High resolution time & frequency counters

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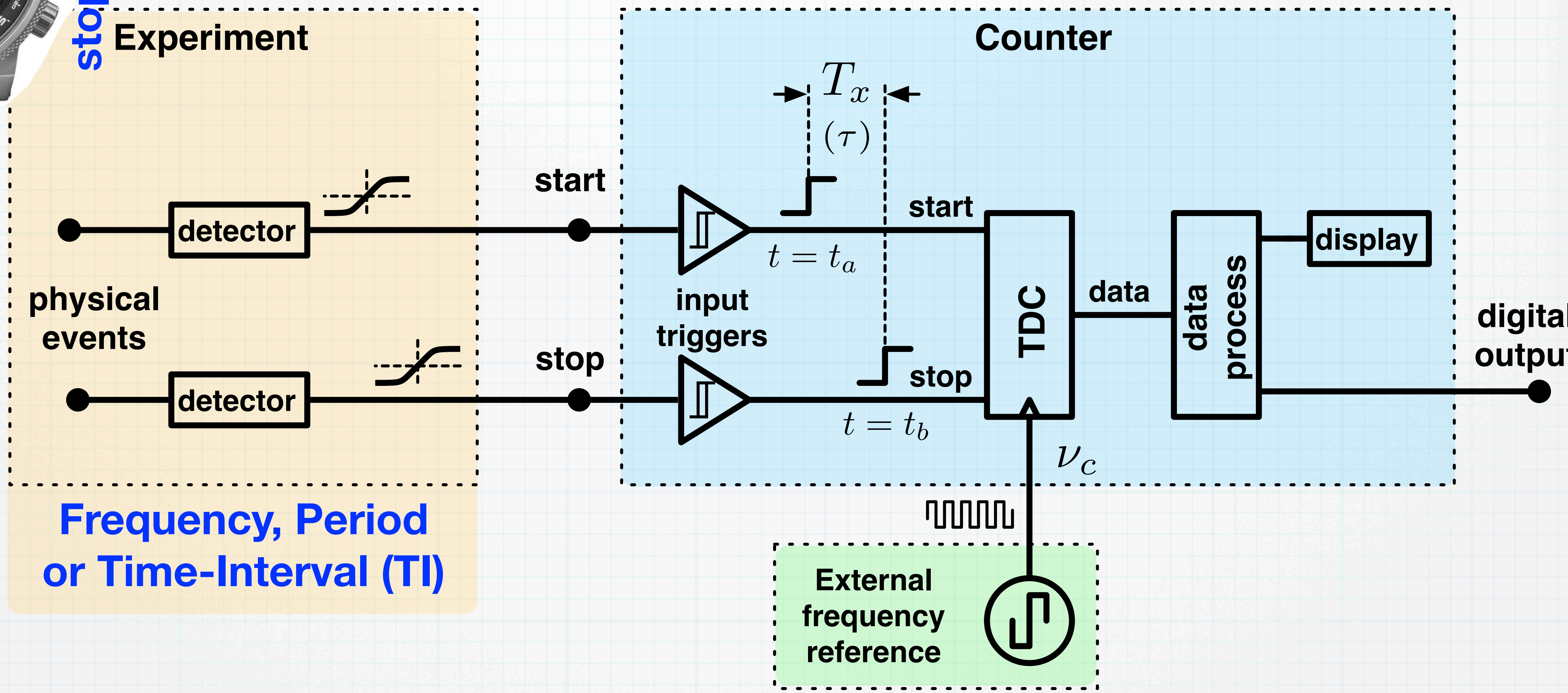
Outline

- Introduction
- Basic counters (RF & microwave)
- The trigger
- Clock interpolation techniques
- Basic statistics
- Advanced statistics

home page http://rubiola.org

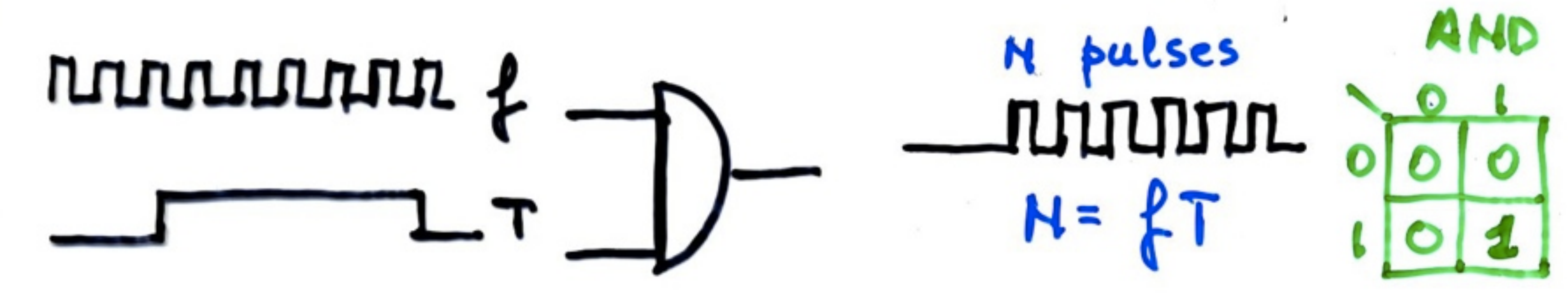
Counter – main purposes



- Compare a physical quantity (frequency, period, time interval) to a frequency reference
- Exploit the full accuracy and precision of the reference, with no degradation

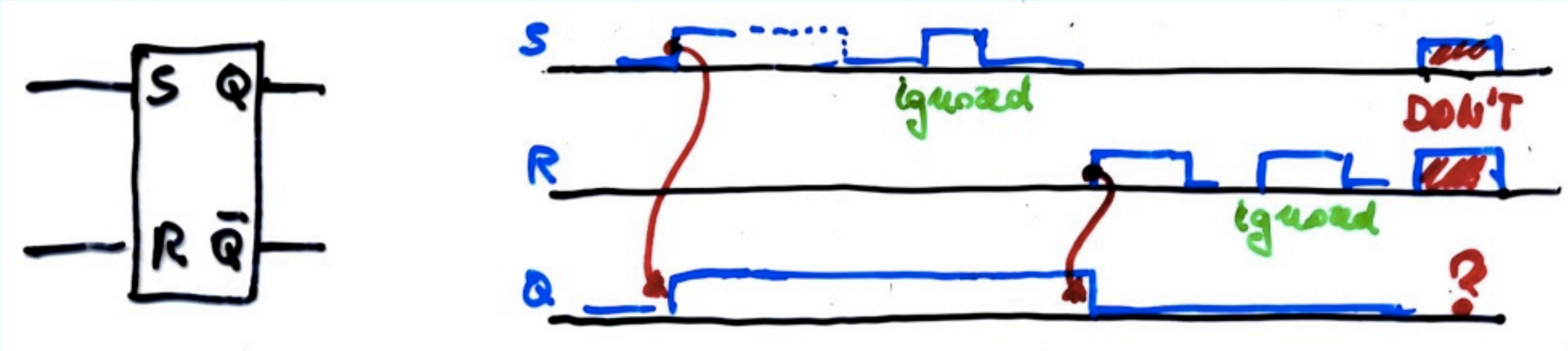
Digital hardware

The AND gate

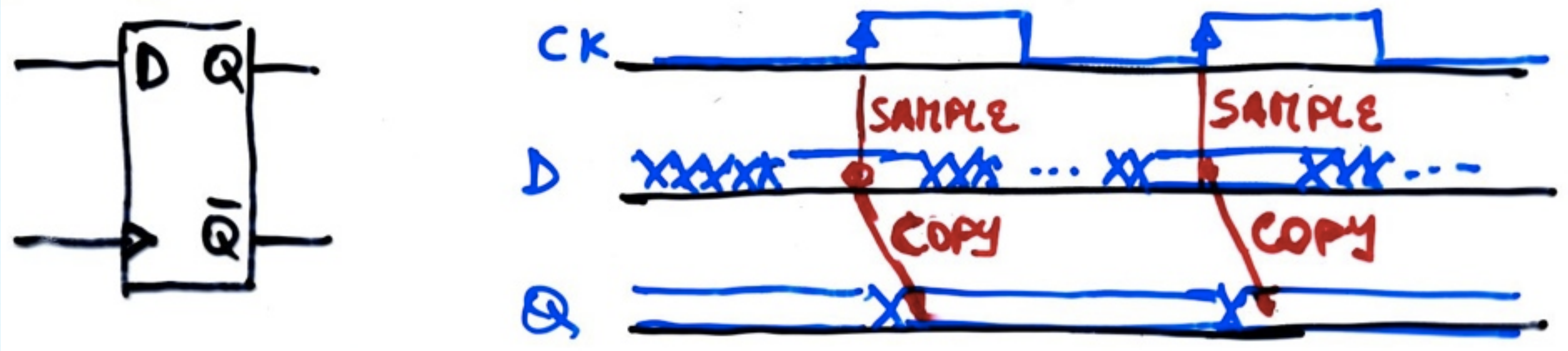


Basic flip-flops

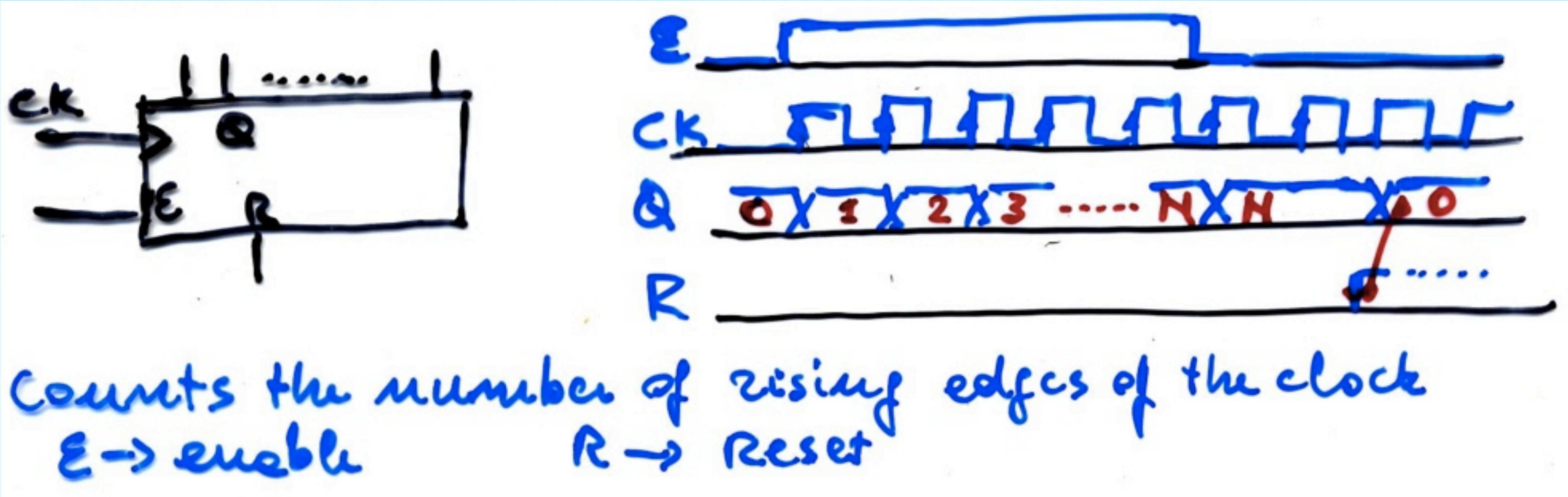
Set-Reset (SR) flip-flop



D-type flip-flop (digital sampler)



Binary counter

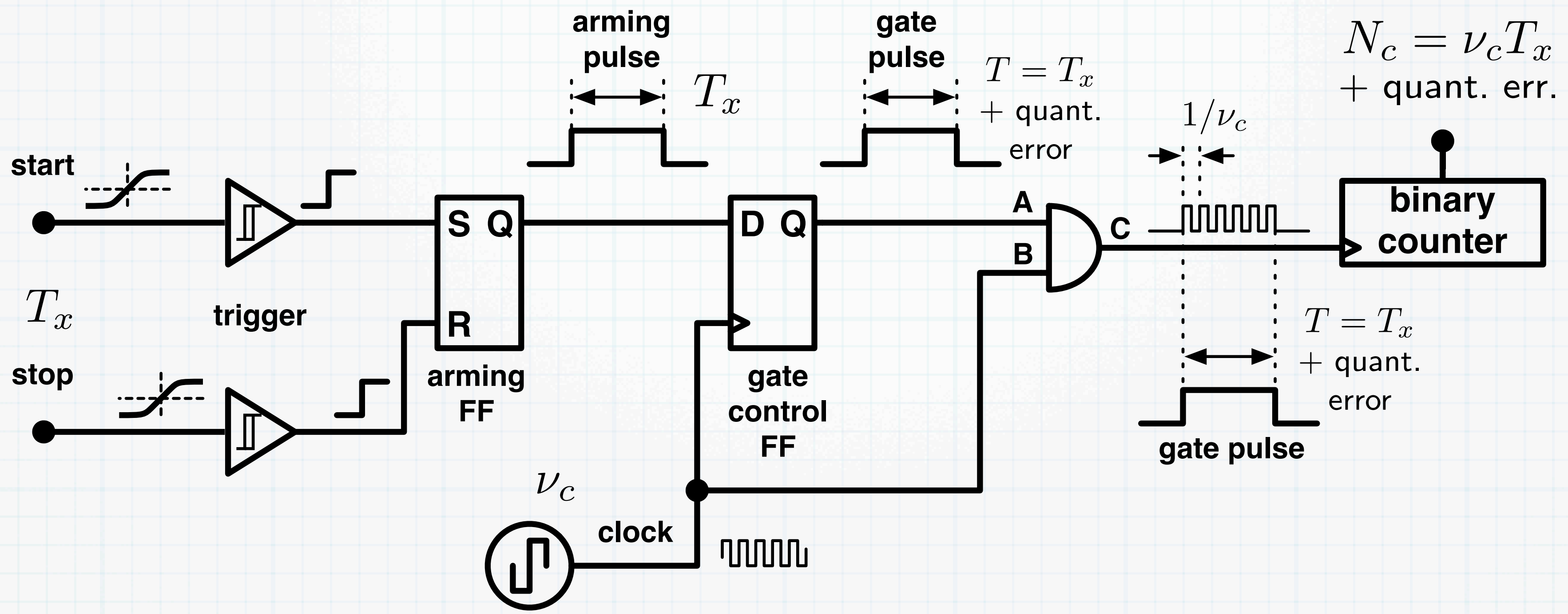


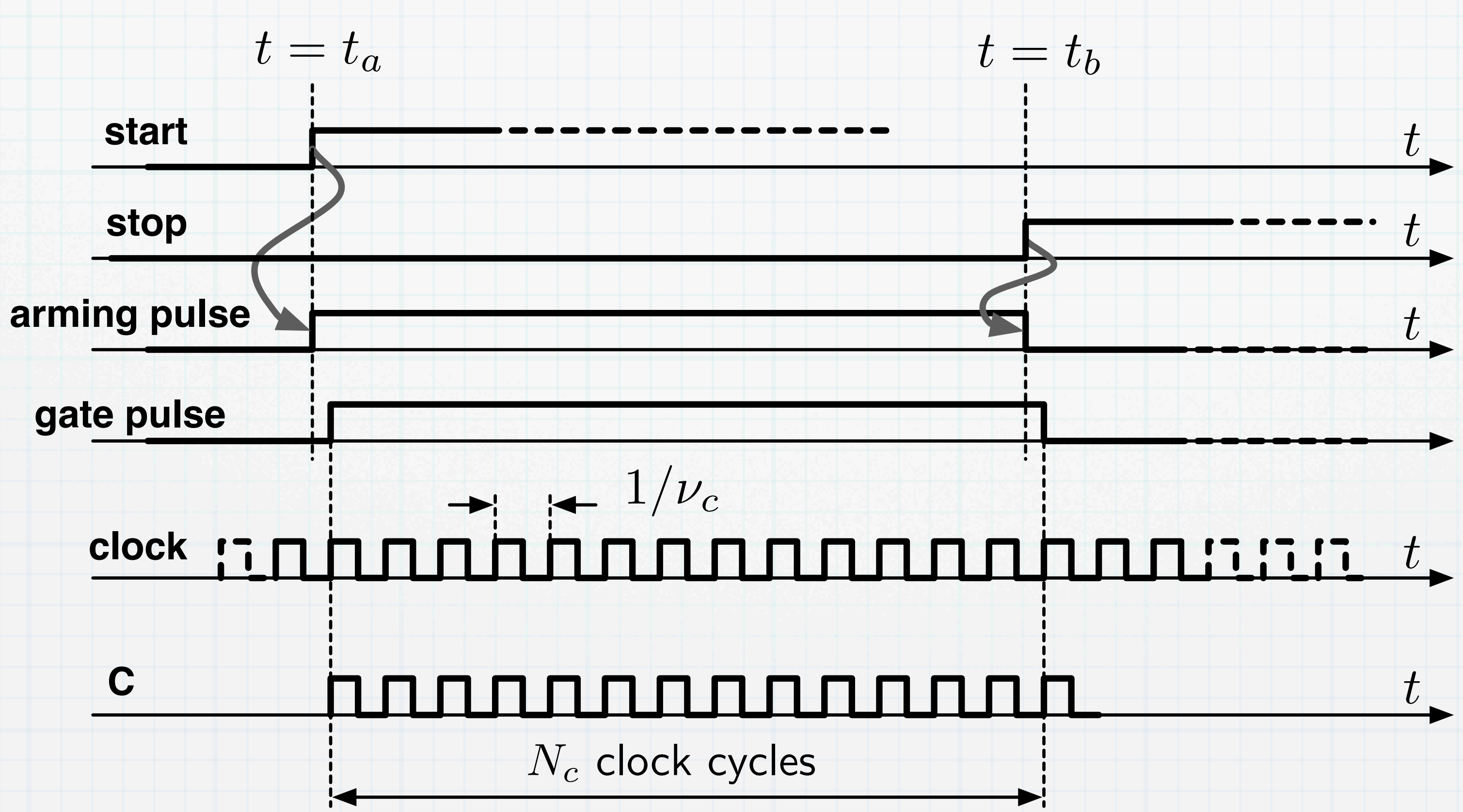
Disambiguation: the word "counter" – is used for both

- the binary / BCD counter – the digital circuit
- the time / frequency counter – the instrument

1 – Basic counters

Time Interval (TI) counter

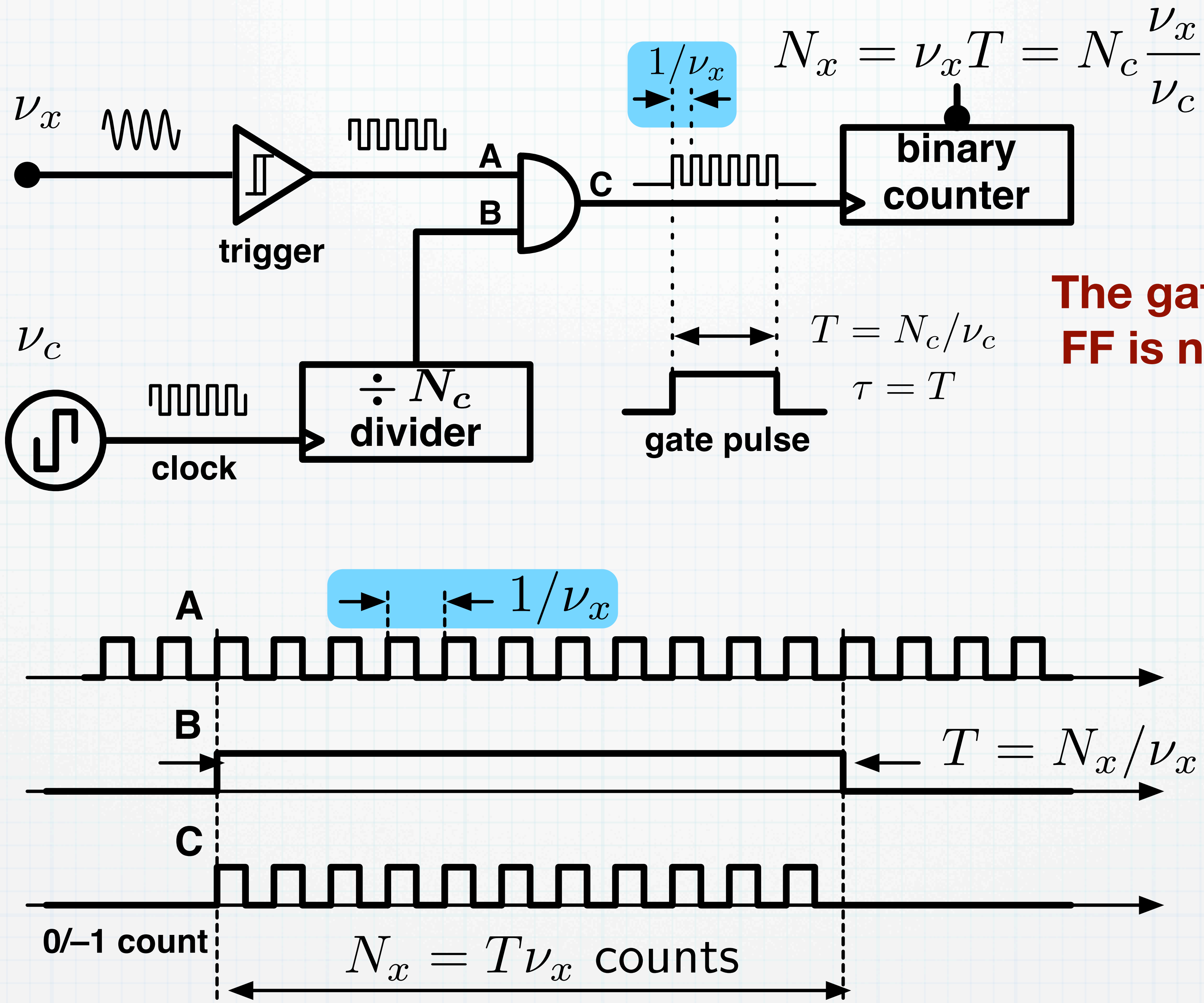




The resolution is set by the clock period $1/\nu_c$

The (old) frequency counter

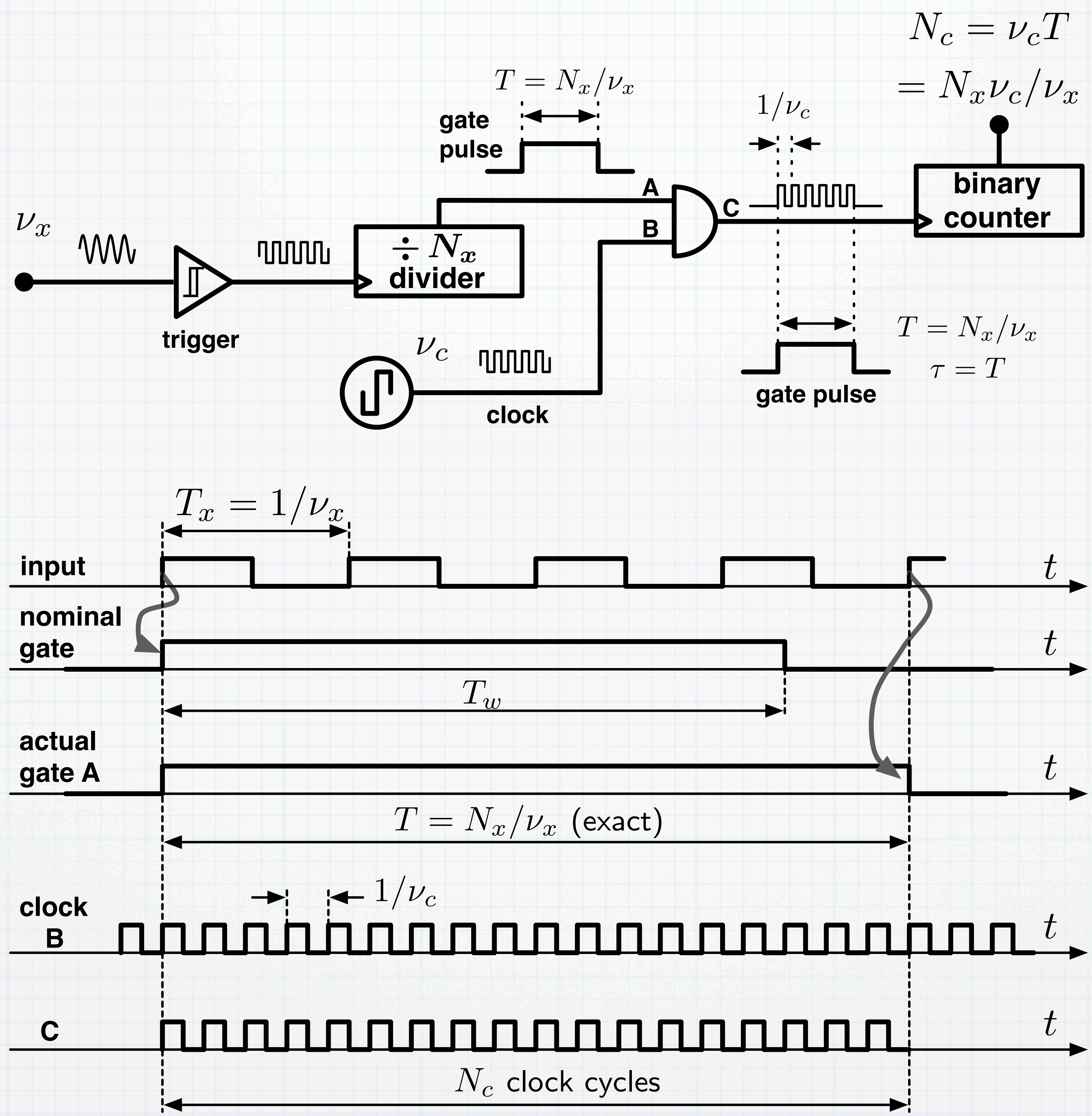
The gate-control FF is not shown



The gate-control FF is not shown

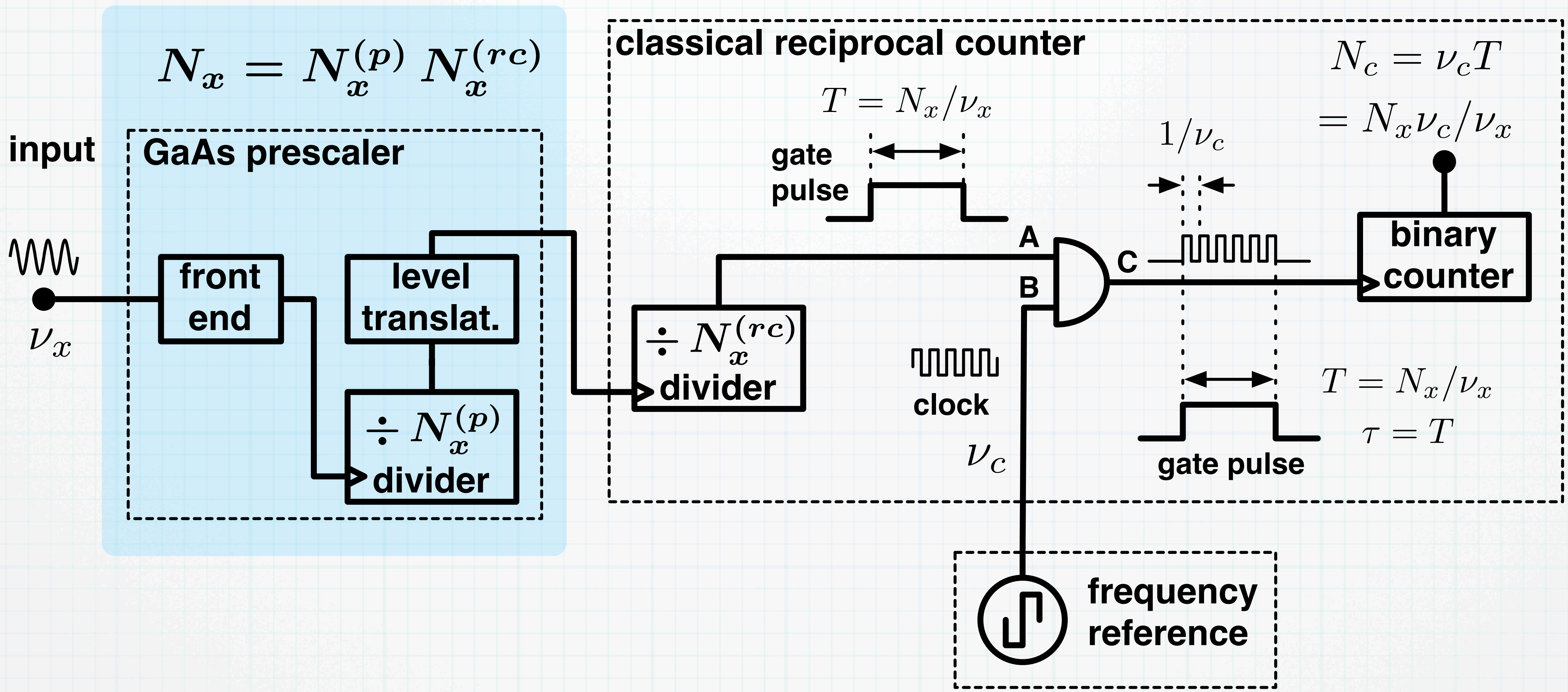
The resolution is set by the input period $1/\nu_x$, which can be poor

Classical reciprocal counter



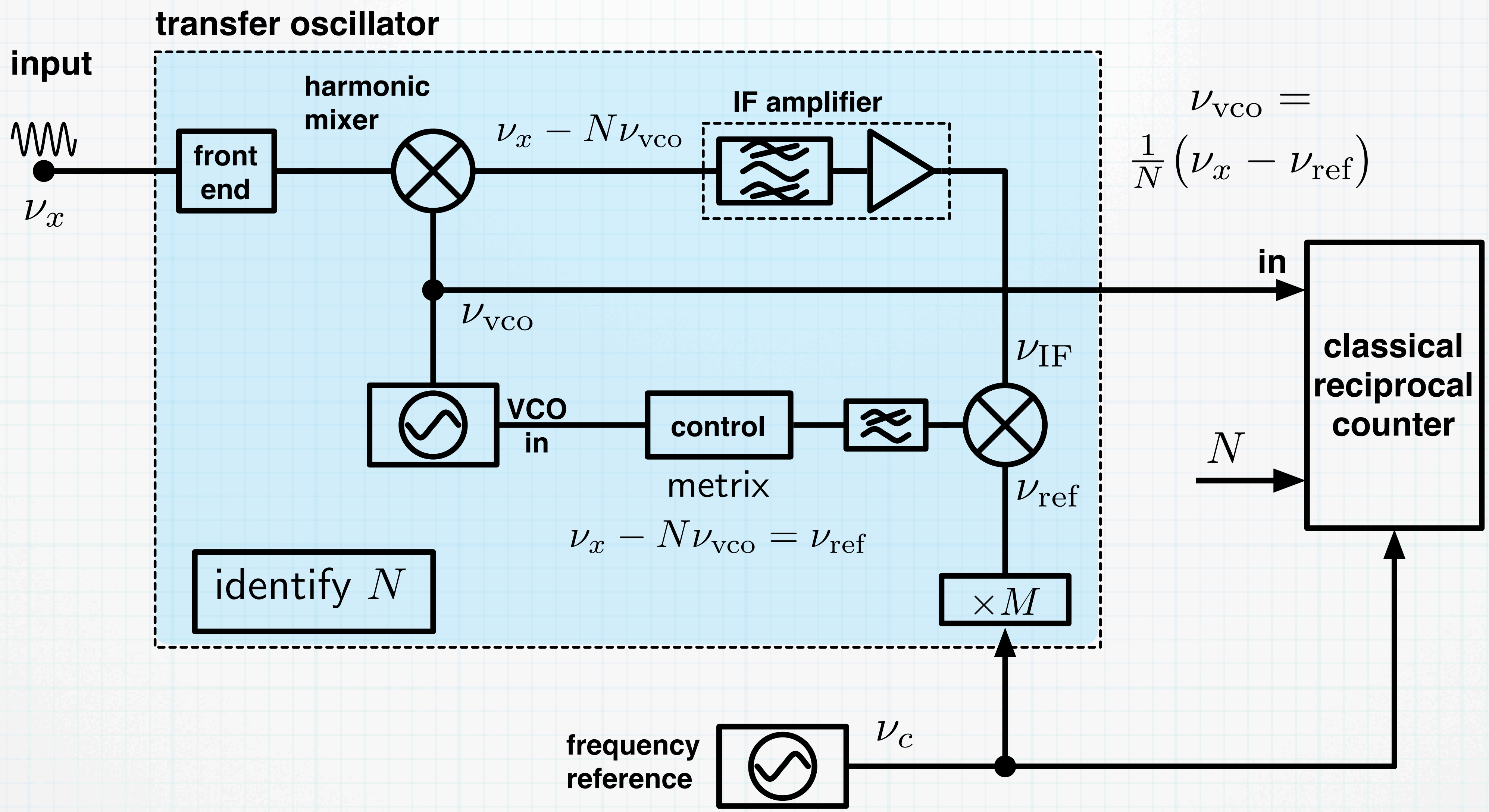
- **Use the highest clock frequency permitted by the hardware**
- **The measurement time is a multiple of the input period**

Prescaler



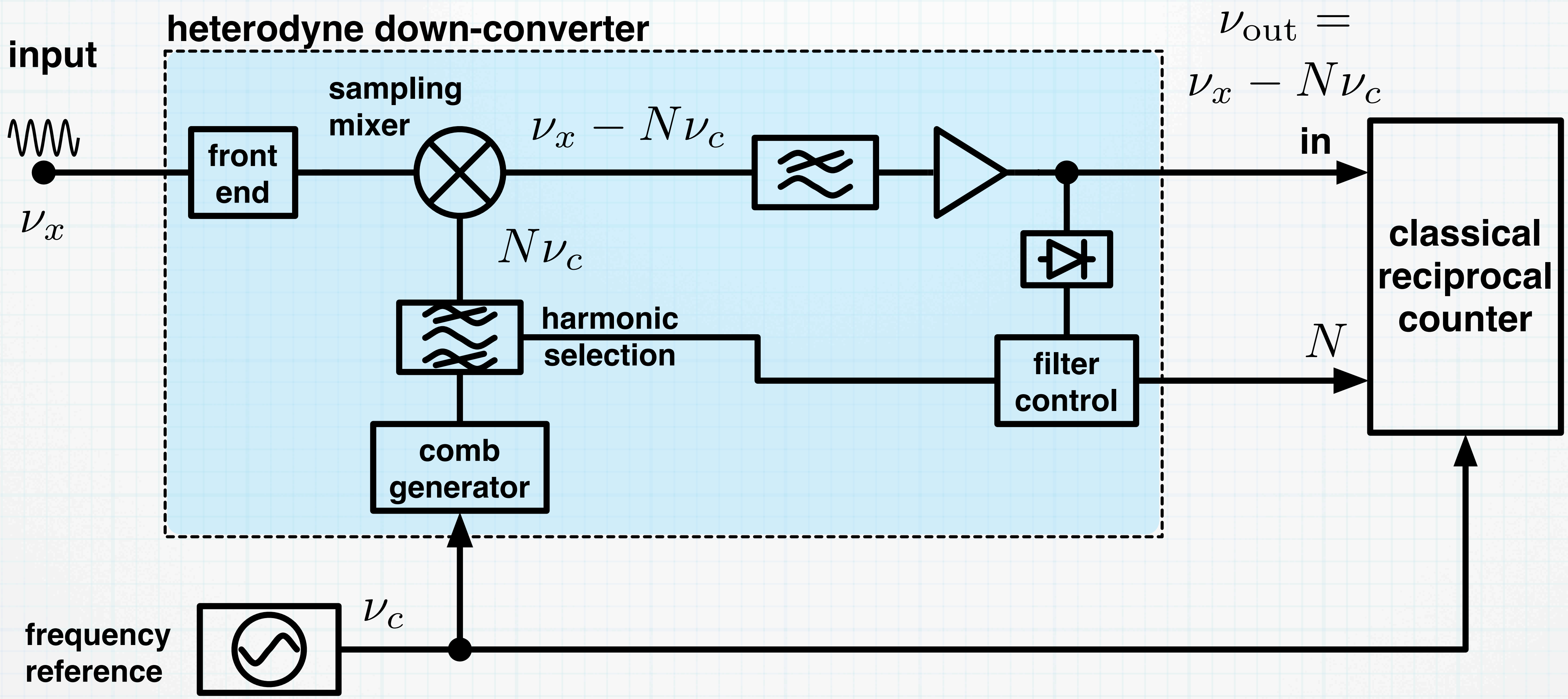
- The prescaler is a n-bit binary divider $\div\ 2^n$
- GaAs dividers work up to at least 20 GHz
- Reciprocal counter => there is no resolution reduction
- Most microwave counters use the prescaler

Transfer-oscillator counter



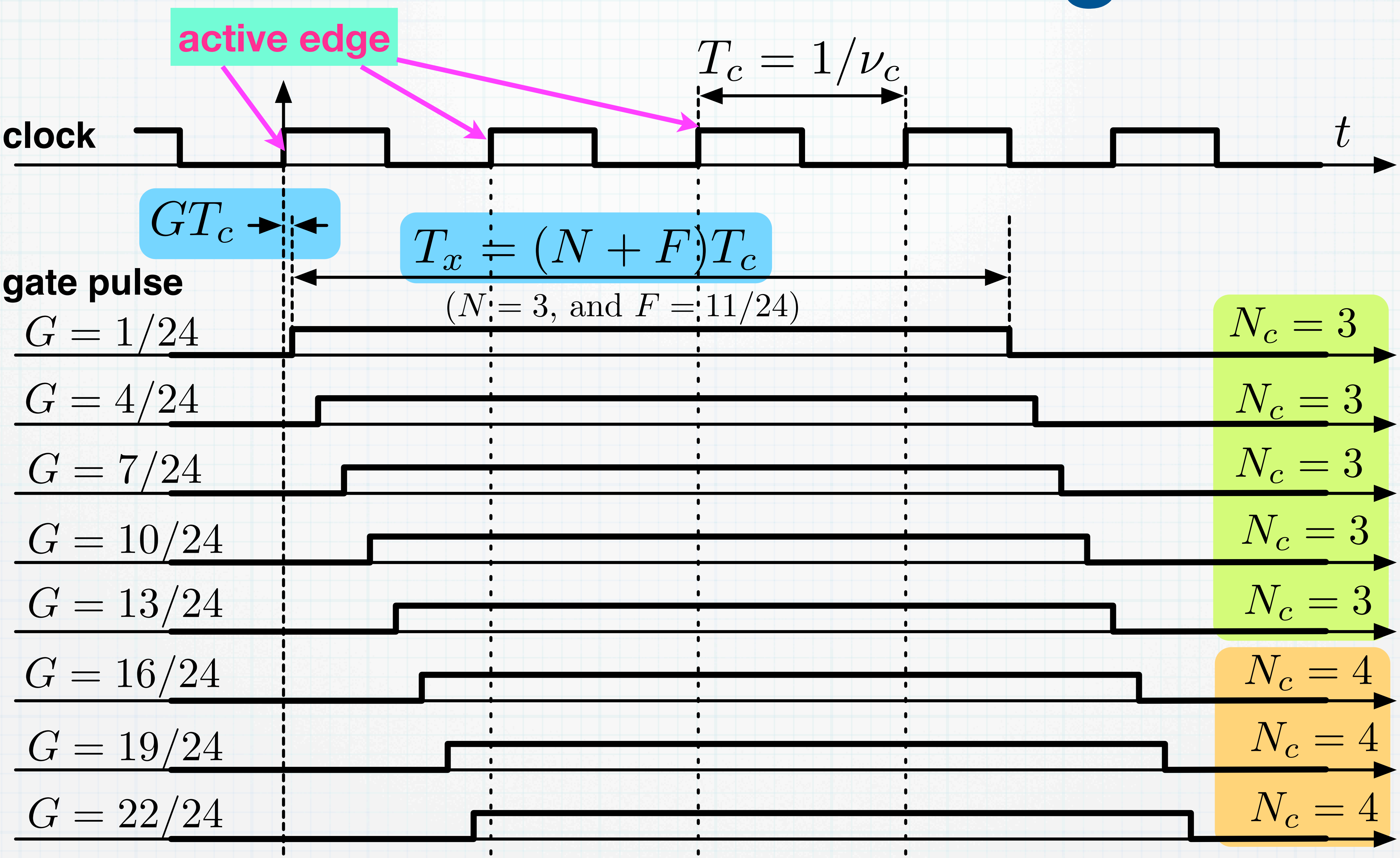
- **The transfer oscillator is a PLL**
- **Harmonics generation takes place inside the mixer**
- **Harmonics locking condition: $N\ \nu_{vco} = \nu_x$**
- **Frequency modulation Δf is used to identify N (a rather complex scheme, $\times N$ => $\Delta\nu$ –> $N\Delta\nu$)**

Heterodyne counter



- **Down-conversion: $f_b = | \nu_x - N \nu_c |$**
- **ν_b is in the range of a classical counter (100–200 MHz max)**
- **no resolution reduction in the case of a classical *frequency* counter (no need of reciprocal counter)**
- **Old scheme, nowadays used only in some special cases (frequency metrology)**

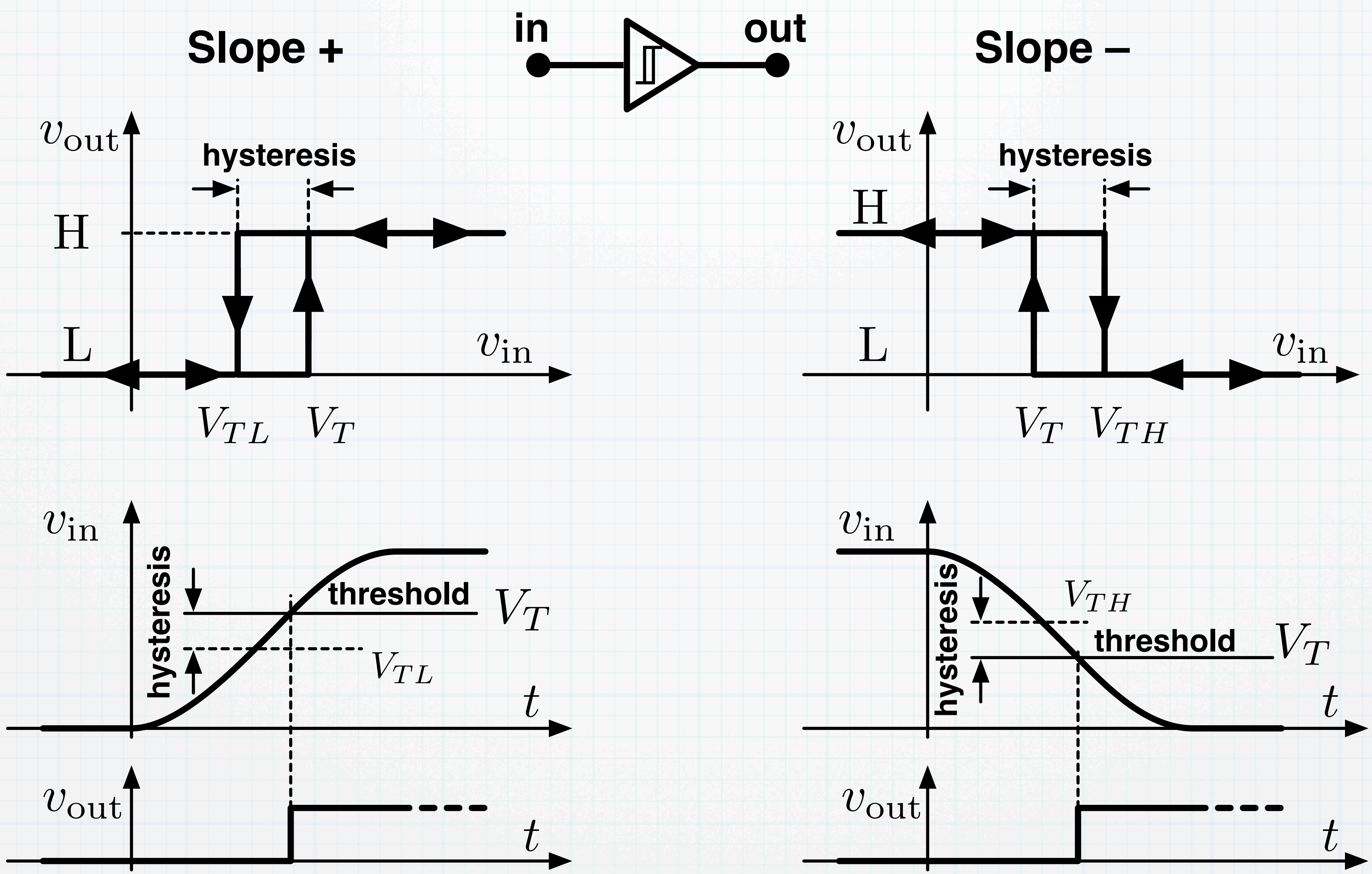
Coarse counting



The integer number N_c of clock cycles that falls in the gate pulse T_x is either *N* or *N*+1, depending on the the fractional part FT_c and on the delay GT_c

2 – Trigger

Trigger hysteresis



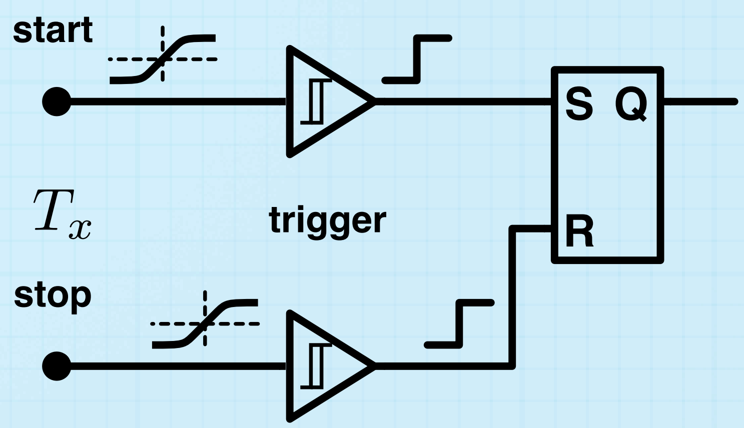
Hysteresis is necessary to avoid chatter in the presence of noise

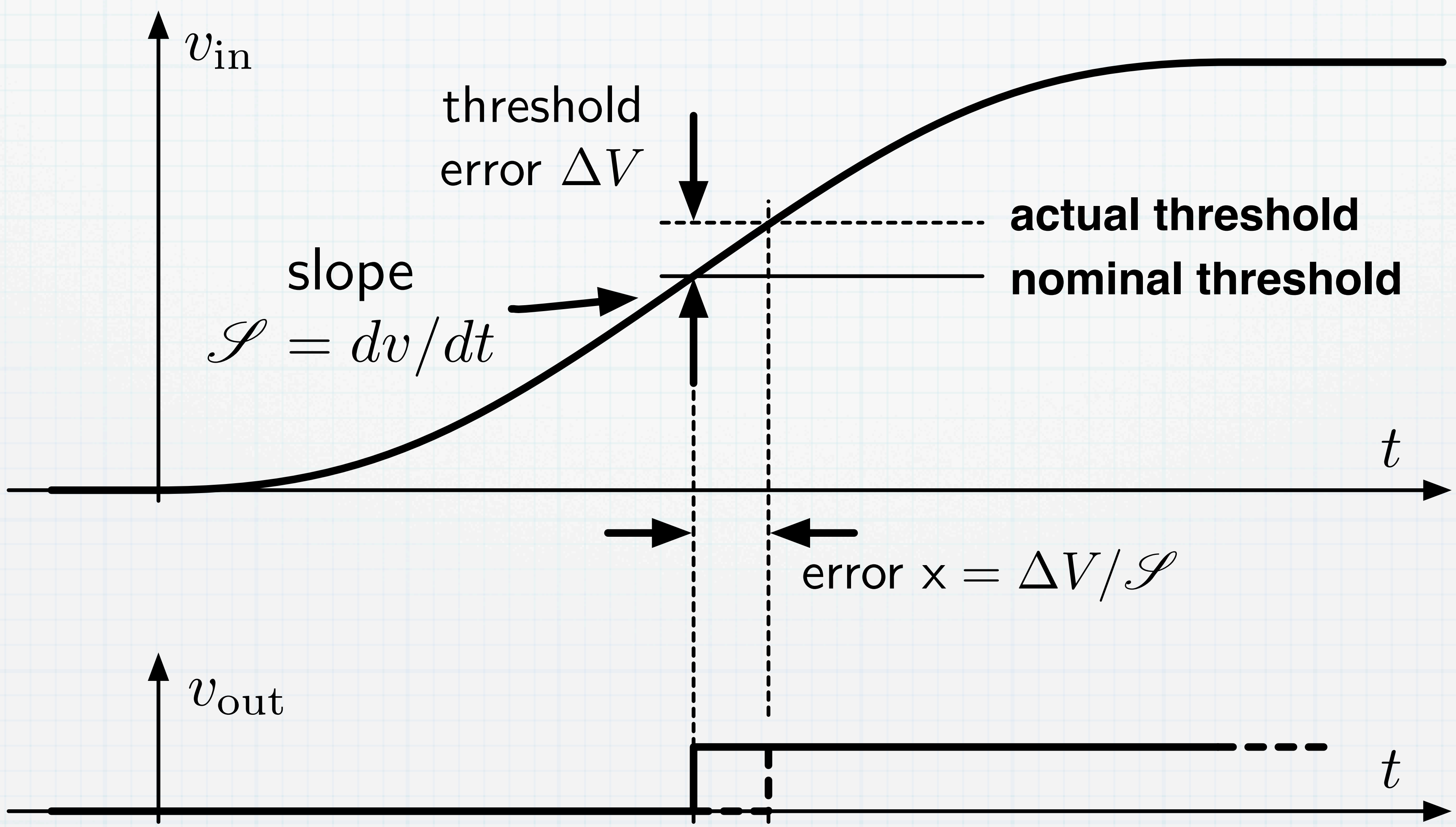
Threshold fluctuation

‘stop’ – ‘start’

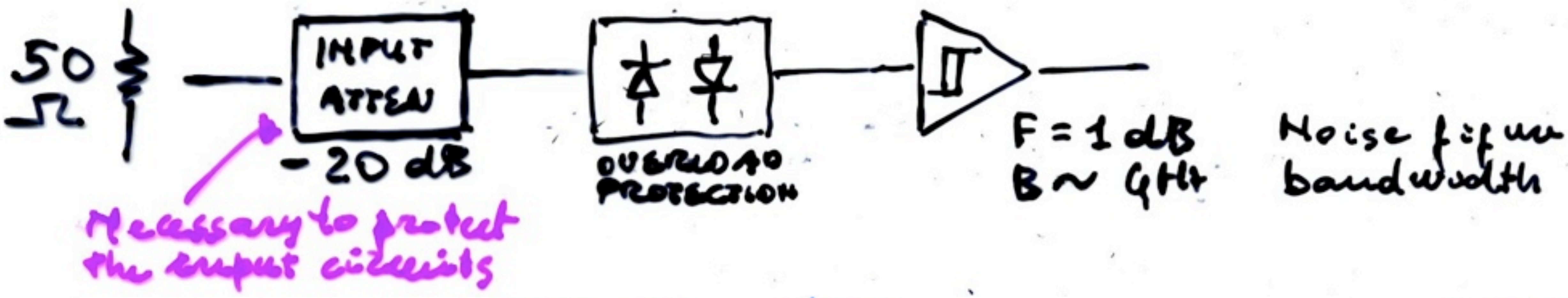
systematic $$\mathsf{x} = \frac{(\Delta V)_b}{\mathscr{S}_b} - \frac{(\Delta V)_a}{\mathscr{S}_a}$$

random $$\sigma_{\mathsf{x}}^2 = \frac{(\sigma_V^2)_a}{\mathscr{S}_a^2} + \frac{(\sigma_V^2)_b}{\mathscr{S}_b^2}$$





DON'T BLAME THE TRIGGER



THERMAL NOISE $\sqrt{4kTR}$ $V/\sqrt{Hz}$ $\longrightarrow$ $0.9\ \mu V/\sqrt{Hz}$

Boltzmann $k = 1.38 \times 10^{-23}$
temperature $T = 300\ K$
resistance $R = 50\ \Omega$

THERMAL NOISE INTEGRATED OVER THE BANDWIDTH $\times\sqrt{B}$

(HP 5370) $B = 225\ MHz \rightarrow V_n = 13.5\ \mu V$
(SR 620) $B = 1.3\ GHz \rightarrow V_n = 32.5\ \mu V$

Account for the loss $\ell = 20\ dB$ (multiply by 10)
and for the noise figure $F = 1\ dB$ (multiply by 1.12)

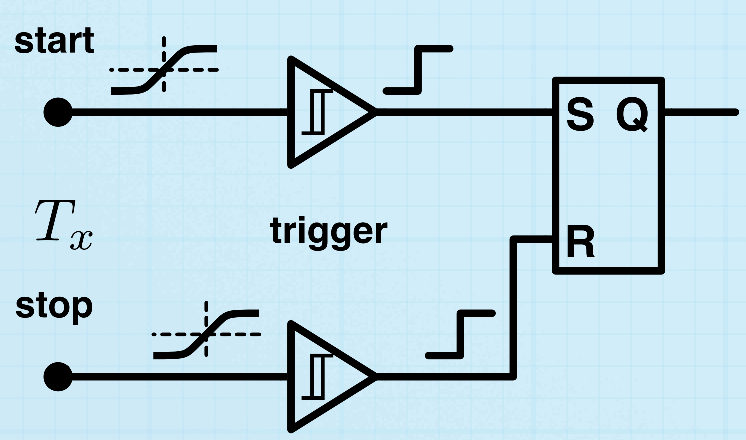
$B = 225\ MHz \rightarrow V_n = 150\ \mu V$
$B = 1.3\ GHz \rightarrow V_n = 355\ \mu V$

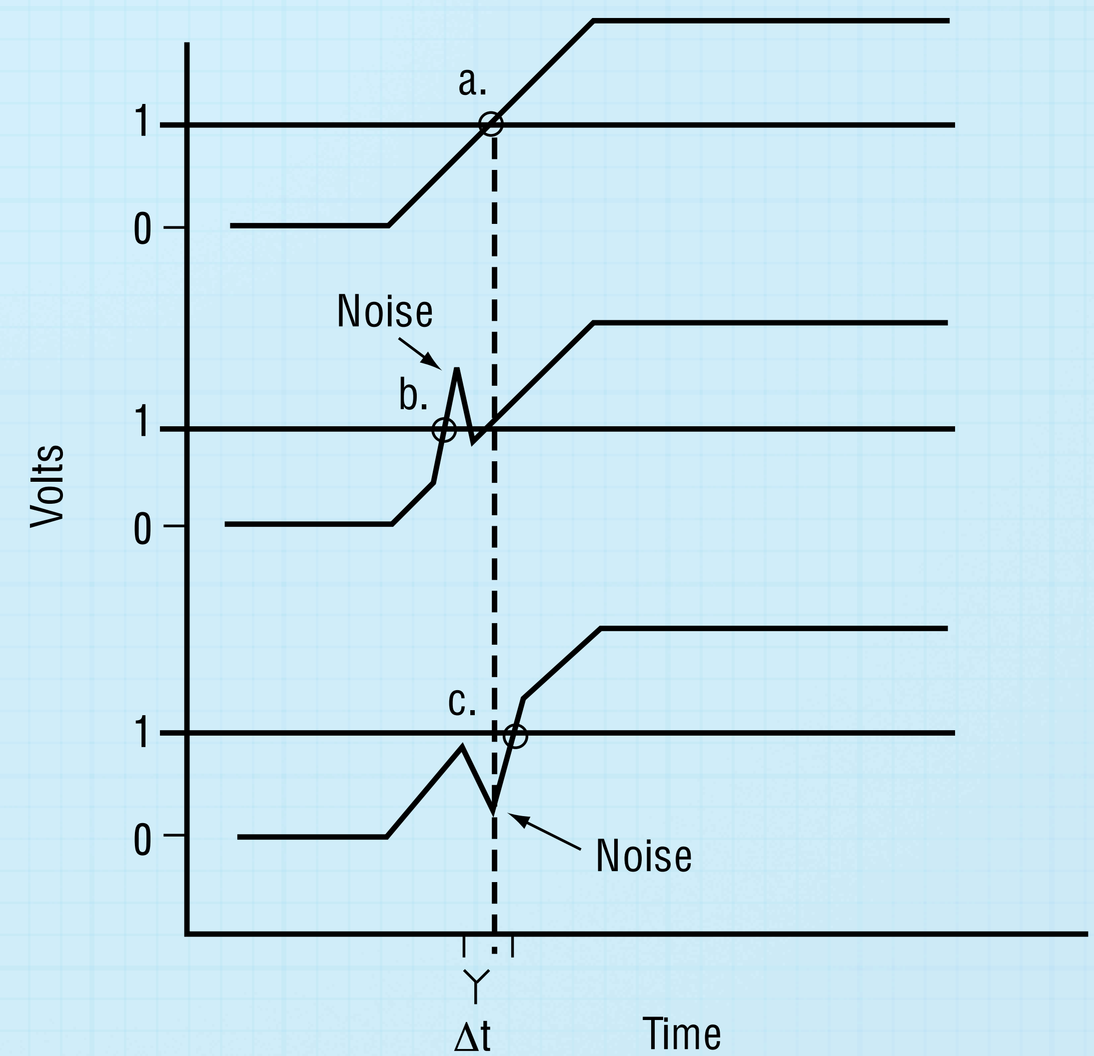
Trigger noise – oversimplified

$$\text{`stop'} - \text{`start'}$$

systematic $$\mathsf{x} = \frac{(\Delta V)_b}{\mathscr{S}_b} - \frac{(\Delta V)_a}{\mathscr{S}_a}$$

random $$\sigma_{\mathsf{x}}^2 = \frac{(\sigma_V^2)_a}{\mathscr{S}_a^2} + \frac{(\sigma_V^2)_b}{\mathscr{S}_b^2}$$





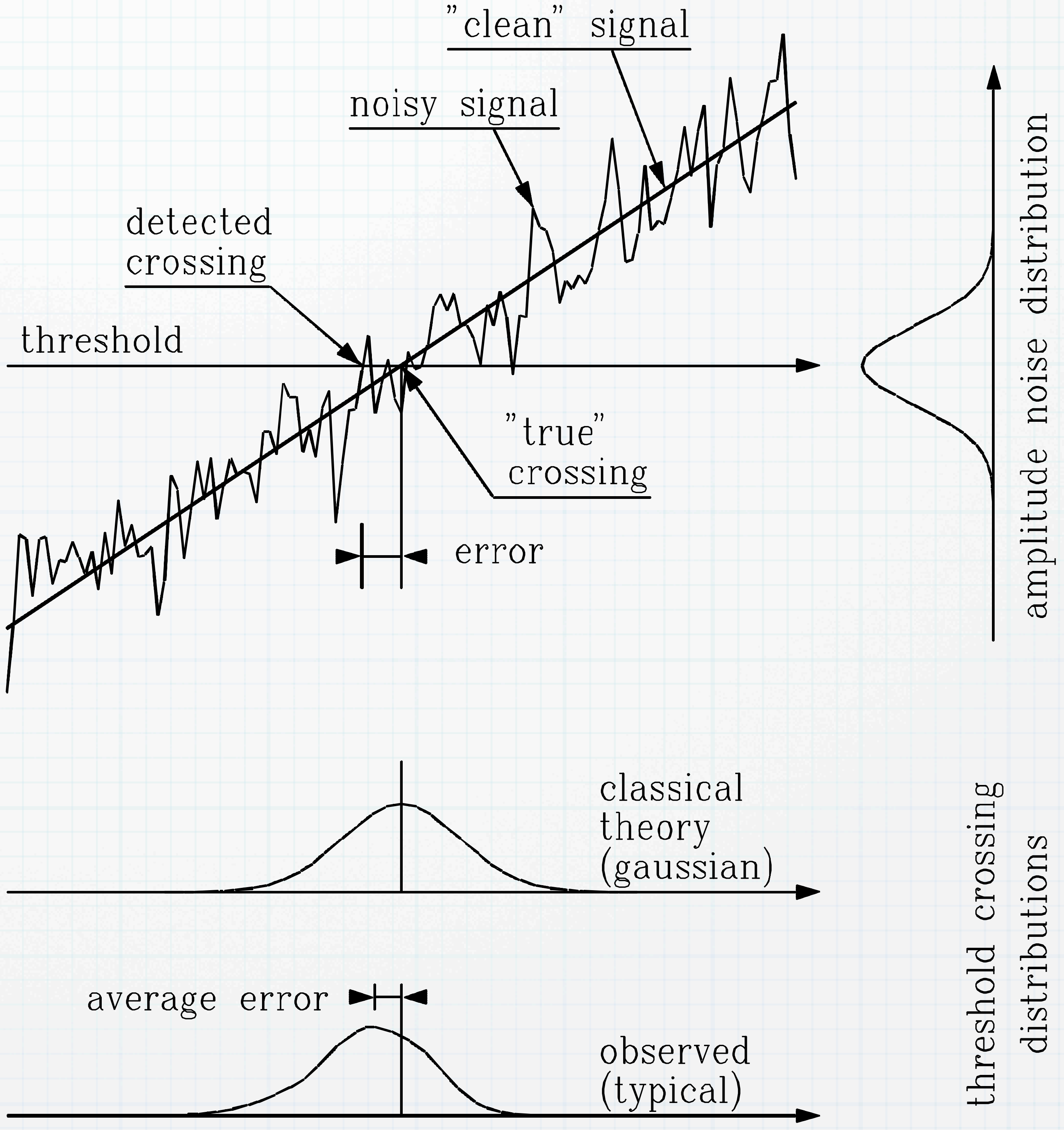
Agilent, Application Note 200-3, 1997

- **The effect of noise is often explained with a plot like this**
- **Yet, the formula holds in the absence of spikes!!!**
- **To the general practitioner, this explanation looks simple**

Effect of (too) wide-band noise

When the rms slope of noise is higher than the signal slope:

- the trigger leads
- systematic error



E. Rubiola & al., Proc. 46 FCS pp. 265-269, May 1992

Trigger behavior vs. bandwidth

Noise rms slope

$$\mathscr{S}_n^2 = 4\pi^2 \int_0^B f^2 S_V(f)\, df$$

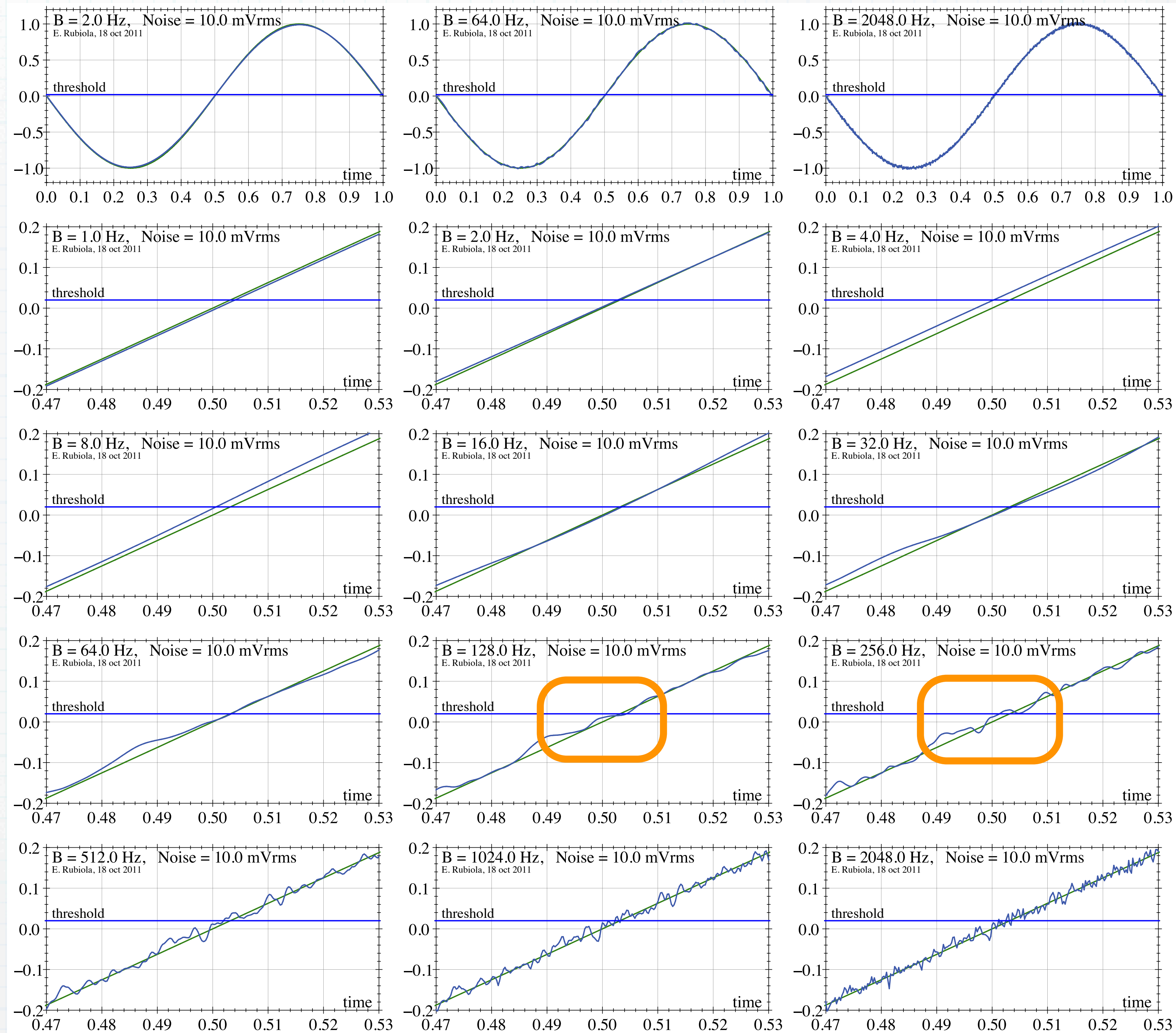
$$\mathscr{S}_n^2 = \frac{4\pi^2}{3} \sigma_V^2 B^2$$

Critical slope

$$\mathscr{S}_s^2 = \frac{4\pi^2}{3} S_V B^3$$

$$\mathscr{S}_s^2 = \frac{4\pi^2}{3} \sigma_V^2 B^2$$

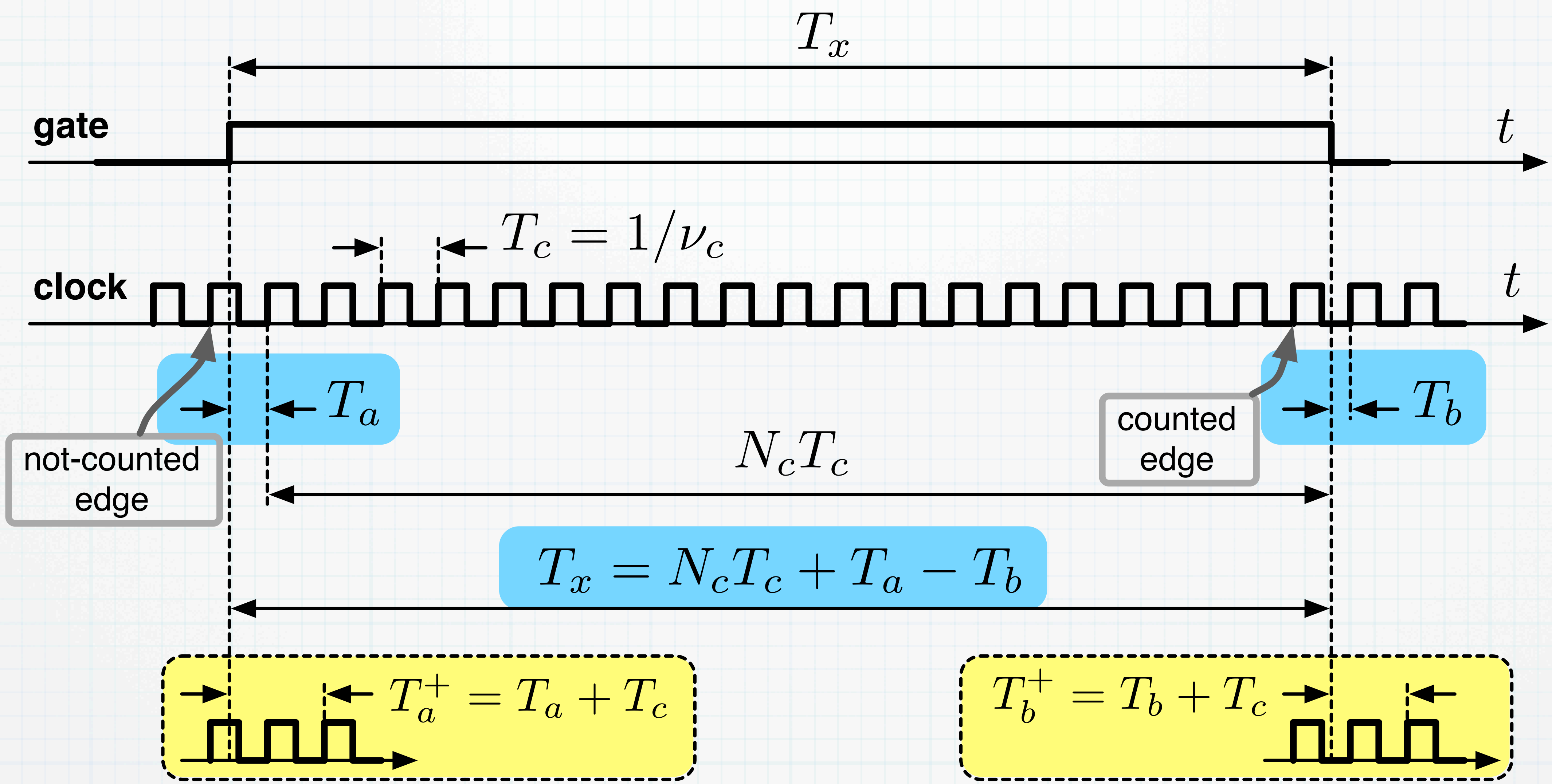
Signal slope equals rms noise slope



- **When the noise slope exceeds the clean-signal slope, the total slope changes sign**
- **There result spikes, and systematic lead error**

3 – Interpolation schemes

Clock interpolation – Main idea

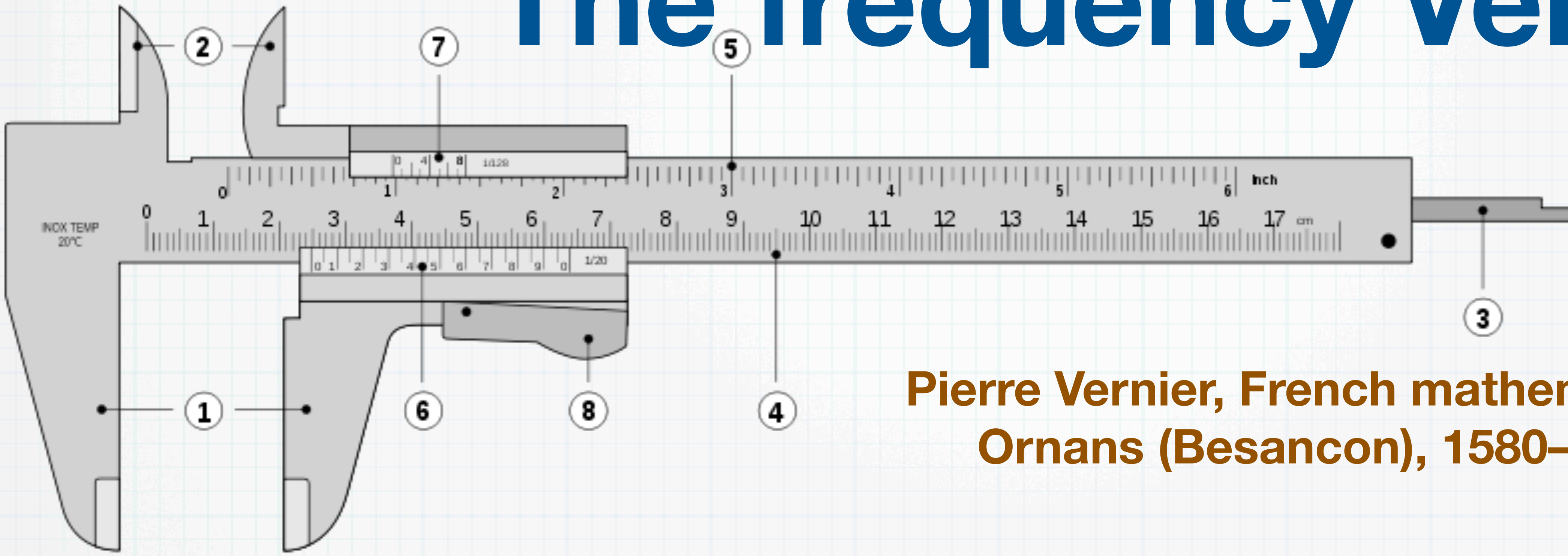


Too short T_a and T_b are difficult to measure, so we add one T_c to each

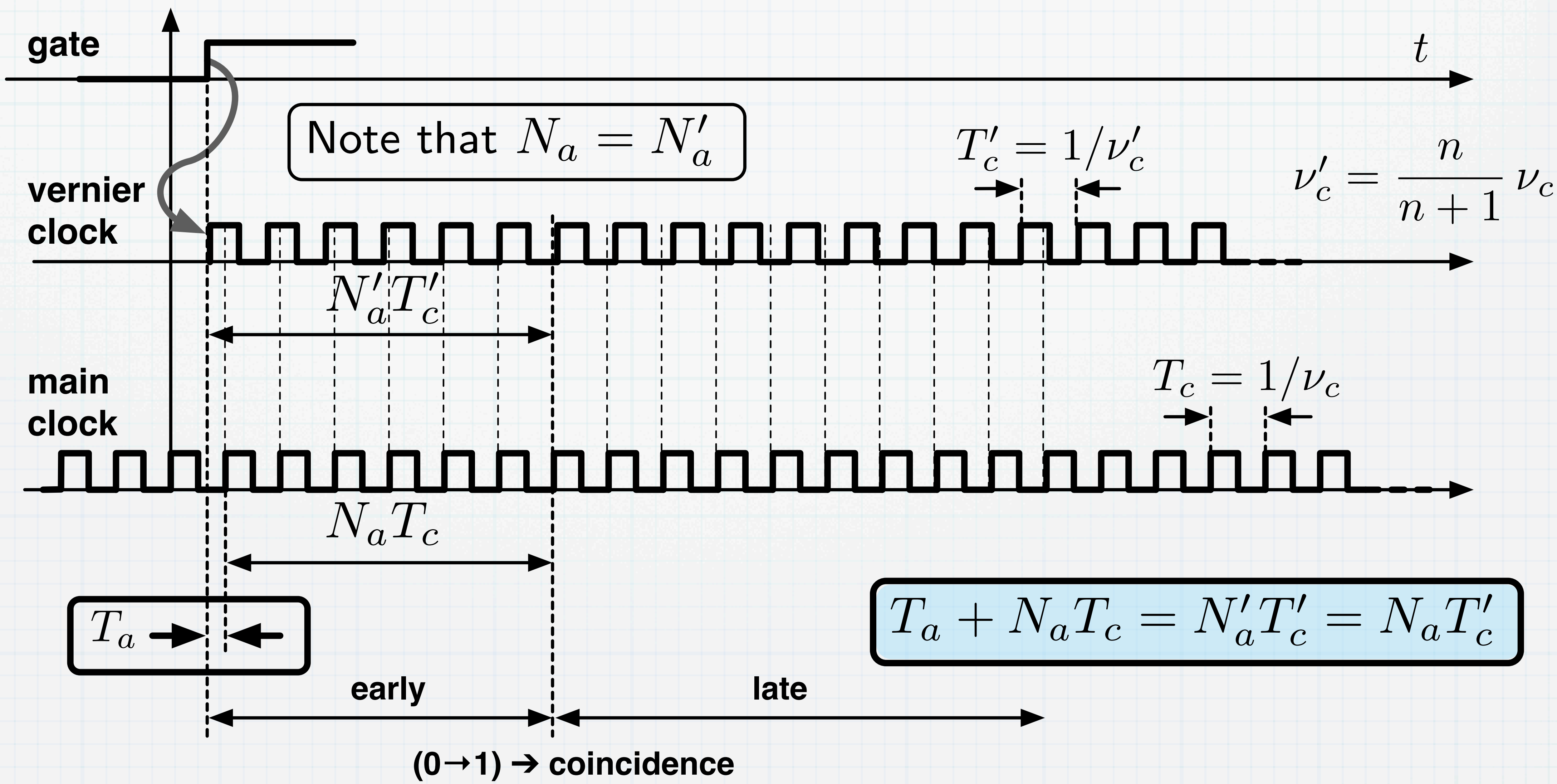
Interpolation is made possible by the fact that

the clock frequency is constant and accurately known

The frequency Vernier

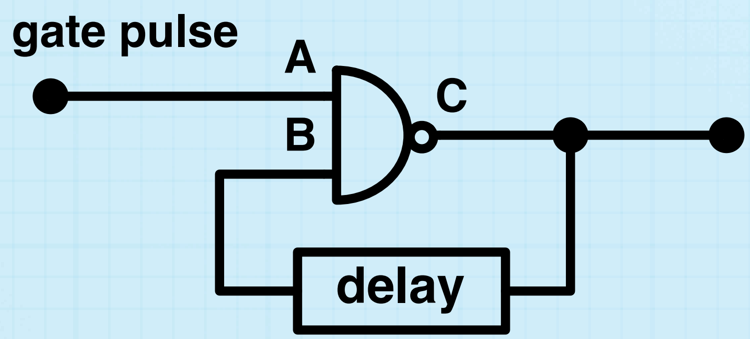


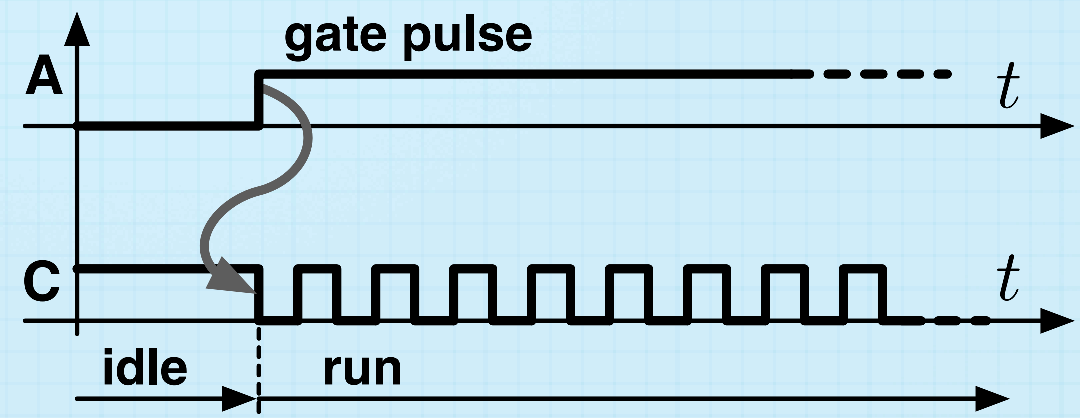
Pierre Vernier, French mathematician
Ornans (Besancon), 1580–1637



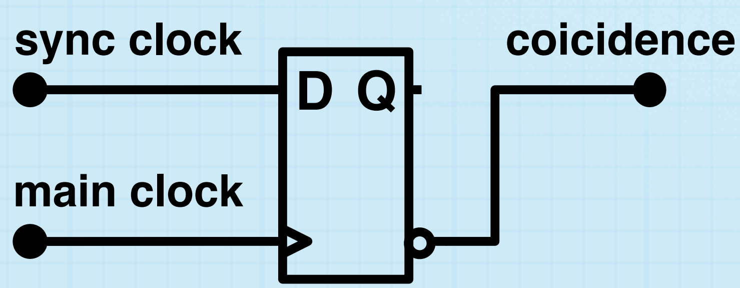
The key elements

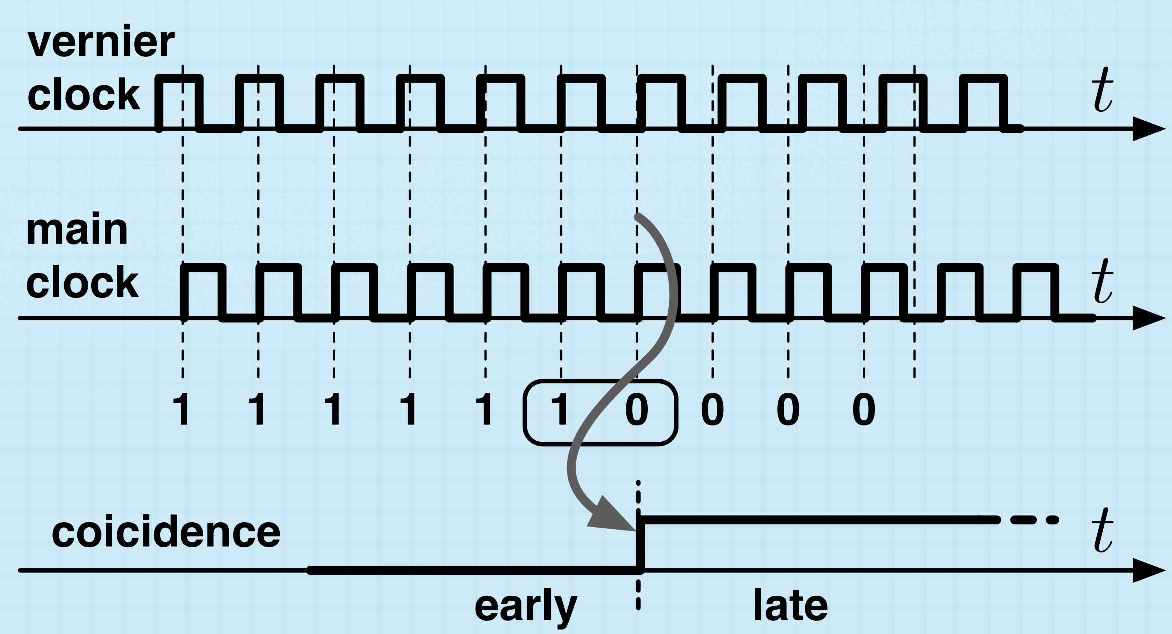
Synchronized oscillator





Coincidence detector





Example: Hewlett Packard 5370A

$f_c = 200$ MHz $\rightarrow$ $\delta T_x = 5$ ns
(ECL Technology)

$n = 256$ $\rightarrow$ $\delta T_a = \delta T_b = \frac{1}{256} \times 5\,\text{ns} = 19.5\,\text{ps}$
($f_v = 199.22$ MHz)

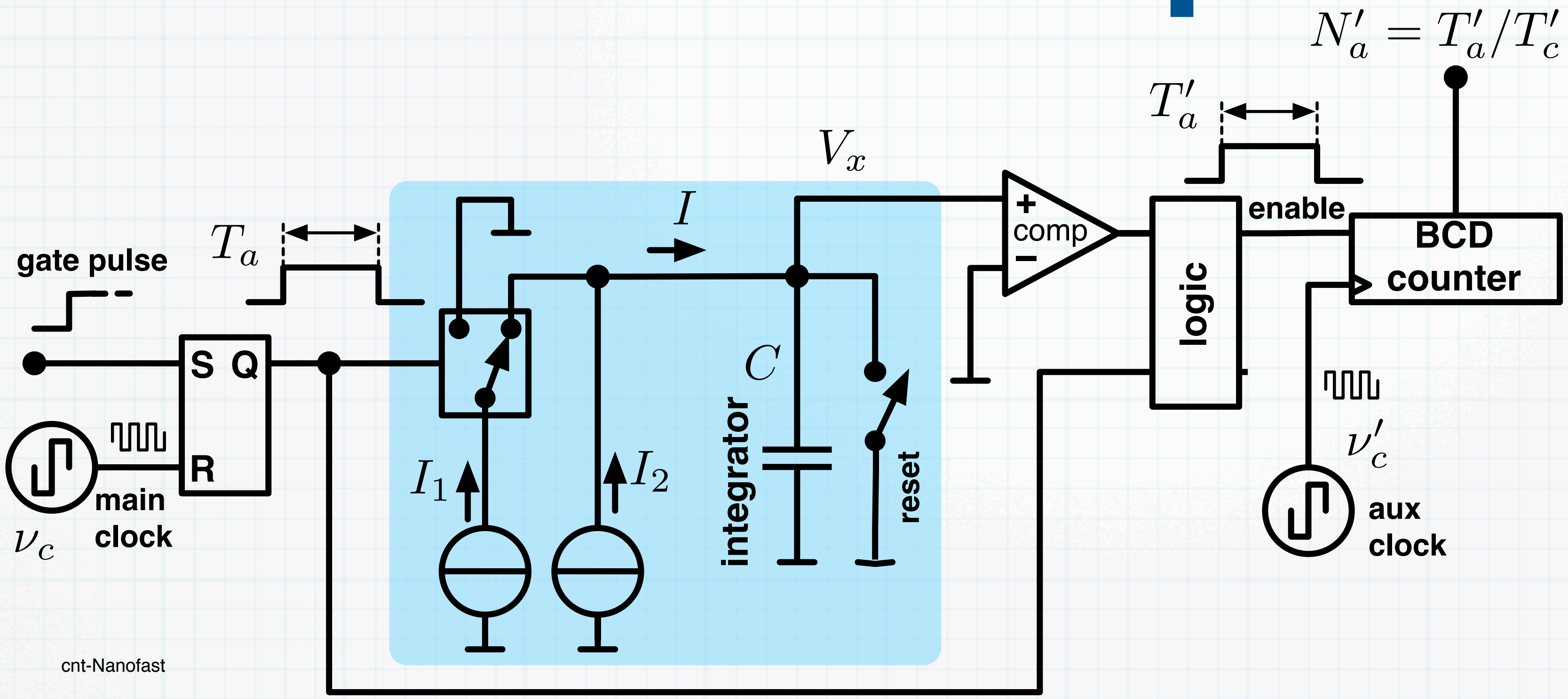
It takes a max. of 257 cycles of f_c for the two clocks to coincide

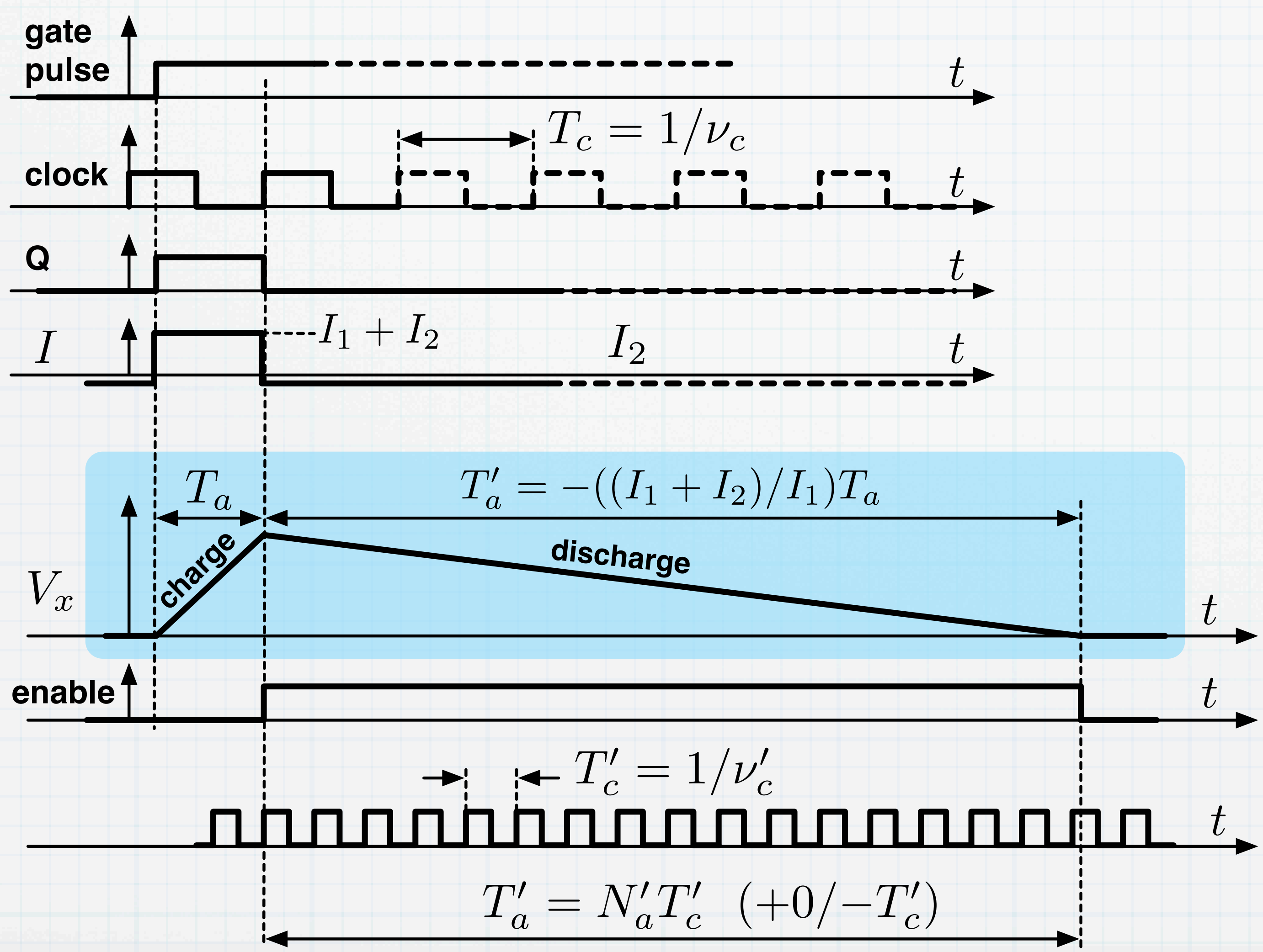
conversion time:
$257 \times 5\,\text{ns} = 1.285\,\mu\text{s}$

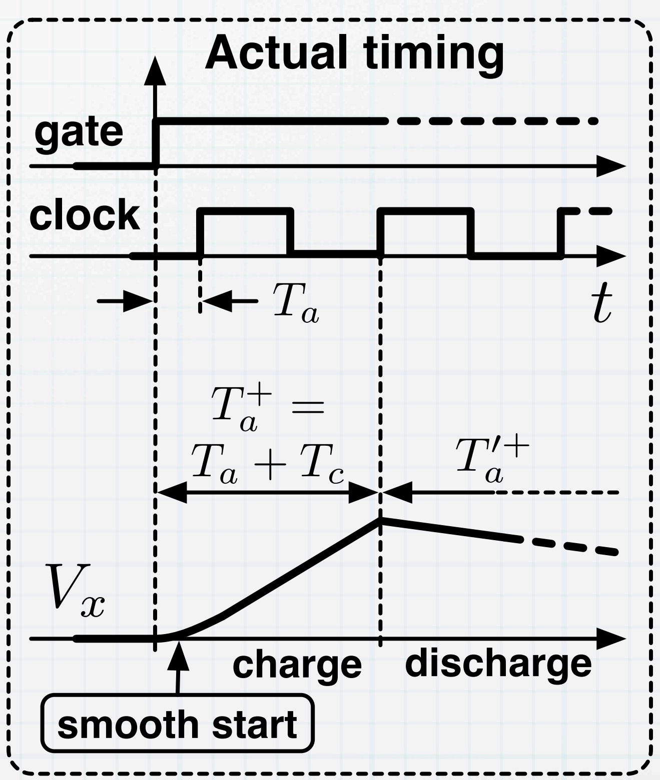
Light speed in a cable $\simeq 0.67c$

$\delta T_a \longleftrightarrow \delta l \simeq 4\,\text{mm}$
(length)

The Nutt dual-slope interpolator







Example: Nanofast 536 B

Smithsonian Astrophysical Laboratory

Main clock $f_c = 10\ \mathrm{MHz} \rightarrow \delta T = T_c = 100\ \mathrm{ns}$

Time Interval amplifier $\frac{I_1}{I_2} = 4000$

$$T'_a \in (200\ \mu s,\ 400\ \mu s)$$

aux. clock 20 MHz for the measurement of T'_a

$$\delta T'_a = T'_c = 50\ \mathrm{ns} \qquad (1/20\ \mathrm{MHz})$$

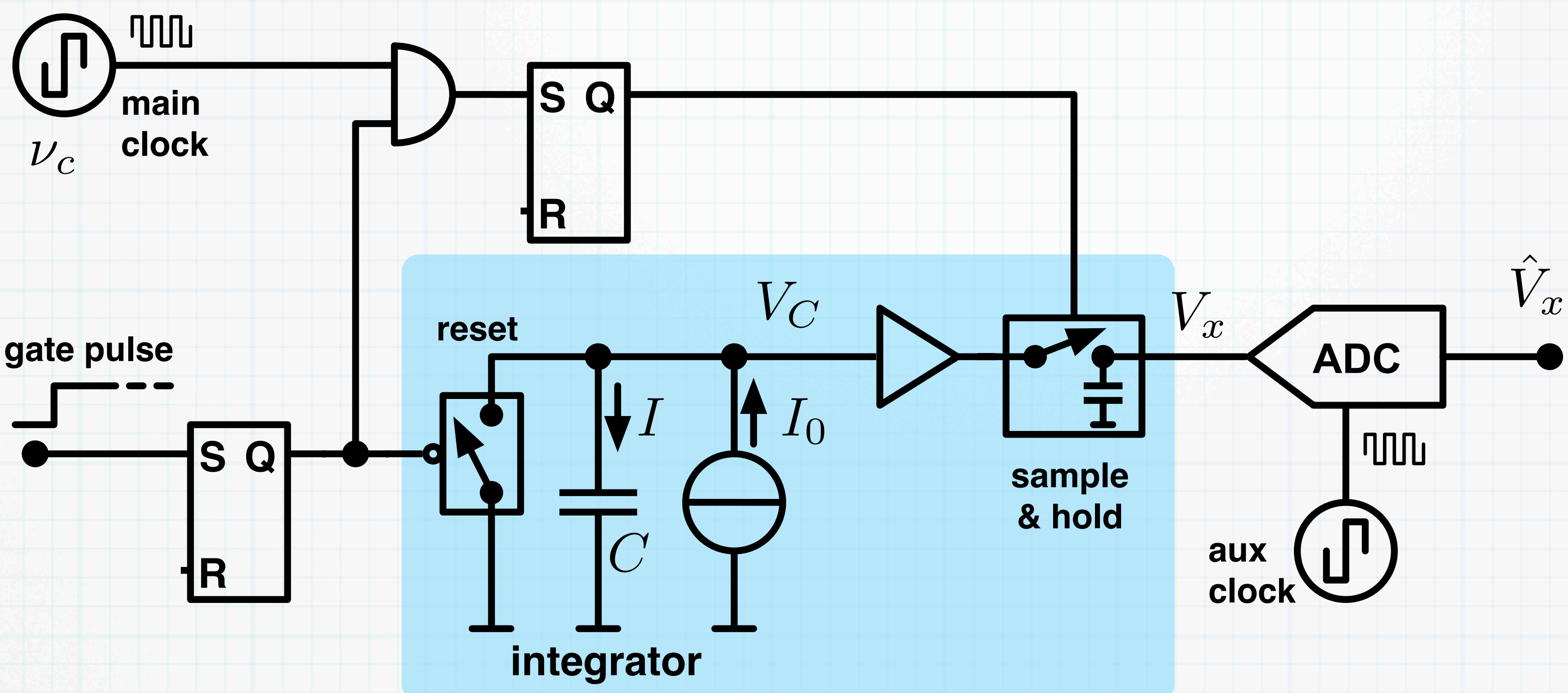
$$\delta T_a = \frac{I_2}{I_1} T'_c \qquad \delta T_a = \frac{1}{4000} \times 50\ \mathrm{ns} = 12.5\ \mathrm{ps}$$

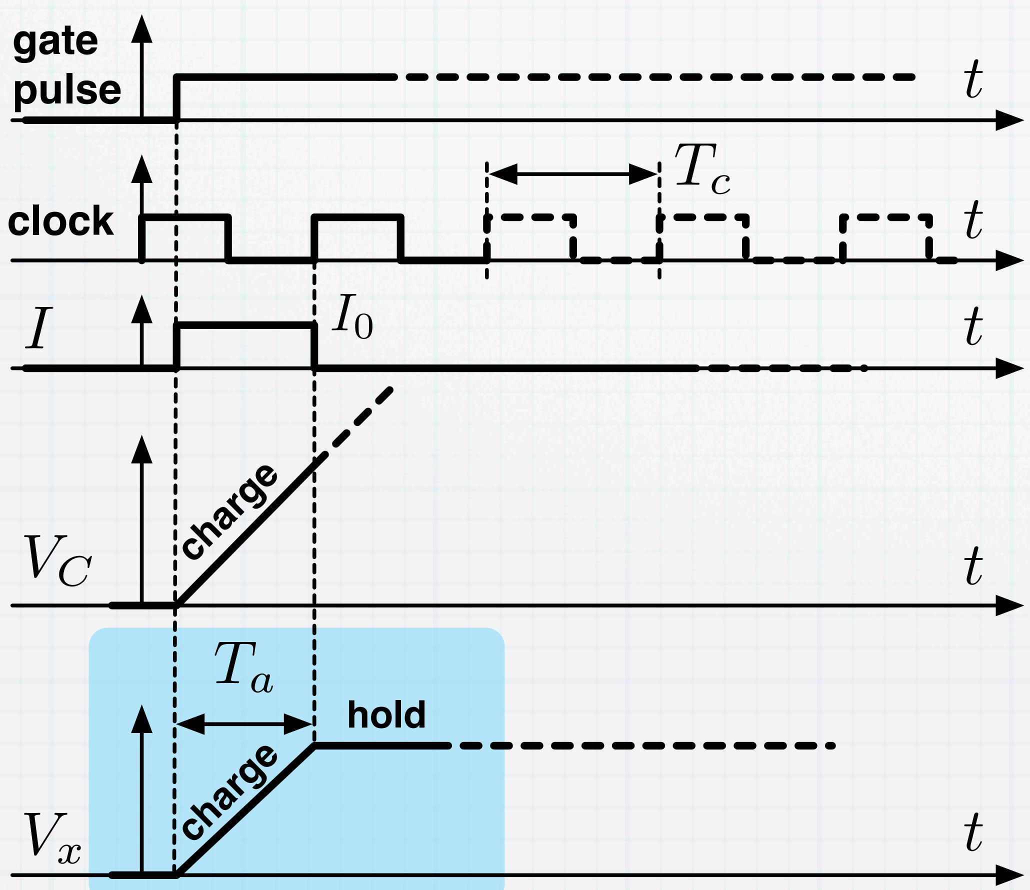
The Nanofast 536 B counter is (was?) a part of the Mark IV system for Very Long Baseline Interferometry (VLBI). Early TTL technology

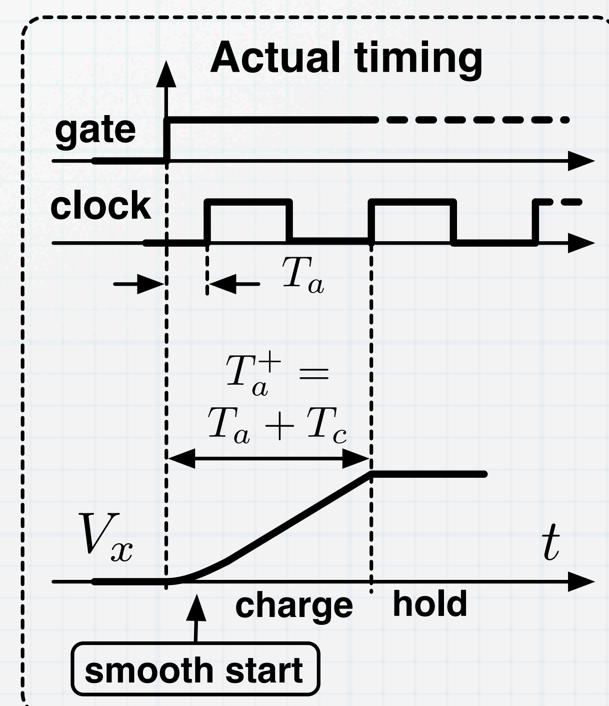
Note: a pulse propagates in a cable at $c' \approx \frac{2}{3} c$

δT_a is equivalent to a length of 2.5 mm

The ramp interpolator







Example: Stanford SR 620

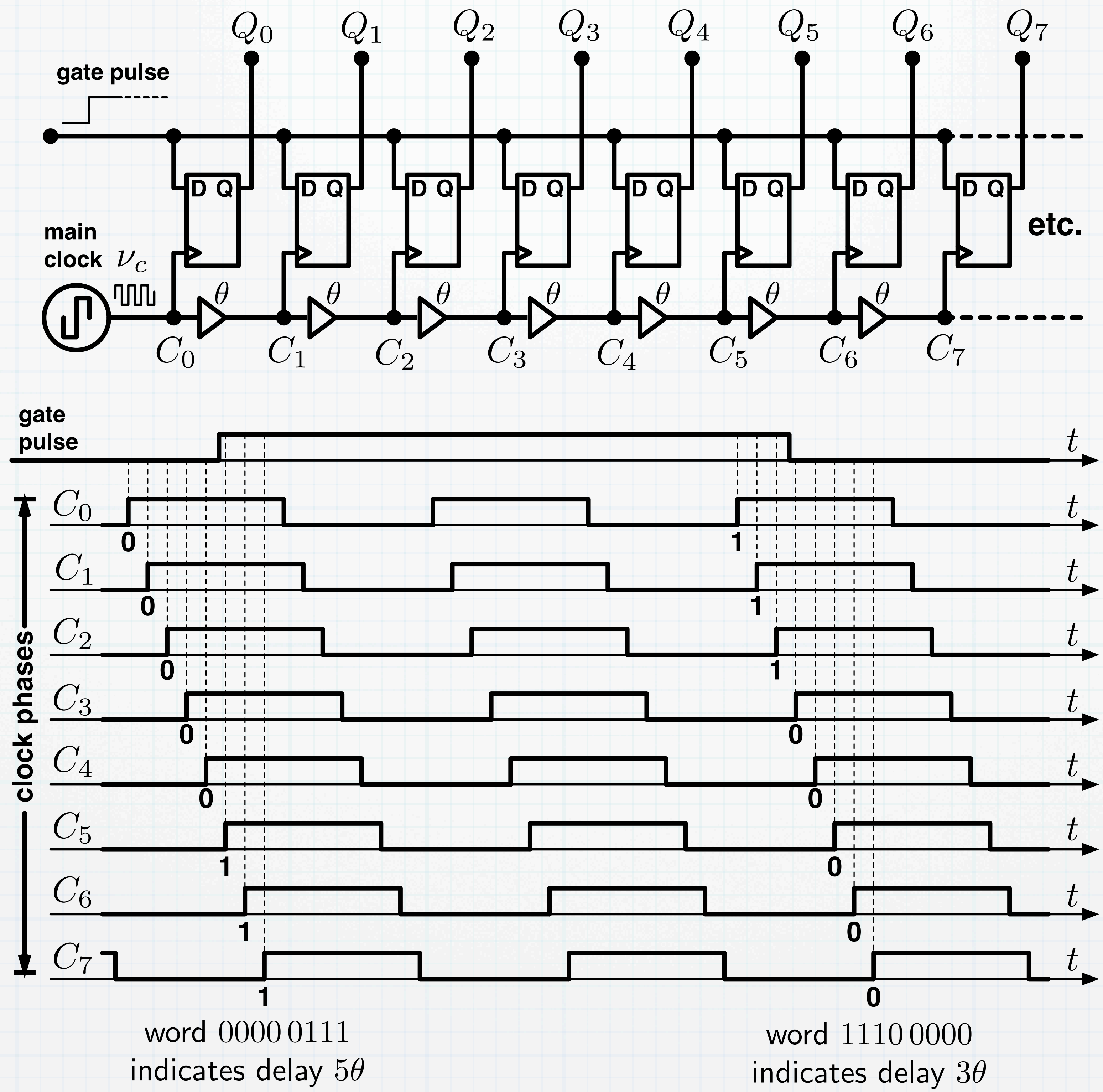
$f_c = 90$ MHz phase-locked to the 10 MHz reference.

$T_c = 11.1$ ns ECL Technology

12 bit converter, 1 bit lost because of the extra T_c

11 bits $\delta T_e = \frac{11.1 \text{ ns}}{2^{11}} = 5.4$ ps

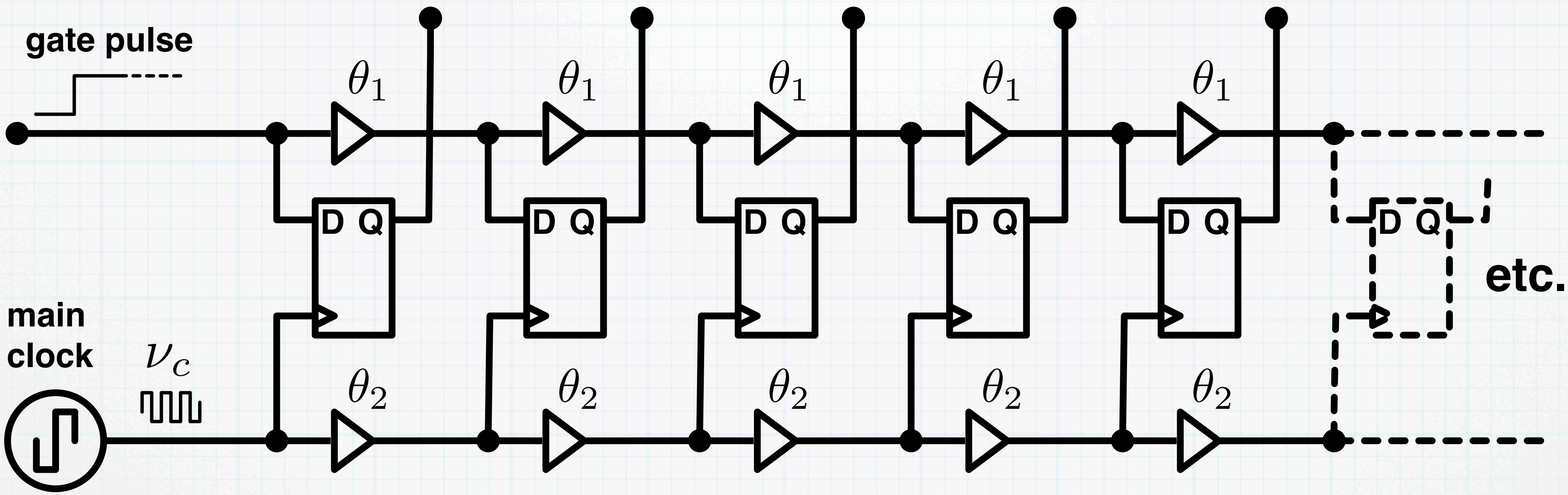
Thermometer-code interpolator



Also called Multi-tapped delay-line interpolator

Review article: J. Kalisz, Metrologia 41 (2004) 17–32

Vernier thermometer-code interpolator

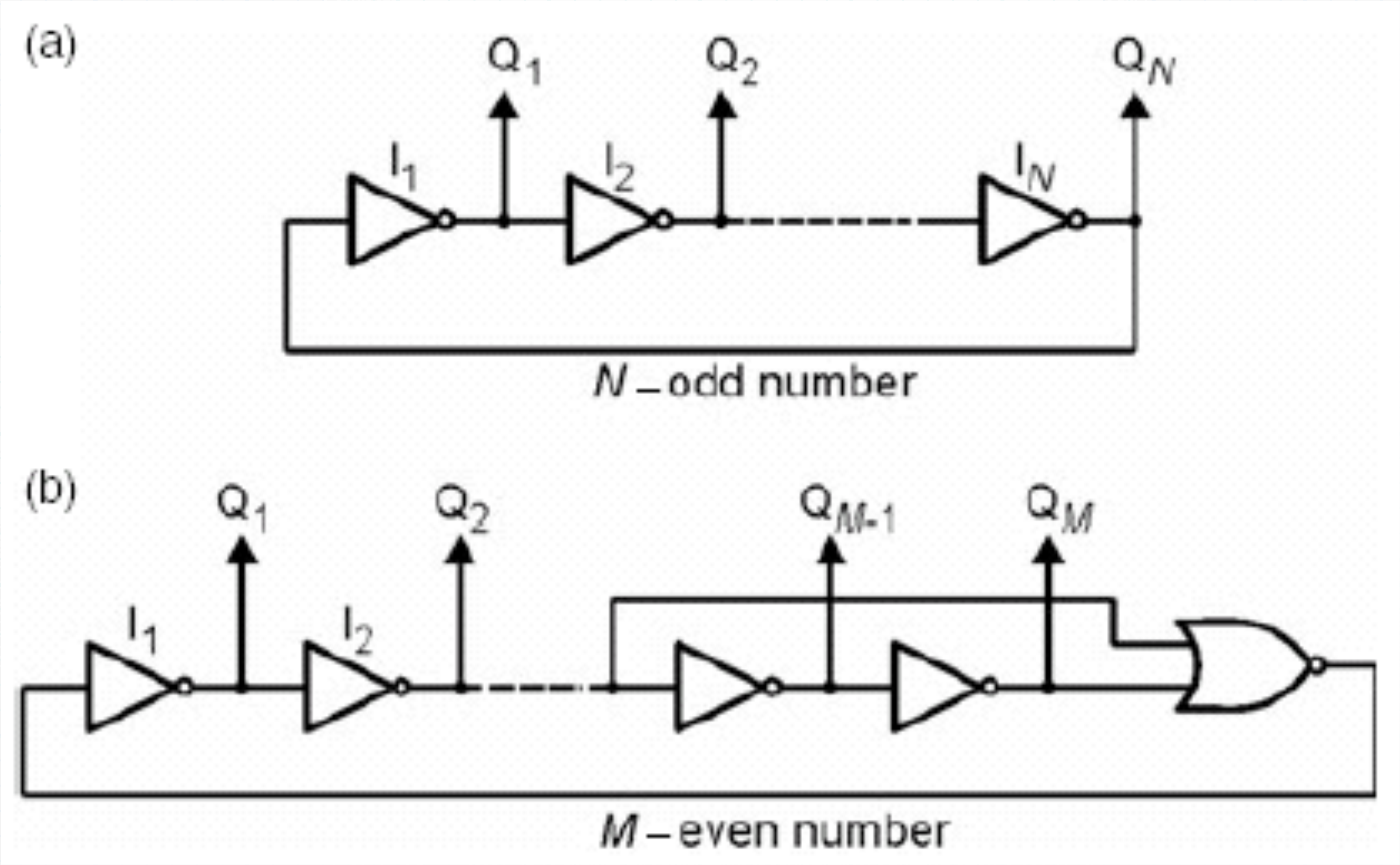


$$\theta_{eq} = \theta_2 - \theta_1$$

Owing to physical size, both θ_1 and θ_2 are always present

Ring Oscillator

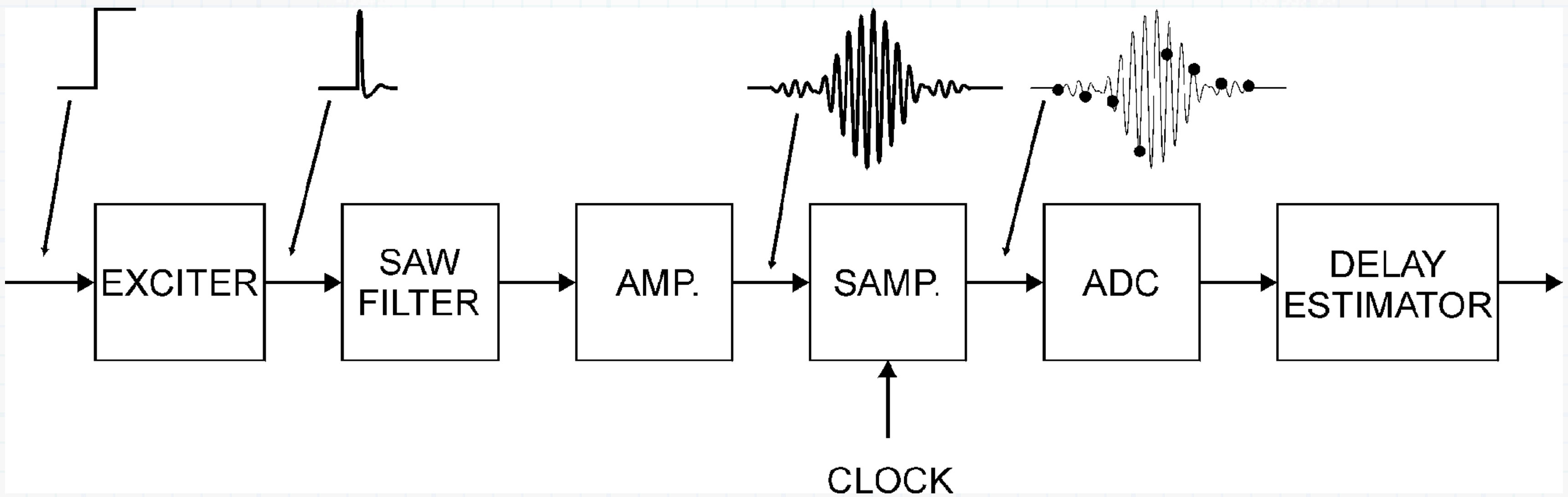
Figure from J. Kalisz, Metrologia 41 (2004) 17–32



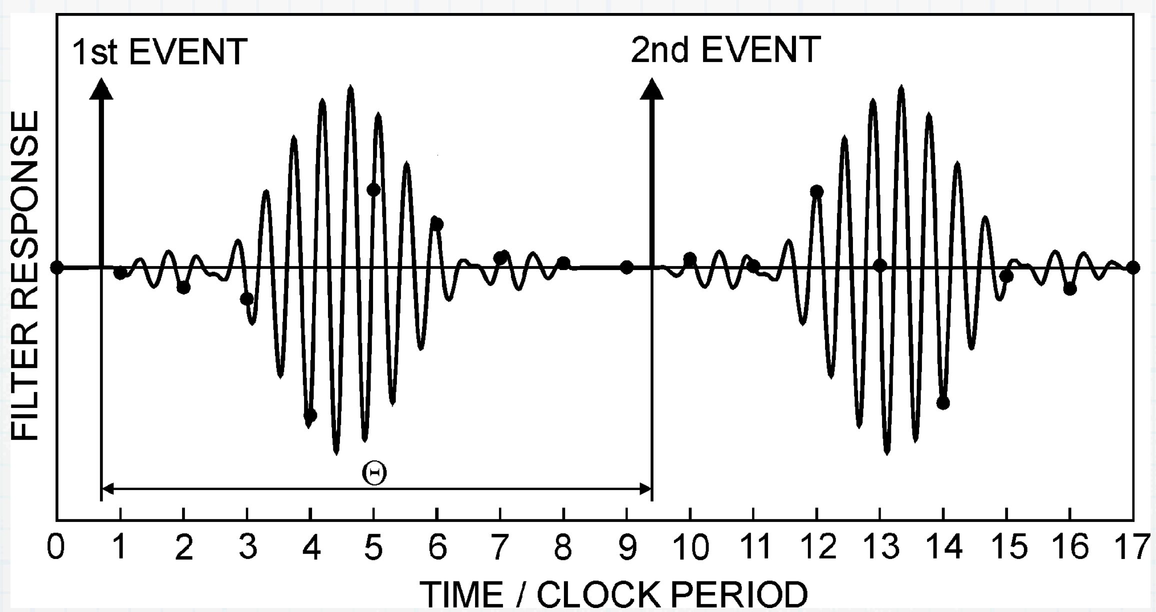
Also used in PLL circuits for clock-frequency multiplication

SAW delay-line interpolator

A – Block diagram



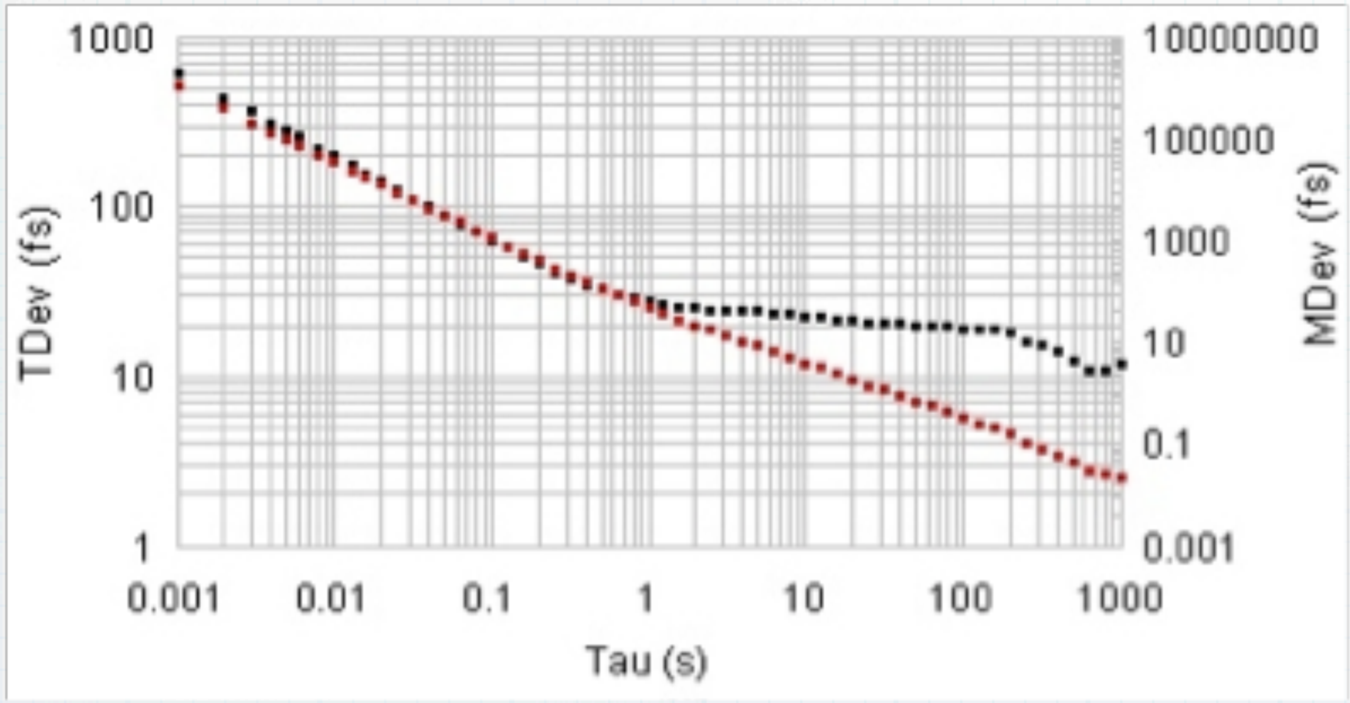
B – Pulse waveforms

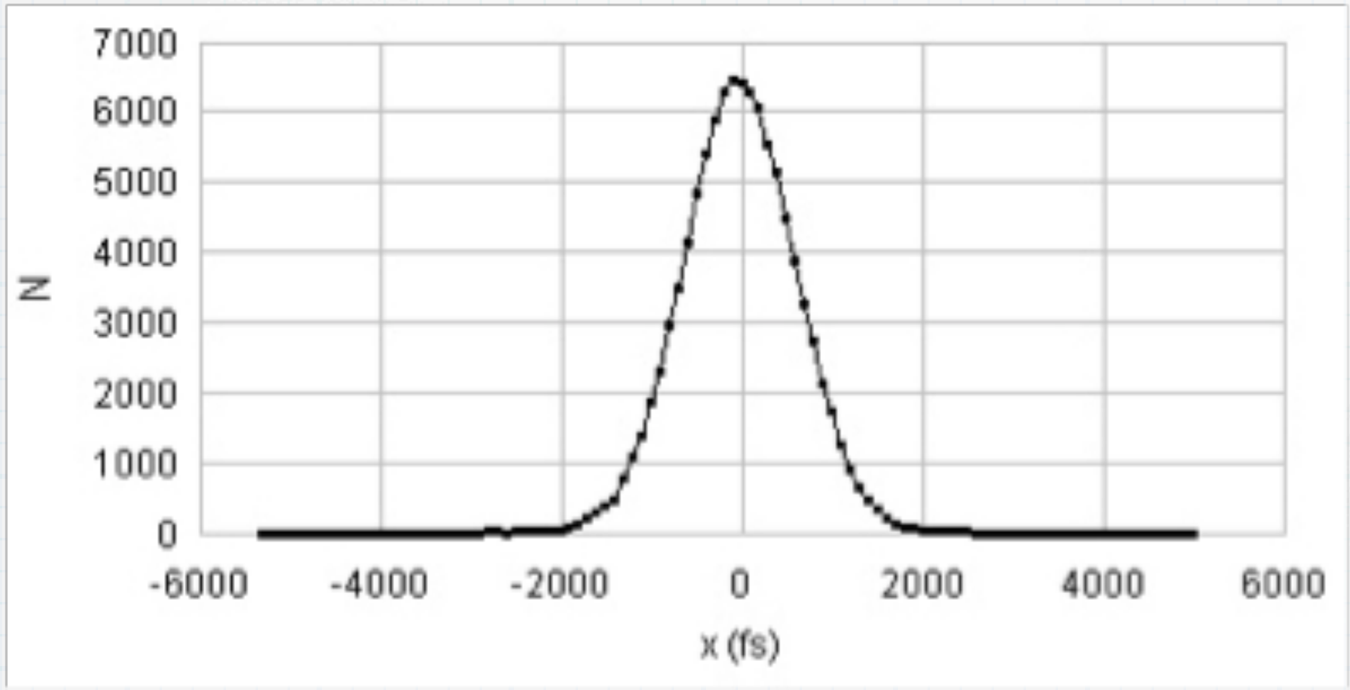


- Dispersion stretches the input pulse
- Sub-sampling and identification of the alias

P. Panek, I. Prochazka, Rev. Sci. Instrum. 78(9):094701, 2007

Sigma Time STX301 counter





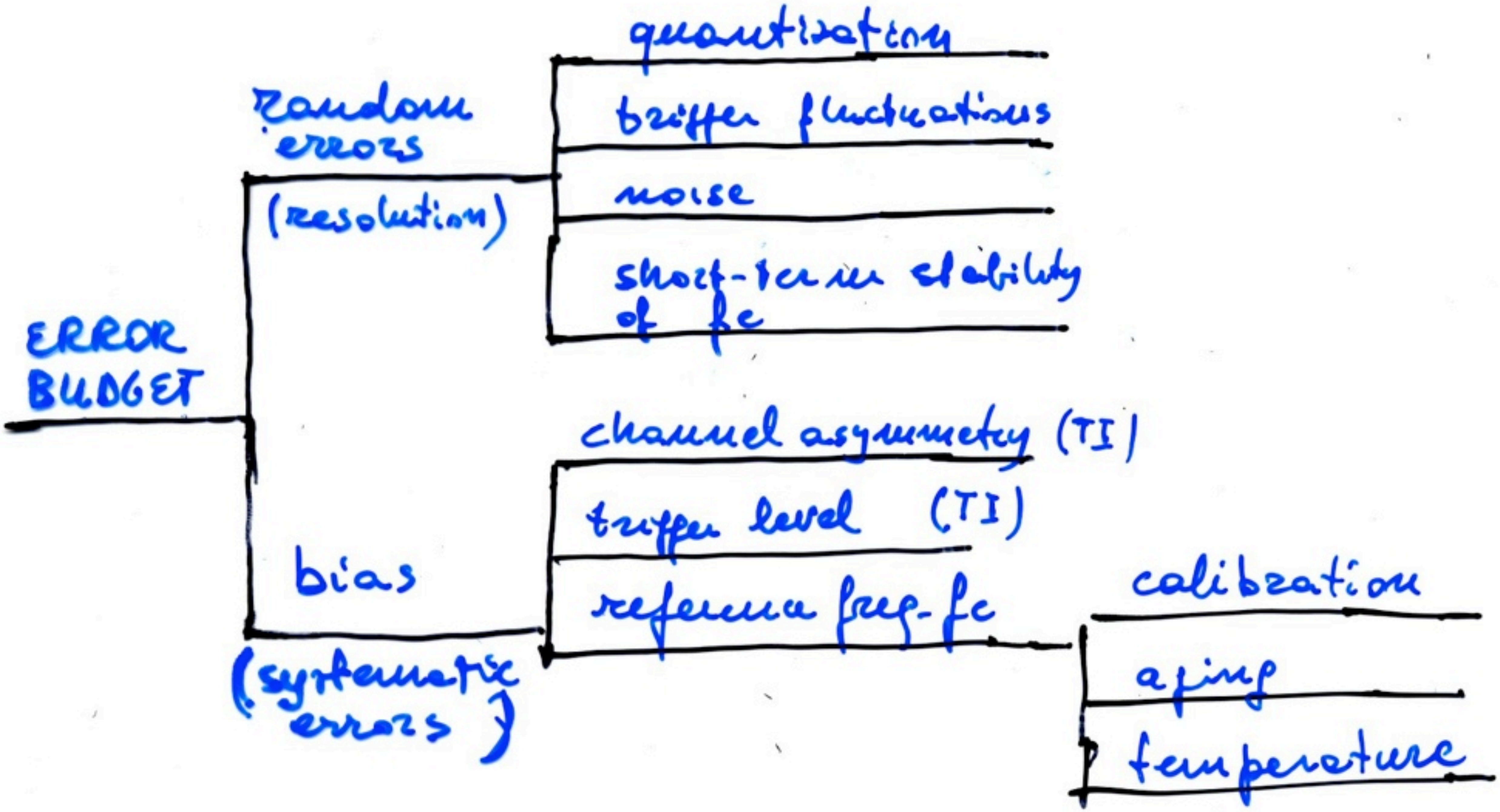
- Gossips report that this is none of the above methods
- No information at all, I'm unable to reverse-engineer

4 – Basic statistics

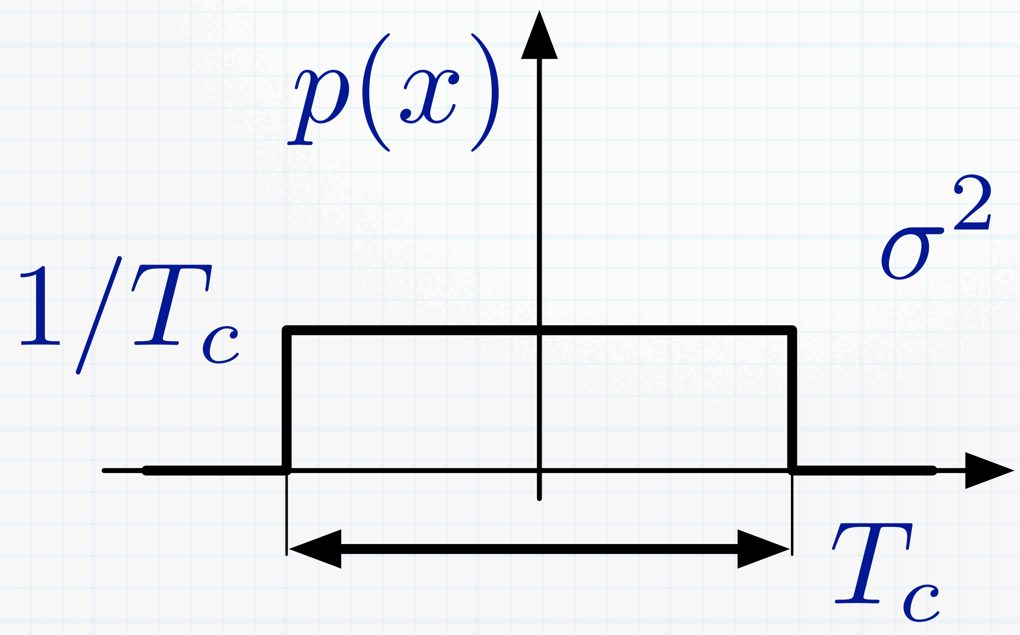
– after all, not that basic! –

ACCURACY AND PRECISION

11a



Quantization uncertainty



$$\sigma^2 = \frac{T_c^2}{12}$$

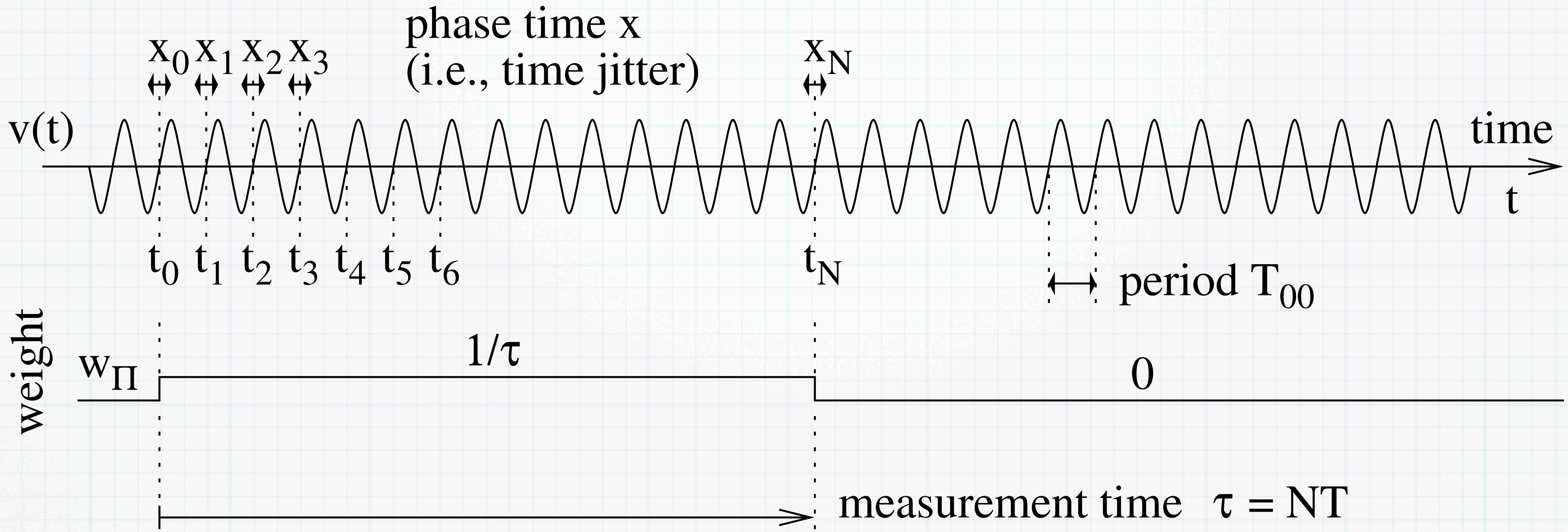
$$1/\sqrt{12} = 0.29$$

Example: 100 MHz clock

Tx = 10 ns

σ = 2.9 ns

Classical (Π) reciprocal counter



the measure is a scalar product

$$\mathbb{E}\{\nu\} = \int_{-\infty}^{+\infty} \nu(t) w_\Pi(t)\, dt$$ Π estimator

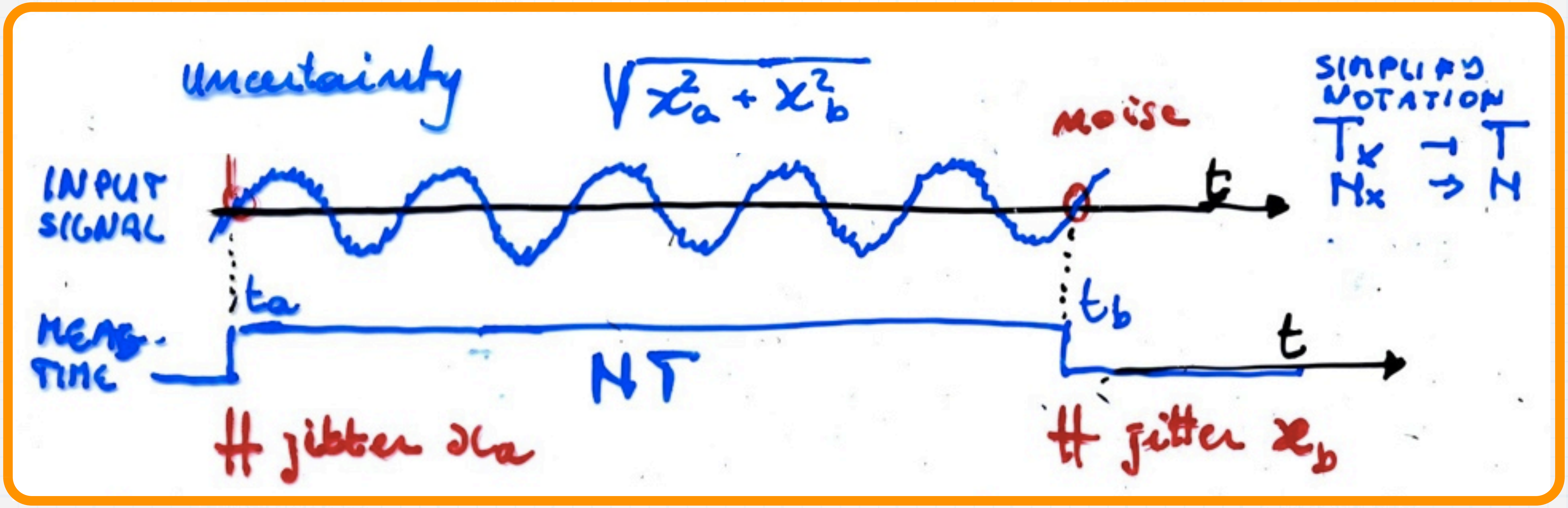
$$w_\Pi(t) = \begin{cases} 1/\tau & 0 < t < \tau \\ 0 & \text{elsewhere} \end{cases}$$ weight

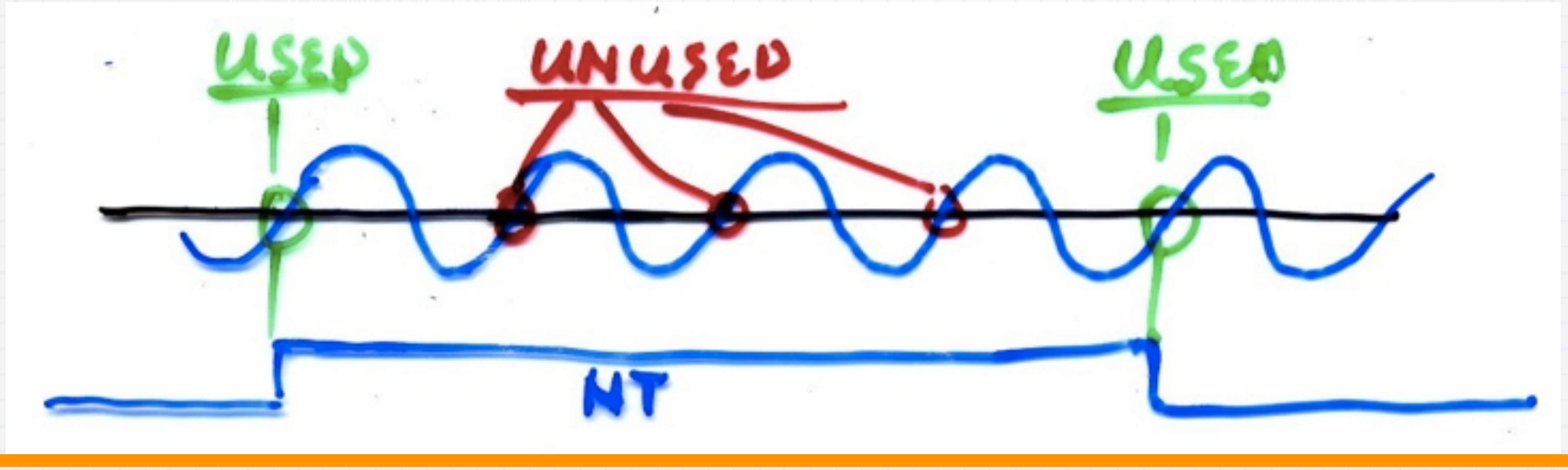
$$\int_{-\infty}^{+\infty} w_\Pi(t)\, dt = 1$$ normalization

variance

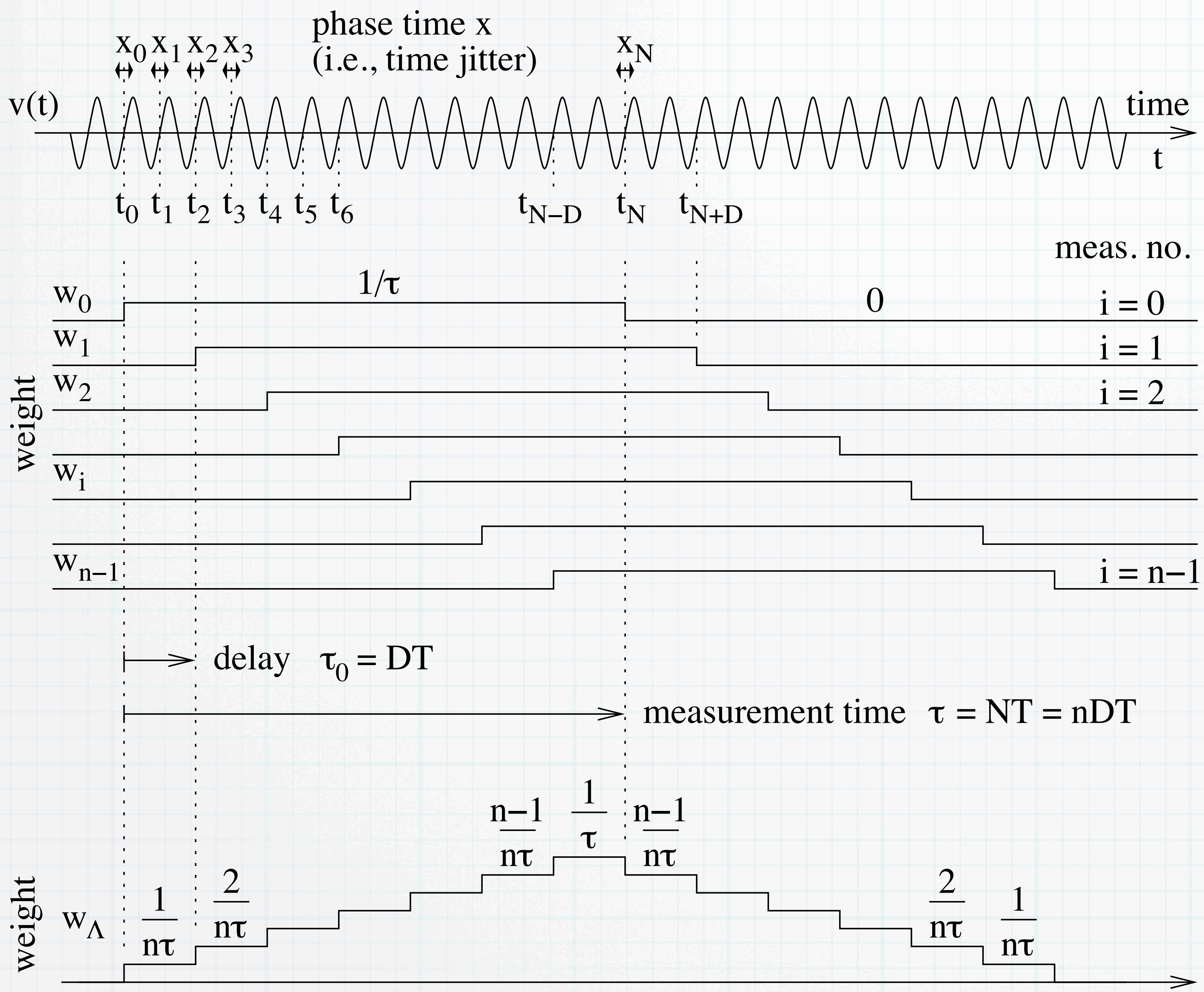
$$\sigma_y^2 = \frac{2\sigma_x^2}{\tau^2}$$ classical variance

From Π to Λ – key concept





Enhanced-resolution (Λ) counter



$$\mathbb{E}\{\nu\} = \frac{1}{n}\sum_{i=0}^{n-1} \overline{\nu}_i \qquad \overline{\nu}_i = N/\tau_i$$

Λ estimator

$$\mathbb{E}\{\nu\} = \int_{-\infty}^{+\infty} \nu(t) w_\Lambda(t)\, dt$$

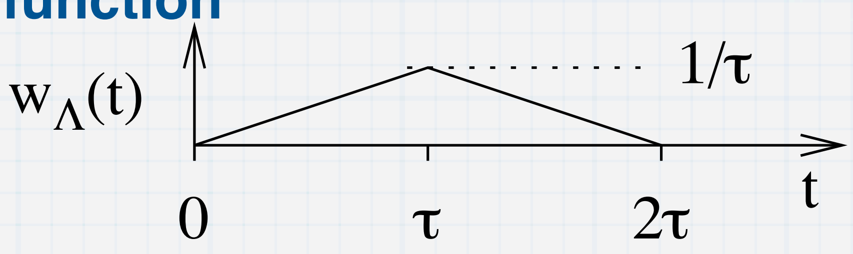
weight

$$w_\Lambda(t) = \begin{cases} t/\tau & 0 < t < \tau \\ 2 - t/\tau & \tau < t < 2\tau \\ 0 & \text{elsewhere} \end{cases}$$

normalization

$$\int_{-\infty}^{+\infty} w_\Lambda(t)\, dt = 1$$

limit τ_0 -> 0 of the weight function



white noise: the autocorrelation function is a narrow pulse, about the inverse of the bandwidth

the variance is divided by n

$$\sigma_y^2 = \frac{1}{n}\,\frac{2\sigma_x^2}{\tau^2}$$

classical variance

Actual formulae look like this

$$(\Pi) \quad \sigma_y = \frac{1}{\tau}\sqrt{2(\delta t)^2_{\text{trigger}} + 2(\delta t)^2_{\text{interpolator}}}$$

$$(\Lambda) \quad \sigma_y = \frac{1}{\tau\sqrt{n}}\sqrt{2(\delta t)^2_{\text{trigger}} + 2(\delta t)^2_{\text{interpolator}}}$$

$$n = \begin{cases} \nu_0\tau & \nu_{00} \leq \nu_I \\ \nu_I\tau & \nu_{00} > \nu_I \end{cases}$$

Understanding technical data

classical reciprocal counter

$$\sigma_y^2 = \frac{2\sigma_x^2}{\tau^2}$$ classical variance

enhanced-resolution counter

$$\sigma_y^2 = \frac{1}{n}\,\frac{2\sigma_x^2}{\tau^2}$$ classical variance

low frequency: full speed

$$\tau_0 = T \quad \Longrightarrow \quad n = \nu_{00}\tau$$

$$\sigma_y^2 = \frac{1}{\nu_{00}}\,\frac{2\sigma_x^2}{\tau^3}$$ classical variance

high frequency: housekeeping takes time

$$\tau_0 = DT \ \text{ with } D>1 \quad \Longrightarrow \quad n = \nu_{00}\tau$$

$$\sigma_y^2 = \frac{1}{\nu_I}\,\frac{2\sigma_x^2}{\tau^3}$$ classical variance

the slope of the classical variance tells the whole story

$$1/\tau^2 \quad \Longrightarrow \quad \Pi \text{ estimator (classical reciprocal)}$$

$$1/\tau^3 \quad \Longrightarrow \quad \Lambda \text{ estimator (enhanced-resolution)}$$

look for formulae and plots in the instruction manual

Examples

Stanford SRS-620

$$\begin{bmatrix}\text{RMS}\\\text{resolution}\\\text{(in Hz)}\end{bmatrix} = \frac{\text{frequency}}{\text{gate time}}\sqrt{\frac{(25\text{ ps})^2 + \left[\begin{pmatrix}\text{short term}\\\text{stability}\end{pmatrix} \times \begin{pmatrix}\text{gate}\\\text{time}\end{pmatrix}\right]^2 + 2\times \begin{bmatrix}\text{trigger}\\\text{jitter}\end{bmatrix}^2}{\text{N}}}$$

RMS resolution	$\sigma_\nu = \nu_{00}\sigma_y$
frequency	ν_{00}
gate time	τ

Agilent 53132A

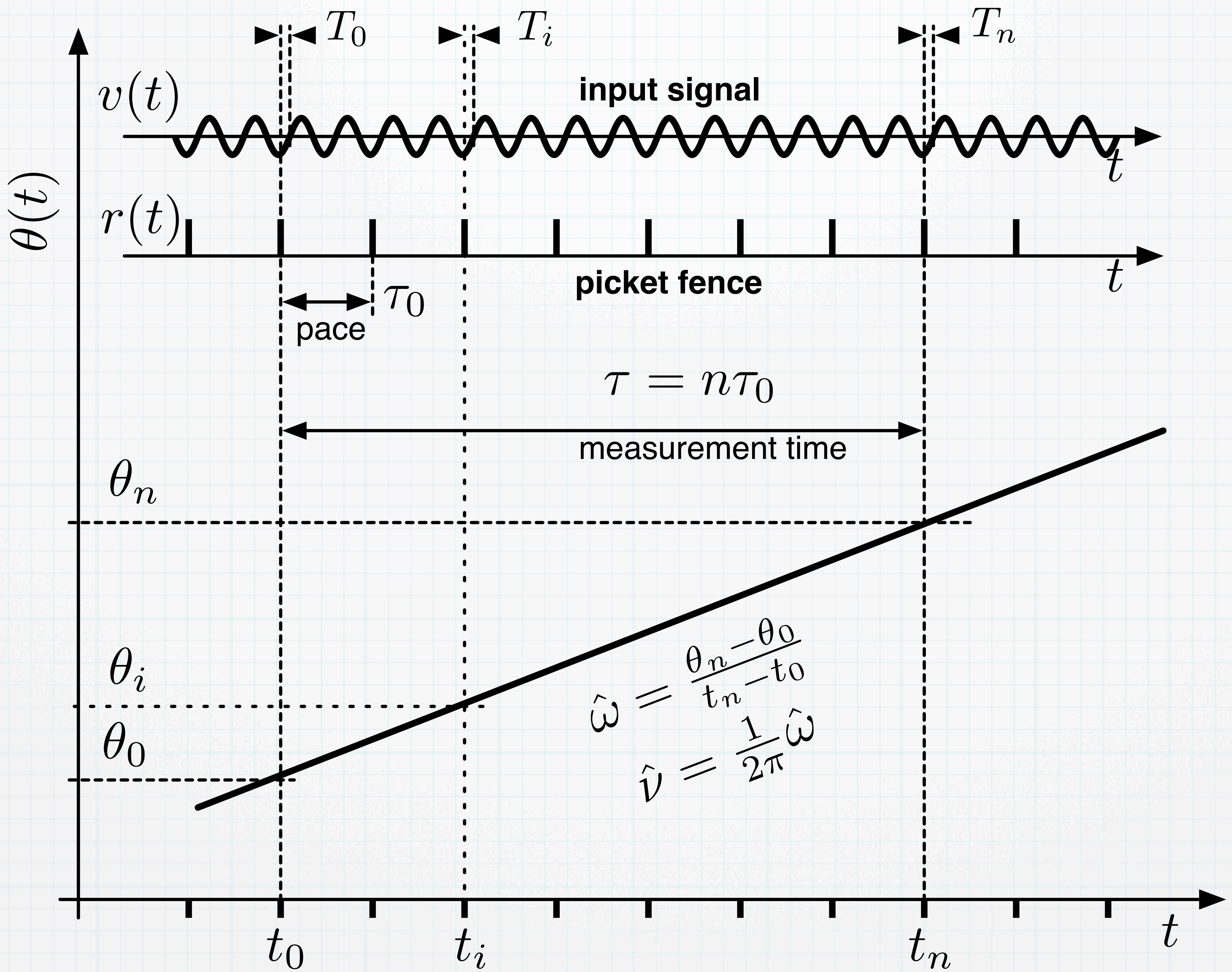
$$\begin{bmatrix}\text{RMS}\\\text{resolution}\end{bmatrix} = \begin{pmatrix}\text{frequency}\\\text{or period}\end{pmatrix} \times \left[\frac{4\times\sqrt{(t_{\text{res}})^2 + 2\times(\text{trigger error})^2}}{(\text{gate time})\times\sqrt{\text{no. of samples}}} + \frac{t_{\text{jitter}}}{\text{gate time}}\right]$$

$t_{\text{res}} = 225$ ps
$t_{\text{jitter}} = 3$ ps

$$\text{number of samples} = \begin{cases}(\text{gate time})\times(\text{frequency}) & \text{for } f < 200\text{ kHz}\\(\text{gate time})\times 2\times10^5 & \text{for } f \geq 200\text{ kHz}\end{cases}$$

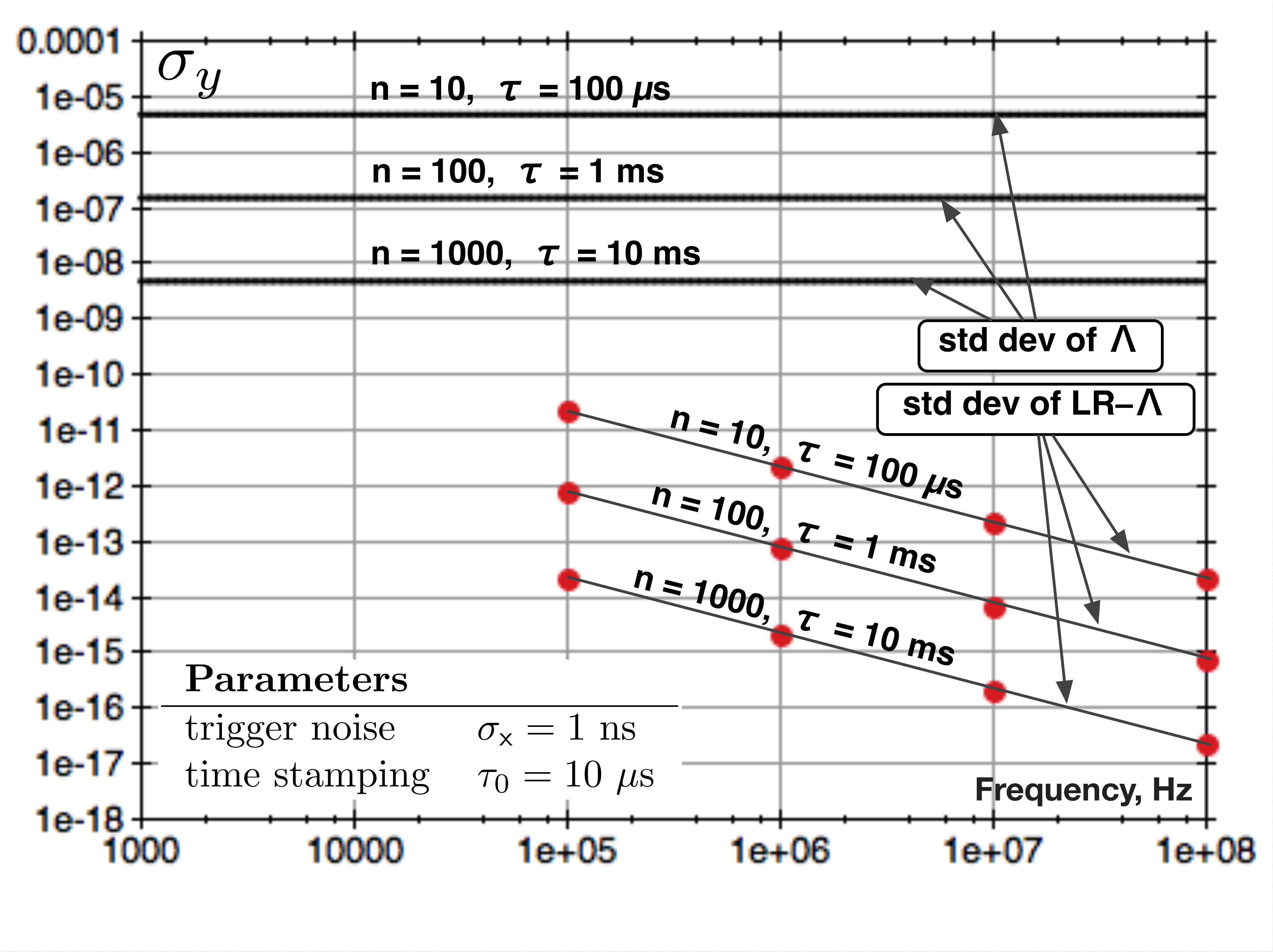
RMS resolution	$\sigma_\nu = \nu_{00}\sigma_y$ or $\sigma_T = T_{00}\sigma_y$
frequency	ν_{00}
period	T_{00}
gate time	τ
no. of samples	$n = \begin{cases}\nu_{00}\tau & \nu_{00} < 200\text{ kHz}\\\tau\times 2\times10^5 & \nu_{00} \geq 200\text{ kHz}\end{cases}$

Linear-regression counter



Linear regression on a sequence of time stamps provides accurate estimation of frequency

Linear regression vs. Λ estimator



The linear regression estimator is asymptotically equivalent to the Λ estimator

5 – Advanced statistics

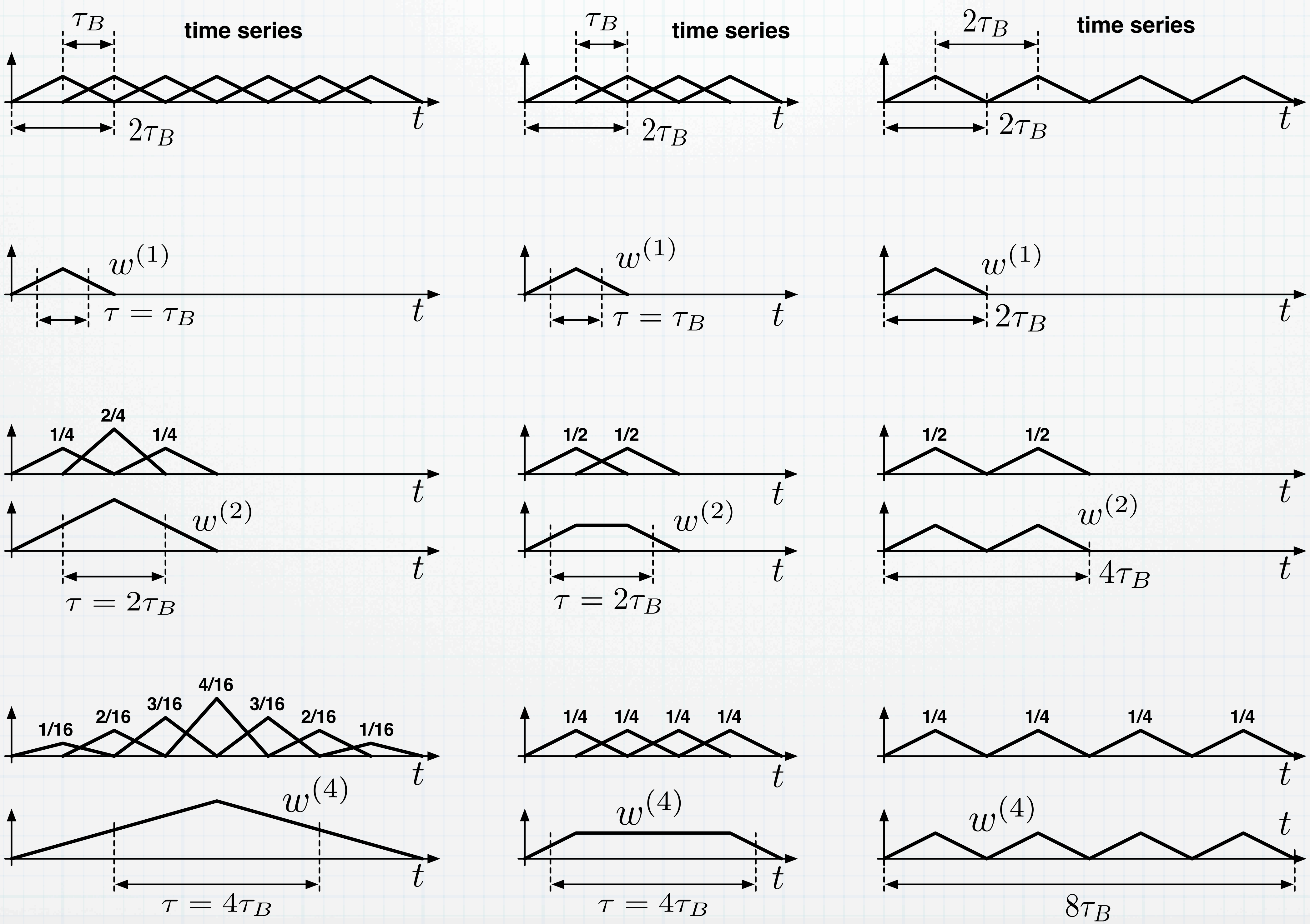
Decimation of Λ estimates

How to combine contiguous Λ measures in a way that makes sense

(A) optimally overlapped, recursive decimation

(B) optimally overlapped, the average converges to a Π estimate

(C) separated measures, multi-triangle average



Allan variance

definition

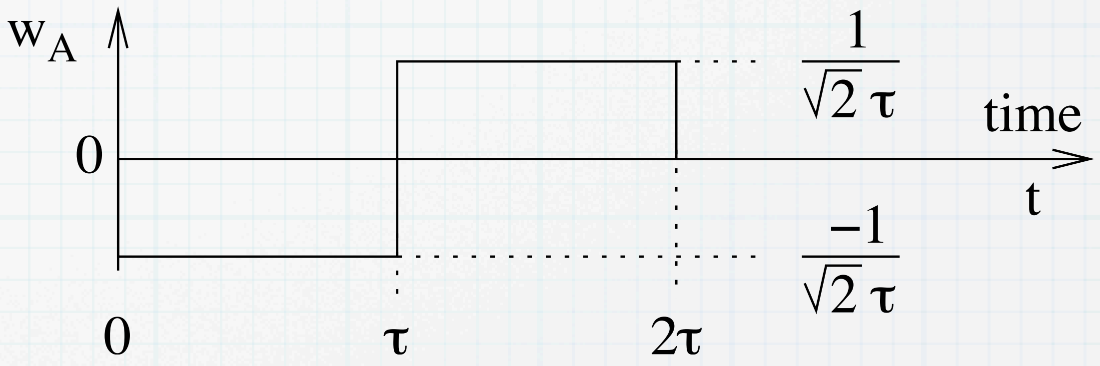
$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\Big[\overline{y}_{k+1} - \overline{y}_k\Big]^2\right\}$$

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\frac{1}{\tau}\int_{(k+1)\tau}^{(k+2)\tau} y(t)\,dt - \frac{1}{\tau}\int_{k\tau}^{(k+1)\tau} y(t)\,dt\right]^2\right\}$$

wavelet-like variance

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\Big[\int_{-\infty}^{+\infty} y(t)\,w_A(t)\,dt\Big]^2\right\}$$

$$w_A = \begin{cases} -\frac{1}{\sqrt{2}\tau} & 0 < t < \tau \\ \frac{1}{\sqrt{2}\tau} & \tau < t < 2\tau \\ 0 & \text{elsewhere} \end{cases}$$



energy

$$E\{w_A\} = \int_{-\infty}^{+\infty} w_A^2(t)\,dt = \frac{1}{\tau}$$

the Allan variance differs from a wavelet variance in the normalization on power, instead of on energy

Phase noise & friends

$$v(t) = V_p\,[1 + \alpha(t)] \cos\,[2\pi\nu_0 t + \varphi(t)]$$

random phase fluctuation

$$S_\varphi(f) = \text{PSD of } \varphi(t)$$

power spectral density

it is measured as

$$S_\varphi(f) = \tfrac{1}{T}\mathbb{E}\left\{\Phi(f)\Phi^*(f)\right\} \quad \text{(expectation)}$$

$$S_\varphi(f) \approx \tfrac{1}{T}\left\langle \Phi(f)\Phi^*(f)\right\rangle_m \quad \text{(average)}$$

$$\mathcal{L}(f) = \frac{1}{2} S_\varphi(f) \quad \text{dBc}$$

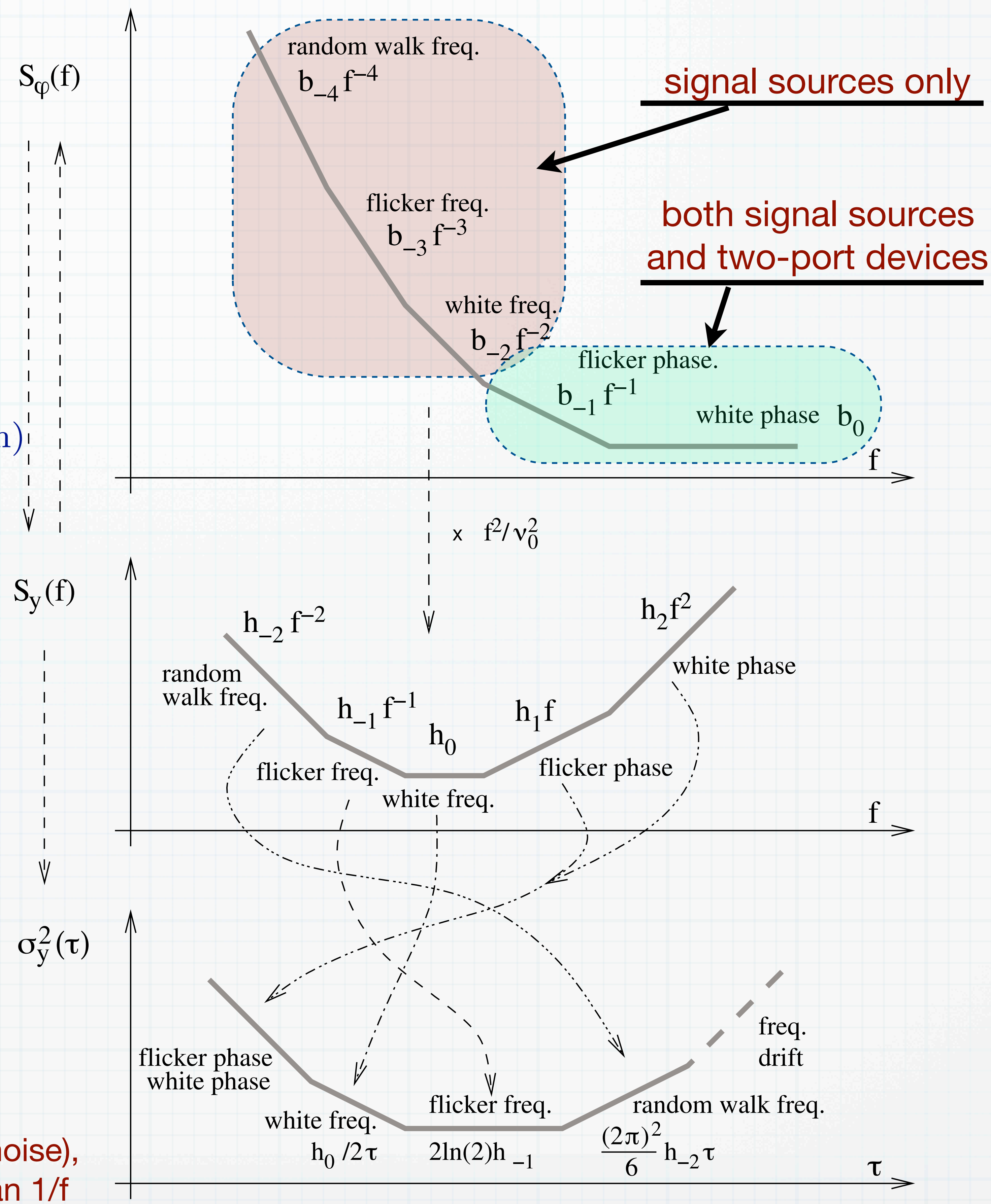
random fractional-frequency fluctuation

$$y(t) = \frac{\dot\varphi(t)}{2\pi\nu_0} \quad \Rightarrow \quad S_y = \frac{f^2}{\nu_0^2} S_\varphi(f)$$

Allan variance
(two-sample wavelet-like variance)

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{y}_{k+1} - \overline{y}_k\right]^2\right\} .$$

approaches a half-octave bandpass filter (for white noise),
hence it converges even with processes steeper than 1/f



Figures are from E. Rubiola, Phase noise and frequency stability in oscillators, © Cambridge University Press

Modified Allan variance

definition

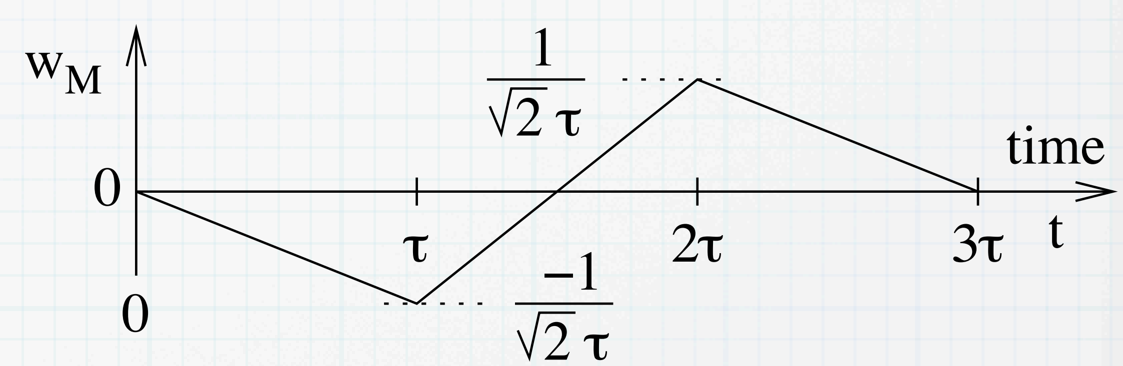
$$\mathrm{mod}\,\sigma_y^2(\tau) = \mathbb{E}\left\{ \frac{1}{2}\left[\frac{1}{n}\sum_{i=0}^{n-1}\left(\frac{1}{\tau}\int_{(i+n)\tau_0}^{(i+2n)\tau_0} y(t)\,dt - \frac{1}{\tau}\int_{i\tau_0}^{(i+n)\tau_0} y(t)\,dt \right)\right]^2 \right\}$$

with $\tau = n\tau_0$.

wavelet-like variance

$$\mathrm{mod}\,\sigma_y^2(\tau) = \mathbb{E}\left\{ \left[\int_{-\infty}^{+\infty} y(t)\,w_M(t)\,dt \right]^2 \right\}$$

$$w_M = \begin{cases} -\frac{1}{\sqrt{2}\tau^2}t & 0 < t < \tau \\ \frac{1}{\sqrt{2}\tau^2}(2t-3) & \tau < t < 2\tau \\ -\frac{1}{\sqrt{2}\tau^2}(t-3) & 2\tau < t < 3\tau \\ 0 & \text{elsewhere} \end{cases}$$



energy

$$E\{w_M\} = \int_{-\infty}^{+\infty} w_M^2(t)\,dt = \frac{1}{2\tau}$$

compare the energy

$$E\{w_M\} = \frac{1}{2}\,E\{w_A\}$$

this explains why the mod Allan variance is always lower than the Allan variance

Spectra and variances

Noise Type	$S_y(f)$	Allan (σ^2_{A})	Modified Allan	Triangle
White PM	$h_2 f^2$	$\frac{3\, f_H}{4\, \pi^2} h_2 \tau^{-2}$ $= \sigma^2_{\mathrm{A}}(\tau)$	$\frac{3}{8\, \pi^2} h_2 \tau^{-3}$ $= \frac{1}{2\, f_H \tau} \sigma^2_{\mathrm{A}}(\tau)$	$\frac{2}{\pi^2} h_2 \tau^{-3}$ $= \frac{8}{3\, f_H \tau} \sigma^2_{\mathrm{A}}(\tau)$
Flicker PM	$h_1 f$	$\frac{1.038+3\, ln(2\, \pi f_H \tau)}{4\, \pi^2}\, h_1 \tau^{-2}$ $= \sigma^2_{\mathrm{A}}(\tau)$	$\frac{3\, ln(\frac{256}{27})}{8\, \pi^2}\, h_1 \tau^{-2}$ $= \frac{3.37}{3.12+3\ln \pi f_H \tau}\, \sigma^2_{\mathrm{A}}(\tau)$	$\frac{6\, ln(\frac{27}{16})}{\pi^2}\, h_1 \tau^{-2}$ $= \frac{12.56}{3.12+3\ln \pi f_H \tau}\, \sigma^2_{\mathrm{A}}(\tau)$
White FM	h_0	$\frac{1}{2} h_0 \tau^{-1}$ $= \sigma^2_{\mathrm{A}}(\tau)$	$\frac{1}{4} h_0 \tau^{-1}$ $= 0.50\, \sigma^2_{\mathrm{A}}(\tau)$	$\frac{2}{3} h_0 \tau^{-1}$ $= 1.33\, \sigma^2_{\mathrm{A}}(\tau)$
Flicker FM	$h_{-1} f^{-1}$	$2\, ln(2)\, h_{-1}$ $= \sigma^2_{\mathrm{A}}(\tau)$	$2\, ln(\frac{3\, 3^{11/16}}{4})\, h_{-1}$ $= 0.67\, \sigma^2_{\mathrm{A}}(\tau)$	$(24\, ln(2) - \frac{27}{2}\, ln(3))\, h_{-1}$ $= 1.30\, \sigma^2_{\mathrm{A}}(\tau)$
Random Walk FM	$h_{-2} f^{-2}$	$\frac{2}{3}\, \pi^2\, h_{-2}\, \tau$ $= \sigma^2_{\mathrm{A}}(\tau)$	$\frac{11}{20}\, \pi^2\, h_{-2}\, \tau$ $= 0.82\, \sigma^2_{\mathrm{A}}(\tau)$	$\frac{23}{30}\, \pi^2\, h_{-2}\, \tau$ $= 1.15\, \sigma^2_{\mathrm{A}}(\tau)$
Frequency Drift ($\dot{y} = D_y$)	-	$\frac{1}{2} D_y^2 \tau^2$	$\frac{1}{2} D_y^2 \tau^2$	$\frac{1}{2} D_y^2 \tau^2$

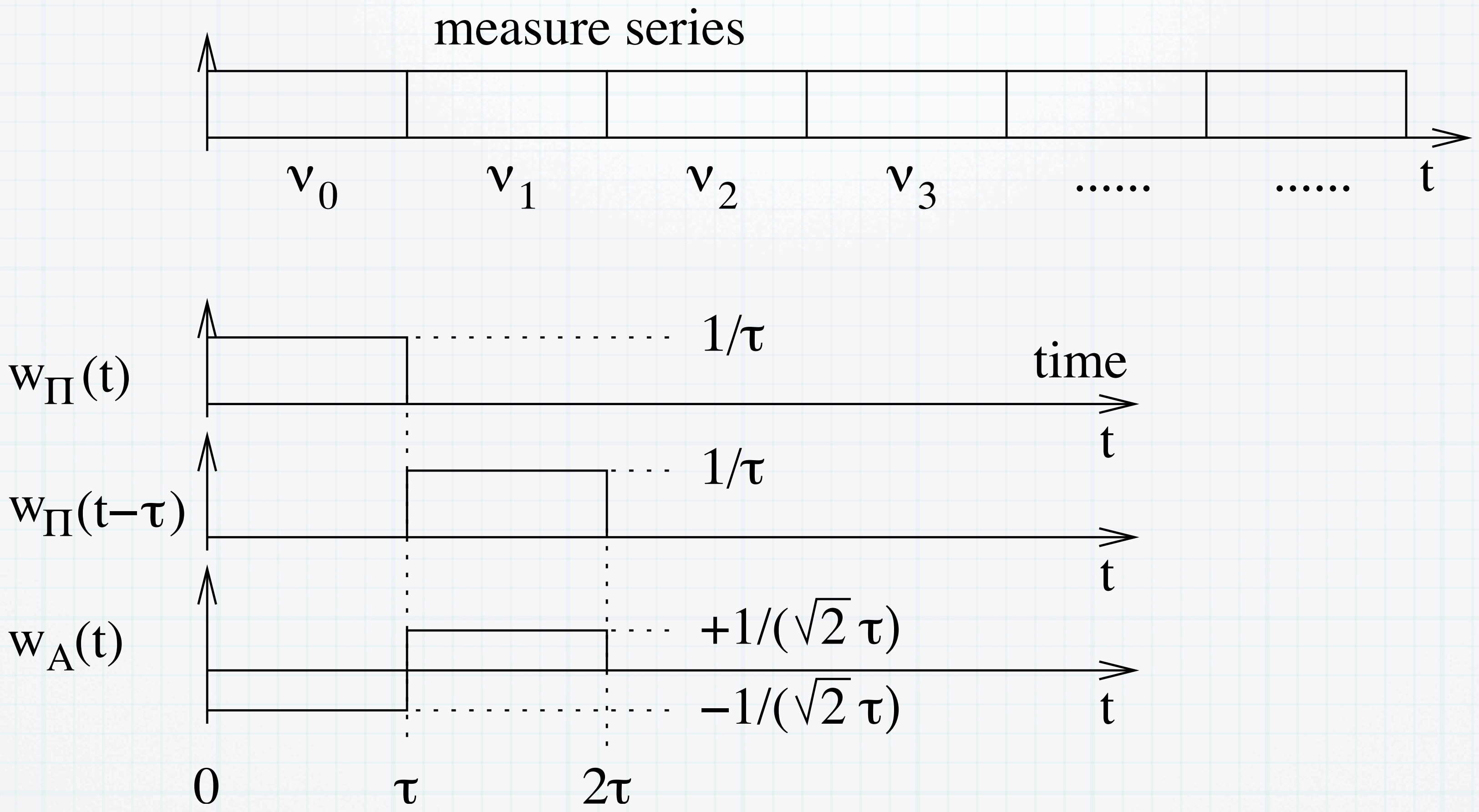
ν_{00} is replaced with ν_0 for consistency with the general literature

f_H is the high cutoff frequency, needed for the noise power to be finite

S.T. Dawkins, J.J. McFerran, A.N. Luiten, IEEE Trans. UFFC 54(5) p.918–925, May 2007

Π estimator —> Allan variance

given a series of contiguous non-overlapped measures



the Allan variance is easily evaluated

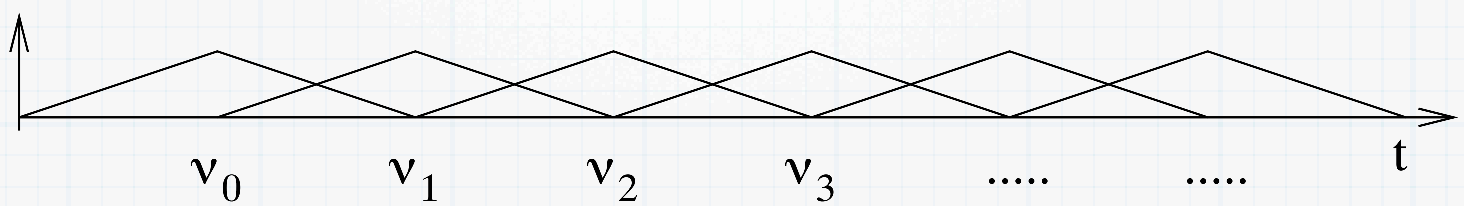
$$\sigma_y^2(\tau) = \mathbb{E}\left\{ \frac{1}{2} \left[\overline{y}_{k+1} - \overline{y}_k \right]^2 \right\}$$

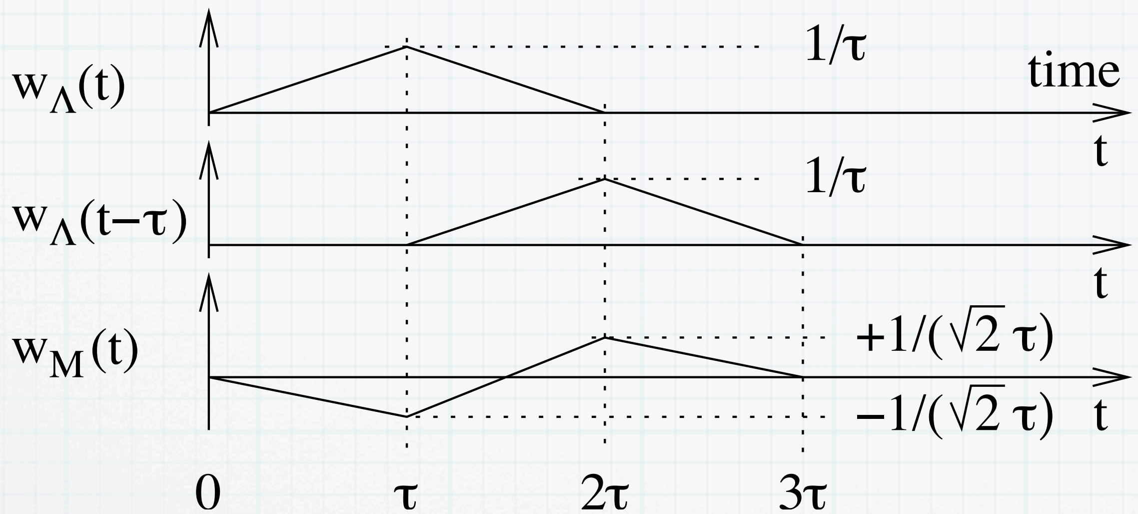
Overlapped Λ estimator —> MVAR

by feeding a series of Λ-estimates of frequency in the formula of the Allan variance

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{y}_{k+1} - \overline{y}_k\right]^2\right\}$$

as they were Π-estimates





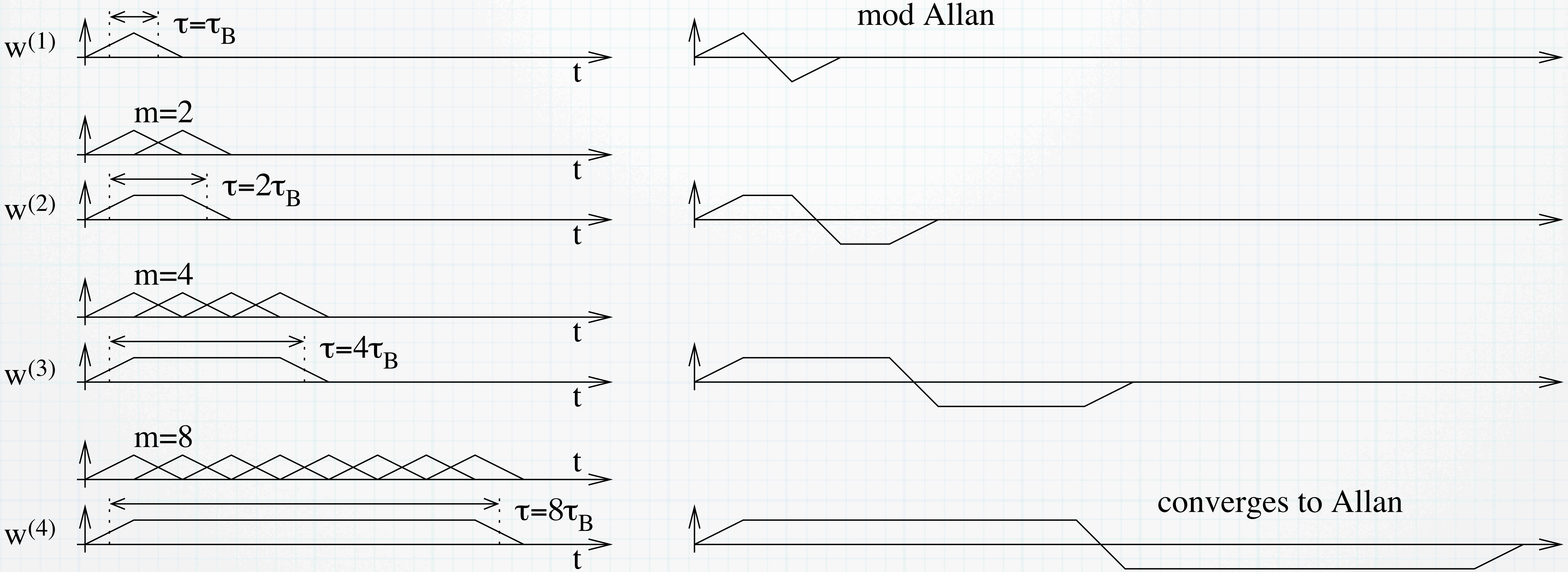
one gets exactly the modified Allan variance!

$$\mathrm{mod}\,\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\frac{1}{n}\sum_{i=0}^{n-1}\left(\frac{1}{\tau}\int_{(i+n)\tau_0}^{(i+2n)\tau_0} y(t)\,dt - \frac{1}{\tau}\int_{i\tau_0}^{(i+n)\tau_0} y(t)\,dt\right)\right]^2\right\}$$

with $\tau = n\tau_0$.

Joining contiguous values to increase τ

graphical proof



m = 1 **mod Allan**
m = 2 **this is not what we expected**
m = 4 **...**
m ≥ 8 **the variance converges to the (non modified) Allan variance**

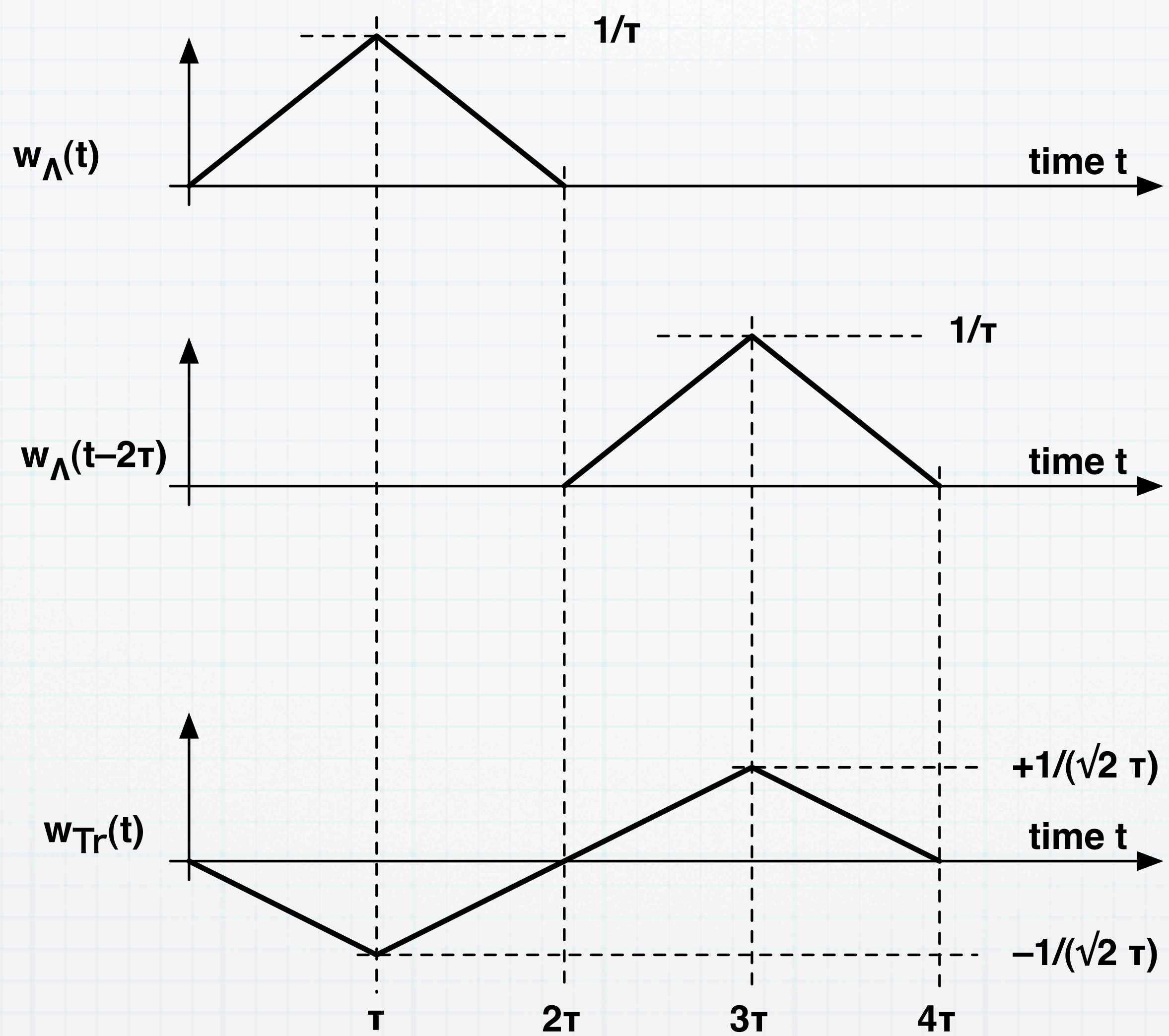
There is a mistake in one of my articles: I believed that in the case of the Agilent counters the contiguous measures were overlapped. They are not.

Non-overlapped Λ estimator —> TrVAR

by feeding a series of Λ-estimates of frequency in the formula of the Allan variance

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{y}_{k+1} - \overline{y}_k\right]^2\right\}$$

as they were Π-estimates



one gets the triangular variance!

S.T. Dawkins, J.J. McFerran, A.N. Luiten, IEEE Trans. UFFC 54(5) p.918–925, May 2007

E. Rubiola
Experimental methods in AM-PM noise metrology
— book project —

Front cover: The Wind Machines
Artist view of the AM and PM noise
Courtesy of Roberto Bergonzo, http://robertobergonzo.com

Conclusions

- Review of general techniques
- The trigger may not what it seems – however, in unusual conditions
- Sophisticated interpolation techniques
- The thermometer-code interpolator is simple with modern FPGAs
- The Λ (triangular) estimator provides higher resolution than the Π (rectangular) estimator, but can be used with periodic phenomena only
- Mistakes are around the corner if the counter inside is not understood
- Some of the reported ideas are suitable to education laboratories and classroom works (I used a bicycle odometer and milestones to demonstrate the Λ estimator)

Thanks to J. Dick (JPL, retired), V. Giordano (Femto-ST), C. Greenhall (JPL), D. Howe (NIST) M. Oxborrow (NPL), F. Vernotte (Observatory of Besancon) for discussions & more

To know more:

1 - http://rubiola.org, slides and articles
2 - http://arxiv.org, document arXiv:physics/0503022v1
3 - Rev. of Sci. Instrum. vol. 76 no. 5, art.no. 054703, May 2005.

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