Seminar given at the
Advanced Photon Source, Argonne National Laboratory
Argonne, IL, USA

The Measurement of AM-PM Noise, and the Origin of Noise in Oscillators

June 8 2010, ANL - APS conference room
Part 1: 11 AM, Part 2: 2 PM

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home page http://rubiola.org

Acknowledgements

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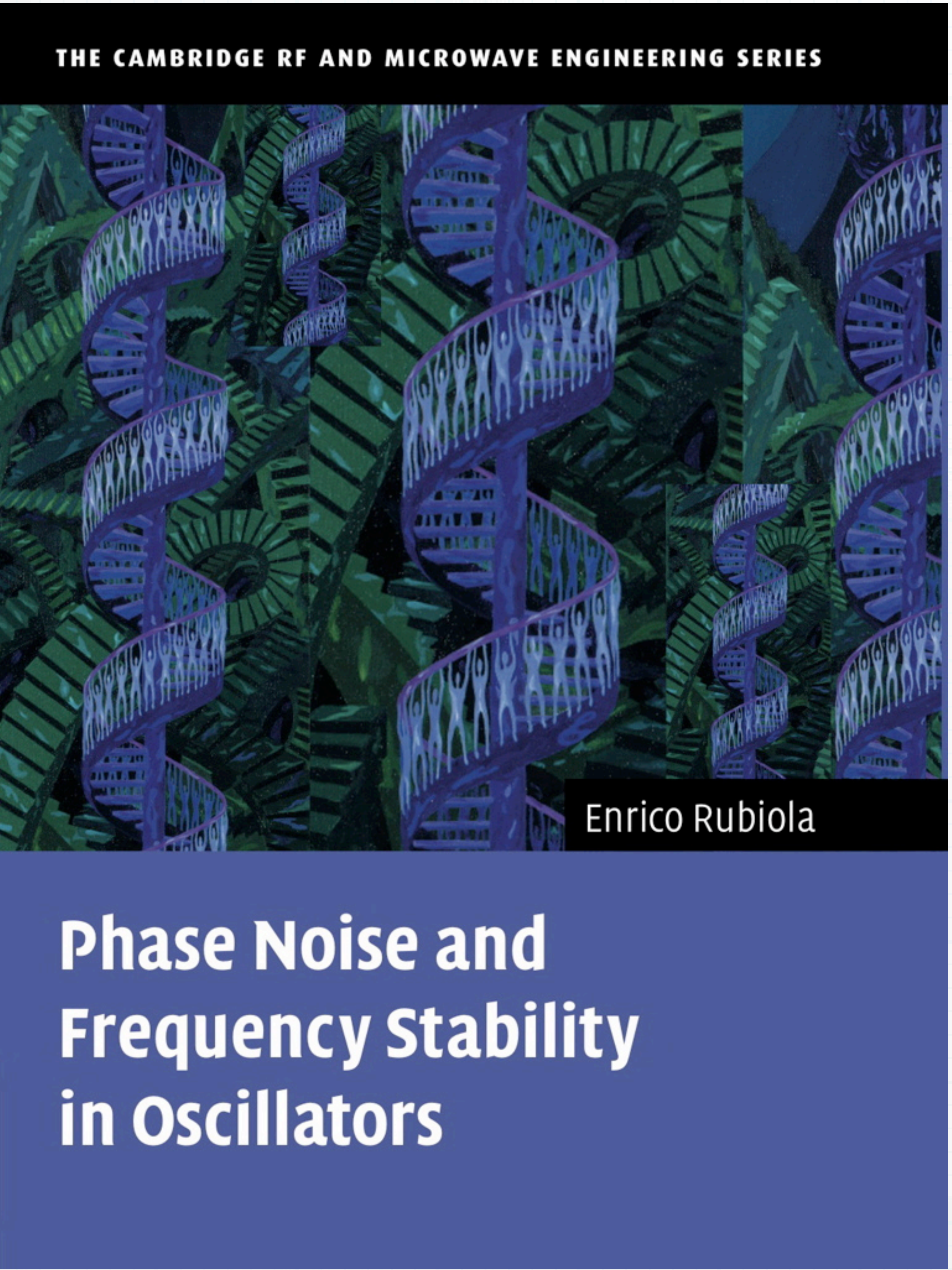
I am grateful to Lute Maleki and to John Dick for numerous discussions during my visits at the NASA JPL, which are the first seed of my approach to the oscillator noise

This material would never have existed without continuous discussions, help and support of Vincent Giordano, FEMTO-ST, over more than a dozen of years

Part 1 is based on the draft book
E. Rubiola, *Experimental methods in AM-PM noise metrology*

Part 2 is based on
E.Rubiola, *Phase noise and frequency stability in oscillators*, Cambridge 2008,
and on the complementary material
E. Rubiola, R. Brendel, A generalization of the Leeson effect, arXiv:1004.5539 [physics.ins-det]

Phase noise and frequency stability in oscillators



Cambridge University Press, November 2008

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Another book is in progress, on the **Experimental methods for the measurement of AM/PM noise**

E. Rubiola
Experimental methods in AM-PM noise metrology
— book project —

The Wind Machines
Artist view of the AM and PM noise
Courtesy of Roberto Bergonzo, http://robertobergonzo.com

AM and PM noise

in RF/microwave and photonic systems

Enrico Rubiola

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- Experiments
- Photonic systems
- Cross-spectum measurements
- Bridge method
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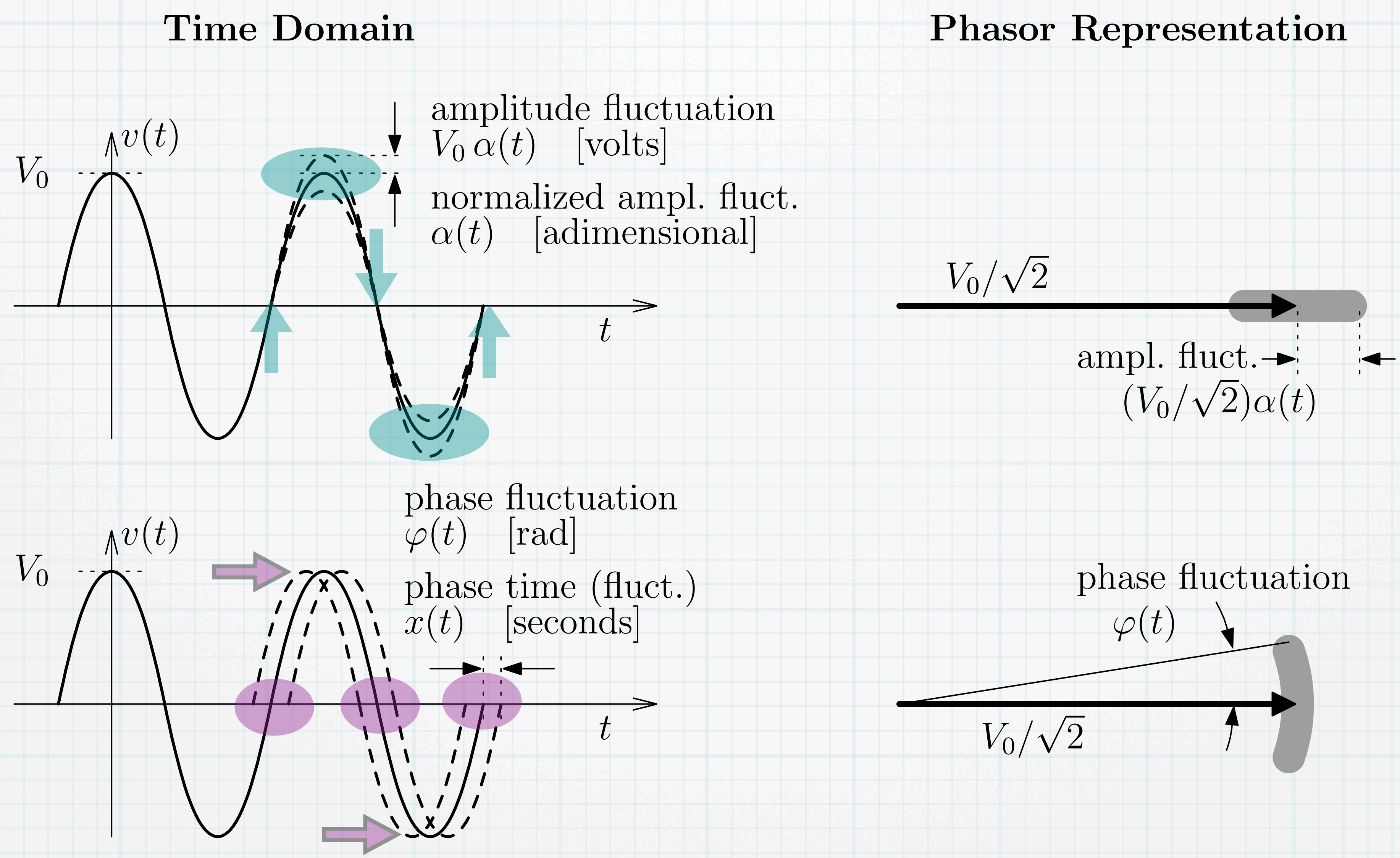
Phase noise & friends

Clock signal affected by noise



polar coordinates $$v(t) = V_0\,[1+\alpha(t)]\cos[\omega_0 t + \varphi(t)]$$

Cartesian coordinates $$v(t) = V_0 \cos\omega_0 t + n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t$$

under low noise approximation

$$|n_c(t)| \ll V_0 \quad \text{and} \quad |n_s(t)| \ll V_0$$

It holds that

$$\alpha(t) = \frac{n_c(t)}{V_0} \quad \text{and} \quad \varphi(t) = \frac{n_s(t)}{V_0}$$

Physical quantities

$$v(t) = V_0\,[1 + \alpha(t)]\cos\,[2\pi\nu_0 t + \varphi(t)]$$

Allow φ(t) to exceed ±π and count the number of turns, so that φ(t) describes the clock fluctuation in full

radian

phase noise $\varphi(t)$

$$(\Delta\nu)(t) = \frac{1}{2\pi}\frac{d\varphi(t)}{dt}$$

Hertz

$(\Delta\nu)(t)$ frequency fluctuation

$$x = \frac{\varphi}{2\pi\nu_0}$$

$$y(t) = \frac{1}{\nu_0}(\Delta\nu)(t)$$

phase-time (fluctuation) $x(t)$

second

$$y(t) = \frac{dx(t)}{dt}$$

$y(t)$ fractional-frequency fluctuation

dimensionless

Phase noise & friends

$$v(t) = V_p\,[1+\alpha(t)]\cos\,[2\pi\nu_0 t + \varphi(t)]$$

random phase fluctuation

$$S_\varphi(f) = \text{PSD of } \varphi(t)$$

power spectral density

it is measured as

$$S_\varphi(f) = \frac{1}{T}\mathbb{E}\left\{\Phi(f)\Phi^*(f)\right\} \quad \text{(expectation)}$$

$$S_\varphi(f) \approx \frac{1}{T}\left\langle\Phi(f)\Phi^*(f)\right\rangle_m \quad \text{(average)}$$

$$\mathcal{L}(f) = \frac{1}{2}S_\varphi(f) \quad \text{dBc}$$

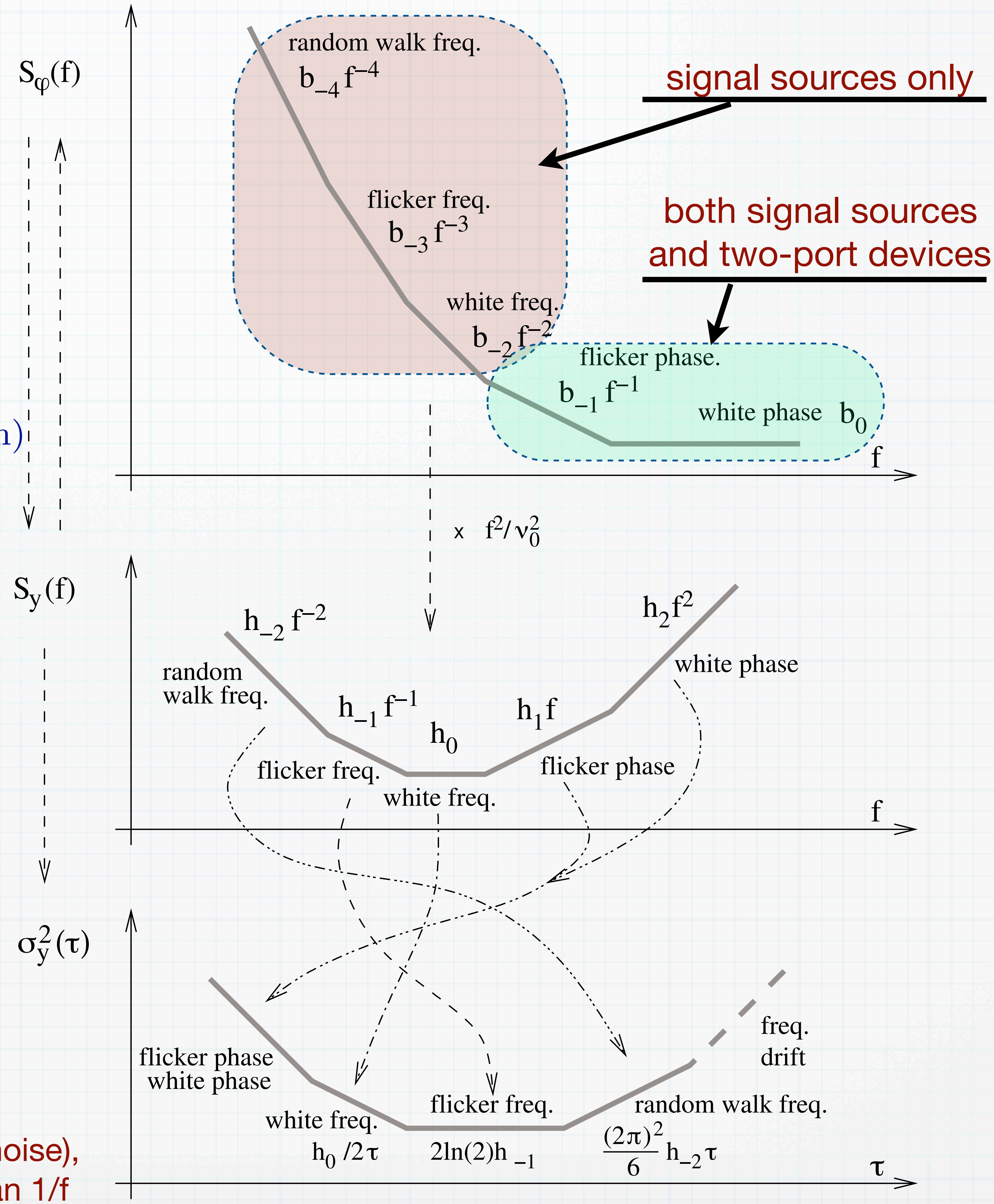
random fractional-frequency fluctuation

$$y(t) = \frac{\dot\varphi(t)}{2\pi\nu_0} \quad \Rightarrow \quad S_y = \frac{f^2}{\nu_0^2}S_\varphi(f)$$

Allan variance
(two-sample wavelet-like variance)

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{y}_{k+1}-\overline{y}_k\right]^2\right\}\,.$$

approaches a half-octave bandpass filter (for white noise),
hence it converges even with processes steeper than 1/f



Flicker never diverges in practice

$$P = \int_a^b S(f)\, df$$

$$P = \int_a^b \frac{h_{-1}}{f}\, df = h_{-1}\, \ln \frac{b}{a}$$

b/a = 10 (1 decade)
ln(b/a) ≈ 2.3 (3.6 dB)

1/a = 3600 s (1h)
b = 2 GHz (max ADC speed)
$\log_2$(b/a) = 42 (bits)
ln(b/a) ≈ 29.6 (14.7 dB)

1/a = 1E9 s (30 years)
b = 500 THz (visible)
$\log_2$(b/a) = 79 (bits)
ln(b/a) ≈ 54.6 (17.4 dB)

1/a = 85400 s (day)
b = 200 GHz (electronics)
$\log_2$(b/a) = 54 (bits)
ln(b/a) ≈ 37.4 (15.7 dB)

1/a = 1E18 s (universe lifetime)
1/b = 1E–44 s (Planck time)
$\log_2$(b/a) = 200 (bits)
ln(b/a) ≈ 143 (21.5 dB)

b/a = 1E6
$\log_2$(b/a) = 20 (bits)
ln(b/a) ≈ 13.8 (11.4 dB)

Allan variance

definition

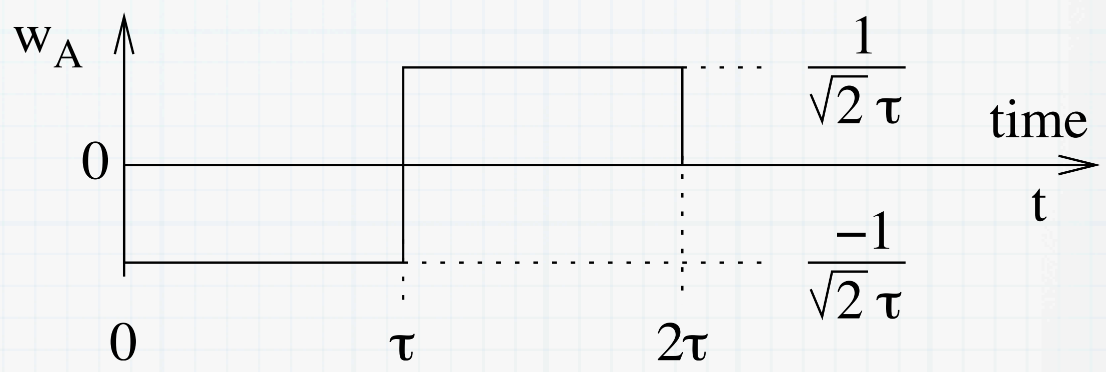
$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{y}_{k+1} - \overline{y}_k\right]^2\right\}$$

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\frac{1}{\tau}\int_{(k+1)\tau}^{(k+2)\tau} y(t)\,dt - \frac{1}{\tau}\int_{k\tau}^{(k+1)\tau} y(t)\,dt\right]^2\right\}$$

wavelet-like variance

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\left[\int_{-\infty}^{+\infty} y(t)\,w_A(t)\,dt\right]^2\right\}$$

$$w_A = \begin{cases} -\frac{1}{\sqrt{2}\tau} & 0 < t < \tau \\ \frac{1}{\sqrt{2}\tau} & \tau < t < 2\tau \\ 0 & \text{elsewhere} \end{cases}$$



energy

$$E\{w_A\} = \int_{-\infty}^{+\infty} w_A^2(t)\,dt = \frac{1}{\tau}$$

the Allan variance differs from a wavelet variance in the normalization on power, instead of on energy

Relationships between spectra and variances

noise type	$S_\varphi(f)$	$S_y(f)$	$S_\varphi \leftrightarrow S_y$	$\sigma_y^2(\tau)$	mod $\sigma_y^2(\tau)$
white PM	b_0	$h_2 f^2$	$h_2 = \dfrac{b_0}{\nu_0^2}$	$\dfrac{3 f_H h_2}{(2\pi)^2}\tau^{-2}$ $2\pi\tau f_H \gg 1$	$\dfrac{3 f_H \tau_0 h_2}{(2\pi)^2}\,\tau^{-3}$
flicker PM	$b_{-1} f^{-1}$	$h_1 f$	$h_1 = \dfrac{b_{-1}}{\nu_0^2}$	$[1.038 + 3\ln(2\pi f_H \tau)]\,\dfrac{h_1}{(2\pi)^2}\,\tau^{-2}$	$0.084\, h_1 \tau^{-2}$ $n \gg 1$
white FM	$b_{-2} f^{-2}$	h_0	$h_0 = \dfrac{b_{-2}}{\nu_0^2}$	$\dfrac{1}{2} h_0\, \tau^{-1}$	$\dfrac{1}{4} h_0\, \tau^{-1}$
flicker FM	$b_{-3} f^{-3}$	$h_{-1} f^{-1}$	$h_{-1} = \dfrac{b_{-3}}{\nu_0^2}$	$2\ln(2)\; h_{-1}$	$\dfrac{27}{20}\ln(2)\; h_{-1}$
random walk FM	$b_{-4} f^{-4}$	$h_{-2} f^{-2}$	$h_{-2} = \dfrac{b_{-4}}{\nu_0^2}$	$\dfrac{(2\pi)^2}{6} h_{-2}\tau$	$0.824\,\dfrac{(2\pi)^2}{6} h_{-2}\,\tau$
linear frequency drift $\dot{y}$				$\dfrac{1}{2}\,(\dot{y})^2\,\tau^2$	$\dfrac{1}{2}\,(\dot{y})^2\,\tau^2$

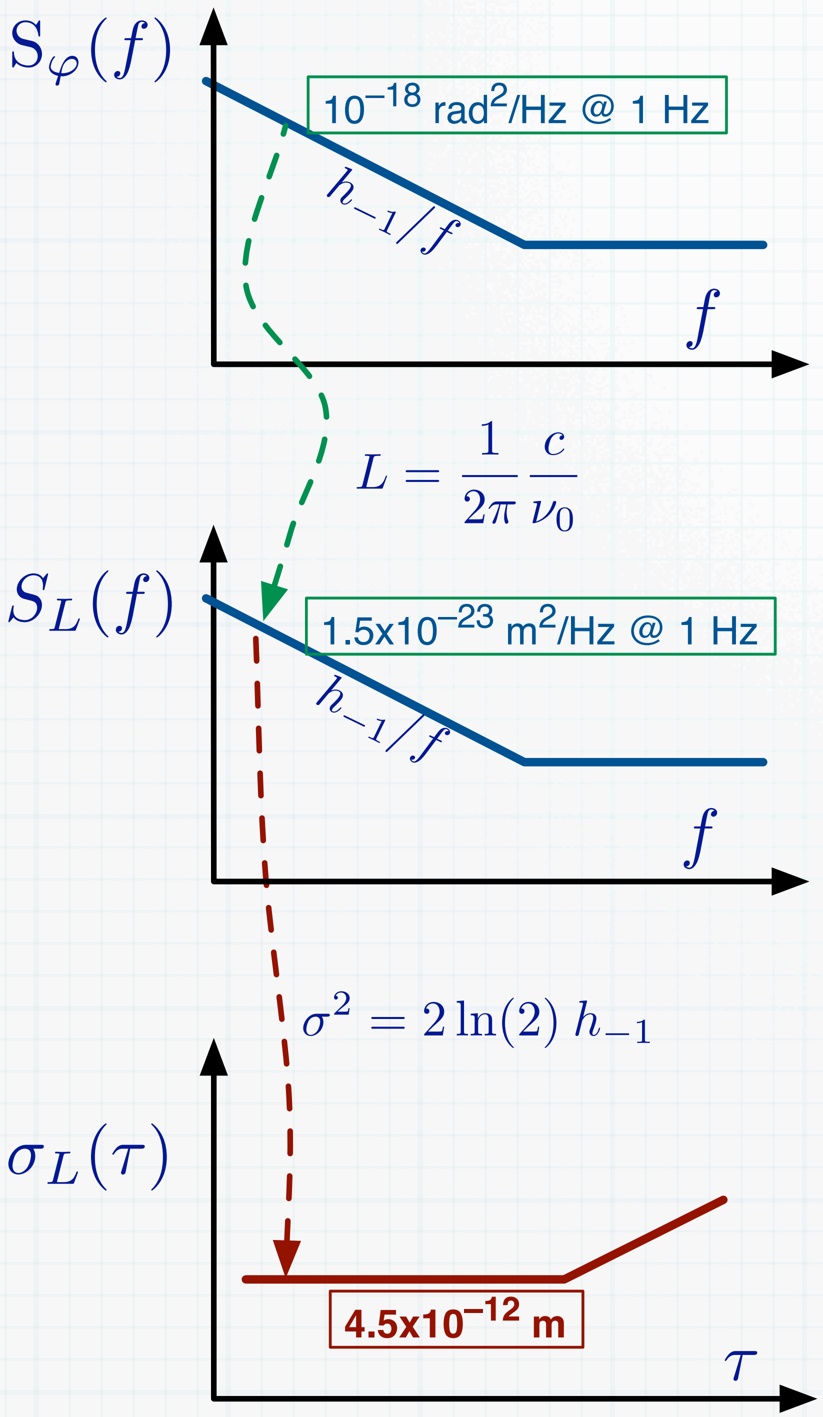
f_H is the high cutoff frequency, needed for the noise power to be finite.

Jitter

- Convert phase noise PSD into time-fluctuation PSD
- Integrate over the suitable bandwidth
- Jitter bandwidth:
  - lower limit is set by the “size” of the system
  - upper limit is set by the circuit bandwidth

Mechanical stability

b_{-1}/f is replaced with h_{-1}/f because the Allan variance formulae are written with h_{-1}



Any phase fluctuation can be converted into length fluctuation

$$L = \frac{\varphi}{2\pi}\frac{c}{\nu_0}$$

$b_{-1} = -180$ dBrad^2/Hz and $\nu_0 = 10$ GHz is equivalent to $S_L = 1.46\text{x}10^{-23}$ m^2/Hz at f = 1 Hz

Any flicker spectrum h_{-1}/f can be converted into a flat Allan variance

$$\sigma_L^2 = 2\ln(2)\,h_{-1}$$

A residual flicker of −180 dBrad^2/Hz at f = 1 Hz off the 10 GHz carrier is equivalent to

$\sigma^2 = 2\text{x}10^{-23}$ m^2 thus $\sigma = 4.5\text{x}10^{-12}$ m

for reference, the Bohr radius of the electron is R = 0.529 Å

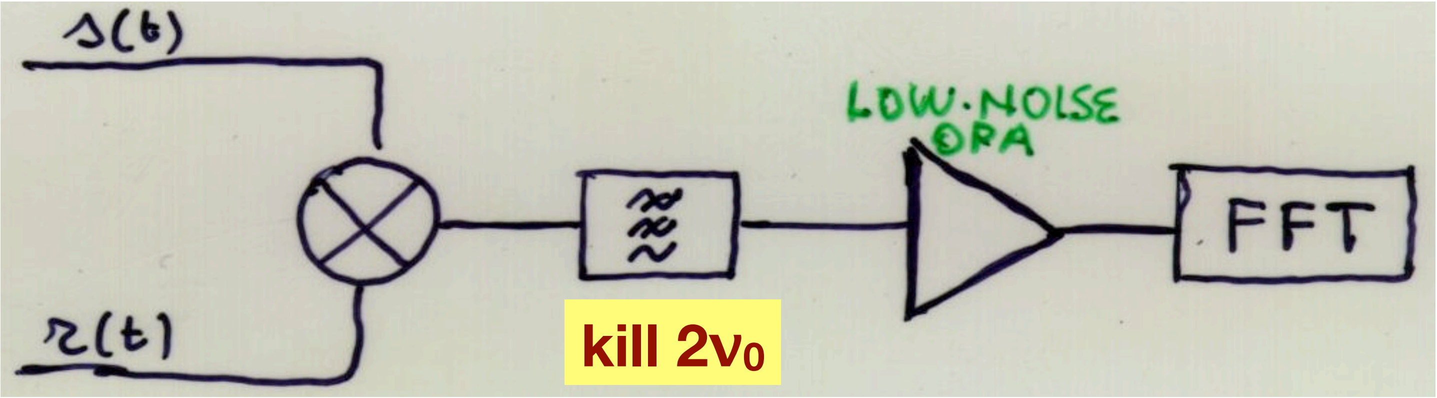
- **Don’t think “this is just engineering” !!!**
- **Learn from non-optical microscopy (bulk matter, $5\text{x}10^{-14}$ m)**
- **Careful DC section (capacitance and piezoelectricity)**
- **The best advice is to be *at least* paranoiac**

Saturated mixer & calibration

Need only basic knowledge because commercial equipment does all the job

Double-balanced mixer

phase-to-voltage detector $v_o(t) = k_\varphi\, \varphi(t)$, $k_\varphi \approx 100 \ldots 500$ mV/rad



1 – Power

narrow power range:
±5 dB around P_{nom} = 7–13 dBm
r(t) and s(t) should have ~ same P

2 – Flicker noise

due to the mixer internal diodes
typical S_φ = –140 dBrad²/Hz at 1 Hz
in average-good conditions

3 – Low gain

k_φ ~ 0.2–0.3 V/rad typ.
–10 to –14 dBV/rad

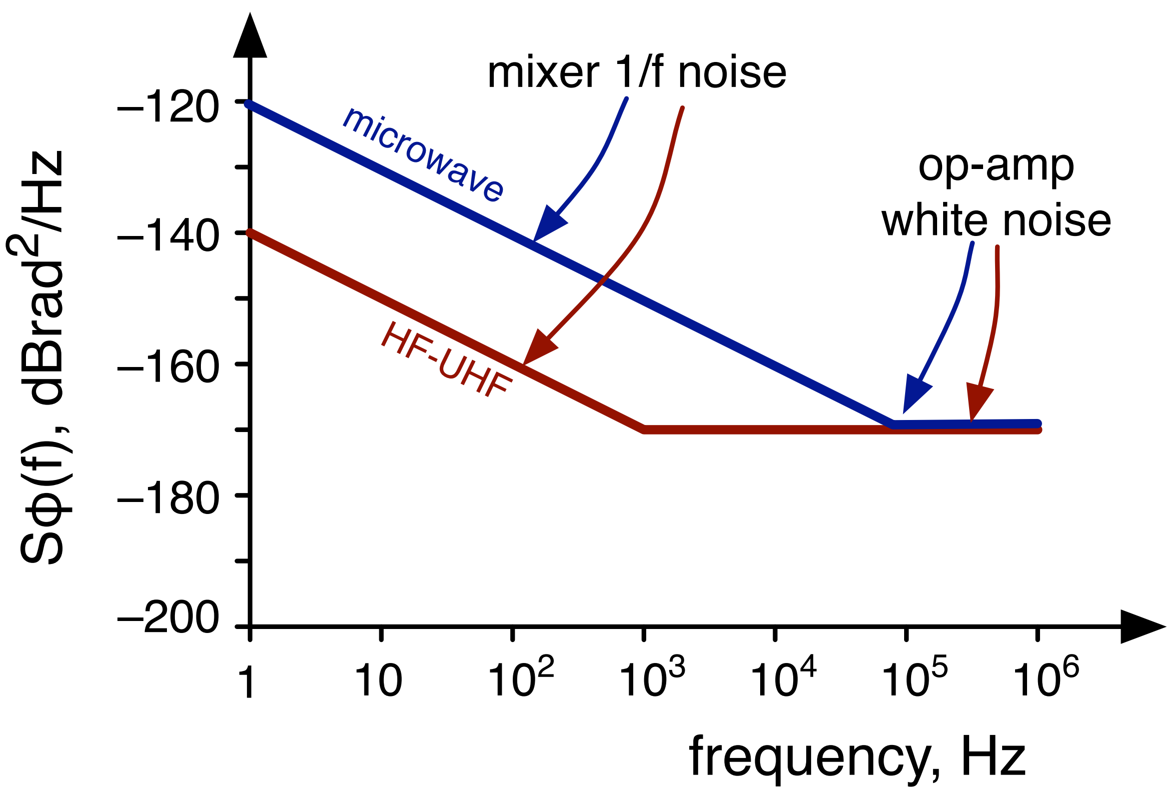
4 – White noise

due to the operational amplifier

5 – Takes in AM noise

due to the residual power-to-offset conversion

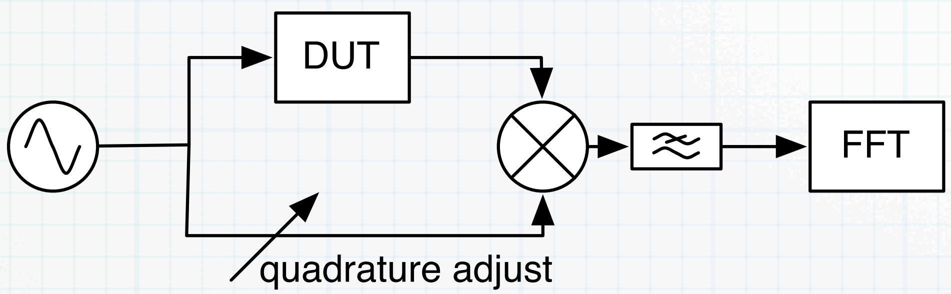
mixer background noise



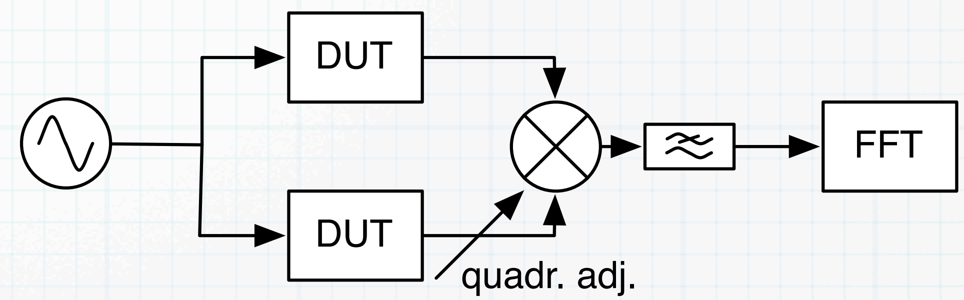
E. Rubiola, Tutorial on the double-balanced mixer, arXiv/physics/0608211

Useful schemes

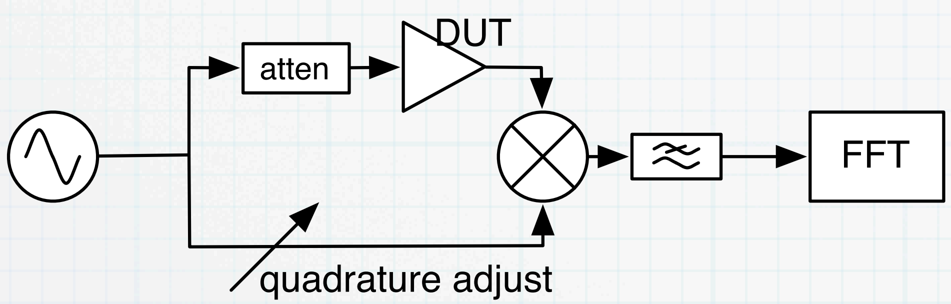
two-port device under test



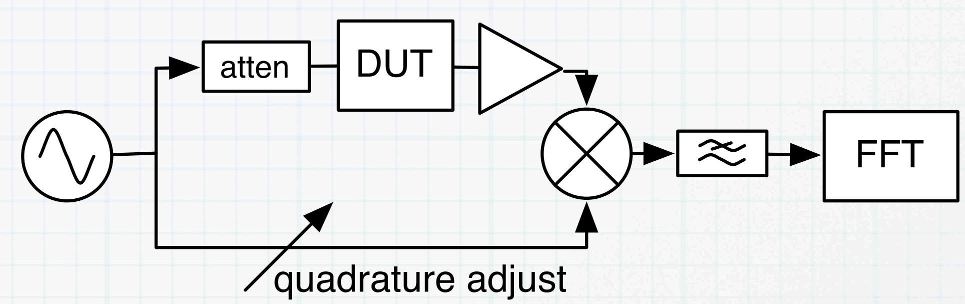
a pair of two-port devices
3 dB improved sensitivity



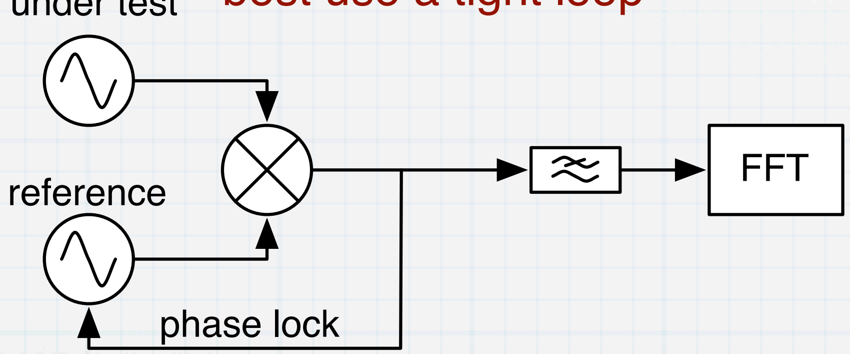
the measurement of an amplifier
needs an attenuator



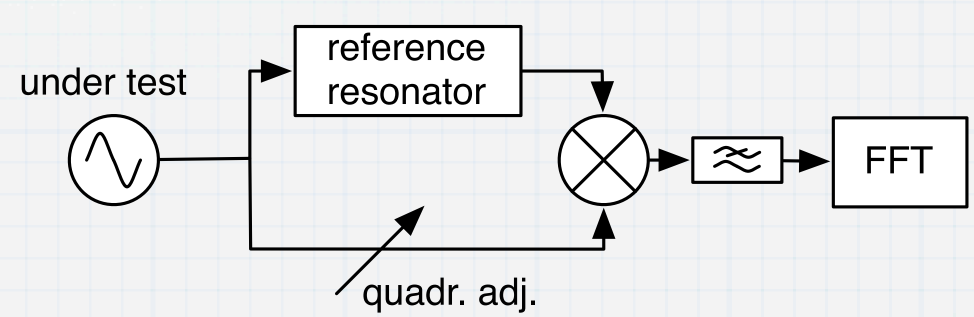
the measurement of a low-power DUT
needs an amplifier, which flickers



measure two oscillators
best use a tight loop

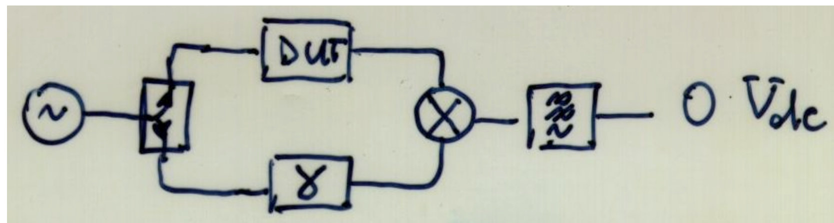


measure an oscillator vs. a resonator



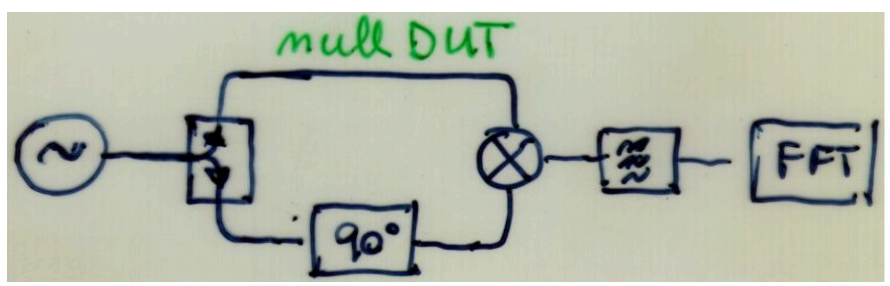
Calibration – general procedure

1 – adjust for proper operation: driving power and quadrature

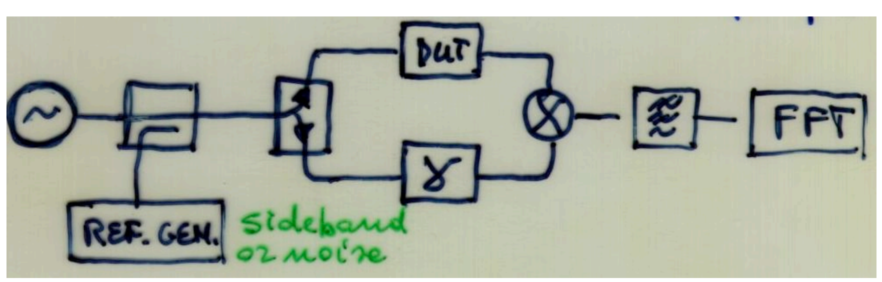


2 – measure the mixer gain k_φ (volts/rad) —> next

3 – measure the residual noise of the instrument



4 – measure the rejection of the oscillator noise



Make sure that the power and the quadrature are the same during all the calibration process

Calibration methods

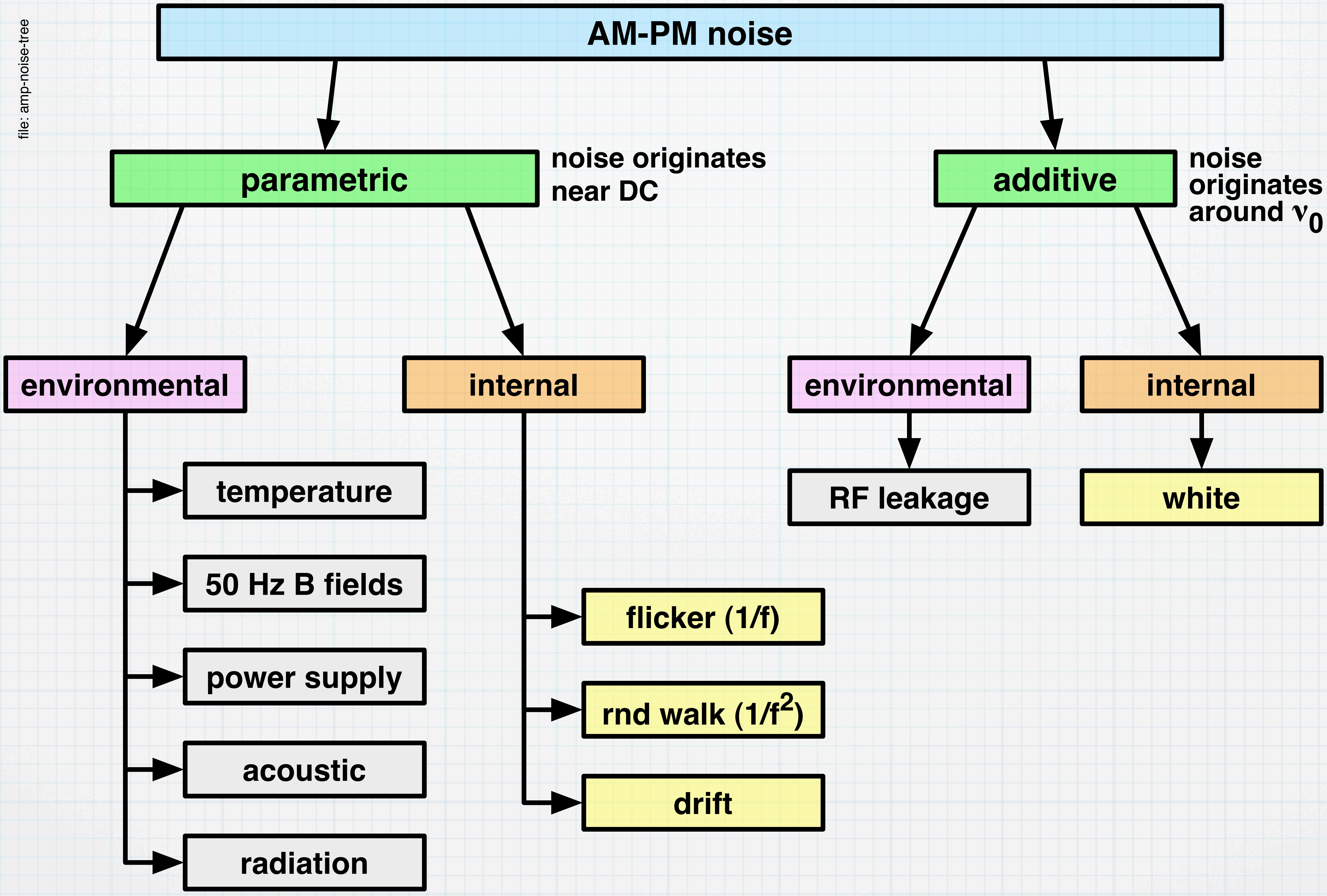
- Send to the mixer two signals with small frequency difference $\Delta\nu$ using two synthesizers driven by the same oscillator
  - read the gain on the oscilloscope using $\Delta\omega = d\varphi/dt$
  - suggested, $\Delta\nu$ = 159 Hz (1 krad/s)
- Add a sideband to the DUT signal, powers P_s and P_0
  - phase modulation $\varphi_{rms} = (P_s/2P_0)^{1/2}$
  - amplitude modulation $\alpha_{rms} = (P_s/2P_0)^{1/2}$
- Use a reference phase modulator, calibrated with a network analyzer

For precision measurements (≤2 dB), be aware of the pollution from AM, which is not the same for DUT and calibration, and also depends on the calibration method

AM-PM noise in amplifier and other devices

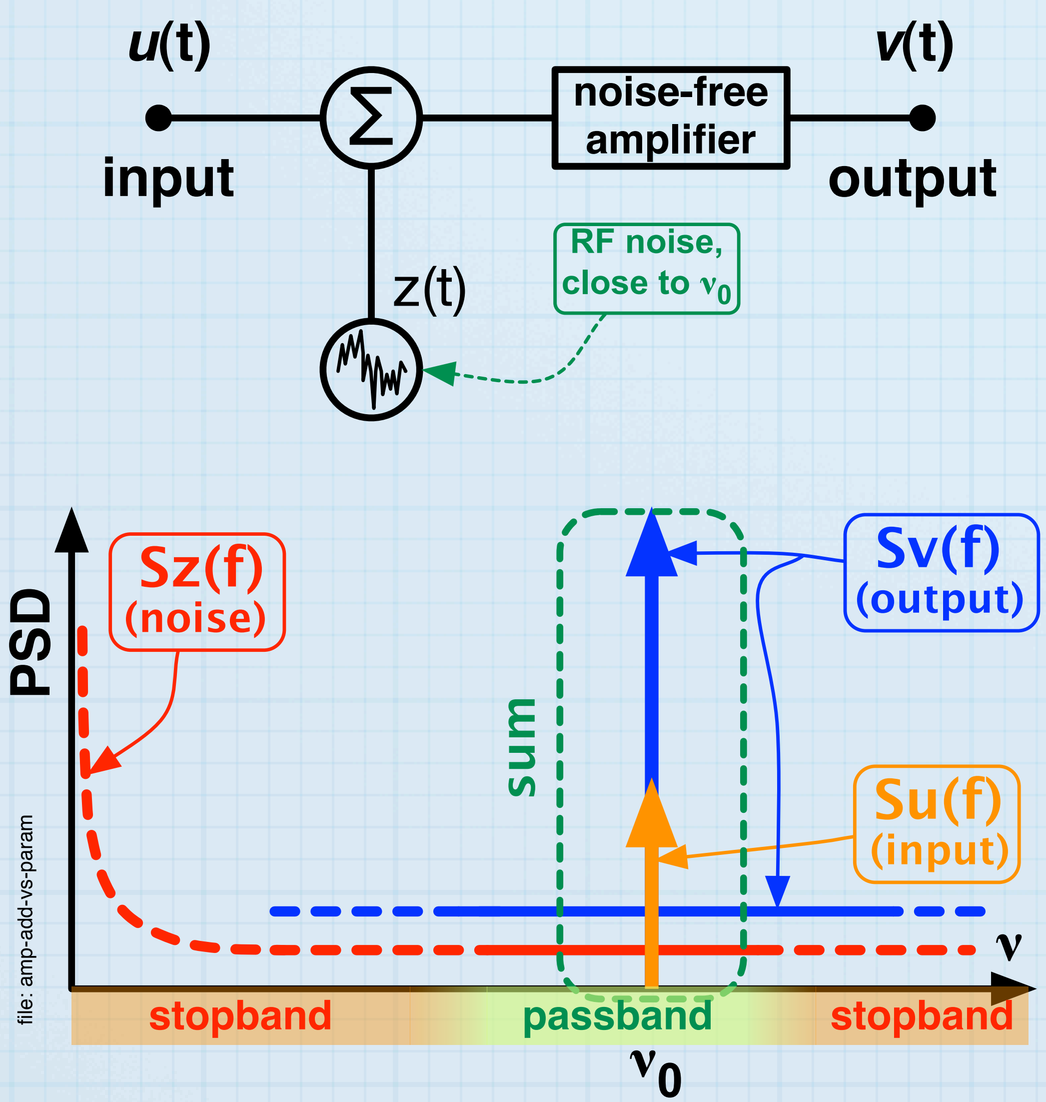
AM-PM noise types

file: amp-noise-tree



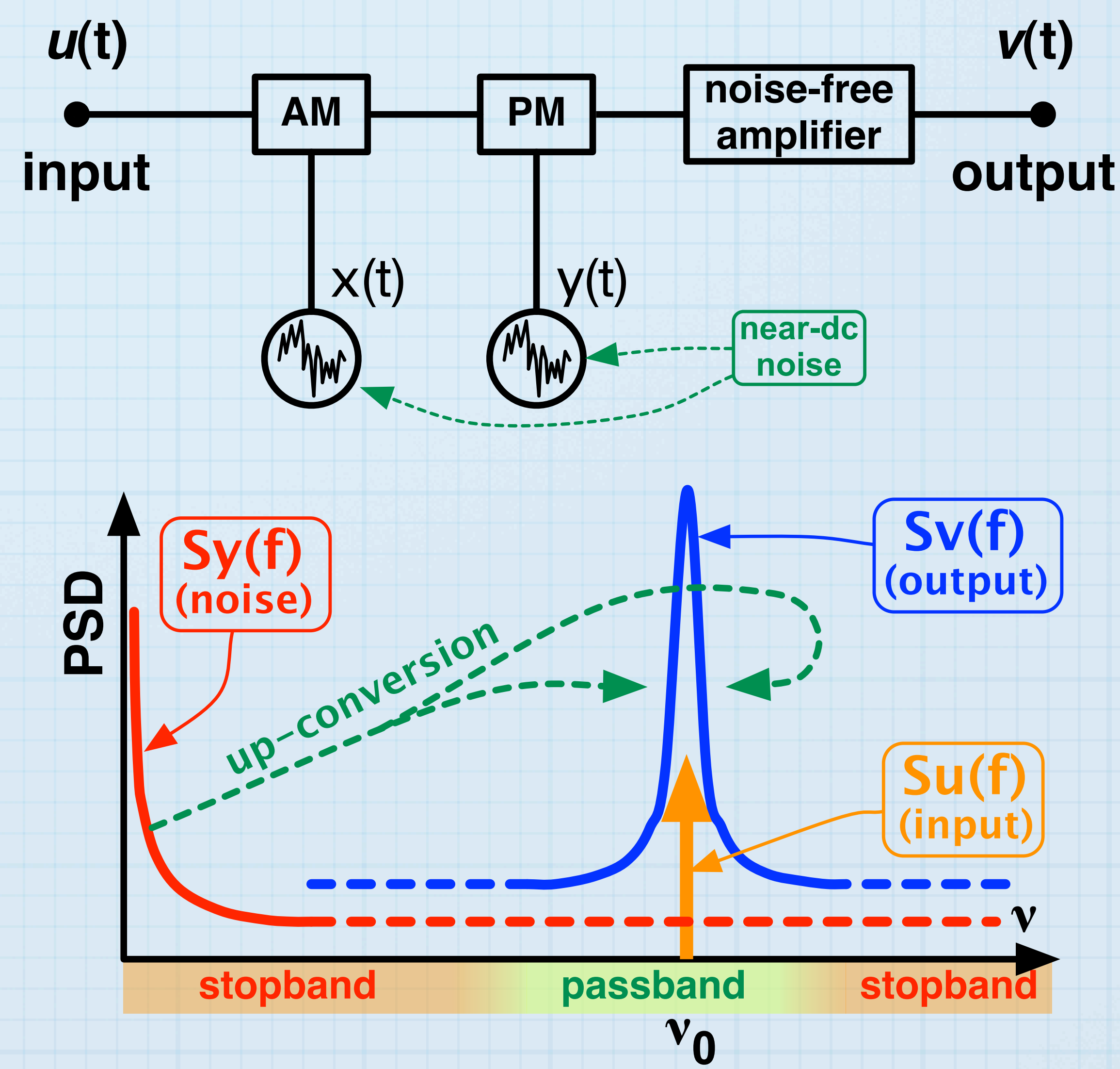
The difference between additive and parametric noise

additive noise



the noise sidebands are
independent of the carrier

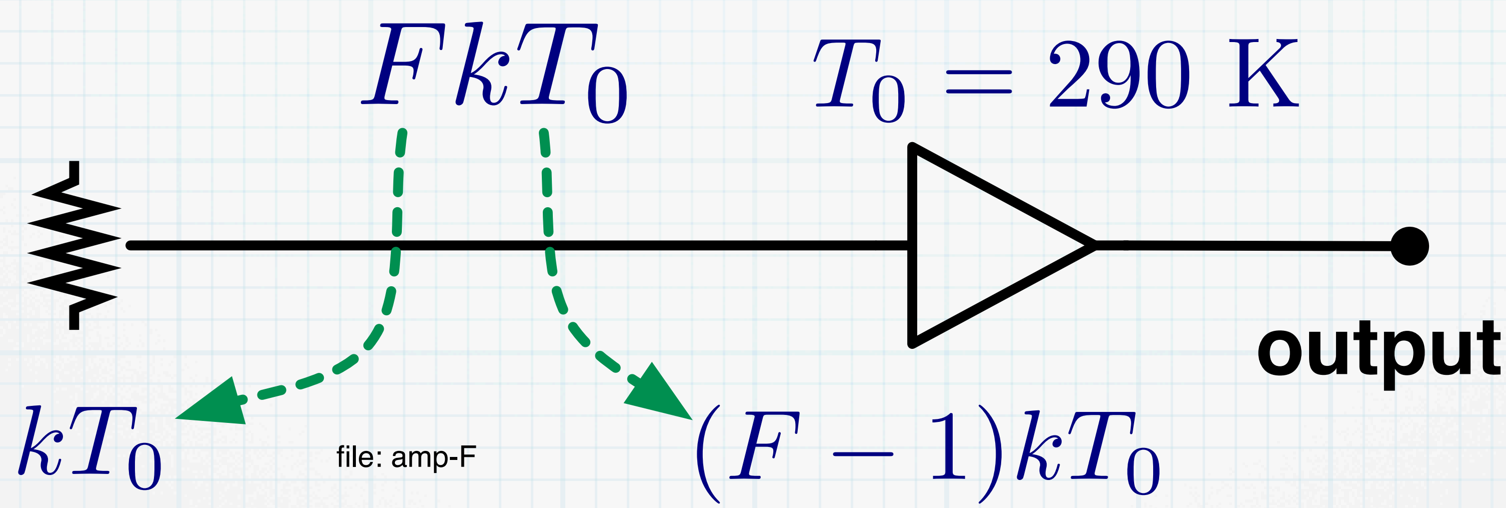
parametric noise



the noise sidebands are
proportional to the carrier

Noise figure

Noise figure $$F = \frac{\mathrm{SNR(out)}}{\mathrm{SNR(in)}}$$ **general definition**



Assume that the whole circuit is at the reference temperature T0 = 290 K (17 ºC)

The total noise referred to the amplifier input is FkT_0

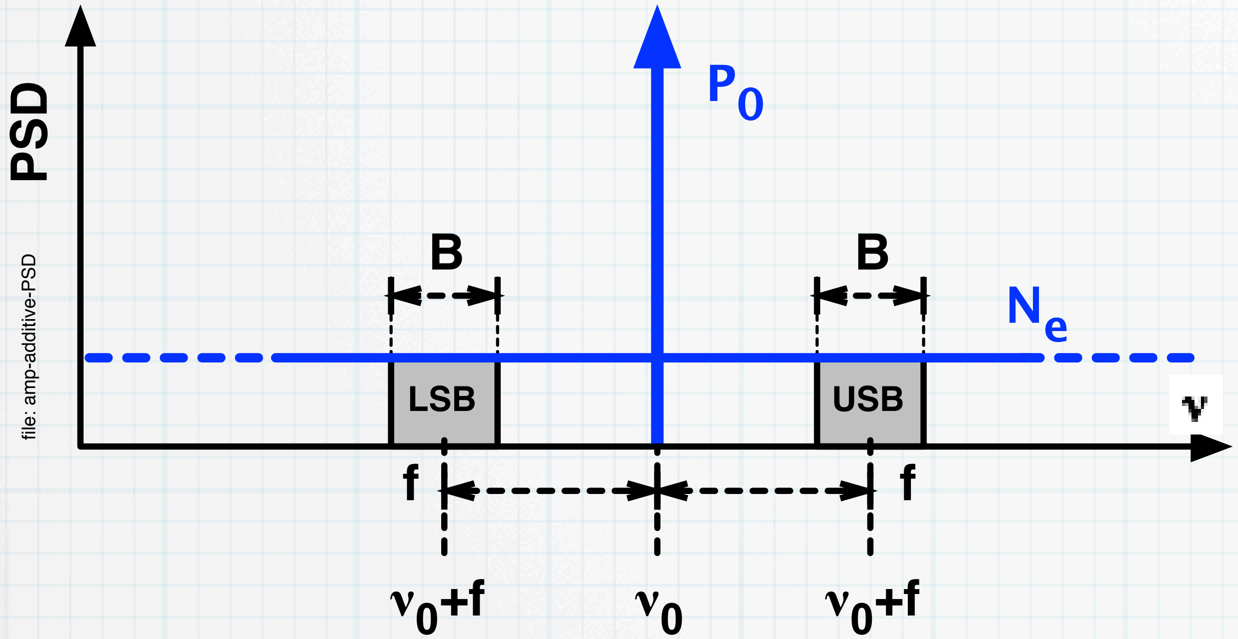
amplifiers and RF/μw devices

$$FkT_0 = kT_e = k(T_a + T_0) \qquad T_0 = 290\,\mathrm{K}$$

$$F = \frac{T_a + T_0}{T_0} \qquad \text{and} \qquad T_a = (F-1)T_0$$

Warning: the noise figure is a radio-engineer concept, can be misleading in optics

Amplifier white phase noise



Noise is equally split between AM and PM

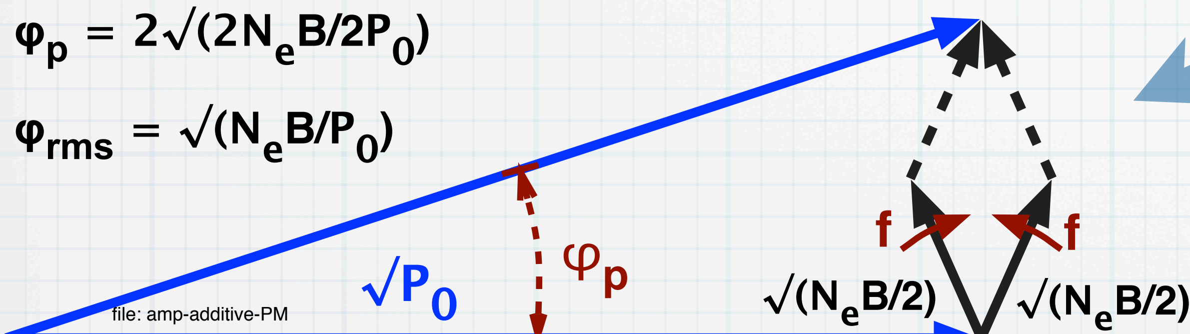
PM (rms)

$$v_{\mathrm{usb}}(t) = \imath\sqrt{N_e B/2}\, e^{\imath 2\pi f t}$$

$$v_{\mathrm{lsb}}(t) = -\imath\sqrt{N_e B/2}\, e^{-\imath 2\pi f t}$$

$\varphi_p = 2\sqrt{(2N_e B/2P_0)}$

$\varphi_{rms} = \sqrt{(N_e B/P_0)}$



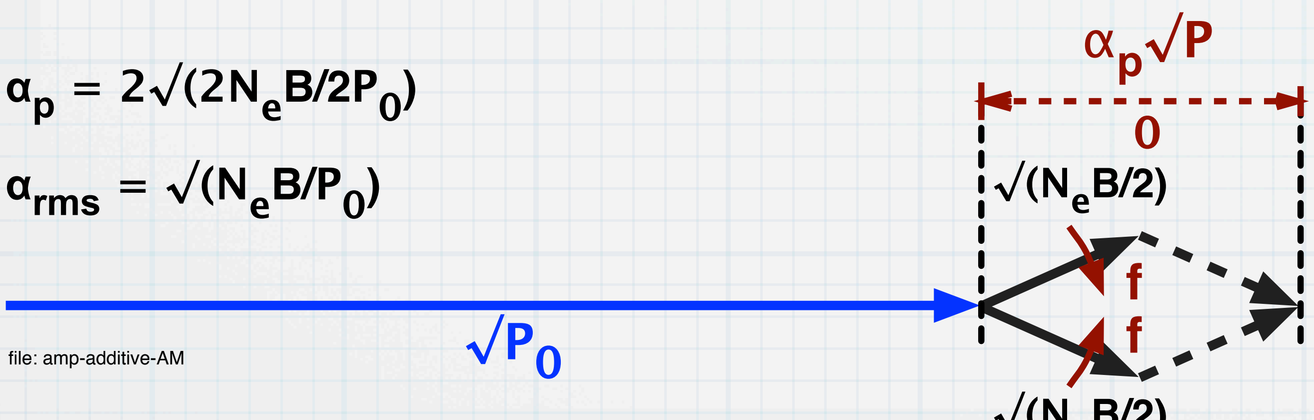
AM (rms)

$$v_{\mathrm{usb}}(t) = \sqrt{N_e B/2}\, e^{\imath 2\pi f t}$$

$$v_{\mathrm{lsb}}(t) = \sqrt{N_e B/2}\, e^{-\imath 2\pi f t}$$

$\alpha_p = 2\sqrt{(2N_e B/2P_0)}$

$\alpha_{rms} = \sqrt{(N_e B/P_0)}$

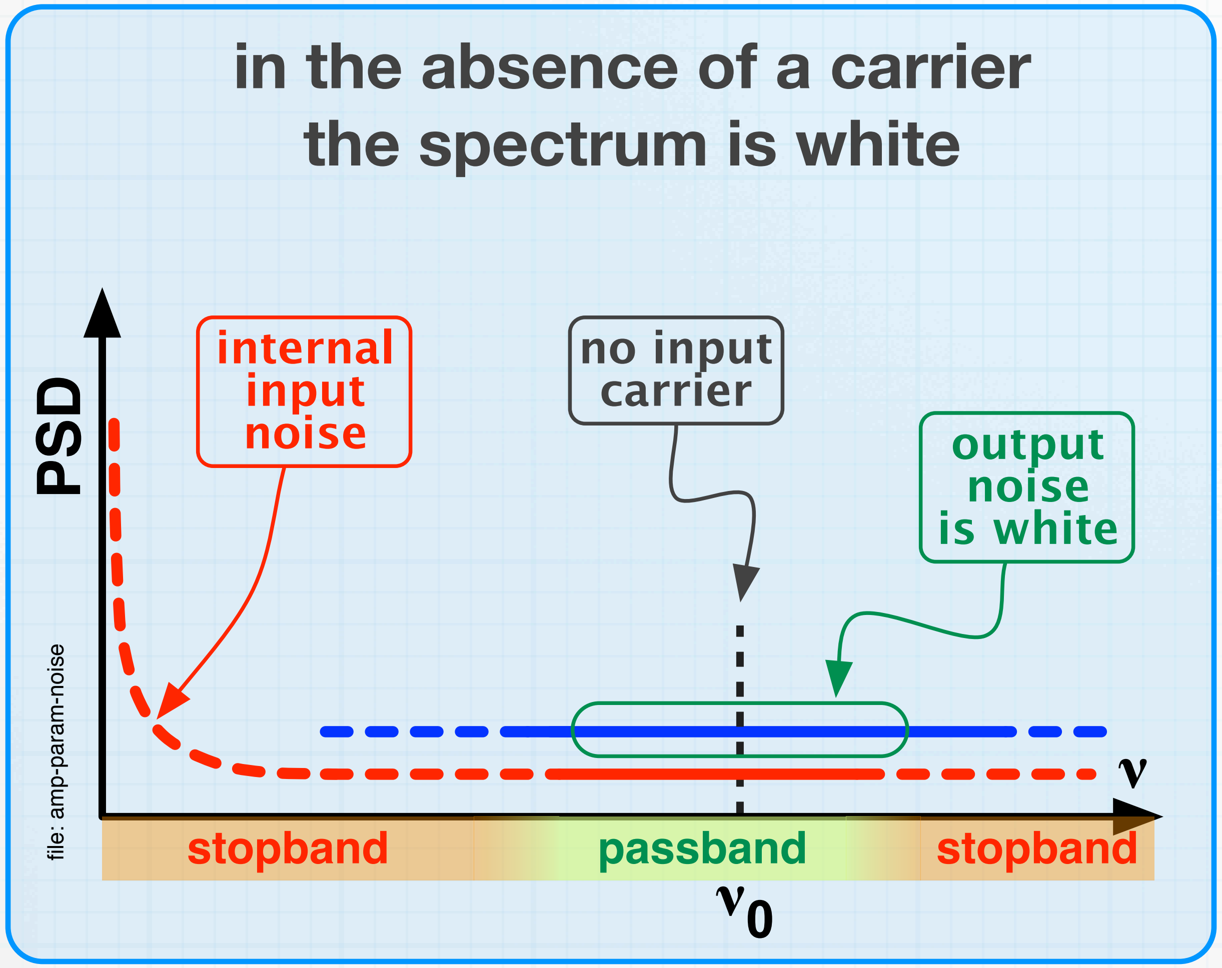


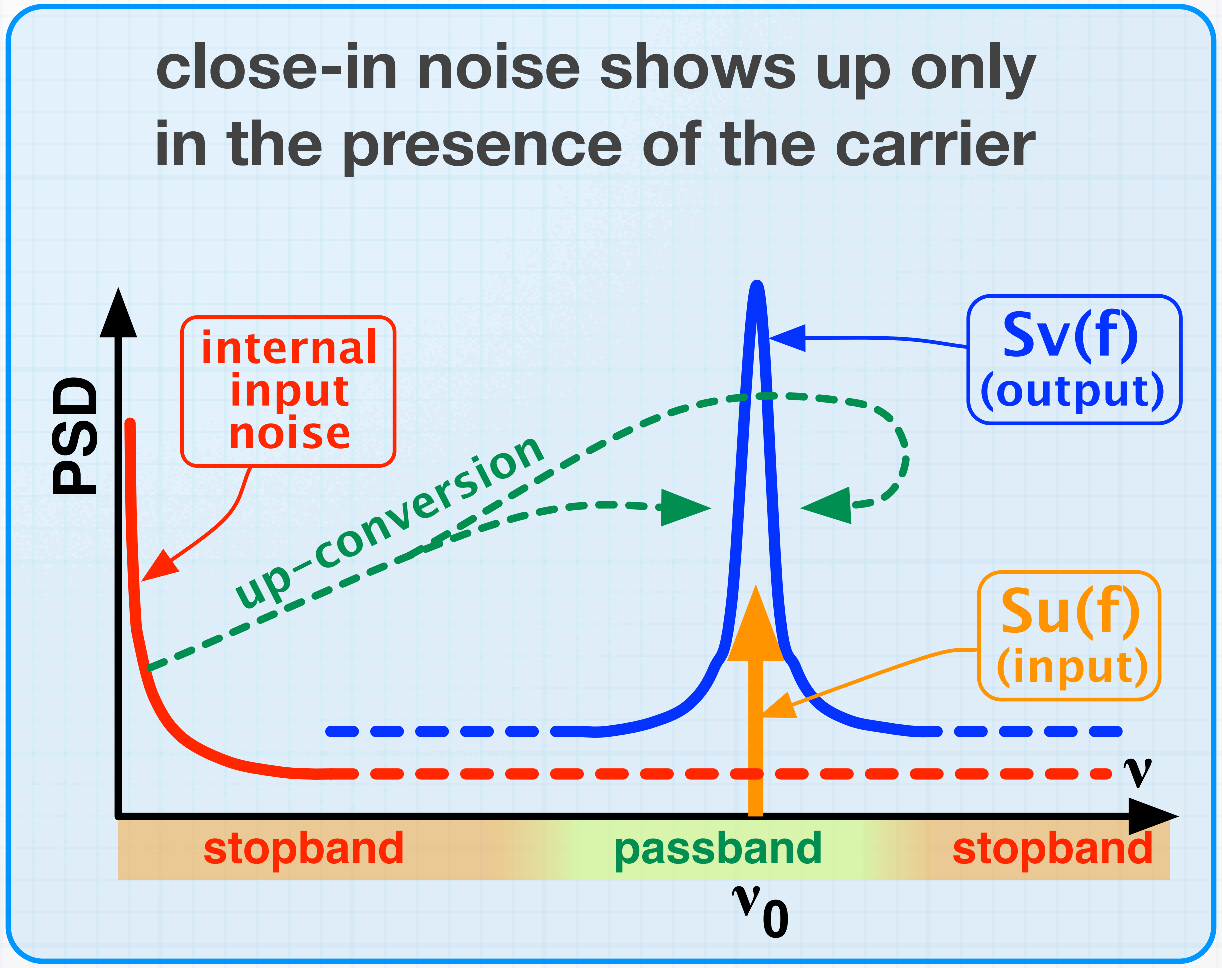
Normalize on B

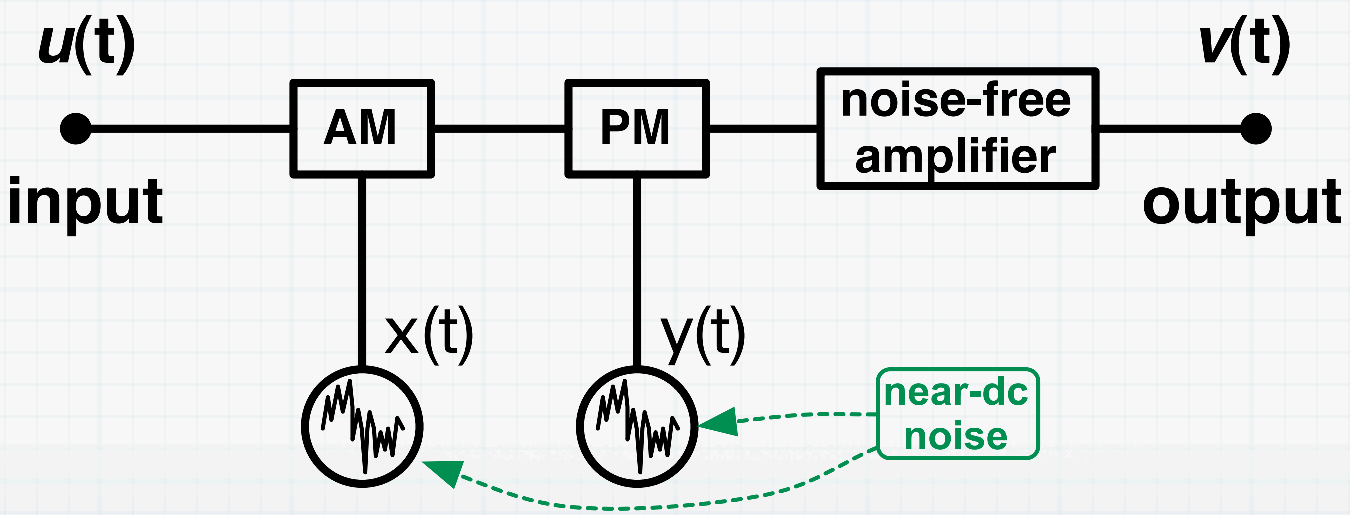
$$S_\varphi(f) = \frac{N_e}{P_0}\,, \qquad S_\alpha(f) = \frac{N_e}{P_0}$$

Amplifier flicker noise

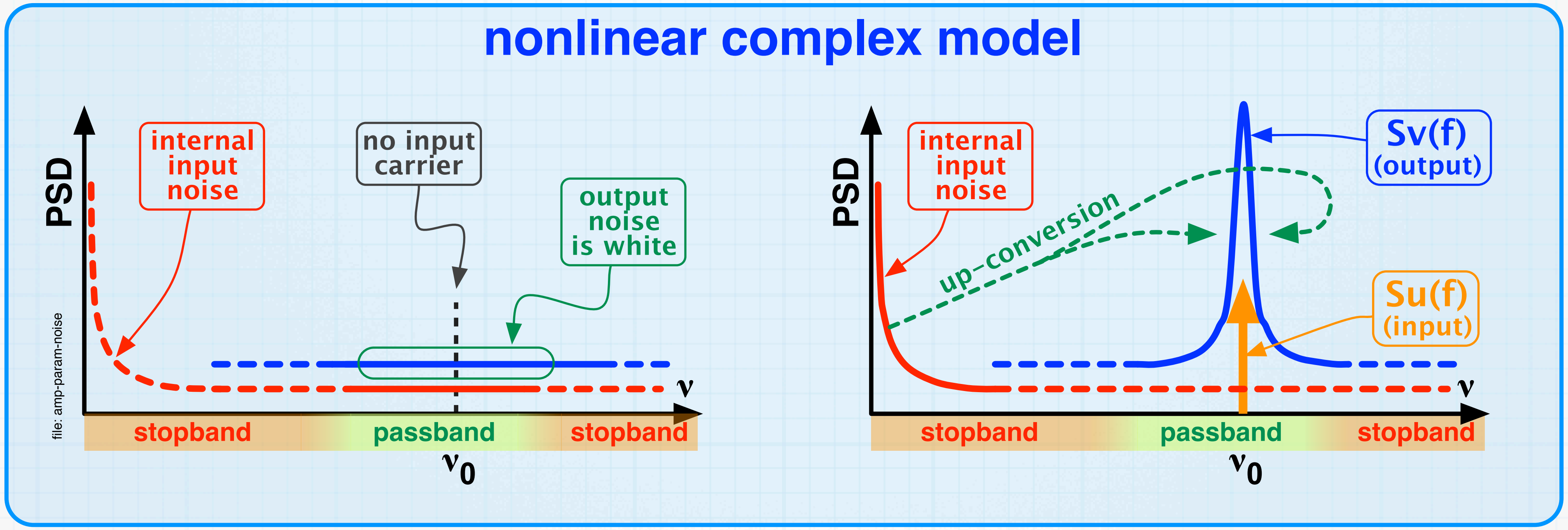
linear parametric model







Amplifier flicker noise



carrier　　near-dc noise

$$v_i(t) = V_i\, e^{\imath\omega_0 t} + n'(t) + \imath n''(t)$$

the parametric nature of 1/f noise is hidden in

substitute
(careful, this hides the down-conversion)

$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + \ldots$$

non-linear (parametric) amplifier

expand and select the ω_0 terms

$$v_o(t) = V_i \Big\{ a_1 + 2a_2 \big[n'(t) + \imath n''(t) \big] \Big\} e^{\imath\omega_0 t}$$

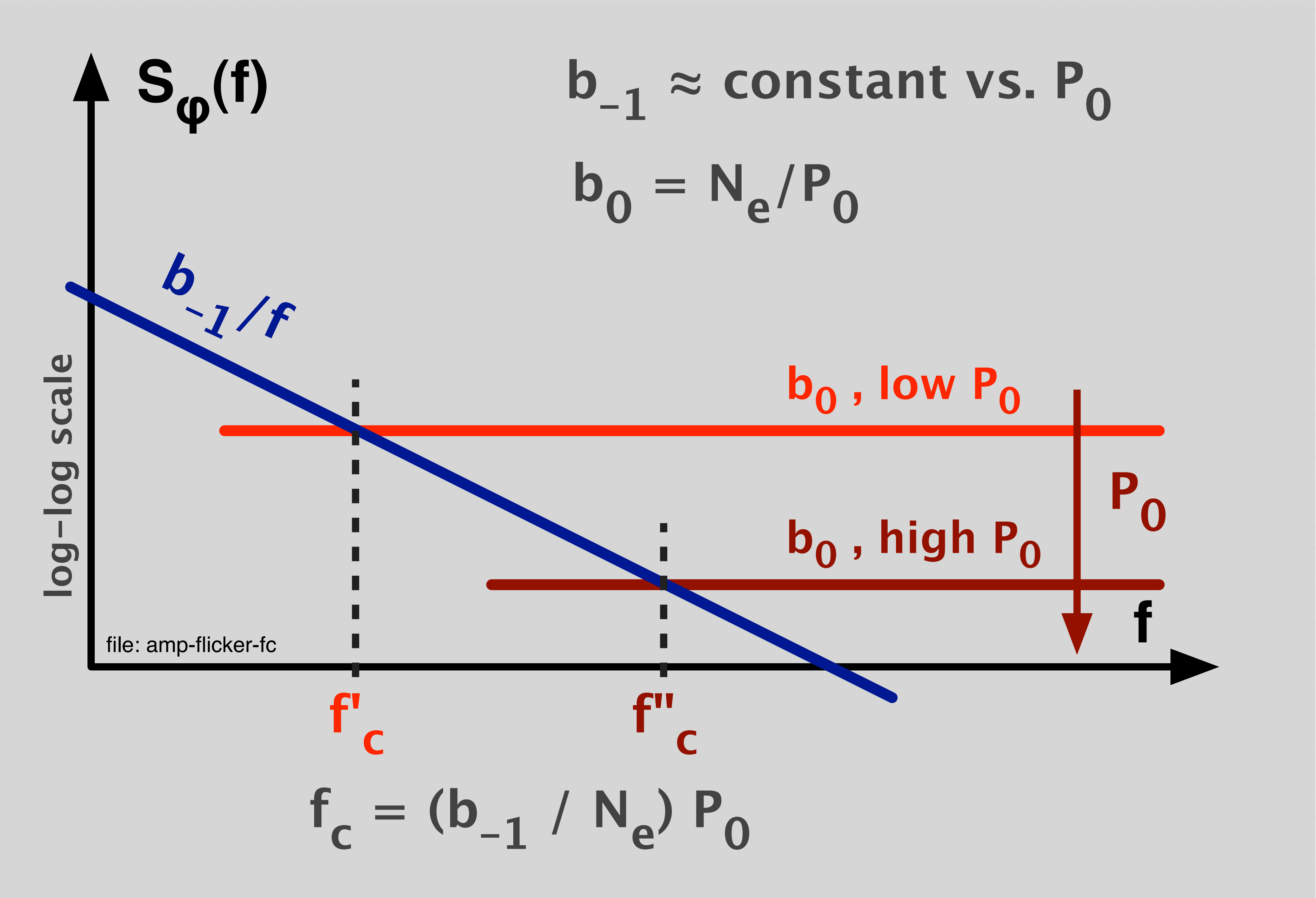
The noise sidebands are proportional to the input carrier

get AM and PM noise

$$\alpha(t) = 2\,\frac{a_2}{a_1}\,n'(t) \qquad \varphi(t) = 2\,\frac{a_2}{a_1}\,n''(t)$$

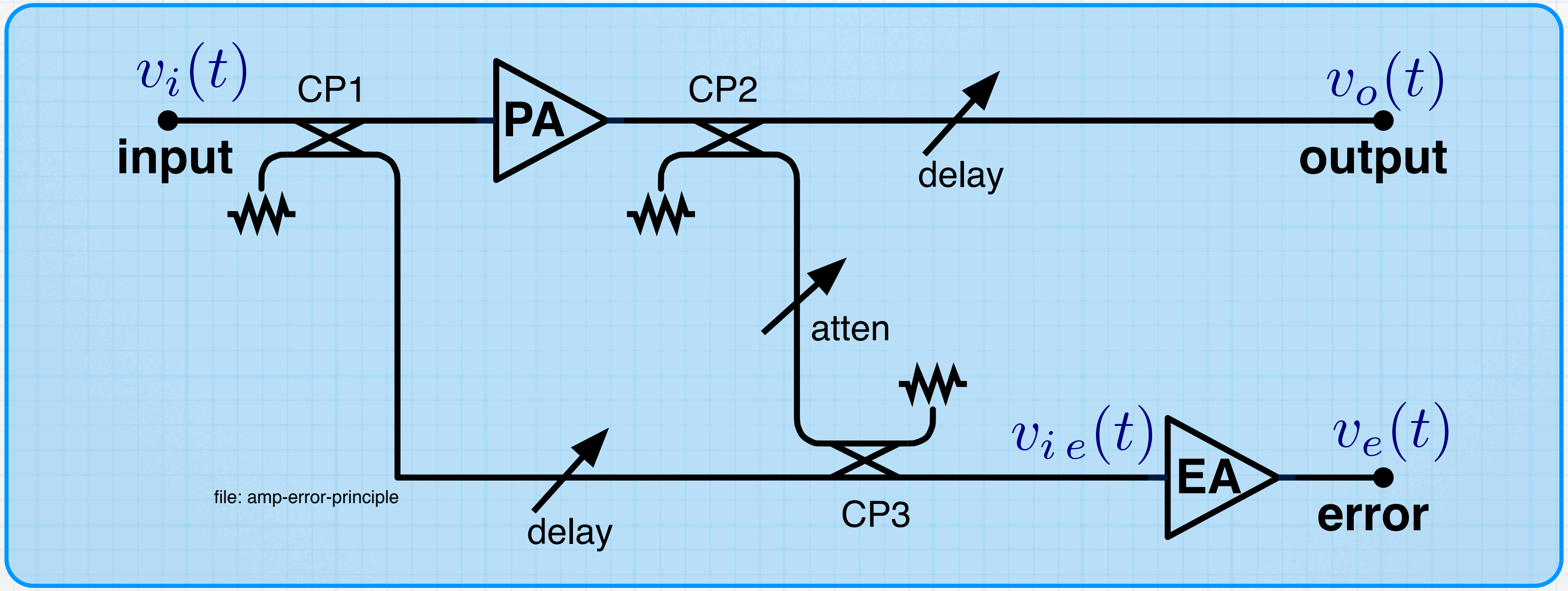
The AM and the PM noise are independent of V_i, thus of power

Amplifier white and flicker noise



The corner frequency f_c, sometimes specified in data sheets is a misleading parameter because it depends on P_0 – ➠examples

The virtues of the error amplifier



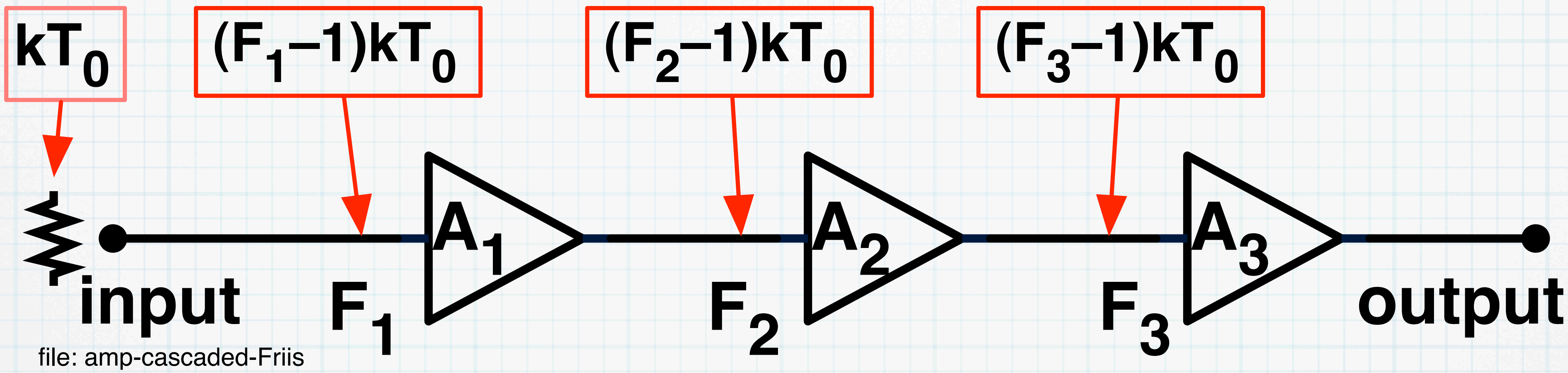
- Use a Power Amplifier (PA) and an Error amplifier (EA)
- The carrier is suppressed (strongly rejected) at the EA input
- Delay matching is needed for wide suppression bandwidth
- Low 1/f sidebands at the EA output because there is no carrier
- $v_e(t)$ is proportional to the PA noise sidebands
- Use $v_e(t)$ for the real-time correction of the PA noise
- feedback or feedforward correction schemes are possible

Noise in amplifier networks & systems

Still not like how this section is organized

White noise in cascaded amplifiers

White noise is chiefly the noise of the first stage



$$N_e = F_1 kT_0 + \frac{(F_2-1)kT_0}{A_1^2} + \frac{(F_3-1)kT_0}{A_2^2 A_1^2} + \ldots$$

$$F = F_1 + \frac{(F_2-1)}{A_1^2} + \frac{(F_3-1)}{A_2^2 A_1^2} + \ldots$$

Friis formulae

H. T. Friis, Proc. IRE 32 p.419-422, jul 1944

Noise is chiefly that of the 1st stage

$$b_0 = \frac{FkT_0}{P_0}$$

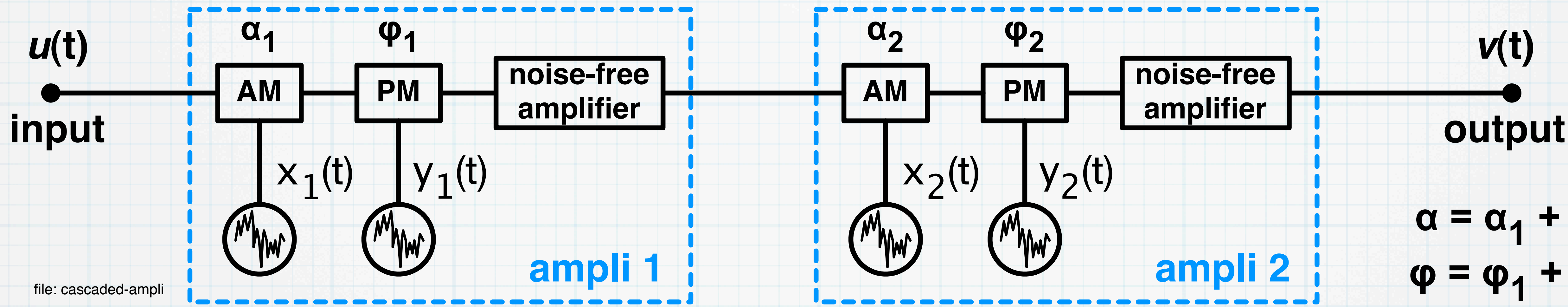
white phase noise

$$b_0 = \frac{F_1 kT_0}{P_0} + \frac{(F_2-1)kT_0}{A_1^2 P_0} + \frac{(F_3-1)kT_0}{A_2^2 A_1^2 P_0} + \ldots$$

Friis formula for phase noise

Parametric noise in cascaded amplifiers

E. Rubiola, *Phase Noise and Frequency Stability in Oscillators*, Cambridge 2008, ISBN 978-0521-88677-2



$\alpha = \alpha_1 + \alpha_2$

$\varphi = \varphi_1 + \varphi_2$

Flicker: the two amplifiers are independent

$$\mathbb{E}\{\alpha^2\} = \mathbb{E}\{\alpha_1^2\} + \mathbb{E}\{\alpha_2^2\} \qquad S_\alpha = S_{\alpha\,1} + S_{\alpha\,2}$$

$$\mathbb{E}\{\varphi^2\} = \mathbb{E}\{\varphi_1^2\} + \mathbb{E}\{\varphi_2^2\} \qquad S_\alpha = S_{\varphi\,1} + S_{\varphi\,2}$$

Environment: a single process drives the two amplifiers

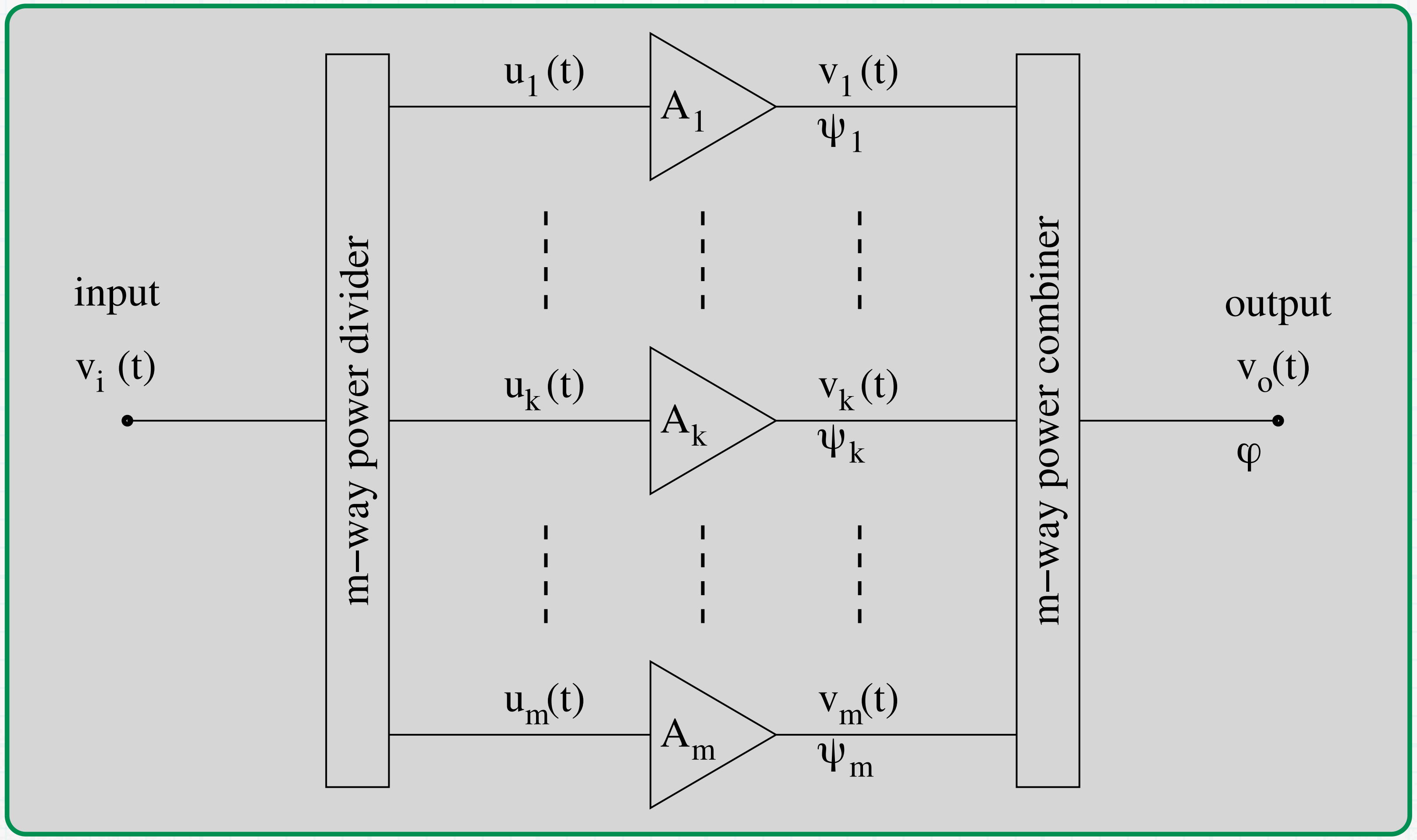
$$\alpha = \alpha_1 + \alpha_2 \qquad \mathbb{E}\{\alpha^2\} = \mathbb{E}\{(\alpha_1 + \alpha_2)^2\}$$

$$\varphi = \varphi_1 + \varphi_2 \qquad \mathbb{E}\{\varphi^2\} = \mathbb{E}\{(\varphi_1 + \varphi_2)^2\}$$

Yet there can be a time constant, not necessarily the same for the two devices

Flicker noise in parallel amplifiers

E. Rubiola, *Phase Noise and Frequency Stability in Oscillators*, Cambridge 2008, ISBN 978-0521-88677-2

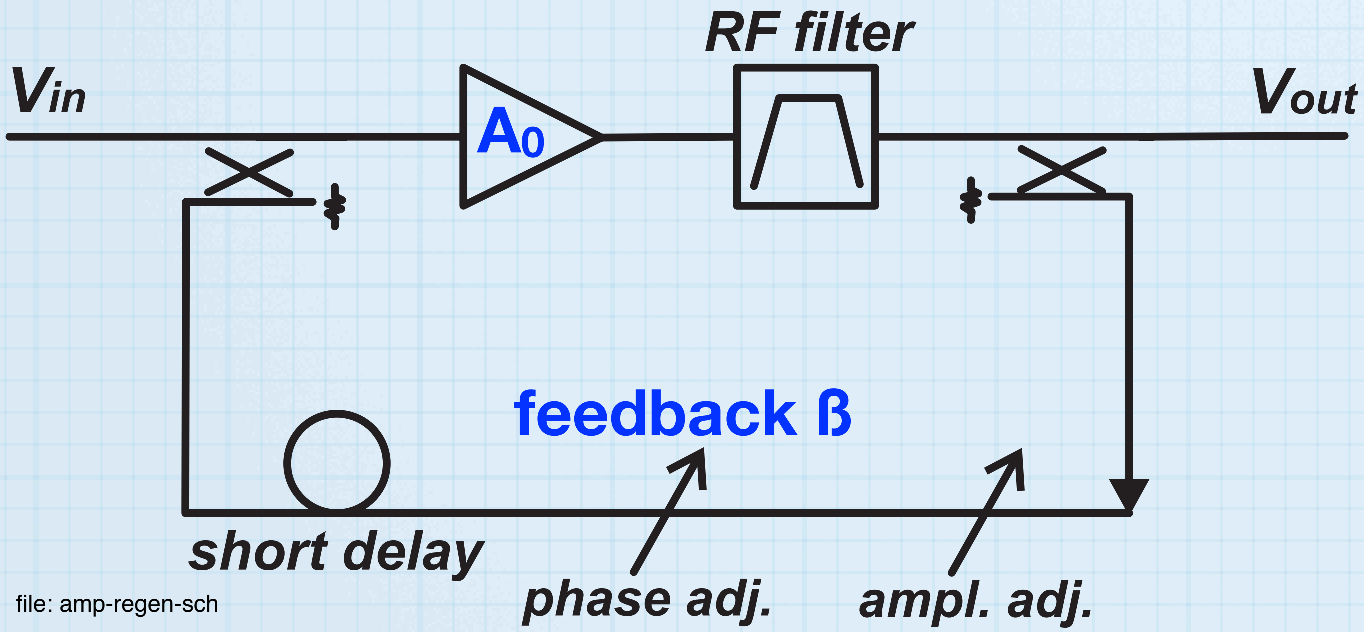


- The phase flicker coefficient b_{-1} is about independent of power
- The flicker of a branch is not increased by splitting the input power
- At the output,
  - the carrier adds up coherently
  - the phase noise adds up statistically
- Hence, the 1/f phase noise is reduced by a factor m
- Only the flicker noise can be reduced in this way

$$b_{-1} = \frac{1}{m}\left[b_{-1}\right]_{\text{cell}}$$

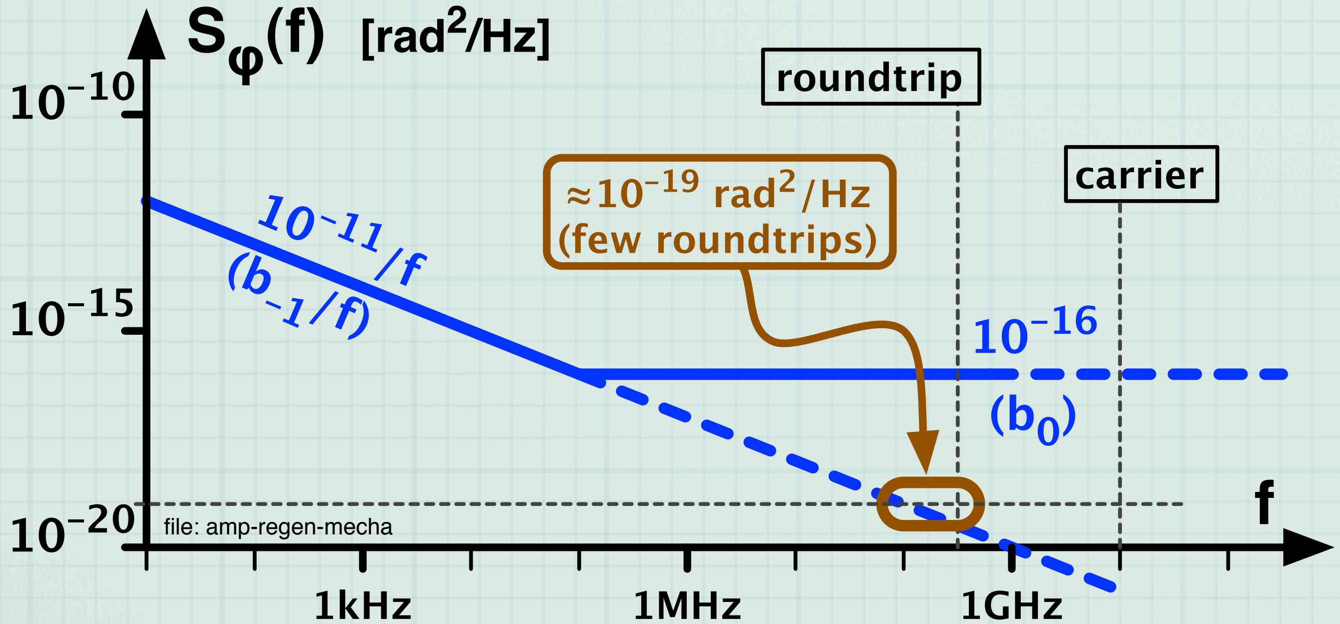
Parametric noise in regenerative amplifiers

R. Boudot, E. Rubiola, arXiv:1001.2047v1, Jan 2010. Submitt. IEEE Transact. MTT



$$A = \frac{A_0}{1 - A_0\beta}$$

$$A = A_0^m \quad \Rightarrow \quad \beta = \frac{A_0^{m-1} - 1}{A_0^m}$$



- **Short roundtrip time, vs. flicker time frame**
- **Quasi-static analysis holds**

$$A \to \frac{A_0\, e^{j\psi}}{1 - A_0\beta\, e^{j\psi}}$$

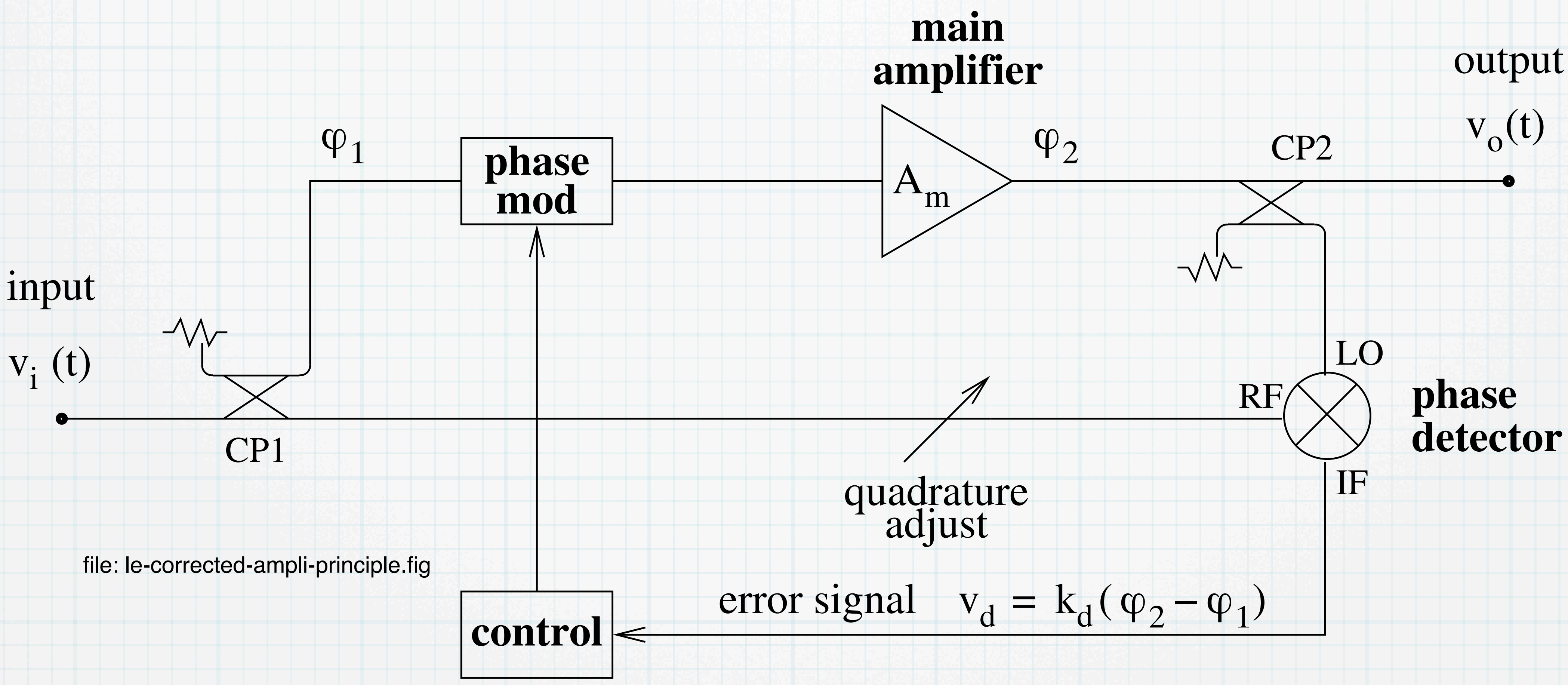
$$A = \frac{A_0}{1 - A_0\beta}\left[1 + j\,\frac{1}{1 - A_0\beta}\,\psi\right] .$$

$$\varphi(t) = \frac{1}{1 - A_0\beta}\,\psi(t)$$

$$(b_{-1})_{\mathrm{RA}} = \left[\frac{1}{1 - A_0\beta}\right]^2 (b_{-1})_{\mathrm{ampli}}$$

$$(b_{-1})_{\mathrm{RA}} = m^2 (b_{-1})_{\mathrm{ampli}}$$

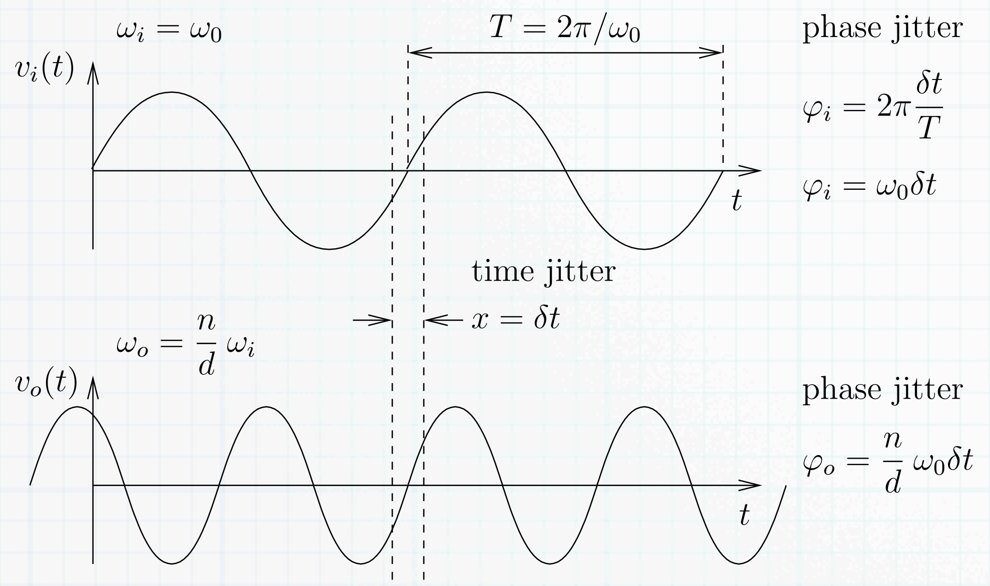
Baseband-feedback amplifier

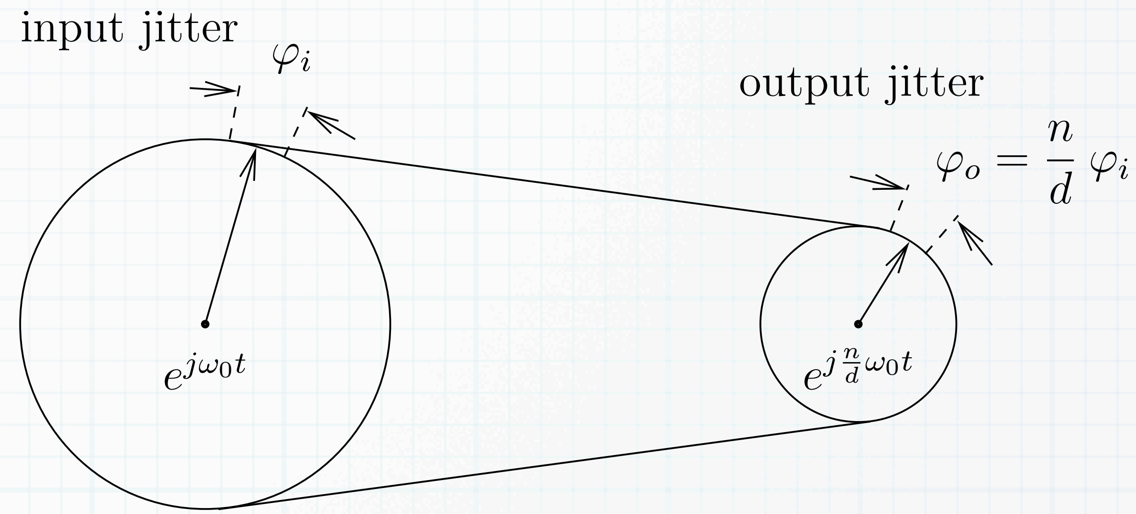


- The detector measures the phase $\varphi_2 - \varphi_1$ across the main amplifier plus phase modulator
- The control stabilizes $\varphi_2 - \varphi_1$ = constant (virtual ground)
- The correction of AM noise is also possible in a similar way

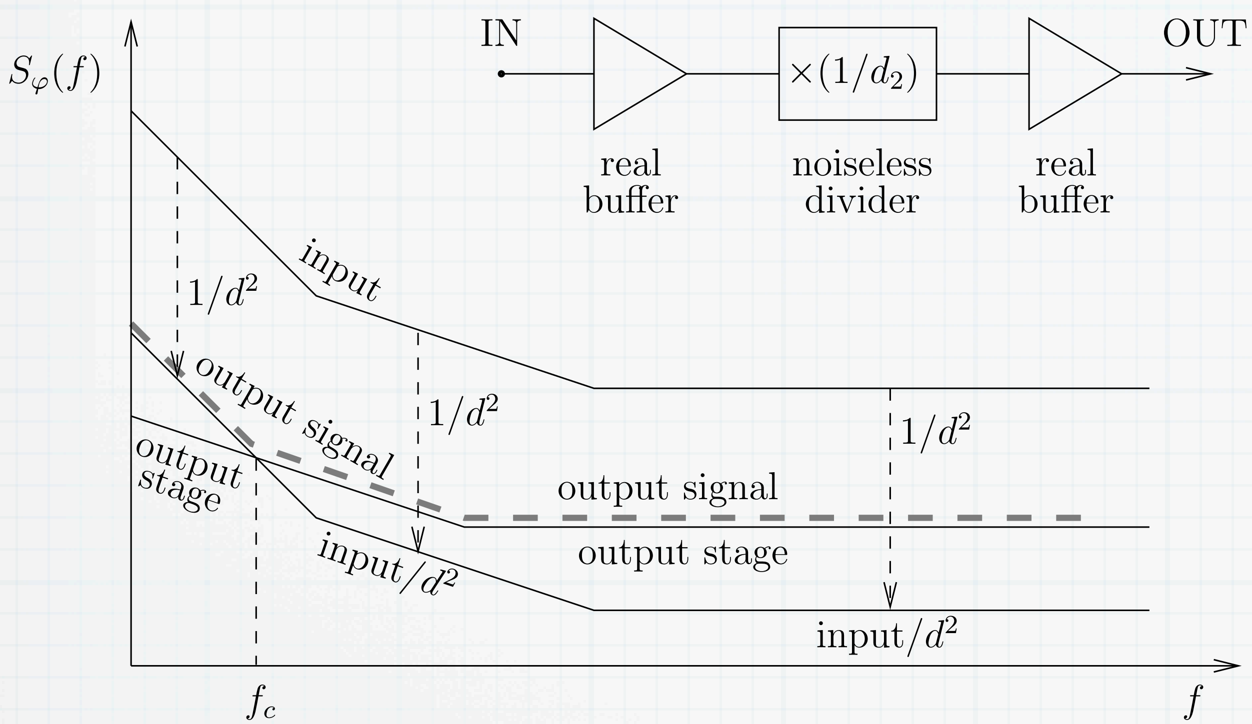
Frequency synthesis

The ideal noise-free frequency synthesizer repeats the input time jitter

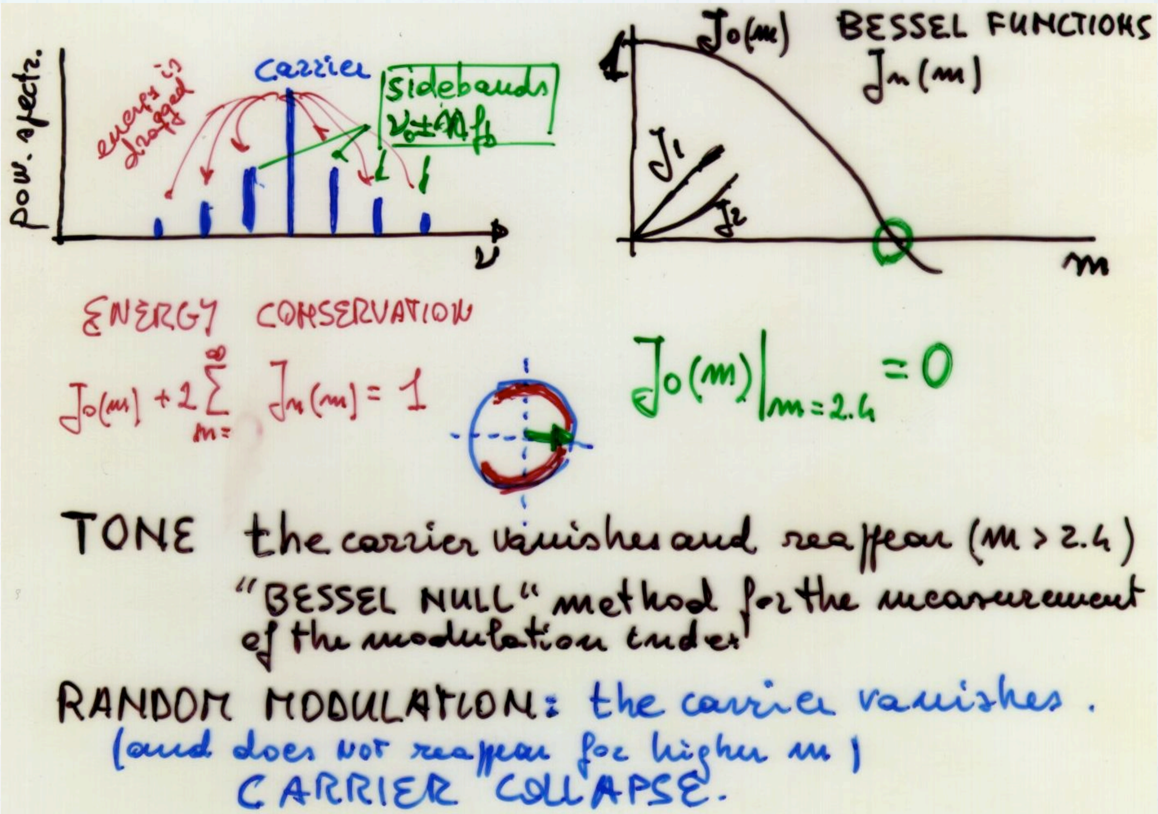




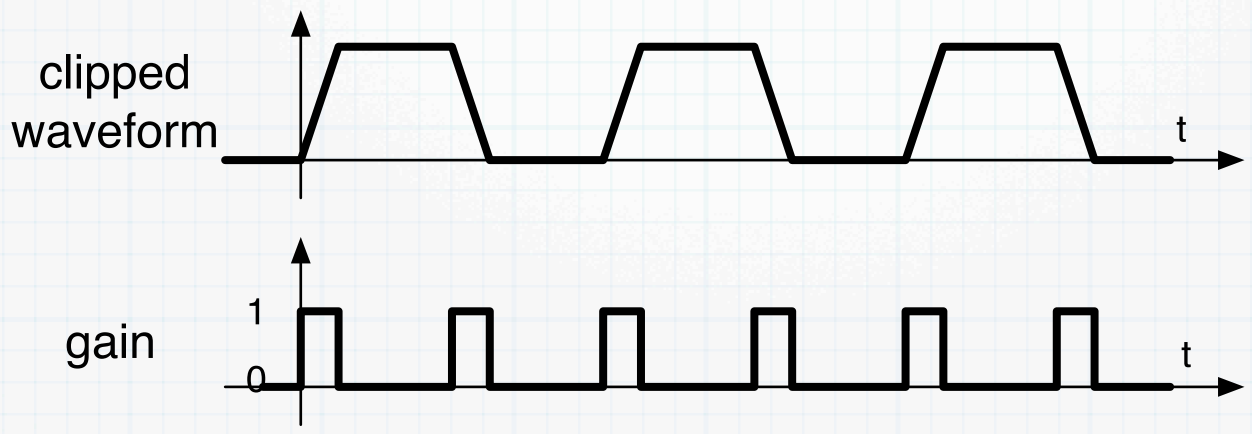
After division, the noise of the output buffer may be larger than the input-noise scaled down



After multiplication, the scaled-up phase noise sinks energy from the carrier. At m ≈ 2.4, the carrier vanishes



Saturation and sampling



Saturation is equivalent to reducing the gain

Digital circuits, for example, amplify (linearly) only during the transitions

Experiments

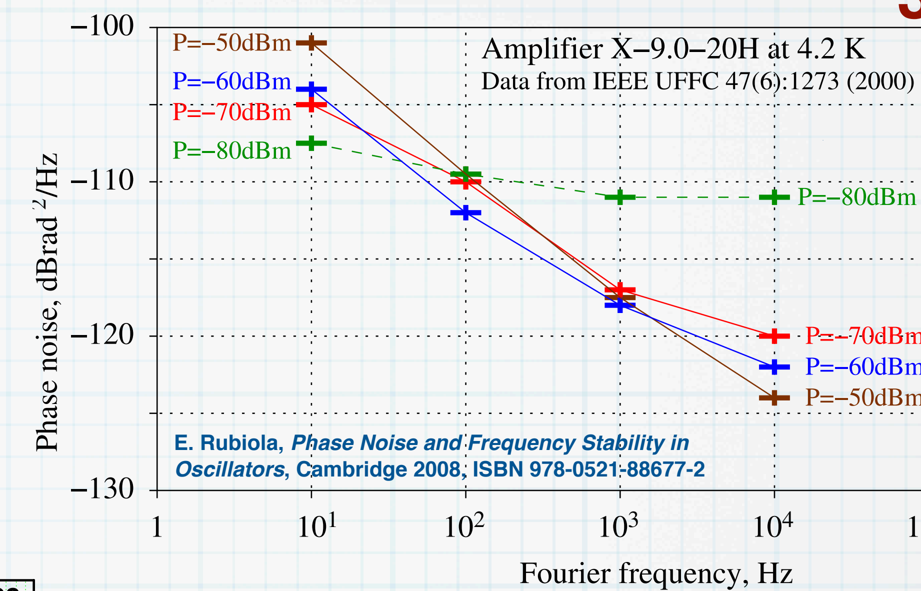
Flicker noise of some amplifiers

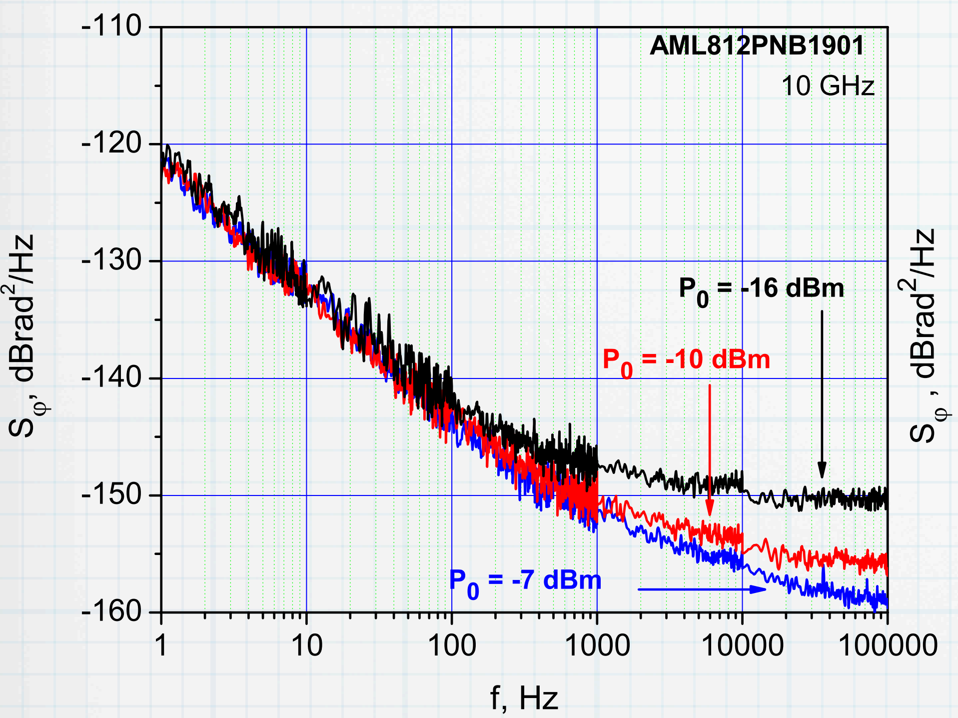
R. Boudot, E. Rubiola, arXiv:1001.2047v1, Jan 2010. Submitt. IEEE Transact. MTT

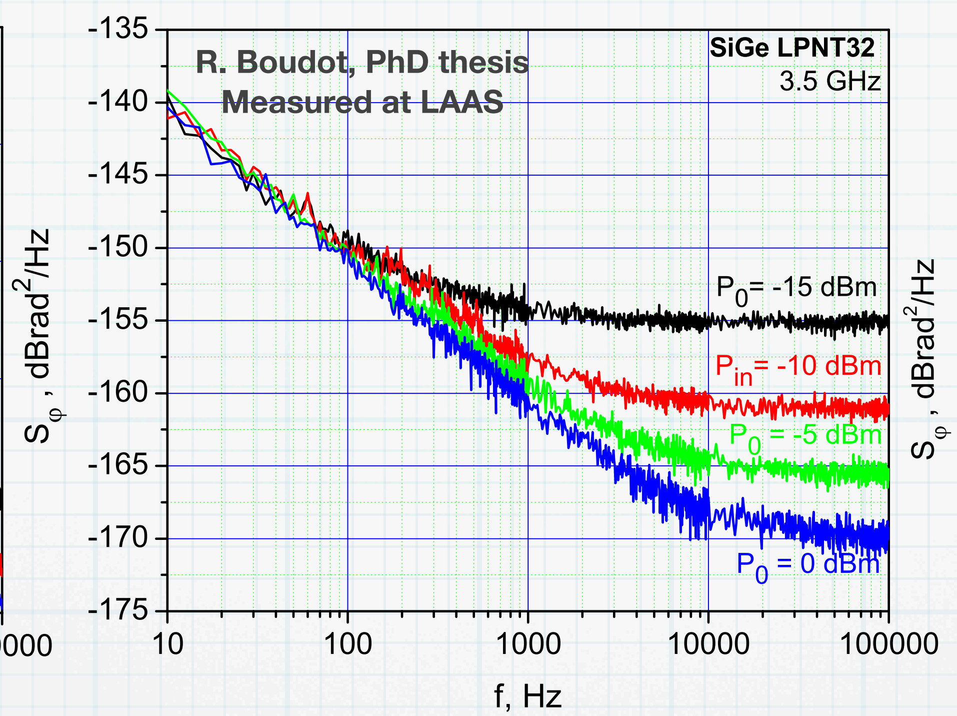
Amplifier	Frequency (GHz)	Gain (dB)	$P_{1\,\mathrm{dB}}$ (dBm)	F (dB)	DC bias	b_{-1} (meas.) (dBrad^2/Hz)
AML812PNB1901	8 – 12	22	17	7	15 V, 425 mA	−122
AML412L2001	4 – 12	20	10	2.5	15 V, 100 mA	−112.5
AML612L2201	6 – 12	22	10	2	15 V, 100 mA	−115.5
AML812PNB2401	8 – 12	24	26	7	15 V, 1.1A	−119
AFS6	8 – 12	44	16	1.2	15 V, 171 mA	−105
JS2	8 – 12	17.5	13.5	1.3	15 V, 92 mA	−106
SiGe LPNT32	3.5	13	11	1	2 V, 10 mA	−130
Avantek UTC573	0.01 – 0.5	14.5	13	3.5	15 V, 100 mA	−141.5
Avantek UTO512	0.005–0.5	21	8	2.5	15 V, 23 mA	−137

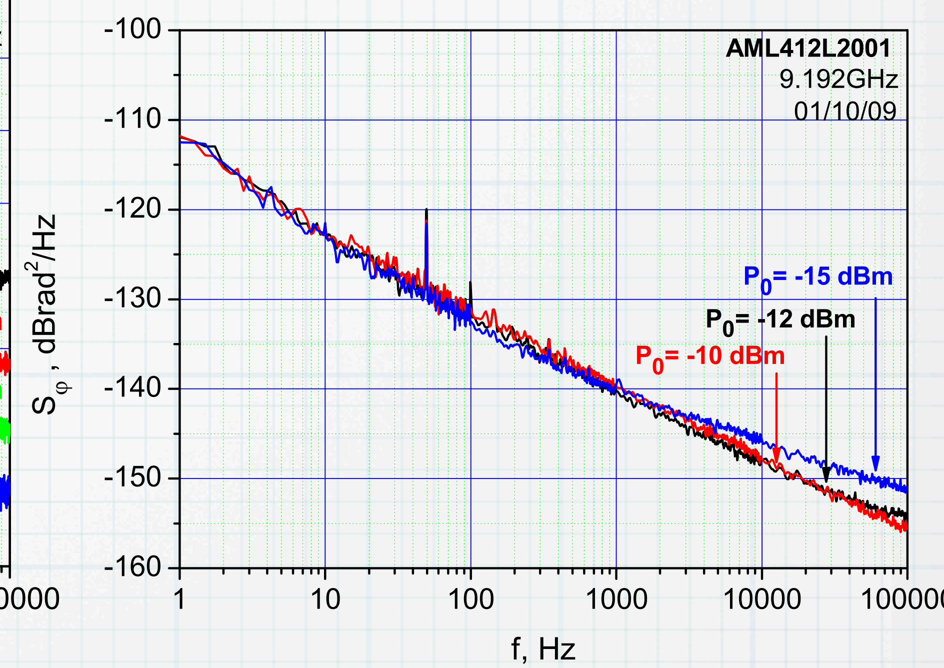
Phase noise vs. power

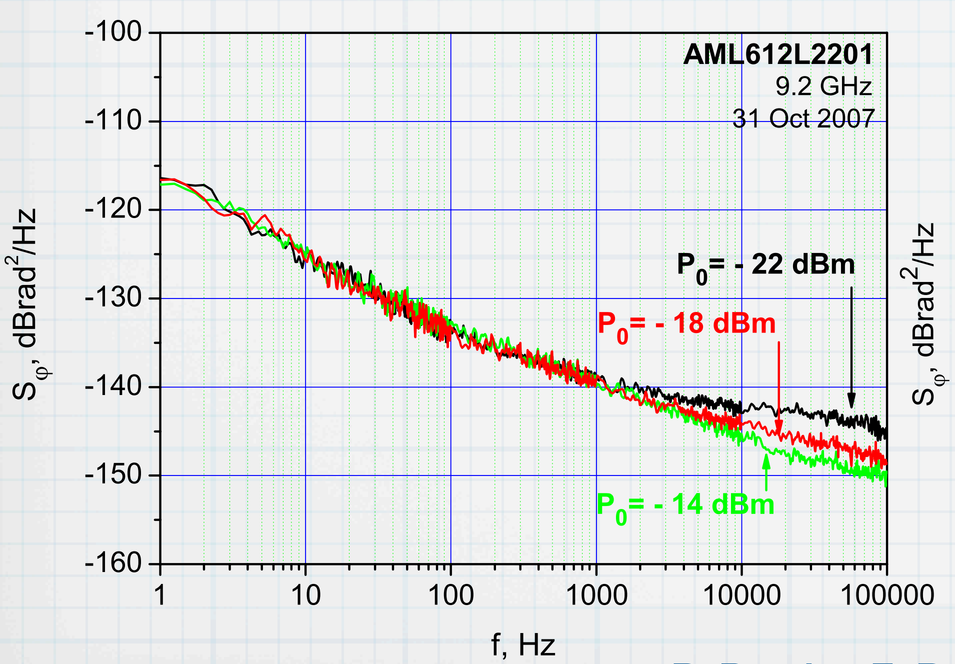
- **The 1/f phase noise b_{-1} is about independent of power**
- **The white noise b_0 scales as the inverse of the power**
- **The corner frequency is misleading because it depends on power**

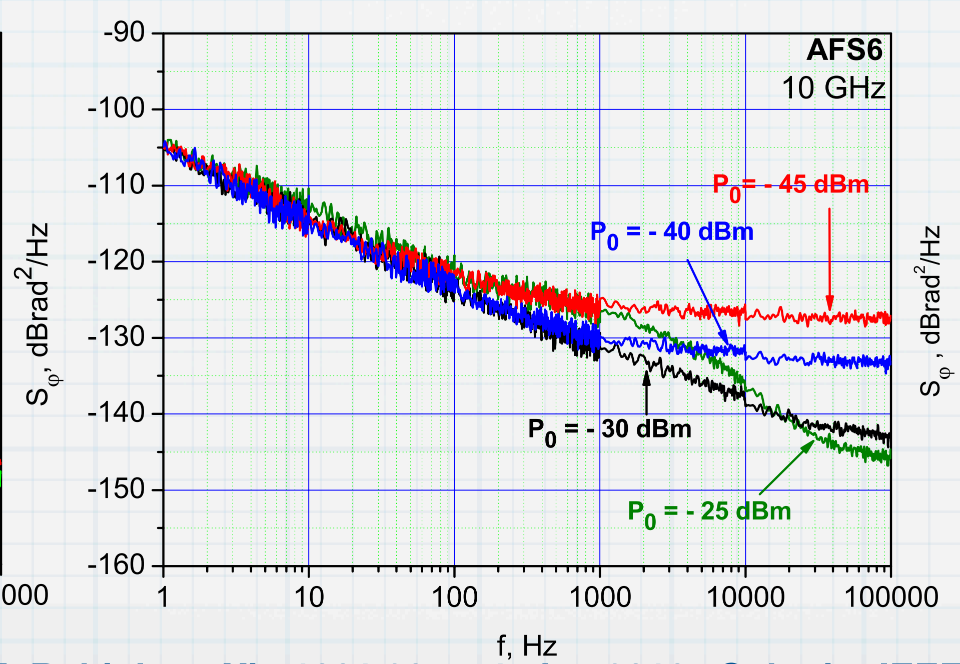


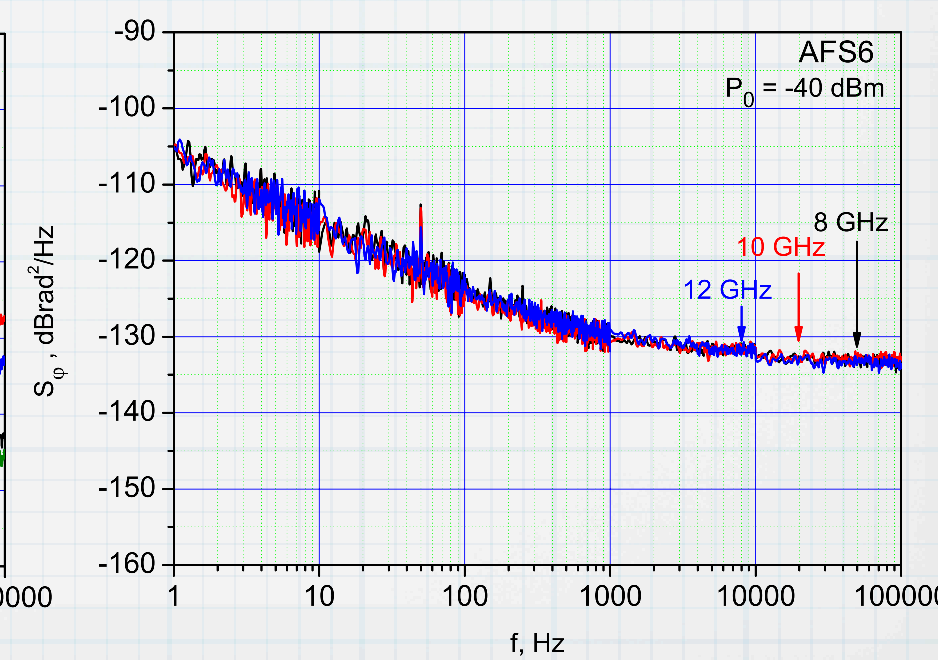








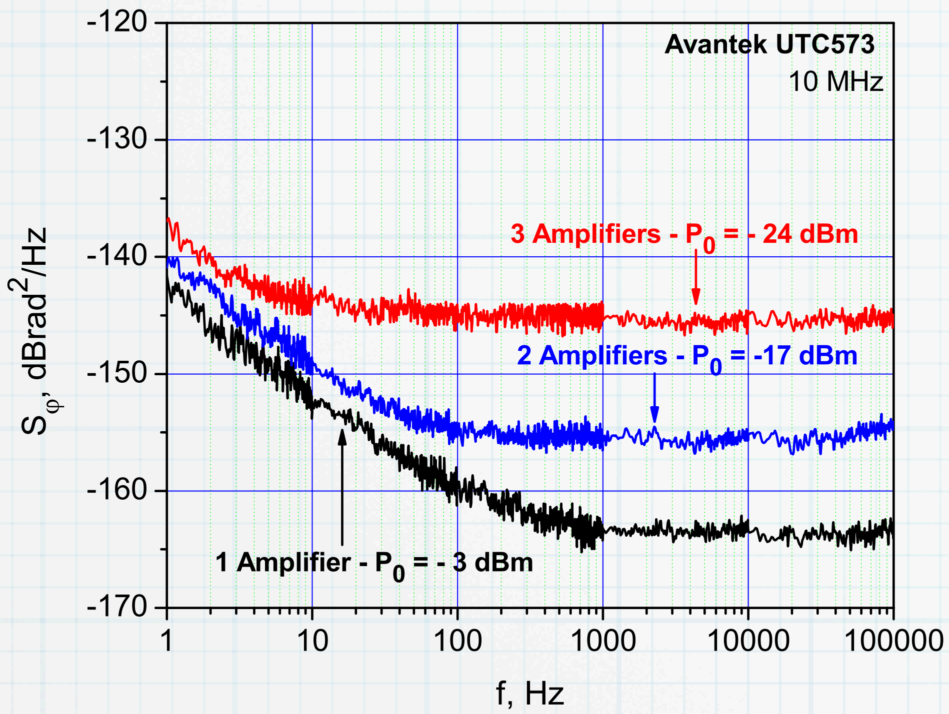


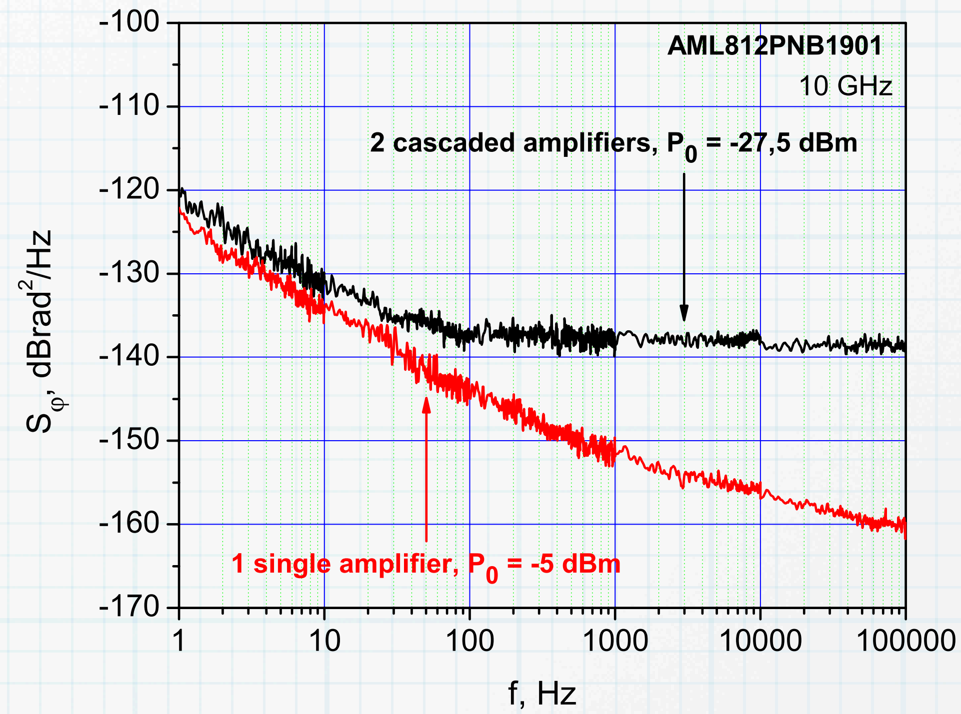


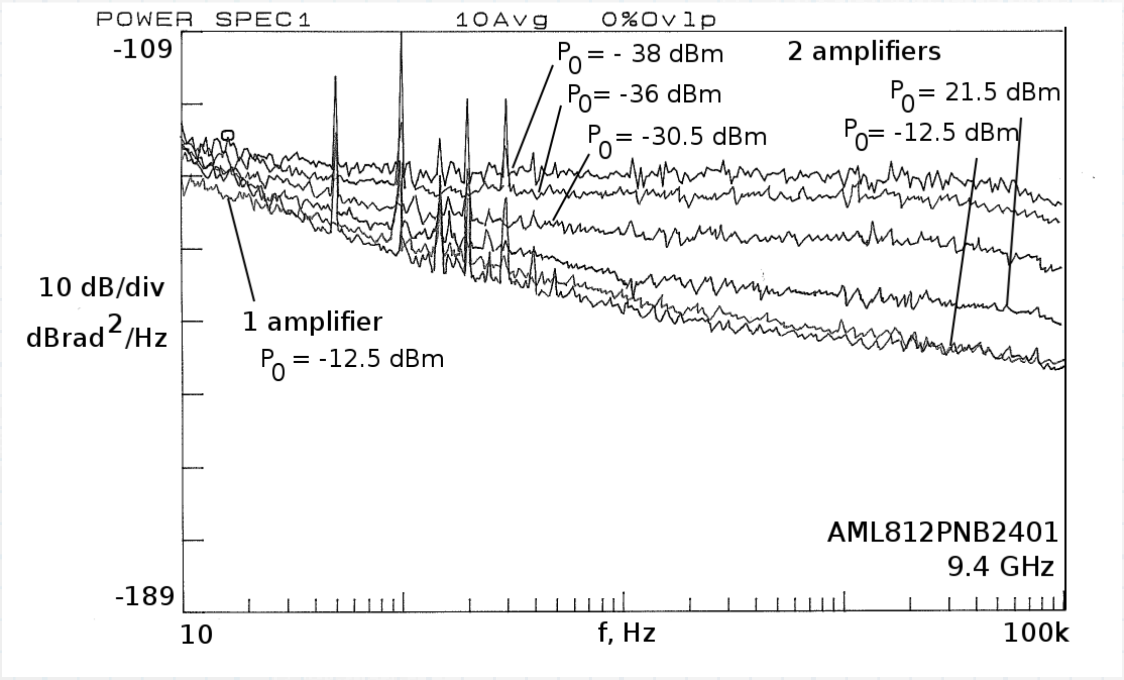
R. Boudot, E. Rubiola, arXiv:1001.2047v1, Jan 2010. Submitt. IEEE Transact. MTT

Phase noise in cascaded amplifiers

R. Boudot, E. Rubiola, arXiv:1001.2047v1, Jan 2010. Submitt. IEEE Transact. MTT







The expected flicker of a cascade increases by:

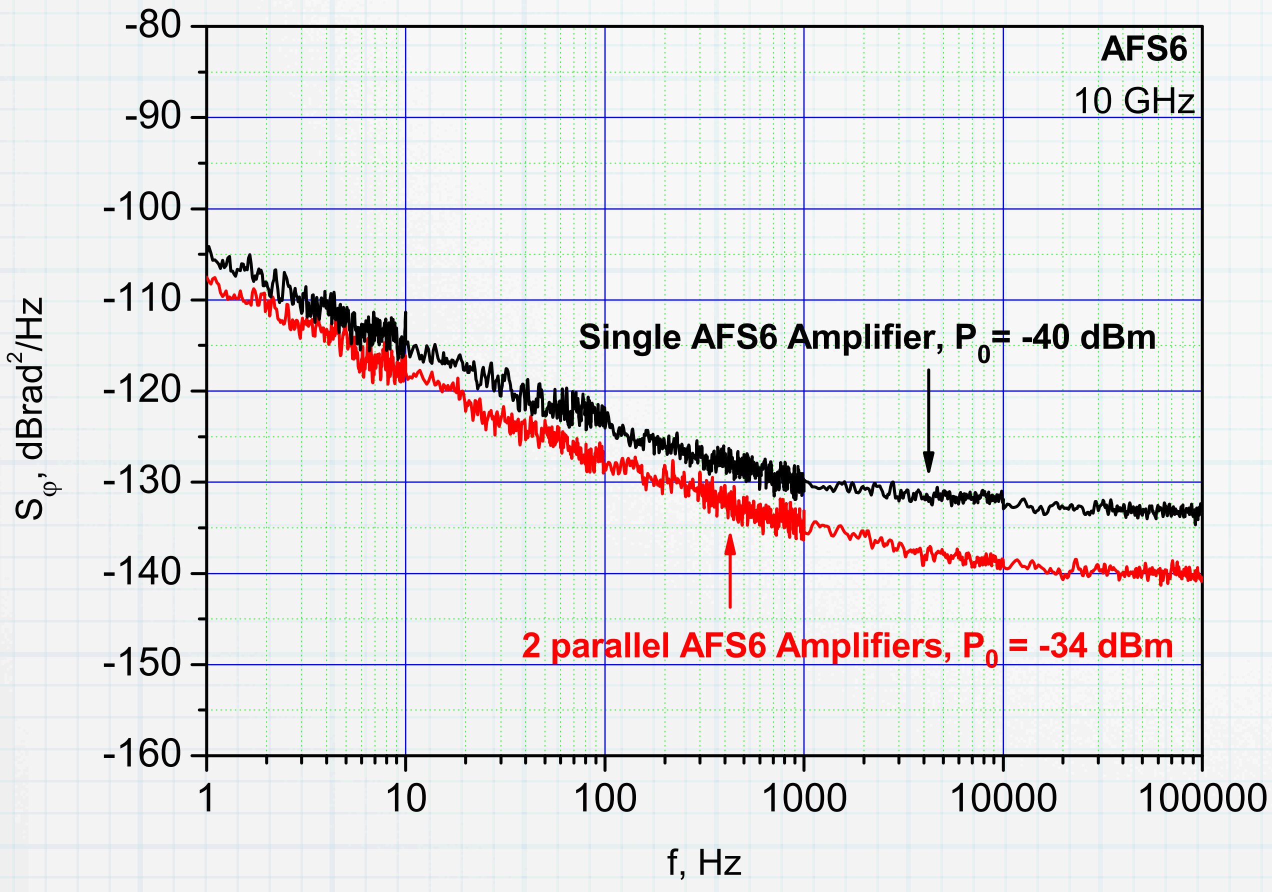
3 dB, with 2 amplifiers

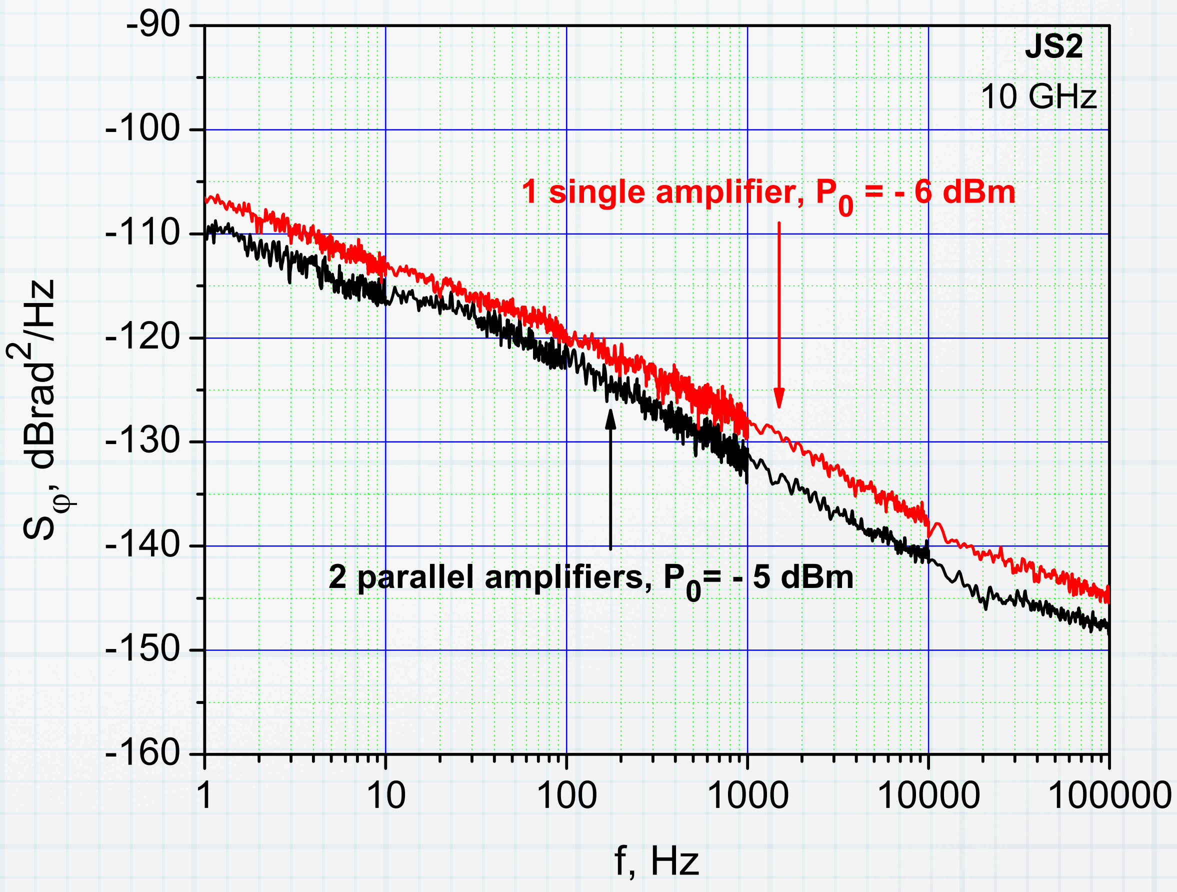
4.8 dB, with 3 amplifiers

White noise is limited by the (small) input power

Phase noise in parallel amplifiers

R. Boudot, E. Rubiola, arXiv:1001.2047v1, Jan 2010. Submitt. IEEE Transact. MTT

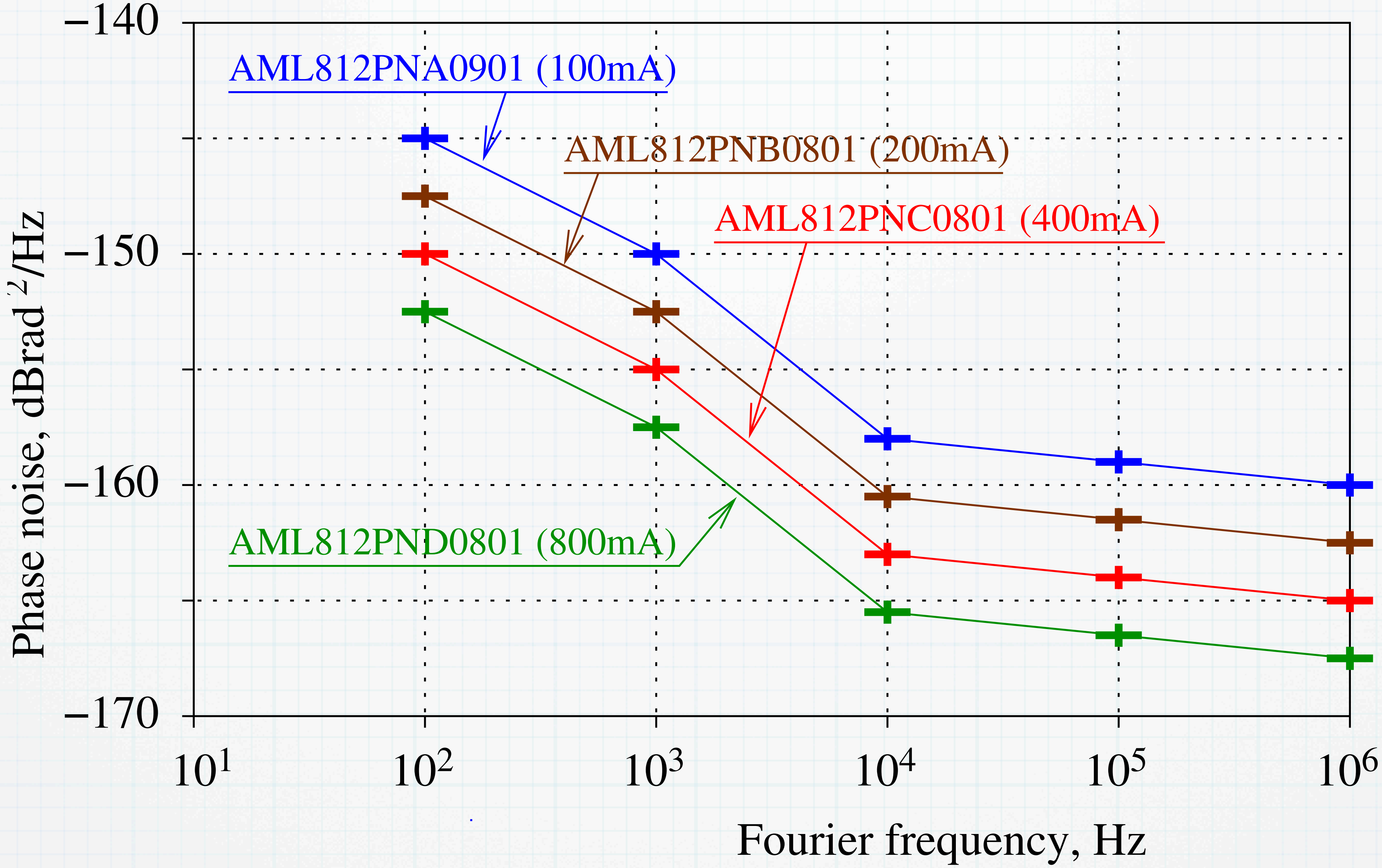




Connecting two amplifier in parallel, a 3 dB reduction of flicker is expected

Flicker noise in parallel amplifiers

E. Rubiola, *Phase Noise and Frequency Stability in Oscillators*, Cambridge 2008, ISBN 978-0521-88677-2

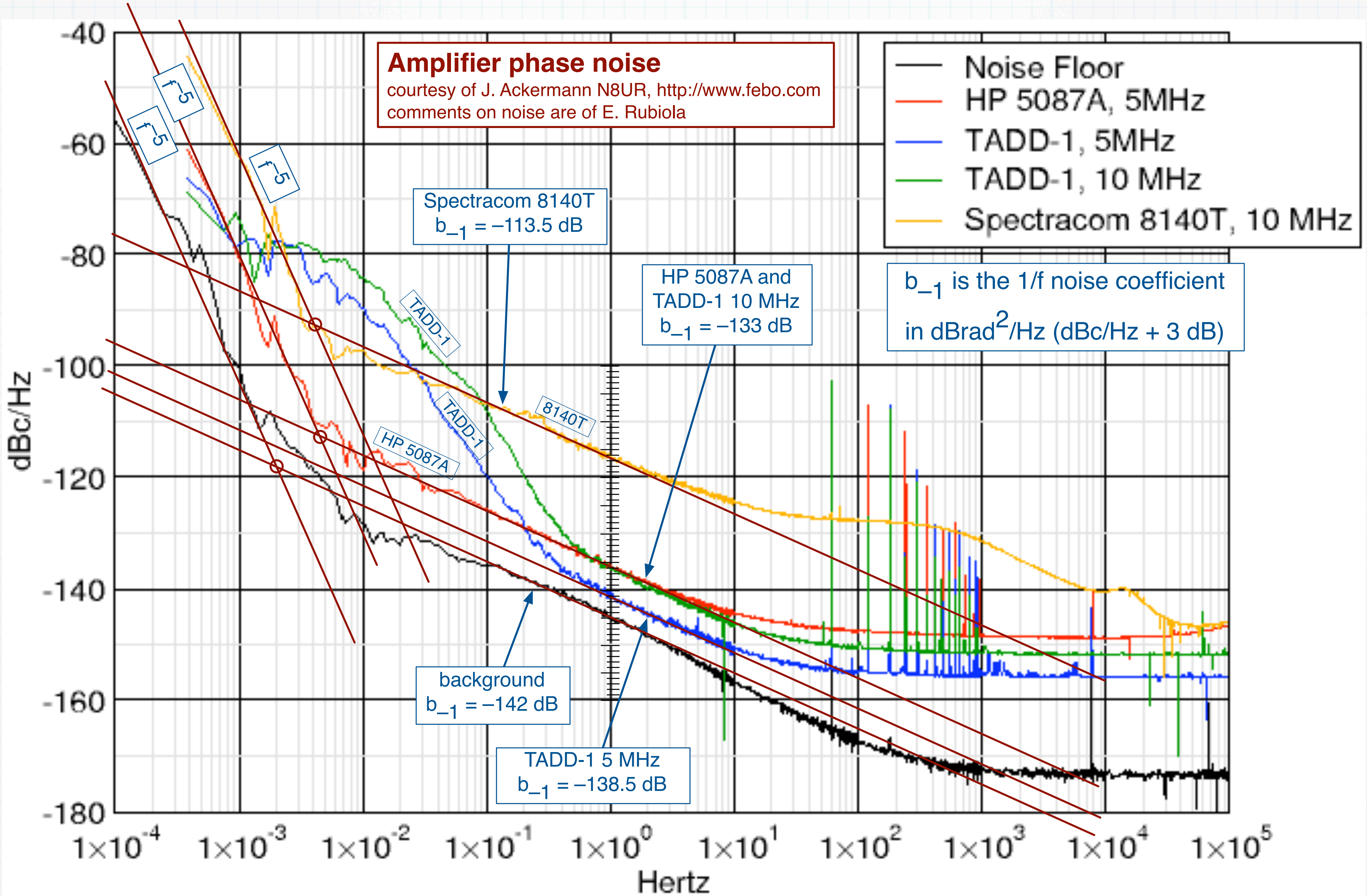


Specification of low phase-noise amplifiers (AML web page)

amplifier	parameters				phase noise vs. f, Hz			
	gain	F	bias	power	10^2	10^3	10^4	10^5
AML812PNA0901	10	6.0	100	9	−145.0	−150.0	−158.0	−159.0
AML812PNB0801	9	6.5	200	11	−147.5	−152.5	−160.5	−161.5
AML812PNC0801	8	6.5	400	13	−150.0	−155.0	−163.0	−164.0
AML812PND0801	8	6.5	800	15	−152.5	−157.5	−165.5	−166.5
unit	dB	dB	mA	dBm	dBrad^2/Hz			

Environmental effects in RF amplifiers

E. Rubiola, *Phase Noise and Frequency Stability in Oscillators*, Cambridge 2008, ISBN 978-0521-88677-2



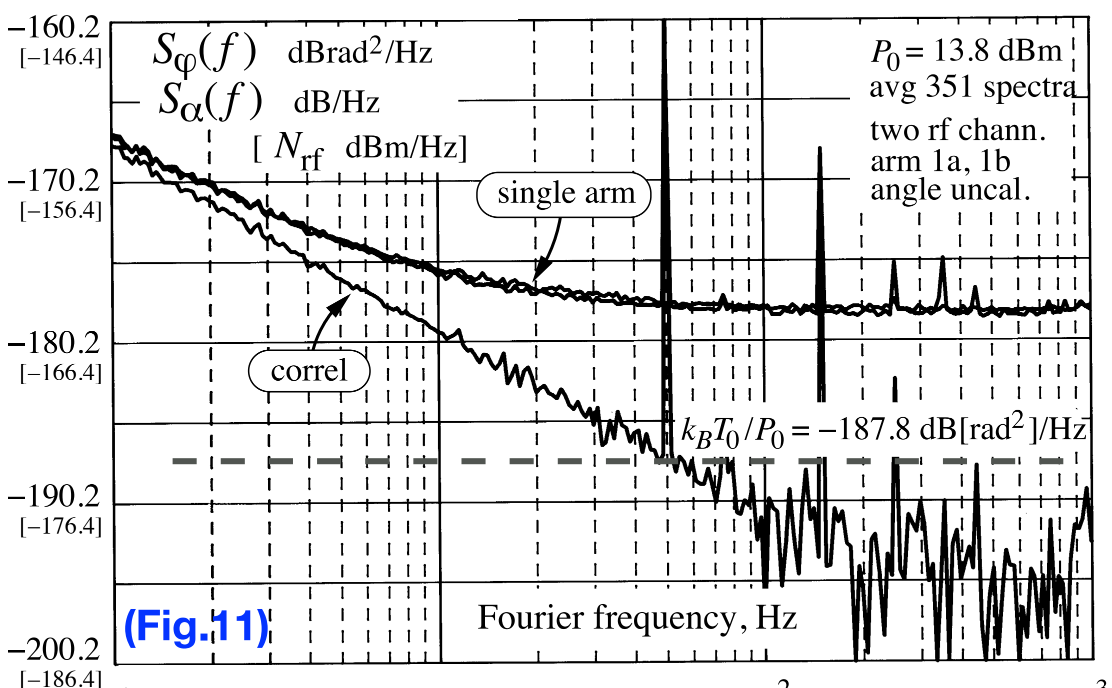
It is experimentally observed that the temperature fluctuations cause a spectrum $S_\alpha(f)$ or $S_\varphi(f)$ of the $1/f^5$ type

Yet, at low frequencies the spectrum folds back to 1/f

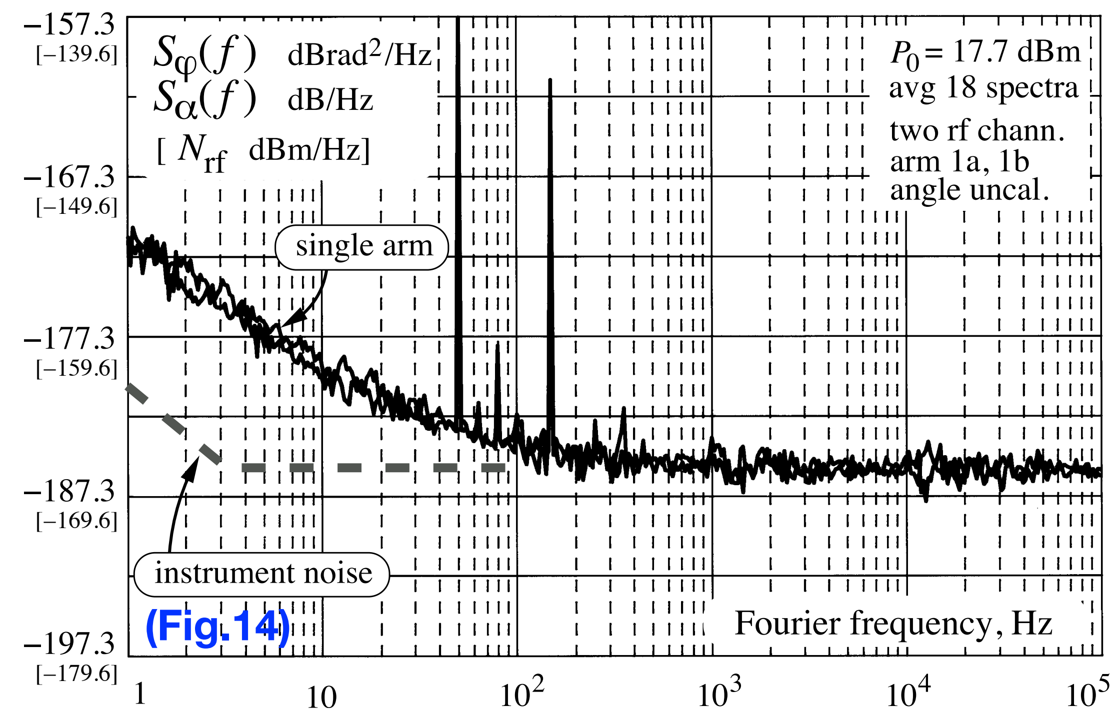
VHF passive devices

E. Rubiola, V. Giordano, RSI 73(6) p.2445-2457, 2002

two by-step attenuators



two ferrite hybrid junctions

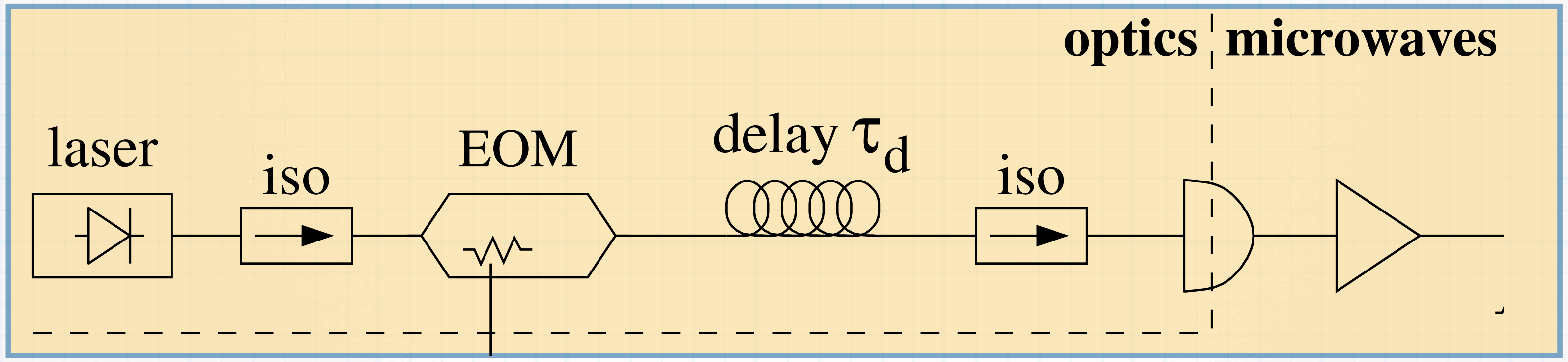


Flicker noise of components

device	PM b_{-1}	AM h_{-1}	frequency	References and comments
Si bipolar HF-UHF amplifier	–135 … –145		5…1000 MHz	(general experience)
SiGe HBT μwave amplifier	–120 … –130		4…20 GHz	(general experience)
GaAs HBT μwave amplifier	–95 … –110		3…10 GHz	(general experience)
Cr^{3+} maser amplifier (0.2 cm^3)	≈ –160		11 GHz	G.J.Dick, private discussion
HF-UHF double-balanced mixer	–135 … –150		5…1000 MHz	(general experience)
μwave double-balanced mixer	–110 … –125		4…20 GHz	(general experience)
μwave circulator (iso port)	–170	–170	9.1 GHz	Rubiola & al, IEEE T.UFFC 51(8) 957-963 (2004)
μwave isolator (terminated circulator)	–150	–150	≈ 10 GHz	Woode & al, MST 9(9) 1593-9 (1998)
HF-UHF ferrite power splitter	–170	–170	100 MHz	Rubiola, Giordano, RSI 73(6) 2445-2457 (2002)
HF-UHF variab. attenuator (potentiometer)	–150		100 MHz	Rubiola, Giordano, RSI 70(1) 220-225 (1999)
HF-UHF by-step attenuator	–170	–170	100 MHz	Rubiola, Giordano, RSI 73(6) 2445-2457 (2002)
μwave variable attenuator (absorber)	–150		9.1 GHz	Rubiola, Giordano, RSI 70(1) 220-225 (1999)
μwave line stretcher	–150		100 MHz	Rubiola, Giordano, RSI 70(1) 220-225 (1999)
μwave power detector (Schottky)	----	–120	10 GHz	Grop, Rubiola, preliminary (in progress)
μwave photodetector	–120	–120	10 GHz	Rubiola & al, TMTT/JLT 54(2) 816-820 (2006)
2-4 km optical-fiber microwave ink	< –110		10 GHz	Volyanskiy & al, JOSAB 25(12) 2140-2150 (2008)

Photonic systems

Opto-electronic delay line



intensity modulation $P(t) = \overline{P}(1 + m\cos\omega_\mu t)$

photocurrent $i(t) = \frac{q\eta}{h\nu}\,\overline{P}(1 + m\cos\omega_\mu t)$

shot noise $N_s = 2\frac{q^2\eta}{h\nu}\,\overline{P}R_0$

microwave power $\overline{P}_\mu = \frac{1}{2}m^2R_0\left(\frac{q\eta}{h\nu}\right)^2 P^2$

thermal noise $N_t = FkT_0$

total white noise

$$S_{\varphi 0} = \frac{2}{m^2}\left[\underbrace{2\frac{h\nu_\lambda}{\eta}\frac{1}{\overline{P}}}_{\text{shot}} + \underbrace{\frac{FkT_0}{R_0}\left(\frac{h\nu_\lambda}{q\eta}\right)^2\left(\frac{1}{\overline{P}}\right)^2}_{\text{thermal}}\right]$$

flicker phase noise

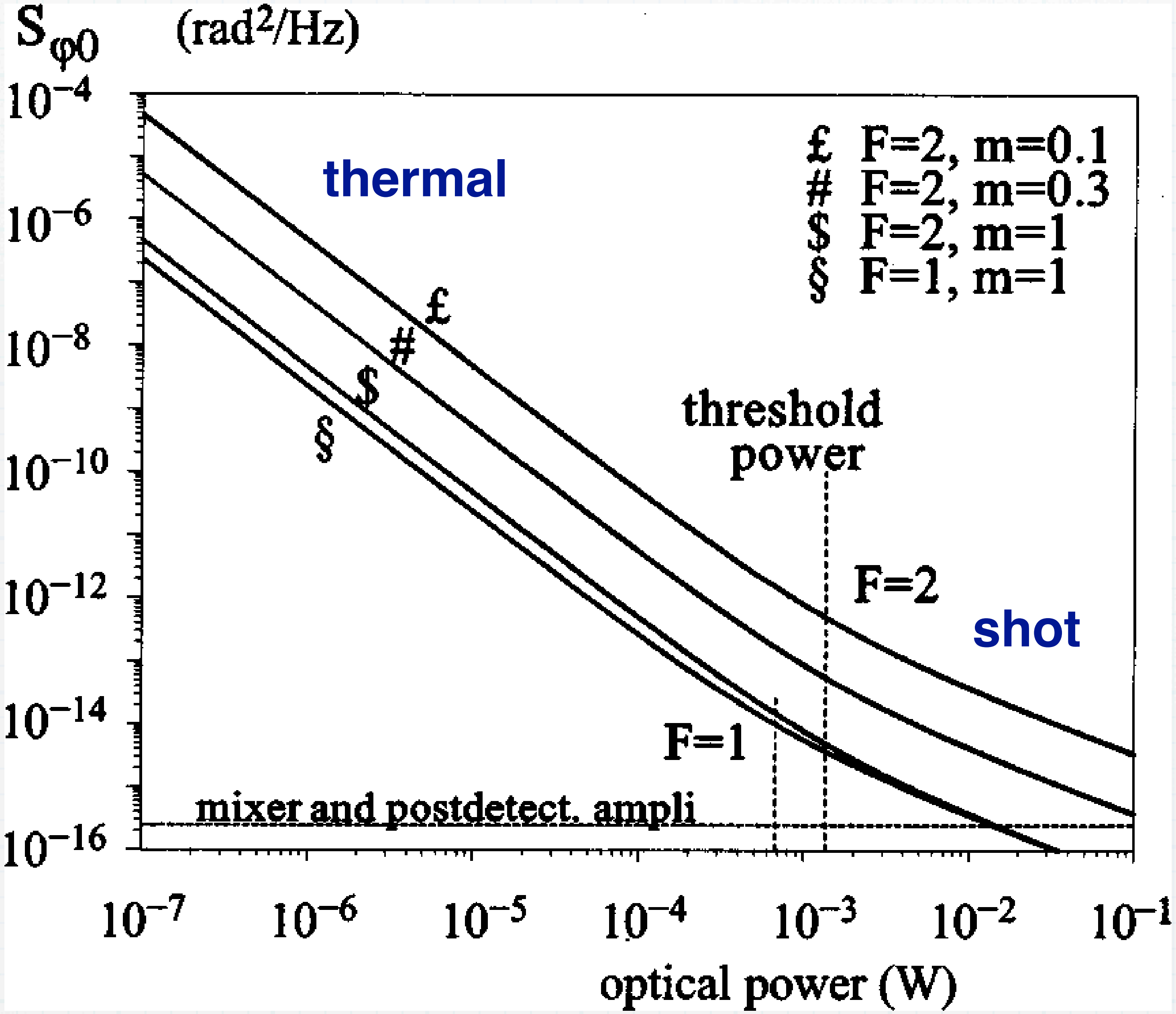
- amplifier GaAs: $b_{-1} \approx -100$ to -110 dBrad²/Hz, SiGe: $b_{-1} \approx -120$ dBrad²/Hz
- photodetector $b_{-1} \approx -120$ dBrad²/Hz [Rubiola & al. MTT/JLT 54(2), feb. 2006
- (mixer $b_{-1} \approx -120$ dBrad²/Hz)
- the phase flicker coefficient b_{-1} is about independent of power
- in a cascade, $(b_{-1})_{tot}$ adds up, regardless of the device order

optical-fiber phase noise? still an experimental parameter

Threshold power

$$S_{\varphi 0} = \frac{16}{m^2}\left[\frac{h\nu_\lambda}{\eta}\,\frac{1}{\overline{\overline{P}}} + \frac{FkT_0}{R_0}\left(\frac{h\nu_\lambda}{q\eta}\right)^2\left(\frac{1}{\overline{\overline{P}}}\right)^2\right]$$

holds for two detectors



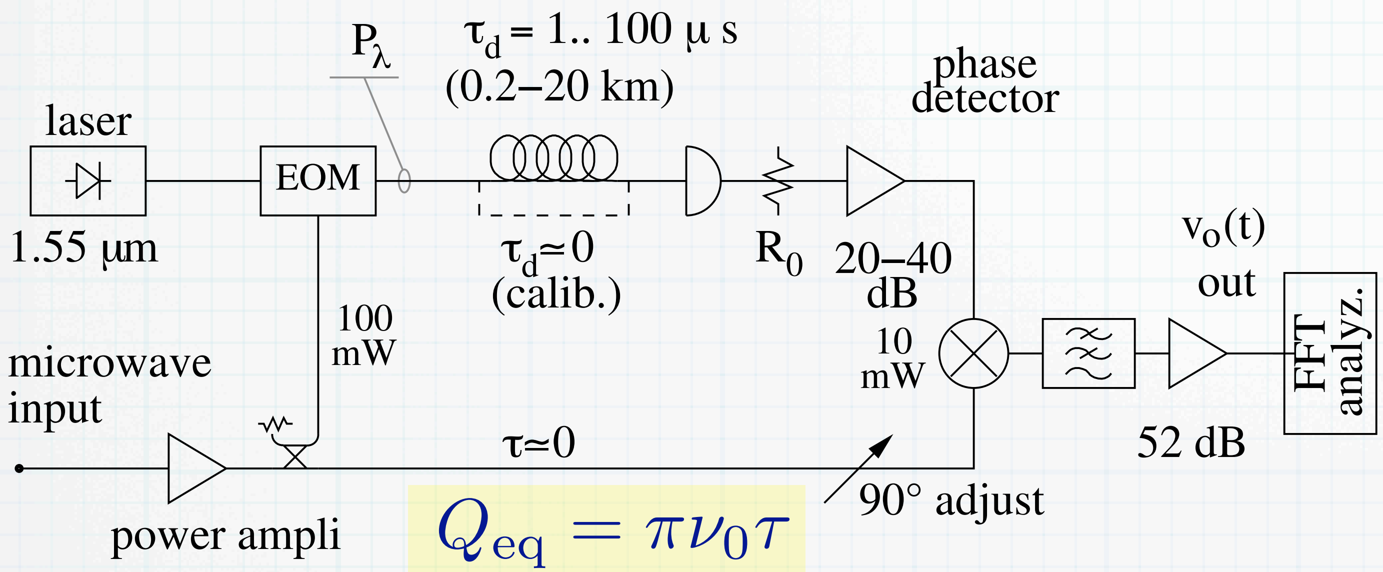
threshold power

$$P_{\lambda,t} = \frac{FkT_0}{R_0}\;\frac{h\nu_\lambda}{q^2\eta}$$

new high-power photodetectors 5–10 mW

Opto-electronic discriminator

Rubiola & al., JOSAB 22(5) p.987–997 (2005) --- Volyanskiy & al., JOSAB 25(12) p.2140–2150 (2008)



$$Q_{\text{eq}} = \pi \nu_0 \tau$$

The short arm can be a microwave cable or a photonic channel

Laplace transforms

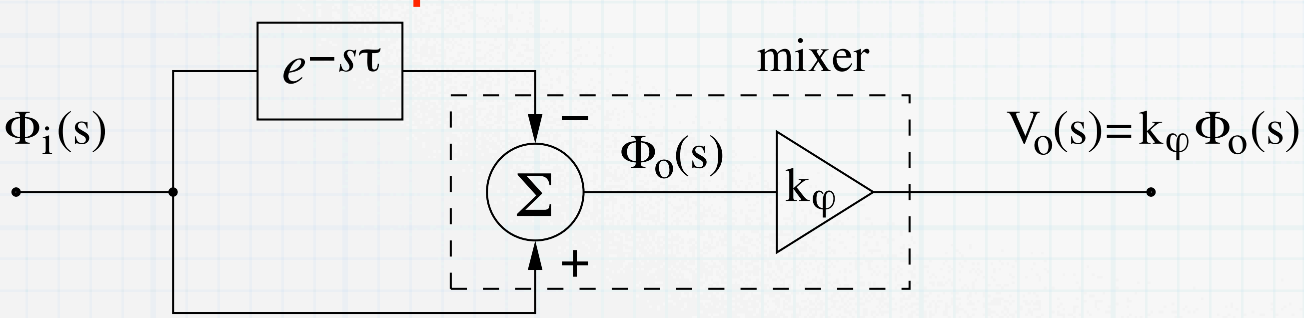
$$\Phi(s) = H_\varphi(s)\Phi_i(s)$$

$$|H_\varphi(f)|^2 = 4\sin^2(\pi f \tau)$$

$$S_y(f) = |H_y(f)|^2\, S_{\varphi\, i}(f)$$

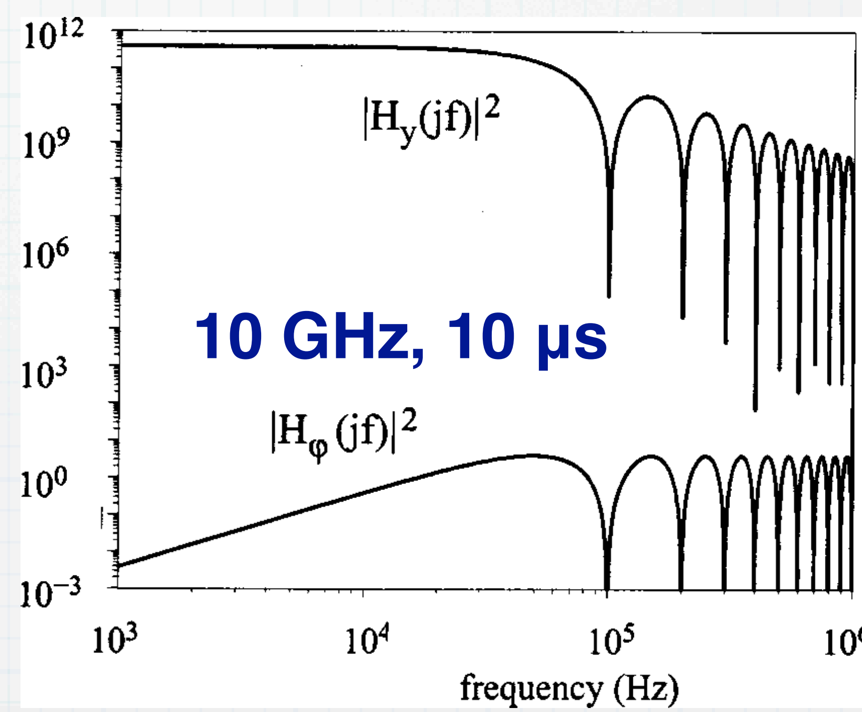
$$|H_y(f)|^2 = \frac{4\nu_0^2}{f^2}\,\sin^2(\pi f \tau)$$

Laplace transforms



$$\Phi_o(s) = (1 - e^{-s\tau})\,\Phi_i(s)$$

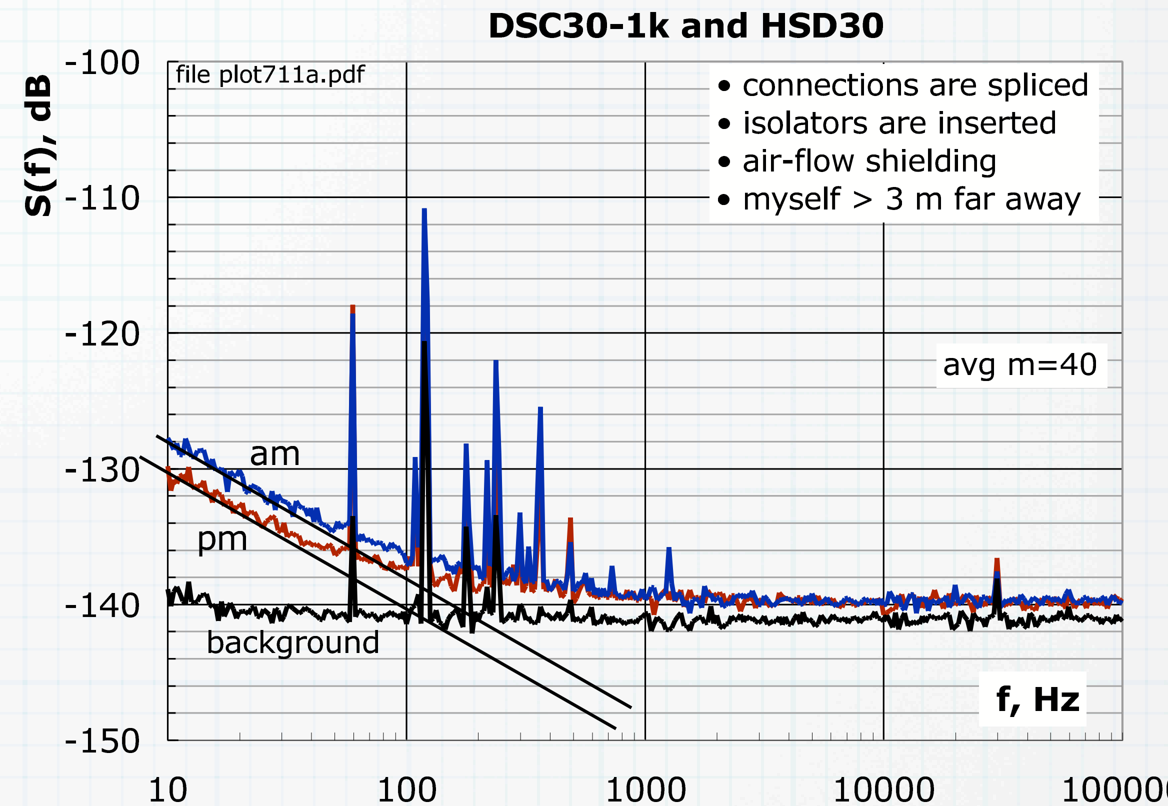
- delay –> frequency-to-phase conversion
- works at any frequency
- long delay (microseconds) is necessary for high sensitivity
- the delay line must be an optical fiber
  fiber: attenuation 0.2 dB/km, thermal coeff. 6.8 10^{-6}/K
  cable: attenuation 0.8 dB/m, thermal coeff. ~ 10^{-3}/K

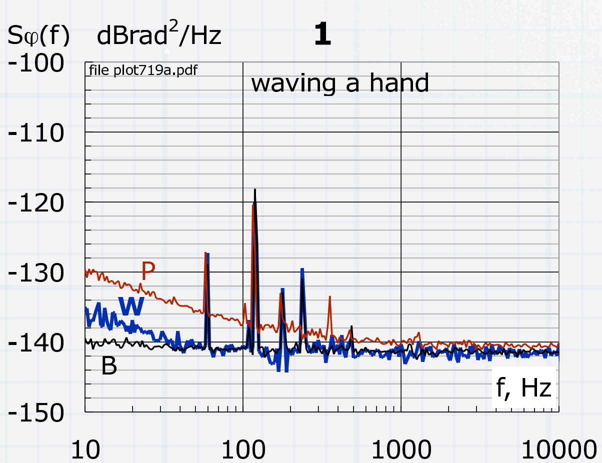


Photodetector 1/f noise

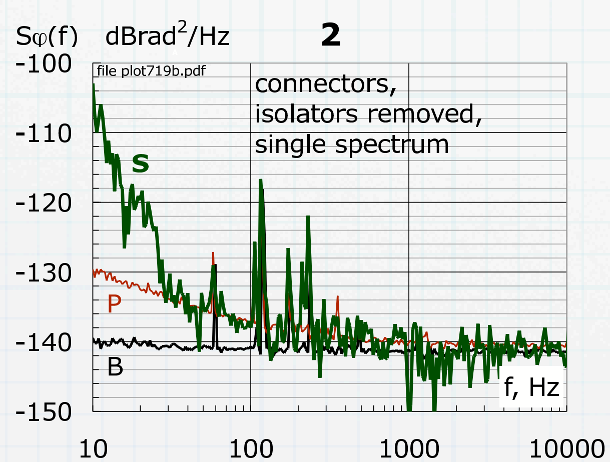
Rubiola, Salik, Yu, Maleki, MTT 54(2) p.816-820, Feb 2006

- the photodetectors we measured are similar in AM and PM 1/f noise
- the 1/f noise is about -120 dB[rad^2]/Hz
- other effects are easily mistaken for the photodetector 1/f noise
- environment and packaging deserve attention in order to take the full benefit from the low noise of the junction

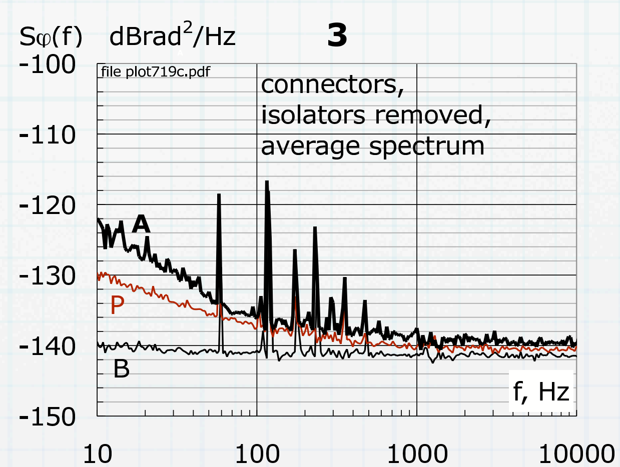




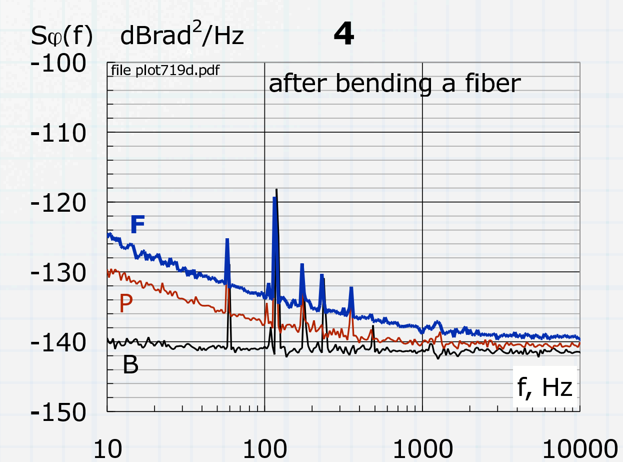
W: waving a hand 0.2 m/s, 3 m far from the system
B: background noise
P: photodiode noise



S: single spectrum, with optical connectors and no isolators
B: background noise
P: photodiode noise



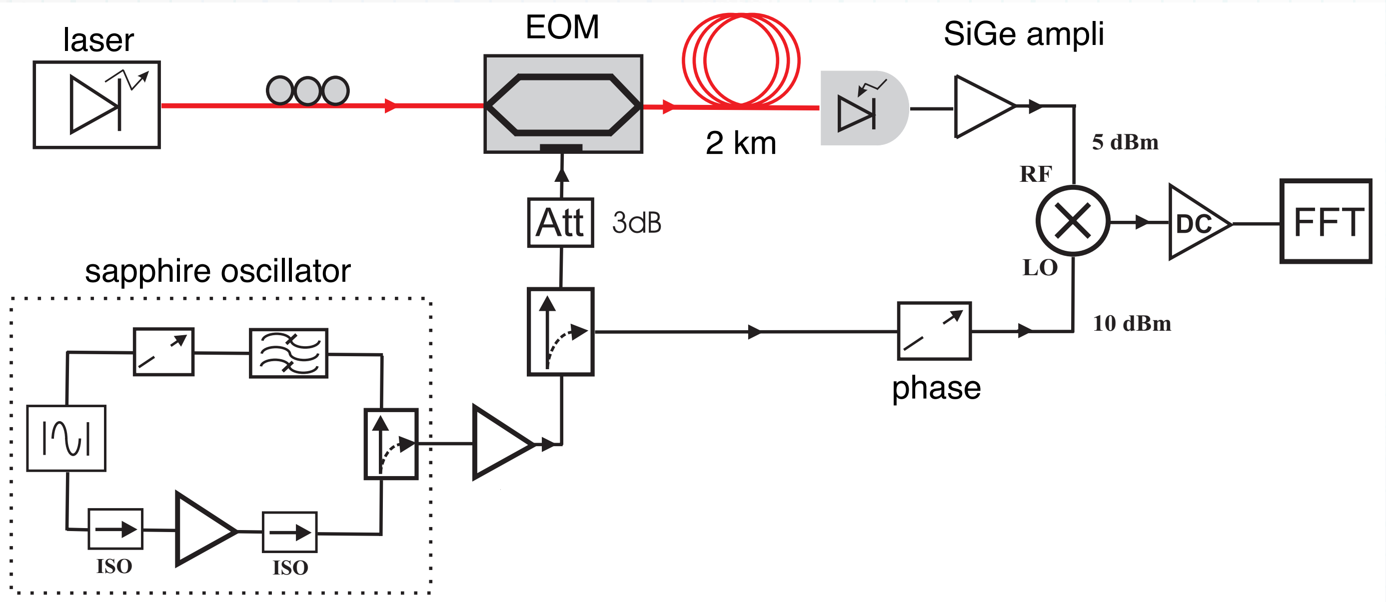
A: average spectrum, with optical connectors and no isolators
B: background noise
P: photodiode noise

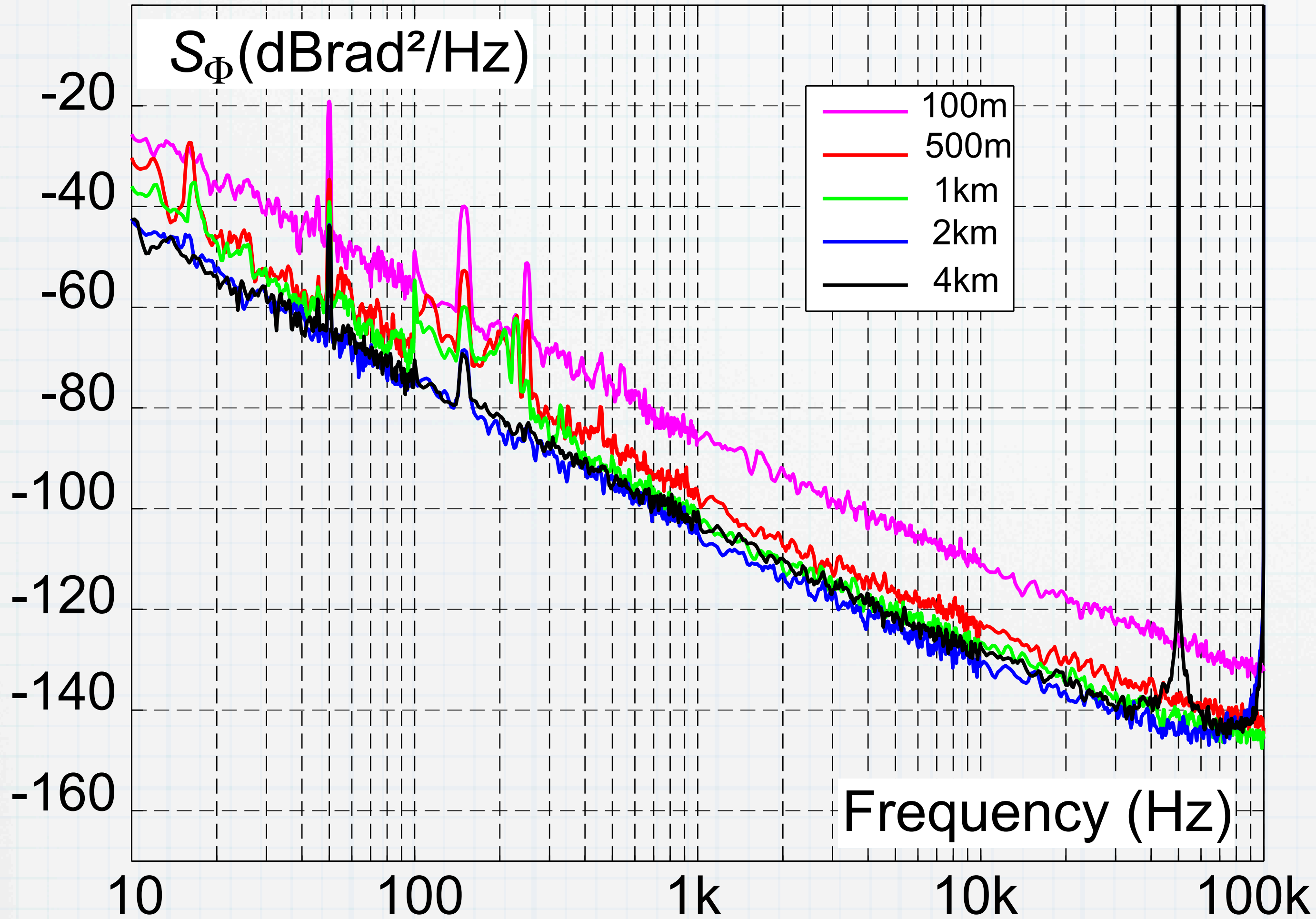


F: after bending a fiber, 1/f noise can increase unpredictably
B: background noise
P: photodiode noise

Measurement of a sapphire oscillator

JOSAB 25 (12) p.2140-2150, Dec 2008. Also arXiv:0807.3494v1 [physics.optics] Jul 2008.

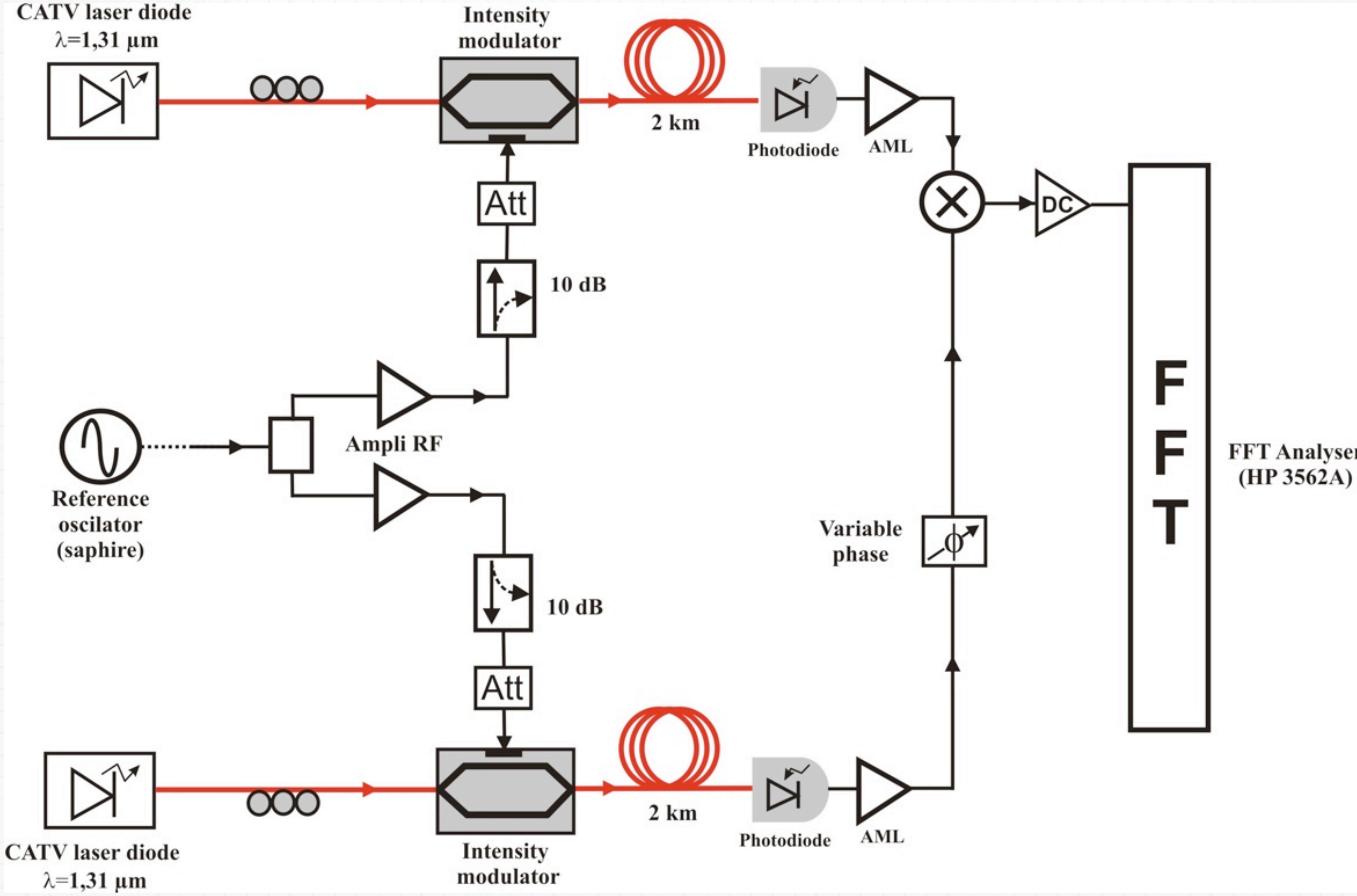




- The instrument noise scales as 1/τ, yet the blue and black plots overlap
  magenta, red, green => instrument noise
  blue, black => noise of the sapphire oscillator under test
- The $1/f^3$ phase noise (frequency flicker) outperforms the 10 GHz sapphire oscillator (the lowest-noise microwave oscillator)
- Low AM noise of the oscillator under test is necessary

Measurement of the optical-fiber noise

Volyanskiy & al., JOSAB 25(12) 2140-2150, Dec.2008. Also arXiv:0807.3494v1 [physics.optics] July 2008.



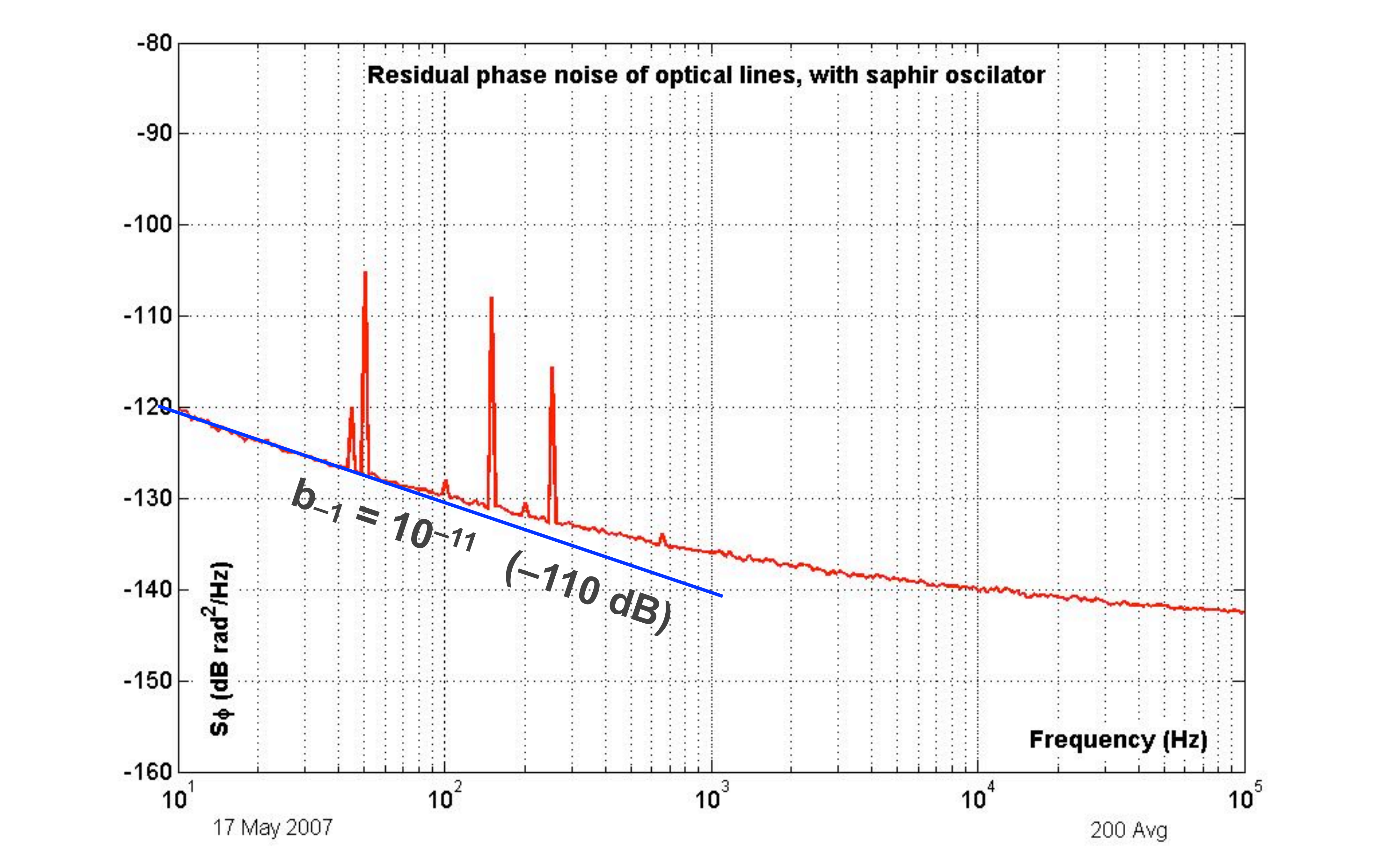
- matching the delays, the oscillator phase noise cancels
- this scheme gives the total noise

  2 × (ampli + fiber + photodiode + ampli) + mixer

  thus it enables only to assess an upper bound of the fiber noise

Measurement of the delay-line noise (2)

Volyanskiy & al., JOSAB 25(12) 2140-2150, Dec.2008. Also arXiv:0807.3494v1 [physics.optics] July 2008.



- The method enables only to assess an **upper bound of the delay-line noise** $b_{-1} \leq 5\times10^{-12}$ rad²/Hz for L = 2 km (–113 dBrad²/Hz)
- We believe that this residual noise is the signature of the two GaAs power amplifier that drives the MZ modulator

Physical phenomena in optical fibers

Birefringence. Common optical fibers are made of amorphous Ge-doped silica, for an ideal fiber is not expected to be birefringent. Nonetheless, actual fibers show birefringent behavior due to a variety of reasons, namely: core ellipticity, internal defects and forces, external forces (bending, twisting, tension, kinks), external electric and magnetic fields. The overall effect is that light propagates through the fiber core in a non-degenerate, orthogonal pair of axes at different speed. Polarization effects are strongly reduced in polarization maintaining (PM) fibers. In this case, the cladding structure stresses the core in order to increase the difference in refraction index between the two modes.

Polarization mode dispersion (PMD). This effect rises from the random birefringence of the optical fiber. The optical pulse can choose many different paths, for it broadens into a bell-like shape bounded by the propagation times determined by the highest and the lowest refraction index. Polarization vanishes exponentially along the light path. It is to be understood that PMD results from the vector sum over multiple forward paths, for it yields a well-shaped dispersion pattern.

Bragg scattering. In the presence of monocromatic light (usually X-rays), the periodic structure of a crystal turns the randomness of scattering into an interference pattern. This is a weak phenomenon at micron wavelengths because the inter-atom distance is of the order of 0.3--0.5 nm. Bragg scattering is not present in amorphous materials.

Brillouin scattering. In solids, the photon-atom collision involves the emission or the absorption of an acoustic phonon, hence the scattered photons have a wavelength slightly different from incoming photons. An exotic form of Brillouin scattering has been reported in optical fibers, due to a transverse mechanical resonance in the cladding, which stresses the core and originates a noise bump on the region of 200--400 MHz.

Raman scattering. This phenomenon is similar to Rayleigh scattering, but it involves the optical branch of phonons.

Rayleigh scattering. This is random scattering due to molecules in a disordered medium, by which light looses direction and polarization. A small fraction of the light intensity is thereby back-scattered one or more times, for it reaches the fiber end after a stochastic to-and-fro path, which originates phase noise. In SM fibers at 1.55 μm it contributes 0.15 dB/km to the optical loss.

Kerr effect. This effect states that an electric field changes the refraction index. So, the electric field of light modulate the refraction index, which originates the 2nd-order nonlinearity.

Discontinuities. Discontinuities cause the wave to be reflected and/or to change polarization. As the pulse can be split into a pulse train depending on wavelength, this effect can turn into noise.

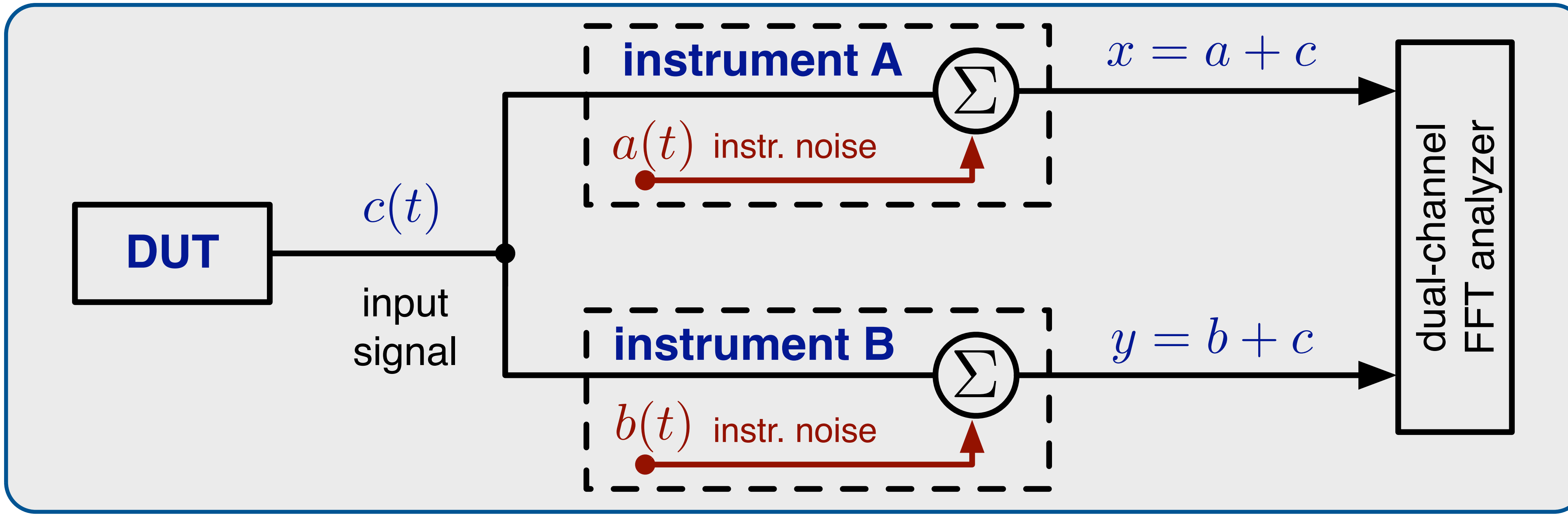
Group delay dispersion (GVD). There exist dispersion-shifted fibers, that have a minimum GVD at 1550 nm. GVD compensators are also available.

PMD-Kerr compensation. In principle, it is possible that PMD and Kerr effect null one another. This requires to launch the appropriate power into each polarization mode, for two power controllers are needed. Of course, this is incompatible with PM fibers.

Which is the most important effect? In the community of optical communications, PMD is considered the most significant effect. Yet, this is related to the fact that excessive PMD increases the error rate and destroys the eye pattern of a channel. In the case of the photonic oscillator, the signal is a pure sinusoid, with no symbol randomness.

Cross-spectrum measurements

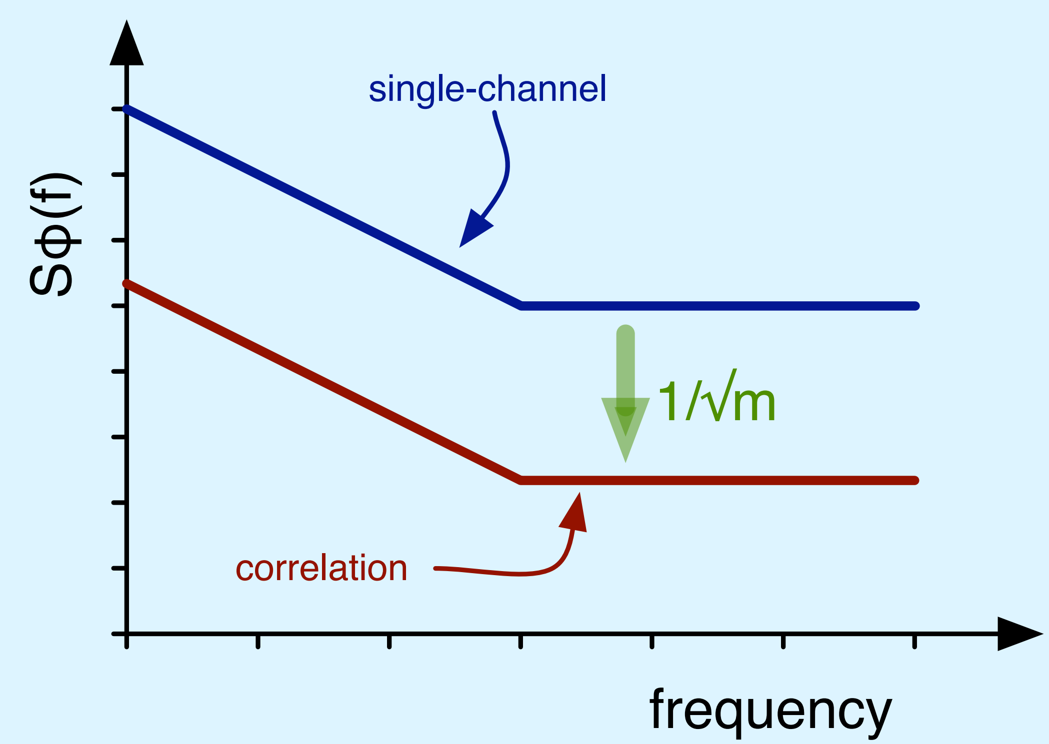
Correlation measurements



Two separate instruments measure the same DUT. Only the DUT noise is common

noise measurements		
DUT noise, normal use	a, b c	instrument noise DUT noise
background, ideal case	a, b c = 0	instrument noise no DUT
background, real case	a, b c ≠ 0	c is the correlated instrument noise

a(t), b(t) –> instrument noise
c(t) –> DUT noise



Boring exercises before playing a Steinway

Power spectral density S_{xx}

X is white Gaussian noise
Take one frequency, S(f) –> S. Same applies to all frequencies

Spectrum $\langle S_{xx} \rangle_m = \frac{1}{T} \langle X X^* \rangle_m$

$$= \frac{1}{T} \langle (X' + \imath X'') \times (X' - \imath X'') \rangle_m$$

$$= \frac{1}{T} \boxed{\left\langle (X')^2 + (X'')^2 \right\rangle_m}$$

white, Gaussian, avg = 0, var = 1/2

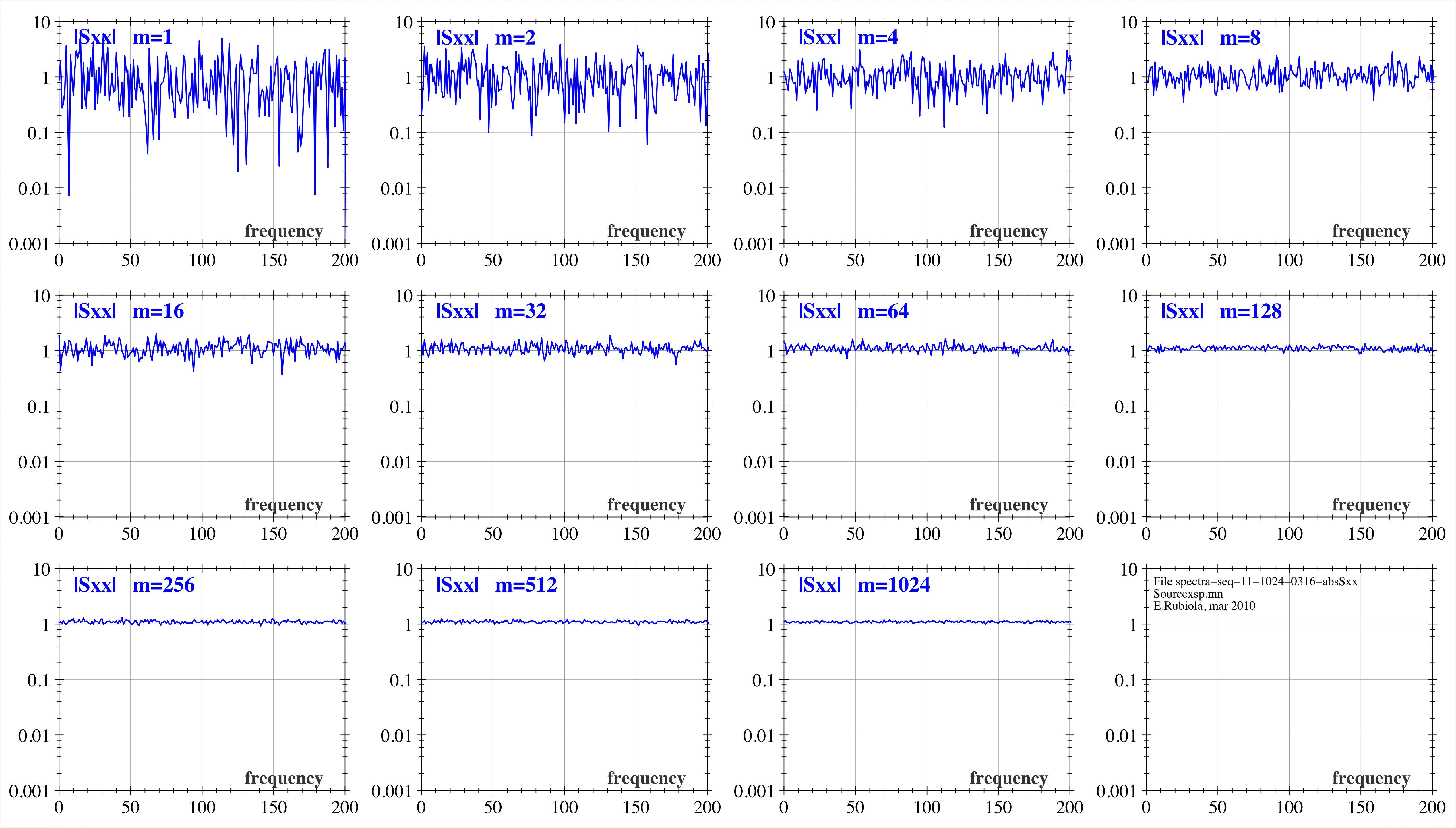
white, χ^2, with 2m degrees of freedom avg = 1, var = 1/m

$$\frac{\text{dev}}{\text{avg}} = \sqrt{\frac{1}{m}}$$

the S_{xx} track on the FFT-SA shrinks as $1/m^{1/2}$

Normalization: in 1 Hz bandwidth
var{X}= 1, and var{X'}= var{X"}= 1/2

Measurement of |Sxx|



Running the measurement, m increases and S_{xx} shrinks => better confidence level

S_{yx} with correlated term (1)

A, B = instrument background
C = DUT noise
channel 1 X = A + C
channel 2 Y = B + C
A, B, C are independent Gaussian noises
Re{ } and Im{ } are independent Gaussian noises

Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ^2
var{A'} = var{A"} = var{B'} = var{B"} = 1/2, and var{C'} = var{C"} = $\kappa^2/2$

Cross-spectrum

$$\langle S_{yx}\rangle_m = \frac{1}{T}\langle YX^*\rangle_m = \frac{1}{T}\langle (Y'+\imath Y'')\times(X'-\imath X'')\rangle_m$$

Expand using

$$X = (A'+\imath A'')+(C'+\imath C'') \qquad \text{and} \qquad Y = (B'+\imath B'')+(C'+\imath C'')$$

Split S_{yx} into three sets

$$\langle S_{yx}\rangle_m = \langle S_{yx}\rangle_m\Big|_{\text{instr}} + \langle S_{yx}\rangle_m\Big|_{\text{mixed}} + \langle S_{yx}\rangle_m\Big|_{\text{DUT}}$$

background only — background and DUT noise — DUT noise only

... and work it out !!!

S_{yx} with correlated term $\kappa \neq 0$ (2)

All the DUT signal goes in Re{S_{yx}}, Im{S_{yx}} contains only noise

Real

var=1/2 | var=1/2 | var= $\kappa^2/2$ | var=1/2 | var= $\kappa^2/2$ | var= $\kappa^2/2$

$$\Re\left\{\langle S_{yx}\rangle_m\right\} = \frac{1}{T}\left\{\langle B'A' + B''A''\rangle_m + \langle B'C' + B''C''\rangle_m + \langle C'A' + C''A''\rangle_m + \langle (C')^2 + (C'')^2\rangle_m\right\}$$

Bessel K_0, avg=0, var=1/4

Bessel K_0, avg=0, var=$\kappa^2/4$

white, χ^2, 2 DF avg = κ^2, var = κ^4

Gaussian, avg = 0, var = 1/2m

Gaussian, avg = 0, var = κ^2/2m

Gaussian, avg = 0, var = κ^2/2m

white, χ^2 2m deg. of freedom avg = κ^2, var = κ^4/m Set $\mathscr{C}$

Set $\mathscr{A}$ Gaussian, avg = 0, var = (1+2κ^2)/2m

Note: DF < 2m See vol.XVI p.56

Imaginary

$$\Im\left\{\langle S_{yx}\rangle_m\right\} = \frac{1}{T}\left\{\langle B''A' + B'A''\rangle_m + \langle B''C' - B'C''\rangle_m + \langle C''A' - C'A''\rangle_m\right\}$$

Bessel K_0, avg = 0, var = 1/4

Bessel K_0, avg = 0, var = $\kappa^2/4$

Gaussian, avg = 0, var = 1/2m

Gaussian, avg = 0, var = κ^2/2m

Gaussian, avg = 0, var = κ^2/2m

Set $\mathscr{B}$ Gaussian, avg = 0, var = (1+2κ^2)/2m

Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ^2
var{A'} = var{A"} = var{B'} = var{B"} = 1/2, and var{C'} = var{C"} = $\kappa^2/2$

A, B, C are independent Gaussian noises
Re{ } and Im{ } are independent Gaussian noises

Expand S_{yx}

$$S_{yx} = \frac{1}{T}\,\mathbb{E}\left\{\mathscr{A} + \imath\mathscr{B} + \mathscr{C}\right\}$$

Bessel K_0,
avg=0, var=1/4

$$\mathscr{A} = B'A' + B''A'' + B'C' + B''C'' + C'A' + C''A''$$
$$\mathscr{B} = B''A' + B'A'' + B''C' - B'C'' + C''A' - C'A''$$

Bessel K_0,
avg=0, var=$\kappa^2/4$

$$\mathscr{C} = C'^{\,2} + C''^{\,2}$$

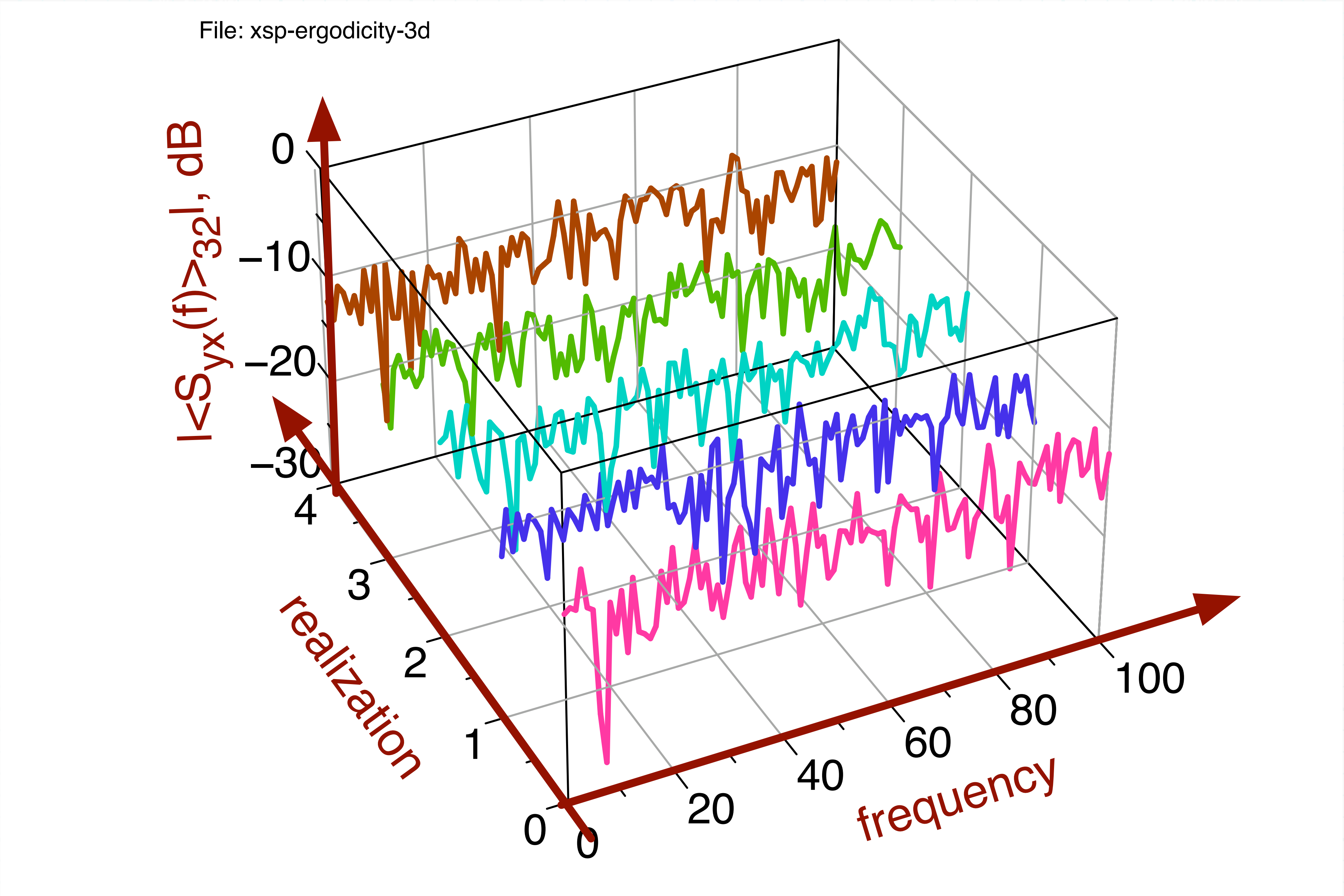
white, χ^2, 2 DF
avg = κ^2, var = κ^4

After averaging, the Bessel K_0 distribution turns into a Gaussian distribution (central limit theorem)

term	$\mathbb{E}$	$\mathbb{V}$	PDF	comment
$\langle\mathscr{A}\rangle_m$	0	$\dfrac{1+2\kappa^2}{2m}$	Gauss	average (sum) of zero-mean
$\langle\mathscr{B}\rangle_m$	0	$\dfrac{1+2\kappa^2}{2m}$	Gauss	Gaussian processes
$\langle\mathscr{C}\rangle_m$	κ^2	κ^4/m	χ^2 $\nu = 2m$	average (sum) of chi-square processes
$\langle\tilde{\mathscr{C}}\rangle_m$	κ^2	κ^4/m	Gauss	approximates $\langle\mathscr{C}\rangle_m$ for large m

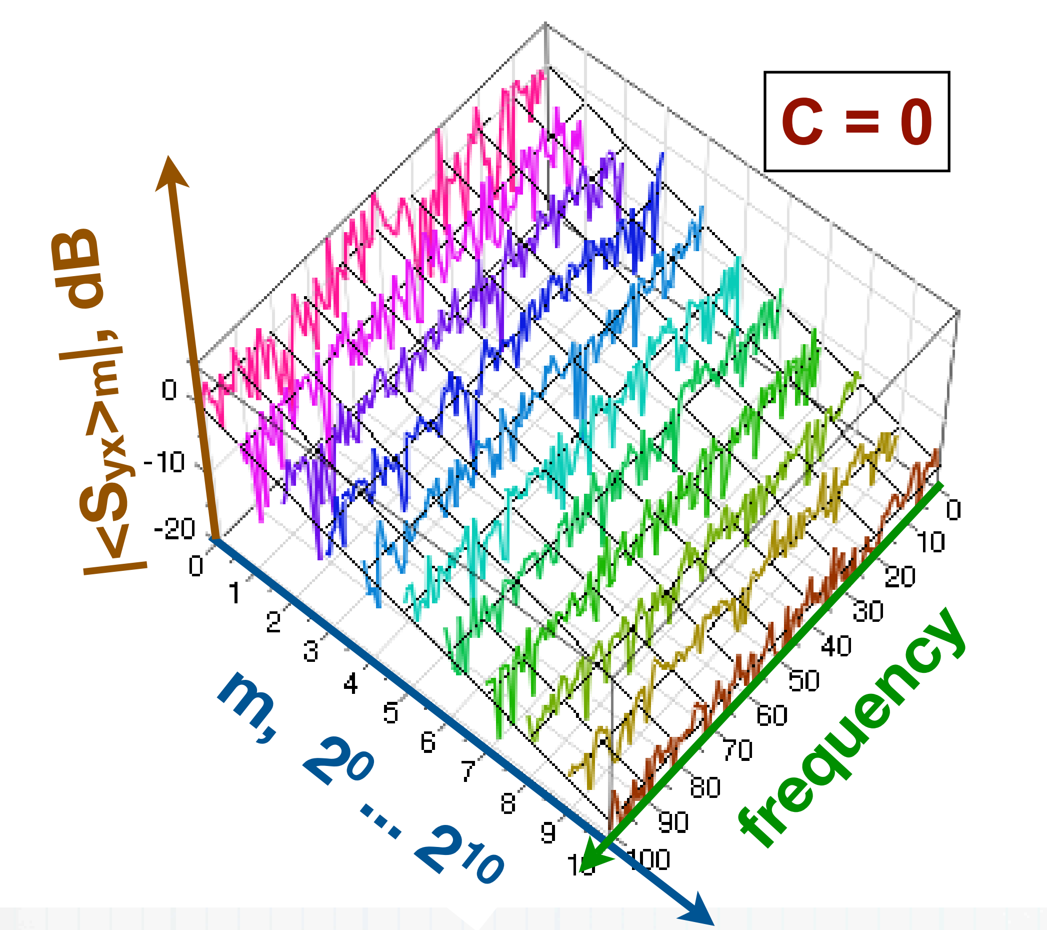
Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ^2
var{A'} = var{A"} = var{B'} = var{B"} = 1/2, and var{C'} = var{C"} = $\kappa^2/2$

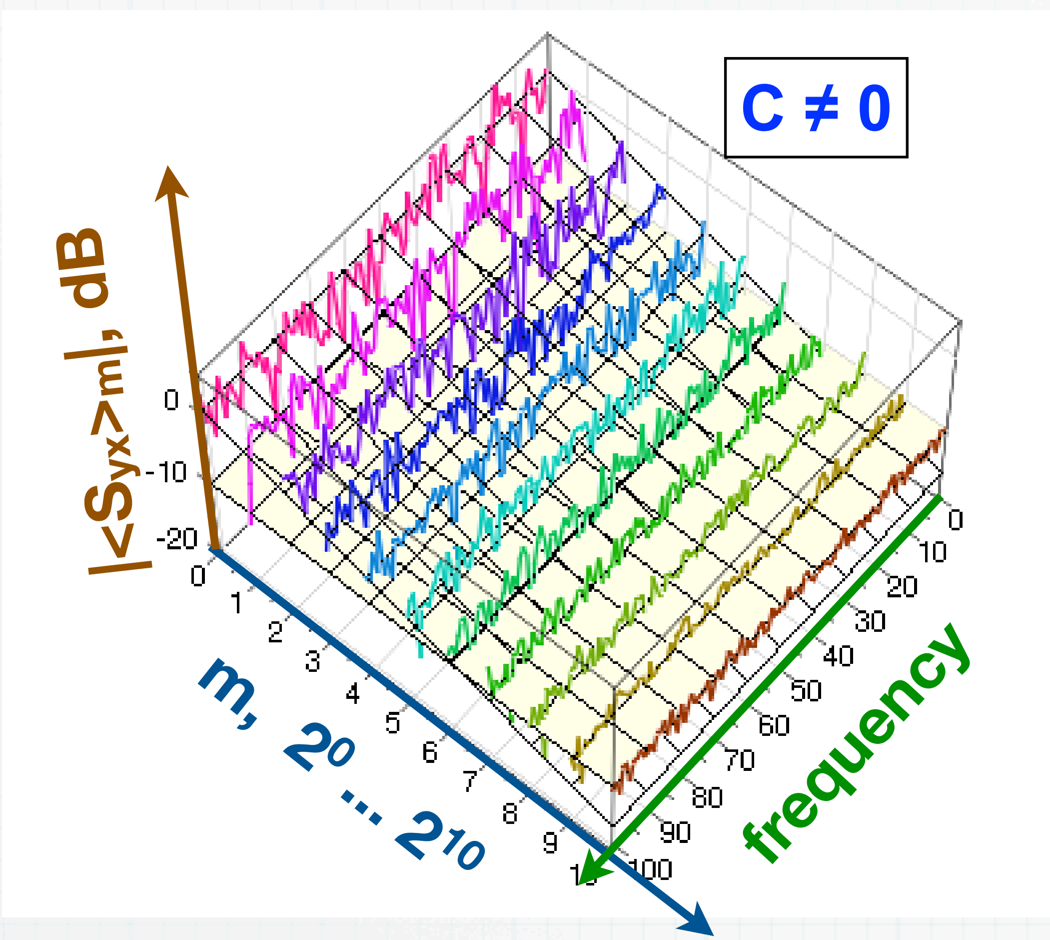
The concept of ergodicity

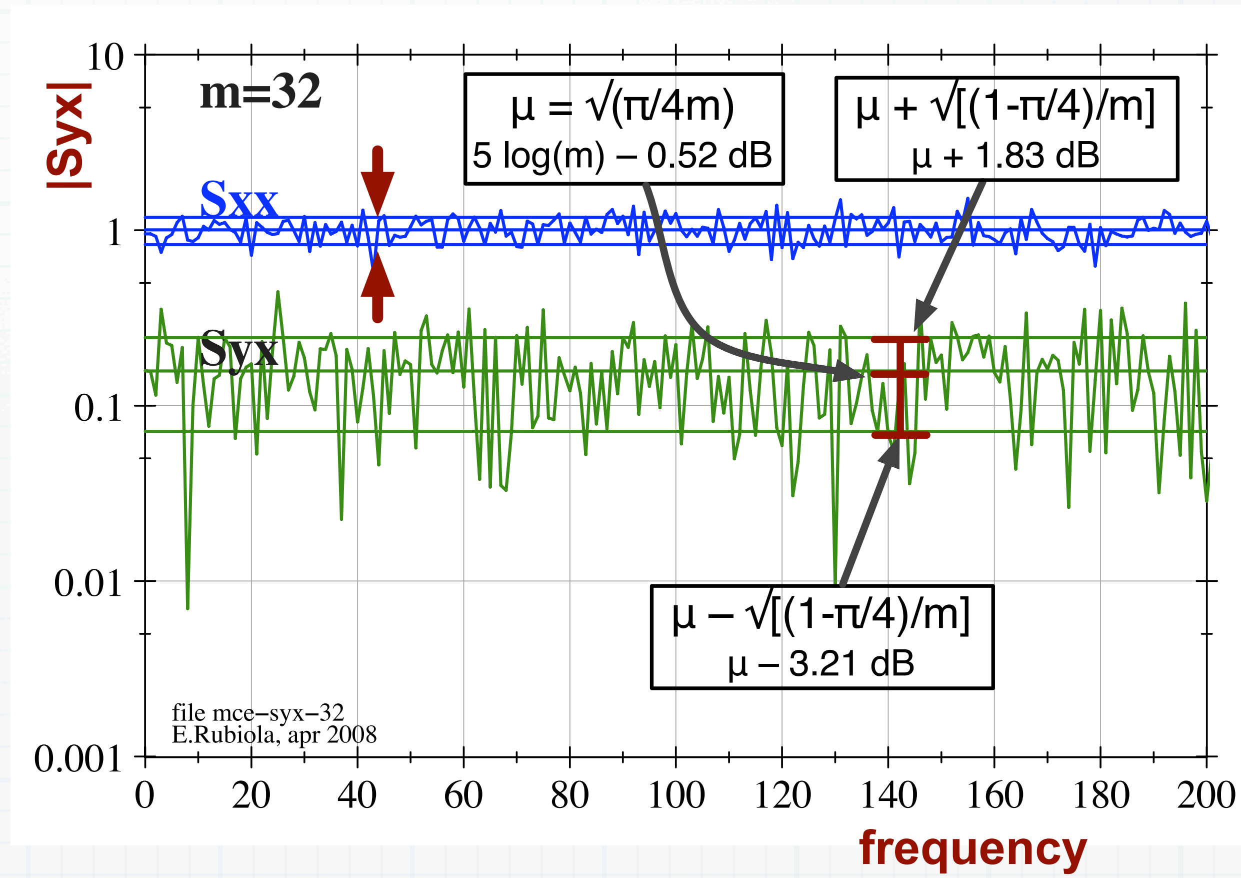


Ergodicity allows to interchange time statistics and ensemble statistics, thus the running index i of the sequence and the frequency f.
The average and the deviation calculated on the frequency axis are the same as the average and the deviation of the time series.

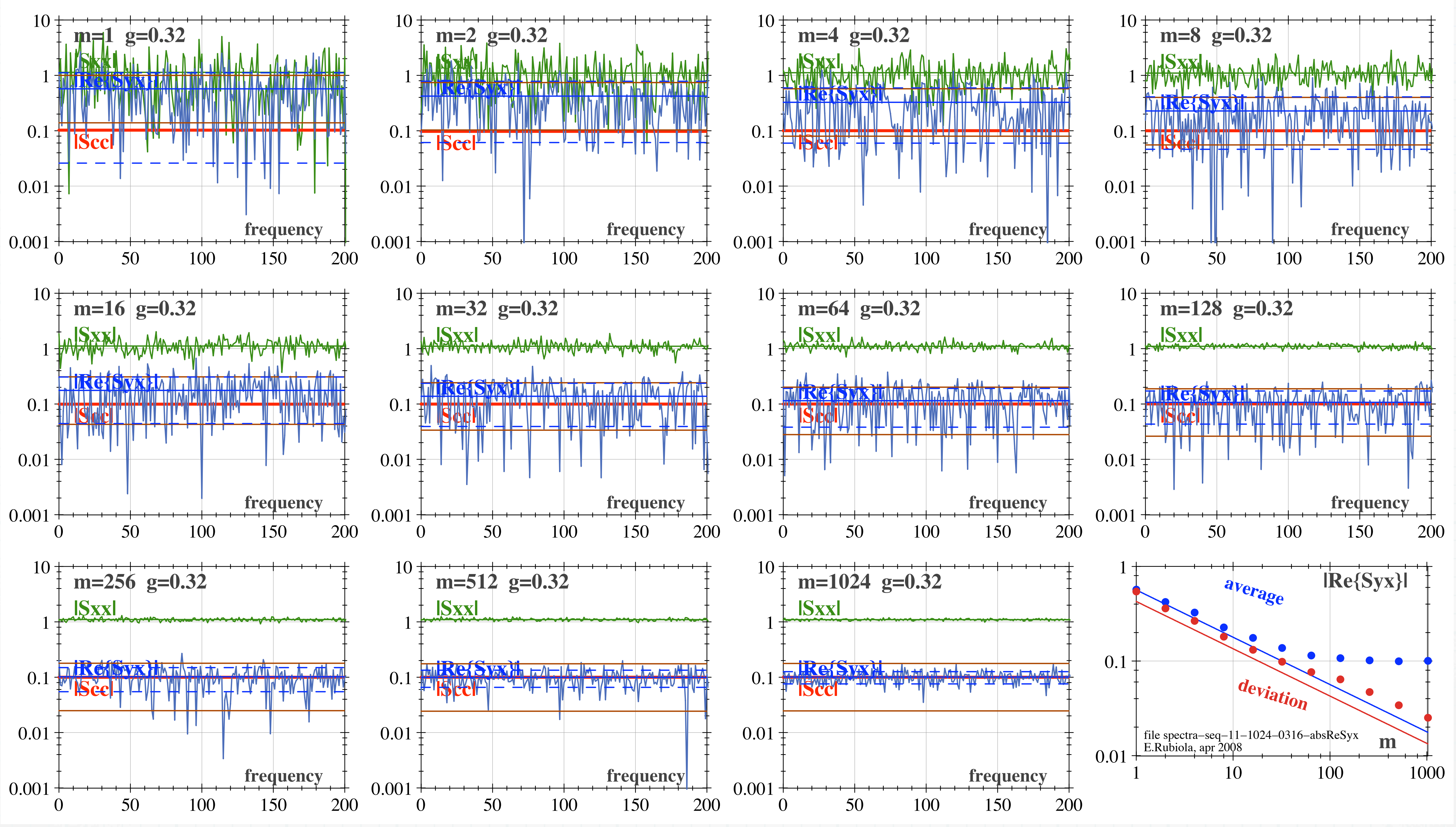
Example: Measurement of |Syx|







Measurement (C≠0), |Re{Syx}|

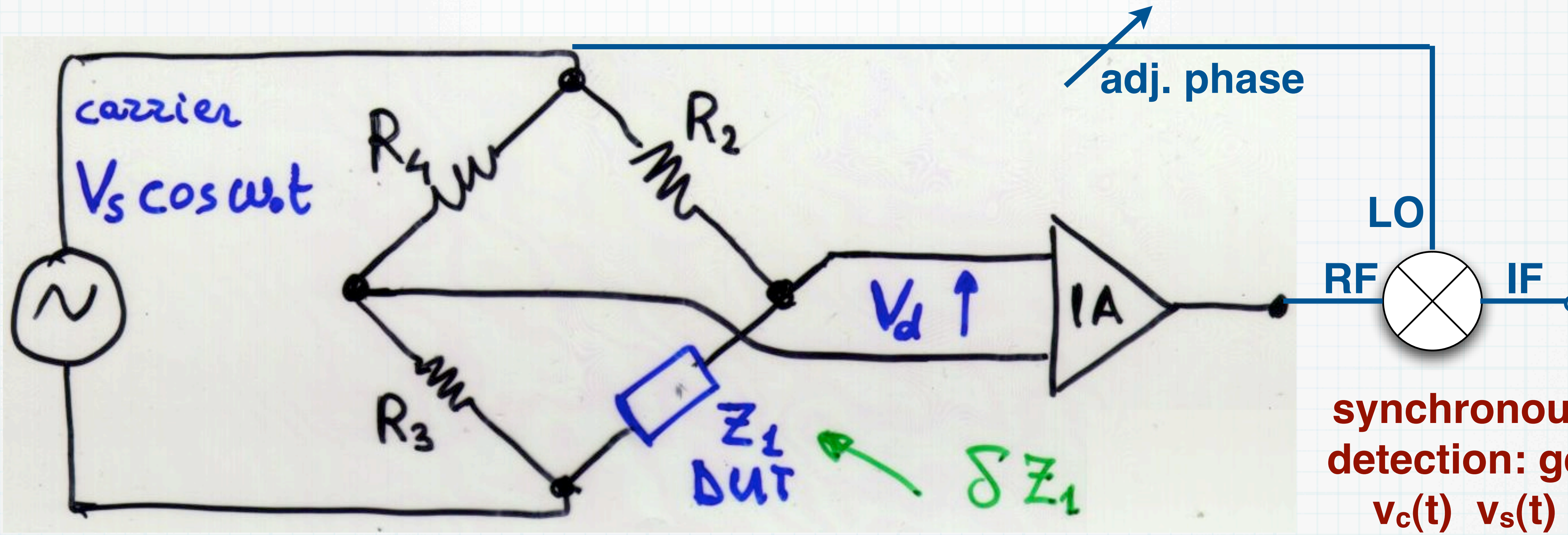


Running the measurement, m increases
S_{xx} shrinks => better confidence level
S_{yx} decreases => higher single-channel noise rejection

Bridge method

real δZ_1 => AM noise $v_c(t)\cos(\omega_0 t)$
imaginary δZ_1 => PM noise $-v_s(t)\sin(\omega_0 t)$

Wheatstone bridge



synchronous detection: get $v_c(t)$ $v_s(t)$ (AM or PM noise)

equilibrium: $V_d = 0$ –> carrier suppression

static error δZ_1 –> some residual carrier

real δZ_1 => in-phase residual carrier $V_{re} \cos(\omega_0 t)$

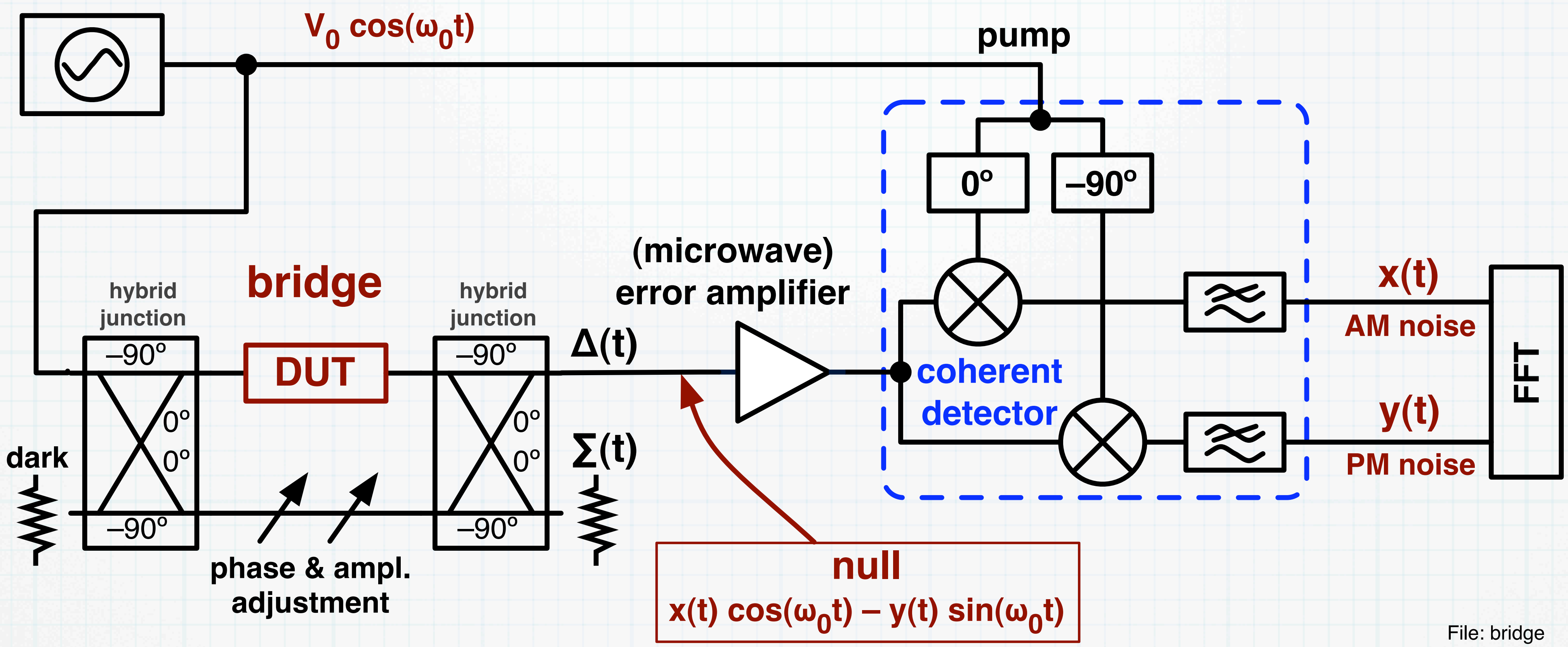
imaginary δZ_1 => quadrature residual carrier $V_{im} \sin(\omega_0 t)$

fluctuating error δZ_1 => noise sidebands

real δZ_1 => AM noise $v_c(t) \cos(\omega_0 t)$

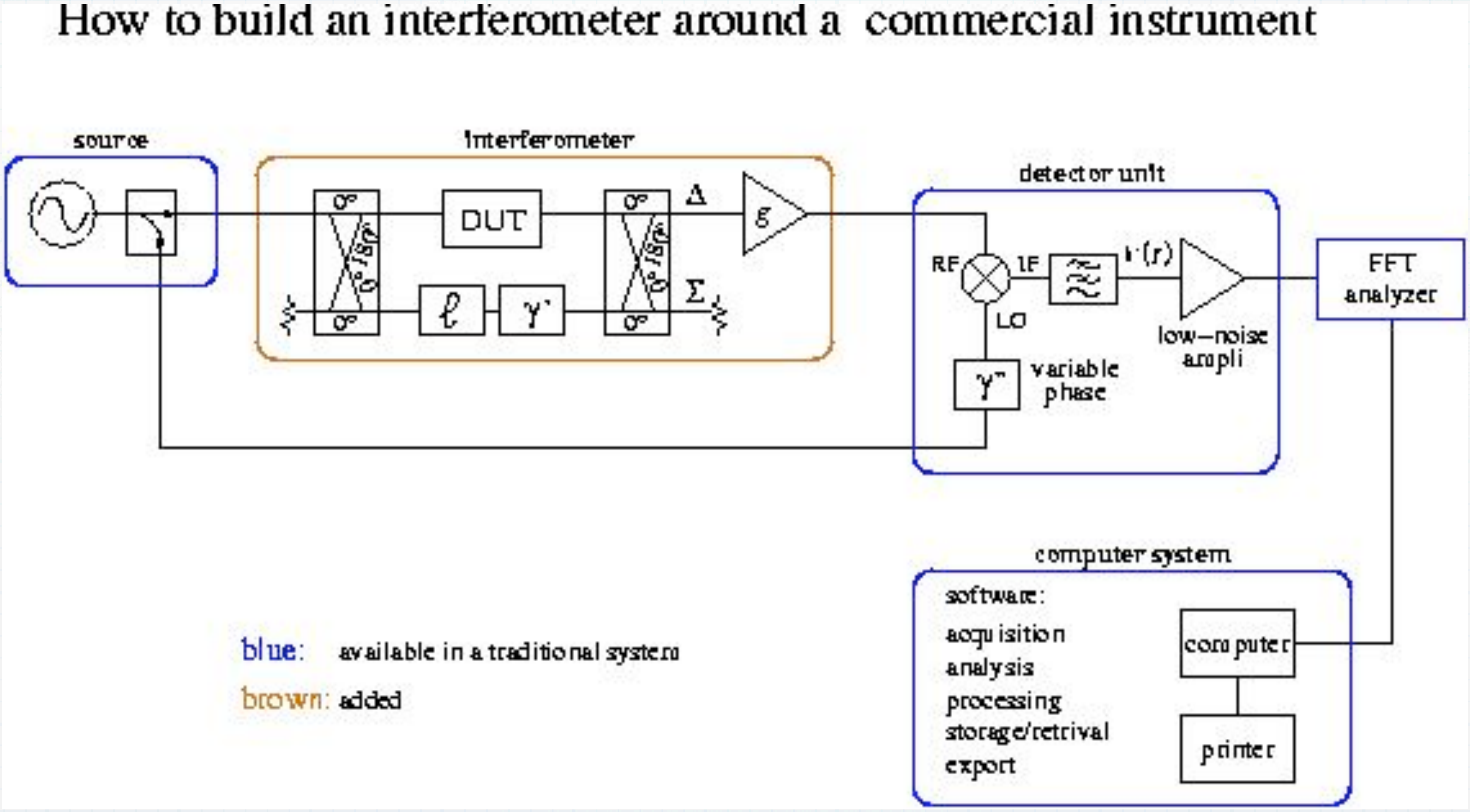
imaginary δZ_1 => PM noise $-v_s(t) \sin(\omega_0 t)$

Bridge PM and AM noise measurement



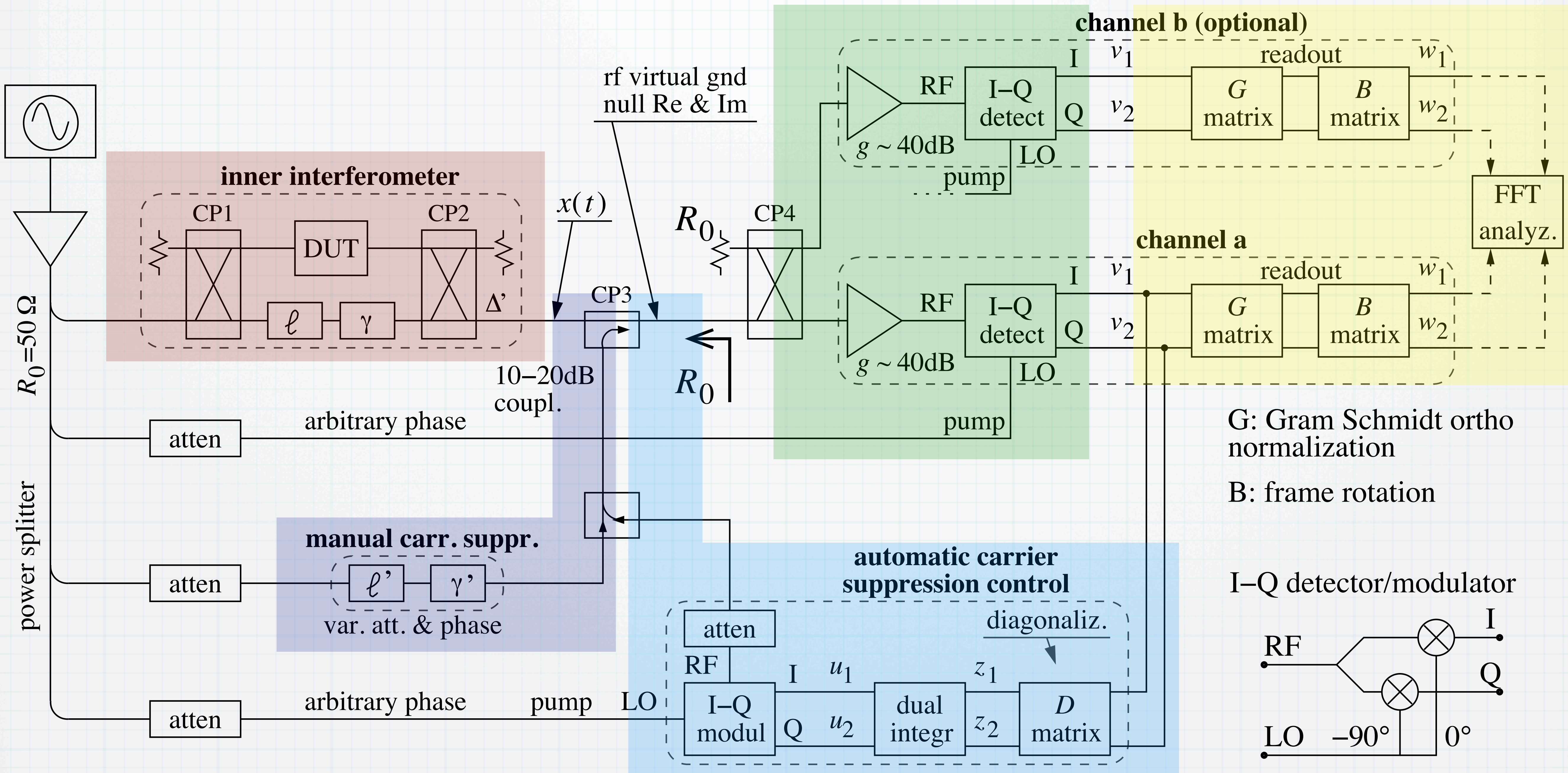
- Bridge => high rejection of the master-oscillator noise
- Amplification and synchronous detection of the noise sidebands
- No carrier => the amplifier can't flicker (no up-conversion of near-dc 1/f)
- High microwave gain before detection => low background
- Low 50-60 Hz residuals because microwave circuits are insensitive to magnetic fields

A bridge (interferometric) instrument can be built around a commercial instrument



You will appreciate the computer interface and the software ready for use

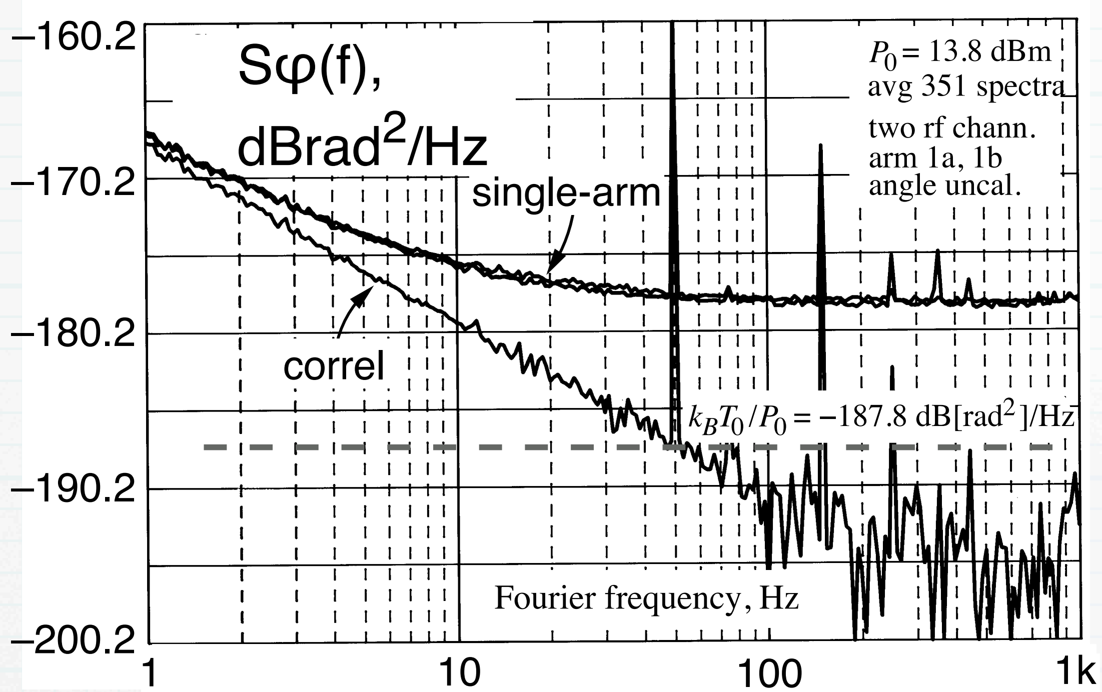
Flicker reduction, correlation, and closed-loop carrier suppression can be combined



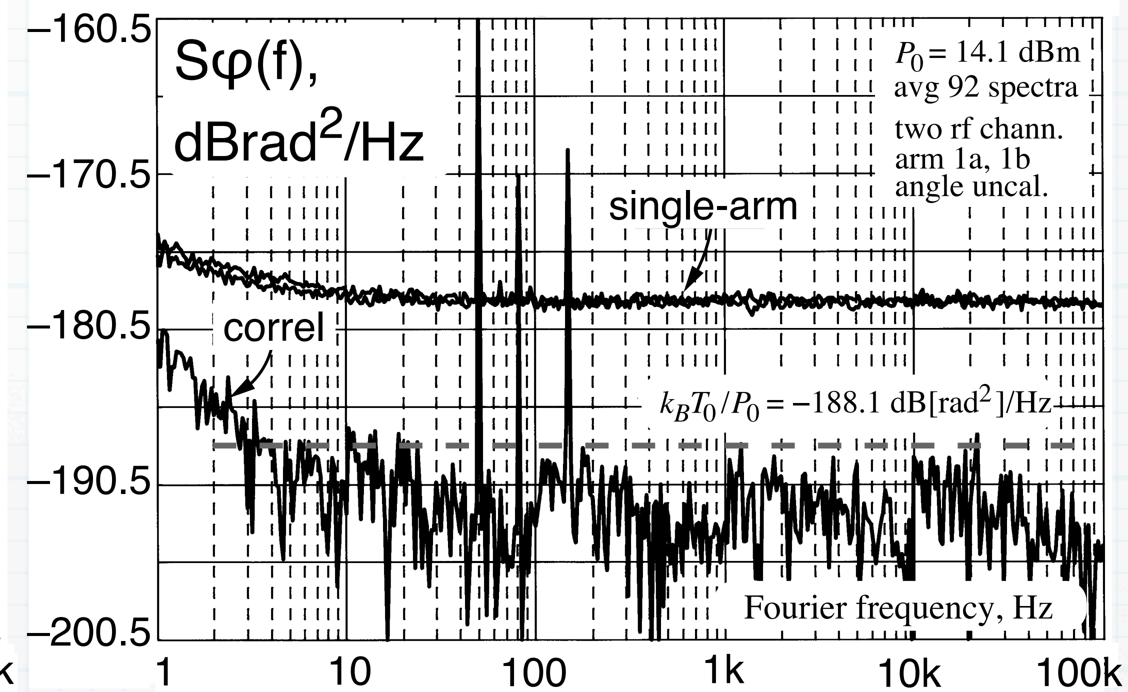
E. Rubiola, V. Giordano, Rev. Sci. Instrum. 73(6) pp.2445-2457, June 2002

Example of results

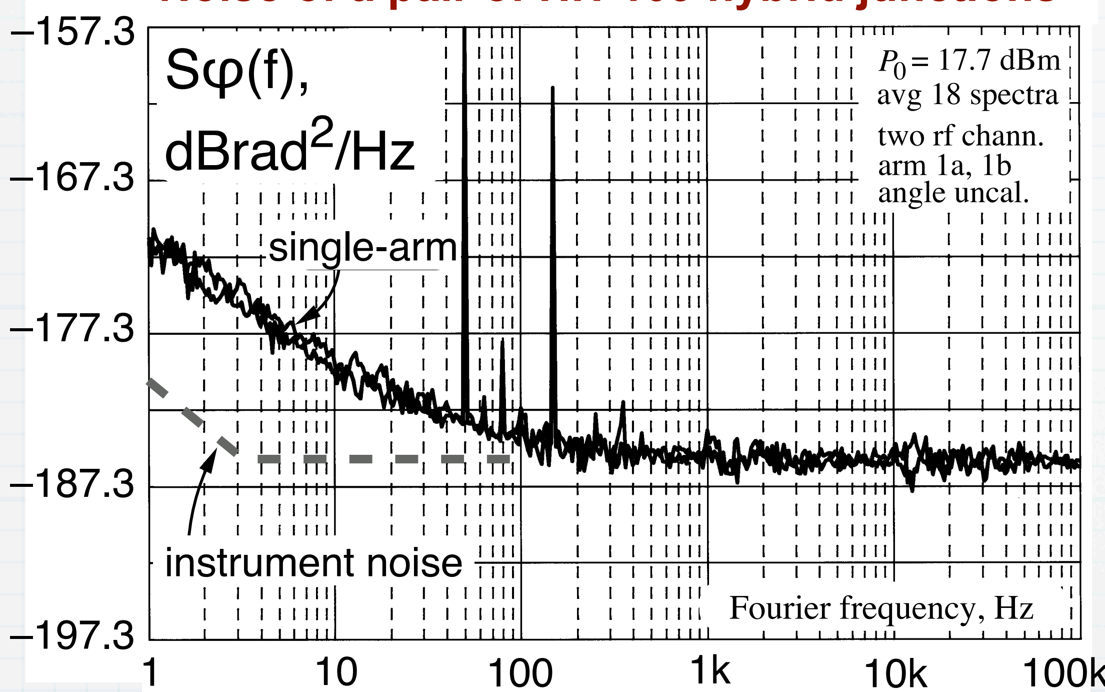
Noise of a by-step attenuator



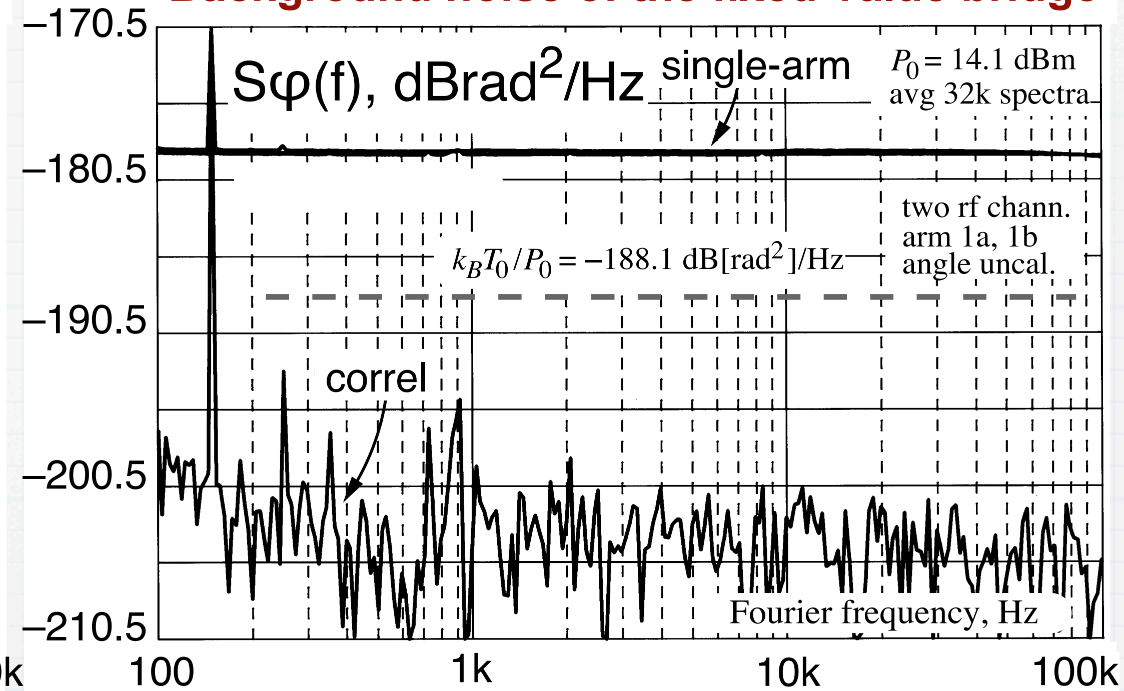
Background noise of the fixed-value bridge



Noise of a pair of HH-109 hybrid junctions



Background noise of the fixed-value bridge

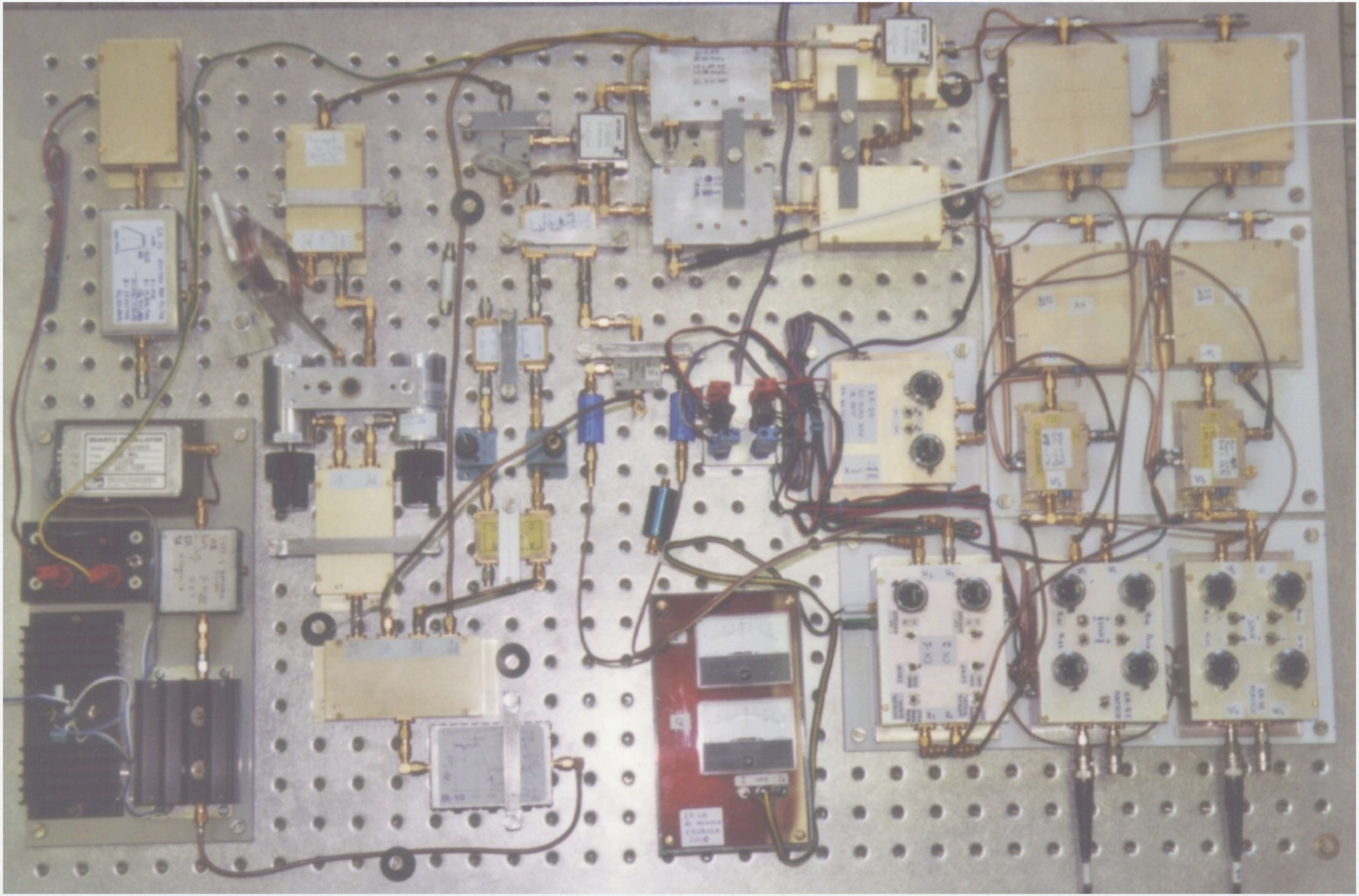


Averaged spectra must be smooth

Average on m spectra: confidence of a point improves by $O(1/m^{1/2})$

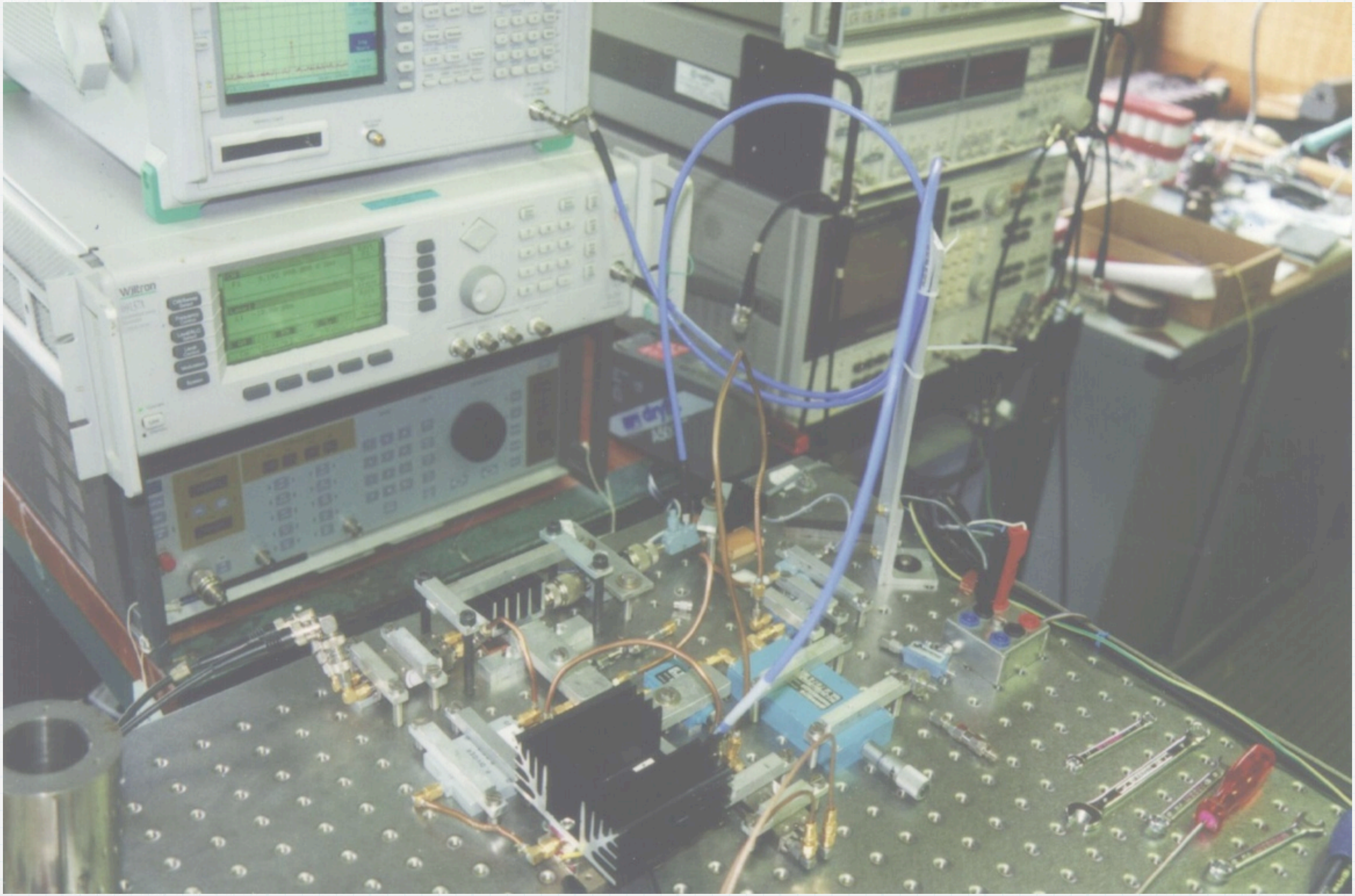
interchange ensemble with frequency: smoothness $O(1/m^{1/2})$

The complete machine (100 MHz)

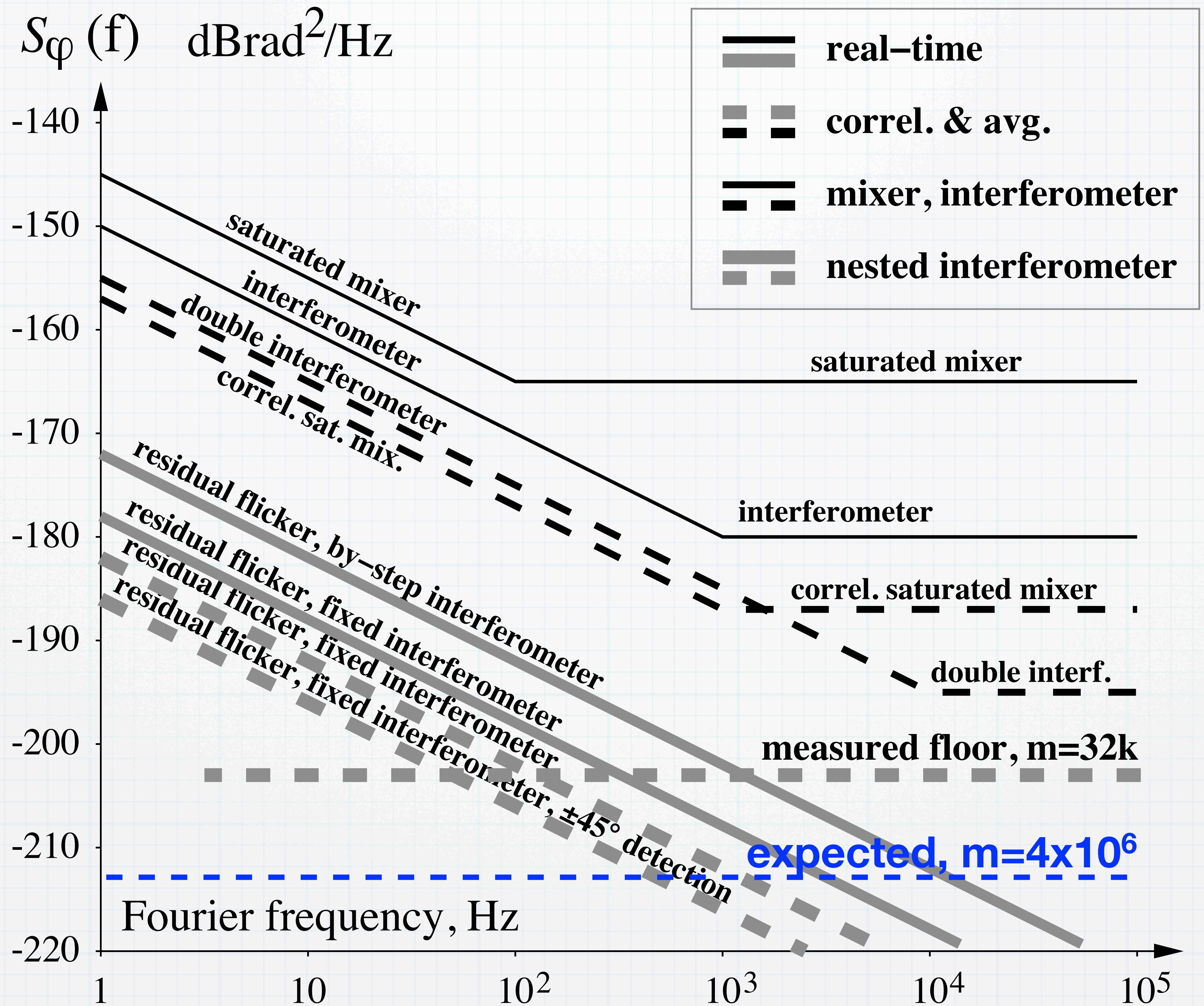


A 9 GHz experiment

(dc circuits not shown)



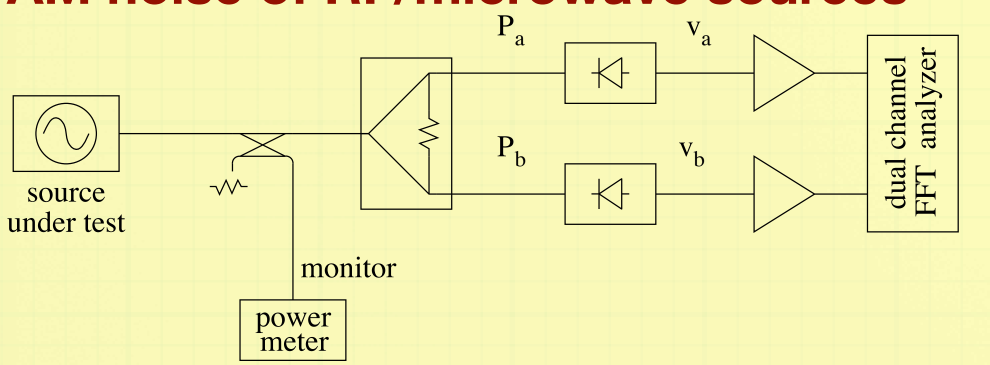
Comparison of the background noise



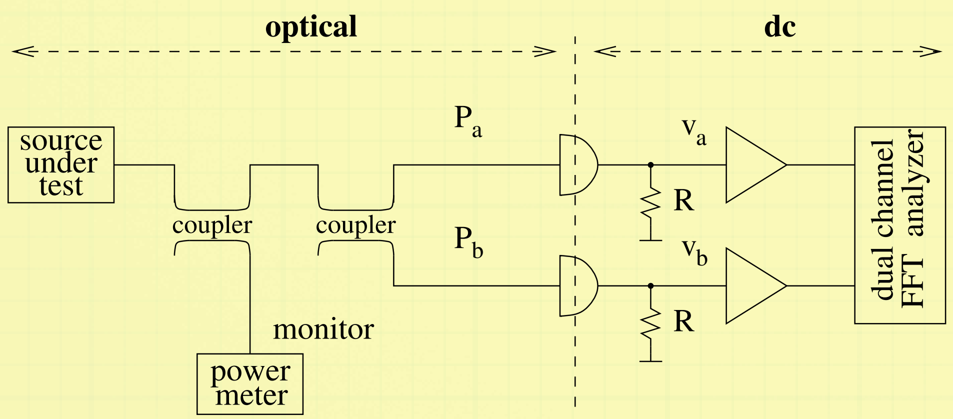
AM noise & RIN

Amplitude noise & laser RIN

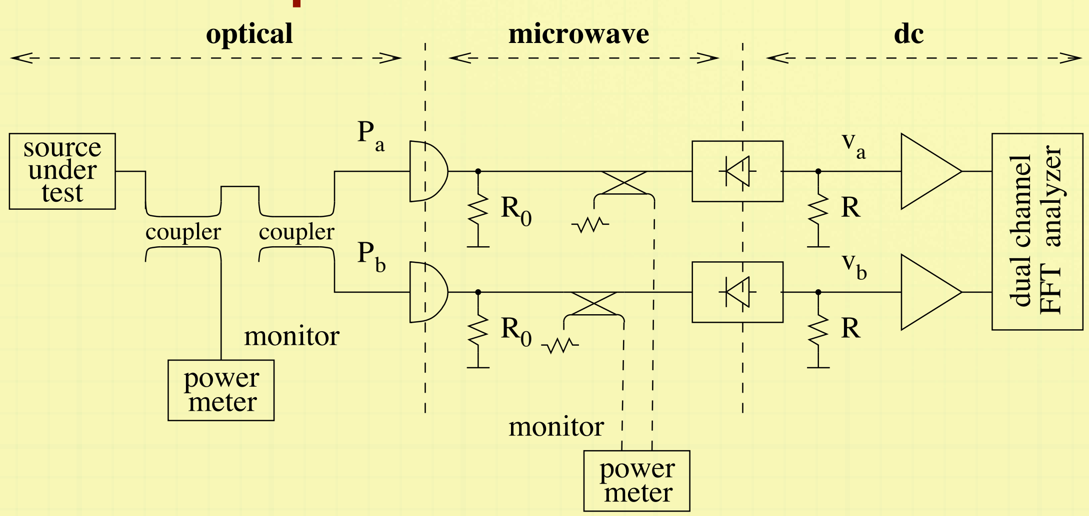
AM noise of RF/microwave sources



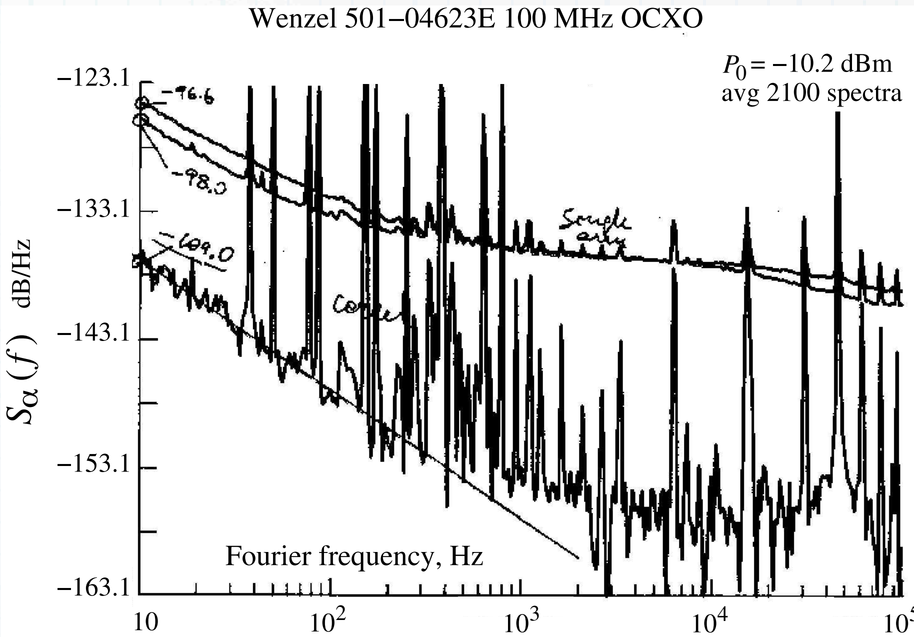
Laser RIN

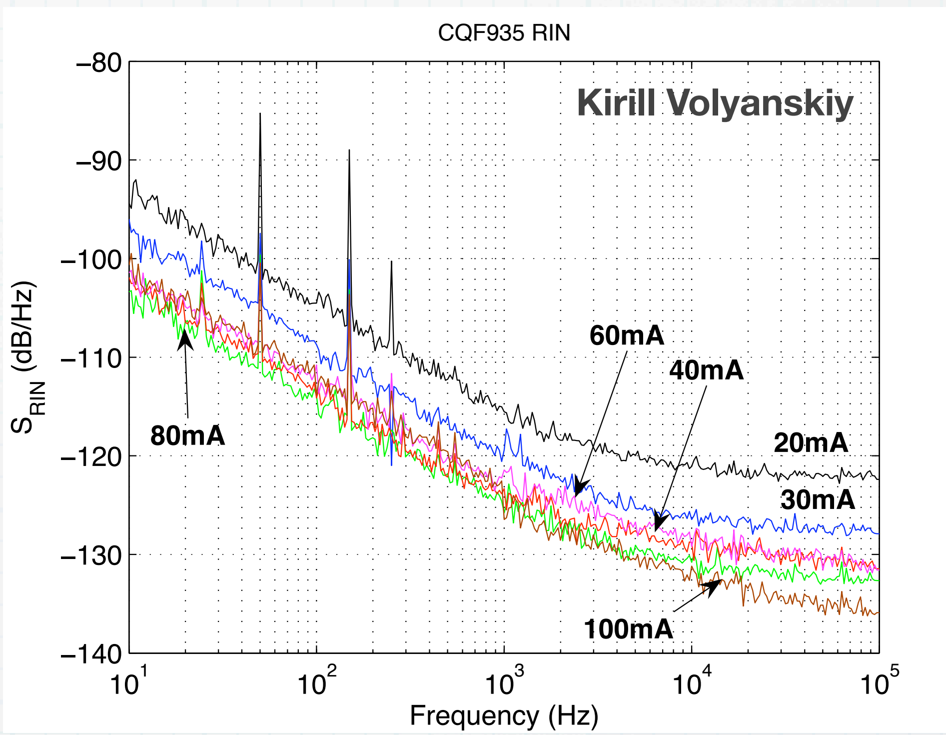


AM noise of photonic RF/microwave sources



- In PM noise measurements, one can validate the instrument by feeding the same signal into the phase detector
- In AM noise this is *not possible* without a lower-noise reference
- Provided the crosstalk was measured otherwise, correlation enables to validate the instrument





E. Rubiola, the measurement of AM noise, dec 2005
arXiv:physics/0512082v1 [physics.ins-det]

AM noise of some sources

source	h_{-1} (flicker)		$(\sigma_\alpha)_{\text{floor}}$
Anritsu MG3690A synthesizer (10 GHz)	2.5×10^{-11}	-106.0 dB	5.9×10^{-6}
Marconi synthesizer (5 GHz)	1.1×10^{-12}	-119.6 dB	1.2×10^{-6}
Macom PLX 32-18 0.1 $\rightarrow$ 9.9 GHz multipl.	1.0×10^{-12}	-120.0 dB	1.2×10^{-6}
Omega DRV9R192-105F 9.2 GHz DRO	8.1×10^{-11}	-100.9 dB	1.1×10^{-5}
Narda DBP-0812N733 amplifier (9.9 GHz)	2.9×10^{-11}	-105.4 dB	6.3×10^{-6}
HP 8662A no. 1 synthesizer (100 MHz)	6.8×10^{-13}	-121.7 dB	9.7×10^{-7}
HP 8662A no. 2 synthesizer (100 MHz)	1.3×10^{-12}	-118.8 dB	1.4×10^{-6}
Fluke 6160B synthesizer	1.5×10^{-12}	-118.3 dB	1.5×10^{-6}
Racal Dana 9087B synthesizer (100 MHz)	8.4×10^{-12}	-110.8 dB	3.4×10^{-6}
Wenzel 500-02789D 100 MHz OCXO	4.7×10^{-12}	-113.3 dB	2.6×10^{-6}
Wenzel 501-04623E no. 1 100 MHz OCXO	2.0×10^{-13}	-127.1 dB	5.2×10^{-7}
Wenzel 501-04623E no. 2 100 MHz OCXO	1.5×10^{-13}	-128.2 dB	4.6×10^{-7}

worst

best

The Leeson effect

Phase noise and frequency stability in oscillators

Enrico Rubiola

FEMTO-ST Institute, CNRS and UFC, Besancon, France

D. B. Leeson, A simple model for feed back oscillator noise, Proc. IEEE 54(2):329 (Feb 1966)

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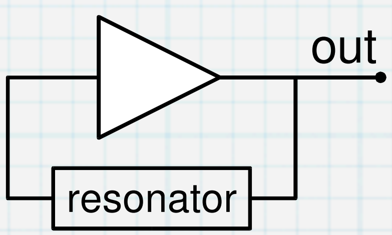
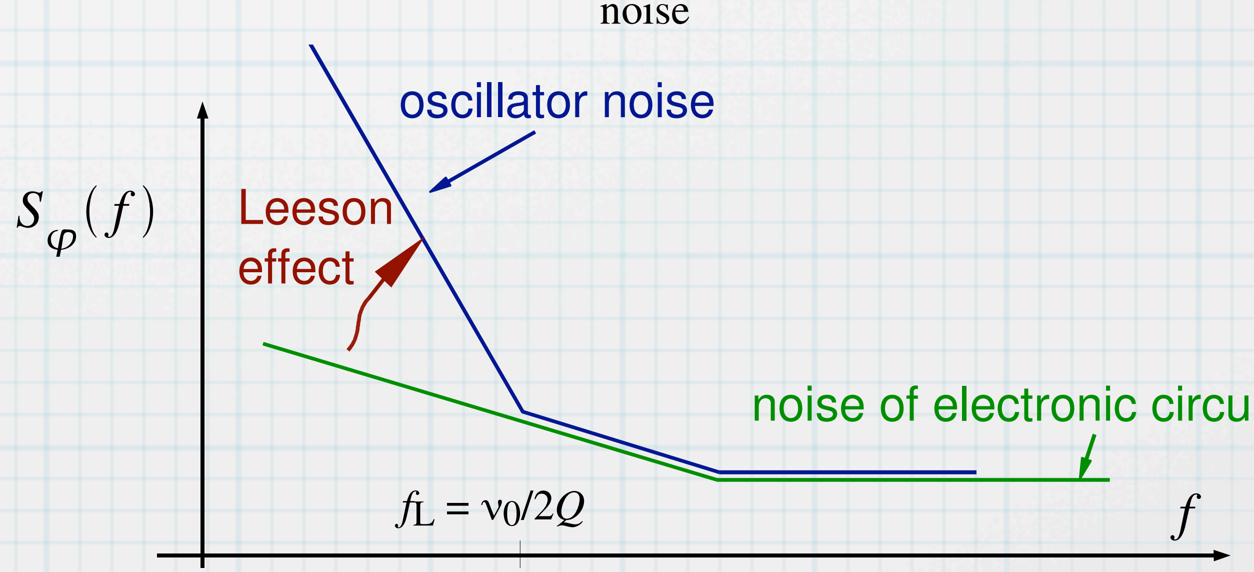


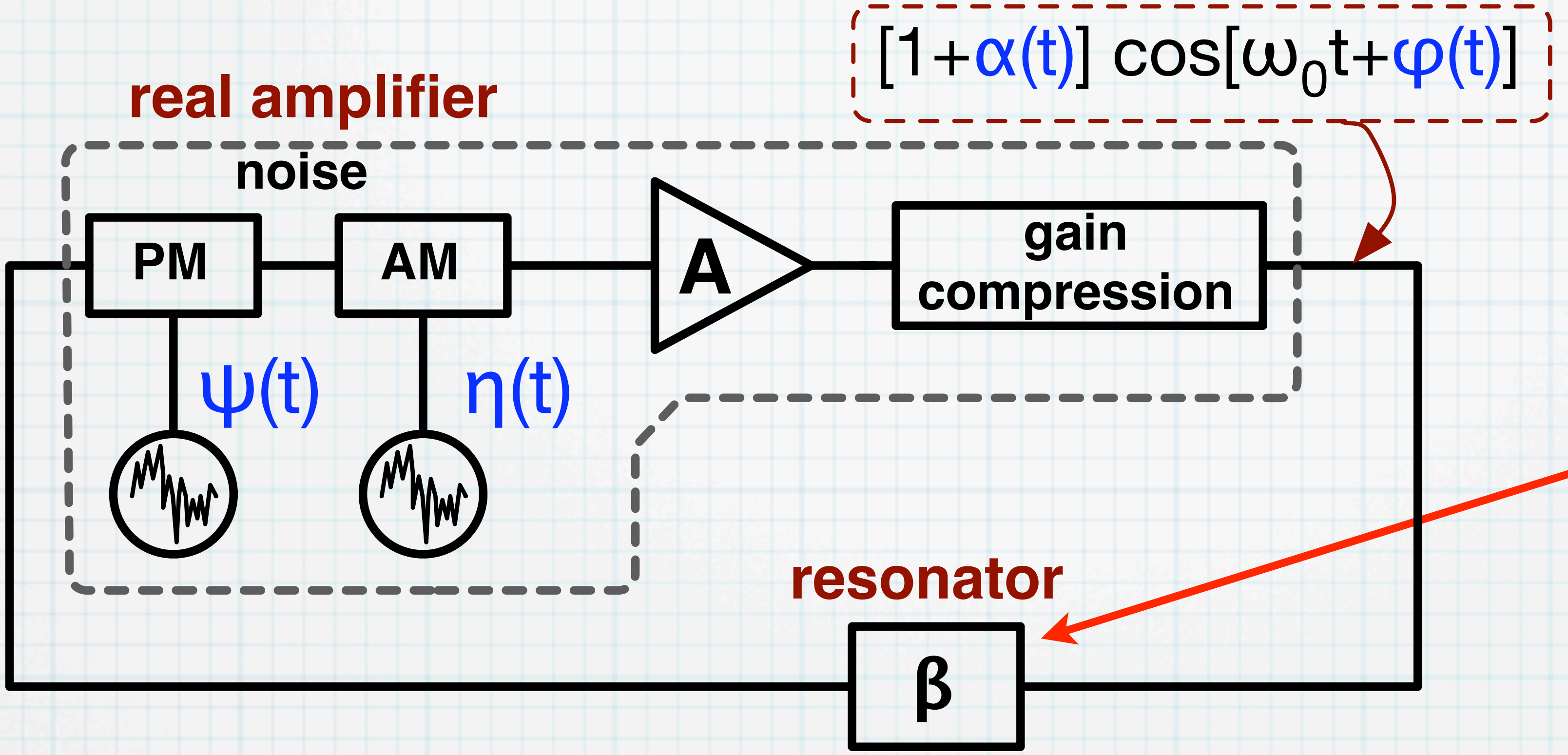
$$\underbrace{S_\varphi(f)}_{\text{oscillator noise}} = \left[1 + \left(\frac{\nu_0}{2Q}\right)^2 \frac{1}{f^2}\right] \underbrace{S_\psi(f)}_{\text{ampli noise}}$$

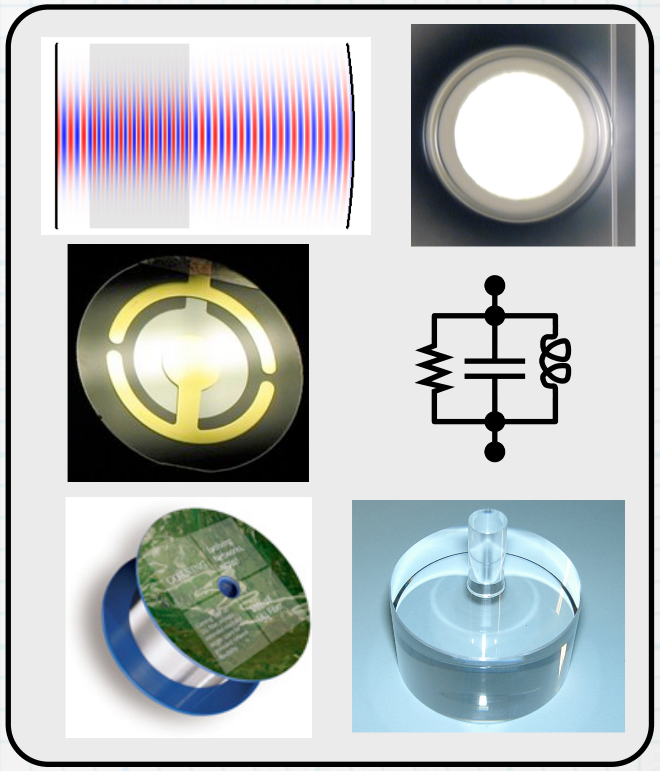


home page http://rubiola.org

Oscillator fundamentals

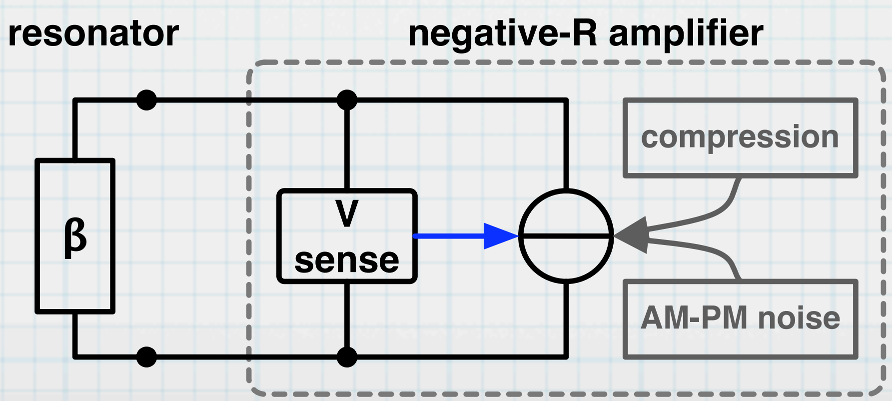
General oscillator model





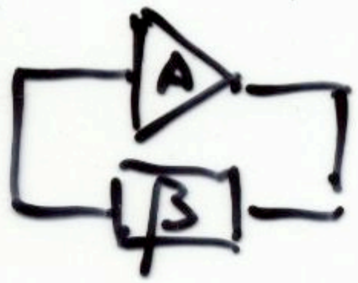
Barkhausen condition $A\beta = 1$ at ω_0
(phase matching)

The model also describes the negative-R oscillator



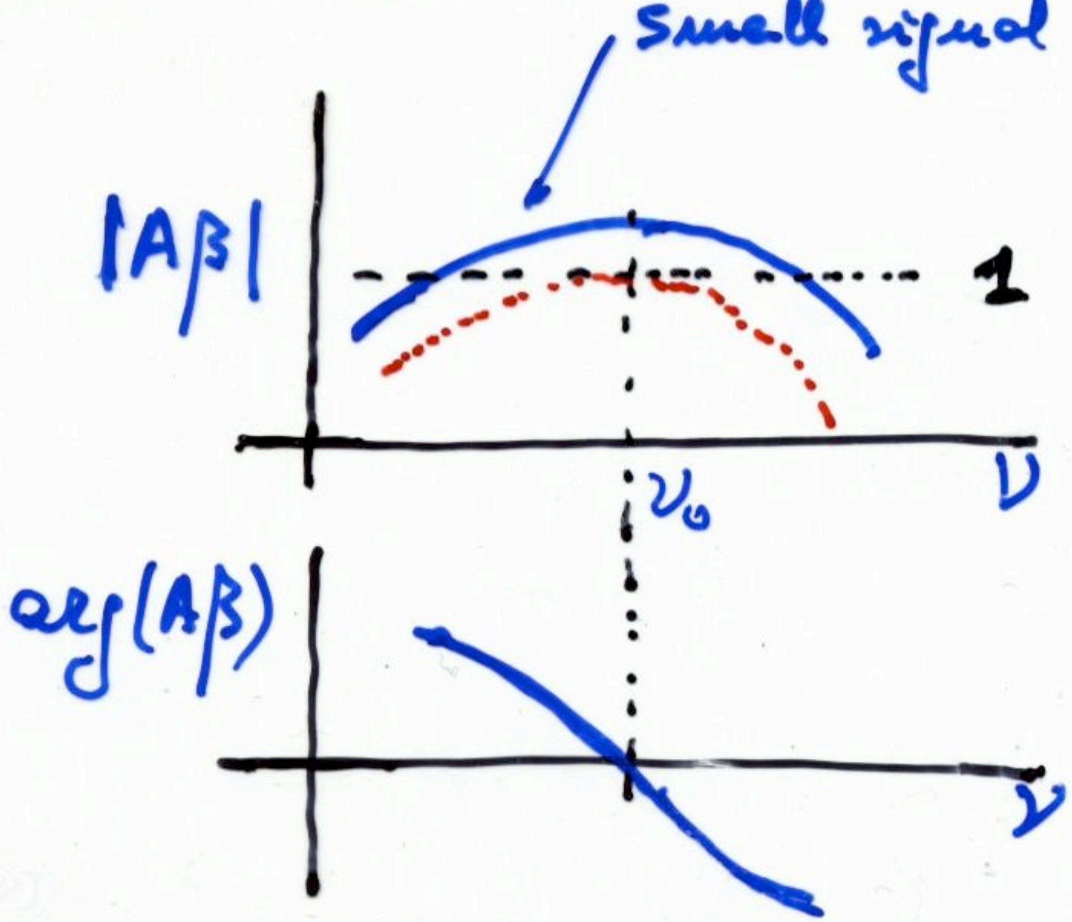
AR

BARKHAUSEN CONDITION



let $A = $ const

β: 2nd order diff. eq resonator



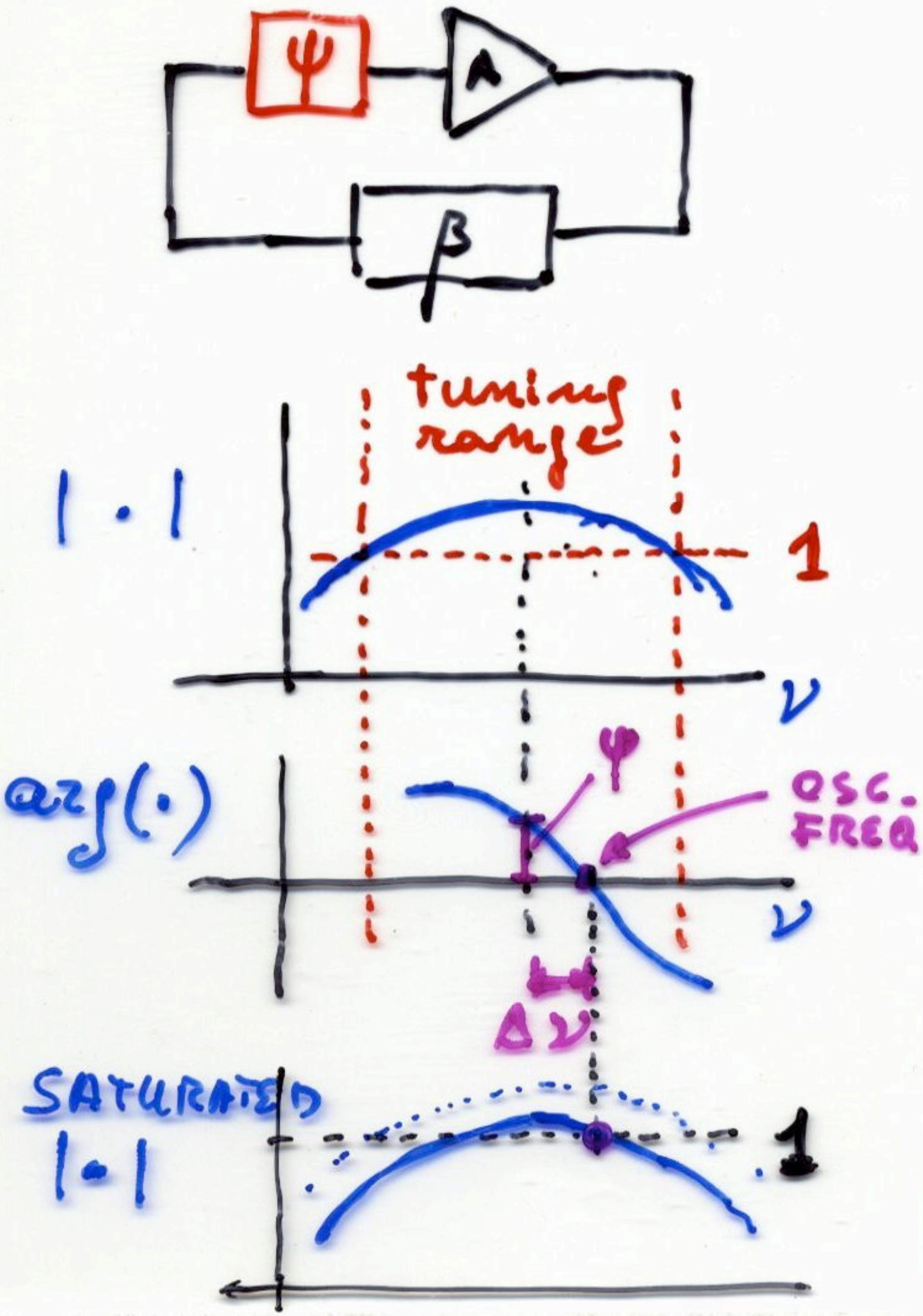
$$\arg(\beta) = -\arctan Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

$$\simeq -2Q\,\frac{\omega - \omega_0}{\omega_0}$$

$$-2Q\,\frac{\Delta\omega}{\omega_0}$$

- $\arg(\beta)$ sets the oscillation frequency
- saturation fixes $|A\beta| = 1$

TUNING AN OSCILLATOR



add a phase ψ

$$+\arg(\beta) + \psi = 0$$

$$+\arg(\beta) = \psi$$

approx:

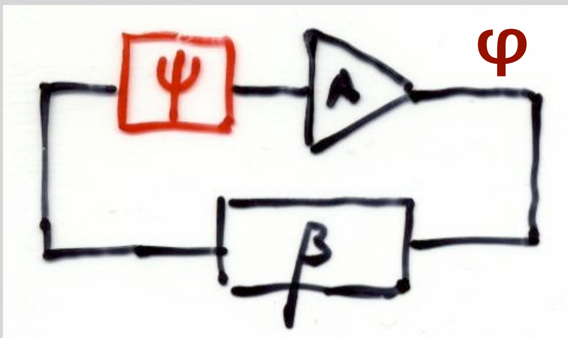
$$2Q \frac{\Delta\omega}{\omega_0} = \psi$$

$$\frac{\Delta\omega}{\omega_0} = \frac{\Delta\nu}{\nu_0} = \frac{\psi}{2Q}$$

Heuristic approach

Heuristic derivation of the Leeson formula

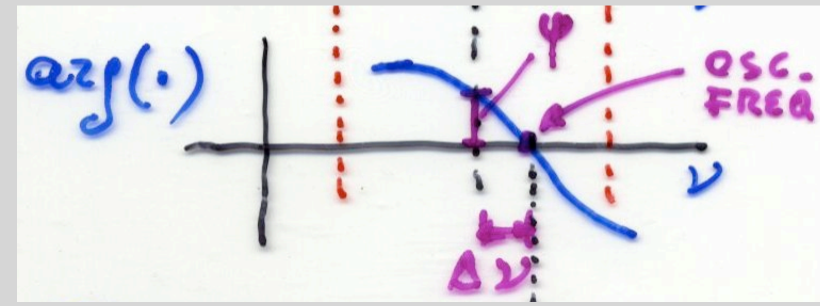
fast fluctuation: no feedback



$$\varphi(t) = \psi(t)$$

$$S_\varphi(f) = S_\psi(f)$$

slow fluctuations: ψ ⇒ Δν conversion



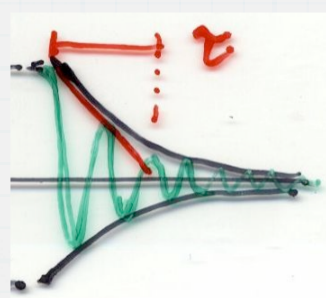
$$\Delta\nu = \frac{\nu_0}{2Q}\,\psi$$ **static**

$$S_{\Delta\nu}(f) = \left(\frac{\nu_0}{2Q}\right)^2 S_\psi(f)$$

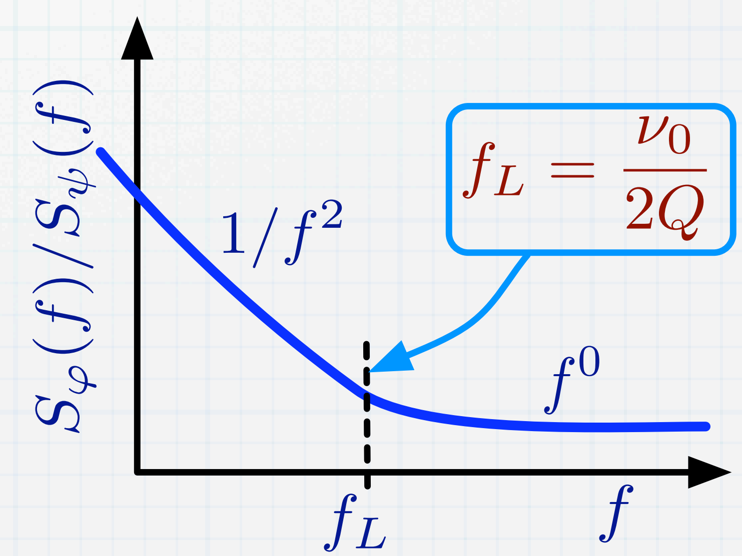
$$S_\varphi(f) = \frac{1}{f^2}\left(\frac{\nu_0}{2Q}\right)^2 S_\psi(f)$$ **integral**

fast or slow?

$$\tau = \frac{Q}{\pi\nu_0}$$

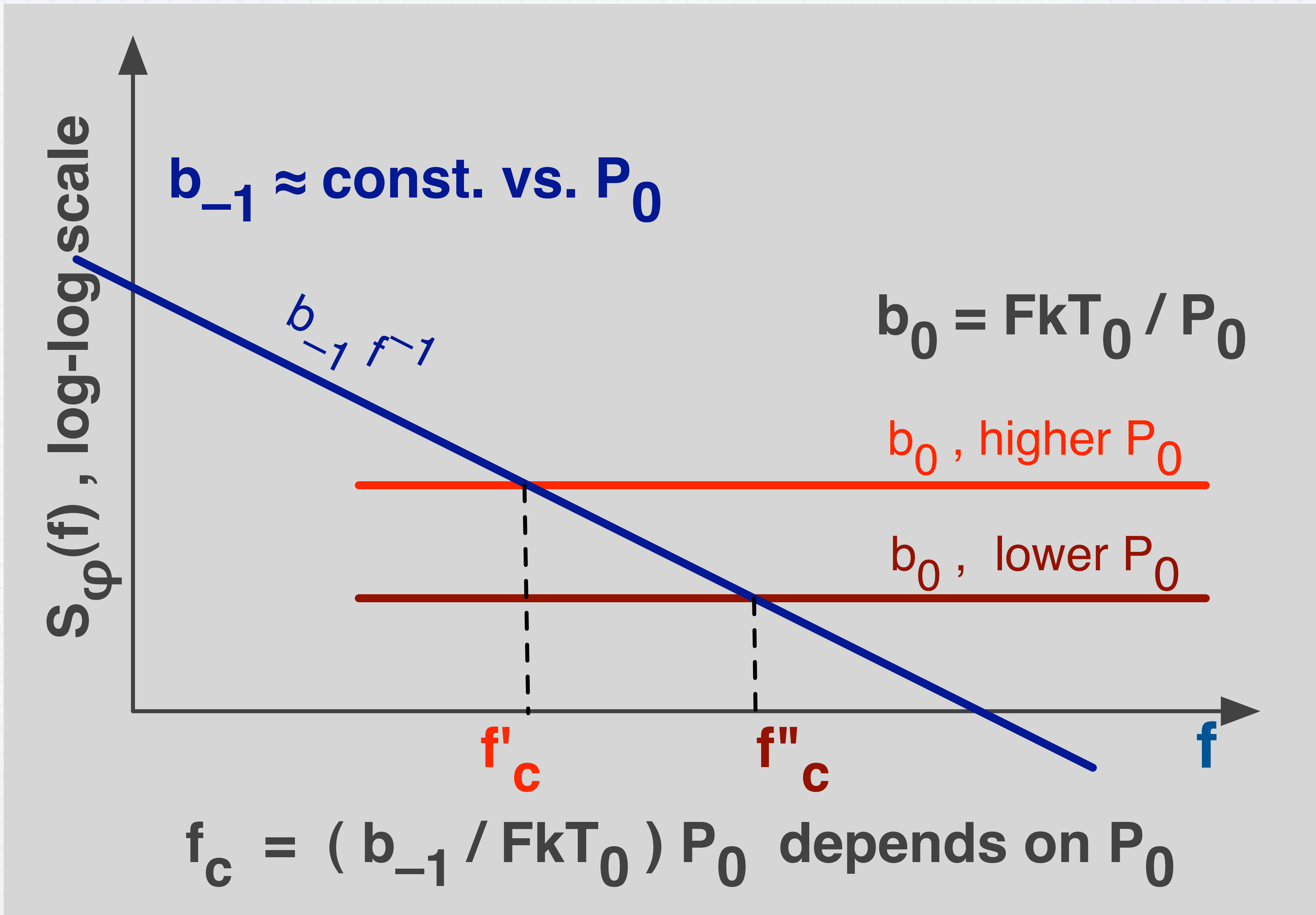


$$S_\varphi(f) = \left[1 + \frac{1}{f^2}\left(\frac{\nu_0}{2Q}\right)^2\right] S_\psi(f)$$



Though obtained with simplifications, this result turns out to be is exact

Amplifier white and flicker noise

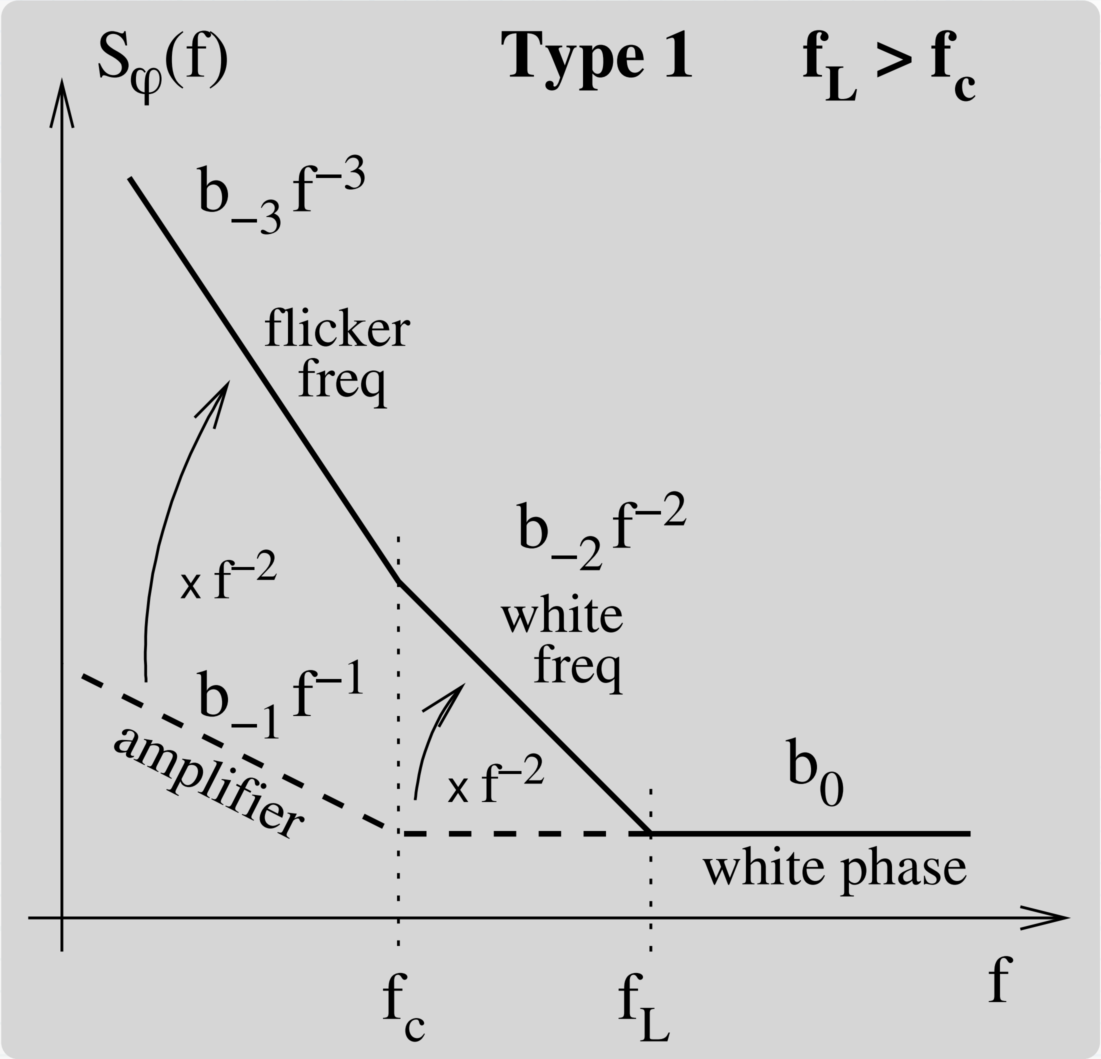


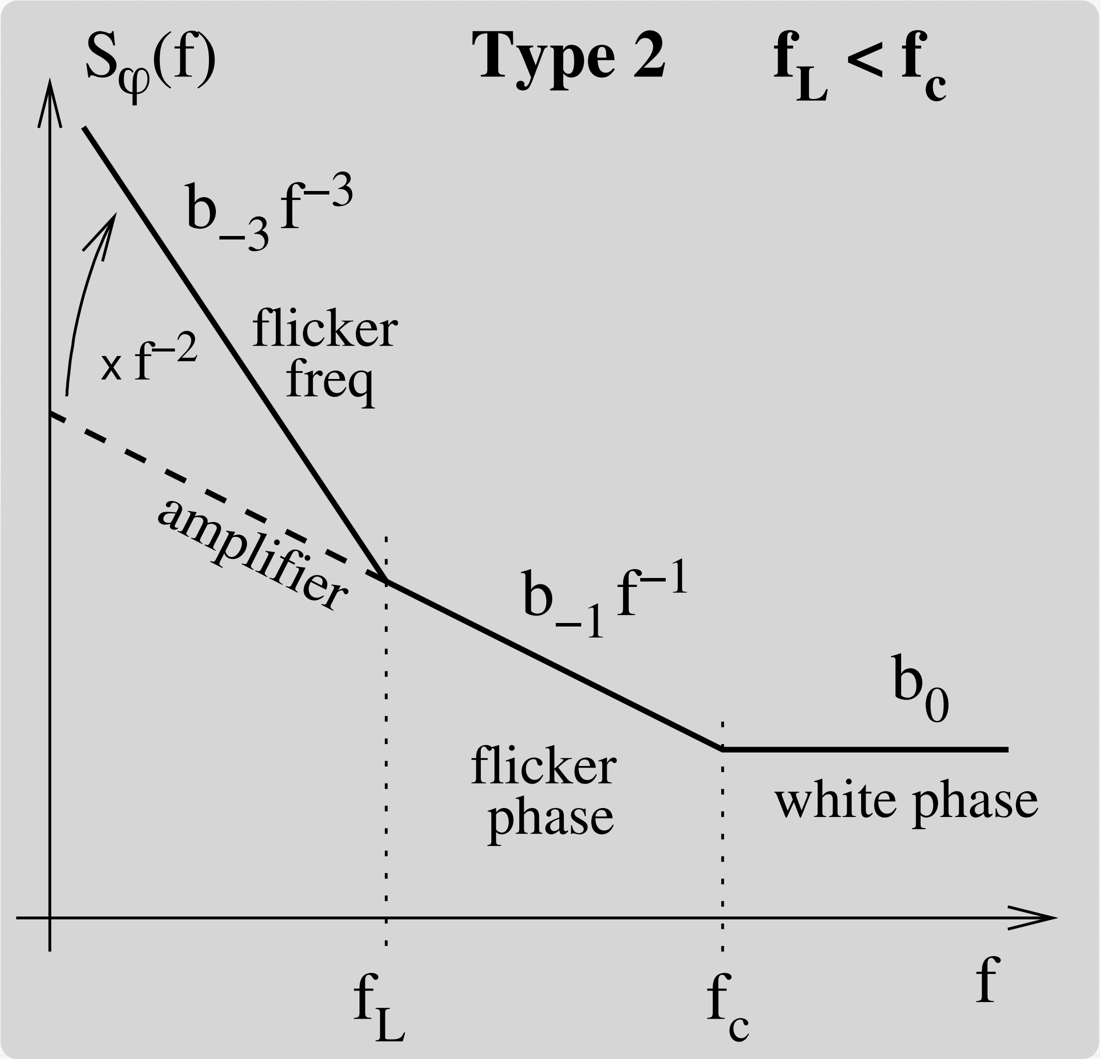
typical amplifier phase noise			
RATE	GaAs HBT microwave	SiGe HBT microwave	Si bipolar HF/UHF
fair	−100		−120
good	−110	−120	−130
best	−120	−130	−150
	unit $\mathrm{dBrad}^2/\mathrm{Hz}$		

photodetector $b_{-1} \approx -120\ \mathrm{dBrad}^2/\mathrm{Hz}$ Rubiola & al. IEEE Trans. MTT (& JLT) 54 (2) p.816–820 (2006)

Including the sustaining-amplifier noise

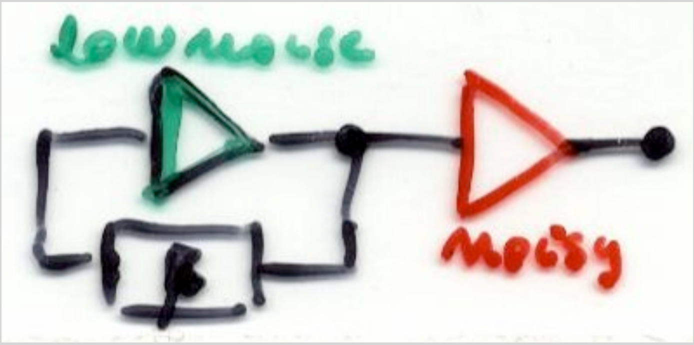
Figures are from E. Rubiola, Phase noise and frequency stability in oscillators, © Cambridge University Press





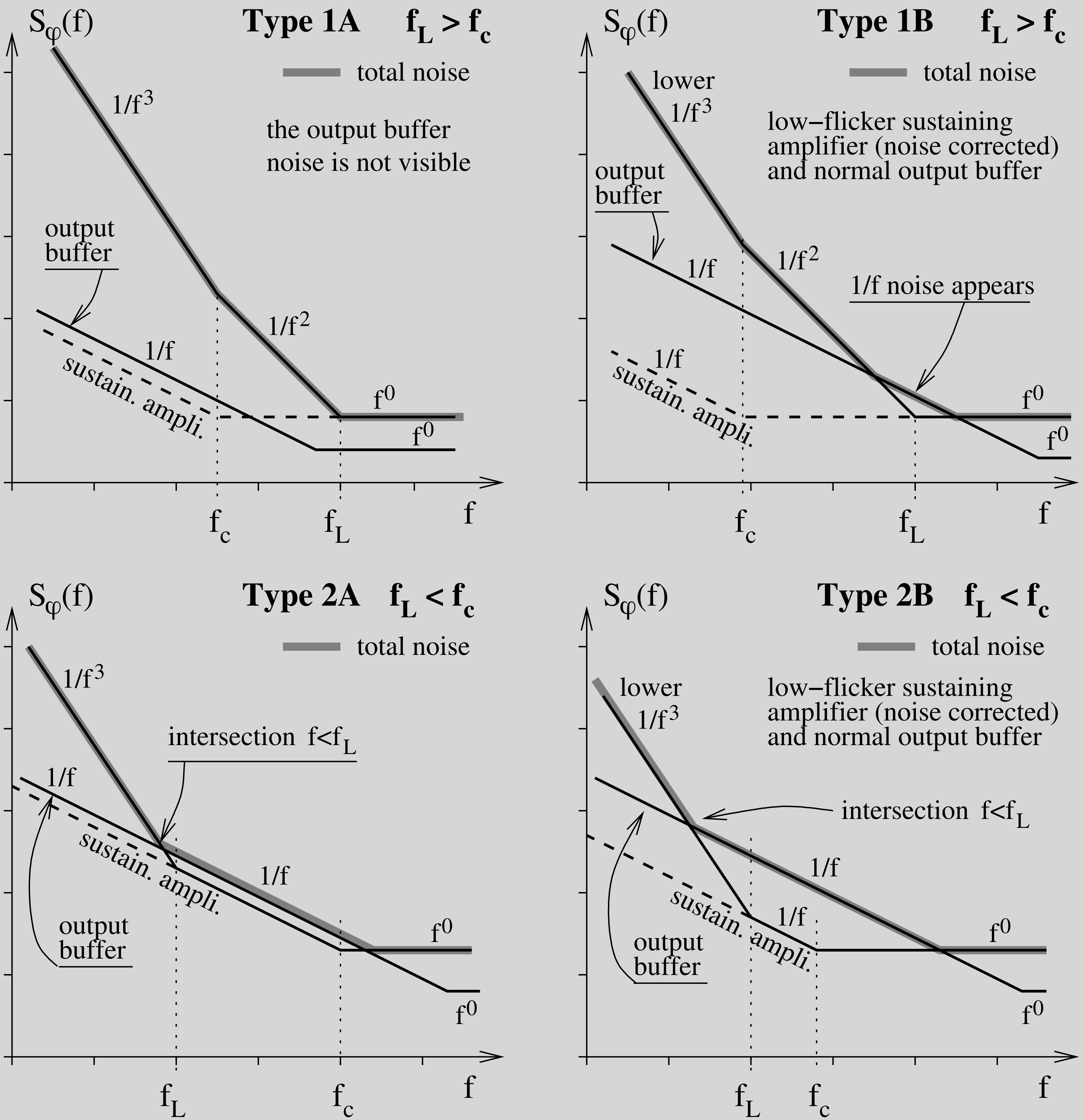
The sustaining-amplifier noise is $S_\varphi(f) = b_0 + b_{-1}/f$ (white and flicker)

The effect of the output buffer



Cascading two amplifiers, flicker noise adds as

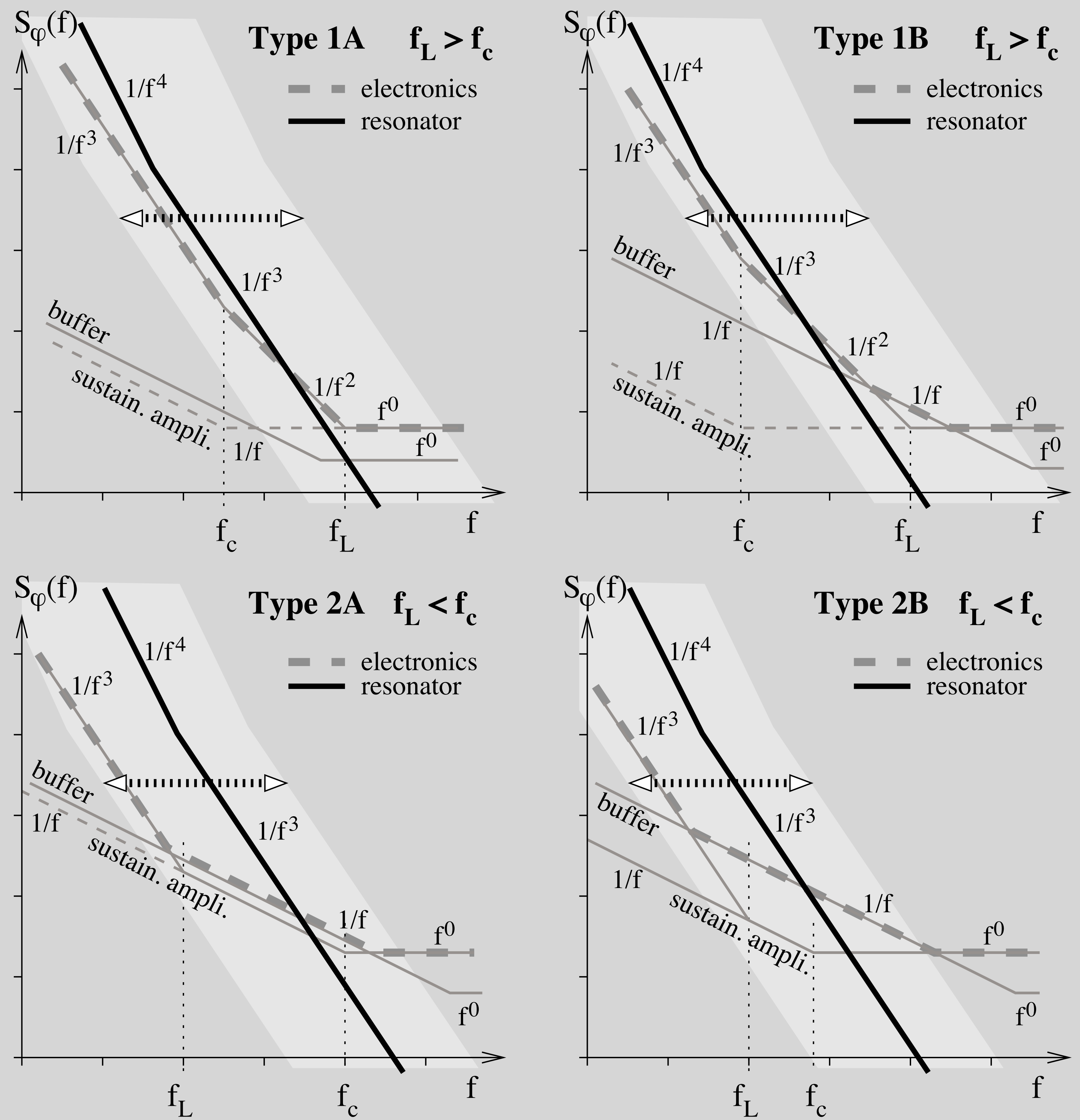
$$S_\varphi(f) = [S_\varphi(f)]_1 + [S_\varphi(f)]_2$$



Figures are from E. Rubiola, Phase noise and frequency stability in oscillators, © Cambridge University Press

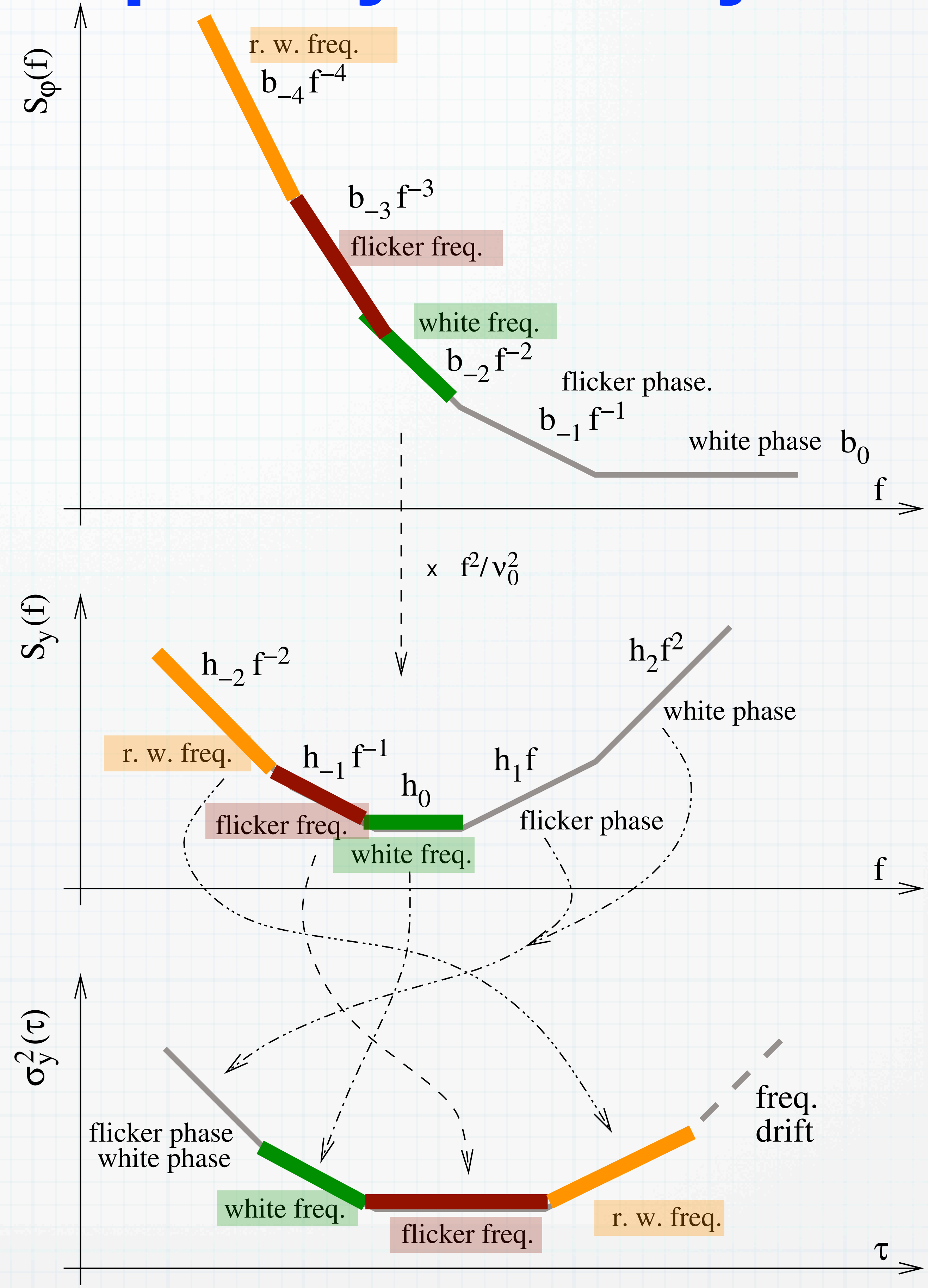
The resonator natural frequency fluctuates

- The oscillator tracks the resonator natural frequency, hence its fluctuations
- The fluctuations of the resonator natural frequency contain **1/f and $1/f^2$** (frequency flicker and random walk), thus **$1/f^3$ and $1/f^4$** of the oscillator phase
- The resonator bandwidth does not apply to the natural-frequency fluctuation. (Tip: an oscillator can be frequency modulated ar a rate >> f_L)



Figures are from E. Rubiola, Phase noise and frequency stability in oscillators, © Cambridge University Press

Phase noise –> frequency stability



phase noise

frequency noise

Allan variance

white	$\sigma^2(\tau) = h_0/2\tau$
flicker	$\sigma^2(\tau) = 2\ln(2)\ h_{-1}$
r.walk	$\sigma^2(\tau) = ((2\pi)^2/6)\ h_0\tau$

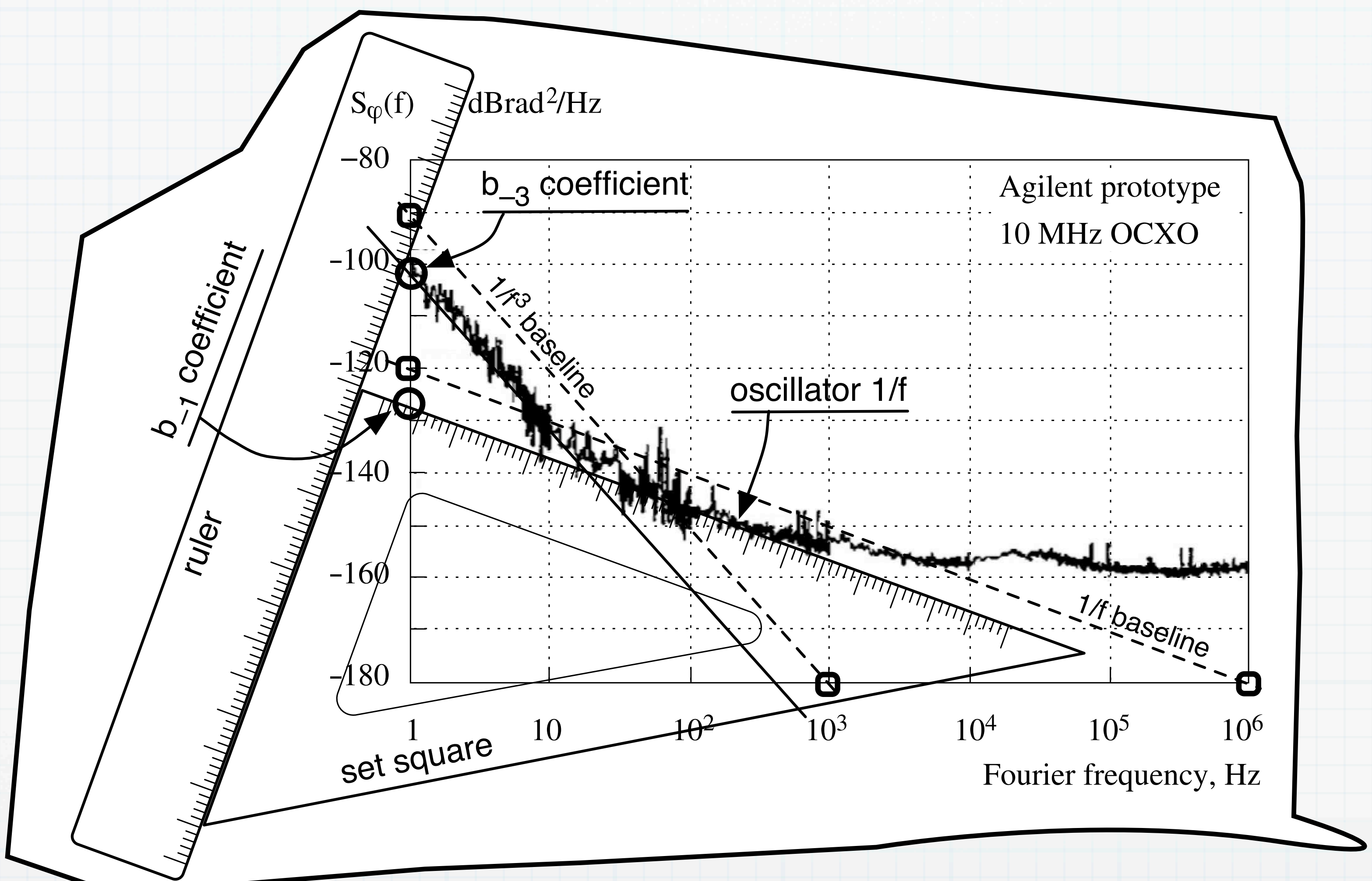
Oscillator Hacking

Analysis of commercial oscillators

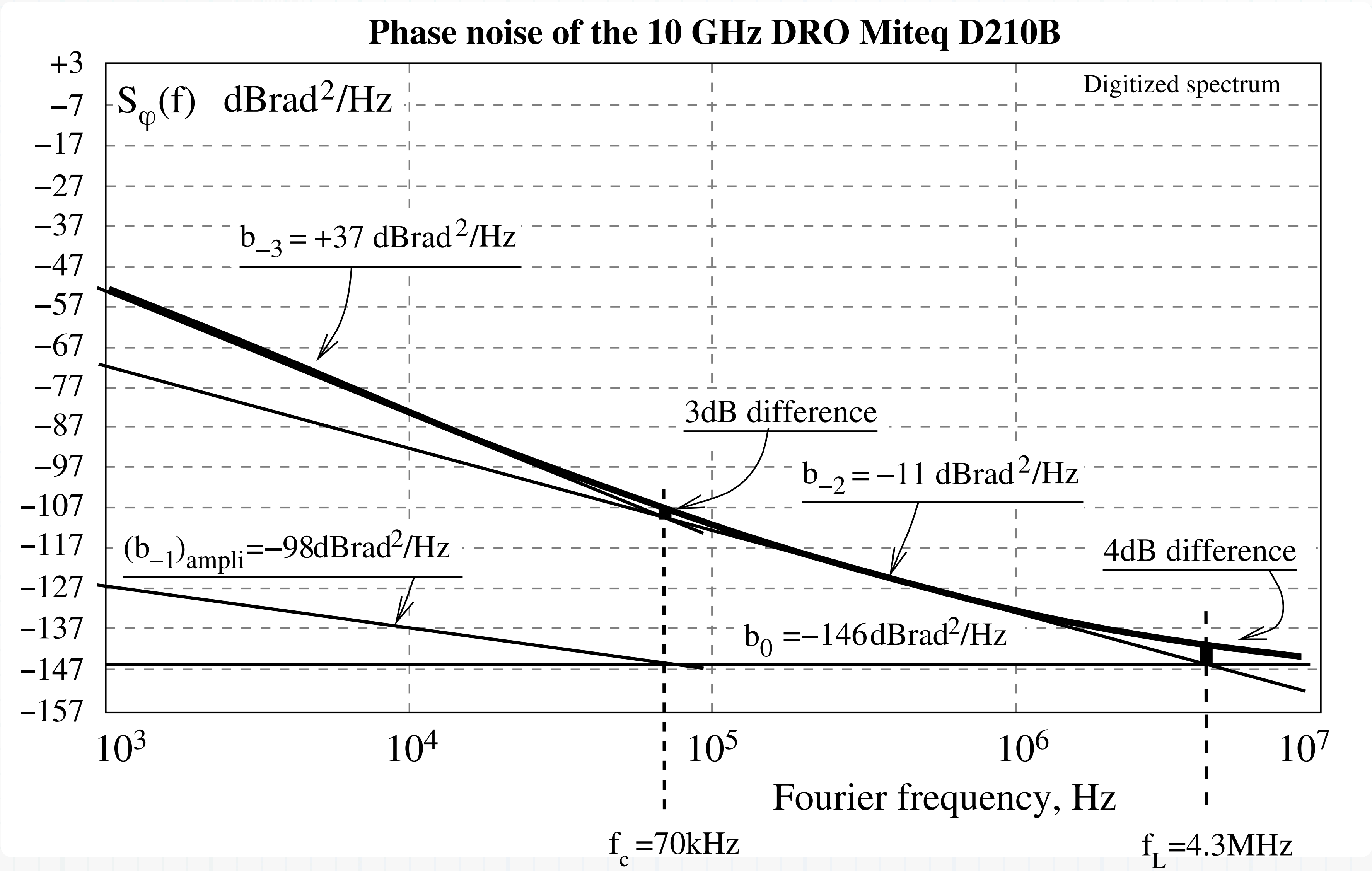
The purpose of this section is to help to understand the oscillator inside from the phase noise spectra, plus some technical information. I have chosen some commercial oscillators as an example.

The conclusions about each oscillator represent only my understanding based on experience and on the data sheets published on the manufacturer web site.

You should be aware that this process of interpretation is not free from errors. My conclusions were not submitted to manufacturers before writing, for their comments could not be included.



Miteq D210B, 10 GHz DRO



tables

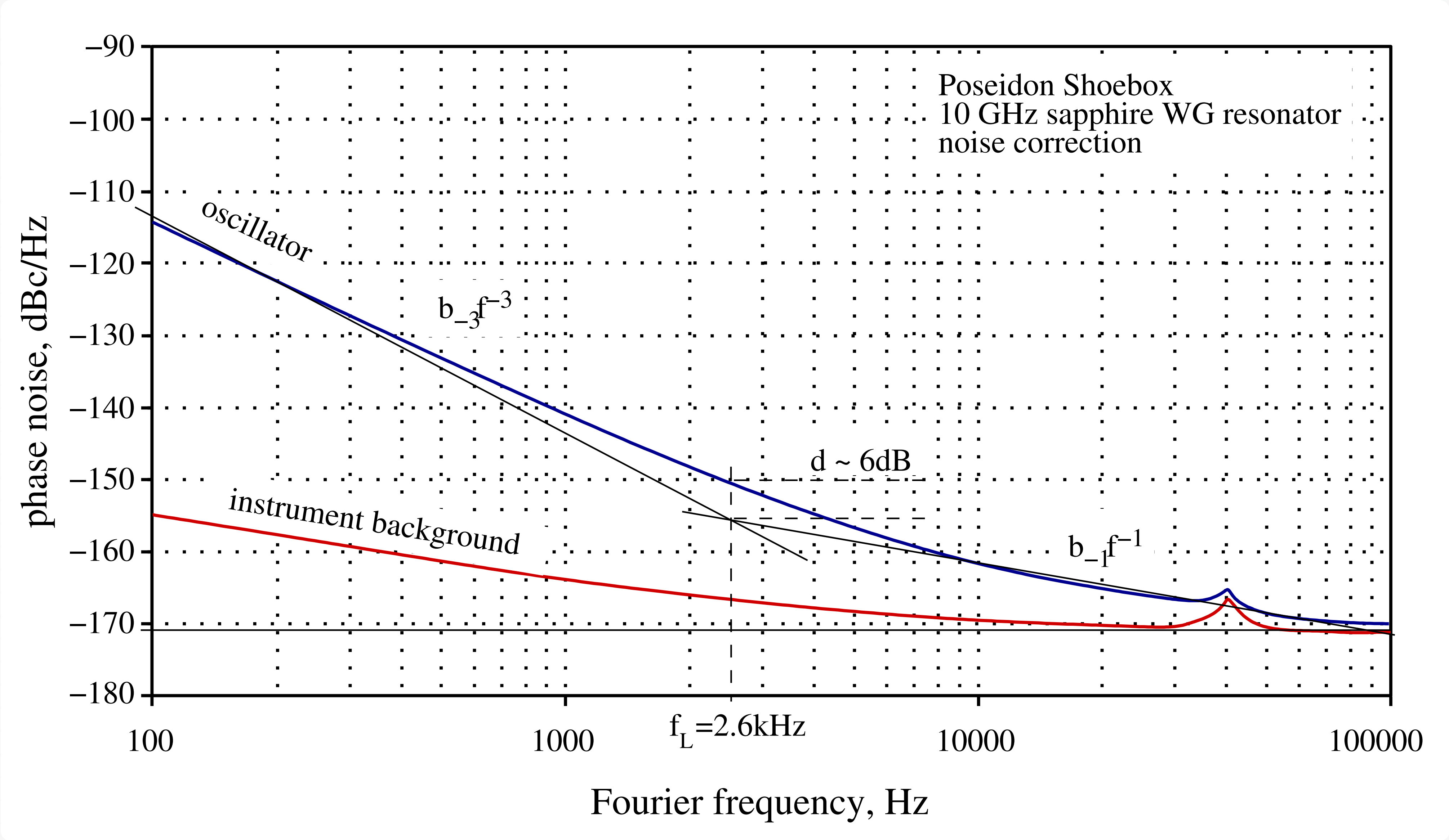
$\sigma^2_y = h_0/2\tau + 2\ln(2)h_{-1}$

$h_0 = b_{-2}/\nu^2_0$

$h_{-1} = b_{-3}/\nu^2_0$

- $kT_0 = 4\times10^{-21}$ W/Hz (–174 dBm/Hz)
- floor –146 dBrad²/Hz, guess F = 1.25 (1 dB) => P_0 = 2 μW (–27 dBm)
- f_L = 4.3 MHz, $f_L = \nu 0/2Q$ => Q = 1160
- f_c = 70 kHz, $b_{-1}/f = b0$ => $b_{-1} = 1.8\times10^{-10}$ (–98 dBrad²/Hz) [sust.ampli]
- $h_0 = 7.9\times10^{-22}$ and $h_{-1} = 5\times10^{-17}$ => $\sigma_y = 2\times10^{-11}/\sqrt{\tau} + 8.3\times10^{-9}$

Poseidon Scientific Instruments – Shoebox 10 GHz sapphire whispering-gallery (1)

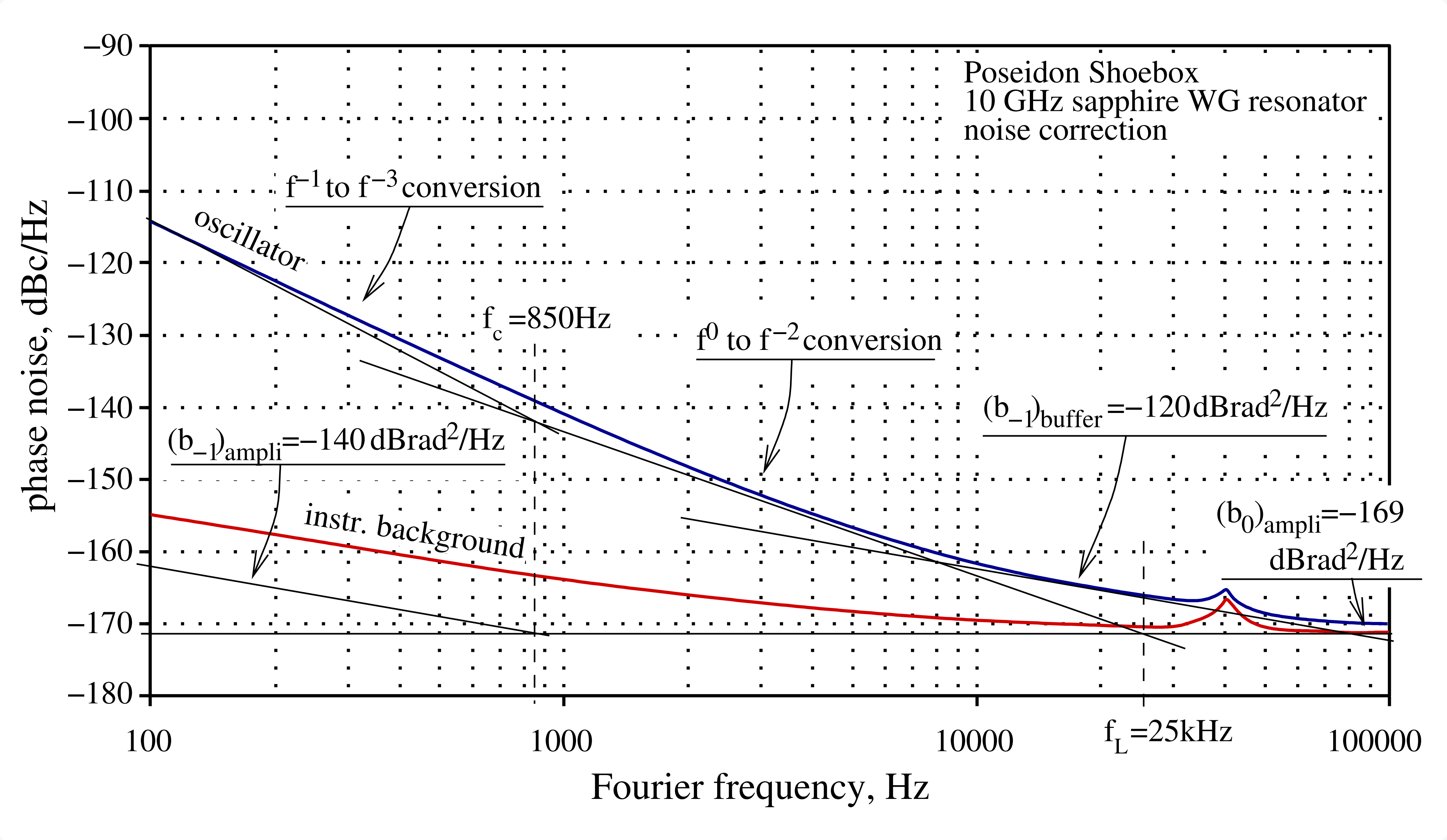


$f_L = \nu_0/2Q = 2.6$ kHz => $Q = 1.8\times10^6$

This incompatible with the resonator technology.
Typical Q of a sapphire whispering gallery resonator:
2×10^5 @ 295K (room temp), 3×10^7 @ 77K (liquid N), 4×10^9 @ 4K (liquid He).
In addition, d ~ 6 dB does not fit the power-law.

The interpretation shown is wrong, and the Leeson frequency is somewhere else

Poseidon Scientific Instruments – Shoebox 10 GHz sapphire whispering-gallery (2)



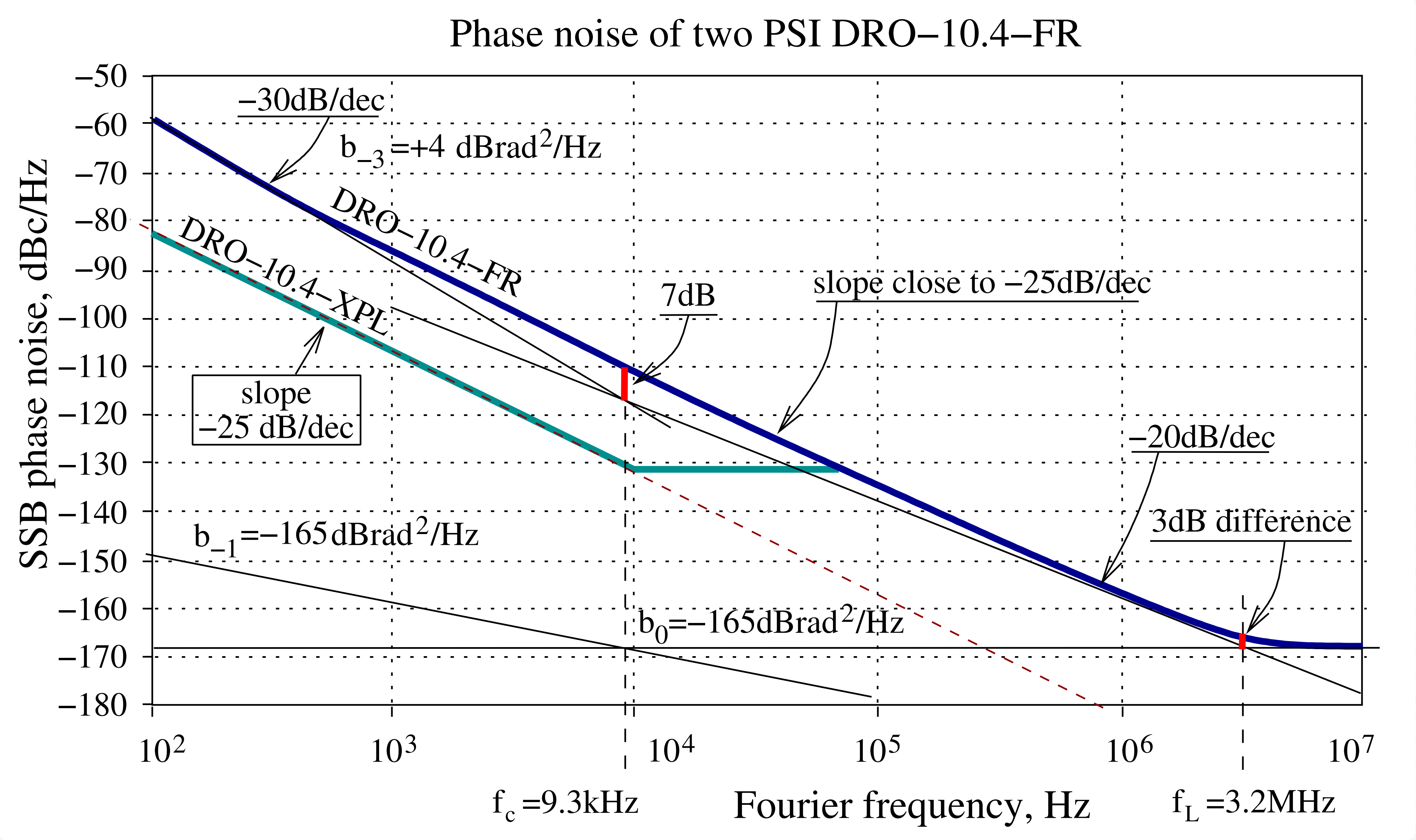
The 1/f noise of the output buffer is higher than that of the sustaining amplifier (a compex amplifier with interferometric noise reduction)

In this case both 1/f and 1/f² are present

white noise –169 dBrad²/Hz, guess F = 5 dB (interferometer) => P_0 = 0 dBm
buffer flicker –120 dBrad²/Hz @ 1 Hz => good microwave amplifier

$f_L = \nu_0/2Q$ = 25 kHz => Q = 2×10⁵ (quite reasonable)
f_c = 850 Hz => flicker of the interferometric amplifier –139 dBrad²/Hz @ 1 Hz

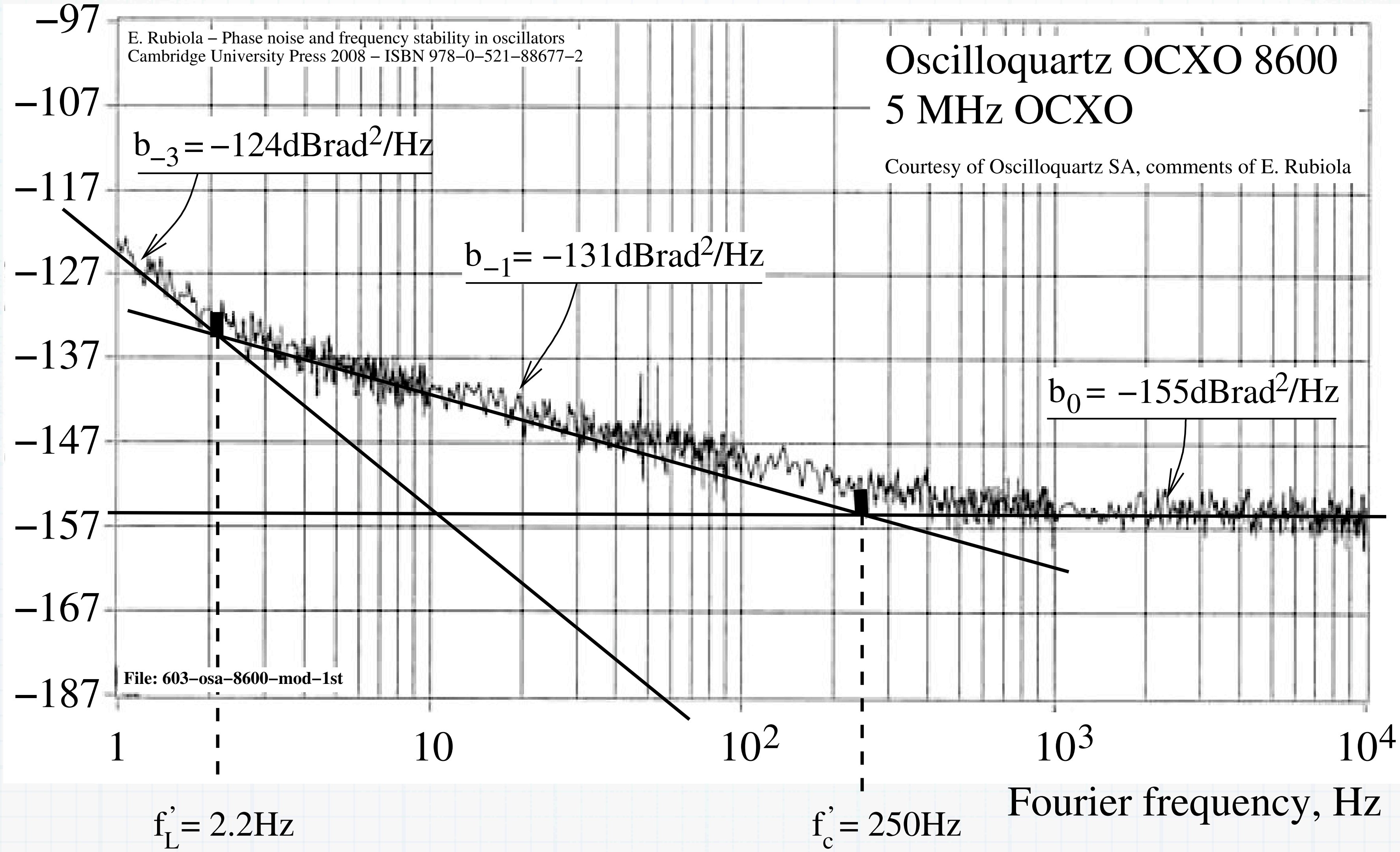
Poseidon Scientific Instruments
10 GHz dielectric resonator oscillator (DRO)



- floor –165 dBrad^2/Hz, guess F = 1.25 (1 dB) => P_0 = 160 μW (–8 dBm)
- f_L = 3.2 MHz, f_L = ν0/2Q => Q = 625
- f_c = 9.3 kHz, $b_{-1}/f = b0$ => $b_{-1} = 2.9{\times}10^{-13}$ (–125 dBrad^2/Hz) [sust.ampli, too low]

Slopes are not in agreement with the theory

Example – Oscilloquartz 8600 (wrong)



ANALYSIS

1 – floor $S_{\varphi 0}$ = –155 dBrad²/Hz, guess F = 1 dB → P_0 = –18 dBm

2 – ampli flicker S_φ = –132 dBrad²/Hz @ 1 Hz → good RF amplifier

3 – merit factor $Q = \nu_0/2f_L = 5\cdot10^6/5 = 10^6$ (seems too low)

4 – take away some flicker for the output buffer:

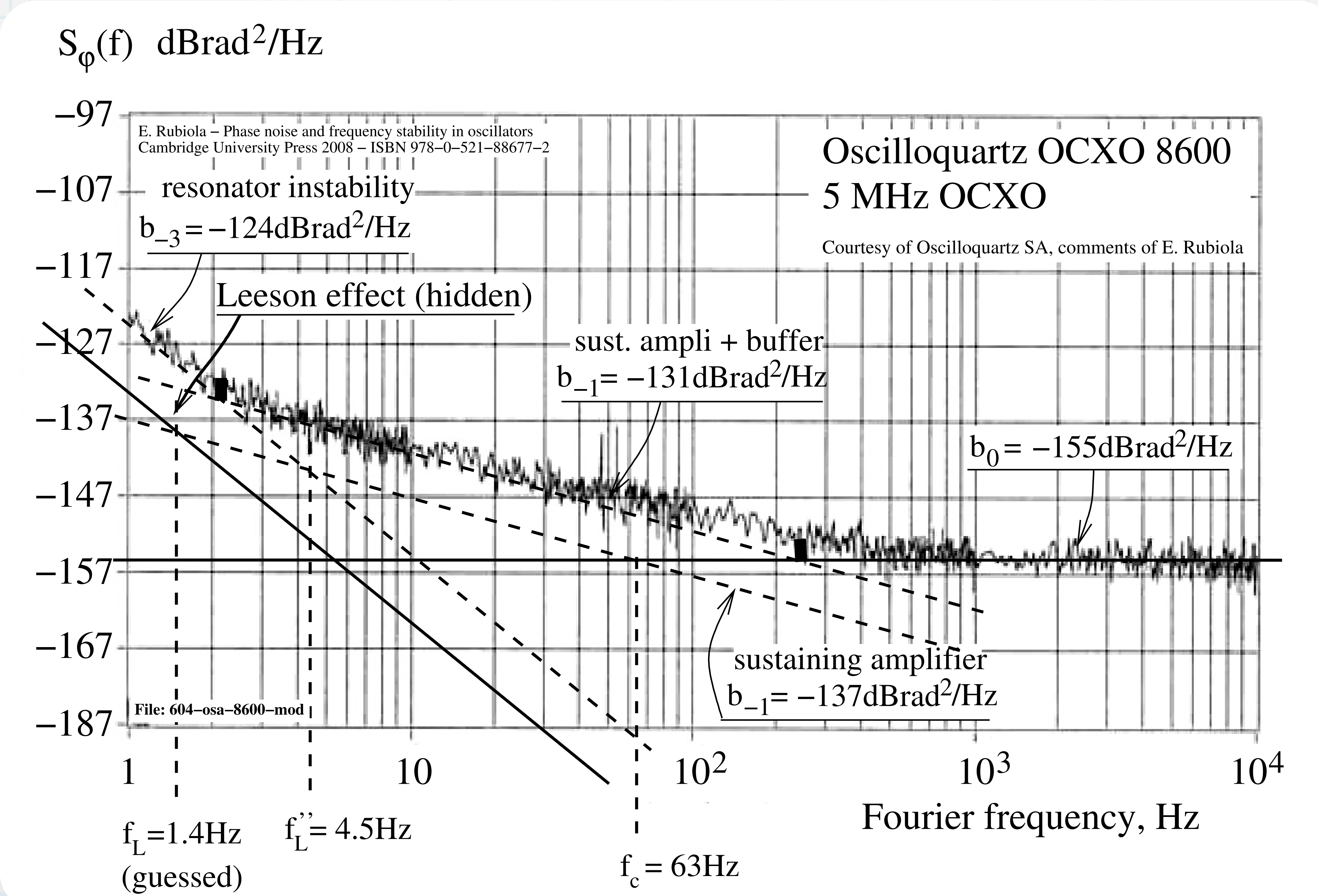
- flicker in the oscillator core is lower than –132 dBrad²/Hz @ 1 Hz
- f_L is higher than 2.5 Hz
- the resonator Q is lower than 10^6

This is inconsistent with the resonator technology (expect Q > 10^6).

The true Leeson frequency is lower than the frequency labeled as f_L

The $1/f^3$ noise is attributed to the fluctuation of the quartz resonant frequency

Example – Oscilloquartz 8600 (right)

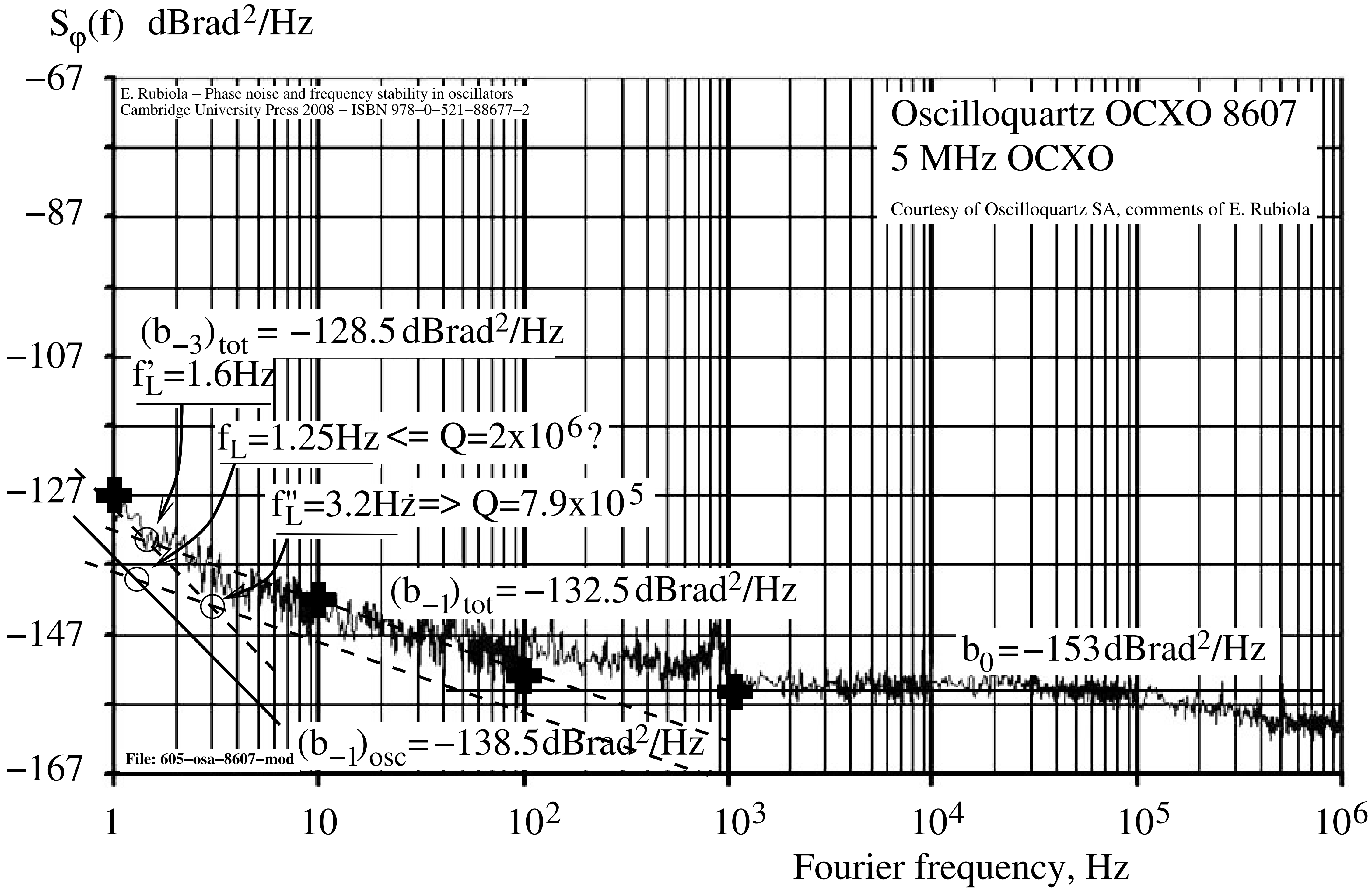


F=1dB b_0 => P_0=–18 dBm

$(b_{-3})_{osc}$ => σ_y=1.5x10^{-13}, Q=5.6x10^5 (too low)

$Q \stackrel{?}{=} 1.8$x10^6 => σ_y=4.6x10^{-14} Leeson (too low)

Example – Oscilloquartz 8607

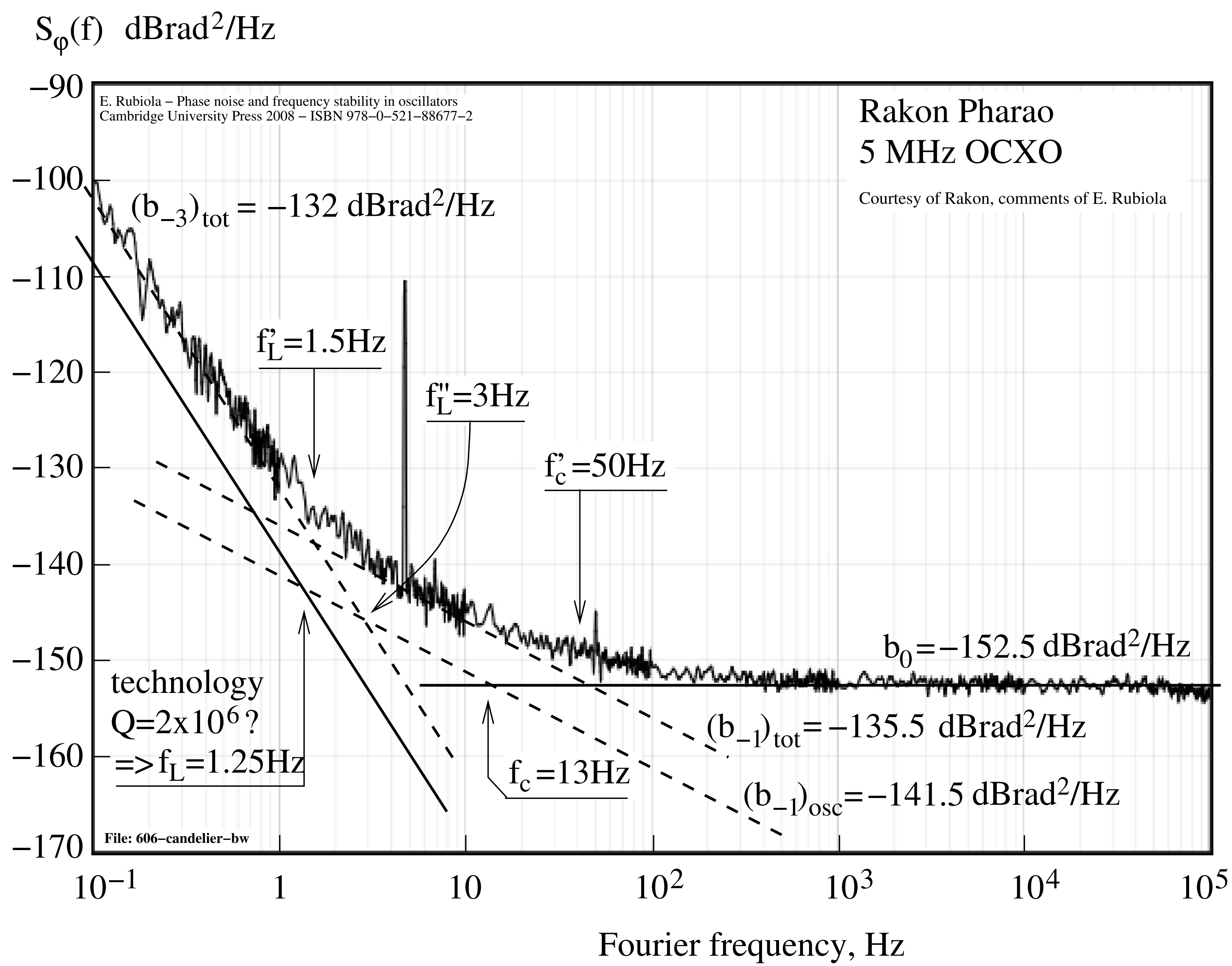


F=1dB b_0 => P_0=–20 dBm

$(b_{-3})_{osc}$ => $\sigma_y=8.8\times10^{-14}$, $Q=7.8\times10^5$ (too low)

$Q \stackrel{?}{=} 2\times10^6$ => $\sigma_y=3.5\times10^{-14}$ Leeson (too low)

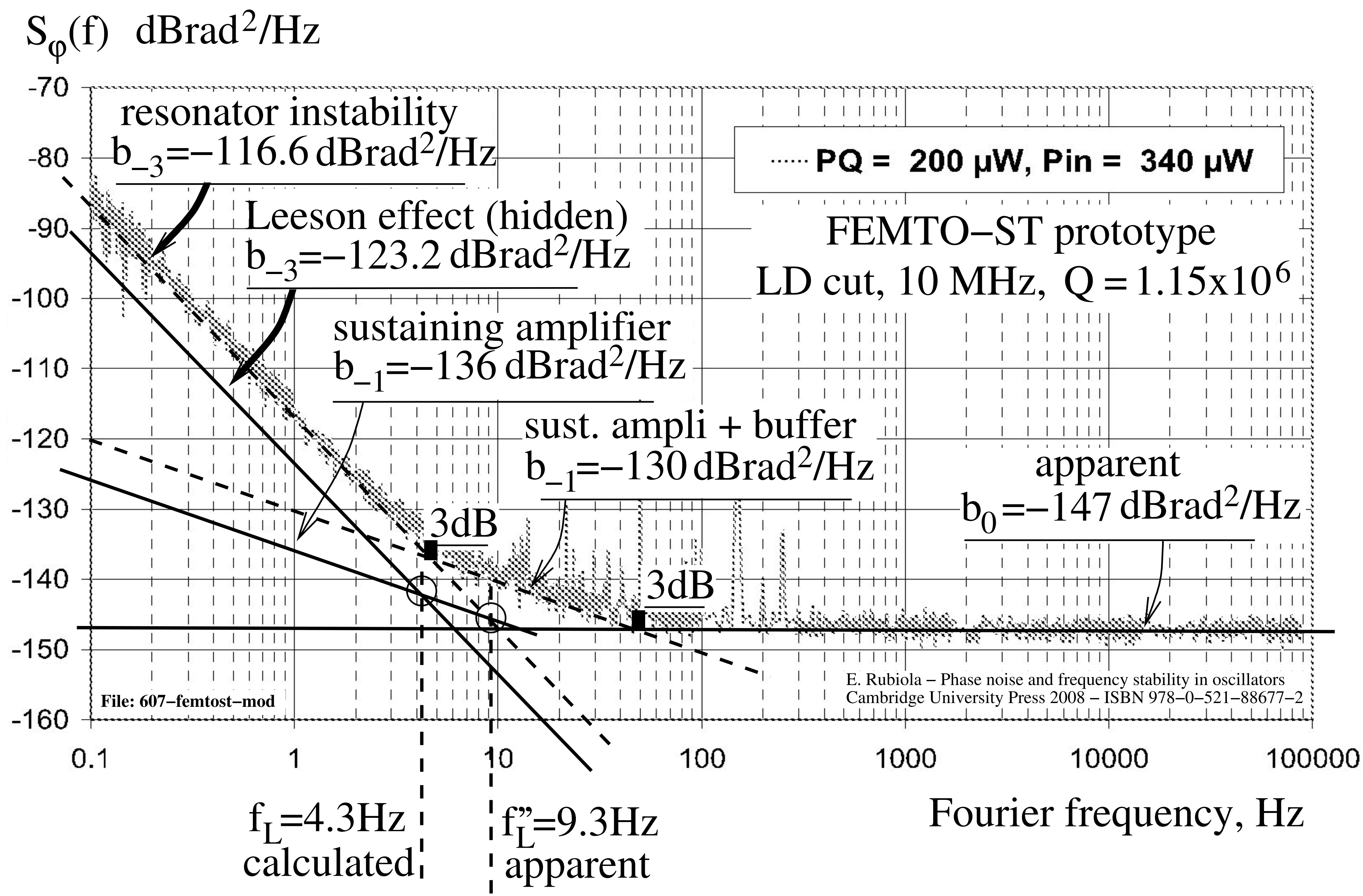
Example – CMAC Pharao



F=1dB b_0 => P_0=–20.5 dBm **$(b_{-3})_{osc}$ => $\sigma_y=5.9\times10^{-14}$, $Q=8.4\times10^5$ (too low)**

$Q \stackrel{?}{=} 2\times10^6$ => $\sigma_y=2.5\times10^{-14}$ Leeson (too low)

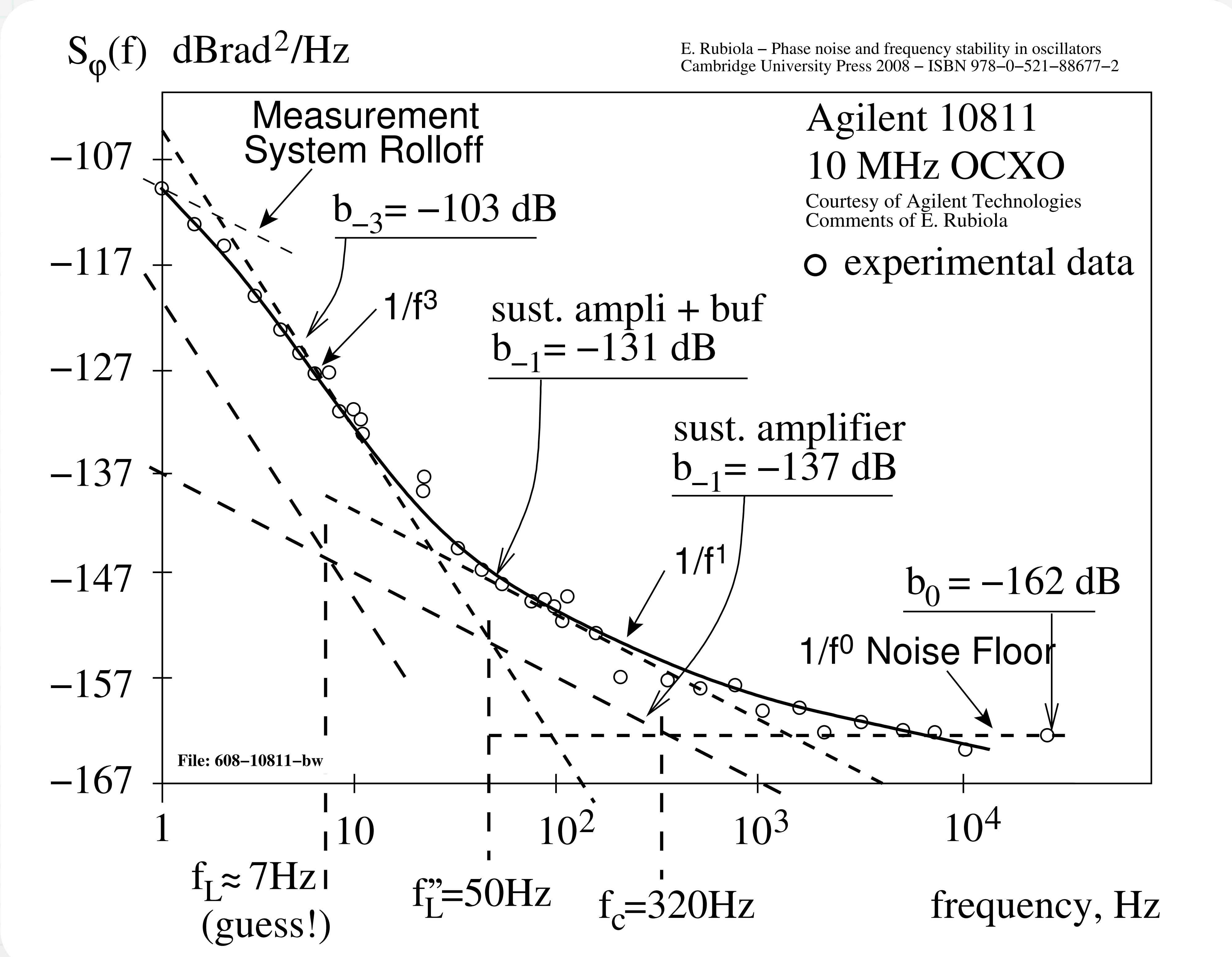
Example – FEMTO-ST prototype



F=1dB b_0 => P_0=–26 dBm — **$(b_{-3})_{osc}$ => σ_y=1.7x10^{-13}, Q=5.4x10^5 (too low)**

(there is a problem) — **Q=1.15x10^6 => σ_y=8.1x10^{-14} Leeson (too low)**

Example – Agilent 10811

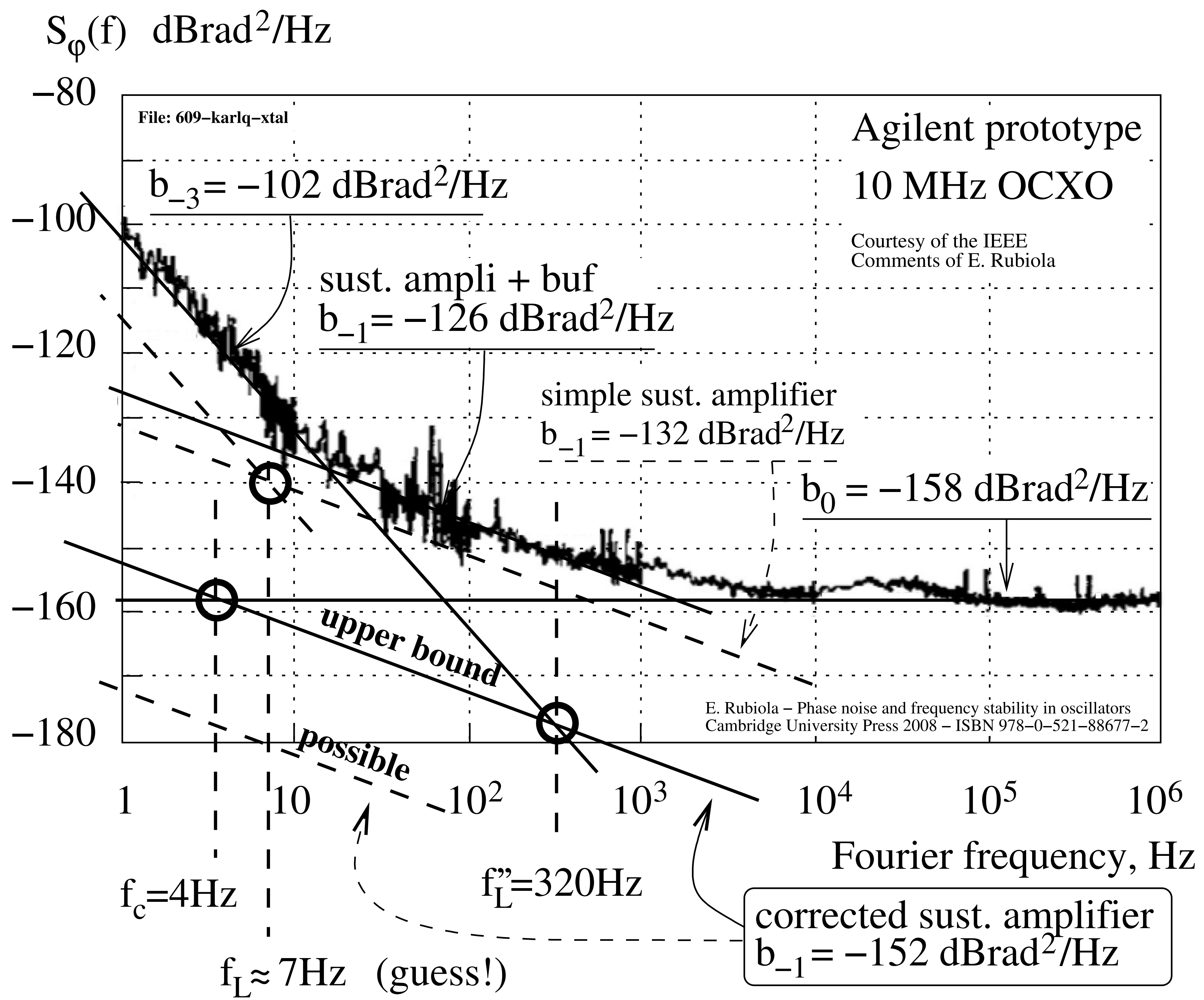


F=1dB b_0 => P_0=–11 dBm

$(b_{-3})_{osc}$ => $\sigma_y=8.3\times10^{-13}$, Q=1x$10^5$ (too low)

$Q \stackrel{?}{=} 7\times10^5$ => $\sigma_y=1.2\times10^{-13}$ Leeson (too low)

Example – Agilent prototype

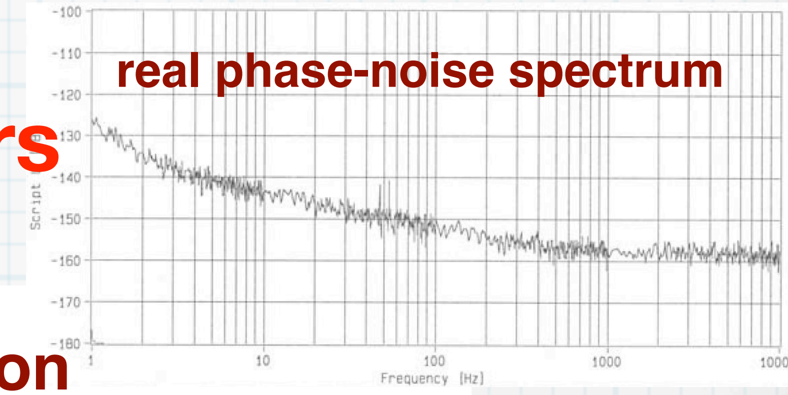


F=1dB b_0 => P_0=–12 dBm

$(b_{-3})_{osc}$ => $\sigma_y = 9.3\times10^{-13}$ Q=1.6x10^5

Q $\stackrel{?}{=}$ 7x10^5 => $\sigma_y = 2.1\times10^{-13}$ (Leeson)

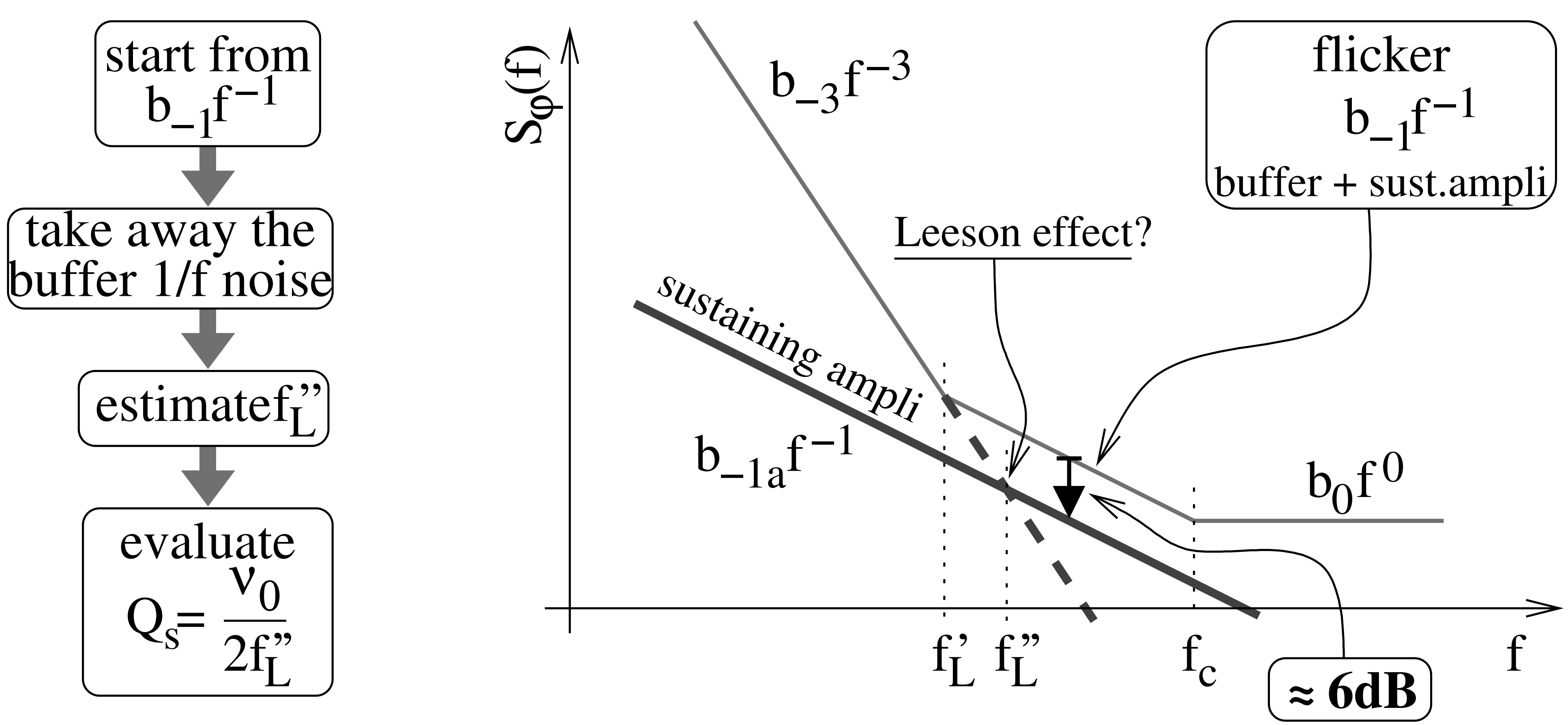
Interpretation of $S_\varphi(f)$ [1]



Only quartz-crystal oscillators

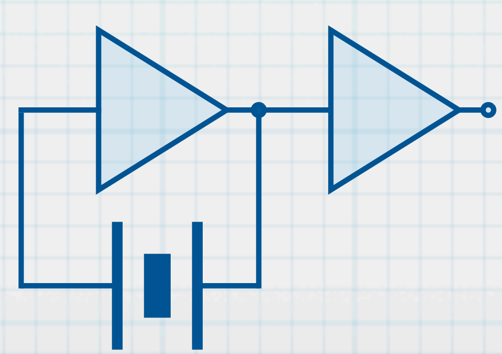
E. Rubiola – Phase noise and frequency stability in oscillators
Cambridge University Press 2008 – ISBN 978–0–521–88677–2

after parametric estimation



File: 602a–xtal–interpretation

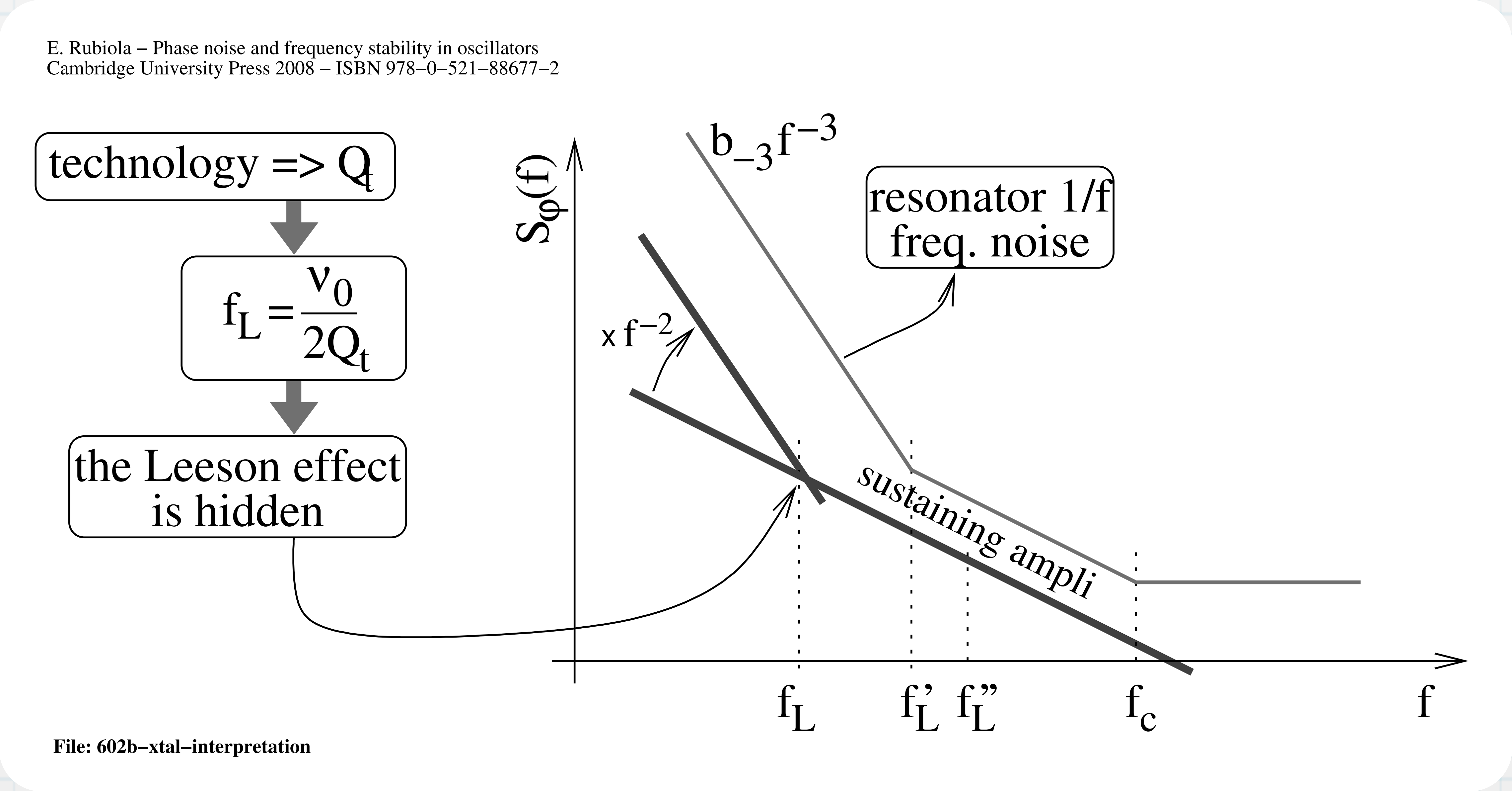
Sanity check:
- power P_0 at amplifier input
- Allan deviation σ_y (floor)



2–3 buffer stages => the sustaining amplifier contributes ≲ 25% of the total 1/f noise

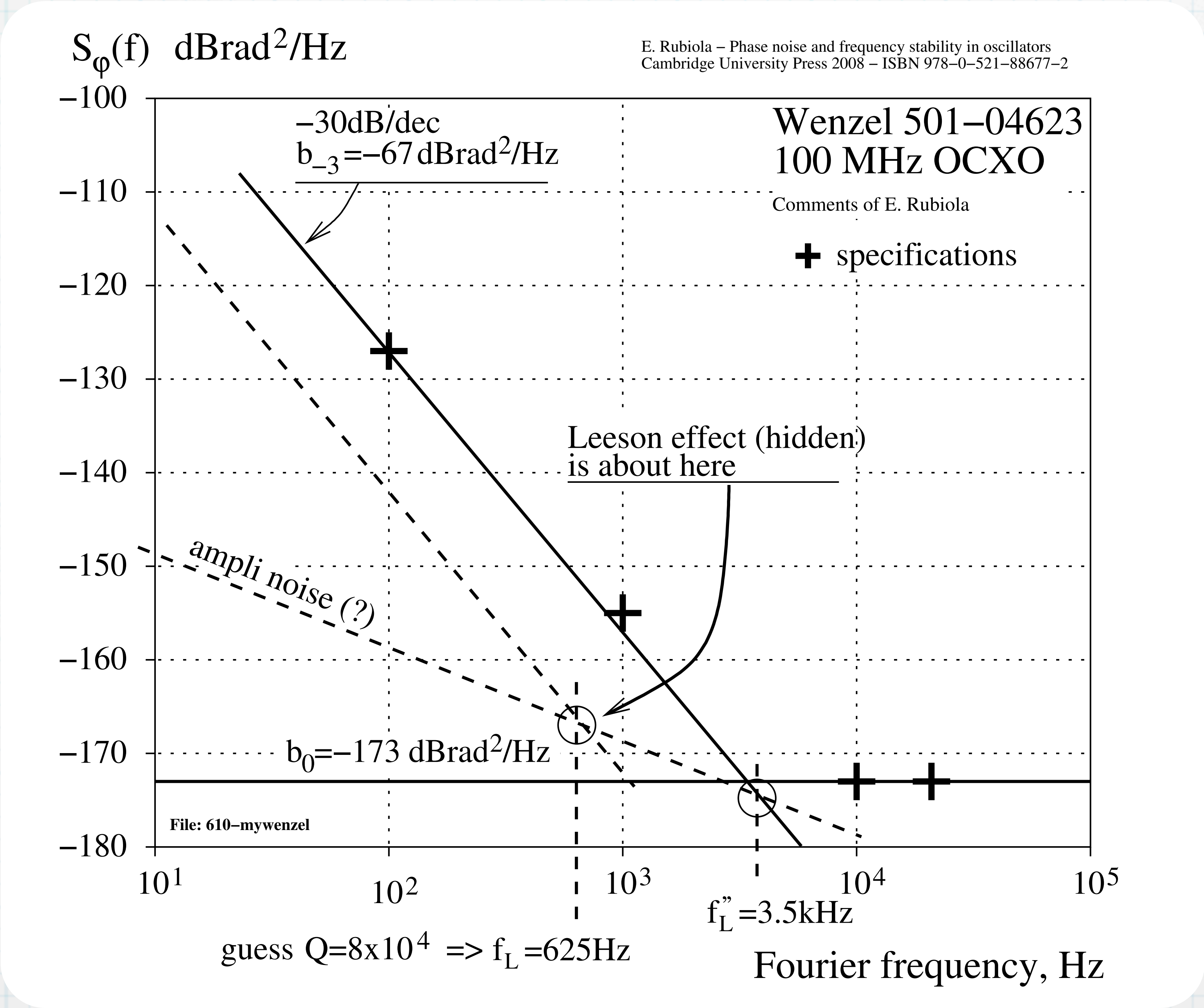
Interpretation of $S_\varphi(f)$ [2]

Only quartz-crystal oscillators



Technology suggests a merit factor Q_t. In all xtal oscillators we find $Q_t \gg Q_s$

Example – Wenzel 501-04623



Data are from the manufacturer web site. Interpretation and mistakes are of the authors.

Estimating $(b_{-1})_{ampli}$ is difficult because there is no visible 1/f region

F=1dB b_0 => P_0=0 dBm

$(b_{-3})_{osc}$ => $\sigma_y=5.3\times10^{-12}$ $Q=1.4\times10^4$

$Q \stackrel{?}{=} 8\times10^4$ => $\sigma_y=9.3\times10^{-13}$ (Leeson)

Quartz-oscillator summary

Oscillator	ν_0	$(b_{-3})_{\text{tot}}$	$(b_{-1})_{\text{tot}}$	$(b_{-1})_{\text{amp}}$	f'_L	f''_L	Q_s	Q_t	f_L	$(b_{-3})_{\text{L}}$	R	Note
Oscilloquartz 8600	5	-124.0	-131.0	-137.0	2.24	4.5	5.6×10^5	1.8×10^6	1.4	-134.1	10.1	(1)
Oscilloquartz 8607	5	-128.5	-132.5	-138.5	1.6	3.2	7.9×10^5	2×10^6	1.25	-136.5	8.1	(1)
Rakon Pharao	5	-132.0	-135.5	-141.1	1.5	3	8.4×10^5	2×10^6	1.25	-139.6	7.6	(2)
FEMTO-ST LD prot.	10	-116.6	-130.0	-136.0	4.7	9.3	5.4×10^5	1.15×10^6	4.3	-123.2	6.6	(3)
Agilent 10811	10	-103.0	-131.0	-137.0	25	50	1×10^5	7×10^5	7.1	-119.9	16.9	(4)
Agilent prototype	10	-102.0	-126.0	-132.0	16	32	1.6×10^5	7×10^5	7.1	-114.9	12.9	(5)
Wenzel 501-04623	100	-67.0	-132 ?	-138 ?	1800	3500	1.4×10^4	8×10^4	625	-79.1	15.1	(6)
unit	MHz	dB rad^2/Hz	dB rad^2/Hz	dB rad^2/Hz	Hz	Hz	(none)	(none)	Hz	dB rad^2/Hz	dB	

Notes
(1) Data are from specifications, full options about low noise and high stability.
(2) Measured by Rakon on a sample. Rakon confirmed that $2\times10^6 < Q < 2.2\times10^6$ in actual conditions.
(3) LD cut, built and measured in our laboratory, yet by a different team. Q_t is known.
(4) Measured by Hewlett Packard (now Agilent) on a sample.
(5) Implements a bridge scheme for the degeneration of the amplifier noise. Same resonator of the Agilent 10811.
(6) Data are from specifications.

$$R = \left.\frac{(\sigma_y)_{\text{oscill}}}{(\sigma_y)_{\text{Leeson}}}\right|_{\text{floor}} = \sqrt{\frac{(b_{-3})_{\text{tot}}}{(b_{-3})_L}} = \frac{Q_t}{Q_s} = \frac{f''_L}{f_L}$$

Opto-electronic oscillator

TIDALwave™

A12x

Ultra-Low Phase Noise Microwave Signal Source

OEwaves

- Imaging
- Digital Radio (QAM)
- Optical Data Communications

magnitude, and high capacity, high frequency future wireless communications systems. This level of performance will enable manufacturers to retrofit current systems as well as architect capabilities to address new markets.

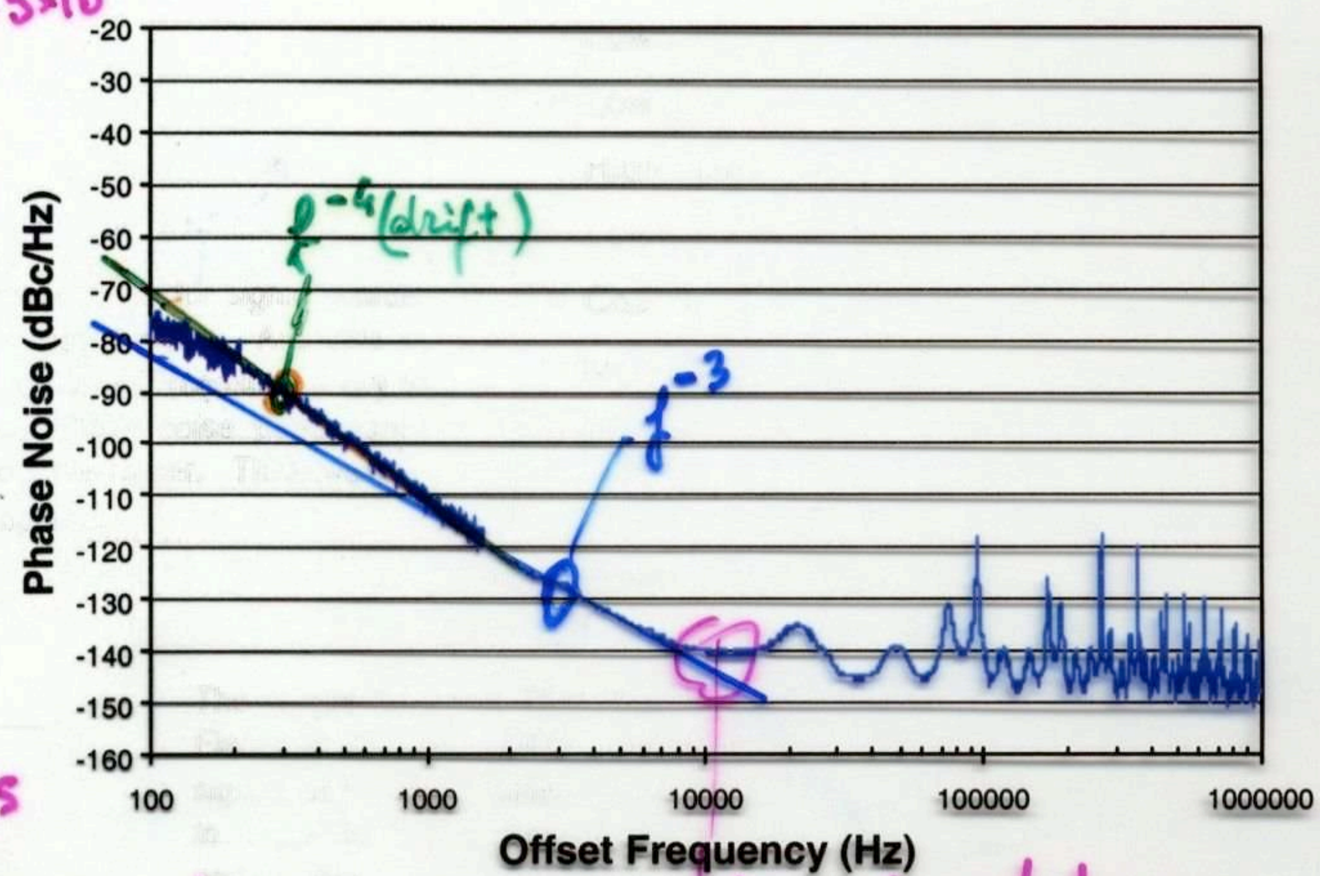
OEwaves is developing *miniaturized* (MINIwave™) and multi-octave *tunable* (TUNEwave™) signal sources based on the performance and specifications of TIDALwave.

$f_L = \frac{\nu_0}{2Q}$ → $Q \simeq \frac{10^{10}}{2\cdot10^4} = 5\times10^5$

Free Running Phase Noise Plot

TIDALwave - 10 GHz

Model: OE1255



delay line

$Q = \pi \nu_0 \tau$

$\tau = \frac{Q}{\pi \nu_0} \sim \frac{5\times10^5}{3.2\cdot10^{10}} = 16\,\mu s$

length $= c\tau =$ 5 km (vacuum)

3.2 km ($n = 1.5$)

f_L about here

Courtesy of OEwaves (handwritten notes are mine). Cut from the oscillator specifications available at the URL http://www.oewaves.com/products/pdf/TDALwave_Datasheet_012104.pdf

Resonator theory

Resonator – time domain

$$\ddot{x} + \frac{\omega_n}{Q}\dot{x} + \omega_n^2 x = \frac{\omega_n}{Q}\dot{v}(t)$$

ω_n natural frequency

Q quality factor

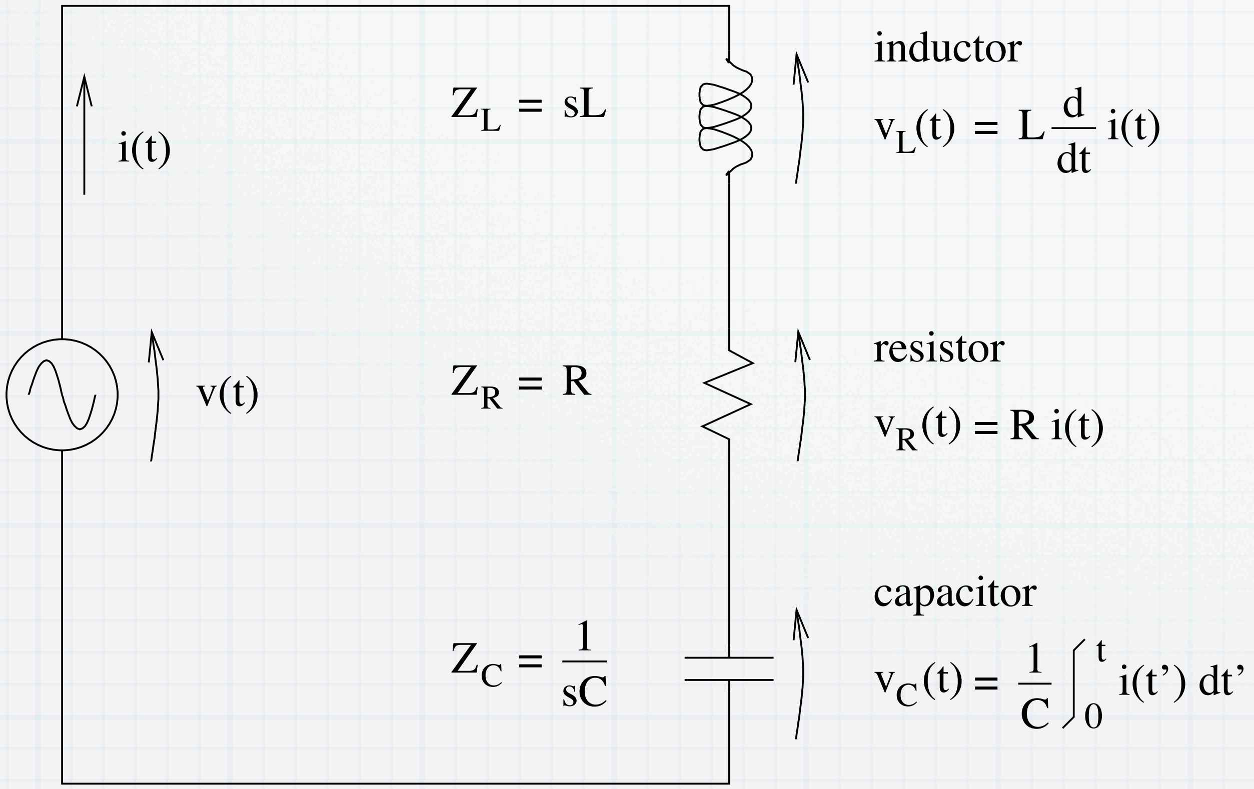
τ relaxation time

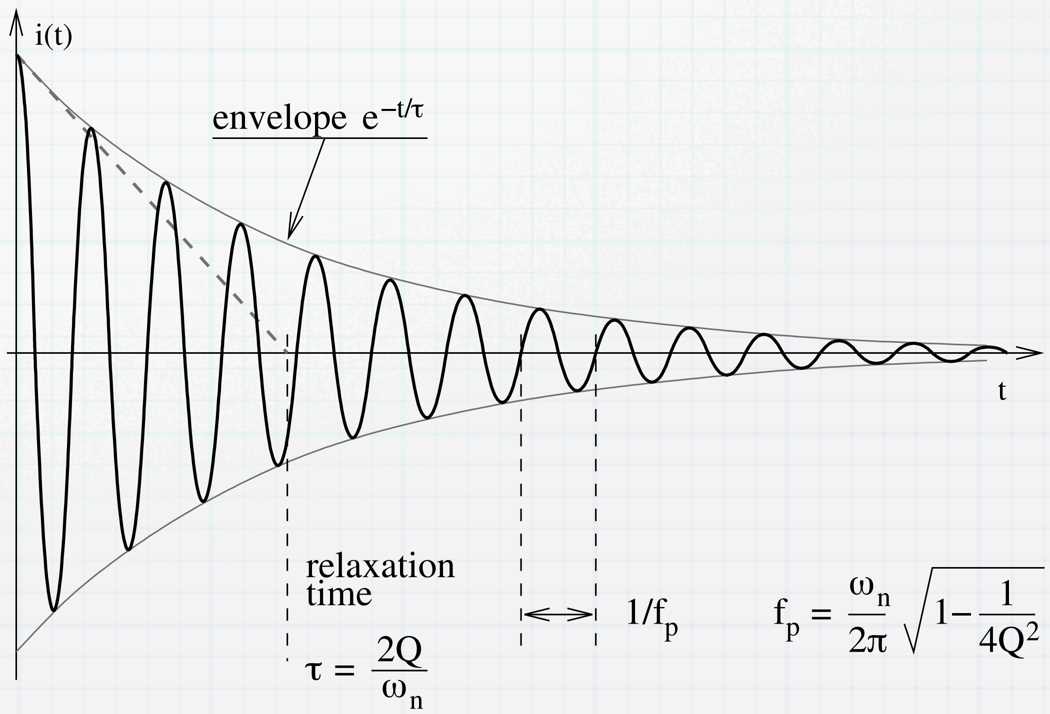
$\tau = \frac{2Q}{\omega_n}$

ω_p free-decay pseudofrequency

$\omega_p = \omega_n\sqrt{1 - 1/4Q^2}$

shorthand: f = ω/2π

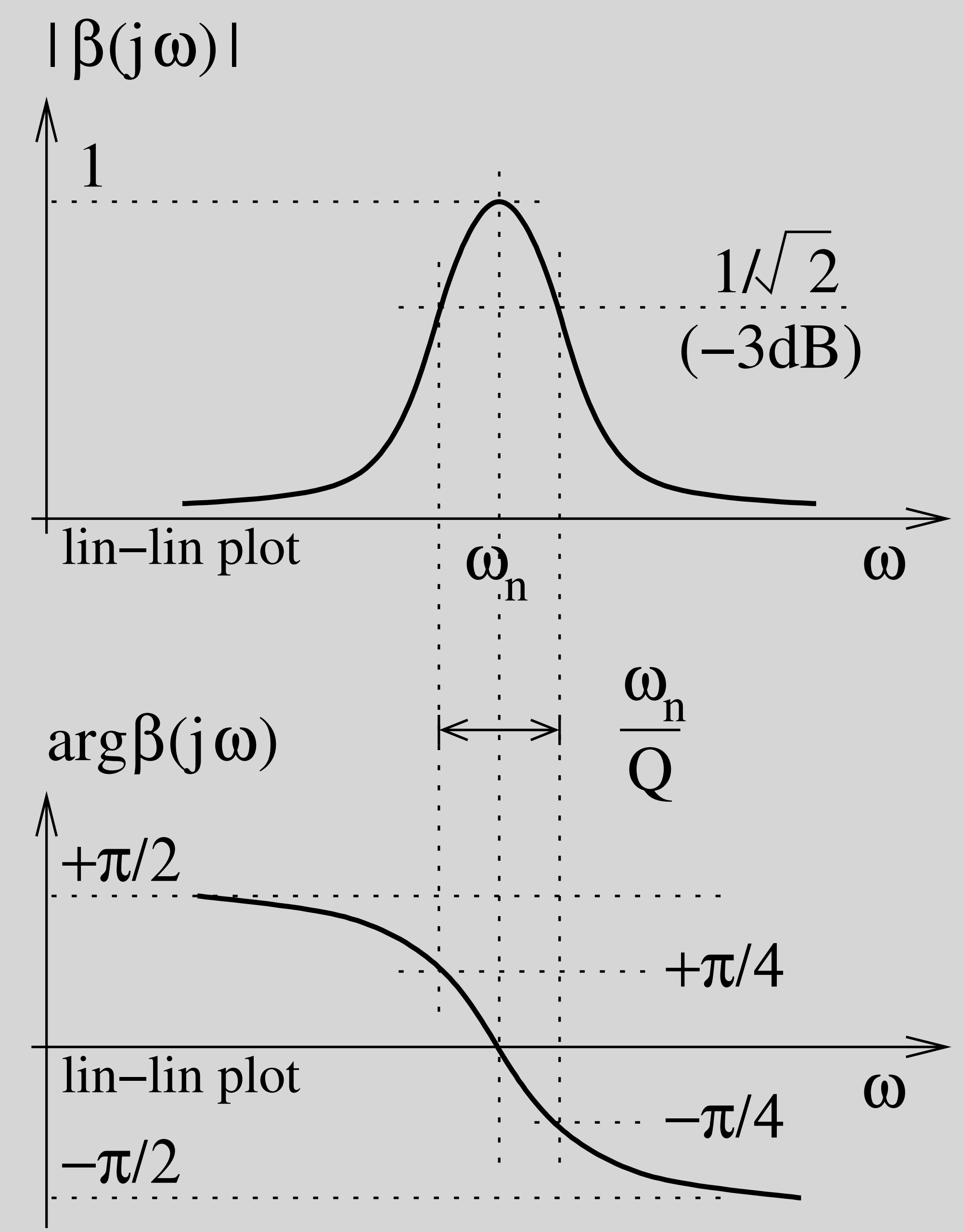




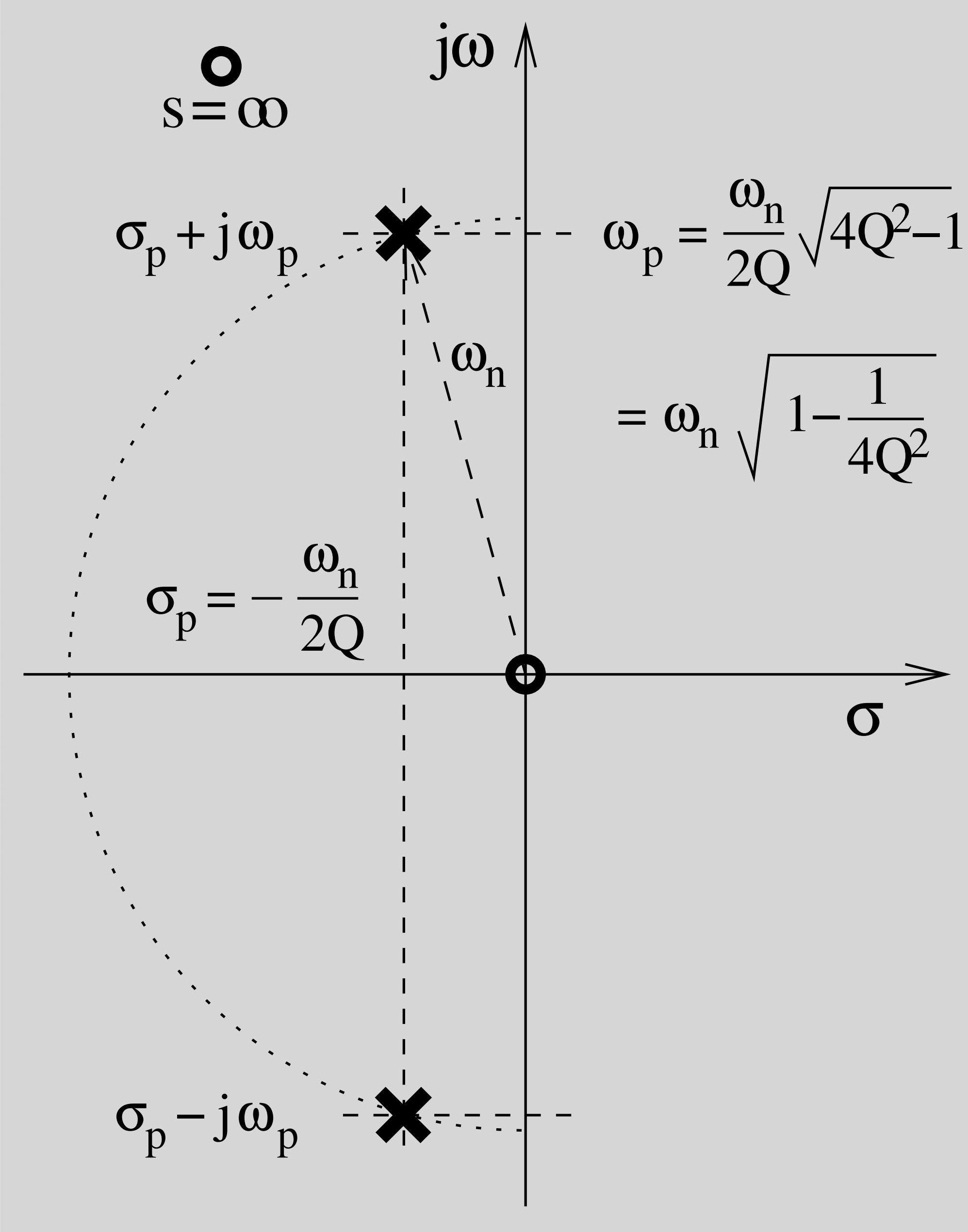
Resonator – frequency domain

$$\beta(s) = \frac{\omega_n}{Q}\,\frac{s}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2} \qquad s = \sigma + j\omega$$

frequency domain



complex plane



B4

RESONATOR

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

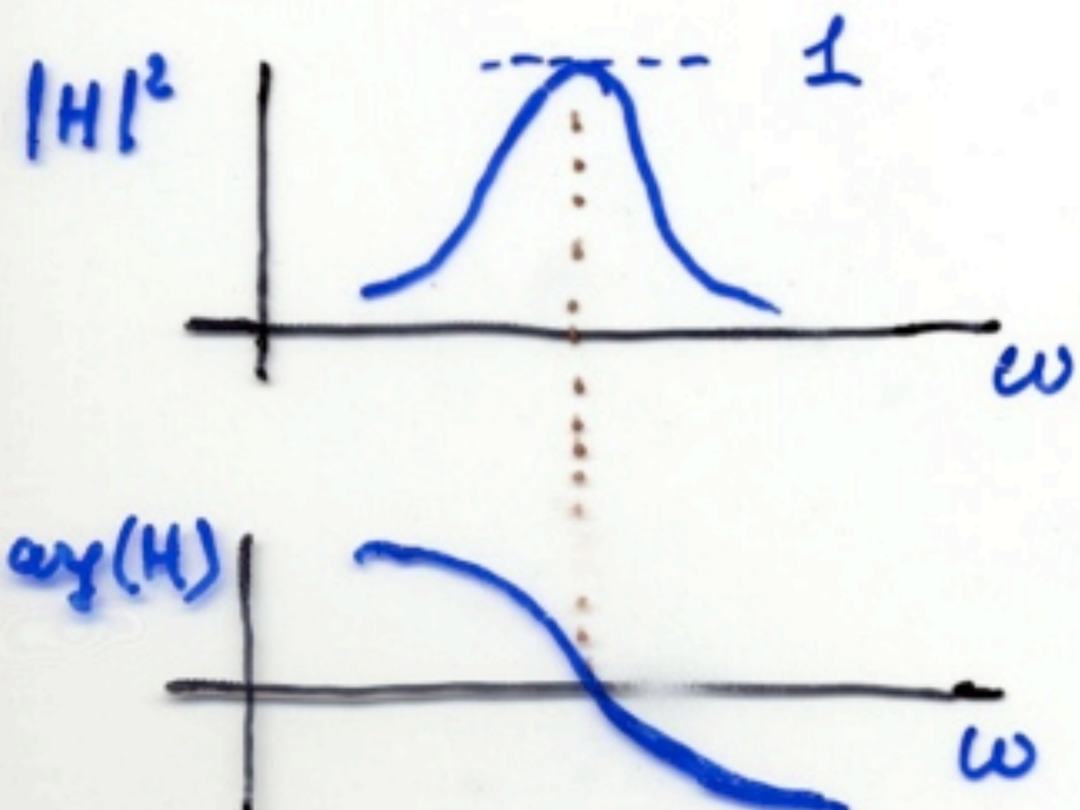
$s = \sigma + j\omega$

$$H(s) = \frac{\omega_0}{Q} \frac{s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

normalization for $H_{max} = 1$

define

$$x = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \xrightarrow{\omega \to \omega_0} 2\frac{\omega - \omega_0}{\omega}$$



$$H(j\omega) = \frac{1}{1 + jQx} = \frac{1 - jQx}{1 + Q^2x^2}$$

Real, Imag

$$R(\omega) = \frac{1}{1 + Q^2x^2}$$

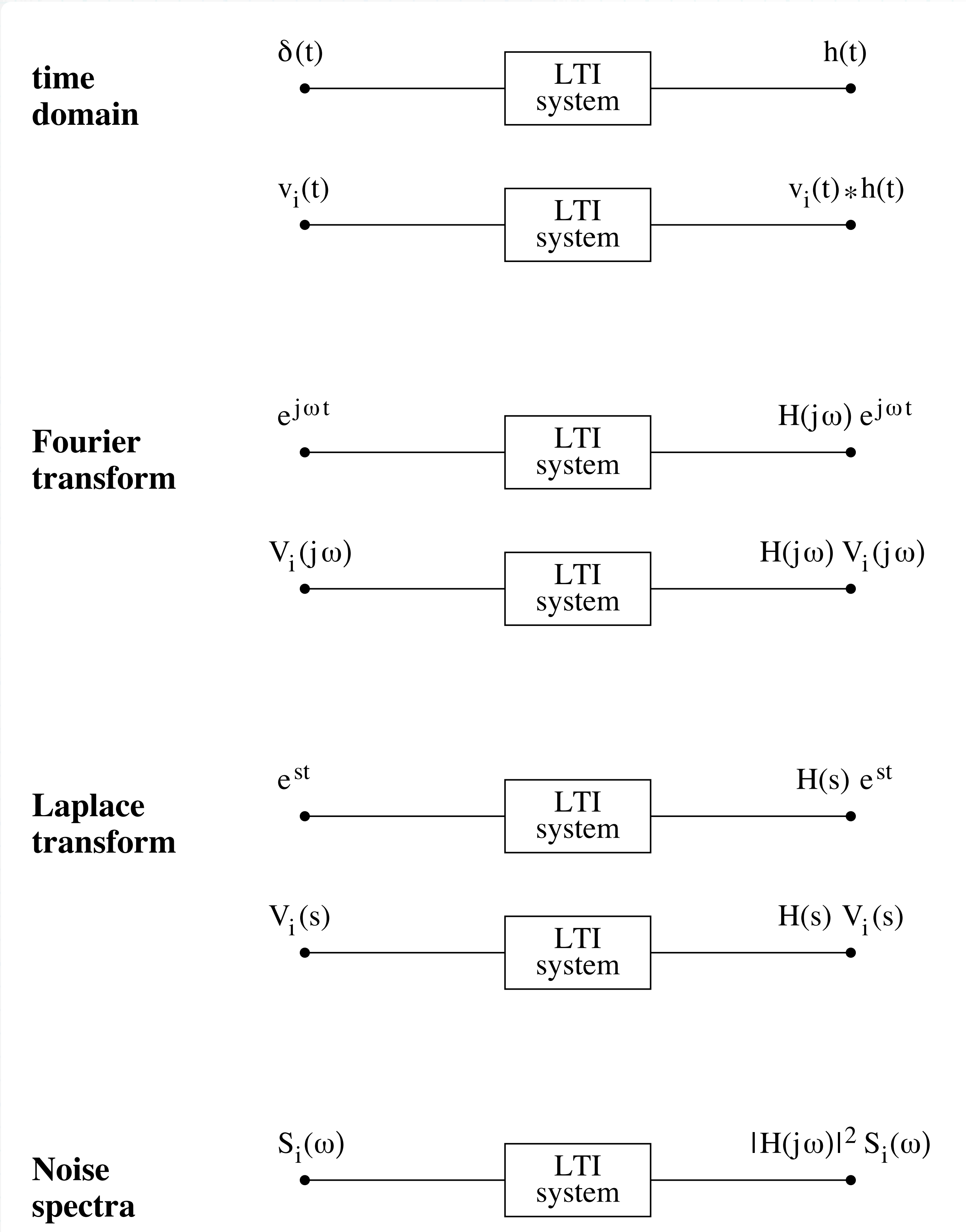
$$I(\omega) = \frac{-Qx}{1 + Q^2x^2}$$

Modulus, phase

$$M(\omega) = \frac{1}{\sqrt{1 + Q^2x^2}}$$

$$\Phi(\omega) = -\arctan Qx$$

Linear time-invariant (LTI) systems



impulse response

response to the generic signal $v_i(t)$

$$H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$

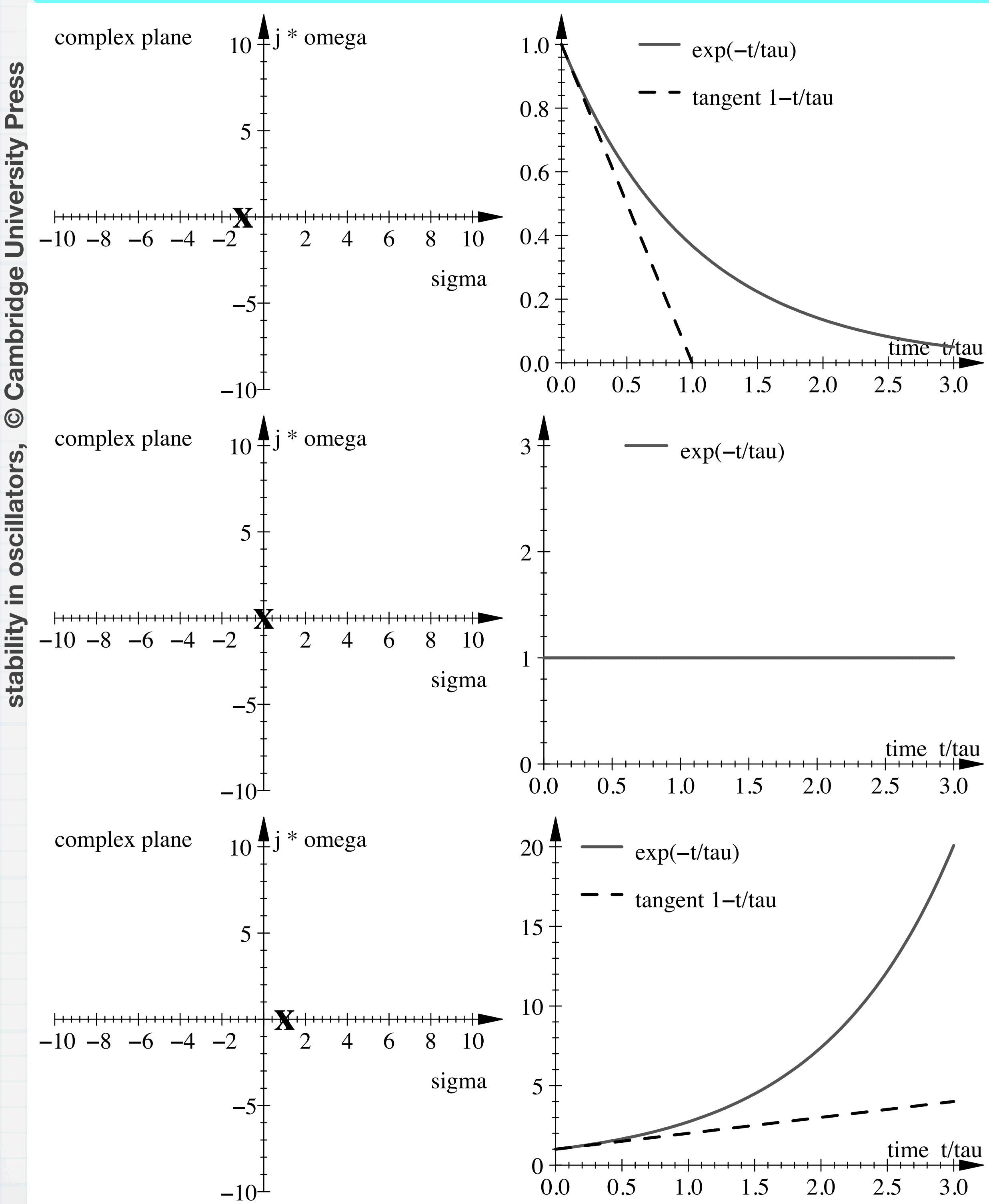
$$H(s) = \int_{0}^{\infty} h(t)\, e^{-st}\, dt$$

H(s), s=σ+jω, is the analytic continuation of H(ω) for causal system, where h(t)=0 for t<0

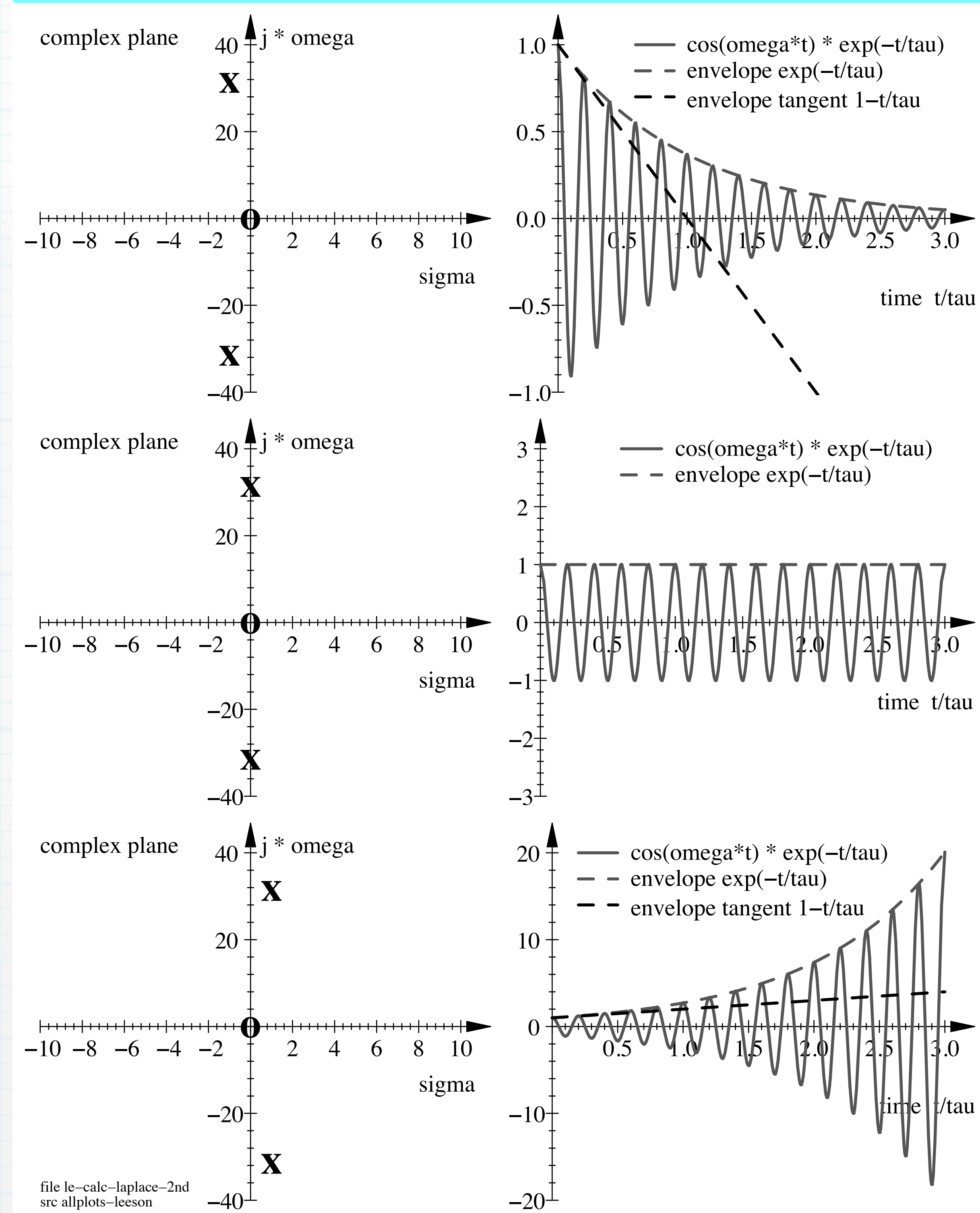
Laplace-transform patterns

Fundamental theorem of complex algebra: F(s) is completely determined by its roots

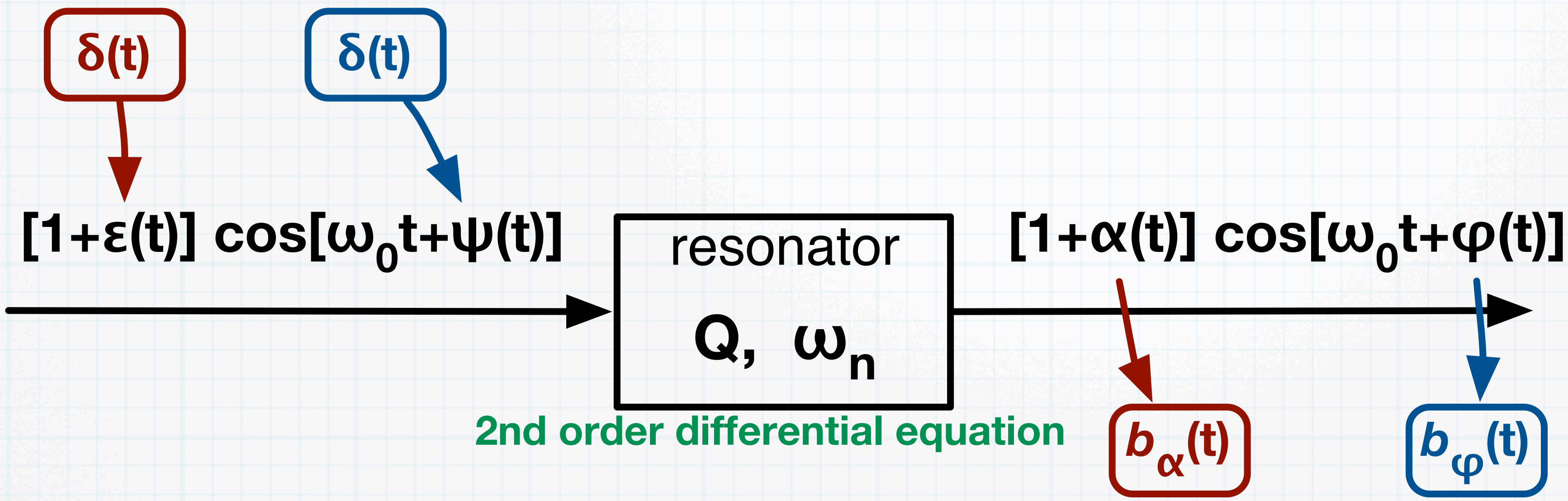
$$F(s) = \frac{1}{s + 1/\tau}$$



$$F(s) = \frac{s}{s^2 + 2s/\tau + \omega_n^2}$$

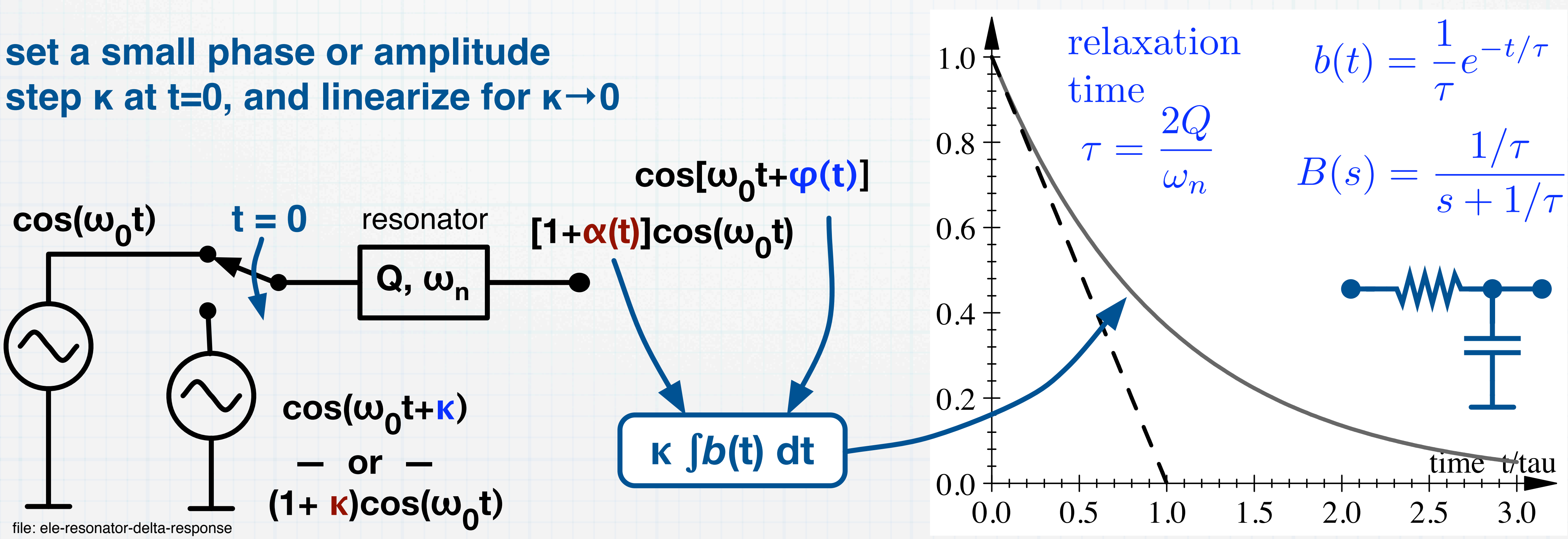


Resonator impulse response



Cannot figure out a δ(t) of phase or amplitude? Use a step and differentiate

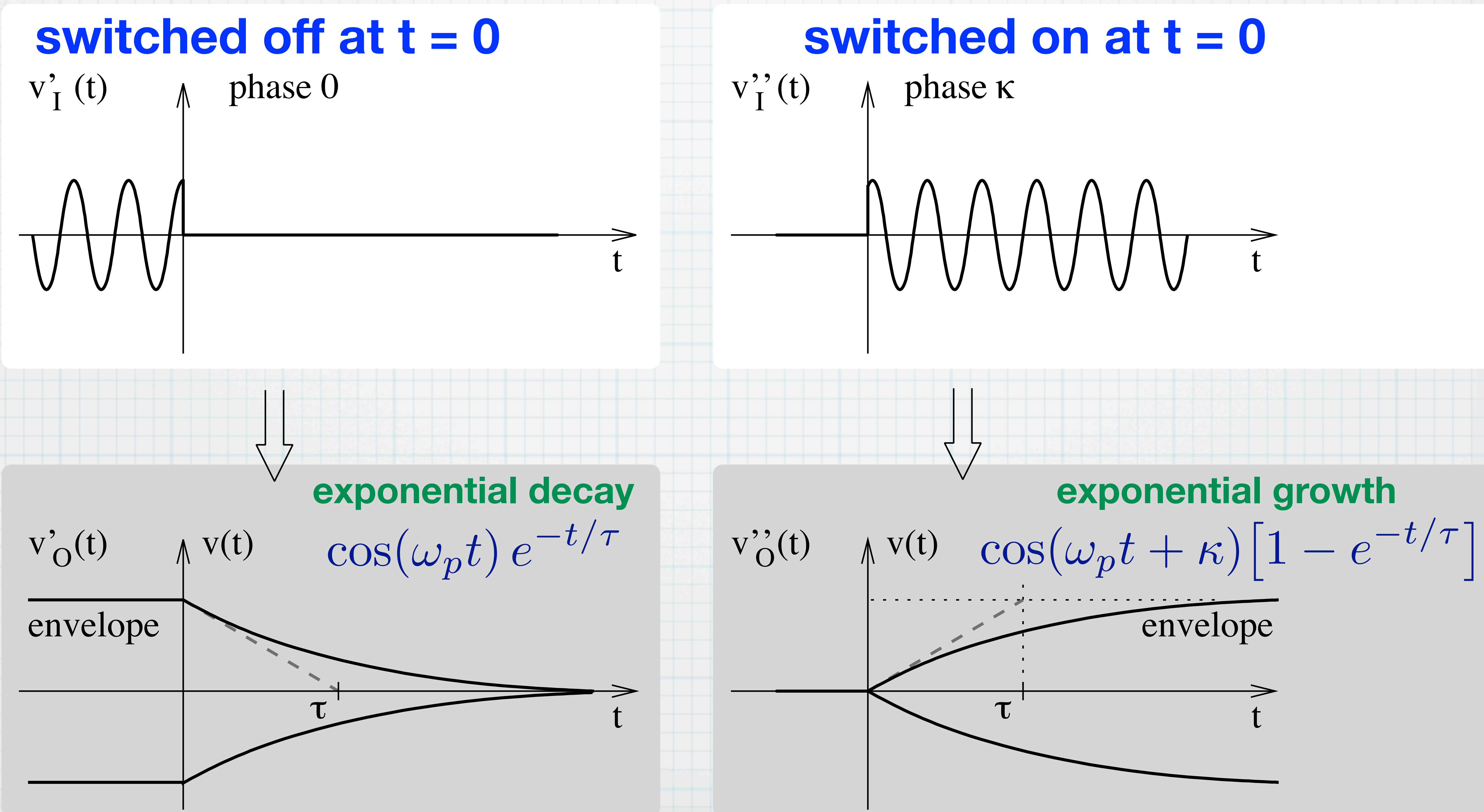
set a small phase or amplitude step κ at t=0, and linearize for κ→0



file: ele-resonator-delta-response

Response to a phase step κ

A phase step is equivalent to switching a sinusoid off at t = 0, and switching a shifted sinusoid on at t=0



Resonator impulse response ($\omega_0=\omega_n$)

$$v_i(t) = \underbrace{\cos(\omega_0 t)\,\mathfrak{u}(-t)}_{\text{switched off at } t=0} + \underbrace{\cos(\omega_0 t + \kappa)\,\mathfrak{u}(t)}_{\text{switched on at } t=0}$$

phase step κ at t=0

$$v_o(t) = \cos(\omega_p t)\, e^{-t/\tau} + \cos(\omega_p t + \kappa)\left[1 - e^{-t/\tau}\right] \quad t > 0$$

output

$$v_o(t) = \cos(\omega_p t) - \kappa \sin(\omega_p t)\left[1 - e^{-t/\tau}\right] \quad \kappa \rightarrow 0$$

linearize

$$v_o(t) = \cos(\omega_0 t) - \kappa \sin(\omega_0 t)\left[1 - e^{-t/\tau}\right] \quad \omega_p \rightarrow \omega_0$$

high Q

$$\mathbf{V_o}(t) = \frac{1}{\sqrt{2}}\left\{1 + j\kappa\left[1 - e^{-t/\tau}\right]\right\}$$

slow-varying phase vector

$$\arctan\left(\frac{\Im\{\mathbf{V_o}(t)\}}{\Re\{\mathbf{V_o}(t)\}}\right) \simeq \kappa\left[1 - e^{-t/\tau}\right]$$

phasor angle

delete κ and differentiate

impulse response

$$\mathrm{b}(t) = \frac{1}{\tau} e^{-s\tau} \quad \leftrightarrow \quad \mathrm{B}(s) = \frac{1/\tau}{s + 1/\tau}$$

Detuned resonator (1)

amplitude
phase

$$\begin{bmatrix}\alpha\\ \varphi\end{bmatrix} = \begin{bmatrix}\mathrm{b}_{\alpha\alpha} & \mathrm{b}_{\alpha\varphi}\\ \mathrm{b}_{\varphi\alpha} & \mathrm{b}_{\varphi\varphi}\end{bmatrix} * \begin{bmatrix}\varepsilon\\ \psi\end{bmatrix} \quad \leftrightarrow \quad \begin{bmatrix}\mathcal{A}\\ \Phi\end{bmatrix} = \begin{bmatrix}\mathrm{B}_{\alpha\alpha} & \mathrm{B}_{\alpha\varphi}\\ \mathrm{B}_{\varphi\alpha} & \mathrm{B}_{\varphi\varphi}\end{bmatrix} \begin{bmatrix}\mathcal{E}\\ \Psi\end{bmatrix}$$

$\Omega = \omega_0 - \omega_n$	**detuning**
$\beta_0 = \lvert\beta(j\omega_0)\rvert$	**mudulus**
$\theta = \arg(\beta(j\omega_0))$	**phase**

$$v_i(t) = \underbrace{\frac{1}{\beta_0}\cos(\omega_0 t - \theta)\,\mathfrak{u}(-t)}_{\text{switched off at } t=0} + \underbrace{\frac{1}{\beta_0}\cos(\omega_0 t - \theta + \kappa)\,\mathfrak{u}(t)}_{\text{switched on at } t=0}$$

phase step κ at t=0

$$= \frac{1}{\beta_0}\cos(\omega_0 t - \theta)\,\mathfrak{u}(-t) + \frac{1}{\beta_0}\Big[\cos(\omega_0 t - \theta)\cos\kappa - \sin(\omega_0 t - \theta)\sin\kappa\Big]\,\mathfrak{u}(t)$$

$$\simeq \frac{1}{\beta_0}\cos(\omega_0 t - \theta)\,\mathfrak{u}(-t) + \frac{1}{\beta_0}\Big[\cos(\omega_0 t - \theta) - \kappa\sin(\omega_0 t - \theta)\Big]\,\mathfrak{u}(t) \qquad \kappa \ll 1.$$

Detuned resonator (2)

$$v_o(t) = \cos(\omega_0 t) - \kappa \sin(\omega_0 t) + \kappa \sin(\omega_n t)\, e^{-t/\tau}$$ **output, large Q (ω_p=ω_n)**

use $\Omega = \omega_0 - \omega_n$

$$v_o(t) = \cos(\omega_0 t)\left[1 - \kappa \sin(\Omega t) e^{-t/\tau}\right] - \kappa \sin(\omega_0 t)\left[1 - \cos(\Omega t) e^{-t/\tau}\right]$$

slow-varying phase vector

$$\mathbf{V_o}(t) = \frac{1}{\sqrt{2}}\left\{1 - \kappa \sin(\Omega t) e^{-t/\tau} + j\kappa\left[1 - \cos(\Omega t) e^{-t/\tau}\right]\right\} \qquad \kappa \ll 1$$

$$\arctan \frac{\Im\{\mathbf{V_o}(t)\}}{\Re\{\mathbf{V_o}(t)\}} = \kappa\left[1 - \cos(\Omega t) e^{-t/\tau}\right]$$ **angle**

$$|\mathbf{V_o}(t)| = |\mathbf{V_o}(0)| - \kappa \sin(\Omega t) e^{-t/\tau}$$ **amplitude**

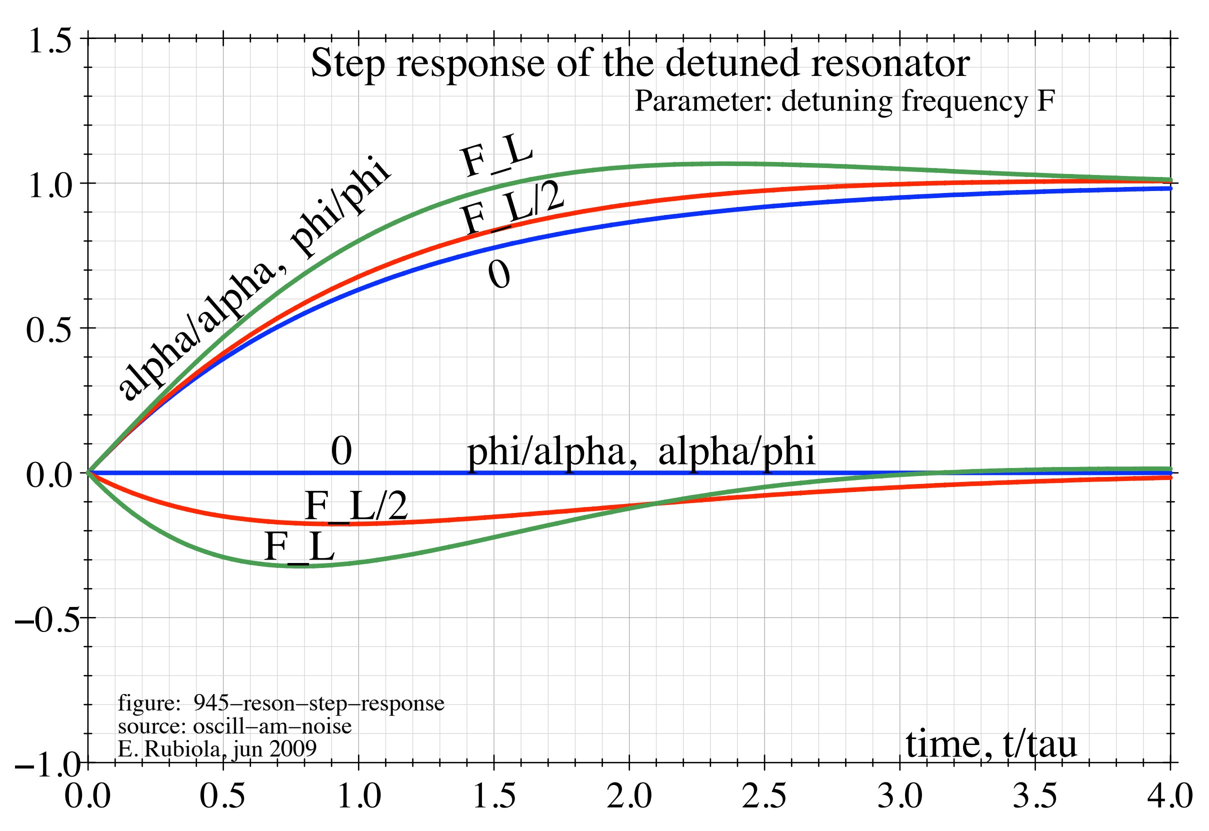
delete κ and differentiate

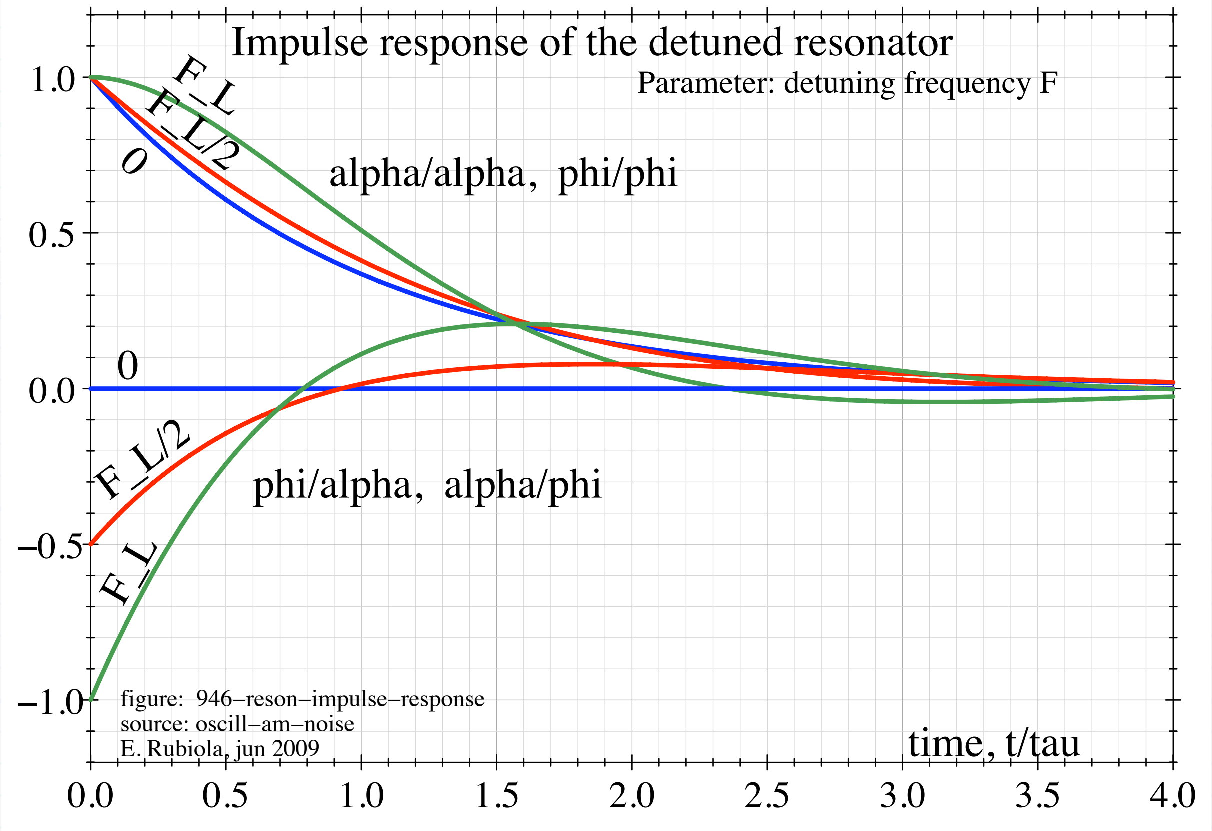
impulse response

$$\mathrm{b}_{\varphi\varphi}(t) = \left[\Omega \sin(\Omega t) + \frac{1}{\tau}\cos(\Omega t)\right] e^{-t/\tau} \qquad \text{phase}$$

$$\mathrm{b}_{\alpha\varphi}(t) = \left[-\Omega \cos(\Omega t) + \frac{1}{\tau}\sin(\Omega t)\right] e^{-t/\tau} \qquad \text{amplitude}$$

Resonator step and impulse response

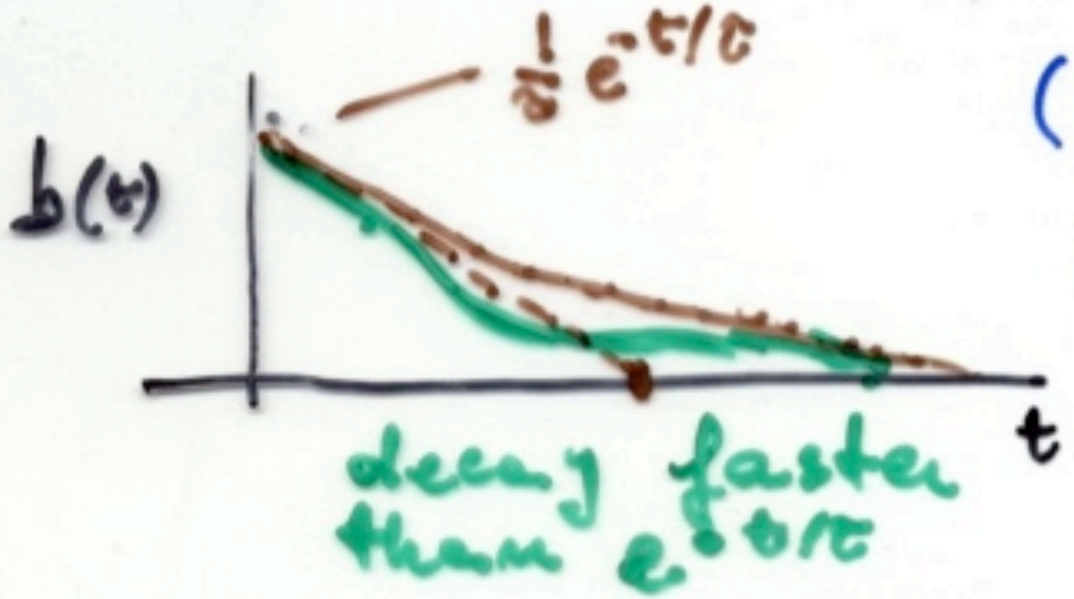




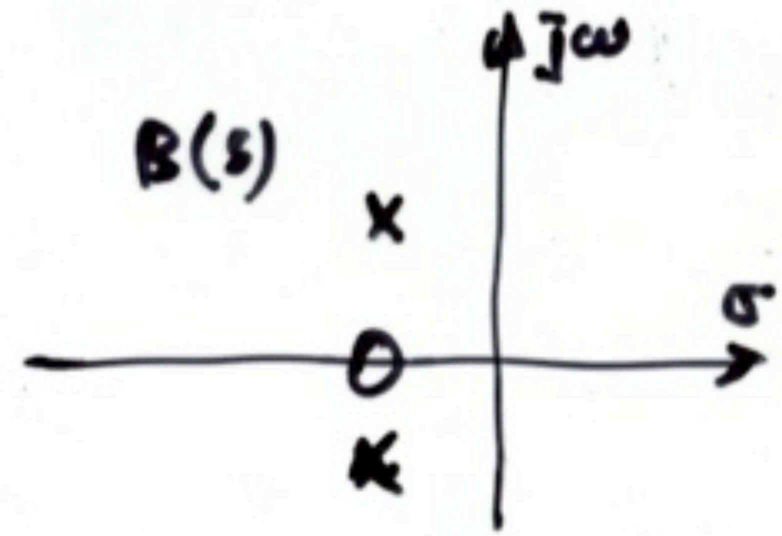
$$[\mathrm{b}](t) = \begin{bmatrix} \left(\Omega \sin \Omega t + \frac{1}{\tau} \cos \Omega t\right) e^{-t/\tau} & \left(-\Omega \cos \Omega t + \frac{1}{\tau} \sin \Omega t\right) e^{-t/\tau} \\ \left(-\Omega \cos \Omega t + \frac{1}{\tau} \sin \Omega t\right) e^{-t/\tau} & \left(\Omega \sin \Omega t + \frac{1}{\tau} \cos \Omega t\right) e^{-t/\tau} \end{bmatrix}$$

DETUNED RESONATOR

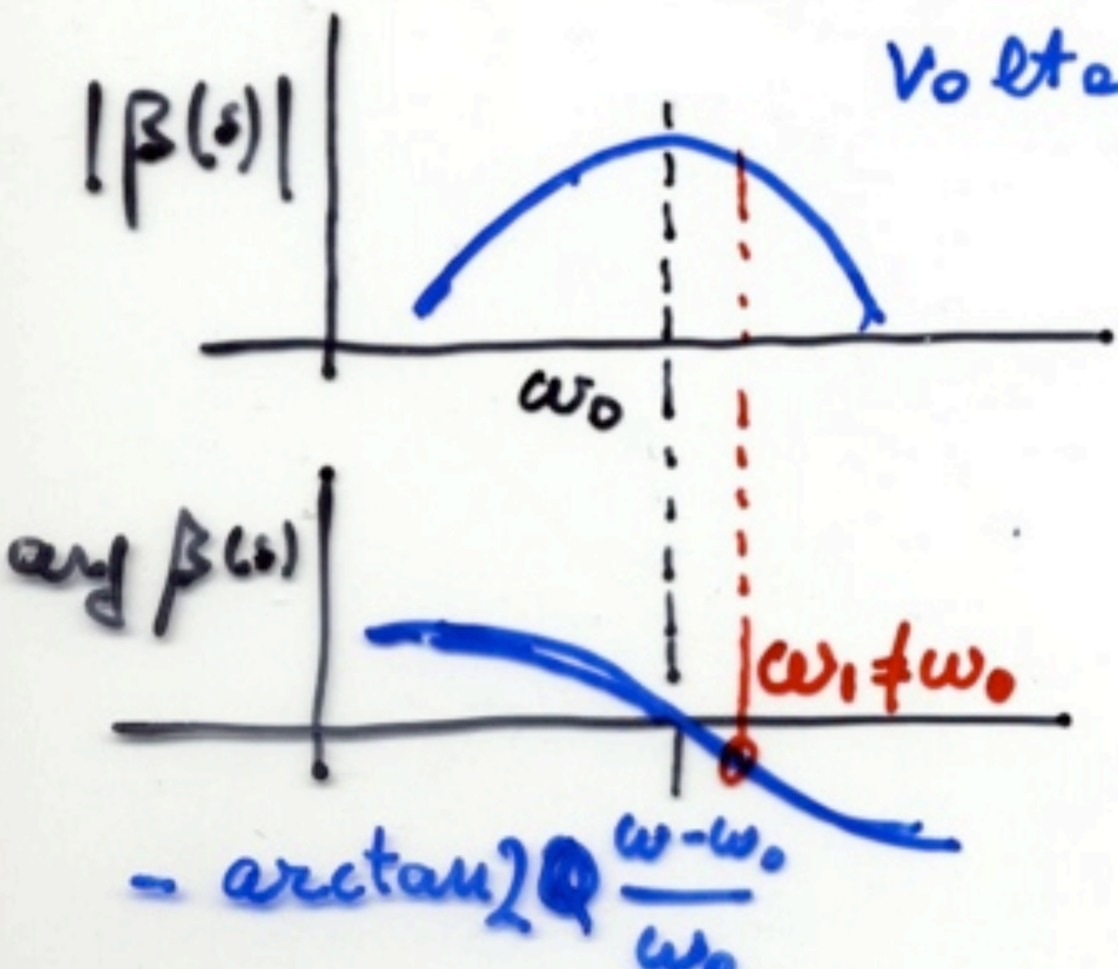
C4



(CONTROLS)
people use 2nd order systems!



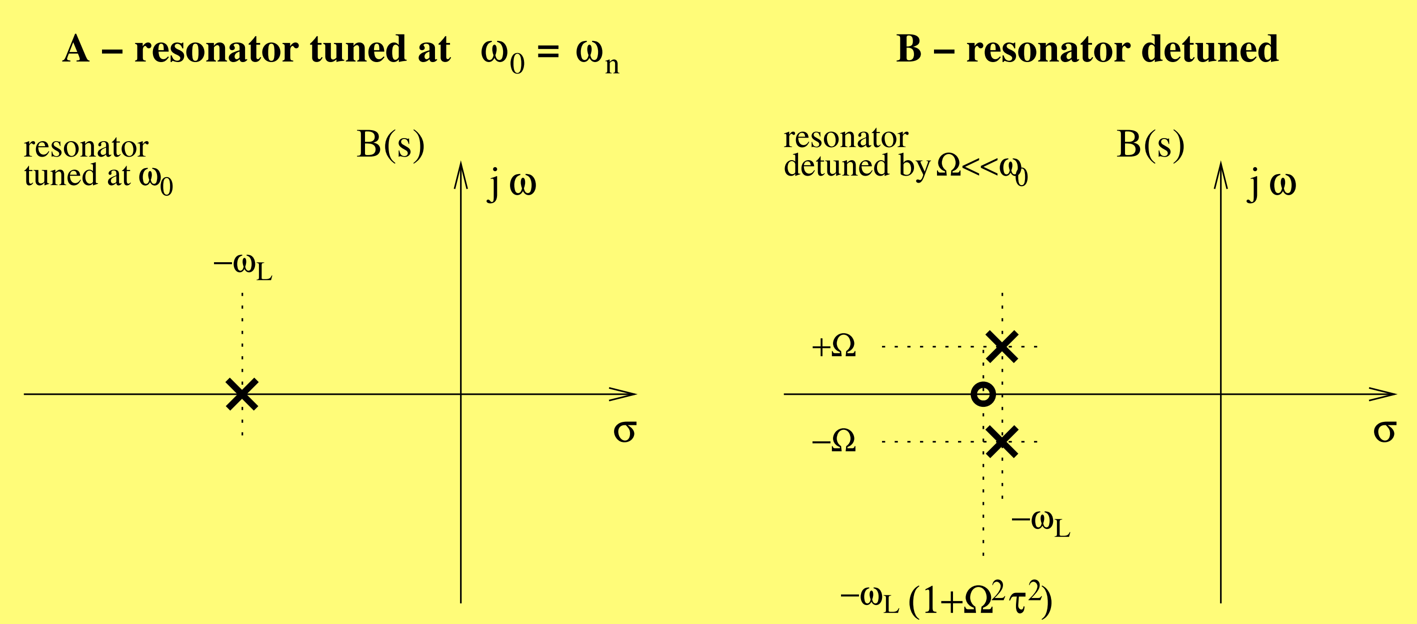
Voltage transfer function



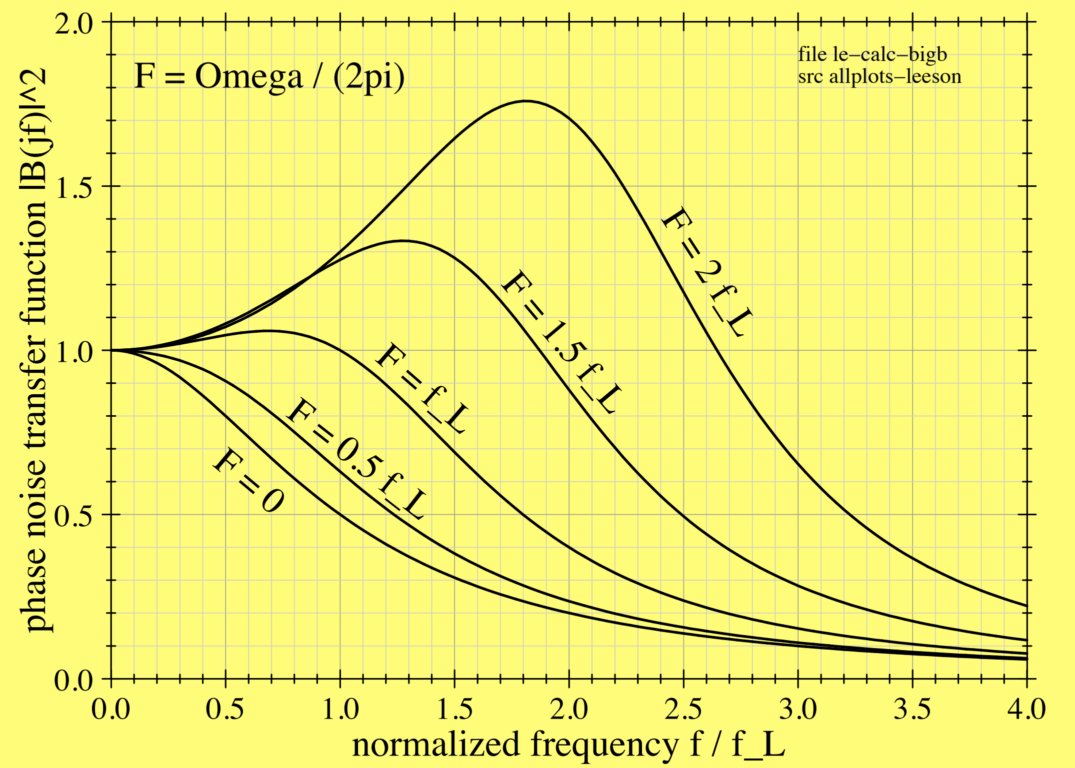
arg (β) has a lower slope at $\omega_1 \neq \omega_0$.

The local behavior (ω_1) is that of a lower-Q resonator.

Frequency response



diagonal terms

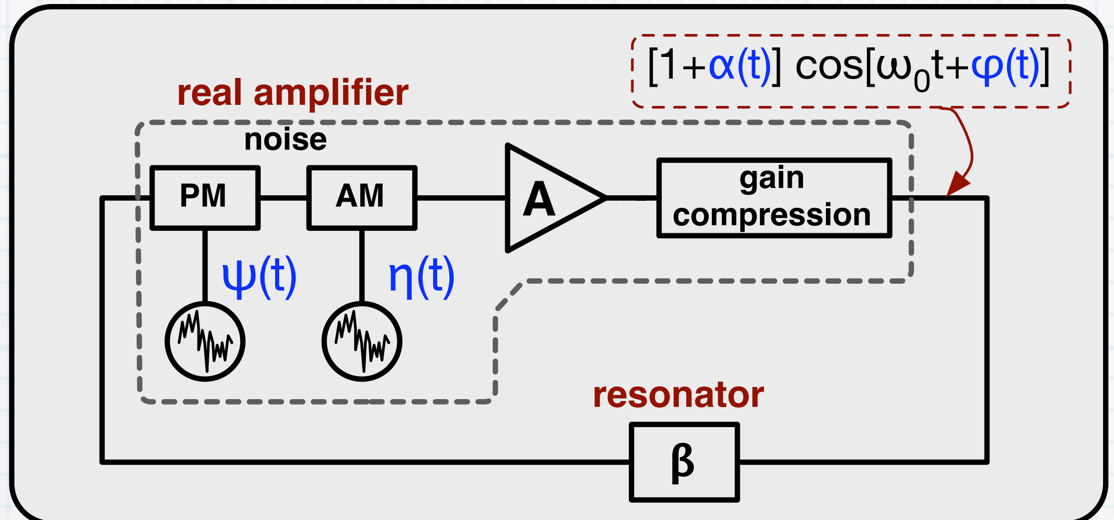


diagonal terms

$$[\mathrm{B}](s) = \begin{bmatrix} \dfrac{1}{\tau}\,\dfrac{s+\frac{1}{\tau}+\Omega^2\tau}{s^2+\frac{2}{\tau}s+\frac{1}{\tau^2}+\Omega^2} & \dfrac{-\Omega s}{s^2+\frac{2}{\tau}s+\frac{1}{\tau^2}+\Omega^2} \\ \dfrac{-\Omega s}{s^2+\frac{2}{\tau}s+\frac{1}{\tau^2}+\Omega^2} & \dfrac{1}{\tau}\,\dfrac{s+\frac{1}{\tau}+\Omega^2\tau}{s^2+\frac{2}{\tau}s+\frac{1}{\tau^2}+\Omega^2} \end{bmatrix}$$

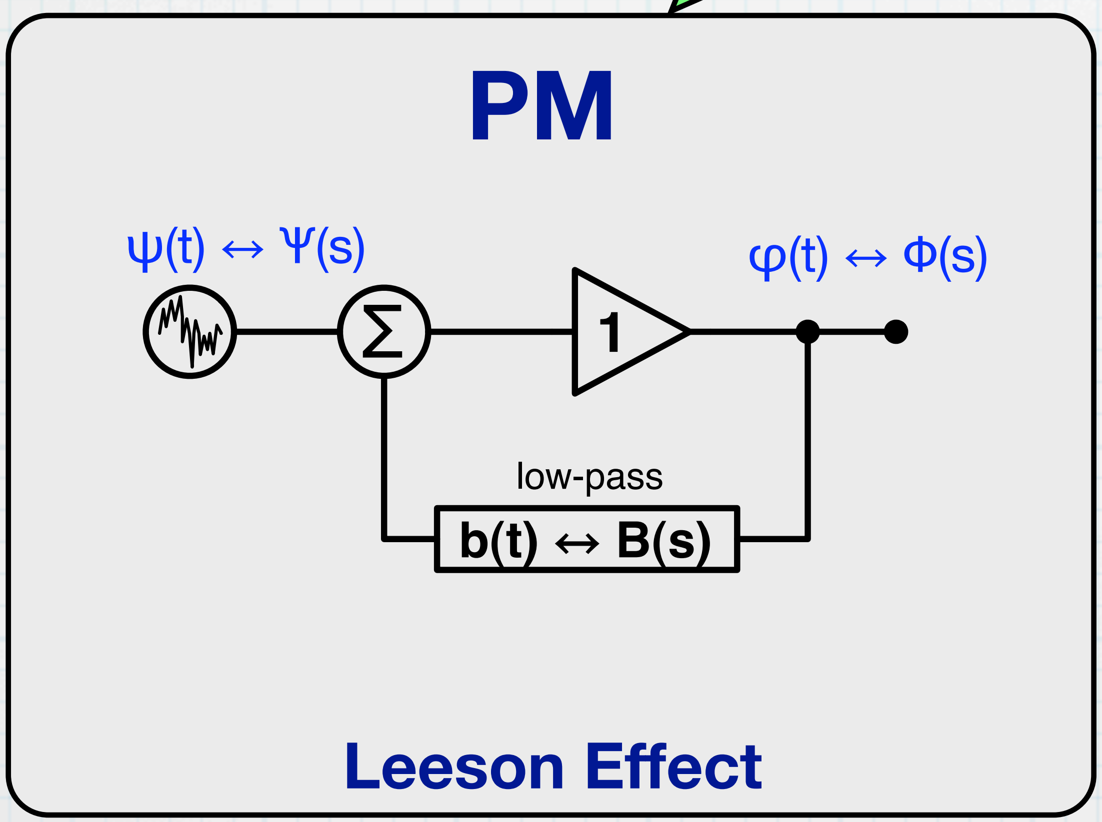
Leeson effect

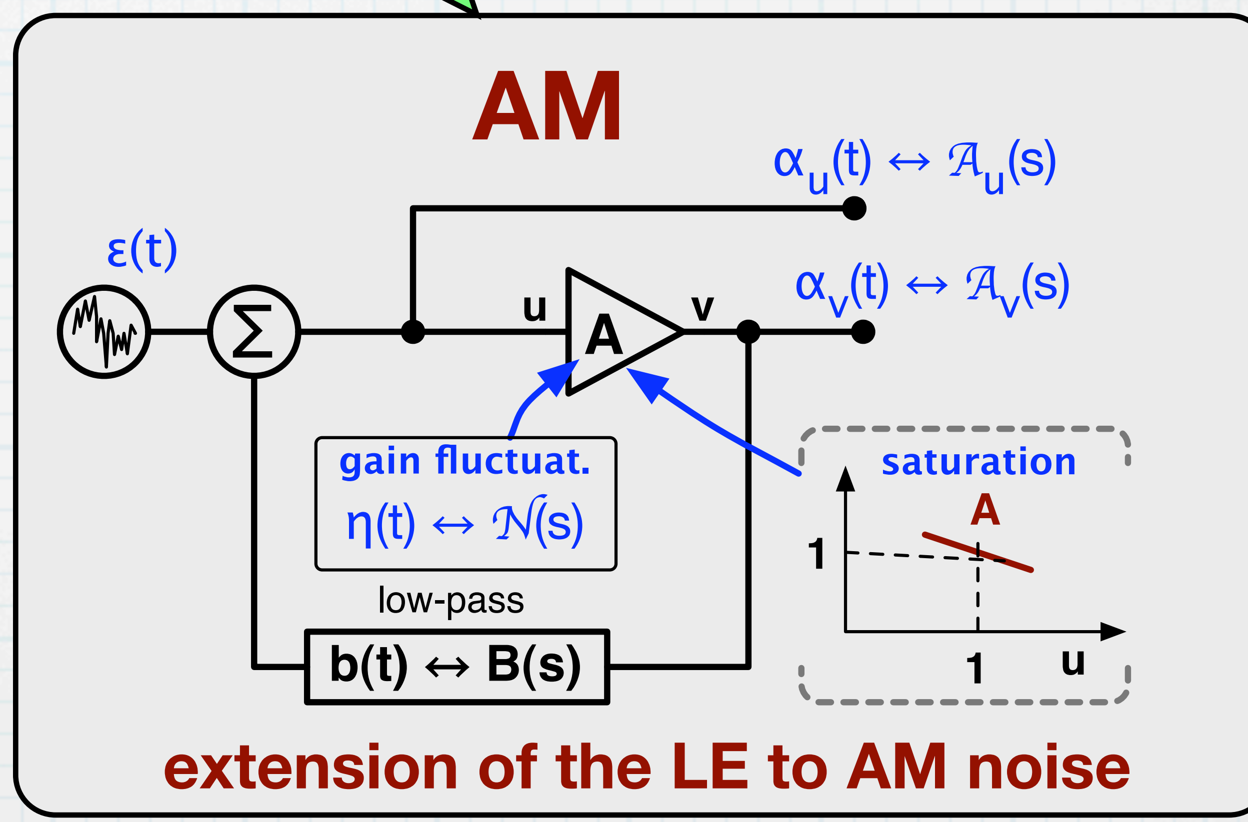
Low-pass representation of AM-PM noise



RF, µwaves or optics

low-pass equivalent



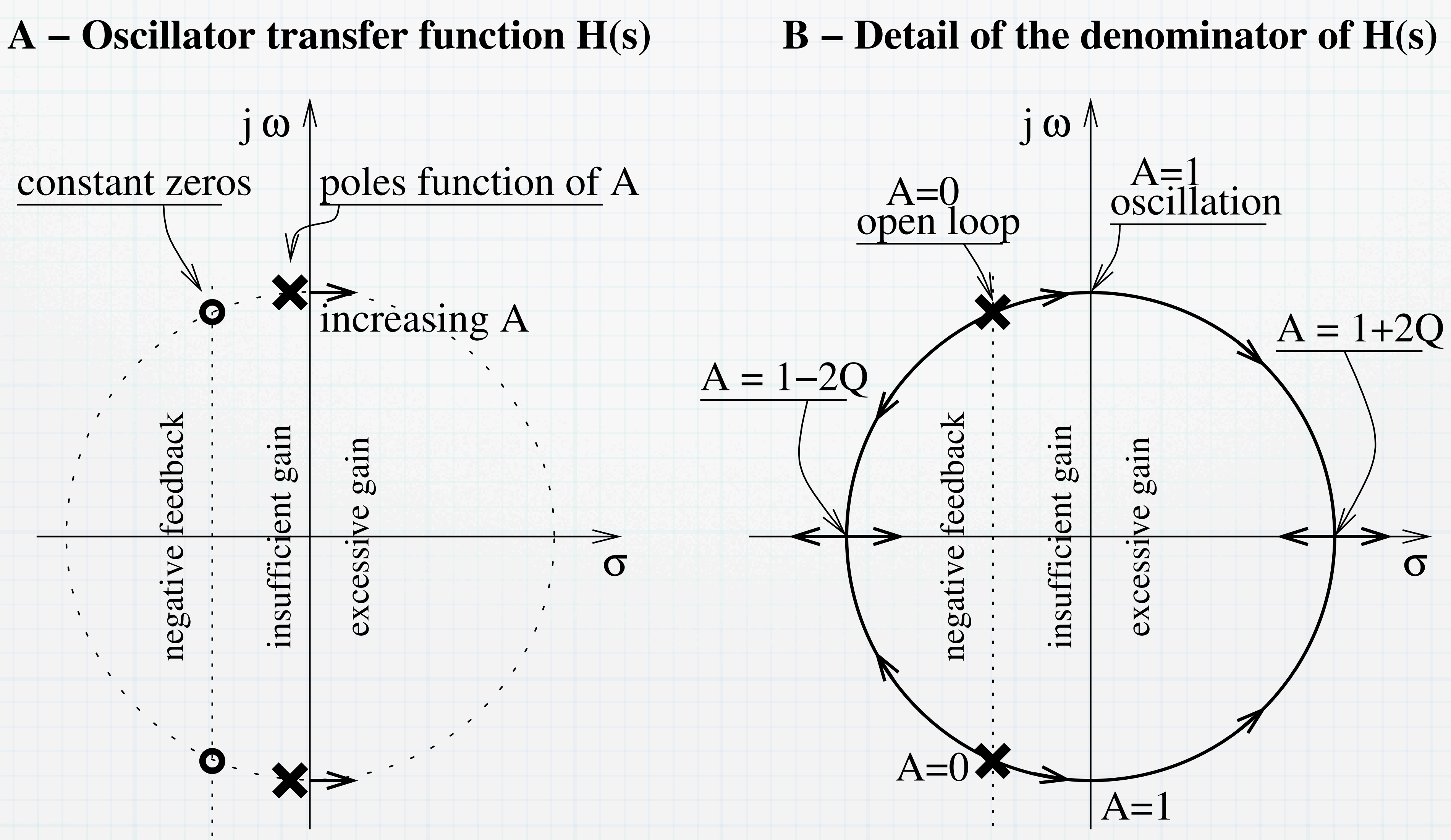


The amplifier
– "copies" the input phase to the out
– adds phase noise

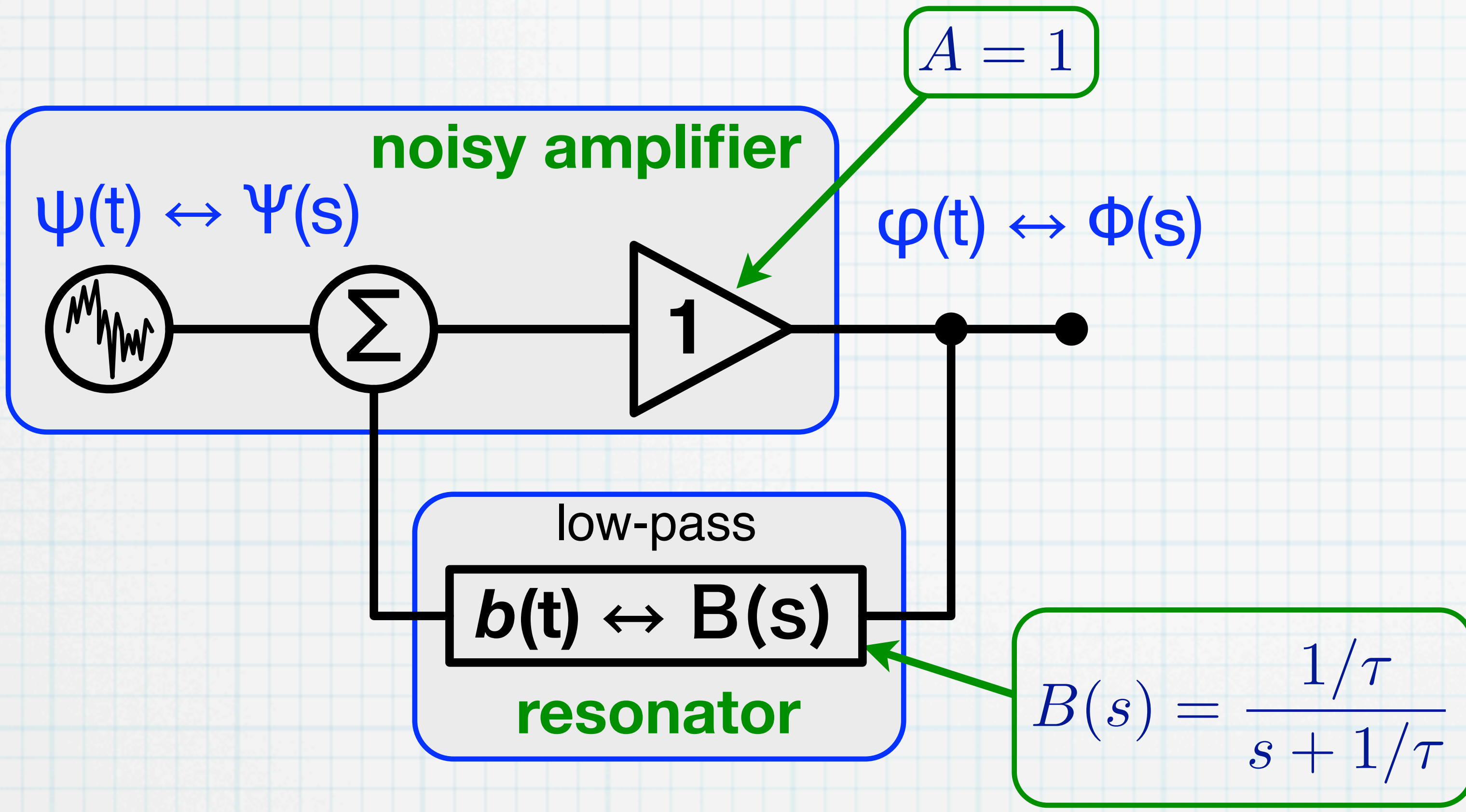
The amplifier
– compresses the amplitude
– adds amplitude noise

Effect of feedback

Oscillator transfer function (RF)



Leeson effect

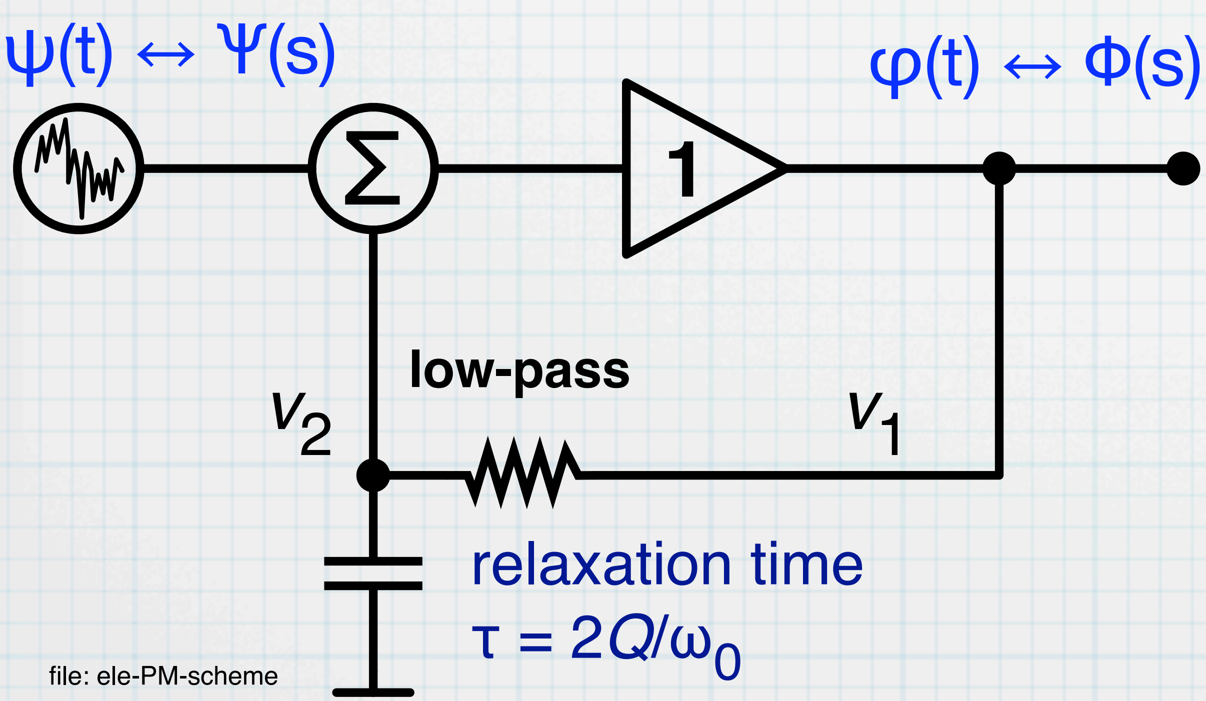


phase-noise transfer function

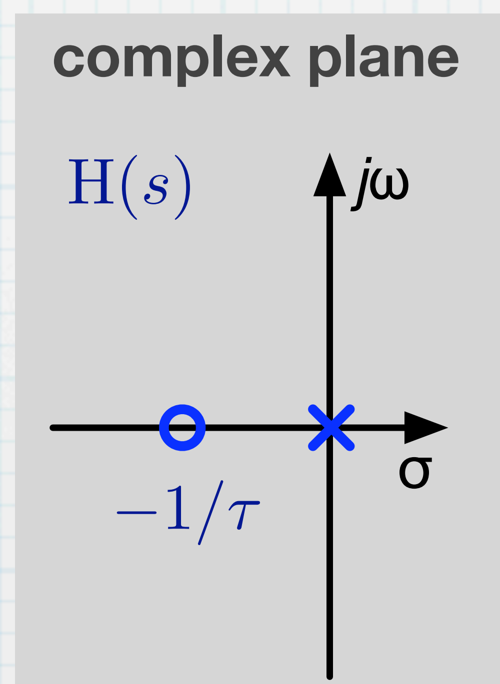
$$\mathrm{H}(s) = \frac{\Phi(s)}{\Psi(s)}$$ **definition**

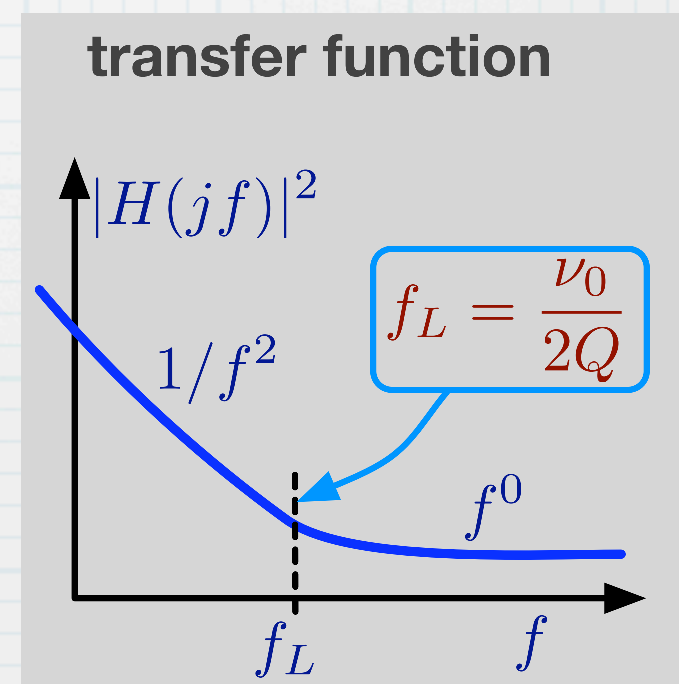
$$\mathrm{H}(s) = \frac{1}{1 + AB(s)}$$ **general feedback theory**

$$\mathrm{H}(s) = \frac{1 + s\tau}{s\tau}$$ **Leeson effect**

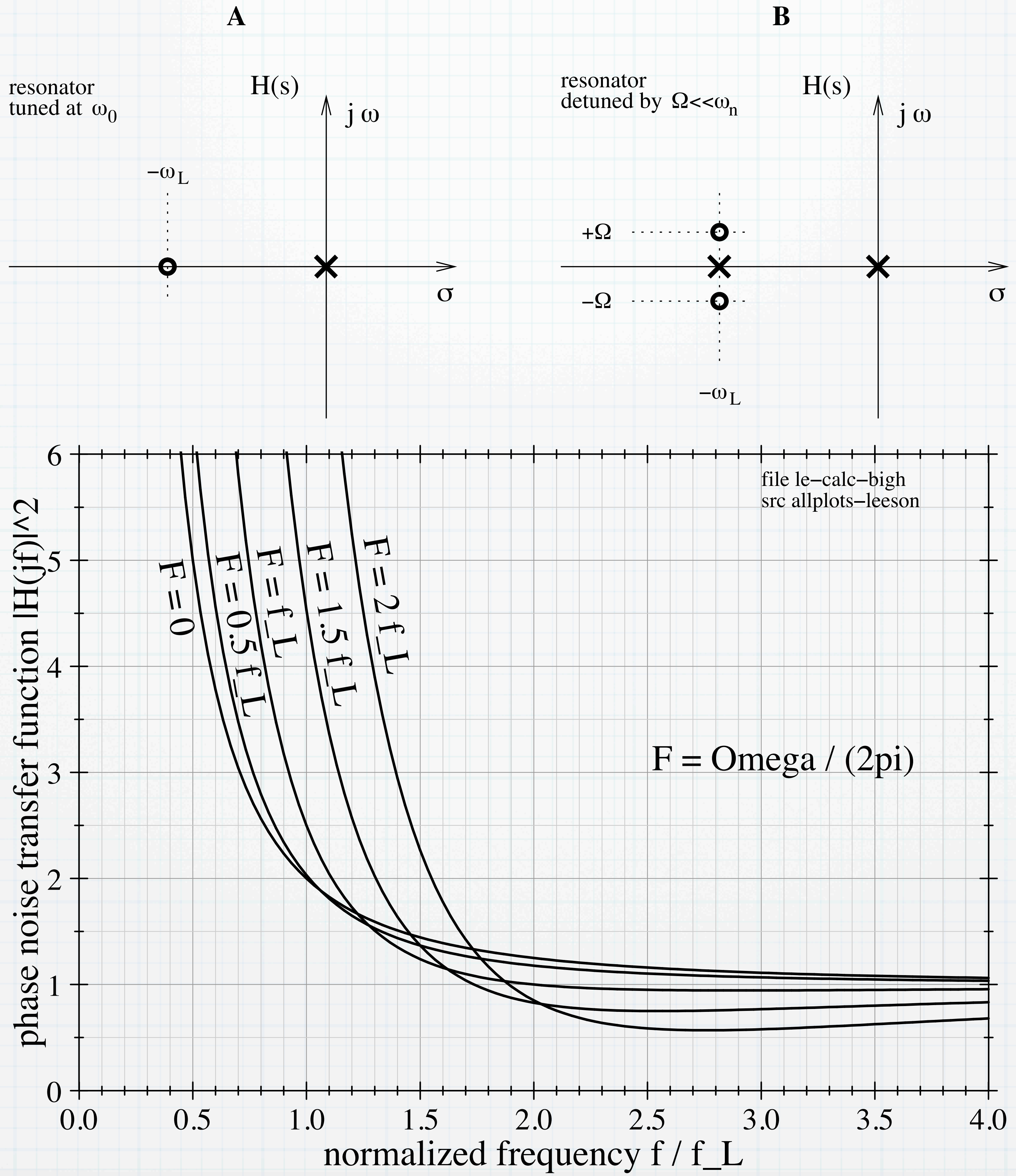


file: ele-PM-scheme



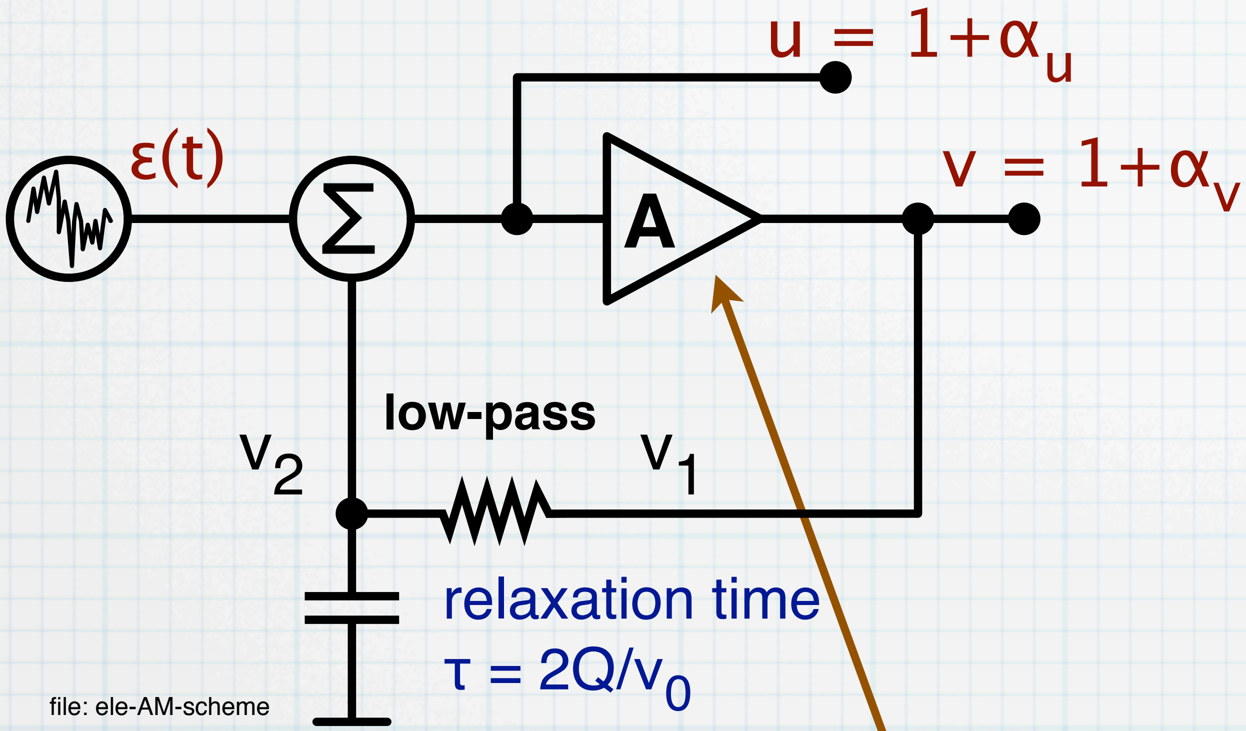


Detuned resonator



Low-pass model of amplitude (1)

First we need to relate the system restoring time τ_r to the relaxation time τ



simple feedback theory

$$u = \epsilon + v_2$$

$$v_2 = \frac{1}{\tau}\int (v_1 - v_2)\, dt$$

$$v_2 = u - \epsilon$$

$$v_1 = v = Au$$

$$u = \epsilon + \frac{1}{\tau}\int (A-1)u + \epsilon\, dt$$

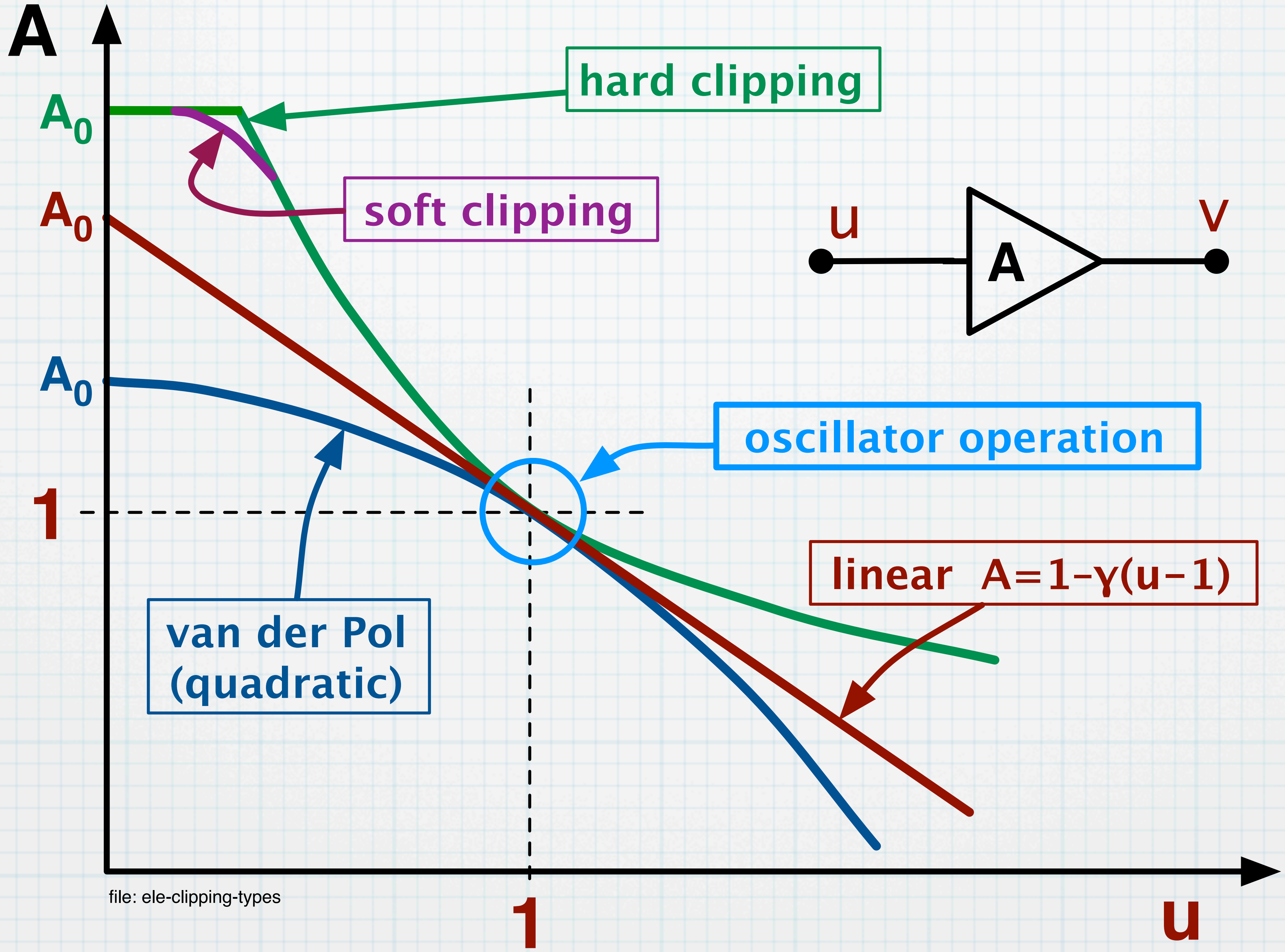
differential equation

$$\dot{u} - \frac{1}{\tau}\,(A-1)\,u = \frac{1}{\tau}\epsilon + \dot{\epsilon}$$

Gain compression is necessary for the oscillation amplitude to be stable

The Laplace / Heaviside formalism cannot be used because the amplifier is non-linear

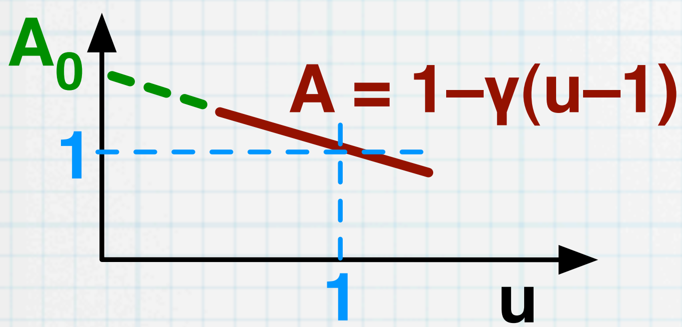
Common types of gain saturation



Gain compression is necessary for the oscillation amplitude to be stable

Low-pass model of amplitude (2)

Gain compression

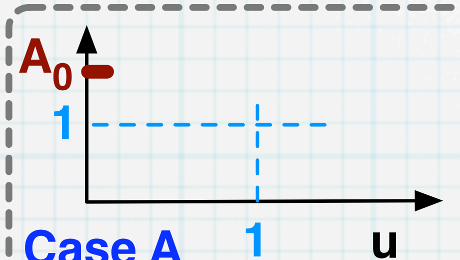


homogeneous differential equation

$$\dot{u} - \frac{1}{\tau}\,(A-1)\,u = 0$$

Three asymptotic cases

At low RF amplitude, let the gain be an arbitrary value denoted with A_0



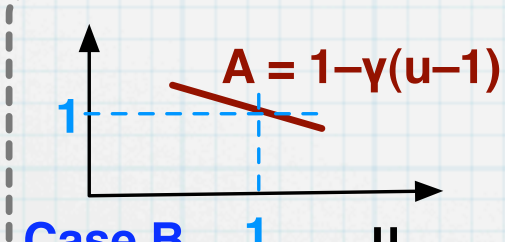
Case A

Startup: u→0, A → A_0 > 1

$$\dot{u} - \frac{1}{\tau}\,(A_0-1)\,u = 0 \quad\Rightarrow\quad \boxed{u = C_1 e^{(A_0-1)\,t/\tau}}$$

rising exponential

For small fluctuation of the stationary RF amplitude, the gain amplitude, the gain varies linearly with V



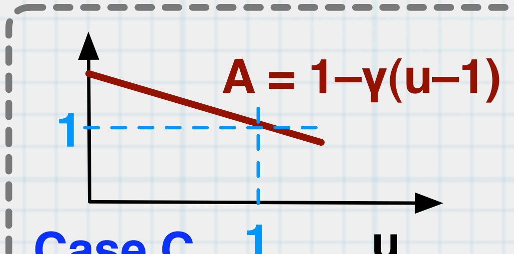
Case B

Regime: u→1, A = 1 – γ (u – 1)

$$\dot{u} + \frac{\gamma}{\tau}\,(u-1)\,u = 0 \quad\Rightarrow\quad \boxed{u = C_2 e^{-\gamma\,t/\tau}}$$

restoring time constant $\tau_r = \tau/\gamma$

Simplification: the gain varies linearly with V in all the input range

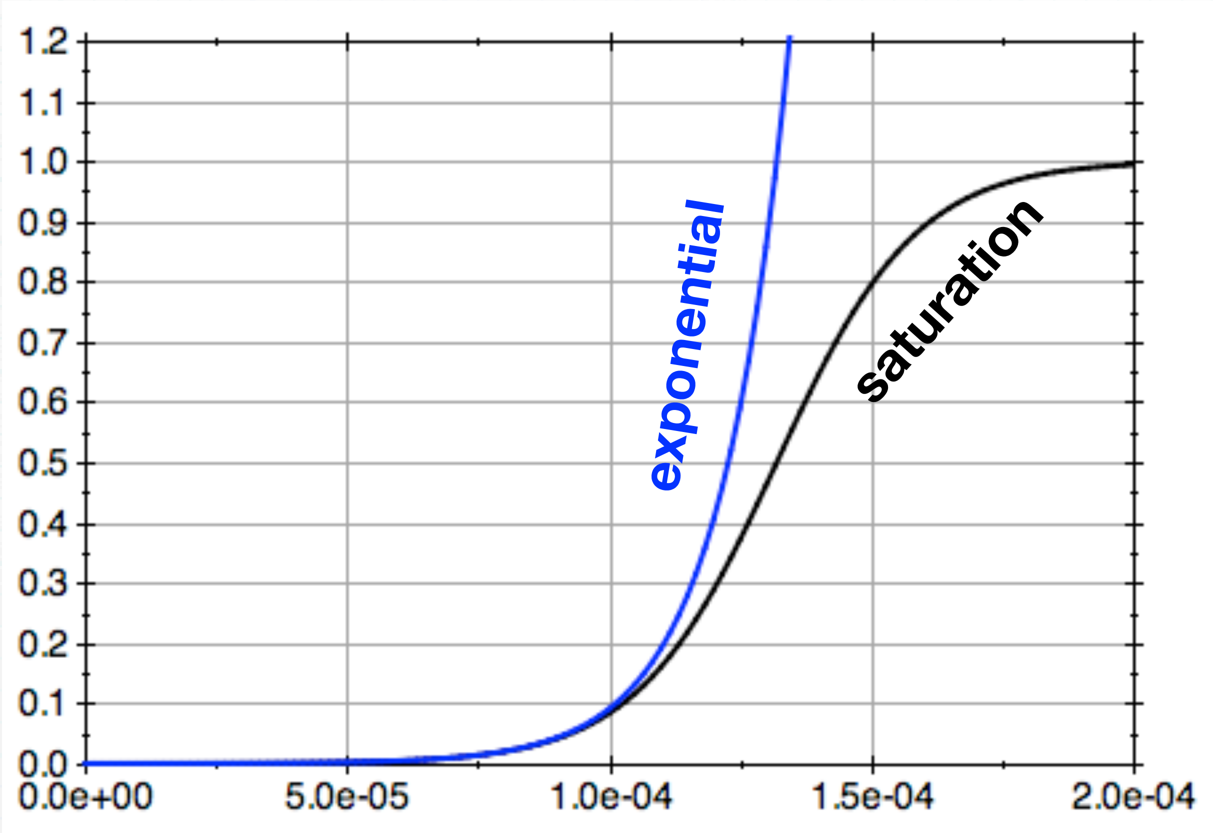


Case C

Linear gain: A = 1 – γ (u – 1)

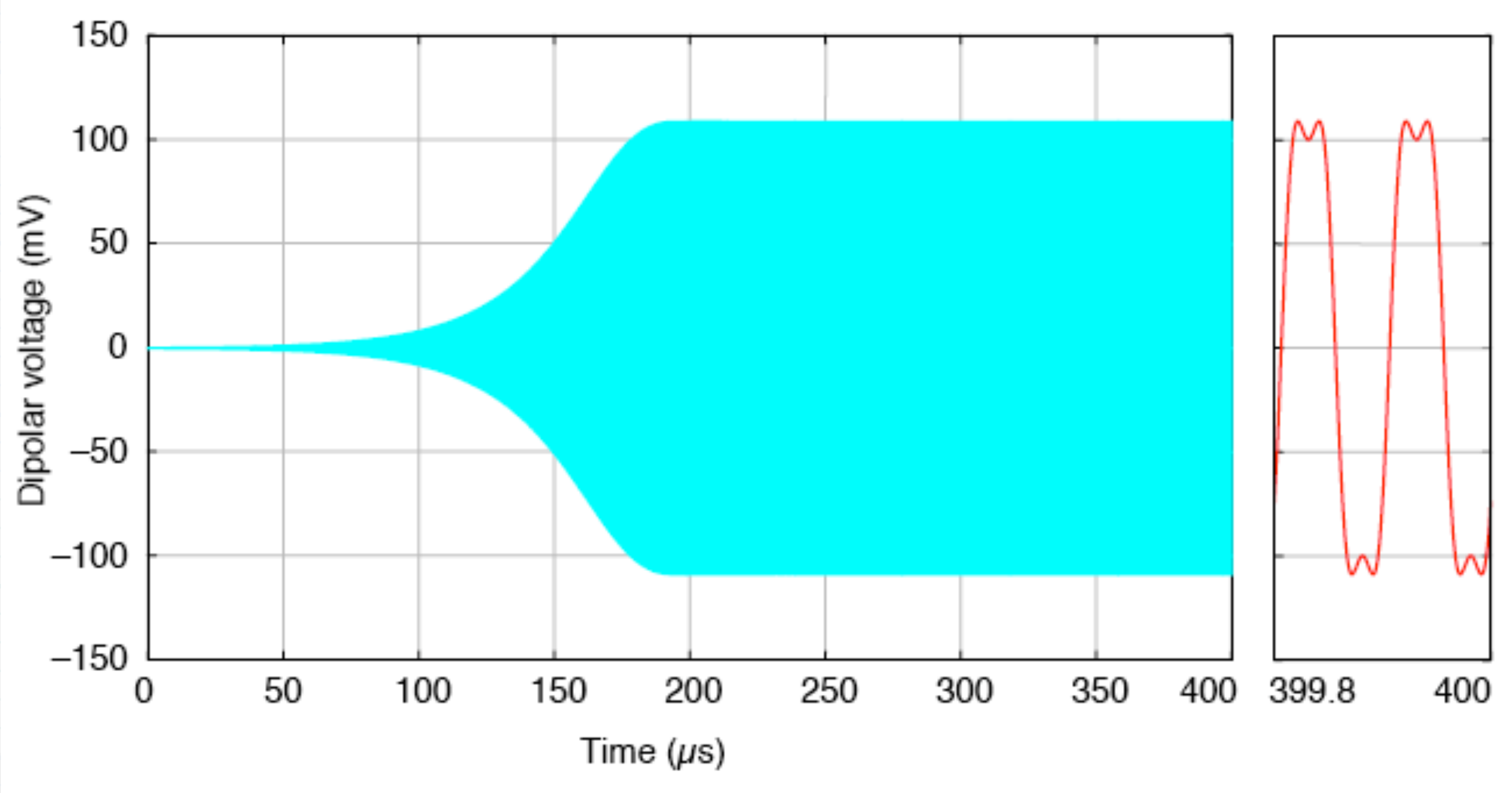
$$u = \frac{1}{\left(\frac{1}{u(0)} - 1\right) e^{-\gamma t/\tau} + 1}$$

Startup – analysis vs. simulation



analytical solution,
A = 1–γ(u–1)

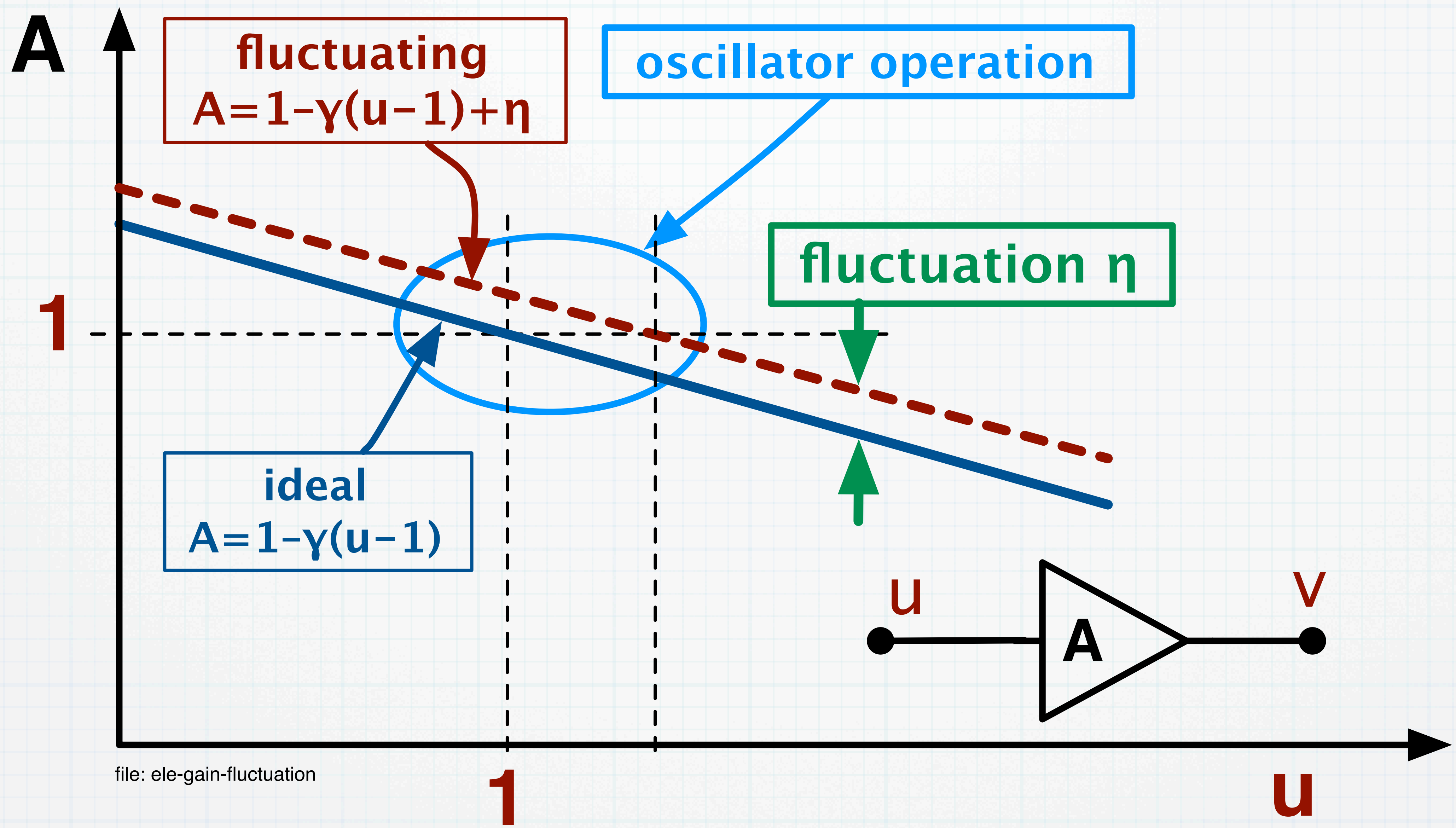
10 MHz oscillator
L = 1 mH
R = 125 Ω
Q ~ 503



van der Pol oscillator
simulated by RB

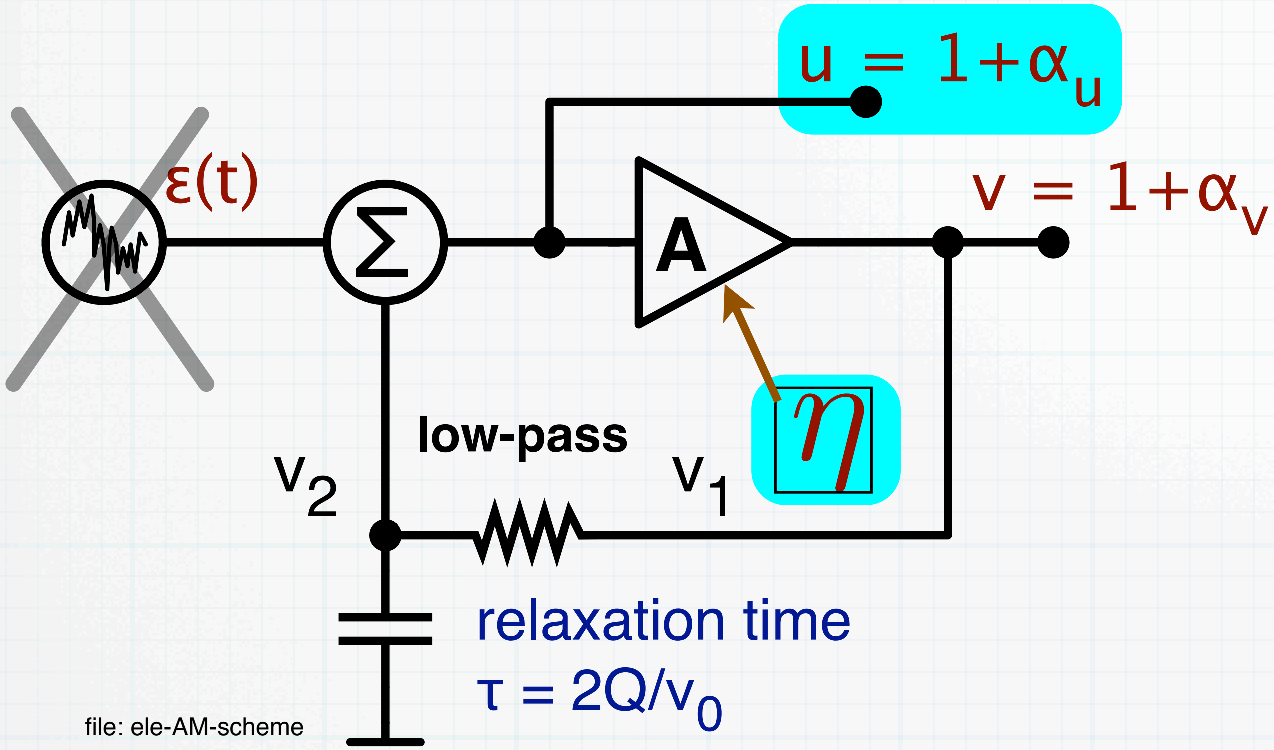
Rising exponential.
We find the same time constant –τ/γ

Gain fluctuations – definition



Gain compression is necessary for the oscillation amplitude to be stable

Gain fluctuations – output is u



$$\dot{u} = \frac{1}{\tau}(A-1)u$$ **non-linear equation**

$$A = 1 - \gamma(u-1) + \eta$$

$$\dot{u} + \frac{\gamma}{\tau}\underline{(u-1)}u = \frac{\eta}{\tau}u$$ **linearization for low noise**

$\dot{u} \to \dot{\alpha}_u$, $(u-1) \to \alpha_u$, $u \to 1$, $u \to 1$

$$\dot{\alpha}_u + \frac{\gamma}{\tau}\alpha_u = \frac{1}{\tau}\eta$$ **linearized equation**

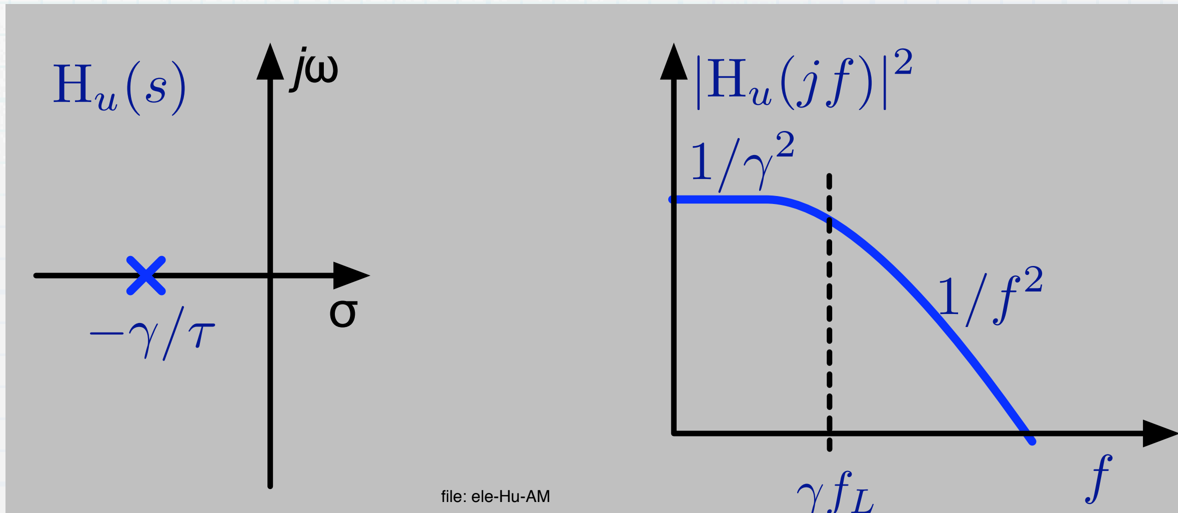
$$\left(s + \frac{\gamma}{\tau}\right)\mathcal{A}_u(s) = \frac{1}{\tau}\mathcal{N}(s)$$ **Laplace transform**

Linearize for low noise and use the Laplace transforms

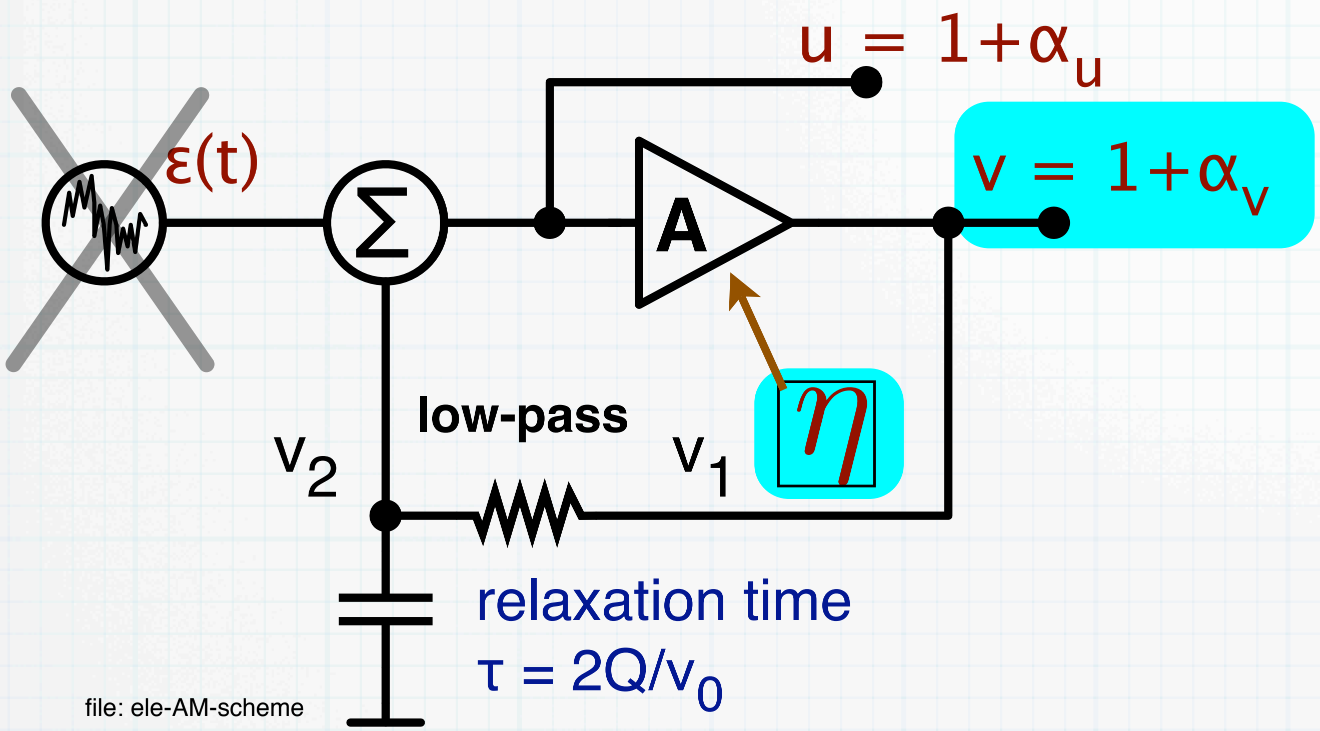
$$\alpha_u(t) \leftrightarrow \mathcal{A}_u(s) \quad \text{and} \quad \eta(t) \leftrightarrow \mathcal{N}(s)$$

$$\mathrm{H}_u(s) = \frac{\mathcal{A}_u(s)}{\mathcal{N}(s)}$$ **definition**

$$\mathrm{H}_u(s) = \frac{1/\tau}{s + \gamma/\tau}$$ **result**



Gain fluctuations – output is v



$$\left(s+\frac{\gamma}{\tau}\right)\mathcal{A}_u(s)=\frac{1}{\tau}\mathcal{N}(s)$$ **starting equation**

$$\mathcal{A}_u(s)=\frac{\mathcal{A}_v(s)-\mathcal{N}(s)}{1-\gamma}$$

$$\left(s+\frac{\gamma}{\tau}\right)\mathcal{A}_v(s)=\left(s+\frac{1}{\tau}\right)\mathcal{N}(s)$$

$$\mathrm{H}(s)=\frac{\mathcal{A}_v(s)}{\mathcal{N}(s)}$$ **definition**

$$\mathrm{H}(s)=\frac{s+1/\tau}{s+\gamma/\tau}$$ **result**

boring algebra relates α_v to α_u

$$v=Au$$

$$A=-\gamma(u-1)+1+\eta$$

$$v=[-\gamma(u-1)+1+\eta]\,u$$

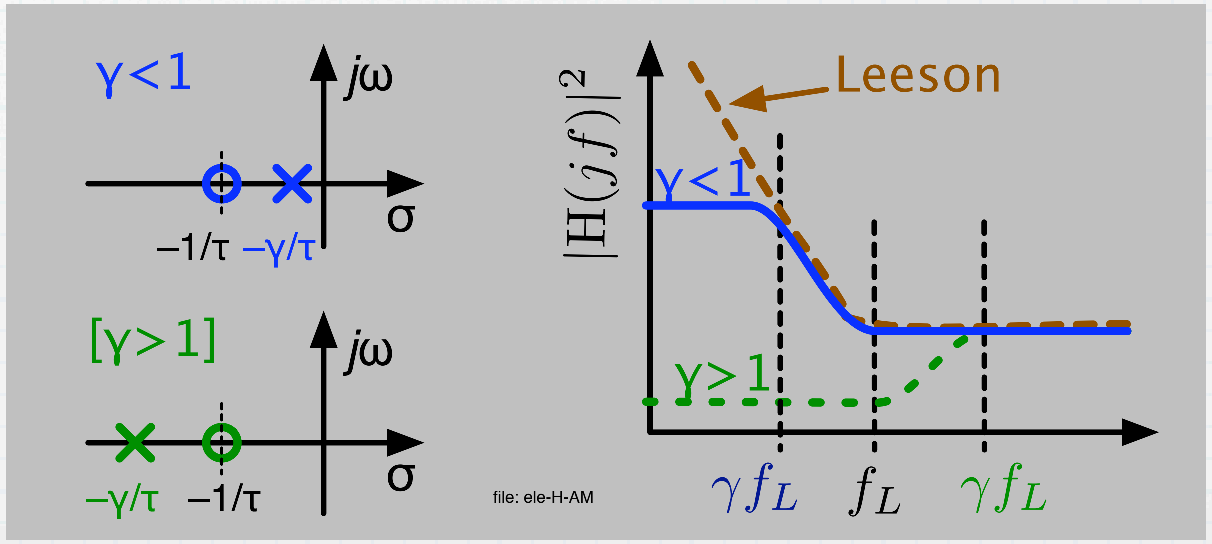
$$v=[-\gamma\alpha_u+1+\eta]\;[1+\alpha_u]$$

$$\cancel{1}+\alpha_v=\cancel{1}+\eta-\gamma\alpha_u+\alpha_u-\cancel{\alpha_u\eta}-\cancel{\gamma\alpha_u^2}$$

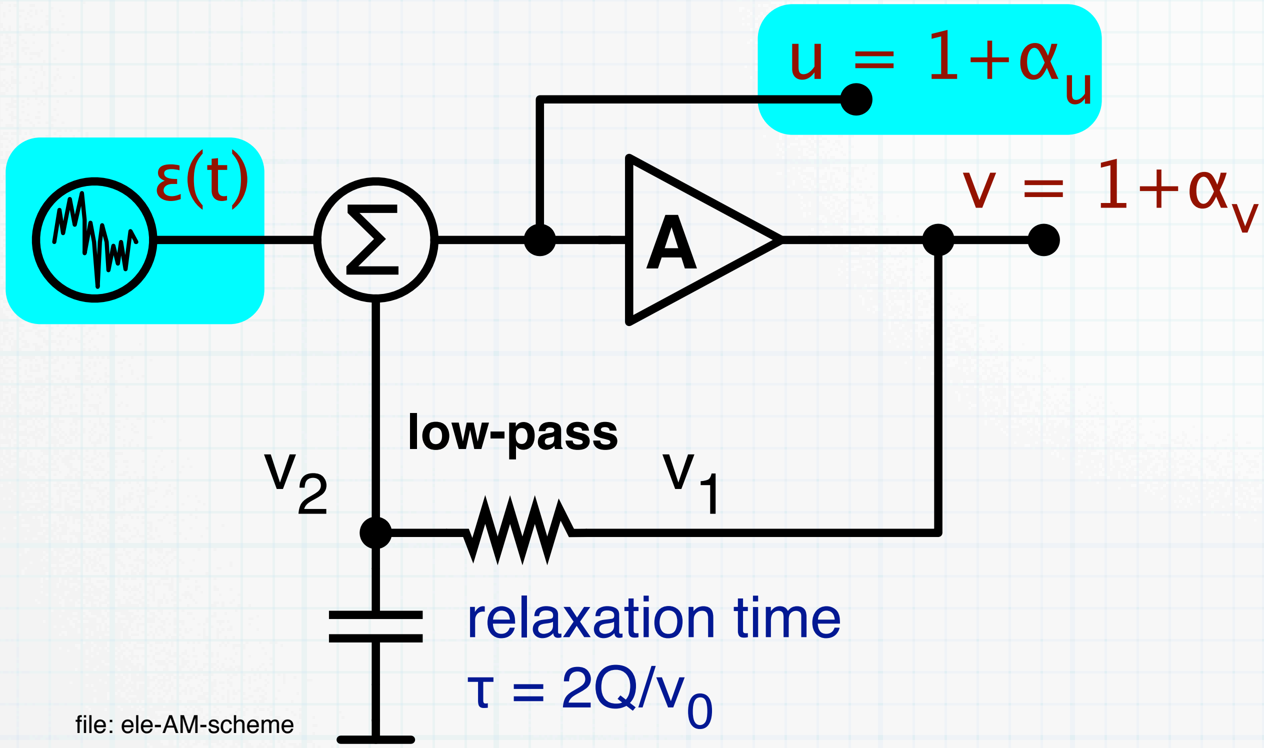
linearization for low noise

$$\alpha_v=(1-\gamma)\alpha_u+\eta$$

$$\alpha_u=\frac{\alpha_v-\eta}{1-\gamma}$$



Additive noise – output is u



$$\dot{u} = \frac{1}{\tau}(A-1)u + \dot{\epsilon} + \frac{1}{\tau}\epsilon$$

$$A = 1 - \gamma(u-1)$$

non-linear equation

$$\underbrace{\dot{u}}_{\dot{\alpha}_u} + \frac{\gamma}{\tau}\underbrace{(u-1)}_{\alpha_u}\underbrace{u}_{1} = \dot{\epsilon} + \frac{1}{\tau}\epsilon$$

lineariz. for low noise

$$\dot{\alpha}_u + \frac{\gamma}{\tau}\alpha_u = \dot{\epsilon} + \frac{1}{\tau}\epsilon$$

linearized equation

$$\left(s + \frac{\gamma}{\tau}\right)\mathcal{A}_u(s) = \left(s + \frac{1}{\tau}\right)\mathcal{E}(s)$$

Laplace transform

Linearize for low noise and use the Laplace transforms

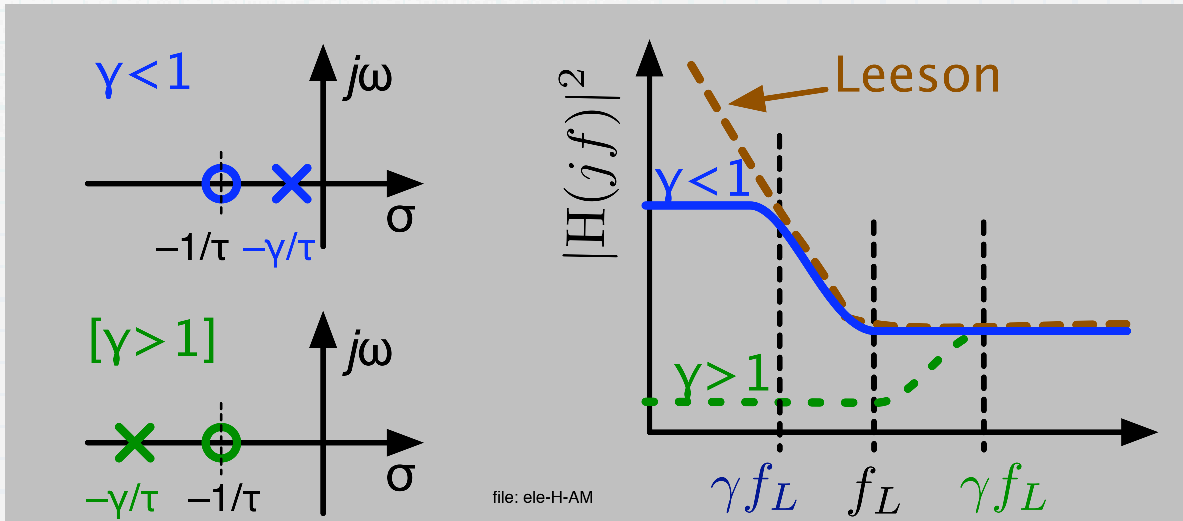
$$\alpha_u(t) \leftrightarrow \mathcal{A}_u(s) \quad \text{and} \quad \epsilon(t) \leftrightarrow \mathcal{E}(s)$$

$$\mathrm{H}_u(s) = \frac{\mathcal{A}_u(s)}{\mathcal{E}(s)}$$

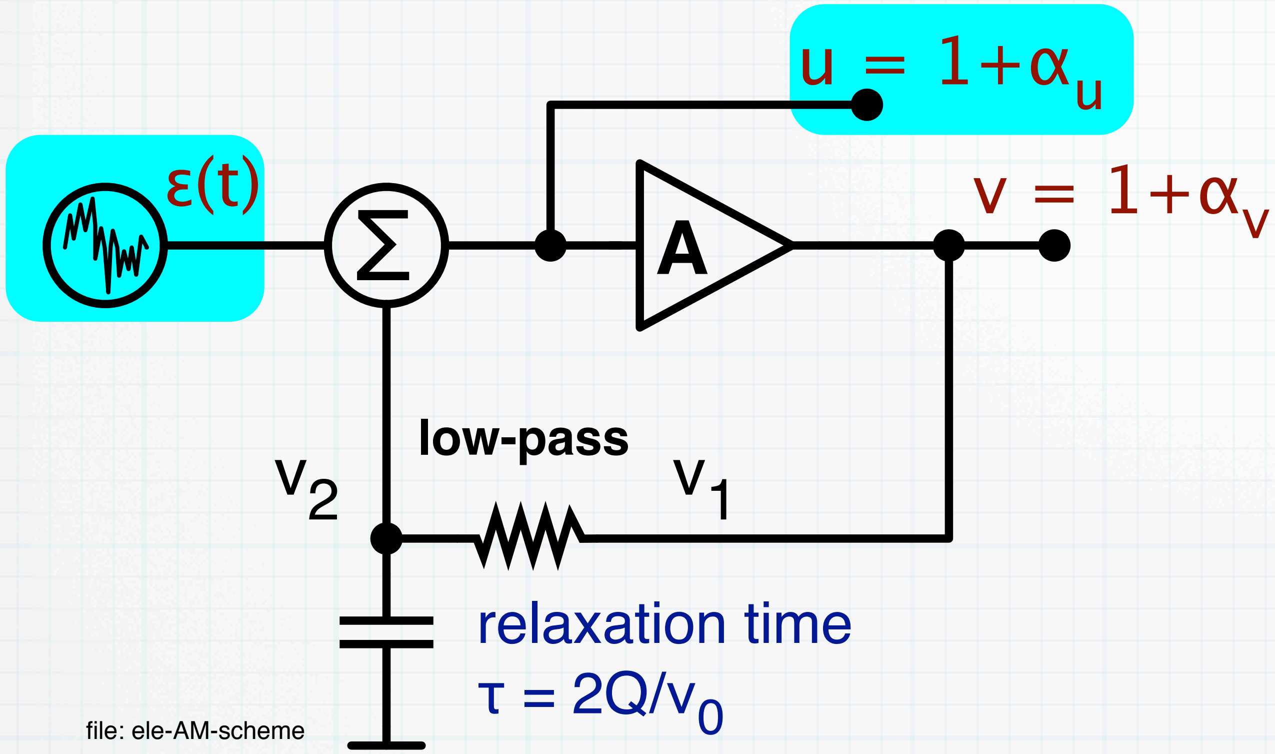
definition

$$\mathrm{H}_u(s) = \frac{s + 1/\tau}{s + \gamma/\tau}$$

result



Additive noise – output is v



$$\dot{\alpha}_u + \frac{\gamma}{\tau}\alpha_u = \dot{\epsilon} + \frac{1}{\tau}\epsilon$$

linearized equation

$$\alpha_u = \alpha_v/(1-\gamma)$$

$$\frac{1}{1-\gamma}\left(\dot{\alpha}_v + \frac{\gamma}{\tau}\alpha_v\right) = \dot{\epsilon} + \frac{1}{\tau}\epsilon$$

$$\frac{1}{1-\gamma}\left(s + \frac{\gamma}{\tau}\right)\mathcal{A}_v(s) = \left(s + \frac{1}{\tau}\right)\mathcal{E}(s)$$

Laplace transform

$$\mathrm{H}(s) = \frac{\mathcal{A}_v(s)}{\mathcal{E}(s)}$$

definition

$$\mathrm{H}(s) = (1-\gamma)\,\frac{s + 1/\tau}{s + \gamma/\tau}$$

result

boring algebra relates α’ to α

$$v = Au$$

$$A = 1 - \gamma(u-1)$$

$$v = [1 - \gamma(u-1)]\,u$$

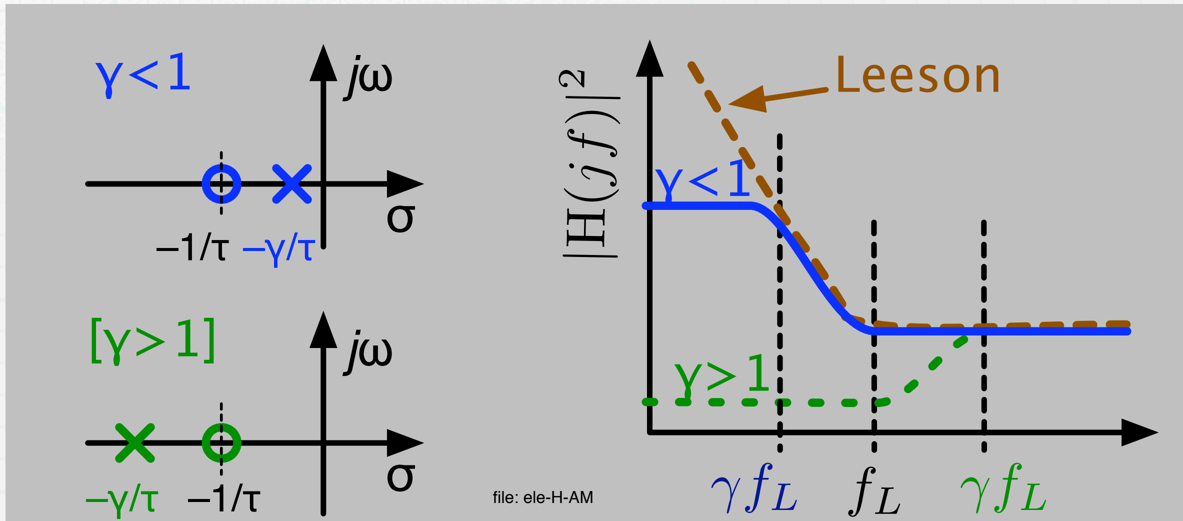
$$1 + \alpha_v = [1 - \gamma\alpha_u]\,[1 + \alpha_u]$$

$$\cancel{1} + \alpha_v = \cancel{1} + \alpha_u - \gamma\alpha_u - \cancel{\gamma\alpha_u^2}$$

linearization for low noise

$$\alpha_v = (1-\gamma)\alpha_u$$

$$\alpha_u = \frac{\alpha_v}{1-\gamma}$$

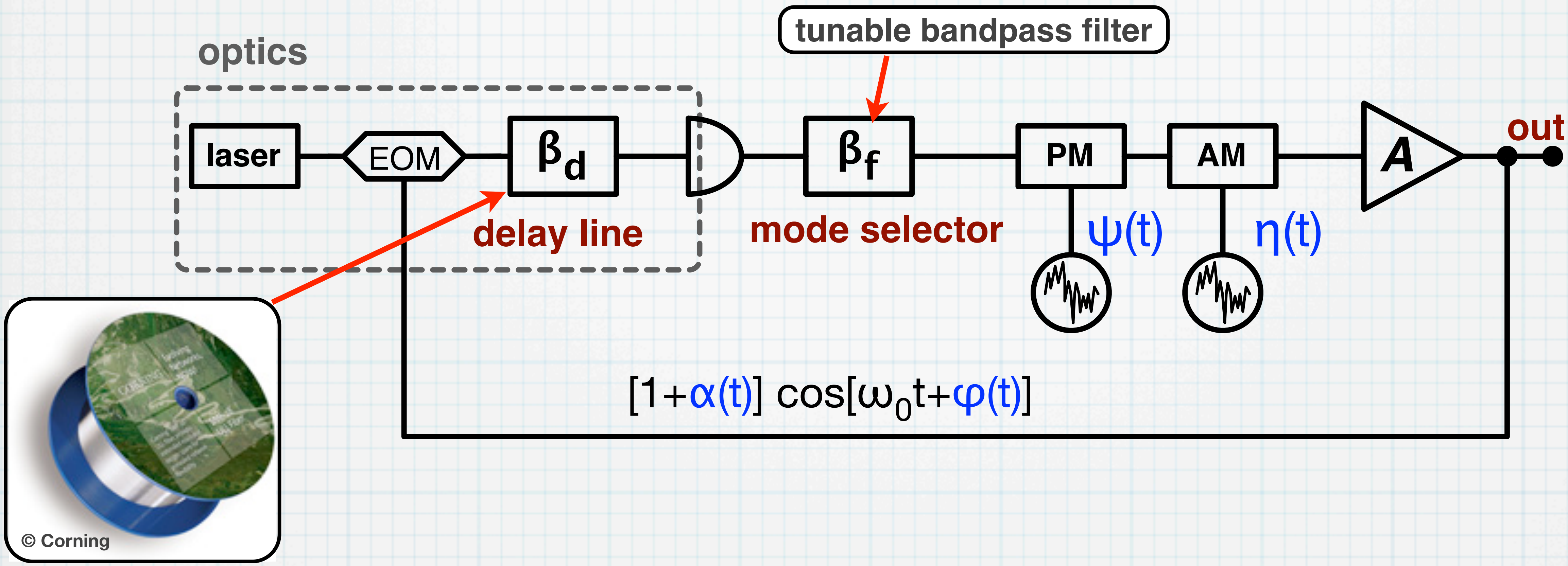


AM-PM noise coupling

Will be shown in the case of the delay-line oscillator

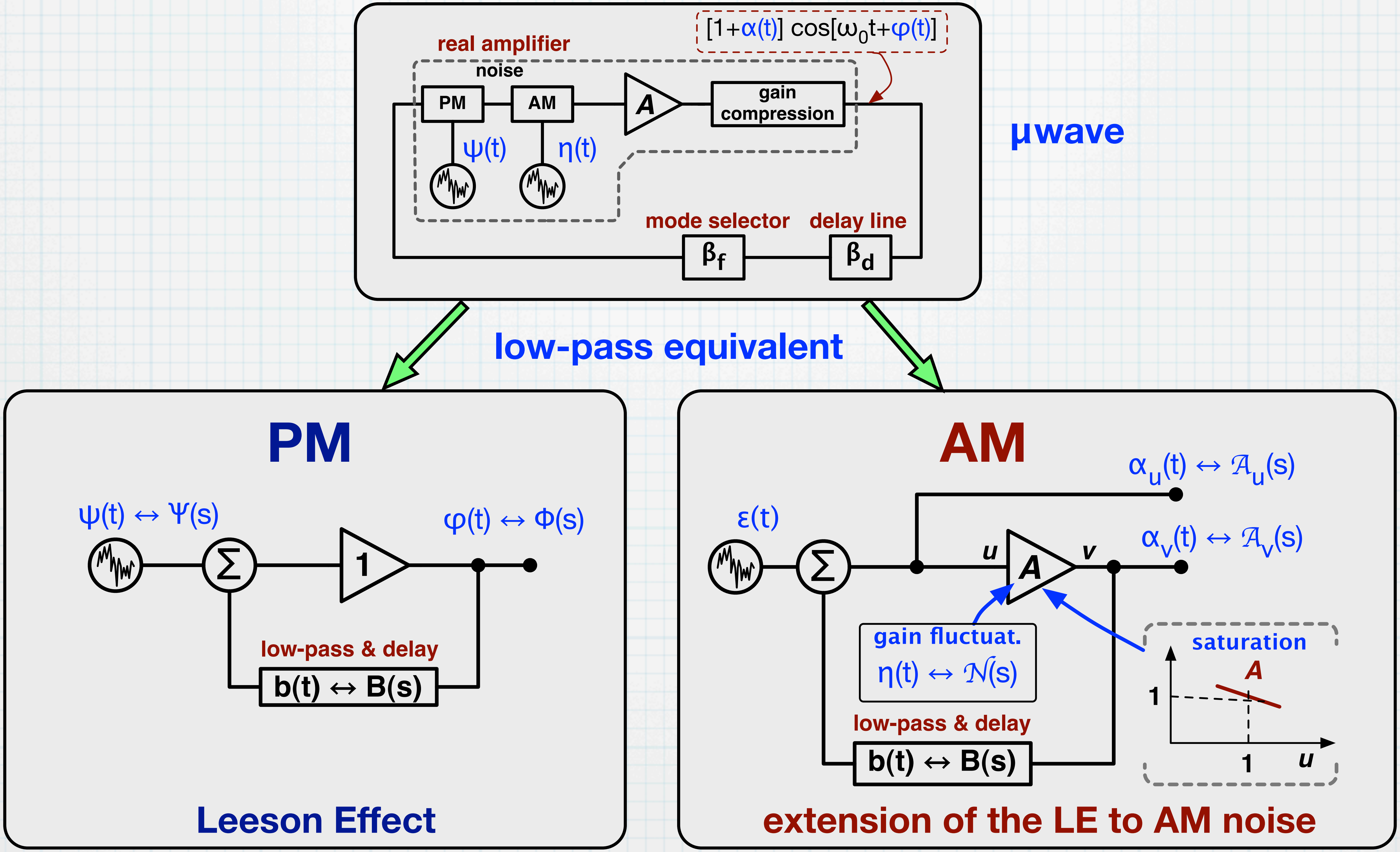
Leeson effect
in delay-line oscillators

Motivations



- Potential for very-low phase noise in the 100 Hz – 1 MHz range
- Invented at JPL, X. S. Yao & L. Maleki, JOSAB 13(8) 1725–1735, Aug 1996
- Early attempt of noise modeling, S. Römisch & al., IEEE T UFFC 47(5) 1159–1165, Sep 2000
- PM-noise analysis, E. Rubiola, *Phase noise and frequency stability in oscillators*, Cambridge 2008 [Chapter 5]
- **Since, no progress in the analysis of noise at system level**
- **Nobody reported on the consequences of AM noise**

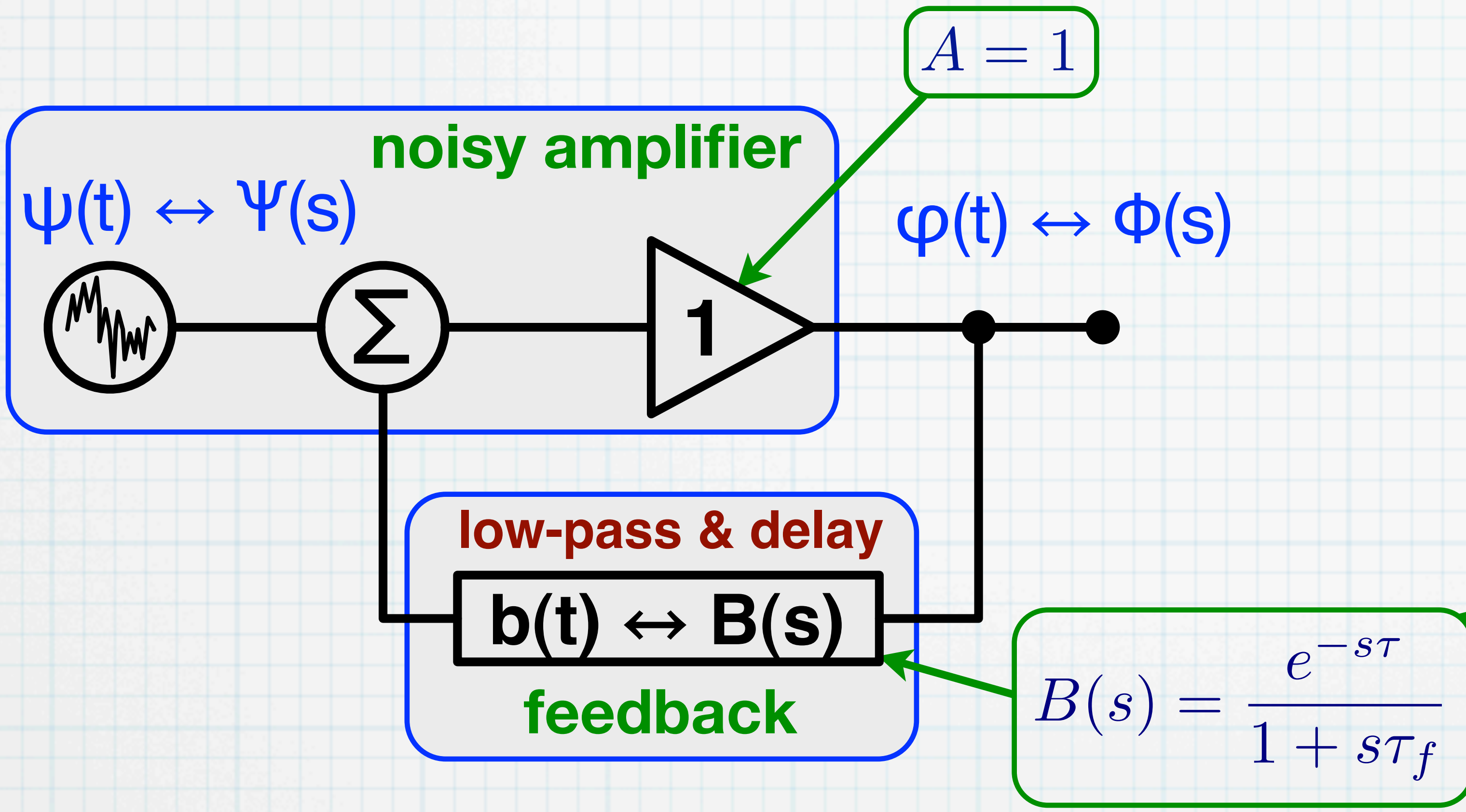
Low-pass representation of AM-PM noise



The amplifier
– “copies” the input phase to the out
– adds phase noise

The amplifier
– compresses the amplitude
– adds amplitude noise

Leeson effect



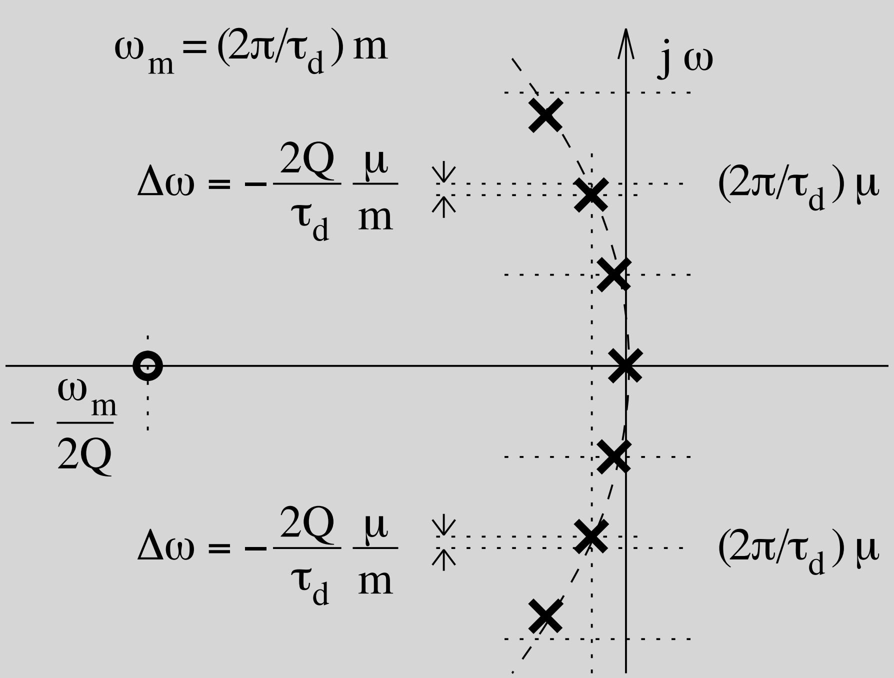
phase-noise transfer function

$$\mathrm{H}(s) = \frac{\Phi(s)}{\Psi(s)}$$ **definition**

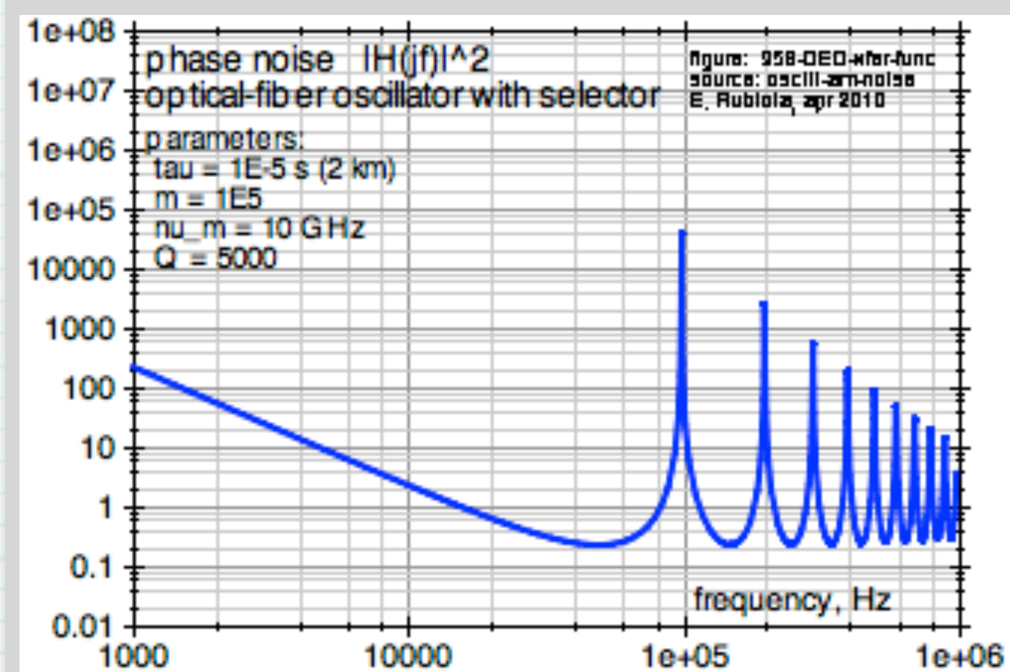
$$\mathrm{H}(s) = \frac{1}{1 + AB(s)}$$ **general feedback theory**

$$\mathrm{H}(s) = \frac{1 + s\tau_f}{1 + s\tau_f - e^{-s\tau}}$$ **Leeson effect**

H, complex plane

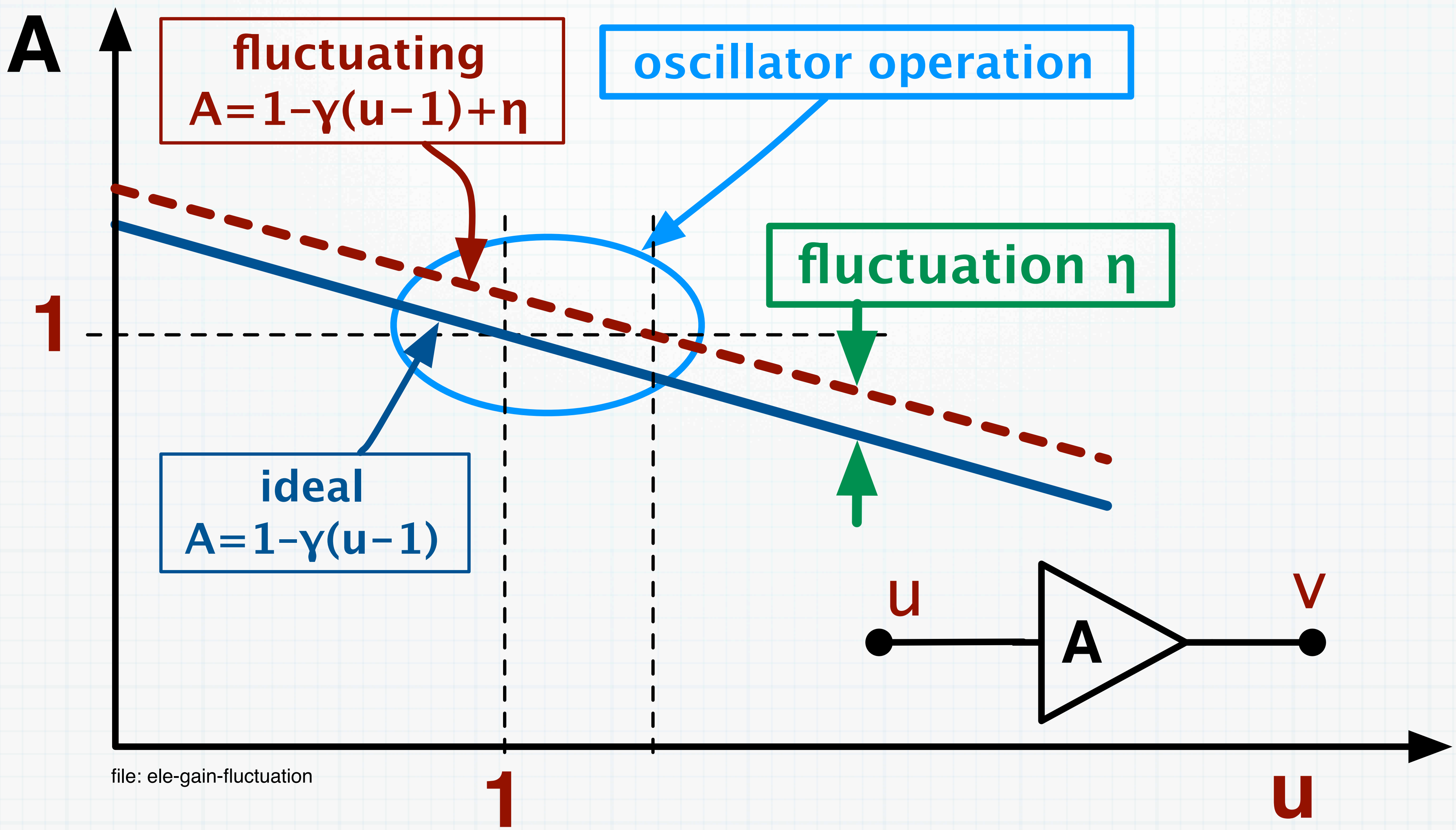


transfer function $|H|^2$



E. Rubiola, Phase noise and frequency stability in oscillators, Cambridge 2008

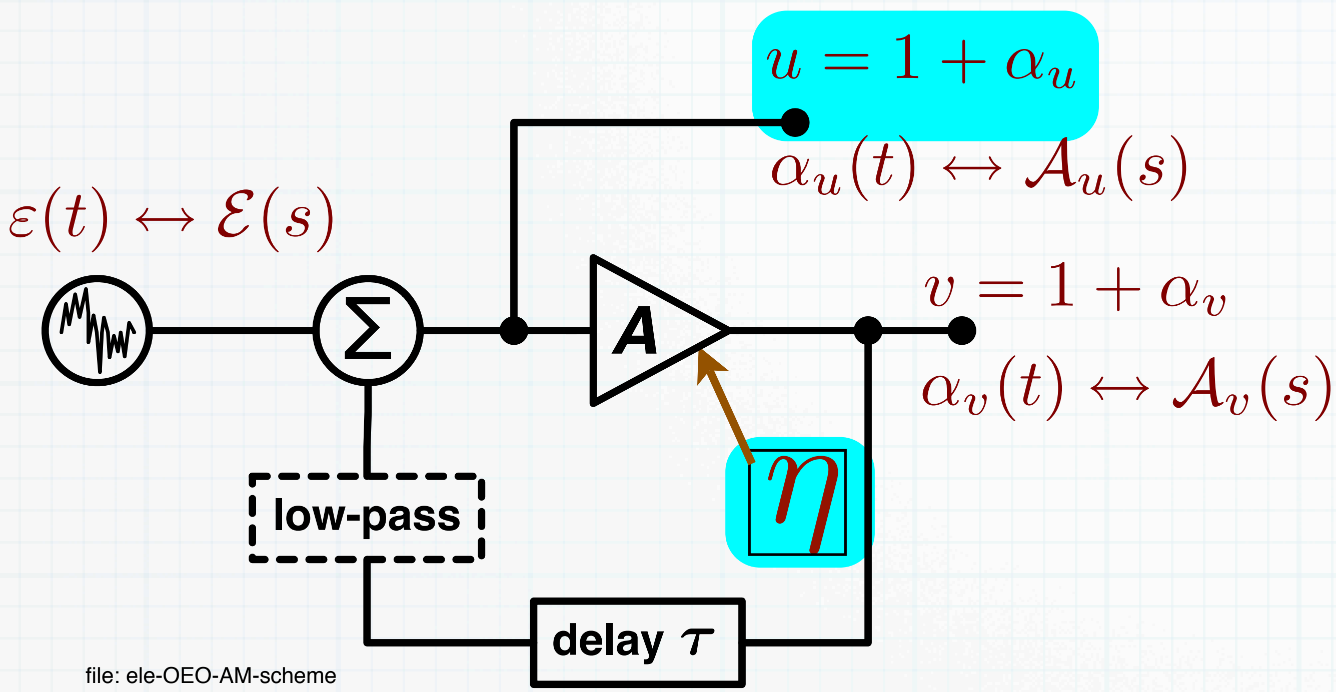
Gain fluctuations – definition



Gain compression is necessary for the oscillation amplitude to be stable

E. Rubiola & R. Brendel, arXiv:1004.5539v1 [physics.ins-det]

Gain fluctuations – output is *u*(*t*)



The low-pass has only 2nd order effect on AM

Linearize for low noise and use the Laplace transform

$$\alpha_u(t) \leftrightarrow \mathcal{A}_u(s) \quad \text{and} \quad \eta(t) \leftrightarrow \mathcal{N}(s)$$

$$\mathrm{H}(s) = \frac{\mathcal{A}_u(s)}{\mathcal{N}(s)}$$ definition

result

$$\mathrm{H}(s) = \frac{1}{1-(1-\gamma)e^{-s\tau}}$$

non-linear equation

$$u = A(t-\tau)\,u(t-\tau)$$

$$A = 1-\gamma(u-1)+\eta$$

use u=α+1, expand and linearize for low noise

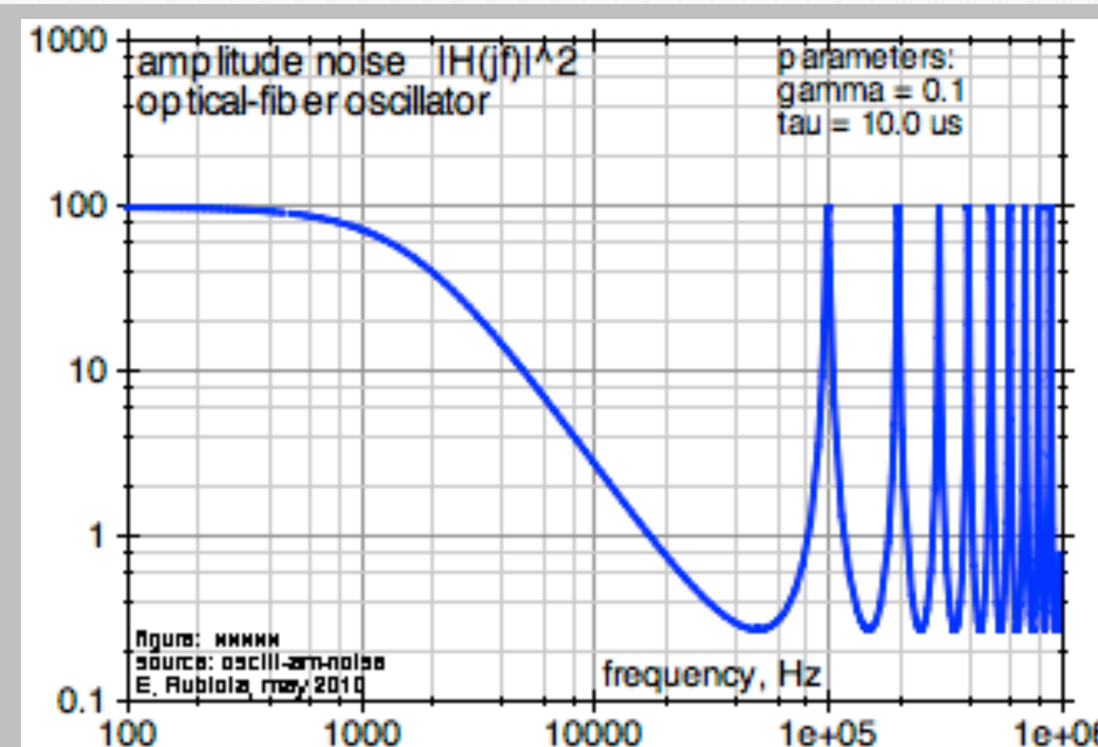
$$\alpha(t) = (1-\gamma)\alpha(t-\tau) - \cancelto{0}{\gamma\alpha^2(t-\tau)}$$
$$+\eta(t-\tau) + \cancelto{0}{\eta(t-\tau)\alpha(t-\tau)}$$

linearized equation

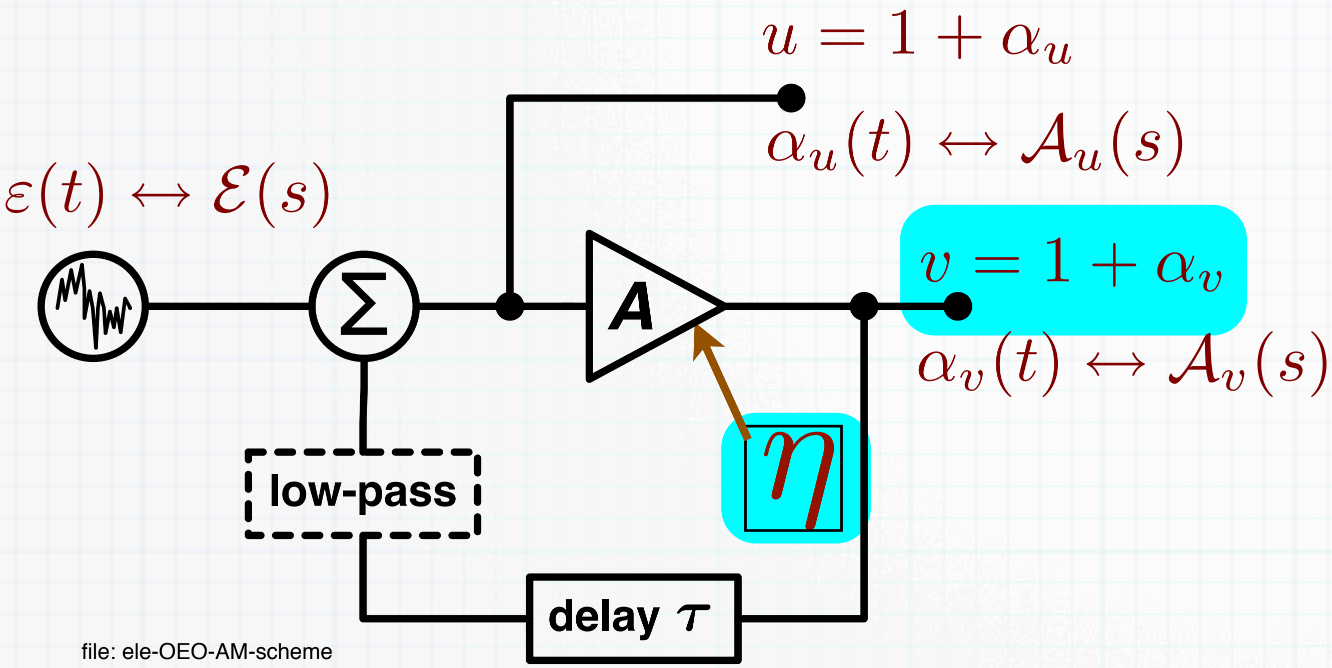
$$\alpha(t) = (1-\gamma)\alpha(t-\tau) + \eta(t-\tau)$$

Laplace transform

$$\mathcal{A}_u(s) = \left[1-(1-\gamma)e^{-s\tau}\right] = \mathcal{N}(s)$$



Gain fluctuations – output is *v*(*t*)



The low-pass has only 2nd order effect on AM

boring algebra relates α_v to α_u

$$v = Au$$

$$A = -\gamma(u-1) + 1 + \eta$$

$$v = [-\gamma(u-1) + 1 + \eta]\, u$$

use u=α+1

$$v = [-\gamma\alpha_u + 1 + \eta]\,[1 + \alpha_u]$$

$$\cancel{1} + \alpha_v = \cancel{1} + \eta - \gamma\alpha_u + \alpha_u - \cancel{\alpha_u\eta} - \cancel{\gamma\alpha_u^2}$$

linearization for low noise

$$\alpha_v = (1-\gamma)\alpha_u + \eta$$

$$\alpha_u = \frac{\alpha_v - \eta}{1-\gamma}$$

$$\mathcal{A}_u(s)\left[1 - (1-\gamma)e^{-\imath\omega\tau}\right] = \mathcal{N}(s)$$

starting equation

$$\mathcal{A}_u(s) = \frac{\mathcal{A}_v(s) - \mathcal{N}(s)}{1-\gamma}$$

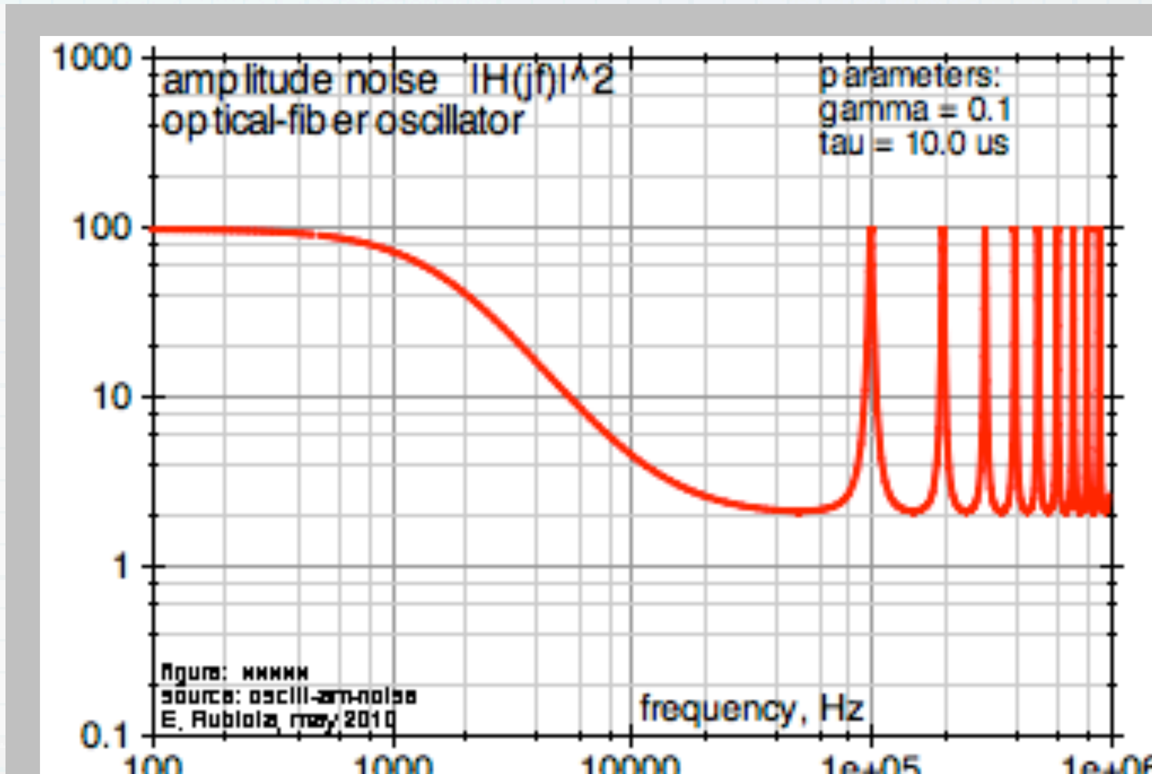
$$\left[1 + (1-\gamma)\left(1 - e^{-s\tau}\right)\right]\mathcal{A}_v(s) = \left[1 - (1-\gamma)e^{-s\tau}\right]\mathcal{N}(s)$$

$$\mathrm{H}(s) = \frac{\mathcal{A}_v(s)}{\mathcal{N}(s)}$$

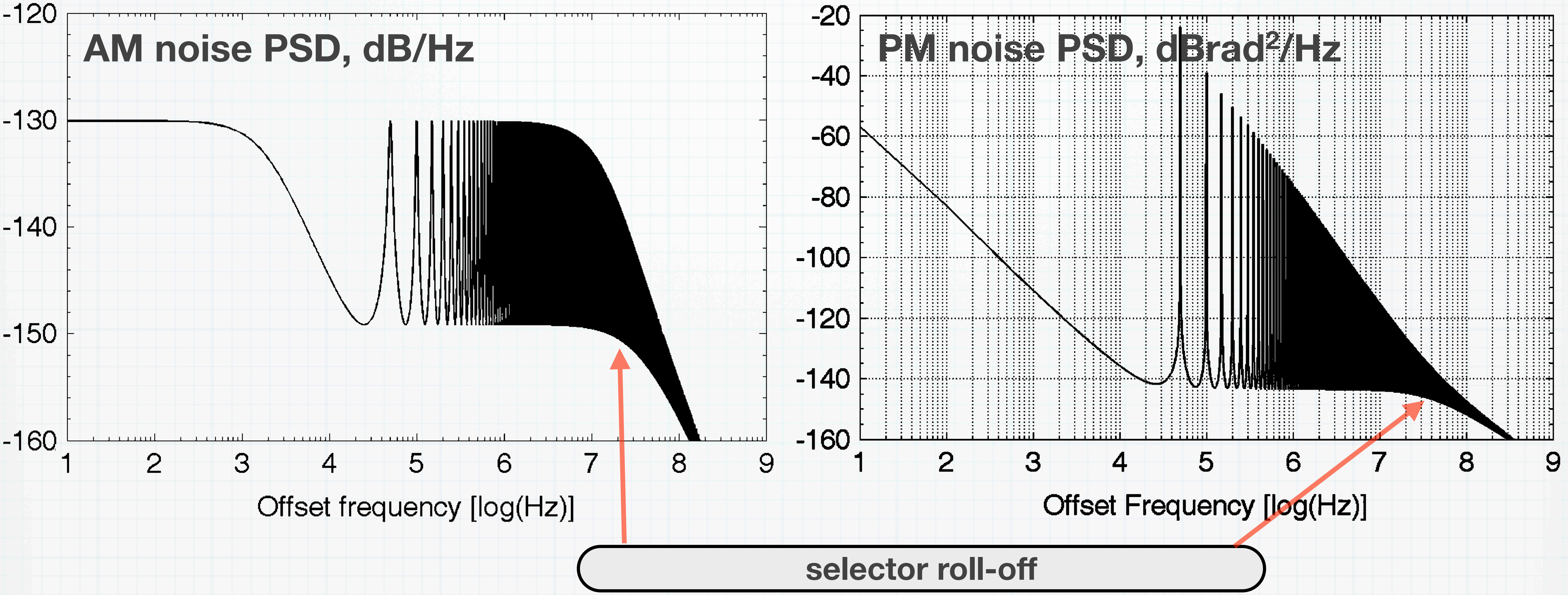
definition

$$\mathrm{H}(s) = \frac{1 + (1-\gamma)\left(1 - e^{-s\tau}\right)}{1 - (1-\gamma)e^{-s\tau}}$$

result



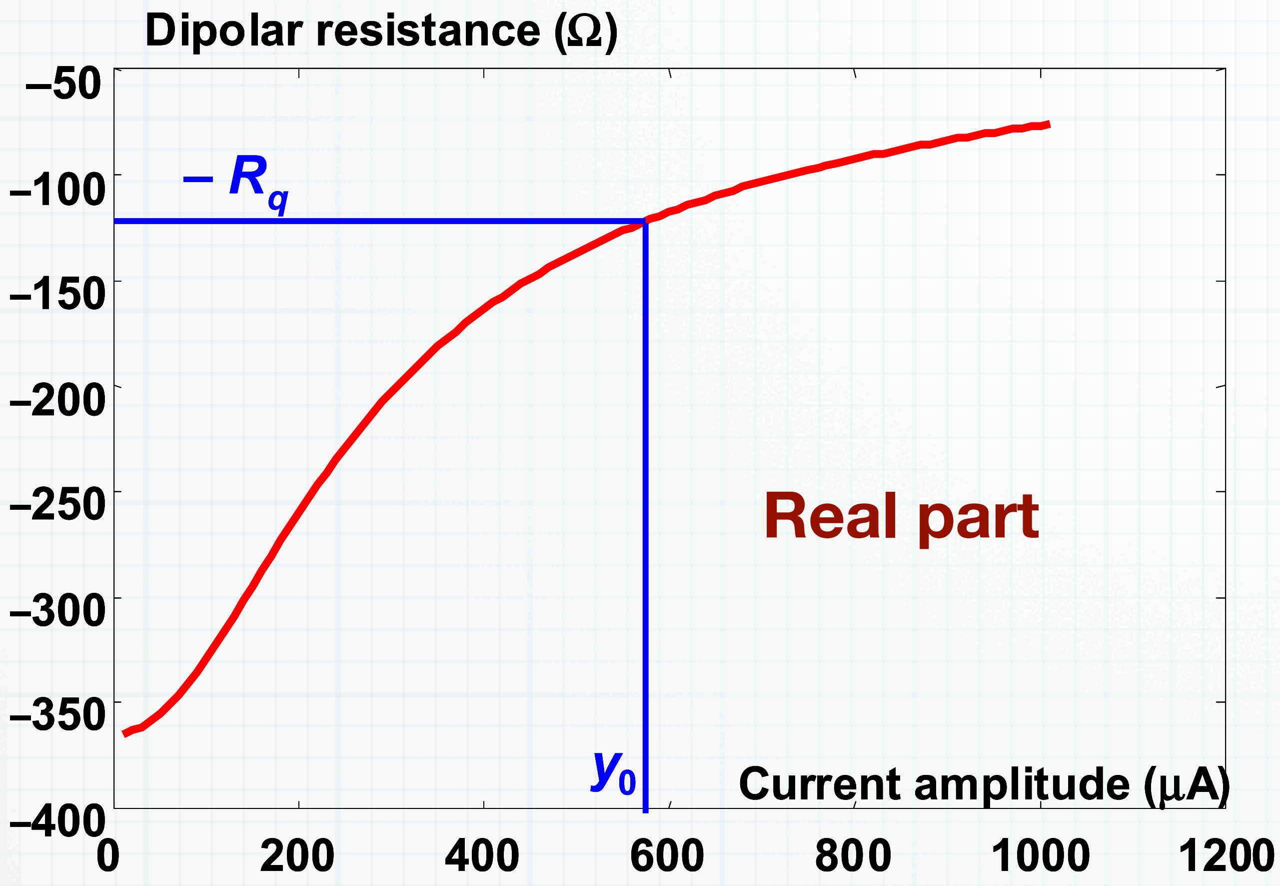
AM & PM spectra were anticipated

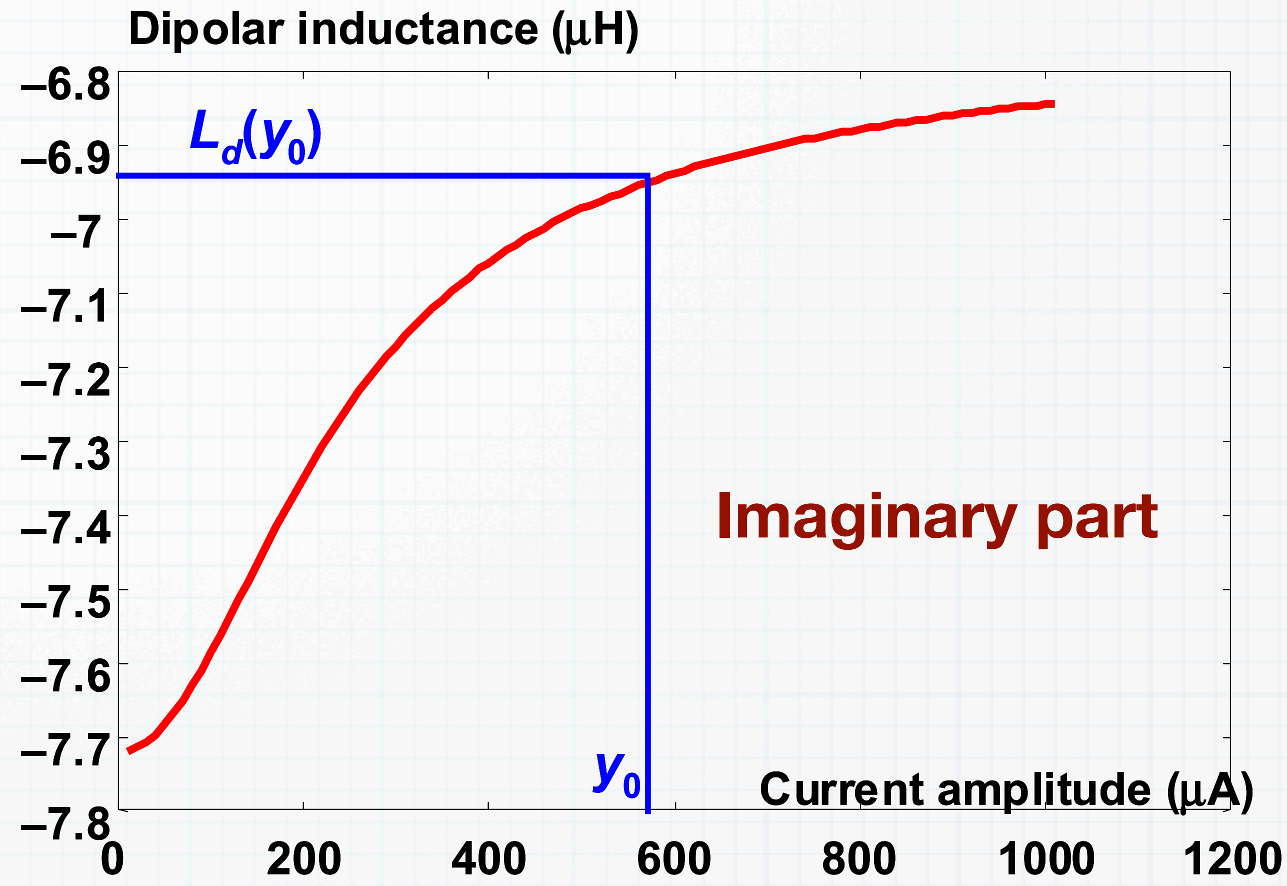


- **Prediction is based on the stochastic diffusion (Langevin) theory**
- **However complex, the Langevin theory provides an independent check**

Y.K. Chembo & al., IEEE J. Quant. Electron. 45(2) p.178-186, Feb 2009

Amplitude-phase coupling in amplifiers





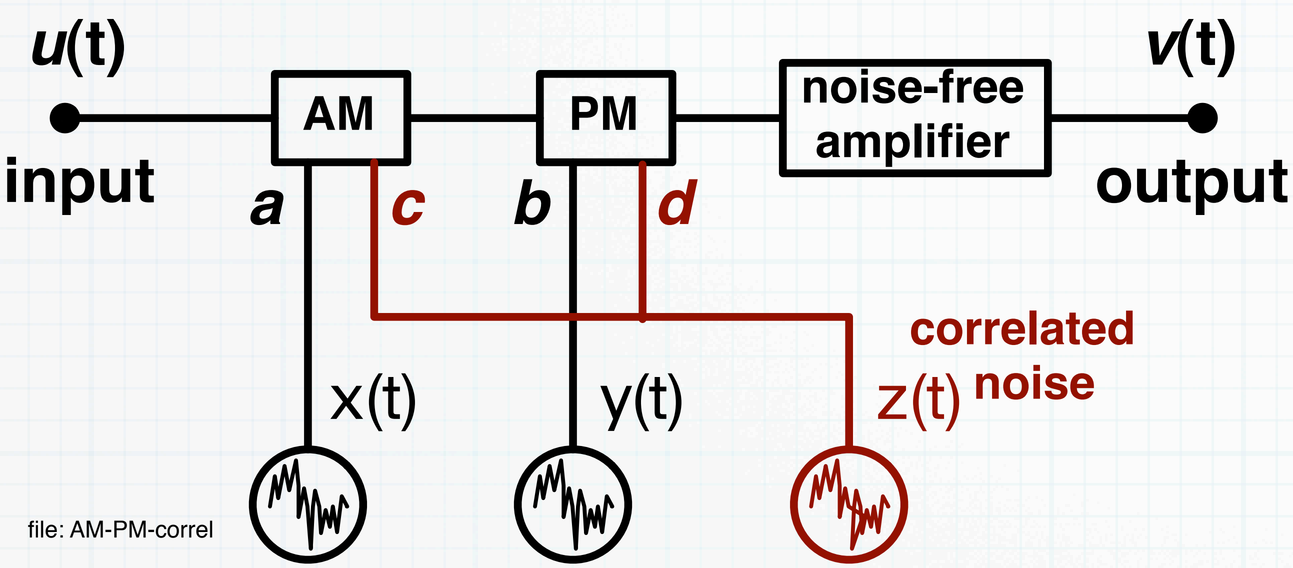
Oscillation amplitude is hidden in the current

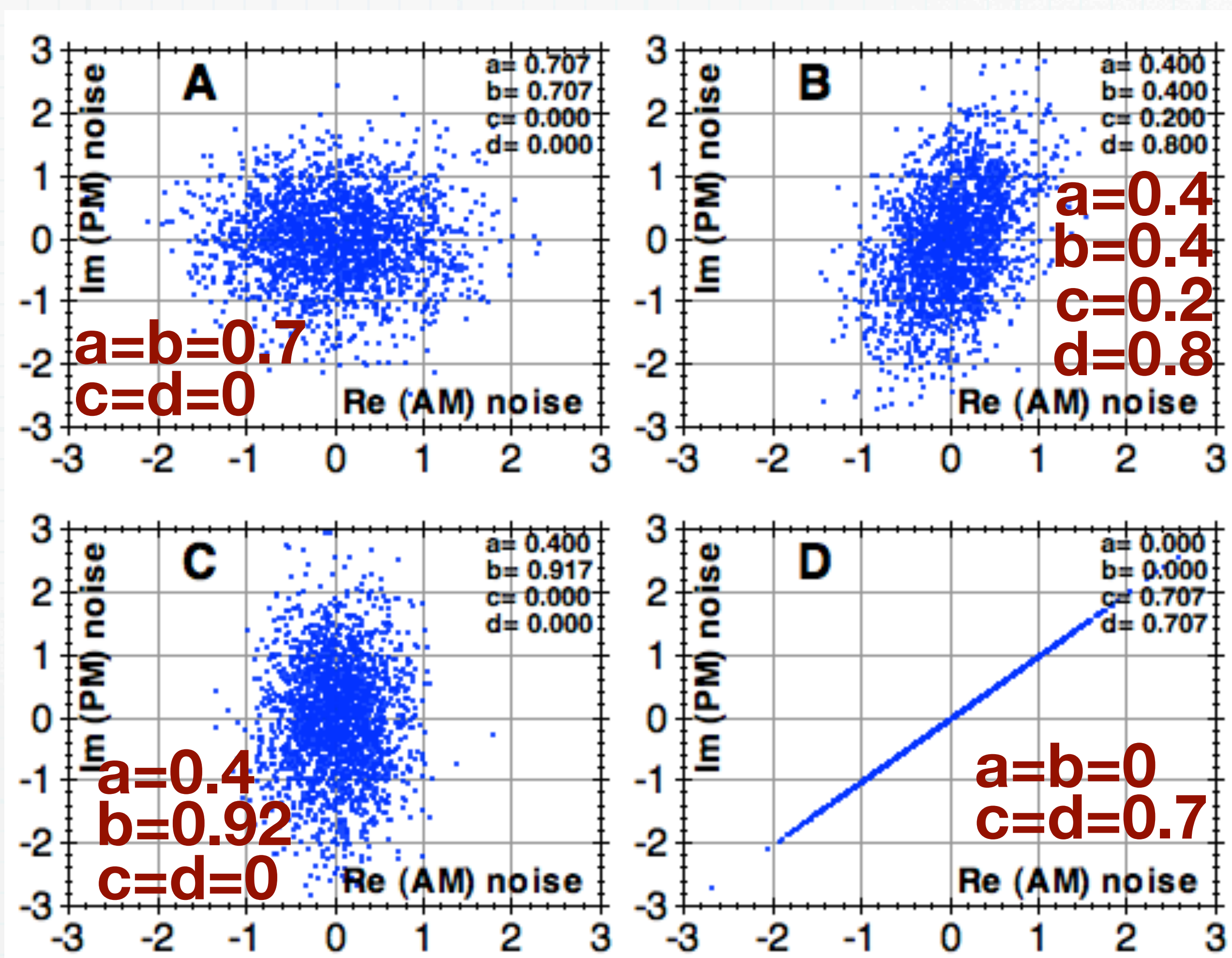
- In the gain-compression region, RF amplitude affects the phase
- The consequence is that AM noise turns into PM noise
- Well established fact in quartz oscillators (Colpitts and other schemes)
- Similar phenomenon occurs in other types of (sustaining) amplifier

R. Brendel & E. Rubiola, Proc. 2007 IFCS p.1099-1105, Geneva CH, 28 may - 1 Jun 2007

Correlation between AM and PM noise

R. Boudot, E. Rubiola, arXiv:1001.2047v1, Jan 2010. Also IEEE T MTT (submitted)





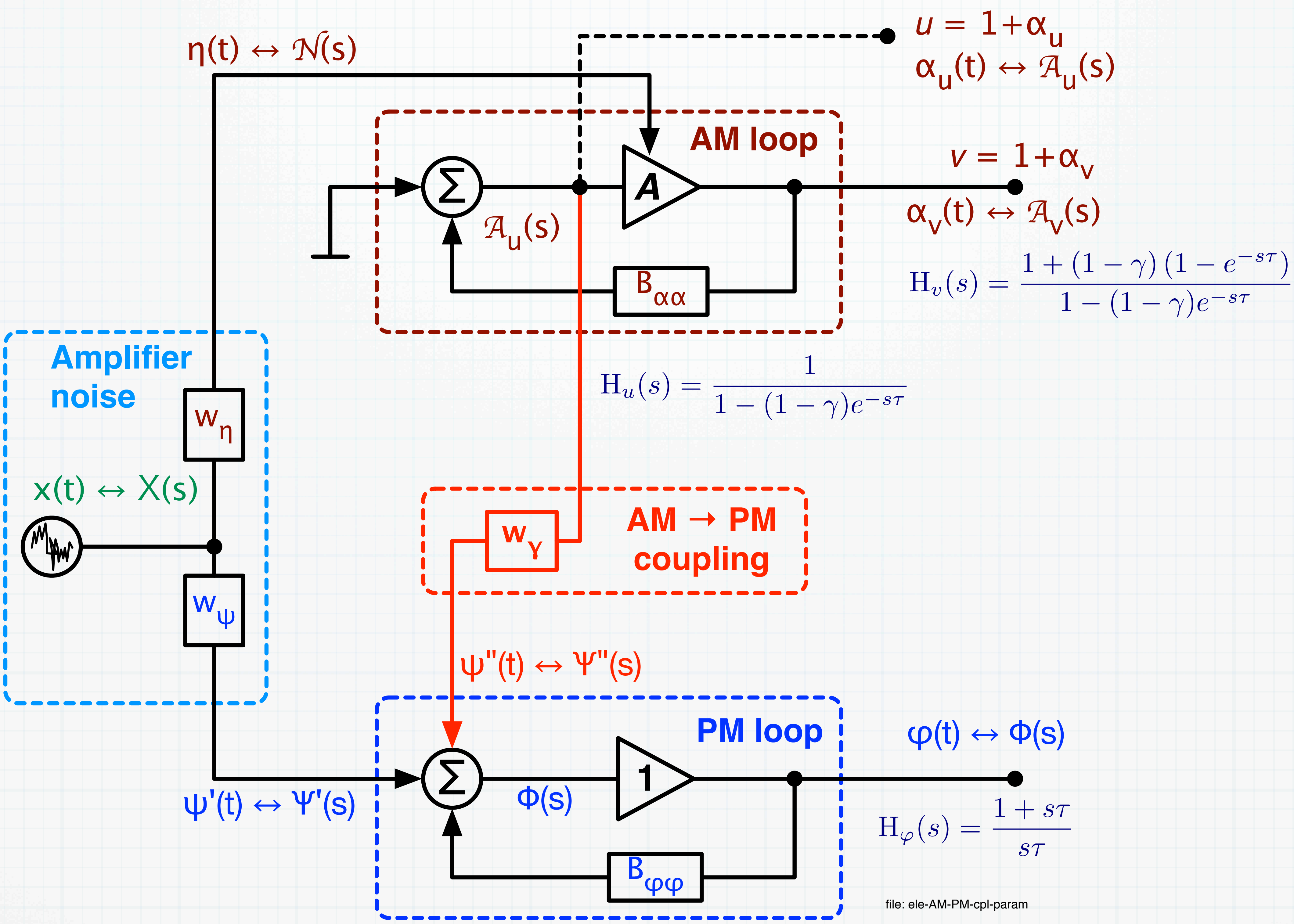
$$a^2 + b^2 + c^2 + d^2 = 1$$

The need for this model comes from the physics of popular amplifiers

- Bipolar transistor. The fluctuation of the carriers in the base region acts on the base thickness, thus on the gain, and on the capacitance of the reverse-biased base-collector junction.
- Field-effect transistor. The fluctuation of the carriers in the channel acts on the drain-source current, and also on the gate-channel capacitance because the distance between the `electrodes' is affected by the channel thickness.
- Laser amplifier. The fluctuation of the pump power acts on the density of the excited atoms, and in turn on gain, on maximum power, and on refraction index.

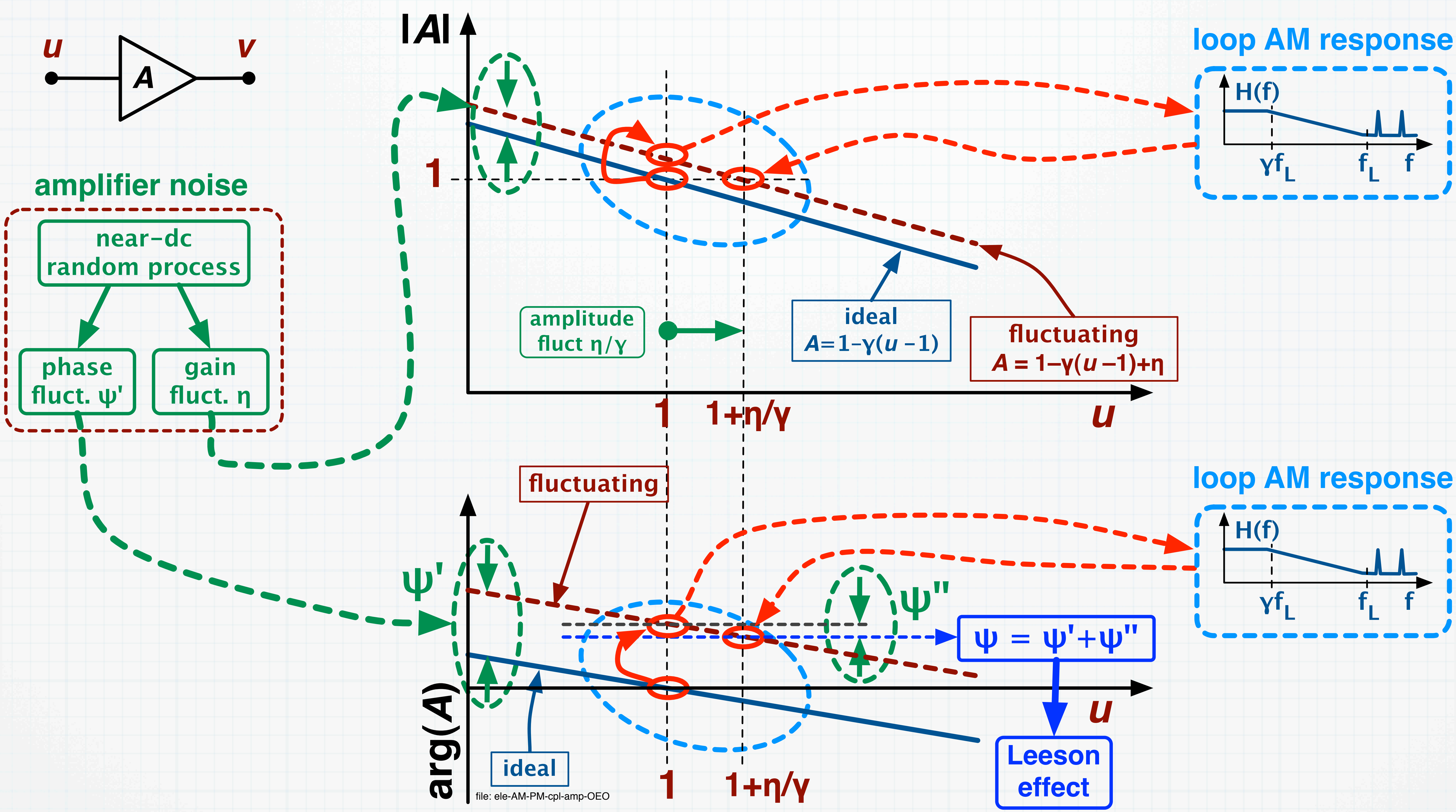
AM and PM fluctuations are correlated because originate from the same near-dc random process

Parametric noise & AM-PM noise coupling



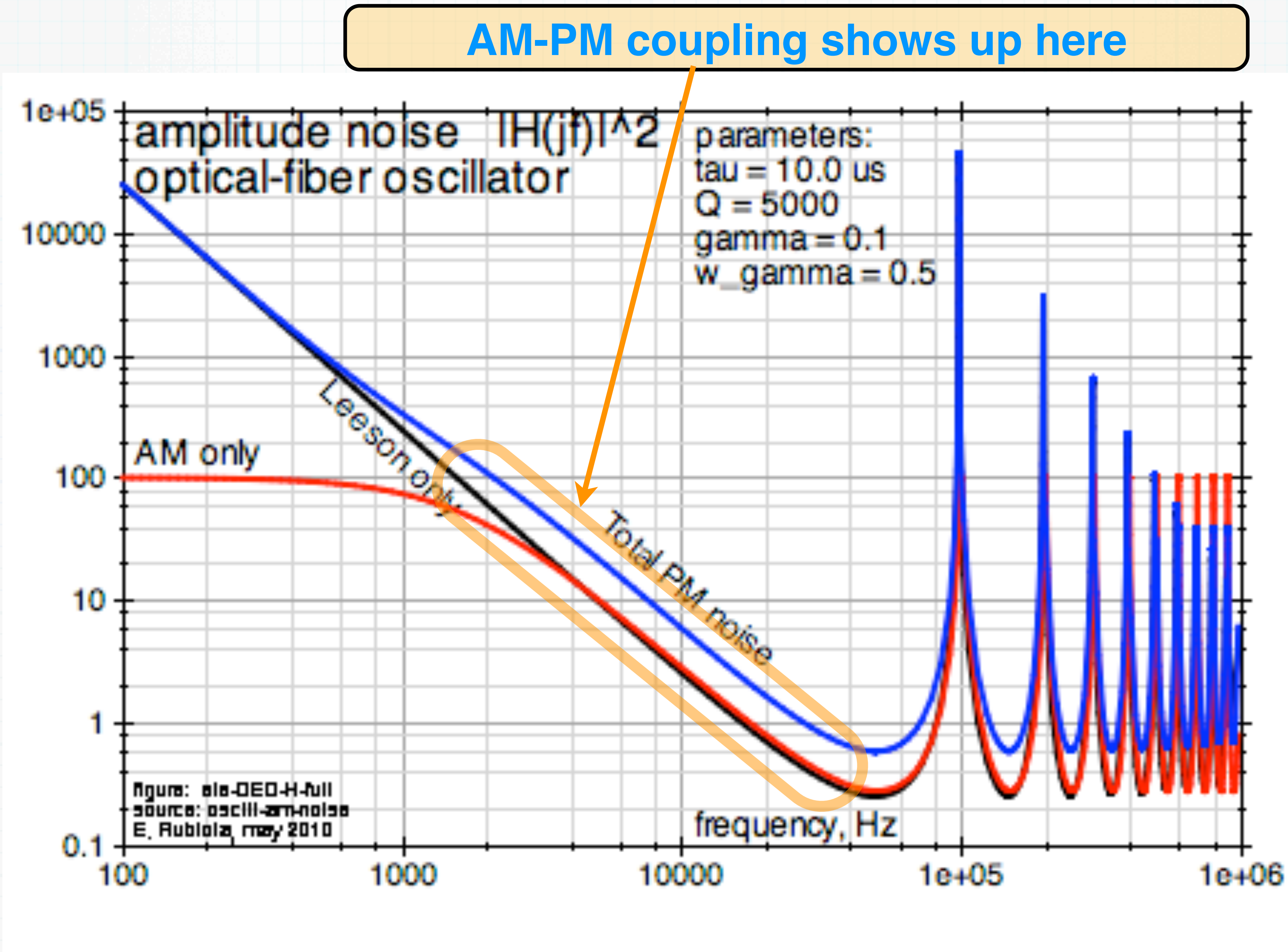
E. Rubiola & R. Brendel, arXiv:1004.5539v1 [physics.ins-det]

Effect of AM-PM noise coupling



E. Rubiola & R. Brendel, arXiv:1004.5539v1 [physics.ins-det]

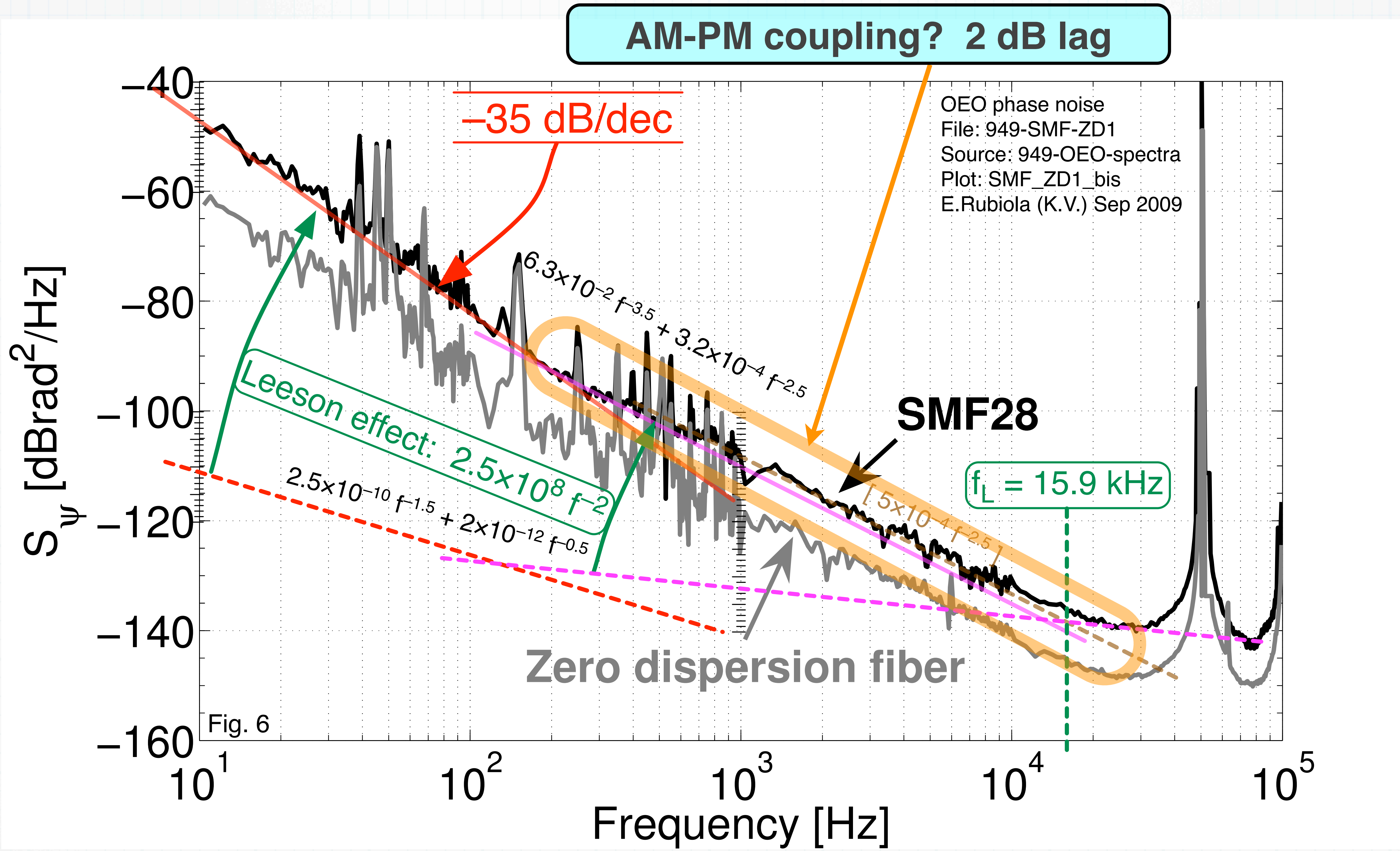
Noise transfer function and spectra



Notice that the AM-PM coupling can increase or decrease the PM noise

In a real oscillator, flicker noise shows up below some 10 kHz
In the flicker region, all plots are multiplied by 1/f

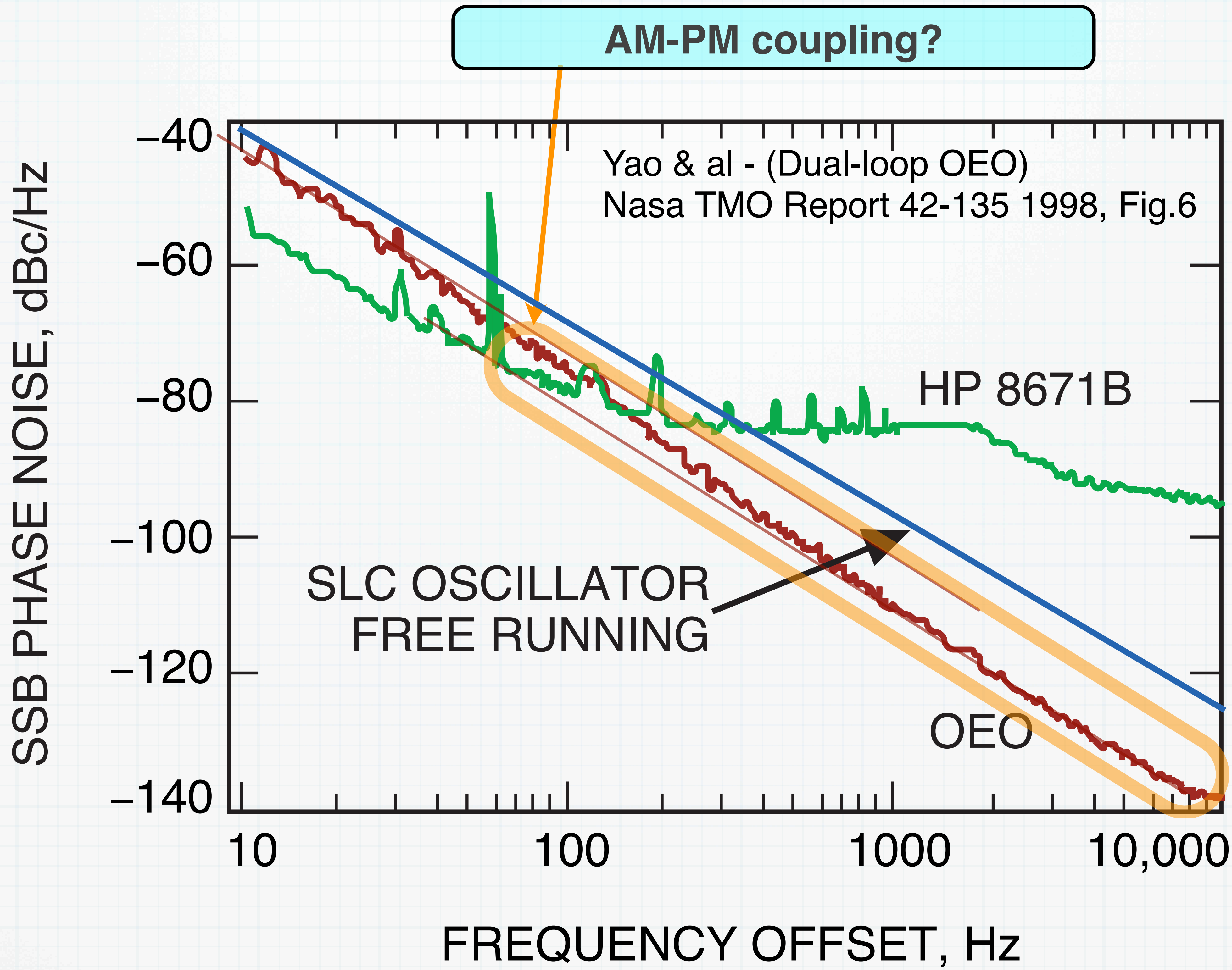
Noise spectra



Unfortunately, the awareness of this model come after the end of the experiments

Spectrum from K. Volyanskiy & al., IEEE JLT (Submitted, Apr. 2010)

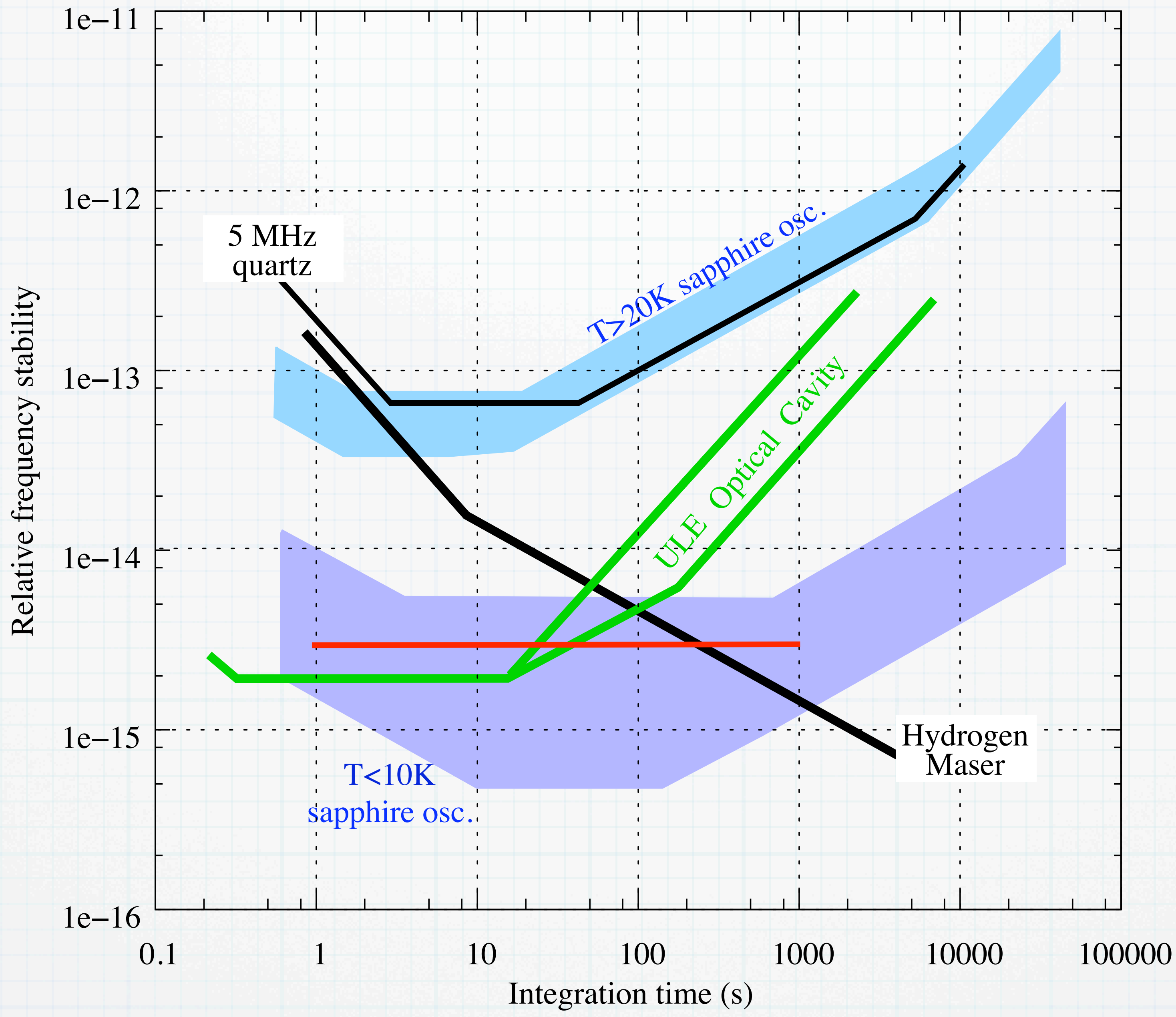
Noise spectra



X.S.Yao & al., NASA TMO Report 42-135 (1998), Fig. 6

Cryogenic oscillator
(Elisa)

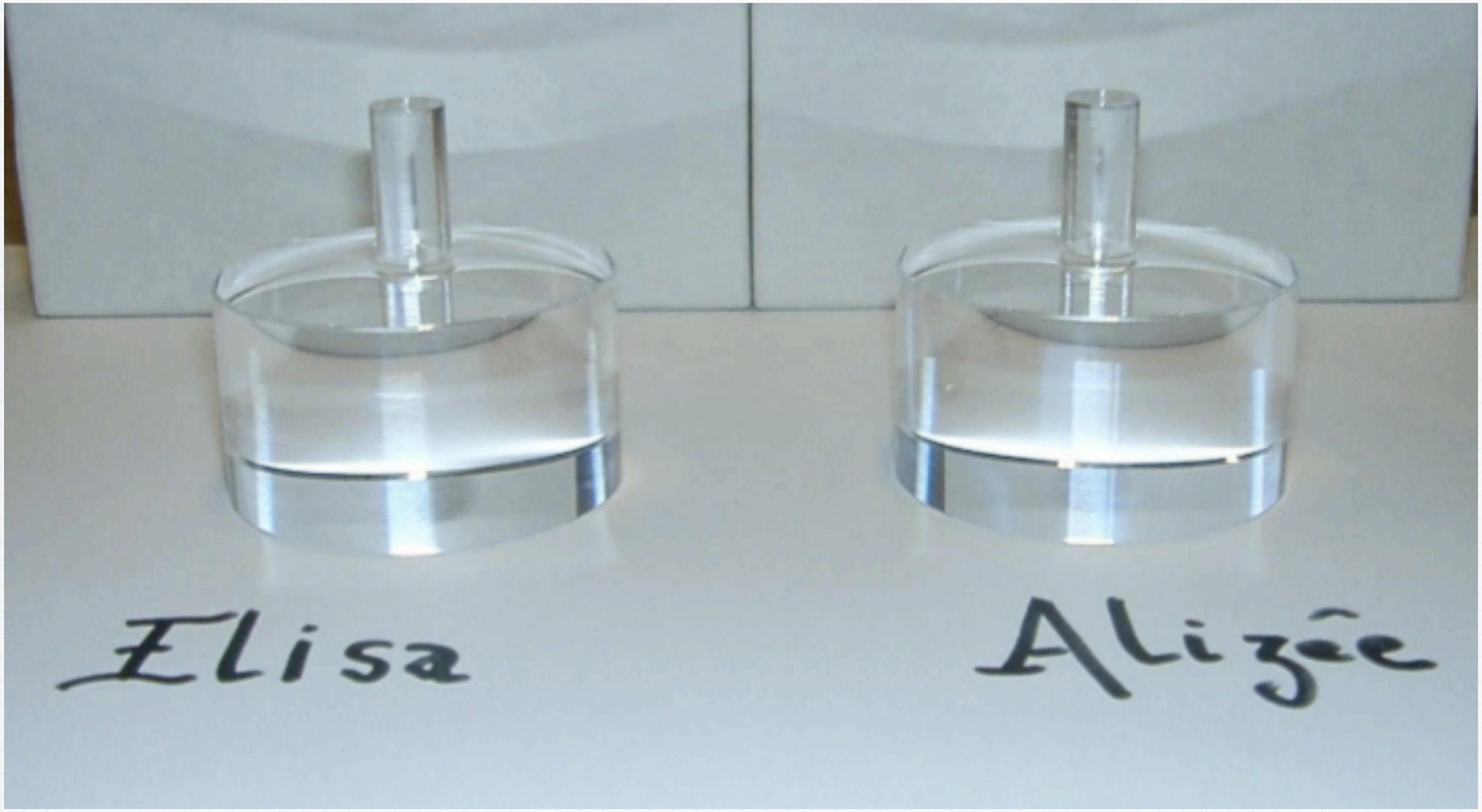
Ultrastable oscillator technologies



Cryogenic Sapphire Oscillator -> frequency stability
Cryocooler -> Autonomy

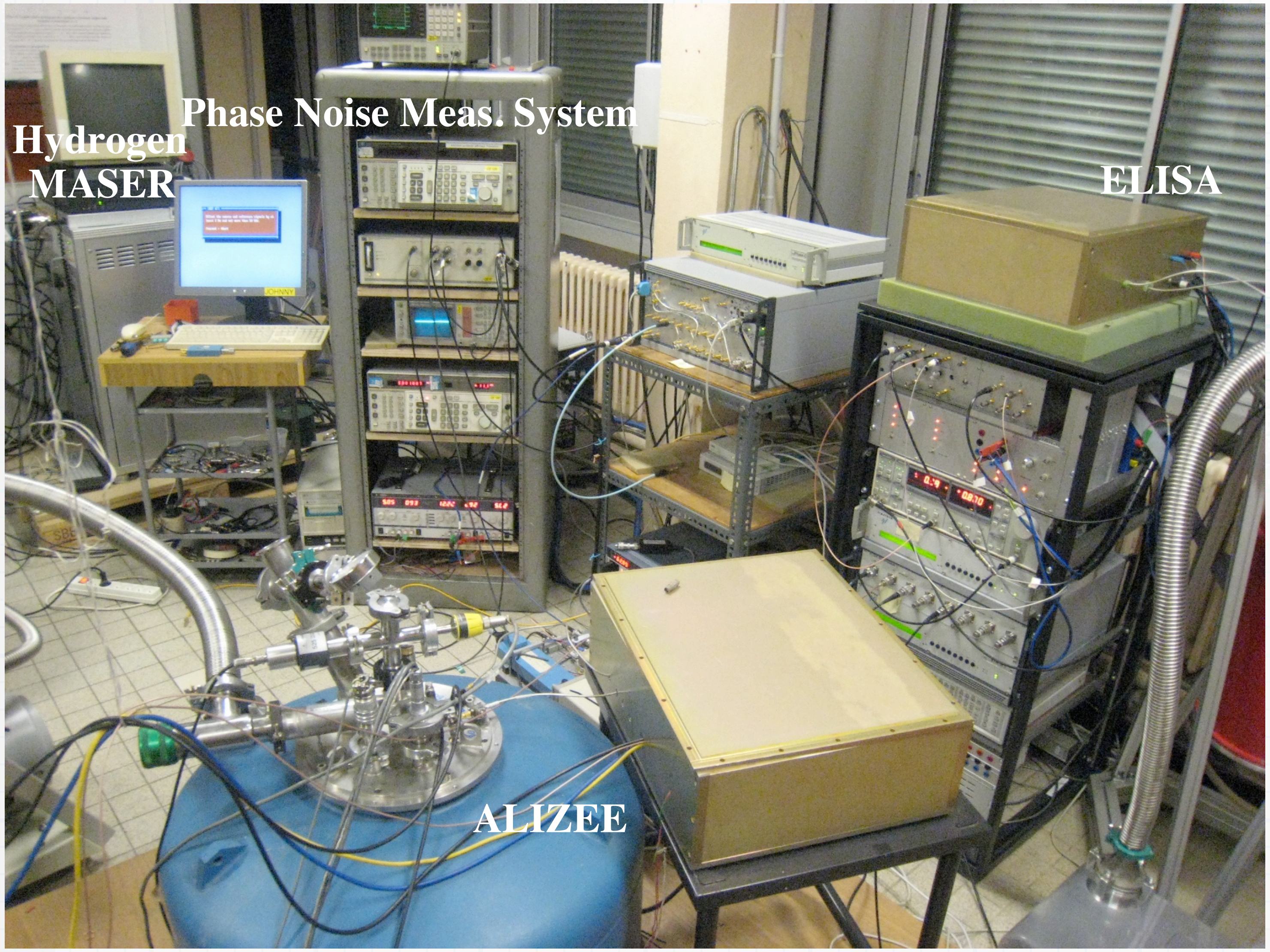
Challenge : vibrations and temperature fluctuations into the cryocooler

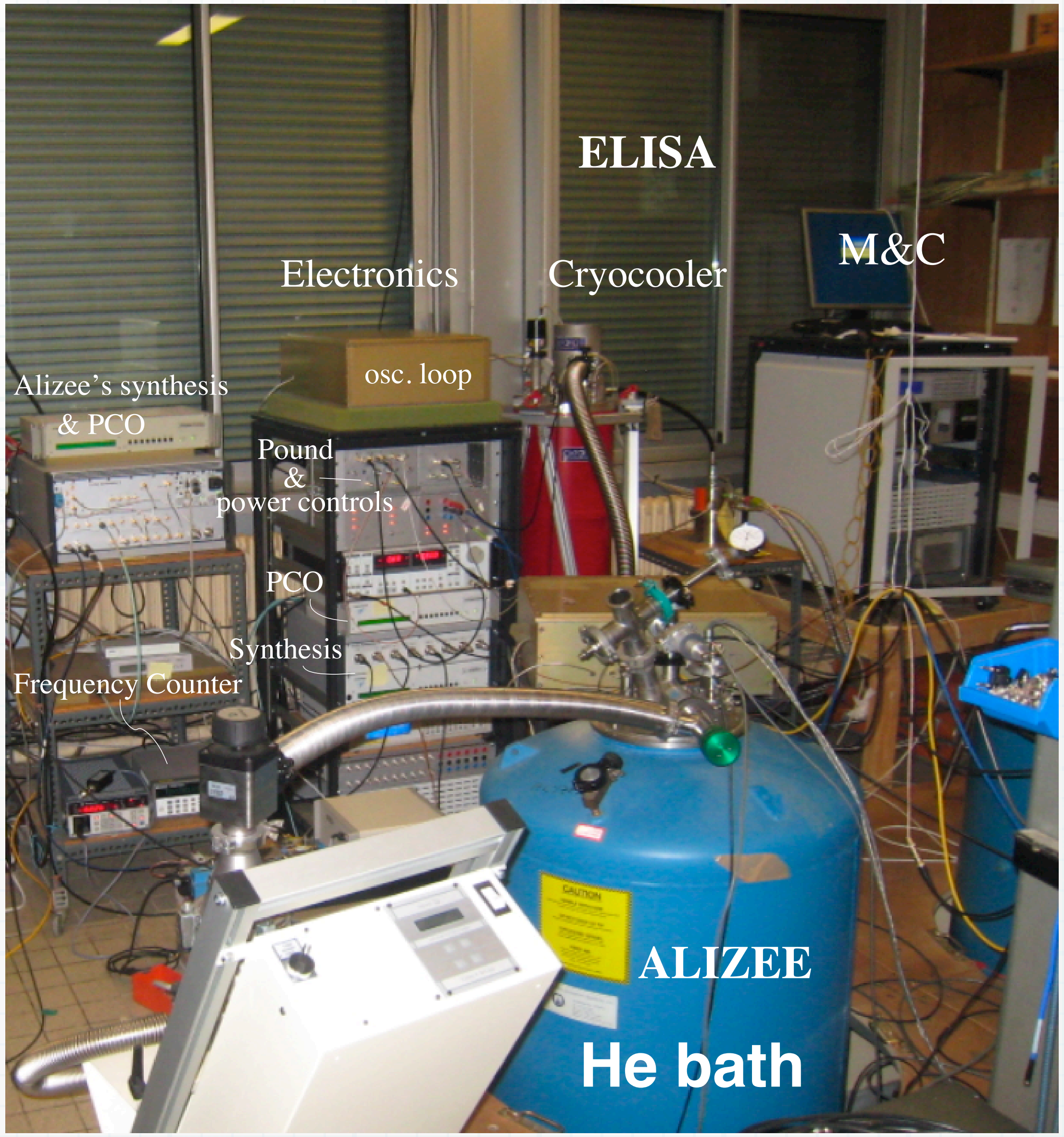
Sapphire resonator



- Hemex grade sapphire monocrystal
- Whispering-gallery mode, frequency 10 GHz
- Sytnthesizer fixes machining tolerance (≈ MHz)
- Quality factor Q ≈ 1E9 at 5-8 K
- Temperature turning point

Elisa and Alizée





Stability comparison