Noise Analysis of the Microwave OEO

P. Salzenstein, R. Brendel, Y. Koumou Chembo, K. Volyanski, L. Larger, E. Rubiola

IFCS, Newport, CA, 1–4 June 2010

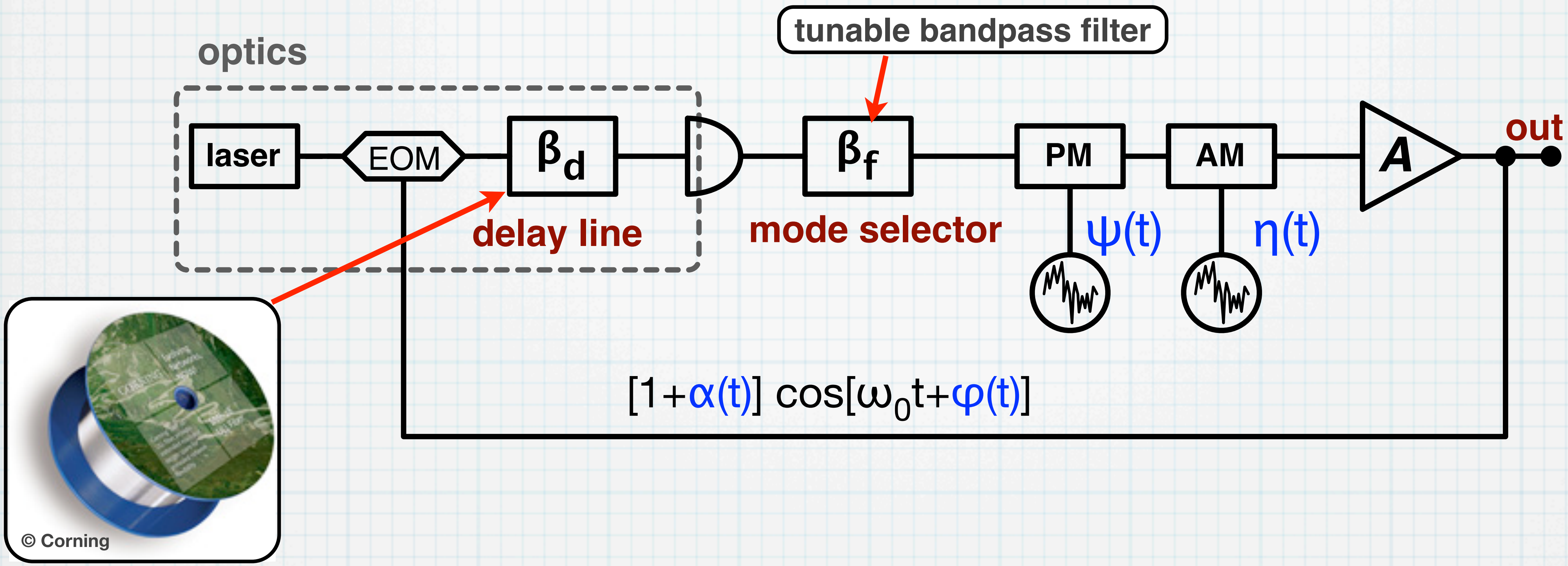
Outline

- Introduction
- Generalization of the Leeson effect
- AM-PM noise coupling
- Noise transfer functions (AM-PM spectra)
- Spectra of real oscillators

This presentation is mostly about parametric noise

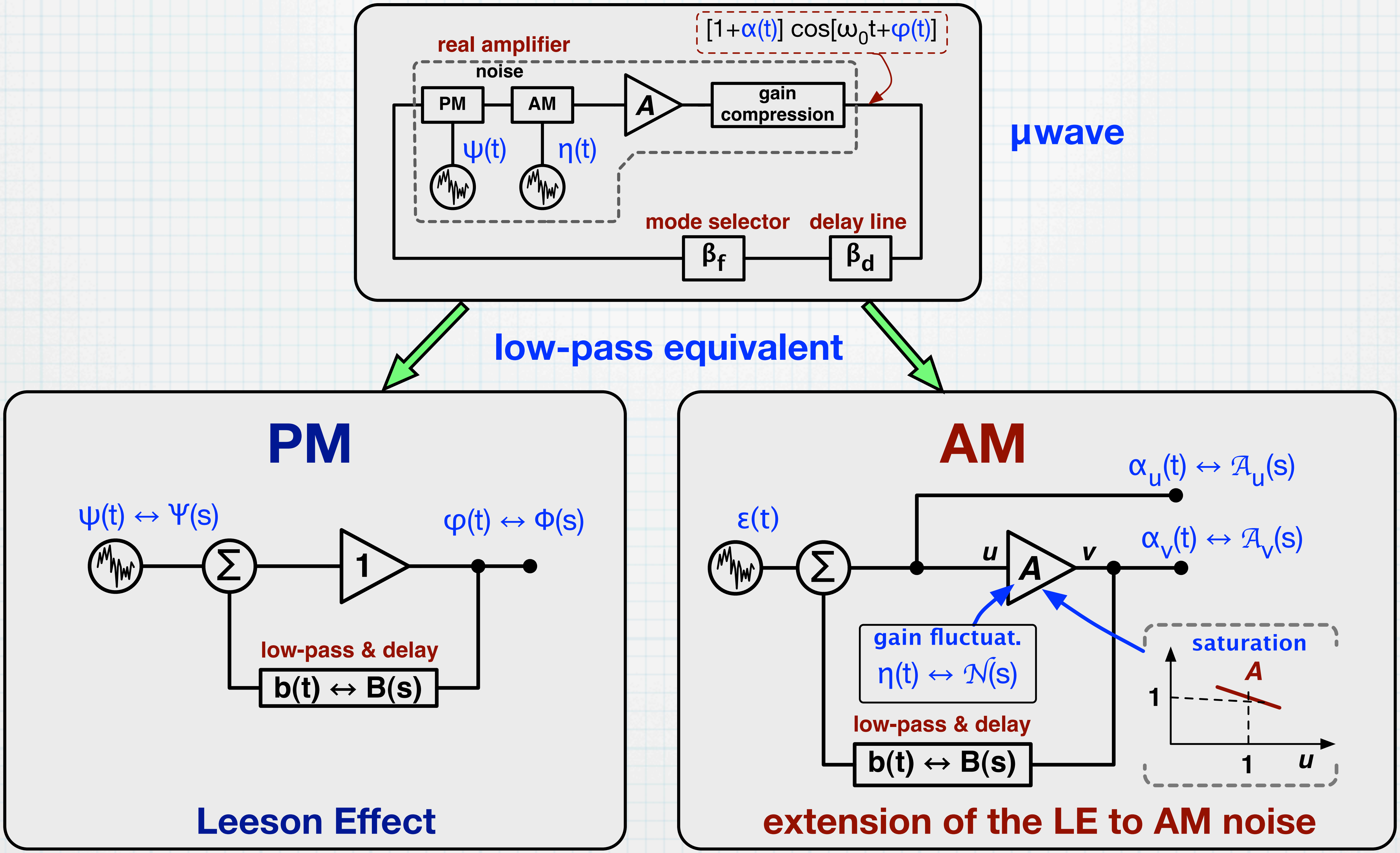
home page http://rubiola.org

Motivations



- **Potential for very-low phase noise in the 100 Hz – 1 MHz range**
- **Invented at JPL, X. S. Yao & L. Maleki, JOSAB 13(8) 1725–1735, Aug 1996**
- **Early attempt of noise modeling, S. Römisch & al., IEEE T UFFC 47(5) 1159–1165, Sep 2000**
- **PM-noise analysis, E. Rubiola, *Phase noise and frequency stability in oscillators*, Cambridge 2008 [Chapter 5]**
- **Since, no progress in the analysis of noise at system level**
- **Nobody reported on the consequences of AM noise**

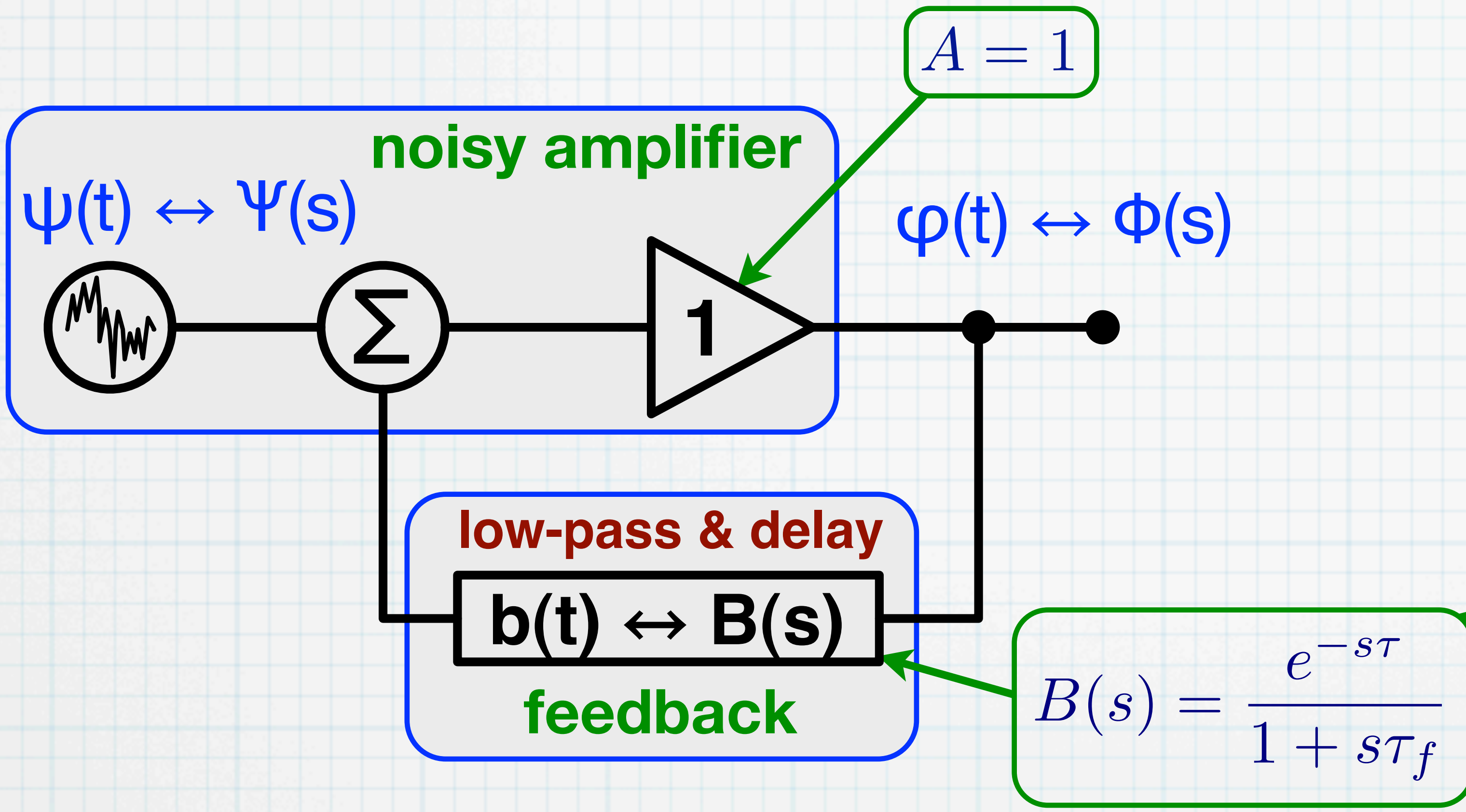
Low-pass representation of AM-PM noise



The amplifier
– "copies" the input phase to the out
– adds phase noise

The amplifier
– compresses the amplitude
– adds amplitude noise

Leeson effect

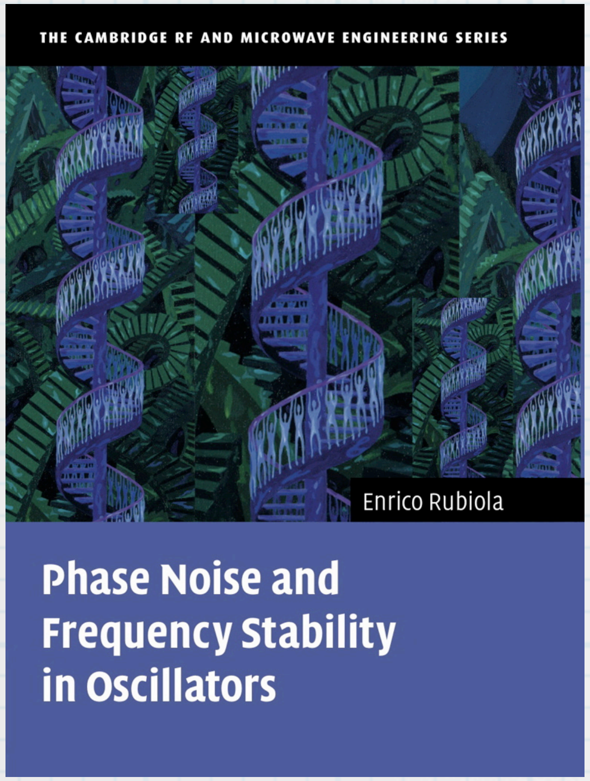


phase-noise transfer function

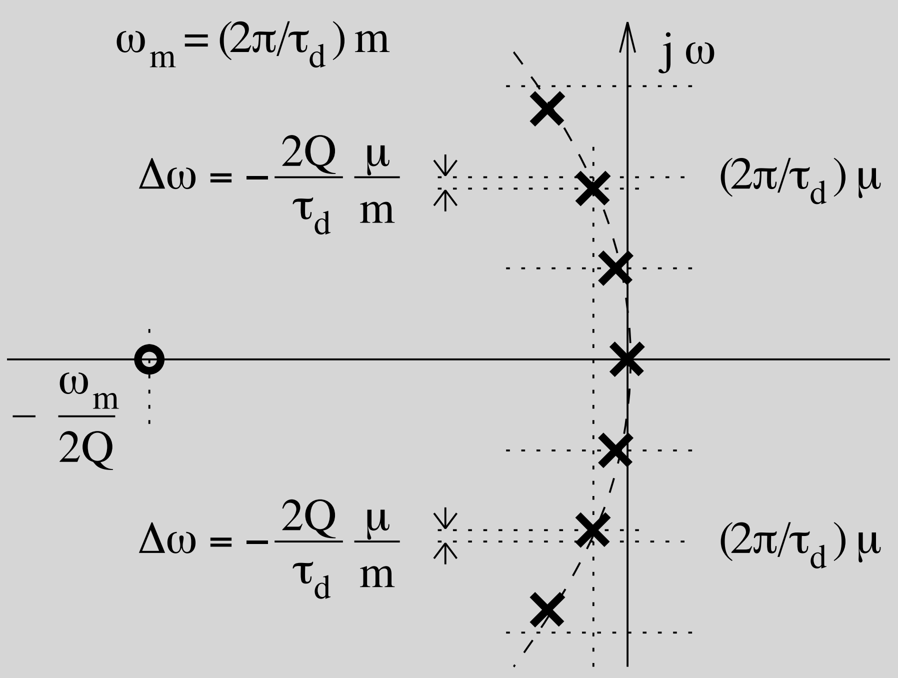
$$H(s) = \frac{\Phi(s)}{\Psi(s)}$$ **definition**

$$H(s) = \frac{1}{1+AB(s)}$$ **general feedback theory**

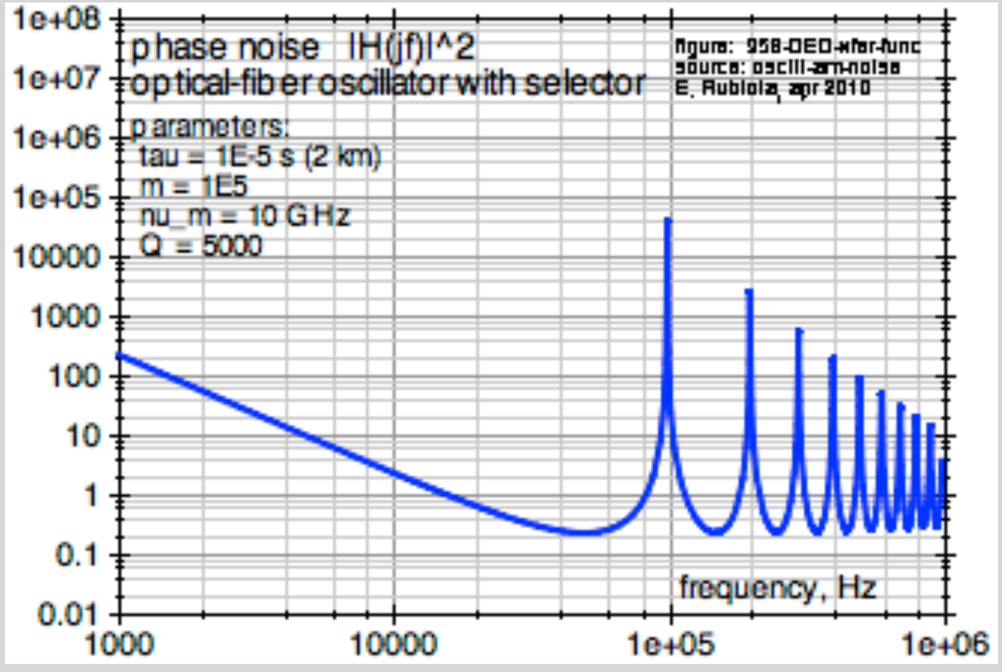
$$H(s) = \frac{1+s\tau_f}{1+s\tau_f - e^{-s\tau}}$$ **Leeson effect**



H, complex plane

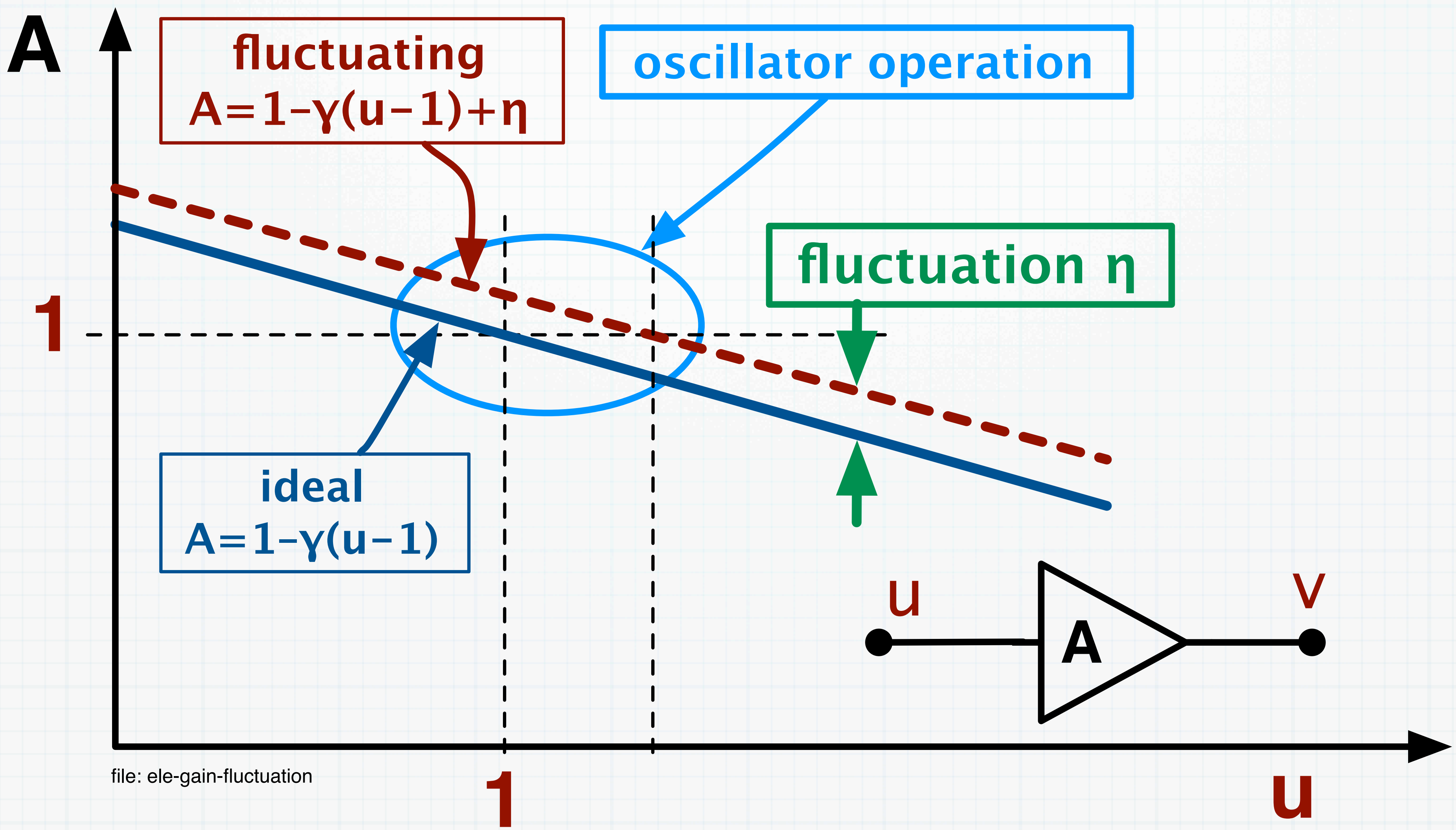


transfer function $|H|^2$



E. Rubiola, Phase noise and frequency stability in oscillators, Cambridge 2008

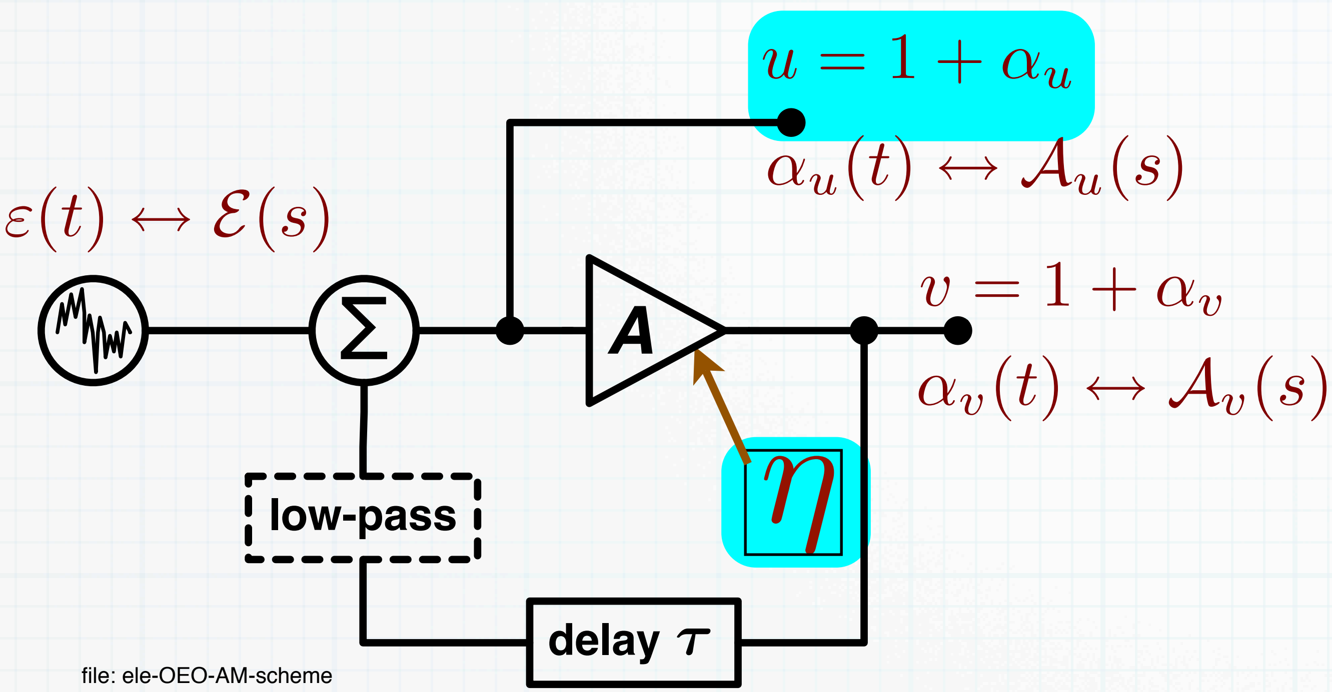
Gain fluctuations – definition



Gain compression is necessary for the oscillation amplitude to be stable

E. Rubiola & R. Brendel, arXiv:1004.5539v1 [physics.ins-det]

Gain fluctuations – output is *u*(*t*)



The low-pass has only 2nd order effect on AM

Linearize for low noise and use the Laplace transform

$$\alpha_u(t) \leftrightarrow \mathcal{A}_u(s) \quad \text{and} \quad \eta(t) \leftrightarrow \mathcal{N}(s)$$

$$\mathrm{H}(s) = \frac{\mathcal{A}_u(s)}{\mathcal{N}(s)}$$ definition

result

$$\mathrm{H}(s) = \frac{1}{1-(1-\gamma)e^{-s\tau}}$$

non-linear equation

$$u = A(t-\tau)\, u(t-\tau)$$

$$A = 1 - \gamma(u-1) + \eta$$

use u=α+1, expand and linearize for low noise

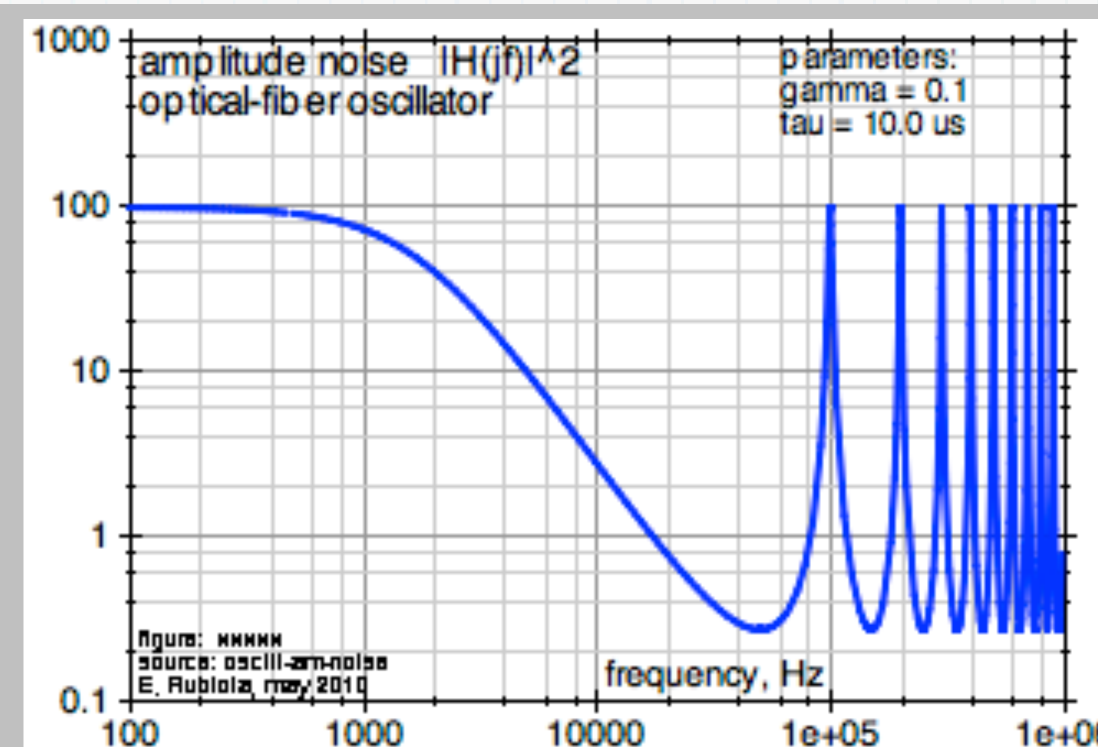
$$\alpha(t) = (1-\gamma)\alpha(t-\tau) - \gamma\alpha^2(t-\tau) \rightarrow 0$$
$$+\,\eta(t-\tau) + \eta(t-\tau)\alpha(t-\tau) \rightarrow 0$$

linearized equation

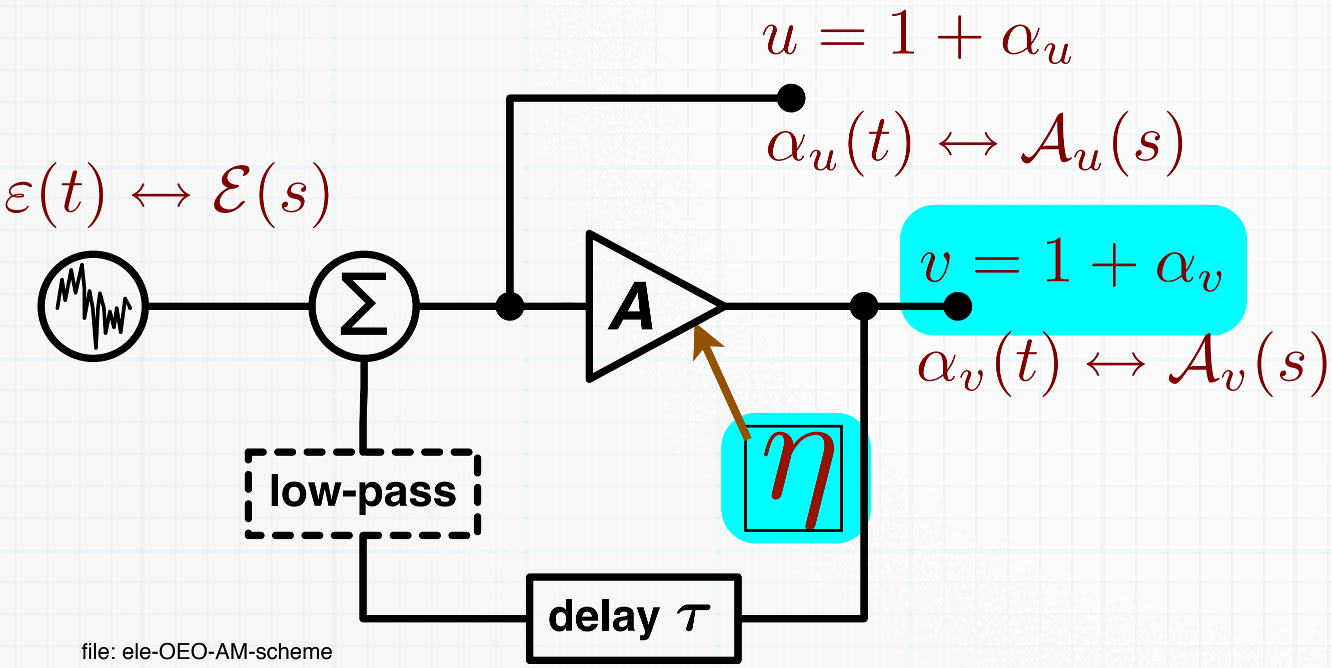
$$\alpha(t) = (1-\gamma)\alpha(t-\tau) + \eta(t-\tau)$$

Laplace transform

$$\mathcal{A}_u(s) = \left[1-(1-\gamma)e^{-s\tau}\right] = \mathcal{N}(s)$$



Gain fluctuations – output is *v*(*t*)



The low-pass has only 2nd order effect on AM

boring algebra relates α_v to α_u

$$v = Au$$

$$A = -\gamma(u-1) + 1 + \eta$$

$$v = [-\gamma(u-1) + 1 + \eta]\, u \qquad \text{use u=α+1}$$

$$v = [-\gamma\alpha_u + 1 + \eta]\,[1 + \alpha_u]$$

$$\not{1} + \alpha_v = \not{1} + \eta - \gamma\alpha_u + \alpha_u - \cancel{\alpha_u\eta} - \cancel{\gamma\alpha_u^2}$$

$$\alpha_v = (1-\gamma)\alpha_u + \eta$$

$$\alpha_u = \frac{\alpha_v - \eta}{1-\gamma}$$

linearization for low noise

$$\mathcal{A}_u(s)\left[1 - (1-\gamma)e^{-\imath\omega\tau}\right] = \mathcal{N}(s)$$

starting equation

$$\mathcal{A}_u(s) = \frac{\mathcal{A}_v(s) - \mathcal{N}(s)}{1-\gamma}$$

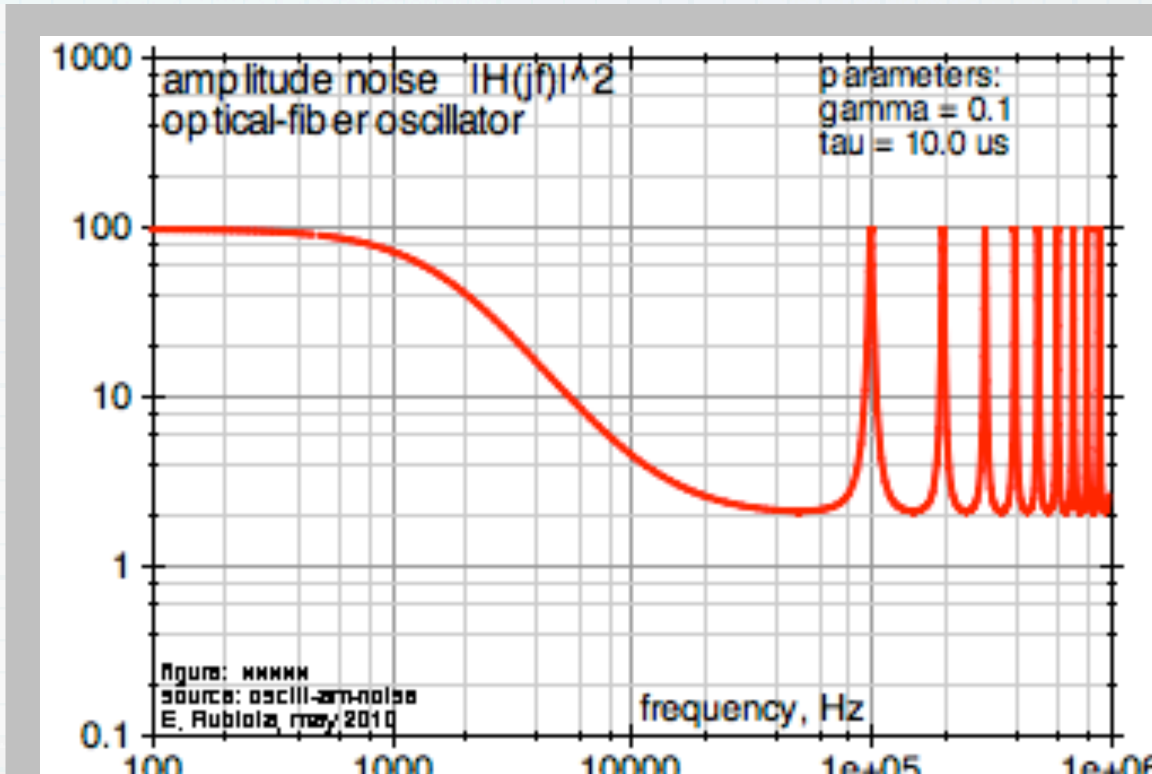
$$\left[1 + (1-\gamma)\left(1 - e^{-s\tau}\right)\right]\mathcal{A}_v(s) = \left[1 - (1-\gamma)e^{-s\tau}\right]\mathcal{N}(s)$$

$$\mathrm{H}(s) = \frac{\mathcal{A}_v(s)}{\mathcal{N}(s)}$$

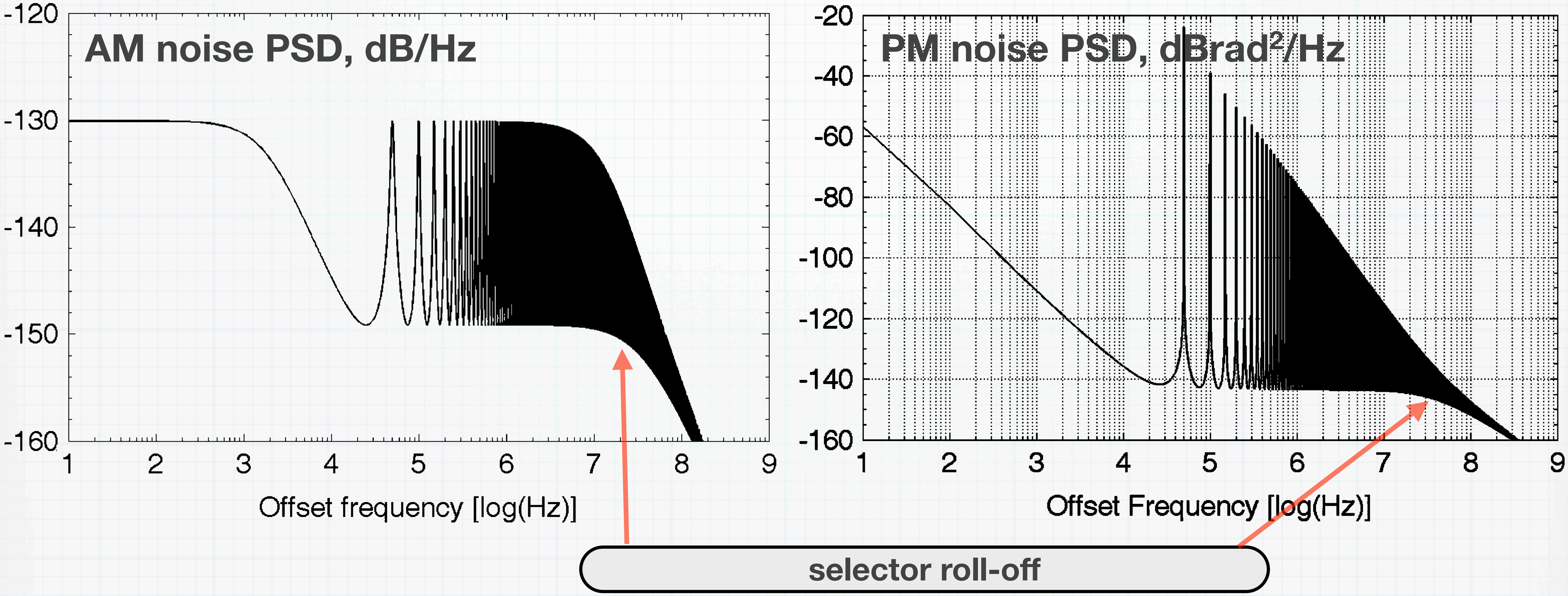
definition

$$\mathrm{H}(s) = \frac{1 + (1-\gamma)\left(1 - e^{-s\tau}\right)}{1 - (1-\gamma)e^{-s\tau}}$$

result



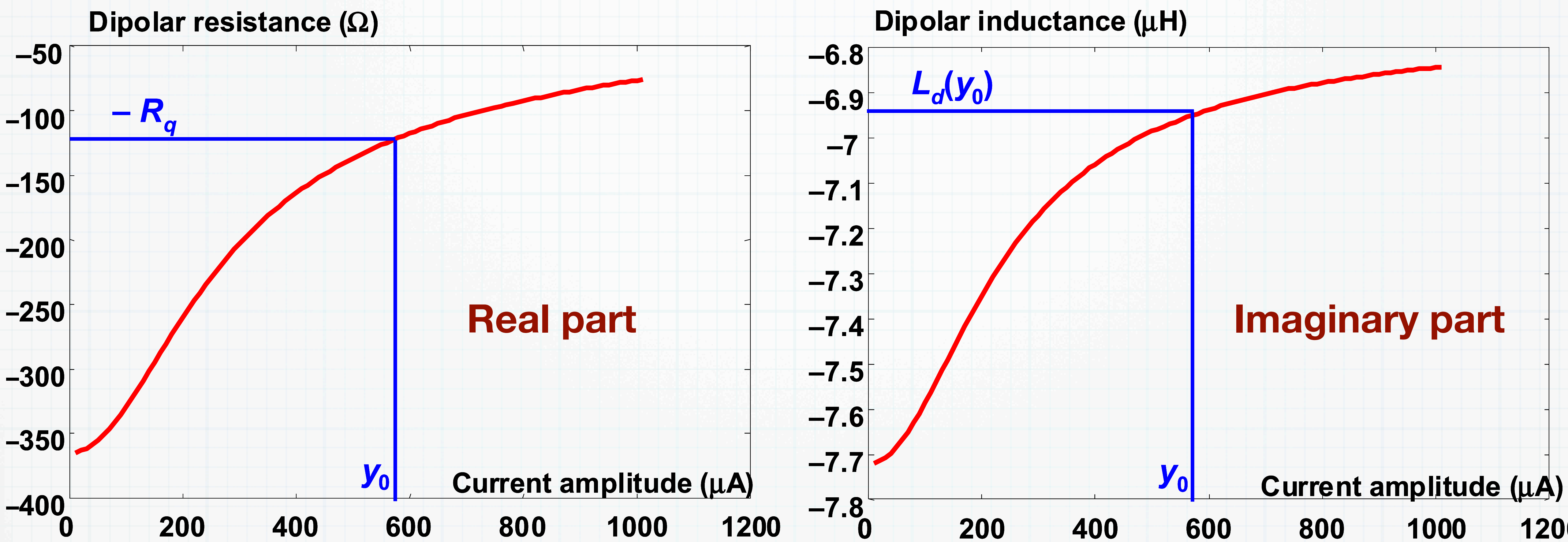
AM & PM spectra were anticipated



- Prediction is based on the stochastic diffusion (Langevin) theory
- However complex, the Langevin theory provides an independent check

Y.K. Chembo & al., IEEE J. Quant. Electron. 45(2) p.178-186, Feb 2009

Amplitude-phase coupling in amplifiers



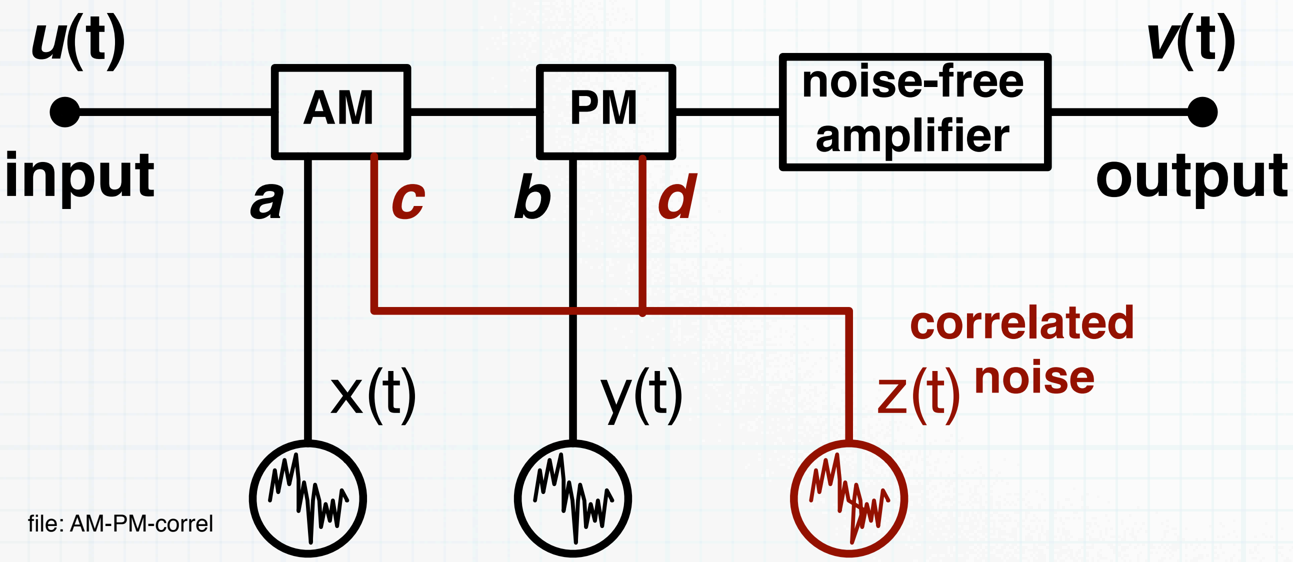
Oscillation amplitude is hidden in the current

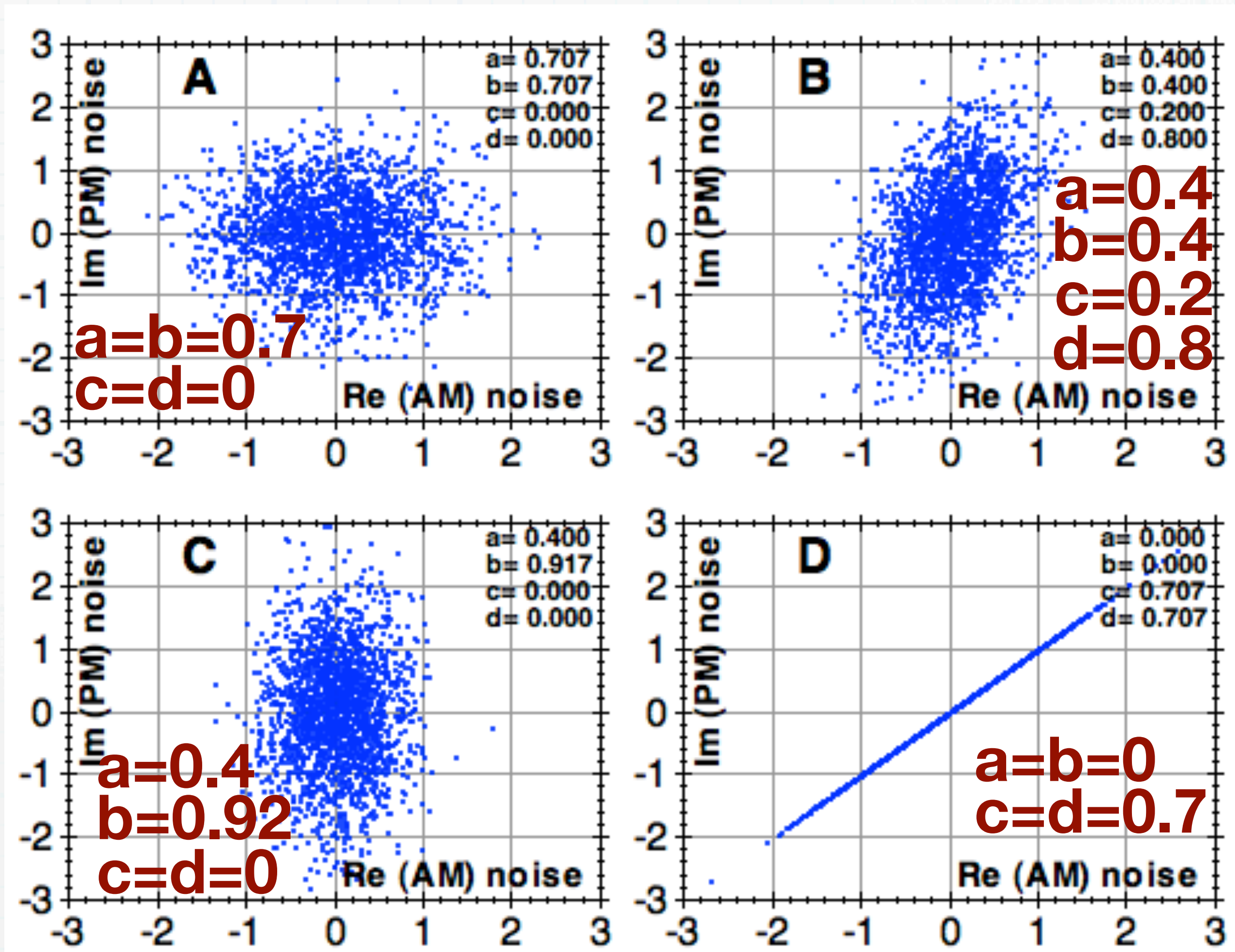
- In the gain-compression region, RF amplitude affects the phase
- The consequence is that AM noise turns into PM noise
- Well established fact in quartz oscillators (Colpitts and other schemes)
- Similar phenomenon occurs in other types of (sustaining) amplifier

R. Brendel & E. Rubiola, Proc. 2007 IFCS p.1099-1105, Geneva CH, 28 may - 1 Jun 2007

Correlation between AM and PM noise

R. Boudot, E. Rubiola, arXiv:1001.2047v1, Jan 2010. Also IEEE T MTT (submitted)





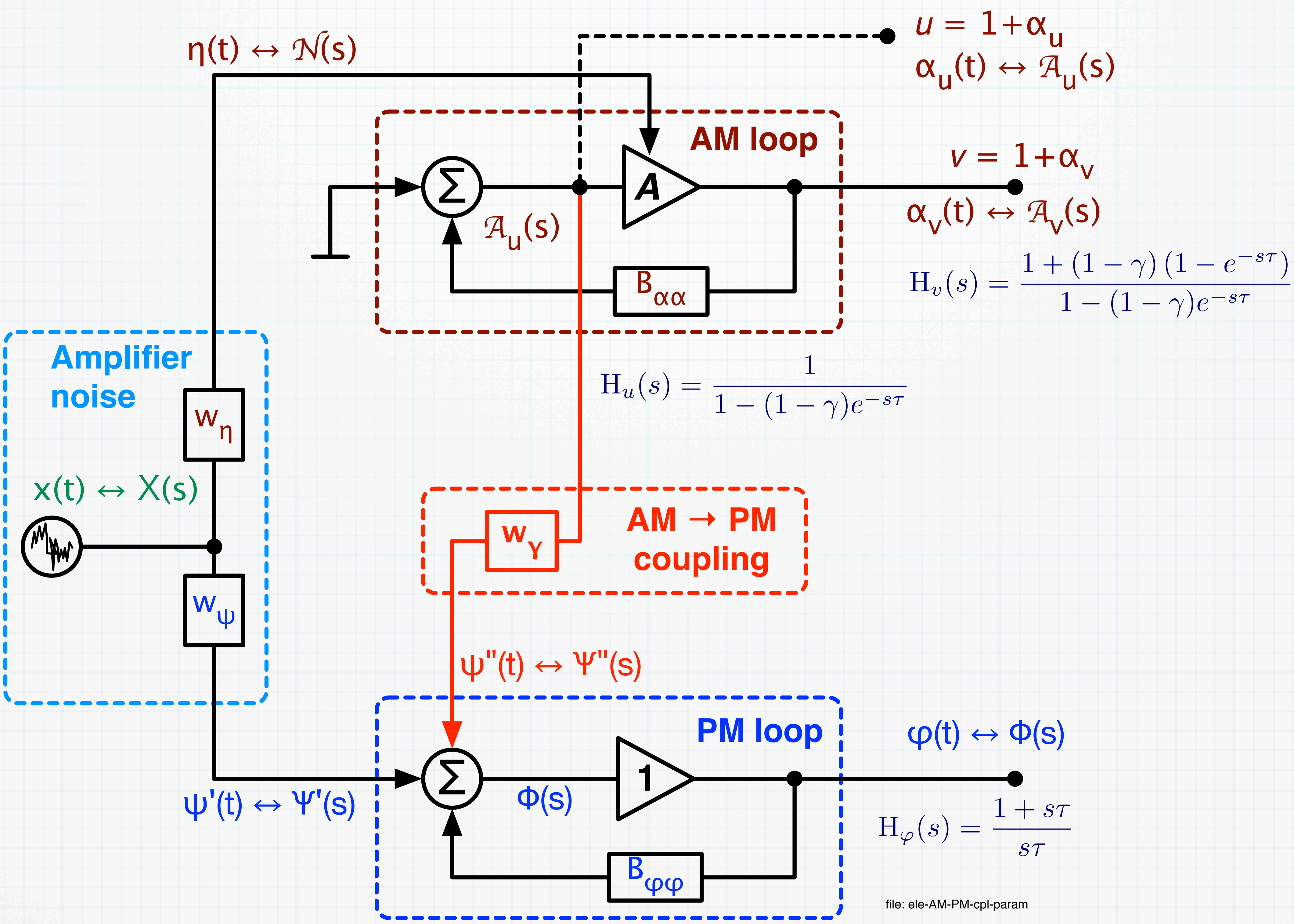
$$a^2 + b^2 + c^2 + d^2 = 1$$

The need for this model comes from the physics of popular amplifiers

- Bipolar transistor. The fluctuation of the carriers in the base region acts on the base thickness, thus on the gain, and on the capacitance of the reverse-biased base-collector junction.
- Field-effect transistor. The fluctuation of the carriers in the channel acts on the drain-source current, and also on the gate-channel capacitance because the distance between the `electrodes' is affected by the channel thickness.
- Laser amplifier. The fluctuation of the pump power acts on the density of the excited atoms, and in turn on gain, on maximum power, and on refraction index.

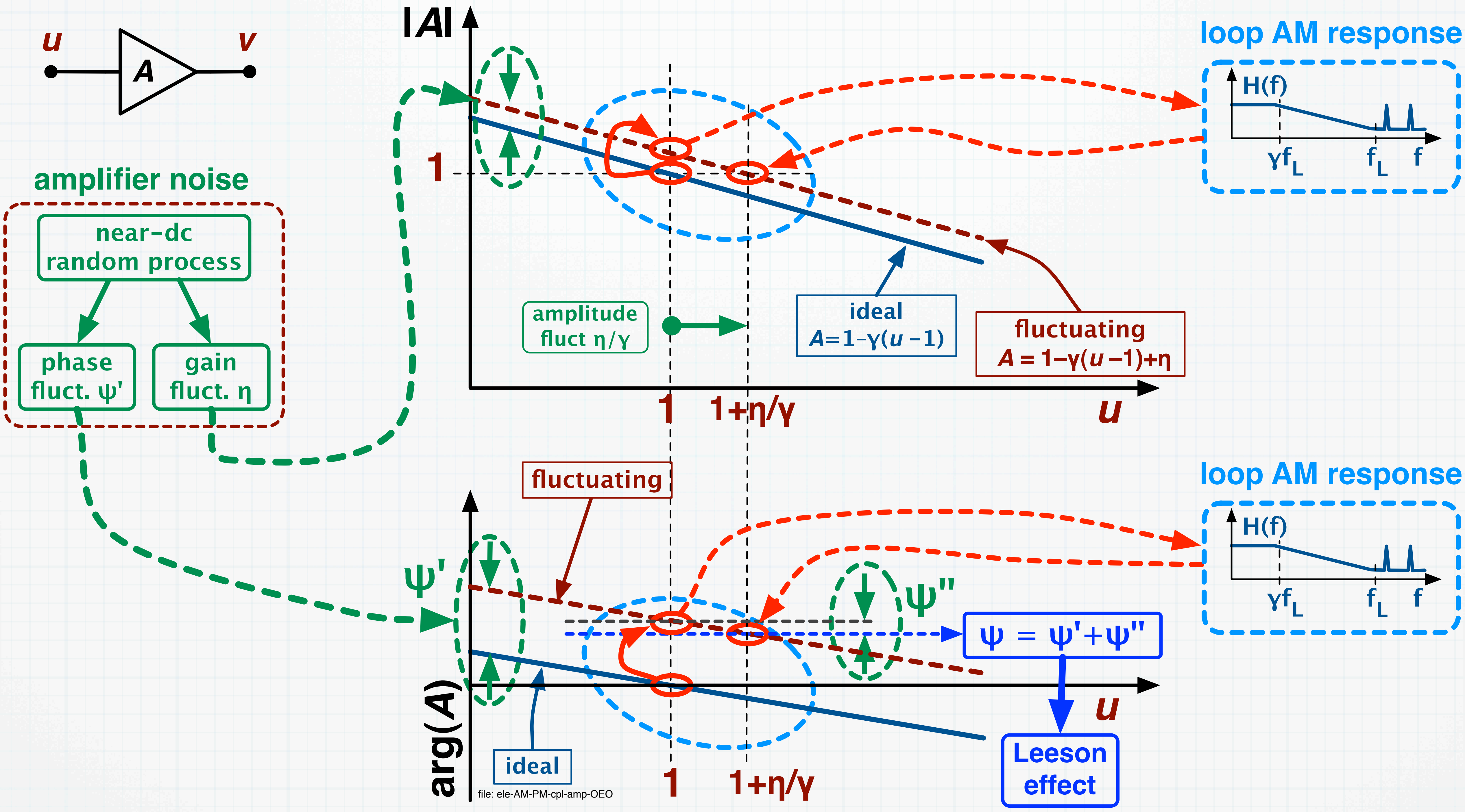
AM and PM fluctuations are correlated because originate from the same near-dc random process

Parametric noise & AM-PM noise coupling



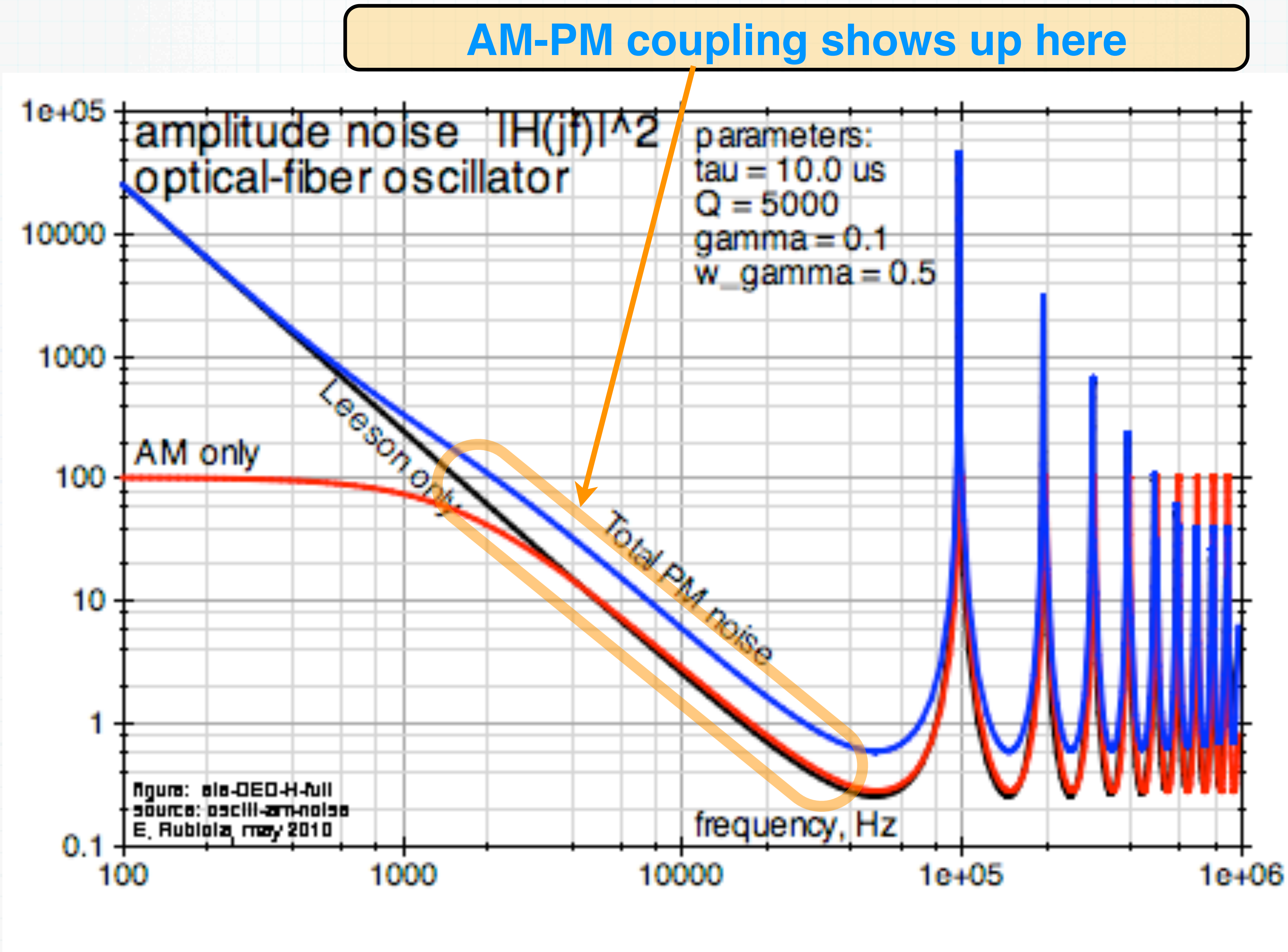
E. Rubiola & R. Brendel, arXiv:1004.5539v1 [physics.ins-det]

Effect of AM-PM noise coupling



E. Rubiola & R. Brendel, arXiv:1004.5539v1 [physics.ins-det]

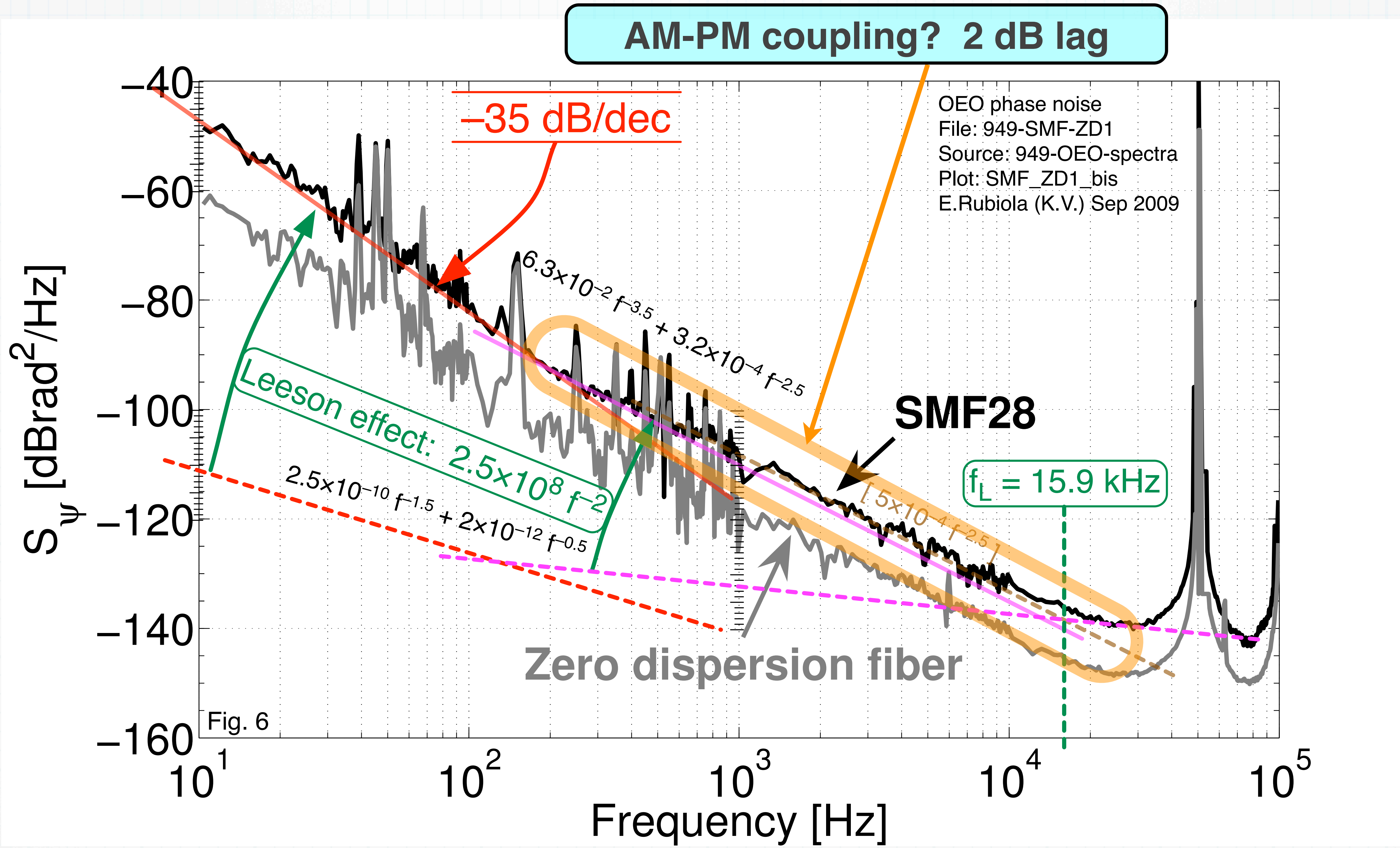
Noise transfer function and spectra



Notice that the AM-PM coupling can increase or decrease the PM noise

In a real oscillator, flicker noise shows up below some 10 kHz
In the flicker region, all plots are multiplied by 1/f

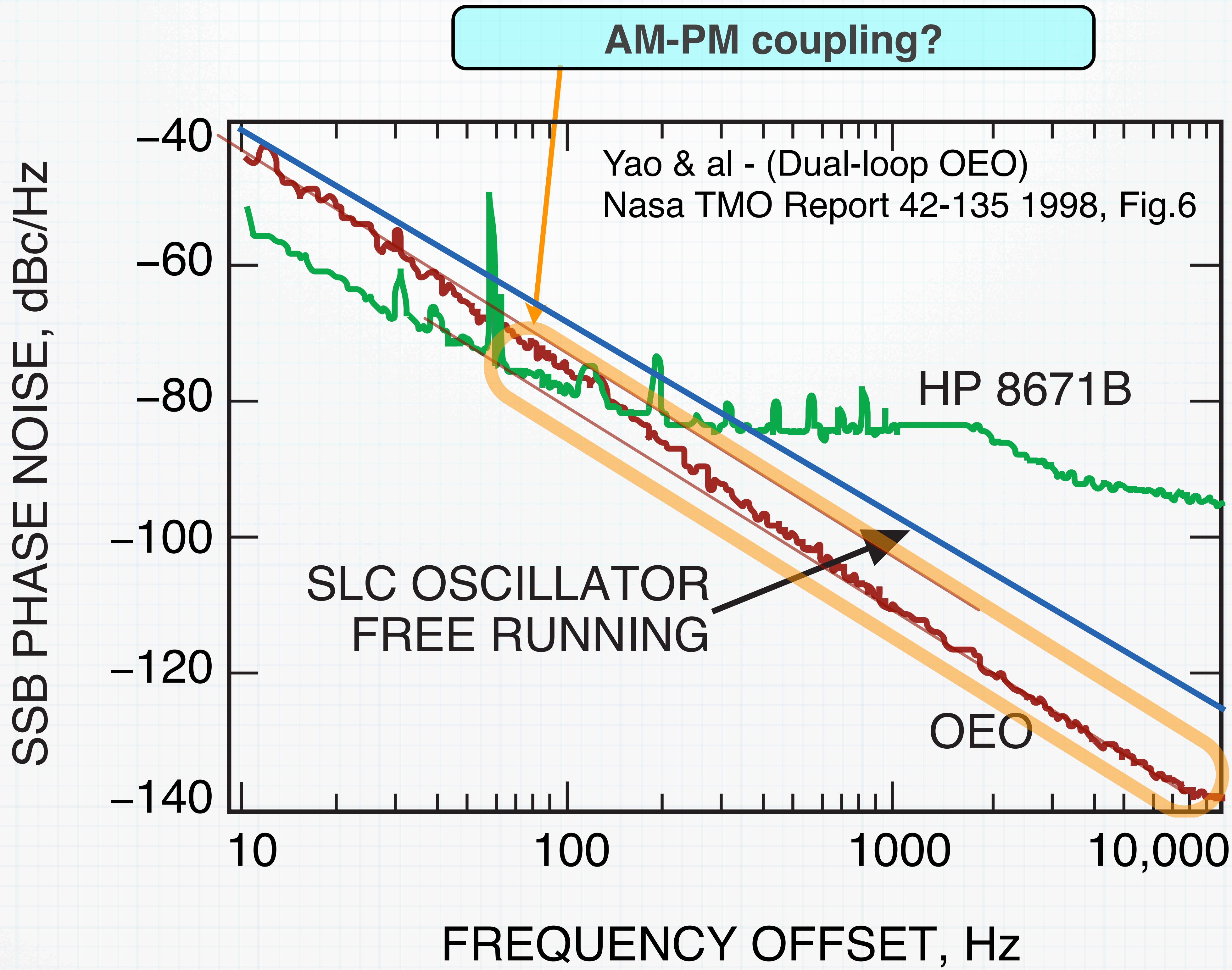
Noise spectra



Unfortunately, the awareness of this model come after the end of the experiments

Spectrum from K. Volyanskiy & al., IEEE JLT (Submitted, Apr. 2010)

Noise spectra



X.S.Yao & al., NASA TMO Report 42-135 (1998), Fig. 6

Conclusions

- **Well-established framework, fully tested with PM noise**
- **Extension of the Leeson effect to the oscillator AM noise**
- **Simple analytical model and theory**
- **The results from the Laplace/Heaviside approach (ER) are in close agreement with the results from the Langevin/diffusion approach (YKC), yet developed independently**
- **Most of the relevant conclusions were anticipated by ER & RB (2007 IFCS, and E. Rubiola & R. Brendel, arXiv:1004.5539v1 [physics.ins-det]), and now adapted to the OEO**
- **AM-PM coupling can *increase* or *decrease* parametric PM noise in the region between γf_L and f_L (≈1 decade)**
- **The theory of AM-PM coupling needs experiments**

This work is partially supported by the contract CNES/60281/00

home page http://rubiola.org