MAX-PLANCK-INSTITUT
FÜR QUANTENOPTIK

MPQ

Short course on stable oscillators

December 2009

Enrico Rubiola

FEMTO-ST Institute, CNRS and UFC, Besancon, France

Contents

home page http://rubiola.org

Acknowledgements

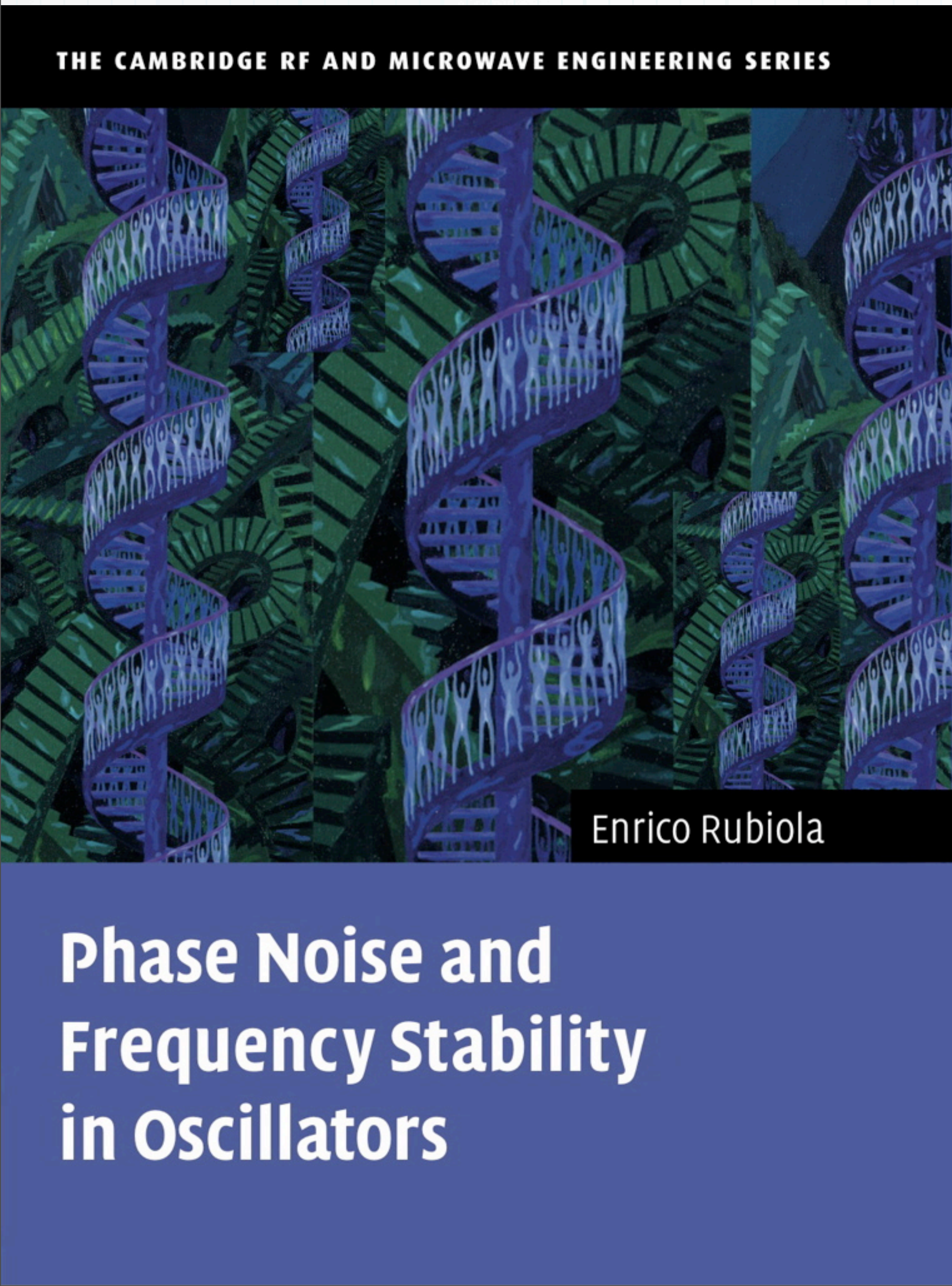
I am indebted to prof. Theodor W. Hänsch for inviting me at MPQ

I owe gratitude to dr. Thomas Udem for having proposed and organized my visits at MPQ, and for a wealth of discussions and experience that followed

I am grateful to Lute Maleki and to John Dick for numerous discussions during my visits at the NASA JPL, which are the first seed of my approach to the oscillator noise

This material would never have existed without continuous help and support of Vincent Giordano, FEMTO-ST, during more than a dozen of years

Phase noise and frequency stability in oscillators



Cambridge University Press, November 2008

ISBN 978-0-521-88677-2

Cambridge announced the paperback edition

Contents

- Forewords (L. Maleki, D. B. Leeson)
- Phase noise and frequency stability
- Phase noise in semiconductors & amplifiers
- Heuristic approach to the Leson effect
- Phase noise and feedback theory
- Noise in delay-line oscillators and lasers
- Oscillator hacking
- Appendix

Another book is in progress, on the **Experimental methods for the measurement of AM/PM noise**

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Short course on Stable oscillators

— Part 1 —

General metrology of amplitude and phase noise

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Outline

- Basics & definitions
- PM noise in systems
- The saturated mixer
- The measurement of the oscillator PM noise
- Advanced topics (including AM noise)

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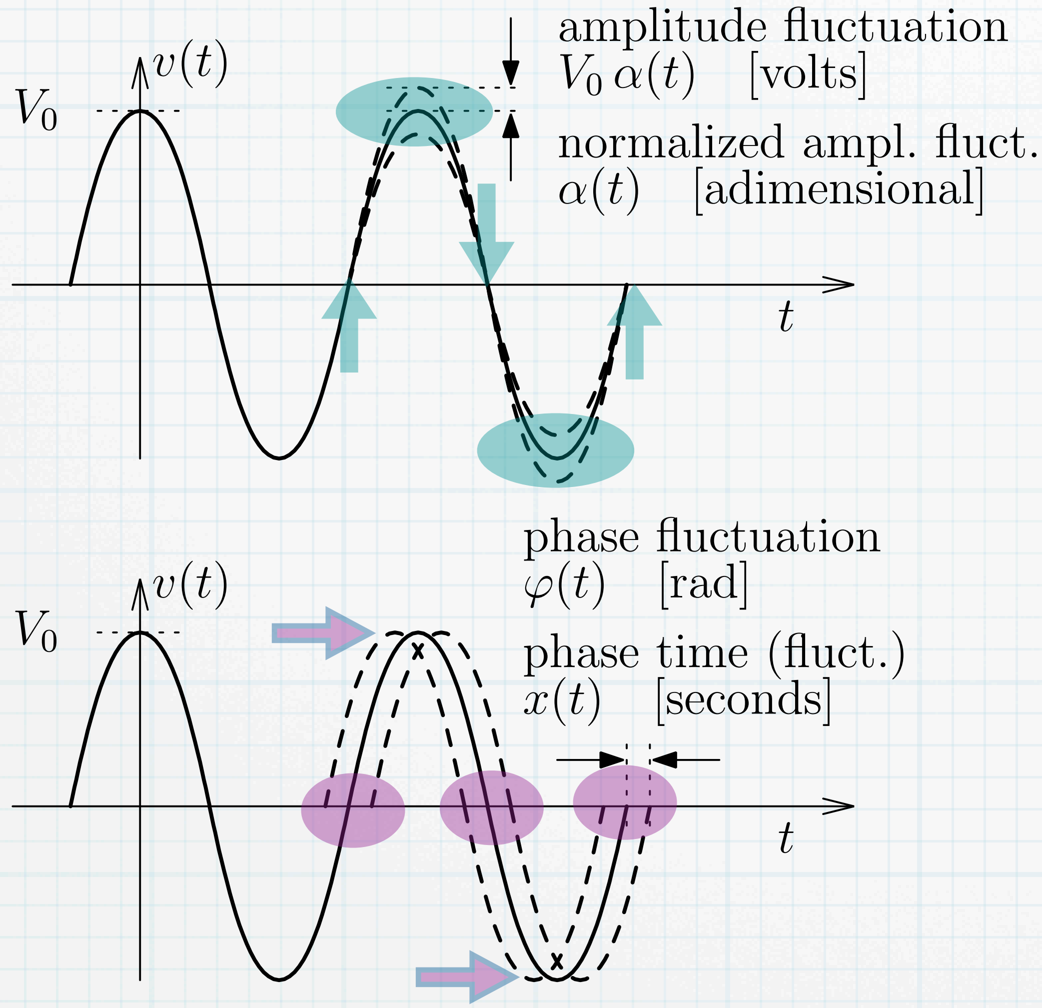
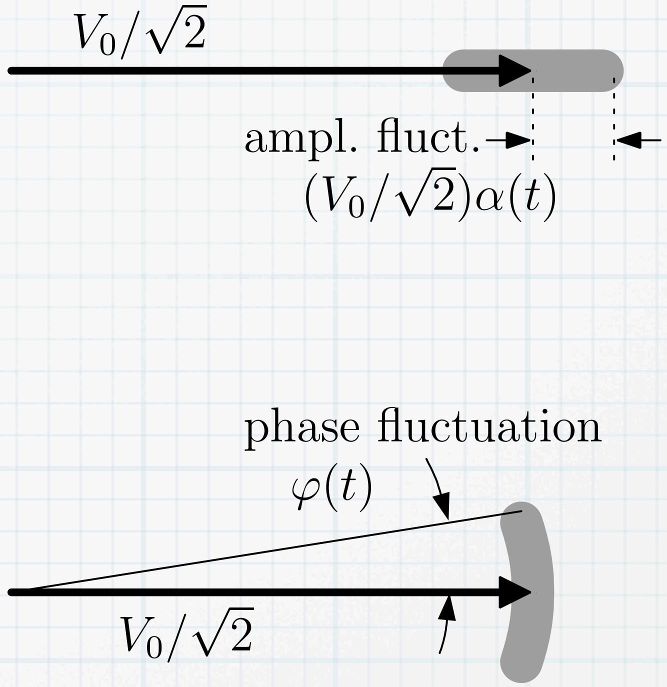
Basics

Clock signal affected by noise

Time Domain



Phasor Representation



polar coordinates $$v(t) = V_0\,[1+\alpha(t)]\cos\left[\omega_0 t + \varphi(t)\right]$$

Cartesian coordinates $$v(t) = V_0 \cos\omega_0 t + n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t$$

under low noise approximation

$$|n_c(t)| \ll V_0 \quad \text{and} \quad |n_s(t)| \ll V_0$$

It holds that

$$\alpha(t) = \frac{n_c(t)}{V_0} \quad \text{and} \quad \varphi(t) = \frac{n_s(t)}{V_0}$$

Physical quantities

$$v(t) = V_0 \left[1 + \alpha(t)\right] \cos\left[2\pi\nu_0 t + \varphi(t)\right]$$

Allow φ(t) to exceed ±π and count the number of turns, so that φ(t) describes the clock fluctuation in full

radian

phase noise $\varphi(t)$

$$(\Delta\nu)(t) = \frac{1}{2\pi}\,\frac{d\varphi(t)}{dt}$$

Hertz

$(\Delta\nu)(t)$ frequency fluctuation

$$x = \frac{\varphi}{2\pi\nu_0}$$

$$y(t) = \frac{1}{\nu_0}(\Delta\nu)(t)$$

$$y(t) = \frac{dx(t)}{dt}$$

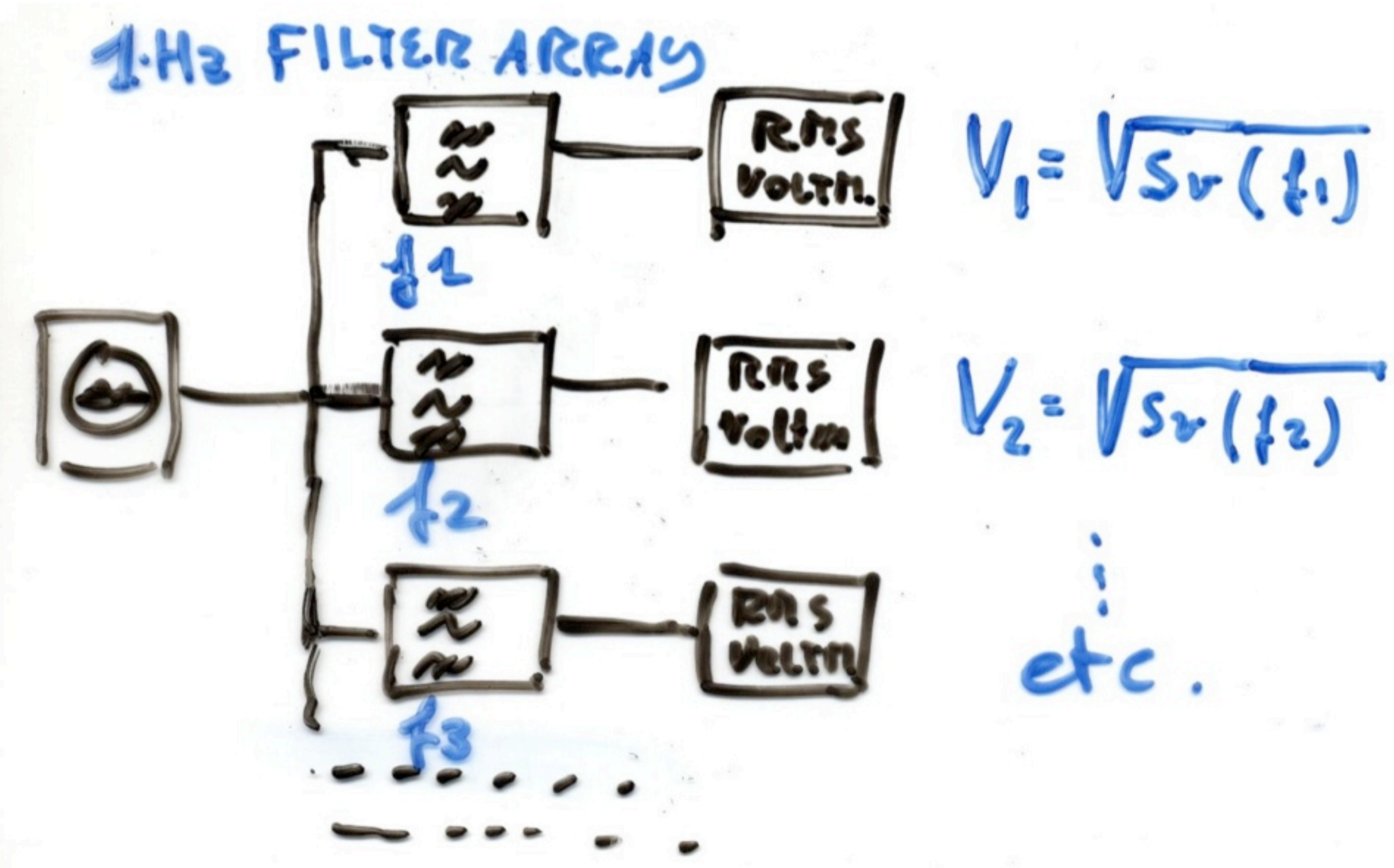
phase-time (fluctuation) $x(t)$

second

$y(t)$ fractional-frequency fluctuation

adimensional

The power spectral density



The power spectral density extends the concept of root-mean-square value to the frequency domain

Phase noise & friends

$$v(t) = V_p\,[1+\alpha(t)]\cos\left[2\pi\nu_0 t + \varphi(t)\right]$$

random phase fluctuation

$$S_\varphi(f) = \text{PSD of } \varphi(t)$$

power spectral density

it is measured as

$$S_\varphi(f) = \mathbb{E}\left\{\Phi(f)\Phi^*(f)\right\} \quad \text{(expectation)}$$

$$S_\varphi(f) \approx \left\langle \Phi(f)\Phi^*(f)\right\rangle_m \quad \text{(average)}$$

$$\mathcal{L}(f) = \frac{1}{2}S_\varphi(f) \ \ \text{dBc}$$

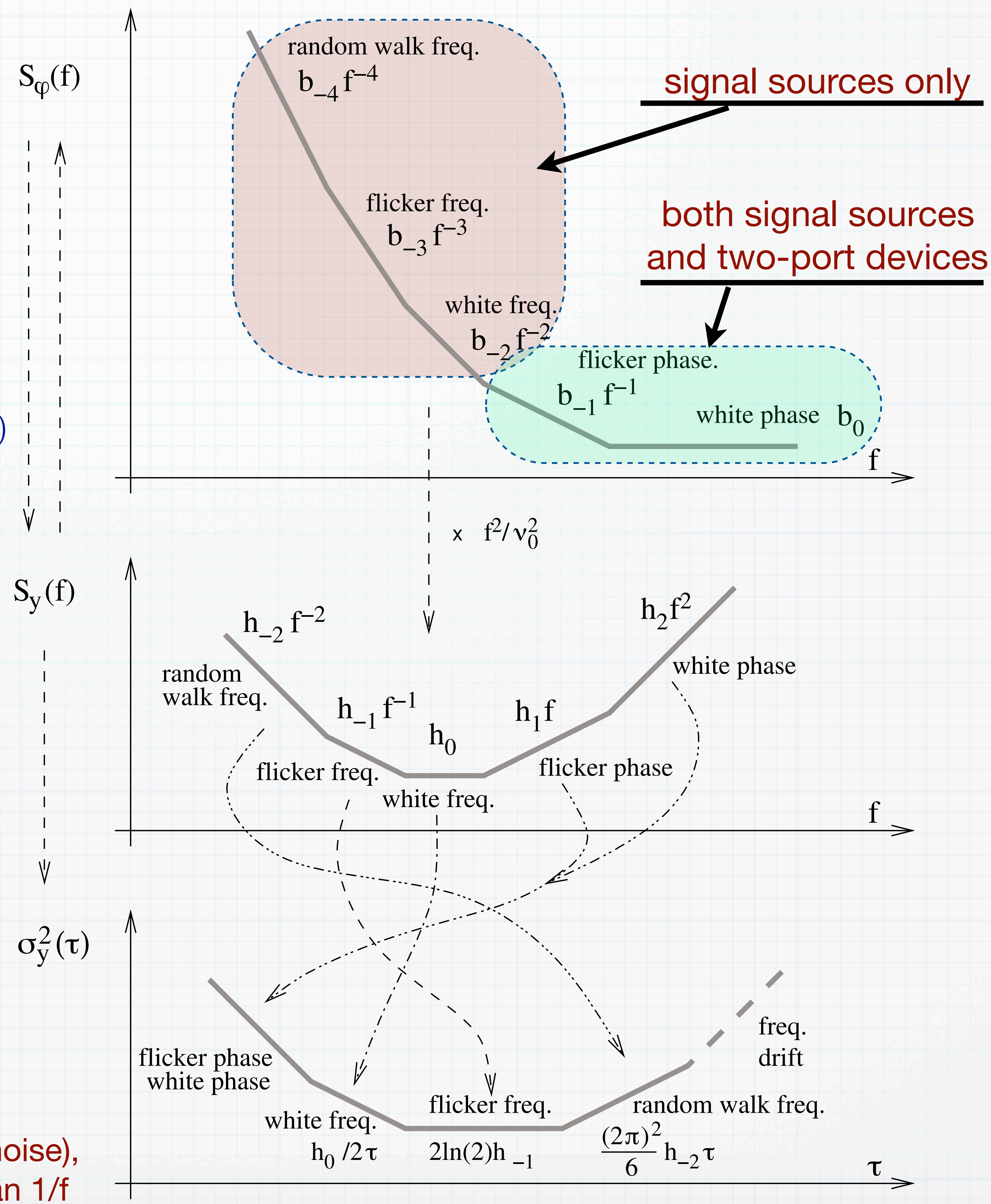
random fractional-frequency fluctuation

$$y(t) = \frac{\dot\varphi(t)}{2\pi\nu_0} \quad \Rightarrow \quad S_y = \frac{f^2}{\nu_0^2}S_\varphi(f)$$

Allan variance
(two-sample wavelet-like variance)

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{y}_{k+1} - \overline{y}_k\right]^2\right\} \ .$$

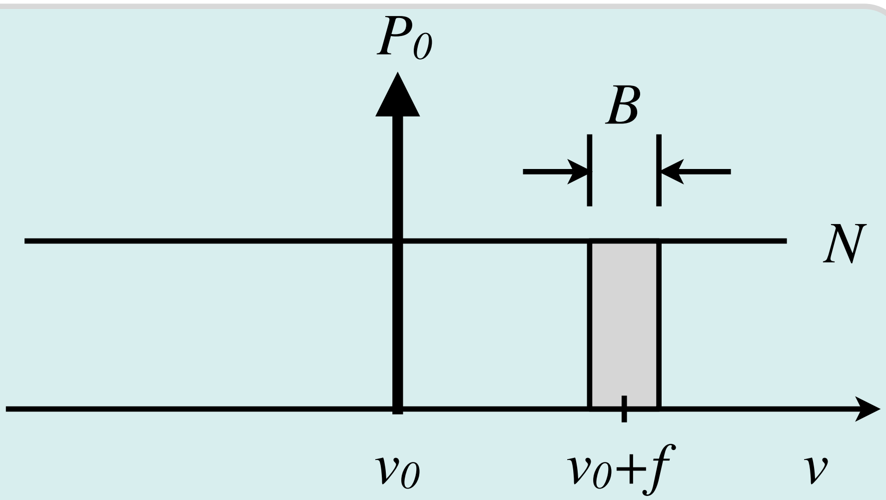
approaches a half-octave bandpass filter (for white noise),
hence it converges even with processes steeper than 1/f



$\mathcal{L}(f)$ (re)defined

The first definition of $\mathcal{L}(f)$ was

$\mathcal{L}(f)$ = (SSB power in 1Hz bandwidth) / (carrier power)



The problem with this definition is that it does not divide AM noise from PM noise, which yields to ambiguous results

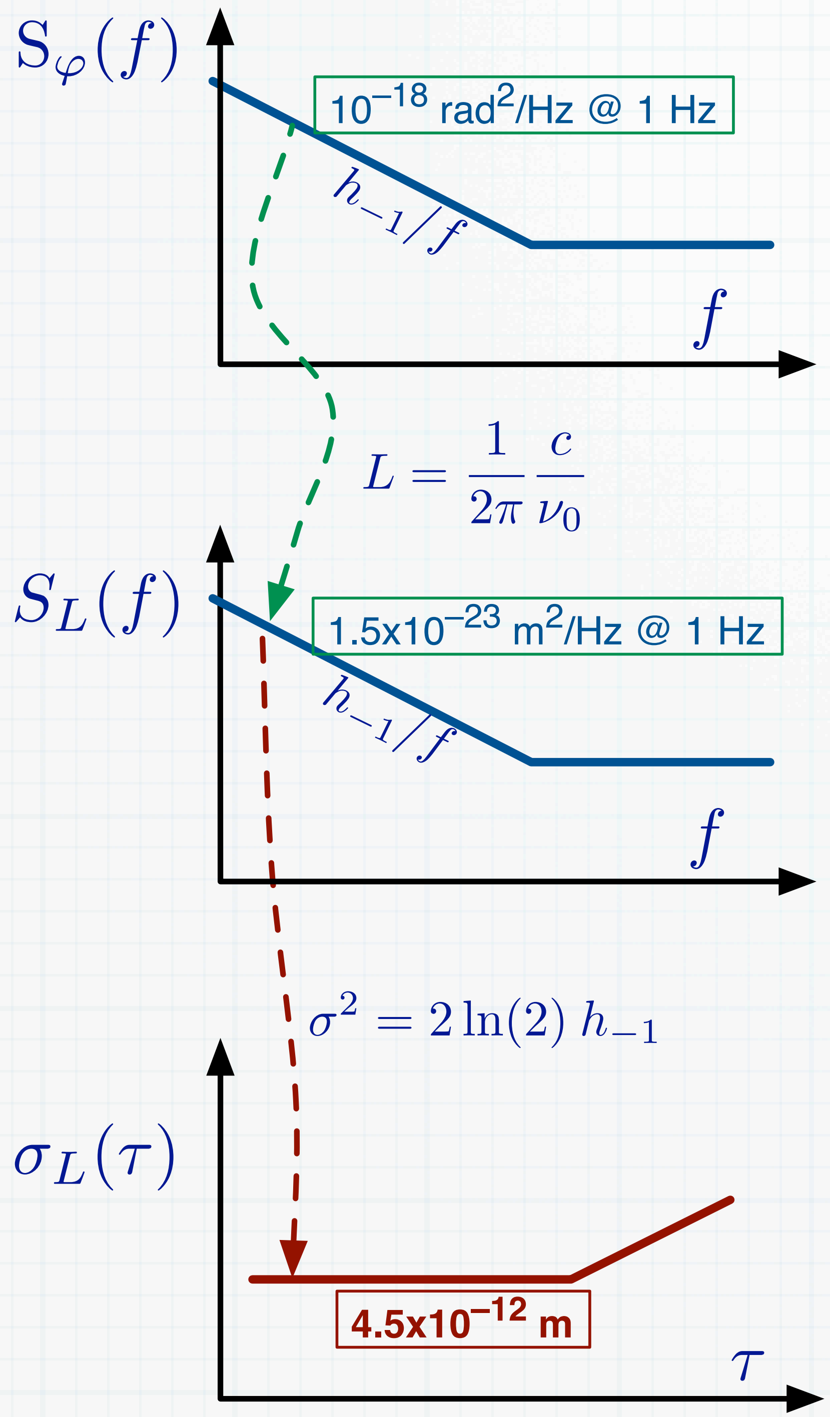
Engineers (manufacturers even more) like $\mathcal{L}(f)$

The IEEE Std 1139-1999 redefines $\mathcal{L}(f)$ as

$$\mathcal{L}(f) = (1/2) \times S_\varphi(f)$$

Mechanical stability

b₋₁/f is replaced with h₋₁/f because the Allan variance formulae are written with h₋₁



Any phase fluctuation can be converted into length fluctuation

$$L = \frac{\varphi}{2\pi}\frac{c}{\nu_0}$$

$b_{-1} = -180$ dBrad^2/Hz and $\nu_0 = 10$ GHz is equivalent to $S_L = 1.46\times10^{-23}$ m^2/Hz at f = 1 Hz

Any flicker spectrum h_{-1}/f can be converted into a flat Allan variance

$$\sigma_L^2 = 2\ln(2)\,h_{-1}$$

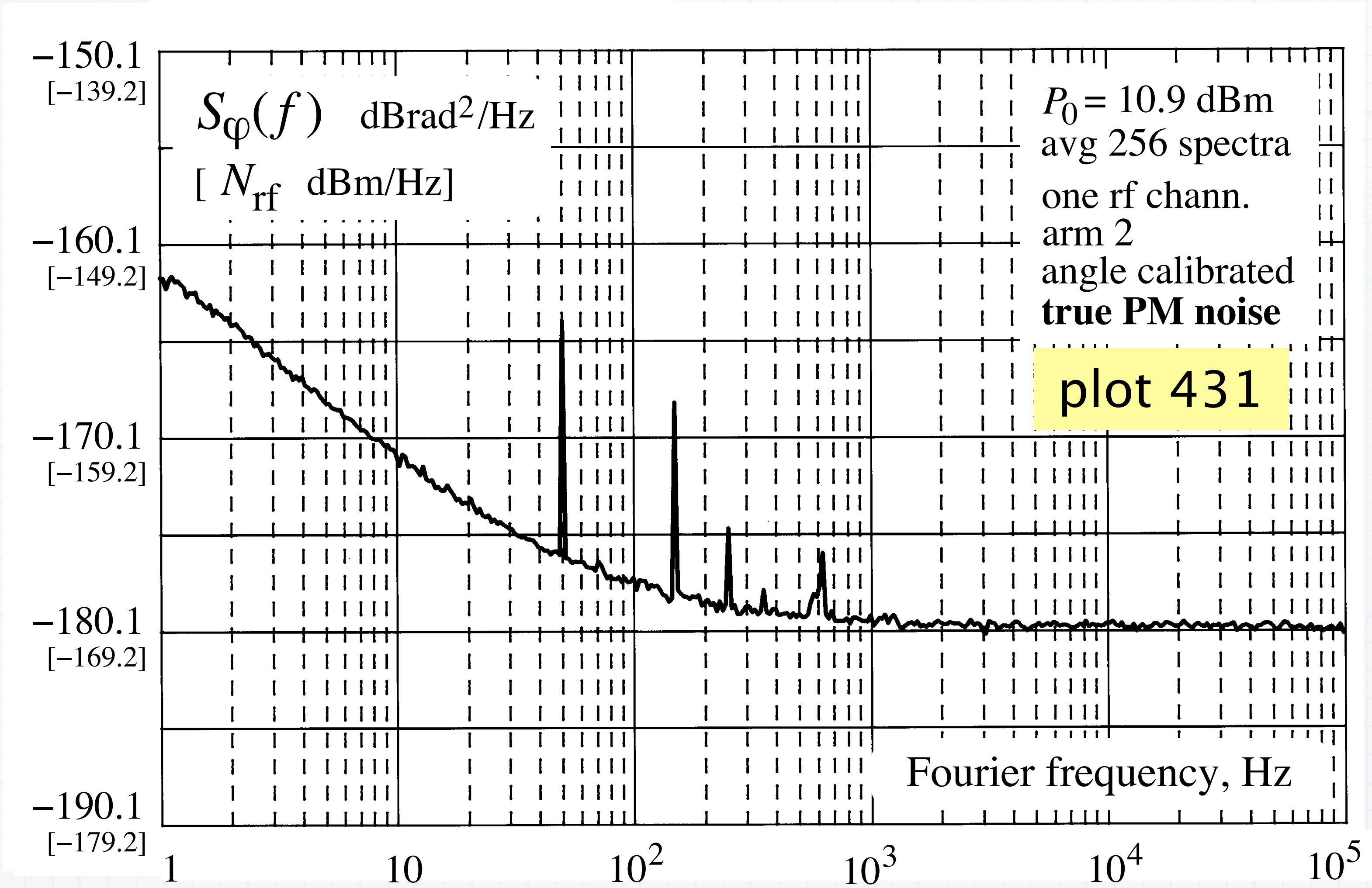
A residual flicker of –180 dBrad^2/Hz at f = 1 Hz off the 10 GHz carrier is equivalent to

$\sigma^2 = 2\times10^{-23}$ m^2 thus $\sigma = 4.5\times10^{-12}$ m

for reference, the Bohr radius of the electron is R = 0.529 Å

- **Don’t think “this is just engineering” !!!**
- **Learn from non-optical microscopy (bulk matter, 5×10^{-14} m)**
- **Careful DC section (capacitance and piezoelectricity)**
- **The best advice is to be *at least* paranoiac**

Averaged spectra must be smooth



Rice representation

$$v(t) = \sum_{n=0}^{\infty} a_n(t)\cos(n\omega_0 t) - b_n(t)\sin(n\omega_0 t)$$

$$S_v(n\omega_0) = \left[a_n^2 + b_n^2\right]/\omega_0$$

$a_n(t)$ and $b_n(t)$ contain the noise in the $\omega_0/2$ band centered at $n\omega_0$

stationary & ergodic process (means repeatable and reproducible): the statistics of all $a_n(t)$ and $b_n(t)$ is the same

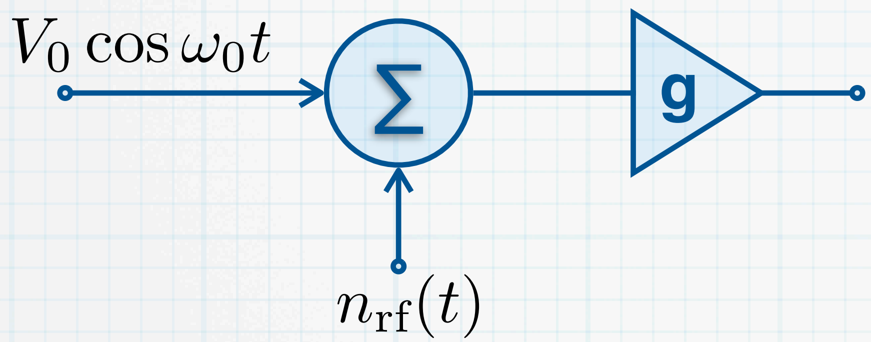
average on m spectra: confidence of a point improves by $1/m^{1/2}$

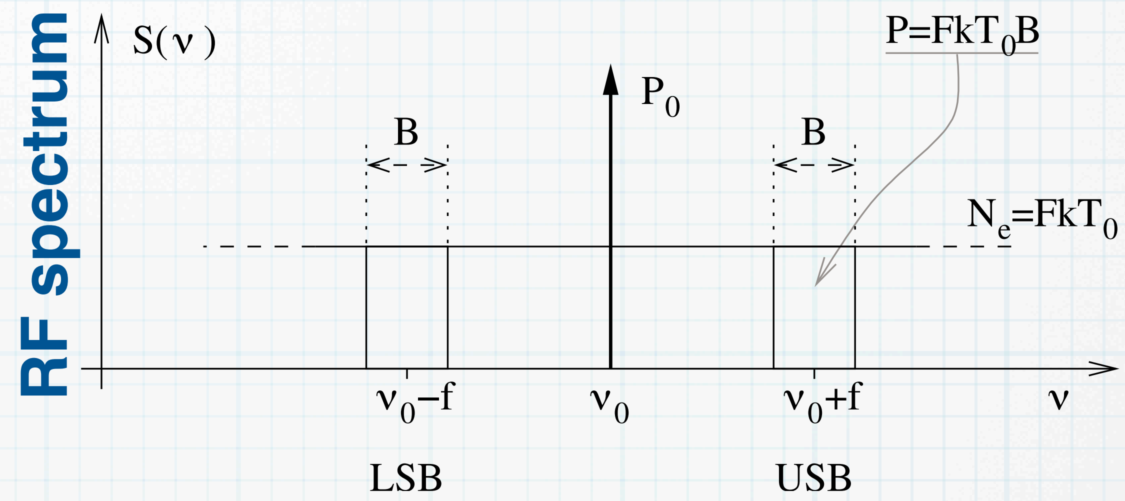
interchange ensemble with frequency: smoothness $1/m^{1/2}$

PM noise in systems

White noise

Noise figure F, Input power P_0



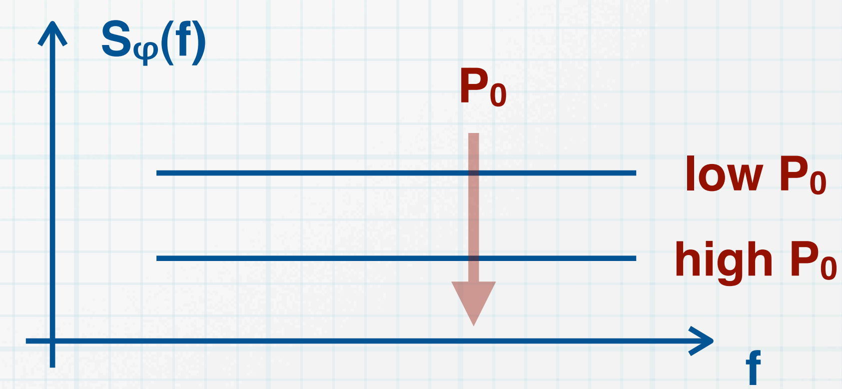


power law

$$S_\varphi = \sum_{i=-4}^{0} b_i f^i$$

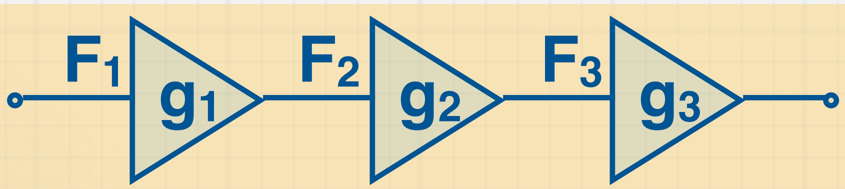
white phase noise

$$b_0 = \frac{FkT_0}{P_0}$$



Cascaded amplifiers (Friis formula)

The (phase) noise is chiefly that of the 1st stage



$$N = F_1 kT_0 + \frac{(F_2 - 1)kT_0}{g_1^2} + \ldots$$

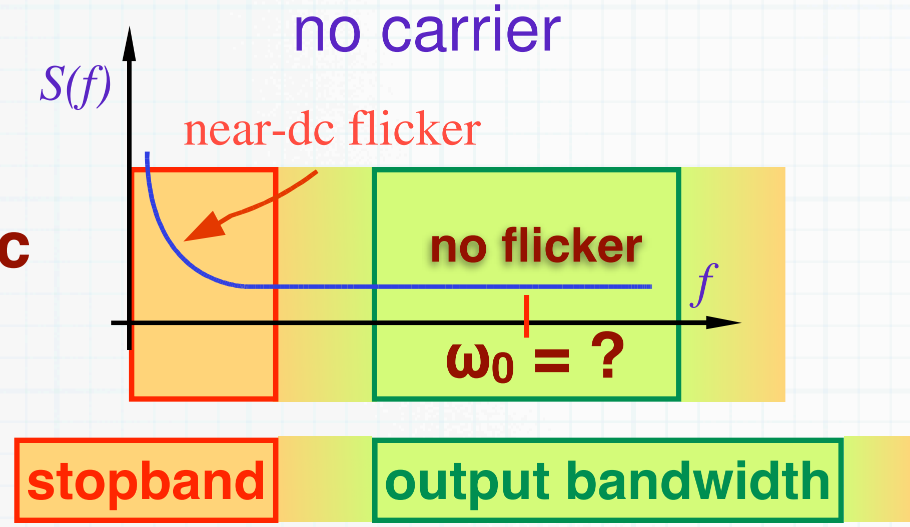
The Friis formula applied to phase noise

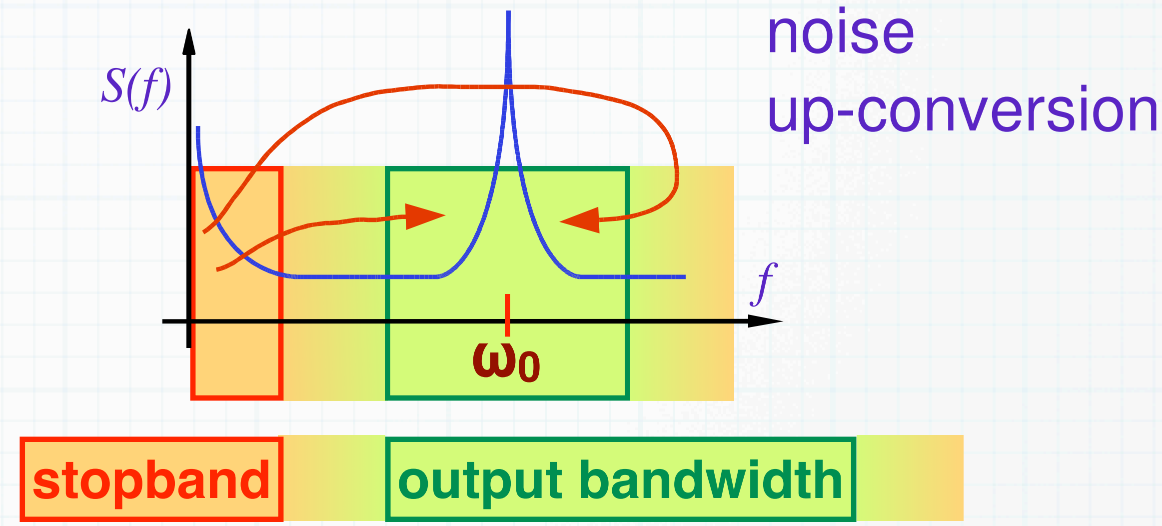
$$b_0 = \frac{F_1 kT_0}{P_0} + \frac{(F_2 - 1)kT_0}{P_0 g_1^2} + \ldots$$

H. T. Friis, Proc. IRE 32 p.419-422, jul 1944

Flicker noise

near-dc noise





carrier **near-dc noise**

$$v_i(t) = V_i\, e^{j\omega_0 t} + n'(t) + jn''(t)$$

the parametric nature of 1/f noise is hidden in n' and n''

substitute
(careful, this hides the down-conversion)

$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + \dots$$

non-linear (parametric) amplifier

expand and select the ω_0 terms

$$v_o(t) = V_i \Big\{ a_1 + 2a_2 \big[n'(t) + jn''(t) \big] \Big\} e^{j\omega_0 t}$$

The noise sidebands are proportional to the input carrier

get AM and PM noise

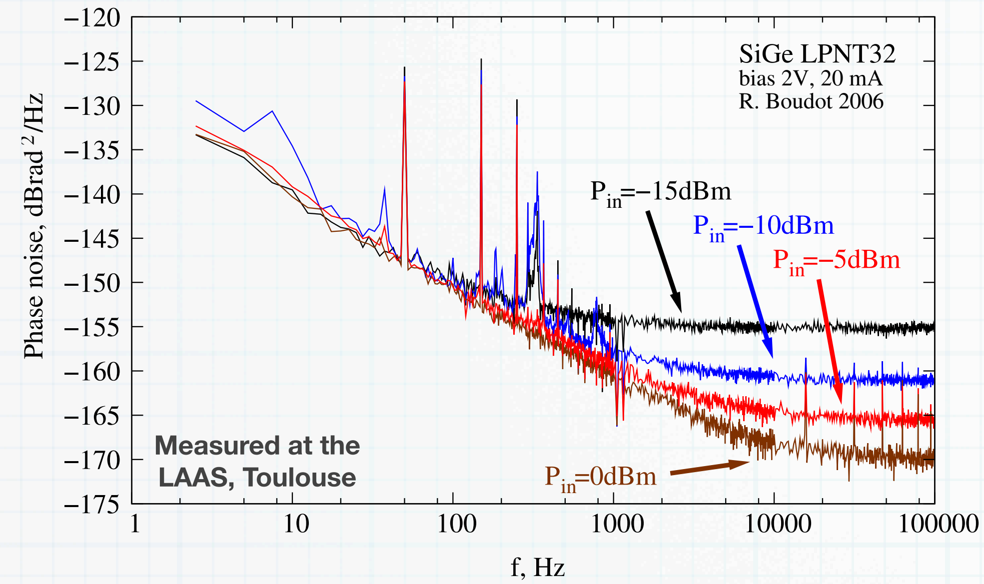
$$\alpha(t) = 2\,\frac{a_2}{a_1}\, n'(t) \qquad \varphi(t) = 2\,\frac{a_2}{a_1}\, n''(t)$$

The AM and the PM noise are independent of V_i , thus of power

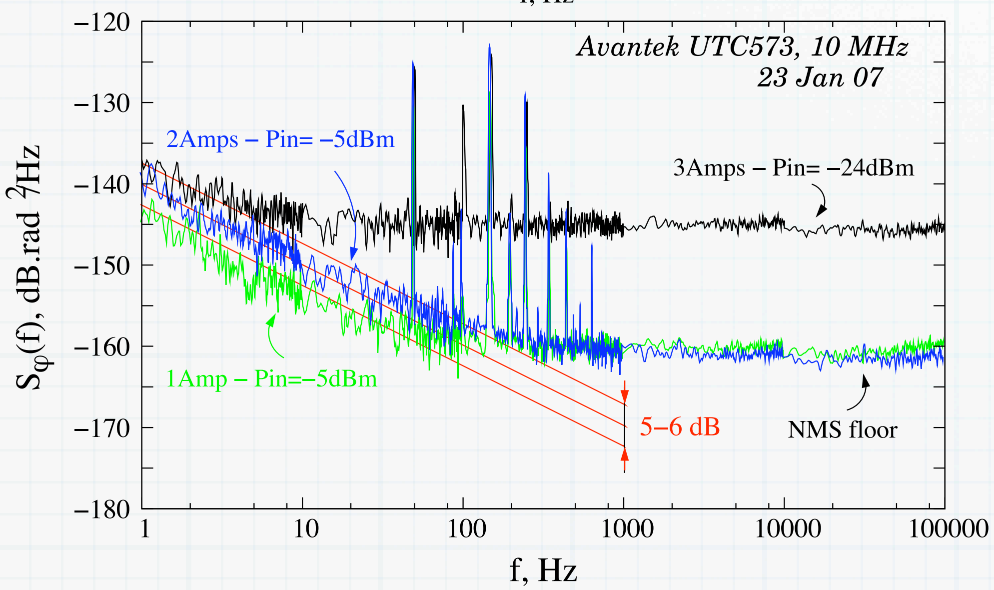
There is also a linear parametric model, which yields the same results

Amplifier flicker noise – experiments

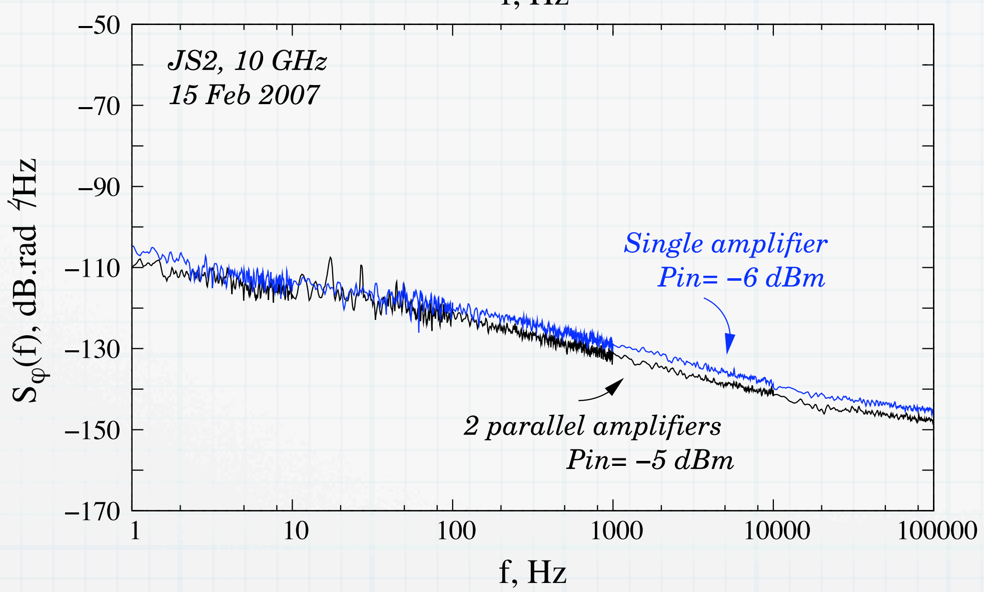
Phase noise vs. power



Cascaded amplifiers



Paralleled amplifiers



Phase noise vs. power

- The 1/f phase noise b_{-1} is about independent of power
- The white noise b_0 scales up/down as $1/P_0$, i.e., the inverse of the carrier power
- Describing the 1/f noise in terms of fc is misleading because fc depends on the input power

Phase noise of cascaded amplifiers

- The expected flicker of a cascade increases by:
  3 dB, with 2 amplifiers
  5 dB, with 3 amplifiers

Regenerative amplifiers

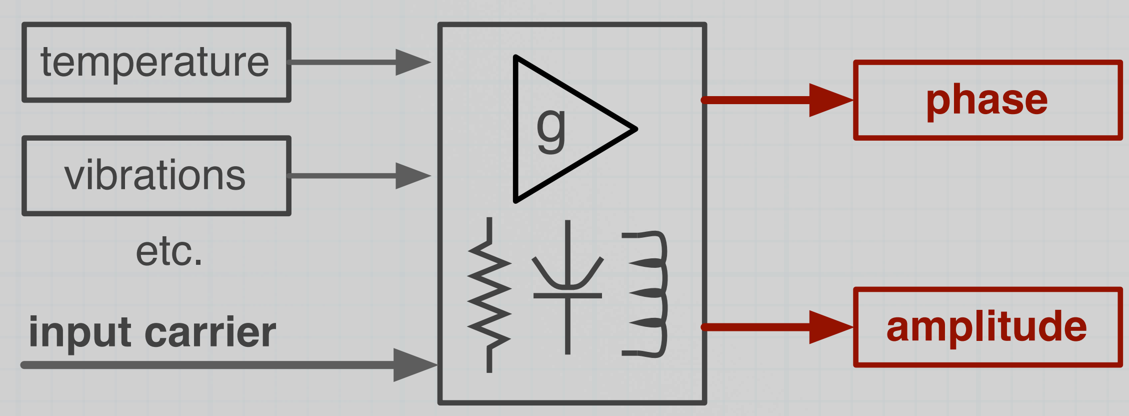
- Phase noise increase as the squared gain because the noise source at each roundtrip is correlated

Phase noise of paralleled amplifiers

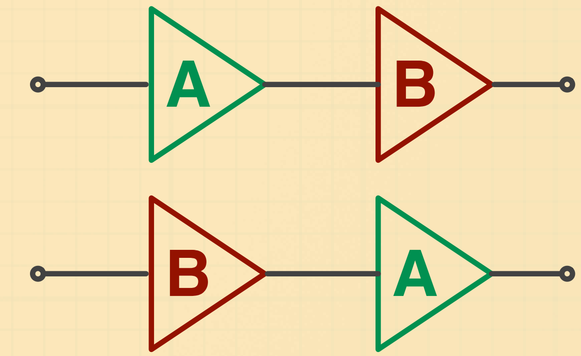
- Connecting two amplifiers in parallel, the phase-noise flicker is expected to decrease by 3 dB

The theory is fully confirmed on more amplifiers (E.Rubiola & R.Boudot)

Environmental noise



A time constant can be present



$\varphi = \varphi_A + \varphi_B$ and $\alpha = \alpha_A + \alpha_B$
regardless of the amplifier order

Cascading *m* equal amplifiers, $S_\alpha(f)$ and $S_\varphi(f)$ increase by a factor m^2.

If the amplifier were independent, $S_\alpha(f)$ and $S_\varphi(f)$ would increase only by a factor *m*.

It is experimentally observed that the temperature fluctuations cause a spectrum $S_\alpha(f)$ or $S_\varphi(f)$ of the $1/f^5$ type

Yet, at lower frequencies the spectrum folds back to 1/f

Cascaded amplifiers

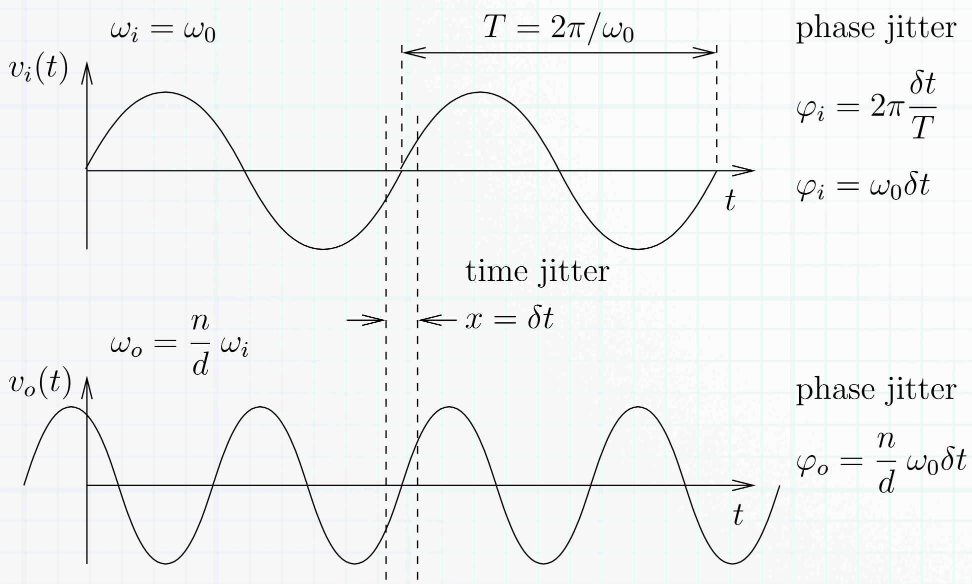
let $\quad z(t) = x(t) + y(t)$

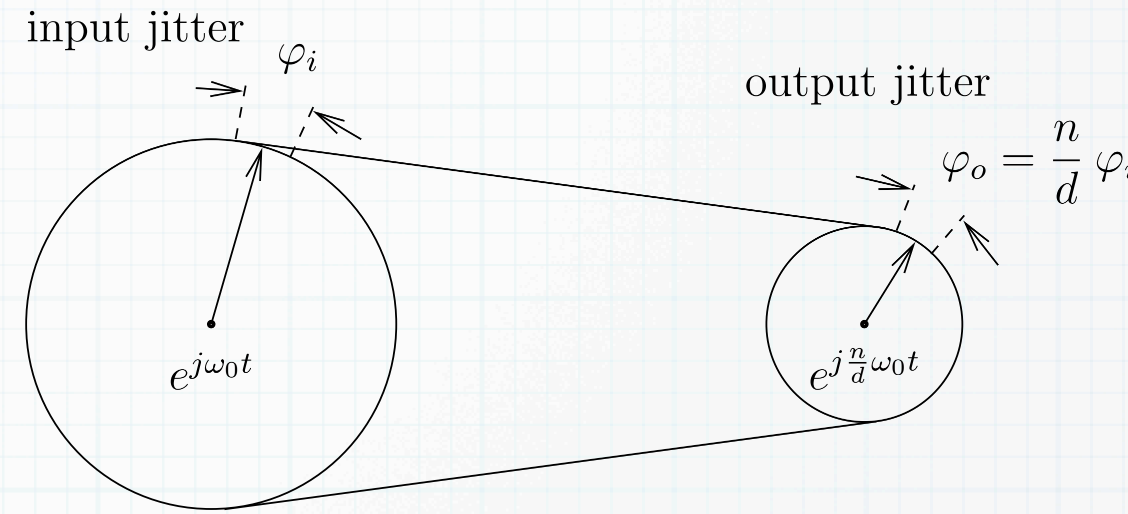
Phase noise

$$\begin{aligned} S_z(f) &= ZZ^* \\ &= (X+Y)\,(X+Y)^* \\ &= XX^* + YY^* + XY^* + YX^* \\ &= S_x + S_y + \underbrace{S_{xy}}_{>0} + \underbrace{S_{yx}}_{>0} \end{aligned}$$

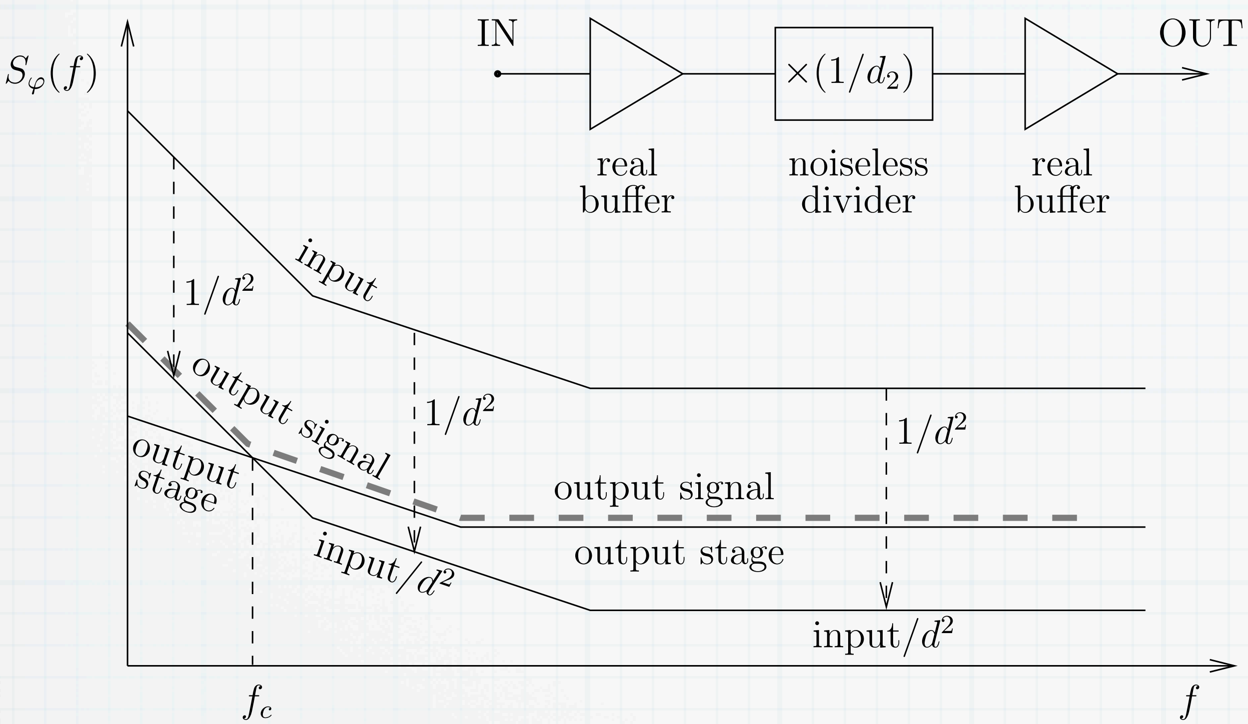
Frequency synthesis

The ideal noise-free frequency synthesizer repeats the input time jitter

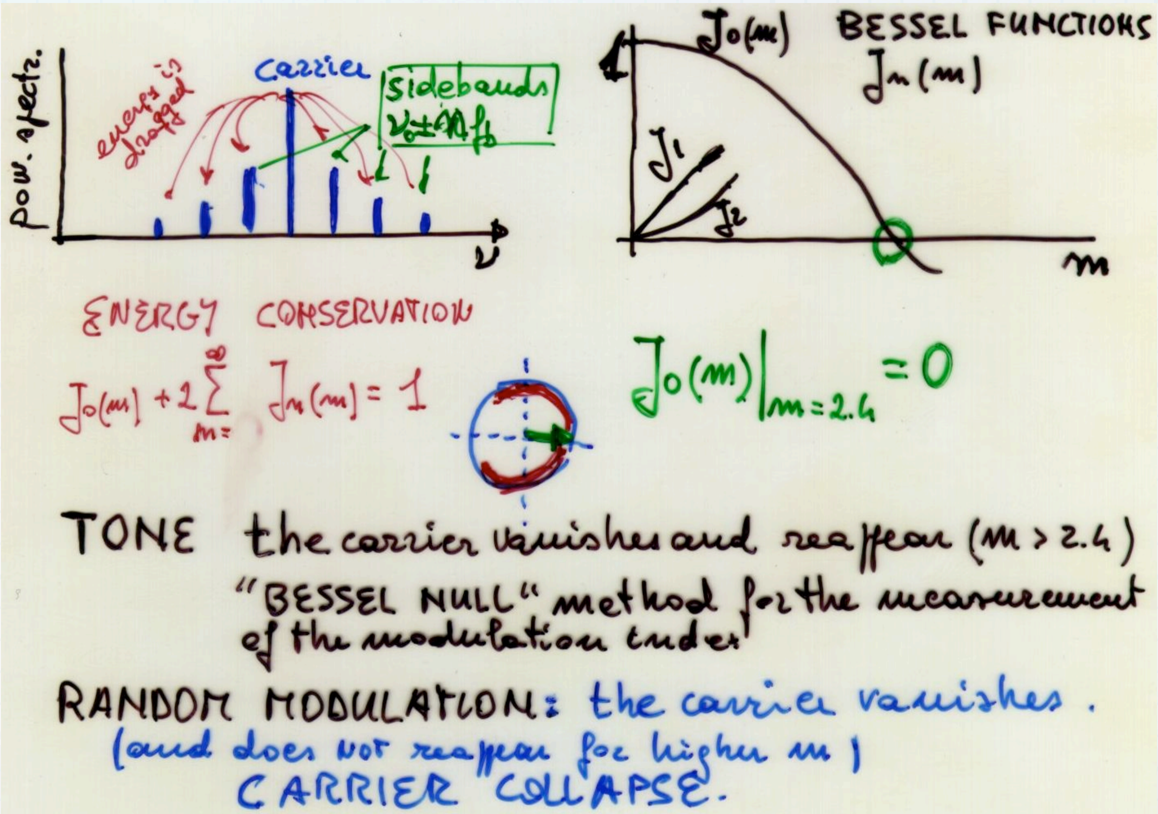




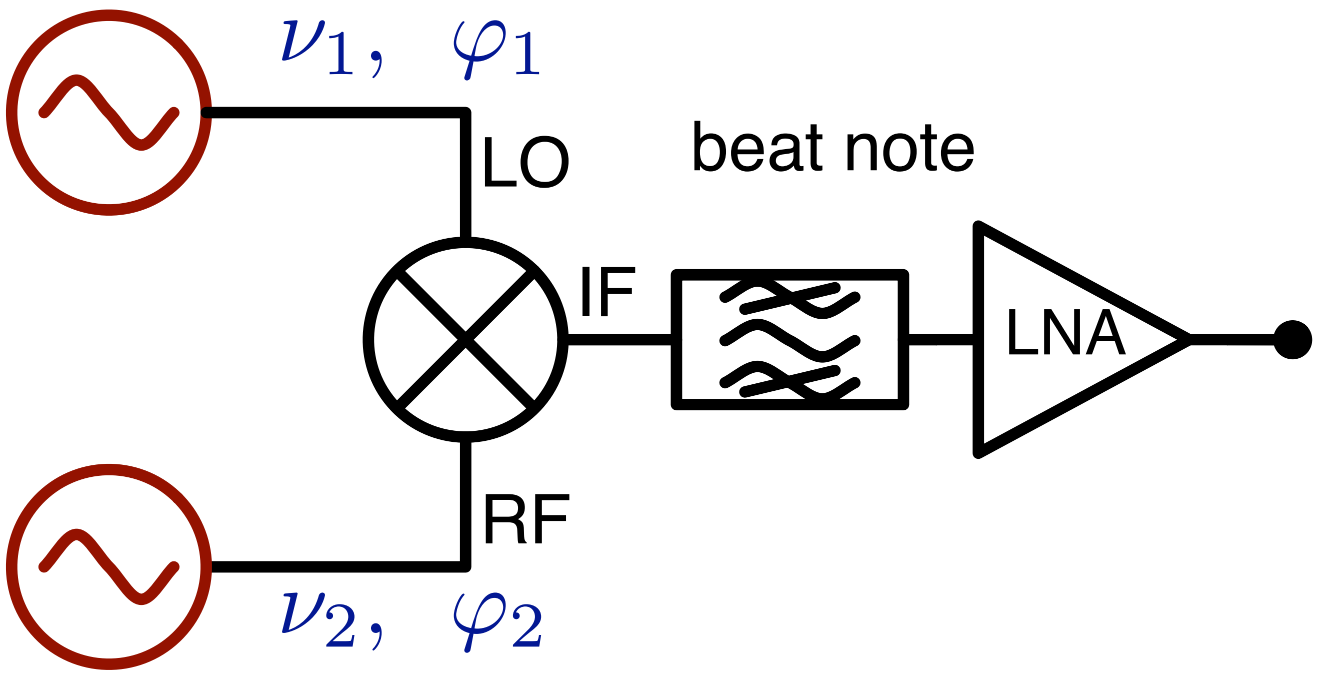
After division, the noise of the output buffer may be larger than the input-noise scaled down



After multiplication, the scaled-up phase noise sinks energy from the carrier. At m ≈ 2.4, the carrier vanishes



Beat note



$$\nu_b = \nu_2 - \nu_1$$

$$\varphi = \varphi_2 - \varphi_1$$

$$S_\varphi = S_{\varphi\,2} + S_{\varphi\,1}$$

Chose $\nu_1 \approx \nu_2$ with a small difference ν_b

The beat stretches the time associated to 1 rad by a factor $\nu_1\nu_b \cong \nu_2\nu_b$

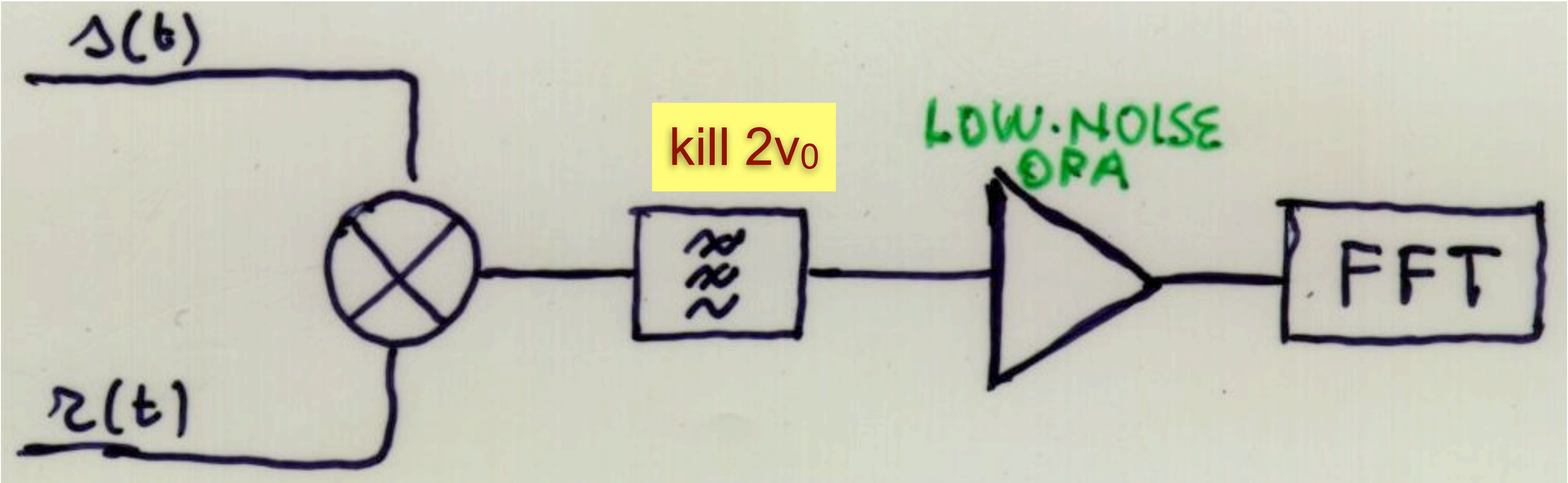
Accordingly, it is easier to measure S_φ at the low frequency ν_b,
or to find a reference with negligible S_φ

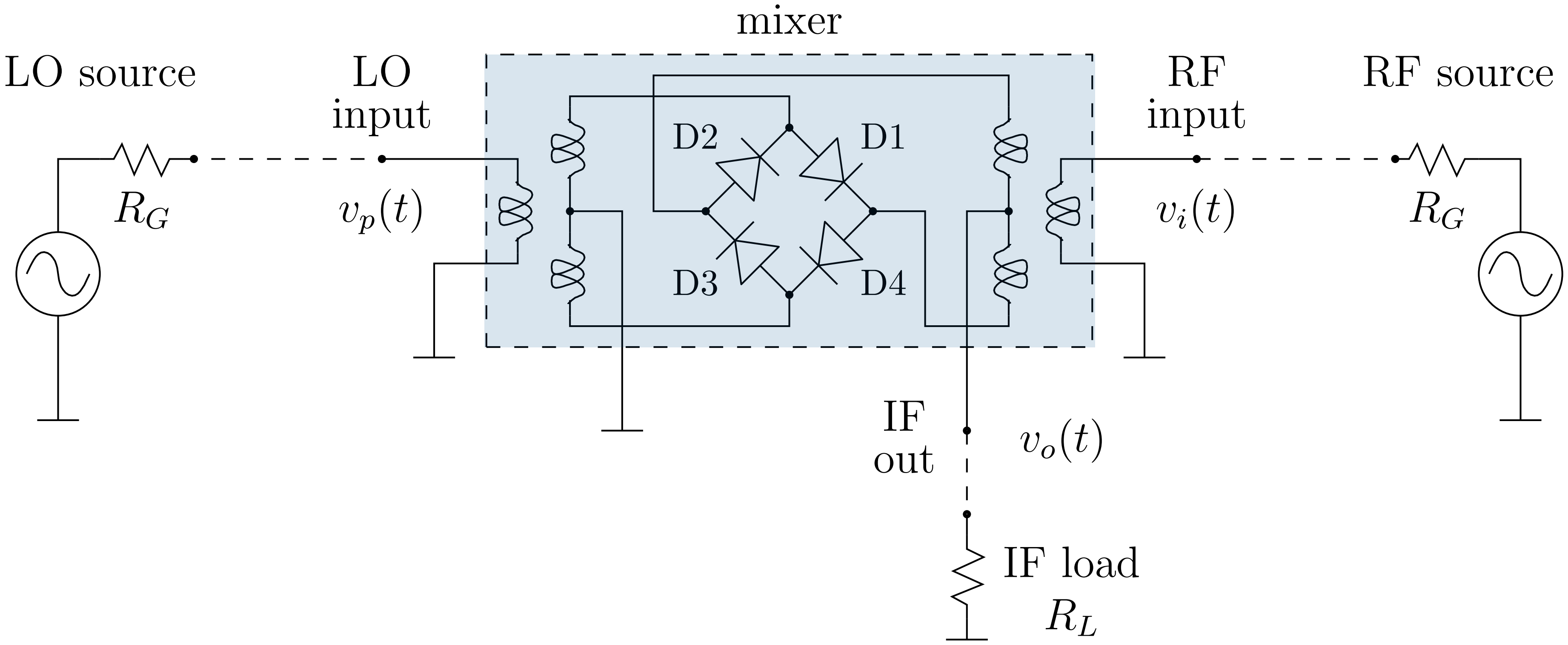
The saturated mixer

Double-balanced mixer

saturated multiplier => phase-to-voltage detector $v_o(t) = k_\varphi \, \varphi(t)$

$k_\varphi \approx 100 \dots 500$ mV/rad

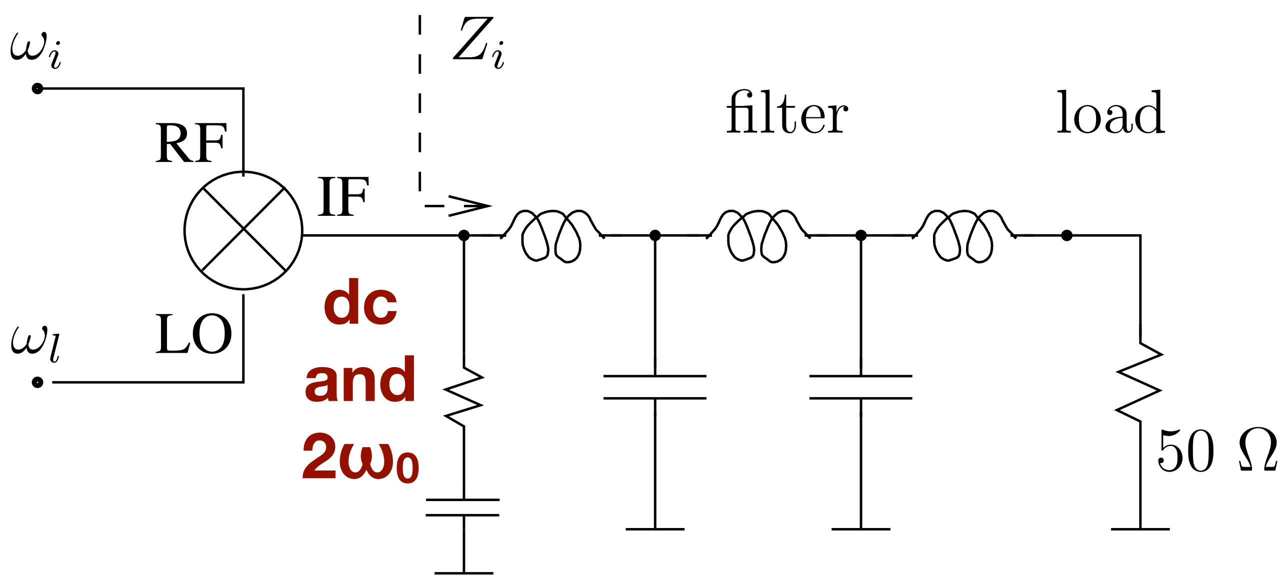


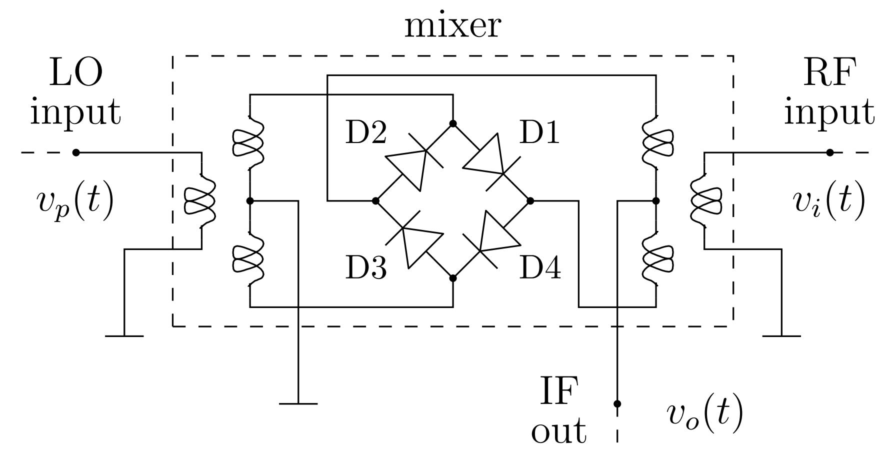


E. Rubiola, Tutorial on the double-balanced mixer, arXiv/physics/0608211

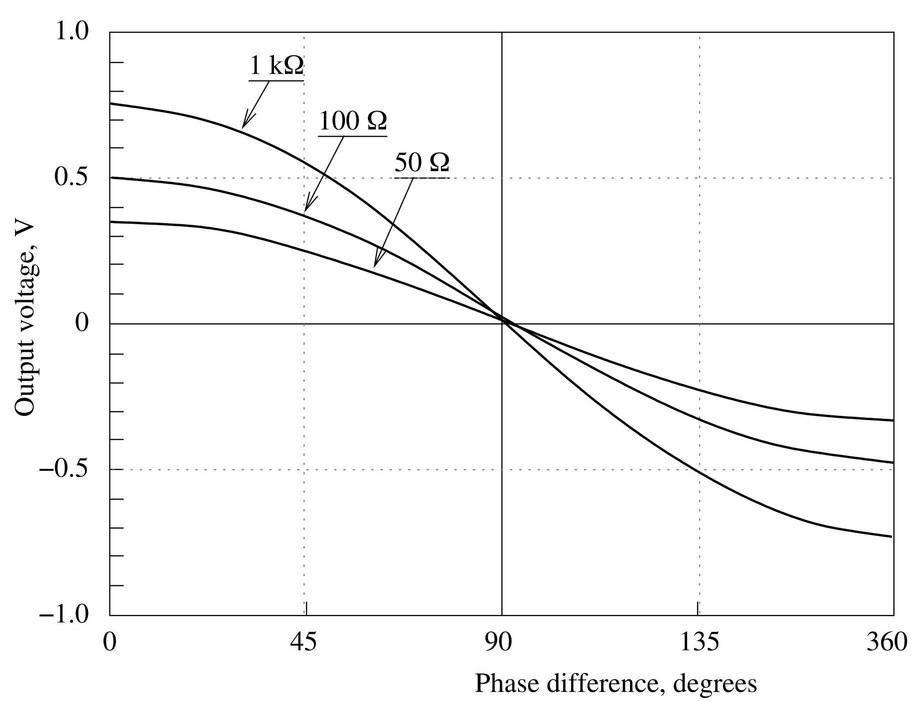
Practical issues

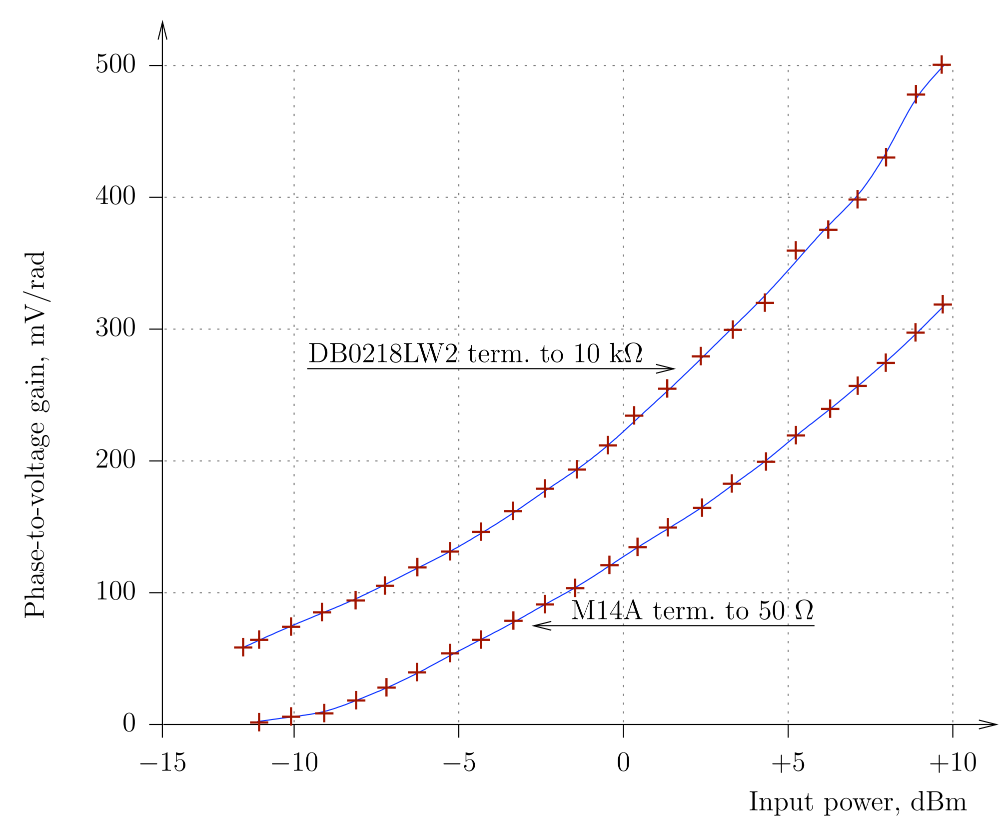
needs a capacitive-input filter to recirculate the $2\omega_0$ output signal





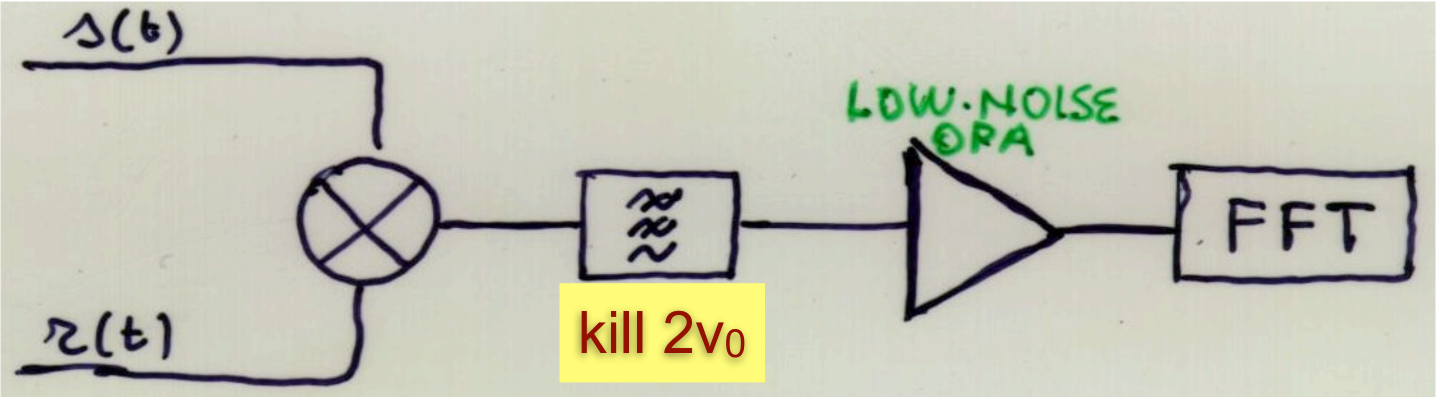
actual phase-to-voltage conversion





E. Rubiola, Tutorial on the double-balanced mixer, arXiv/physics/0608211

Mixer limitations



1 – Power

narrow power range:
±5 dB around P_{nom} = 7–13 dBm
r(t) and s(t) should have ~ same P

2 – Flicker noise

due to the mixer internal diodes
typical S_φ = –140 dBrad²/Hz at 1 Hz
in average-good conditions

3 – Low gain

k_φ ~ 0.2–0.3 V/rad typ.
–10 to –14 dBV/rad

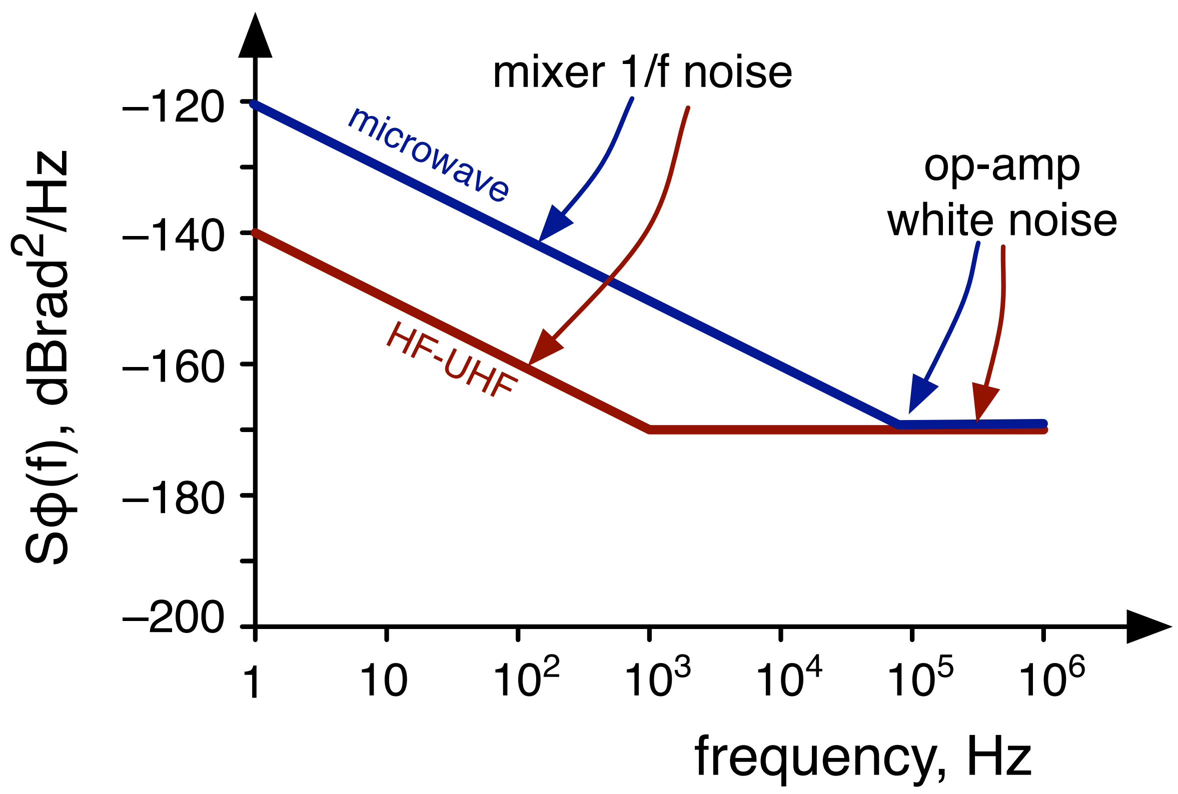
4 – White noise

due to the operational amplifier

5 – Takes in AM noise

due to the residual power-to-offset conversion

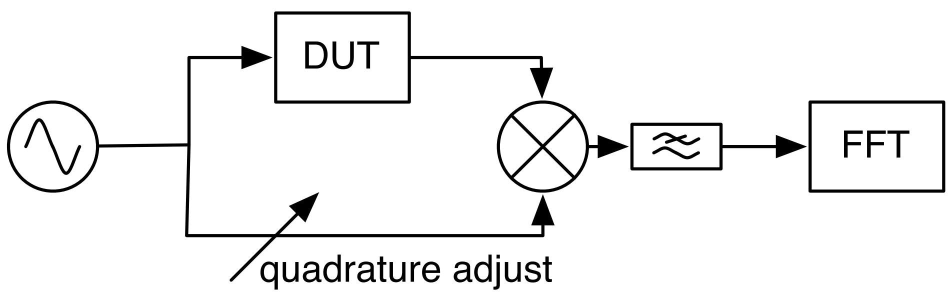
mixer background noise



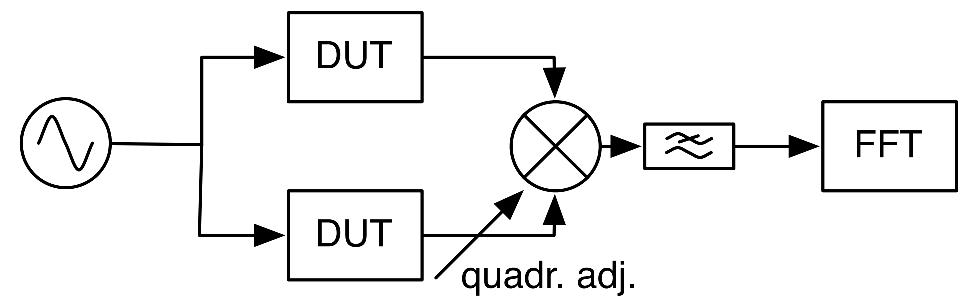
E. Rubiola, Tutorial on the double-balanced mixer, arXiv/physics/0608211

Useful schemes

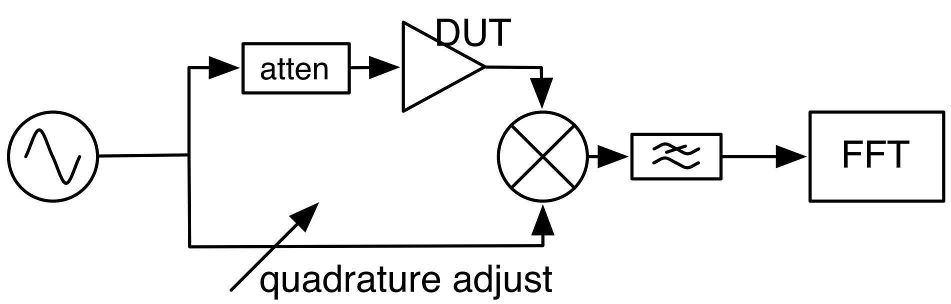
two-port device under test



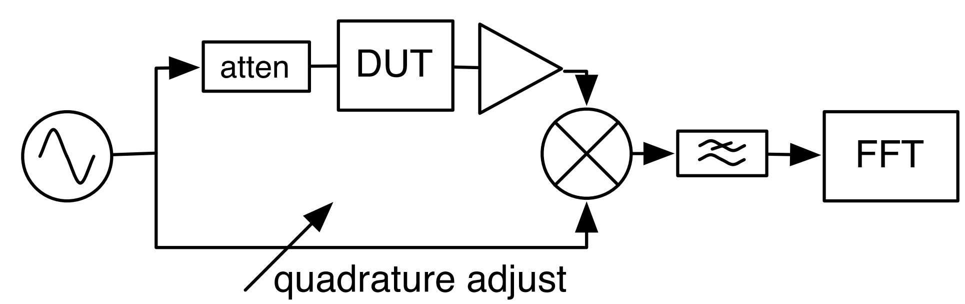
a pair of two-port devices
3 dB improved sensitivity



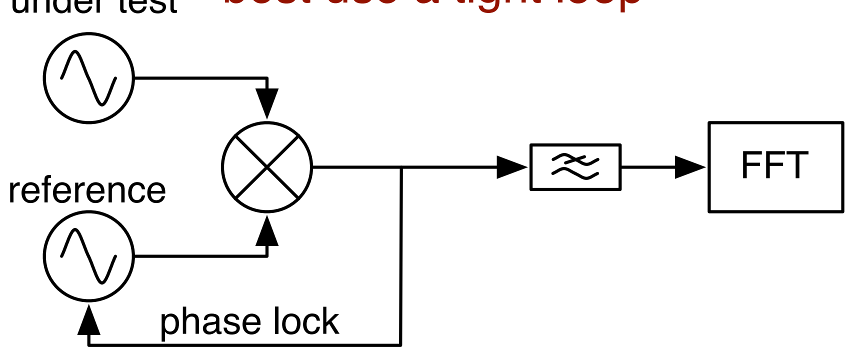
the measurement of an amplifier needs an attenuator



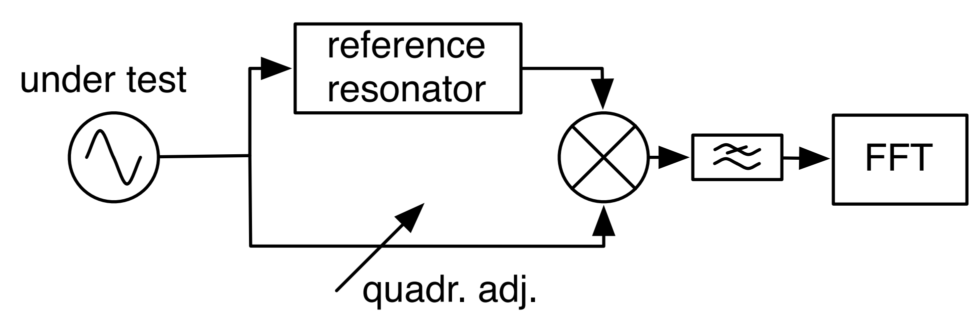
the measurement of a low-power DUT needs an amplifier, which flickers



measure two oscillators
best use a tight loop

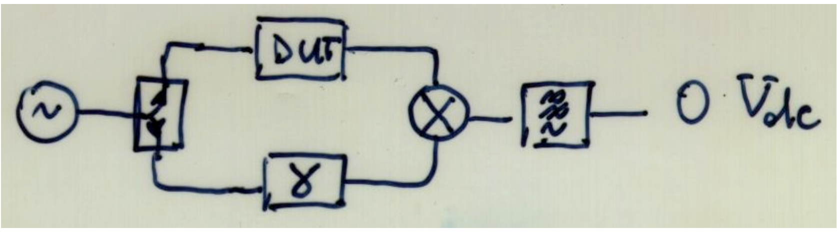


measure an oscillator vs. a resonator



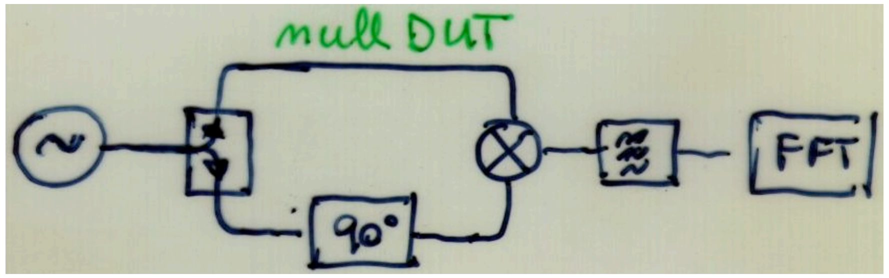
Calibration – general procedure

1 – adjust for proper operation: driving power and quadrature

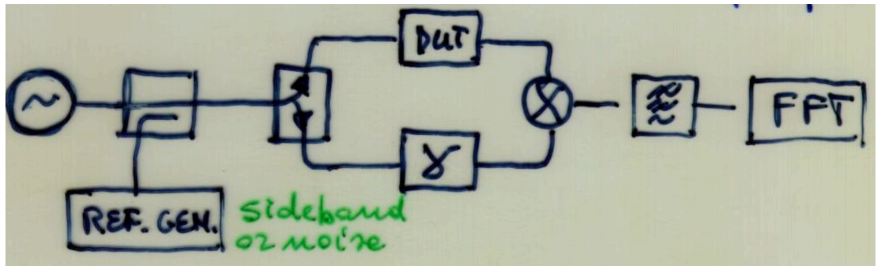


2 – measure the mixer gain k_φ (volts/rad)
- offset 159 Hz (1 krad/s),
  measure the slope with an oscilloscope
- reference phase modulator
- other methods

3 – measure the residual noise of the instrument



4 – measure the rejection of the oscillator noise



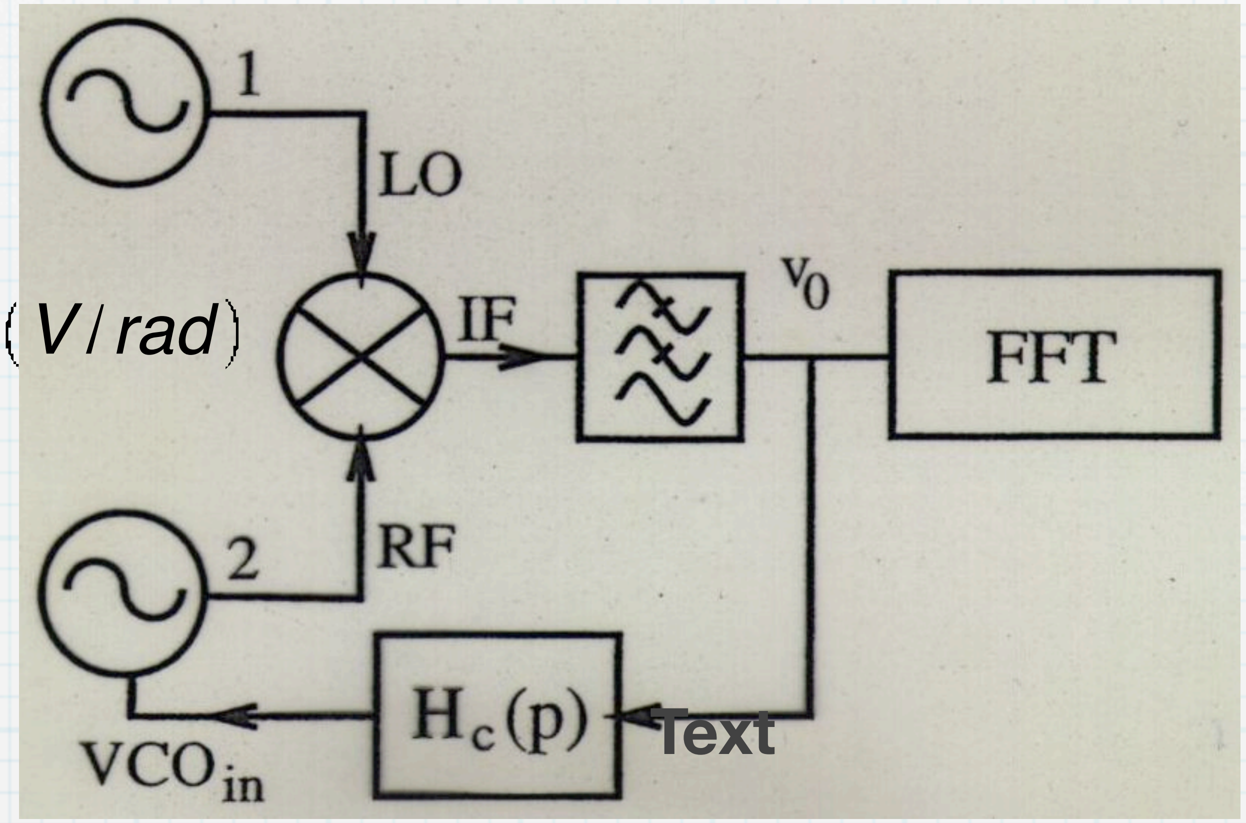
Make sure that the power and the quadrature are the same during all the process

The measurement of the oscillator PM noise

Phase Locked Loop (PLL)

$k_{\varphi} \; (V/rad)$

$k_o \; (rad/s/V)$



Phase: the PLL is a low-pass filter

$$\frac{S_{\varphi 2}(f)}{S_{\varphi 1}(f)} = \frac{\left|k_o k_{\varphi} H_c(f)\right|^2}{4\pi^2 f^2 + \left|k_o k_{\varphi} H_c(f)\right|^2}$$

Output voltage: the PLL is a high-pass filter

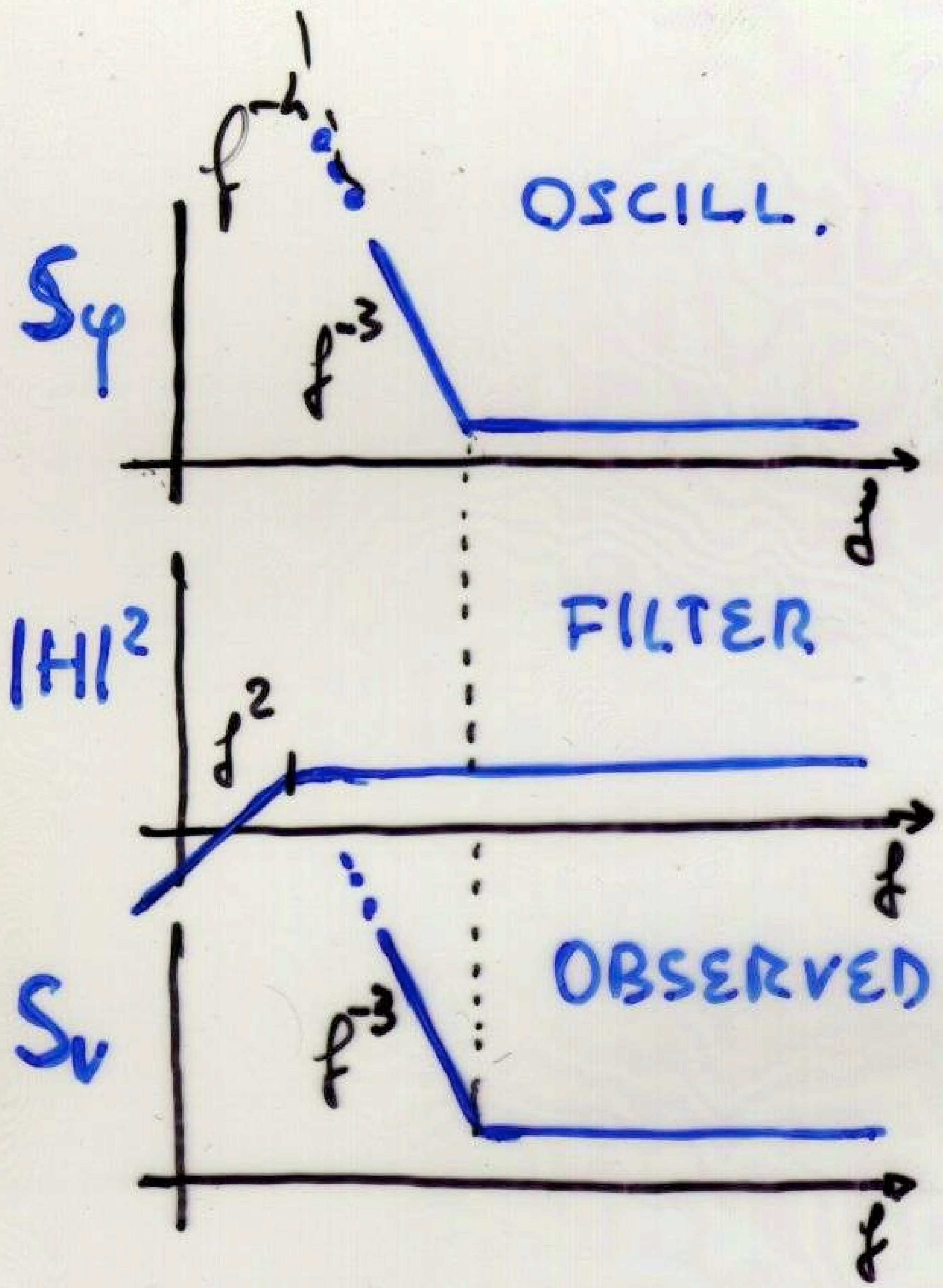
$$\frac{S_{vo}(f)}{S_{\varphi 1}(f)} = \frac{4\pi f^2 k_{\varphi}^2}{4\pi^2 f^2 + \left|k_o k_{\varphi} H_c(f)\right|^2}$$

compare an oscillator under test to a reference low-noise oscillator

– or –

compare two equal oscillators and divide the spectrum by 2 (take away 3 dB)

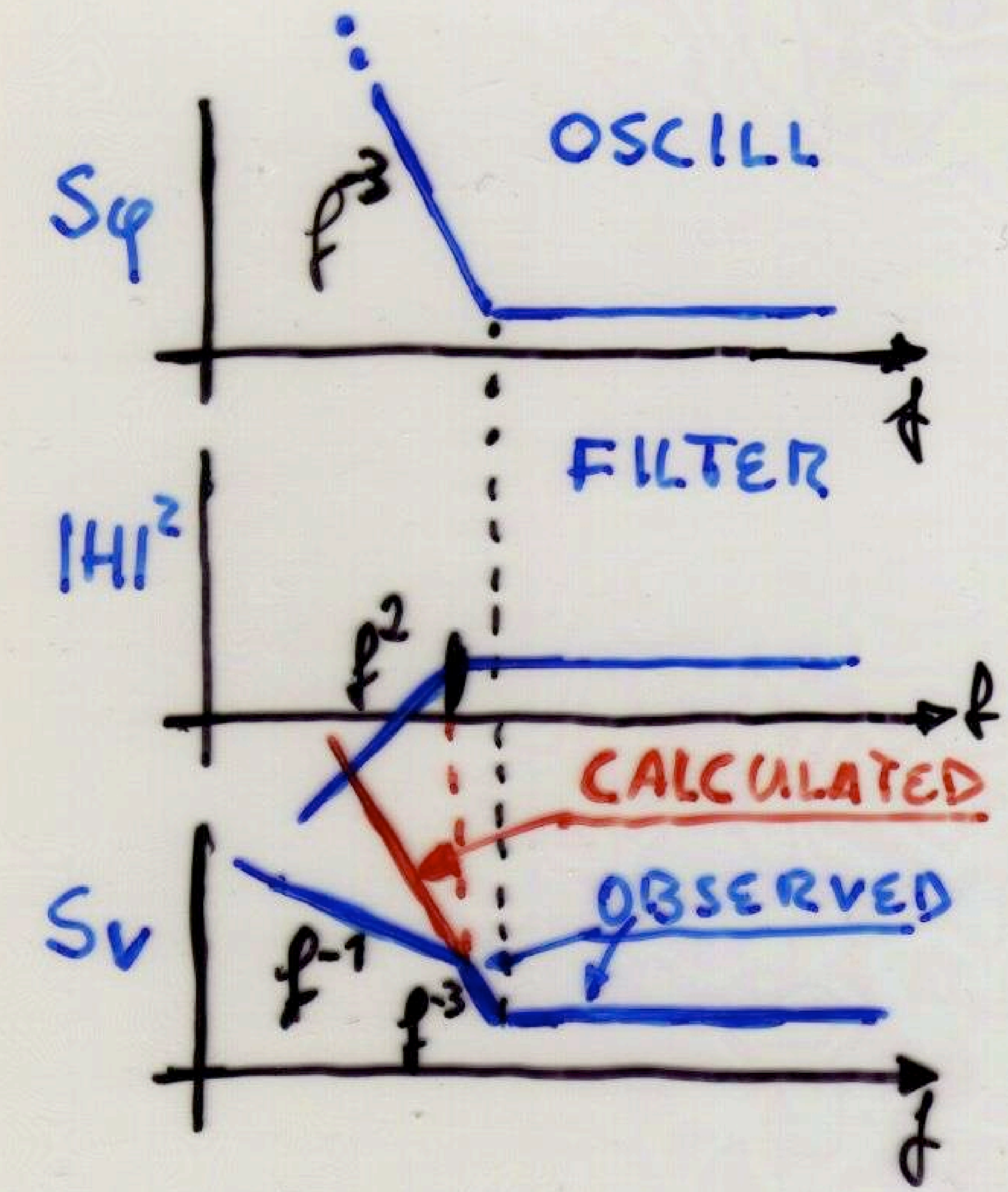
Loose Phase Locked Loop (PLL)



H_c is a constant
(1st order PLL)

A large dynamics is required because of the f^{-3} slope

A tight PLL shows many advantages



Still 1st order PLL

increased gain

a narrower dynamics is sufficient

slope: $f^{-3} \rightarrow f^{-1}$

but you have to correct the spectrum for the PLL transfer function

Practical measurement of $S_\varphi(f)$ with a PLL

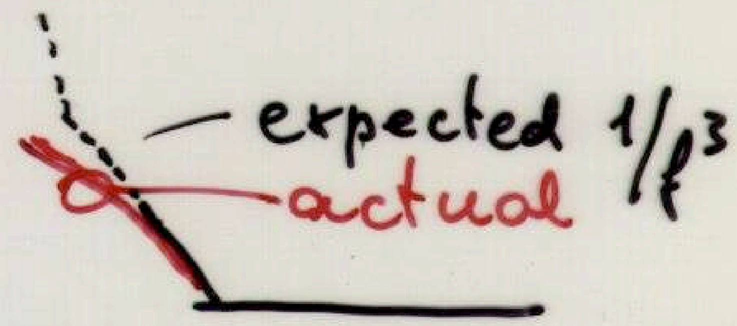
1. **Set the circuit for proper electrical operation**
    a. power level
    b. lock condition (there is no beat note at the mixer out)
    c. zero dc error at the mixer output (a small V can be tolerated)
2. **Choose the appropriate time constant**
3. **Measure the oscillator noise**
4. **At end, measure the background noise**

Warning: a PLL may not be what it seems

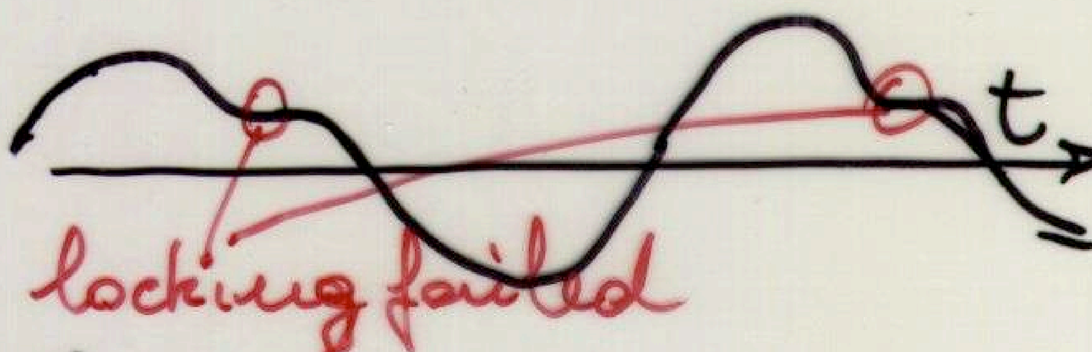
Parasitic locking or coupling of the oscillators may impair the result

BAD SYMPTOMS:

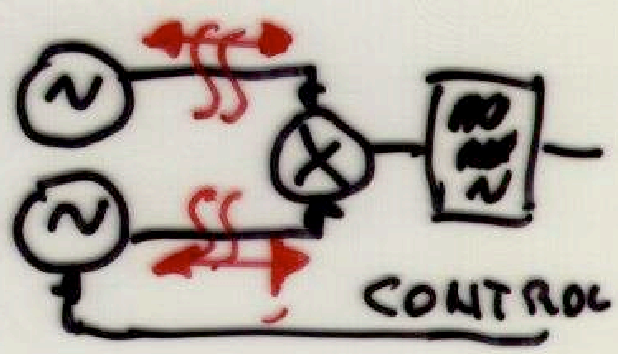
- odd slope S_φ



- open-loop waveforms IFout

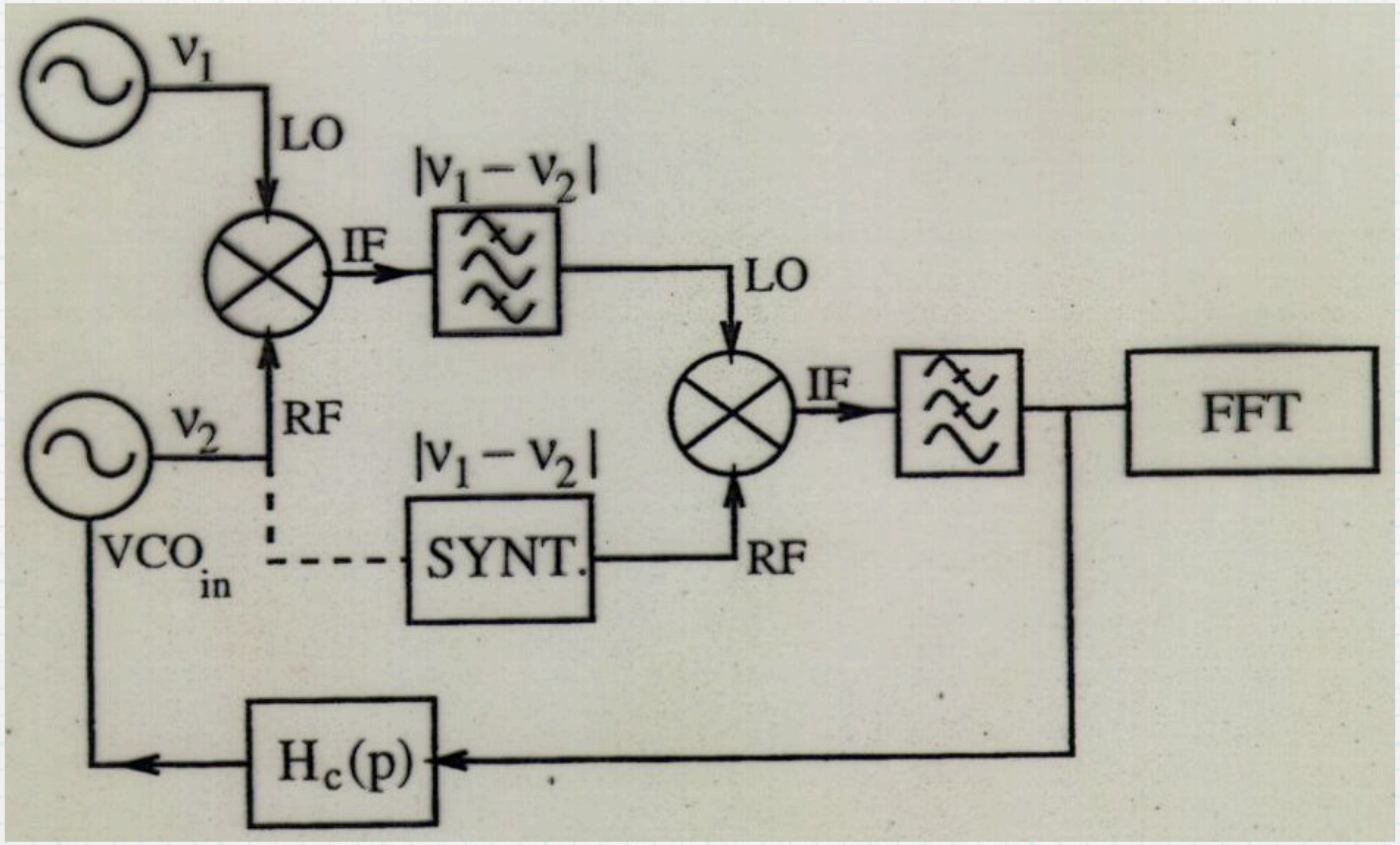


- results (S_φ) depend on the cable length



PLL – beat method

With low-noise microwave oscillators (like whispering gallery) the noise of a microwave synthesizer at the oscillator output can not be tolerated.

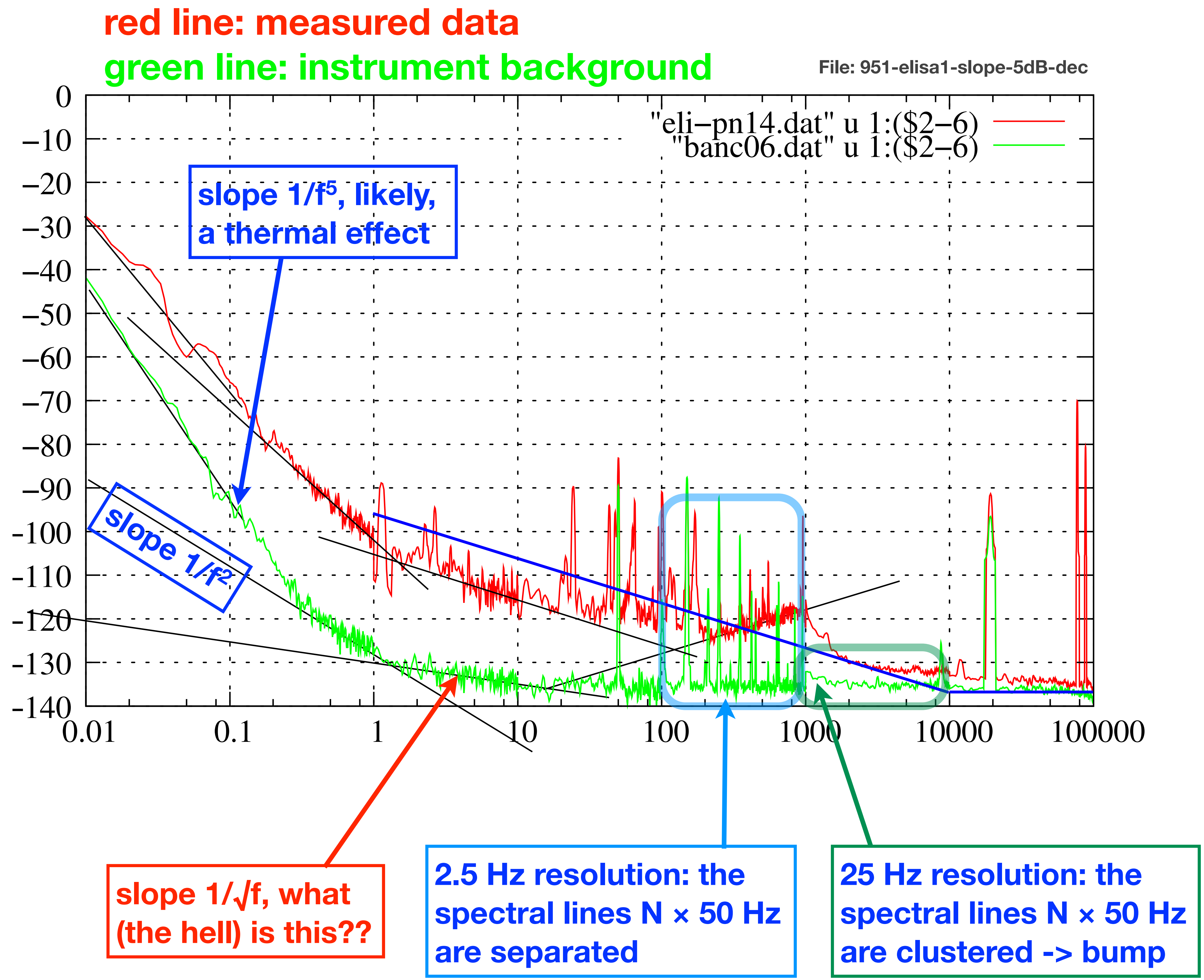


Due to the lower carrier frequency, the noise of a VHF synthesizer is lower than the noise of a microwave synthesizer.

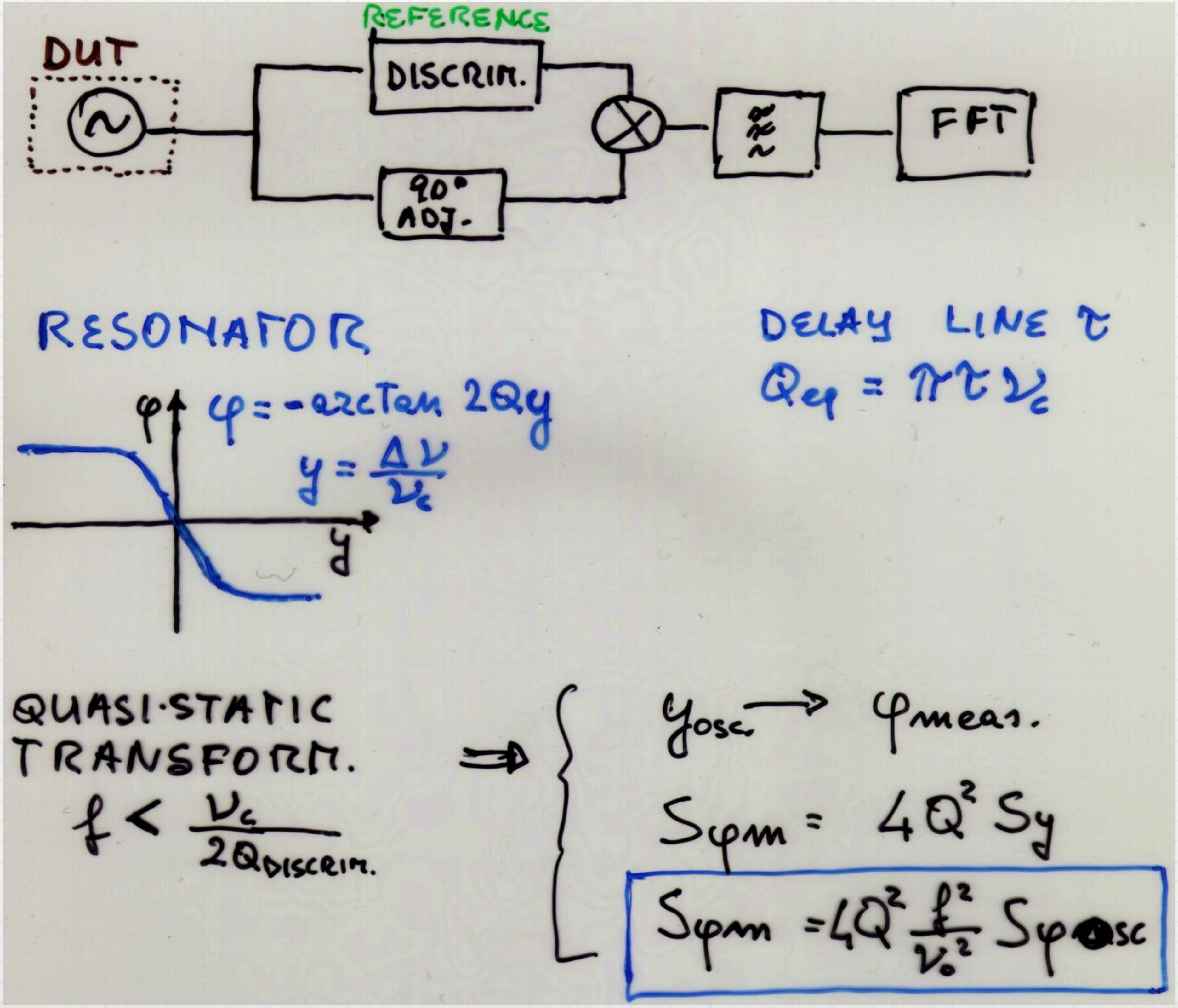
This scheme is useful

- **with narrow tuning-range oscillator, which cannot work at the same freq.**
- **to prevent injection locking due to microwave leakage**

A weird example

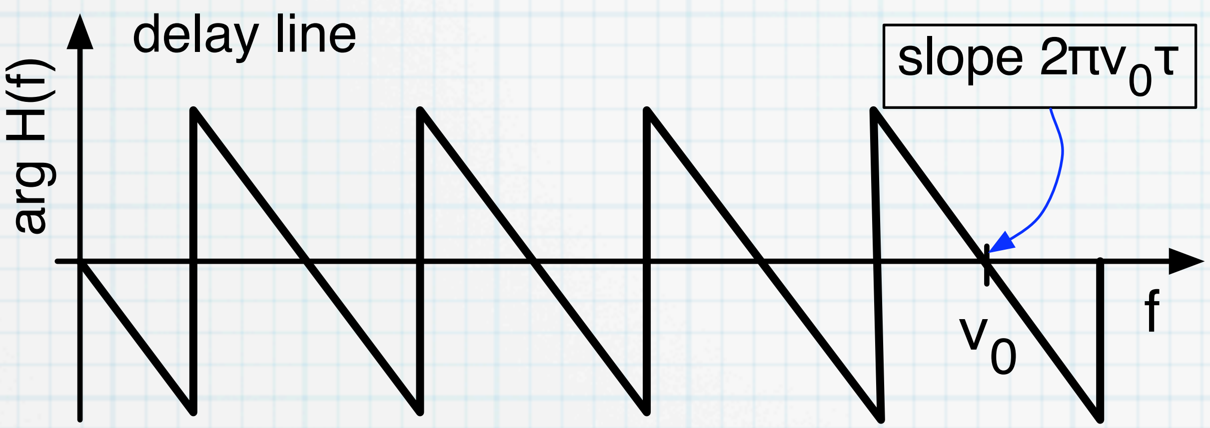


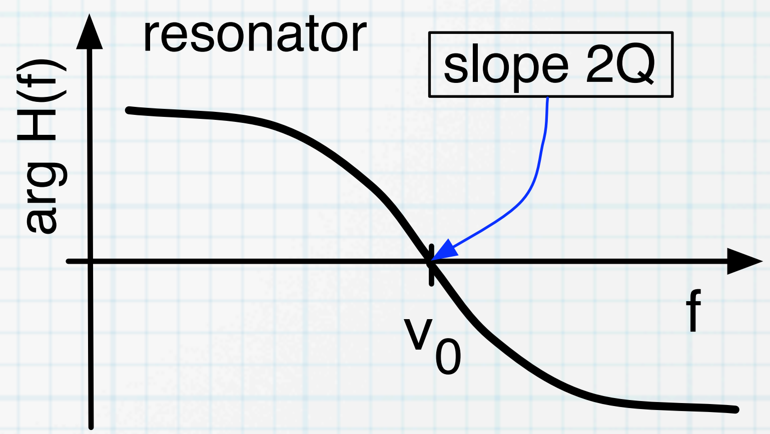
A frequency discriminator can be used to measure the phase noise of an oscillator



The delay-line as a discriminator

The delay line turns a frequency into a phase





comparing the slope: $Q_{\text{eq}} = \pi\nu_0\tau$

Virtues

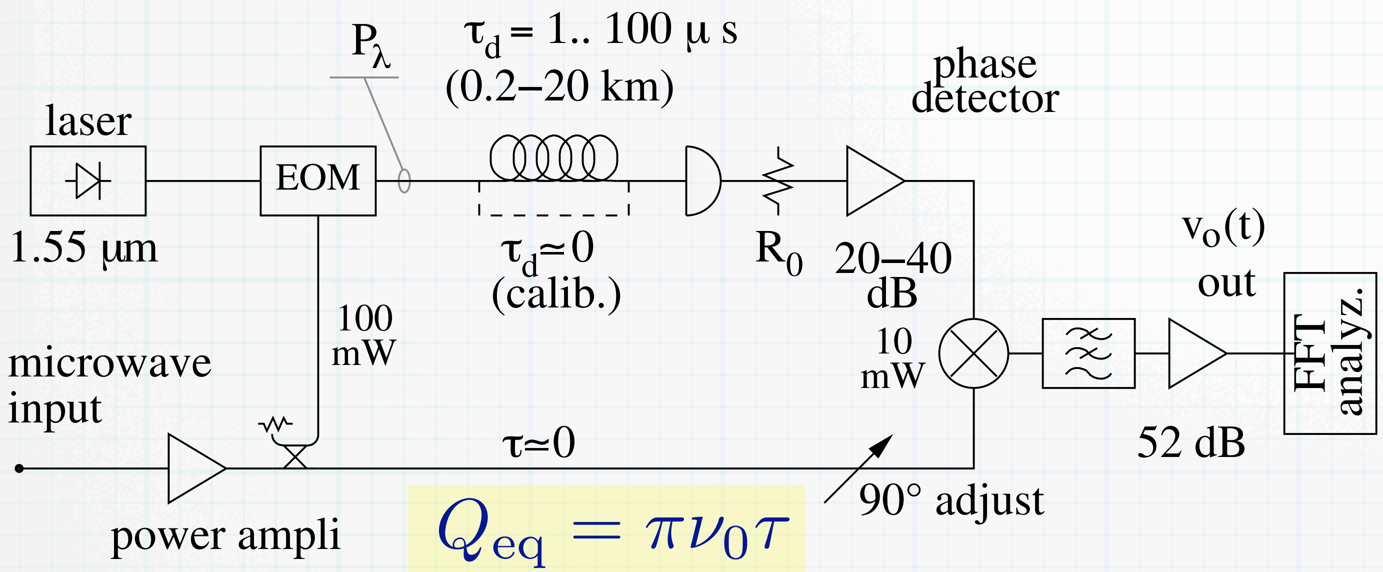
- **Works at any frequency ν = n/τ, integer τ (the resonator does not)**
- **Sφ measurement of an oscillator**
- **Dual-channel Sφ measurement of an oscillator**
- **Stabilization of an oscillator**
- **Opto-electronic oscillator**

Problems & solution

- **Coax cable: 50 dB attenuation limits to**
  - **950 ns @ 1 GHz (Q=3000) - RG213**
  - **300 ns @ 10 GHz (Q=11500) - RG402**
- **Optical fiber:**
  - **max delay is not limited by the attenuation**
  - **1-100 μs delay is possible ($Q=10^5$–10^7 @ 31 GHz)**

Opto-electronic discriminator

Rubiola & al., JOSAB 22(5) p.987–997 (2005) --- Volyanskiy & al., JOSAB 25(12) p.2140–2150 (2008)



The short arm can be a microwave cable or a photonic channel

Laplace transforms

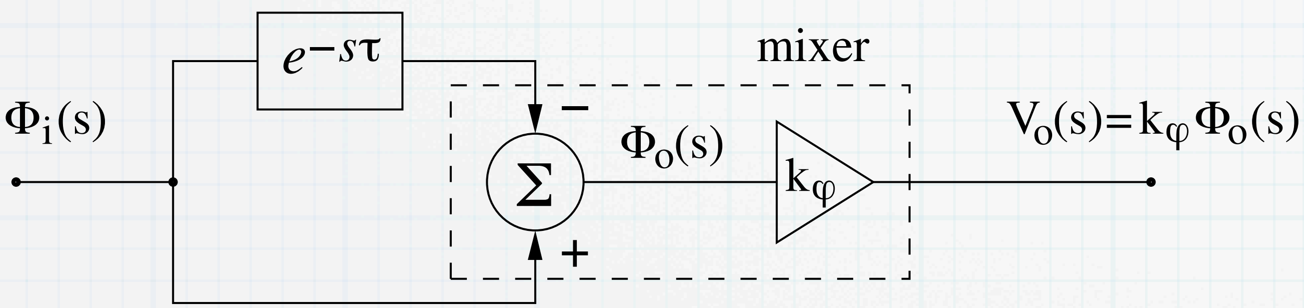
$$\Phi(s) = H_\varphi(s)\Phi_i(s)$$

$$|H_\varphi(f)|^2 = 4\sin^2(\pi f \tau)$$

$$S_y(f) = |H_y(f)|^2\, S_{\varphi\, i}(f)$$

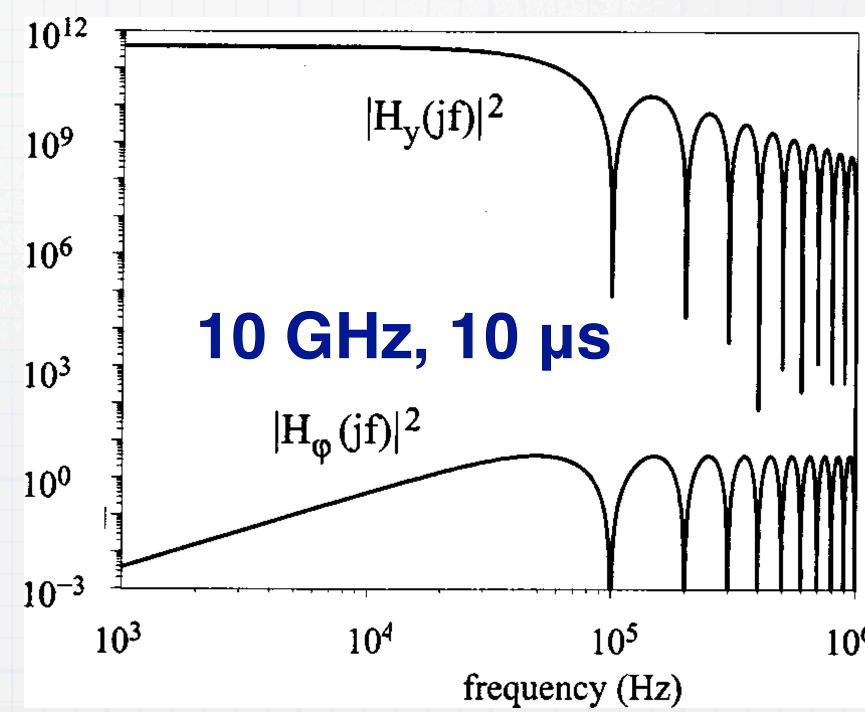
$$|H_y(f)|^2 = \frac{4\nu_0^2}{f^2}\,\sin^2(\pi f \tau)$$

Laplace transforms



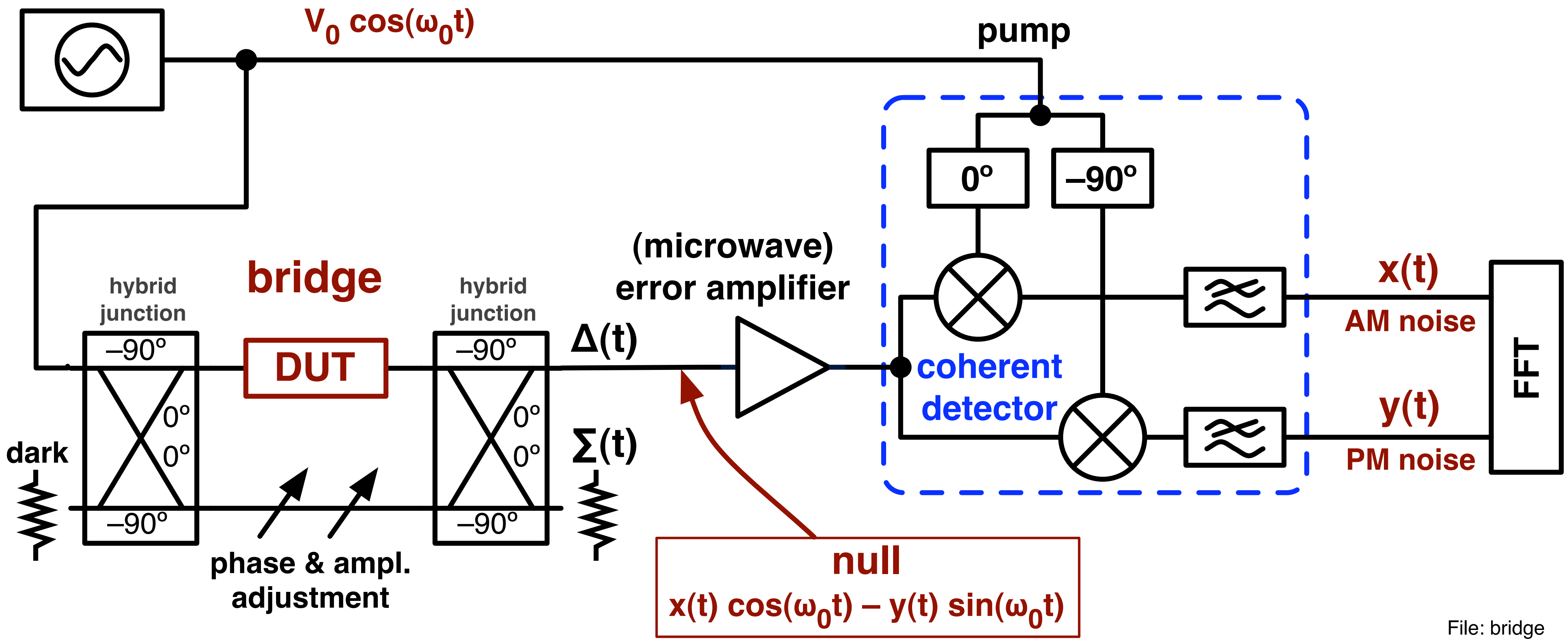
$$\Phi_o(s) = (1 - e^{-s\tau})\,\Phi_i(s)$$

- delay –> frequency-to-phase conversion
- works at any frequency
- long delay (microseconds) is necessary for high sensitivity
- the delay line must be an optical fiber
  fiber: attenuation 0.2 dB/km, thermal coeff. 6.8 10^{-6}/K
  cable: attenuation 0.8 dB/m, thermal coeff. ~ 10^{-3}/K



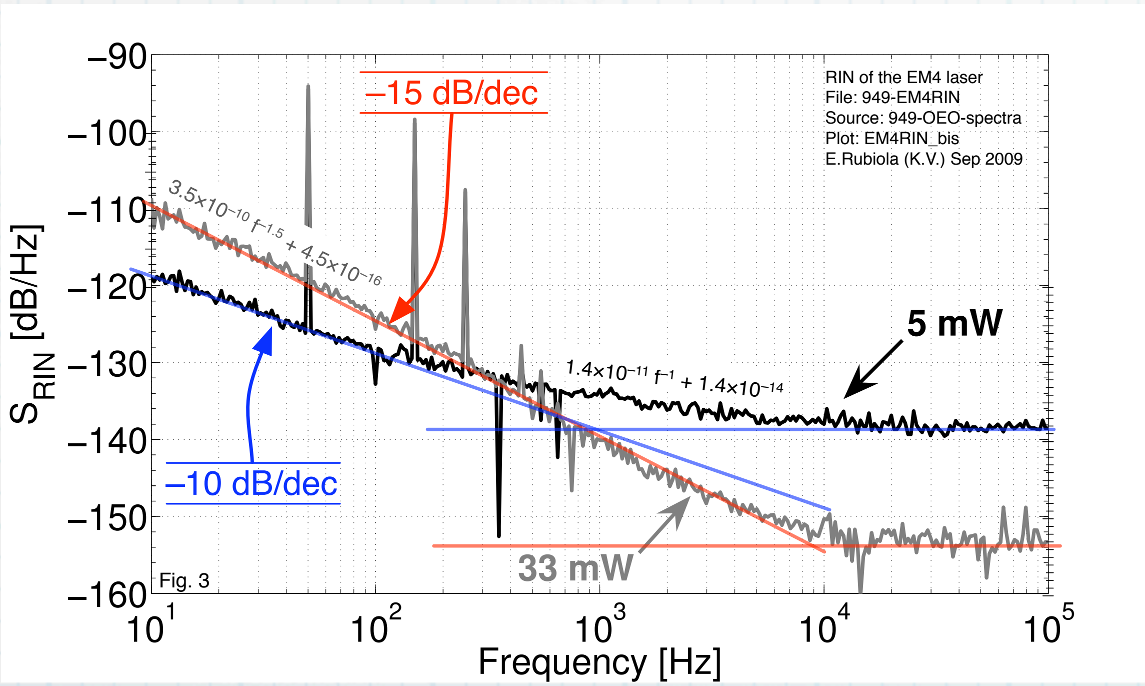
Advanced topics (including AM noise)

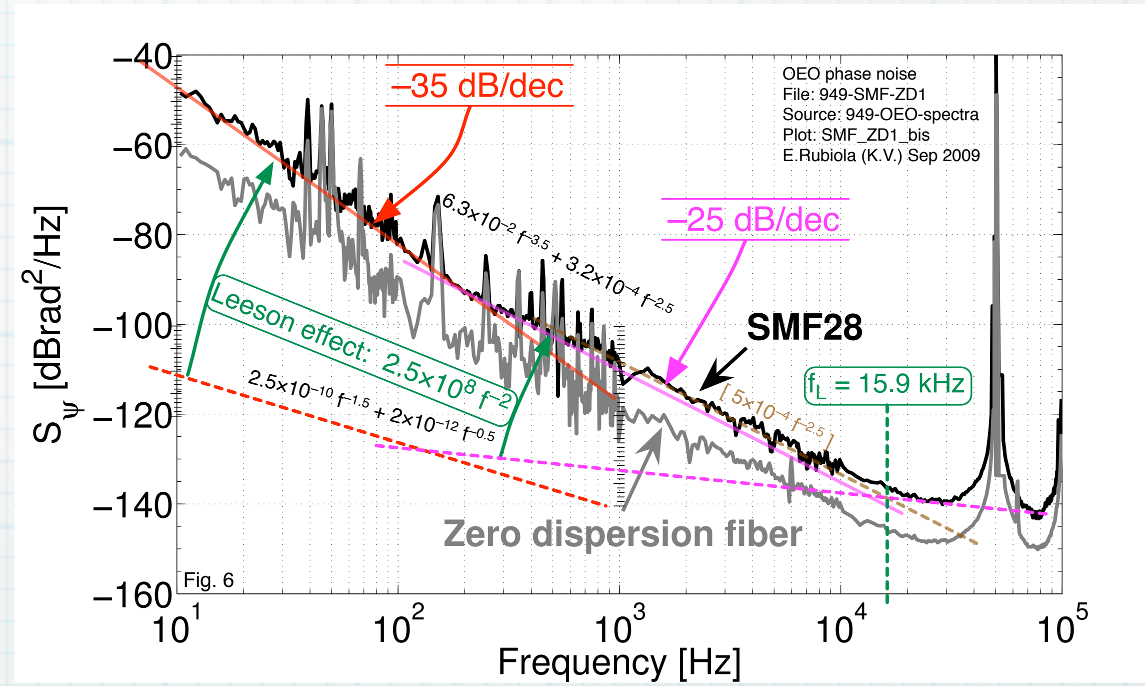
Bridge PM and AM noise measurement

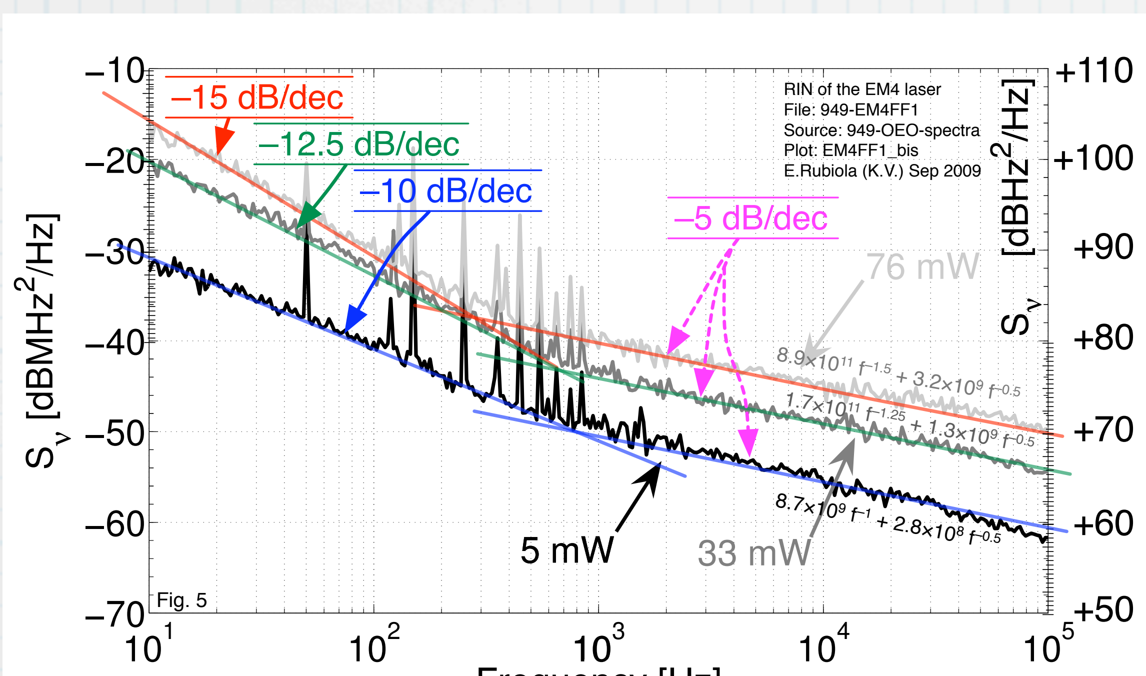


- Bridge => high rejection of the master-oscillator noise
- Amplification and synchronous detection of the noise sidebands
- No carrier => the amplifier can't flicker (no up-conversion of near-dc 1/f)
- High microwave gain before detection => low background
- Low 50-60 Hz residuals because microwave circuits are insensitive to magnetic fields

Fractional noise (5, 15, 25, 35 dB/dec)







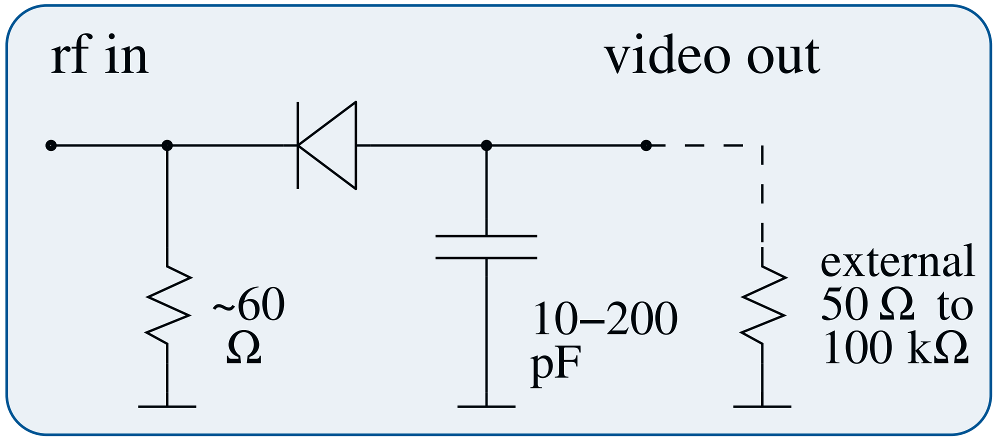
However heretic it seems, we have observed these slopes in optical systems.

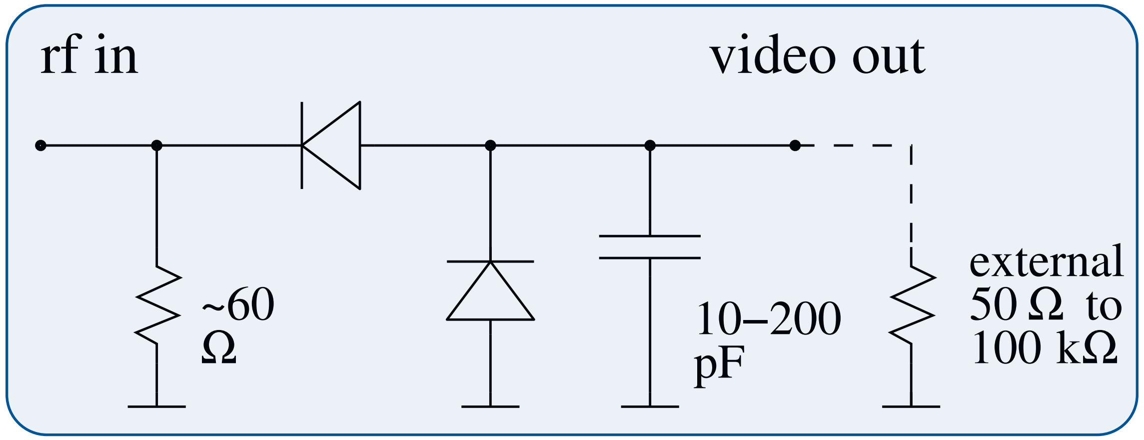
Other researchers report on similar issues, yet without pointing out the problem (exception, D. Eliyahu)

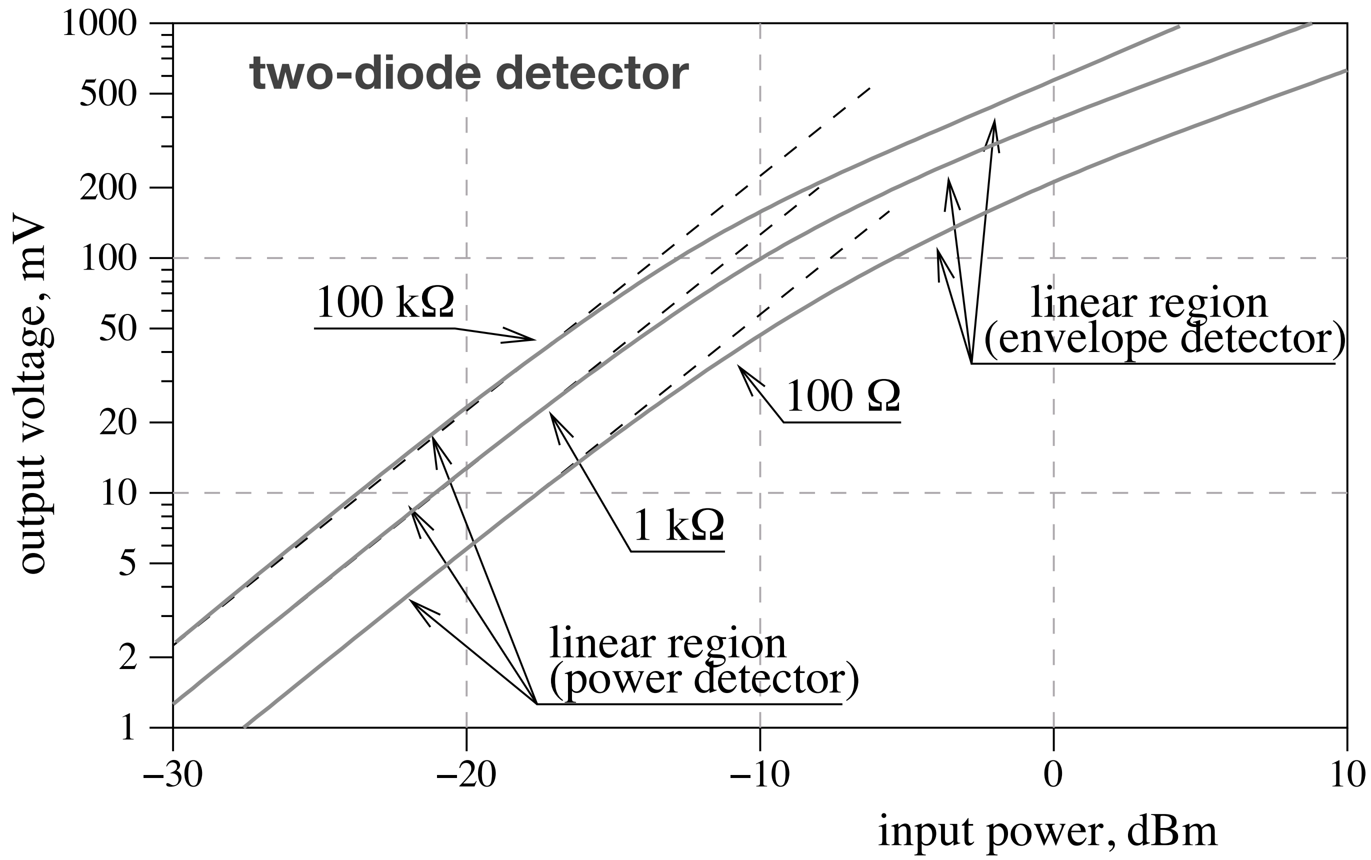
AM noise – The diode power detector

law: v = k_d P

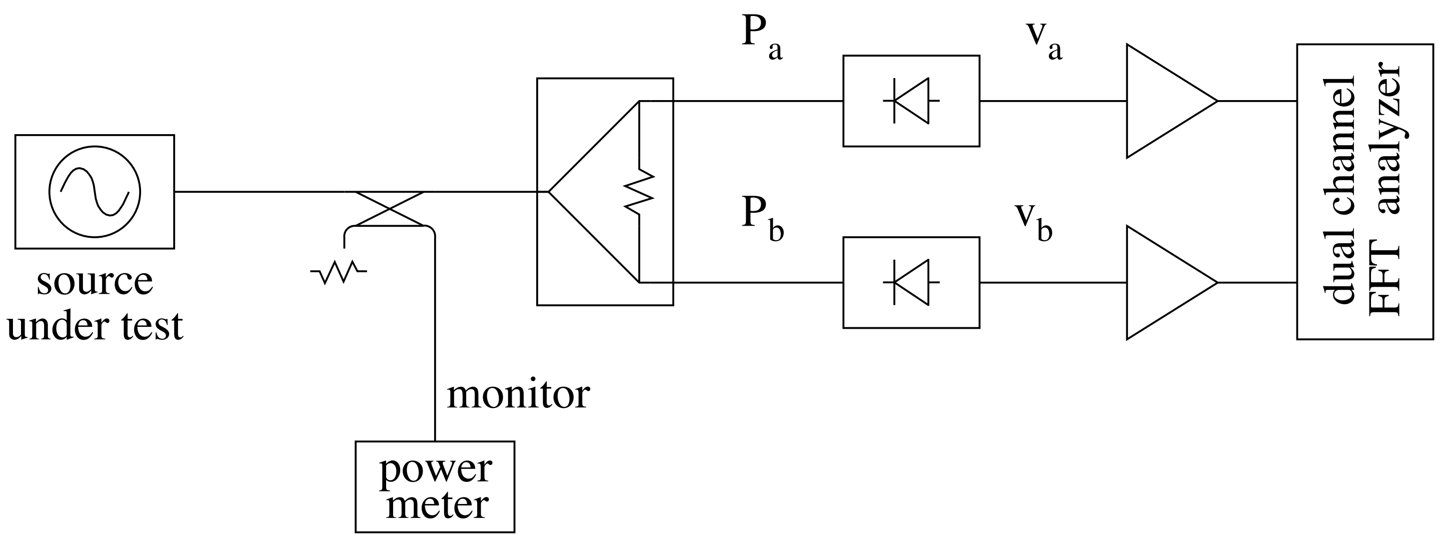
differential resistance $R_d = \dfrac{V_T}{I_0}$ $V_T = kT/q \simeq 25$ mV thermal voltage







AM noise – Cross-spectrum method

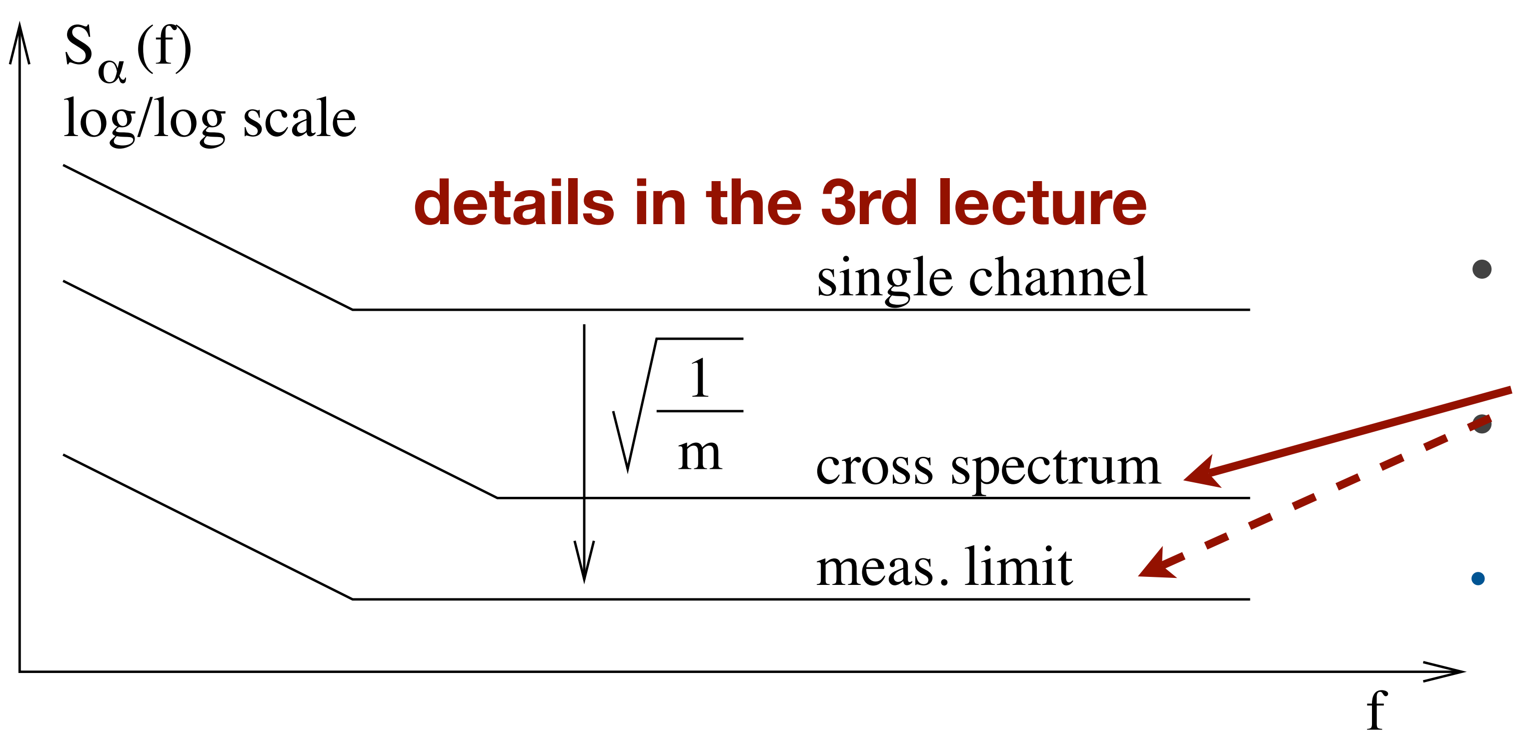


$$v_a(t) = 2k_a P_a \alpha(t) + \text{noise}$$
$$v_b(t) = 2k_a P_b \alpha(t) + \text{noise}$$

The cross spectrum $S_{ba}(f)$ rejects the single-channel noise because the two channels are independent.

$$S_{ba}(f) = \frac{1}{4k_a k_b P_a P_b} S_\alpha(f)$$

The problem with single-channel measurement is that the background noise cannot be measured without a reference source



details in the 3rd lecture

- Averaging on m spectra, the single-channel noise is rejected by $\sqrt{1/2m}$
- A cross-spectrum higher than the averaging limit validates the measure
- The knowledge of the single-channel noise is not necessary

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Short course on Stable oscillators

— Part 2 —

The origin of frequency instability and noise in oscillators

December 2009

Enrico Rubiola

FEMTO-ST Institute, CNRS and UFC, Besancon, France

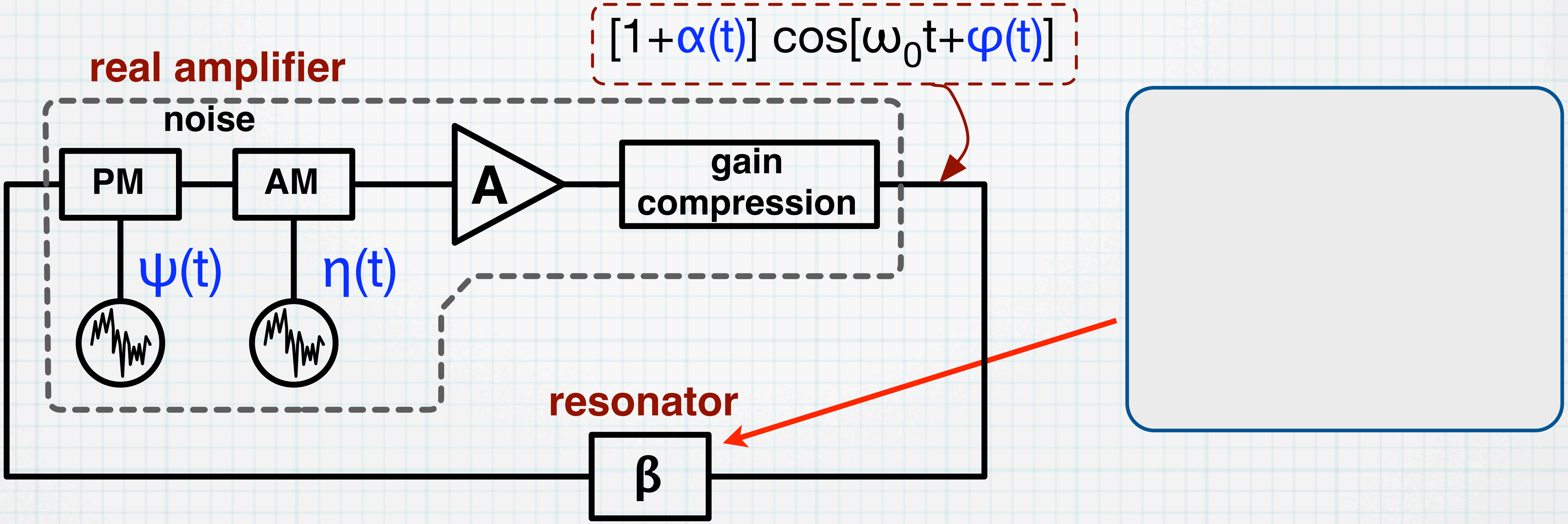
Outline

- Basics
- Oscillator hacking
- The Leeson effect (theory)
- Extension to AM noise
- Delay-line oscillator (and laser)

home page http://rubiola.org

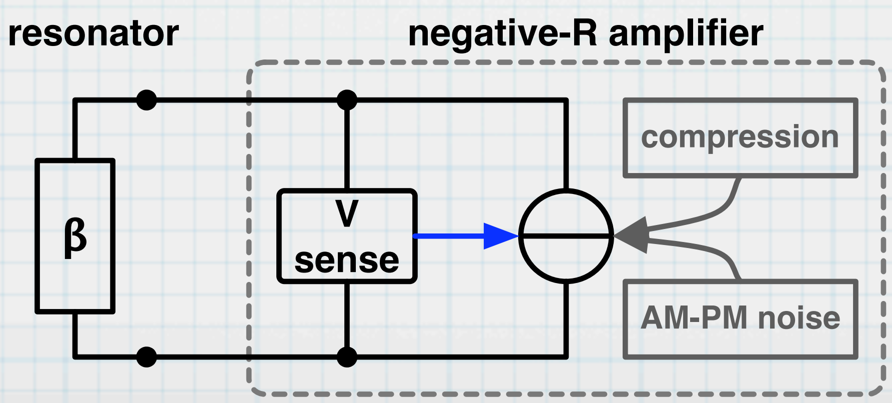
Basics

General oscillator model



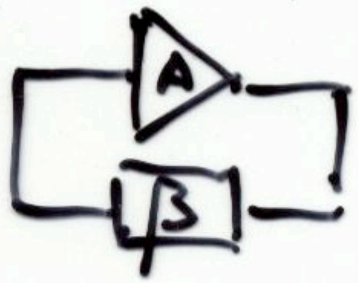
Barkhausen condition $A\beta = 1$ at ω_0
(phase matching)

The model also describes the negative-R oscillator



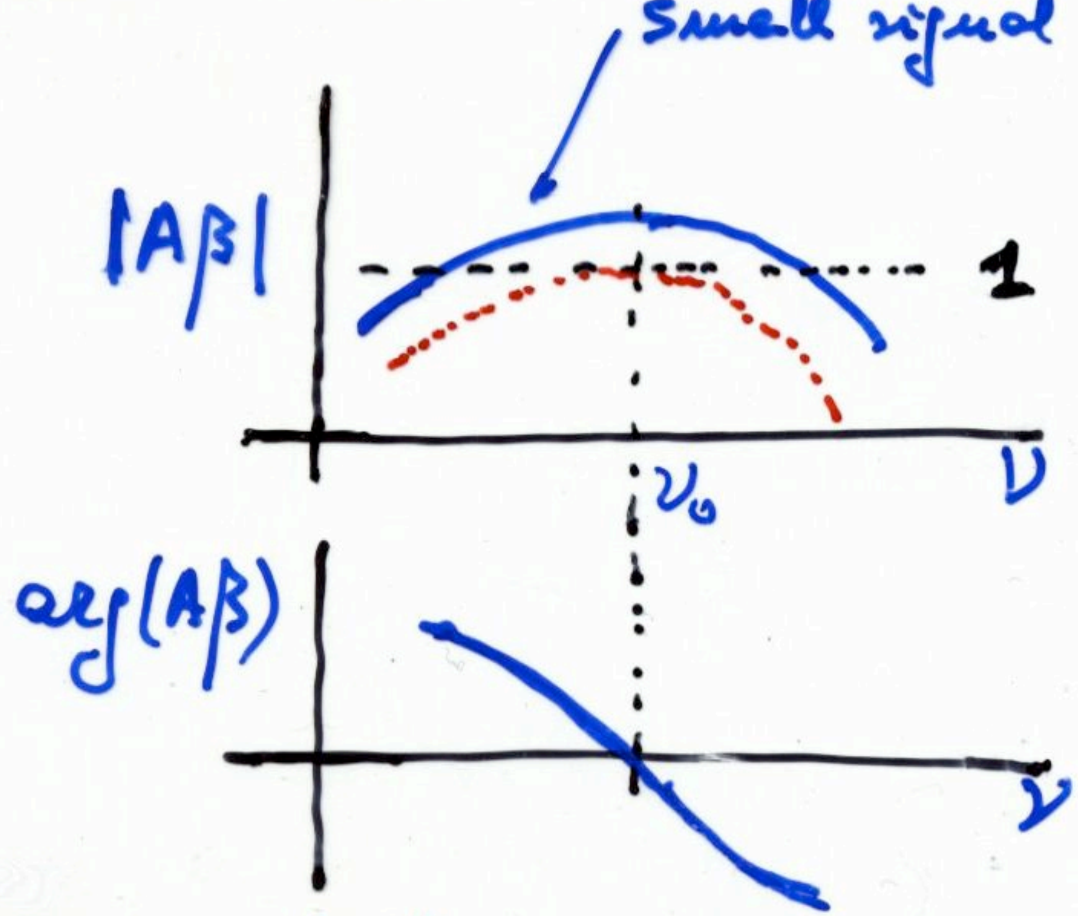
AR

BARKHAUSEN CONDITION



let $A = \text{const}$

β: 2nd order diff. eq
resonator



$$\omega = 2\pi\nu$$

$$\arg(\beta) = -\arctan Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

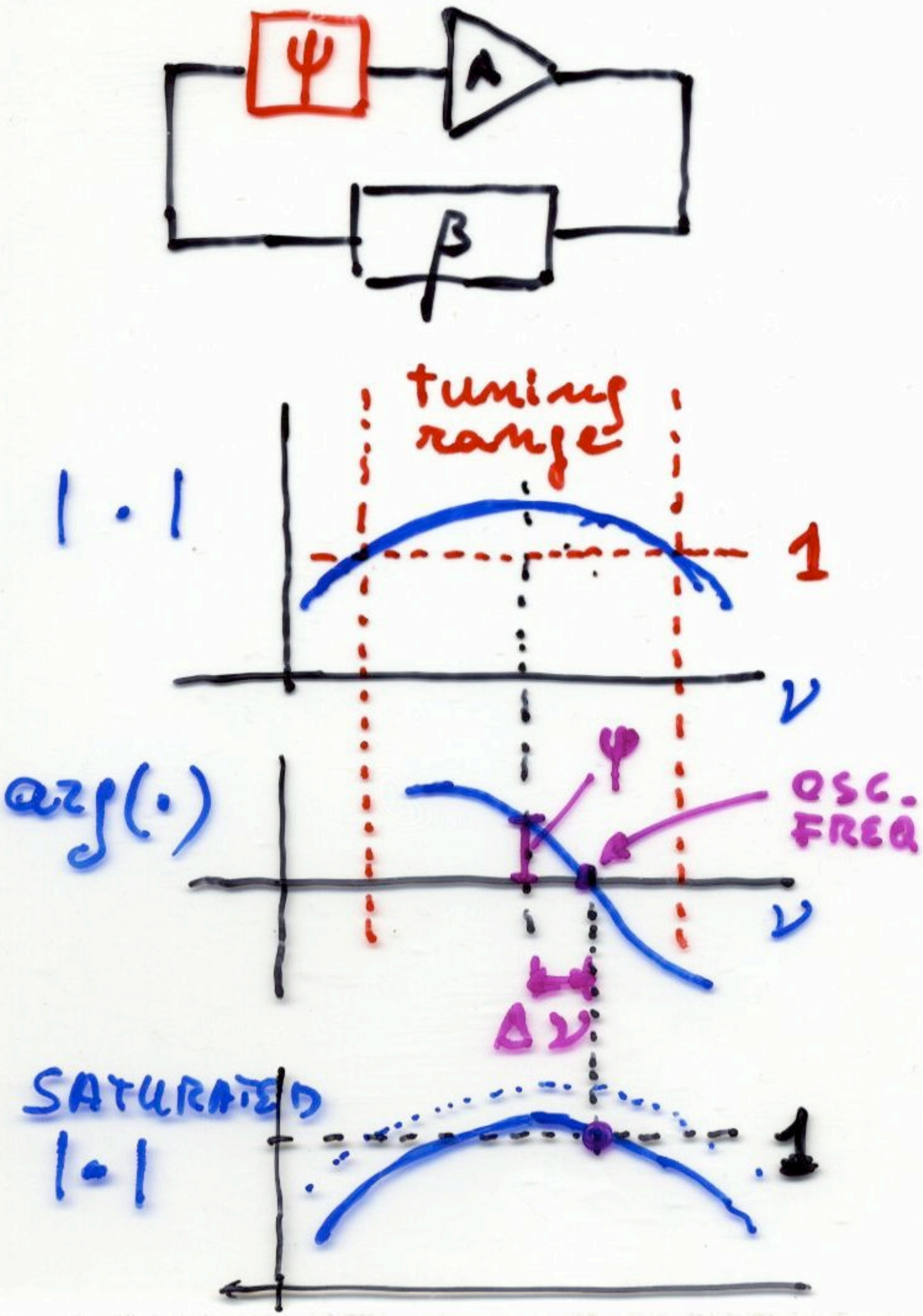
$$\simeq -2Q\,\frac{\omega-\omega_0}{\omega_0}$$

$$-2Q\,\frac{\Delta\omega}{\omega_0}$$

- $\arg(\beta)$ sets the oscillation frequency
- saturation fixes $|A\beta| = 1$

A02b

TUNING AN OSCILLATOR



add a phase ψ

$$+\arg(\beta)+\psi=0$$

$$+\arg(\beta)=-\psi$$

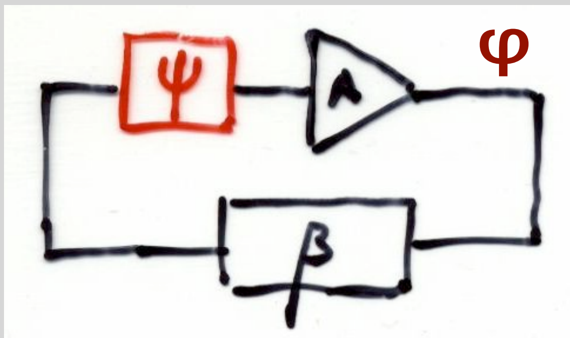
approx:

$$2Q\,\frac{\Delta\omega}{\omega_0}=\psi$$

$$\frac{\Delta\omega}{\omega_0}=\frac{\Delta\nu}{\nu_0}=\frac{\psi}{2Q}$$

Heuristic derivation of the Leeson formula

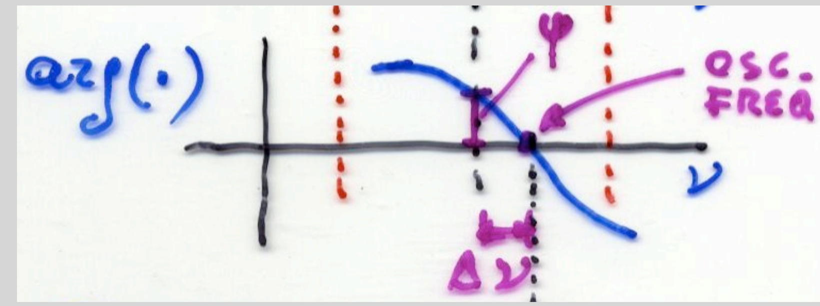
fast fluctuation: no feedback



$$\varphi(t) = \psi(t)$$

$$S_\varphi(f) = S_\psi(f)$$

slow fluctuations: ψ ⇒ Δν conversion

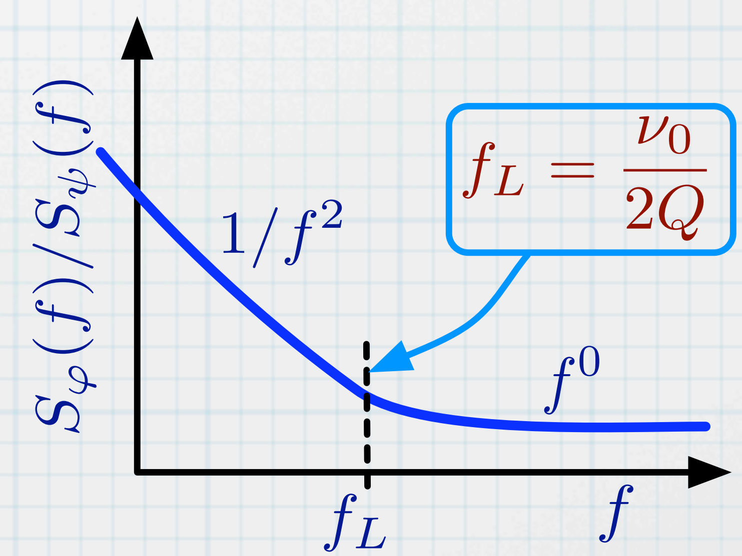


$$\Delta\nu = \frac{\nu_0}{2Q}\,\psi \qquad \textbf{static}$$

$$S_{\Delta\nu}(f) = \left(\frac{\nu_0}{2Q}\right)^2 S_\psi(f)$$

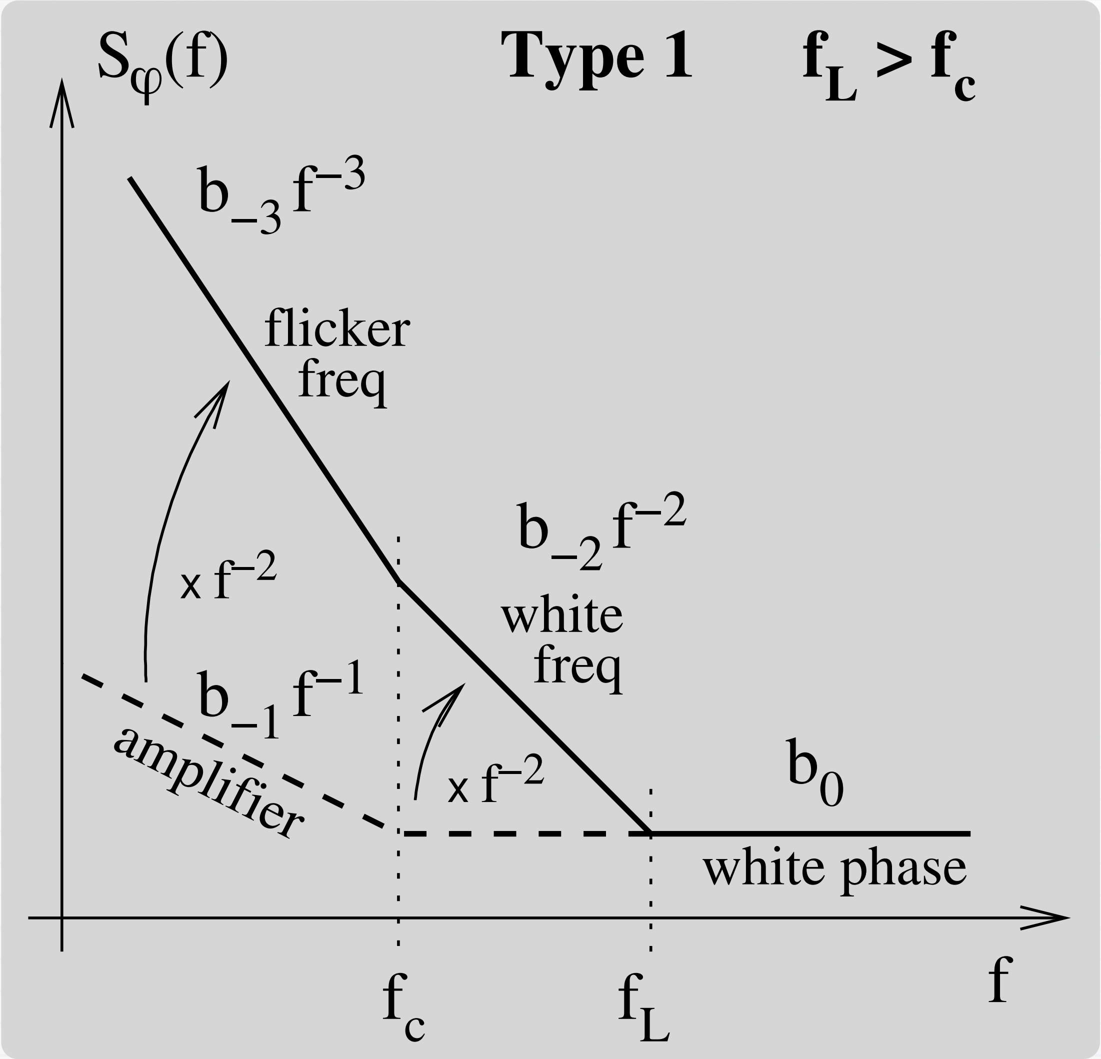
$$S_\varphi(f) = \frac{1}{f^2}\left(\frac{\nu_0}{2Q}\right)^2 S_\psi(f) \qquad \textbf{integral}$$

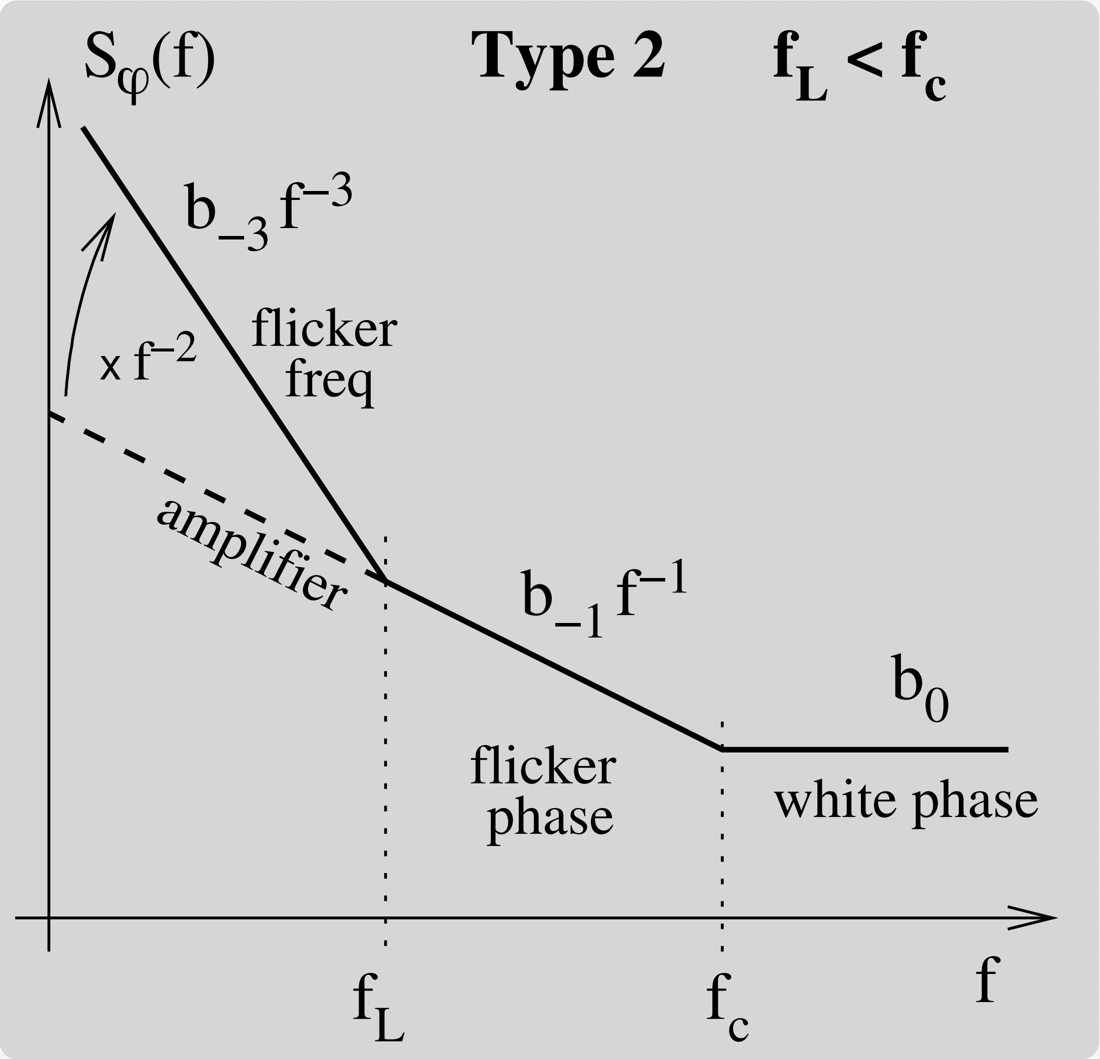
$$S_\varphi(f) = \left[1 + \frac{1}{f^2}\left(\frac{\nu_0}{2Q}\right)^2\right] S_\psi(f)$$



Though obtained with simplifications, this result turns out to be is exact

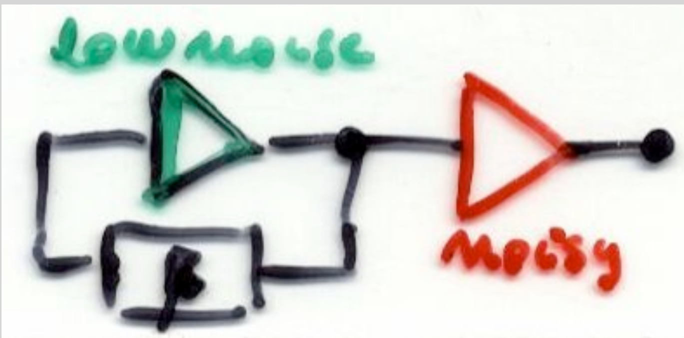
Including the sustaining-amplifier noise





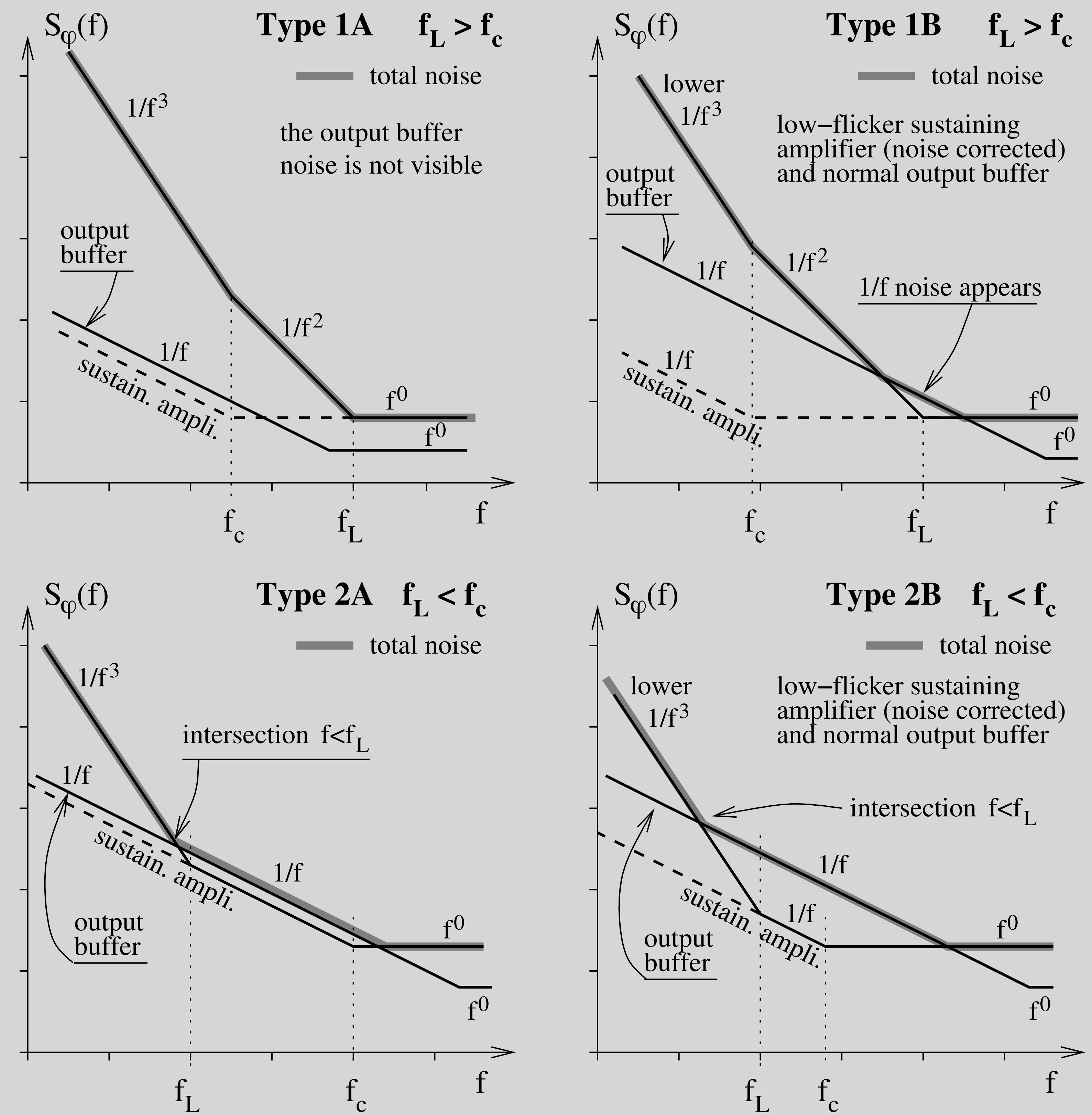
The sustaining-amplifier noise is $S_\varphi(f) = b_0 + b_{-1}/f$ (white and flicker)

The effect of the output buffer



Cascading two amplifiers, flicker noise adds as

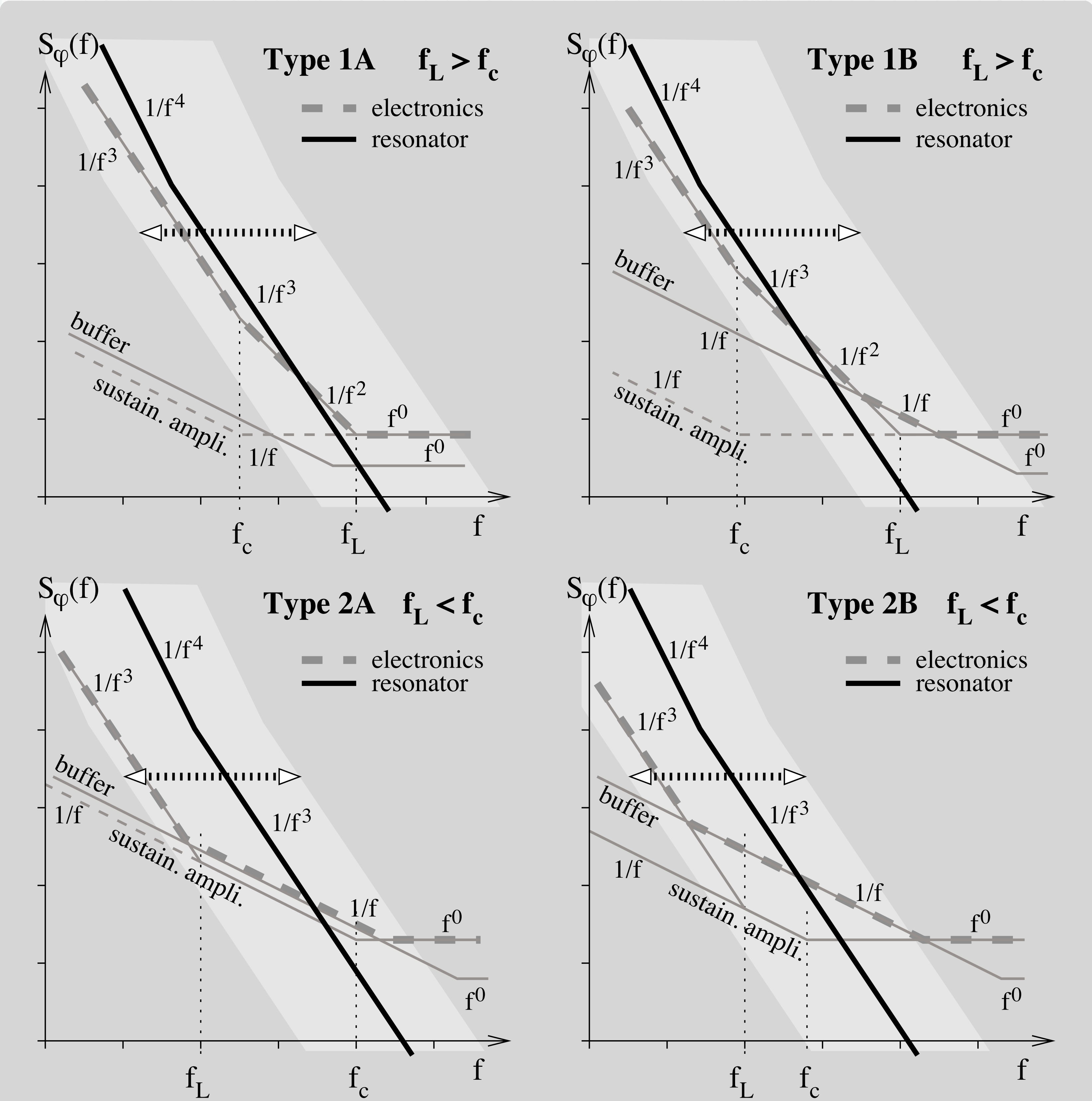
$$S_\varphi(f) = [S_\varphi(f)]_1 + [S_\varphi(f)]_2$$



Figures are from E. Rubiola, Phase noise and frequency stability in oscillators, © Cambridge University Press

The resonator natural frequency fluctuates

- The oscillator tracks the resonator natural frequency, hence its fluctuations
- The fluctuations of the resonator natural frequency contain **$1/f$ and $1/f^2$** (frequency flicker and random walk), thus **$1/f^3$ and $1/f^4$** of the oscillator phase
- The resonator bandwidth does not apply to the natural-frequency fluctuation. (Tip: an oscillator can be frequency modulated ar a rate >> f_L)



Figures are from E. Rubiola, Phase noise and frequency stability in oscillators, © Cambridge University Press

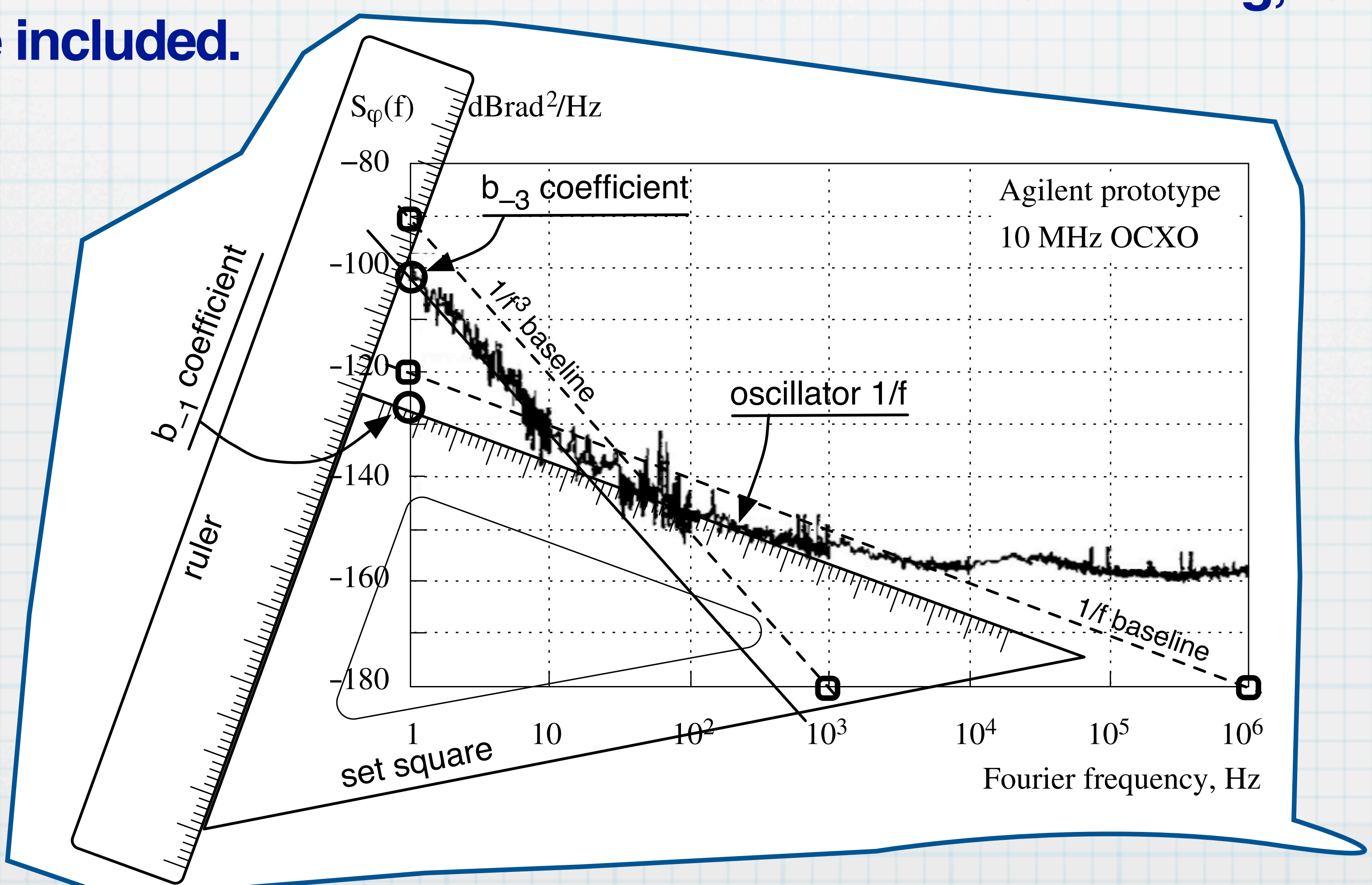
Oscillator hacking

Analysis of some commercial oscillators

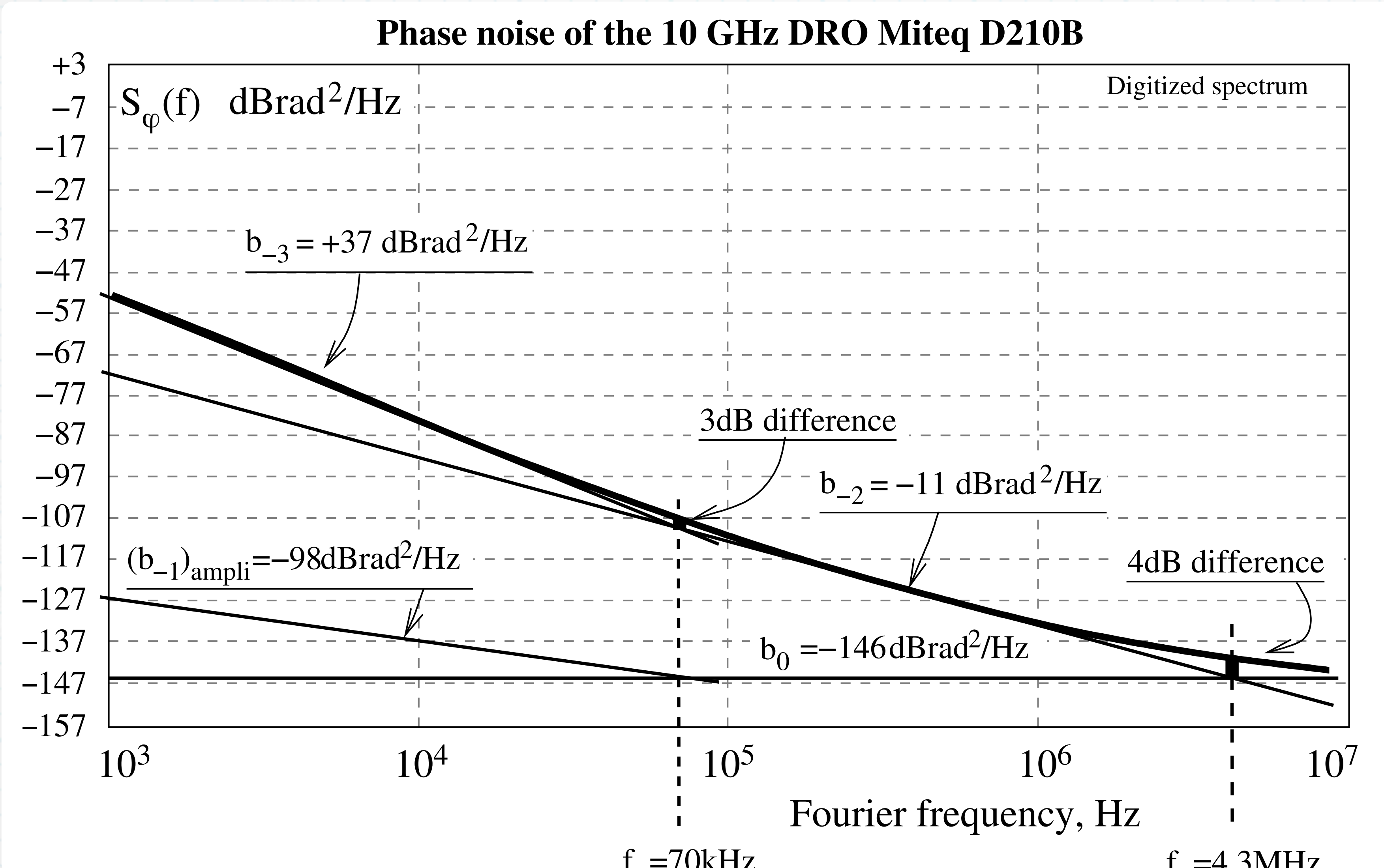
The purpose of this section is to help to understand the oscillator inside from the phase noise spectra, plus some technical information. I have chosen some commercial oscillators as an example.

The conclusions about each oscillator represent only my understanding based on experience and on the data sheets published on the manufacturer web site.

You should be aware that this process of interpretation is not free from errors. My conclusions were not submitted to manufacturers before writing, for their comments could not be included.



Miteq D210B, 10 GHz DRO



tables

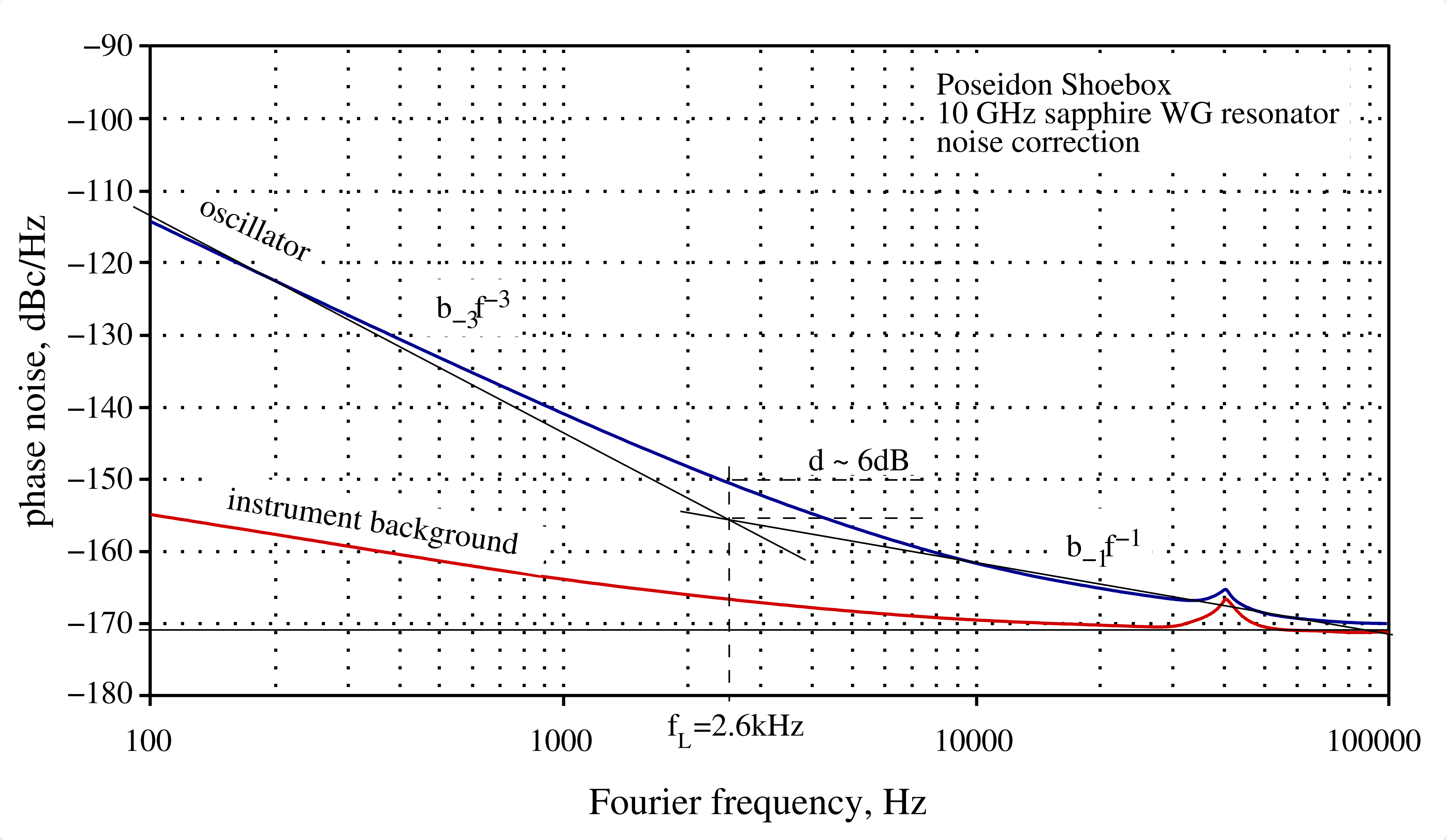
$\sigma^2_y = h_0/2\tau+2\ln(2)h_{-1}$

$h_0 = b_{-2}/\nu^2_0$

$h_{-1} = b_{-3}/\nu^2_0$

- $kT_0 = 4\times10^{-21}$ W/Hz (–174 dBm/Hz)
- floor –146 dBrad²/Hz, guess F = 1.25 (1 dB) => P_0 = 2 µW (–27 dBm)
- f_L = 4.3 MHz, $f_L = \nu0/2Q$ => Q = 1160
- f_c = 70 kHz, $b_{-1}/f = b0$ => $b_{-1} = 1.8\times10^{-10}$ (–98 dBrad²/Hz) [sust.ampli]
- $h_0 = 7.9\times10^{-22}$ and $h_{-1} = 5\times10^{-17}$ => $\sigma_y = 2\times10^{-11}/\sqrt{\tau} + 8.3\times10^{-9}$

Poseidon Scientific Instruments – Shoebox 10 GHz sapphire whispering-gallery (1)

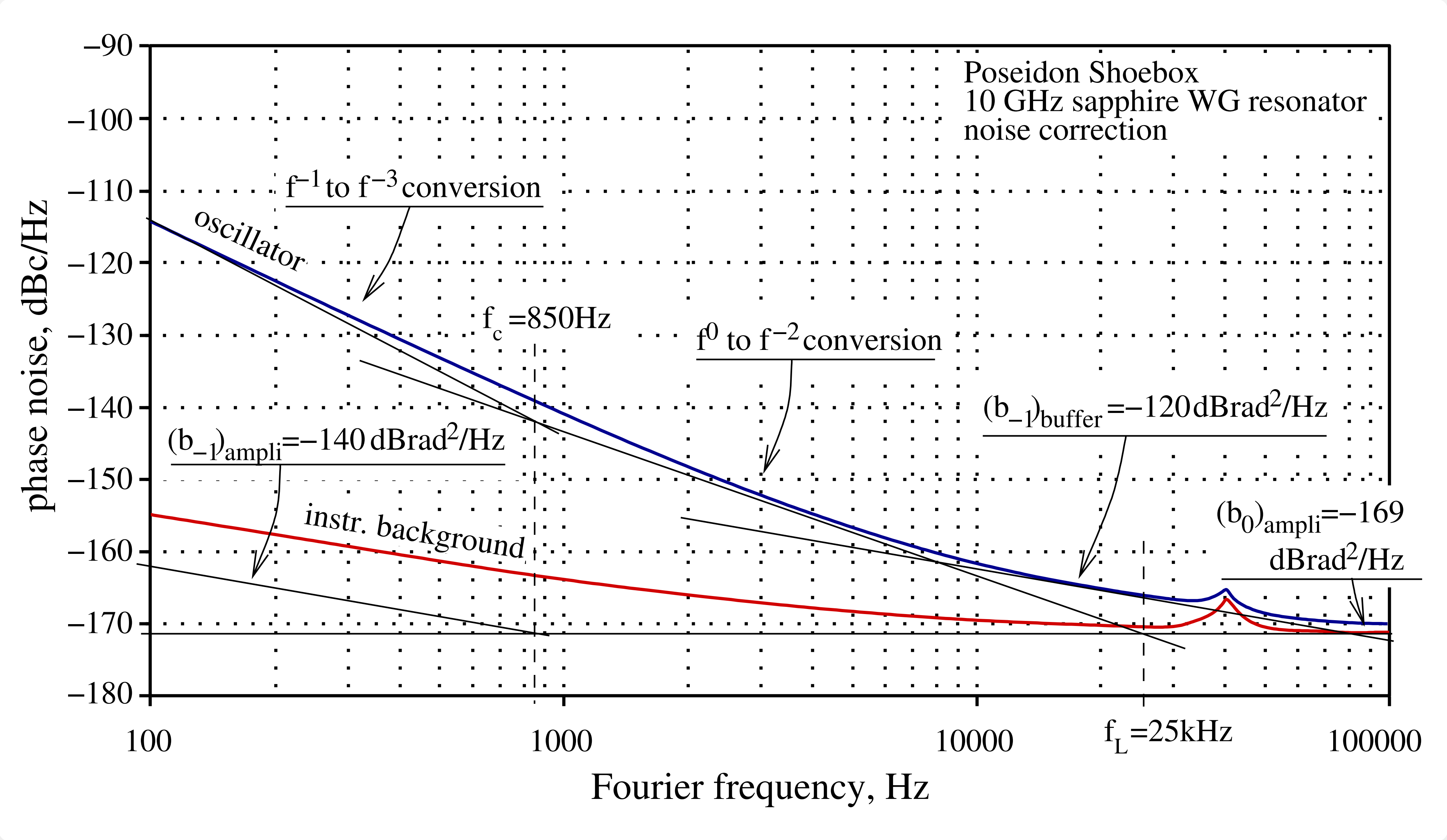


$f_L = \nu_0/2Q = 2.6$ kHz => $Q = 1.8\times10^6$

This incompatible with the resonator technology.
Typical Q of a sapphire whispering gallery resonator:
2×10^5 @ 295K (room temp), 3×10^7 @ 77K (liquid N), 4×10^9 @ 4K (liquid He).
In addition, d ~ 6 dB does not fit the power-law.

The interpretation shown is wrong, and the Leeson frequency is somewhere else

Poseidon Scientific Instruments – Shoebox 10 GHz sapphire whispering-gallery (2)



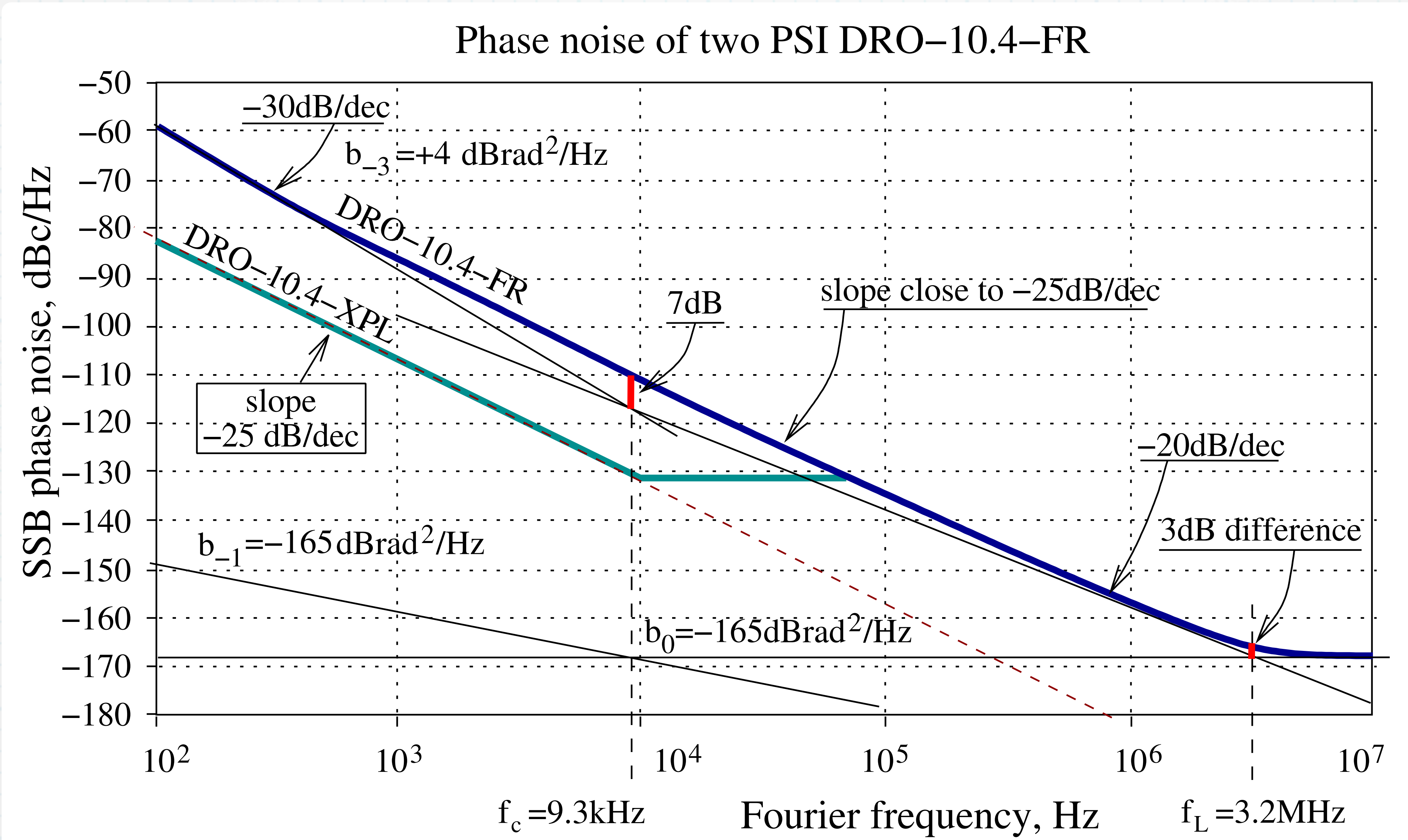
The 1/f noise of the output buffer is higher than that of the sustaining amplifier (a compex amplifier with interferometric noise reduction)

In this case both 1/f and 1/f² are present

white noise –169 dBrad²/Hz, guess F = 5 dB (interferometer) => P_0 = 0 dBm
buffer flicker –120 dBrad²/Hz @ 1 Hz => good microwave amplifier

$f_L = \nu_0/2Q$ = 25 kHz => Q = 2×10⁵ (quite reasonable)
f_c = 850 Hz => flicker of the interferometric amplifier –139 dBrad²/Hz @ 1 Hz

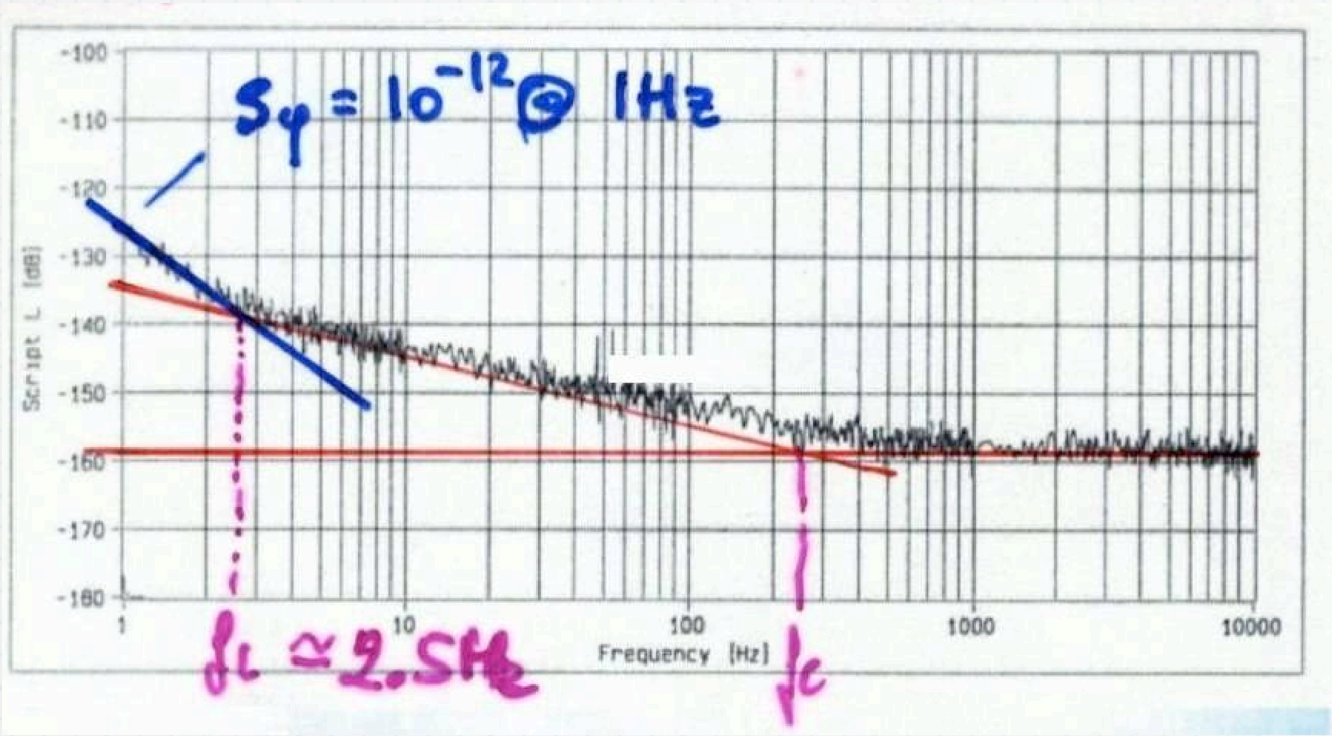
Poseidon Scientific Instruments
10 GHz dielectric resonator oscillator (DRO)



- floor –165 dBrad^2/Hz, guess F = 1.25 (1 dB) => P_0 = 160 μW (–8 dBm)
- f_L = 3.2 MHz, f_L = ν0/2Q => Q = 625
- f_c = 9.3 kHz, b_{-1}/f = b0 => $b_{-1} = 2.9\times10^{-13}$ (–125 dBrad^2/Hz) [sust.ampli, too low]

Slopes are not in agreement with the theory

OCXO 8600
Technical Specification



Oscilloquartz OCXO 8600
outstanding stability oscillator based on a
5 MHz AT-cut BVA (electrodless) resonator
stability $\sigma_y(\tau) = 3\times10\text{-}13$ for $\tau = 0.2 \div 30$ s
aging 3×10^{-12}/day

Courtesy of Oscilloquartz (handwritten notes are mine).
The specifications, which include this spectrum, are available at the URL http://www.oscilloquartz.com/file/pdf/8600.pdf

ANALYSIS

1 – floor $S_{\varphi 0}$ = –155 dBrad^2/Hz, guess F = 1 dB ➜ P_0 = –18 dBm

2 – ampli flicker S_φ = –132 dBrad^2/Hz @ 1 Hz ➜ good RF amplifier

3 – merit factor $Q = \nu_0/2f_L = 5\cdot10^6/5 = 10^6$ (seems too low)

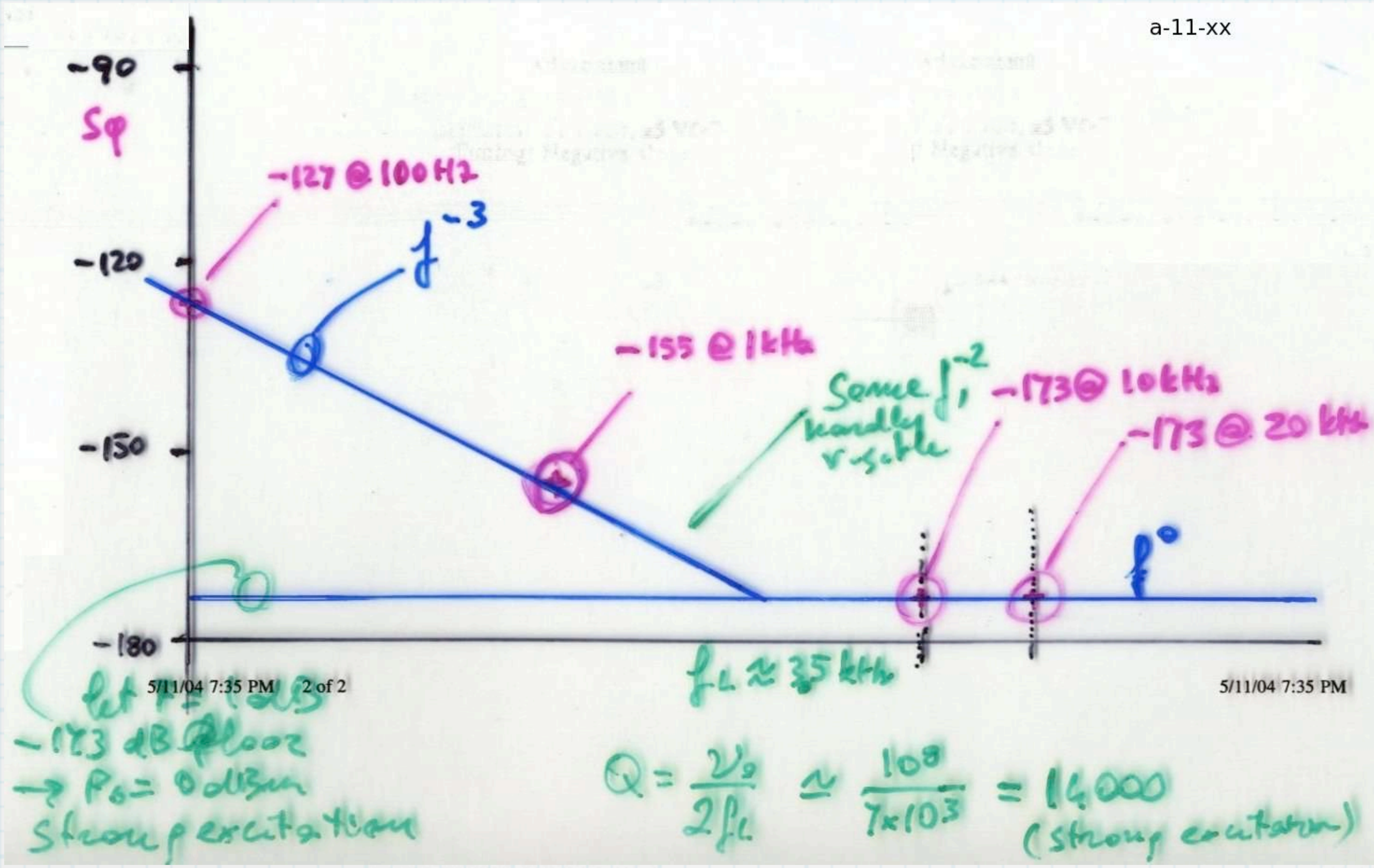
4 – take away some flicker for the output buffer:
- * flicker in the oscillator core is lower than –132 dBrad^2/Hz @ 1 Hz
- * f_L is higher than 2.5 Hz
- * the resonator Q is lower than 10^6

This is inconsistent with the resonator technology (expect $Q > 10^6$).

The true Leeson frequency is lower than the frequency labeled as f_L

The $1/f^3$ noise is attributed to the fluctuation of the quartz resonant frequency

Wenzel 501-04623 G - Lowest phase noise 100 MHz SC-cut oscillator



manufacturer specs,
phase noise

-130 dBc/Hz	@ 100 Hz
-158 dBc/Hz	@ 1 kHz
-176 dBc/Hz	@ 10 kHz
-176 dBc/Hz	@ 20 kHz

1 – floor $S_{\varphi 0}$ = –173 dBrad^2/Hz, guess F = 1 dB → P_0 = 0 dBm

2 – merit factor $Q = \nu_0/2f_L = 10^8/7\times 103 = 1.4\times 10^4$ (seems too low)

From the literature, one expects $Q \sim 10^5$.

The true Leeson frequency is lower than the frequency labeled as f_L

The $1/f^3$ noise is attributed to the fluctuation of the quartz resonant frequency

A12x

TIDALwave™

Ultra-Low Phase Noise Microwave Signal Source

OEwaves

- Imaging
- Digital Radio (QAM)
- Optical Data Communications

magnitude, and high capacity, high frequency future wireless communications systems. This level of performance will enable manufacturers to retrofit current systems as well as architect capabilities to address new markets.

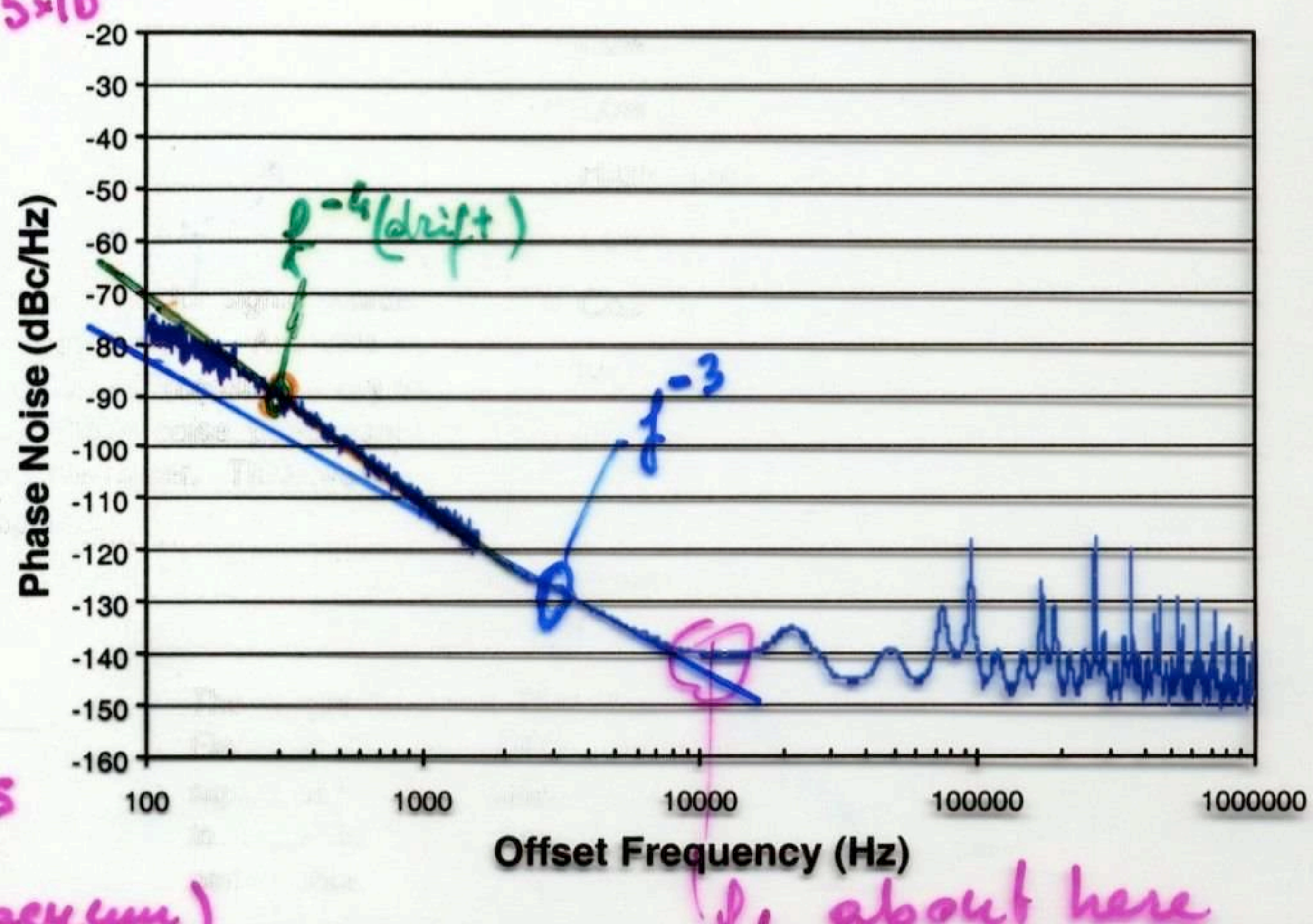
OEwaves is developing *miniaturized* (MINIwave™) and multi-octave *tunable* (TUNEwave™) signal sources based on the performance and specifications of TIDALwave.

$f_c = \frac{\nu_0}{2Q} \rightarrow \quad Q \simeq \frac{10^{10}}{2 \times 10^4} = 5 \times 10^5$

Free Running Phase Noise Plot

TIDALwave - 10 GHz

Model: OE1255



delay line

$Q = \pi \nu_0 \tau$

$\tau = \frac{Q}{\pi \nu_0} \sim \frac{5 \times 10^5}{3.2 \cdot 10^{10}} = 16\ \mu s$

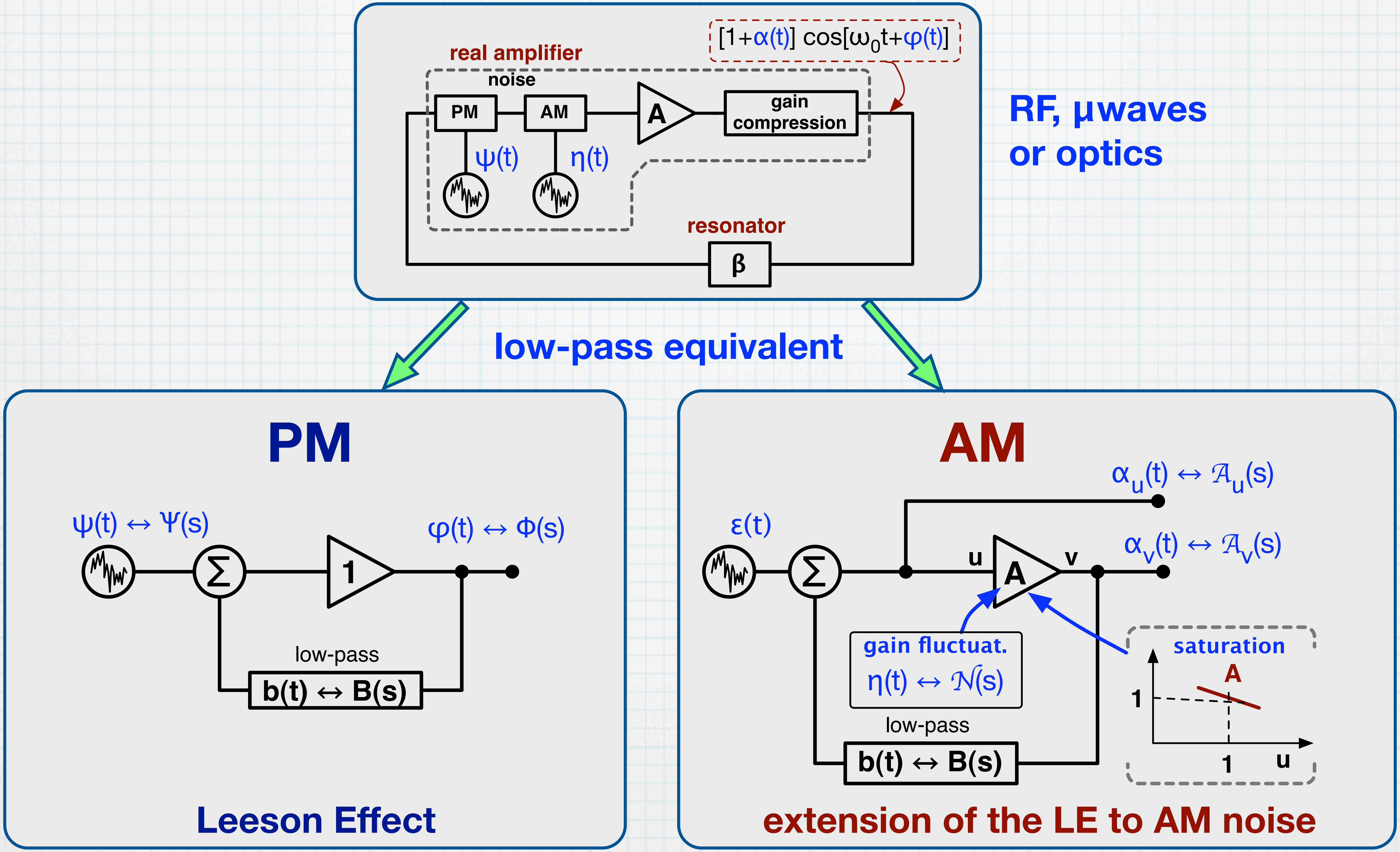
length $= c\tau =$ 5 km (vacuum)

3.2 km ($n = 1.5$)

Courtesy of OEwaves (handwritten notes are mine). Cut from the oscillator specifications available at the URL http://www.oewaves.com/products/pdf/TDALwave_Datasheet_012104.pdf

The Leeson effect

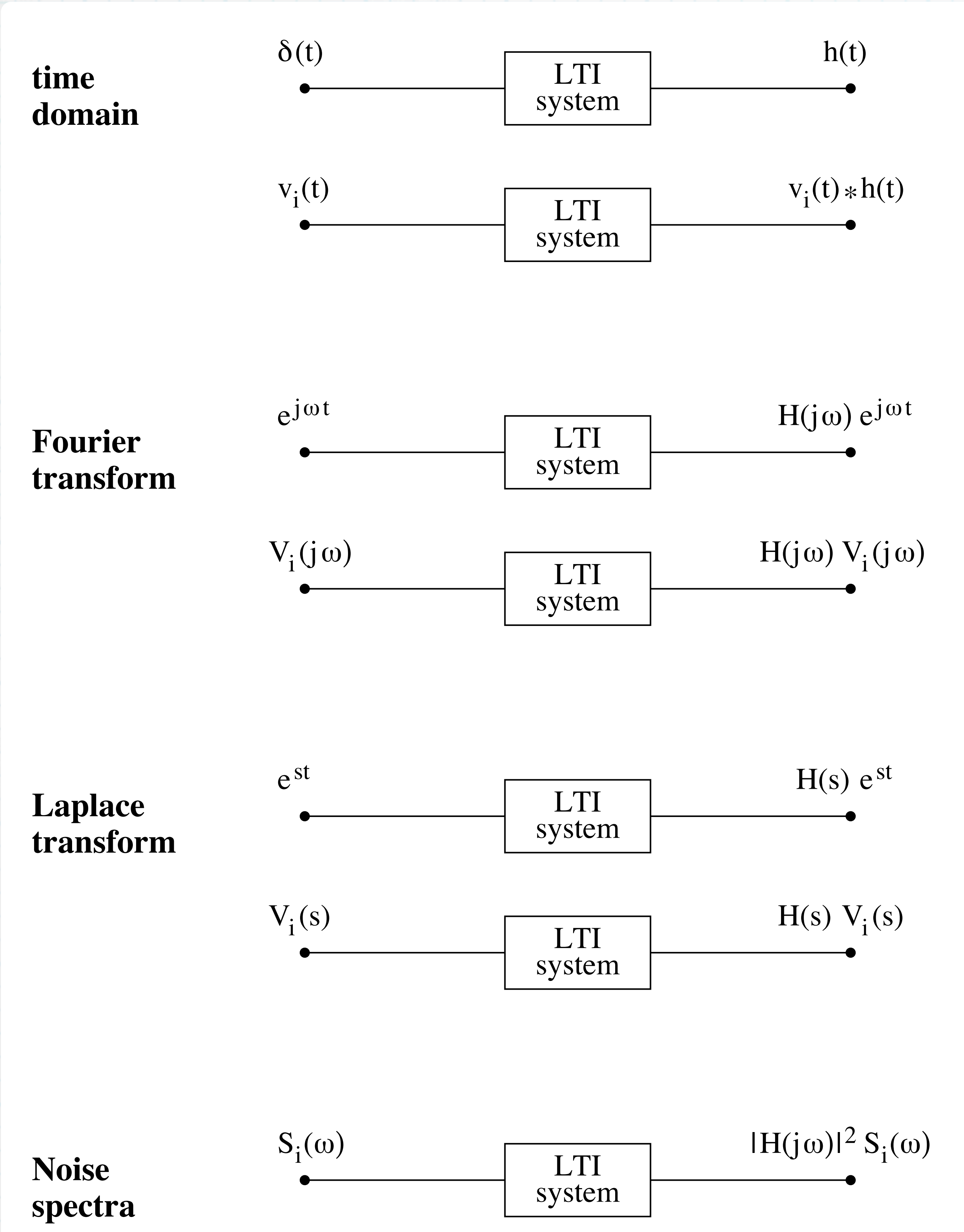
Low-pass representation of AM-PM noise



The amplifier
- **"copies" the input phase to the out**
- **adds phase noise**

The amplifier
- **compresses the amplitude**
- **adds amplitude noise**

Linear time-invariant (LTI) systems



impulse response

response to the generic signal $v_i(t)$

$$H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$

$$H(s) = \int_{0}^{\infty} h(t)\, e^{-st}\, dt$$

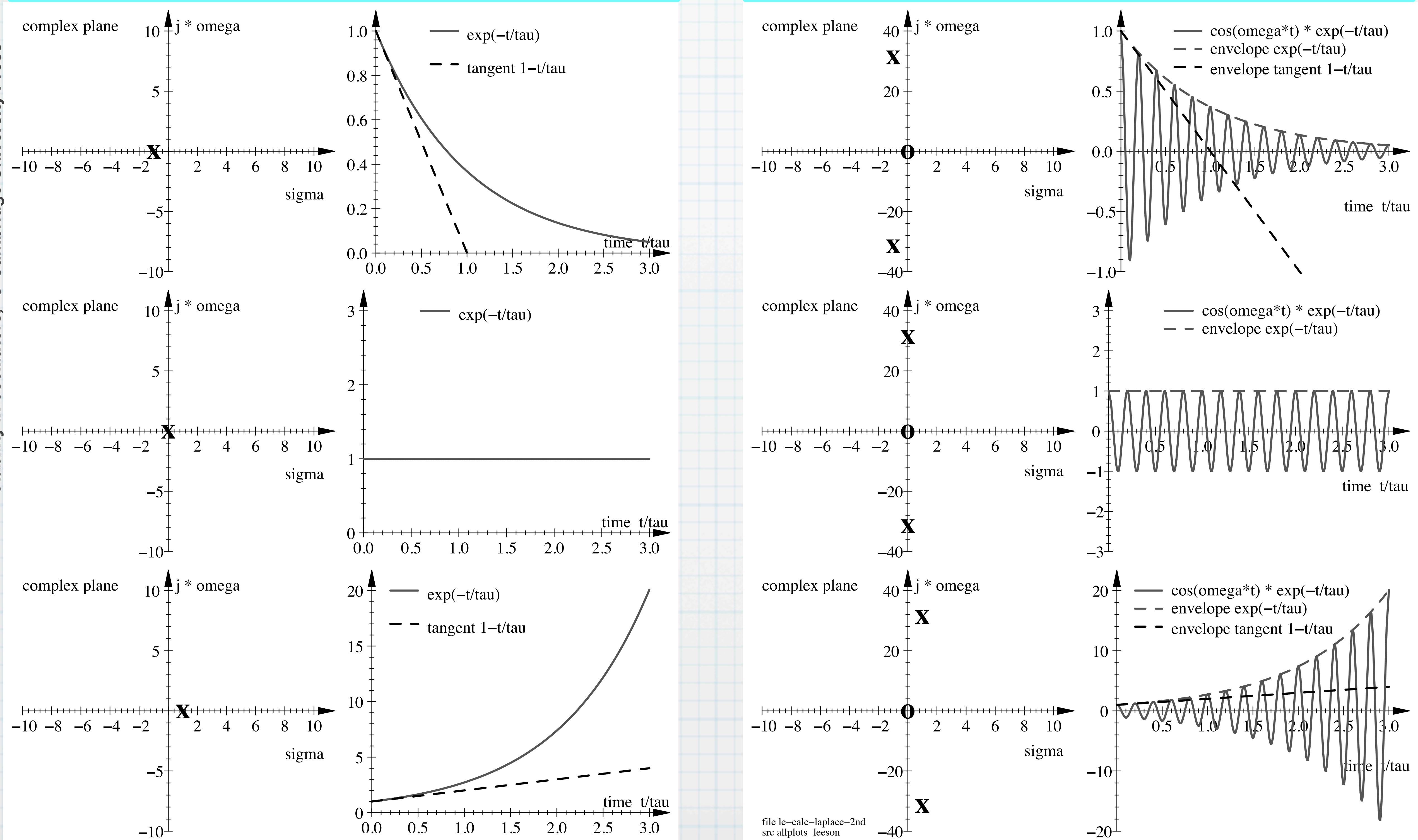
H(s), s=σ+jω, is the analytic continuation of H(ω) for causal system, where h(t)=0 for t<0

Laplace-transform patterns

Fundamental theorem: F(s) is completely determined by its roots (poles and zeros)

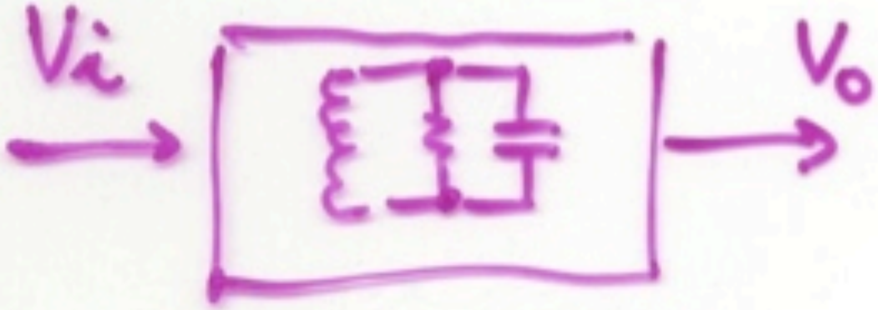
$$F(s) = \frac{1}{s + 1/\tau}$$

$$F(s) = \frac{s}{s^2 + 2s/\tau + \omega_n^2}$$



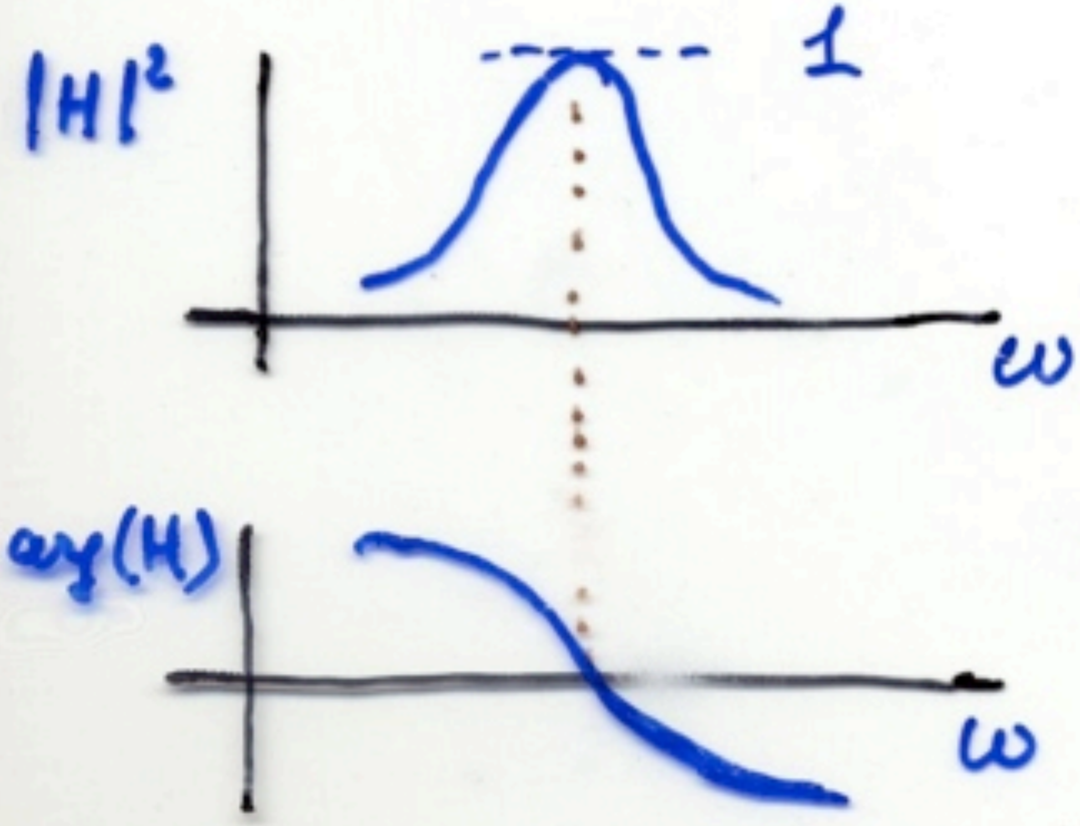
RESONATOR

B4



$$H(s) = \frac{V_0(s)}{V_i(s)}$$

$s = \sigma + j\omega$



$$H(s) = \frac{\omega_0}{Q} \, \frac{s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

normalization for $H_{max} = 1$

define

$$x = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \quad \xrightarrow{\omega \to \omega_0} \quad 2\,\frac{\omega - \omega_0}{\omega}$$

$$H(j\omega) = \frac{1}{1 + jQx} = \frac{1 - jQx}{1 + Q^2 x^2}$$

Real, Imag

$$R(\omega) = \frac{1}{1 + Q^2 x^2}$$

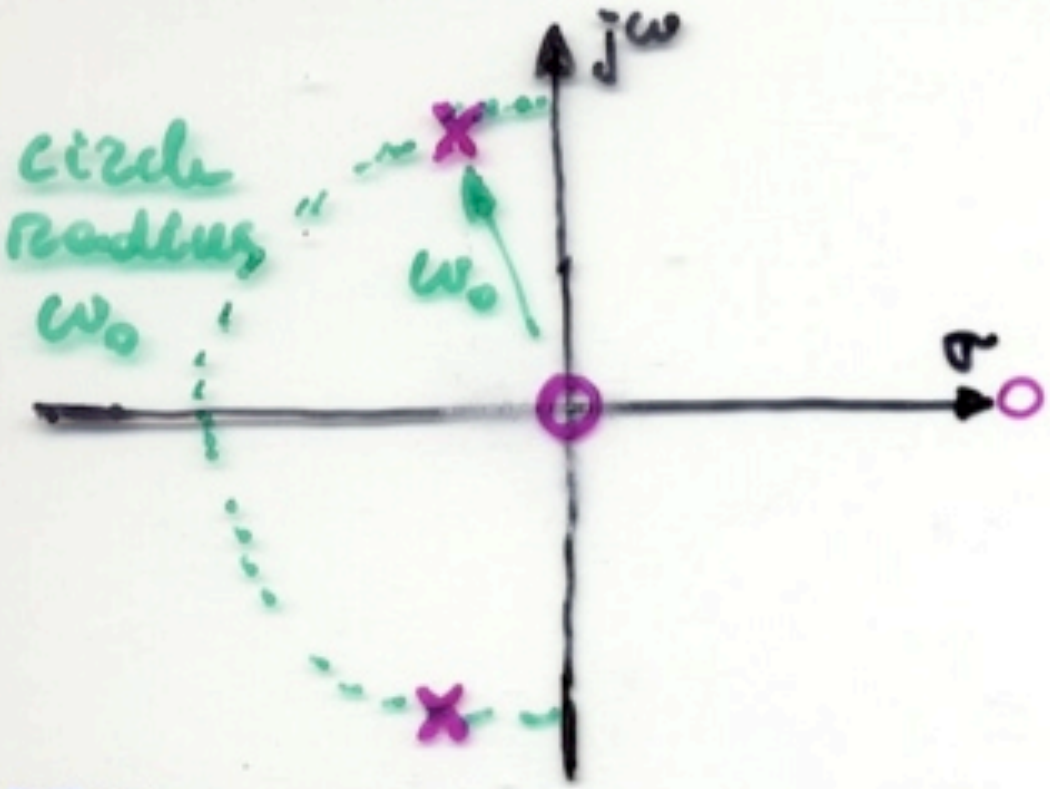
$$I(\omega) = \frac{-Qx}{1 + Q^2 x^2}$$

Modulus, phase

$$M(\omega) = \frac{1}{\sqrt{1 + Q^2 x^2}}$$

$$\Phi(\omega) = -\arctan Qx$$

RESONATOR - COMPLEX PLANE REPRESENTATION



$$H(s) = \frac{\omega_0}{Q} \frac{s}{(s-s_p)(s-s_p^*)}$$

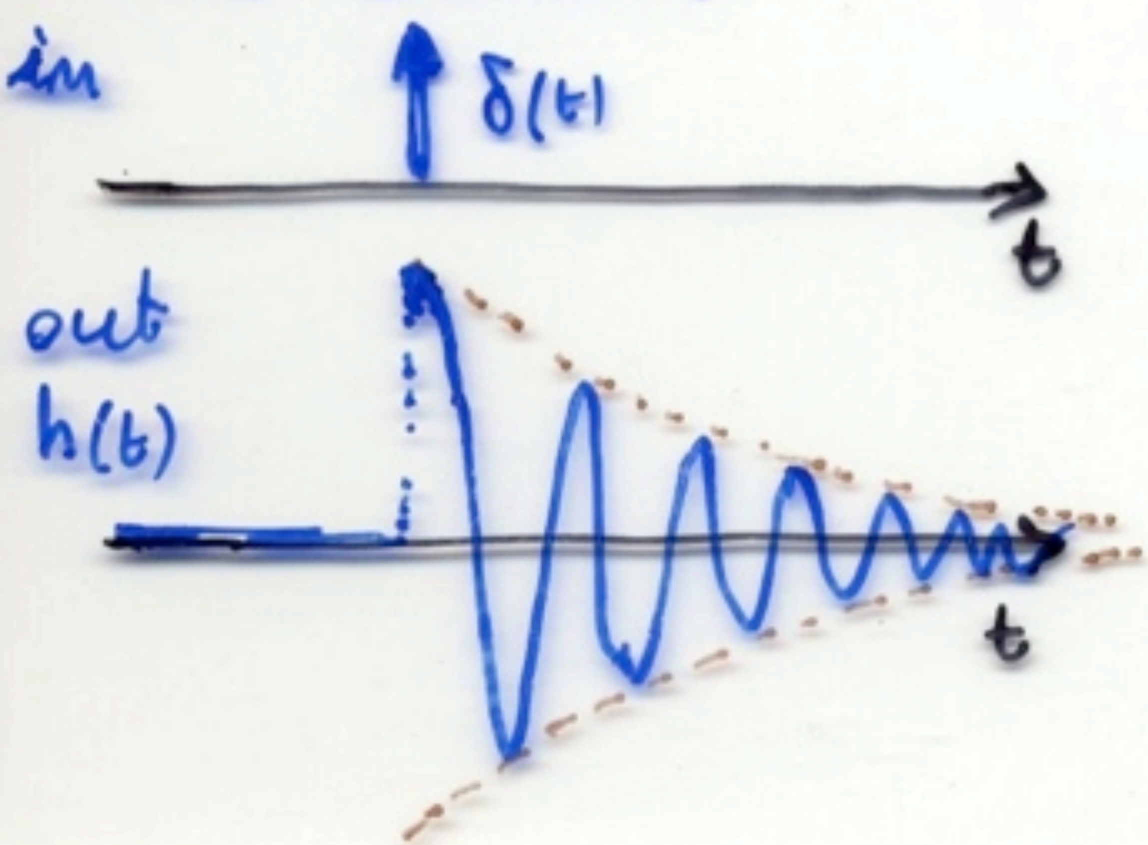
Hurwitz polynomial → poles in the left half plane

$s_p = \sigma_p + j\omega_p$ pole

$$\sigma_p = -\frac{\omega_0}{2Q}$$

$$\omega_p = \frac{\omega_0}{2Q}\sqrt{4Q^2-1}$$

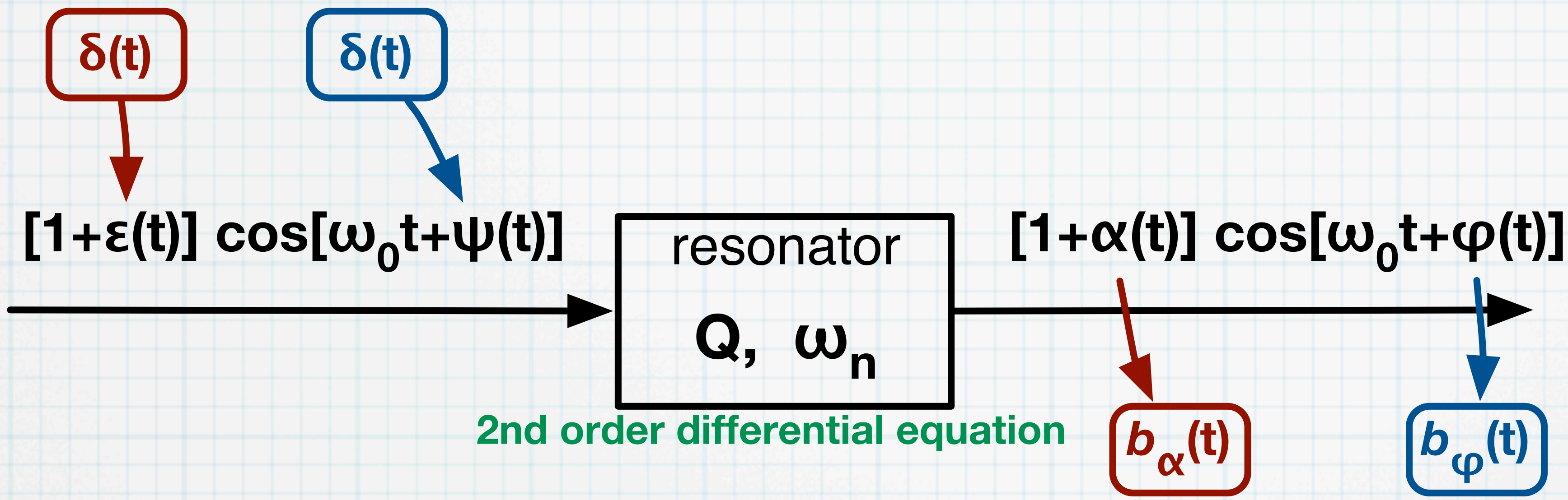
TIME DOMAIN



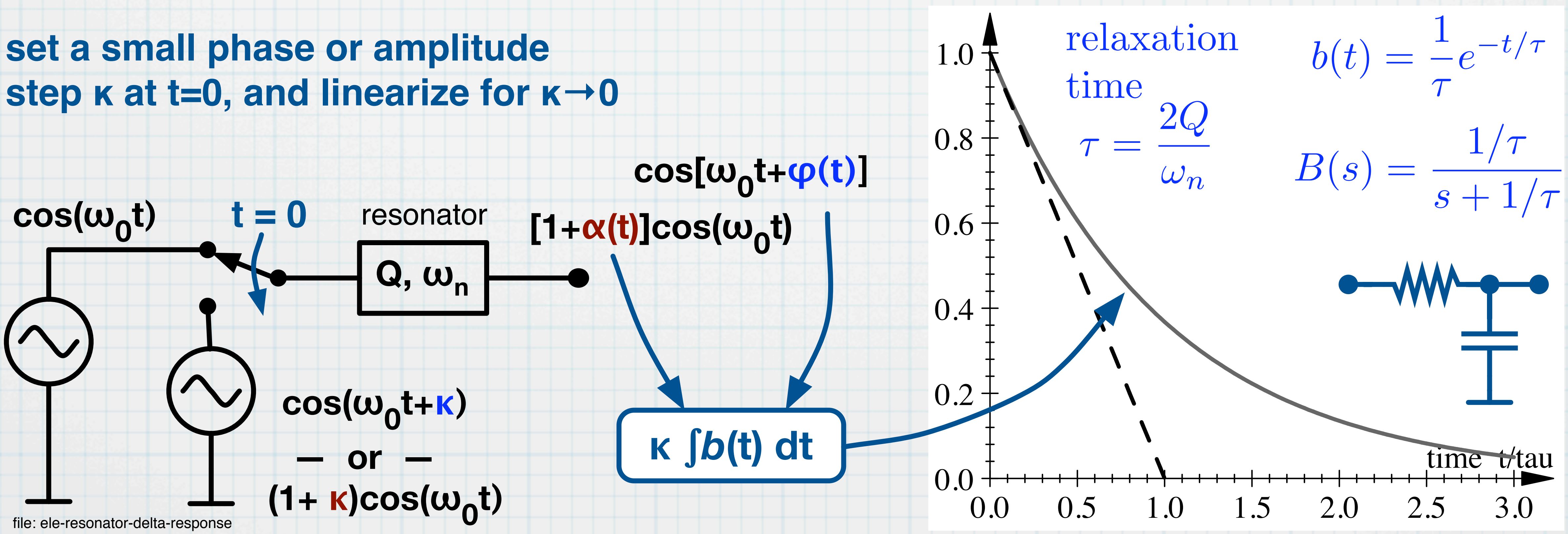
$$h(t) = e^{\sigma_p t}\cos(\omega_p t)$$

	response	(poles:)
$\sigma<0$	decaying	(left)
$\sigma=0$	steady	(imag)
$\sigma>0$	growing	right

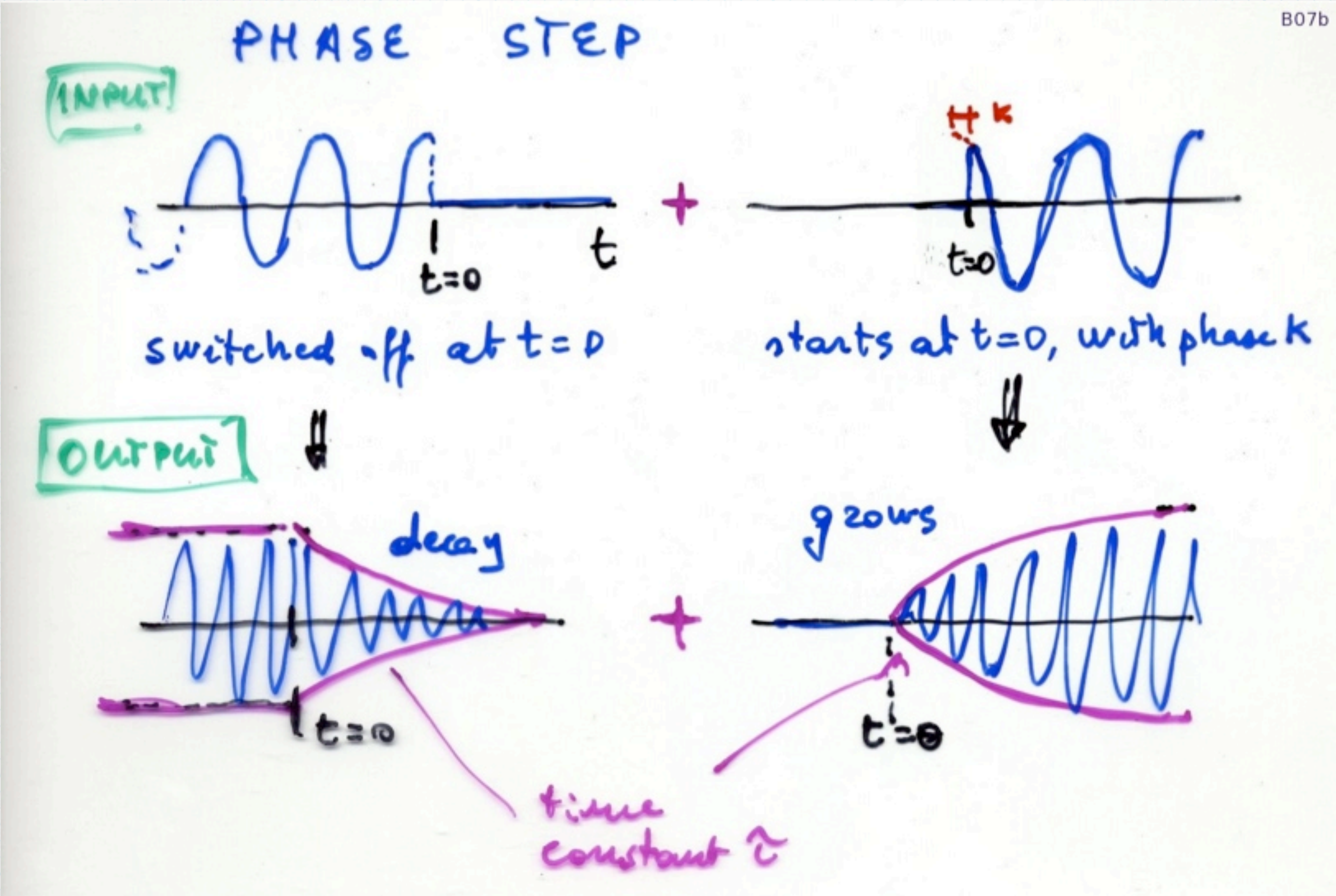
Resonator in the phase space



set a small phase or amplitude step κ at t=0, and linearize for κ→0



Resonator impulse response (proof)



Resonator impulse response (proof)

B8

DETAILS

$v(t)$ resonator output voltage

$$v(t) = \underbrace{\cos(\omega_0 t)\, e^{-t/\tau}}_{\text{decay}} + \cos(\omega_0 t + \kappa)\, \underbrace{\left[1 - e^{-t/\tau}\right]}_{\text{growth}}$$

$$v(t) = \cos(\omega_0 t)\, e^{-t/\tau} + \left[\cos(\omega_0 t)\cos(\kappa) - \sin(\omega_0 t)\sin(\kappa)\right]\left[1 - e^{-t/\tau}\right]$$

use $\kappa \ll 1$ $\cos\kappa = 1$ $\sin\kappa = \kappa$

$$v(t) = \cos(\omega_0 t)\, e^{-t/\tau} + \left[\cos\omega_0 t - \kappa \sin\omega_0 t\right]\left[1 - e^{-t/\tau}\right]$$

$$= \cos(\omega_0 t)\, e^{-t/\tau} + \cos(\omega_0 t)\left[1 - e^{-t/\tau}\right] - \kappa \sin(\omega t)\left[1 - e^{-t/\tau}\right]$$

$$v(t) = \underbrace{\cos(\omega_0 t)}_{\text{Re}} - \underbrace{\kappa \sin(\omega_0 t)\left[1 - e^{-t/\tau}\right]}_{\text{Im}}$$

Fresnel vector

Resonator impulse response (proof)

B08b

FRESNEL VECTOR V

$\kappa\left[1-e^{-t/\tau}\right]$

$$\arg(V) = \kappa\left[1-e^{-t/\tau}\right]$$

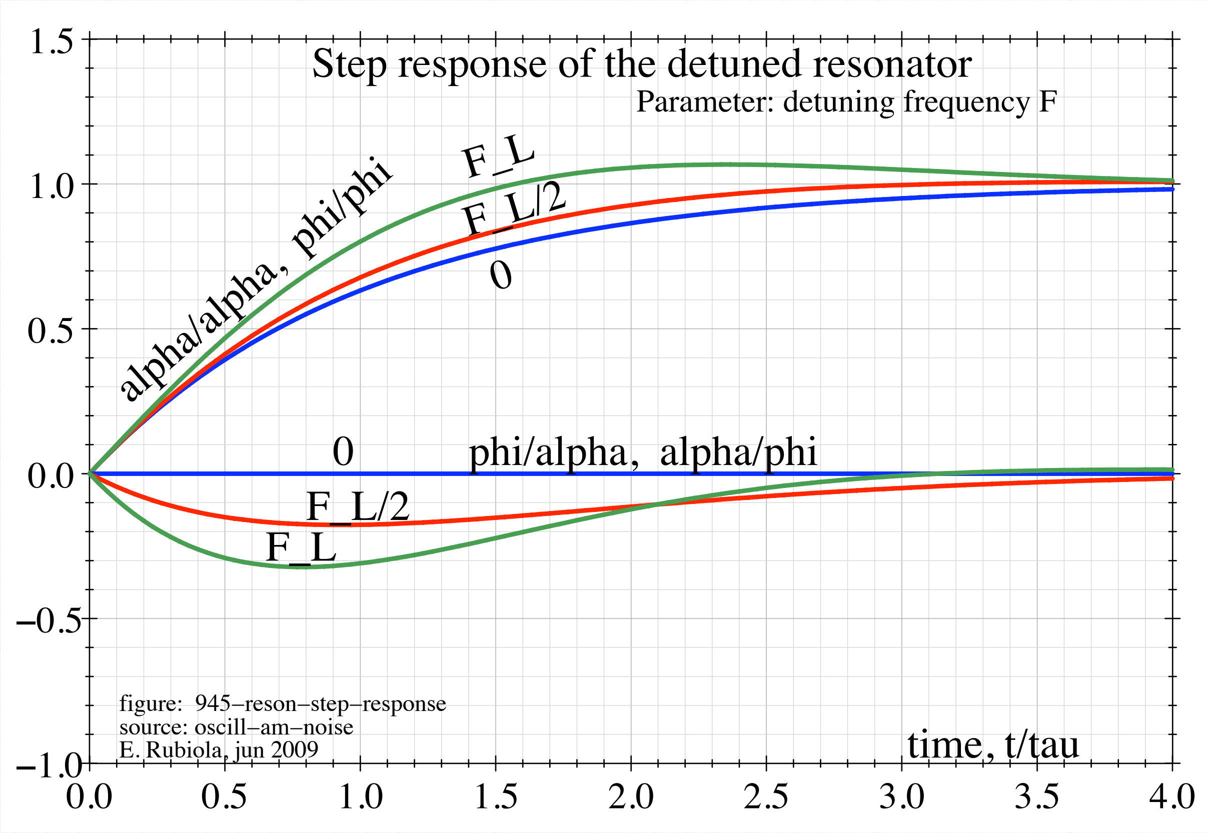
unity step → • replace $\kappa u(t) \to u(t)$
• don't forget that all this holds for $\kappa \ll 1$

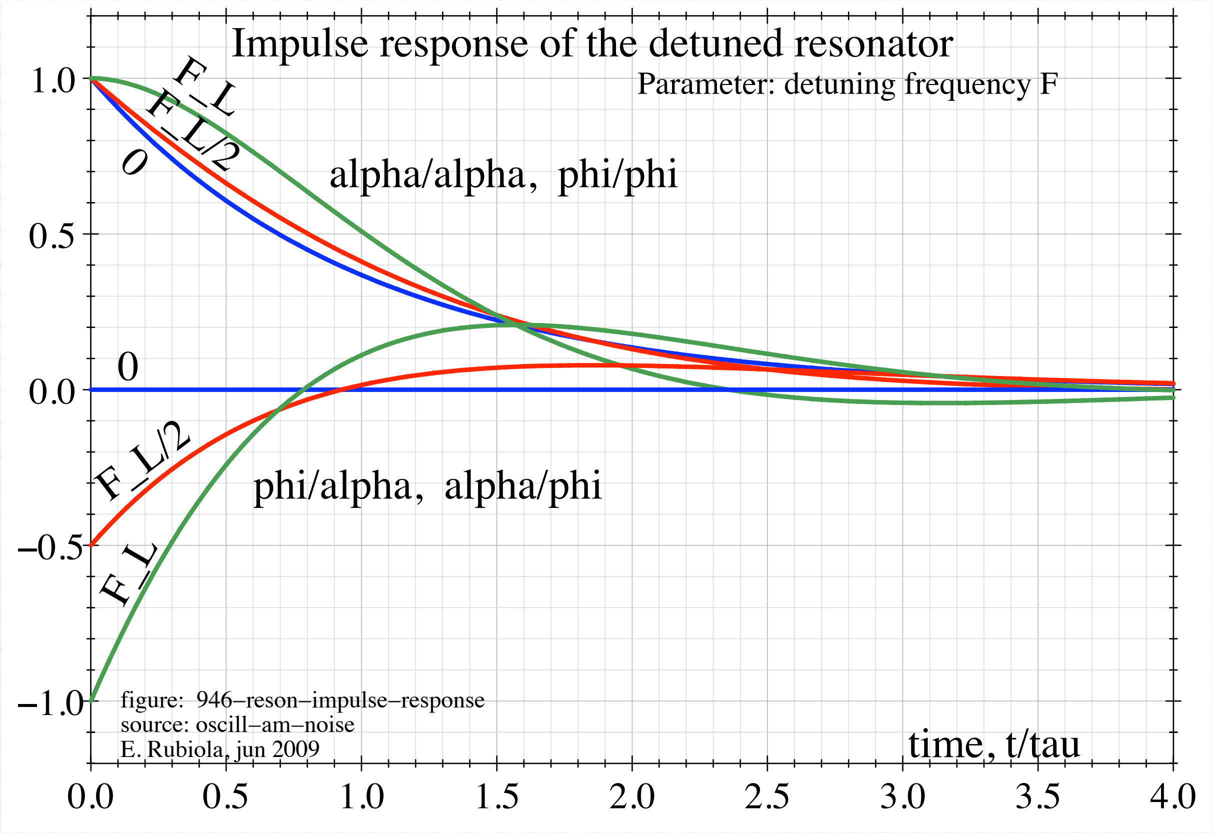
step response $\quad b_u(t) = 1 - e^{-t/\tau}$

impulse response $\quad b(t) = \frac{1}{\tau} e^{-t/\tau}$ $\quad$ derivative

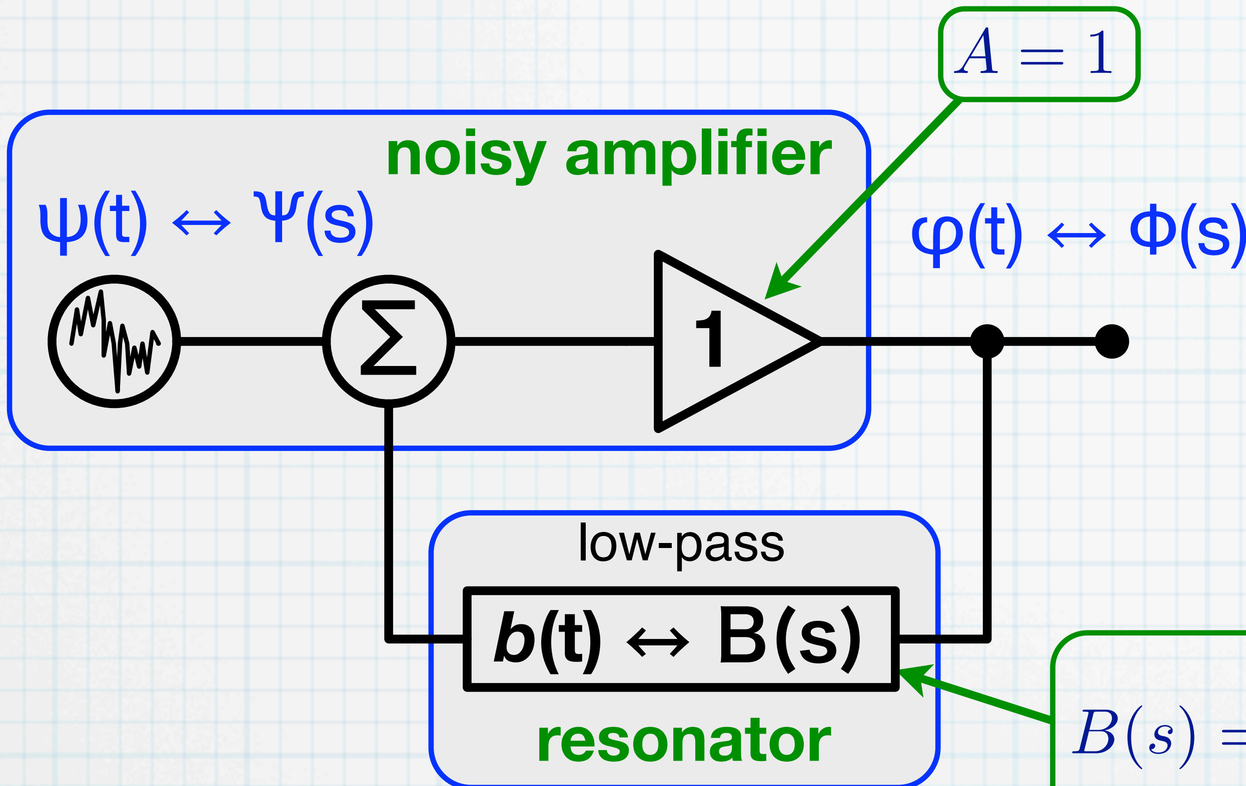
$$B(s) = \frac{1/\tau}{s + 1/\tau} = \frac{1}{s\tau + 1}$$

Resonator step and impulse response





Leeson effect



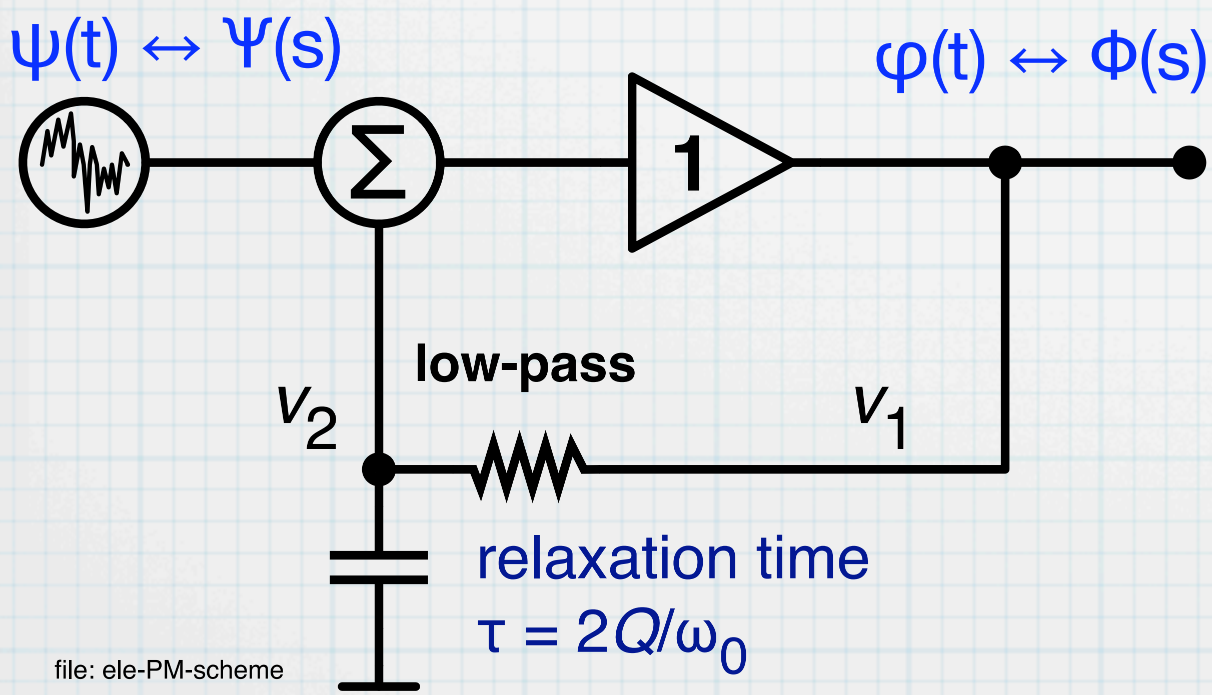
$$B(s) = \frac{1/\tau}{s + 1/\tau}$$

phase-noise transfer function

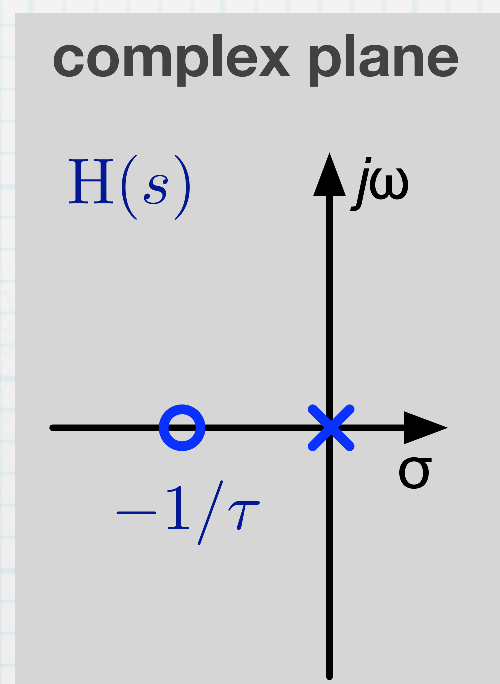
$$\mathrm{H}(s) = \frac{\Phi(s)}{\Psi(s)}$$ **definition**

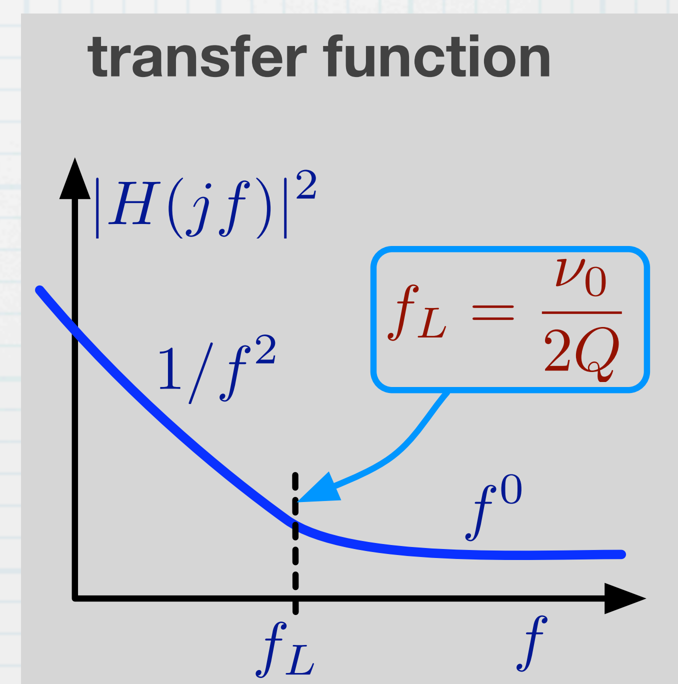
$$\mathrm{H}(s) = \frac{1}{1 + AB(s)}$$ **general feedback theory**

$$\mathrm{H}(s) = \frac{1 + s\tau}{s\tau}$$ **Leeson effect**



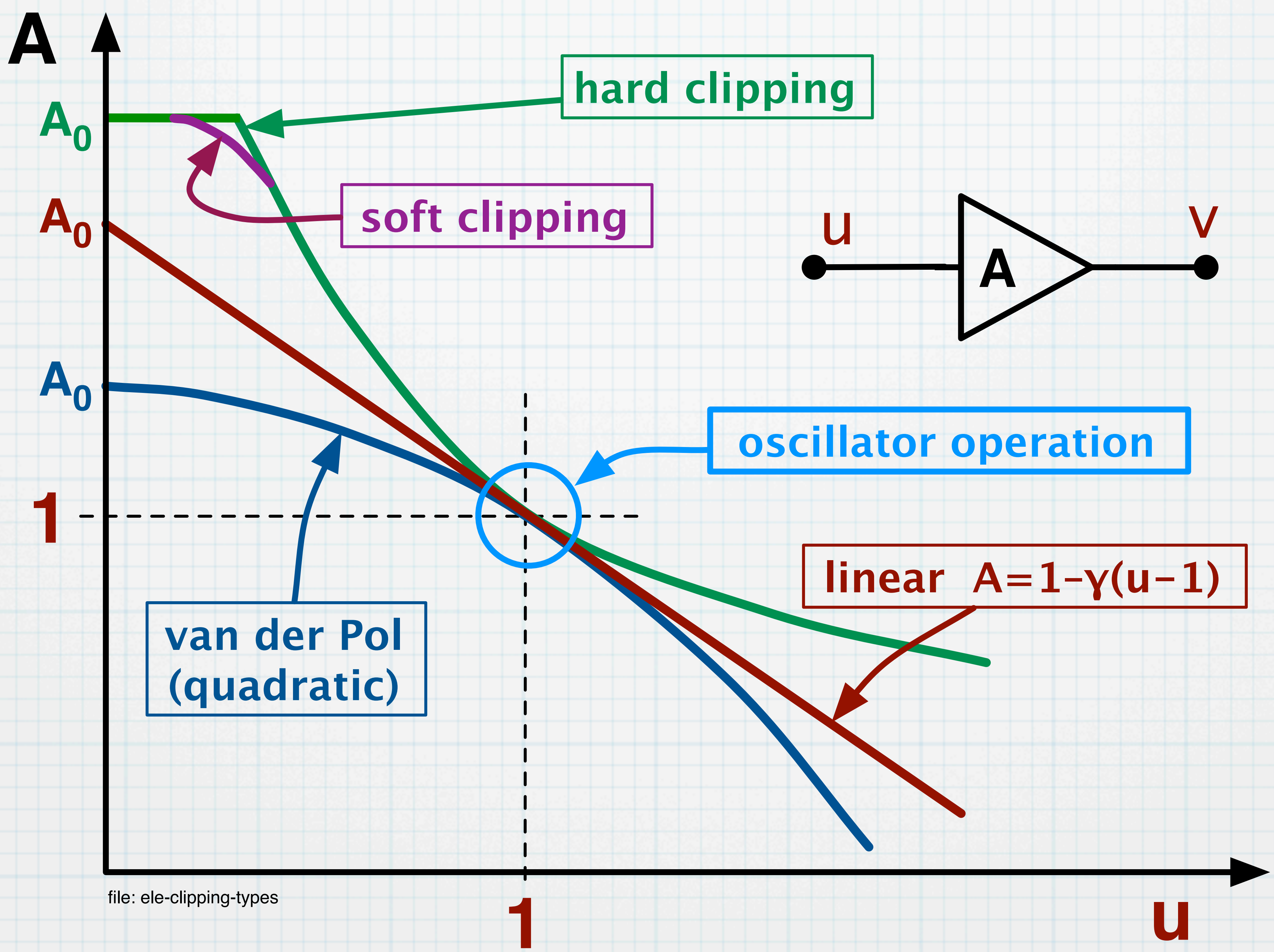
file: ele-PM-scheme





Extension to AM noise

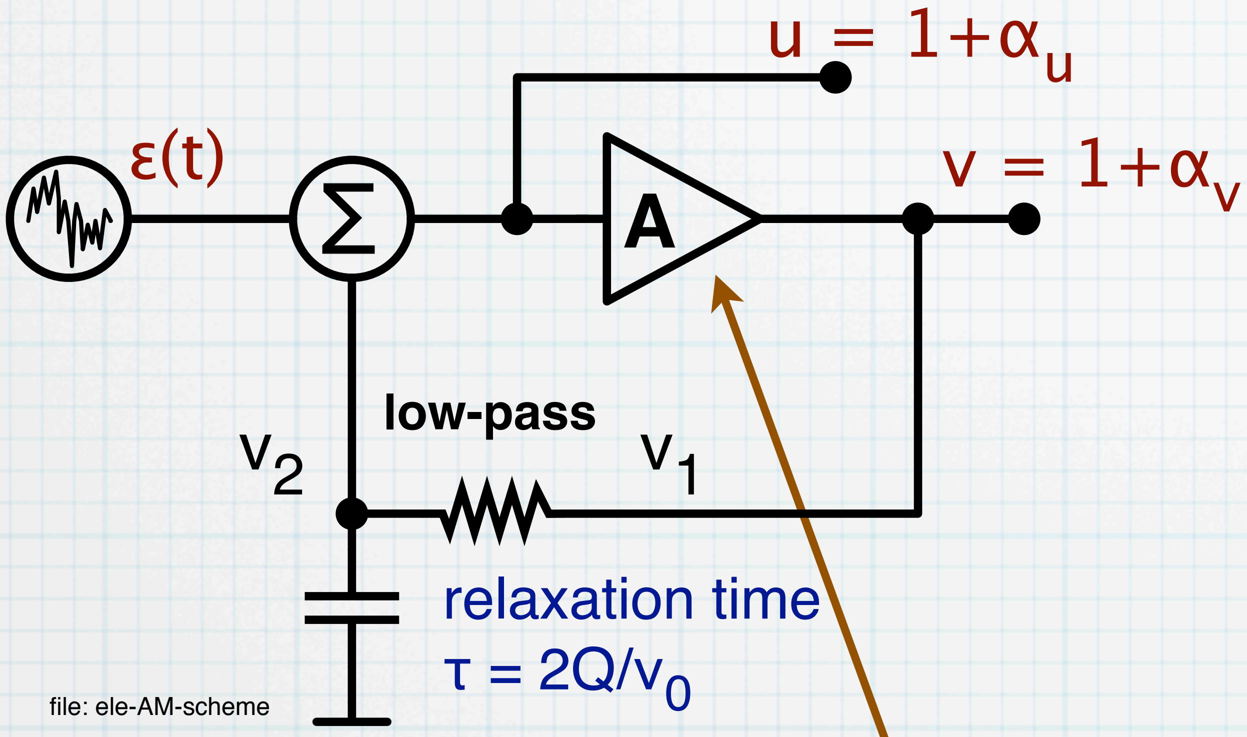
Common types of gain saturation



Gain compression is necessary for the oscillation amplitude to be stable

Low-pass model of amplitude (1)

First we need to relate the system restoring time τ_r to the relaxation time τ



simple feedback theory

$$u = \epsilon + v_2$$

$$v_2 = \frac{1}{\tau}\int (v_1 - v_2)\, dt$$

$$v_2 = u - \epsilon$$

$$v_1 = v = Au$$

$$u = \epsilon + \frac{1}{\tau}\int (A-1)u + \epsilon\, dt$$

differential equation

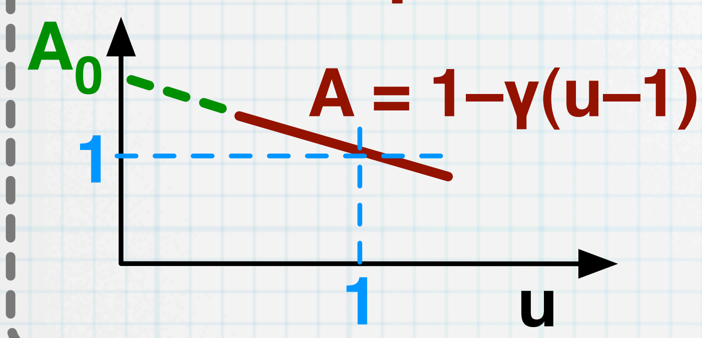
$$\dot{u} - \frac{1}{\tau}\left(A-1\right)u = \frac{1}{\tau}\epsilon + \dot{\epsilon}$$

Gain compression is necessary for the oscillation amplitude to be stable

The Laplace / Heaviside formalism cannot be used because the amplifier is non-linear

Low-pass model of amplitude (2)

Gain compression

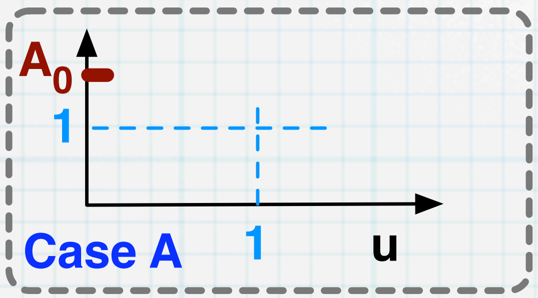


homogeneous differential equation

$$\dot{u} - \frac{1}{\tau}\,(A-1)\,u = 0$$

Three asymptotic cases

At low RF amplitude, let the gain be an arbitrary value denoted with A_0

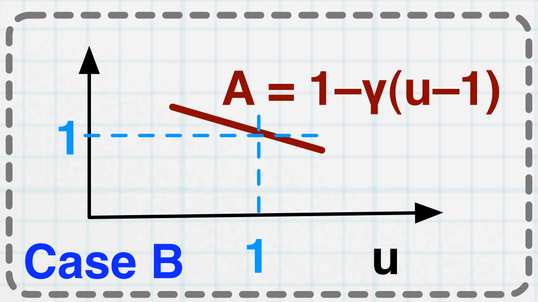


Startup: u→0, A → A_0 > 1

$$\dot{u} - \frac{1}{\tau}\,(A_0-1)\,u = 0 \quad\Rightarrow\quad \boxed{u = C_1 e^{(A_0-1)\,t/\tau}}$$

rising exponential

For small fluctuation of the stationary RF amplitude, the gain amplitude, the gain varies linearly with V

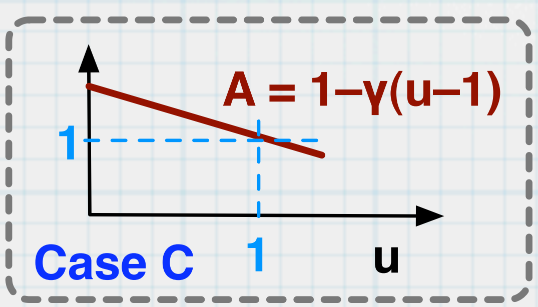


Regime: u→1, A = 1 – γ (u – 1)

$$\dot{u} + \frac{\gamma}{\tau}\,(u-1)\,u = 0 \quad\Rightarrow\quad \boxed{u = C_2 e^{-\gamma\,t/\tau}}$$

restoring time constant $\tau_r = \tau/\gamma$

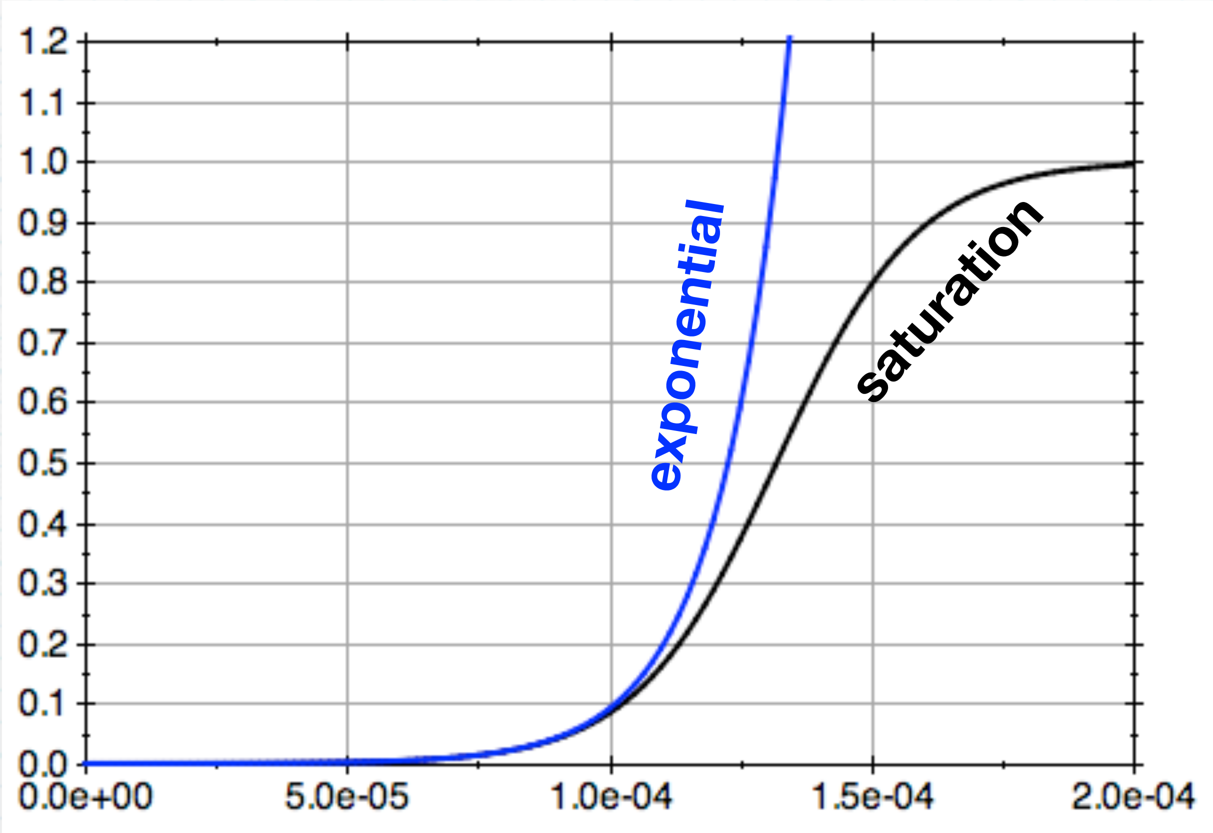
Simplification: the gain varies linearly with V in all the input range



Linear gain: A = 1 – γ (u – 1)

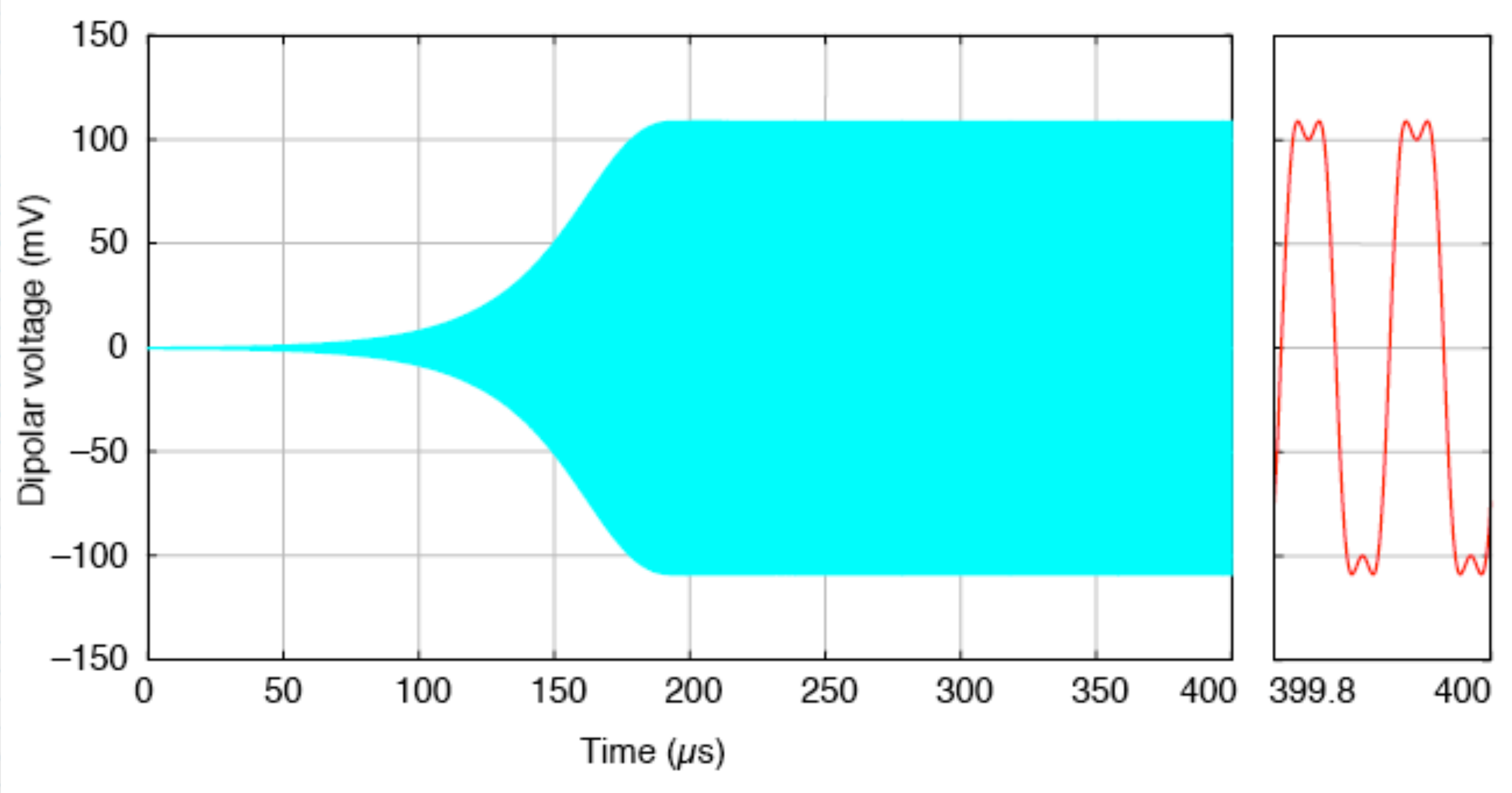
$$u = \frac{1}{\left(\frac{1}{u(0)} - 1\right) e^{-\gamma t/\tau} + 1}$$

Startup – analysis vs. simulation



analytical solution,
A = 1–γ(u–1)

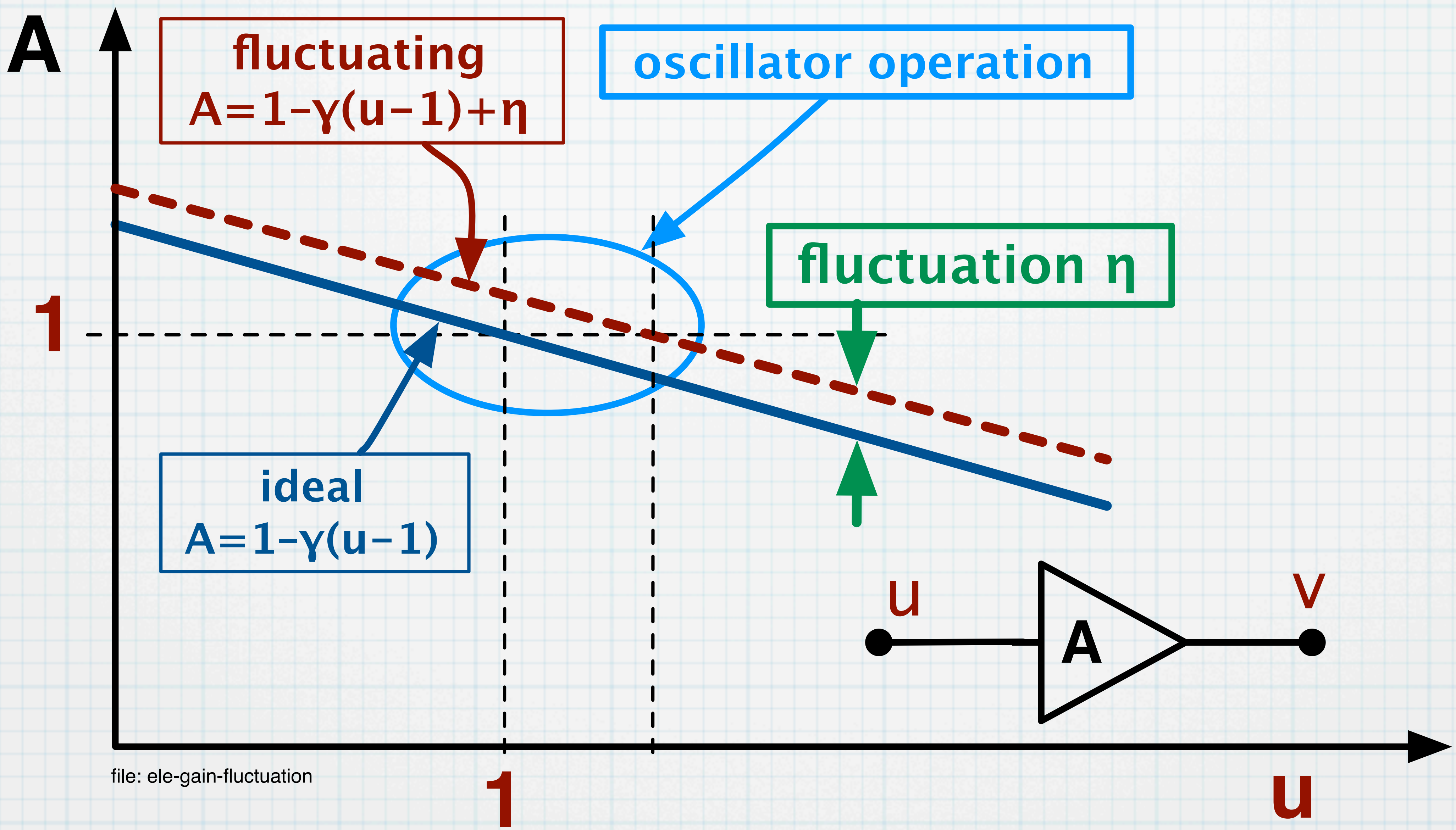
10 MHz oscillator
L = 1 mH
R = 125 Ω
Q ~ 503



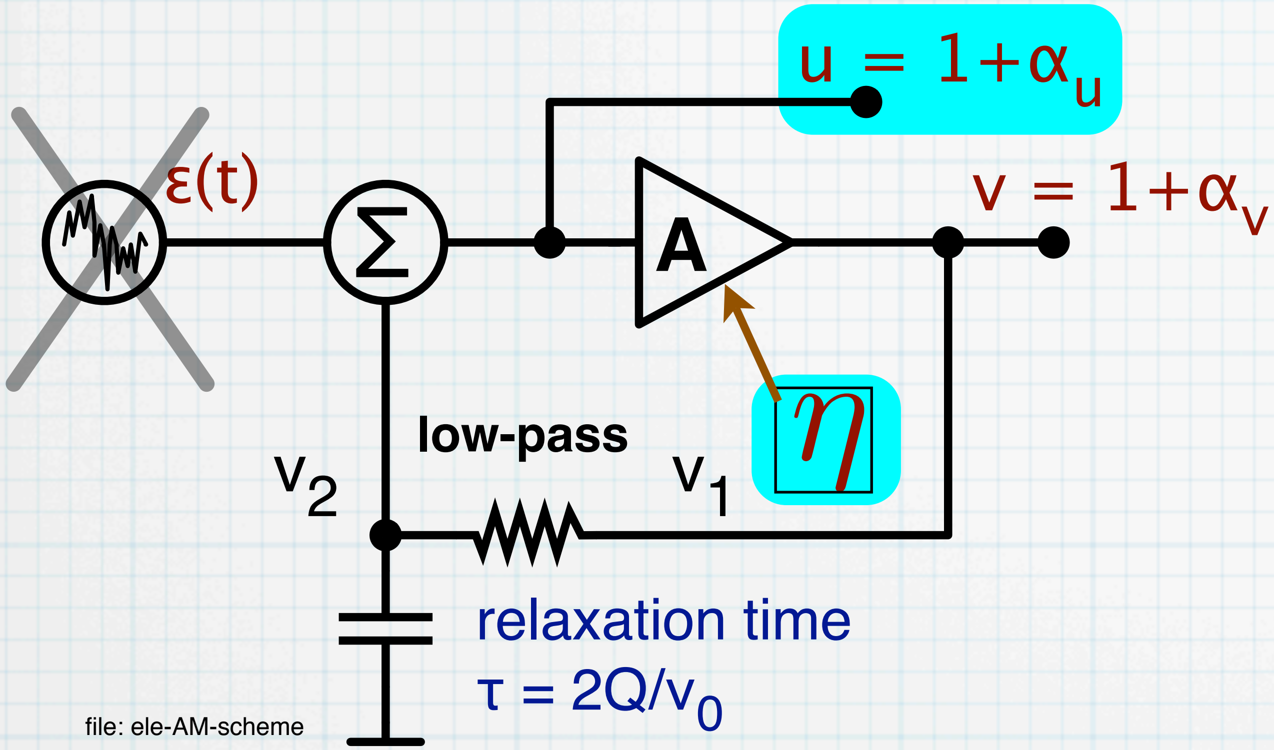
van der Pol oscillator
(simulation by R. Brendel)

Rising exponential.
We find the same
time constant –τ/γ

Gain fluctuations – definition



Gain fluctuations – output is u



$$\dot{u} = \frac{1}{\tau}(A-1)u$$ **non-linear equation**

$$A = 1 - \gamma(u-1) + \eta$$

$$\underset{\dot{\alpha}_u}{\dot{u}} + \frac{\gamma}{\tau}\underbrace{(u-1)}_{\alpha_u}\underset{1}{u} = \frac{\eta}{\tau}\underset{1}{u}$$ **linearization for low noise**

$$\dot{\alpha}_u + \frac{\gamma}{\tau}\alpha_u = \frac{1}{\tau}\eta$$ **linearized equation**

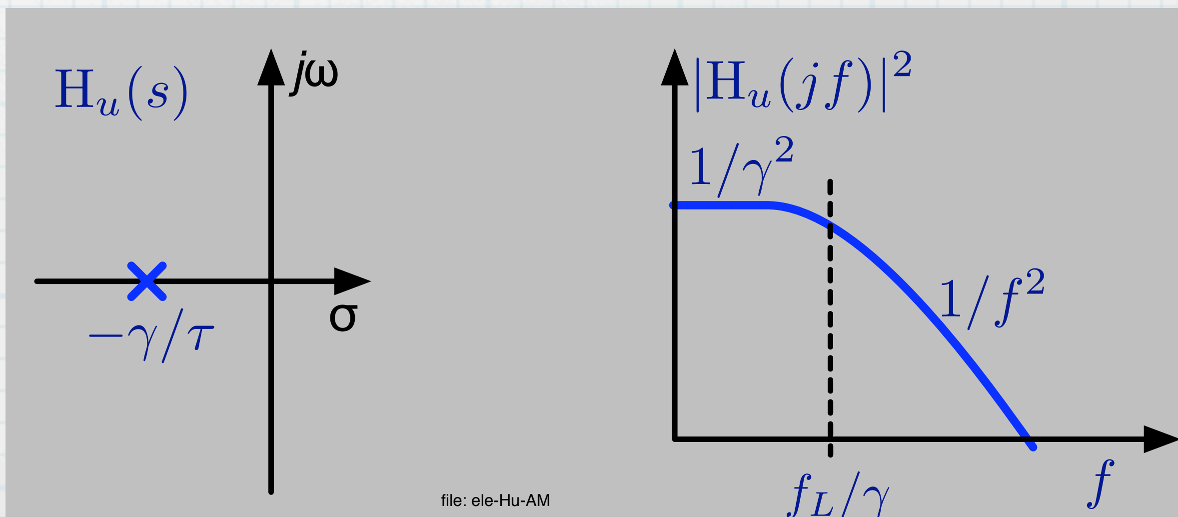
$$\left(s + \frac{\gamma}{\tau}\right)\mathcal{A}_u(s) = \frac{1}{\tau}\mathcal{N}(s)$$ **Laplace transform**

Linearize for low noise and use the Laplace transforms

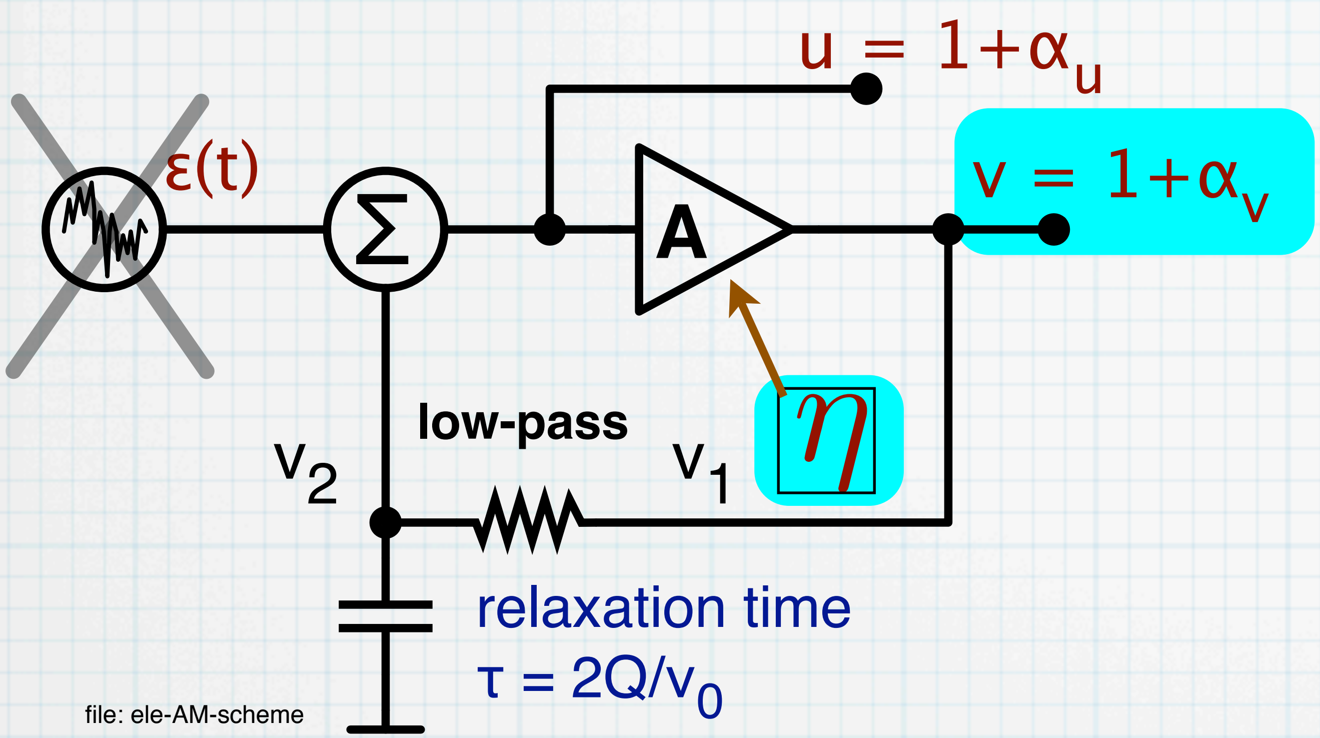
$$\alpha_u(t) \leftrightarrow \mathcal{A}_u(s) \quad \text{and} \quad \eta(t) \leftrightarrow \mathcal{N}(s)$$

$$\mathrm{H}_u(s) = \frac{\mathcal{A}_u(s)}{\mathcal{N}(s)}$$ **definition**

$$\mathrm{H}_u(s) = \frac{1/\tau}{s + \gamma/\tau}$$ **result**



Gain fluctuations – output is v



$$\left(s+\frac{\gamma}{\tau}\right)\mathcal{A}_u(s)=\frac{1}{\tau}\mathcal{N}(s)$$ **starting equation**

$$\mathcal{A}_u(s)=\frac{\mathcal{A}_v(s)-\mathcal{N}(s)}{1-\gamma}$$

$$\left(s+\frac{\gamma}{\tau}\right)\mathcal{A}_v(s)=\left(s+\frac{1}{\tau}\right)\mathcal{N}(s)$$

$$\mathrm{H}(s)=\frac{\mathcal{A}_v(s)}{\mathcal{N}(s)}$$ **definition**

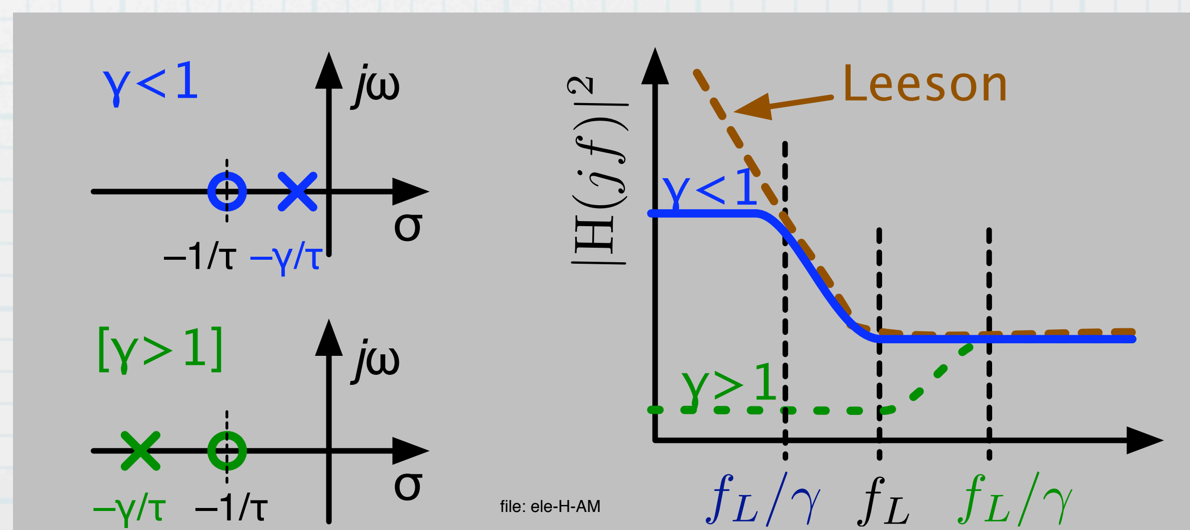
$$\mathrm{H}(s)=\frac{s+1/\tau}{s+\gamma/\tau}$$ **result**

boring algebra relates α_v to α_u

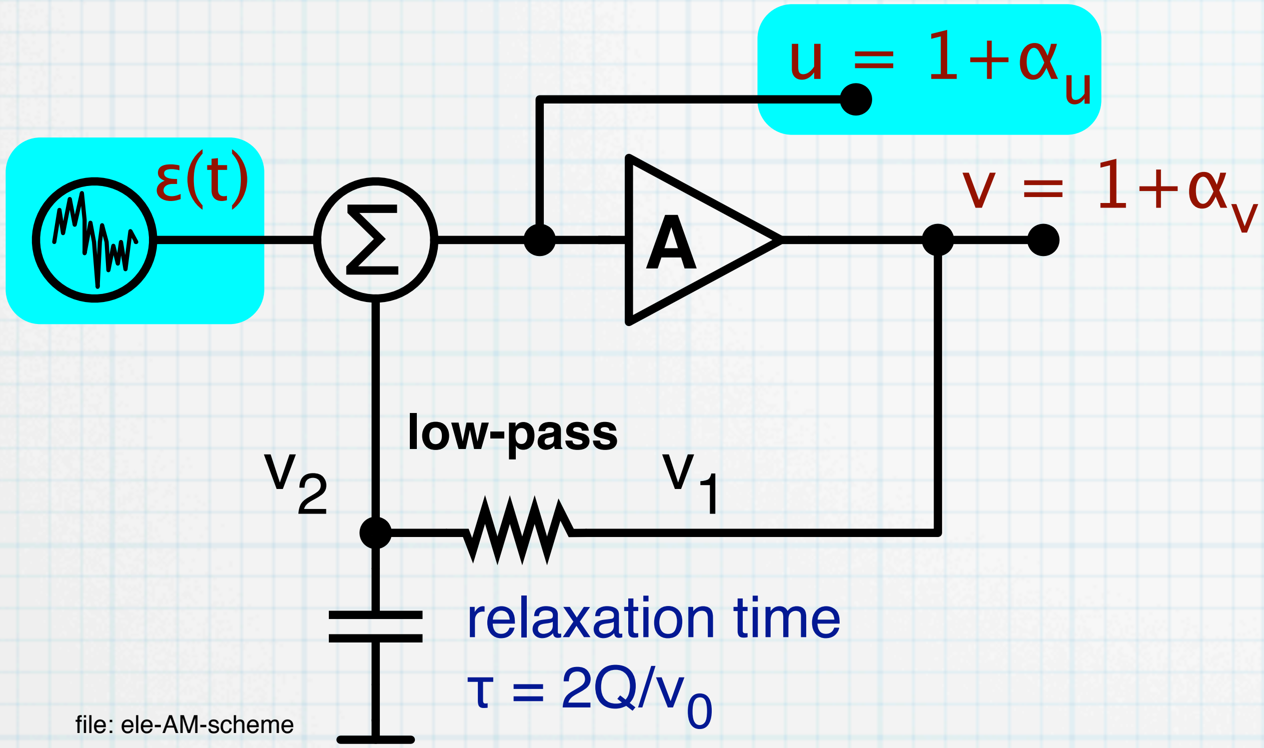
$$v=Au$$
$$A=-\gamma(u-1)+1+\eta$$
$$v=[-\gamma(u-1)+1+\eta]\,u$$
$$v=[-\gamma\alpha_u+1+\eta]\;[1+\alpha_u]$$
$$\cancel{1}+\alpha_v=\cancel{1}+\eta-\gamma\alpha_u+\alpha_u-\cancel{\alpha_u\eta}-\cancel{\gamma\alpha_u^2}$$

linearization for low noise

$$\alpha_v=(1-\gamma)\alpha_u+\eta$$
$$\alpha_u=\frac{\alpha_v-\eta}{1-\gamma}$$



Additive noise – output is u



$$\dot u = \frac{1}{\tau}(A-1)u + \dot\epsilon + \frac{1}{\tau}\epsilon$$
$$A = 1-\gamma(u-1)$$
non-linear equation

$$\underset{\dot\alpha_u}{\dot u} + \frac{\gamma}{\tau}(\underset{\alpha_u}{\underline{u-1}})\underset{1}{u} = \dot\epsilon + \frac{1}{\tau}\epsilon$$
linearız. for low noise

$$\dot\alpha_u + \frac{\gamma}{\tau}\alpha_u = \dot\epsilon + \frac{1}{\tau}\epsilon$$
linearized equation

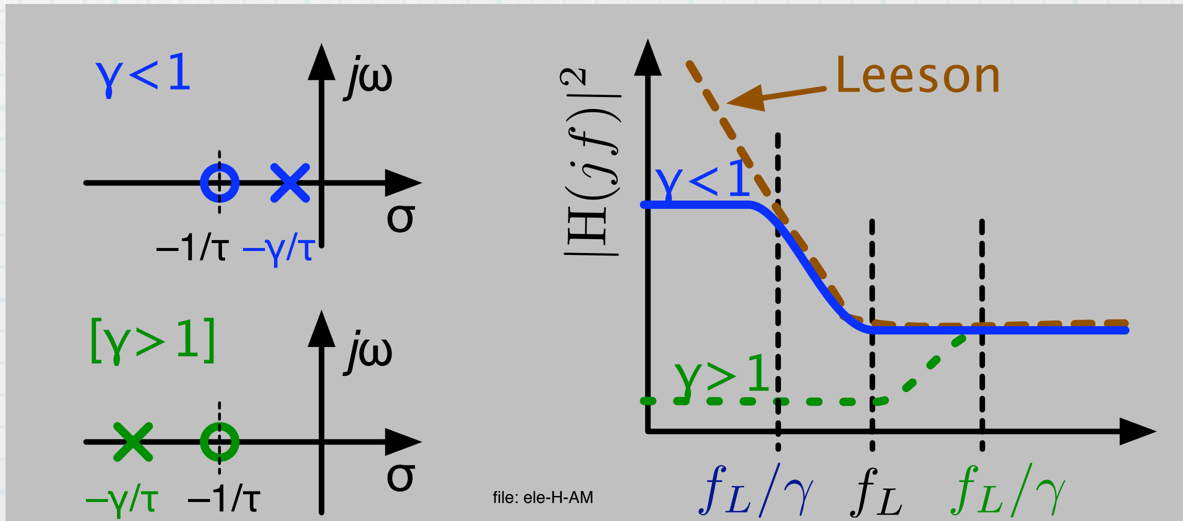
$$\left(s+\frac{\gamma}{\tau}\right)\mathcal{A}_u(s) = \left(s+\frac{1}{\tau}\right)\mathcal{E}(s)$$
Laplace transform

Linearize for low noise and use the Laplace transforms

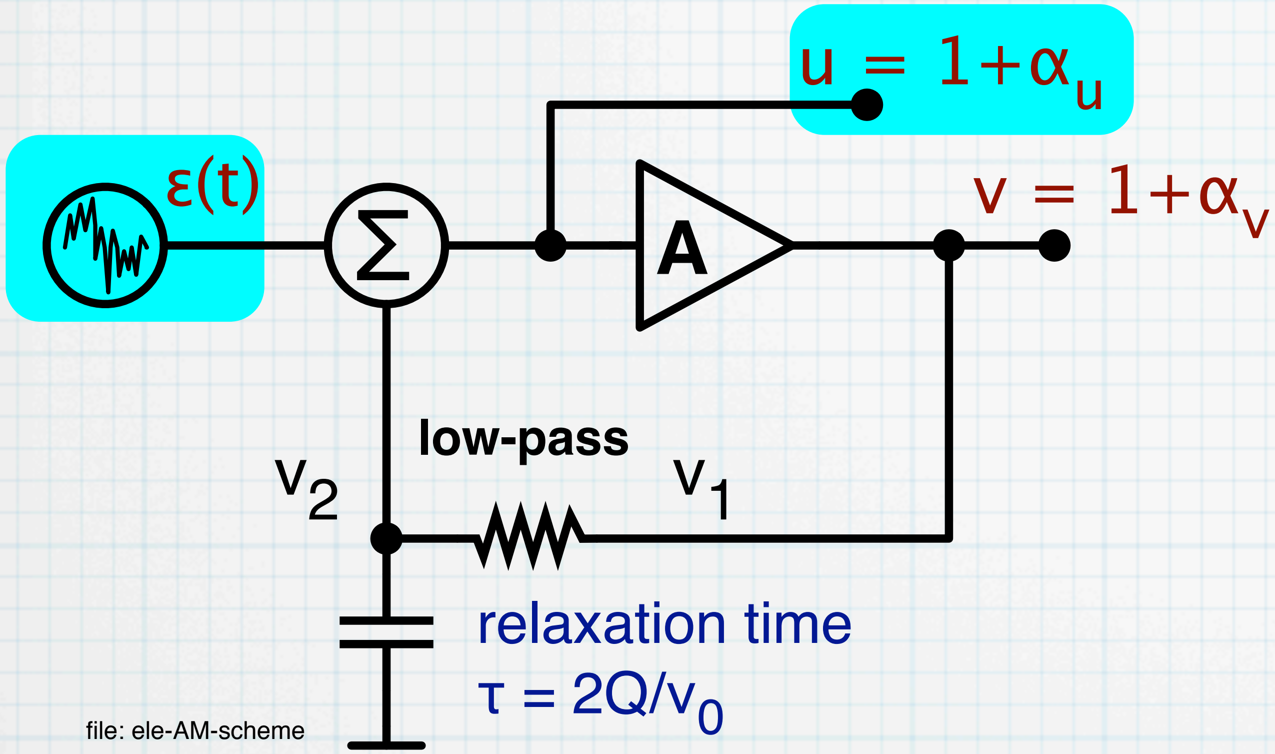
$$\alpha_u(t) \leftrightarrow \mathcal{A}_u(s) \quad \text{and} \quad \epsilon(t) \leftrightarrow \mathcal{E}(s)$$

$$\mathrm{H}_u(s) = \frac{\mathcal{A}_u(s)}{\mathcal{E}(s)}$$
definition

$$\mathrm{H}_u(s) = \frac{s+1/\tau}{s+\gamma/\tau}$$
result



Additive noise – output is v



$$\dot{\alpha}_u + \frac{\gamma}{\tau}\alpha_u = \dot{\epsilon} + \frac{1}{\tau}\epsilon$$

linearized equation

$$\alpha_u = \alpha_v/(1-\gamma)$$

$$\frac{1}{1-\gamma}\left(\dot{\alpha}_v + \frac{\gamma}{\tau}\alpha_v\right) = \dot{\epsilon} + \frac{1}{\tau}\epsilon$$

$$\frac{1}{1-\gamma}\left(s + \frac{\gamma}{\tau}\right)\mathcal{A}_v(s) = \left(s + \frac{1}{\tau}\right)\mathcal{E}(s)$$

Laplace transform

$$\mathrm{H}(s) = \frac{\mathcal{A}_v(s)}{\mathcal{E}(s)}$$

definition

$$\mathrm{H}(s) = (1-\gamma)\,\frac{s+1/\tau}{s+\gamma/\tau}$$

result

boring algebra relates α’ to α

$$v = Au$$

$$A = 1 - \gamma(u-1)$$

$$v = [1-\gamma(u-1)]\,u$$

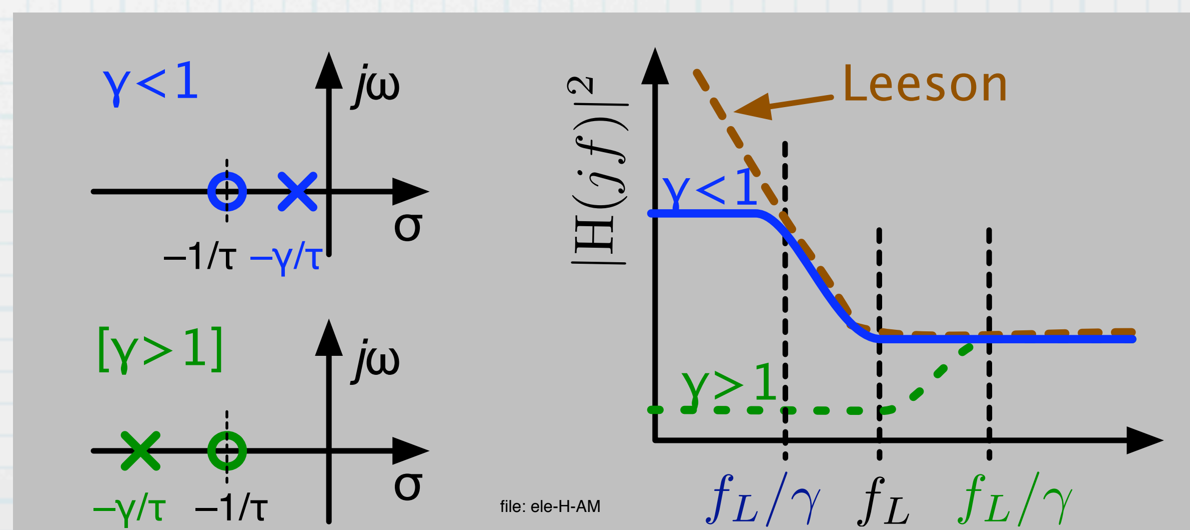
$$1+\alpha_v = [1-\gamma\alpha_u]\;[1+\alpha_u]$$

$$\cancel{1}+\alpha_v = \cancel{1}+\alpha_u - \gamma\alpha_u - \cancel{\gamma\alpha_u^2}$$

linearization for low noise

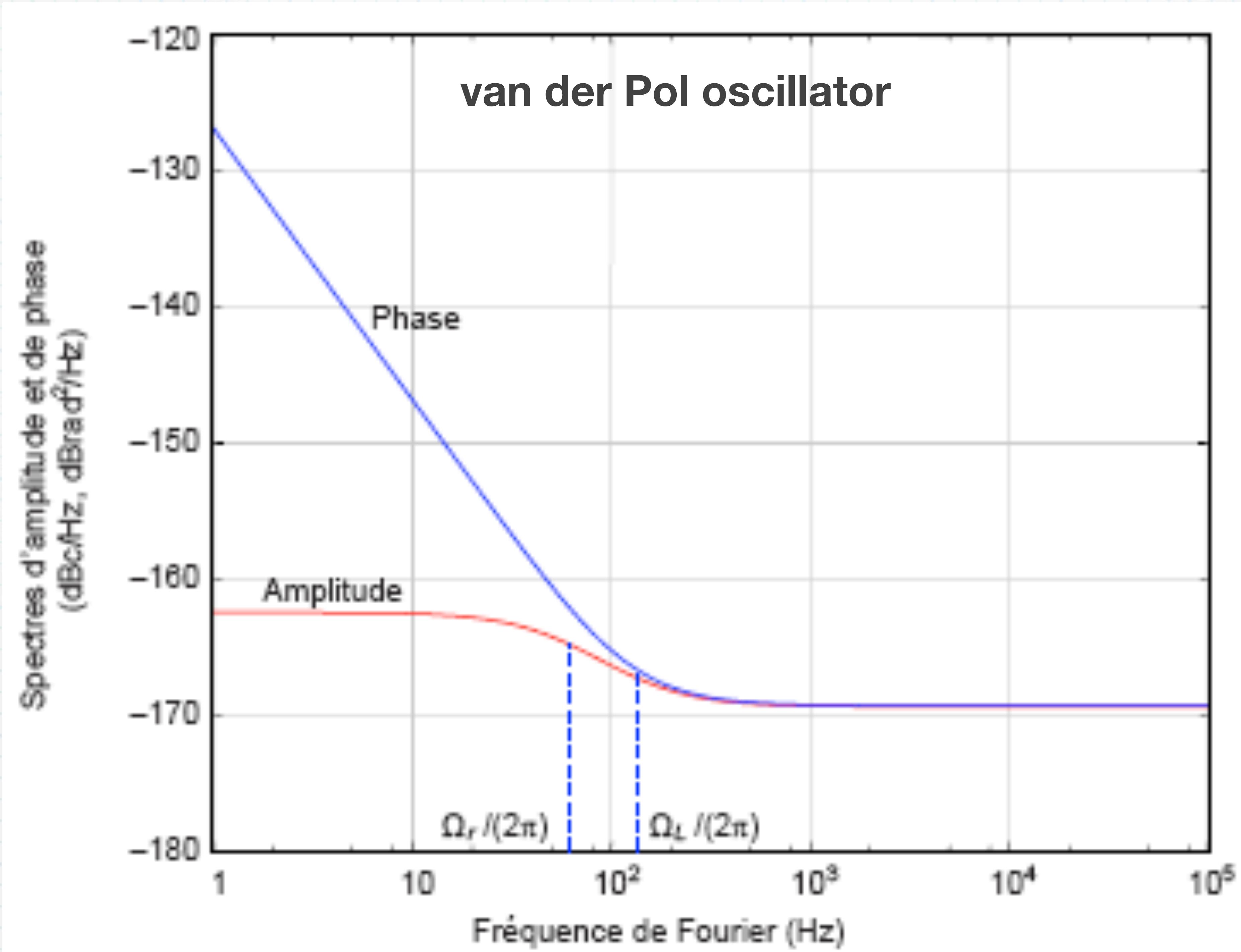
$$\alpha_v = (1-\gamma)\alpha_u$$

$$\alpha_u = \frac{\alpha_v}{1-\gamma}$$



Simulation

van der Pol oscillator
(simulation by R. Brendel)



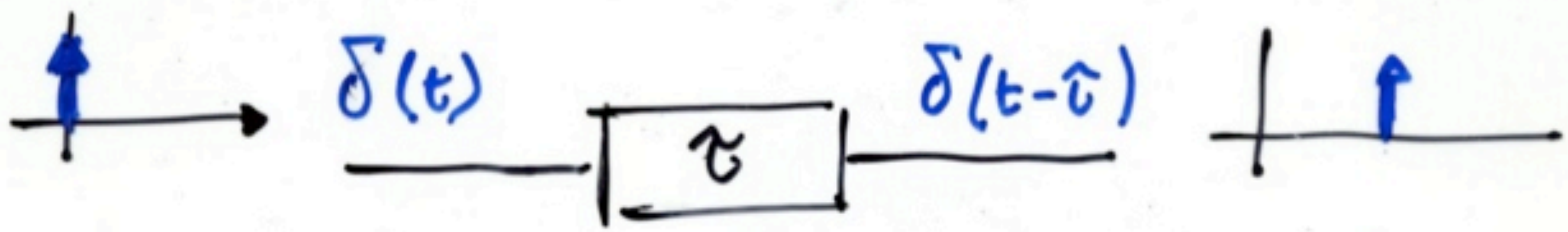
Analytic model and numeric simulation yield same time constants and slopes

Delay-line oscillators (and lasers)

Delay-line oscillator

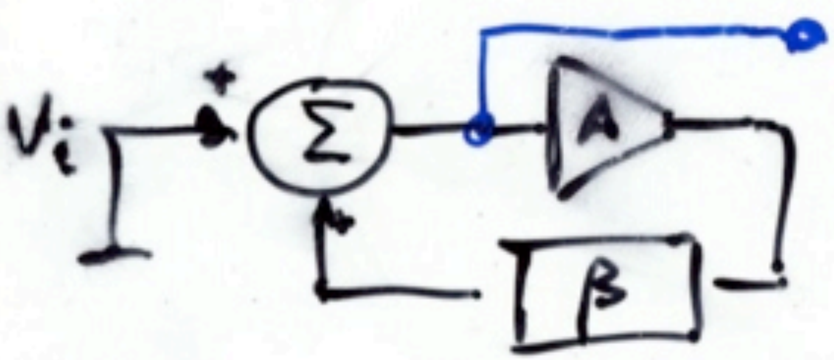
C7

DELAY LINE



COMPLEX VOLTAGE space

$$\beta = e^{-s\tau}$$



$$H(s) = \frac{1}{1 - A\beta} \quad \text{general}$$

$$H(s) = \frac{1}{1 - Ae^{-s\tau}}$$

POLES

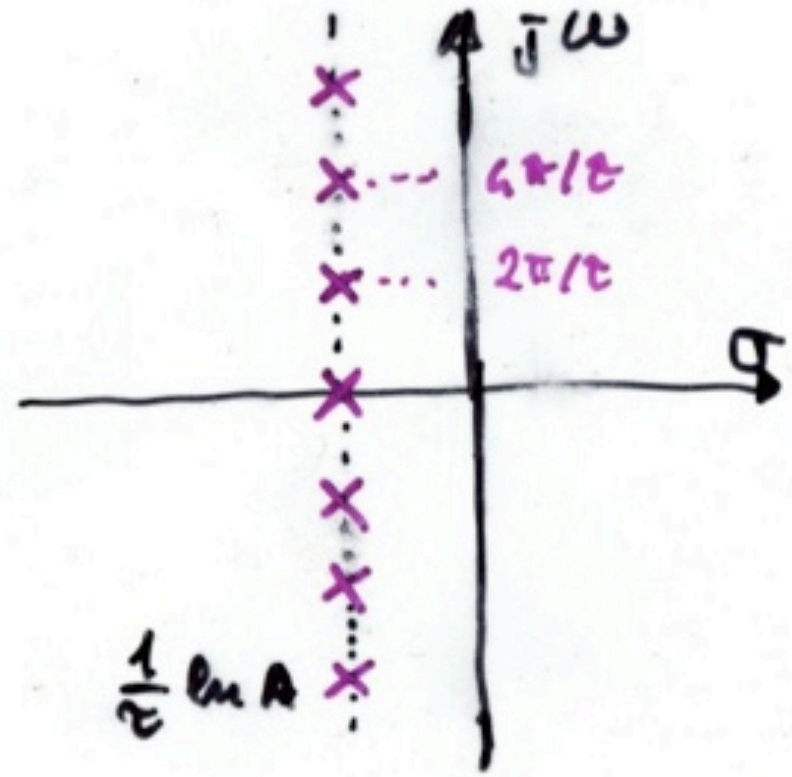
$$1 - Ae^{-s\tau} = 0$$

$$Ae^{-\sigma\tau} e^{-j\omega\tau} = 1 \rightsquigarrow \begin{cases} \sigma = \frac{1}{\tau} \ln(A) \\ \omega = 0 \ \mathrm{mod} \ \frac{2\pi}{\tau} \end{cases}$$

Delay-line oscillator – complex plane

C07b

COMPLEX PLANE



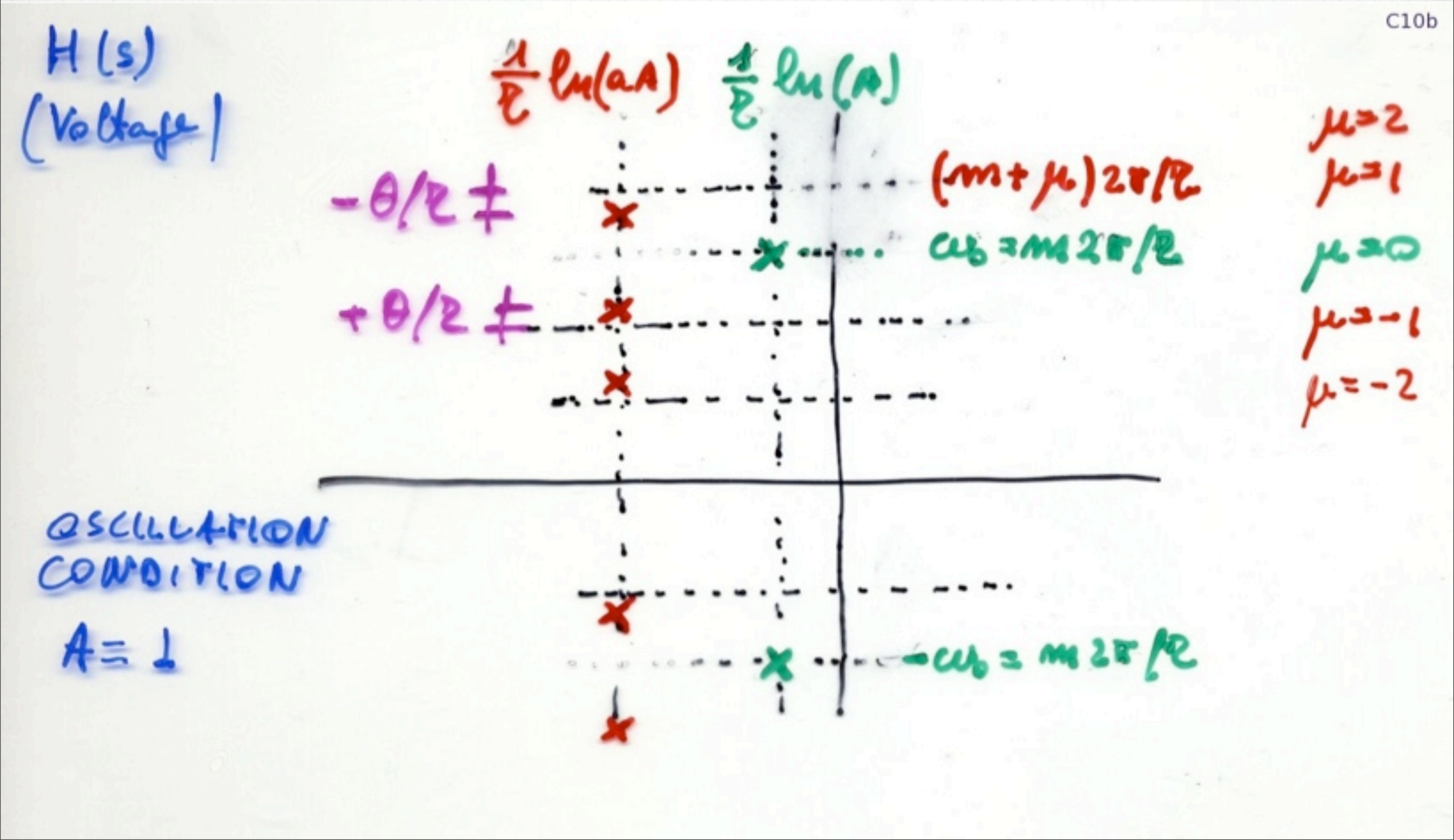
each pole pair is equivalent to a resonator

→ add the frequency response

→ add the δ response

- $A \to 1$ all the poles are on the $j\omega$ axis.
- equivalent: infinite series of undamped resonators.
- the loop sustains any periodic waveform
- ALL IMAGINARY POLES: LINK BETWEEN F SERIES ↔ TRANSFORM
- Infinite amplification of noise ↔ V_i

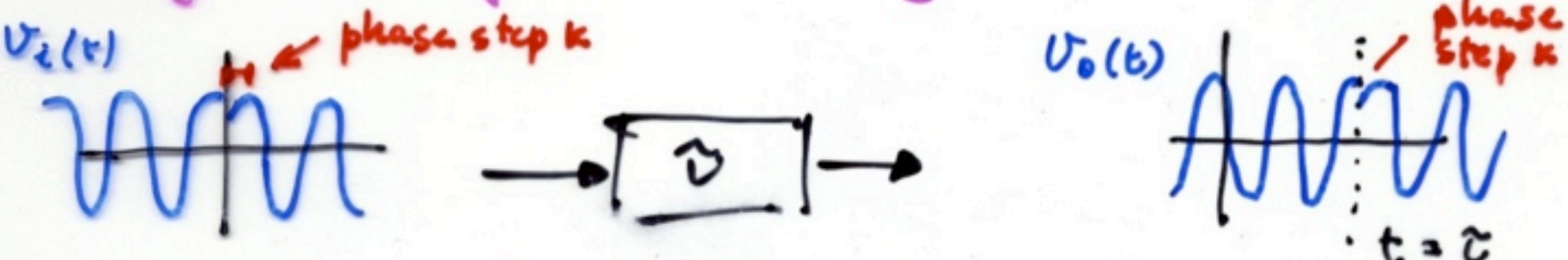
A bandpass filter is necessary



Delay-line oscillator – phase space

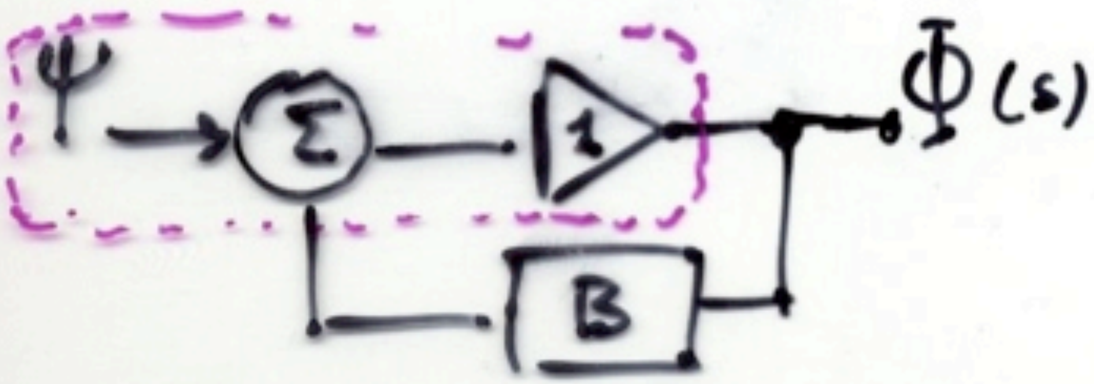
DELAY LINE - PHASE SPACE C8

Assume that the oscillator has choosen a frequency ν_0 by virtue of some dirty trick.



δ response $\quad b(t) = \delta(t-\tau)$

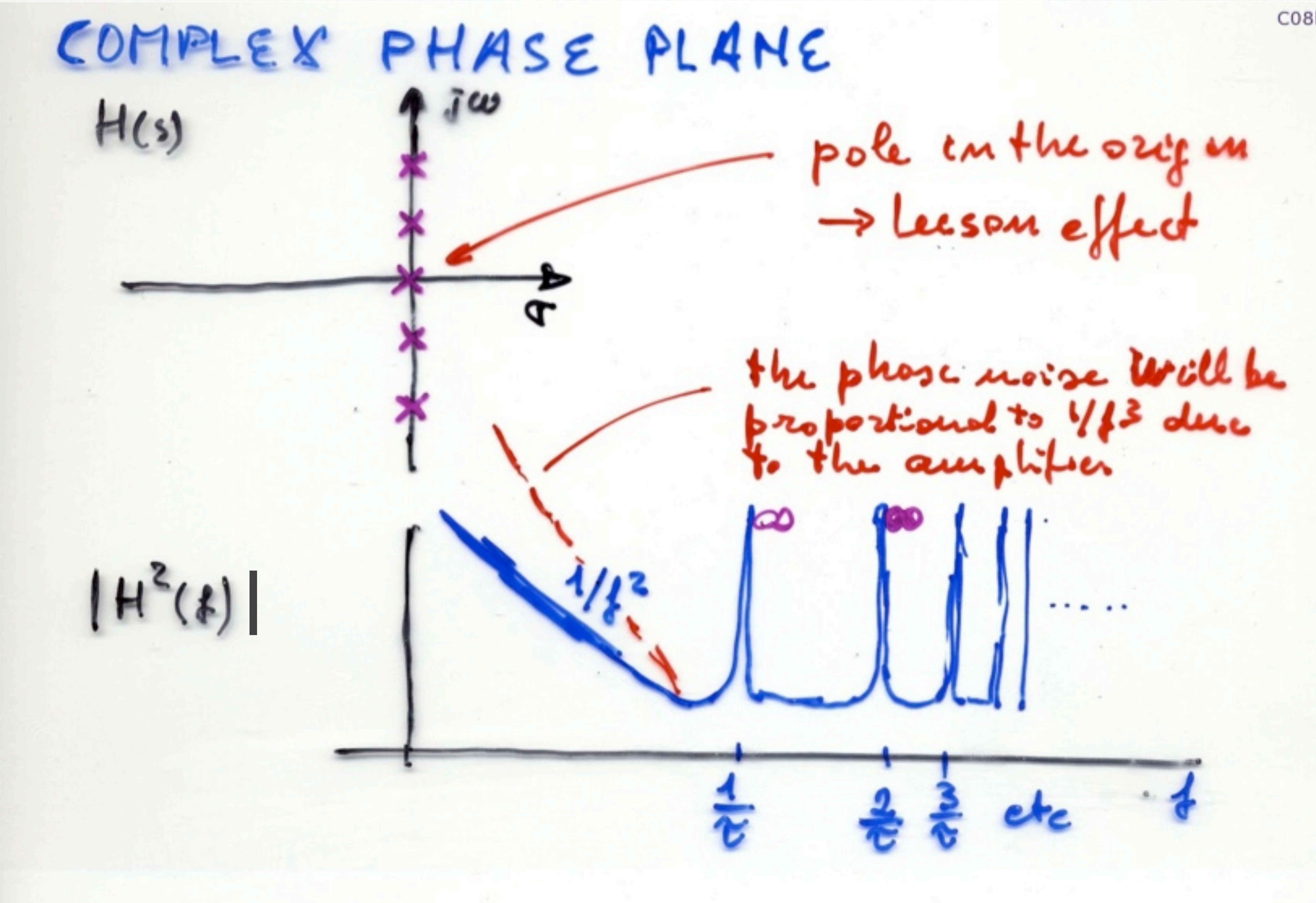
$$B(s) = e^{-s\tau}$$



$$H_\varphi(s) = \frac{1}{1-B} \quad \text{general}$$

$$H_\varphi(s) = \frac{1}{1-e^{-s\tau}}$$

Phase space – complex-plane



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Short course on Stable oscillators

— Part 3 —

The Cross-spectrum experimental method

December 2009

Enrico Rubiola

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Outline

- Statistics
- The FFT analyzer
- Theory
- Applications

home page http://rubiola.org

Statistics

Two reasons to use normal-distributed white noise

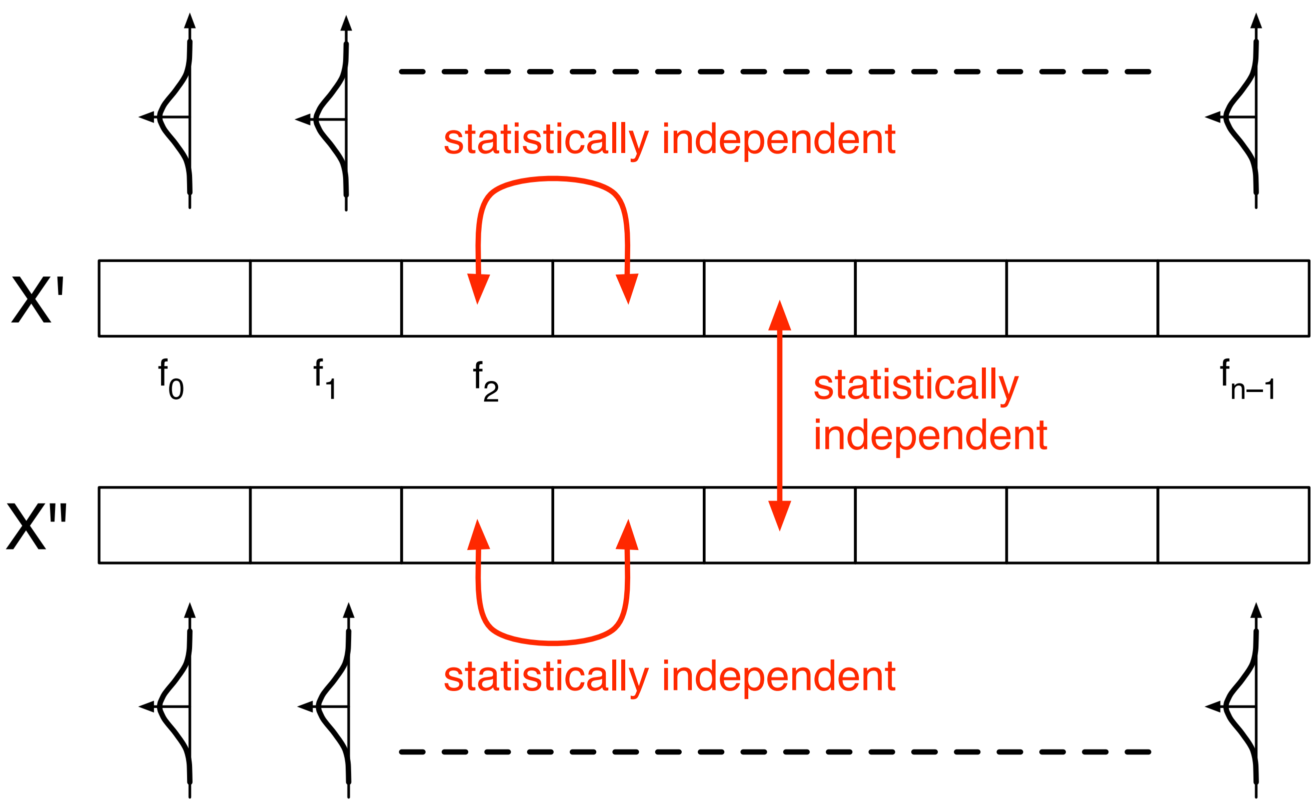
Central limit theorem: many random variables => normal distribution

Noise can (in most cases) be whitened, then unwhitened after processing

Properties of white zero-mean Gaussian noise

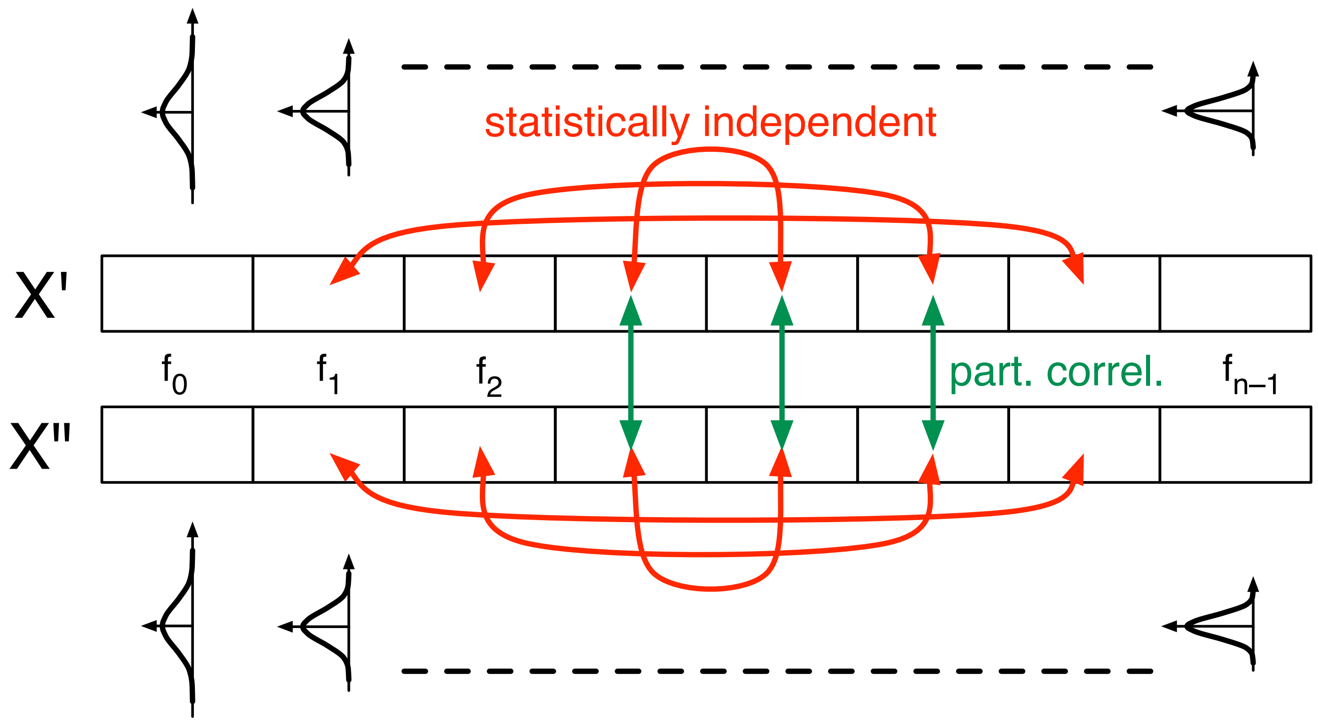
$$x(t) \iff X(\imath f) = X'(\imath f) + \imath X''(\imath f)$$

1. $x(t) \iff X(\imath f)$ are gaussian
2. $X(\imath f_1)$ and $X(\imath f_2)$, $f_1 \neq f_2$
   1. are statistically independent,
   2. $\mathrm{var}\{X(\imath f_1)\} = \mathrm{var}\{X(\imath f_2)\}$
3. real and imaginary part:
   1. X' and X'' are statistically independent
   2. $\mathrm{var}\{X'\} = \mathrm{var}\{X''\} = \mathrm{var}\{X\}/2$
4. $Y = X_1 + X_2$
   1. Y is Gaussian
   2. $\mathrm{var}\{Y\} = \mathrm{var}\{X_1\} + \mathrm{var}\{X_2\}$
5. $Y = X_1 \times X_2$
   1. is Gaussian
   2. $\mathrm{var}\{Y\} = \mathrm{var}\{X_1\}\,\mathrm{var}\{X_2\}$



Properties of flicker noise

$x(t) \iff X(\imath f) = X'(\imath f) + \imath X''(\imath f)$



1. Pair $x(t) \iff X(\imath f)$
   1. there is no a-priori relation between the distribution of $x(t)$ and $X(\imath f)$ (theorem)
   2. **Central limit theorem: $x(t)$ and $X(\imath f)$ end up to be Gaussian**
2. $X(\imath f_1)$ and $X(\imath f_2)$
   1. are statistically independent
   2. $\mathrm{var}\{X(\imath f_2)\} < \mathrm{var}\{X(\imath f_1)\}$ for $f_2 > f_1$
3. Real and imaginary part
   1. X' and X'' can be correlated
   2. $\mathrm{var}\{X'\} \neq \mathrm{var}\{X''\} \neq \mathrm{var}\{X\}/2$
4. $Y = X_1 + X_2$, zero-mean Gaussian r.v.
   $\mathrm{var}\{Y\} = \mathrm{var}\{X_1\} + \mathrm{var}\{X_2\}$
5. If X_1 and X_2 are zero-mean Gaussian r.v.
   1. **$Y = X_1 \times X_2$ is zero-mean Gaussian**
   2. **$\mathrm{var}\{Y\} = \mathrm{var}\{X_1\}\ \mathrm{var}\{X_2\}$**

Normal (Gaussian) distribution

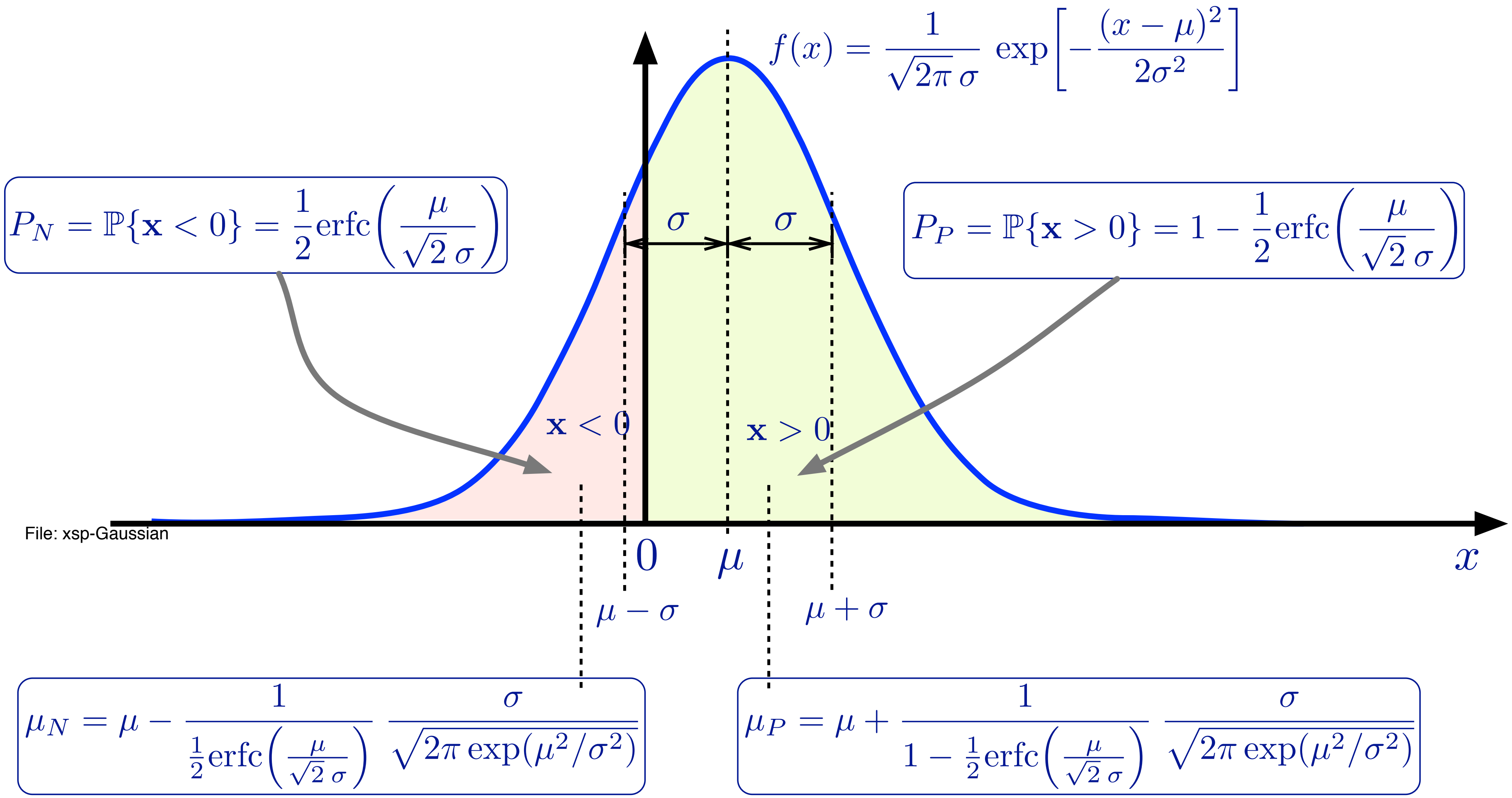
x is normal distributed with zero mean μ and variance σ²

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\,\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

$$\mathbb{E}\{f(x)\} = \mu$$

$$\mathbb{E}\{f^2(x)\} = \mu^2 + \sigma^2$$

$$\mathbb{E}\{|f(x) - \mathbb{E}\{f(x)\}|^2\} = \sigma^2$$



One-sided Gaussian distribution

x is normal distributed with zero mean and variance σ²

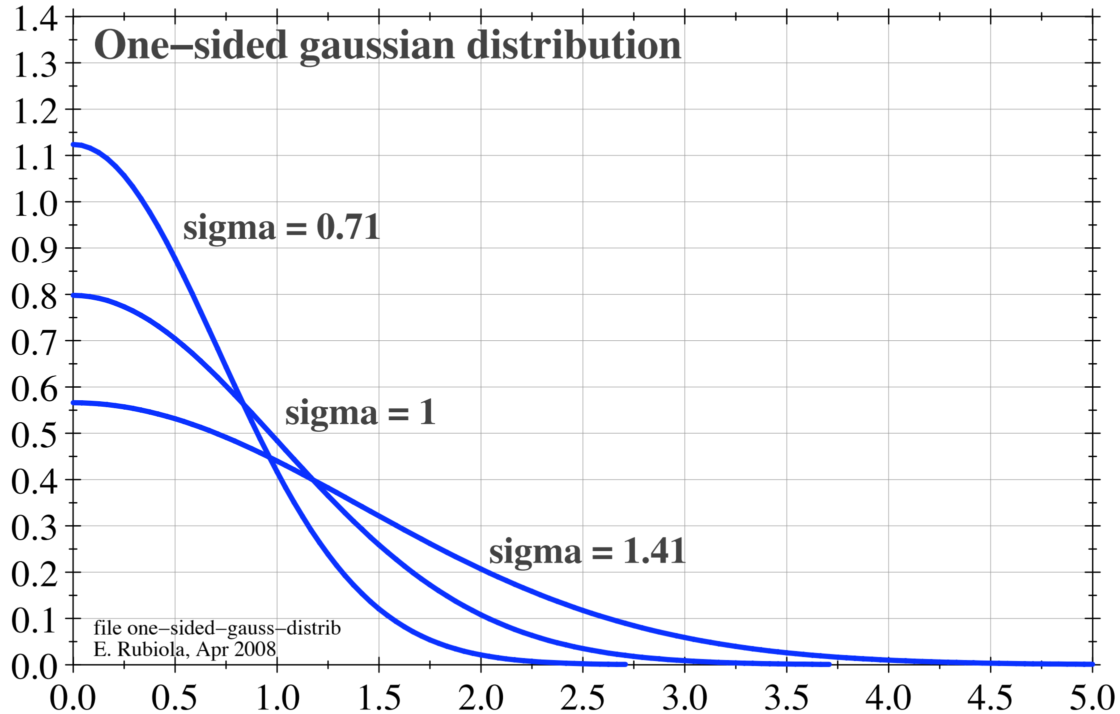
$$y = |x|$$

$$f(x) = 2\frac{1}{\sqrt{2\pi}\,\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$\mathbb{E}\{f(x)\} = \sqrt{\frac{2}{\pi}}\,\sigma$$

$$\mathbb{E}\{f^2(x)\} = \sigma^2$$

$$\mathbb{E}\{|f(x) - \mathbb{E}\{f(x)\}|^2\} = \left(1 - \frac{2}{\pi}\right)\sigma^2$$



one-sided Gaussian distribution with $\sigma^2 = 1/2$	
quantity with $\sigma^2 = 1/2$	value [10 log( ), dB]
average $= \sqrt{\frac{1}{\pi}}$	0.564 [−2.49]
deviation $= \sqrt{\frac{1}{2} - \frac{1}{\pi}}$	0.426 [−3.70]
$\frac{\text{dev}}{\text{avg}} = \sqrt{\frac{\pi}{2} - 1}$	0.756 [−1.22]
$\frac{\text{avg} + \text{dev}}{\text{avg}} = 1 + \sqrt{\frac{1}{2} - \frac{1}{\pi}}$	1.756 [+2.44]
$\frac{\text{avg} - \text{dev}}{\text{avg}} = 1 - \sqrt{\frac{1}{2} - \frac{1}{\pi}}$	0.244 [−6.12]
$\frac{\text{avg} + \text{dev}}{\text{avg} - \text{dev}} = \frac{1 + \sqrt{1/2 - 1/\pi}}{1 - \sqrt{1/2 - 1/\pi}}$	7.18 [8.56]

Chi-square distribution

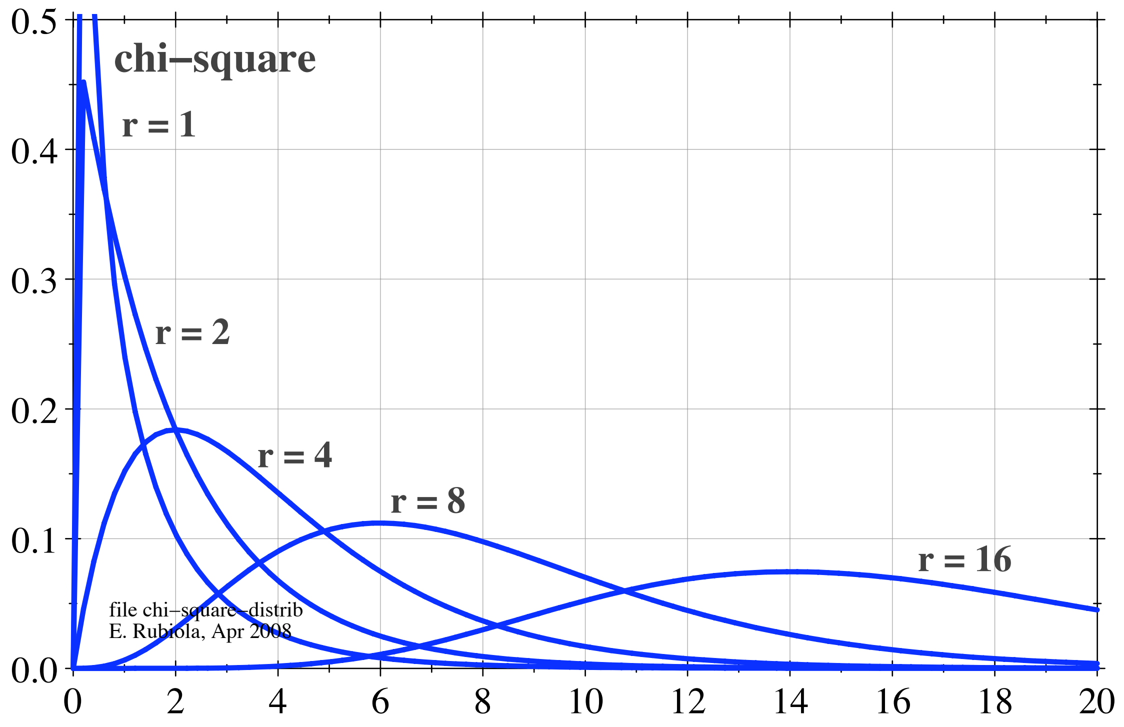
x_i are normal distributed with zero mean and equal variance σ^2

$$\chi^2 = \sum_{i=1}^{r} x_i^2$$

is χ^2 distributed with r degrees of freedom

Notice that the sum of χ^2 is a χ^2 distribution

$$\chi^2 = \sum_{j=1}^{m} \chi_j^2 \,, \qquad r = \sum_{j=1}^{m} r_j$$



$$f(x) = \frac{x^{\frac{r}{2}-1}\, e^{-\frac{x^2}{2}}}{\Gamma\left(\frac{1}{2}r\right) 2^{\frac{r}{2}}} \qquad x \geq 0$$

$$\mathbb{E}\{f(x)\} = \sigma^2 r$$

$$\mathbb{E}\{[f(x)]^2\} = \sigma^4 r(r+2)$$

$$\mathbb{E}\{|f(x) - \mathbb{E}\{f(x)\}|^2\} = 2\sigma^4 r$$

$$z! = \Gamma(z+1), \qquad z \in \mathbb{N}$$

Averaging chi-square distributions

averaging m variables $|X|^2$, complex $X=X'+\imath X''$, yields a χ^2 distribution with r = 2m

$$\frac{1}{m}\chi^2 = \frac{1}{m}\sum_{j=1}^{m}(X'_j)^2+(X''_j)^2$$

$$\frac{\text{dev}}{\text{avg}} = \frac{1}{\sqrt{m}}$$

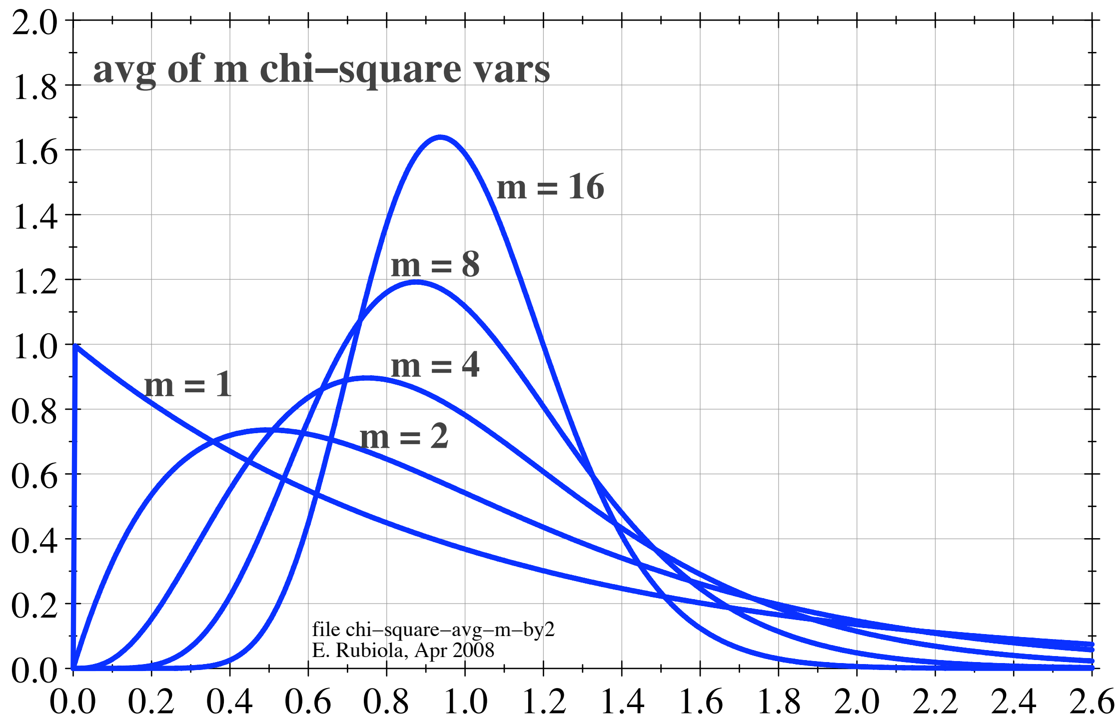
$$\mathbb{E}\left\{\frac{1}{m}f(x)\right\} = \frac{\sigma^2 r}{m} = 2\sigma^2$$

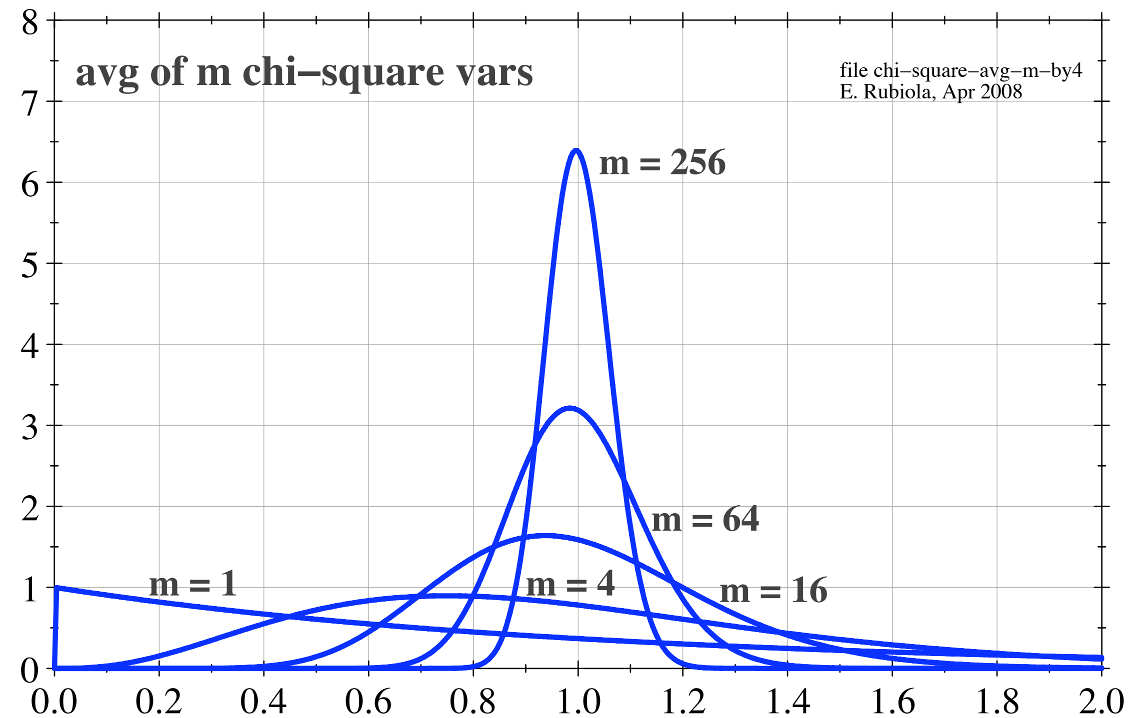
$$\mathbb{E}\left\{\left|\frac{1}{m}f(x)-\mathbb{E}\left\{\frac{1}{m}f(x)\right\}\right|^2\right\} = \frac{2\sigma^4 r}{m^2} = \frac{4\sigma^4}{m}$$

relevant case: $\sigma^2 = 1/2$

$\text{avg} = 1$

$\text{dev} = \dfrac{1}{\sqrt{m}}$

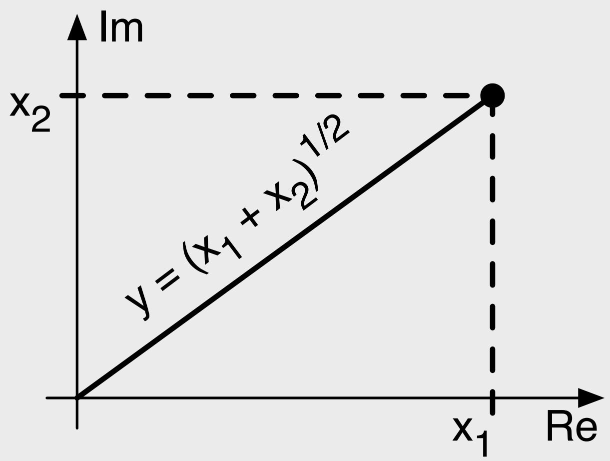




Rayleigh distribution

x_1 and x_2 are normal distributed with zero mean and equal variance σ^2

$$x = \sqrt{x_1^2 + x_2^2}$$



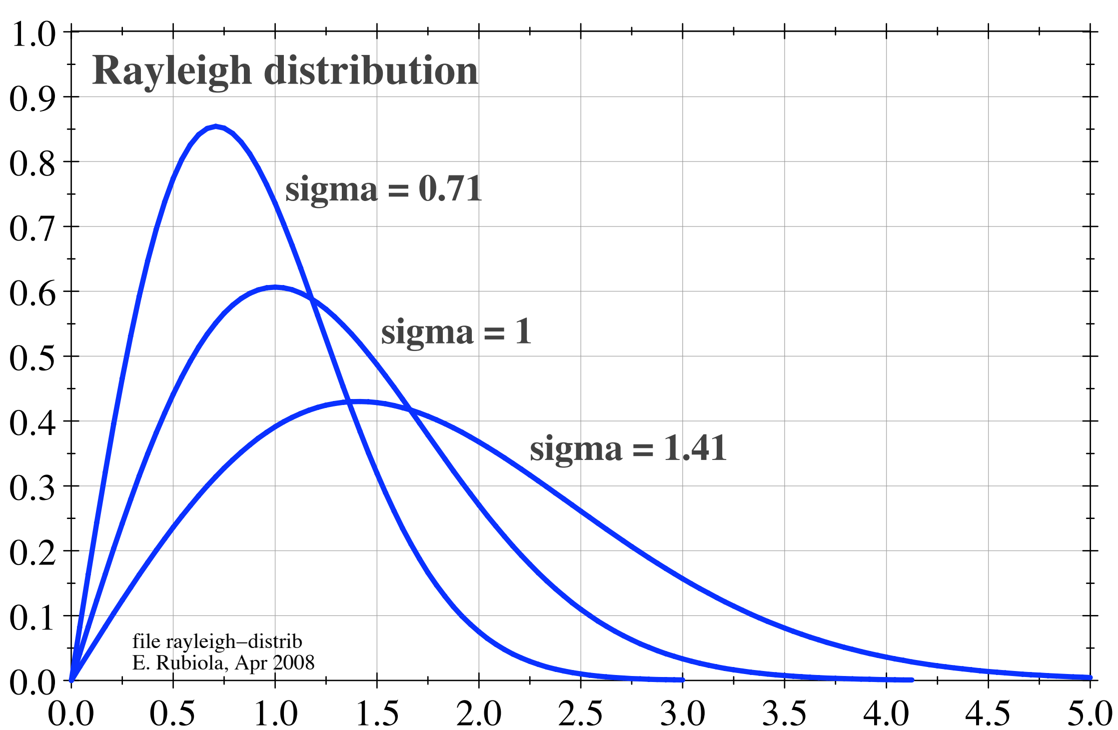
x is Rayleigh-distributed

$$f(x) = \frac{x}{\sigma^2}\,\exp\left(-\frac{x^2}{2\sigma^2}\right) \qquad x \geq 0$$

$$\mathbb{E}\{f(x)\} = \sqrt{\frac{\pi}{2}}\,\sigma$$

$$\mathbb{E}\{f^2(x)\} = 2\sigma^2$$

$$\mathbb{E}\{|f(x) - \mathbb{E}\{f(x)\}|^2\} = \frac{4-\pi}{2}\sigma^2$$



Rayleigh distribution with $\sigma^2 = 1/2$	
quantity with $\sigma^2 = 1/2$	value [10 log( ), dB]
average $= \sqrt{\frac{\pi}{4}}$	0.886 [−0.525]
deviation $= \sqrt{1 - \frac{\pi}{4}}$	0.463 [−3.34]
$\frac{\text{dev}}{\text{avg}} = \sqrt{\frac{4}{\pi} - 1}$	0.523 [−2.82]
$\frac{\text{avg} + \text{dev}}{\text{avg}} = 1 + \sqrt{\frac{4}{\pi} - 1}$	1.523 [+1.83]
$\frac{\text{avg} - \text{dev}}{\text{avg}} = 1 - \sqrt{\frac{4}{\pi} - 1}$	0.477 [−3.21]
$\frac{\text{avg} + \text{dev}}{\text{avg} - \text{dev}} = \frac{1 + \sqrt{4/\pi - 1}}{1 - \sqrt{4/\pi - 1}}$	3.19 [5.04]

The FFT analyzer

Normalization

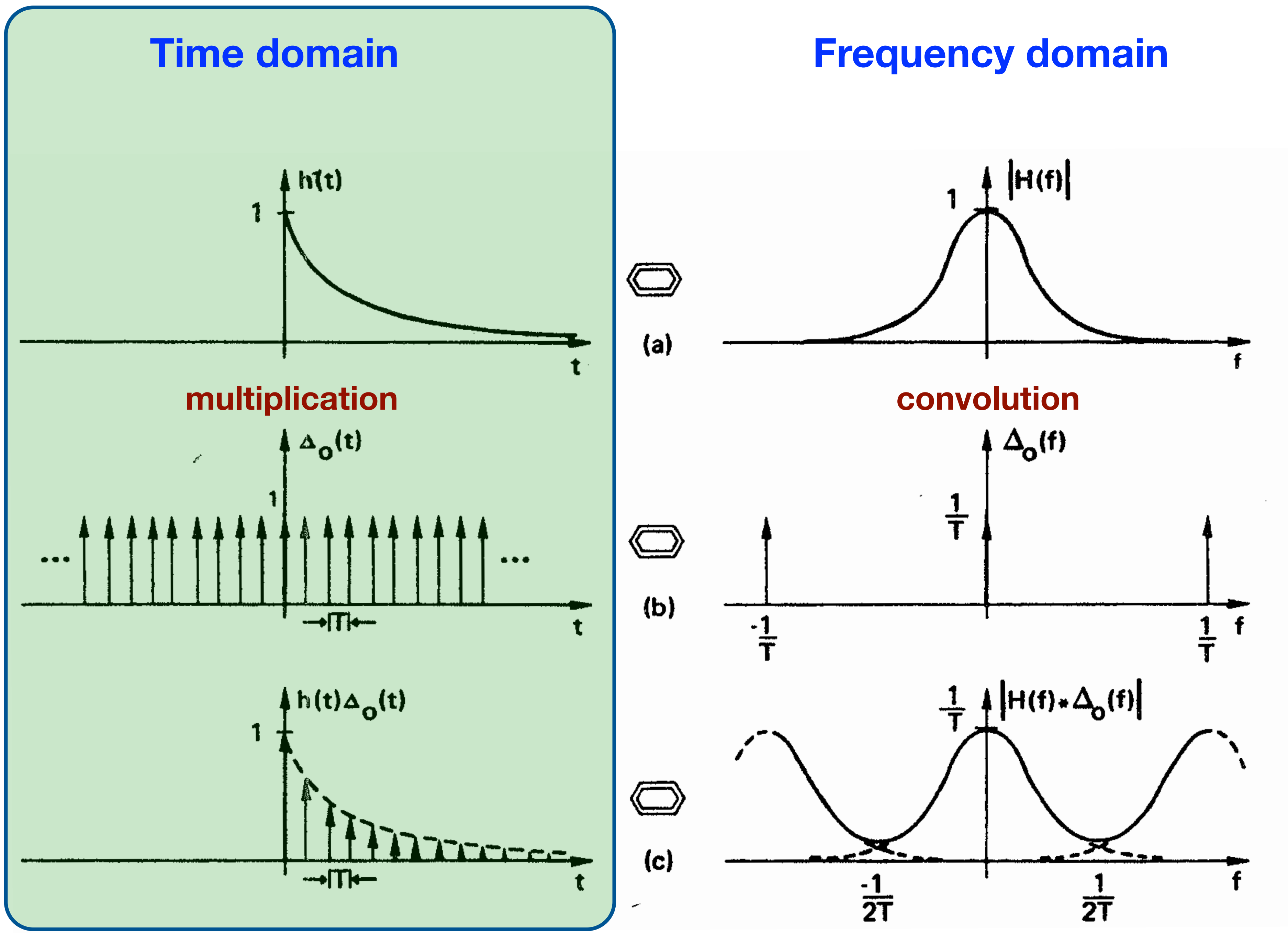
Commonly used quantities

quantity	physical dimension	purpose
$X_T(\imath f)$	V/Hz	Two-sided FT Theoretical issues
$S^I(f) =$ $\frac{2}{T}\lvert X_T(\imath f)\rvert^2, \quad f>0$	V^2/Hz or W/Hz	One-sided PSD Measurement of noise level (power spectral density)
$\frac{1}{T}S^I(f) =$ $\frac{2}{T^2}\lvert X_T(\imath f)\rvert^2, \quad f>0$	V^2 or W	One-sided PS Power measurement of carriers (sinusoidal signals)

Truncated signal

$$X_T(\imath f) = \int_{-T/2}^{T/2} x(t)\, e^{-\imath 2\pi f t}\, dt$$

Sampling and aliasing



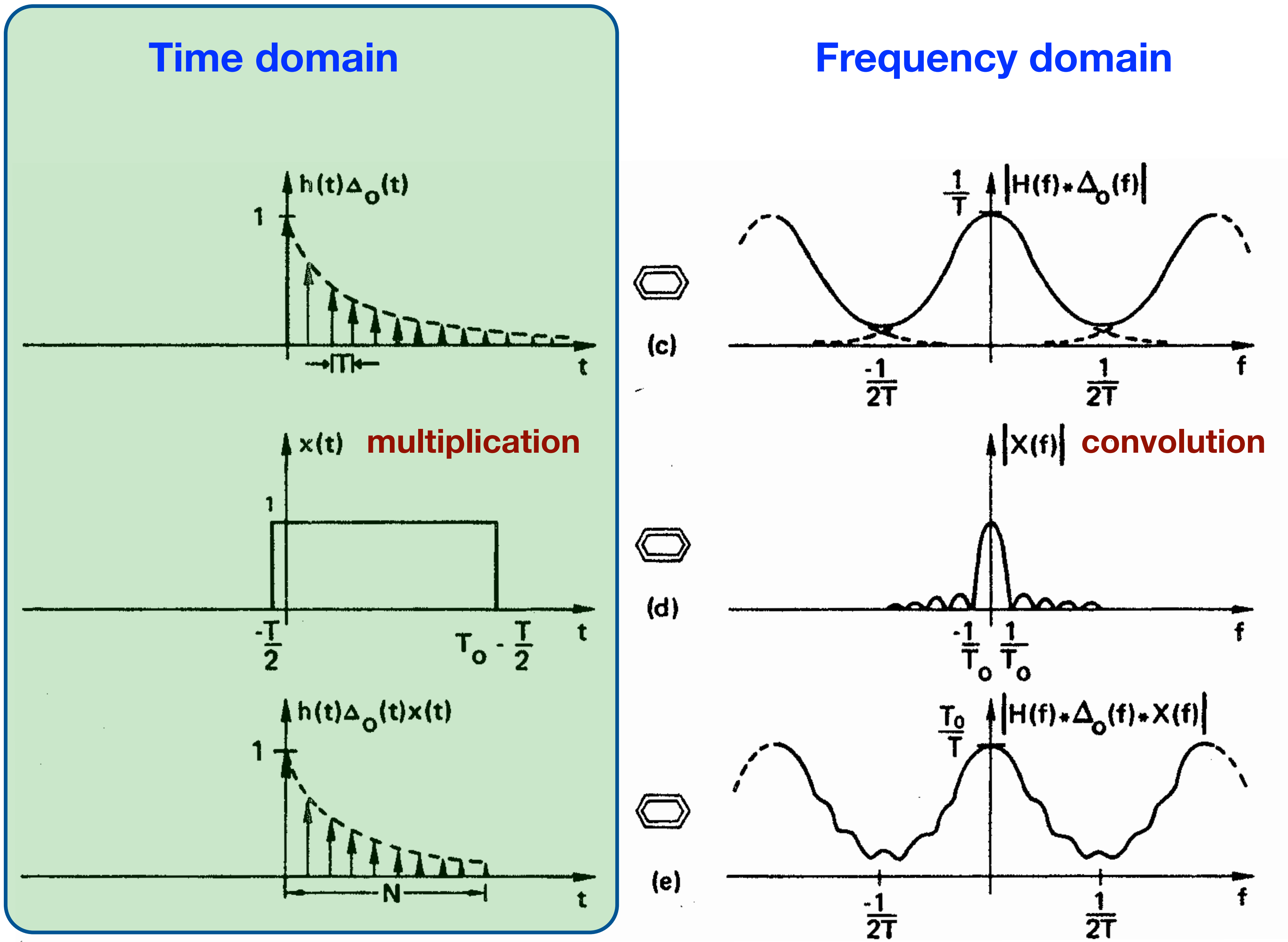
Input signal

(Time-domain) sampling

Sampled signal (and aliasing)

E. Oran Brigham, The fast Fourier Transform, Prentice Hall, 1988

Truncation and energy leakage



Sampled signal & aliasing

Truncation

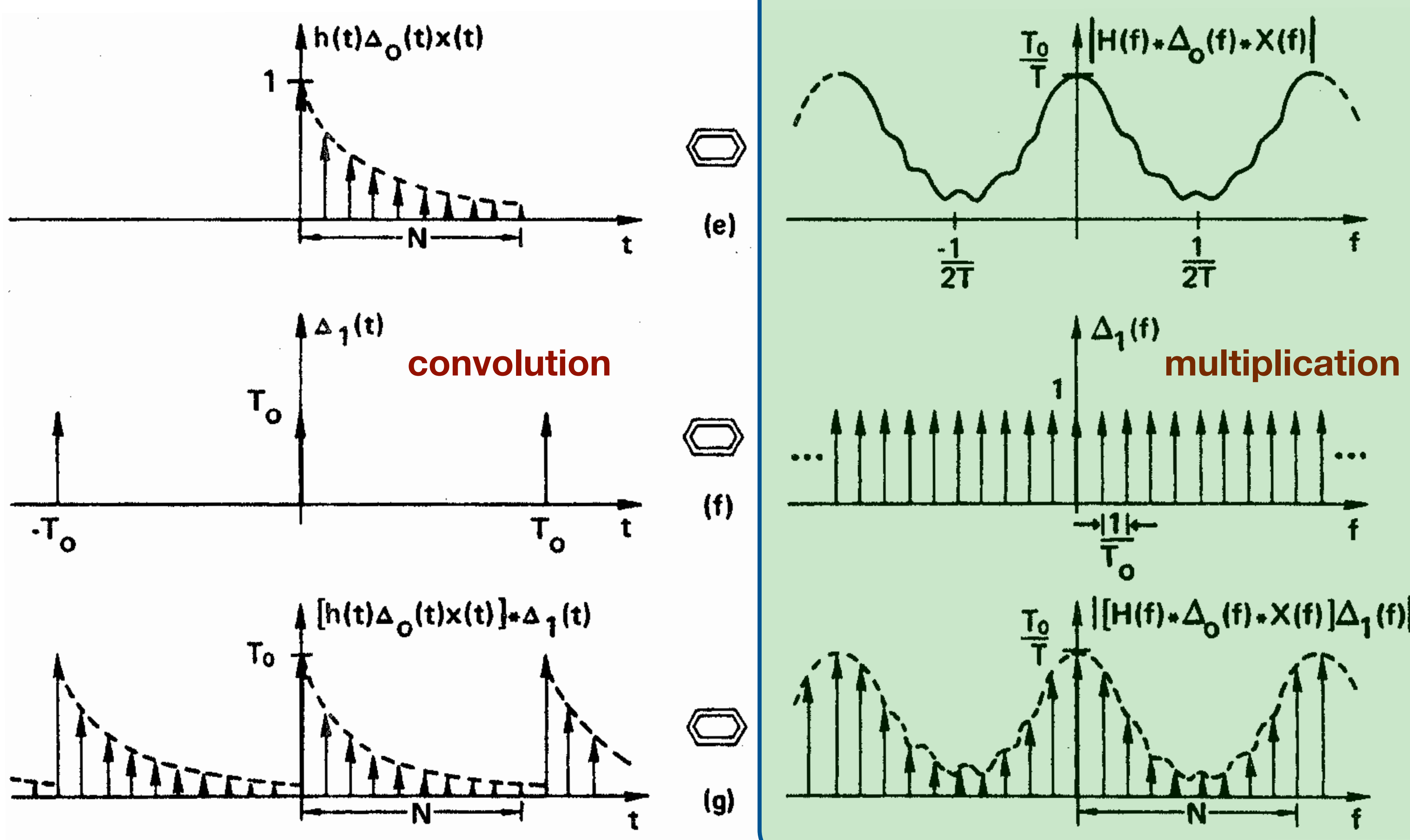
Truncated signal & energy leakage

(need windowing)

E. Oran Brigham, The fast Fourier Transform, Prentice Hall, 1988

Fitting the Fourier transform into a computer memory

Time domain | **Frequency domain**



Truncated signal

Frequency-domain sampling

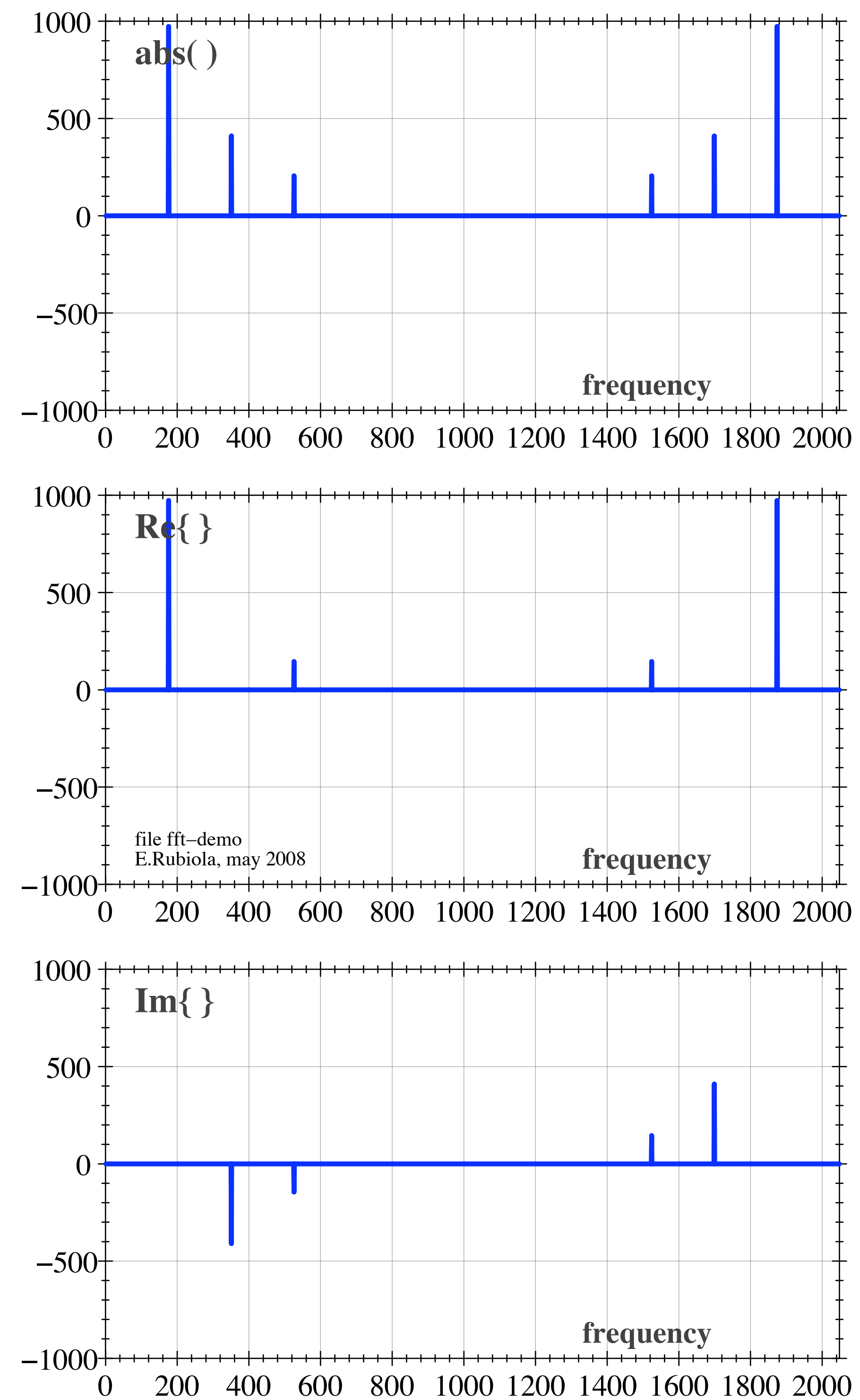
Final DFT
(Time-domain aliasing)

E. Oran Brigham, The fast Fourier Transform, Prentice Hall, 1988

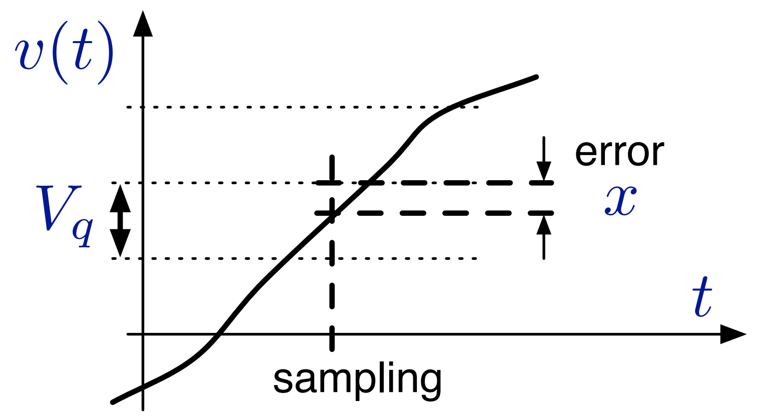
FFT – How long does it take?

total time = acquisition time + computation time

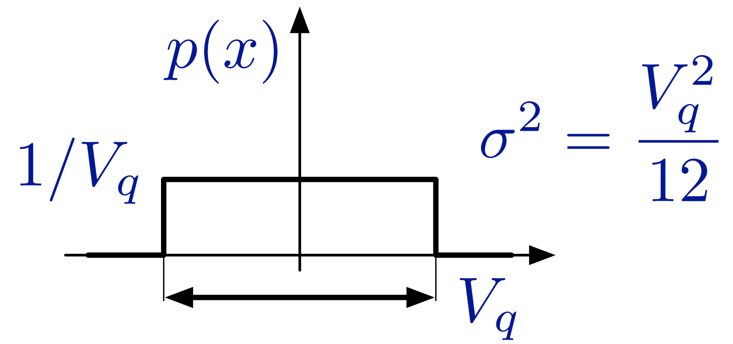
- The FFT of a N-sample time series has N complex points (Re and Im)
- The FFT is symmetric with respect to N/2
  Re{X} has even symmetry
  Im{X} has odd symmetry
- If the samples (time series) are real, all the information is contained in the first N/2 complex points
- The upper part of the spectrum is polluted by aliasing and distorted by the anti-aliasing filter, thus it is not used.
- Keeping the upper ξN/2 points (ξ≈0.8), the displayed points are $N' = \xi N/2$ ($N' \approx 0.4\,N$)
- The acquisition of N samples at the sampling frequency f_s takes a time $T_a = N/f_s$
- The frequency span is $f_{span} = \xi f_s/2$ ($f_{span} \approx 0.4 f_s$)
  The acquisition time is $T_a = N' / f_{span}$
- The FFT algorithm takes $N \log_2(N)$ complex additions and $(N/2) \log_2(N)$ complex multiplications
- **The computation time is proportional to N log(N)**



Spectrum of the quantization noise

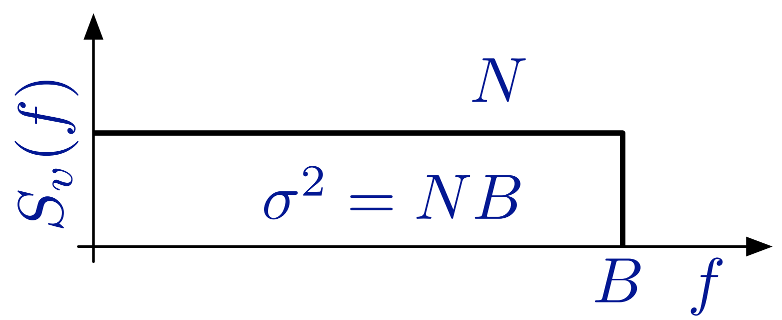


The analog-to-digital converter introduces a quantization error x, $-V_q/2 \leq x \leq +V_q/2$



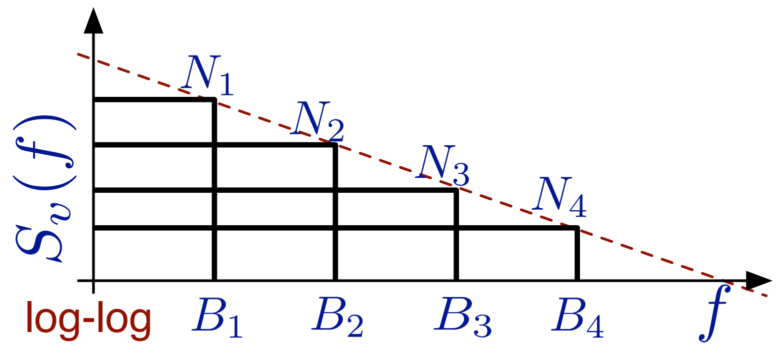
Ergodicity suggests that the quantization noise can be calculated statistically

$$\sigma^2 = \frac{V_q^2}{12}$$



The Parseval theorem states that energy and power can be evaluated by integrating the spectrum

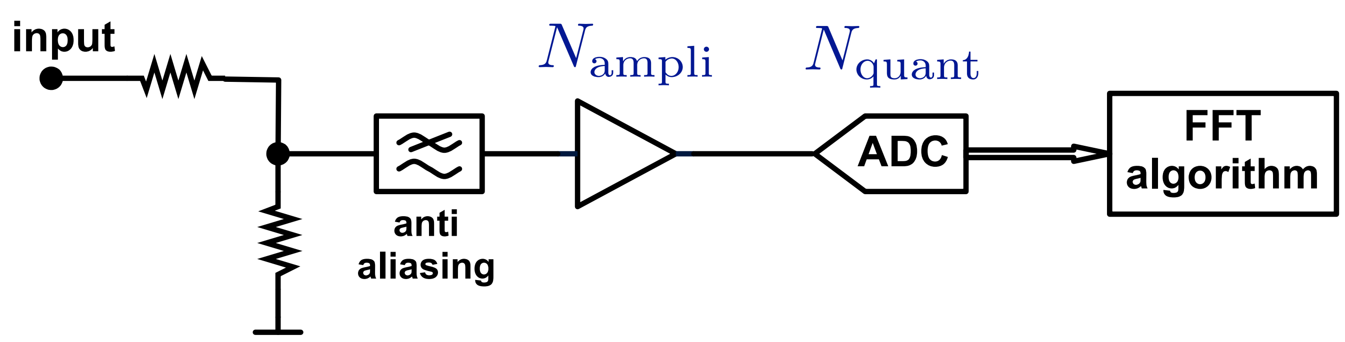
$$NB = \frac{V_q^2}{12}$$

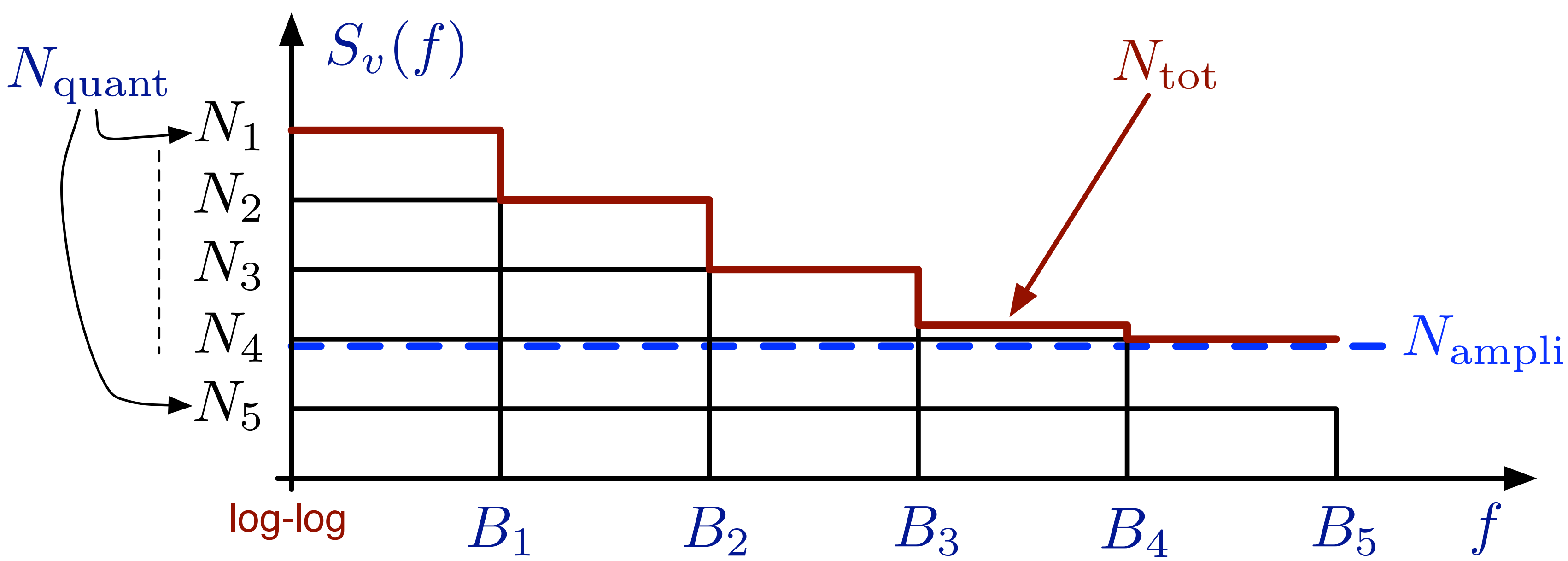


Changing B in geometric progression (decades) yields naturally 1/B (flicker) noise

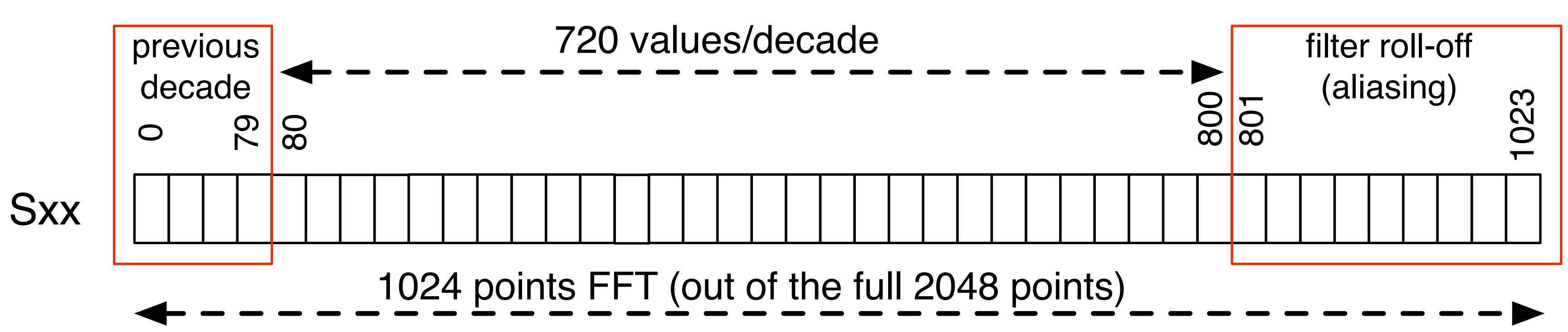
$$N = \frac{V_q^2}{12B}$$

Noise of the real FFT analyzer





The quantization noise scales with the frequency span, the front-end noise is constant



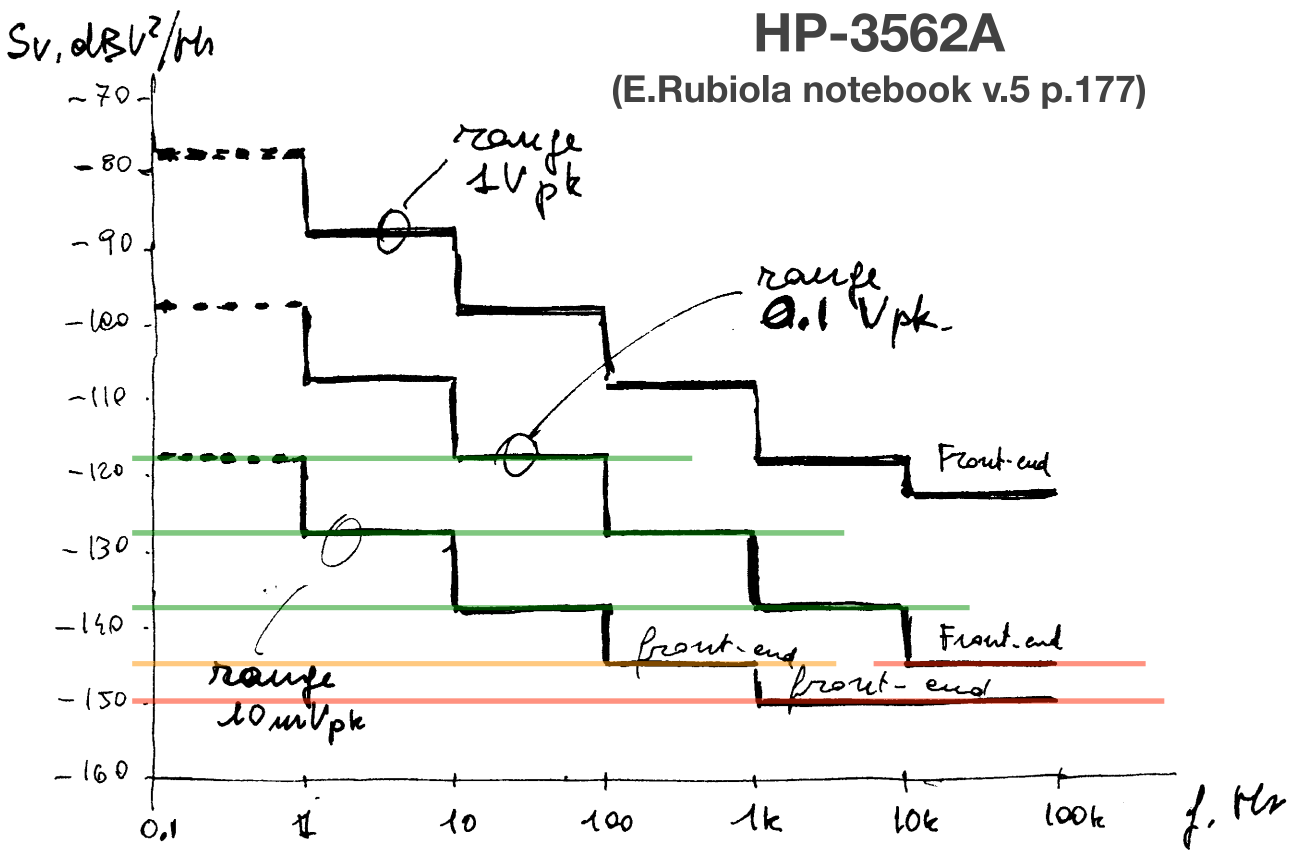
The energy is equally spread in the full FFT bandwidth, including the upper region not displayed because of aliasing

Example of FFT analyzer noise

Experimental observation

HP-3562A

(E.Rubiola notebook v.5 p.177)



Theoretical evaluation

DAC 12 bit resolution, including sign

range 10 mV_{peak}

V_{fsr} = 20 mV (±10 mV)

resolution

$V_q = V_{fsr} / 2^{12}$

$= 4.88\ \mu V$

total noise

$\sigma^2 = (4.88\ \mu V)^2 / 12$

$= 2{\times}10^{-12}\ V^2$ (–117 dB)

quantization noise PSD

$S_v = \sigma^2 / B$

$= -117\ dBV^2/Hz$ with B = 1 Hz (etc.)

Front-end noise, evaluated from the plot

$S_v = 2{\times}10^{-15}\ V^2$ (–150 dB), at 10–100 kHz

or 45 $nV/Hz^{1/2}$

use Sv = 4kTR

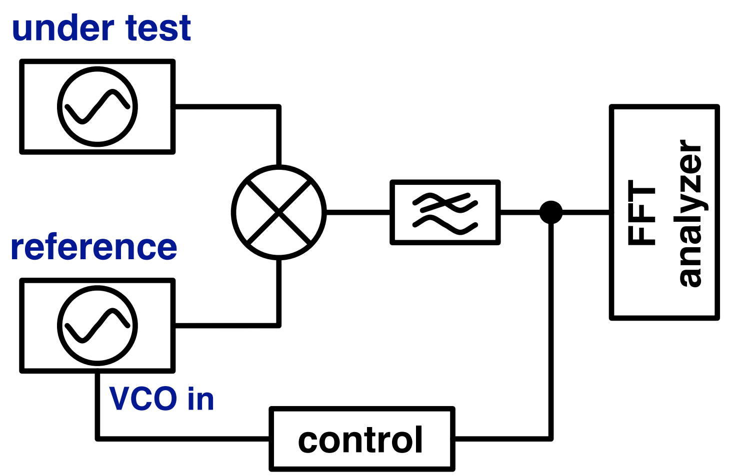
R = 125 kΩ

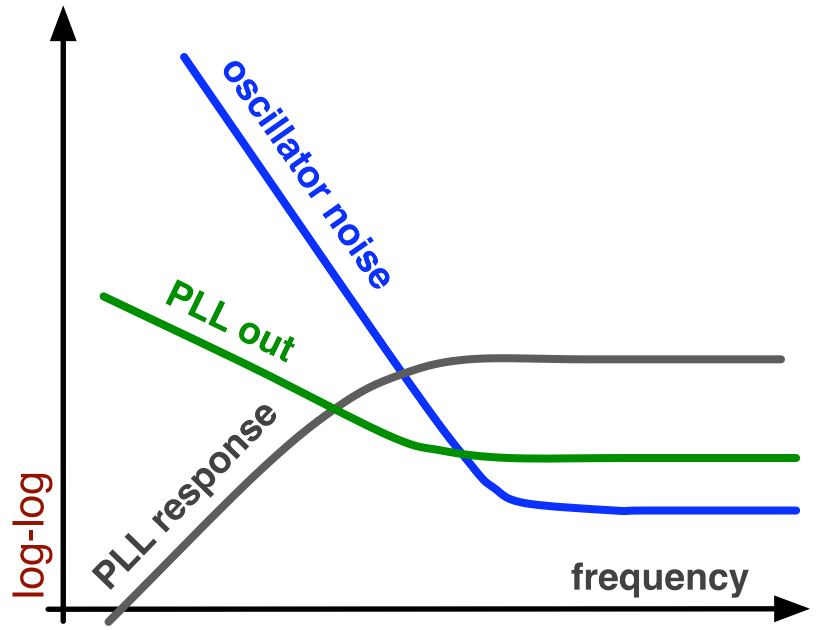
or R = 100 kΩ and F = 1 dB (noise figure)

Oscillator noise measurement

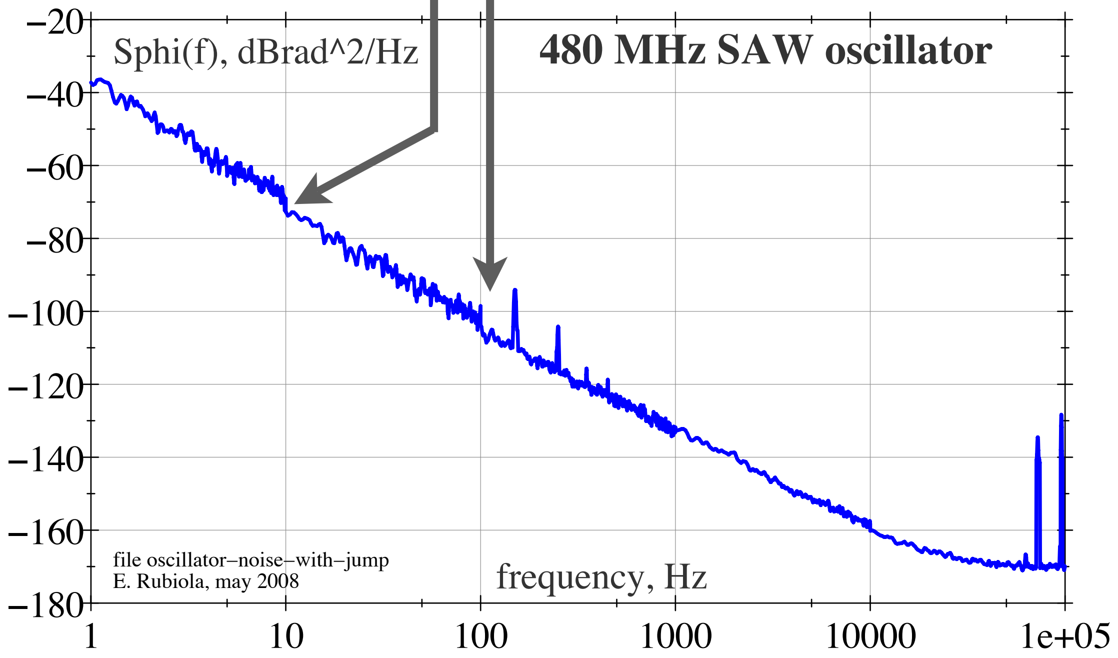
A tight loop is preferred because:

- reduces the required dynamic range
- overrides (parasitic) injection locking





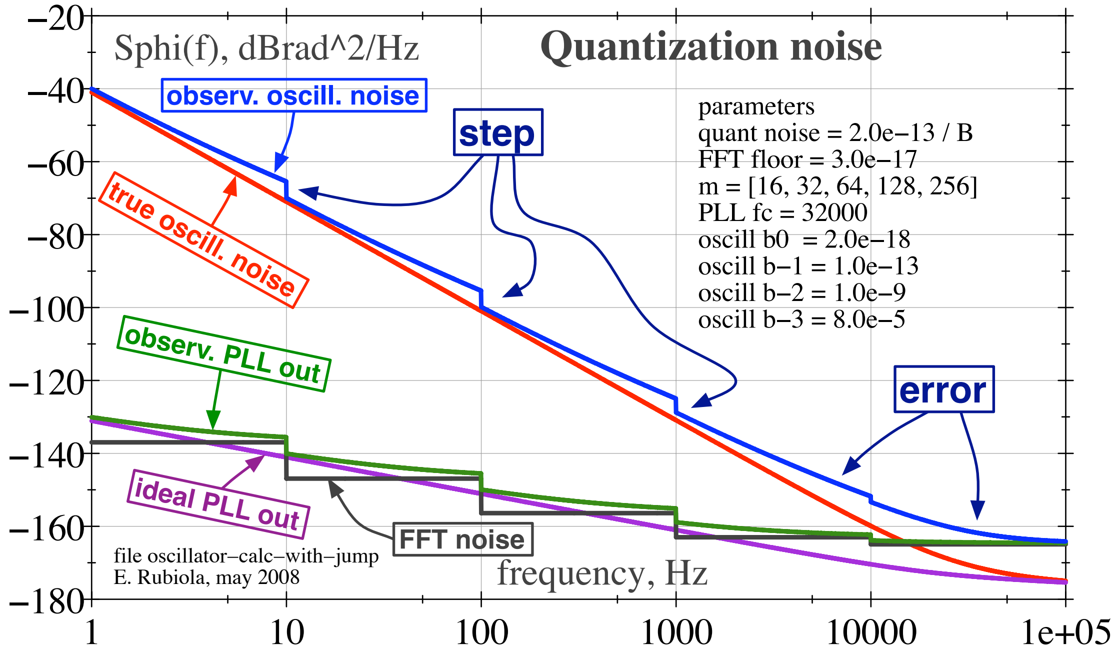
Steps are sometimes observed, due to the FFT quantization noise



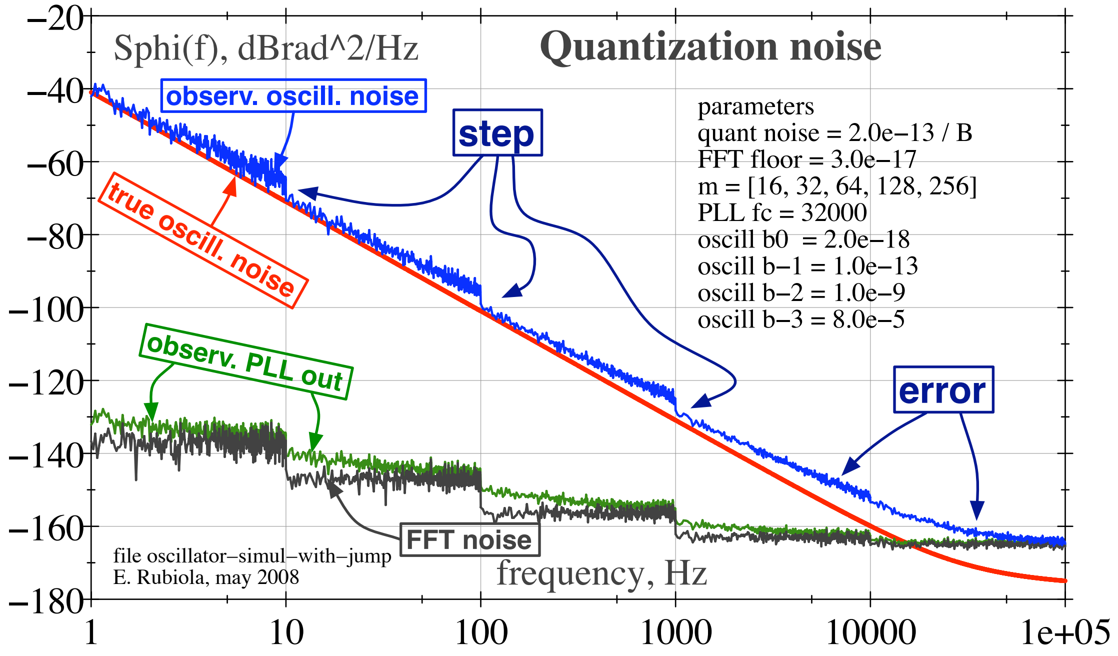
FFT noise in oscillator measurements

Explanation: the steps occurring at the transition between decades are due the quantization noise, when the resolution is insufficient

calculated



simulated



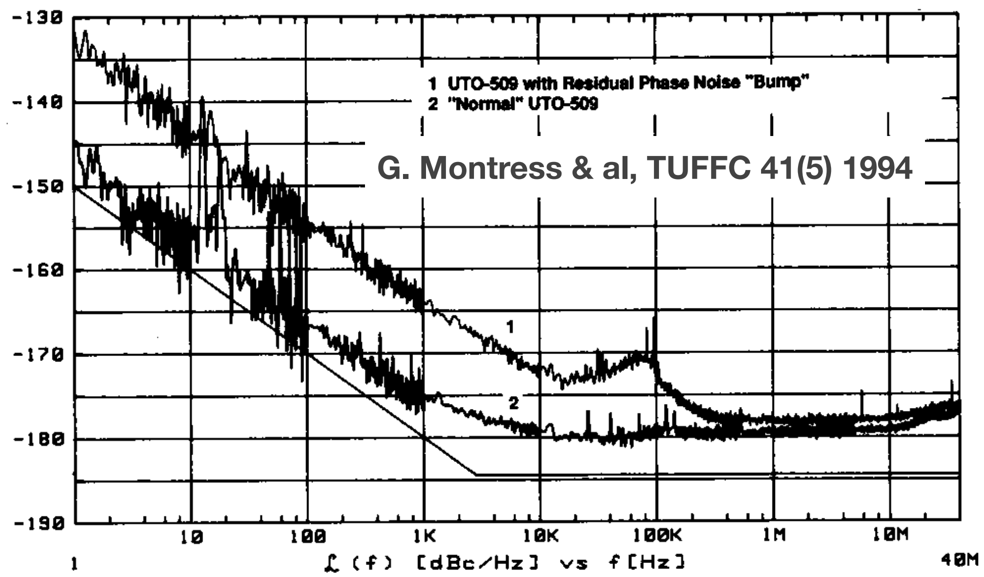
The steps are due to the FFT quantization noise

The problem shows up when the dynamic range is insufficient, often in the presence of large stray signals

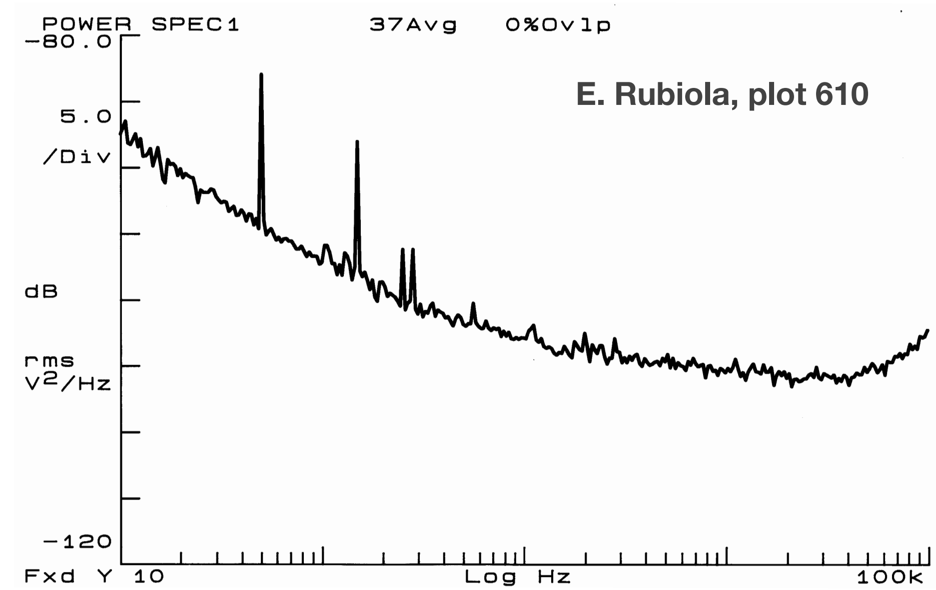
Systematic errors are also possible at high Fourier frequencies

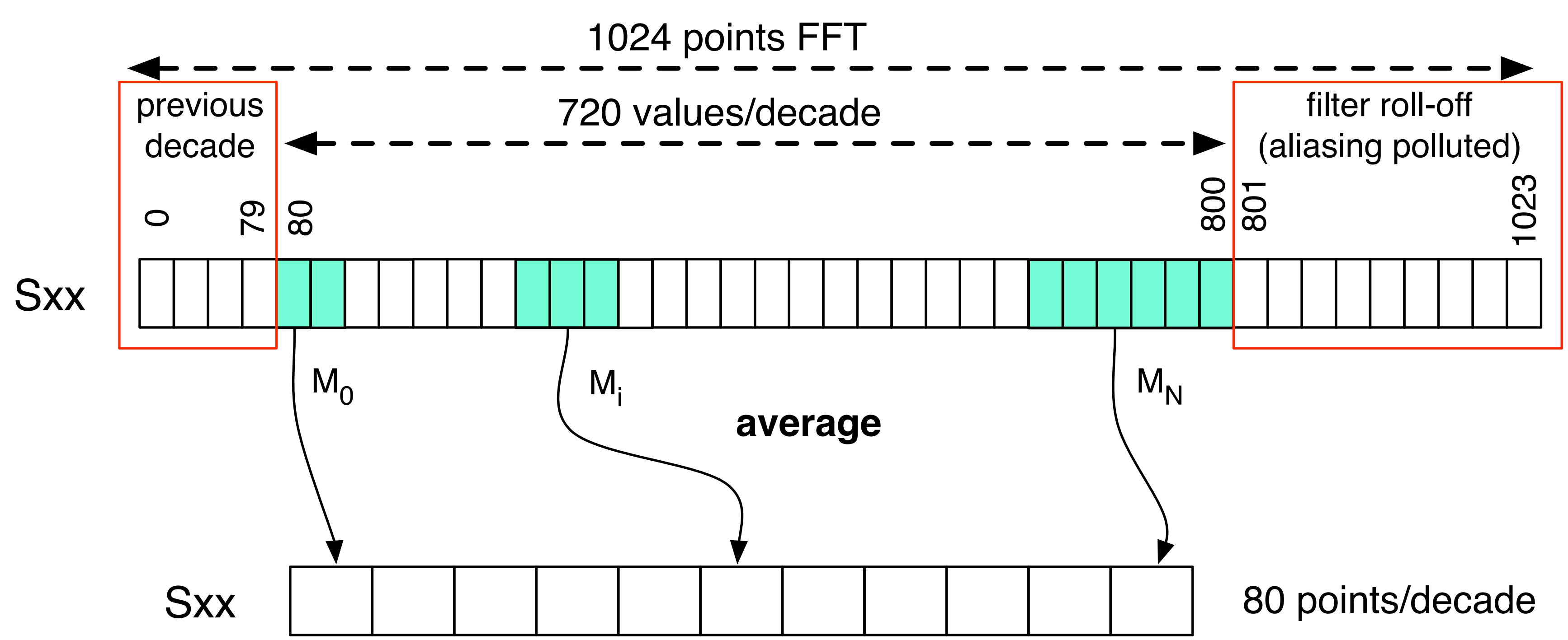
Linear vs. logarithmic resolution

Linear resolution



Logarithmic resolution (80 pt/dec)

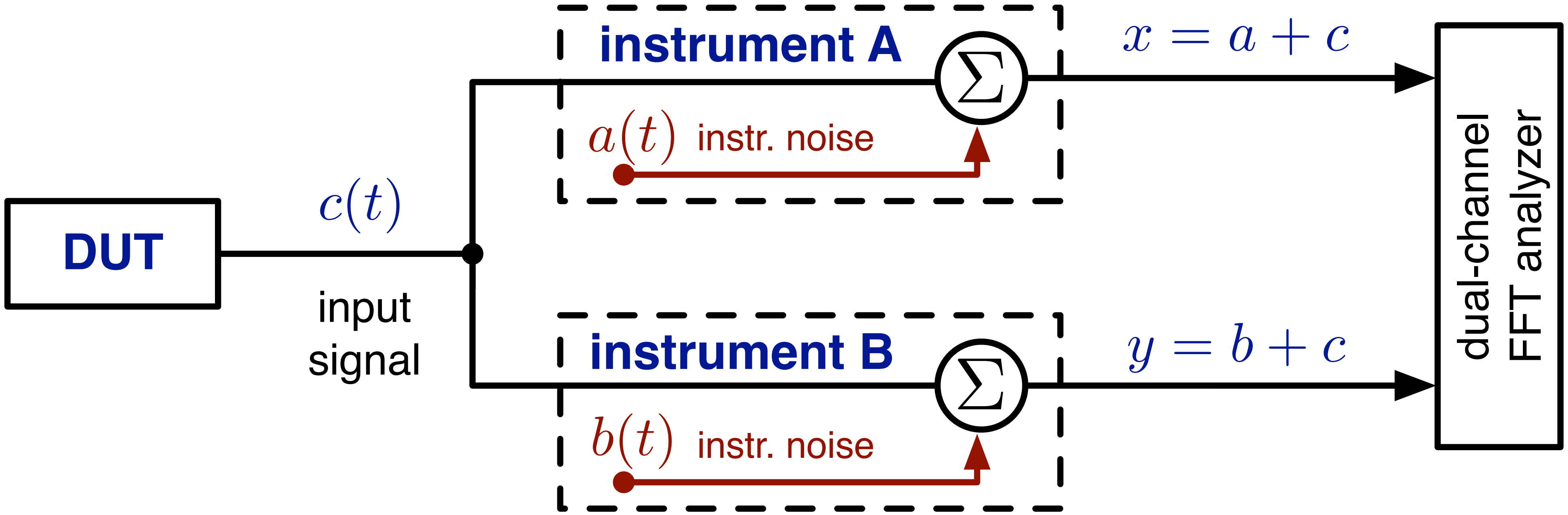




Combining M independent values, the confidence interval is reduced by sqrt(M), (5 dB left-right in one decade)

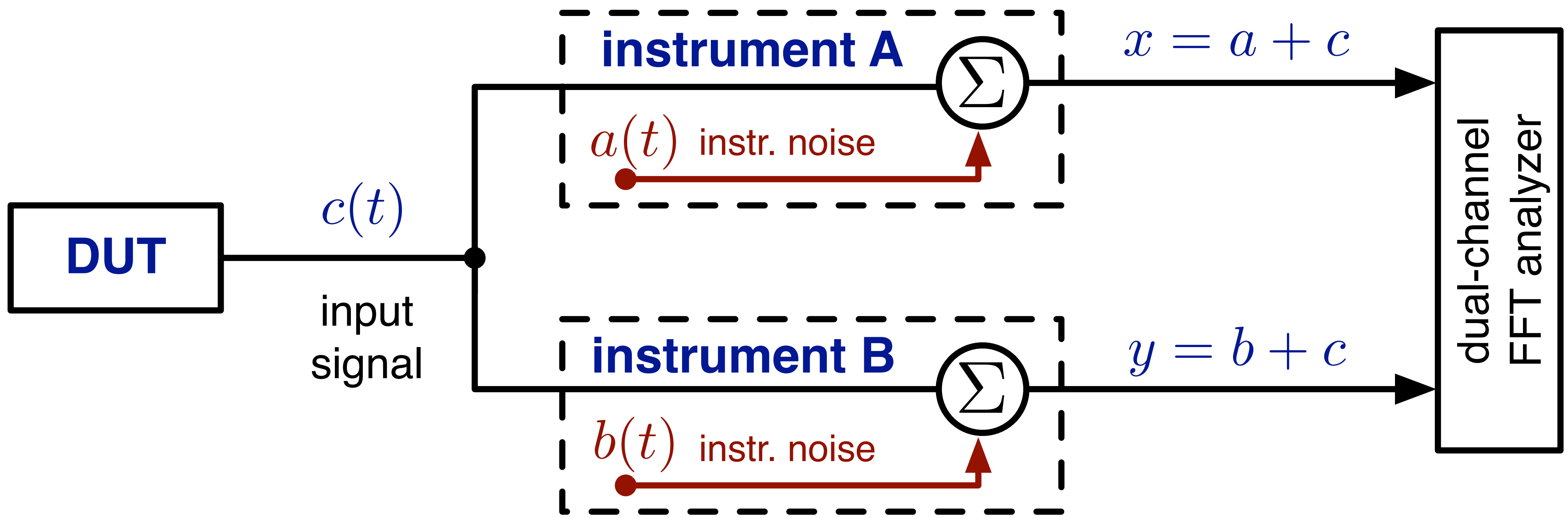
A weighted average is also possible

Theory



DUT
$c(t)$
input signal
instrument A
$a(t)$ instr. noise
instrument B
$b(t)$ instr. noise
$x = a + c$
$y = b + c$
dual-channel FFT analyzer

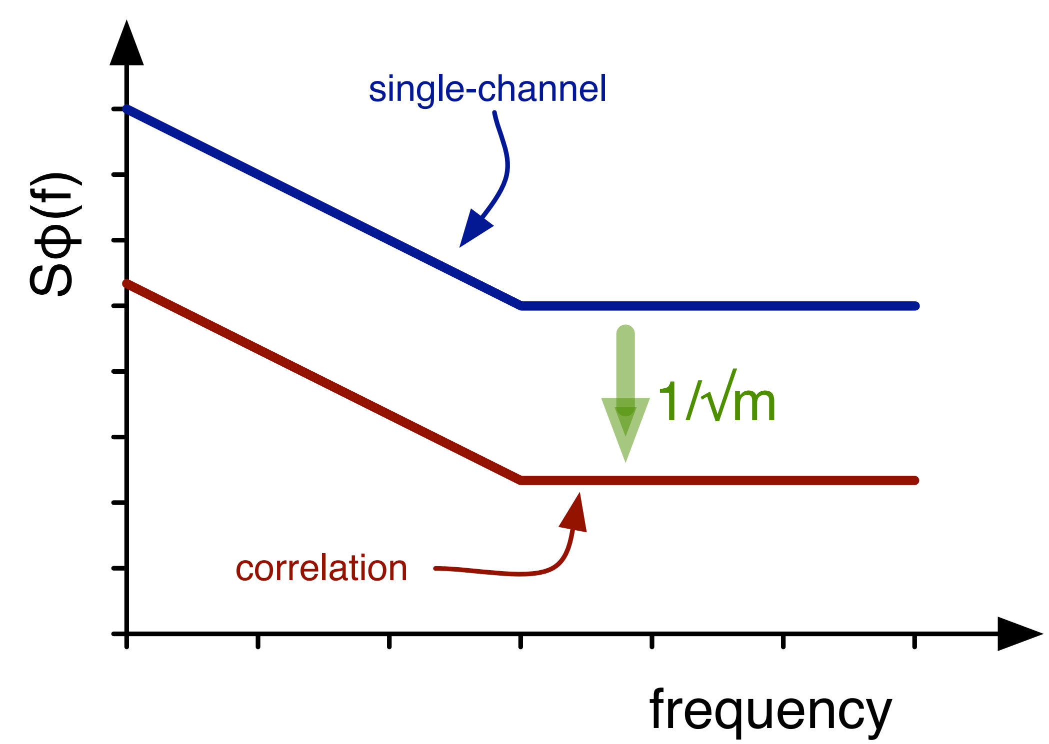
Correlation measurements



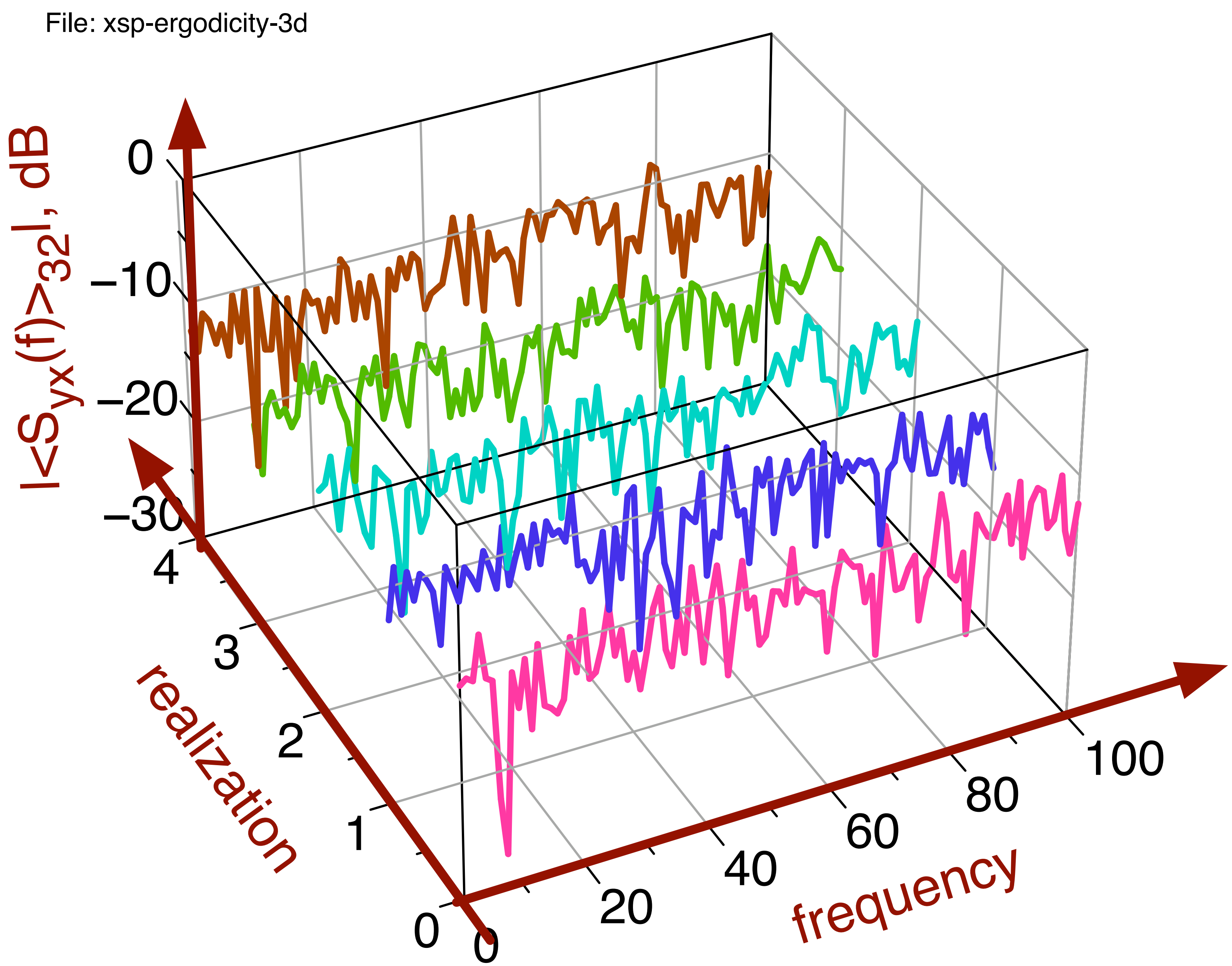
Two separate instruments measure the same DUT. Only the DUT noise is common

a(t), b(t) –> instrument noise
c(t) –> DUT noise

noise measurements		
DUT noise, normal use	a, b c	instrument noise DUT noise
background, ideal case	a, b c = 0	instrument noise no DUT
background, real case	a, b c ≠ 0	c is the correlated instrument noise Zero DUT noise



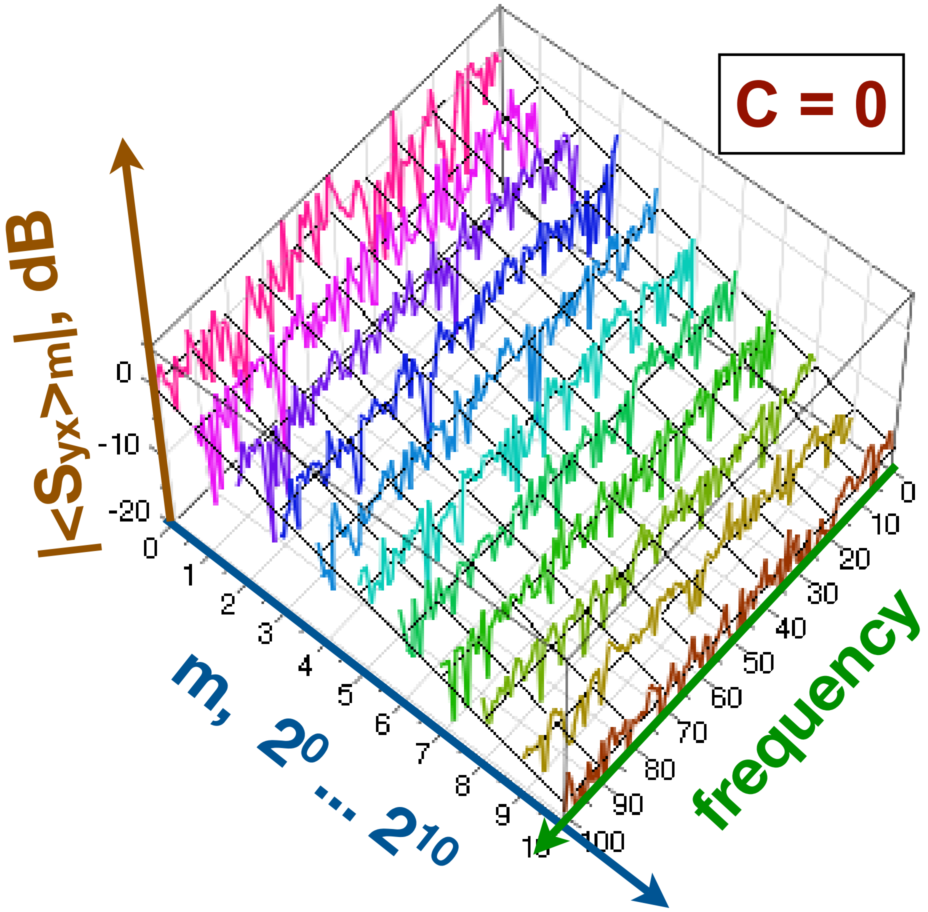
The concept of ergodicity

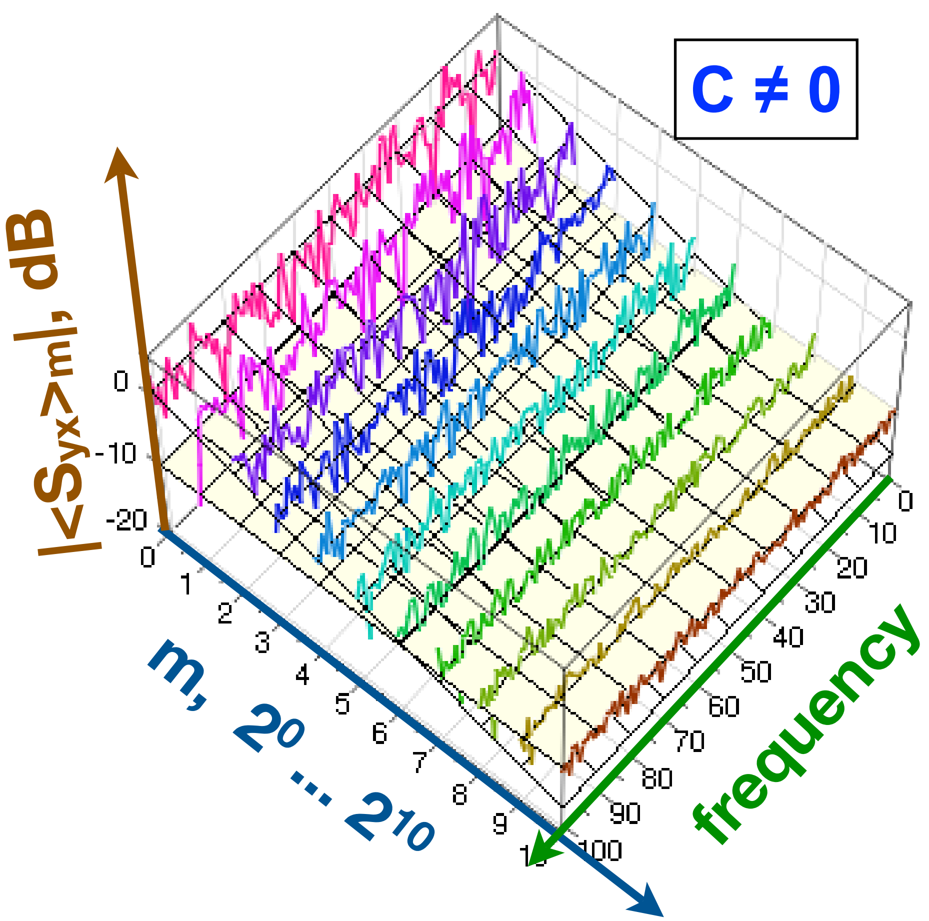


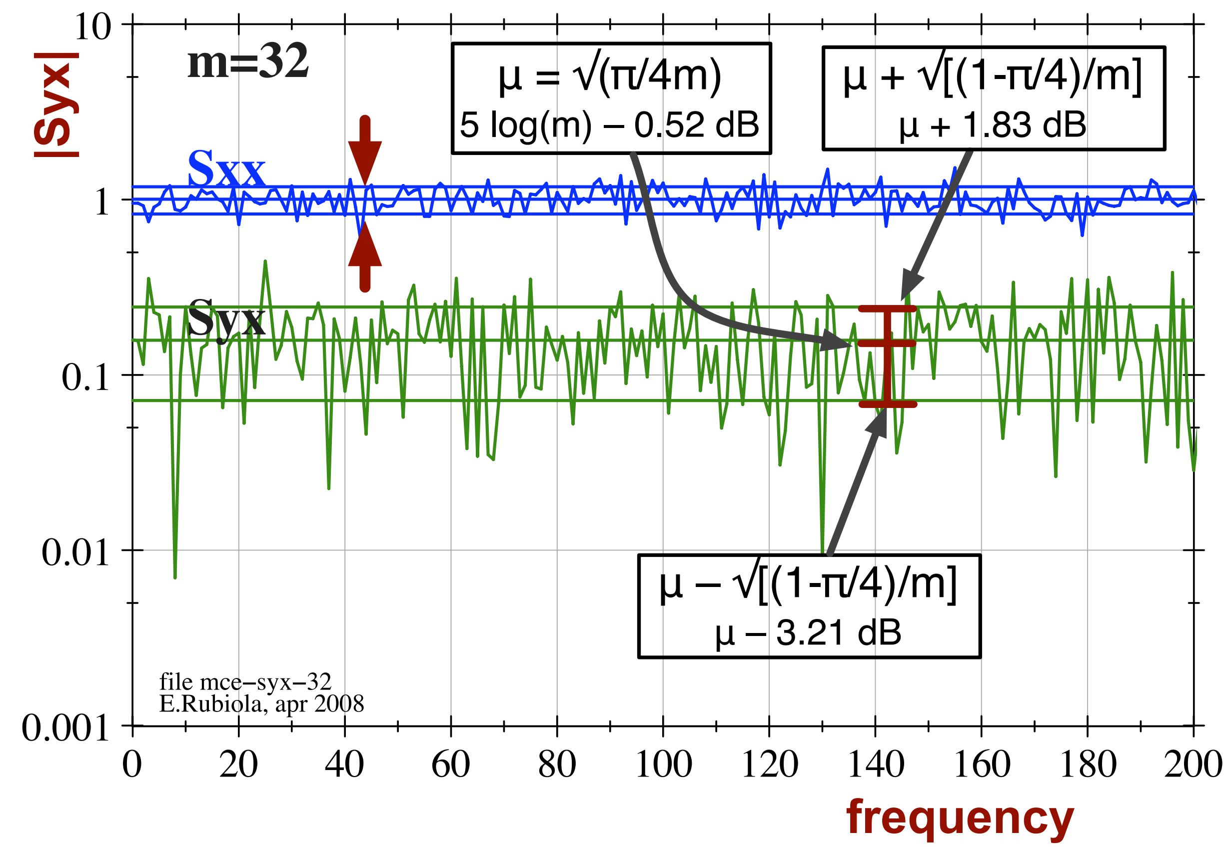
Ergodicity allows to interchange time statistics and ensemble statistics, thus the running index i of the sequence and the frequency f.

The average and the deviation calculated on the frequency axis are the same as the average and the deviation of the time series.

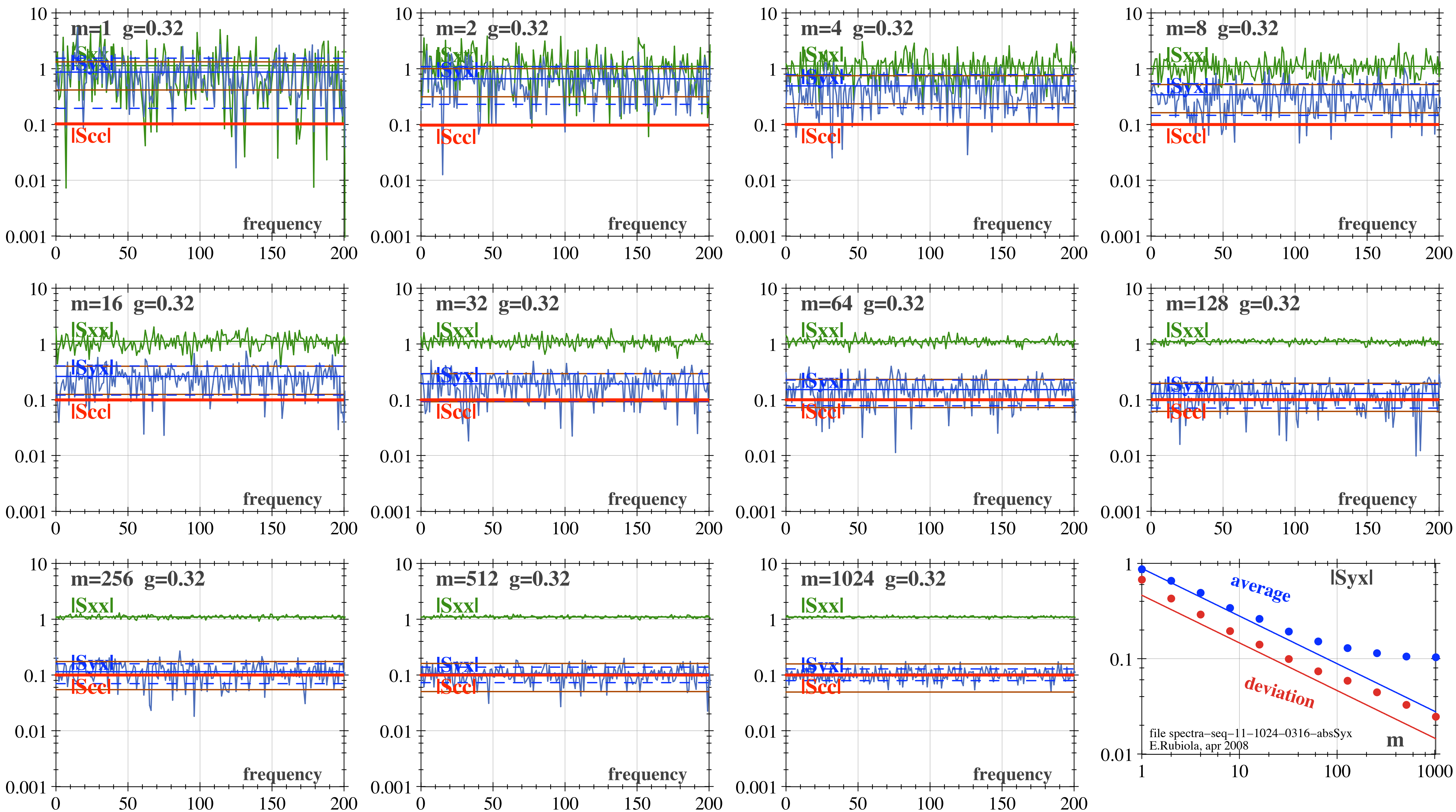
Example: Measurement of |Syx|





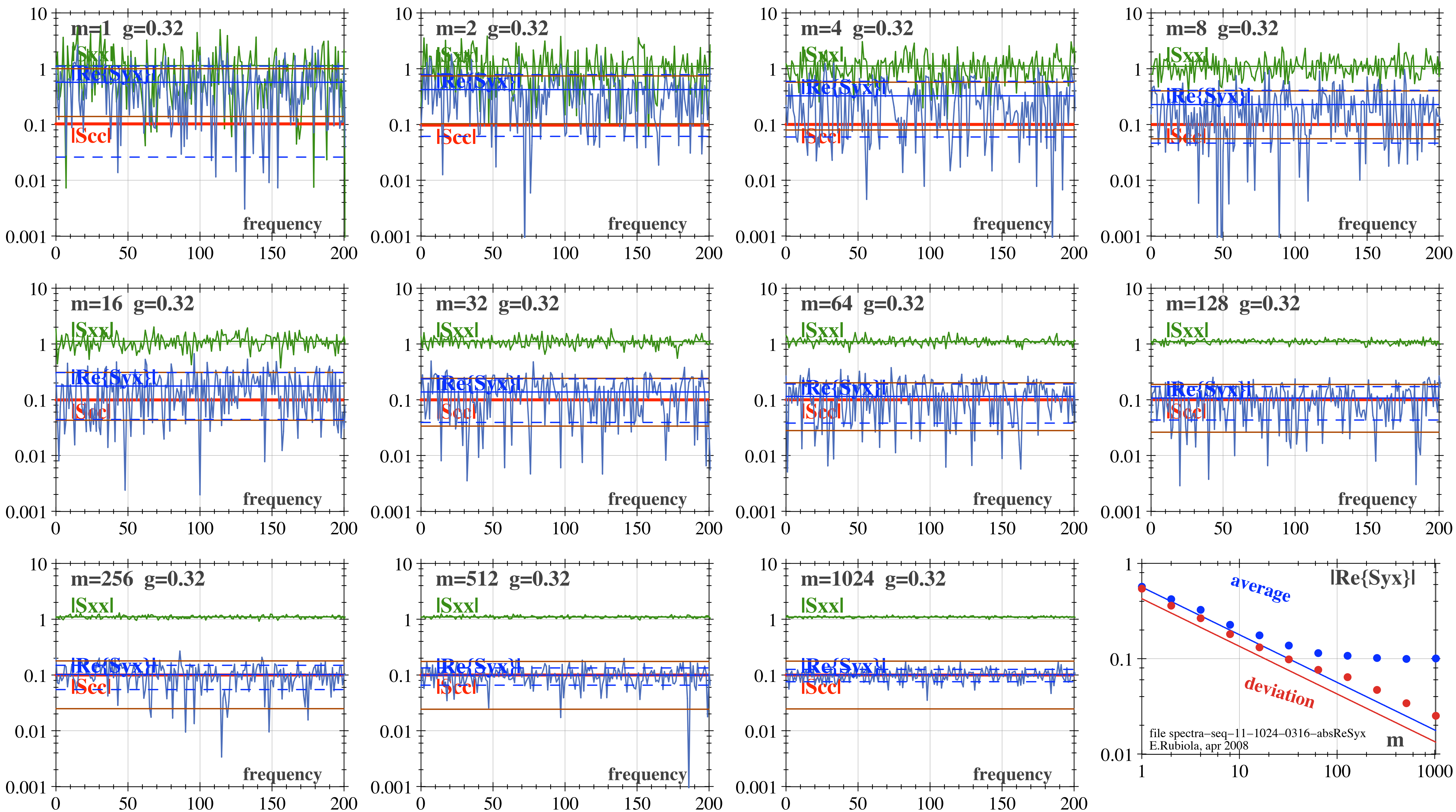


Measurement (C≠0), |Syx|



Running the measurement, m increases
S_{xx} shrinks => better confidence level
S_{yx} decreases => higher single-channel noise rejection

Measurement (C≠0), |Re{Syx}|



Running the measurement, m increases

S_{xx} shrinks => better confidence level

S_{yx} decreases => higher single-channel noise rejection

Boring exercises before playing a Steinway

Single-channel spectrum S_{xx}

Gaussian X with independent Re and Im

Spectrum

$$\begin{aligned}\langle S_{xx}\rangle_m &= \frac{1}{T}\,\langle X X^*\rangle_m \\ &= \frac{1}{T}\,\langle (X' + \imath X'') \times (X' - \imath X'')\rangle_m \\ &= \frac{1}{T}\,\boxed{\langle (X')^2 + (X'')^2\rangle_m}\end{aligned}$$

white, gaussian,
avg = 0, var = 1/2

white, χ^2, with 2m degrees of freedom
avg = 1, var = 1/m

$$\frac{\text{dev}}{\text{avg}} = \sqrt{\frac{1}{m}}$$

the S_{xx} track on the FFT-SA shrinks as $1/m^{1/2}$

Normalization: in 1 Hz bandwidth
var{X} = 1, and var{X'} = var{X"} = 1/2

S_{yx} with correlated term (1)

A, B = instrument background
C = DUT noise
channel 1 X = A + C
channel 2 Y = B + C
A, B, C are independent Gaussian noises
Re{ } and Im{ } are independent Gaussian noises

Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ^2
var{A'} = var{A"} = var{B'} = var{B"} = 1/2, and var{C'} = var{C"} = $\kappa^2/2$

Cross-spectrum

$$\langle S_{yx}\rangle_m = \frac{1}{T}\langle YX^*\rangle_m = \frac{1}{T}\langle (Y'+\imath Y'')\times(X'-\imath X'')\rangle_m$$

Expand using

$$X = (A'+\imath A'')+(C'+\imath C'') \qquad \text{and} \qquad Y = (B'+\imath B'')+(C'+\imath C'')$$

Split S_{yx} into three sets

$$\langle S_{yx}\rangle_m = \langle S_{yx}\rangle_m\Big|_{\text{instr}} + \langle S_{yx}\rangle_m\Big|_{\text{mixed}} + \langle S_{yx}\rangle_m\Big|_{\text{DUT}}$$

background only — **background and DUT noise** — **DUT noise only**

... and work it out !!!

S_{yx} with correlated term κ≠0 (2)

All the DUT signal goes in Re{S_{yx}}, Im{S_{yx}} contains only noise

Real

var=1/2 | var=1/2 | var= κ²/2 | var= κ²/2

$$\Re\left\{\langle S_{yx}\rangle_m\right\} = \frac{1}{T}\left\{\langle B'A' + B''A''\rangle_m + \langle B'C' + B''C''\rangle_m + \langle C'A' + C''A''\rangle_m + \langle (C')^2 + (C'')^2\rangle_m\right\}$$

Gaussian, avg=0, var=1/4

Gaussian, avg=0, var=κ²/4

white, χ², 2 DF
avg = κ², var = κ⁴

Gaussian, avg = 0, var = 1/2m

Gaussian, avg = 0, var = κ²/2m

Gaussian, avg = 0, var = κ²/2m

white, χ²
2m deg. of freedom
avg = κ², var = κ⁴/m
Set $\mathscr{C}$

Set $\mathscr{A}$ Gaussian, avg = 0, var = (1+2κ²)/2m

Imaginary

$$\Im\left\{\langle S_{yx}\rangle_m\right\} = \frac{1}{T}\left\{\langle B''A' + B'A''\rangle_m + \langle B''C' - B'C''\rangle_m + \langle C''A' - C'A''\rangle_m\right\}$$

white, Gaussian, avg = 0, var = 1/4

white, Gaussian, avg = 0, var = κ²/4

Gaussian, avg = 0, var = 1/2m

Gaussian, avg = 0, var = κ²/2m

Gaussian, avg = 0, var = κ²/2m

Set $\mathscr{B}$ Gaussian, avg = 0, var = (1+2κ²)/2m

Note: DF < 2m
See vol.XVI p.56

Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ²
var{A'} = var{A"} = var{B'} = var{B"} = 1/2, and var{C'} = var{C"} = κ²/2

A, B, C are independent Gaussian noises
Re{ } and Im{ } are independent Gaussian noises

Expand S_{yx}

$$S_{yx} = \frac{1}{T}\,\mathbb{E}\,\{\mathscr{A} + \imath\mathscr{B} + \mathscr{C}\}$$

Gaussian, avg=0, var=1/4

$$\mathscr{A} = B'A' + B''A'' + B'C' + B''C'' + C'A' + C''A''$$

$$\mathscr{B} = B''A' + B'A'' + B''C' - B'C'' + C''A' - C'A''$$

Gaussian, avg=0, var=$\kappa^2/4$

$$\mathscr{C} = C'^{\,2} + C''^{\,2}$$

white, χ^2, 2 DF
avg = κ^2, var = κ^4

term	$\mathbb{E}$	$\mathbb{V}$	PDF	comment
$\langle\mathscr{A}\rangle_m$	0	$\dfrac{1+2\kappa^2}{2m}$	Gauss	average (sum) of zero-mean
$\langle\mathscr{B}\rangle_m$	0	$\dfrac{1+2\kappa^2}{2m}$	Gauss	Gaussian processes
$\langle\mathscr{C}\rangle_m$	κ^2	κ^4/m	χ^2 $\nu = 2m$	average (sum) of chi-square processes
$\langle\tilde{\mathscr{C}}\rangle_m$	κ^2	κ^4/m	Gauss	approximates $\langle\mathscr{C}\rangle_m$ for large m

Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ^2
var{A'} = var{A"} = var{B'} = var{B"} = 1/2, and var{C'} = var{C"} = $\kappa^2/2$

Estimator $\hat{S} = |\langle S_{yx}\rangle_m|$

$$
\left|\left\langle S_{yx}\right\rangle_m\right| = \frac{1}{T}\sqrt{\left[\Re\left\{\left\langle YX^*\right\rangle_m\right\}\right]^2 + \left[\Im\left\{\left\langle YX^*\right\rangle_m\right\}\right]^2}
$$
$$
= \frac{1}{T}\sqrt{\left[\left\langle\mathscr{A}\right\rangle_m + \left\langle\tilde{\mathscr{C}}\right\rangle_m\right]^2 + \left[\left\langle\mathscr{B}\right\rangle_m\right]^2}\ .
$$

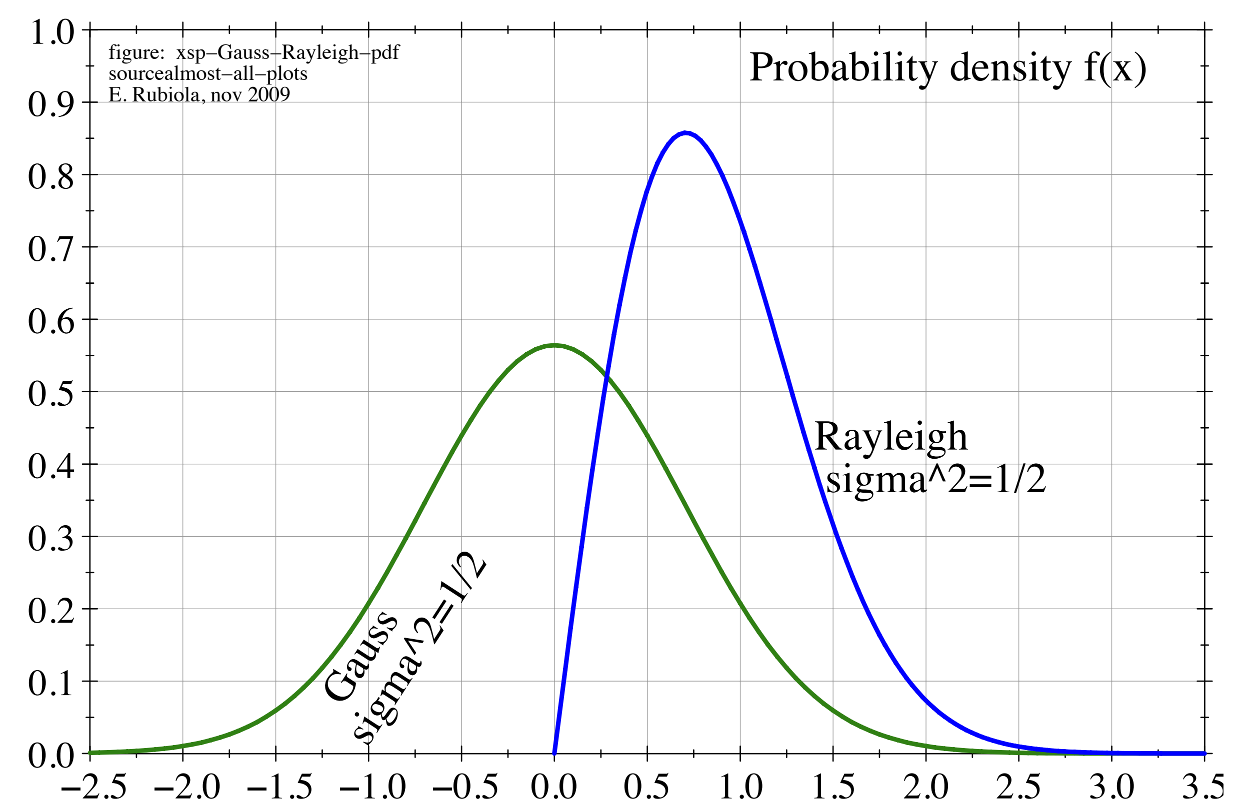
κ → 0 Rayleigh distribution

$$
\left\langle\mathscr{Z}\right\rangle_m = \sqrt{\left[\left\langle\mathscr{A}\right\rangle_m\right]^2 + \left[\left\langle\mathscr{B}\right\rangle_m\right]^2}\ .
$$

$$
\mathbb{E}\{\langle\mathscr{Z}\rangle_m\} = \sqrt{\frac{\pi}{4m}} = \frac{0.886}{\sqrt{m}}
$$

$$
\mathbb{V}\{\langle\mathscr{Z}\rangle_m\} = \frac{1}{m}\left(1 - \frac{\pi}{4}\right) = \frac{0.215}{m}
$$

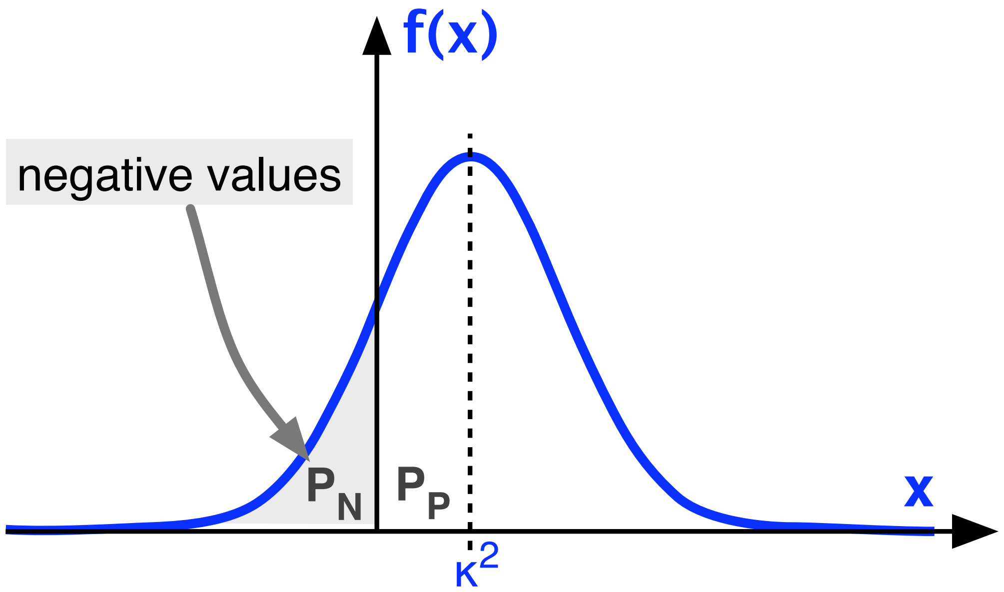
$$
\frac{\operatorname{dev}\{|\langle S_{yx}\rangle_m|\}}{\mathbb{E}\{|\langle S_{yx}\rangle_m|\}} = \sqrt{\frac{4}{\pi} - 1} = 0.523
$$



Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ^2
var{A'} = var{A"} = var{B'} = var{B"} = 1/2, and var{C'} = var{C"} = $\kappa^2/2$

Estimator $\hat{S} = \mathrm{Re}\{\langle S_{yx}\rangle_m\}$

$$\langle \mathscr{Z} \rangle_m = \langle \mathscr{A} \rangle_m + \langle \tilde{\mathscr{C}} \rangle_m$$



$$P_N = \mathbb{P}\{\mathbf{x} < 0\} = \frac{1}{2}\mathrm{erfc}\left(\frac{\kappa^2}{\sqrt{2}\,\sigma}\right)$$

$$\mathbb{E}\left\{\langle \mathscr{Z} \rangle_m\right\} = \kappa^2$$

$$\mathbb{V}\left\{\langle \mathscr{Z} \rangle_m\right\} = \frac{1 + 2\kappa^2 + 2\kappa^4}{2m}$$

$$\mathrm{dev}\left\{\langle \mathscr{Z} \rangle_m\right\} = \sqrt{\frac{1 + 2\kappa^2 + 2\kappa^4}{2m}} \approx \frac{1+\kappa^2}{\sqrt{2m}}$$

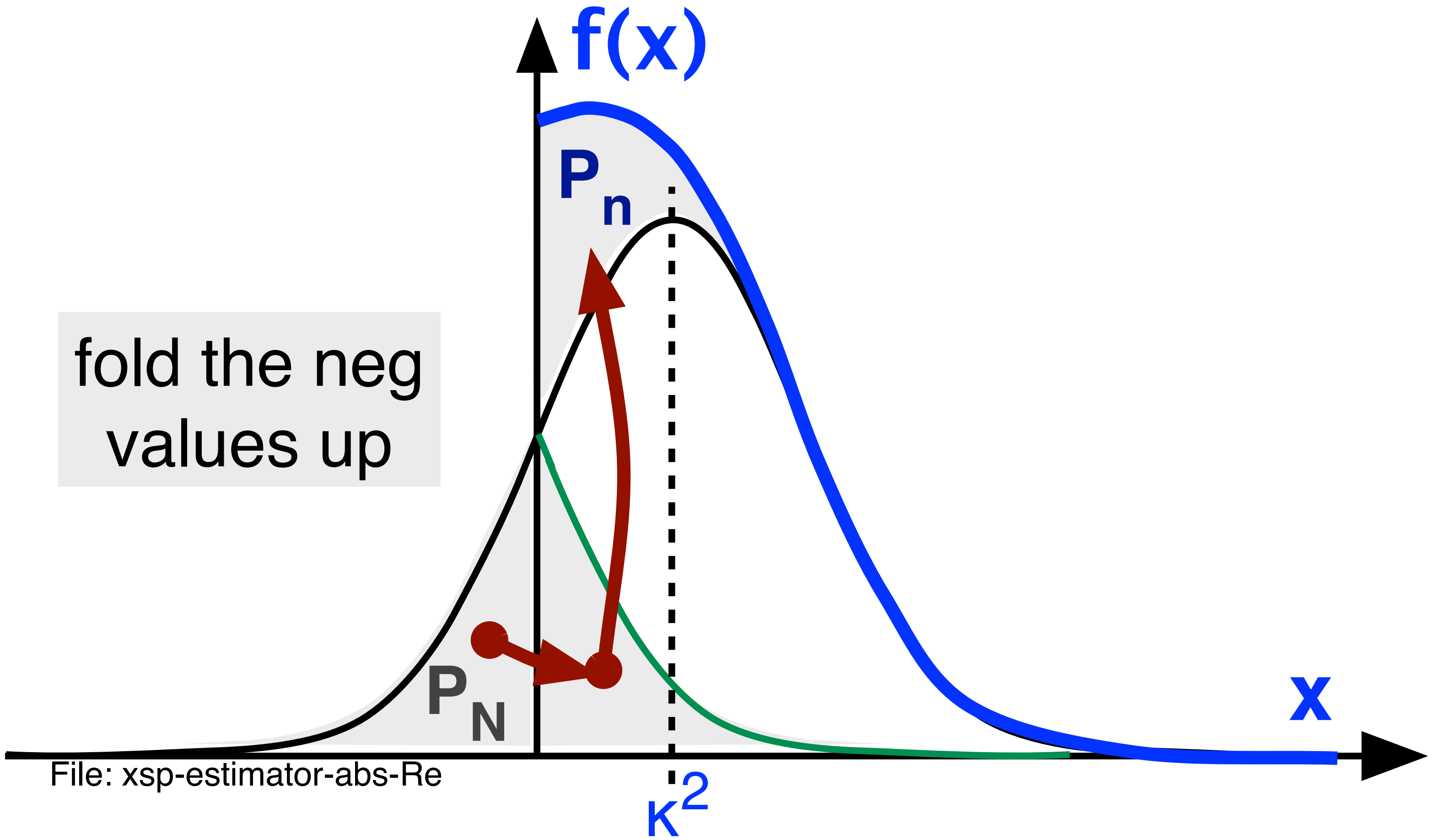
$$\frac{\mathrm{dev}\left\{\langle \mathscr{Z} \rangle_m\right\}}{\mathbb{E}\left\{\langle \mathscr{Z} \rangle_m\right\}} = \frac{\sqrt{1 + 2\kappa^2 + 2\kappa^4}}{\kappa^2\sqrt{2m}} \approx \frac{1+\kappa^2}{\kappa^2\sqrt{2m}}$$

0 dB SNR requires that $m=1/2\kappa^4$.
Example $\kappa=0.1$ (DUT noise 20 dB lower than single-channel background) averaging on 5×10^3 spectra is necessary to get SNR = 0 dB.

Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ^2
var{A’} = var{A”} = var{B’} = var{B”} = 1/2, and var{C’} = var{C”} = $\kappa^2/2$

Estimator $\hat{S} = |\mathrm{Re}\{\langle S_{yx}\rangle_m\}|$

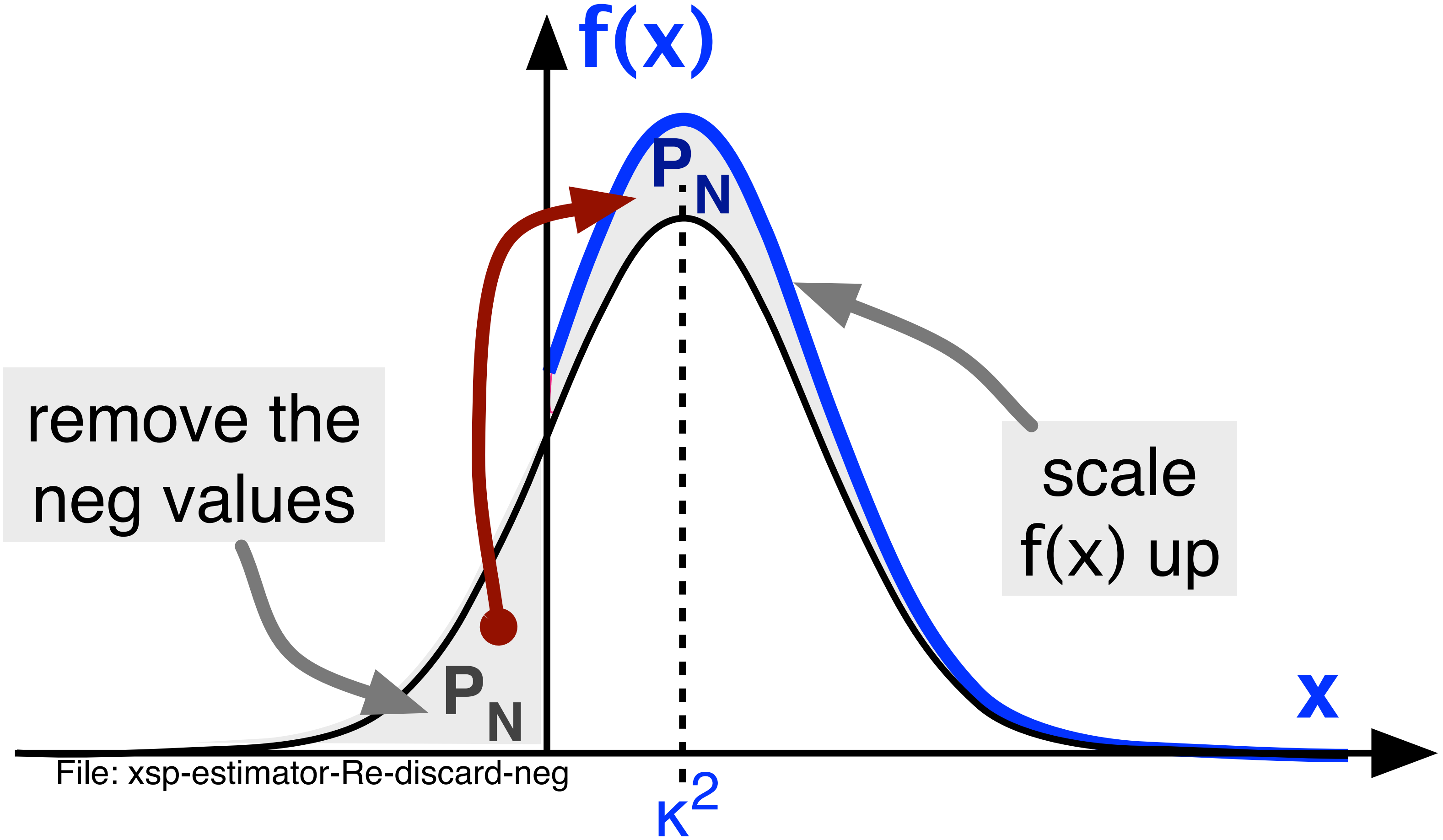
$$\left|\Re\left\{\langle S_{yx}\rangle_m\right\}\right| = \frac{1}{T}\left|\langle\mathscr{A}\rangle_m + \langle\tilde{\mathscr{C}}\rangle_m\right|$$



$$P_N = \mathbb{P}\{\mathbf{x} < 0\} = \frac{1}{2}\mathrm{erfc}\left(\frac{\kappa^2}{\sqrt{2}\,\sigma}\right)$$

Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ^2
var{A'} = var{A"} = var{B'} = var{B"} = 1/2, and var{C'} = var{C"} = $\kappa^2/2$

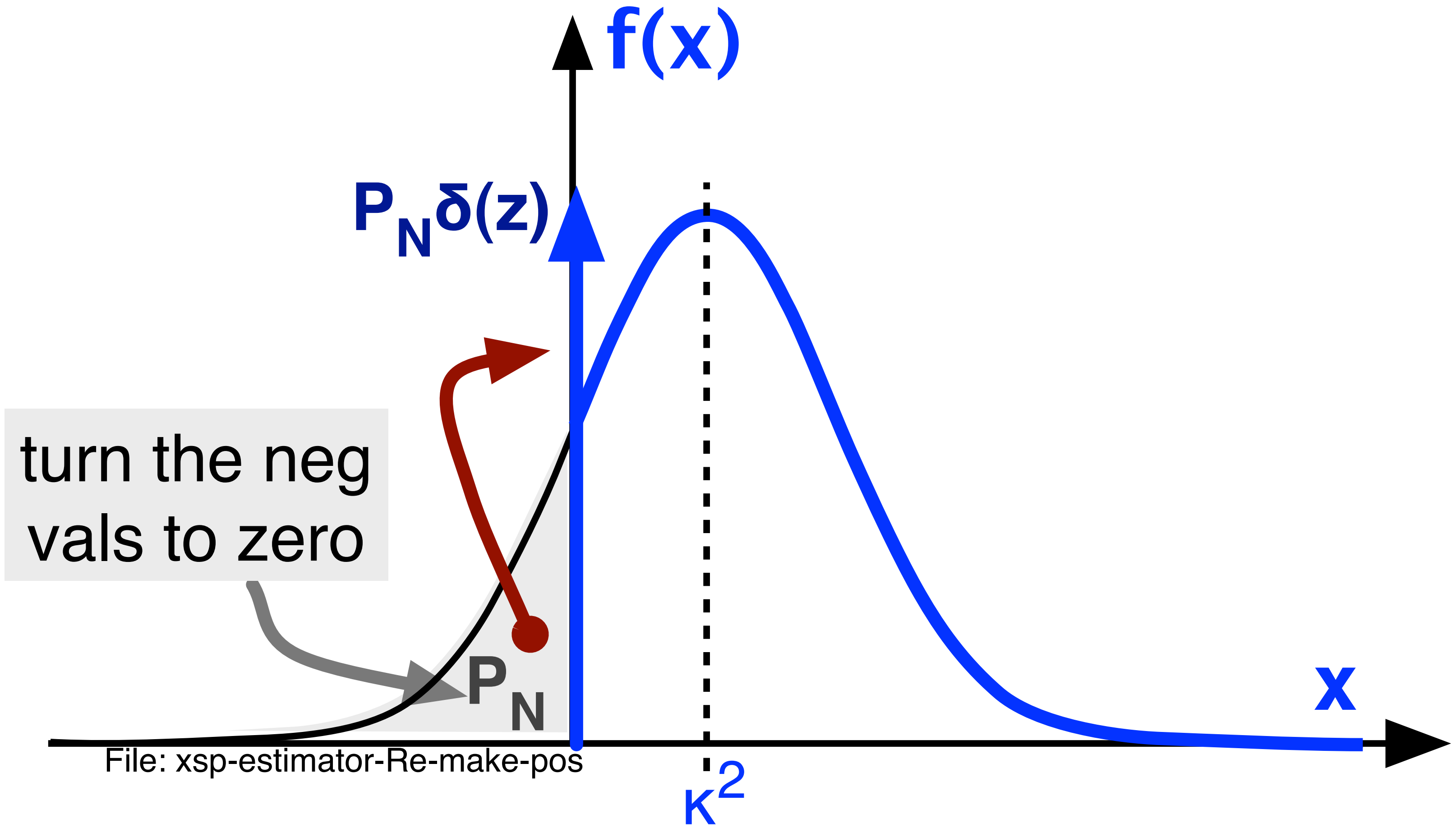
Estimator $\hat{S} = \mathrm{Re}\{\langle S_{yx} \rangle_{m'}\}$ averaging on the m' positive values



$$P_N = \mathbb{P}\{\mathbf{x} < 0\} = \frac{1}{2}\mathrm{erfc}\left(\frac{\kappa^2}{\sqrt{2}\,\sigma}\right)$$

Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ^2
var{A'} = var{A"} = var{B'} = var{B"} = 1/2, and var{C'} = var{C"} = $\kappa^2/2$

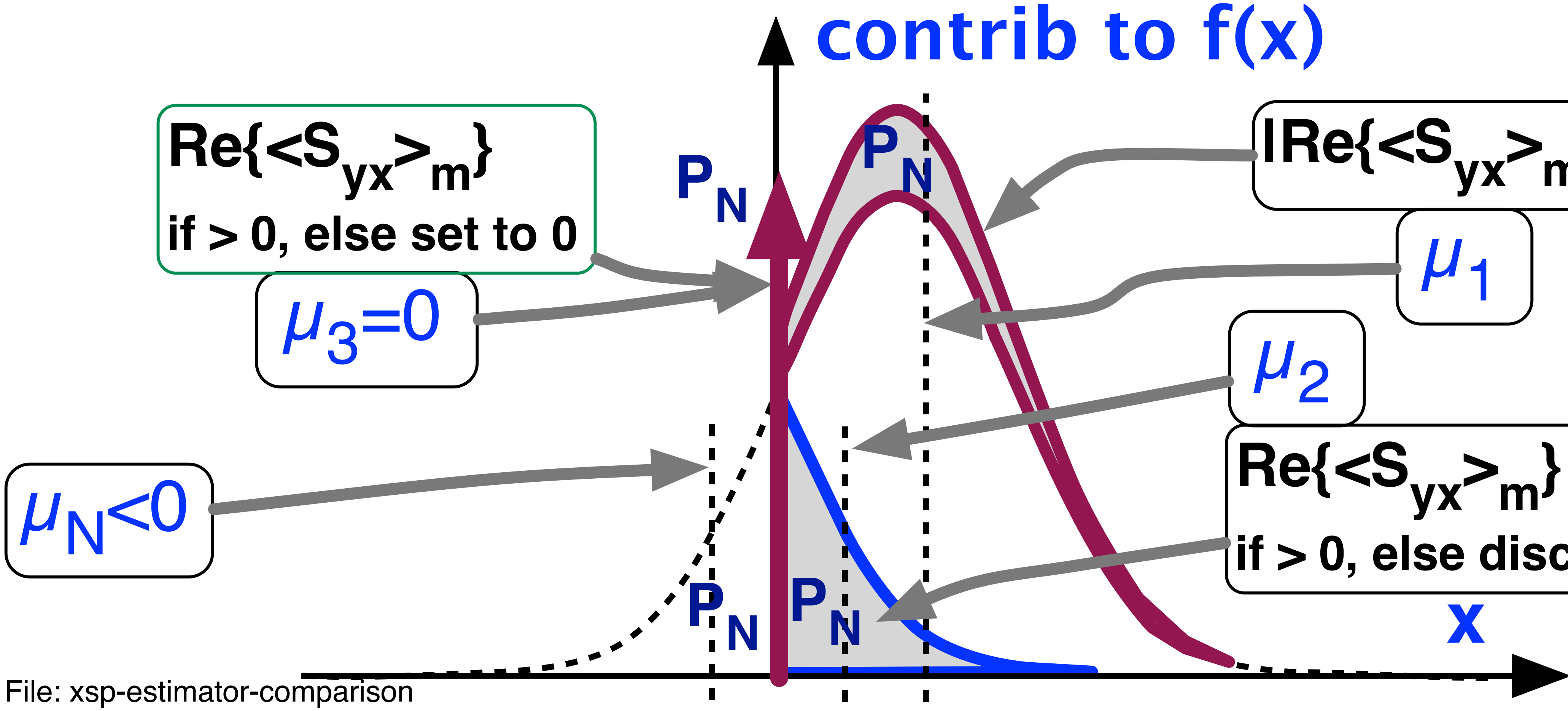
Estimator $\hat{S} = \langle \max(\mathrm{Re}\{ S_{yx}\}, 0_+) \rangle_m$



Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ^2
var{A'} = var{A"} = var{B'} = var{B"} = 1/2, and var{C'} = var{C"} = $\kappa^2/2$

Estimator $\hat{S}$ = <max(Re{ S_{yx}}, 0_+)>$_m$

$$\mu_1 > \mu_2 > \mu_3 \ .$$



File: xsp-estimator-comparison

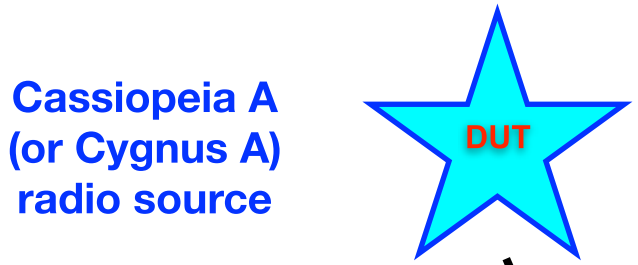
preferred estimator

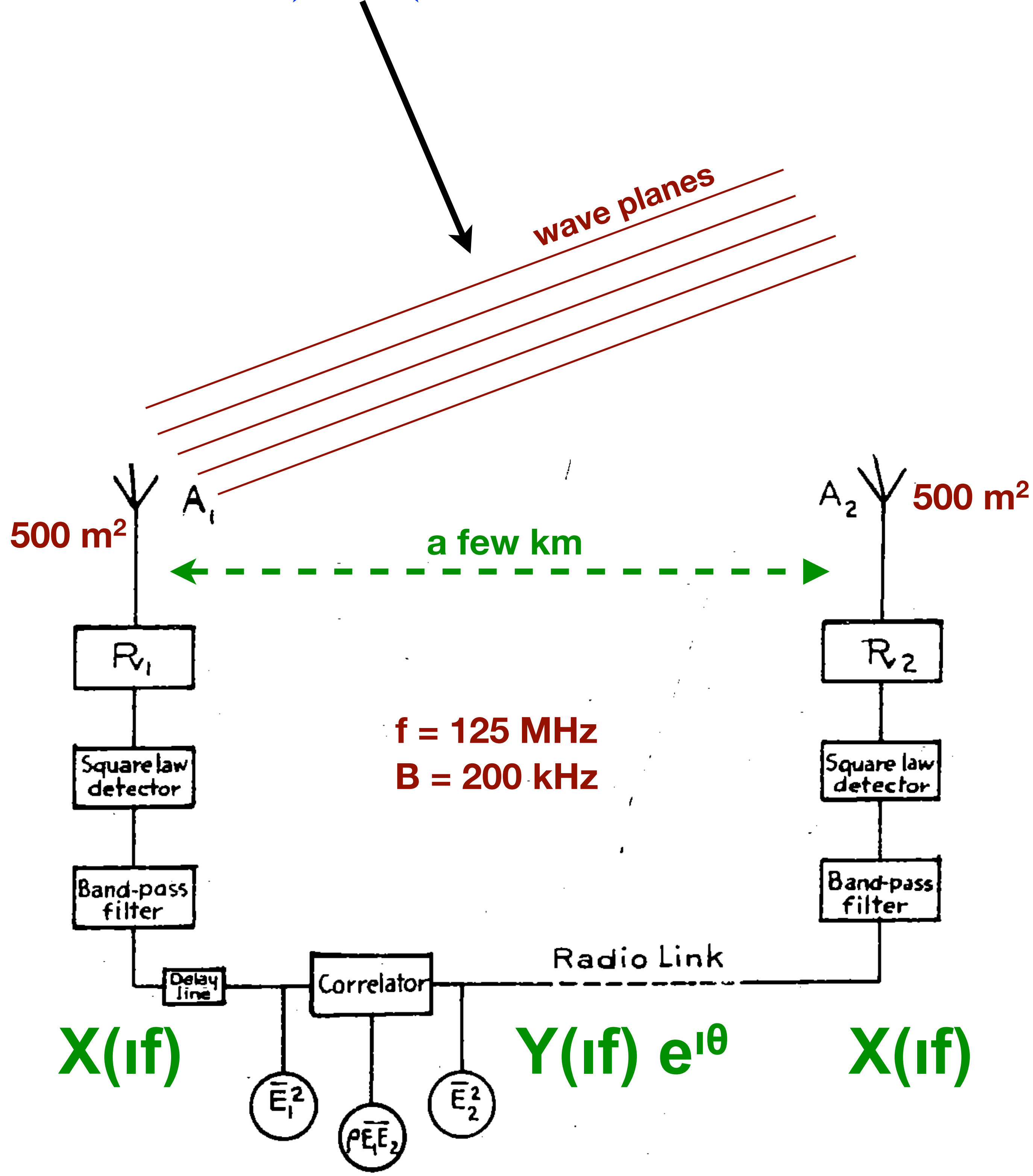
$$\hat{S}_{yx} = \Re\left\{\langle\max(S_{yx}, 0_+)\rangle_m\right\}$$

Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ^2
var{A'} = var{A"} = var{B'} = var{B"} = 1/2, and var{C'} = var{C"} = $\kappa^2/2$

Applications

Radio-astronomy

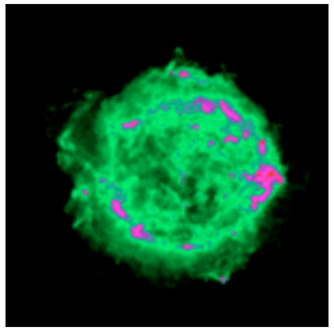




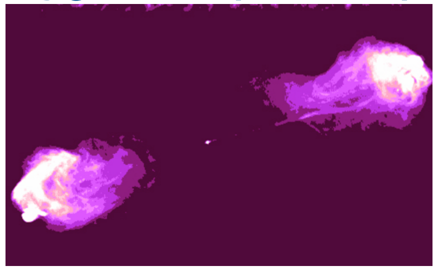
Measurement of the apparent angular size of stellar radio sources

Jodrell Bank, Manchester

Cassiopeia A (Harvard)



Cygnus A (Harvard)

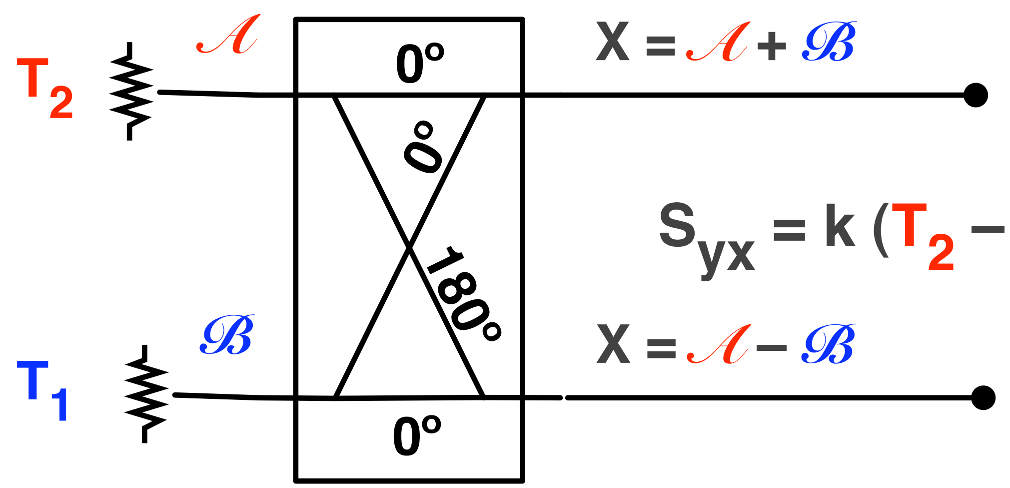


- The radio link breaks the hypothesis of symmetry of the two channels, introducing a phase θ
- The cross spectrum is complex
- The the antenna directivity results from the phase relationships
- The phase of the cross spectrum indicates the direction of the radio source

R. Hanbury Brown & al., Nature 170(4338) p.1061-1063, 20 Dec 1952
R. Hanbury Brown, R. Q. Twiss, Phyl. Mag. ser.7 no.366 p.663-682

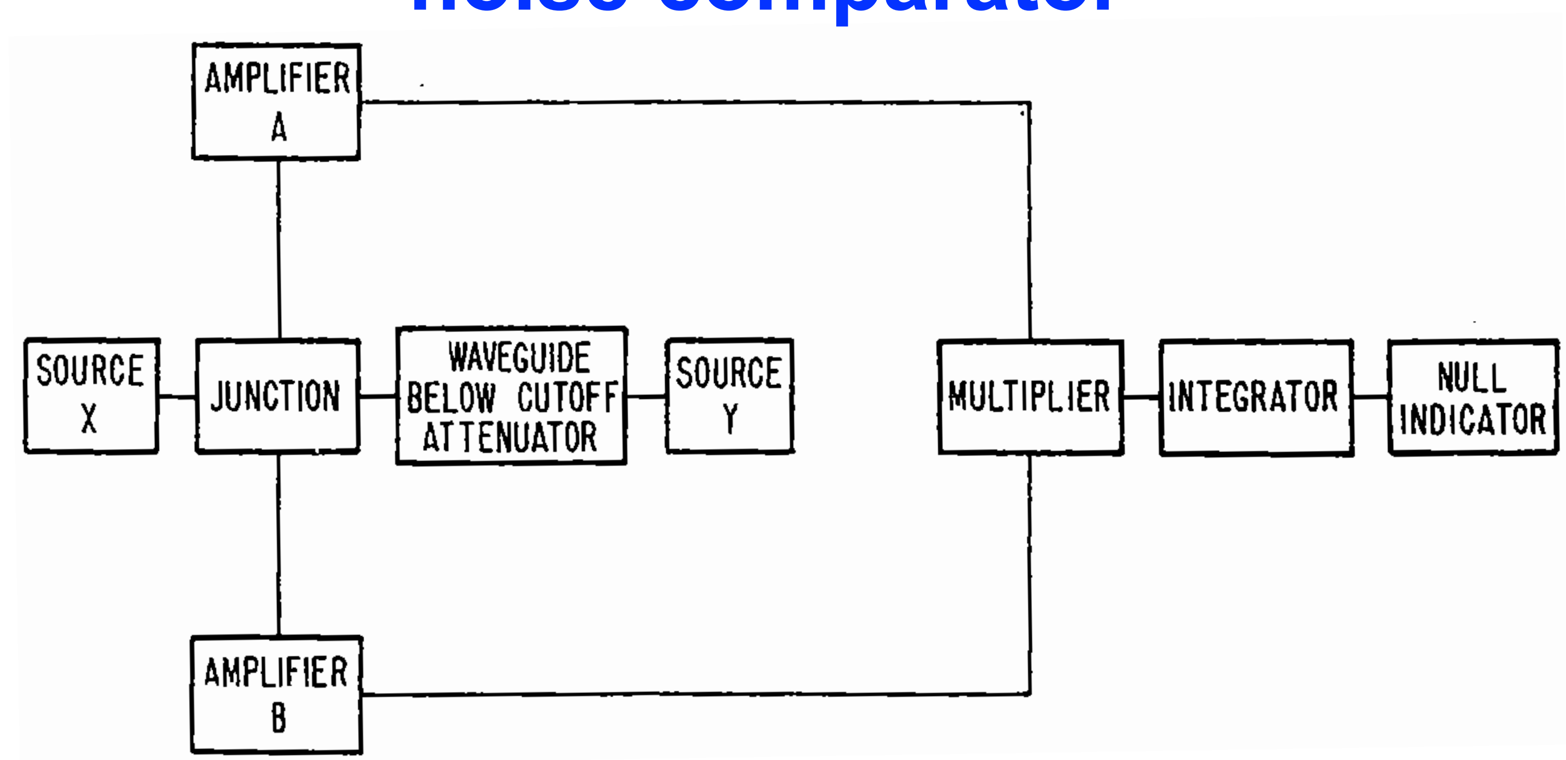
Radiometry -- Johnson thermometry

correlation and anti-correlation



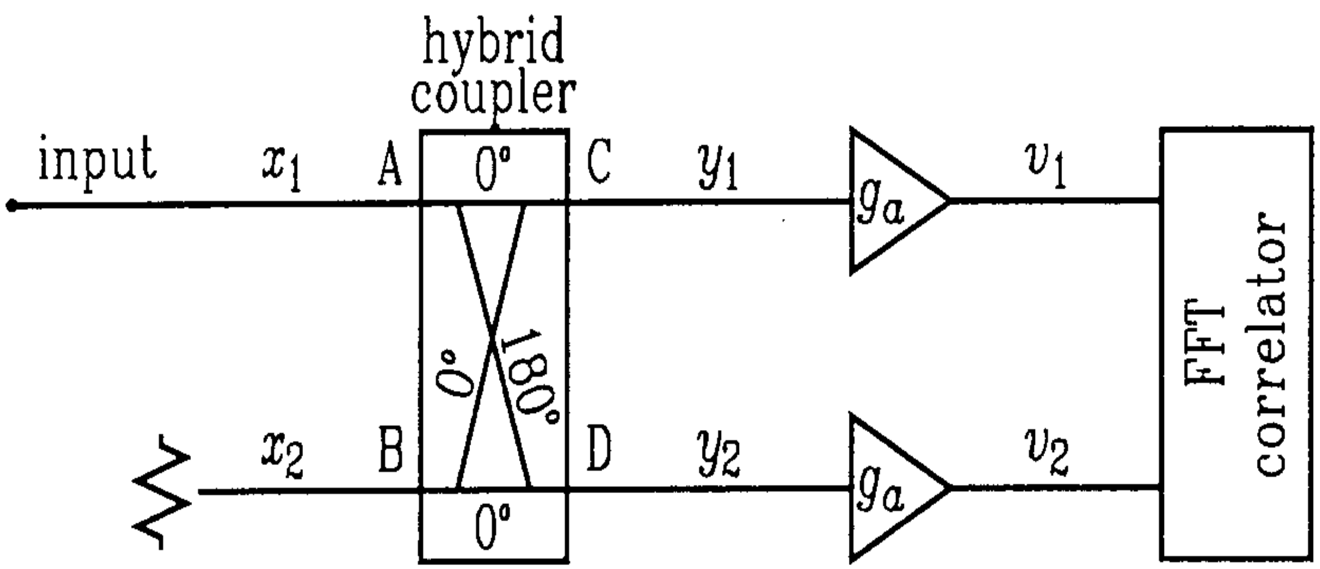
$S_{yx} = k\,(T_2 - T_1)\,/\,2$

noise comparator



C. M. Allred, A precision noise spectral density comparator, J. Res. NBS 66C no.4 p.323-330, Oct-Dec 1962

Thermal noise compensation

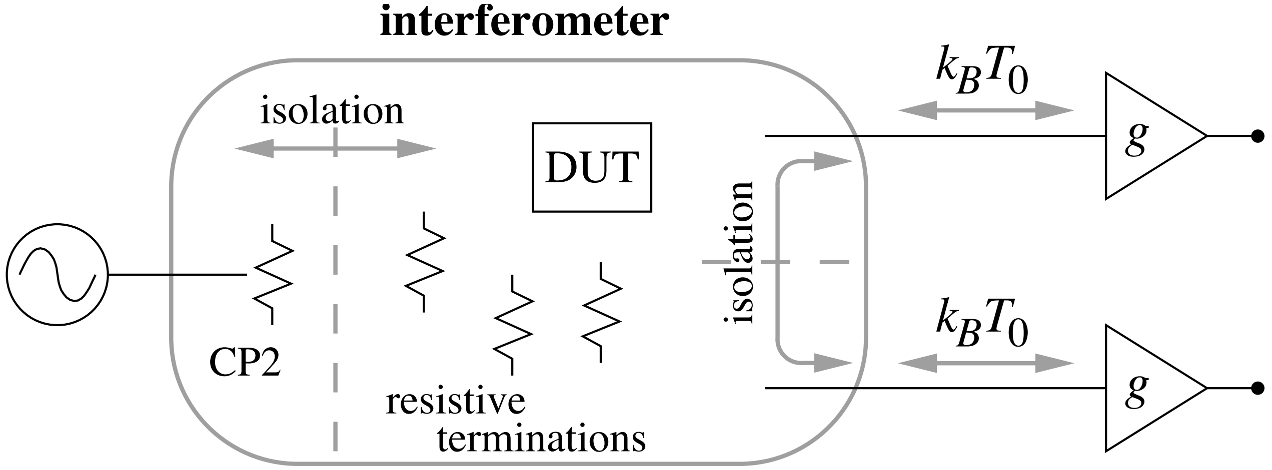


hybrid output

$$y_1(t) = \frac{1}{\sqrt{2}}\,x_2(t) + \frac{1}{\sqrt{2}}\,x_1(t)$$
$$y_2(t) = \frac{1}{\sqrt{2}}\,x_2(t) - \frac{1}{\sqrt{2}}\,x_1(t)$$

correlation

$$\mathcal{R}_{y_1y_2}(\tau) = \lim_{\theta\to\infty}\frac{1}{\theta}\int_\theta y_1(t)\,y_2^*(t-\tau)\,\mathrm{d}t = \frac{1}{2}\,\mathcal{R}_{x_2x_2}(\tau) - \frac{1}{2}\,\mathcal{R}_{x_1x_1}(\tau)$$



Correlation-and-averaging rejects the thermal noise

Fourier transform and thermal noise

$$S_{y_1y_2}(f) = \frac{1}{2}\,S_{x_2}(f) - \frac{1}{2}\,S_{x_1}(f)$$
$$S_{y_1y_2}(f) = \frac{k_B(T_2-T_1)}{2}$$

Noise calibration

thermal noise $S = kT$

shot noise $S = 2qI_{avg}R$

high accuracy of I_{avg} with a dc instrument

Compare shot and thermal noise with a noise bridge

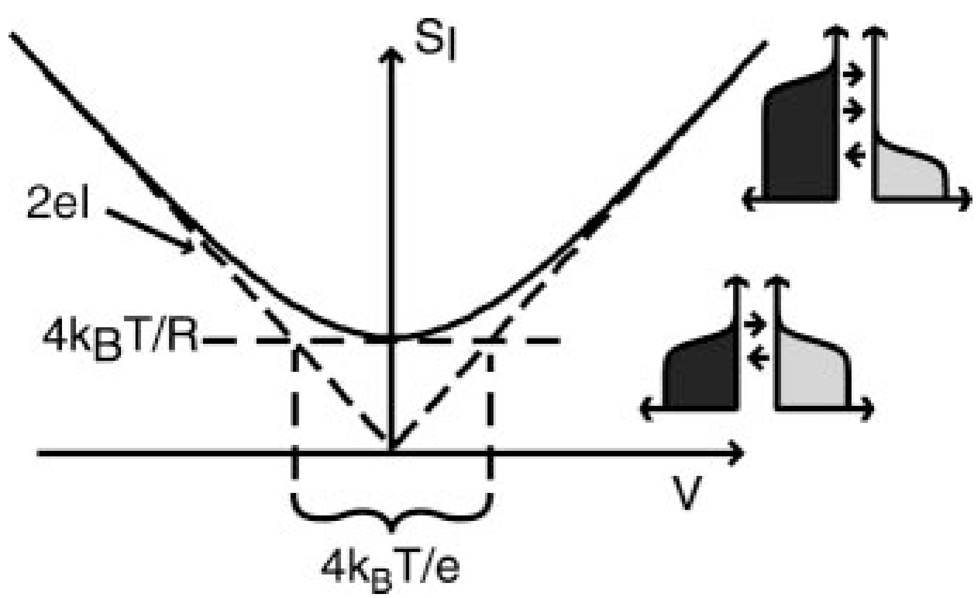


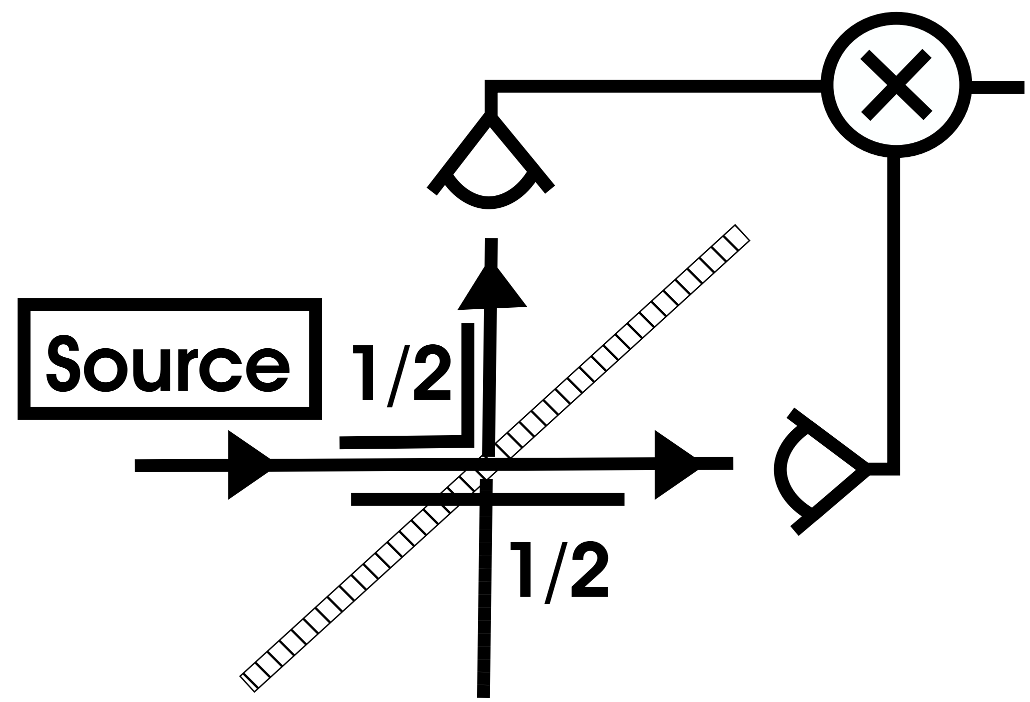
Fig. 1. Theoretical plot of current spectral density of a tunnel junction (Eq. 3) as a function of dc bias voltage. The diagonal dashed lines indicate the shot noise limit, and the horizontal dashed line indicates the Johnson noise limit. The voltage span of the intersection of these limits is $4k_BT/e$ and is indicated by vertical dashed lines. The bottom inset depicts the occupancies of the states in the electrodes in the equilibrium case, and the top inset depicts the out-of-equilibrium case where $eV \gg k_BT$.

This idea could turn into a re-definition of the temperature

In a tunnel junction, theory predicts the amount of shot and thermal noise

L. Spietz & al., Primary electronic thermometry using the shot noise of a tunnel junction, Science 300(20) p. 1929-1932, jun 2003

Hanbury Brown - Twiss effect

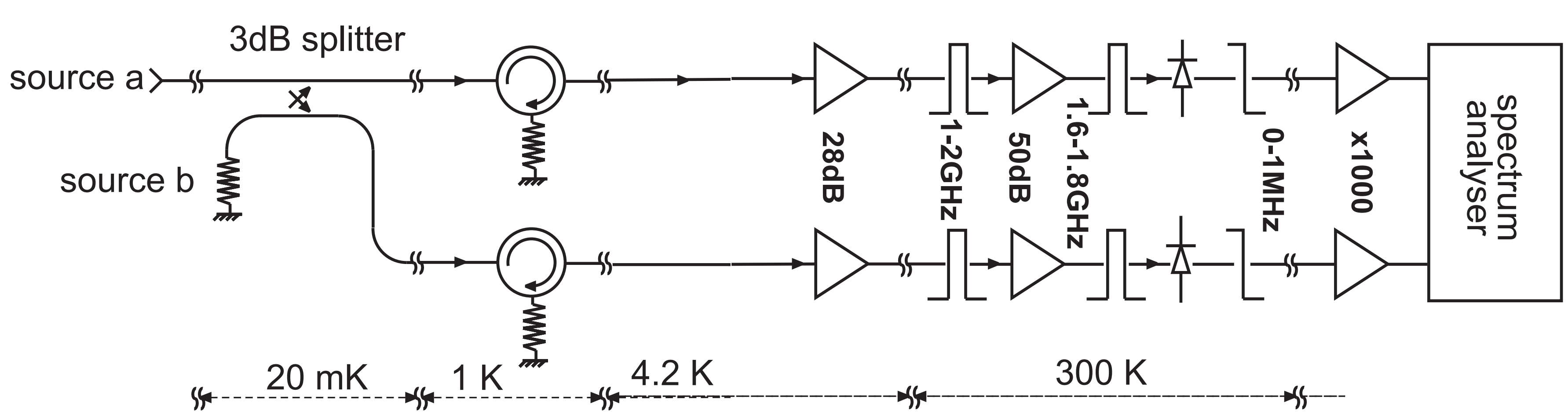


in single-photon regime, anti-correlation shows up

R. Hanbury Brown, R. Q. Twiss, Correlation between photons in two coherent beams of light, Nature 177 (1956) 27-29

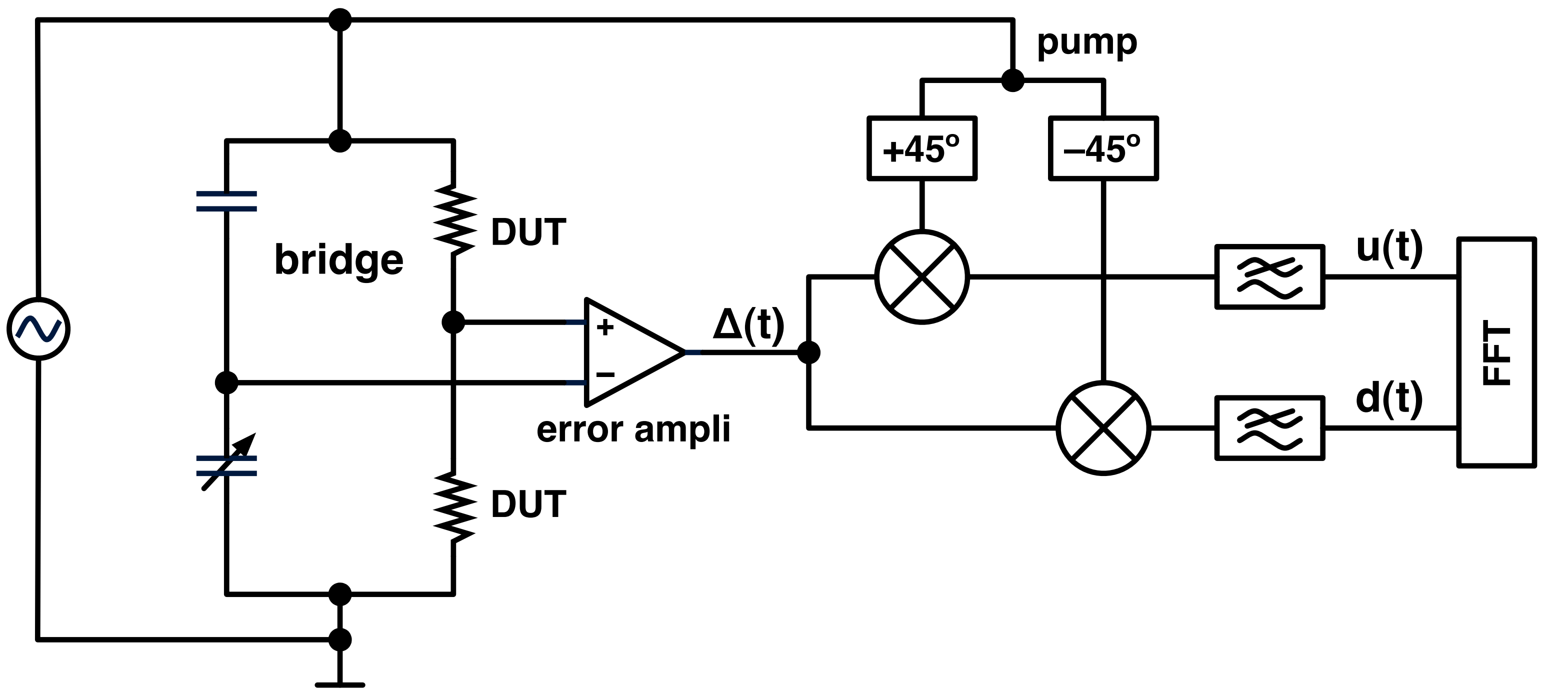
Also observed at microwave frequencies

C. Glattli & al. (2004), PRL 93(5) 056801, Jul 2004



$kT = 2.7\times10^{-25}$ J, $h\nu = 1.12\times10^{-24}$ J, $kT/h\nu = -6.1$ dB

Electromigration in thin films



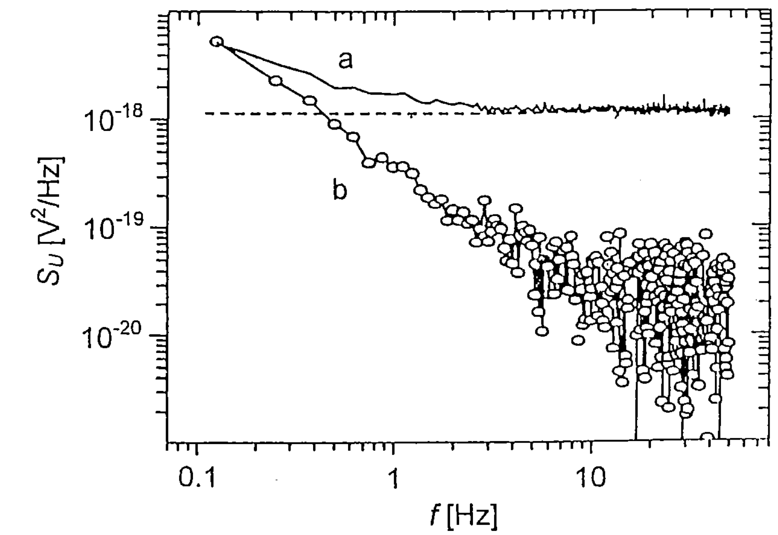
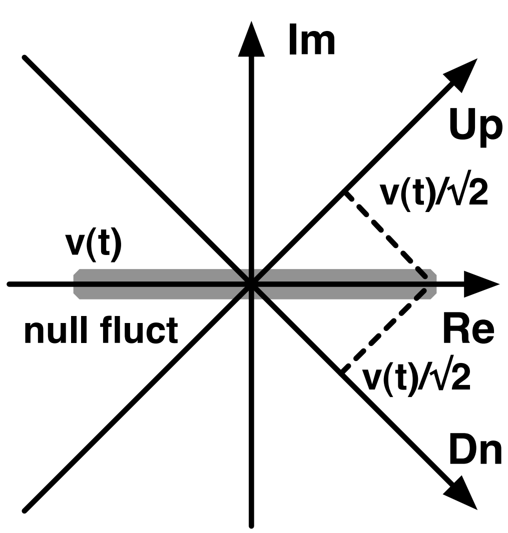


Fig. 1 $1/f$ noise of an $AlSi_{0.01}Cu_{0.002}$ thin film measured at room temperature (a) without and (b) with the phase-sensitive ac correlation technique. The Johnson noise level is indicated by the dashed line.



- Random noise: X' and X" (real and imag part) of a signal are statistically independent
- The detection on two orthogonal axes eliminates the amplifier noise.
  This work with a single amplifier!
- The DUT noise is detected

$$S_{ud}(f) = \frac{1}{2}\left[S_\alpha(f) - S_\varphi(f)\right]$$

A. Seeger, H. Stoll, 1/f noise and defects in thin metal films, proc. ICNF p.162-167, Hong Kong 23-26 aug 1999
RF/microwave version: E. Rubiola, V. Giordano, H. Stoll, IEEE Transact. IM 52(1) pp.182-188, feb 2003

Measurement of noise in semiconductors

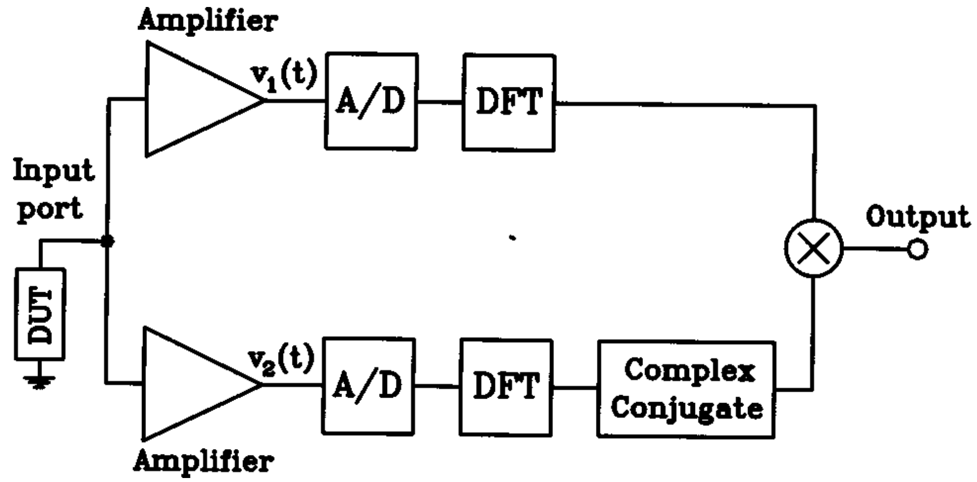


FIG. 2. Schematics of the building blocks of our correlation spectrum analyzer performing the suppression of the uncorrelated input noises by a digital processing of sampled data.

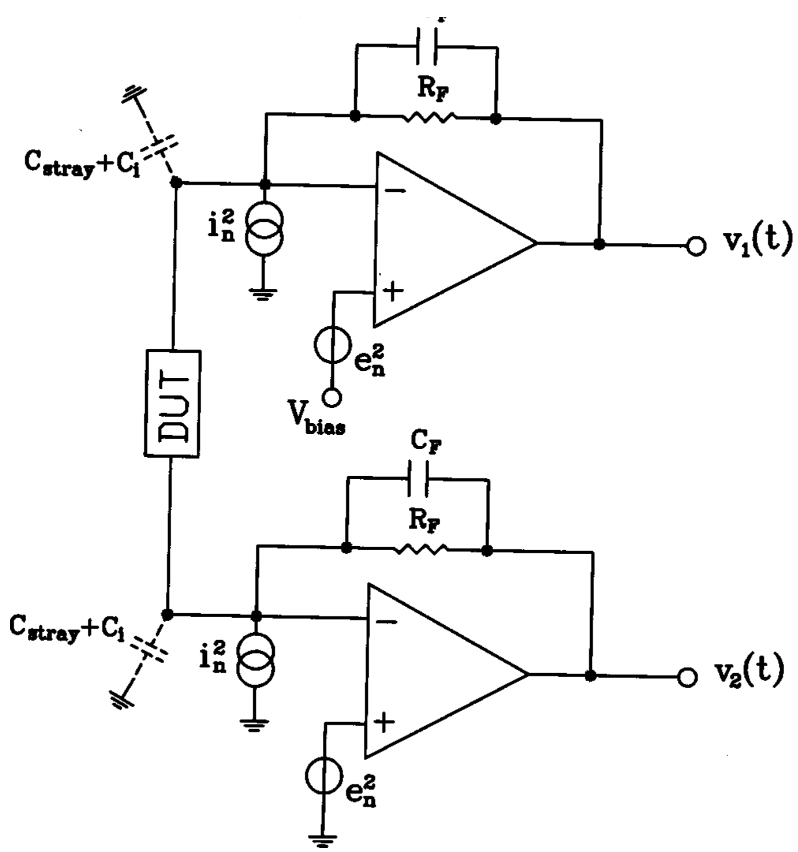


FIG. 3. Schematics of the active test fixture for current noise measurements.

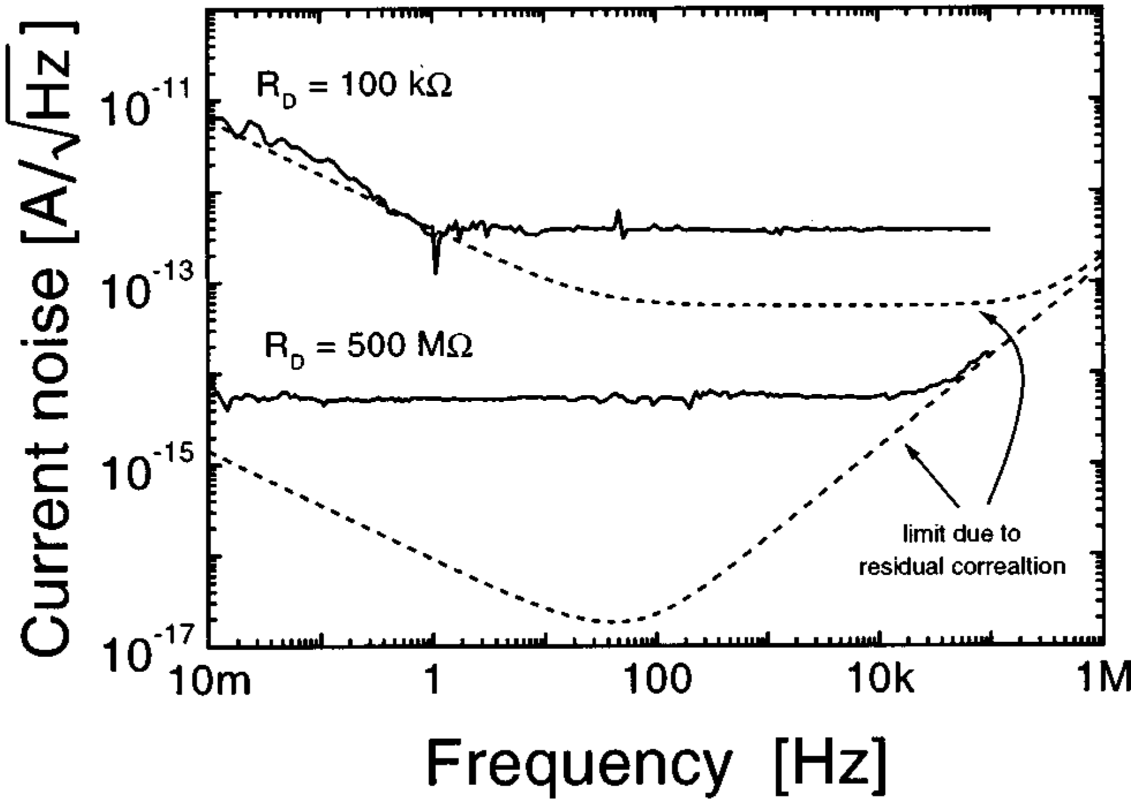
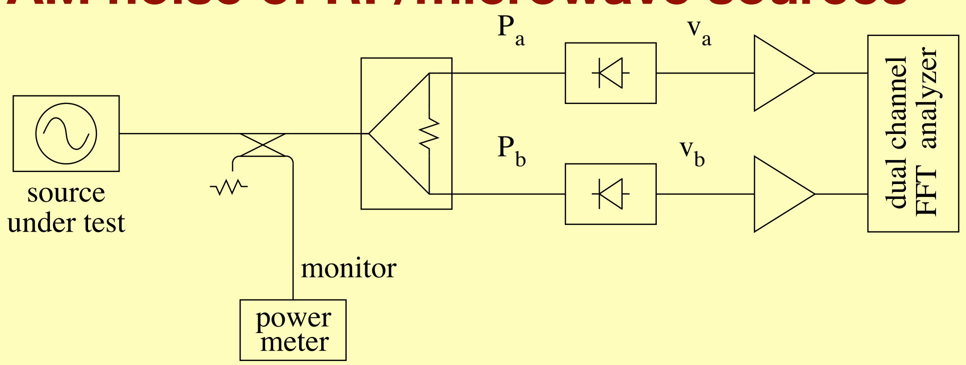


FIG. 9. Experimental frequency spectrum of the current noise from DUT resistances of 100 kΩ and 500 MΩ (continuous line) compared with the limits (dashed line) given by the instrument and set by residual correlated noise components.

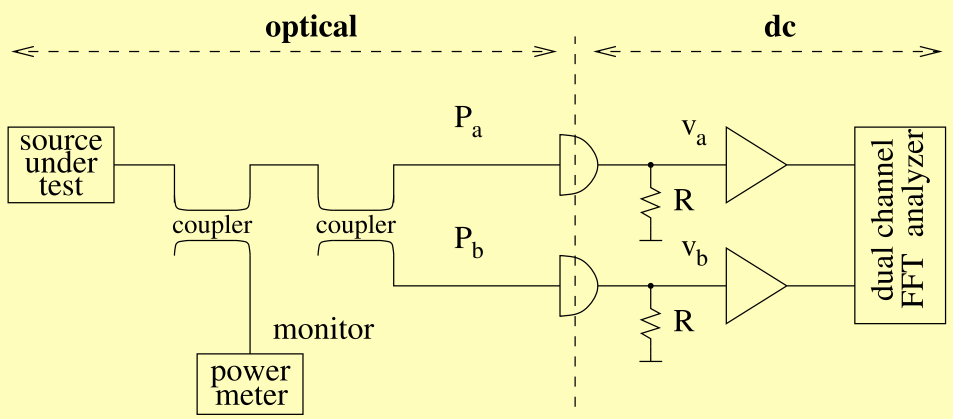
M. Sampietro & al, Rev. Sci. Instrum 70(5) p.2520-2525, may 1999

Amplitude noise & laser RIN

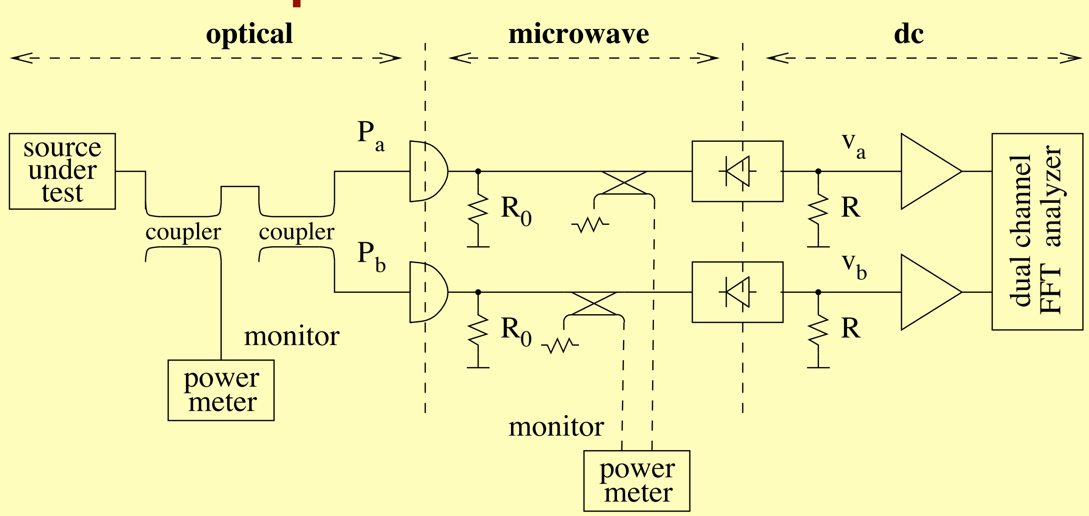
AM noise of RF/microwave sources



Laser RIN

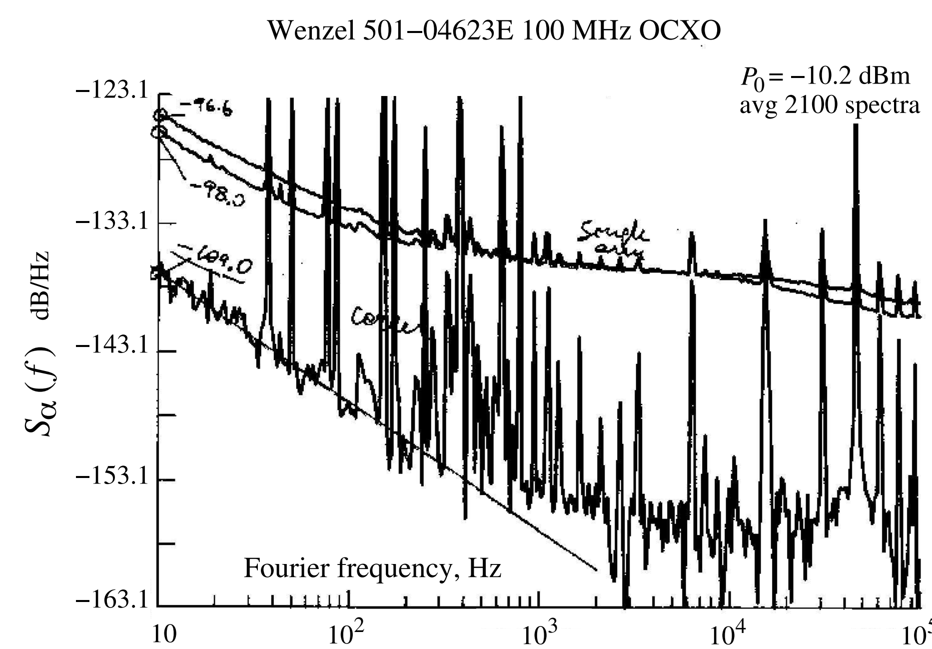


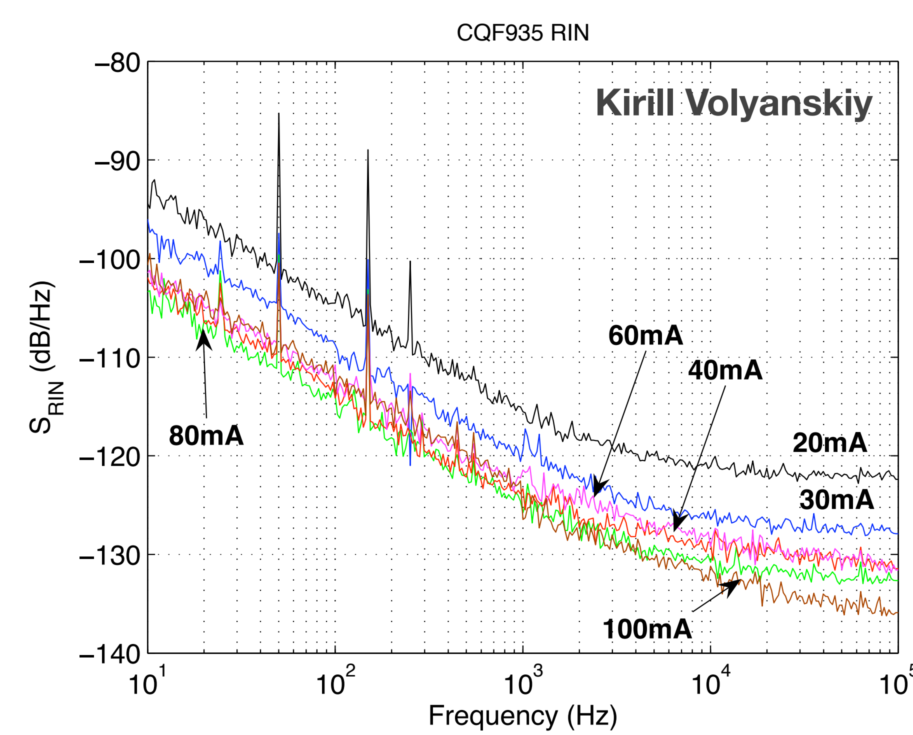
AM noise of photonic RF/microwave sources



E. Rubiola, the measurement of AM noise, dec 2005
arXiv:physics/0512082v1 [physics.ins-det]

- In PM noise measurements, one can validate the instrument by feeding the same signal into the phase detector
- In AM noise this is *not possible* without a lower-noise reference
- Provided the crosstalk was measured otherwise, correlation enables to validate the instrument

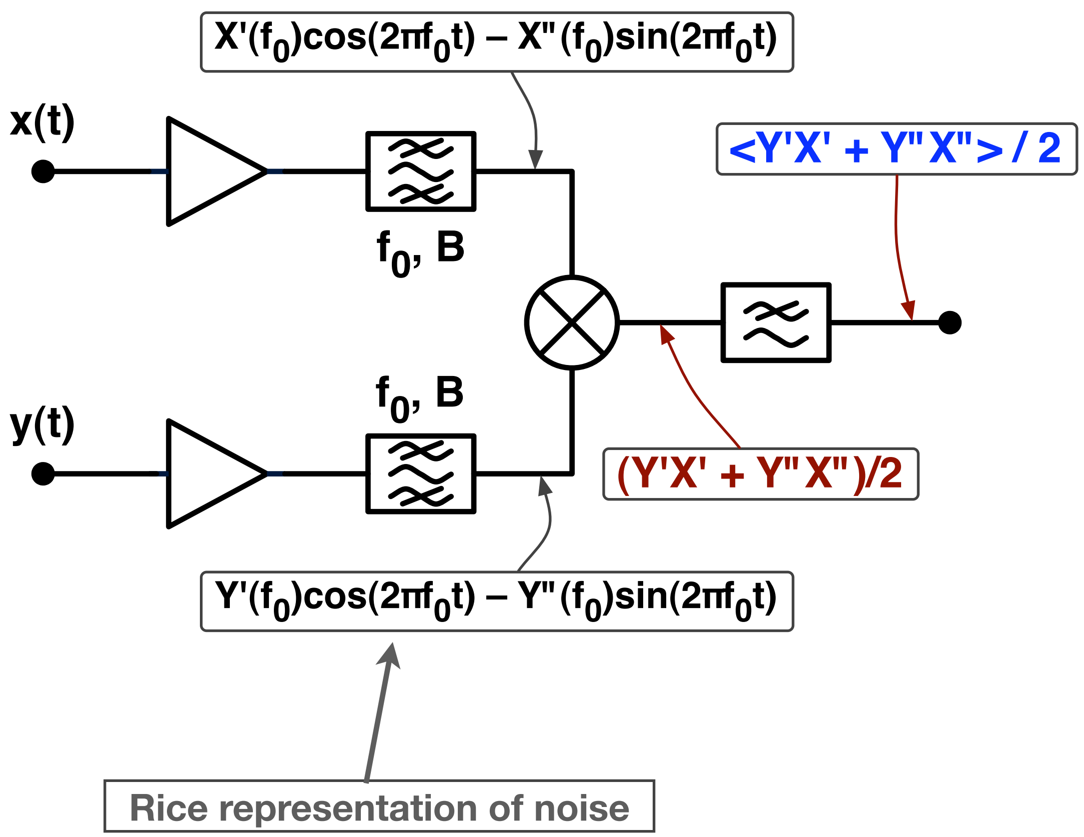




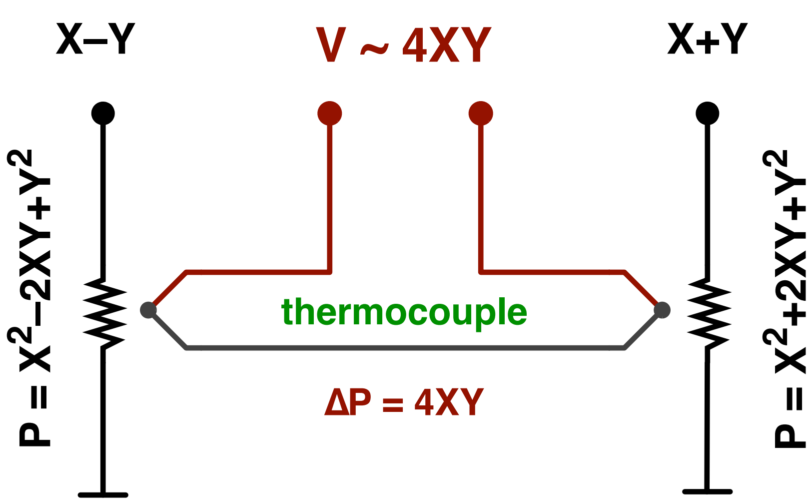
Early implementations

1940-1950 technology

Analog correlator

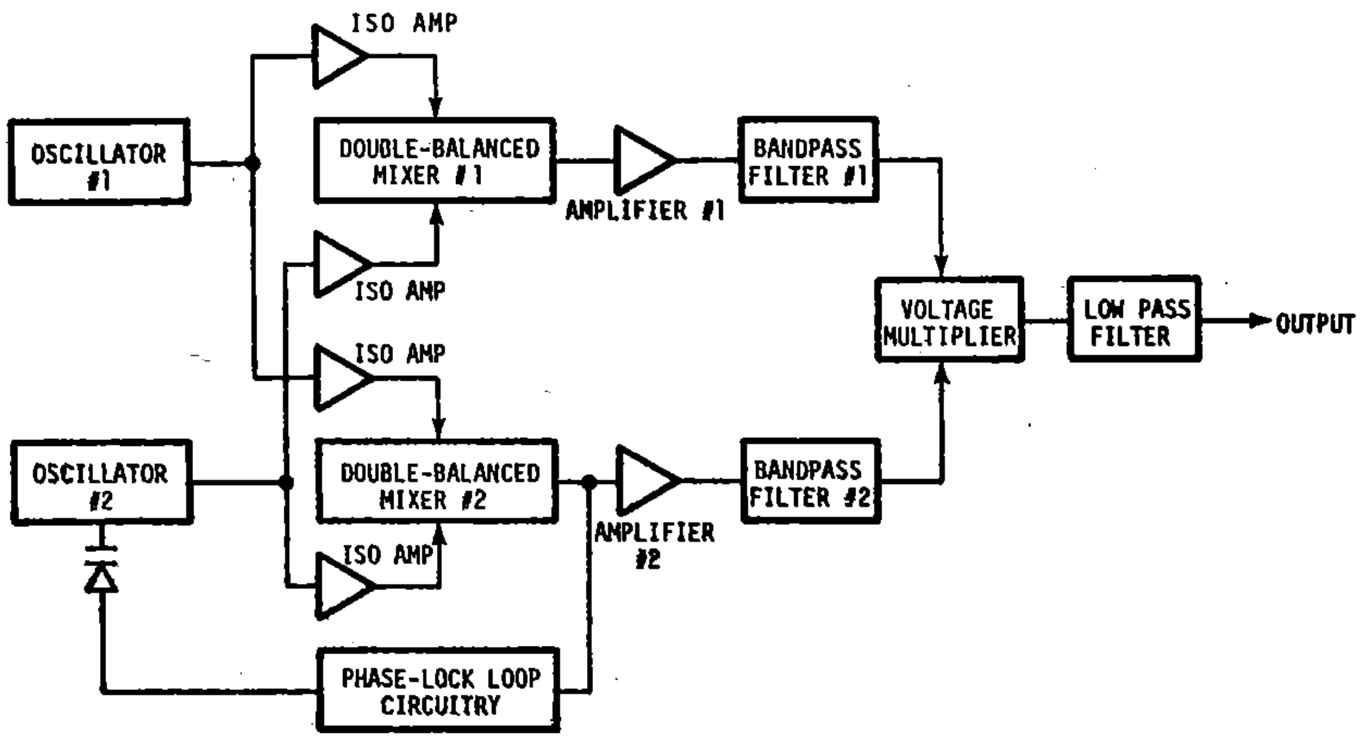


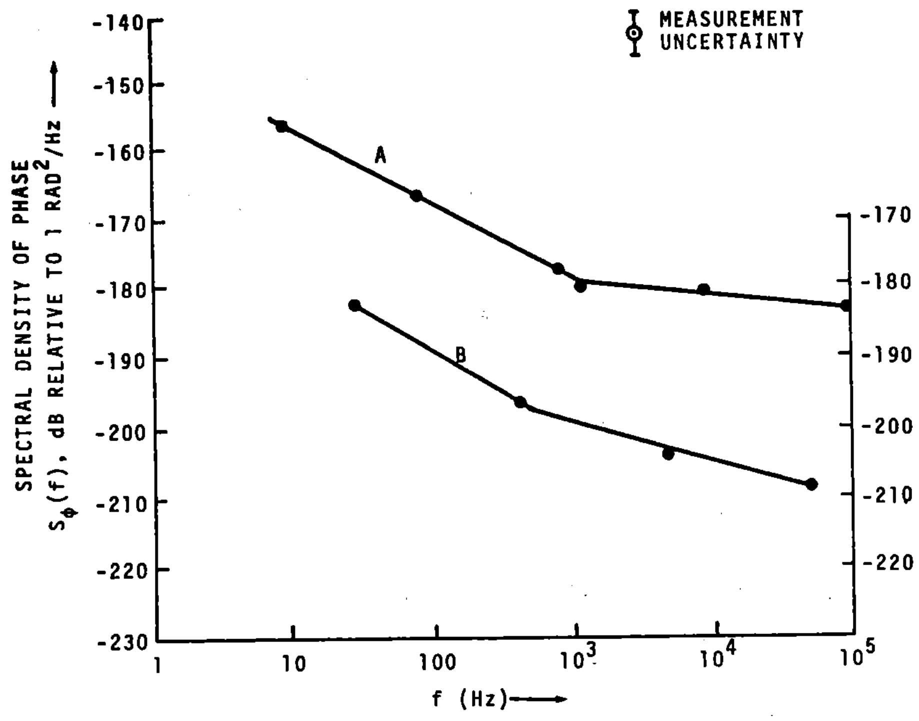
Analog multiplier



Spectral analysis at the single frequency f_0, in the bandwidth B
Need a filter pair for each Fourier frequency

Phase noise measurement

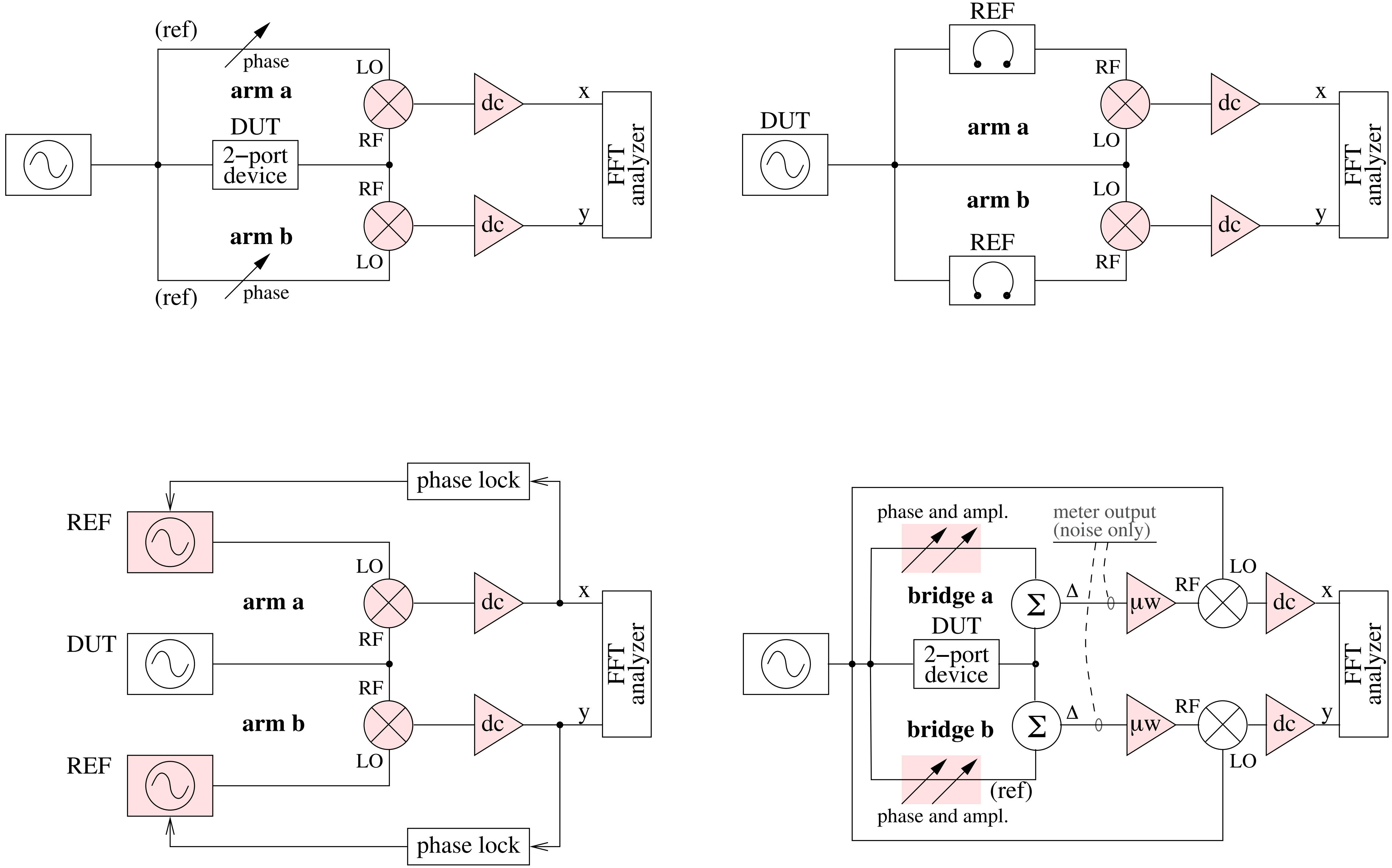




(relatively) large correlation bandwidth provides low noise floor in a reasonable time

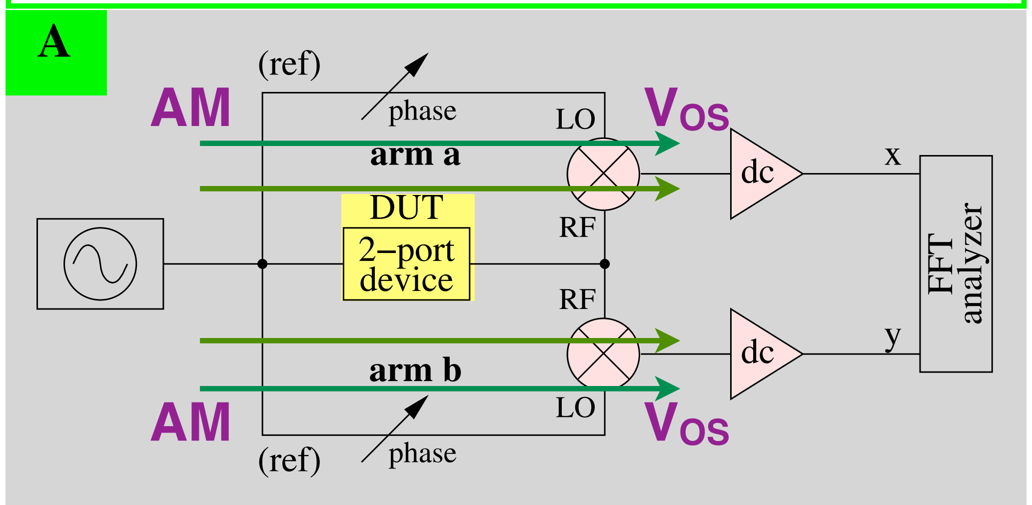
F.L. Walls & al, Proc. 30th FCS pp.269-274, 1976
popular after W. Walls, Proc. 46th FCS pp.257-261, 1992

Phase noise

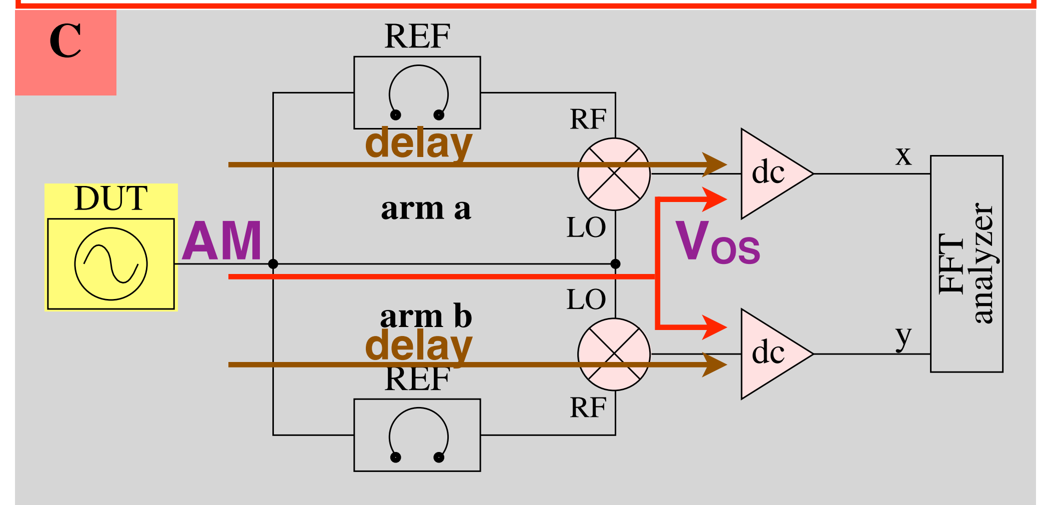

(ref)
phase
arm a
LO
DUT
2–port device
RF
RF
dc
x
FFT analyzer
y
arm b
LO
(ref)
phase
REF
RF
DUT
arm a
LO
LO
arm b
REF
RF
phase lock
REF
LO
arm a
DUT
RF
RF
arm b
REF
LO
phase lock
phase and ampl.
meter output (noise only)
bridge a
Σ
Δ
μw
LO
bridge b
(ref)
phase and ampl.

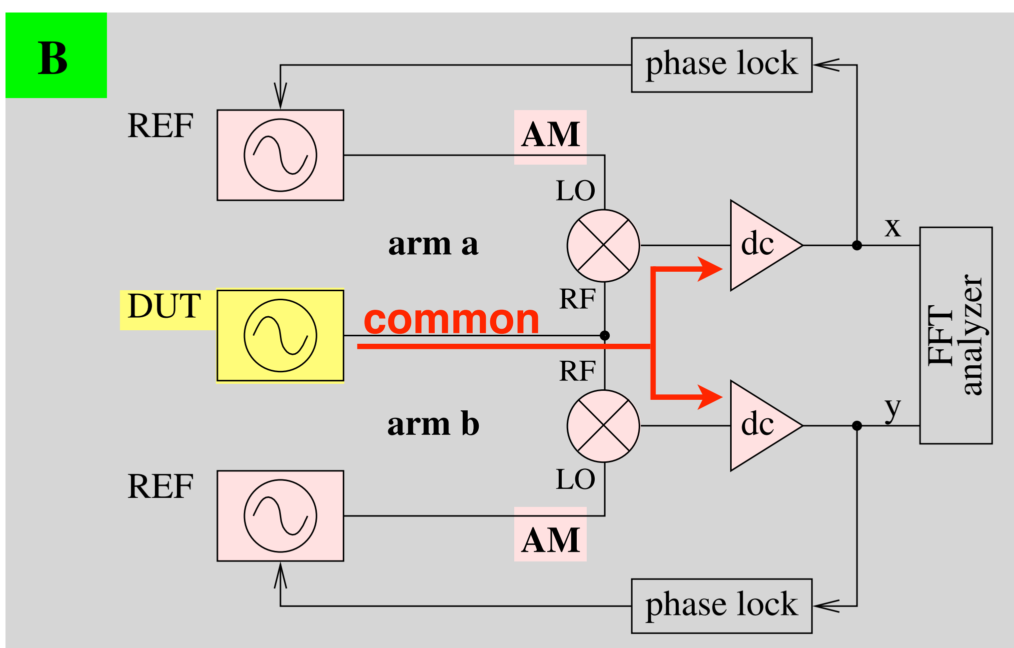
Effect of amplitude noise

Should set both channels at the sweet point, if exists

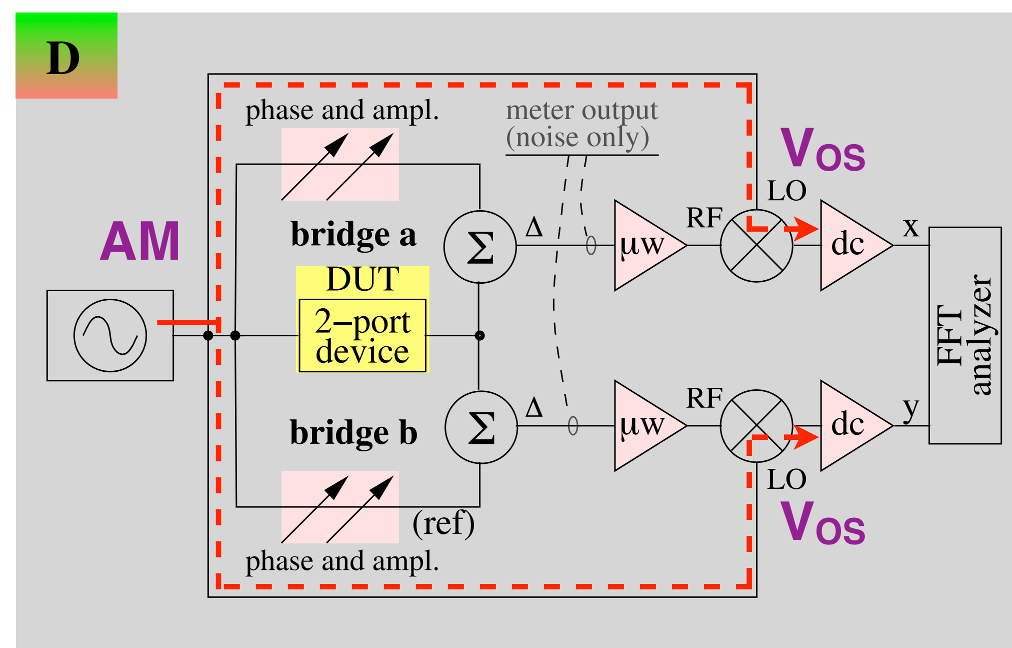


The delay de-correlates the two inputs, so there is no sweet point





Should set both channels at the sweet point of the RF input, if exists, by offsetting the PLL or by biasing the IF

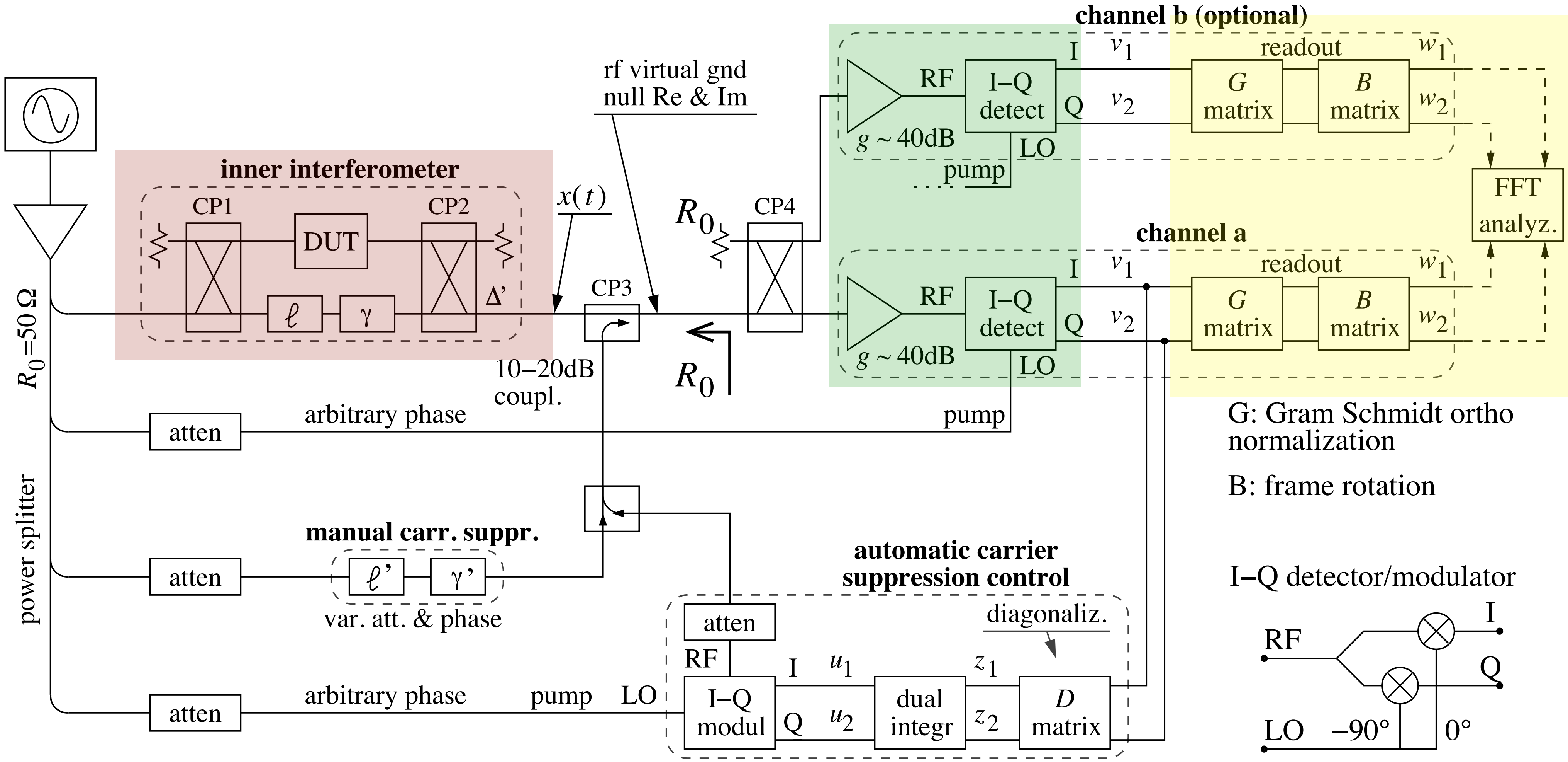


The effect of the AM noise is strongly reduced by the RF amplification

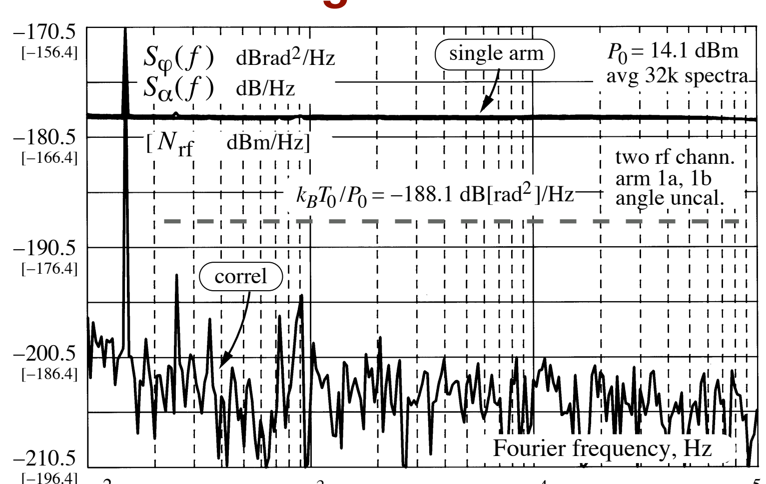
pink: noise rejected by correlation and averaging

E. Rubiola, R. Boudot, IEEE Transact. UFFC 54(5) pp.926-932, may 2007

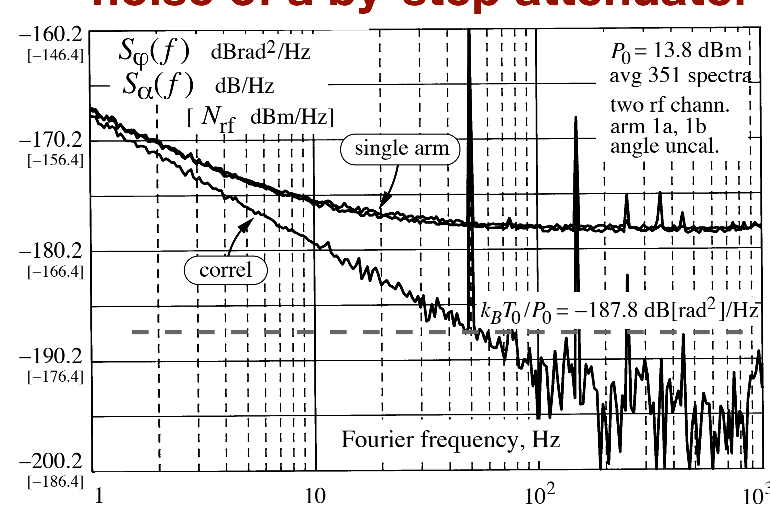
Phase noise measurement



background noise



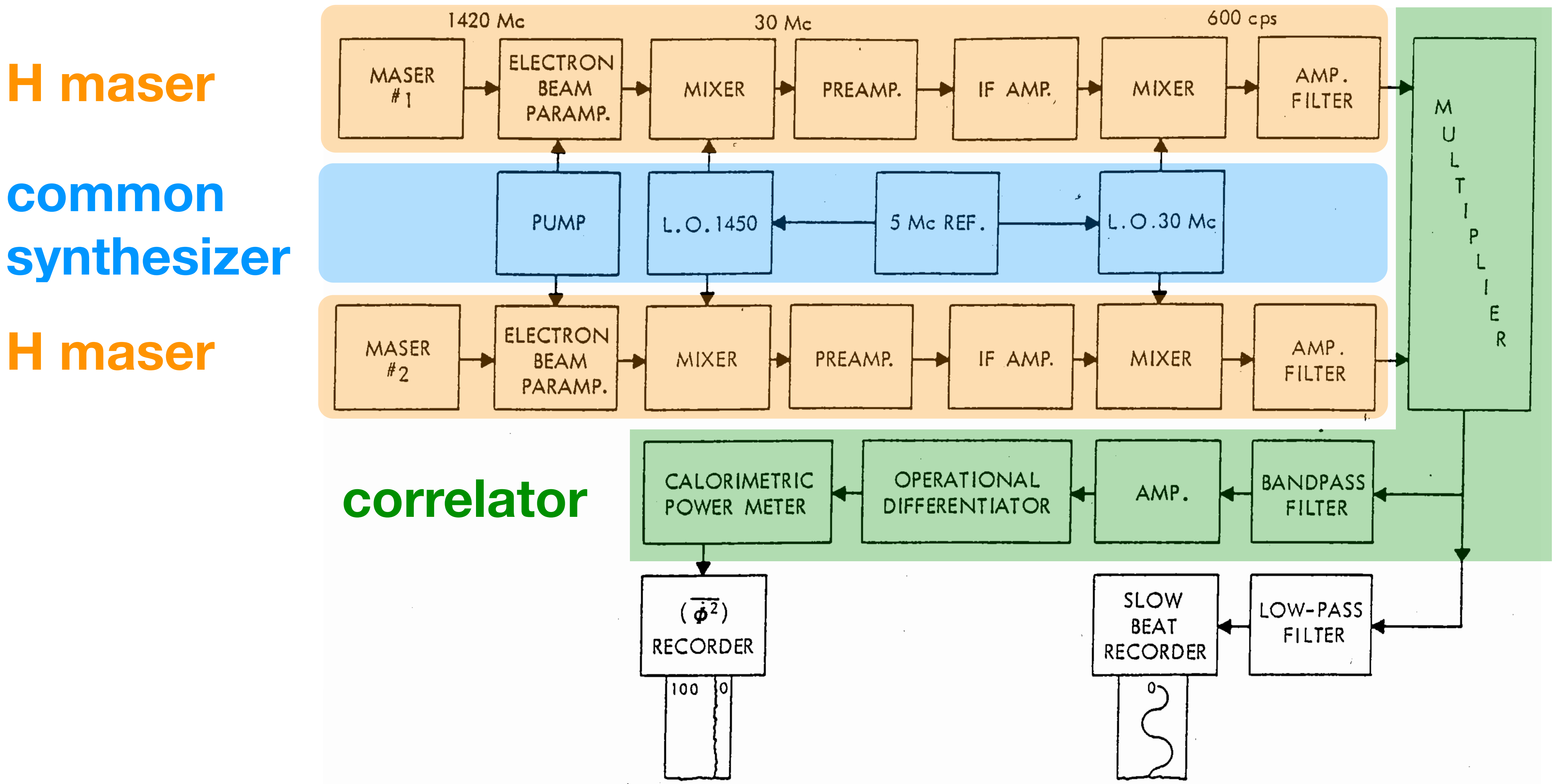
noise of a by-step attenuator



E. Rubiola, V. Giordano, Rev. Sci. Instrum. 71(8) p.3085-3091, aug 2000

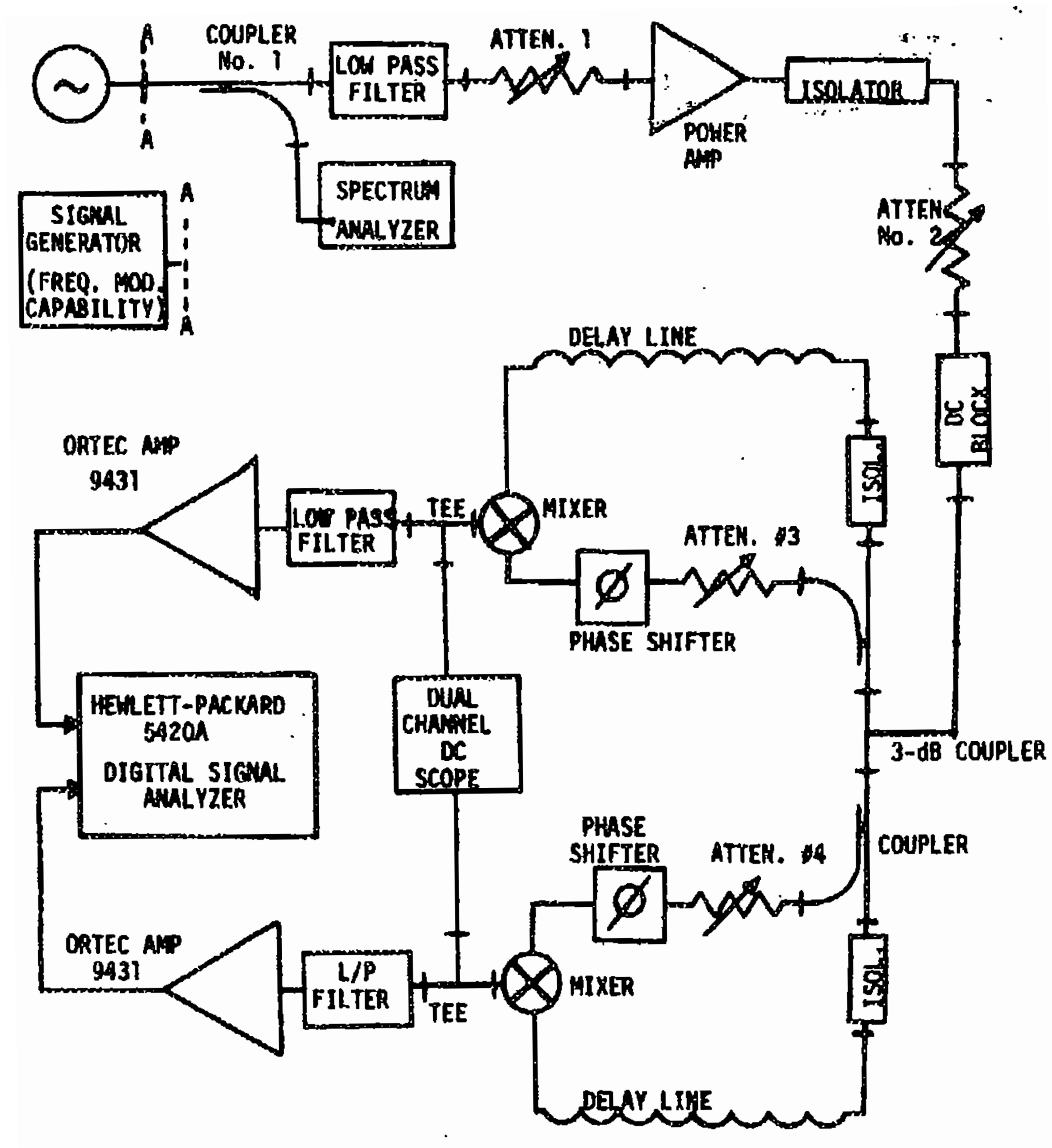
E. Rubiola, V. Giordano, Rev. Sci. Instrum. 73(6) pp.2445-2457, jun 2002

Measurement of H-maser frequency noise



R. F. C. Vessot, Proc. Nasa Symp. on Short Term Frequency Stability p.111-118, Greenbelt, MD, 23-24 Nov 1964

Oscillator phase noise measurement



Original idea:
D. Halford's NBS notebook
F10 p.19-38, apr 1975

First published: A. L. Lance & al, CPEM Digest, 1978

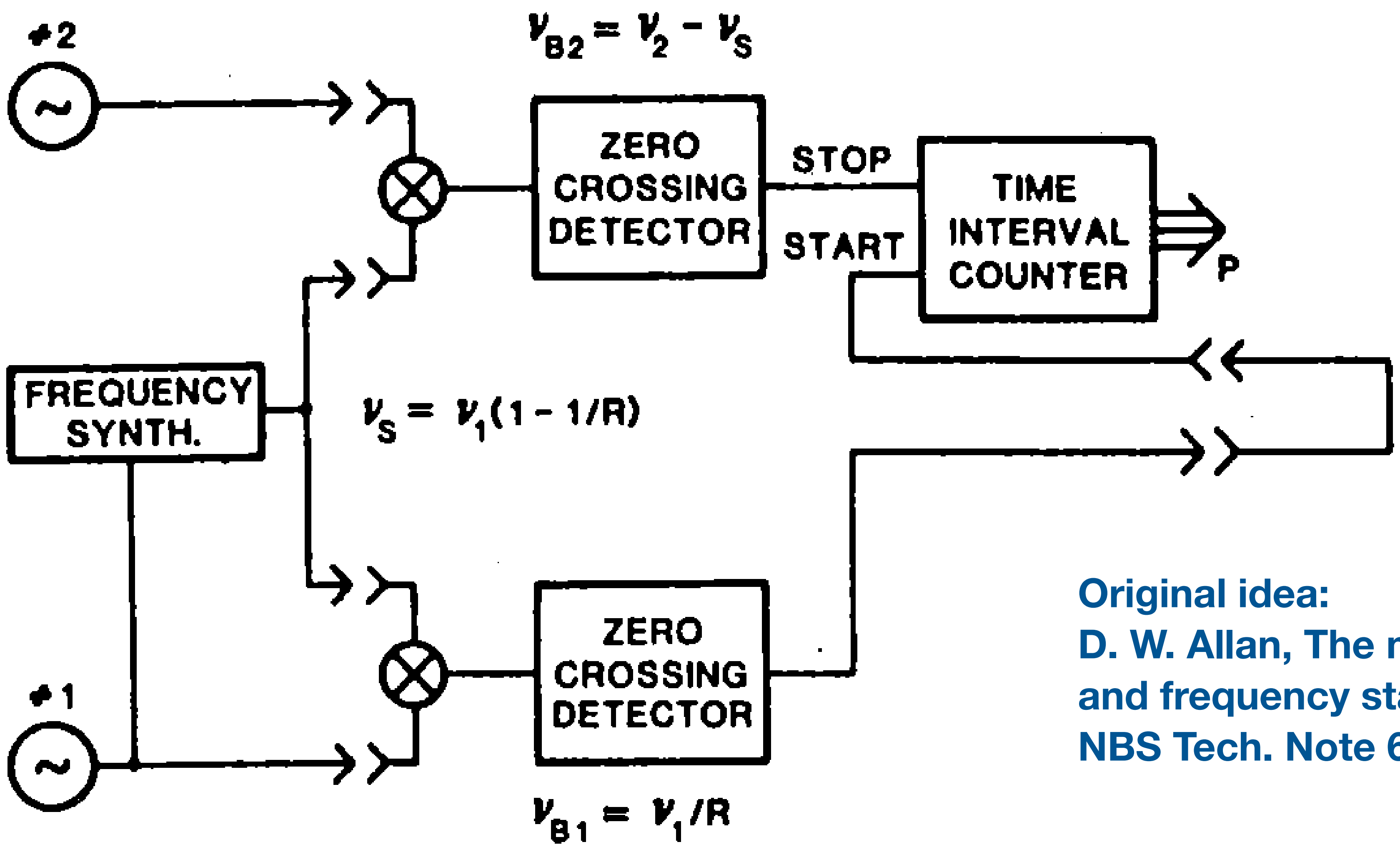
The delay line converts the frequency noise into phase noise

The high loss of the coaxial cable limits the maximum delay

Updated version:
The optical fiber provides long delay with low attenuation (0.2 dB/km or 0.04 dB/μs)

A.L. Lance, W.D. Seal, F. Labaar ISA Transact.21 (4) p.37-84, Apr 1982

Dual-mixer time-domain instrument



Original idea:
D. W. Allan, The measurement of frequency and frequency stability of precision oscillators, NBS Tech. Note 669, 1975

The average process rejects the mixer noise

This scheme is equivalent to the correlation method

S. Stein & al., IEEE Transact. IM 32(1) p.227-230, mar 1983