Phase noise, oscillators etc.

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Outline

- Phase noise & friends
- Amplifier noise
- Correlation
- AM noise
- Bridge (interferometric) noise measurements
- Advanced methods
- Delay-line instrument
- Optical resonators
- Non-linear AM oscillations

home page http://rubiola.org

1 - Phase noise & friends

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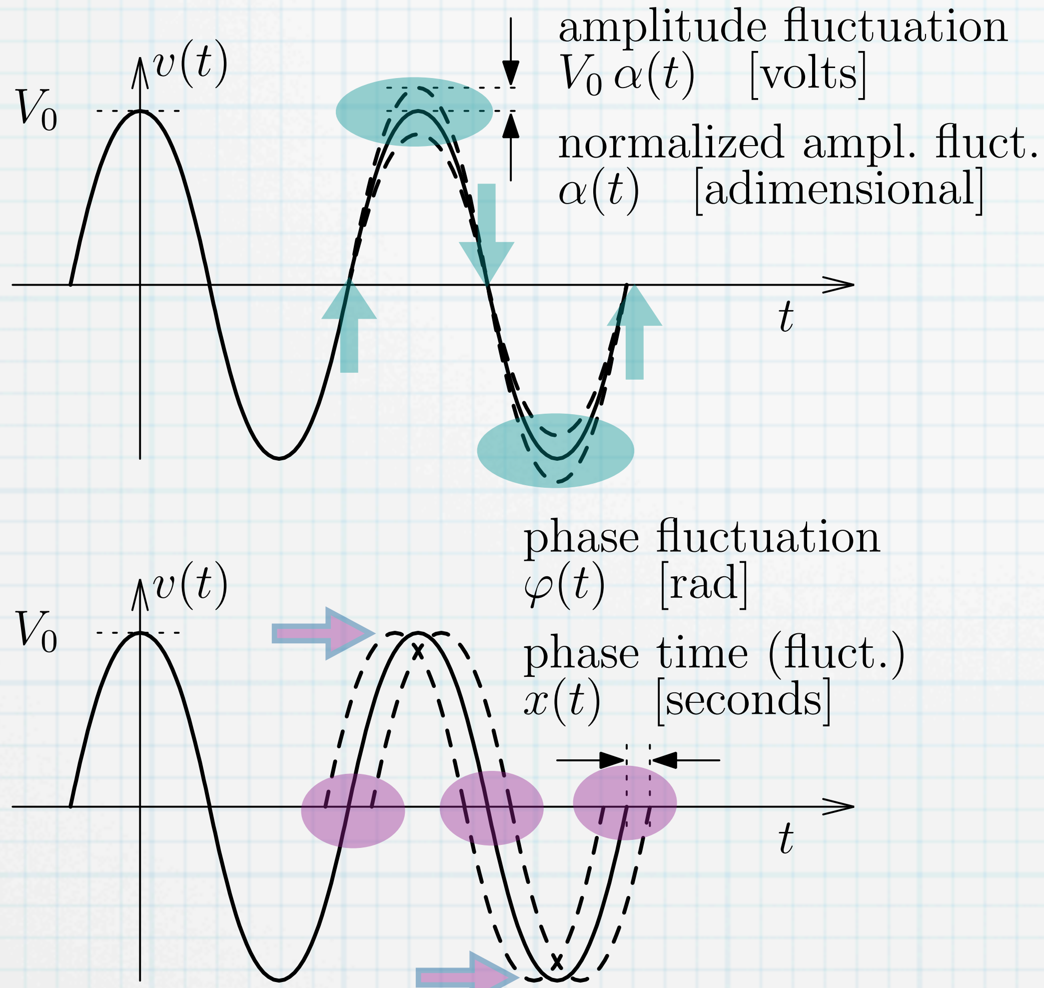
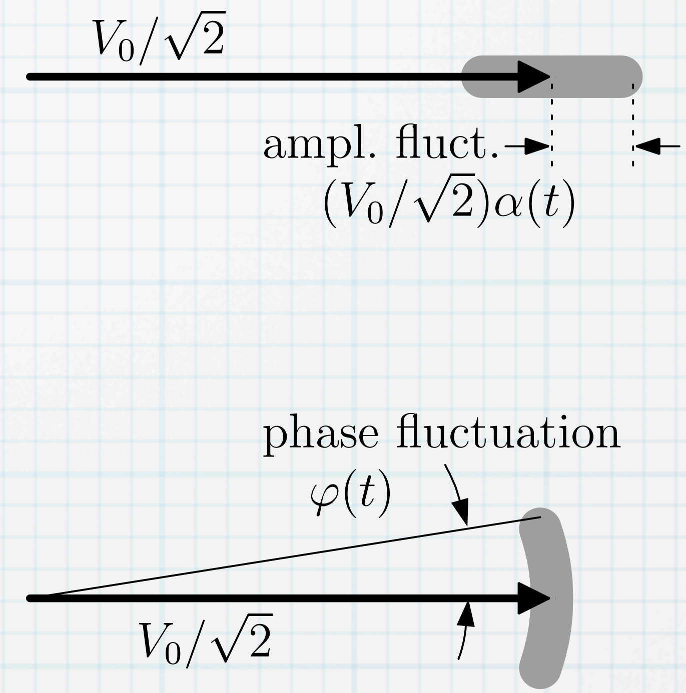
Clock signal affected by noise

Time Domain



Phasor Representation



polar coordinates $$v(t) = V_0\,[1+\alpha(t)]\cos[\omega_0 t + \varphi(t)]$$

Cartesian coordinates $$v(t) = V_0 \cos\omega_0 t + n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t$$

under low noise approximation

$$|n_c(t)| \ll V_0 \quad \text{and} \quad |n_s(t)| \ll V_0$$

It holds that

$$\alpha(t) = \frac{n_c(t)}{V_0} \quad \text{and} \quad \varphi(t) = \frac{n_s(t)}{V_0}$$

Phase noise & friends

random phase fluctuation

$$S_\varphi(f) = \text{PSD of } \varphi(t)$$

power spectral density

it is measured as

$$S_\varphi(f) = \mathbb{E}\left\{\Phi(f)\Phi^*(f)\right\} \quad \text{(expectation)}$$

$$S_\varphi(f) \approx \left\langle \Phi(f)\Phi^*(f)\right\rangle_m \quad \text{(average)}$$

$$\mathcal{L}(f) = \frac{1}{2} S_\varphi(f) \ \text{ dBc}$$

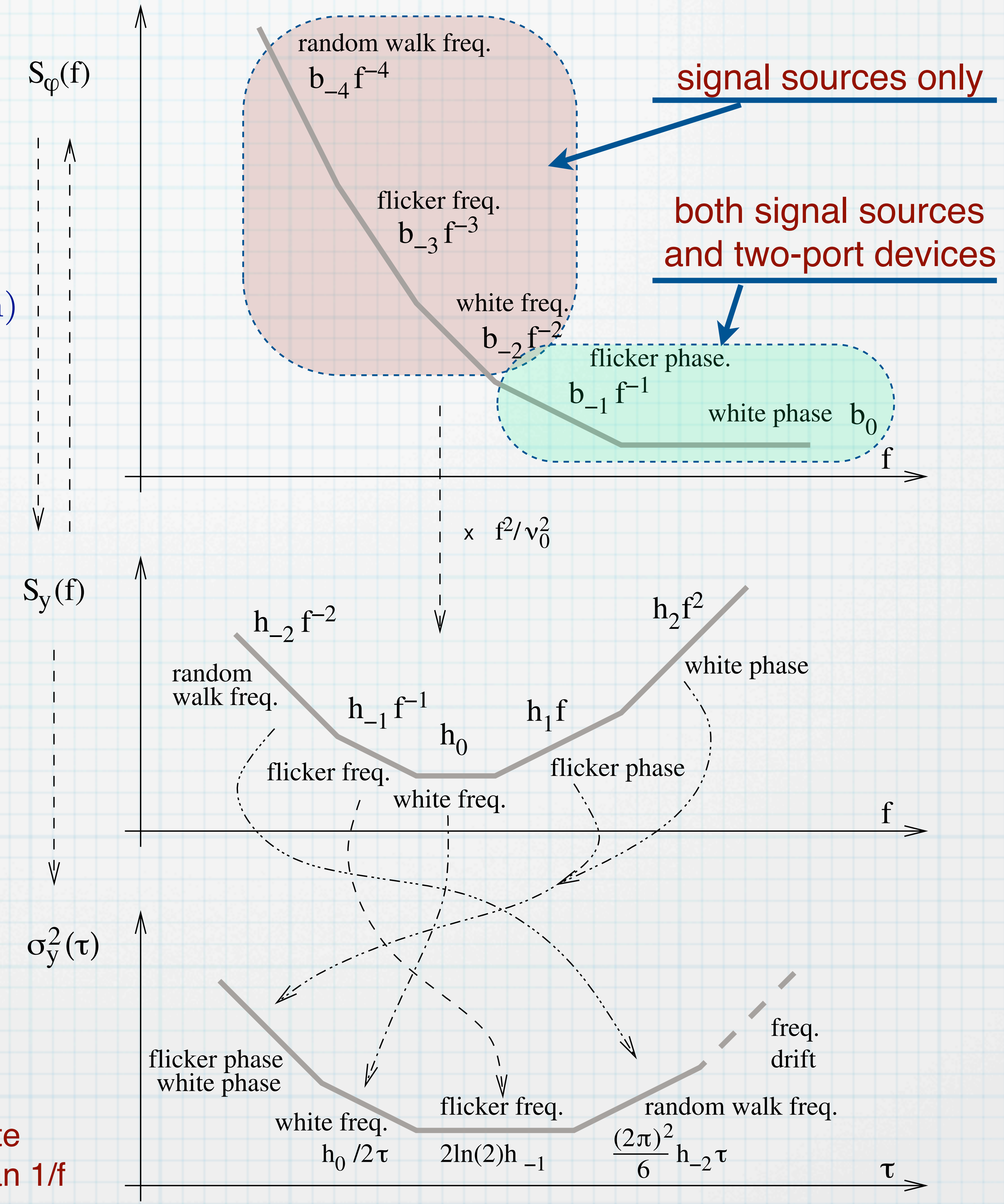
random fractional-frequency fluctuation

$$y(t) = \frac{\dot{\varphi}(t)}{2\pi\nu_0} \quad \Rightarrow \quad S_y = \frac{f^2}{\nu_0^2} S_\varphi(f)$$

Allan variance
(two-sample wavelet-like variance)

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{y}_{k+1} - \overline{y}_k\right]^2\right\} .$$

approaches a half-octave bandpass filter (for white noise), hence it converges for processes steeper than 1/f



Relationships between spectra and variances

noise type	$S_\varphi(f)$	$S_y(f)$	$S_\varphi \leftrightarrow S_y$	$\sigma_y^2(\tau)$	mod $\sigma_y^2(\tau)$
white PM	b_0	$h_2 f^2$	$h_2 = \dfrac{b_0}{\nu_0^2}$	$\dfrac{3 f_H h_2}{(2\pi)^2}\tau^{-2}$ $2\pi\tau f_H \gg 1$	$\dfrac{3 f_H \tau_0 h_2}{(2\pi)^2}\,\tau^{-3}$
flicker PM	$b_{-1} f^{-1}$	$h_1 f$	$h_1 = \dfrac{b_{-1}}{\nu_0^2}$	$[1.038+3\ln(2\pi f_H \tau)]\,\dfrac{h_1}{(2\pi)^2}\,\tau^{-2}$	$0.084\,h_1\,\tau^{-2}$ $n \gg 1$
white FM	$b_{-2} f^{-2}$	h_0	$h_0 = \dfrac{b_{-2}}{\nu_0^2}$	$\dfrac{1}{2} h_0\,\tau^{-1}$	$\dfrac{1}{4} h_0\,\tau^{-1}$
flicker FM	$b_{-3} f^{-3}$	$h_{-1} f^{-1}$	$h_{-1} = \dfrac{b_{-3}}{\nu_0^2}$	$2\ln(2)\;h_{-1}$	$\dfrac{27}{20}\ln(2)\;h_{-1}$
random walk FM	$b_{-4} f^{-4}$	$h_{-2} f^{-2}$	$h_{-2} = \dfrac{b_{-4}}{\nu_0^2}$	$\dfrac{(2\pi)^2}{6} h_{-2}\tau$	$0.824\,\dfrac{(2\pi)^2}{6} h_{-2}\,\tau$
linear frequency drift $\dot{y}$				$\dfrac{1}{2}\,(\dot{y})^2\,\tau^2$	$\dfrac{1}{2}\,(\dot{y})^2\,\tau^2$

f_H is the high cutoff frequency, needed for the noise power to be finite.

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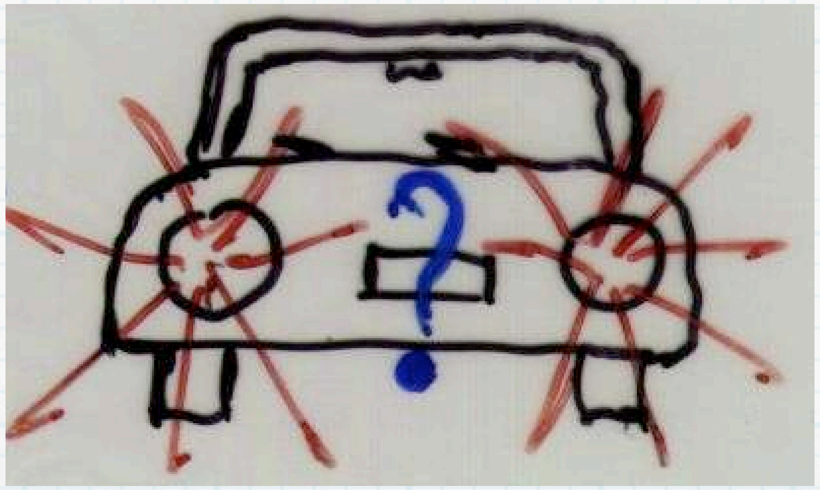
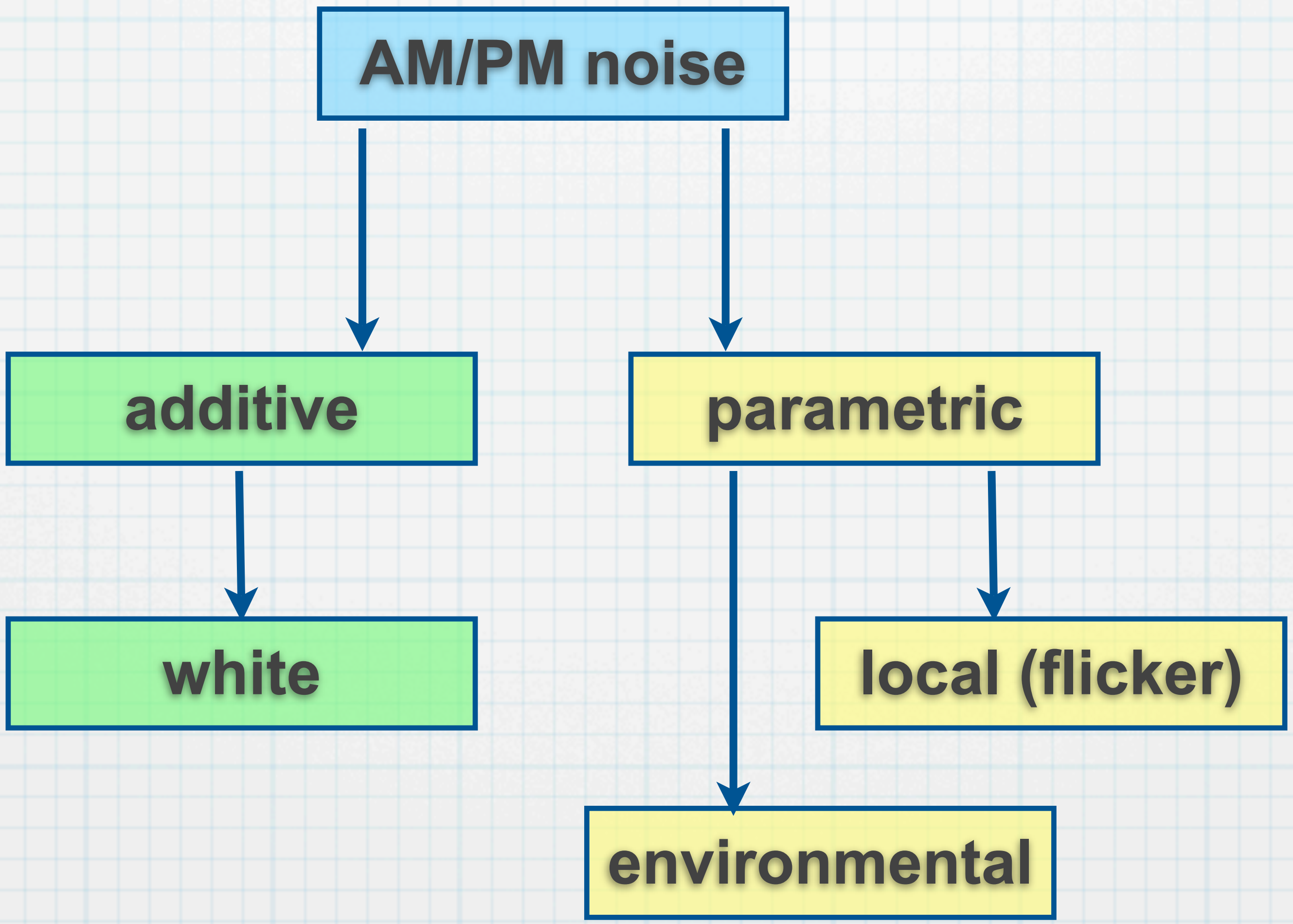
Basic problem: how can we measure a low random signal (noise sidebands) close to a strong dazzling carrier?



solution(s): suppress the carrier and measure the noise

convolution (low-pass)	$s(t) * h_{lp}(t)$	distorsiometer, audio-frequency instruments
time-domain product	$s(t) \times r(t - T/4)$	traditional instruments for phase-noise measurement (saturated mixer)
vector difference	$s(t) - r(t)$	bridge (interferometric) instruments

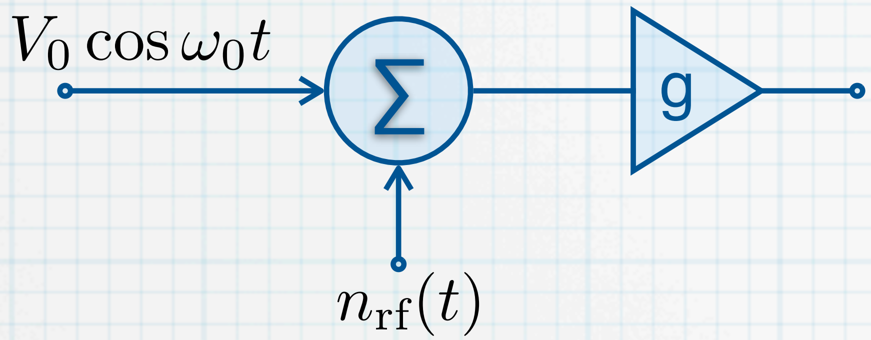
2 - Amplifier noise

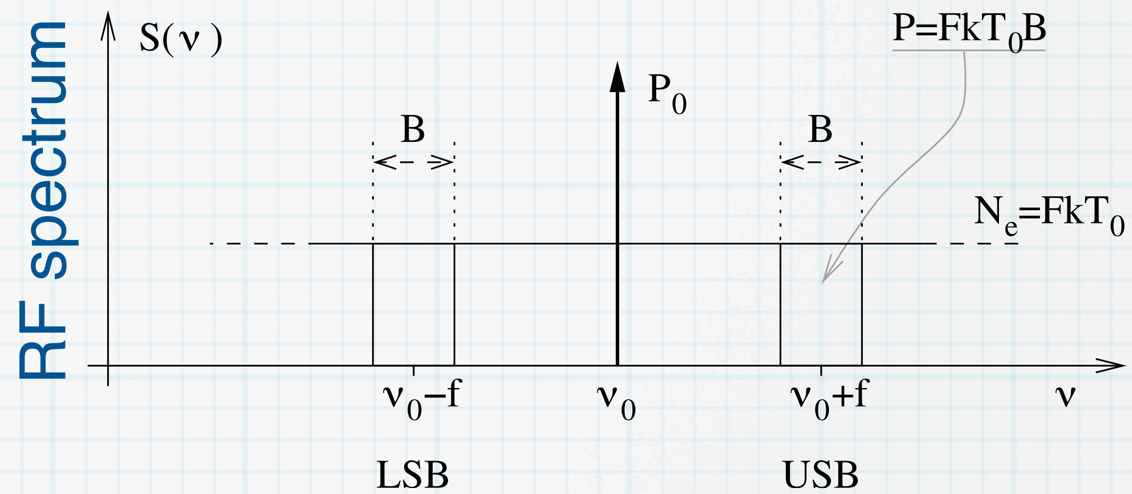


E. Rubiola, *Phase Noise and Frequency Stability in Oscillators*, Cambridge 2008, ISBN13 9780521886772

Amplifier white noise

Noise figure F, Input power P_0

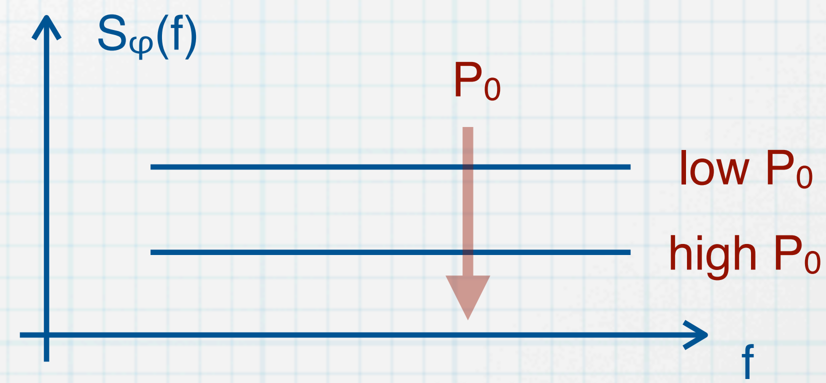




power law

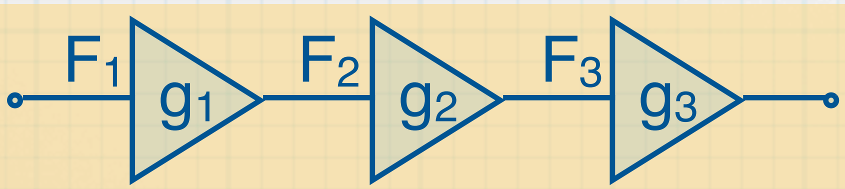
$$S_\varphi = \sum_{i=-4}^{0} b_i f^i$$

white phase noise $$b_0 = \frac{FkT_0}{P_0}$$



Cascaded amplifiers (Friis formula)

The (phase) noise is chiefly that of the 1st stage

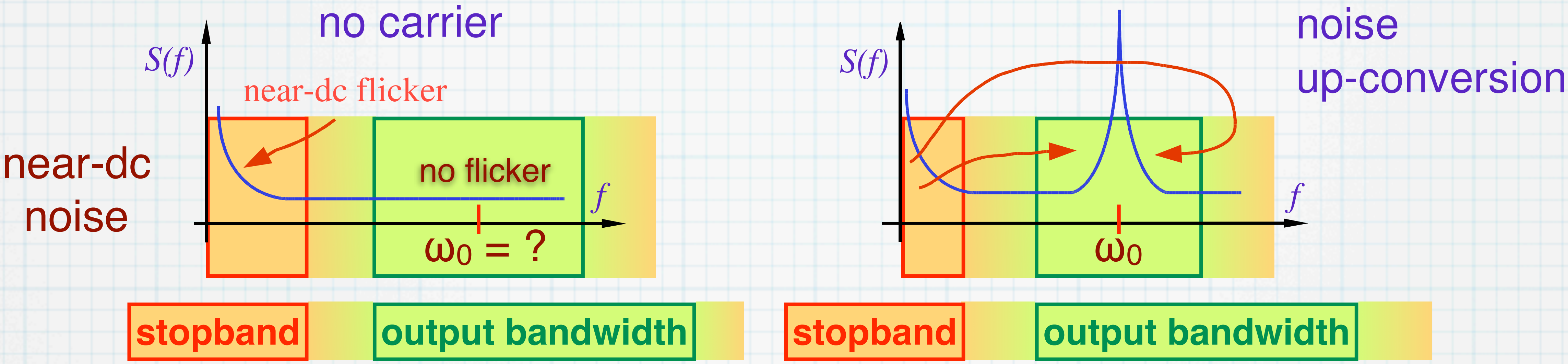


$$N = F_1 kT_0 + \frac{(F_2-1)kT_0}{g_1^2} + \ldots$$

The Friis formula applied to phase noise

$$b_0 = \frac{F_1 kT_0}{P_0} + \frac{(F_2-1)kT_0}{P_0 g_1^2} + \ldots$$

Amplifier flicker noise



carrier near-dc noise

$$v_i(t) = V_i\, e^{j\omega_0 t} + n'(t) + jn''(t)$$

the parametric nature of 1/f noise is hidden in n' and n''

substitute

(careful, this hides the down-conversion)

$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + \ldots$$

non-linear (parametric) amplifier

expand and select the ω_0 terms

$$v_o(t) = V_i \Big\{ a_1 + 2a_2 \big[n'(t) + jn''(t) \big] \Big\} e^{j\omega_0 t}$$

The noise sidebands are proportional to the input carrier

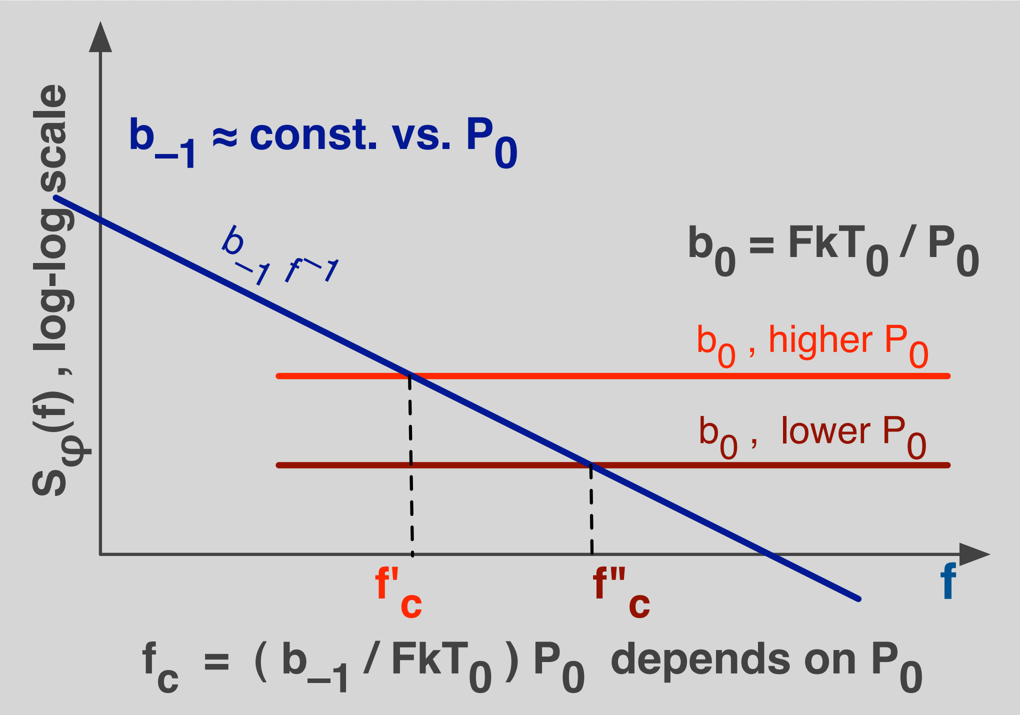
get AM and PM noise

$$\alpha(t) = 2\,\frac{a_2}{a_1}\, n'(t) \qquad \varphi(t) = 2\,\frac{a_2}{a_1}\, n''(t)$$

The AM and the PM noise are independent of V_i, thus of power

There is also a linear parametric model, which yields the same results

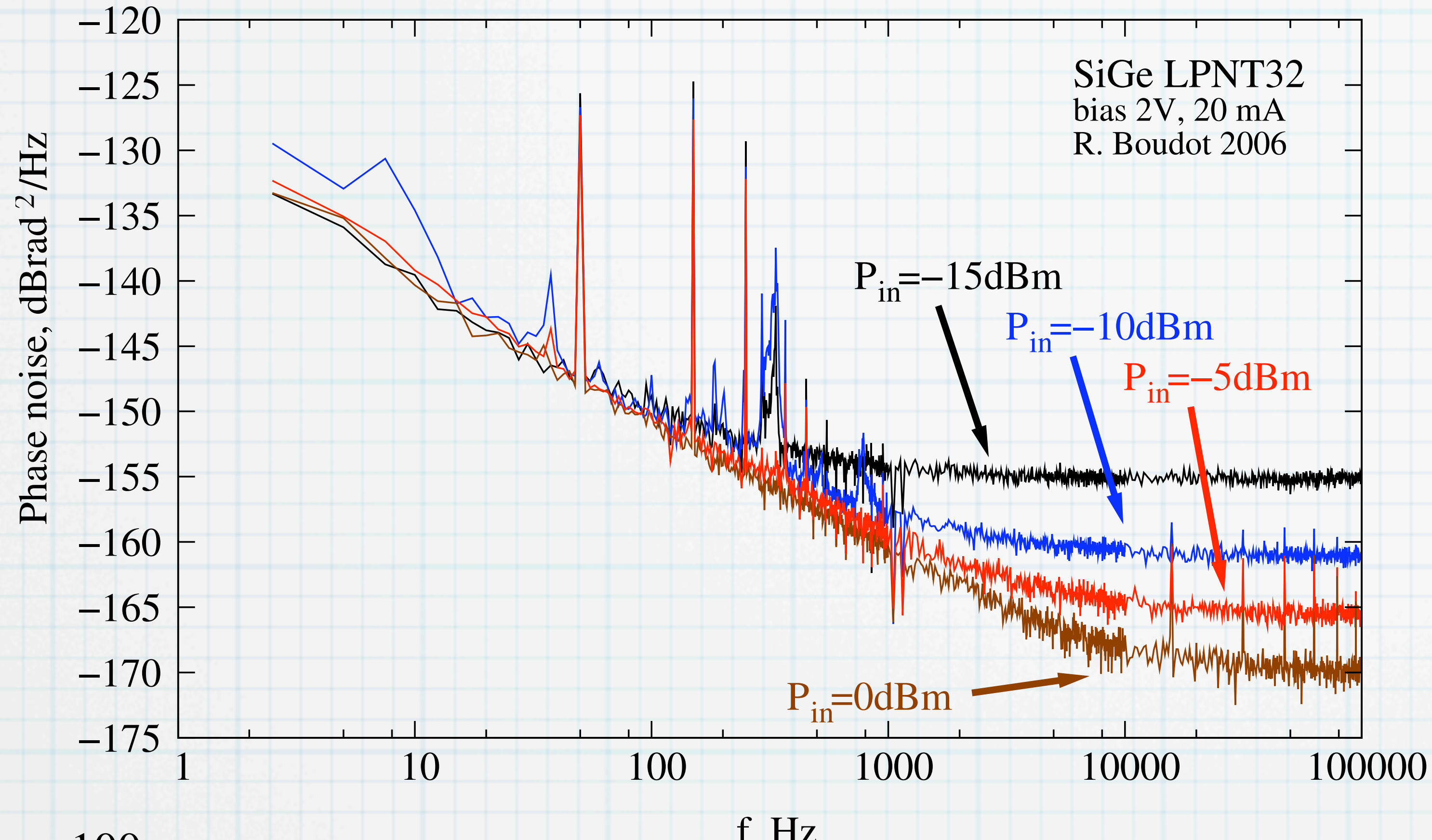
Amplifier flicker noise

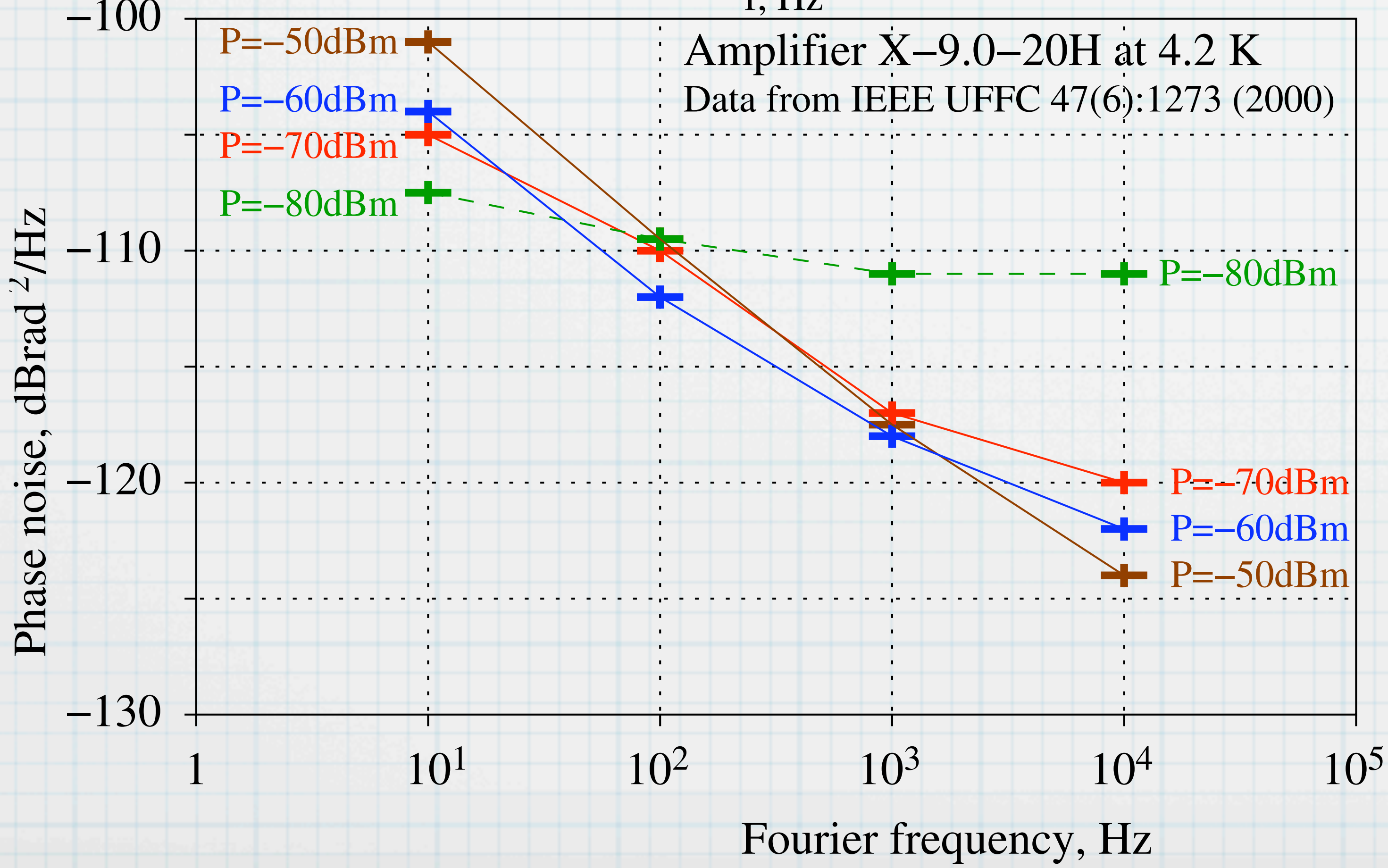


typical amplifier phase noise			
RATE	GaAs HBT microwave	SiGe HBT microwave	Si bipolar HF/UHF
fair	−100		−120
good	−110	−120	−130
best	−120	−130	−150
	unit dBrad^2/Hz		

- **The phase flicker coefficient b_{-1} is about independent of power.**
- Describing the 1/f noise in terms of f_c is misleading because f_c depends on the input power
- In a cascade, $(b_{-1})_{tot}$ does not depend of the amplifier order. Each stage contributes about equally
- b_{-1} is roughly proportional to the gain through the number of stages
- Paralleling m amplifier, $(b_{-1})_{tot}$ is divided by m

Amplifier flicker noise – experiments





- The 1/f phase noise b_{-1} is about independent of power
- The white noise b_0 scales up/down as $1/P_0$, i.e., the inverse of the carrier power

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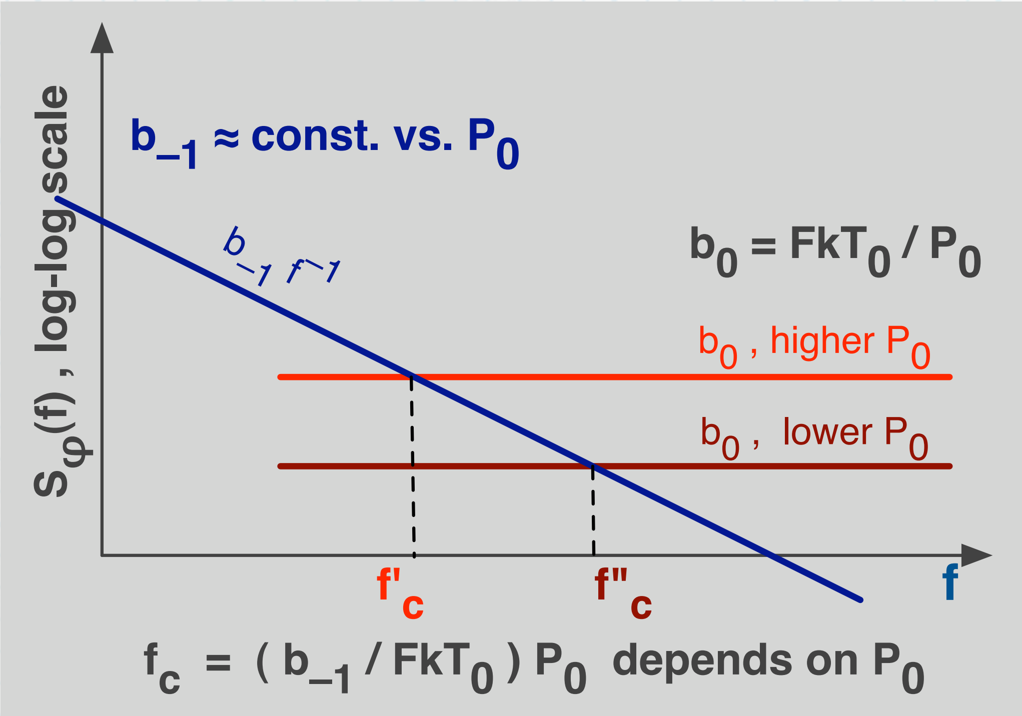
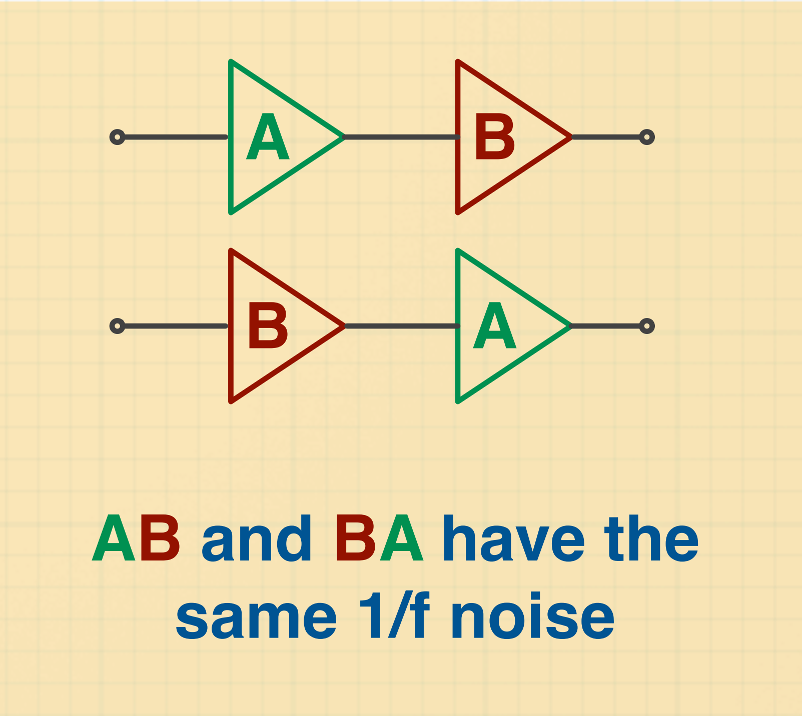
Flicker noise in cascaded amplifiers





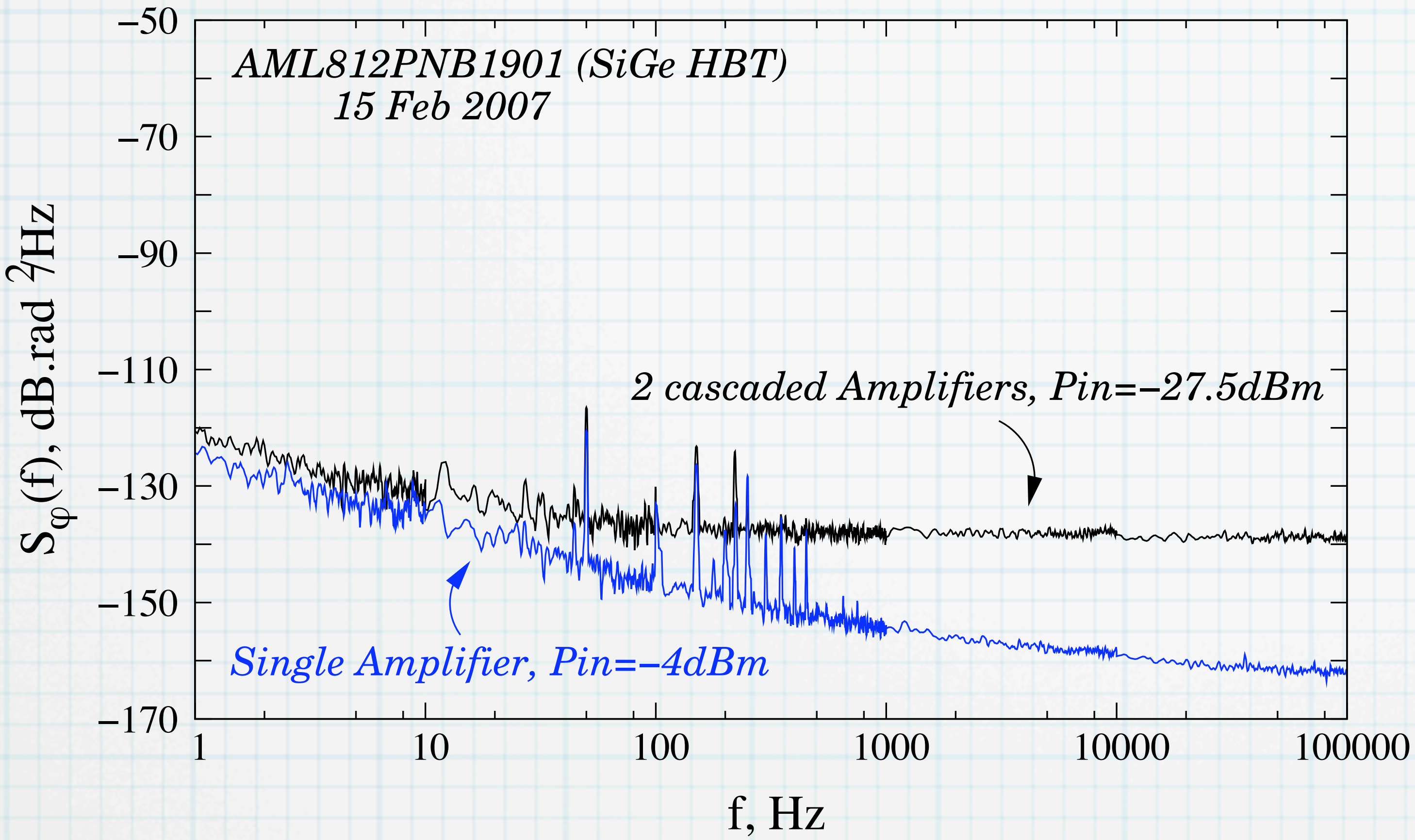
The phase flicker coefficient b_{-1} is about independent of power. Hence:

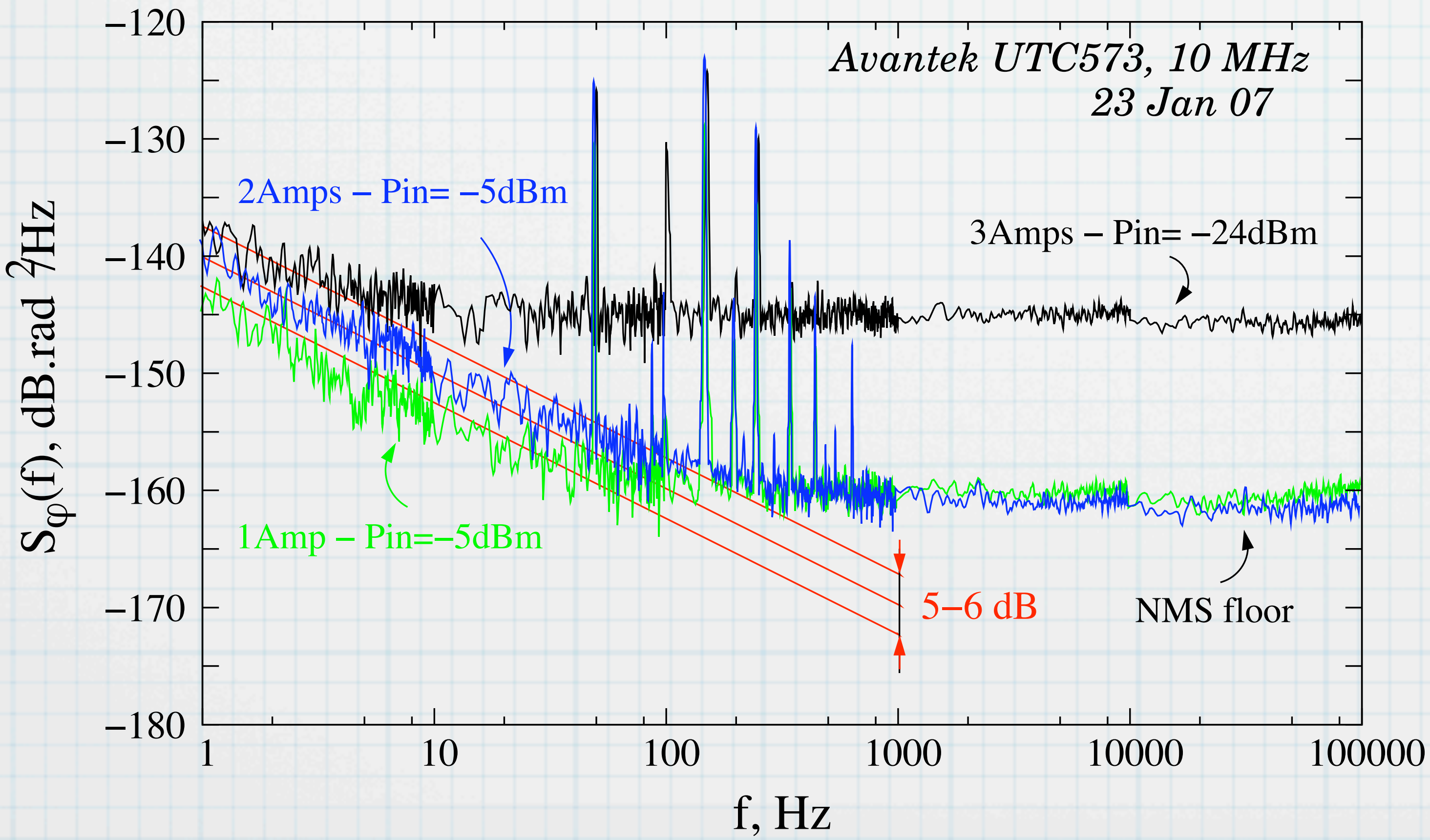
- in a cascade, $(b_{-1})_{tot}$ does not depend of the amplifier order
- in practice, in a cascade each stage contributes about equally

$$(b_{-1})_{\text{tot}} = \sum_{i=1}^{m} (b_{-1})_i$$

- b_{-1} is roughly proportional to the gain through the number of stages

Flicker in cascaded amplifiers – experiments





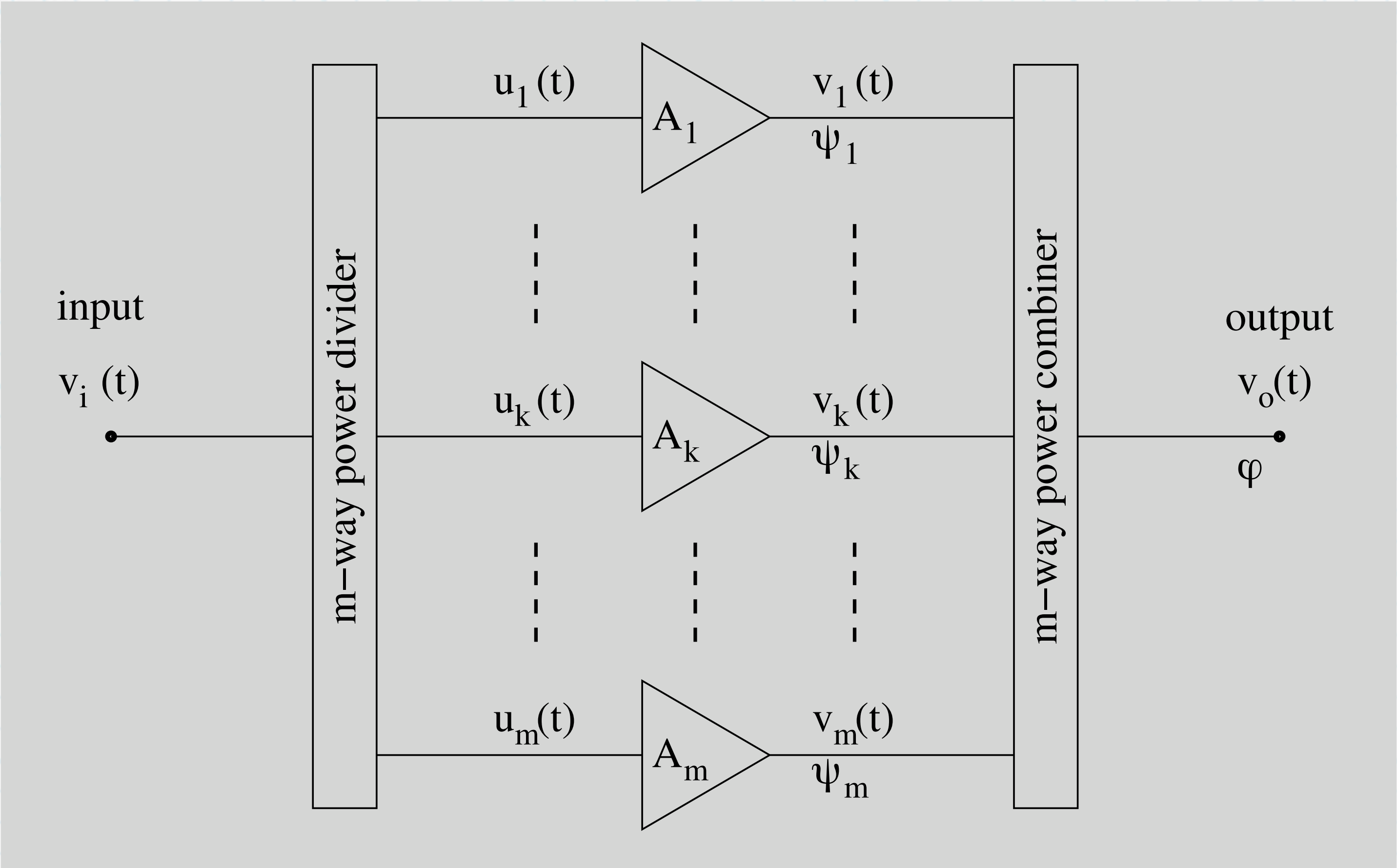
The expected flicker of a cascade increases by:

3 dB, with 2 amplifiers

5 dB, with 3 amplifiers

X

Flicker noise in parallel amplifiers



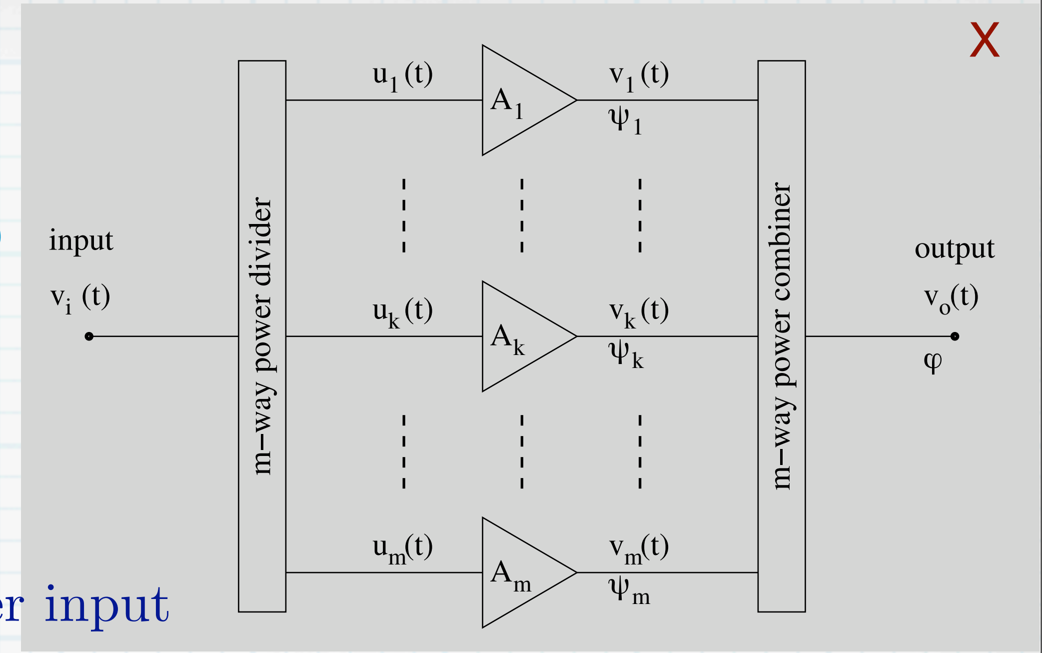
- The phase flicker coefficient b_{-1} is about independent of power
- The flicker of a branch is not increased by splitting the input power
- At the output,
  - the carrier adds up coherently
  - the phase noise adds up statistically
- Hence, the 1/f phase noise is reduced by a factor m

$$b_{-1} = \frac{1}{m}\left[b_{-1}\right]_{\text{branch}}$$

- Only the flicker noise can be reduced in this way

Gedankenexperiment: join the m branches of a parallel amplifier forming a single large active device: the phase flickering is proportional to the inverse physical size of the amplifier active region

Parallel amplifiers, mathematics



$$u_k(t) = \frac{1}{\sqrt{m}} v_i(t)$$ branch-amplifier input

$$v_o(t) = \frac{1}{\sqrt{m}} \sum_{k=1}^{m} v_k(t)$$ main output

$$v_k(t) = \frac{1}{\sqrt{m}} V_i \Big\{ a_1 + 2a_2 \big[n'_k(t) + j n''_k(t) \big] \Big\} e^{j2\pi\nu_0 t}$$ branch $\rightarrow$ output

$$\psi_k(t) = 2\frac{a_2}{a_1} n''_k(t)$$ branch

$$\varphi_k(t) = \frac{\frac{1}{m} V_i \, 2a_2 n''_k(t) \, e^{j2\pi\nu_0 t}}{a_1 V_i \, e^{j2\pi\nu_0 t}}$$ branch $\rightarrow$ output

$$= \frac{1}{m} \, 2\frac{a_2}{a_1} \, n''_k(t)$$

$$S_\varphi(f) = \sum_{k=1}^{m} \frac{1}{m^2} \, 4\frac{a_2^2}{a_1^2} \, S_{n''_k}(f)$$ $\sum$ branches $\rightarrow$ output

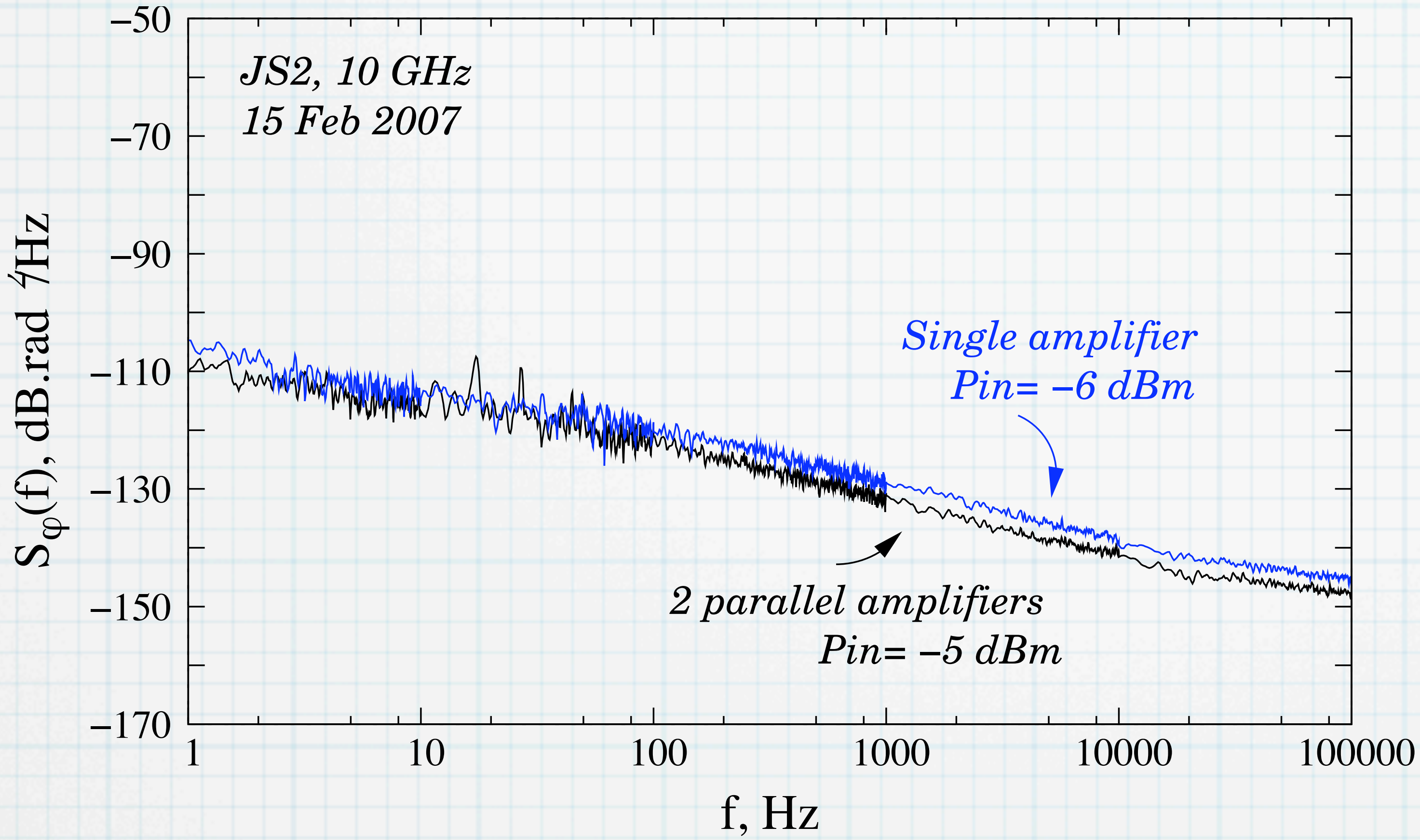
$$S_\varphi(f) = \frac{1}{m} \, 4\frac{a_2^2}{a_1^2} \, S_{n''}(f)$$

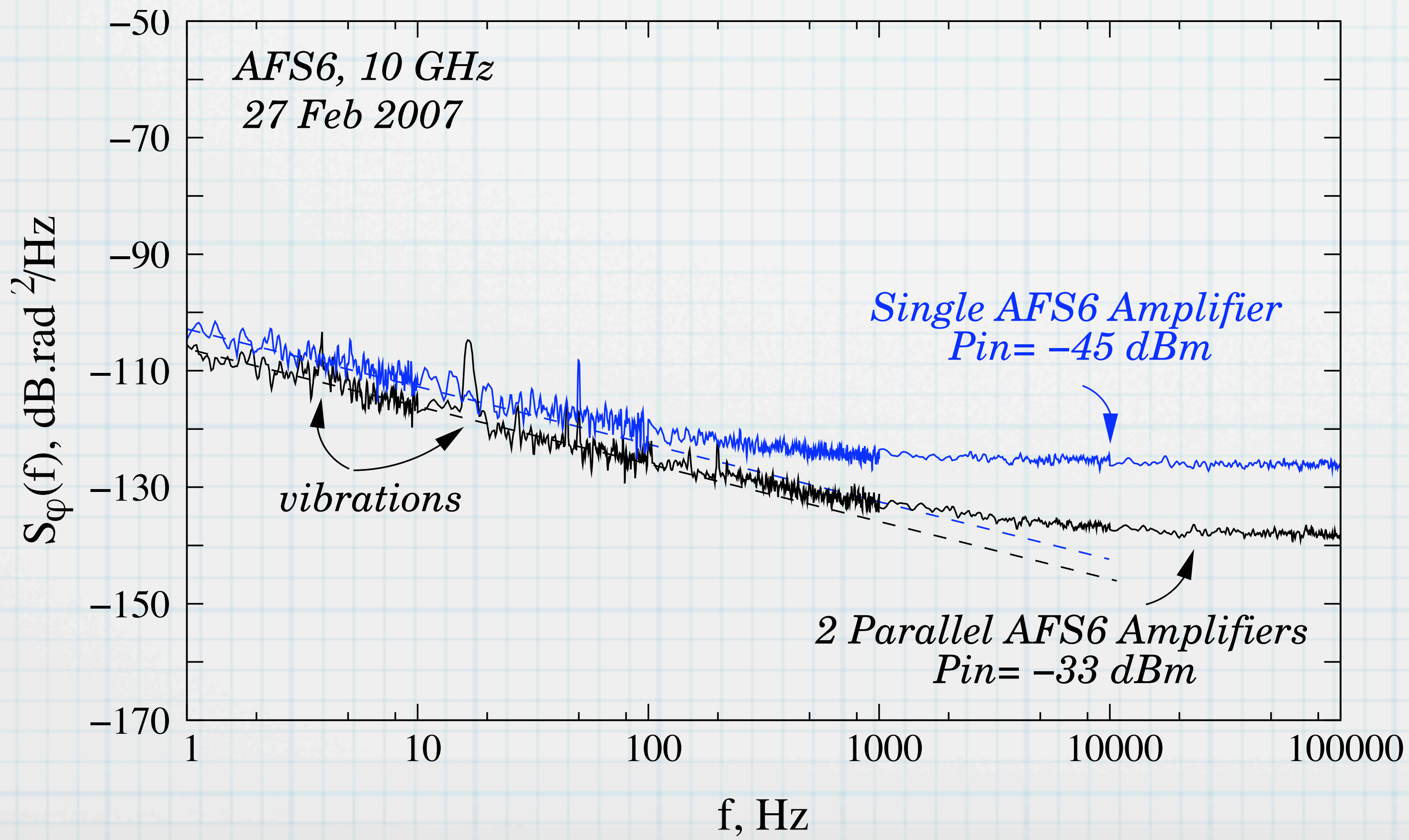
$$S_\varphi(f) = \frac{1}{m} S_\psi(f)$$ m equal branches $\rightarrow$ output

$$b_{-1} = \frac{1}{m} \big[b_{-1} \big]_{\text{branch}}$$

X

Flicker noise in parallel amplifiers





Connecting two amplifiers in parallel, the expected flicker is reduced by 3 dB

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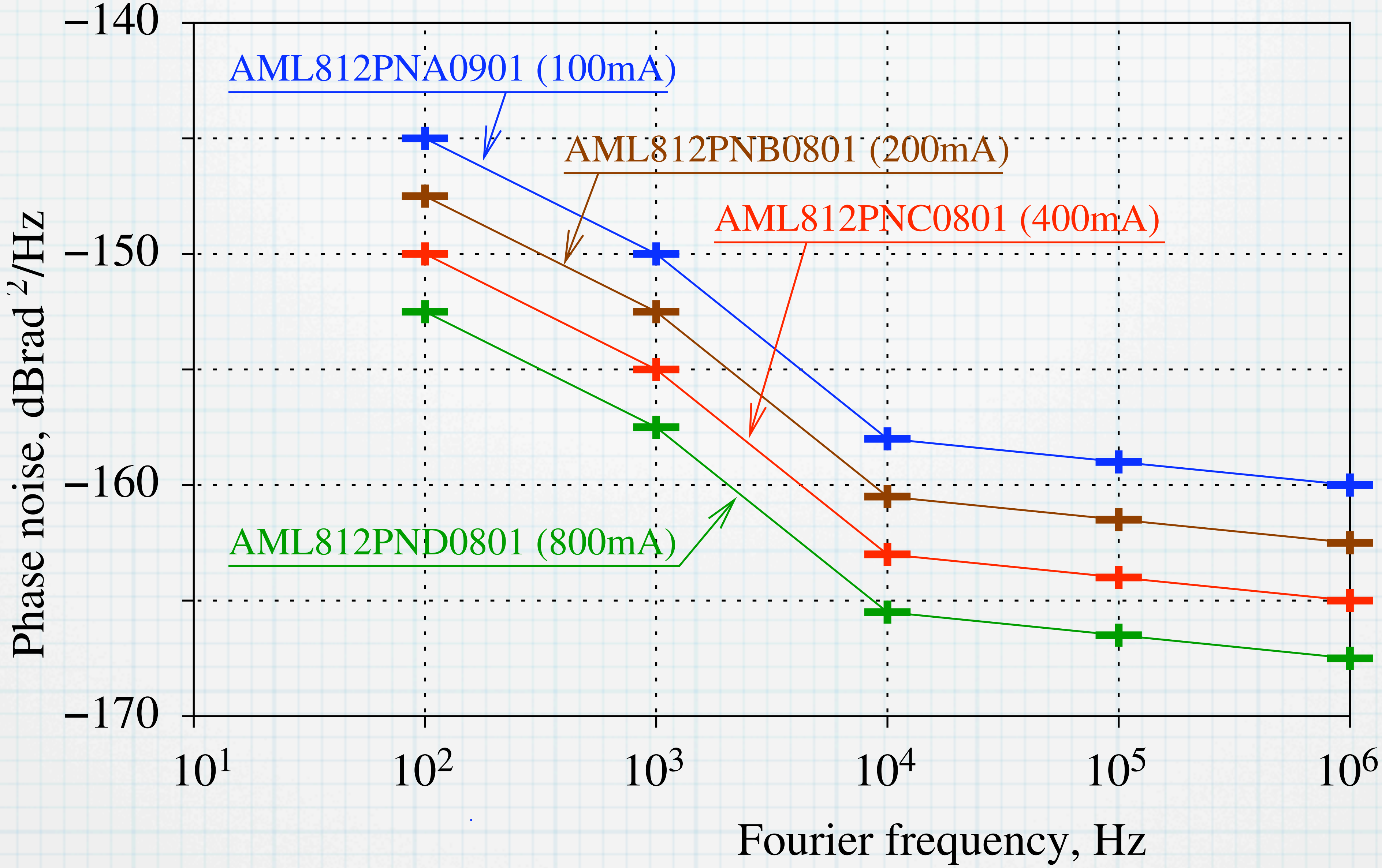
Flicker noise in parallel amplifiers



Specification of low phase-noise amplifiers (AML web page)

amplifier	parameters				phase noise vs. f, Hz			
	gain	F	bias	power	10^2	10^3	10^4	10^5
AML812PNA0901	10	6.0	100	9	−145.0	−150.0	−158.0	−159.0
AML812PNB0801	9	6.5	200	11	−147.5	−152.5	−160.5	−161.5
AML812PNC0801	8	6.5	400	13	−150.0	−155.0	−163.0	−164.0
AML812PND0801	8	6.5	800	15	−152.5	−157.5	−165.5	−166.5
unit	dB	dB	mA	dBm	dBrad^2/Hz			

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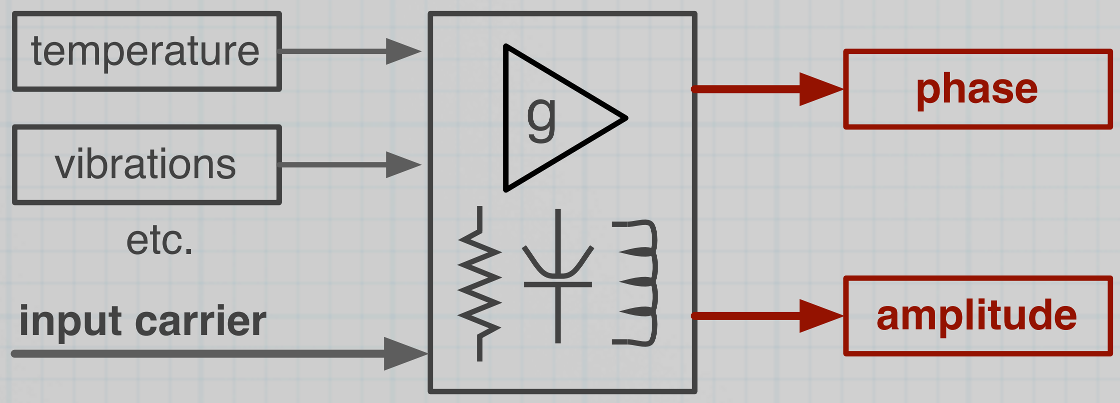
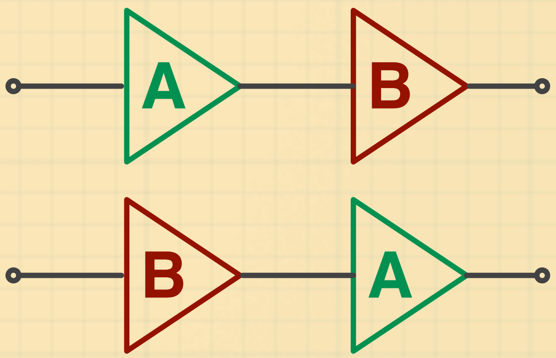
Environmental (parametric) noise in amplifiers



A time constant can be present



$\varphi = \varphi_A + \varphi_B$ and $\alpha = \alpha_A + \alpha_B$
regardless of the amplifier order

Cascading *m* equal amplifiers, $S_\alpha(f)$ and $S_\varphi(f)$ increase by a factor m^2.

If the amplifier were independent, $S_\alpha(f)$ and $S_\varphi(f)$ would increase only by a factor *m*.

Cascaded amplifiers

let $\quad z(t) = x(t) + y(t)$

Phase noise

$$\begin{aligned} S_z(f) &= ZZ^* \\ &= (X+Y)\,(X+Y)^* \\ &= XX^* + YY^* + XY^* + YX^* \\ &= S_x + S_y + \underbrace{S_{xy}}_{>0} + \underbrace{S_{yx}}_{>0} \end{aligned}$$

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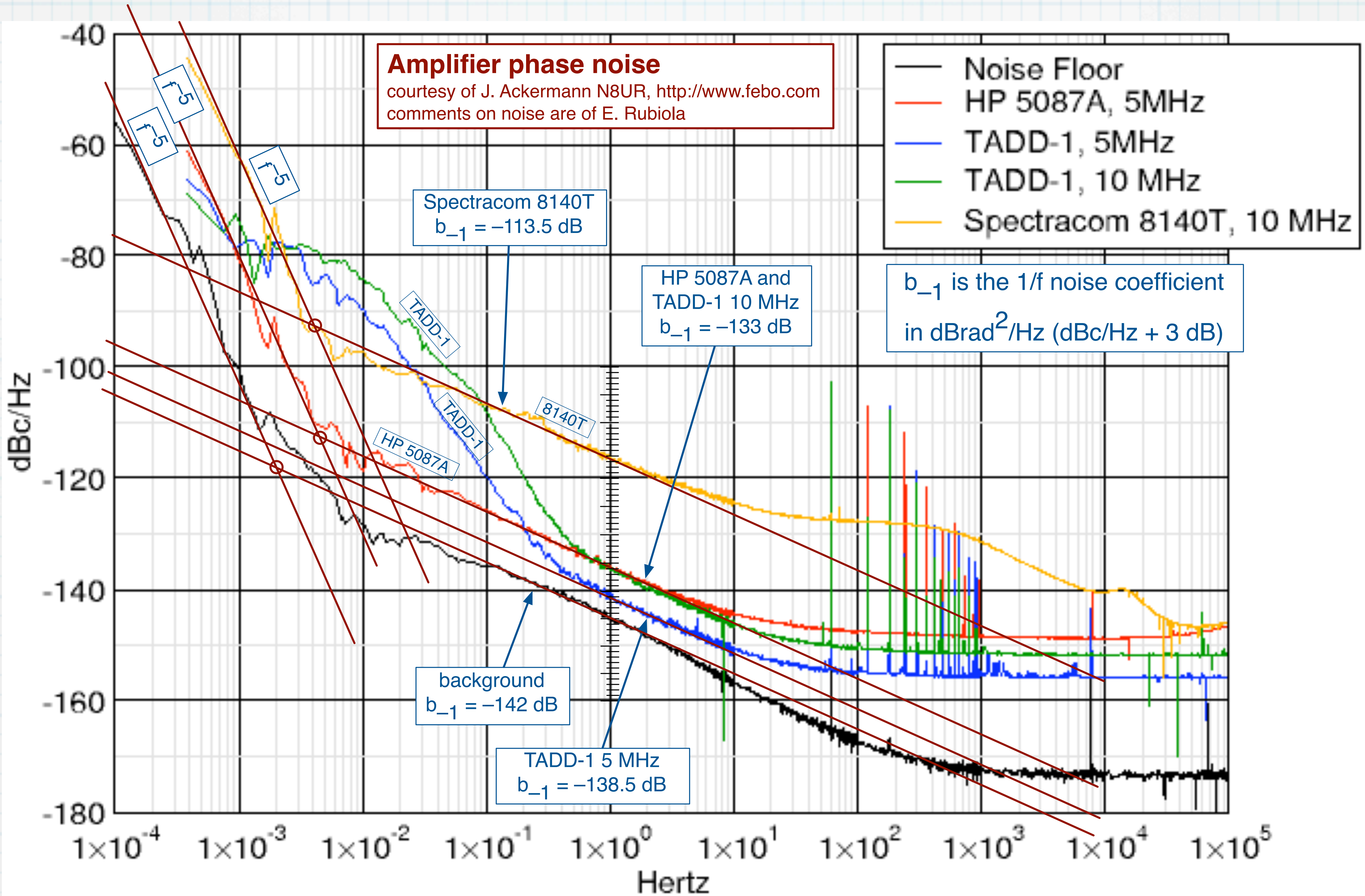
Environmental effects in RF amplifiers

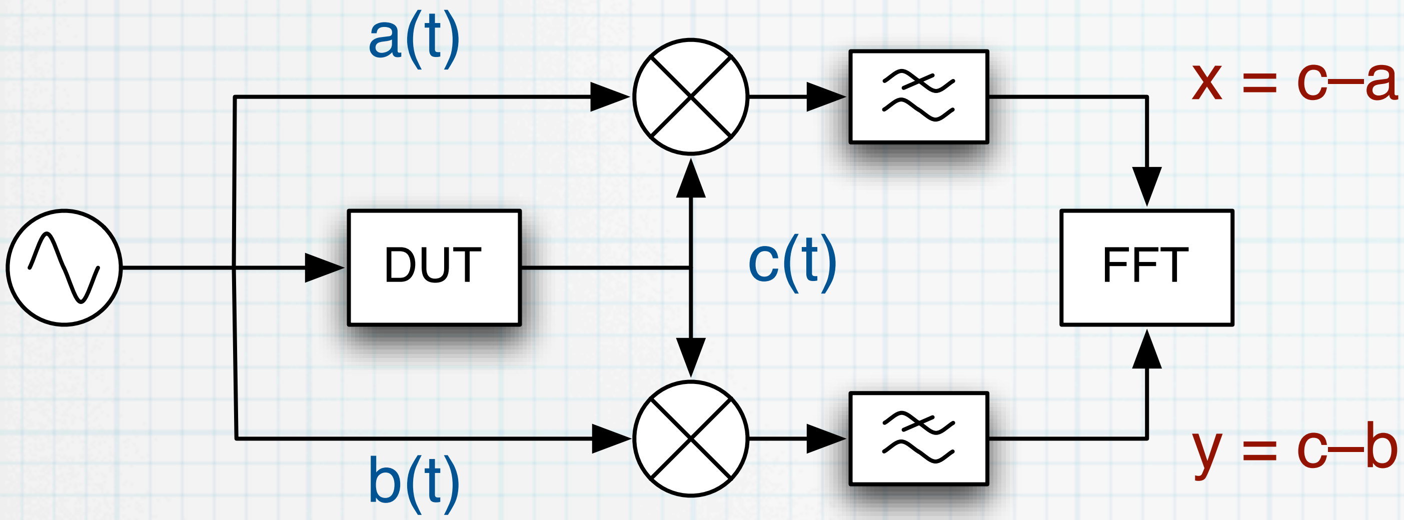


It is experimentally observed that the temperature fluctuations cause a spectrum $S_\alpha(f)$ or $S_\varphi(f)$ of the $1/f^5$ type

Yet, at lower frequencies the spectrum folds back to 1/f

3 - Correlation

Correlation measurements



Two separate mixers measure the same DUT. Only the DUT noise is common

a(t), b(t) –> mixer noise
c(t) –> DUT noise

phase noise measurements		
DUT noise, normal use	a, b c	instrument noise DUT noise
background, ideal case	a, b c = 0	instrument noise no DUT
background, with AM noise	a, b c ≠ 0	instrument noise AM-to-DC noise

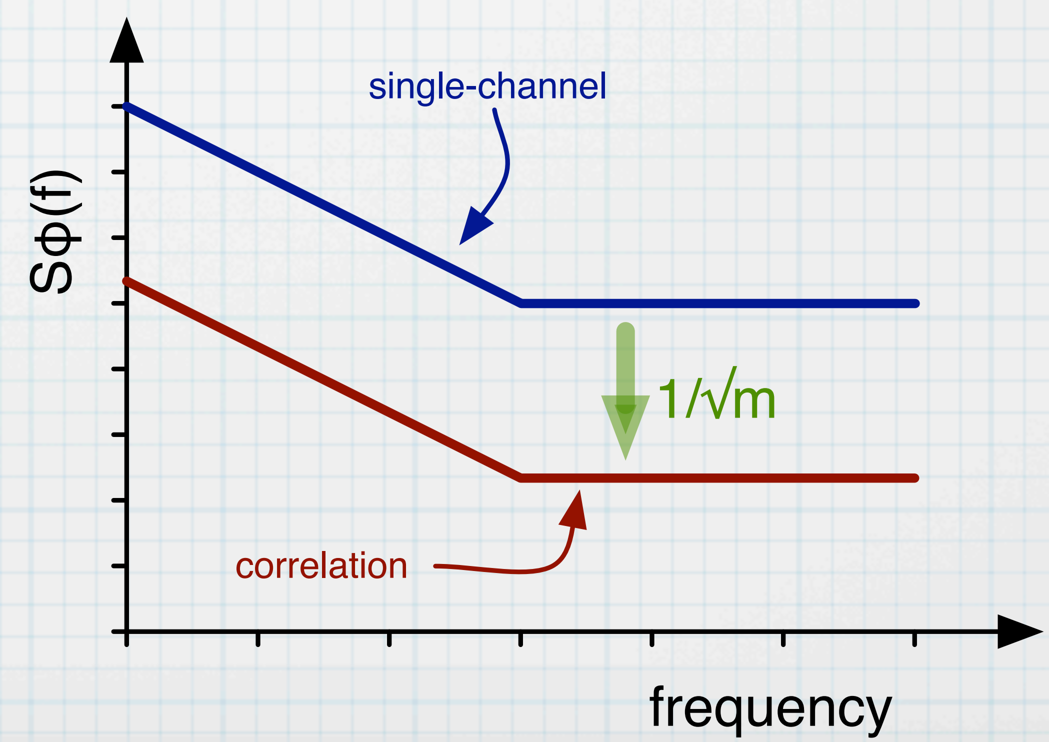
basics of correlation

$$
\begin{aligned}
S_{yx}(f) &= \mathbb{E}\left\{Y(f)X^*(f)\right\} \\
&= \mathbb{E}\left\{(C-A)(C-B)^*\right\} \\
&= \mathbb{E}\left\{CC^* - \underbrace{AC^*}_{0} - \underbrace{CB^*}_{0} + \underbrace{AB^*}_{0}\right\} \\
&= \mathbb{E}\left\{CC^*\right\}
\end{aligned}
$$

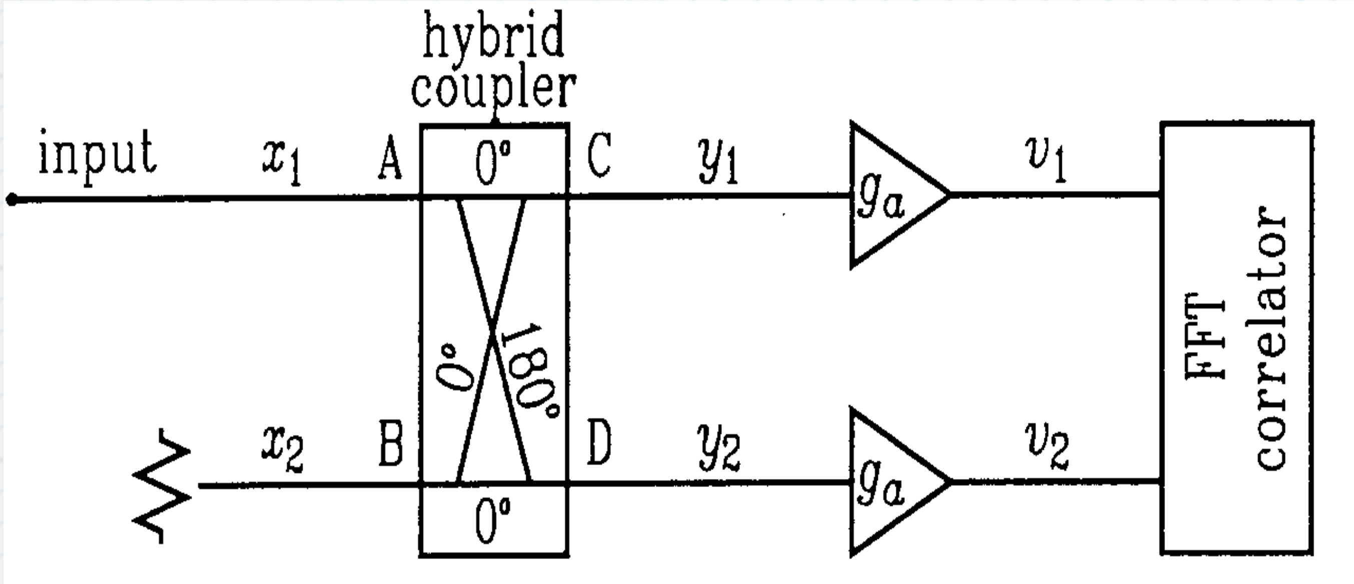
$$
S_{yx}(f) = S_{cc}(f)
$$

in practice, average on m realizations

$$
\begin{aligned}
S_{yx}(f) &= \left\langle Y(f)X^*(f)\right\rangle_m \\
&= \left\langle CC^* \underbrace{- AC^* - CB^* + AB^*}_{\to\ 0 \text{ as } 1/\sqrt{m}}\right\rangle_m \\
&= \left\langle CC^*\right\rangle_m + O(1/m)
\end{aligned}
$$



Thermal noise compensation



hybrid output

$$y_1(t) = \frac{1}{\sqrt{2}}\, x_2(t) + \frac{1}{\sqrt{2}}\, x_1(t)$$
$$y_2(t) = \frac{1}{\sqrt{2}}\, x_2(t) - \frac{1}{\sqrt{2}}\, x_1(t)$$

correlation

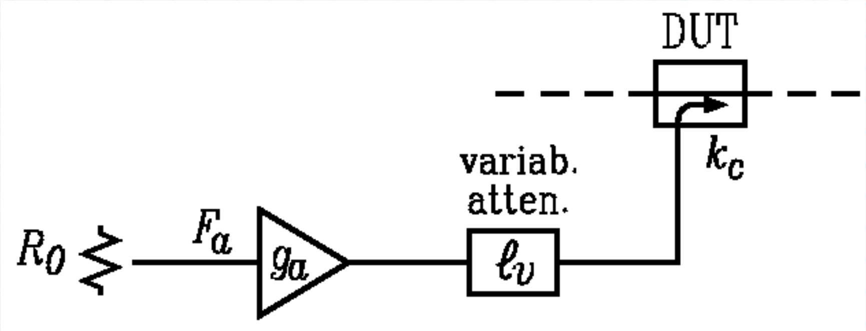
$$\mathcal{R}_{y_1y_2}(\tau) = \lim_{\theta\to\infty} \frac{1}{\theta} \int_\theta y_1(t)\, y_2^*(t-\tau)\, \mathrm{d}t$$
$$= \frac{1}{2}\, \mathcal{R}_{x_2x_2}(\tau) - \frac{1}{2}\, \mathcal{R}_{x_1x_1}(\tau)$$

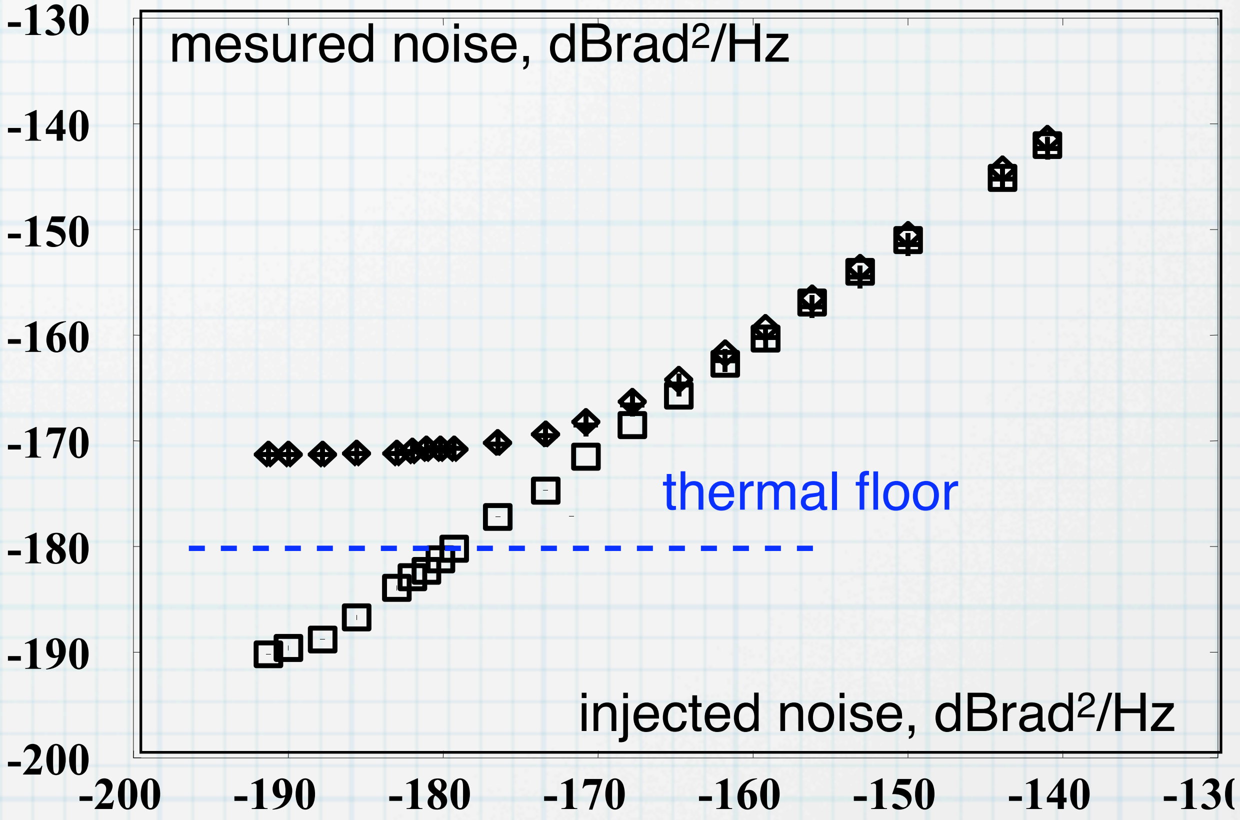
Fourier transform and thermal noise

$$S_{y_1y_2}(f) = \frac{1}{2}\, S_{x_2}(f) - \frac{1}{2}\, S_{x_1}(f)$$
$$S_{y_1y_2}(f) = \frac{k_B(T_2 - T_1)}{2}$$

Thermal noise compensation

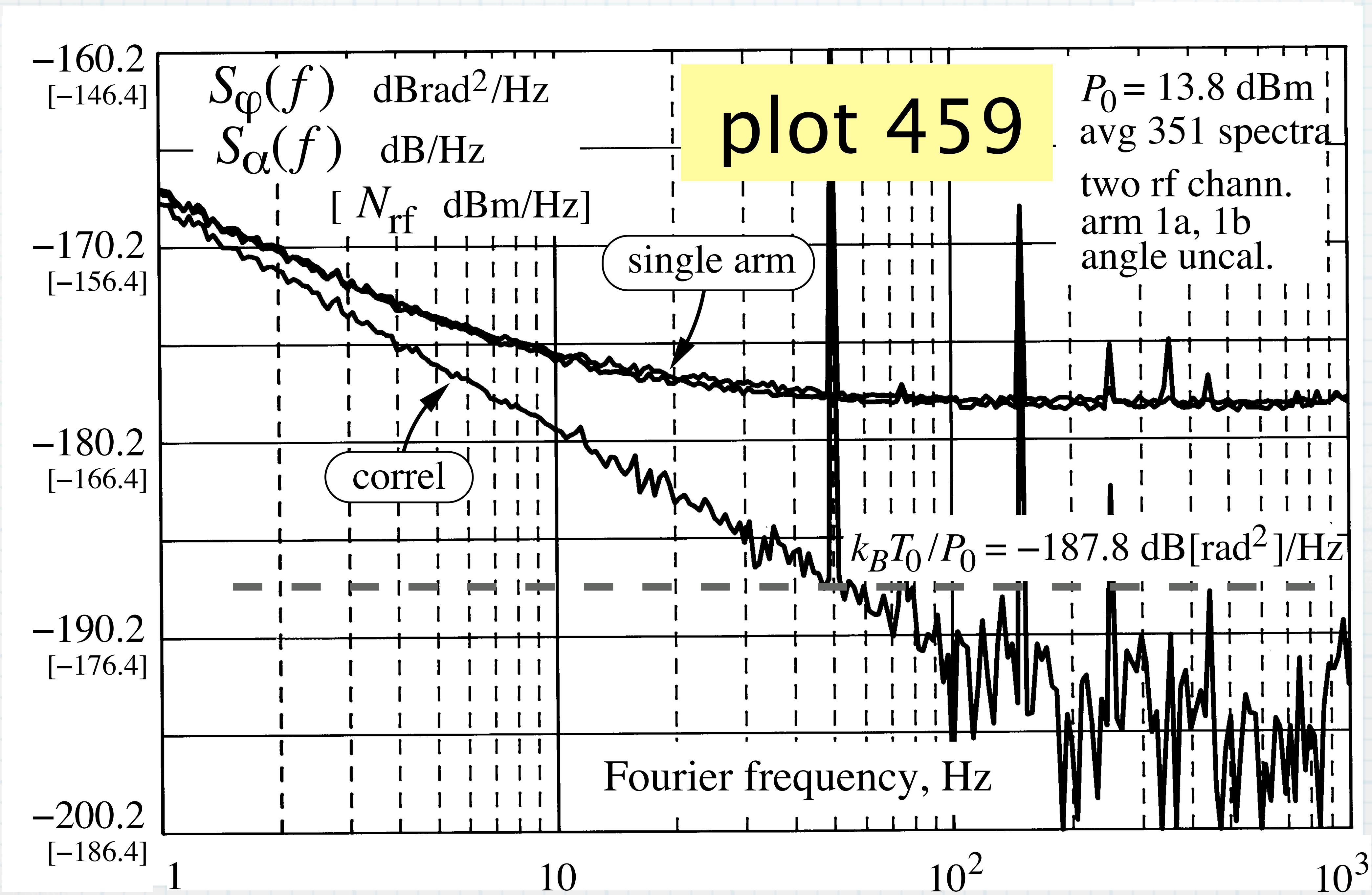
100 MHz prototype, carrier power P_o = 8 dBm





Example of correlation measurement

100 MHz carrier



Noise of a by-step attenuator, measured at 100 MHz by correlation. The mixer is replaced with a bridge.

Useful schemes

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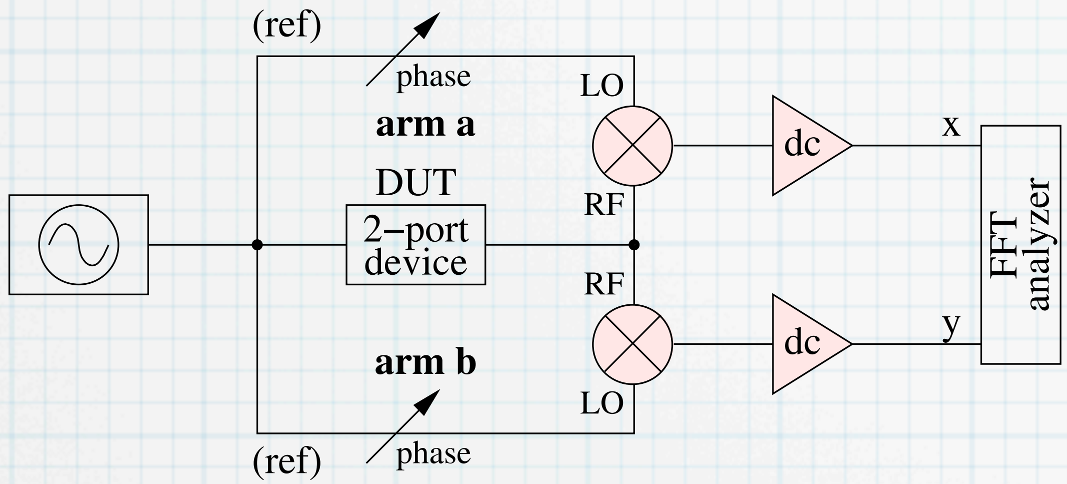
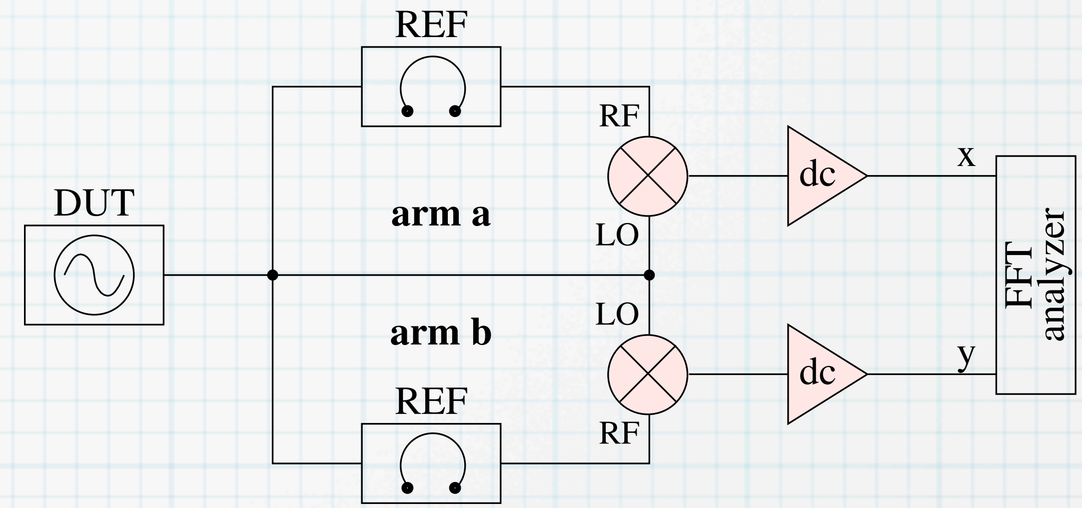
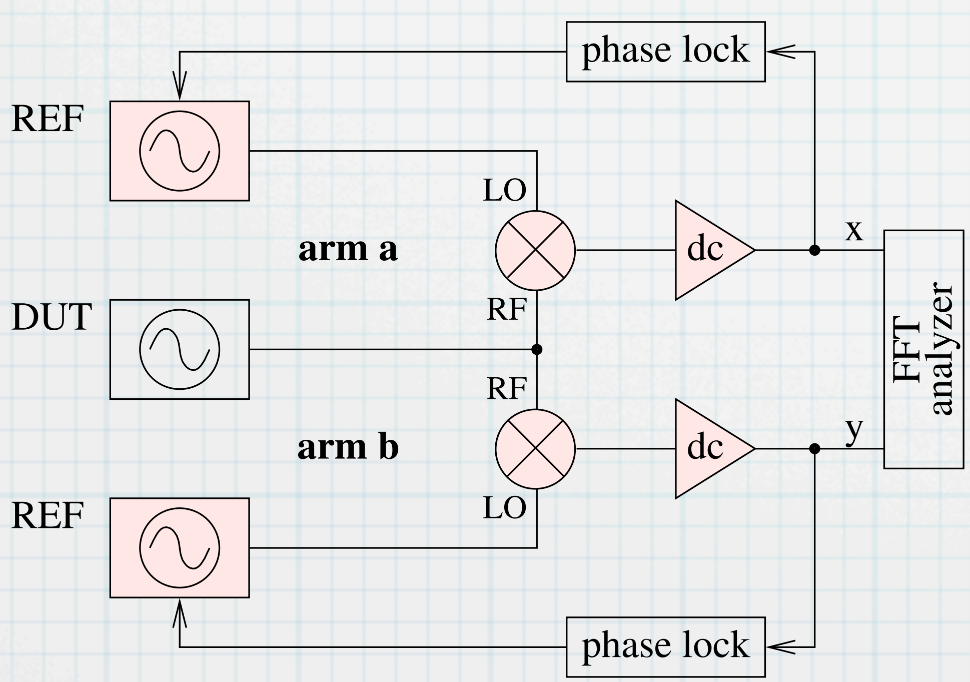
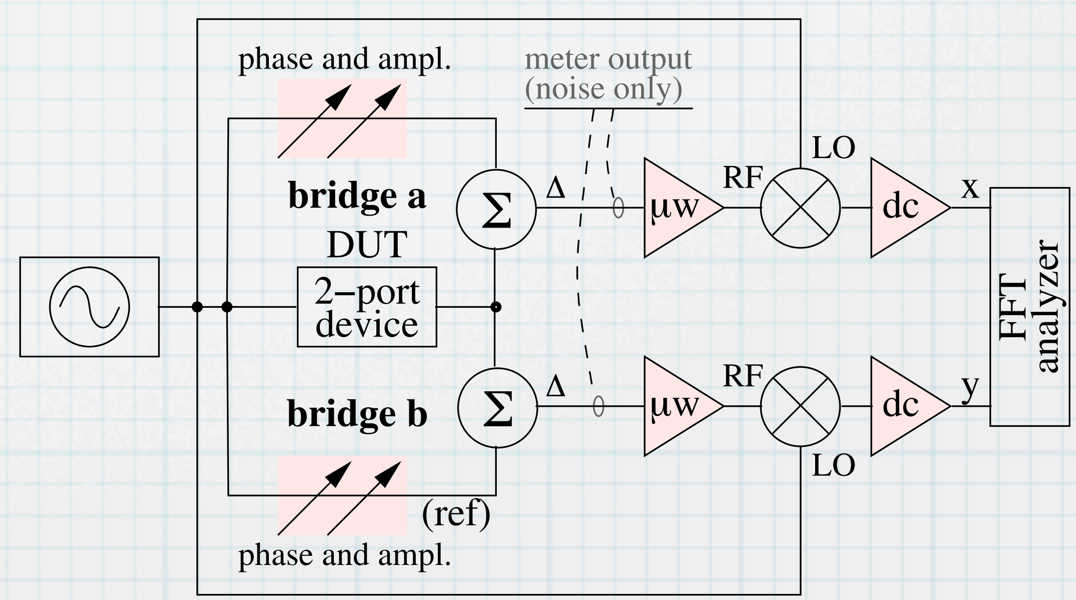
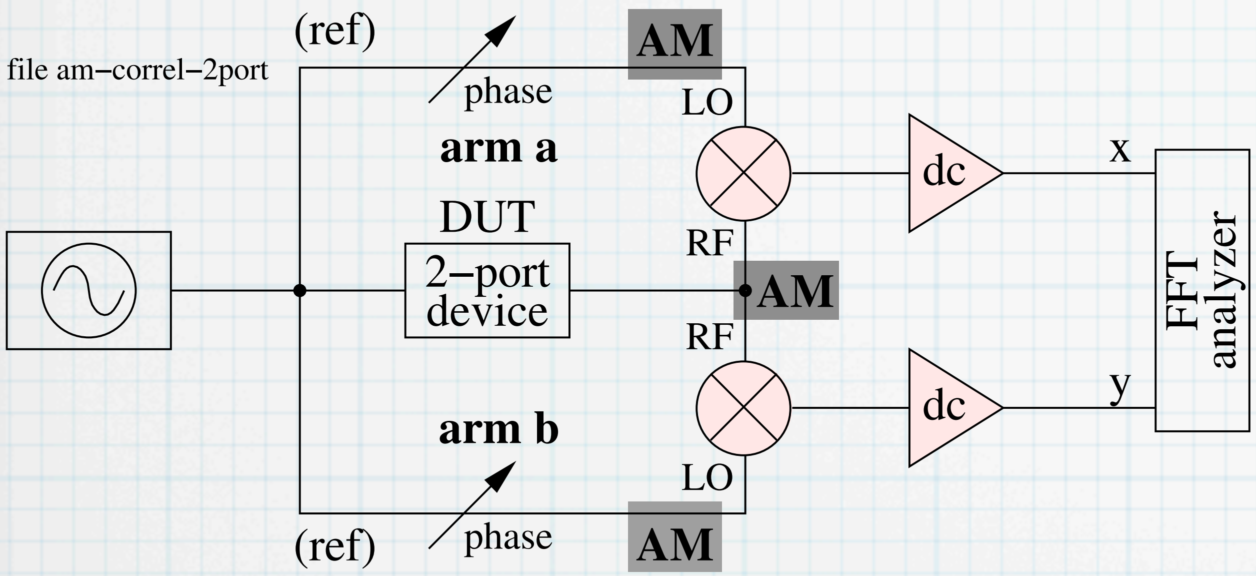








Pollution from AM noise

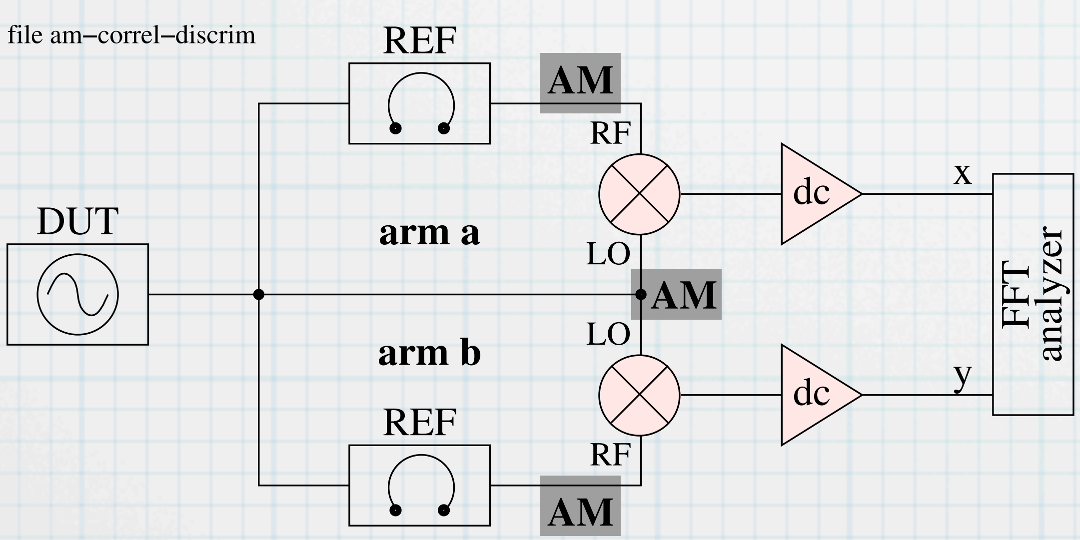


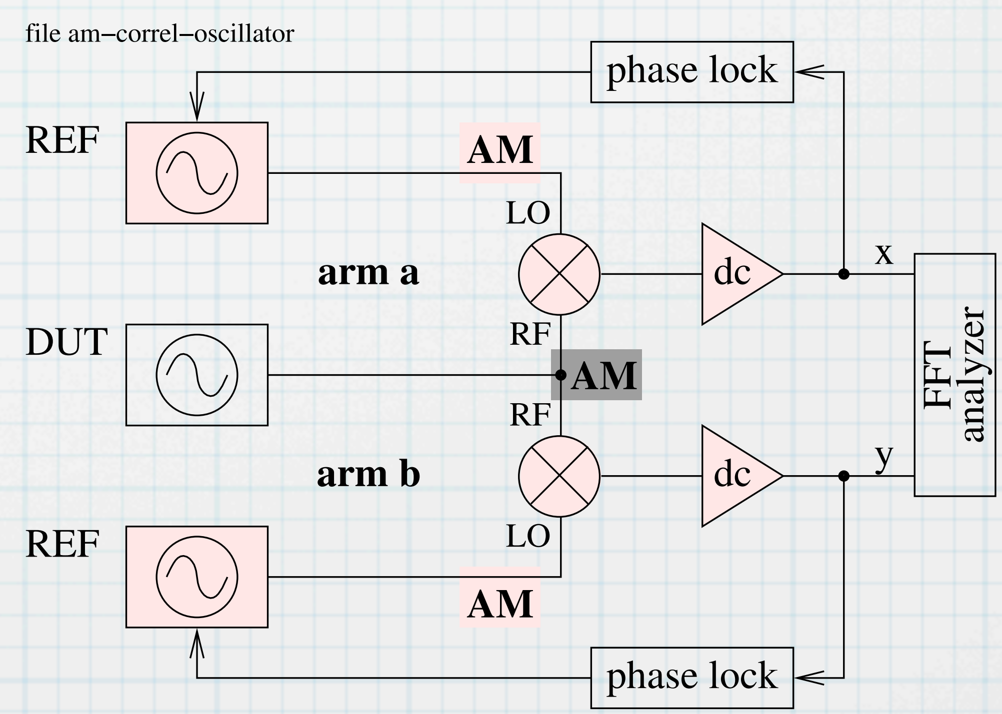
The mixer converts power into dc-offset, thus AM noise into dc-noise, which is mistaken for PM noise

$$v(t) = k_\phi\, \phi(t) + k_{LO}\, \alpha_{LO} + k_{RF}\, \alpha_{RF}$$

rejected by correlation and avg

not rejected by correlation and avg



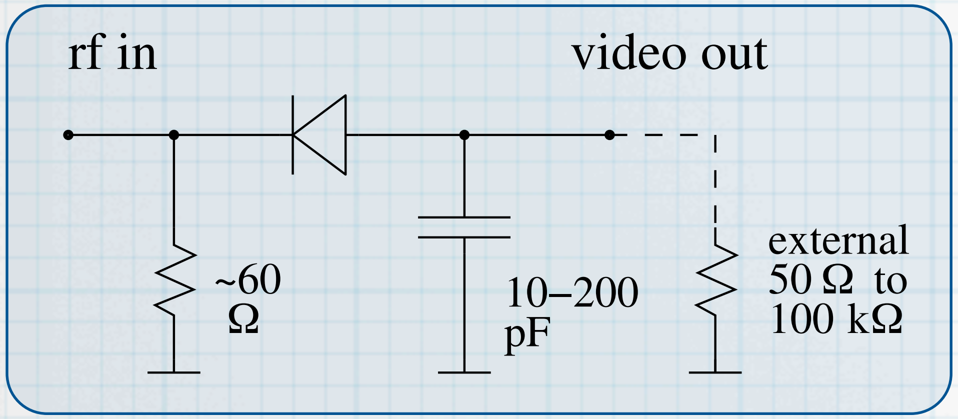


E. Rubiola, R. Boudot, *The effect of AM noise on correlation phase noise measurements*, IEEE Tr.UFFC 54(5):926–932 May 2007, and arXiv/physics/0609147

4 - AM noise

Tunnel and Schottky power detectors

law: $v = k_d\, P$

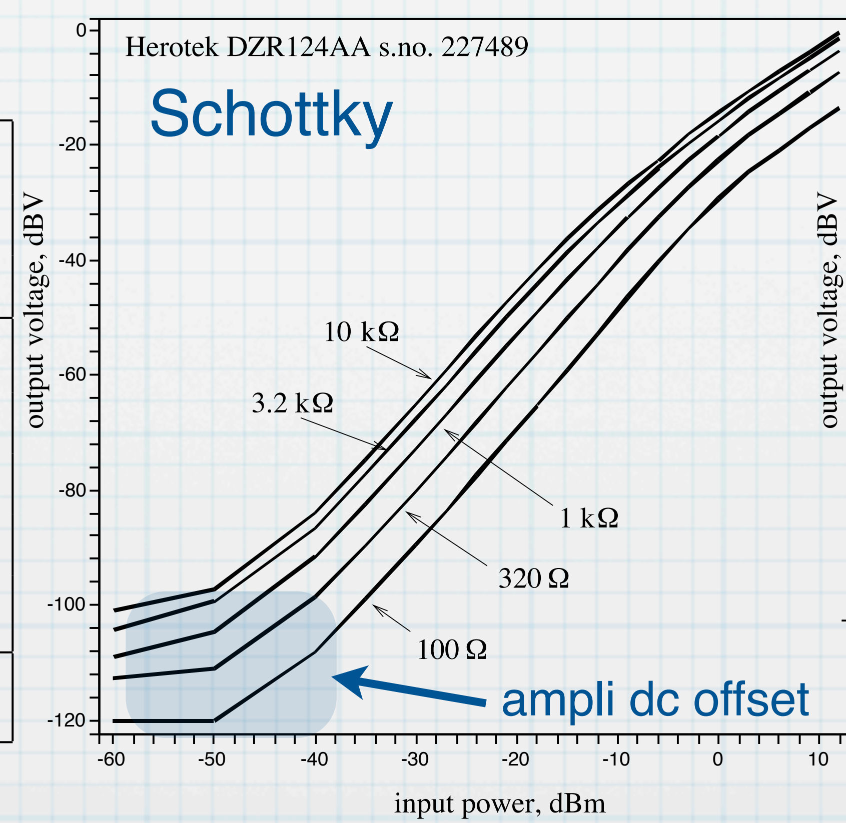


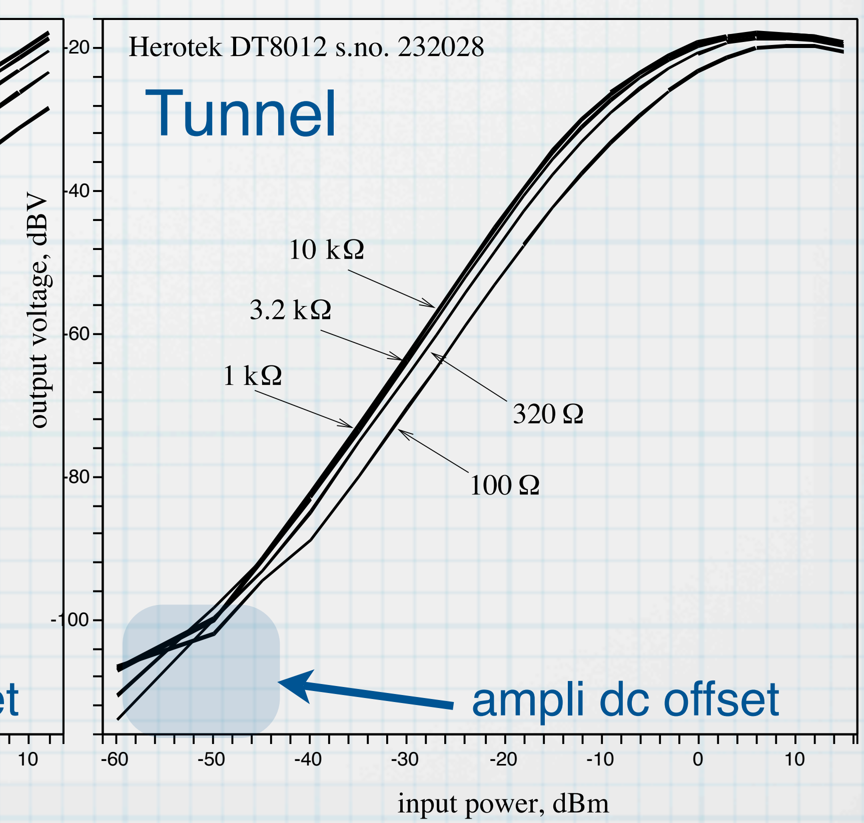
The "tunnel" diode is actually a backward diode. The negative resistance region is absent.

parameter	Schottky	tunnel
input bandwidth	up to 4 decades 10 MHz to 20 GHz	1–3 octaves up to 40 GHz
VSVR max.	1.5:1	3.5:1
max. input power (spec.)	−15 dBm	−15 dBm
absolute max. input power	20 dBm or more	20 dBm
output resistance	1–10 kΩ	50–200 Ω
output capacitance	20–200 pF	10–50 pF
gain	300 V/W	1000 V/W
cryogenic temperature	no	yes
electrically fragile	no	yes

Measured

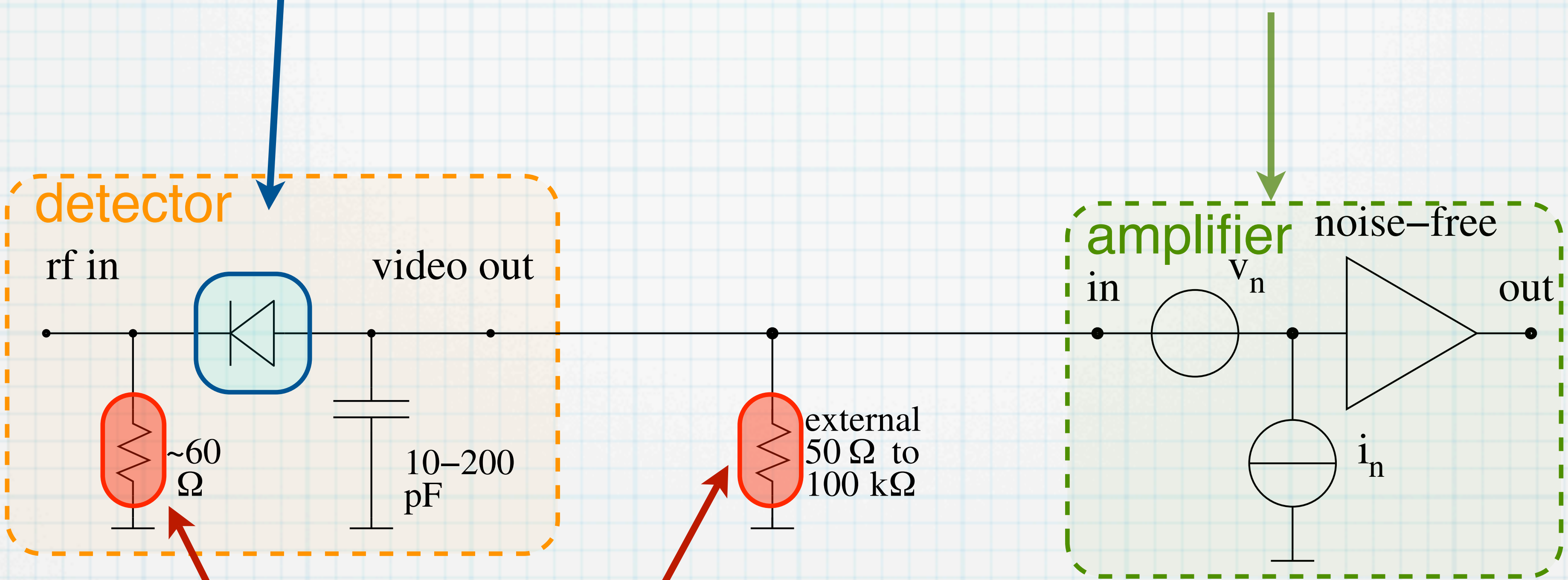
load resistance, Ω	detector gain, A^{-1} DZR124AA (Schottky)	detector gain, A^{-1} DT8012 (tunnel)
1×10^2	35	292
3.2×10^2	98	505
1×10^3	217	652
3.2×10^3	374	724
1×10^4	494	750
conditions: power −50 to −20 dBm		





Noise mechanisms

Shot noise $S_I(f) = 2qI_0$



Thermal noise
$S_V(f) = 4k_BT_0R$

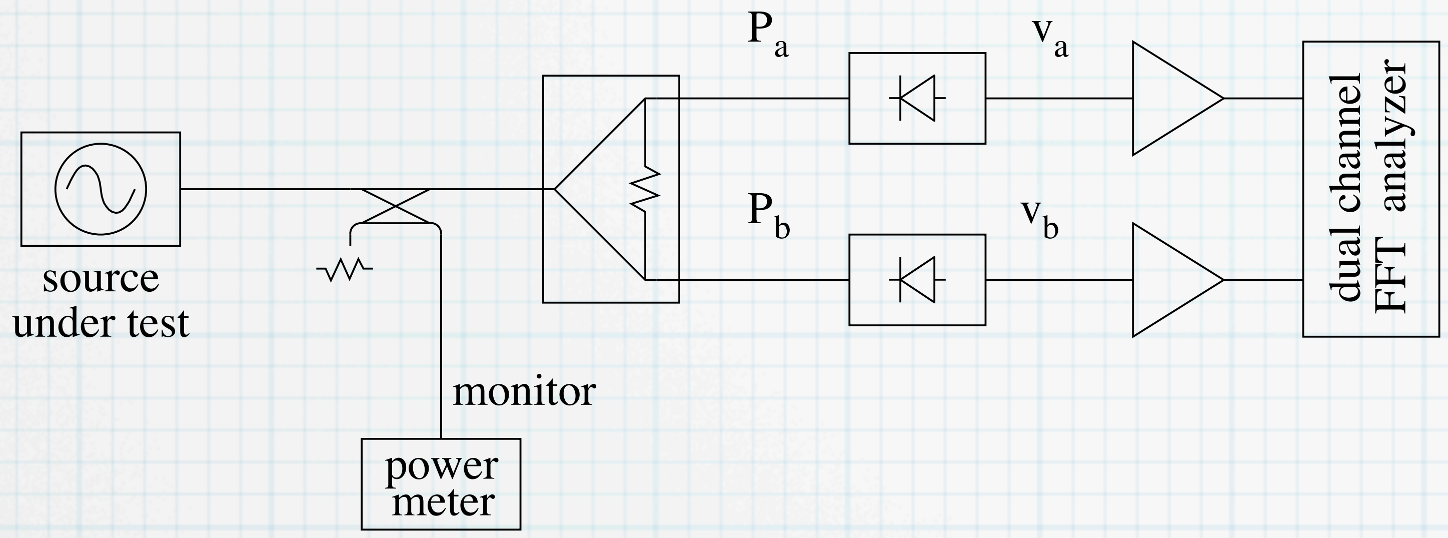
Flicker (1/*f*) noise is also present

Never say that it's *not fundamental*,
unless you know how to remove it

In practice

the amplifier white noise turns out to be higher than the detector noise
and the amplifier flicker noise is even higher

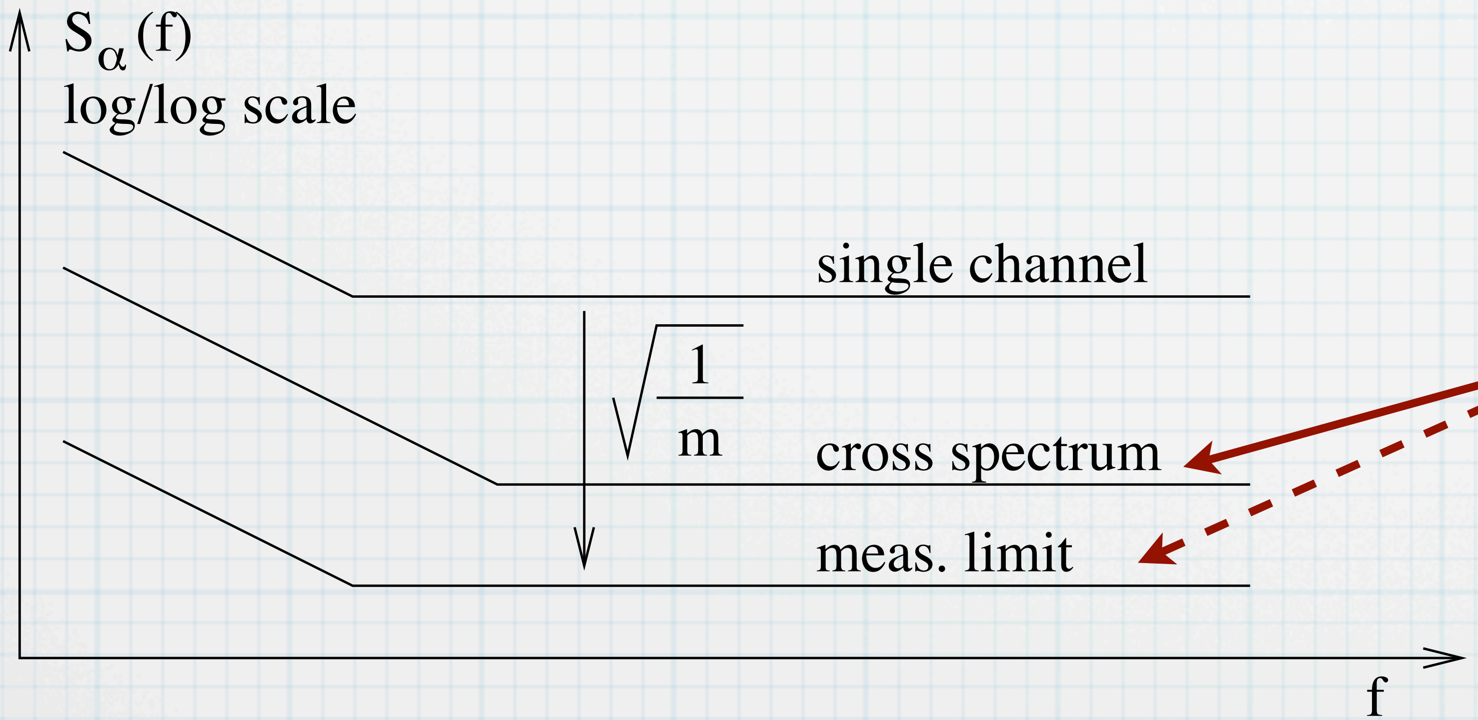
Cross-spectrum method



$$v_a(t) = 2k_a P_a \alpha(t) + \text{noise}$$
$$v_b(t) = 2k_a P_b \alpha(t) + \text{noise}$$

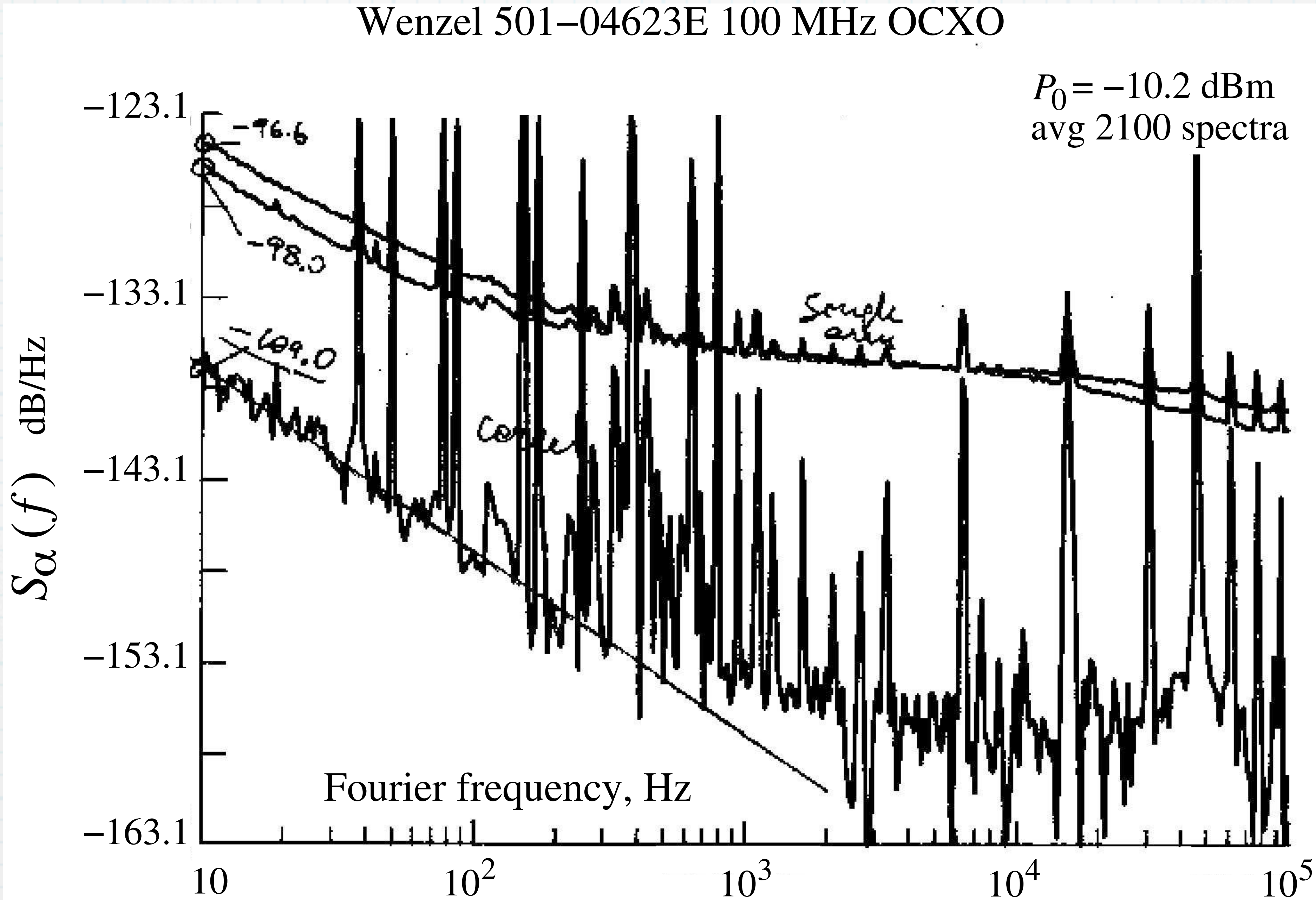
The cross spectrum $S_{ba}(f)$ rejects the single-channel noise because the two channels are independent.

$$S_{ba}(f) = \frac{1}{4k_a k_b P_a P_b} S_\alpha(f)$$



- Averaging on m spectra, the single-channel noise is rejected by √1/2m
- A cross-spectrum higher than the averaging limit validates the measure
- The knowledge of the single-channel noise is not necessary

Example of AM noise spectrum



flicker: $h_{-1} = 1.5\times10^{-13}\ \mathrm{Hz}^{-1}\ (-128.2\ \mathrm{dB}) \quad \Rightarrow \quad \sigma_\alpha = 4.6\times10^{-7}$

Single-arm 1/f noise is that of the dc amplifier
(the amplifier is still not optimized)

AM noise of some sources

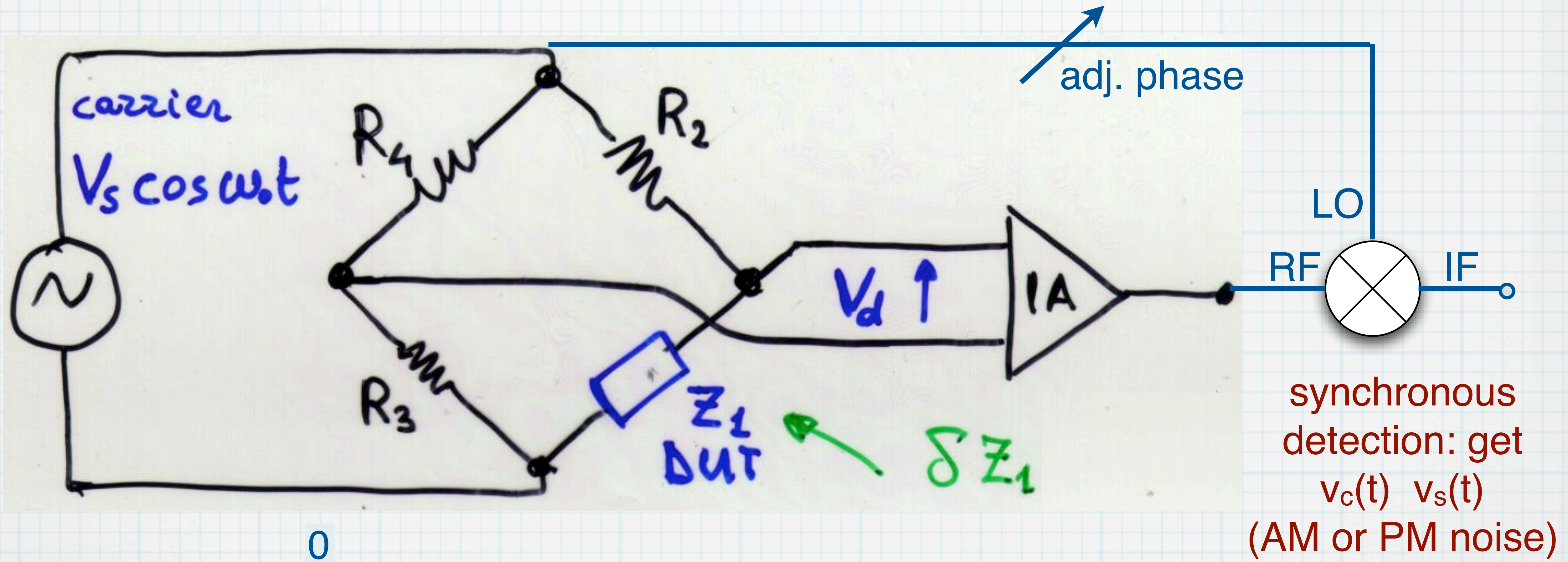
source	h_{-1} (flicker)		$(\sigma_\alpha)_{\text{floor}}$
Anritsu MG3690A synthesizer (10 GHz)	2.5×10^{-11}	-106.0 dB	5.9×10^{-6}
Marconi synthesizer (5 GHz)	1.1×10^{-12}	-119.6 dB	1.2×10^{-6}
Macom PLX 32-18 0.1 $\rightarrow$ 9.9 GHz multipl.	1.0×10^{-12}	-120.0 dB	1.2×10^{-6}
Omega DRV9R192-105F 9.2 GHz DRO	8.1×10^{-11}	-100.9 dB	1.1×10^{-5}
Narda DBP-0812N733 amplifier (9.9 GHz)	2.9×10^{-11}	-105.4 dB	6.3×10^{-6}
HP 8662A no. 1 synthesizer (100 MHz)	6.8×10^{-13}	-121.7 dB	9.7×10^{-7}
HP 8662A no. 2 synthesizer (100 MHz)	1.3×10^{-12}	-118.8 dB	1.4×10^{-6}
Fluke 6160B synthesizer	1.5×10^{-12}	-118.3 dB	1.5×10^{-6}
Racal Dana 9087B synthesizer (100 MHz)	8.4×10^{-12}	-110.8 dB	3.4×10^{-6}
Wenzel 500-02789D 100 MHz OCXO	4.7×10^{-12}	-113.3 dB	2.6×10^{-6}
Wenzel 501-04623E no. 1 100 MHz OCXO	2.0×10^{-13}	-127.1 dB	5.2×10^{-7}
Wenzel 501-04623E no. 2 100 MHz OCXO	1.5×10^{-13}	-128.2 dB	4.6×10^{-7}

worst

best

5 - Bridge method

Wheatstone bridge



equilibrium: $V_d = 0$ –> carrier suppression

static error δZ_1 –> some residual carrier

real δZ_1 => in-phase residual carrier $V_{re} \cos(\omega_0 t)$

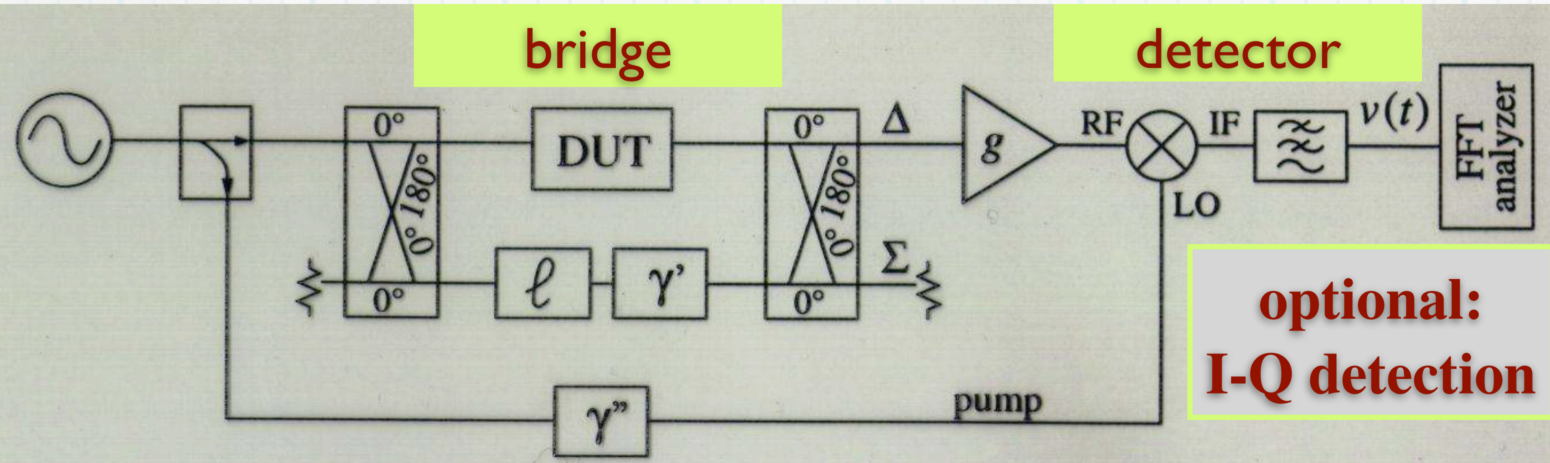
imaginary δZ_1 => quadrature residual carrier $V_{im} \sin(\omega_0 t)$

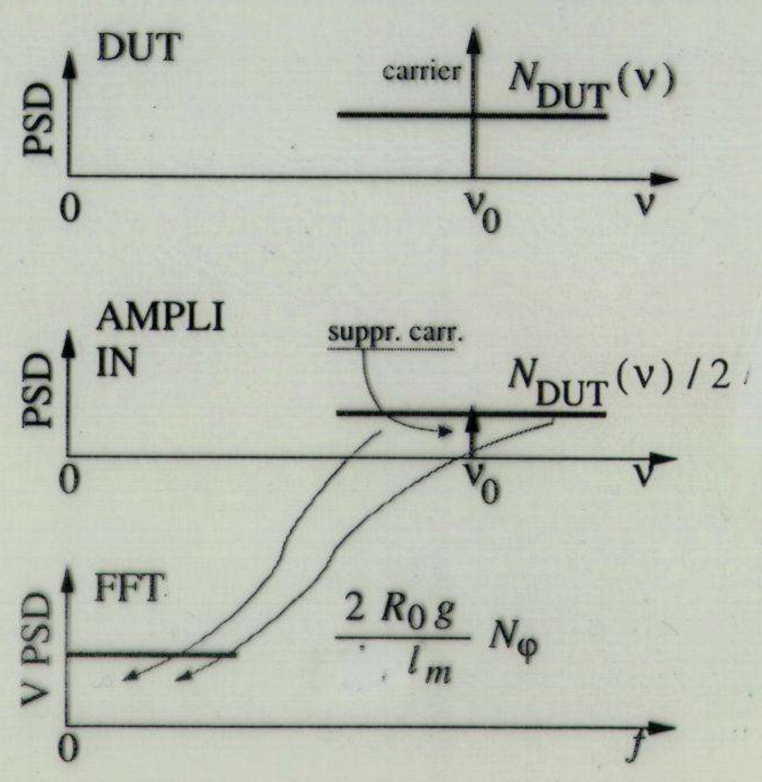
fluctuating error δZ_1 => noise sidebands

real δZ_1 => AM noise $v_c(t) \cos(\omega_0 t)$

imaginary δZ_1 => PM noise $-v_s(t) \sin(\omega_0 t)$

Bridge (interferometric) PM and AM noise measurement





High carrier suppression:

no carrier ⇒ the amplifier can't flicker

High gain:

$$k_\varphi = \frac{v(t)}{\varphi(t)} = \sqrt{\frac{R_0 g P_0}{l_m}} - \textit{dissip. losses}$$

Low Noise Floor:

$$S_{\varphi 0} = \frac{2Fk_B T_0}{P_0} + \textit{dissip. losses}$$

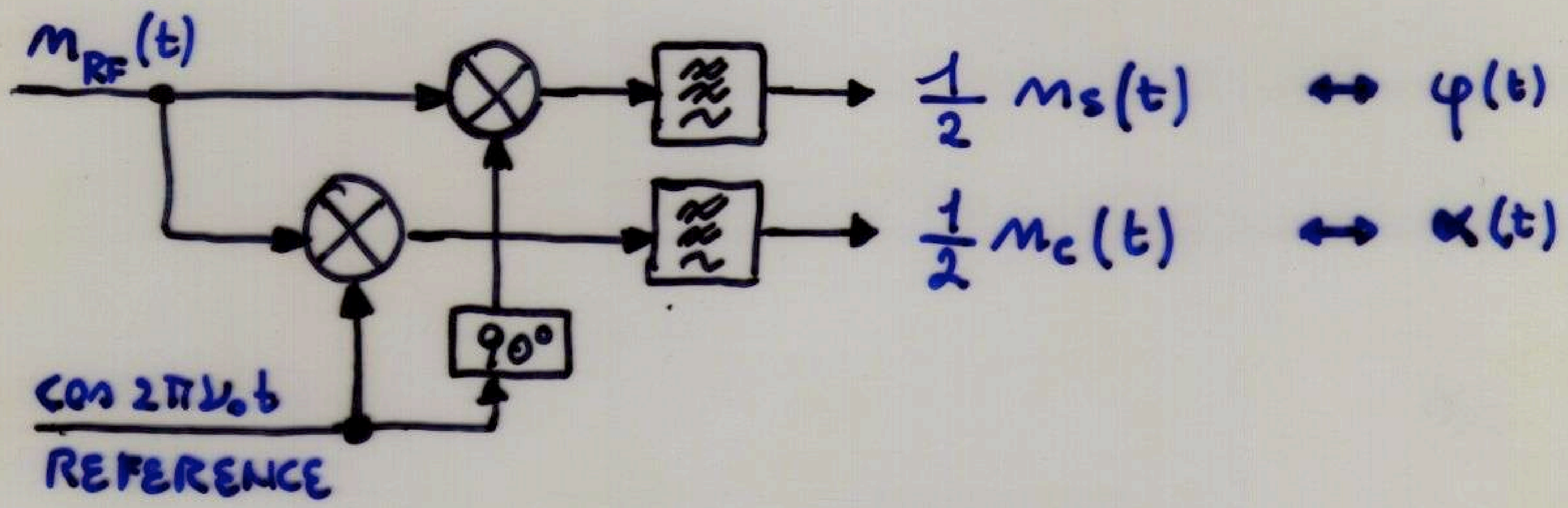
High immunity to low-f magnetic fields

Improved, from RSI **70** 1 pp. 220-225, Jan 1999

and rejection of the master-oscillator noise

yet, difficult for the measurement of oscillators

Synchronous detection



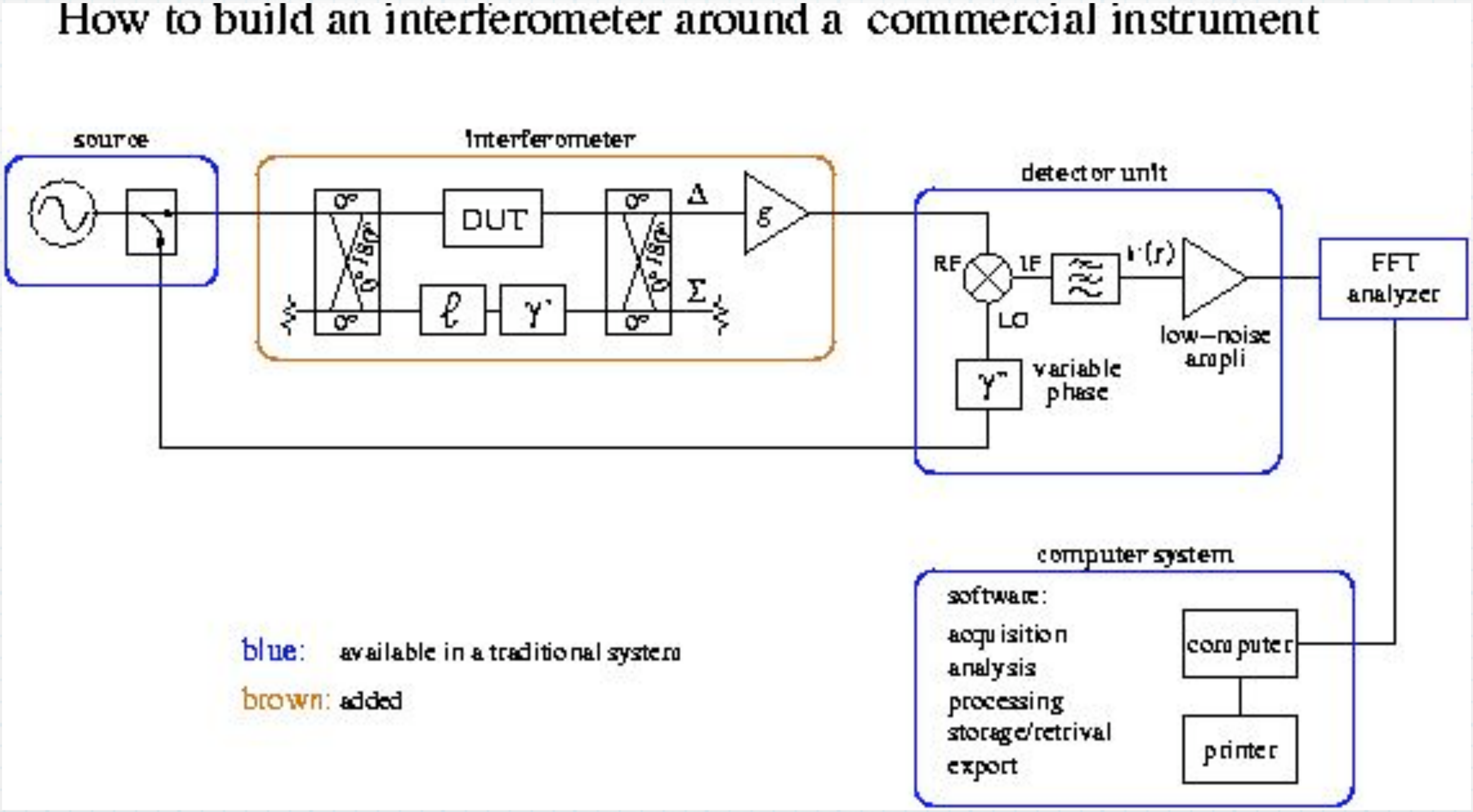
REMEMBER:

$$\cos^2 \omega t = \frac{1}{2}(1 + \cos 2\omega t)$$

$$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t)$$

$$\sin \omega t \cos \omega t = \frac{1}{2} \sin 2\omega t$$

A bridge (interferometric) instrument can be built around a commercial instrument

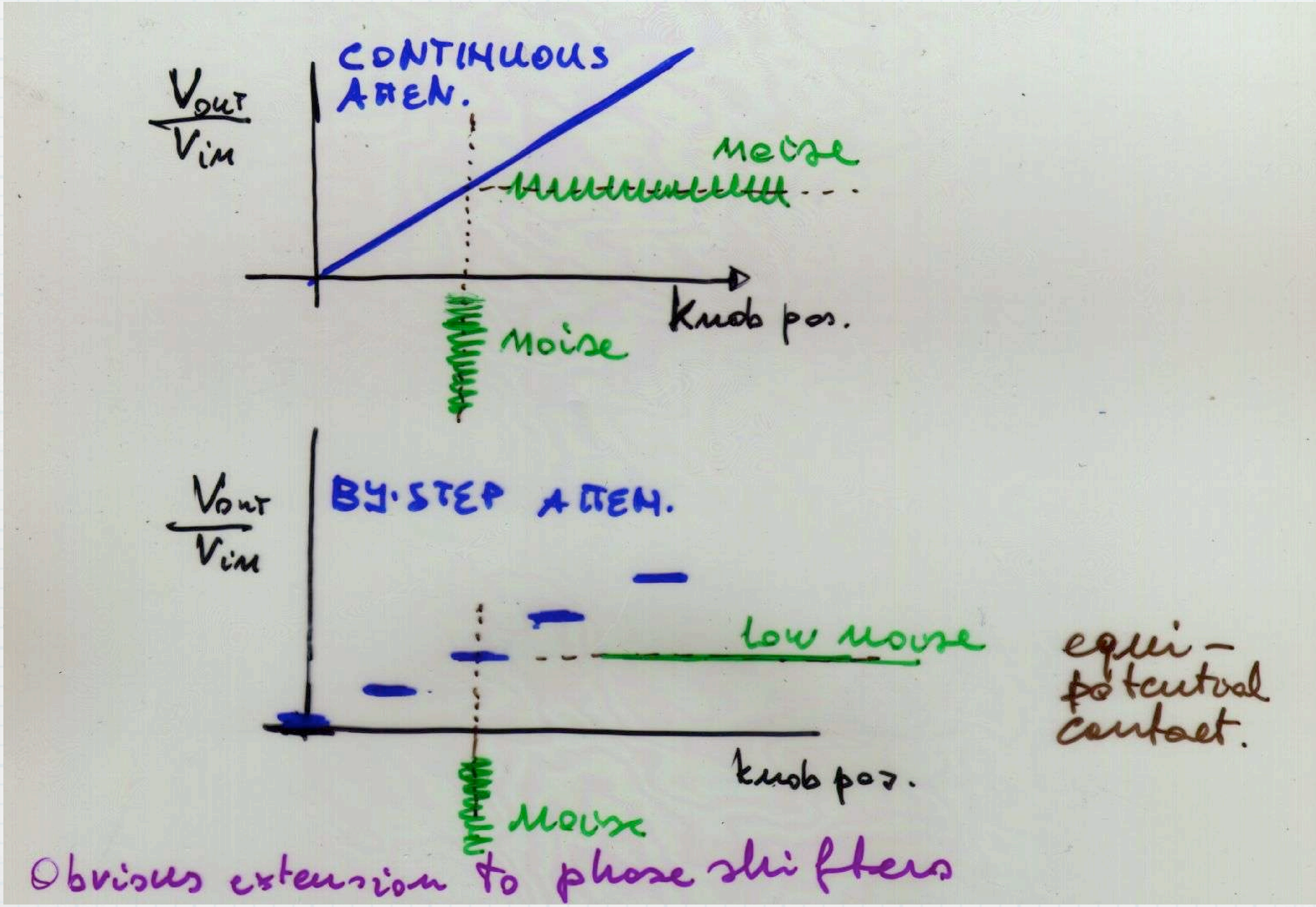


You will appreciate the computer interface and the software ready for use

6 - Advanced methods

Origin of flicker in the bridge

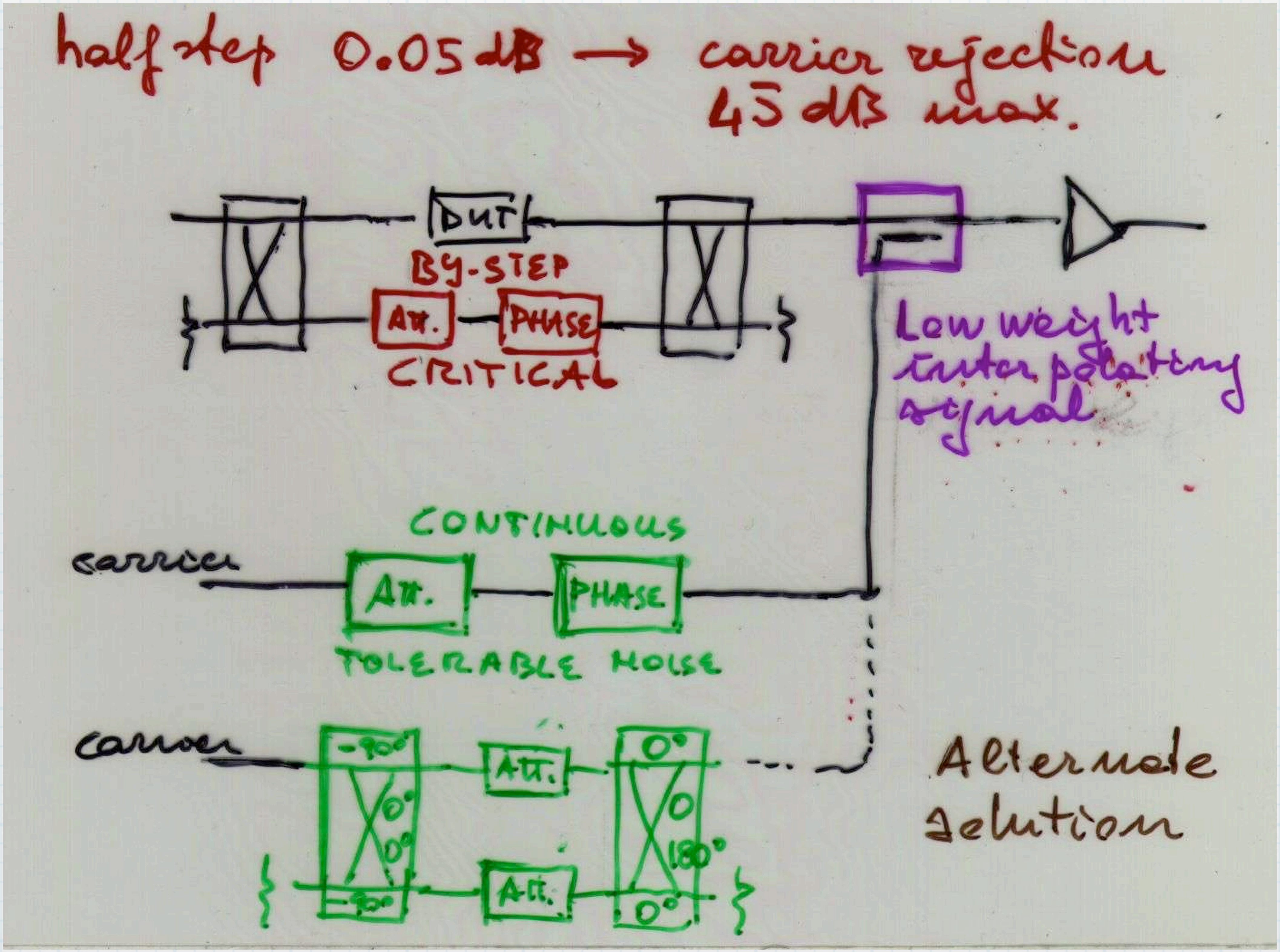
In the early time of electronics, flicker was called “contact noise”



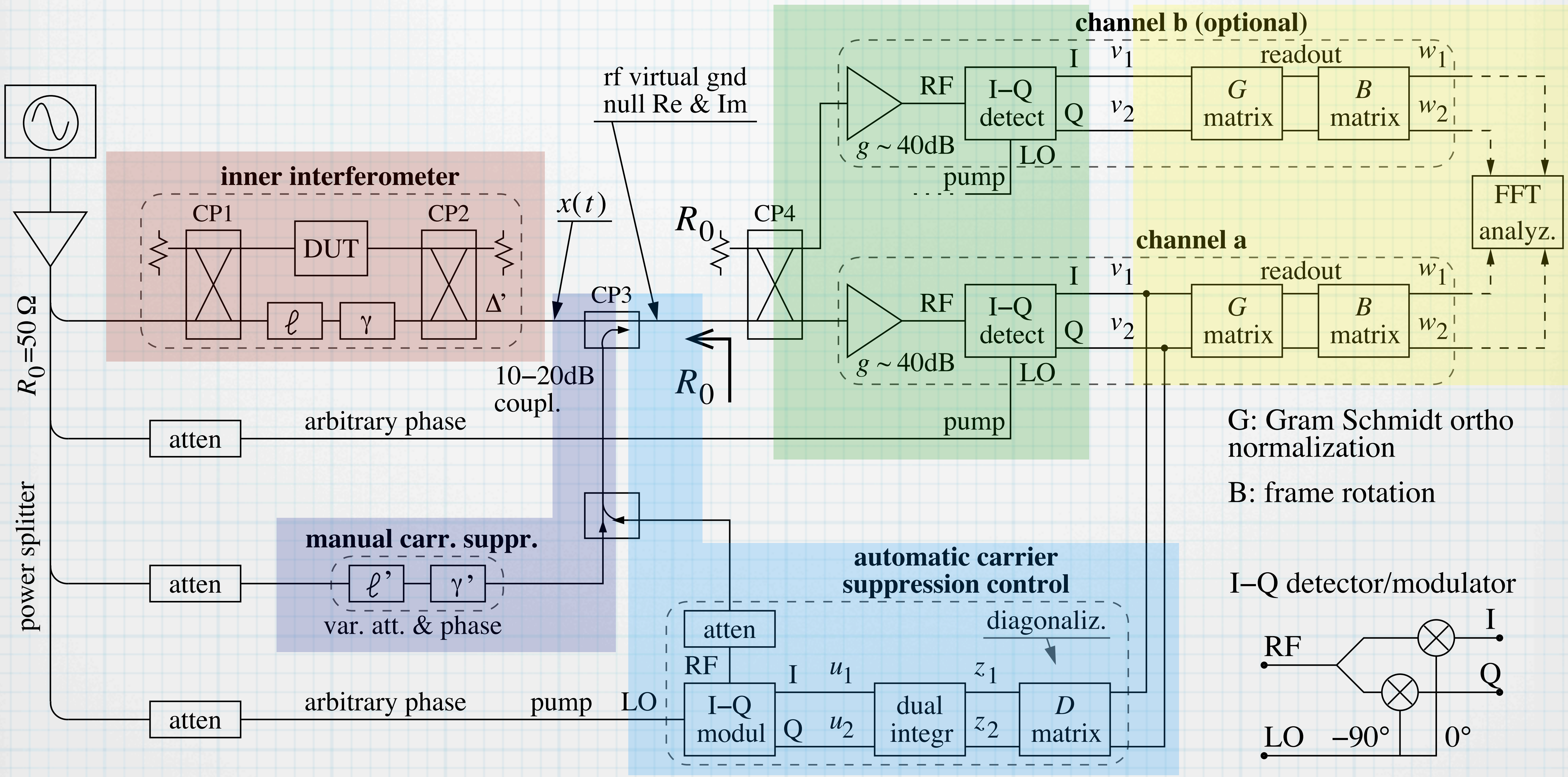
Coarse (by-step) and fine (continuous) adjustment of the bridge null are necessary

X

Coarse and fine adjustment of the bridge null are necessary

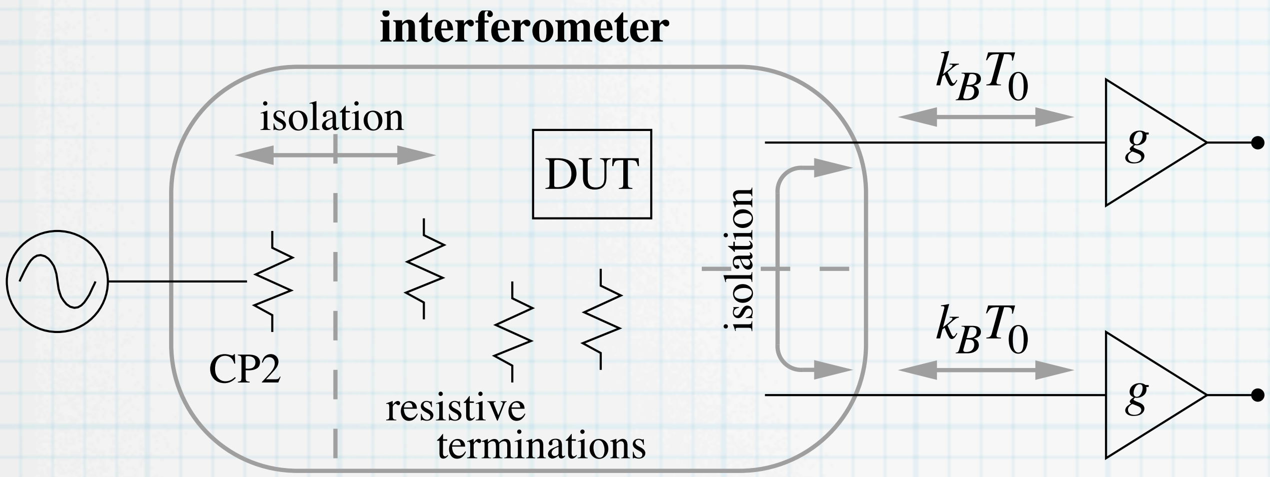


Flicker reduction, correlation, and closed-loop carrier suppression can be combined

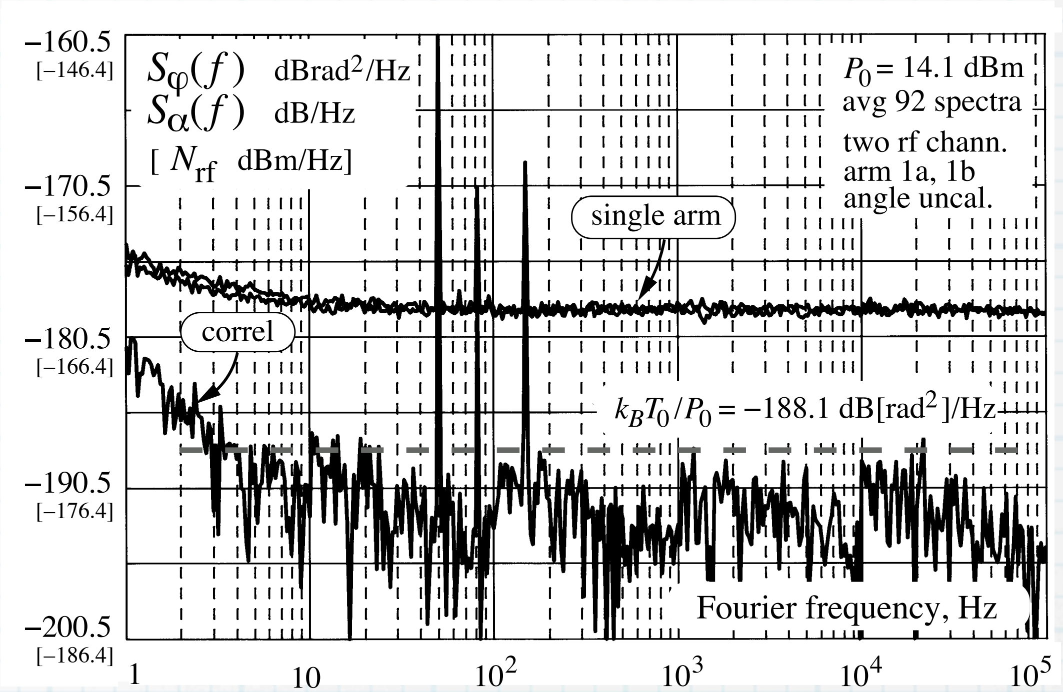


E. Rubiola, V. Giordano, Rev. Sci. Instrum. 73(6) pp.2445-2457, June 2002

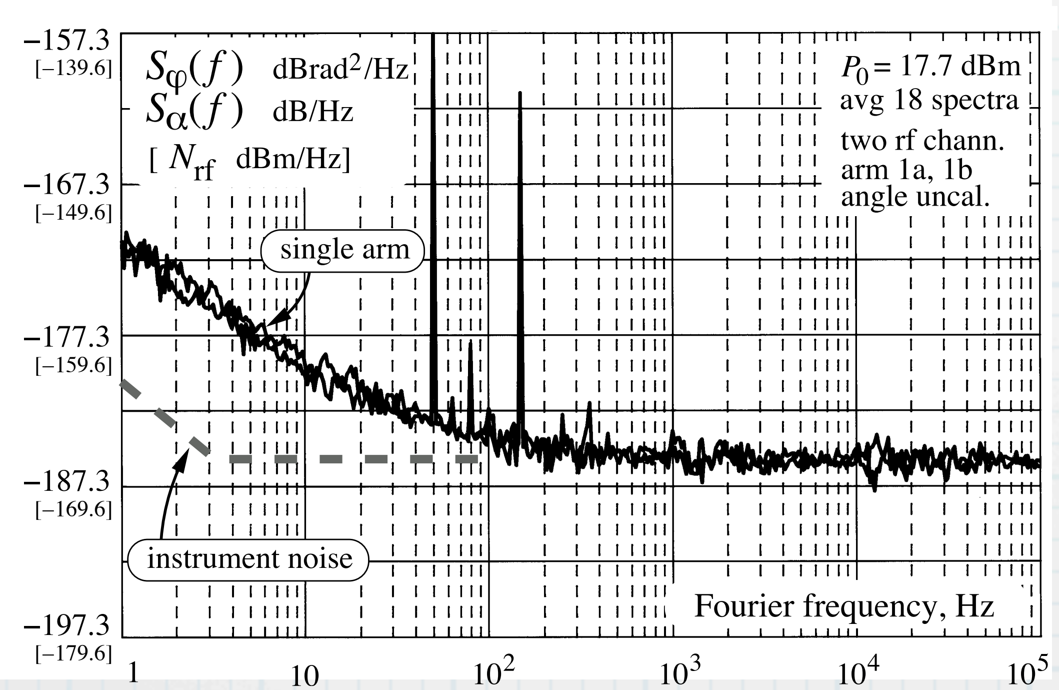
Example of results



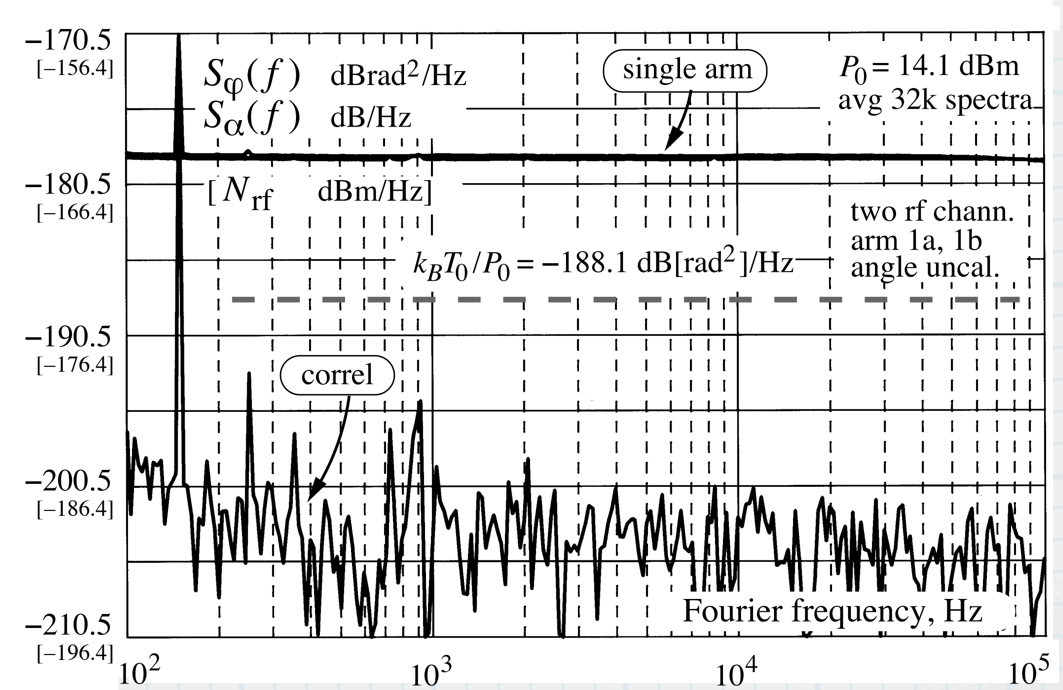
Correlation-and-averaging rejects the thermal noise



Residual noise of the fixed-value bridge, in the absence of the DUT

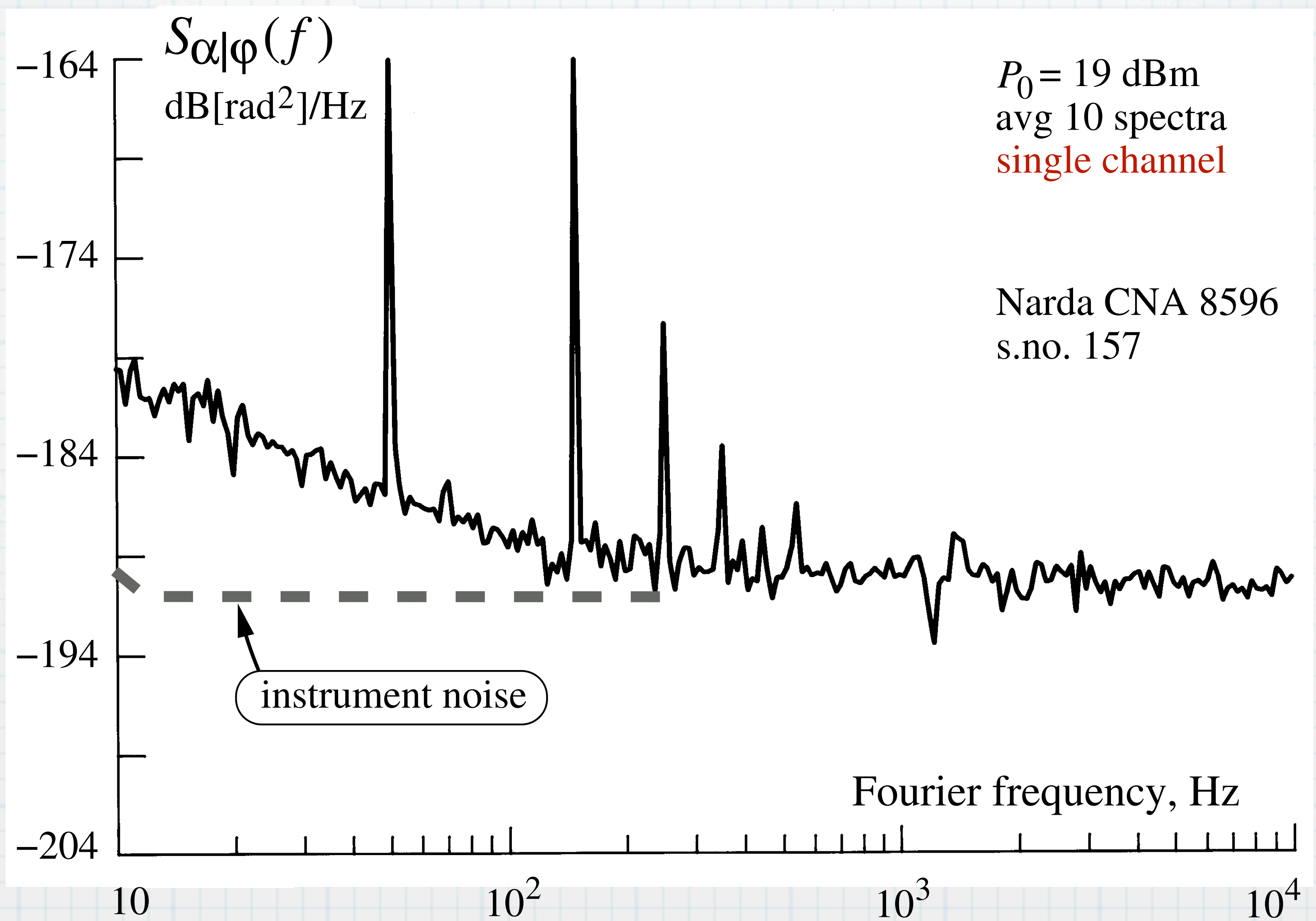


Noise of a pair of HH-109 hybrid couplers measured at 100 MHz



Residual noise of the fixed-value bridge. Same as above, but larger m

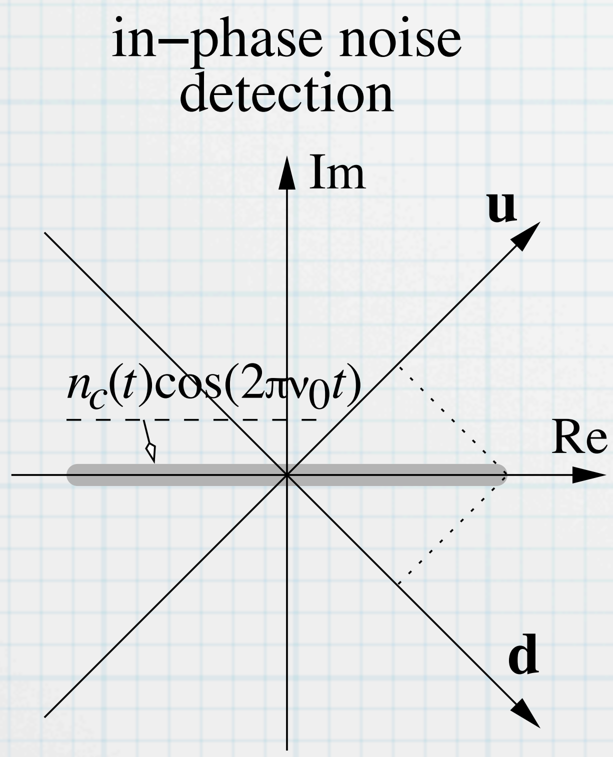
Microwave circulator in reverse mode (refers to the Pound scheme)

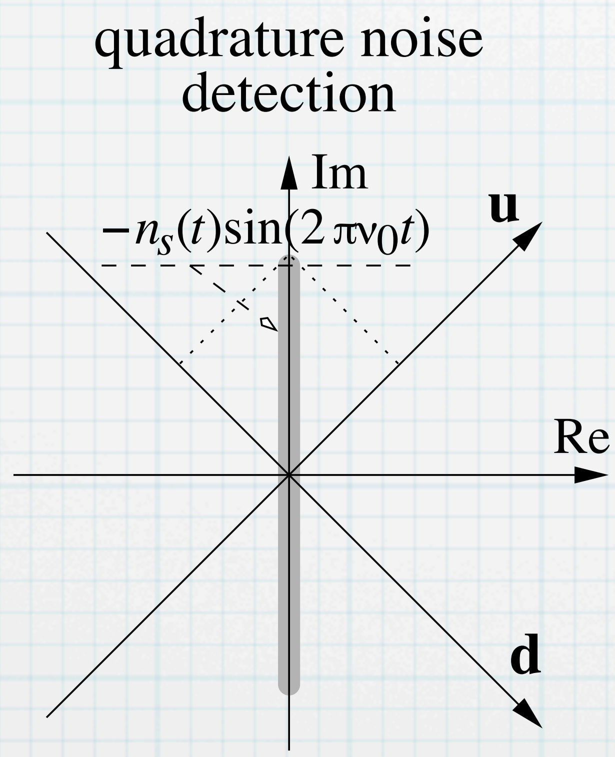


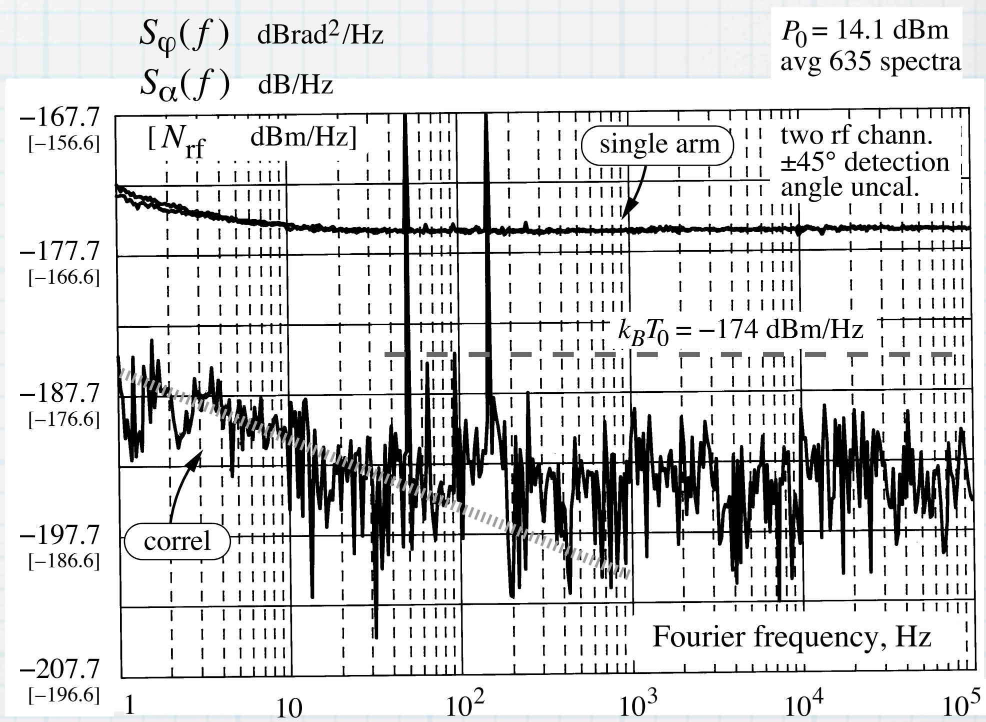
no post-processing is used to hide stray signals, like vibrations or the mains

±45° detection

DUT noise without carrier	$n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t$
UP reference	$u(t) = V_P\cos(\omega_0 t - \pi/4)$
DOWN reference	$d(t) = V_P\cos(\omega_0 t + \pi/4)$
cross spectral density	$S_{ud}(f) = \frac{1}{2}\left[S_\alpha(f) - S_\varphi(f)\right]$



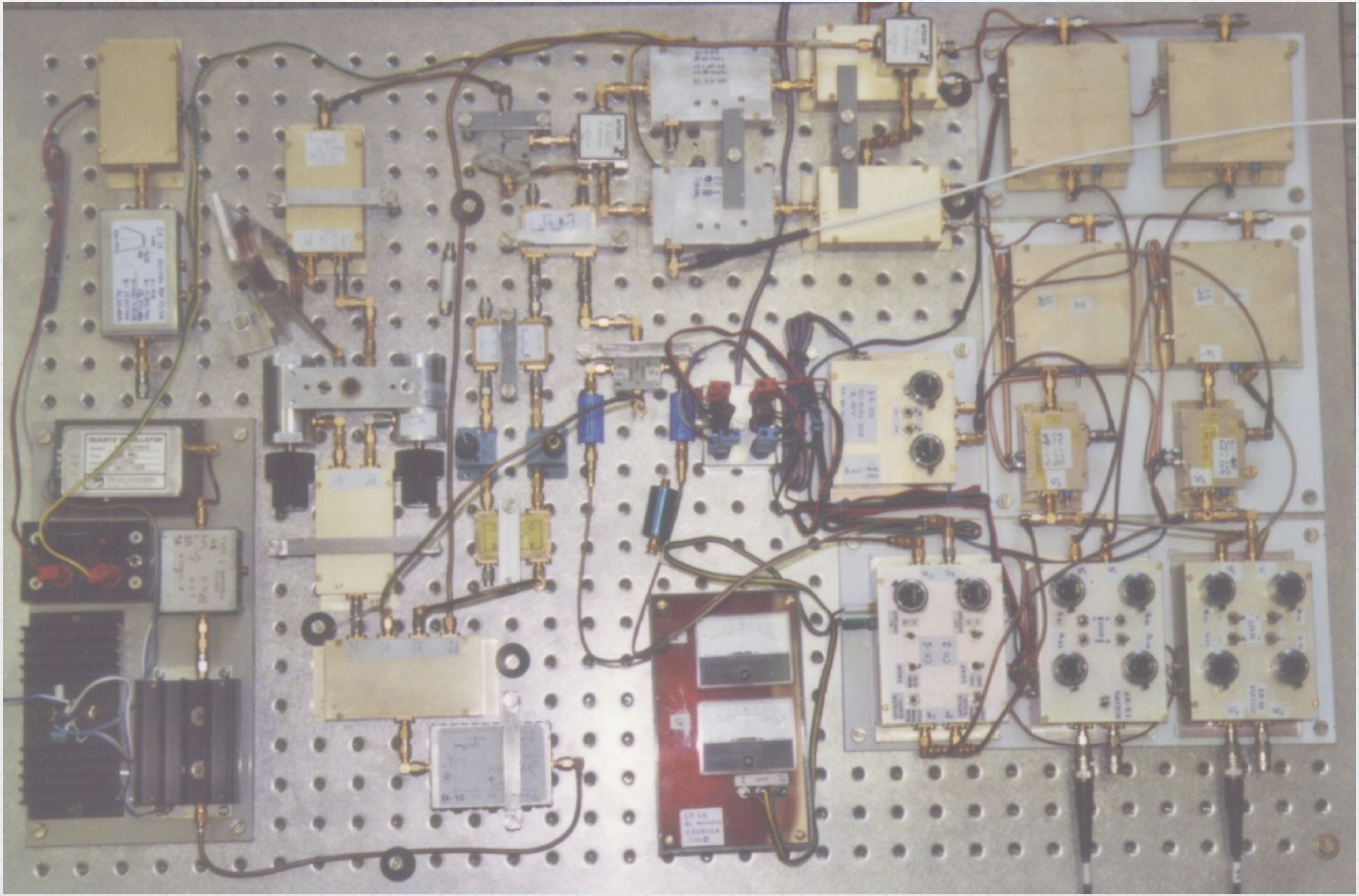




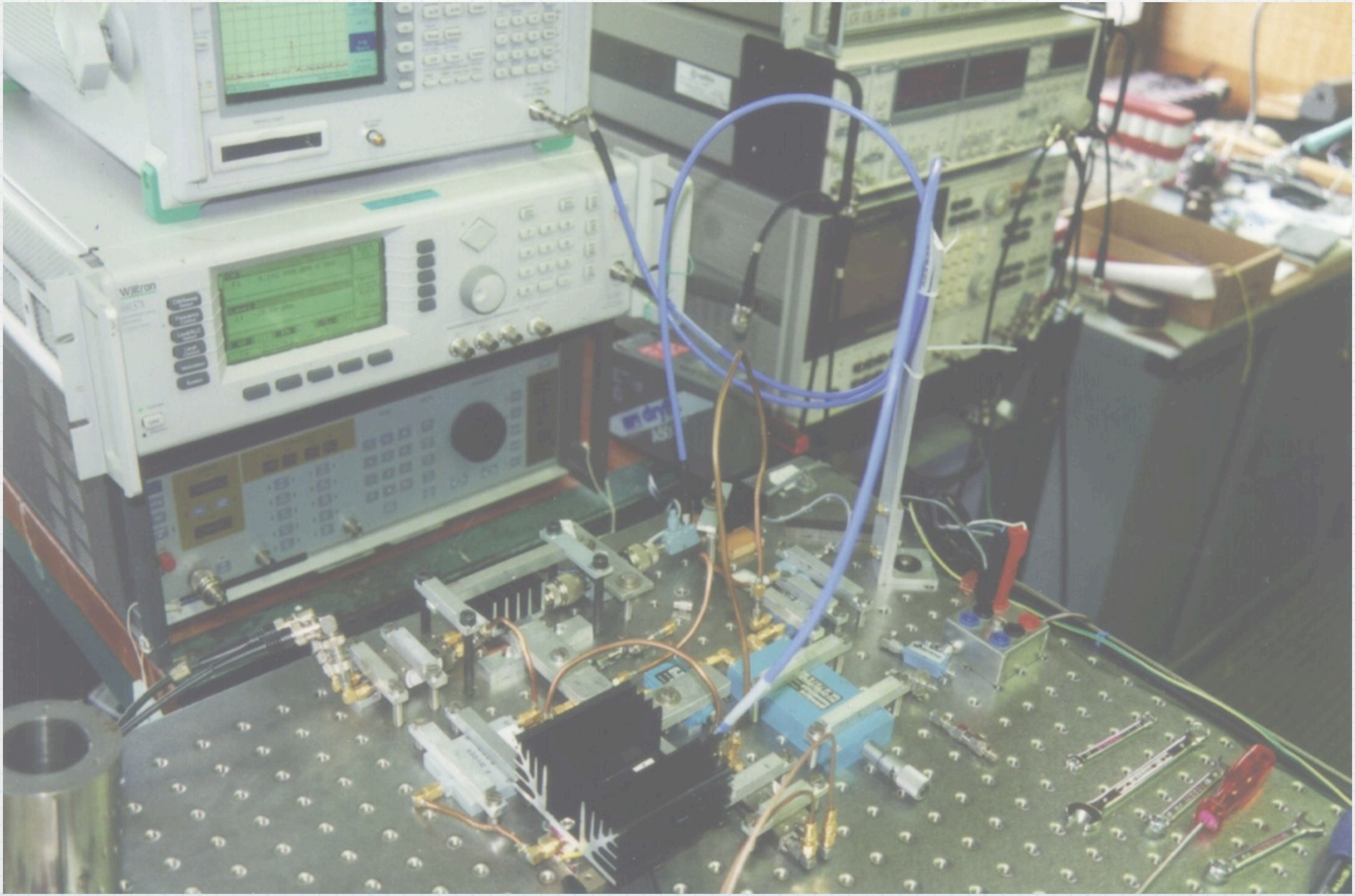
Residual noise, in the absence of the DUT

Smart and nerdish, yet of scarce practical usefulness
First used at 2 kHz to measure electromigration on metals (H. Stoll, MPI)

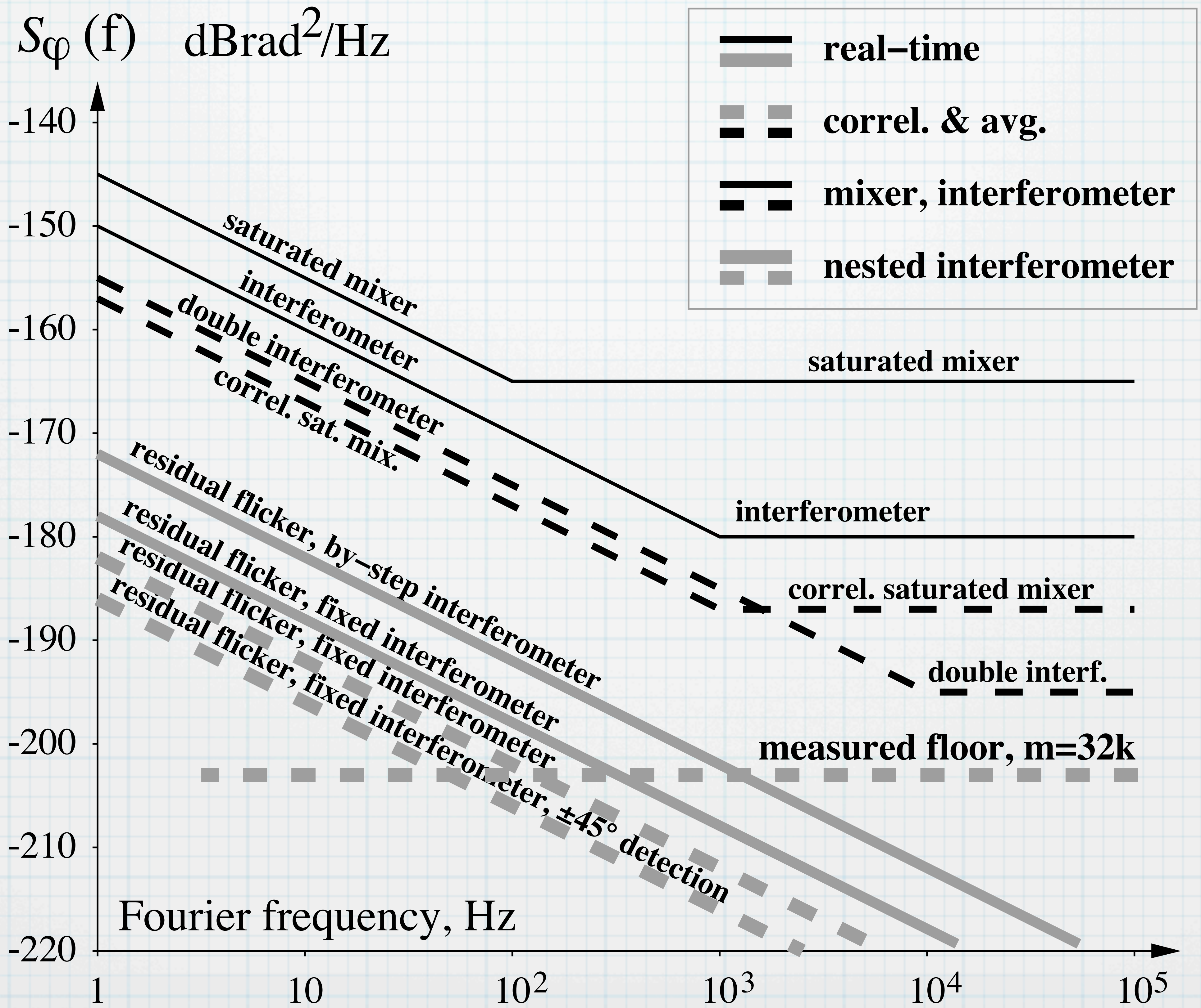
The complete machine (100 MHz)



A 9 GHz experiment (dc circuits not shown)



Comparison of the background noise



Mechanical stability

any flicker spectrum $S(f) = h_{-1}/f$ can be transformed

into the Allan variance $\sigma^2 = 2\ln(2)\, h_{-1}$

(roughly speaking, the integral over one octave)

a phase fluctuation is equivalent to a length fluctuation

$$L = \frac{\varphi}{2\pi}\lambda = \frac{\varphi}{2\pi}\,\frac{c}{\nu_0} \qquad S_L(f) = \frac{1}{4\pi^2}\,\frac{c^2}{\nu_0^2}\,S_\varphi(f)$$

-180 dBrad^2/Hz at $f = 1$ Hz and $\nu_0 = 9.2$ GHz ($c = 0.8\,c_0$) is equivalent to

$S_L = 1.73{\times}10^{-23}$ m^2/Hz ($\sqrt{S_L} = 4.16{\times}10^{-12}$ m/$\sqrt{\mathrm{H}z}$)

a residual flicker of -180 dBrad^2/Hz at $f = 1$ Hz off the $\nu_0 = 9.2$ GHz carrier

($h_{-1} = 1.73{\times}10^{-23}$) is equivalent to a mechanical stability

$$\sigma_L = \sqrt{1.38 \times 1.73{\times}10^{-23}} = 4.9{\times}10^{-12} \quad \mathrm{m}$$

 # don't think "that's only engineering" !!!

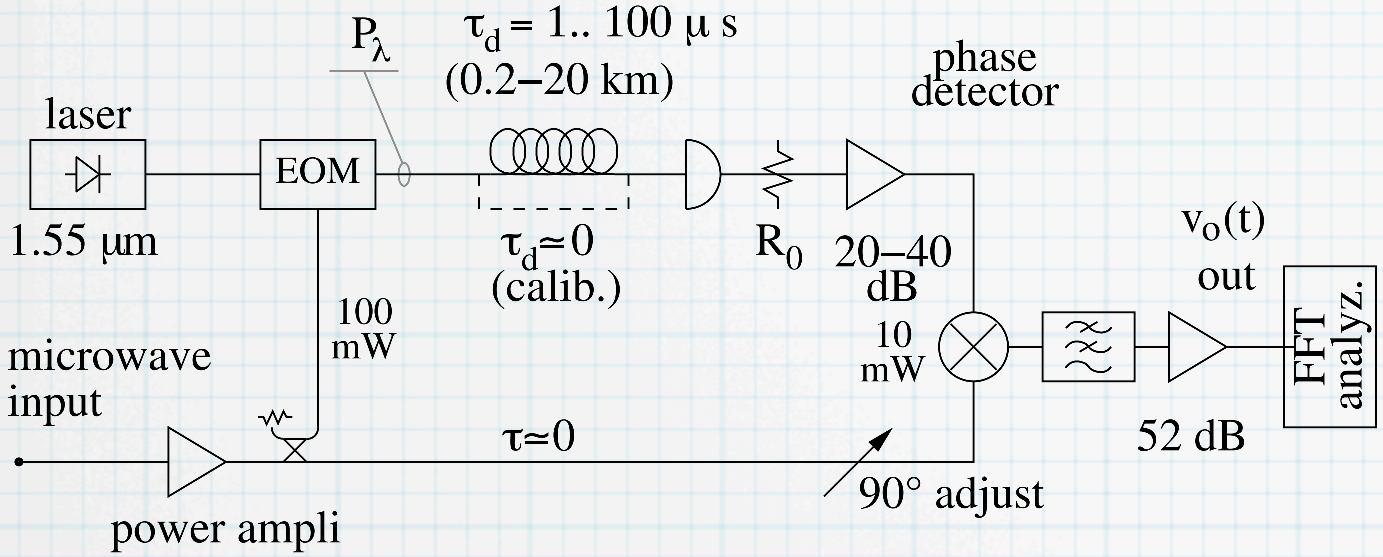
I learned a lot from non-optical microscopy

bulk solid matter is that stable

7 - Optical delay line

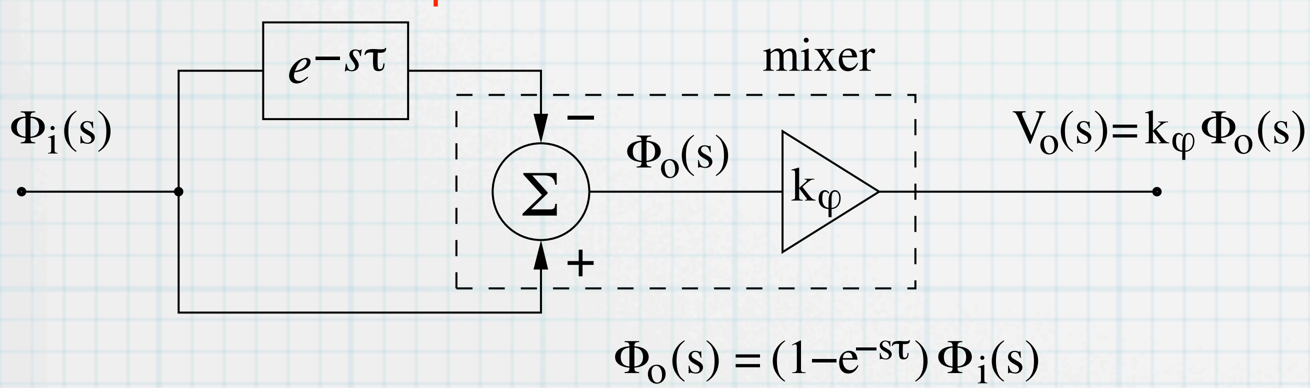
Delay line theory

Rubiola-Salik-Huang-Yu-Maleki, JOSA-B 22(5) p.987–997 (2005)



Note that here one arm is a microwave cable

Laplace transforms



- **delay –> frequency-to-phase conversion**
- **works at any frequency**
- **long delay (microseconds) is necessary for high sensitivity**
- **the delay line must be an optical fiber**
  fiber: attenuation 0.2 dB/km, thermal coeff. 6.8 10^{-6}/K
  cable: attenuation 0.8 dB/m, thermal coeff. ~ 10^{-3}/K

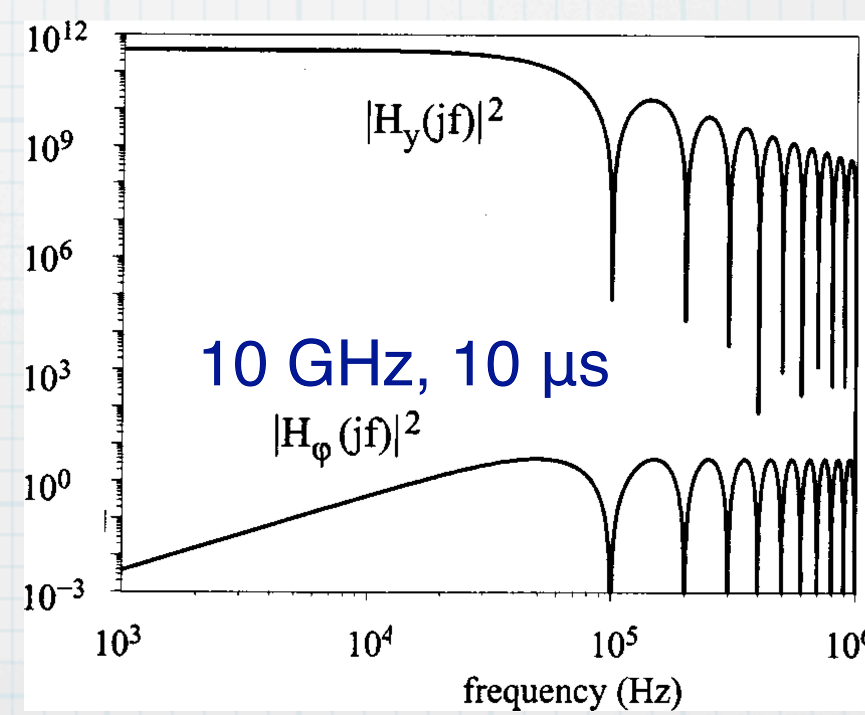
Laplace transforms

$$\Phi(s) = H_\varphi(s)\Phi_i(s)$$

$$|H_\varphi(f)|^2 = 4\sin^2(\pi f \tau)$$

$$S_y(f) = |H_y(f)|^2\, S_{\varphi\, i}(s)$$

$$|H_y(f)|^2 = \frac{4\nu_0^2}{f^2}\sin^2(\pi f \tau)$$



White noise

intensity modulation $$P(t) = \overline{P}(1 + m\cos\omega_\mu t)$$

photocurrent $$i(t) = \frac{q\eta}{h\nu}\,\overline{P}(1 + m\cos\omega_\mu t)$$

microwave power $$\overline{P}_\mu = \frac{1}{2}\,m^2 R_0 \left(\frac{q\eta}{h\nu}\right)^2 P^2$$

shot noise $$N_s = 2\frac{q^2\eta}{h\nu}\,\overline{P}R_0$$

thermal noise $$N_t = FkT_0$$

total white noise (one detector)

$$S_{\varphi 0} = \frac{2}{m^2}\left[\underbrace{2\frac{h\nu_\lambda}{\eta}\,\frac{1}{\overline{P}}}_{\text{shot}} + \underbrace{\frac{FkT_0}{R_0}\left(\frac{h\nu_\lambda}{q\eta}\right)^2\left(\frac{1}{\overline{P}}\right)^2}_{\text{thermal}}\right]$$

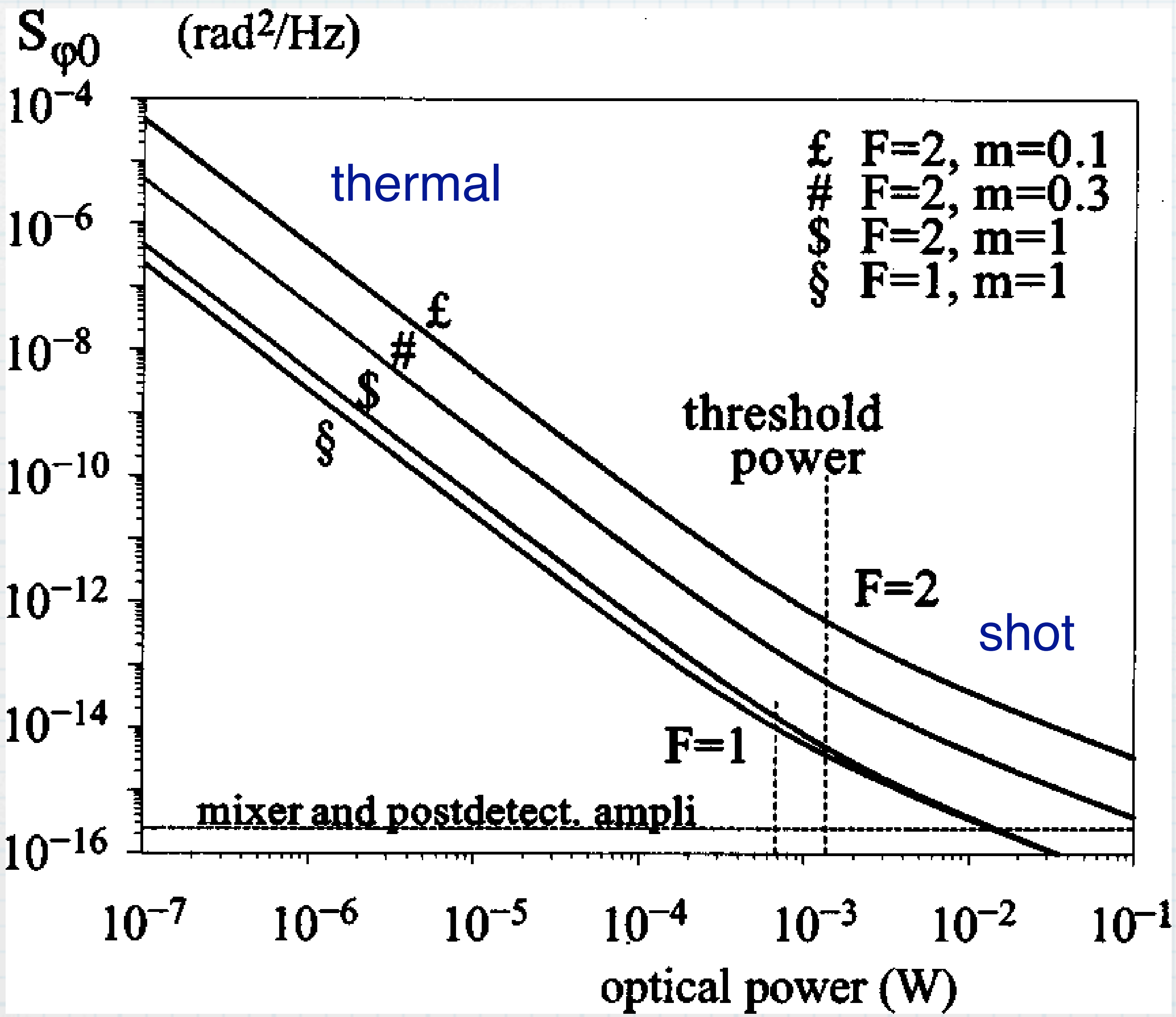
total white noise (P/2 each detector)

$$S_{\varphi 0} = \frac{16}{m^2}\left[\frac{h\nu_\lambda}{\eta}\,\frac{1}{\overline{P}} + \frac{FkT_0}{R_0}\left(\frac{h\nu_\lambda}{q\eta}\right)^2\left(\frac{1}{\overline{P}}\right)^2\right]$$

Threshold power

$$S_{\varphi 0} = \frac{16}{m^2}\left[\frac{h\nu_\lambda}{\eta}\,\frac{1}{\overline{\overline{P}}} + \frac{FkT_0}{R_0}\left(\frac{h\nu_\lambda}{q\eta}\right)^2\left(\frac{1}{\overline{\overline{P}}}\right)^2\right]$$

holds for two detectors

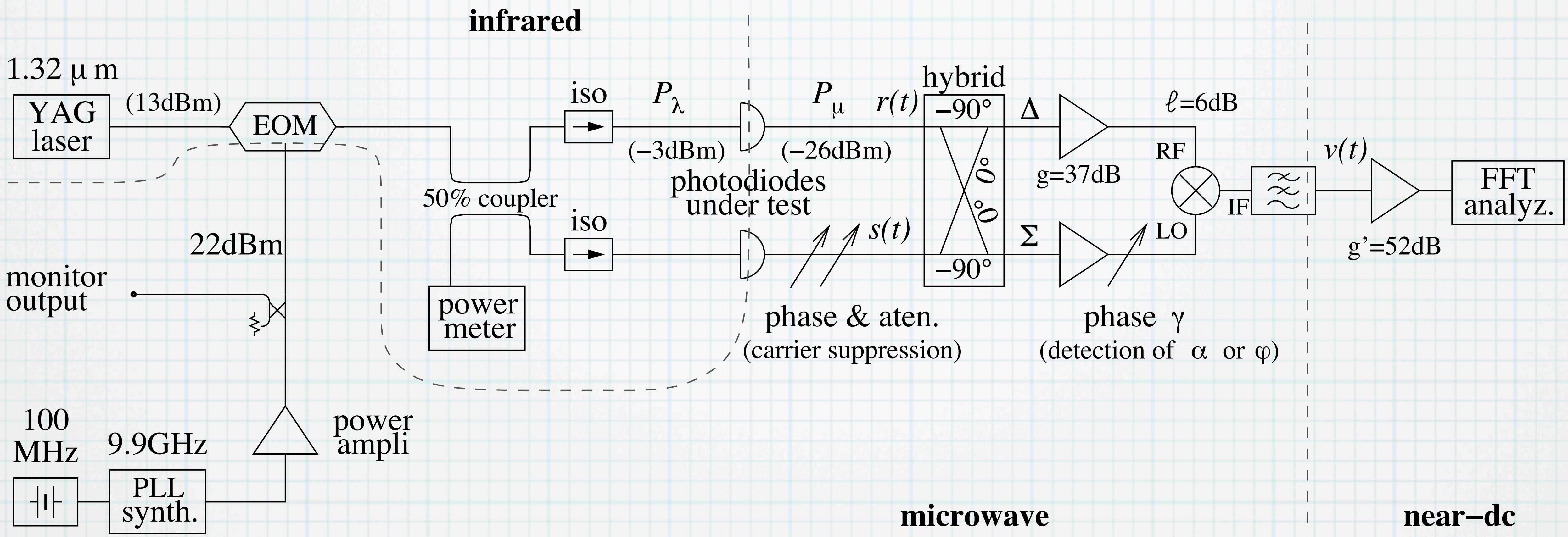


threshold power

$$P_{\lambda,t} = \frac{FkT_0}{R_0}\,\frac{h\nu_\lambda}{q^2\eta}$$

new high-power photodetectors 5–10 mW

Photodetector 1/f noise (1)



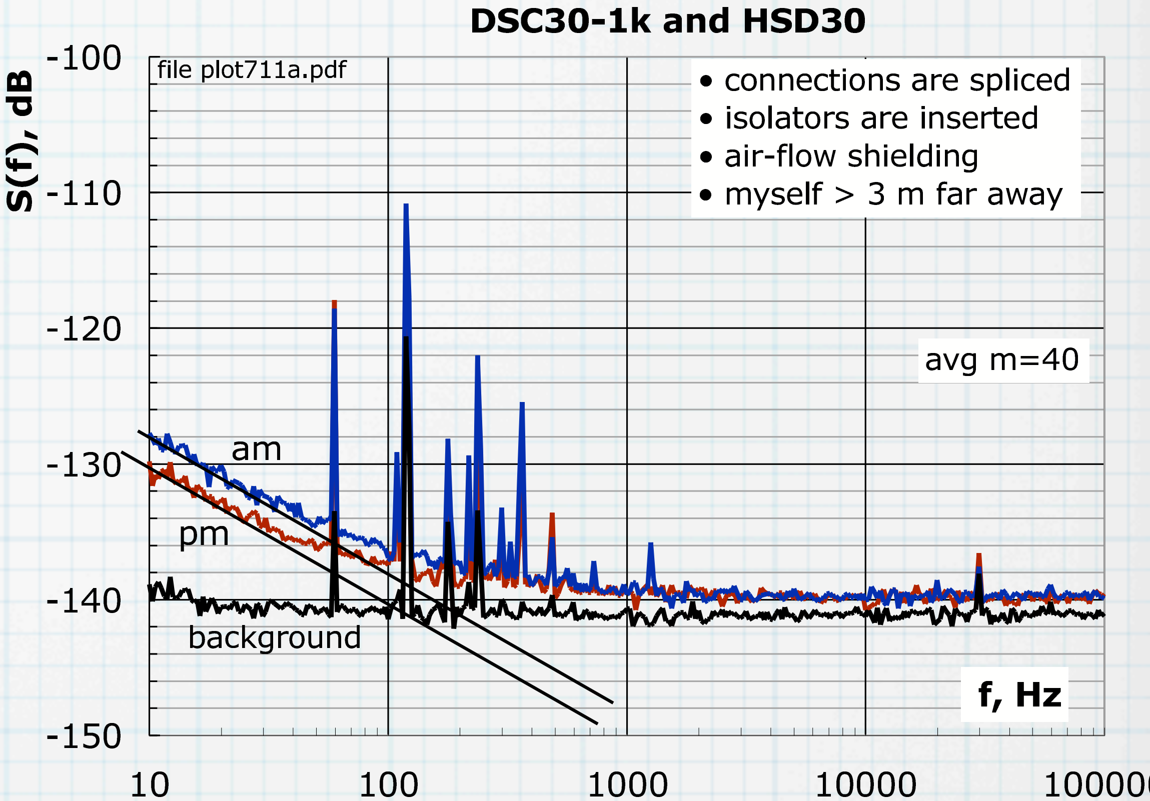
photodiode	$S_\alpha(1\,\text{Hz})$ estimate	$S_\alpha(1\,\text{Hz})$ uncertainty	$S_\varphi(1\,\text{Hz})$ estimate	$S_\varphi(1\,\text{Hz})$ uncertainty
HSD30	−122.7	−7.1 +3.4	−127.6	−8.6 +3.6
DSC30-1K	−119.8	−3.1 +2.4	−120.8	−1.8 +1.7
QDMH3	−114.3	−1.5 +1.4	−120.2	−1.7 +1.6
unit	dB/Hz	dB	dBrad^2/Hz	dB

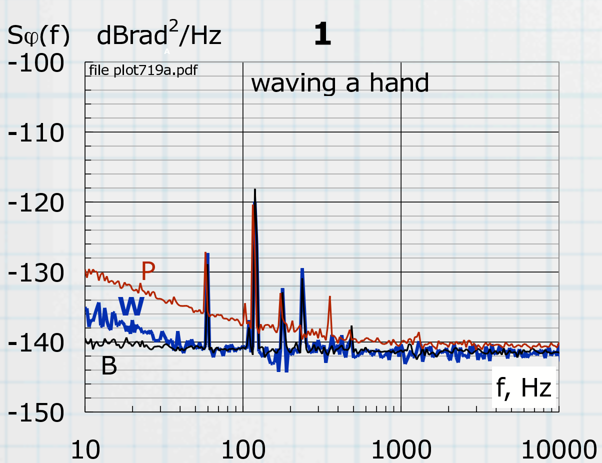
The noise of the Σ amplifier is not detected Electron. Lett. **39** 19 p. 1389 (2003)

X

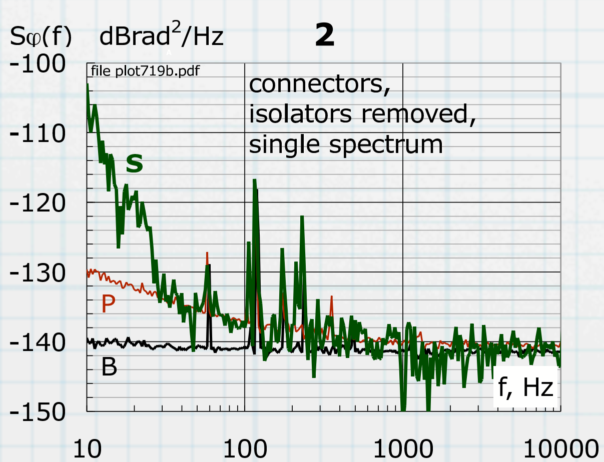
Photodetector 1/f noise (2)

- the photodetectors we measured are similar in AM and PM 1/f noise
- the 1/f noise is about -120 dB[rad²]/Hz
- other effects are easily mistaken for the photodetector 1/f noise
- environment and packaging deserve attention in order to take the full benefit from the low noise of the junction

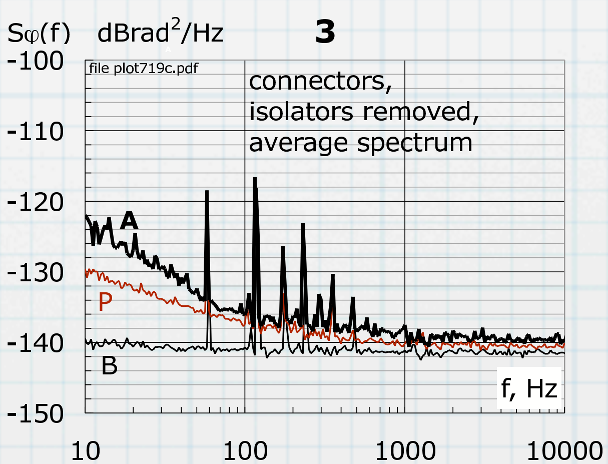




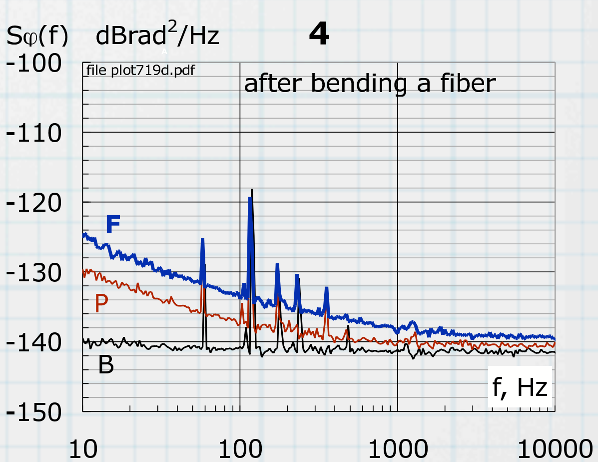
W: waving a hand 0.2 m/s, 3 m far from the system
B: background noise
P: photodiode noise



S: single spectrum, with optical connectors and no isolators
B: background noise
P: photodiode noise



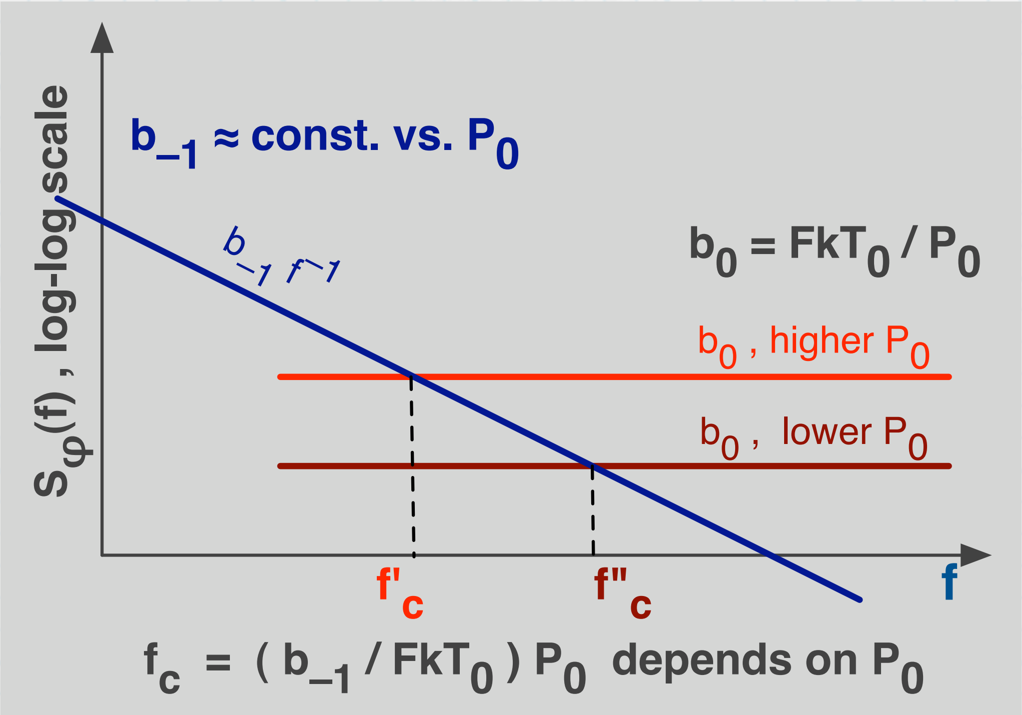
A: average spectrum, with optical connectors and no isolators
B: background noise
P: photodiode noise



F: after bending a fiber, 1/f noise can increase unpredictably
B: background noise
P: photodiode noise

Flicker (1/f) noise

- **experimentally determined (takes skill, time and patience)**
- **amplifier GaAs: $b_{-1} \approx -100$ to -110 dBrad²/Hz,**
  SiGe: $b_{-1} \approx -120$ dBrad²/Hz
- **photodetector $b_{-1} \approx -120$ dBrad²/Hz**
  Rubiola & al. IEEE Trans. MTT (& JLT) 54(2) p.816–820 (2006)
- **mixer $b_{-1} \approx -120$ dBrad²/Hz**
- **contamination from AM noise (delay => de-correlation => no sweet point**
  (Rubiola-Boudot, IEEE Transact UFFC 54(5) p.926–932 (2007)
- **optical fiber**
- **The phase flicker coefficient b_{-1} is about independent of power**
- **in a cascade, $(b_{-1})_{tot}$ adds up, regardless of the device order**



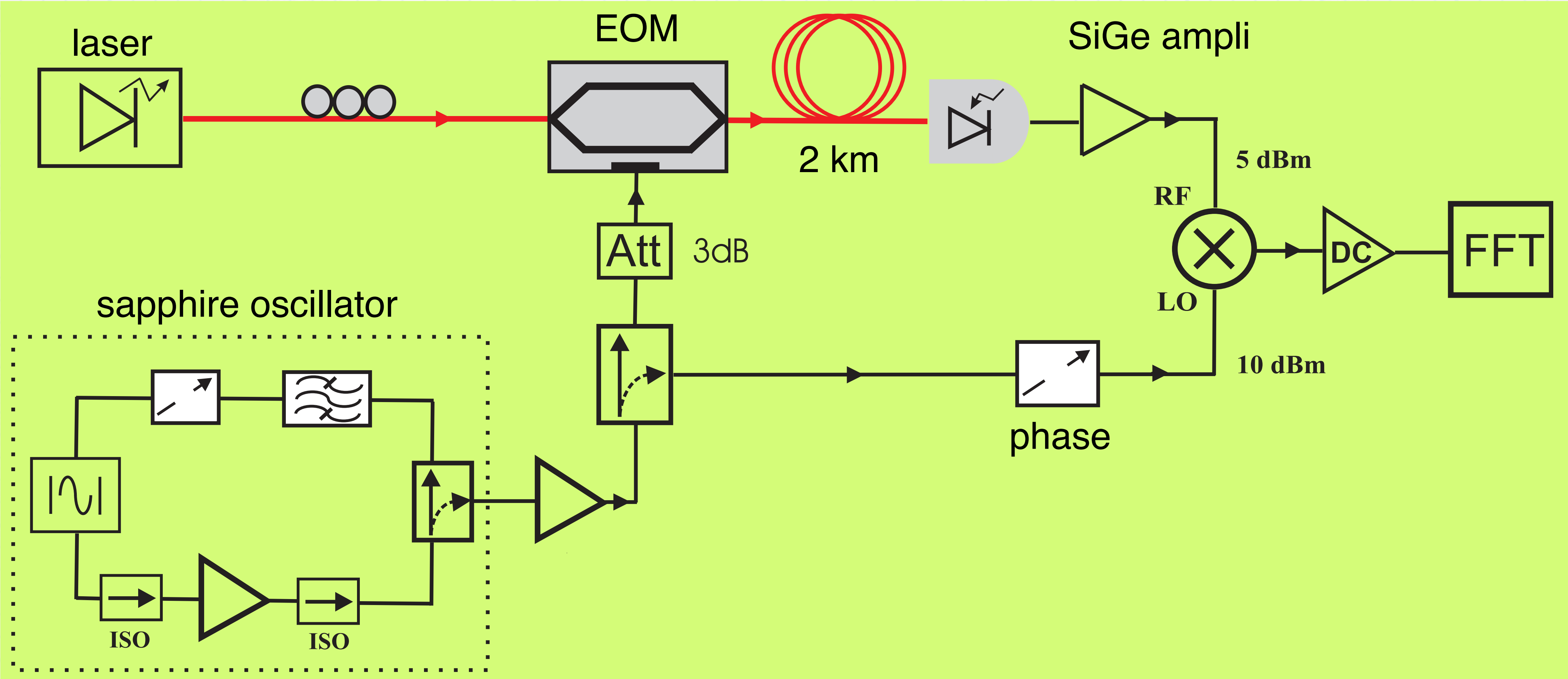
The Friis formula applies to white phase noise

$$b_0 = \frac{F_1 k T_0}{P_0} + \frac{(F_2 - 1) k T_0}{P_0 g_1^2} + \dots$$

In a cascade, the 1/f noise just adds up

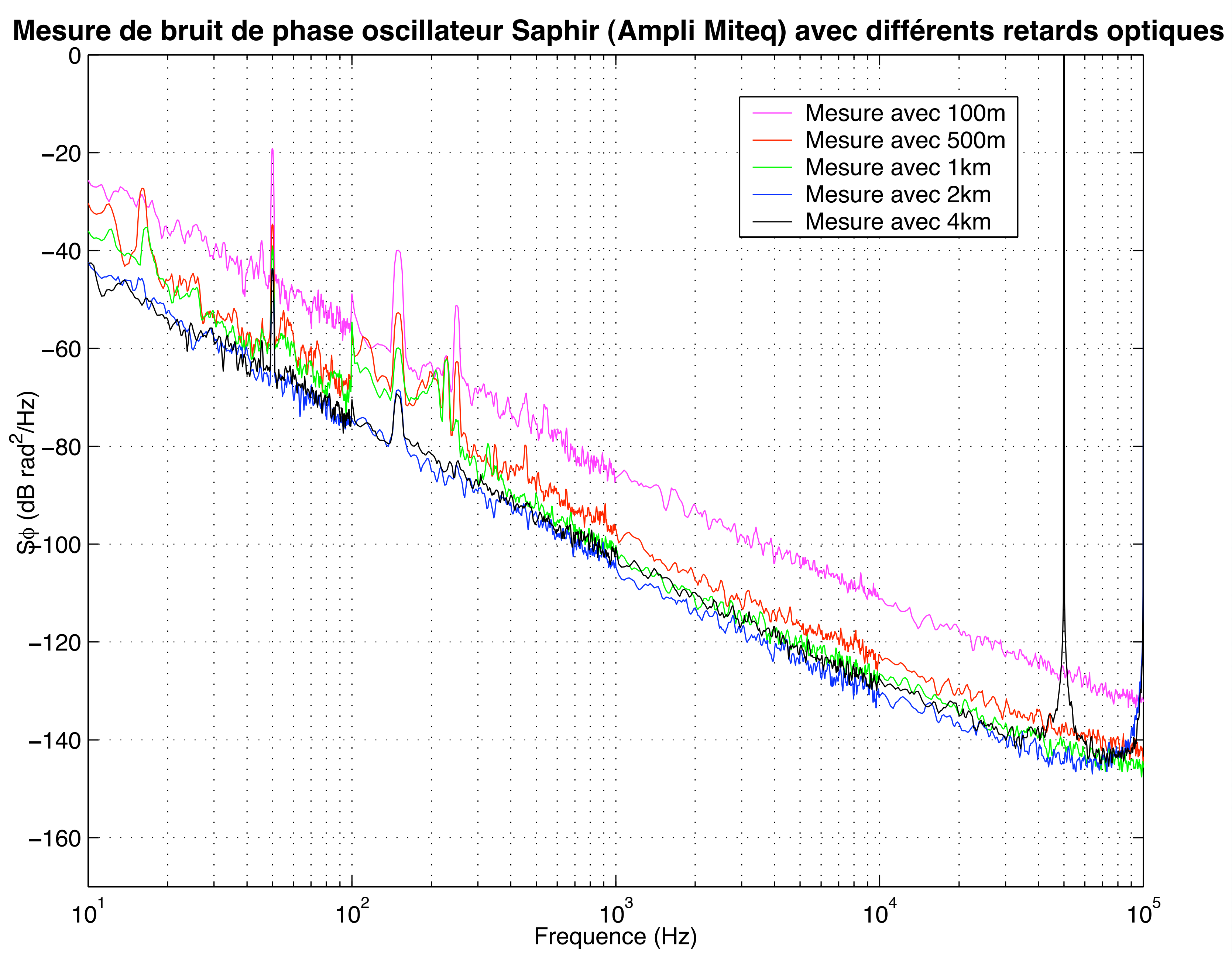
$$(b_{-1})_{\text{tot}} = \sum_{i=1}^{m} (b_{-1})_i$$

Single-channel instrument



- The laser RIN can limit the instrument sensitivity
- In some cases, the AM noise of the oscillator under test turns into a serious problem (got in trouble with an Anritsu synthesizer)

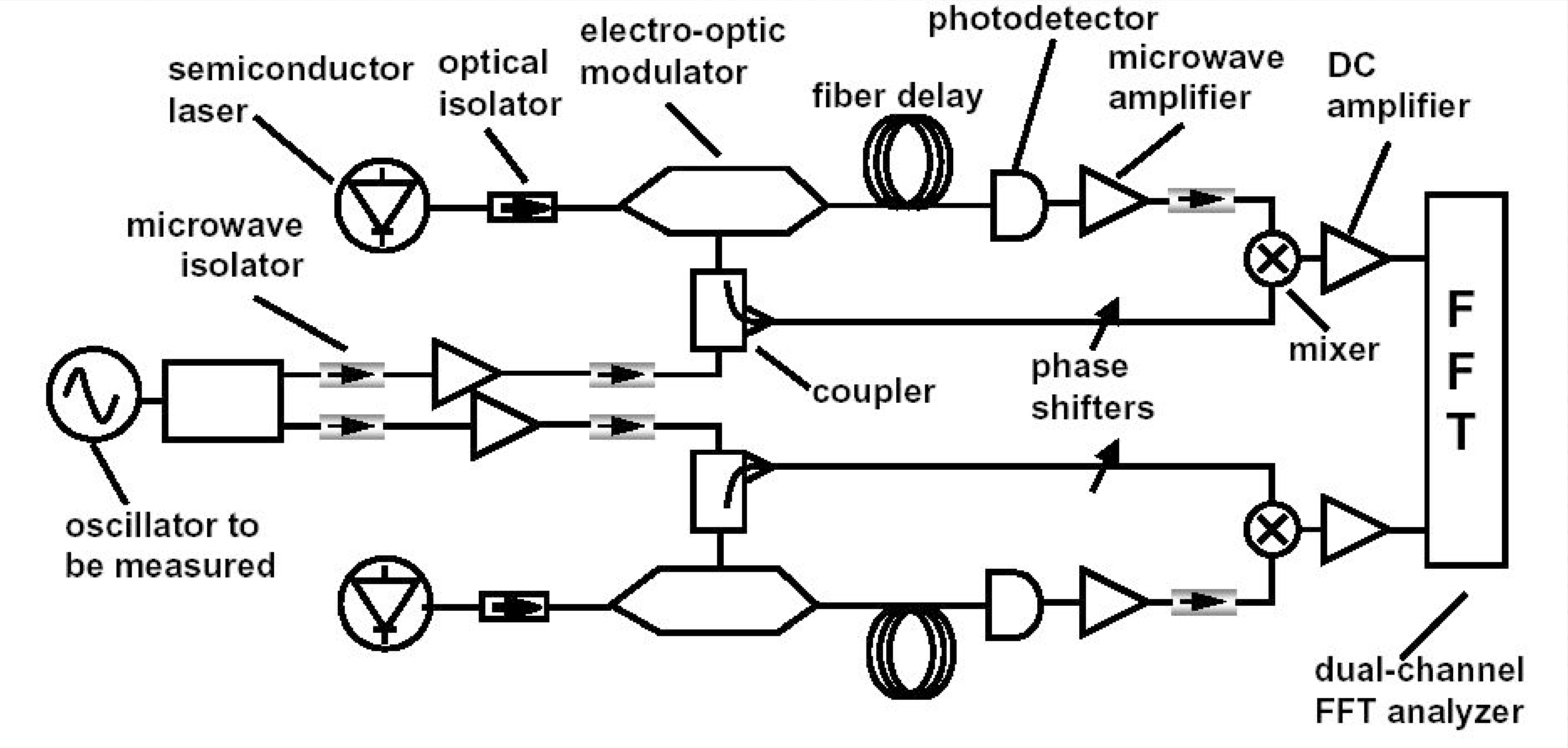
Measurement of a sapphire oscillator



- The instrument noise scales as 1/τ, yet the blue and black plots overlap
  magenta, red, green => instrument noise
  blue, black => noise of the sapphire oscillator under test
- We can measure the $1/f^3$ phase noise (frequency flicker) of a 10 GHz sapphire oscillator (the lowest-noise microwave oscillator)
- Low AM noise of the oscillator under test is necessary

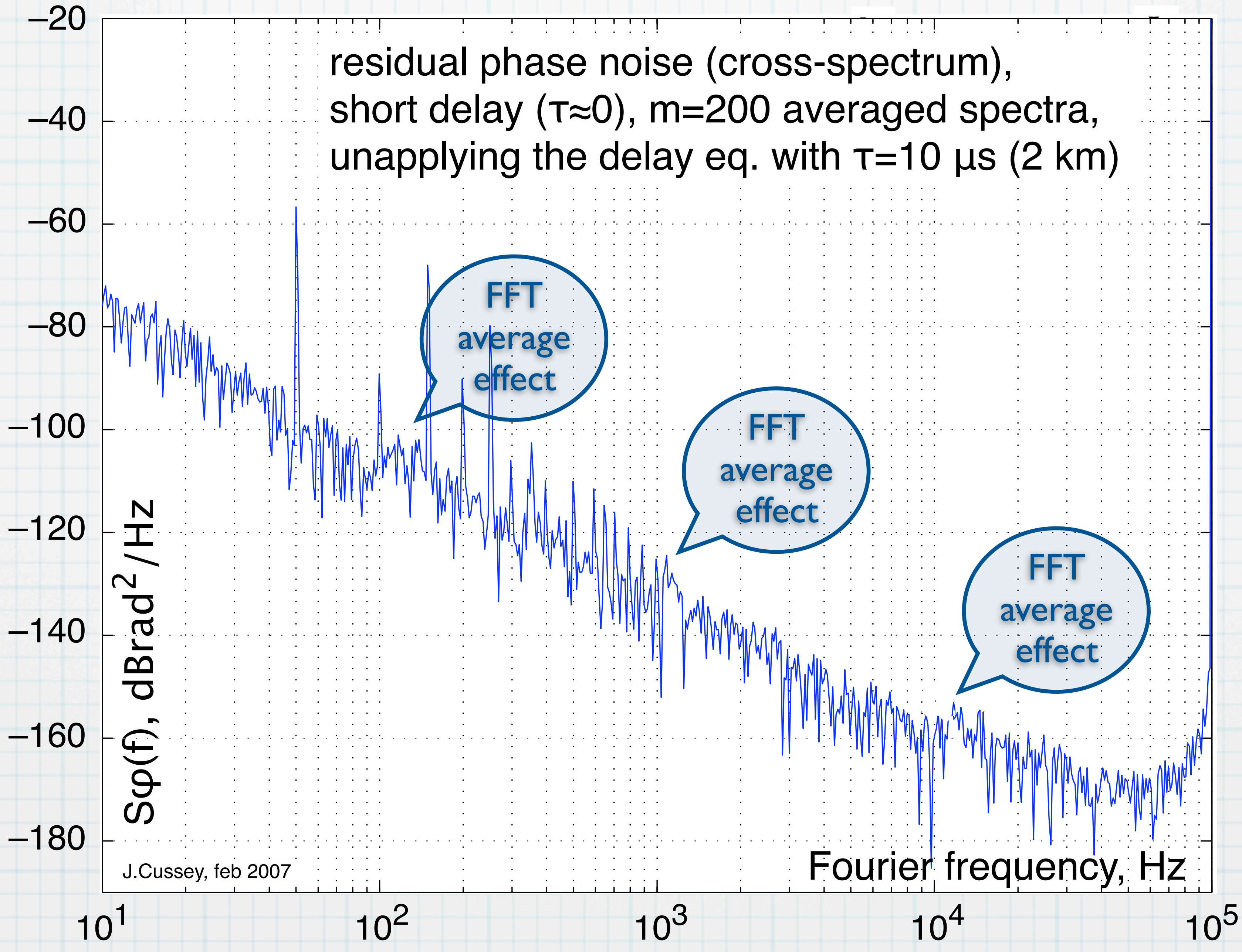
Dual-channel (correlation) instrument

Salik, Yu, Maleki, Rubiola, Proc. Ultrasonics-FCS Joint Conf., Montreal, Aug 2004 p.303-306



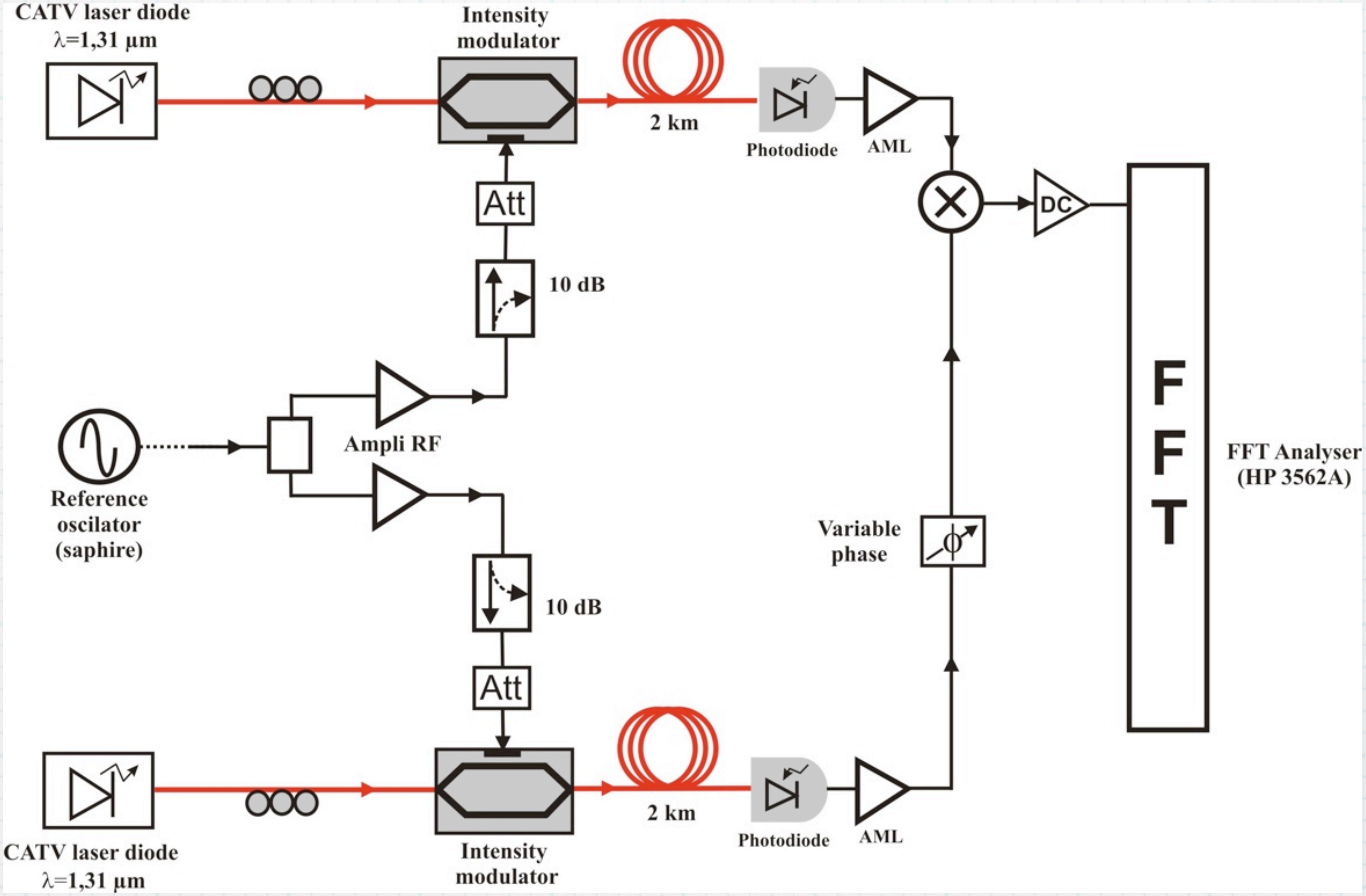
* uses cross spectrum to reduce the background noise
* requires two fully independent channels
* separate lasers for RIN rejection
* optical-input version is not useful because of the insufficient rejection of AM noise
* implemented at the FEMTO-ST Institute

Dual-channel (correlation) measurement



the residual noise is clearly limited by the number of averaged spectra, m=200

Measurement of the optical-fiber noise

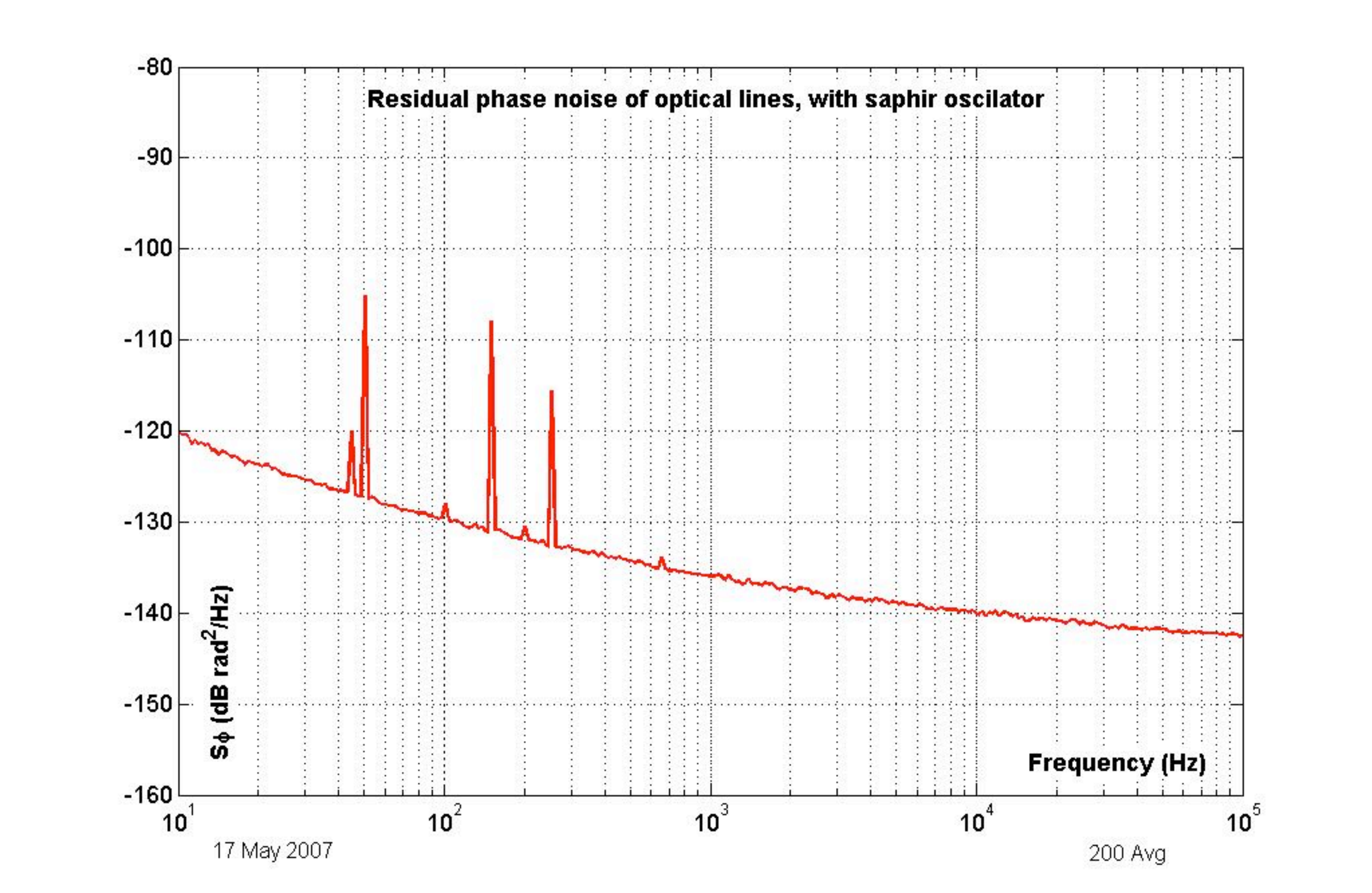


- matching the delays, the oscillator phase noise cancels
- this scheme gives the total noise

  2 × (ampli + fiber + photodiode + ampli) + mixer

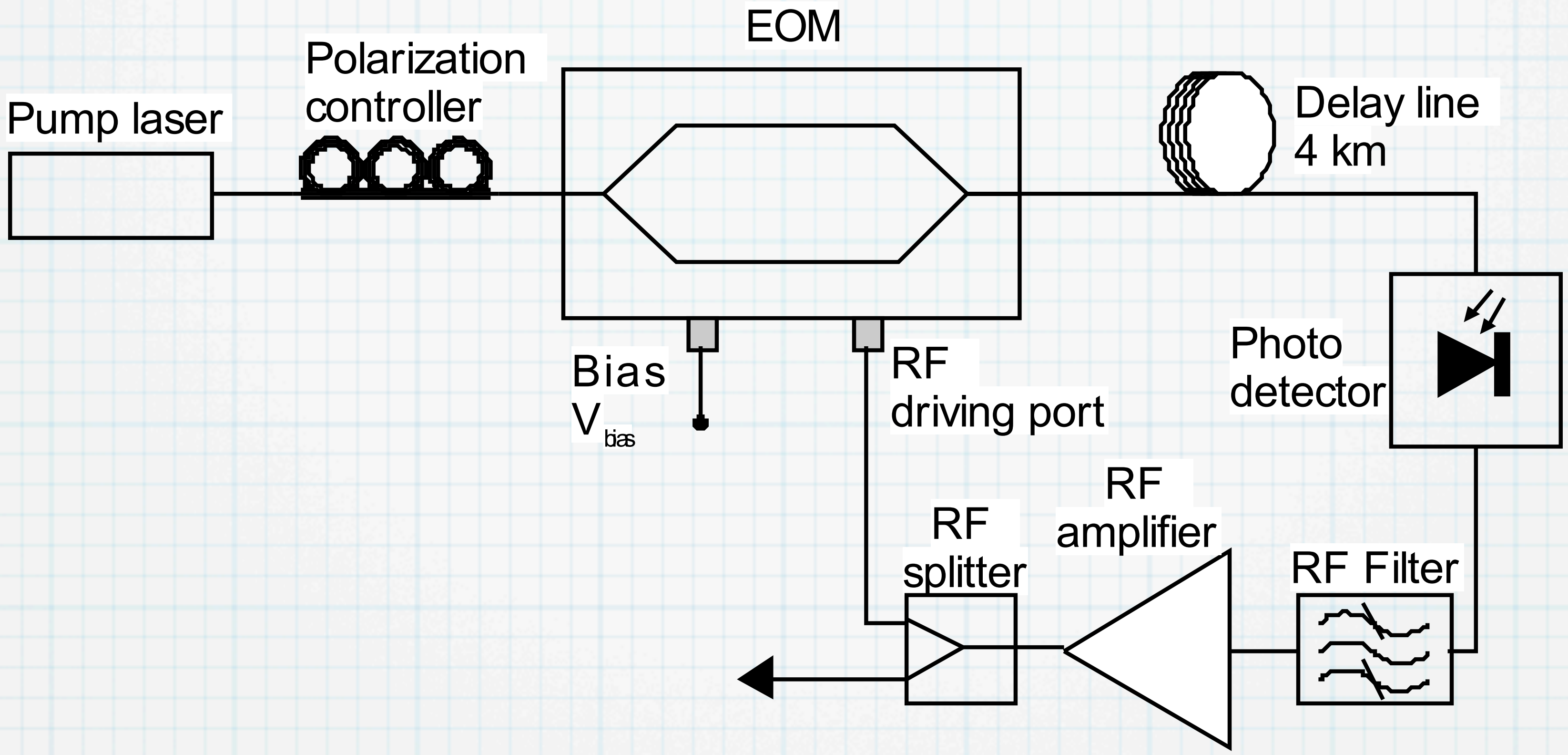
  thus it enables only to assess an upper bound of the fiber noise

Phase noise of the optical fiber



- **The method enables only to assess an upper bound of the fiber noise $b_{-1} \leq 5{\times}10^{-12}$ rad²/Hz for L = 2 km (–113 dBrad²/Hz)**
- **We believe that this residual noise is the signature of the two GaAs power amplifier that drives the MZ modulator**

Delay-line oscillator



$$f_L = \frac{\nu_0}{2Q}$$

$$Q_{eq} = \pi\nu_0\tau$$

$Q_{eq}=3\times10^5 \leftarrow L=4\text{km}$

$$f_L = \frac{1}{4\pi^2\tau^2}$$

$f_L=8\text{kHz}$

$$h_{-1} = b_{-3}/\nu_0^2$$ 6.3×10^{-24}

$$\sigma_y^2 = 2\ln(2)\,h_{-1}$$ 8.8×10^{-24}

10^{-11}

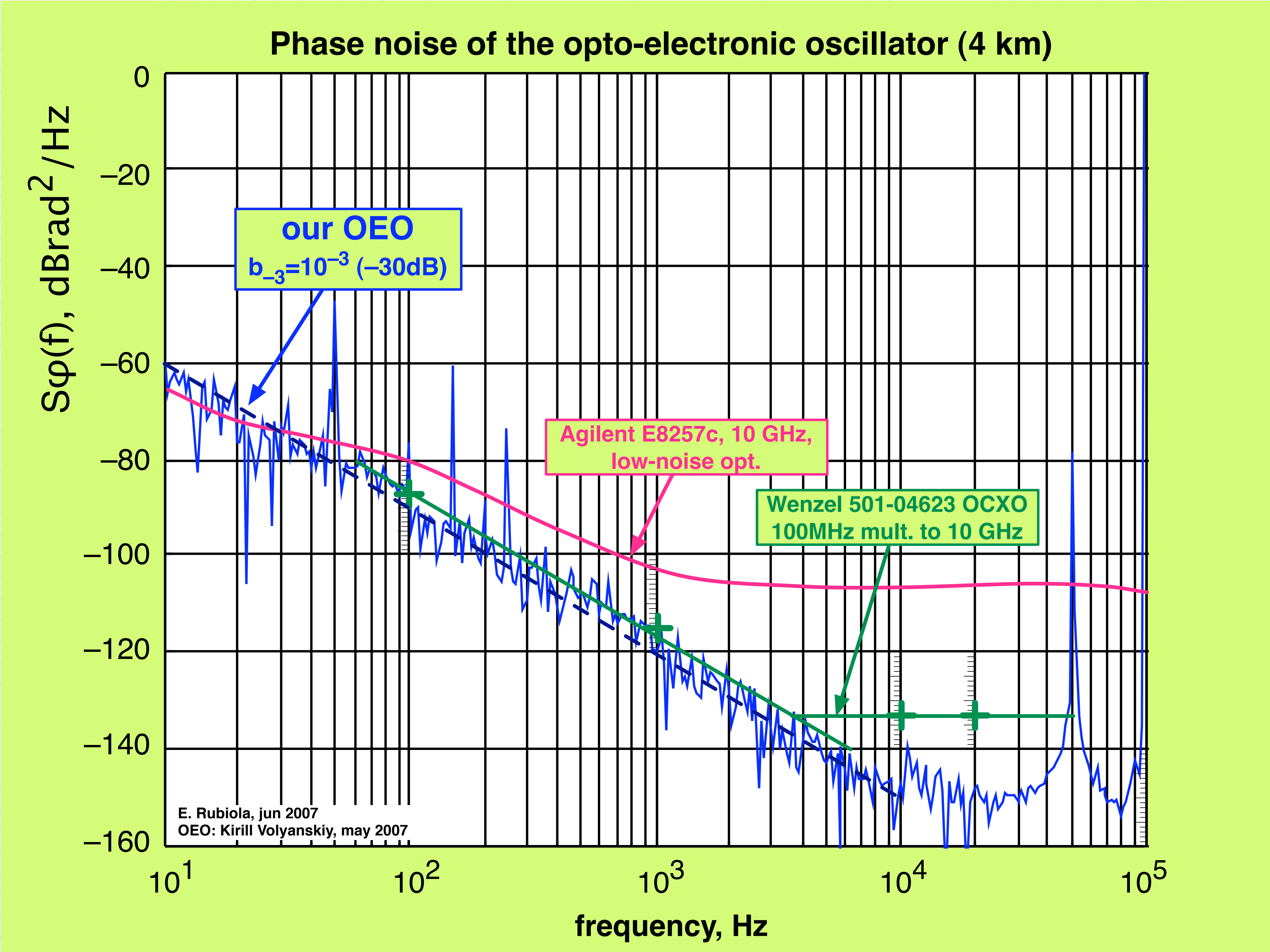
Leeson formula

$$S_\varphi(f) \simeq \frac{f_L^2}{f^2}\,S_\psi(f) \quad \text{for } f \ll f_L$$

$$\sigma_y \simeq 2.9\times10^{-12}$$ **Allan deviation**

$b_{-3} = 6.3\times10^{-4}$ (–32 dB)

Delay-line oscillator

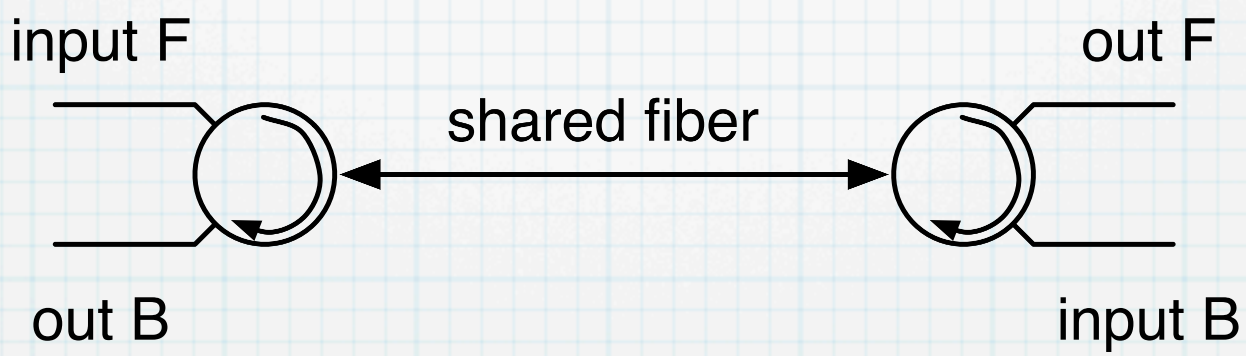


- 1.310 nm DFB CATV laser
- Photodetector DSC 402 (R = 371 V/W)
- RF filter ν_0 = 10 GHz, Q = 125
- RF amplifier AML812PNB1901 (gain +22dB)

expected phase noise
$b_{-3} \approx 6.3\times10^{-4}$ (–32 dB)

In progress

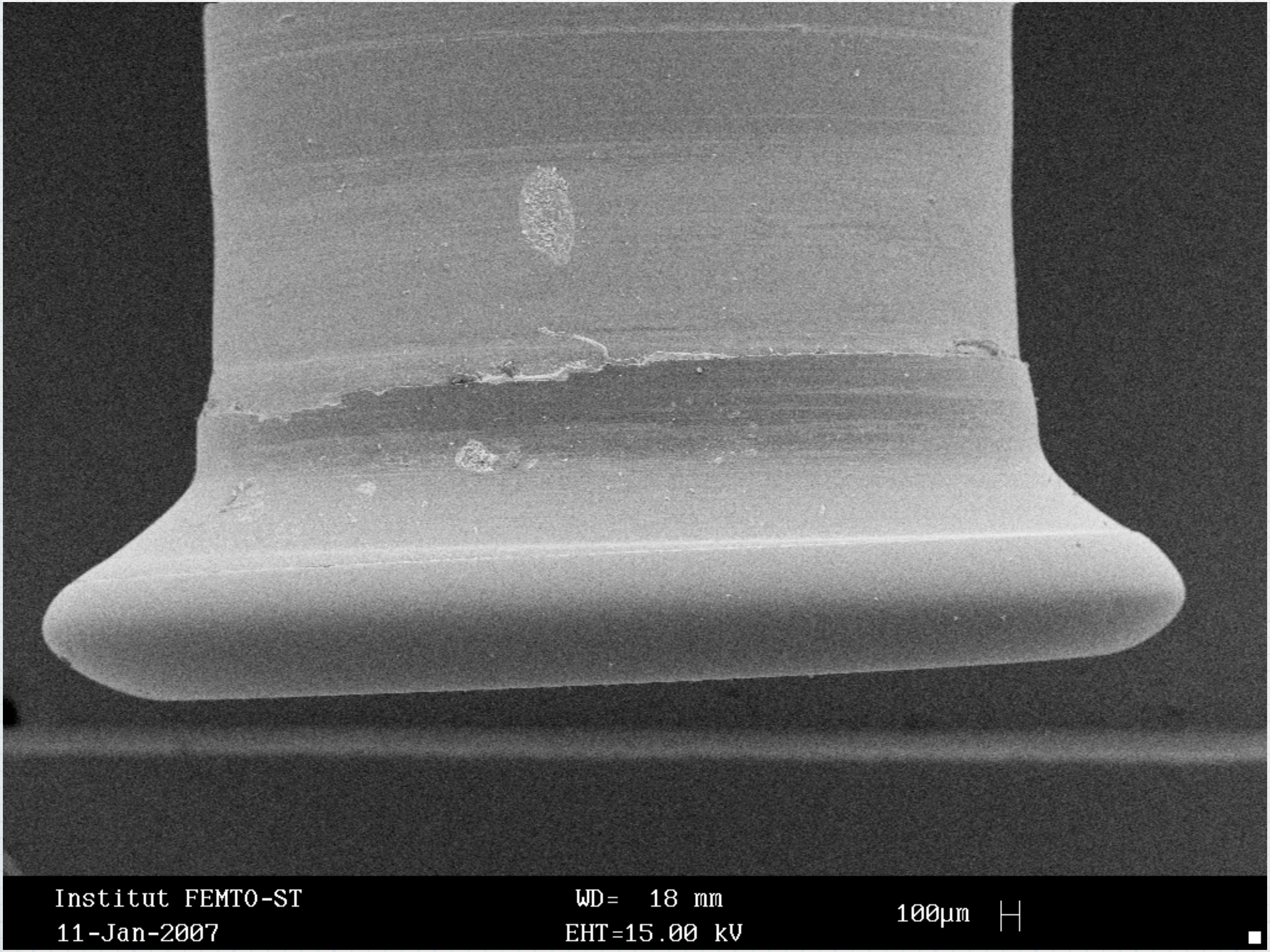
x



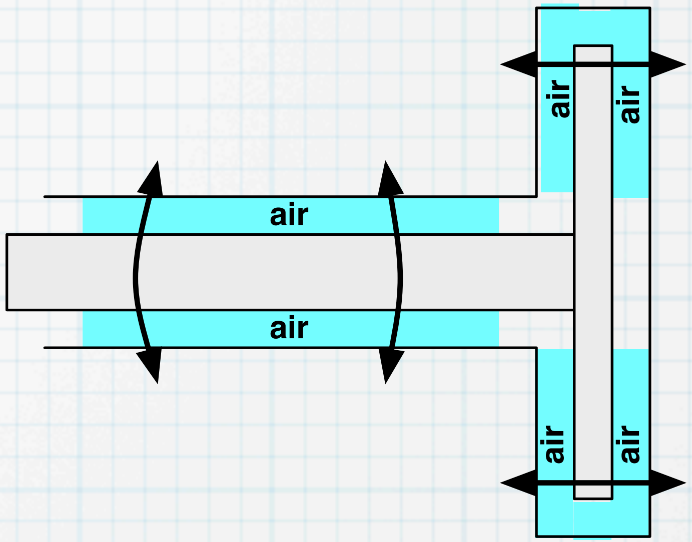
* a single fiber is used for both directions
* environmental effects are the same
* some random effects are independent

8 - Optical resonators

Example of quartz small resonator

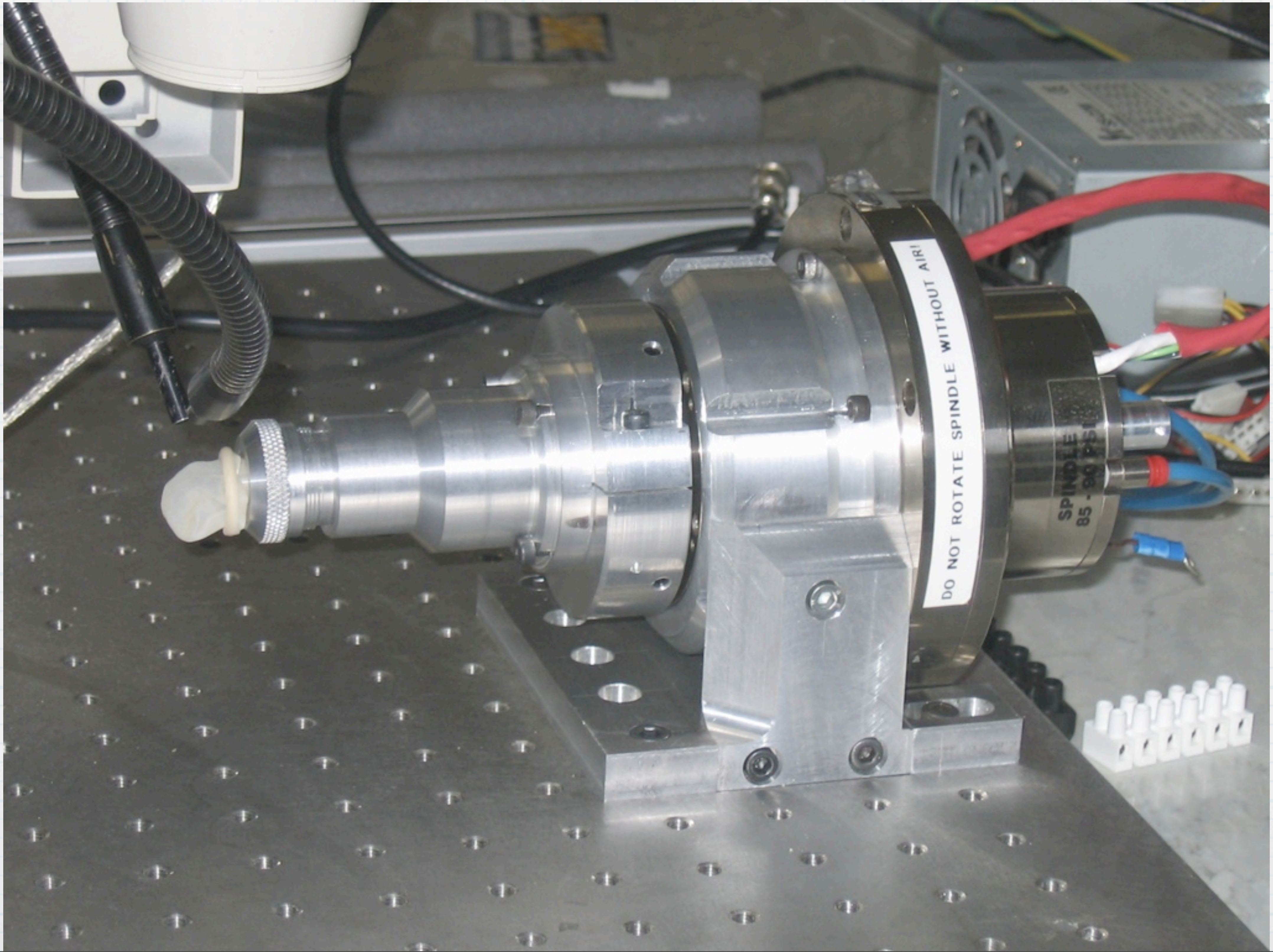


Small resonators



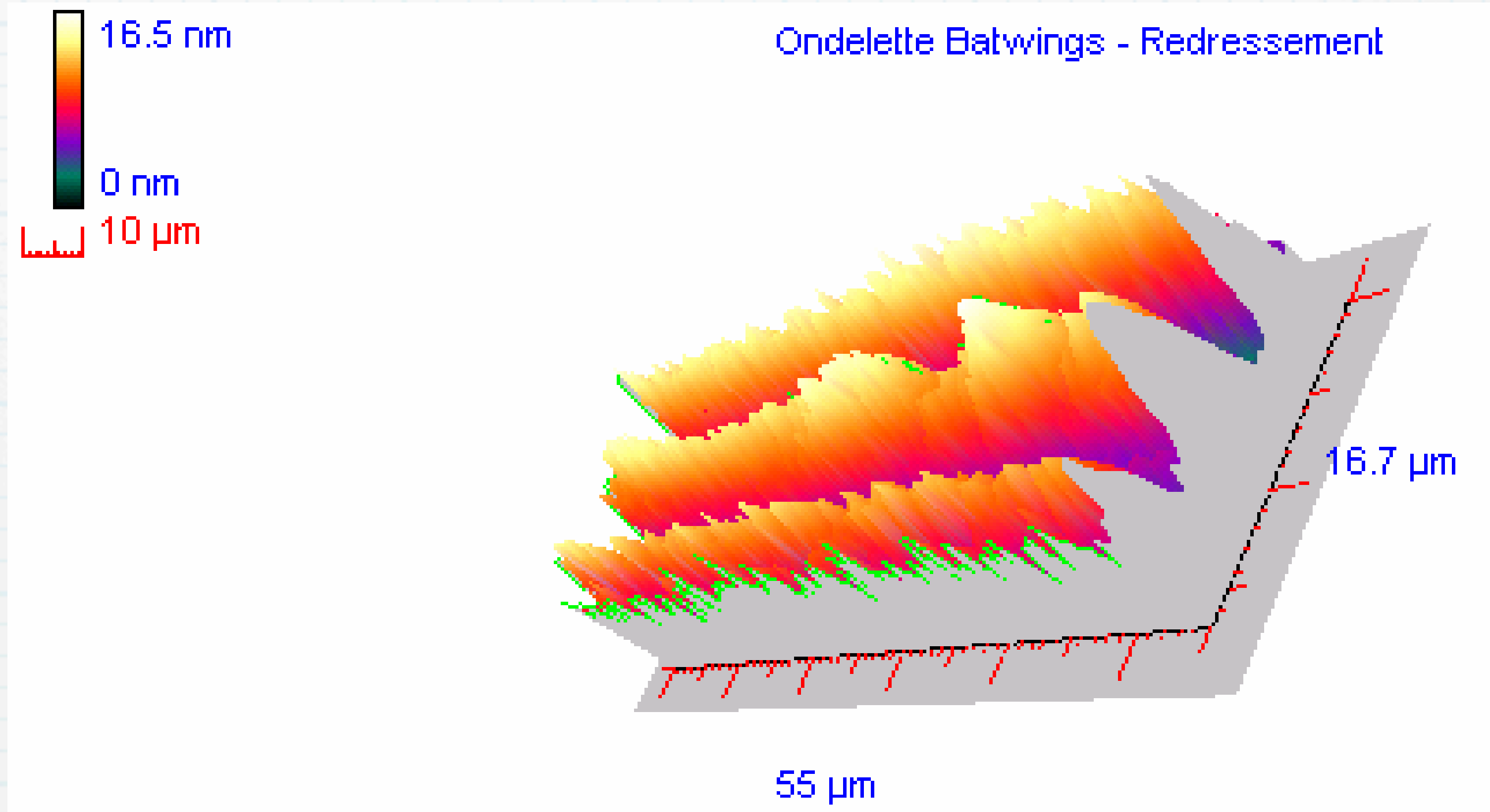
- **Technology: dedicated leathe**
  - an air-spindle motor for lowest vibration
    (from a hard-disk test equipment)
  - btw, can you figure out what a hard disk is?
    - 3.5” & 7200 rpm => ~ 200 km/h
    - 1 $(\mu m)^2$ bit area, 50 nm head–disk distance
- **Surface metrology: ready**
- **A few resonator already made**
  - quartz, 7 °Mohs (technology training, not for serious oscillators)
  - CaF_2 4 °Mohs, too soft for serious precision machining
  - MgF_2 (~6 °Mohs) harder than CaF_2, more suitable to machining
- **Achieved Q=3x10^8 with MgF_2 resonator**
  (still low, but it goes with tapered-fiber coupling)
- **Achieved stable coupling with tapered fiber**

Dedicated leathe

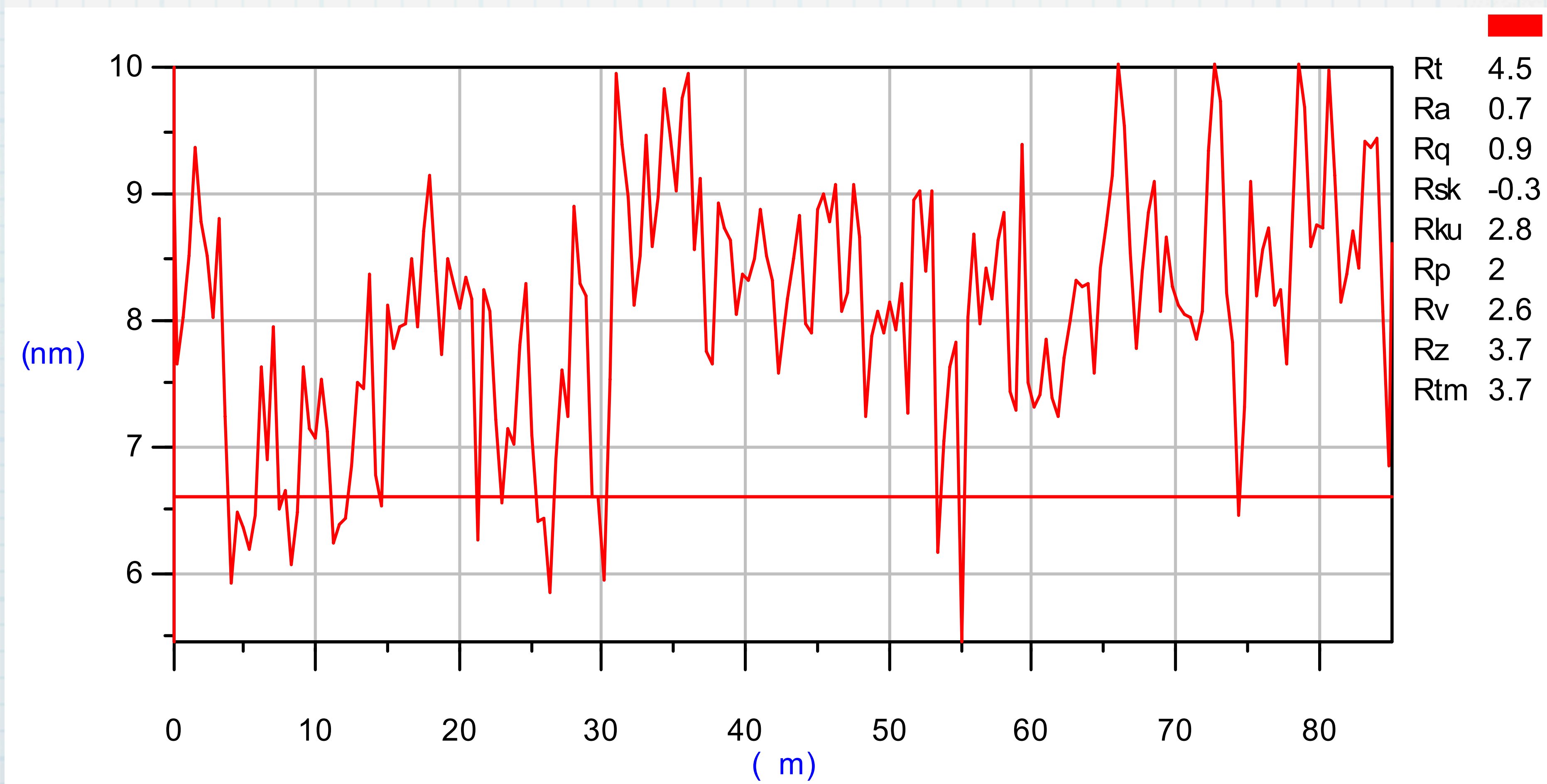


Disk resonator - surface characterization

disk surface

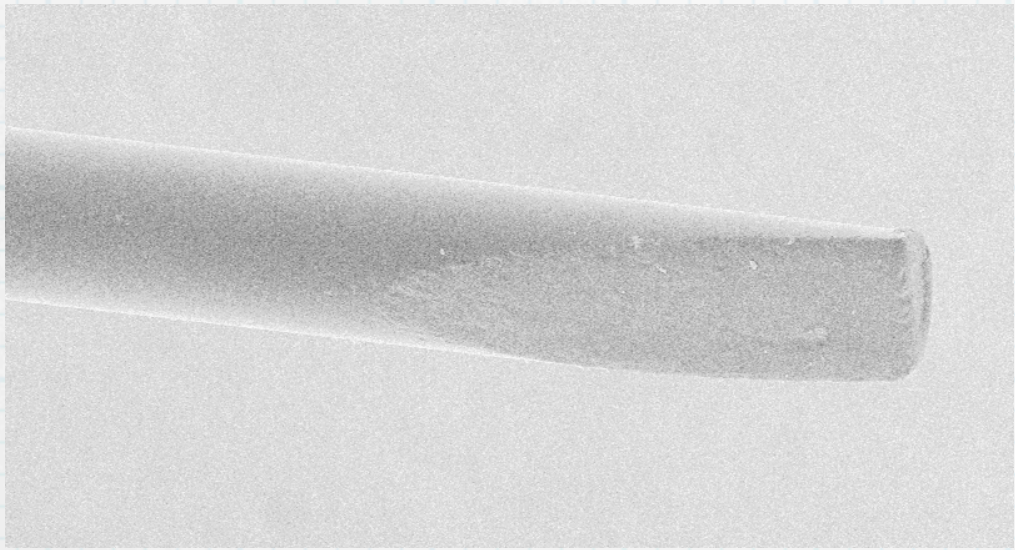


surface rugosity



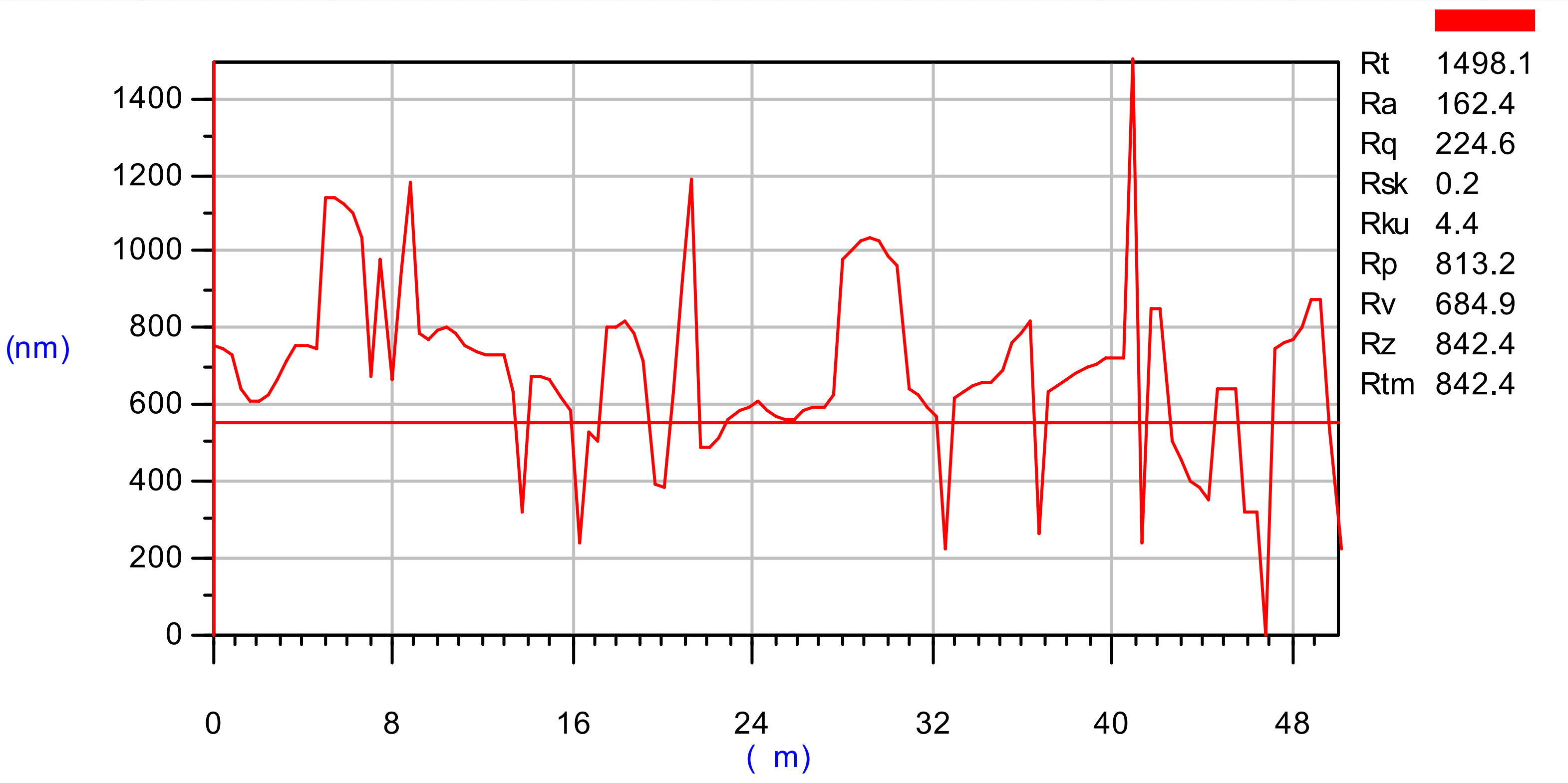
coupling: prism-shaped optical fiber

electron-beam microscope photo



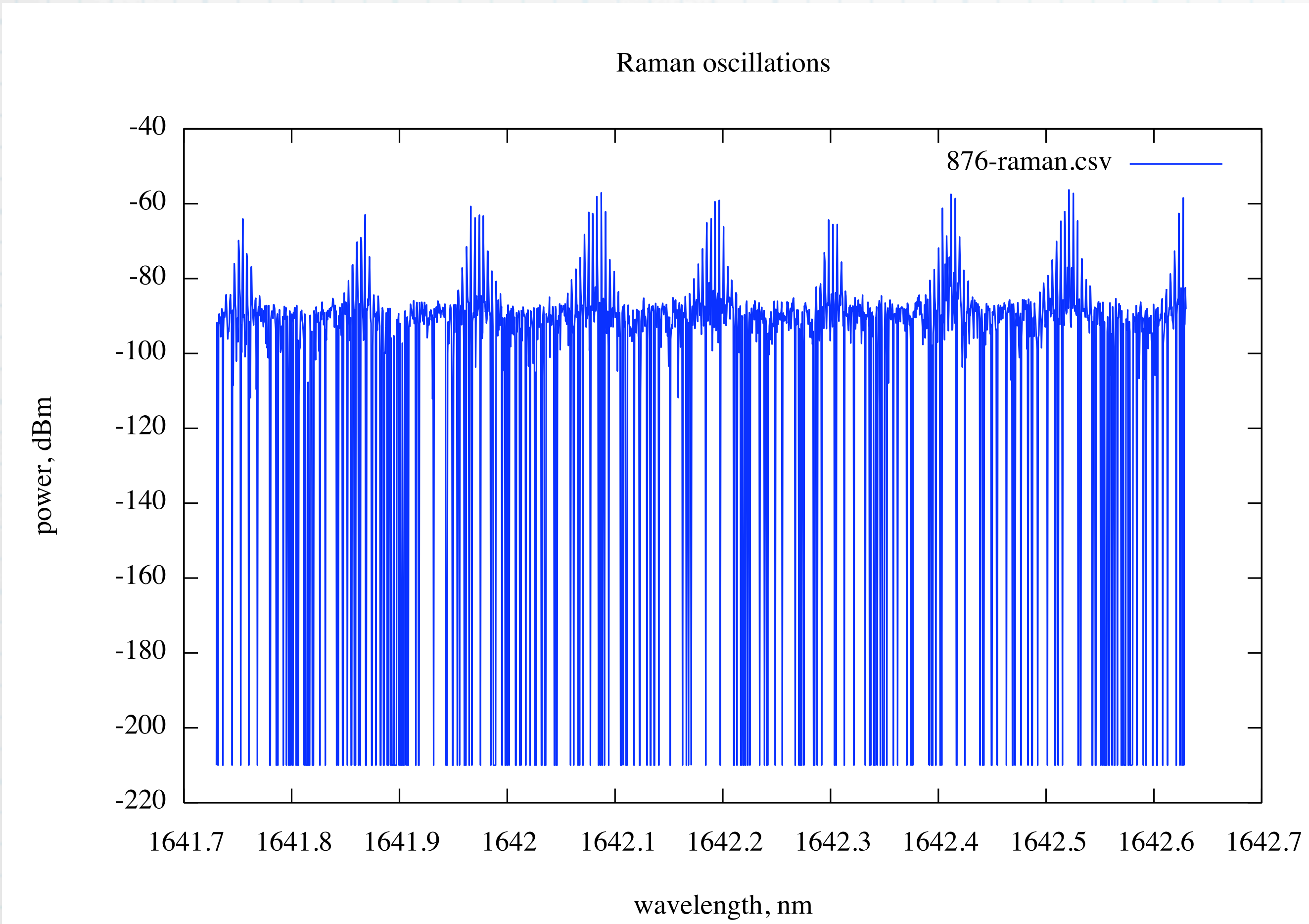
precision saw

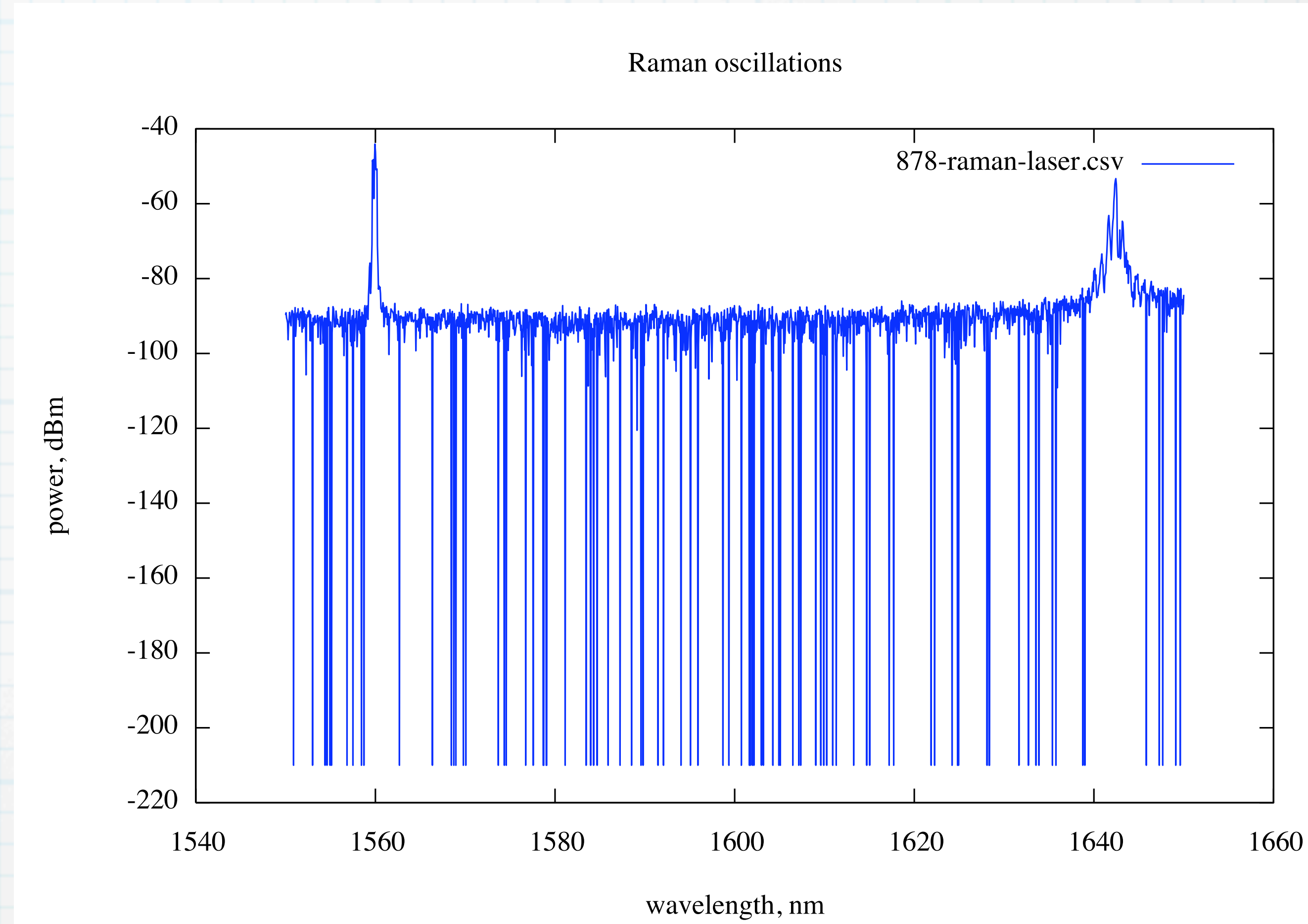
surface rugosity



Let technologists have fun with their weird equipment
I don't think that the fiber machining is that critical (also experience)

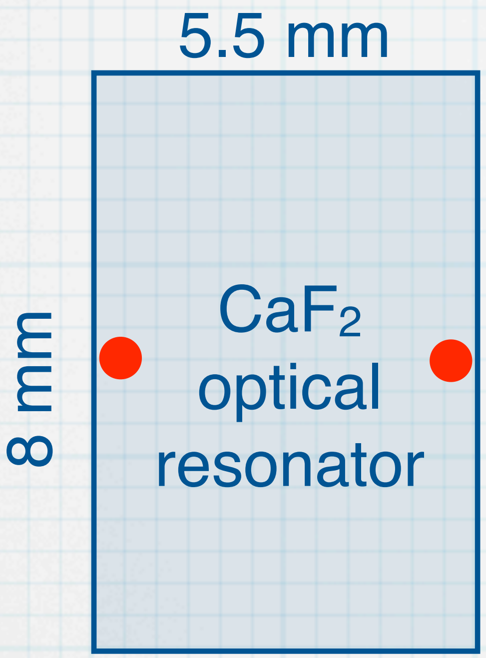
Raman oscillations





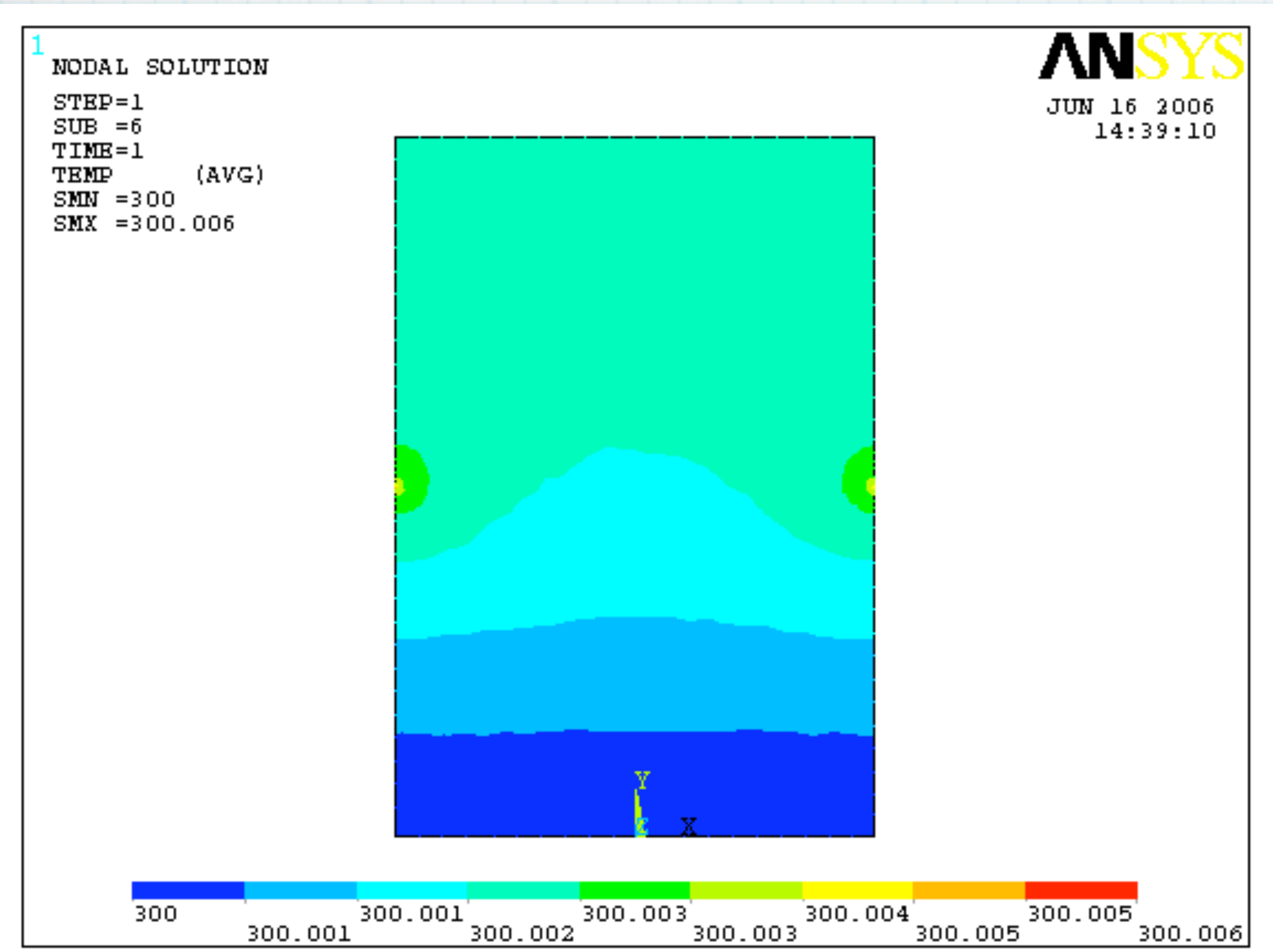
- **The Raman amplification is a quantum phenomenon of nonlinear origin that involves optical phonons.**
- **An amplifier inserted in a high-Q cavity turns into an oscillator, like masers and lasers.**
- **Oscillation threshold ~ $1/Q^2$**
- **In CaF2 pumped at 1.56 μm, Raman oscillation occurs at 1.64 μm**
- **Due to the large linewidth, the Raman oscillation appears as a bunch of (noisy) spectral lines spaced by the FSR (12 GHz, or 100 pm in our case)**
- **Raman phonons modulate the optical properties of the crystal, which induces noise at the pump frequency (1.56 μm)**

High temperature gradient

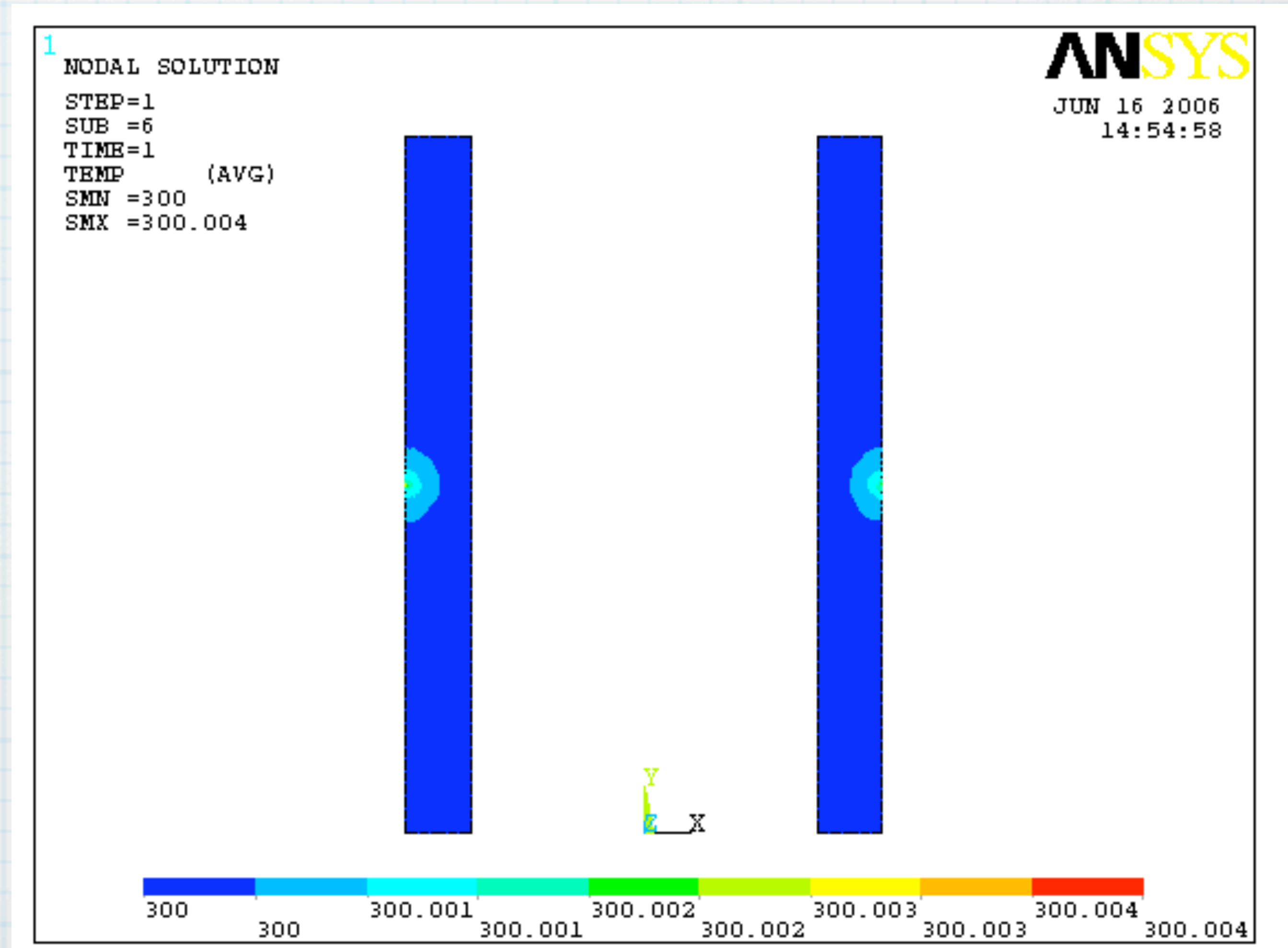


- cross section of the field region 1 µm^2
- CaF_2 thermal conductivity 9.5 W/mK
- dissipated power 300 µW
- wavelength 1.56 µm
- air temperature 300 K
- still air thermal conductivity 10 W/m^{2}K
- simplification: the heat flow from the mode region is uniform

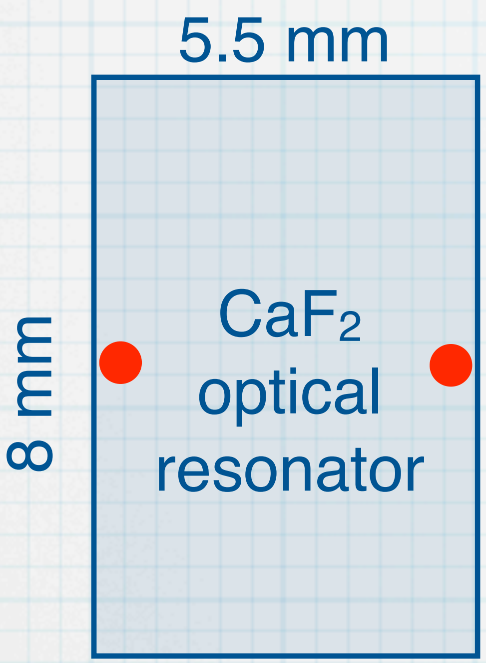
bottom plane at a reference temperature



inner bore at a reference temperature

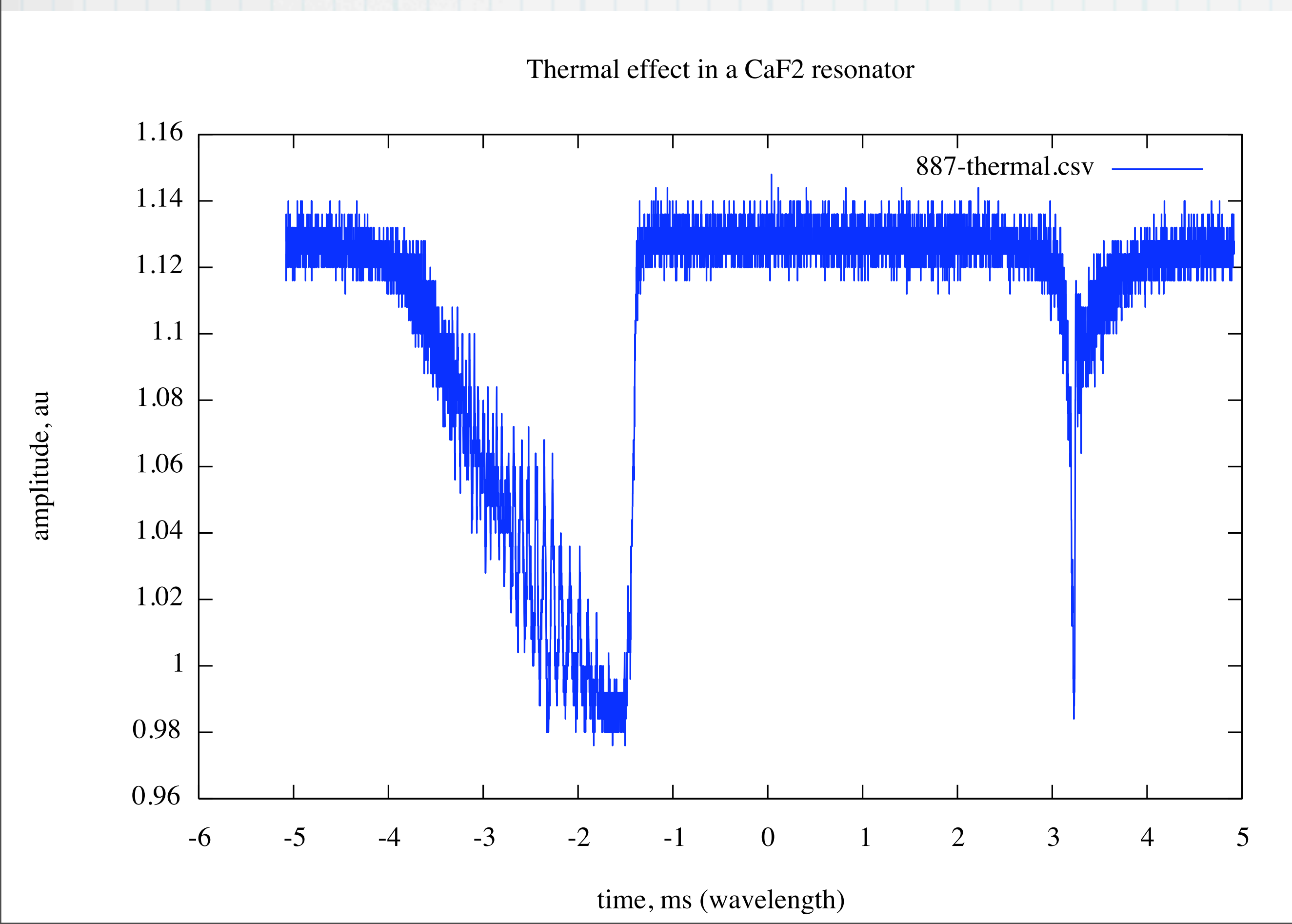


Thermal effect on frequency

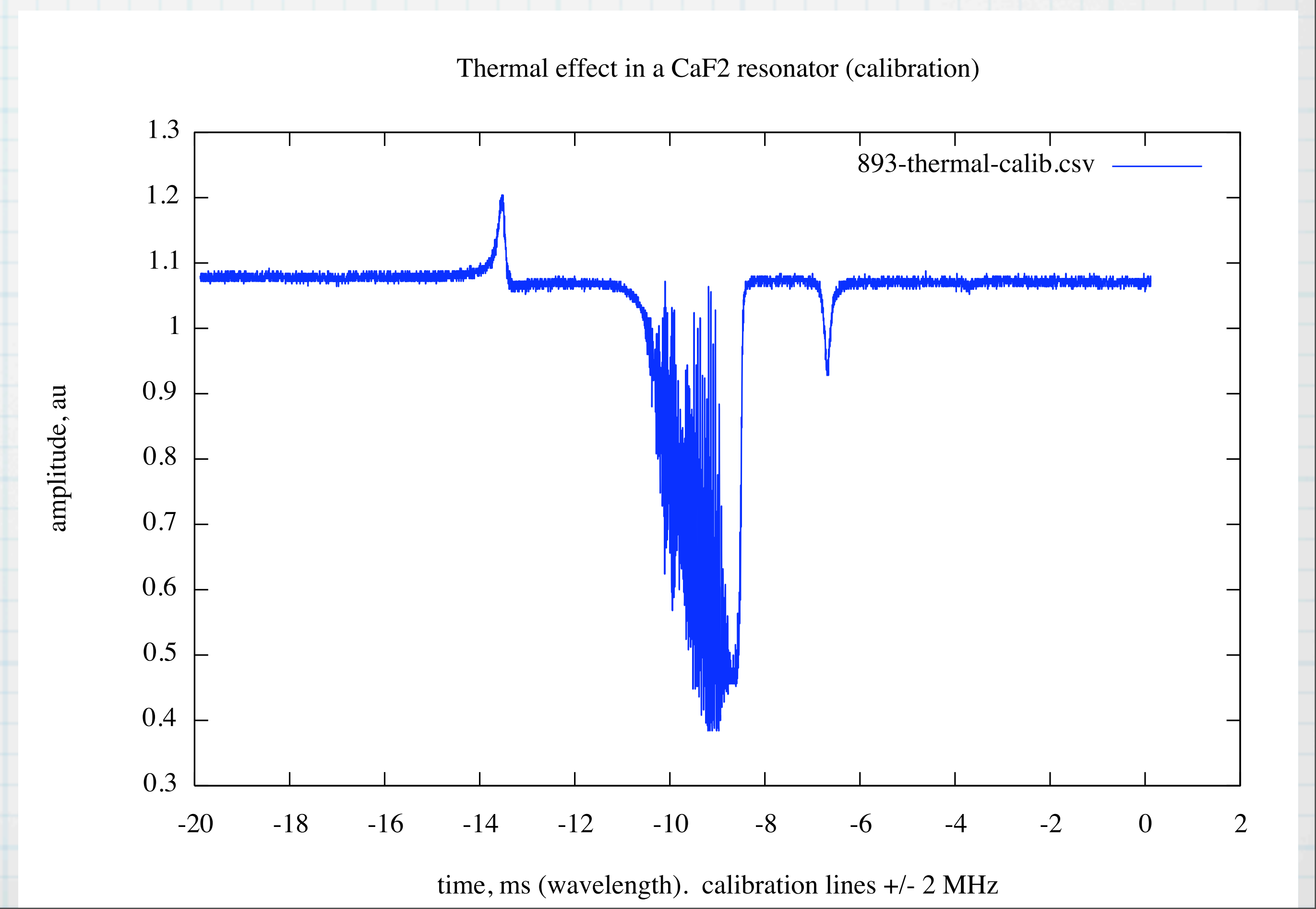


- wavelength 1.56 µm   (ν_0=192 THz)
- Q=5×10^9 –> BW=40 kHz
- a dissipated power of 300 µW shifts the resonant frequency by 1.2 MHz (6×10^{-9}), i.e., 37.5 x BW
- time scale about 60 µs
- Q>10^{11} is possible with CaF_2 and other crystals!!

laser scan



calibration (2 MHz phase modulation)



Low-power oscillator operation

Assume: $\lambda = 1560$ nm, $\rho = 0.8$ A/W, R = 50 Ohm, $(P_\lambda)_{peak} = 2\times10^{-5}$ W (20 µW)

Shot noise (m=1)

$$I_{RMS} = \frac{1}{\sqrt{2}}\,\rho\overline{P}_\lambda$$

$$S_I = 2q\overline{I} = 2q\,\rho\overline{P}_\lambda$$

$$SNR = \frac{1}{4}\,\frac{\rho\overline{P}_\lambda}{q}$$

In practice, –131 dBrad²/Hz

Thermal noise (m=1)

$$I_{RMS} = \frac{1}{\sqrt{2}}\,\rho\overline{P}_\lambda$$

$$S_I = \frac{4kT}{R} \quad \text{or} \quad \frac{4FkT}{R}$$

$$SNR = \frac{1}{8}\,\frac{\rho^2\overline{P}_\lambda^2 R}{kT}$$

In practice, –110 dBrad²/Hz with F=0 dB (!!!)

- Thermal noise is dominant: below threshold, SNR ~ $1/P_\lambda^2$
- Thermal noise can be reduced (10 dB or more?) using VGND amplifiers
- What about flicker of photodetectors with integrated VGND amplifier?
- Dramatic impact on the (phase) noise floor

Small resonators

- **Let us dream**
  - diamond: probably chemical purity may be a problem (insufficient transparence)
  - sapphire: think more about it (we can learn a lot from the microwave technology)

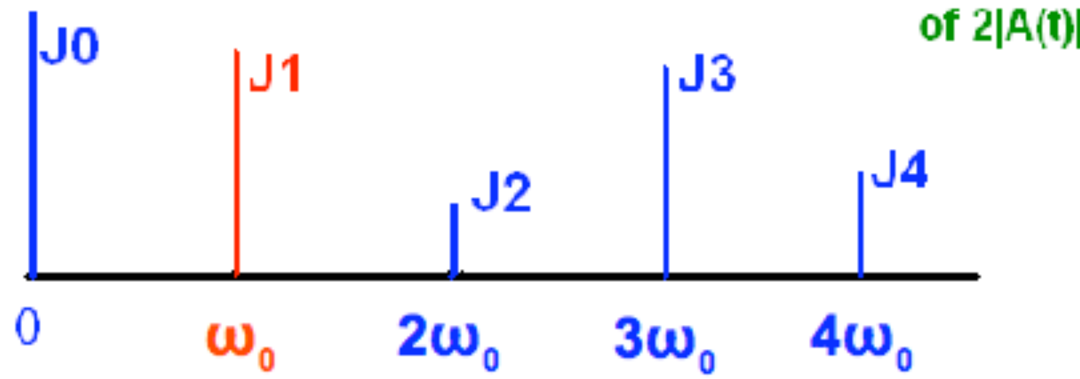
- **Last-minute news**
  MgF_2 seems to have a turning point of the thermorefractive index
  - 74 °C, extraordinary wave
  - 176 °C, ordinary wave

9 - Non-linear AM oscillations

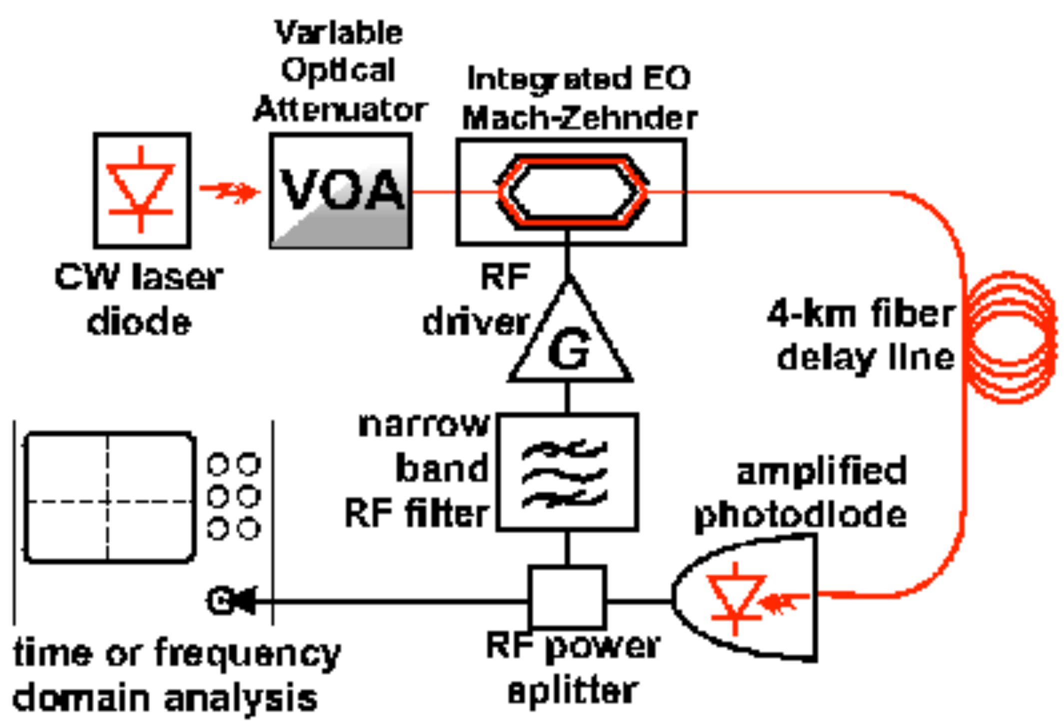
Nonlinear model

Anger-Jacobi expansion

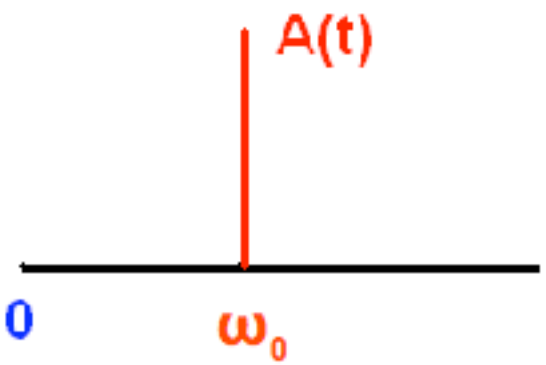
$$e^{iz\cos\alpha} = \sum_{n=-\infty}^{+\infty} i^n J_n(z) e^{in\alpha}$$



$\cos^2[\, x(t) + \phi \,]$



Delaying and dephasing by $\sigma_n = n\,\omega_0\,T$



EQUATION

Filtering around ω_0

A complex envelope equation

X

The complex envelope amplitude of the microwave obeys the equation

$$\dot{\mathcal{A}} = -\mu\mathcal{A} - 2\mu\gamma e^{-i\sigma} \cdot \mathrm{Jc}_1\left[2|\mathcal{A}_T|\right]\mathcal{A}_T$$

where $\mathrm{Jc}_1(x) = \mathrm{J}_1(x)/x$ is the Bessel-cardinal function

Looks like sinus cardinal, but the maximum is ½ instead of 1

$\mu = \Delta\omega/2$ = half-bandwith of the filter (= $2\pi \times 10$ MHz)
$\gamma = \beta \sin 2\phi$ = effective normalized gain (can vary from -5 to 5)
$\sigma = \Omega_0 T$=microwave round-trip phase shift

The solutions of interest are:

- ***A(t)*** $\equiv$ ***0*** **(no oscillations)**
- ***A(t)*** $\equiv$ ***C*** $^{te} \neq$ ***0*** **(pure monochromatic)**

These states are *fixed points* of the envelope equation.

We have to study the existence and the stability of the fixed point solutions, particularly for the solution A(t)=C $\neq$ 0 which is of great technological interest.

Stability of the oscillating solution

It corresponds to the solution ***A(t)* ≡ *A*$_o$ ≠ *0*** with

$$\mathrm{Jc}_1[2|\mathcal{A}_o|] = -\frac{1}{2\gamma}e^{i\sigma}$$

Perturbation equation

$$\delta\dot{\mathcal{A}} = -\mu\cdot\delta\mathcal{A} - 2\mu\gamma\{\mathrm{Jc}_1[2|\mathcal{A}_o|] + 2|\mathcal{A}_o|\mathrm{Jc}_1'[2|\mathcal{A}_o|]\}\delta\mathcal{A}_T$$

Stability condition

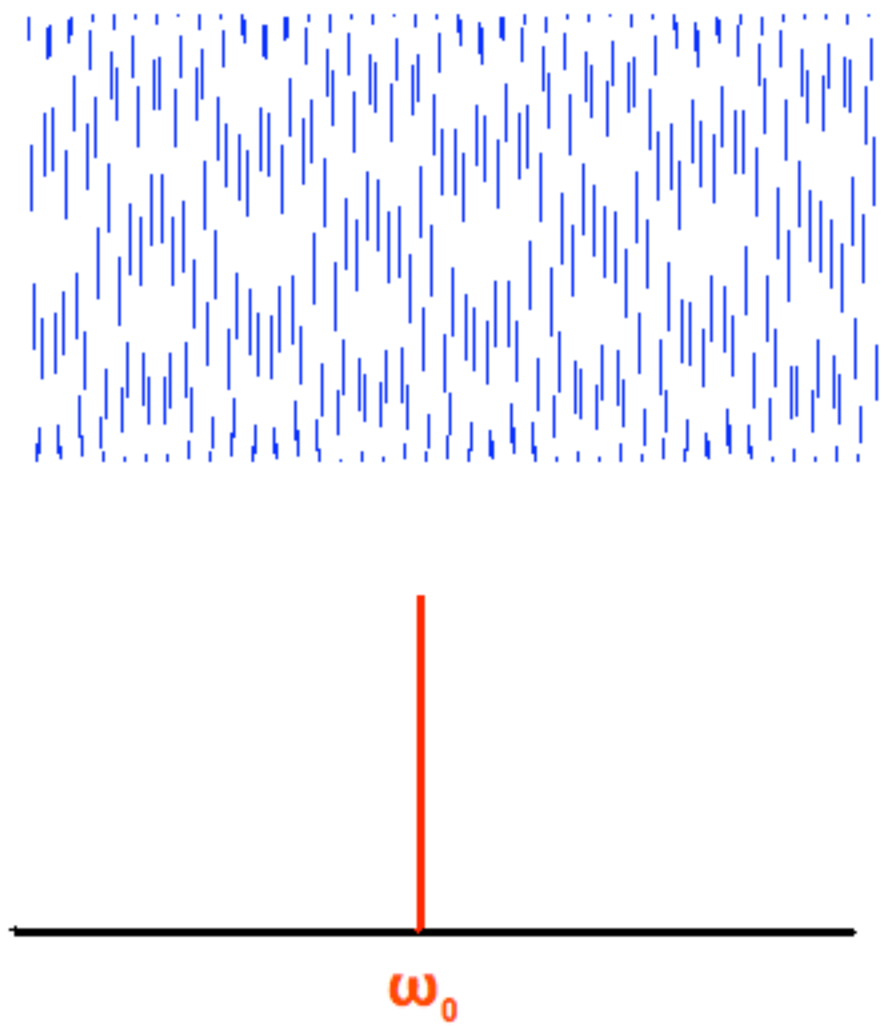
$$\left|\frac{1}{2} + \frac{|\mathcal{A}_o|\mathrm{Jc}_1'[2|\mathcal{A}_o|]}{\mathrm{Jc}_1[2|\mathcal{A}_o|]}\right| < \frac{1}{2}$$ fulfilled when $1 < \gamma < 2.3$, when $e^{-i\sigma} = -1$

What does occur beyond 2.3 ???

A Hopf bifurcation

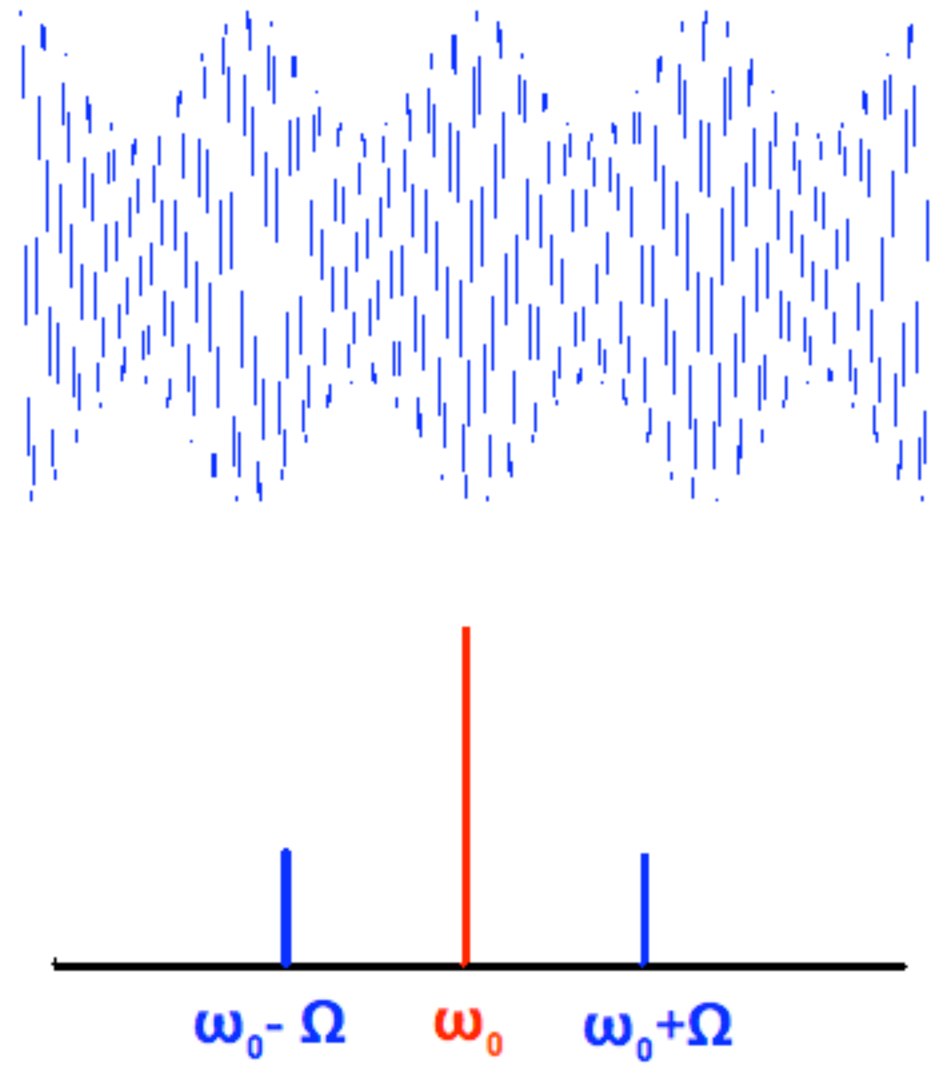
Gain < 2.3

$A = A_o$



Gain > 2.3

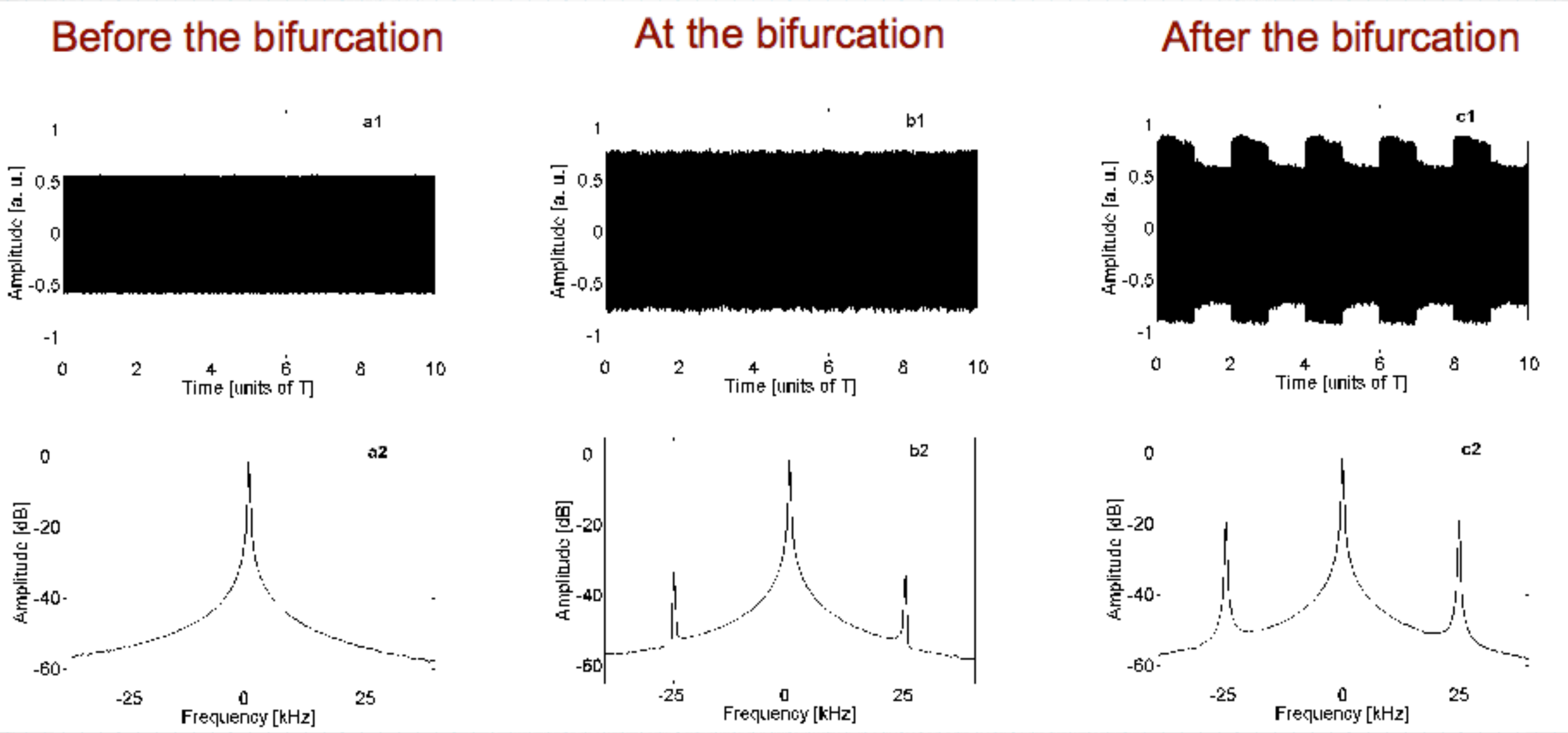
$A = A_o + a_o \exp[i\,\Omega\, t]$



The bifurcation at γ=2.3 should qualitatively modify the Fourier spectrum of OEOs

Hopf bifurcation, observed

X



The Hopf bifurcation leads to the emergence of robust modulation side-peaks in the Fourier spectrum, which may drastically affect the phase noise performance of OEOs