rice representation of noise
Measurement of the frequency noise of a H-maser

H maser

common synthesizer

H maser

correlator
Phase noise measurement

(relatively) large correlation bandwidth provides low noise floor in a reasonable time

Phase noise measurement

Effect of amplitude noise

Should set both channels at the sweet point, if exists

The delay de-correlates the two inputs, so there is no sweet point

Should set both channels at the sweet point of the RF input, if exists, by offsetting the PLL or by biasing the IF

The effect of the AM noise is strongly reduced by the RF amplification

pink: noise rejected by correlation and averaging

E. Rubiola, R. Boudot, IEEE Transact. UFFC 54(5) pp.926-932, may 2007
Dual-delay-line method

The delay line converts the frequency noise into phase noise.

The high loss of the coaxial cable limits the maximum delay.

Updated version:
The optical fiber provides long delay with low attenuation (0.2 dB/km or 0.04 dB/μs)

Original idea:
D. Halford’s NBS notebook
F10 p.19-38, apr 1975

First published: A. L. Lance & al, CPEM Digest, 1978
Optical version of the dual-delay-line method

Two completely separate systems measure the same oscillator under test

The only common part of the setup is the power splitter.

Frequency stability of a resonator

• Bridge in equilibrium
  • The amplifier cannot flicker around $\omega_0$, which it does not know
• The fluctuation of the resonator natural frequency is estimated from phase noise
• Q matching prevents the master-oscillator noise from being taken in
• Correlation removes the noise of the instruments and the reference resonators

Enrico’s weird brain
– however, with the cryogenic sapphire oscillators we can do way better –

Now obsolete, 3E–16 stability from cryogenic oscillator
Amplitude noise & laser RIN

- In PM noise measurements, one can validate the instrument by feeding the same signal into the phase detector
- In AM noise this is \textit{not possible} without a lower-noise reference
- Provided the crosstalk was measured otherwise, correlation enables to validate the instrument

Measurement of the detector noise

Basic ideas

- Remove the noise of the source by balancing C–A and C–B
  - Use a lock-in amplifier to get a sharp null measurement
- Channels A and B are independent → noise is averaged out
- Two separate JFET amplifiers are needed in the C channel
  - JFETs have virtually no bias-current noise
- Only the noise of the detector C remains

In all previous experiments, the amplifier noise was higher than the detector noise
Noise in chemical batteries

- Do not waste DAC bits for a constant DC, \( V = V_{B2} - V_{B1} \) has (almost) zero mean.
- Two separate amplifiers measure the same quantity \( V \).
- Correlation rejects the amplifier noise, and the FFT noise as well.

Noise in semiconductors

FIG. 2. Schematics of the building blocks of our correlation spectrum analyzer performing the suppression of the uncorrelated input noises by a digital processing of sampled data.

FIG. 3. Schematics of the active test fixture for current noise measurements.

FIG. 9. Experimental frequency spectrum of the current noise from DUT resistances of 100 kΩ and 500 MΩ (continuous line) compared with the limits (dashed line) given by the instrument and set by residual correlated noise components.
Electro-migration in thin films

- Random noise: $X'$ and $X''$ (real and imag part) of a signal are statistically independent
- The detection on two orthogonal axes eliminates the amplifier noise.
  This work with a single amplifier!
- The DUT noise is detected

$$S_{ud}(f) = \frac{1}{2} \left[ S_\alpha(f) - S_\varphi(f) \right]$$

A. Seeger, H. Stoll, 1/f noise and defects in thin metal films, proc. ICNF p.162-167, Hong Kong 23-26 aug 1999
Hanbury Brown - Twiss effect

in single-photon regime, anti-correlation shows up
R. Hanbury Brown, R. Q. Twiss, Correlation between photons in two coherent beams of light, Nature 177 (1956) 27-29

Also observed at microwave frequencies
C. Glattli & al. (2004), PRL 93(5) 056801, Jul 2004

\[ kT = 2.7 \times 10^{-25} \text{ J at 20 mK}, \quad hv = 1.12 \times 10^{-24} \text{ J at 1.7 GHz}, \quad kT/hv = -6.1 \text{ dB} \]
Conclusions

• Rejection of the instrument noise

• AM noise, RIN, etc. -> validation of the instrument without a reference low-noise source

• Display quantities
  \( \langle \text{Re}\{\text{Syx}\} \rangle_m \) is faster and more accurate
  \( \langle \text{Im}\{\text{Syx}\} \rangle_m \) gives the background noise
  \( \max\{\langle \text{Syx}\rangle_m, 0_+\} \) provide easier readout

• Applications in many fields of metrology

The cross spectrum method is magic
Correlated noise sometimes makes magic difficult

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