Tutorial seminar

Part 1: Tuesday March 16 2010, 10:30 AM (introduction)

Part 2: Thursday March 18 2010, 10:30 AM (bulk matter)

Observatoire de Besançon conference room, 41 Avenue de l'Observatoire

The magic of cross-correlation in measurements from dc to optics

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The measurement of power spectra is an important issue many branches of science, like astronomy, biology, chemistry, electrical engineering, mechanics, optics, physics, . . .), for it deserves be included in the general background of the experimentalist. Ultimate sensitivity is obtained by correlating two independent measurements of the same phenomenon.

Part 1: The FFT analyzer and the measurement of power spectra

Introductory material, to the benefit of technicians, PhD students and young scientists

Inside the FFT analyzer: general wisdom, amplitude and frequency resolution, background noise, acquisition time, taper (window) functions, averaging, etc. However there is nothing new in this introduction, it took me ten years to learn this stuff from experiments. If you can calculate the instrument background noise in three lines of algebra go straight to Part 2.

Part 2: The cross-spectrum method and its applications

A physical quantity $c(t)$ is measured with two separate instruments, each of which adds its noise. Thus, the available signals are $x(t) = c(t) + a(t)$ and $y(t) = c(t) + b(t)$, where $a(t)$ and $b(t)$ are the instrument noise. All the signals are assumed to be stationary and ergodic, which means that the physical experiment is repeatable and reproducible. By correlating and averaging the two outputs $x(t)$ and $y(t)$, and assuming that the two instruments are independent, it is possible to extract the statistical properties of $c(t)$ and to reject the instrument noise. Thanks to the Wiener-Khinchin theorem, the average product of the Fourier transform of $x(t)$ and $y(t)$ converges to the power spectrum of $c(t)$.

The single-channel noise is rejected proportionally to the square root of the number m of averages, and ultimately to the square root of the measurement time. Of course, the two channels must be independent. The background noise is limited by the thermal inhomogeneity of the system instead of the absolute temperature. The observation of the cross-spectrum as a function of m enables the validation of the result in some weird cases, in which a low-noise reference is not available (AM noise, laser RIN, etc.).

The cross-spectrum method is the basis of the correlation receiver used in radio-astronomy, with which R. Hanbury-Brown measured the first radio sources in the Cassiopeia and Cygnus constellations. The the correlation radiometer followed, opening the way to the re-definition of the temperature in terms of fundamental constants. Batteries and other dc references has been measured in this way, and of course the PM and AM noise of RF/microwave signals, microwave photonic signals, and laser RIN. In semiconductors, small random signals reveal impurities, defects and energy traps of a sample. Another exotic application is the measurement of electromigration in metals at high current density, through the asymmetry between AM and PM $1/f$ noise, which impacts on VLSI technology.

This is a part of my book project on experimental method in AM-PM noise and frequency stability.
Full text: E. Rubiola, F. Vernotte, arXiv:1003.0113v1 [physics.ins-det], http://arxiv.org/abs/1003.0113.
Slides available on http://rubiola.org.