CENTRE NATIONAL
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UNIVERSITÉ DE FRANCHE-COMTÉ

High resolution frequency counters

E. Rubiola
FEMTO-ST Institute, CNRS and Université de Franche Comté

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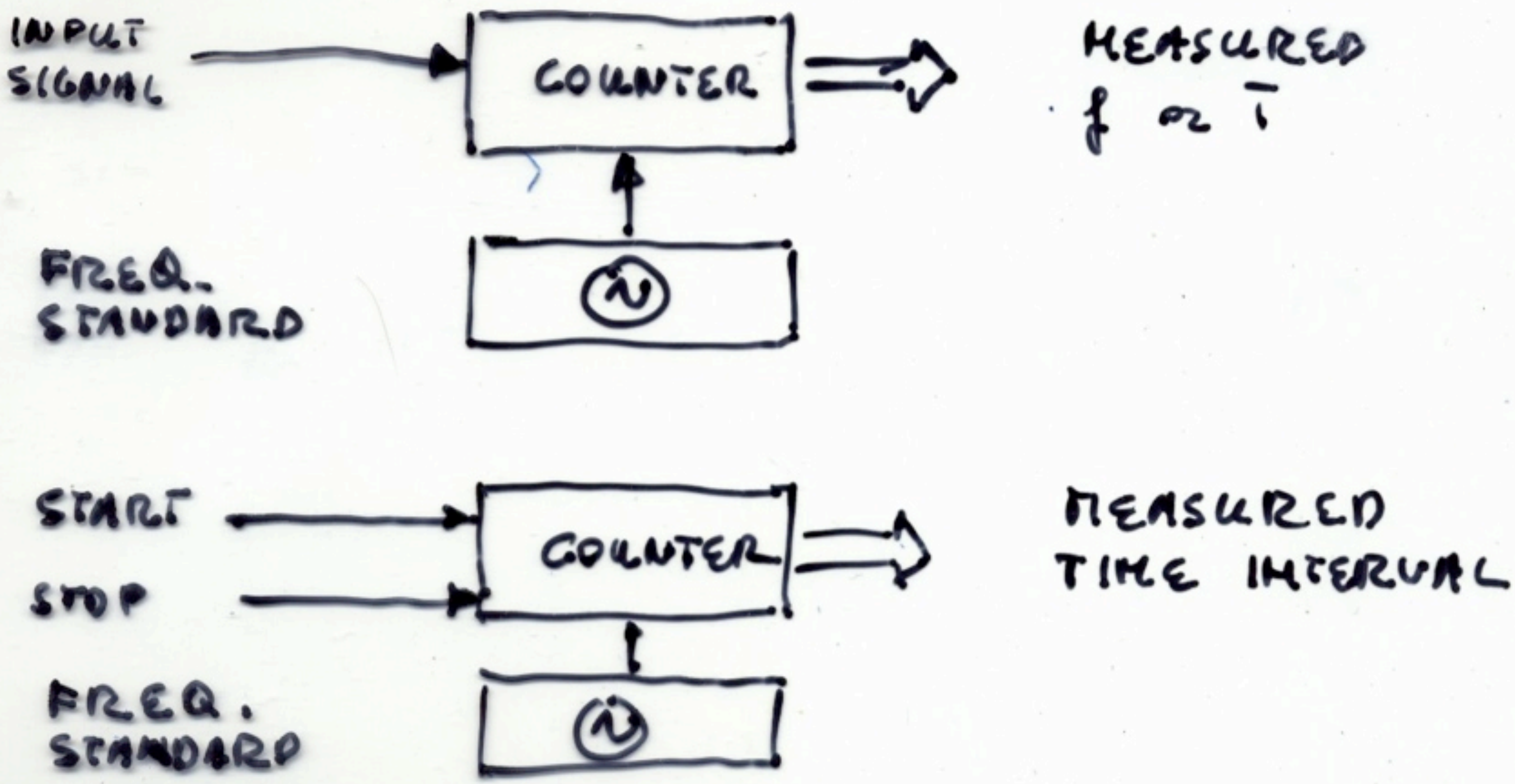
Outline

1. Digital hardware
2. Basic counters
3. Microwave counters
4. Interpolation
   - time-interval amplifier
   - frequency vernier
   - time-to-voltage converter
   - multi-tap delay line
5. Basic statistics
6. Advanced statistics

home page http://rubiola.org

COUNTER

1b

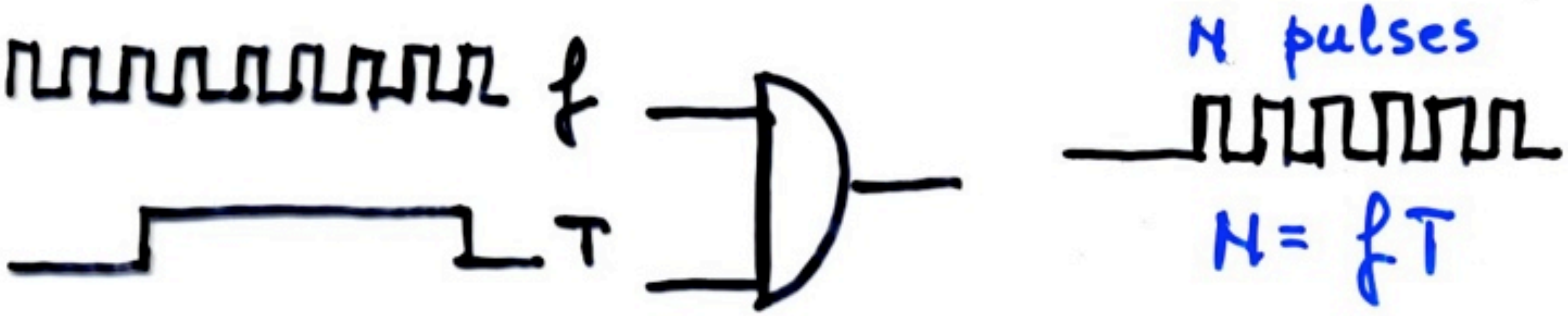


1 How to compare the input signal (f, T, TI) to the frequency standard

2 How to avoid degradation of the standard's metrological performances

1 – Digital hardware

BASIC CELL



AND

	0	1
0	0	0
1	0	1

Count the number N of pulses (± 1)

$$N = fT$$

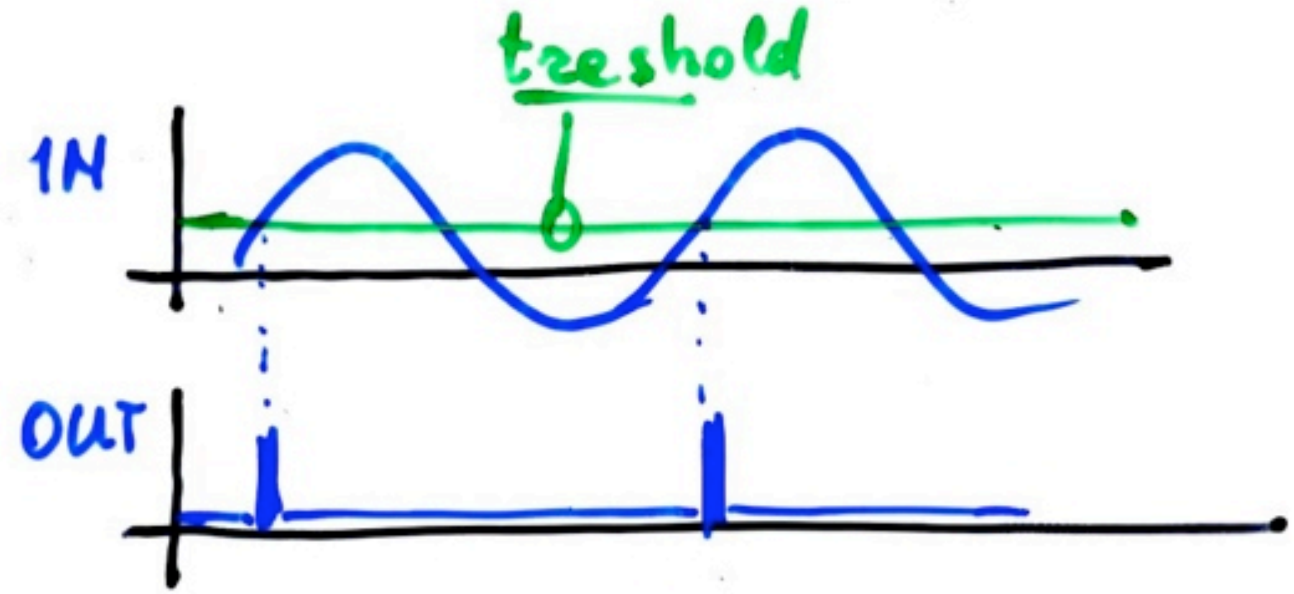
reference	measure	resolution
f	T	$\frac{\delta T}{T} = \frac{1}{N}$
T	f	$\frac{\delta f}{f} = \frac{1}{N}$

The resolution is limited by the measurement time T, and by the maximum switching frequency

200 MHz typ. $\rightarrow$ resolution 5 ns

HIGHER RESOLUTION $\rightarrow$ INTERPOLATION

TRIGGER



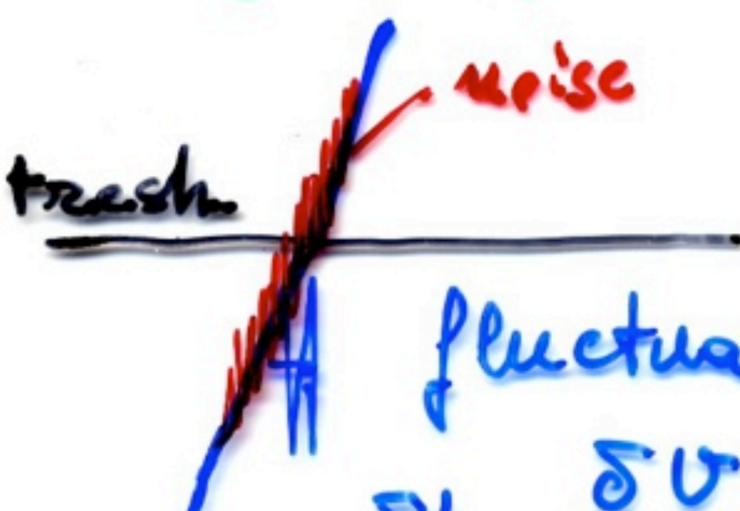
TRIVIAL FUNCTIONS
- 50 Ω , AC/DC
- level, slope

HYSTERESIS

TRIGGER → fast device → wide noise bandwidth

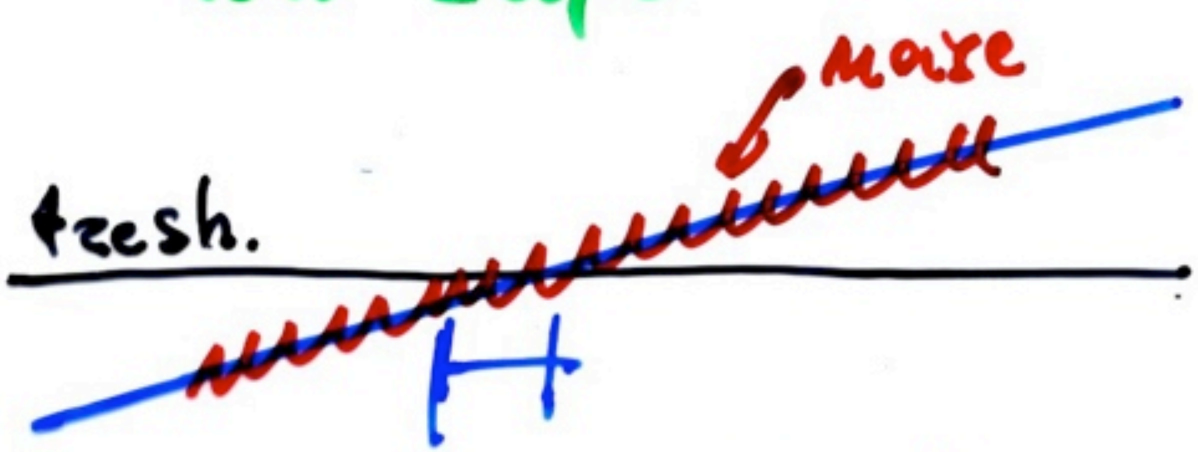
high slope

low slope



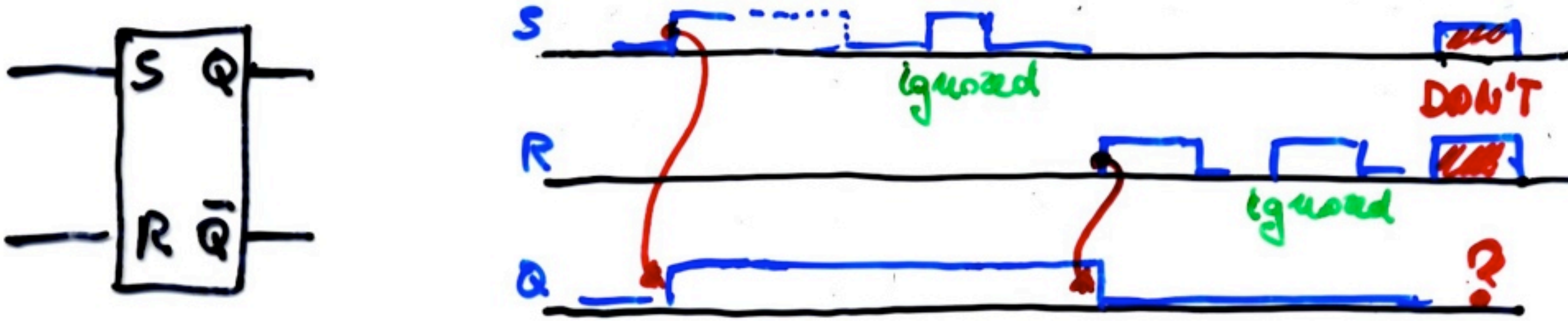
fluctuation

$$\delta t = \frac{\delta v}{dv/dt}$$



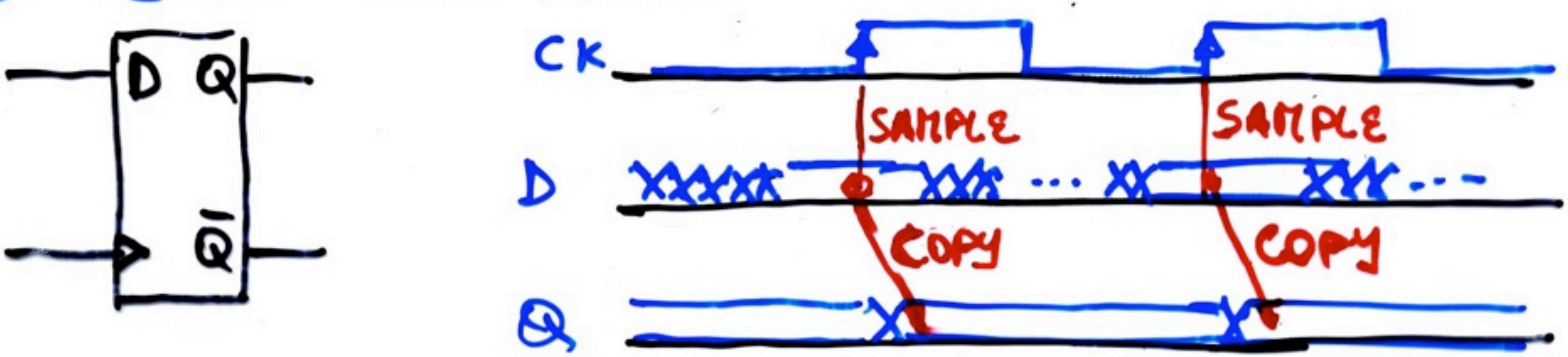
FIRST CROSSING of a random process: difficult mathematical problem

SET-RESET FLIP-FLOP



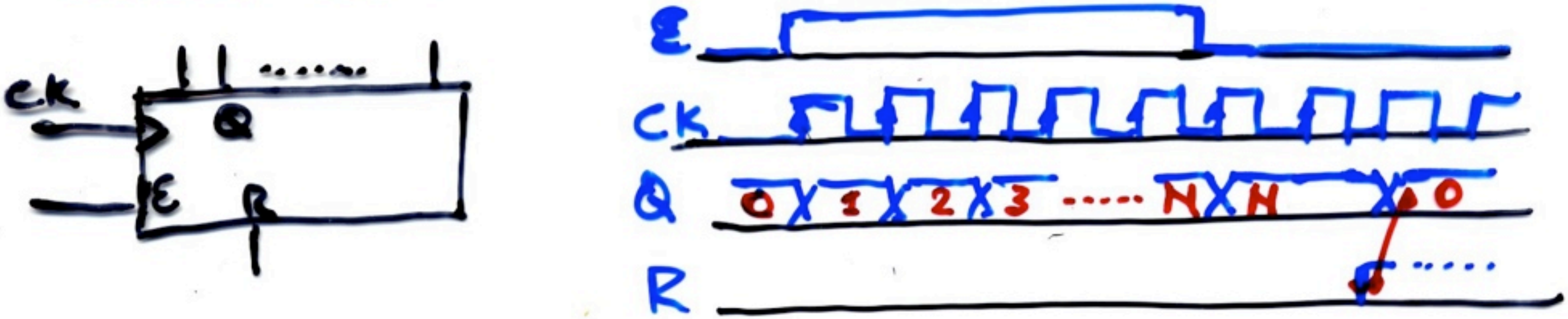
The SR flip-flop is a dual-button switch

D-TYPE FLIP-FLOP



The D-type flip-flop samples the input D during the rising edge ↗ of the clock, and copies D → Q

COUNTER

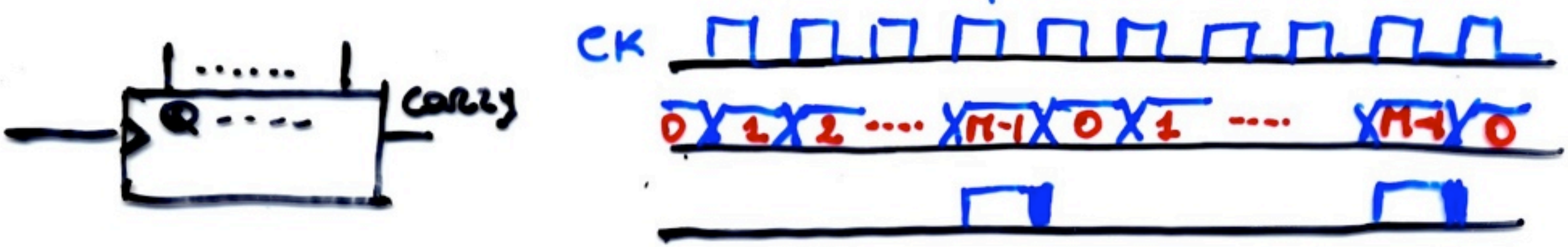


counts the number of rising edges of the clock

E → enable   R → Reset

DIVIDER

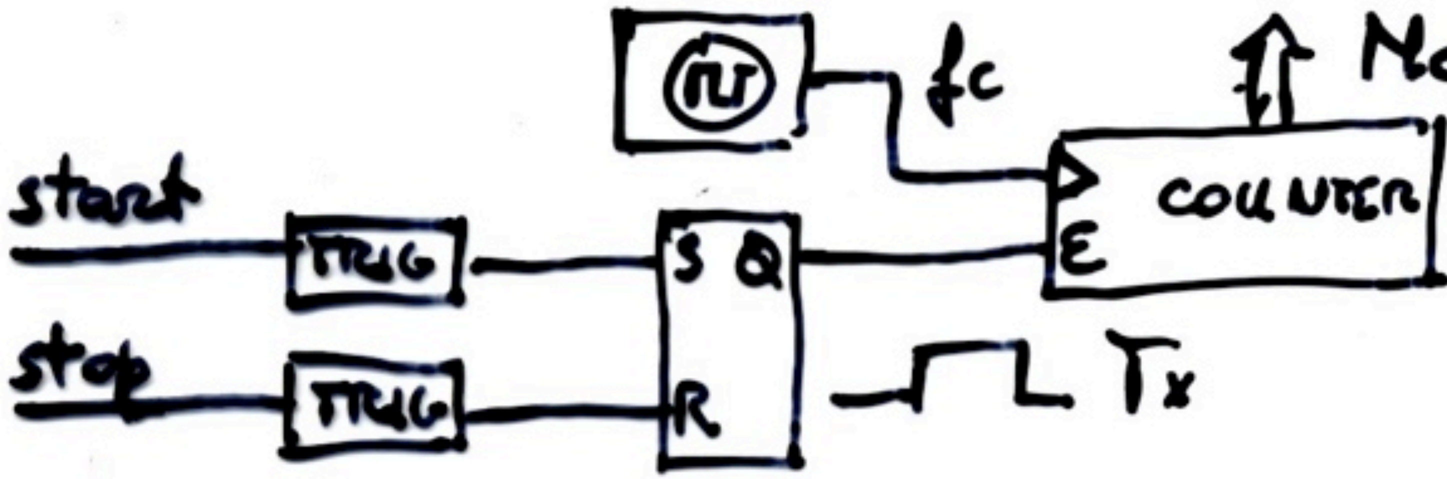
counts modulo M from 0 to M-1

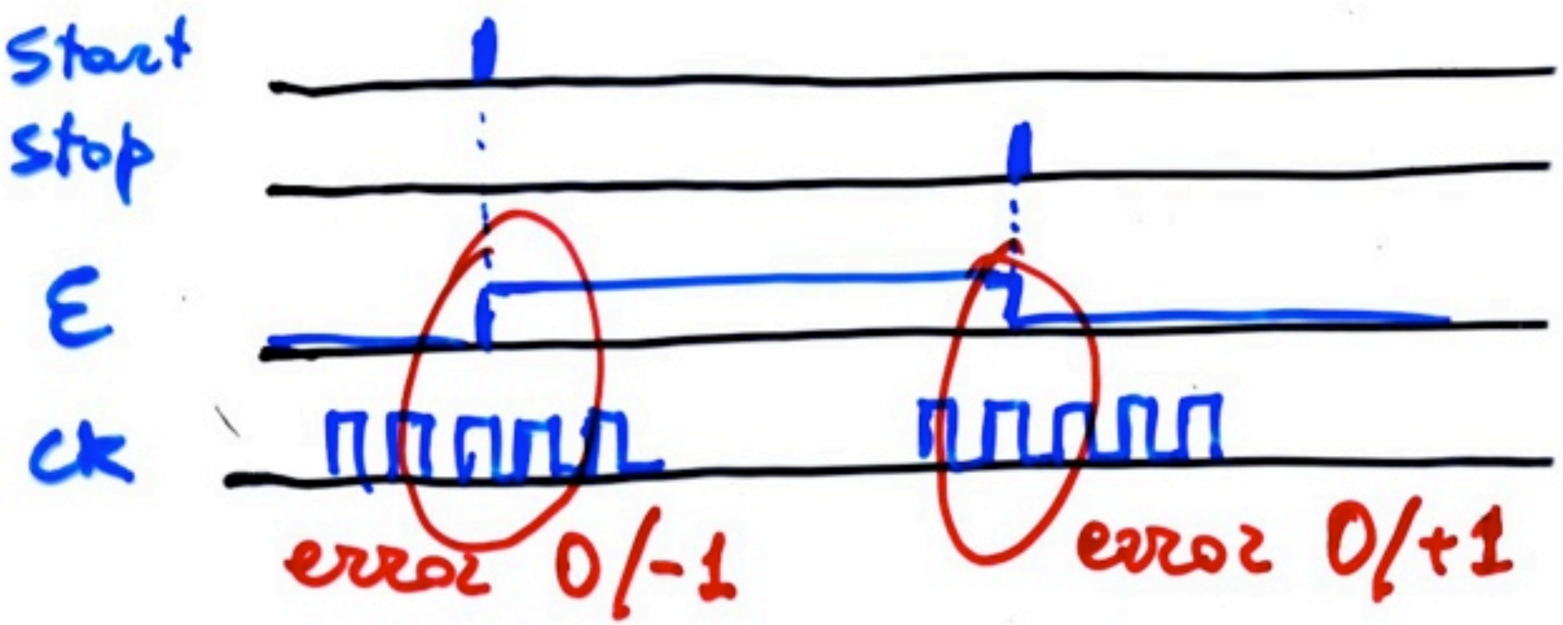


A counter and a divider are almost the same thing
Often a counter has a sufficiently large no. of bits, for it does not overflow.

2 – Basic counters

TIME INTERVAL COUNTER





Nc clock pulses are counted

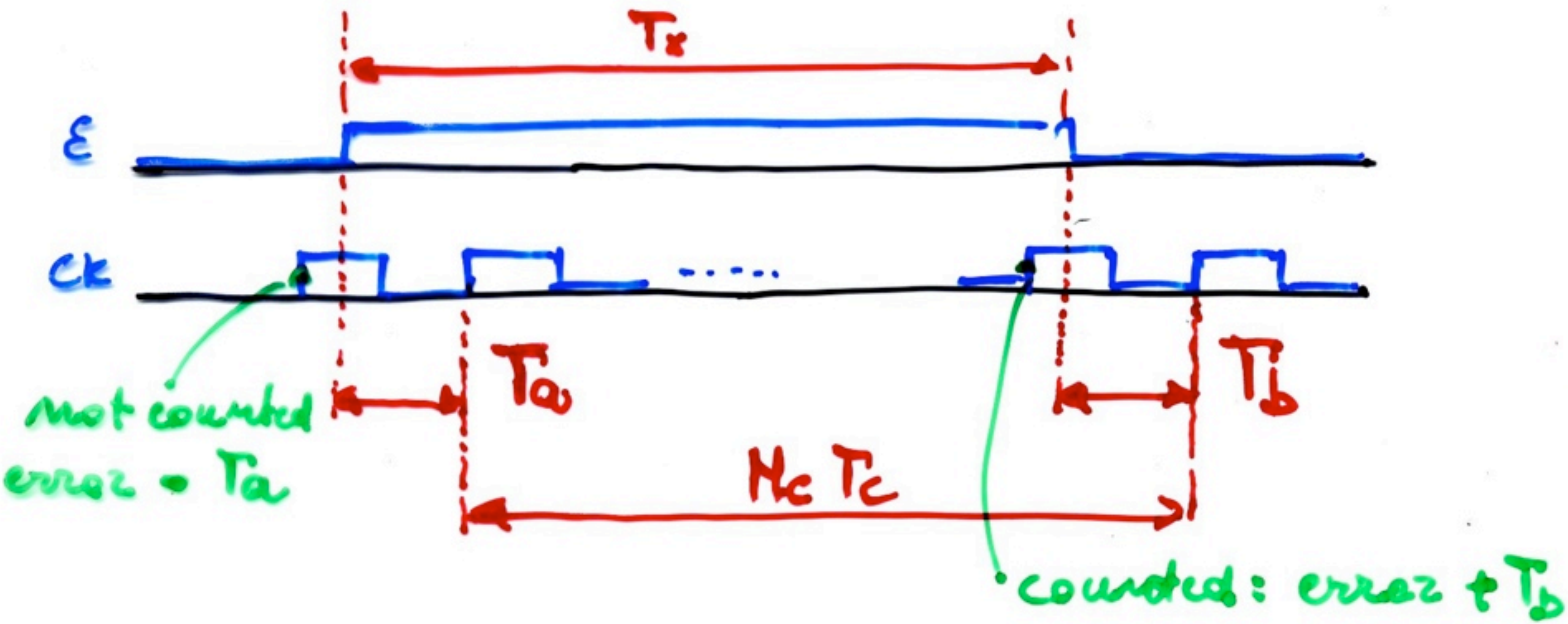
estimated T_x $\quad T_x = N_c T_c = N_c / f_c$

RESOLUTION ↔ QUANTIZATION $\quad \delta N_c = 1 \quad (\pm 1)$

$$\frac{\delta T_x}{T_x} = \frac{1}{N_c} \iff \delta T_x = T_c$$

use the highest possible f_c

INTERPOLATION PROBLEM



$$T_x = N_c T_c + T_a - T_b$$

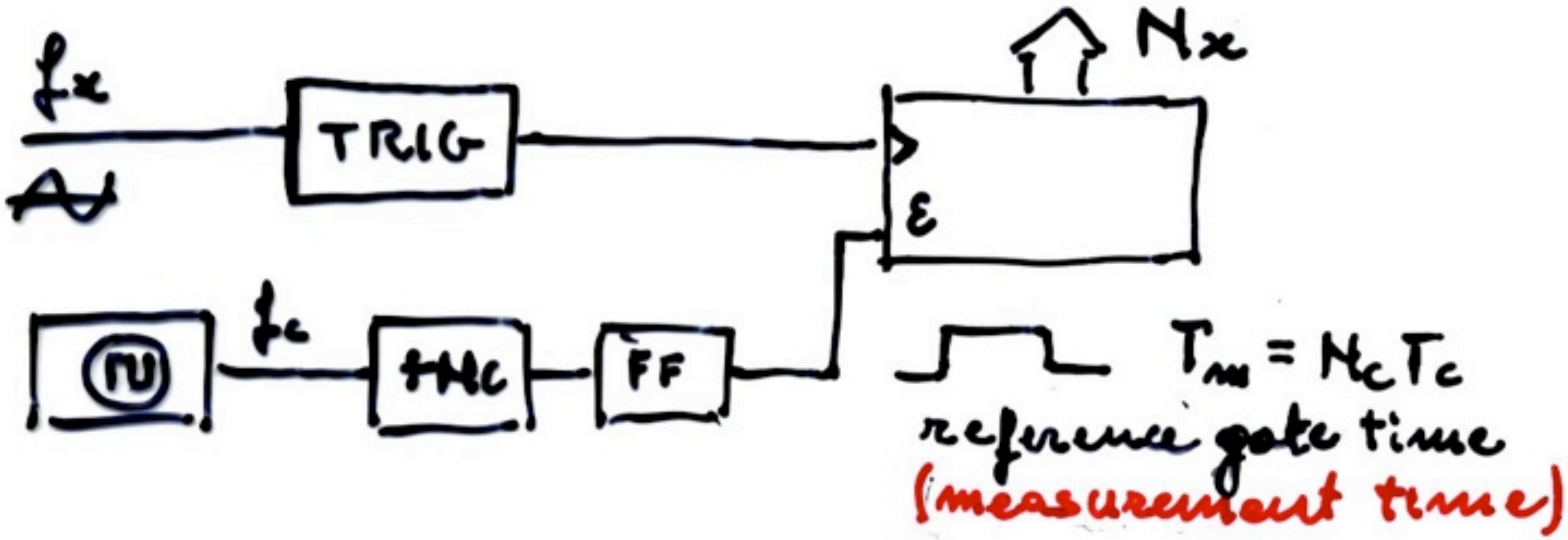
Measure $N_c T_c$, T_a, and T_b

RESOLUTION $\delta T_x = \delta T_a + \delta T_b$ (or $\sqrt{(\cdot)^2 + (\ )^2}$)

ACCURACY $\delta T_a \rightarrow \frac{\delta T_x}{T_x} \approx \frac{\delta T_a}{T_x}$ because $T_a \ll T_x$

relative accuracy $\delta T_a / T_a$ is not (very) critical

FREQUENCY COUNTER



$$N_x T_x = N_c T_c$$

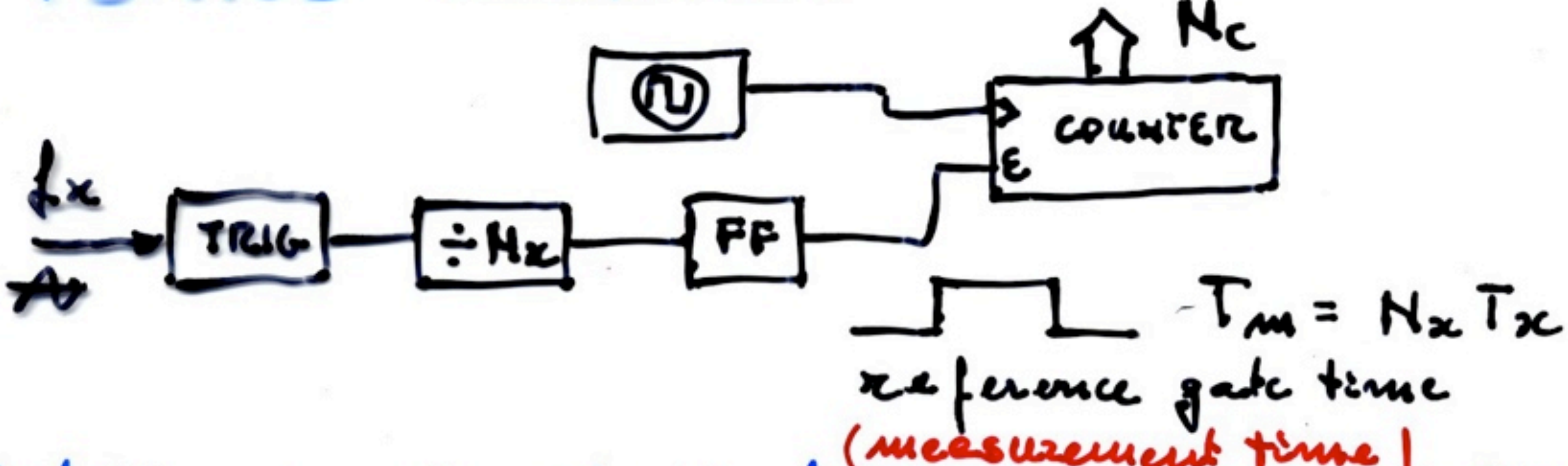
Estimate $f_x = 1/T_x \longrightarrow f_x = \frac{N_x}{N_c} f_c$

QUANTIZATION $\longleftrightarrow \delta N_x = 1$

$$\frac{\delta f_x}{f_x} = \frac{1}{N_x} \iff \frac{\delta f_x}{f_x} = \frac{1}{f_x T_m}$$

- poor resolution at low f_x
- don't even think to interpolate the period T_x (variable in a wide range)

PERIOD COUNTER



Set N_x according to the desired measurement time T_m

$$N_x T_x = N_c T_c$$

estimate f_x $\longrightarrow$ $f_x = \frac{N_x}{N_c} f_c$

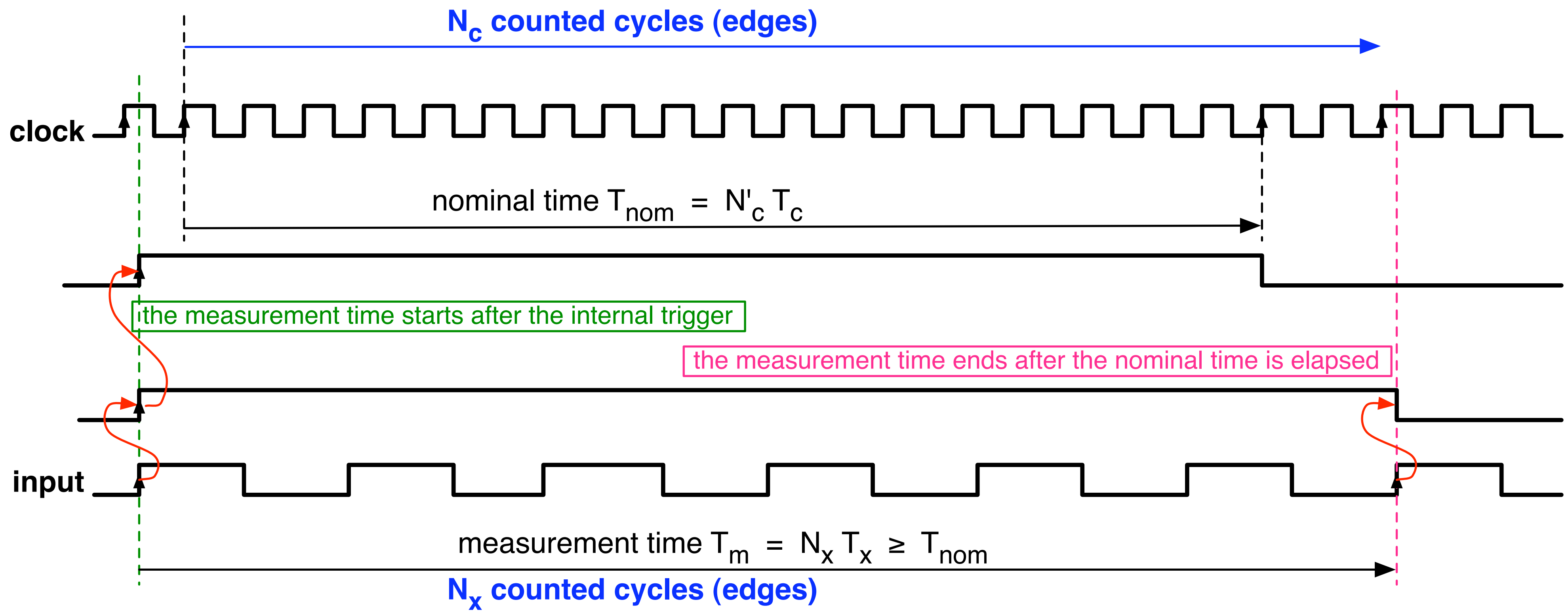
QUANTIZATION $\longrightarrow$ $\delta N_c = 1$

$$\frac{\delta f_x}{f_x} = \frac{1}{N_c} \longleftrightarrow \frac{\delta f_x}{f_x} = \frac{1}{f_c T_m}$$

Choose f_c as the highest frequency for the available technology (or a round no. just below it)

fixed f_c $\rightarrow$ interpolation is possible.

Practical measurement

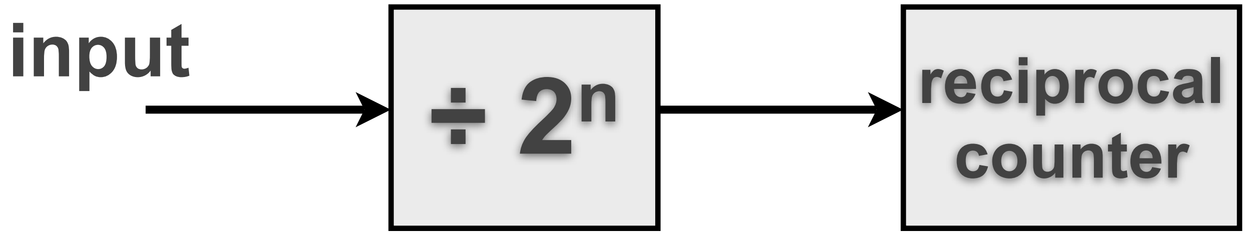


measurement equation: $N_x T_x = N_c T_c$

or $N_x T_x = (N_c \pm 1)\, T_c$, including quantization uncertainty

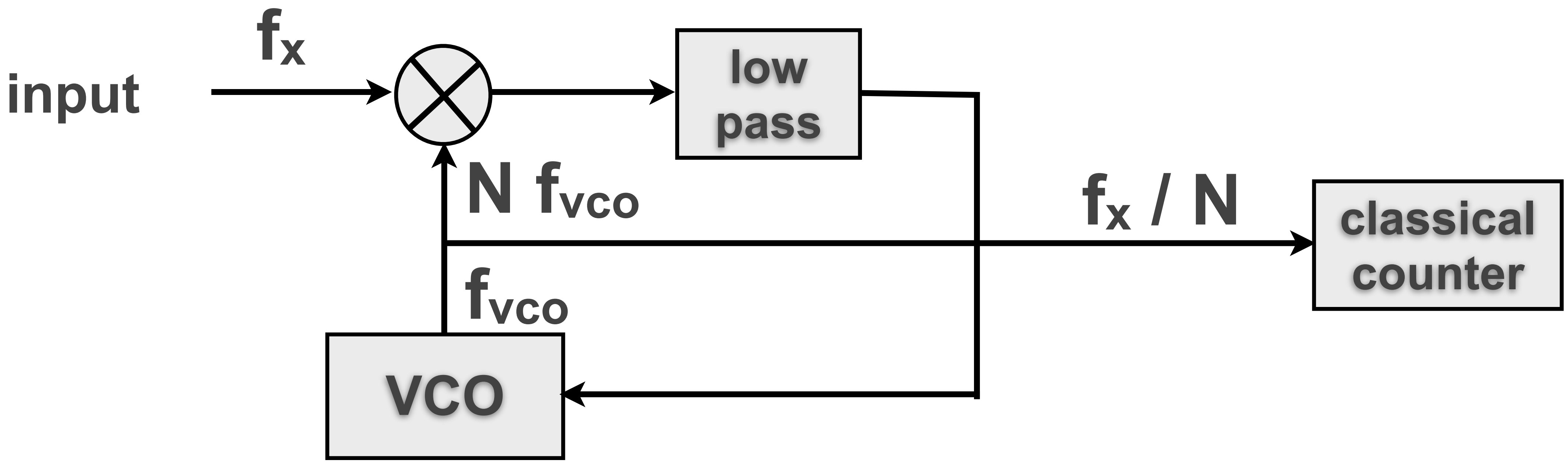
3 – Microwave counters

Prescaler



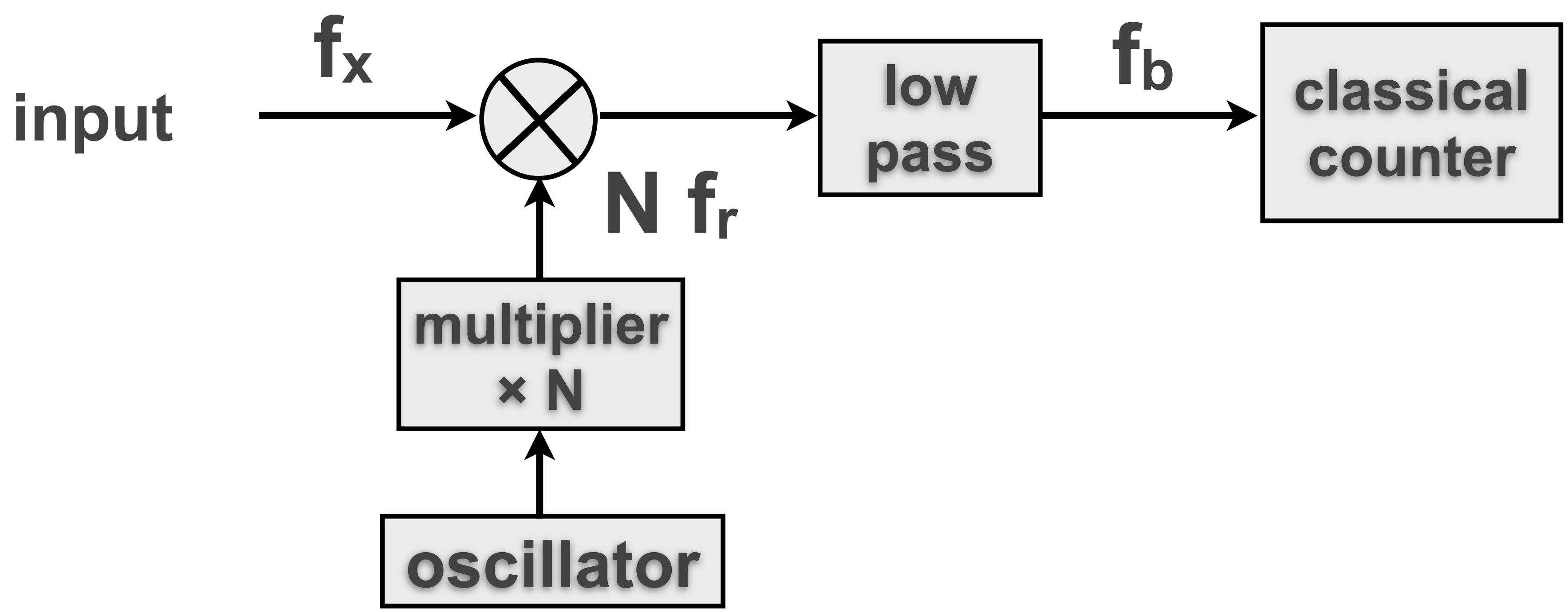
- a prescaler is a n-bit binary divider $\div 2^n$
- GaAs dividers work up to ≈ 20 GHz
- reciprocal counter => there is no resolution reduction
- Most microwave counters use the prescaler

Transfer-oscillator counter



- The transfer oscillator is a PLL
- Harmonics generation takes place inside the mixer
- Harmonics locking condition: $N f_{vco} = f_x$
- Frequency modulation Δf is used to identify N (a rather complex scheme, ×N => Δf -> $N\Delta f$)

Heterodyne counter

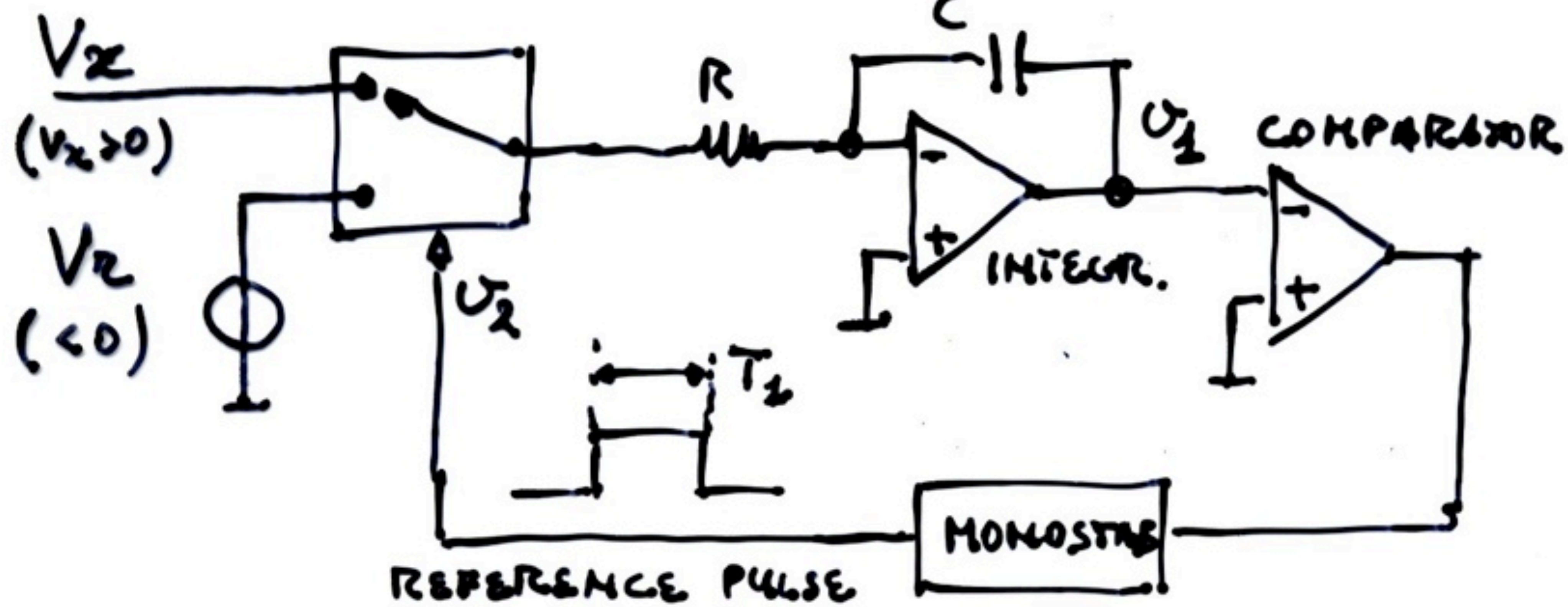


- Down-conversion: $f_b = | f_x - N f_c |$
- f_b is in the range of a classical counter (100-200 MHz max)
- no resolution reduction in the case of a classical *frequency* counter (no need of reciprocal counter)
- Old scheme, nowadays used only in some special cases (frequency metrology)

4 – Interpolation

6a

DUAL-SLOPE VOLTMETER



charge conservation

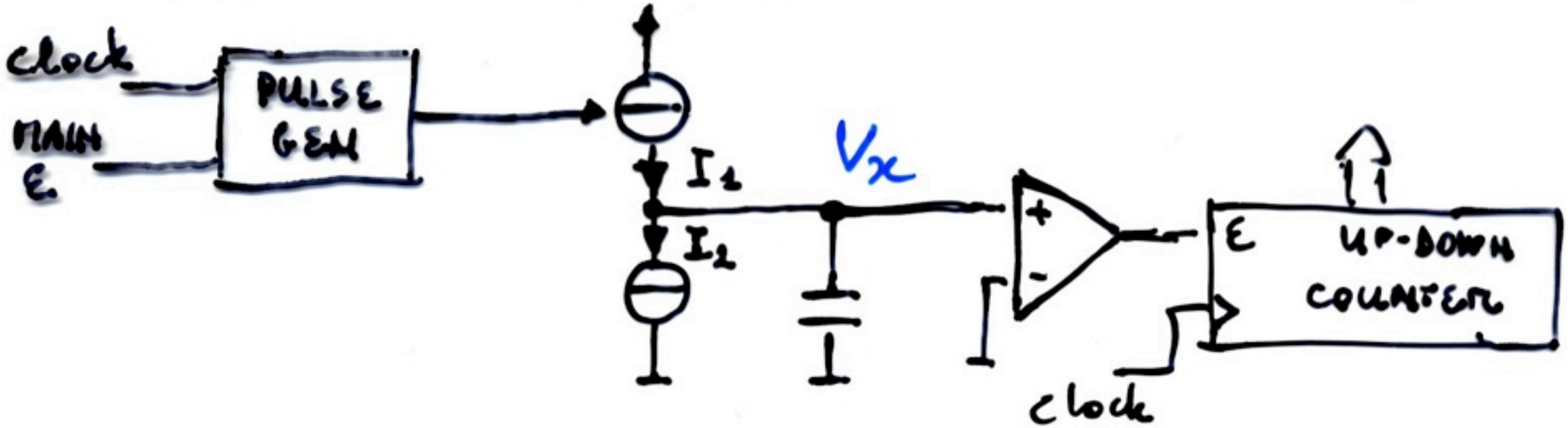
$$\frac{V_x}{RC} T_1 + \frac{V_2}{RC} T_2 = 0$$

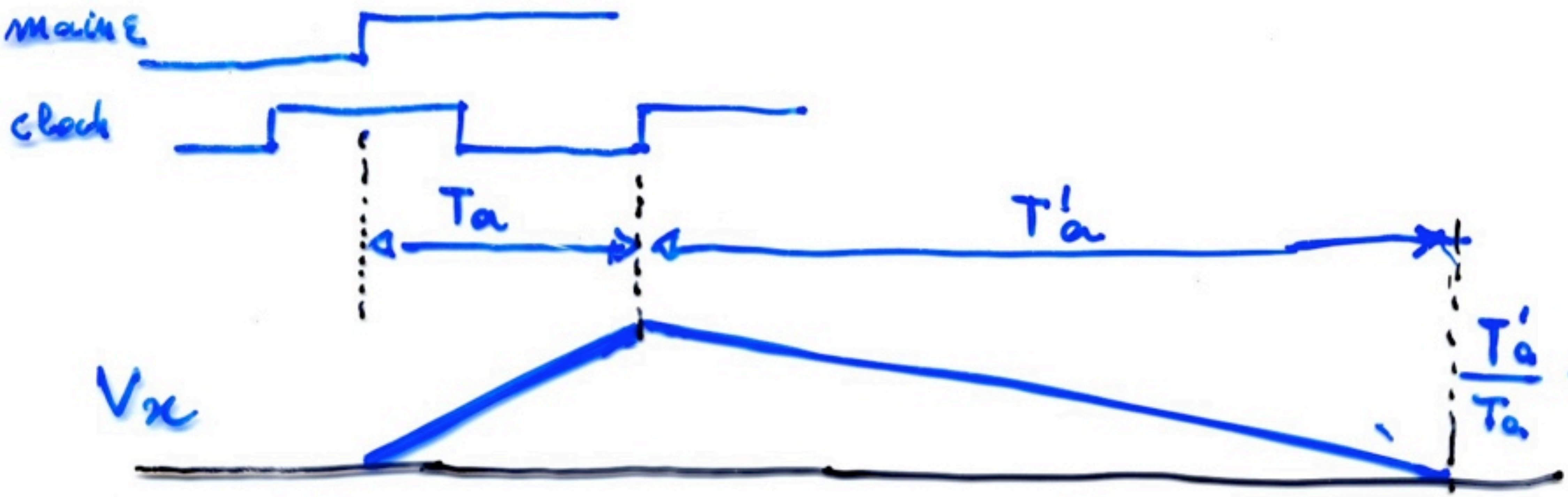
ESTIMATE V_x:

$$V_x = -\frac{T_2}{T_1} V_2$$

TIME INTERVAL AMPLIFIER

66





$$\frac{T'_a}{T_a} = \frac{I_1}{I_2} \gg 1$$

(can be 10^4)

- amplify T_a, increase resolution
- up/down counter → measure $T_a - T_b$

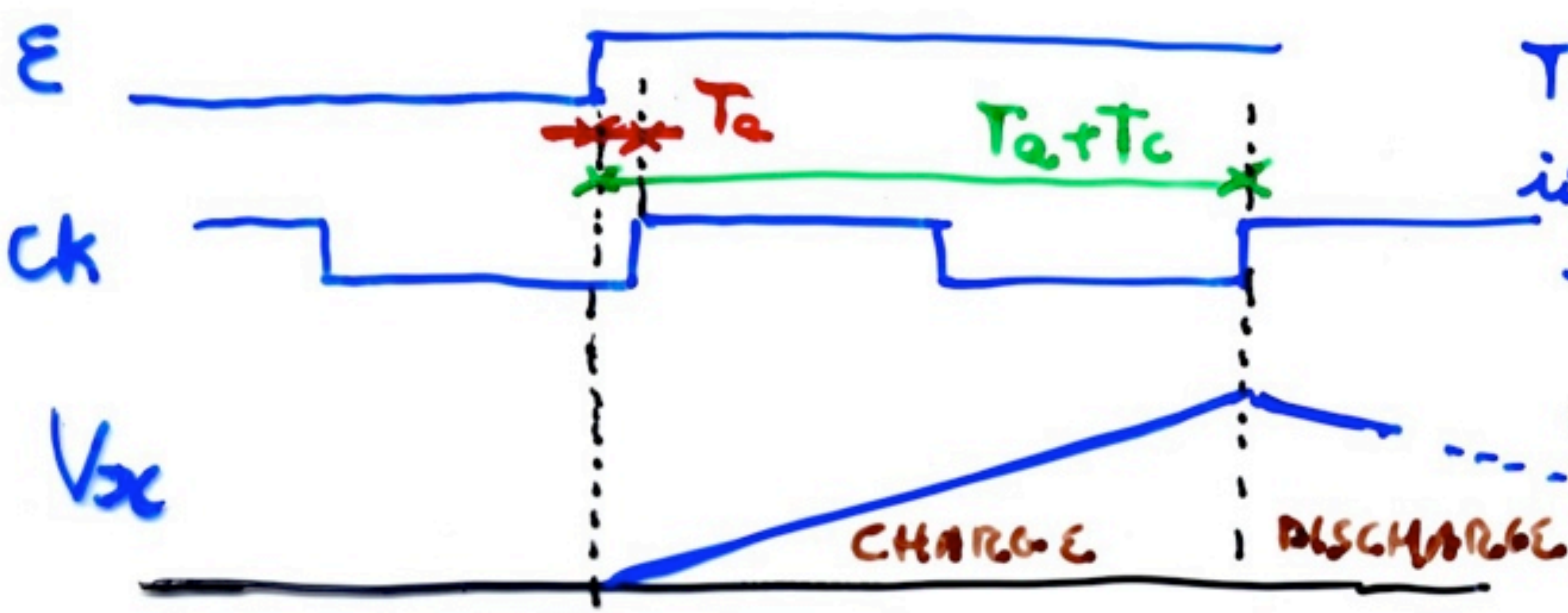
T_a

MINOR TECHNICAL DETAIL

T_a (and T_b) may be a very short time

- ARBITRATION
- ERROR due to the rising edges

SOLUTION



The current pulse is integrated over T_a+T_c ($\geq T_c$)

one T_c is added

ALSO T_b

Measuring $T_a - T_b$, the added T_c rubs out.

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EXAMPLE: NANOFAST 536 B (Smithsonian Astro. lab.)

Main clock $f_c = 10\ \mathrm{MHz} \rightarrow \delta T = T_c = 100\ \mathrm{ns}$

Time Interval amplifier $\frac{I_1}{I_2} = 4000$

$$T'_a \in (200\ \mu s,\ 400\ \mu s)$$

aux. clock 20 MHz for the measurement of T'_a

$$\delta T'_a = T'_c = 50\ \mathrm{ns} \qquad (1/20\ \mathrm{MHz})$$

$$\delta T_a = \frac{I_2}{I_1} T'_c \qquad \delta T_a = \frac{1}{4000} \times 50\ \mathrm{ns} = 12.5\ \mathrm{ps}$$

The Nanofast 536 B counter is (was?) a part of the Mark IV system for Very Long Baseline Interferometry (VLBI). Early TTL technology

Note: a pulse propagates in a cable at $c' \approx \frac{2}{3} c$

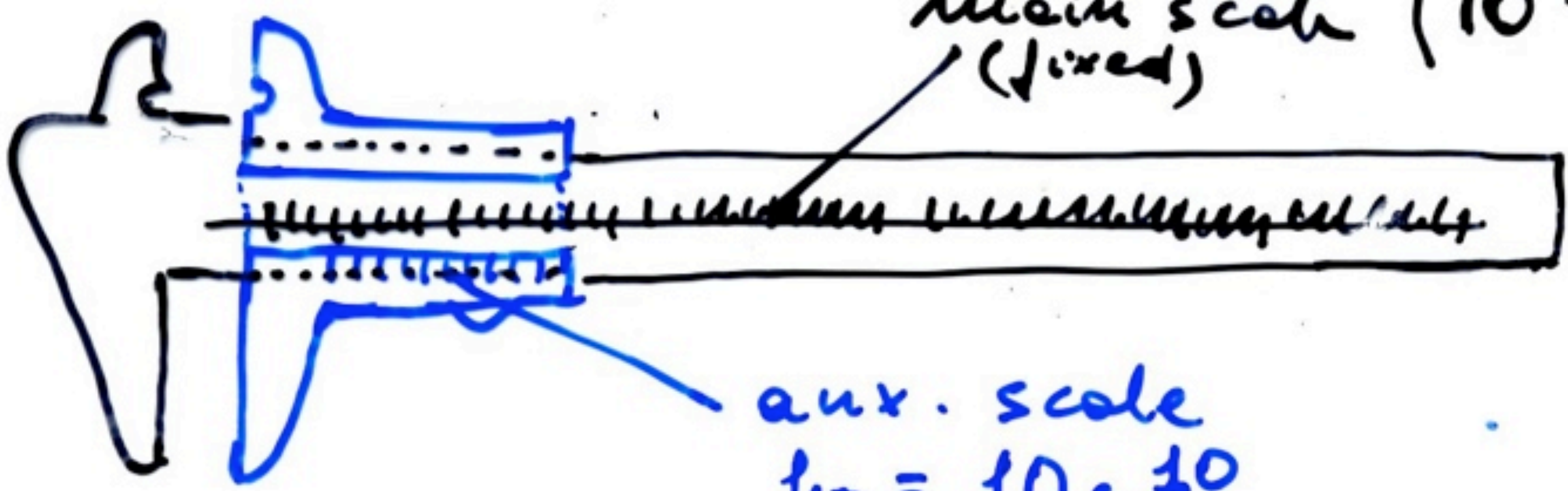
δT_a is equivalent to a length of 2.5 mm

VERNIER CALIPER

8e

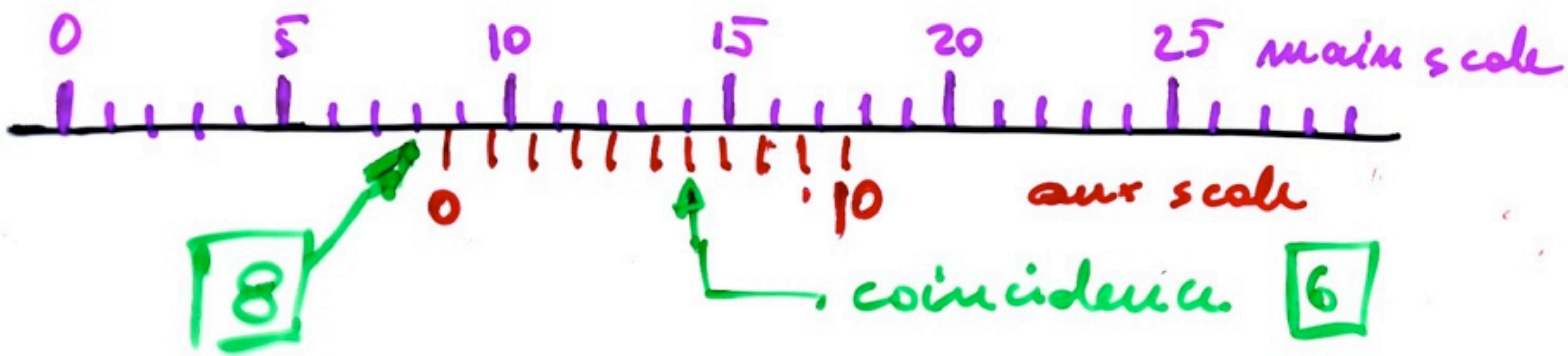
$f_c = 10$

main scale (10 ticks/cm)
(fixed)



aux. scale

$f_v = 10 \cdot \frac{10}{9}$ $\left(\frac{10 \text{ ticks}}{9 \text{ mm}}\right)$

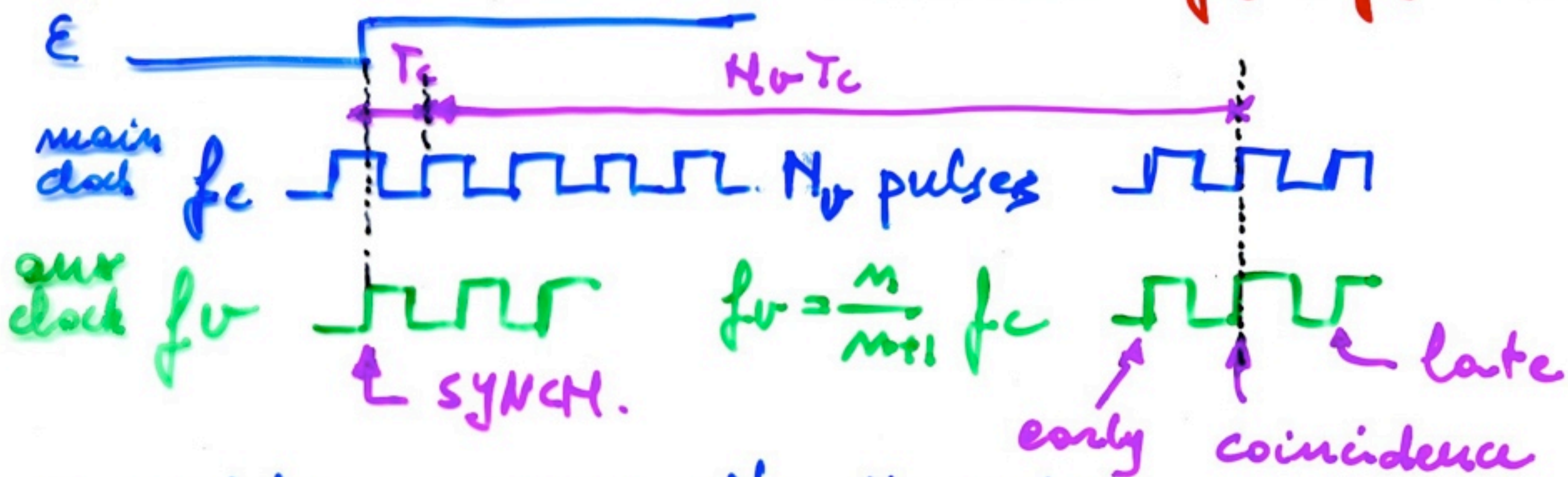


read: 8.6

main aux

FREQUENCY VERNIER

note that $f_v < f_c$ here



Coincidence occurs after N_v pulses

$$T_a + N_v T_c = N_v T_v \longrightarrow T_a = N_v \left[T_v - T_c \right]$$

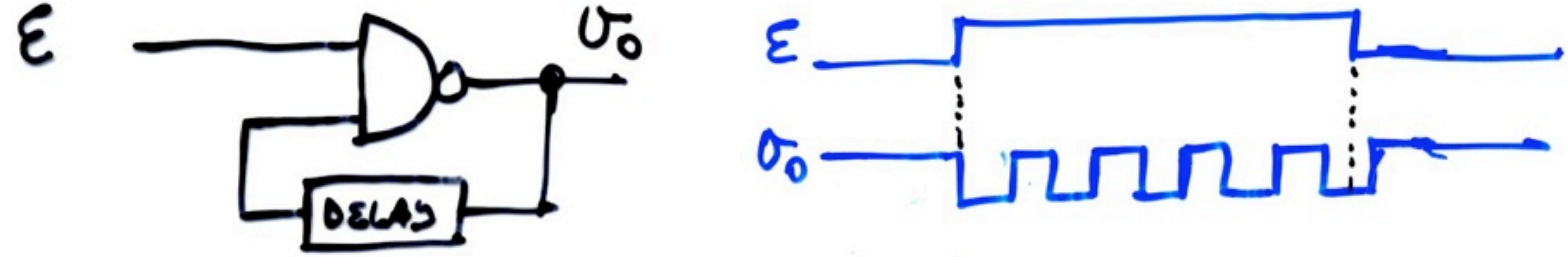
$$T_v = \frac{M+1}{M} T_c$$

$$T_a = N_v \frac{1}{M} T_c$$

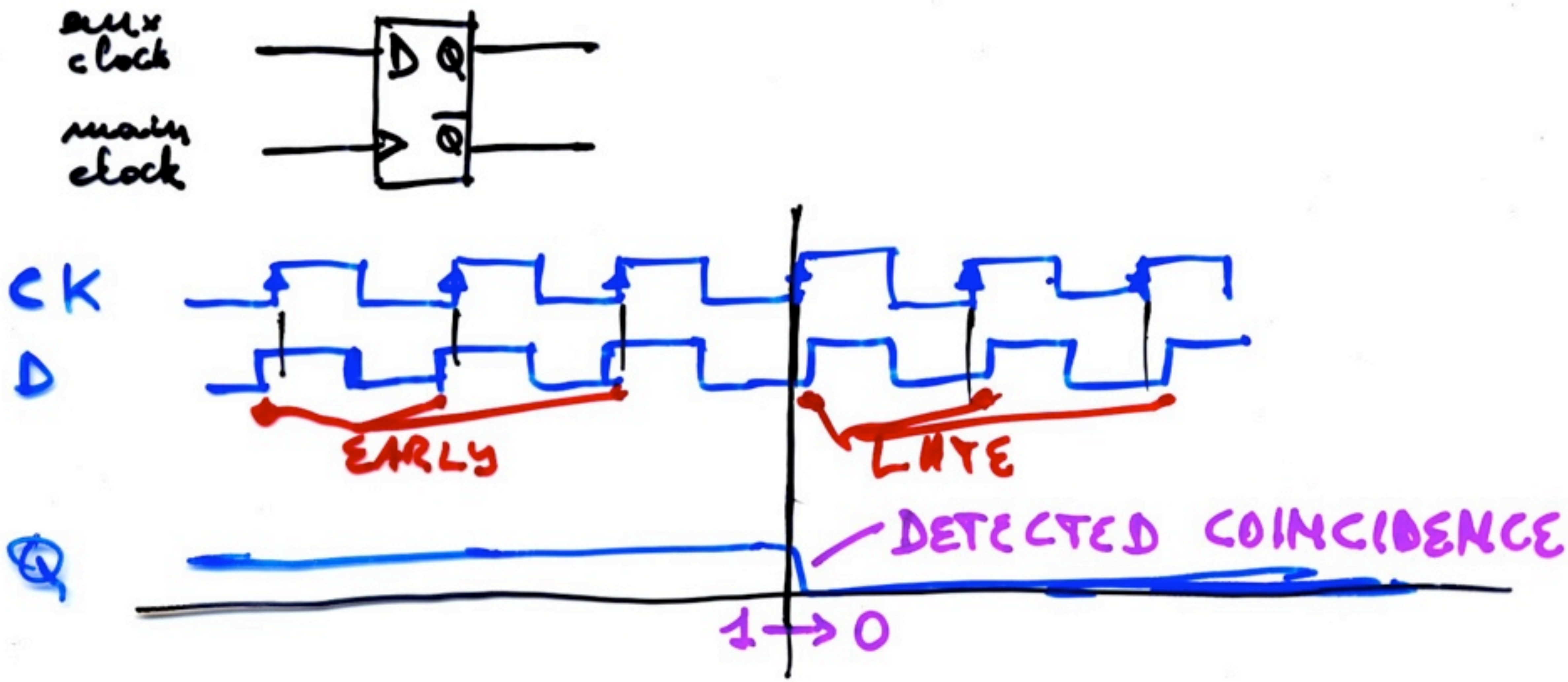
RESOLUTION $\delta N_v = 1 \Rightarrow \delta T_a = \frac{1}{M} T_c$

9a

SYNCHRONIZED OSCILLATOR



COINCIDENCE DETECTOR



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EXAMPLE. HP-5370A

$f_c = 200$ MHz $\rightarrow$ $\delta T_x = 5$ ns
(ECL Technology)

$n = 256$ $\rightarrow$ $\delta T_a = \delta T_b = \frac{1}{256} \times 5\,\text{ns} = 19.5\,\text{ps}$
($f_v = 199.22$ MHz)

It takes a max. of 257 cycles of f_c for the two clocks to coincide

conversion time:

$$257 \times 5\,\text{ns} = 1.285\,\mu\text{s}$$

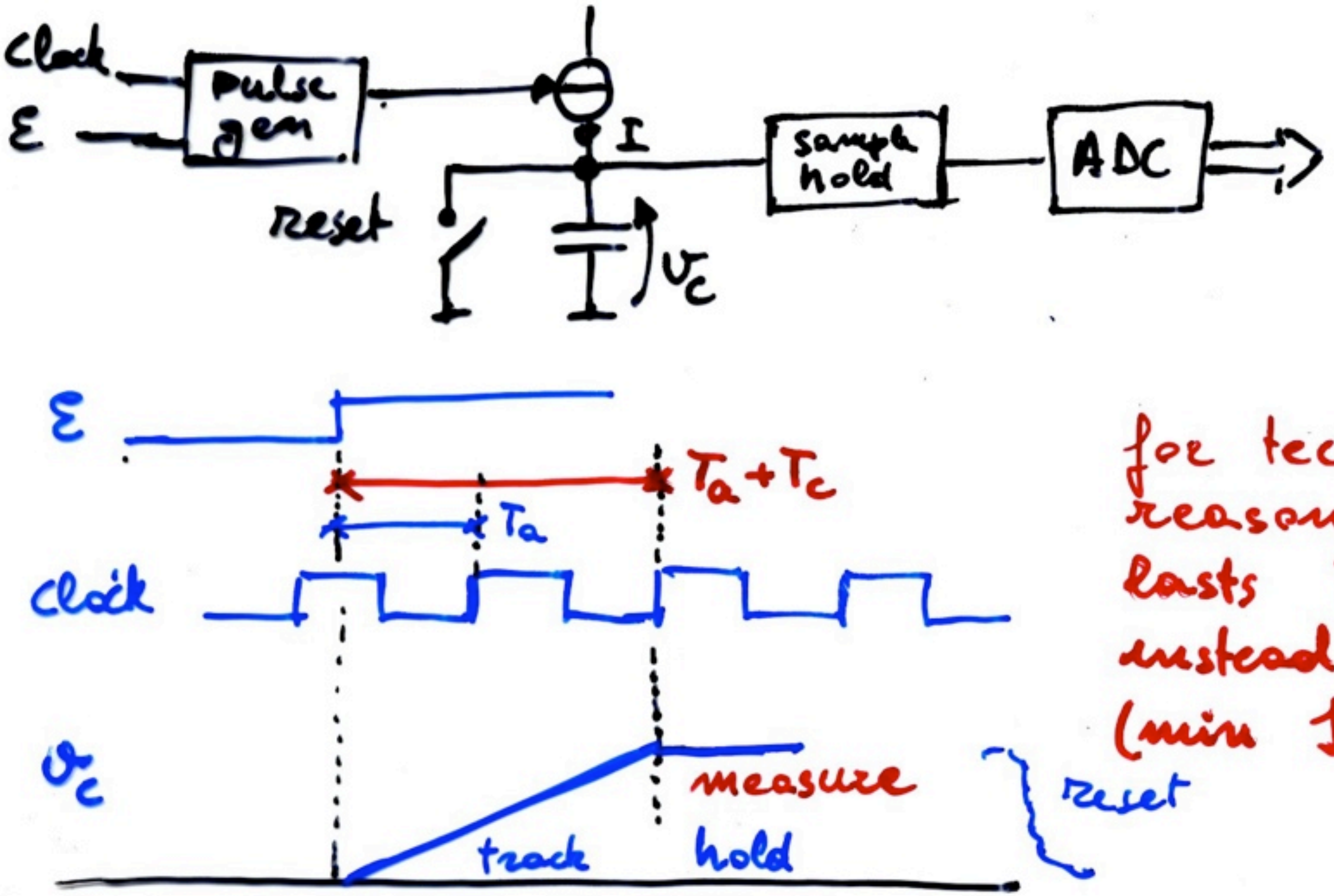
Light speed in a cable $\approx 0.67c$

$$\delta T_a \leftrightarrow \delta l \approx 4\,\text{mm}$$

(length)

TIME-TO-VOLTAGE CONVERTER

10a



for technical reason, integration lasts $T_a + T_c$ instead of T_a (min 1 clock cycle)

- The method is equivalent to the TI amplifier
- Successive approx. conversion is faster then dual-slope

EXAMPLE STANFORD SR·620

101

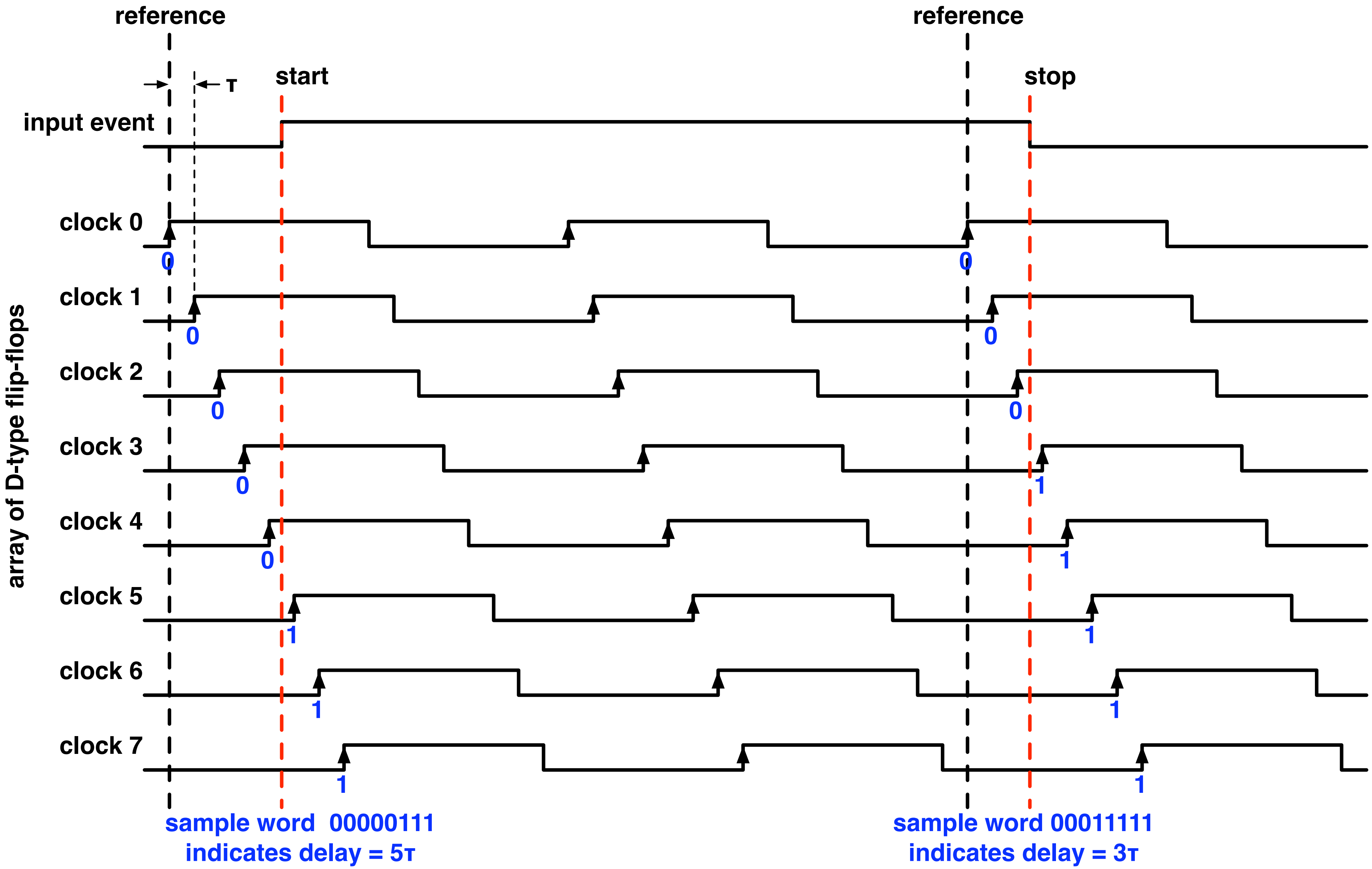
$f_c = 90$ MHz — phase-locked to the 10 MHz reference.

$T_c = 11.1$ ns — ECL Technology

12 bit converter, 1 bit lost because of the extra T_c

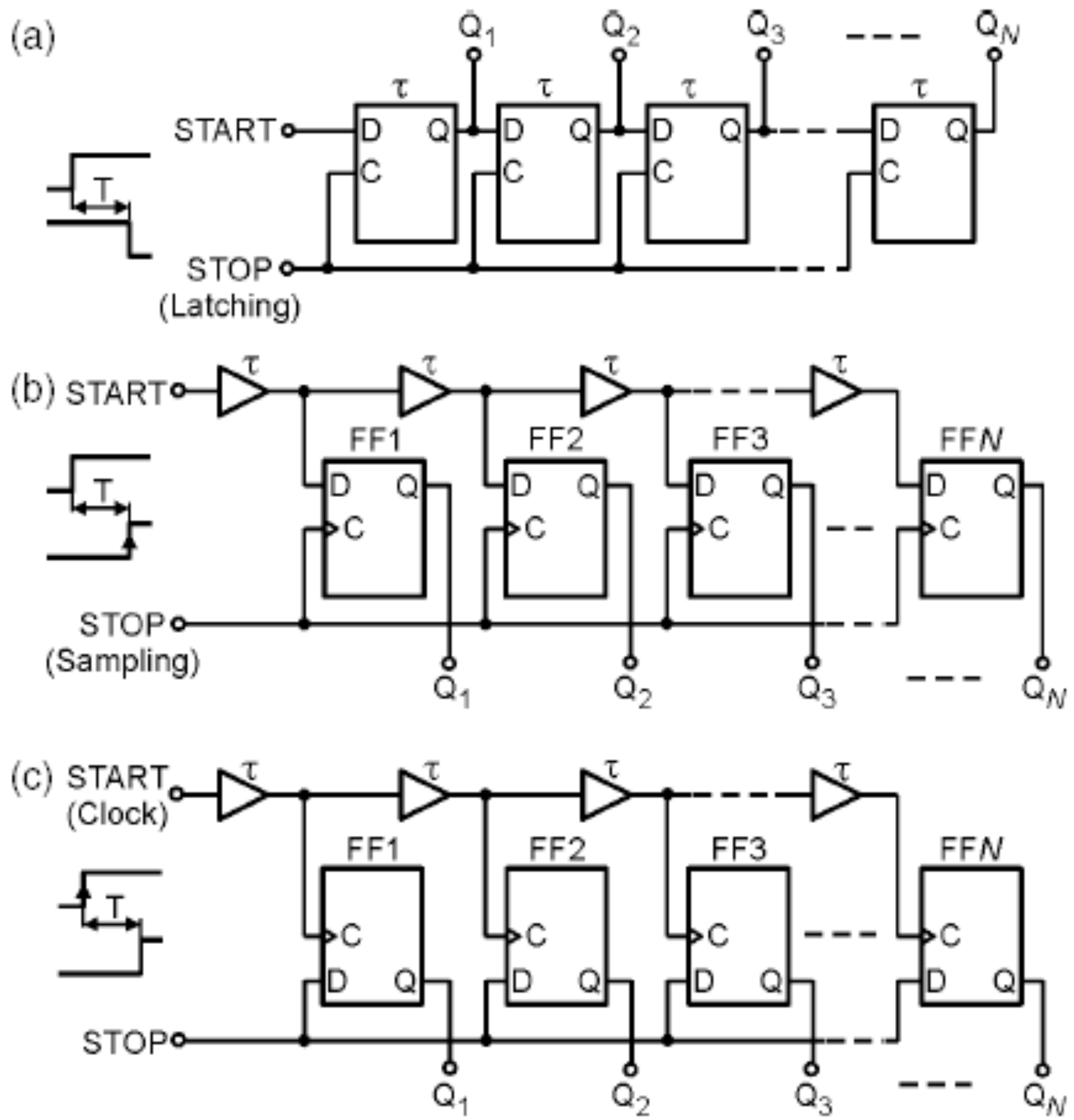
11 bits $\quad \delta T_e = \dfrac{11.1\ \text{ns}}{2^{11}} = 5.4\ \text{ps}$

Interpolation by sampling delayed copies of the clock or of the stop signal



The resolution is determined by the delay τ, instead of by the toggling speed of the flip-flops

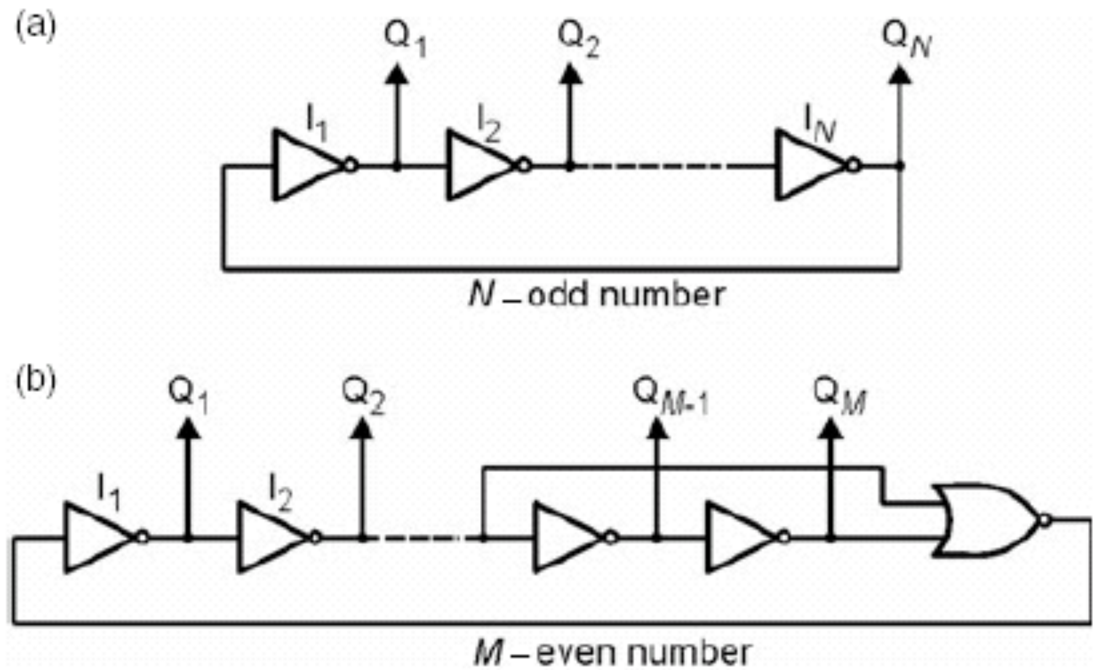
Sampling circuits



J. Kalisz, Metrologia 41 (2004) 17–32

Ring Oscillator

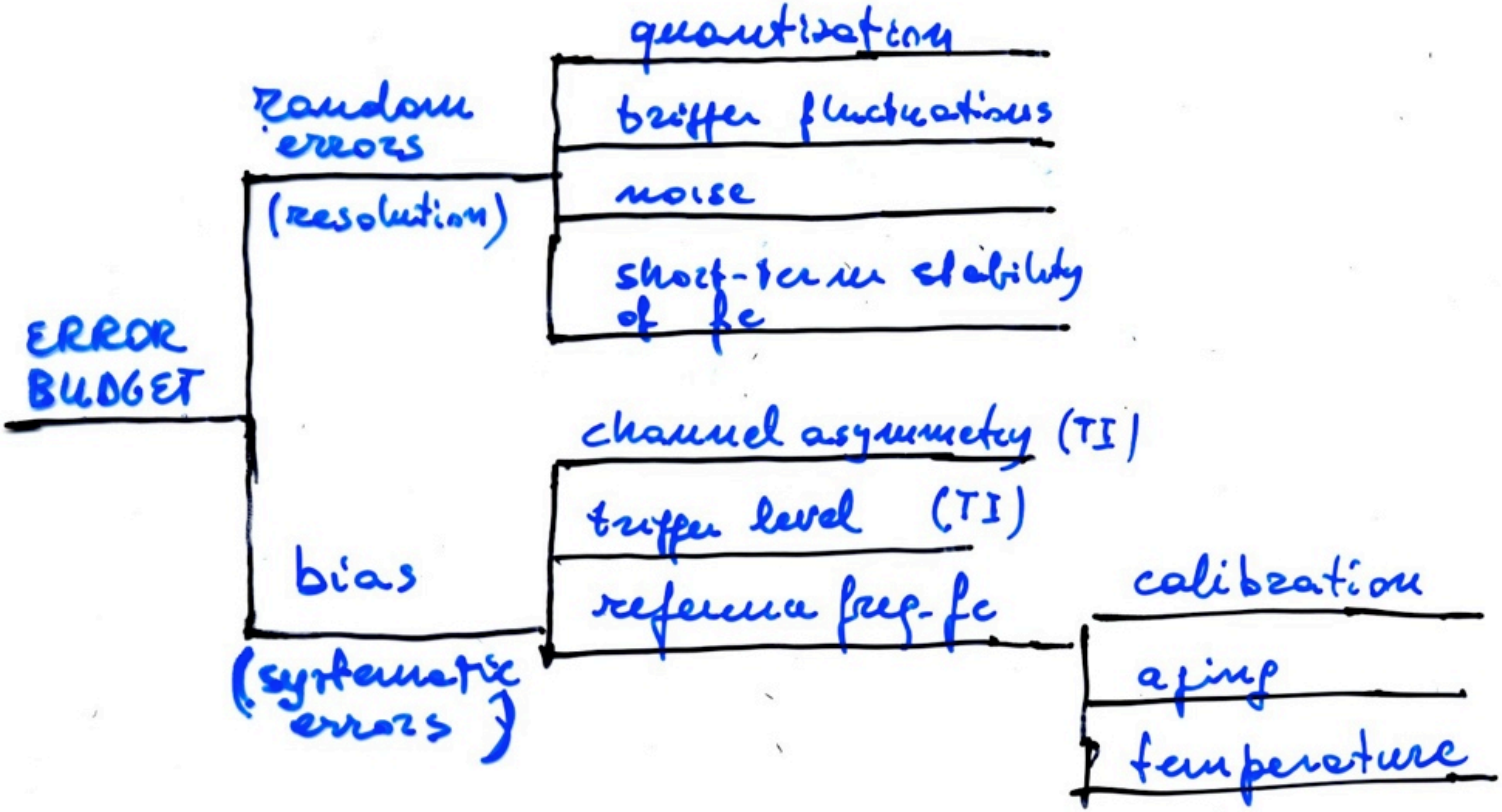
used in PLL circuits for clock-frequency multiplication



J. Kalisz, Metrologia 41 (2004) 17–32

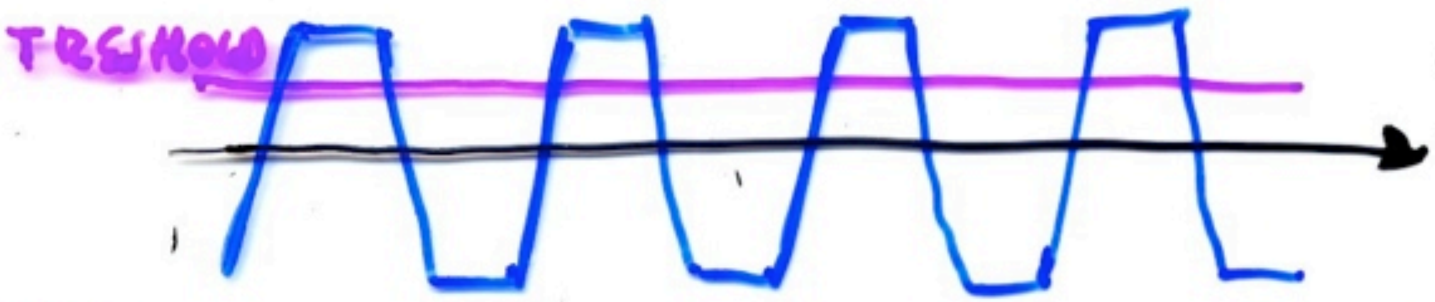
5 – Basic statistics

ACCURACY AND PRECISION

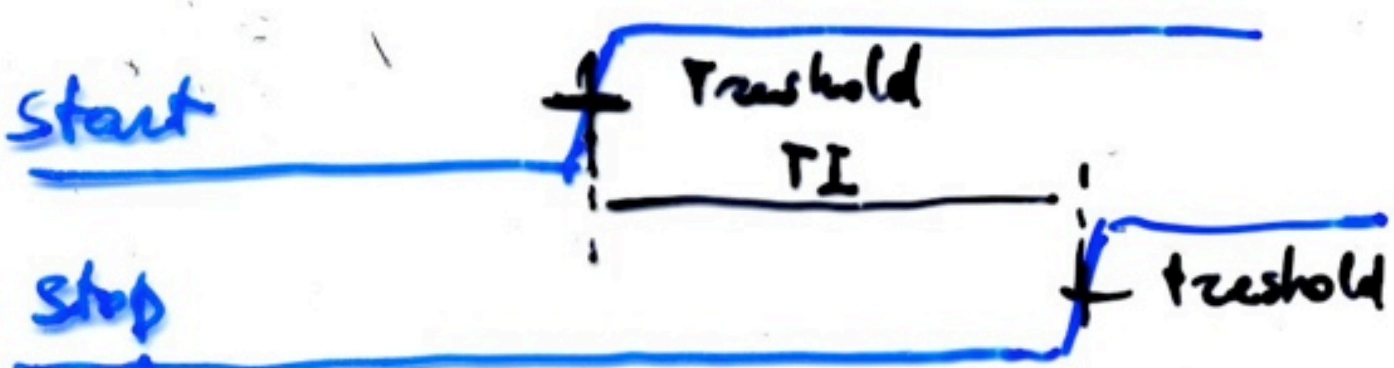


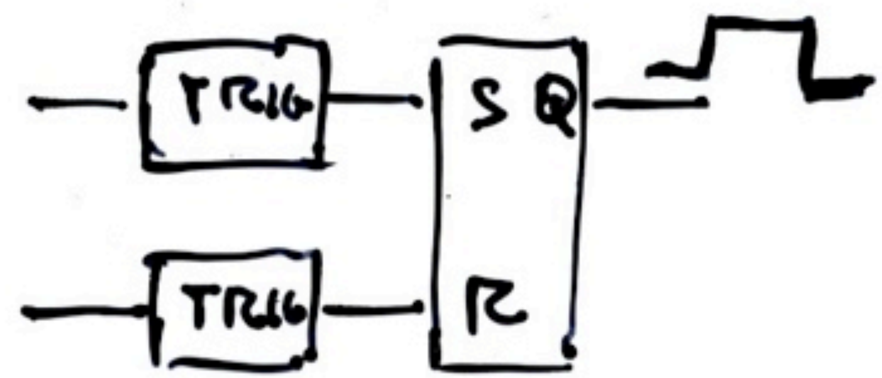
Old Hewlett Packard application notes

TRIGGER



The measured frequency/period is (almost) independent of the trigger treshold, and also on slow fluctuations





Accuracy and stability of the trigger tresholds are critical parameters on Time-Interval measurements

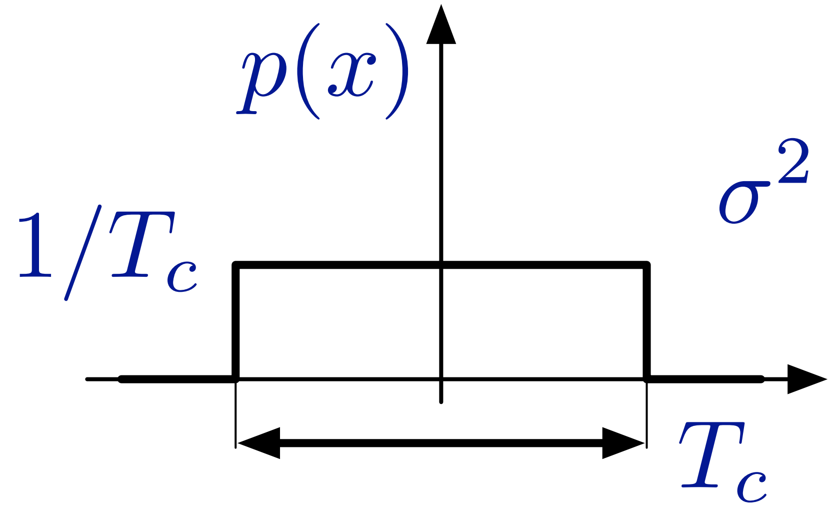
JITTER

$$\delta t = \frac{\delta V}{dV/dt}$$

voltage noise

slew rate

Quantization uncertainty

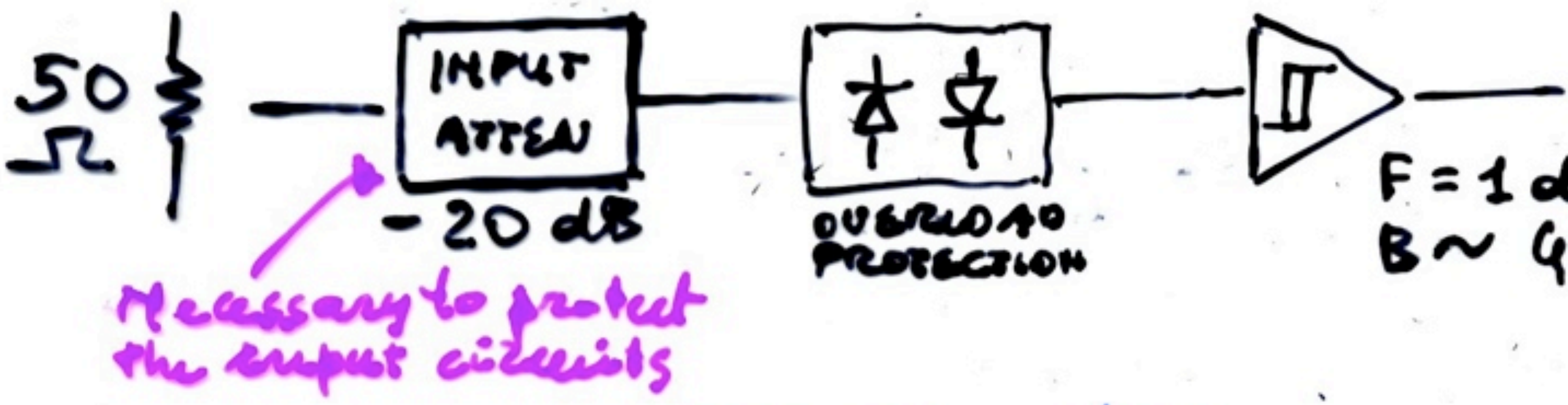


$$\sigma^2 = \frac{T_c^2}{12}$$

$$1/\sqrt{12} = 0.29$$

Example: 100 MHz clock
Tx = 10 ns
σ = 2.9 ns

DON'T BLAME THE TRIGGER



F = 1 dB   Noise figure
B ~ 4 GHz   bandwidth

THERMAL NOISE $\sqrt{4kTR}$ $V/\sqrt{Hz}$ $\longrightarrow$ $0.9\ \mu V/\sqrt{Hz}$

Boltzmann $k = 1.38 \times 10^{-23}$
temperature $T = 300$ K
resistance $R = 50\ \Omega$

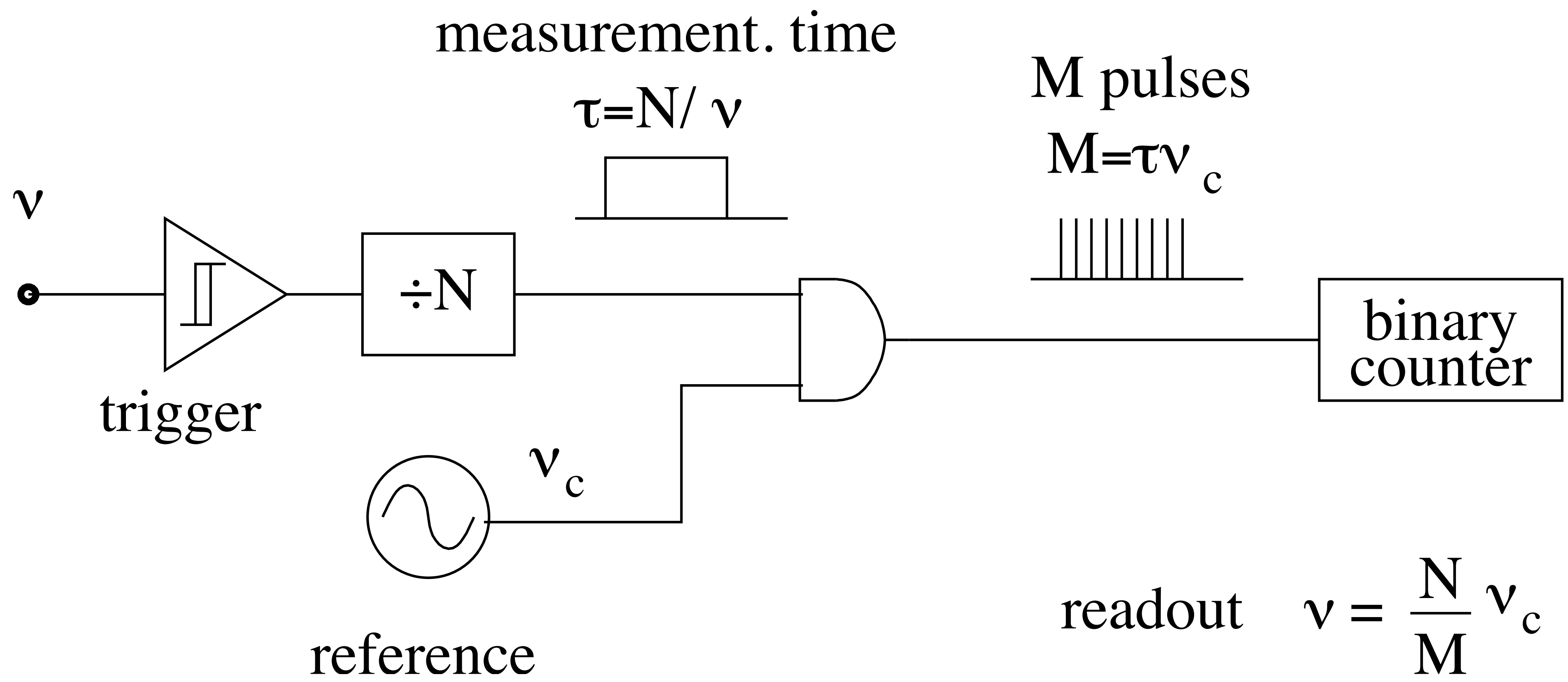
THERMAL NOISE INTEGRATED OVER THE BANDWIDTH $\times\sqrt{B}$

(HP 5370) $B = 225$ MHz $\rightarrow$ $V_n = 13.5\ \mu V$
(SR 620) $B = 1.3$ GHz $\rightarrow$ $V_n = 32.5\ \mu V$

Account for the loss $\ell = 20$ dB (multiply by 10)
and for the noise figure $F = 1$ dB (multiply by 1.12)

$B = 225$ MHz $\rightarrow$ $V_n = 150\ \mu V$
$B = 1.3$ GHz $\rightarrow$ $V_n = 355\ \mu V$

classical reciprocal counter (1)

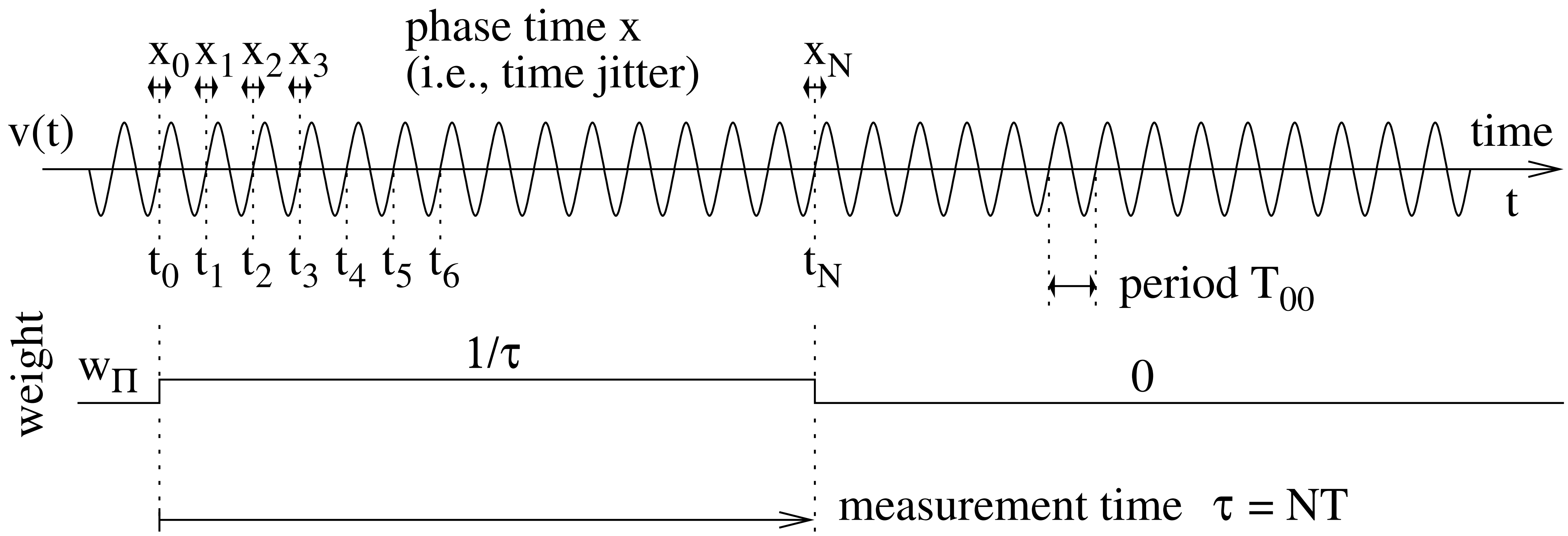


$$\nu = \frac{N}{M}\nu_c$$

period measurement (count the clock pulses) is preferred to frequency measurement (count the input pulses) because:

- it provides higher resolution in a given measurement time tau (the clock frequency can be close to the maximum switching speed)
- interpolation (M is rational instead of integer) can be used to reduce the quantization (interpolators only work at a fixed frequency, thus at the clock freq.)

classical reciprocal counter (2)



measure: scalar product

$$\mathbb{E}\{\nu\} = \int_{-\infty}^{+\infty} \nu(t) w_\Pi(t)\, dt$$ Π estimator

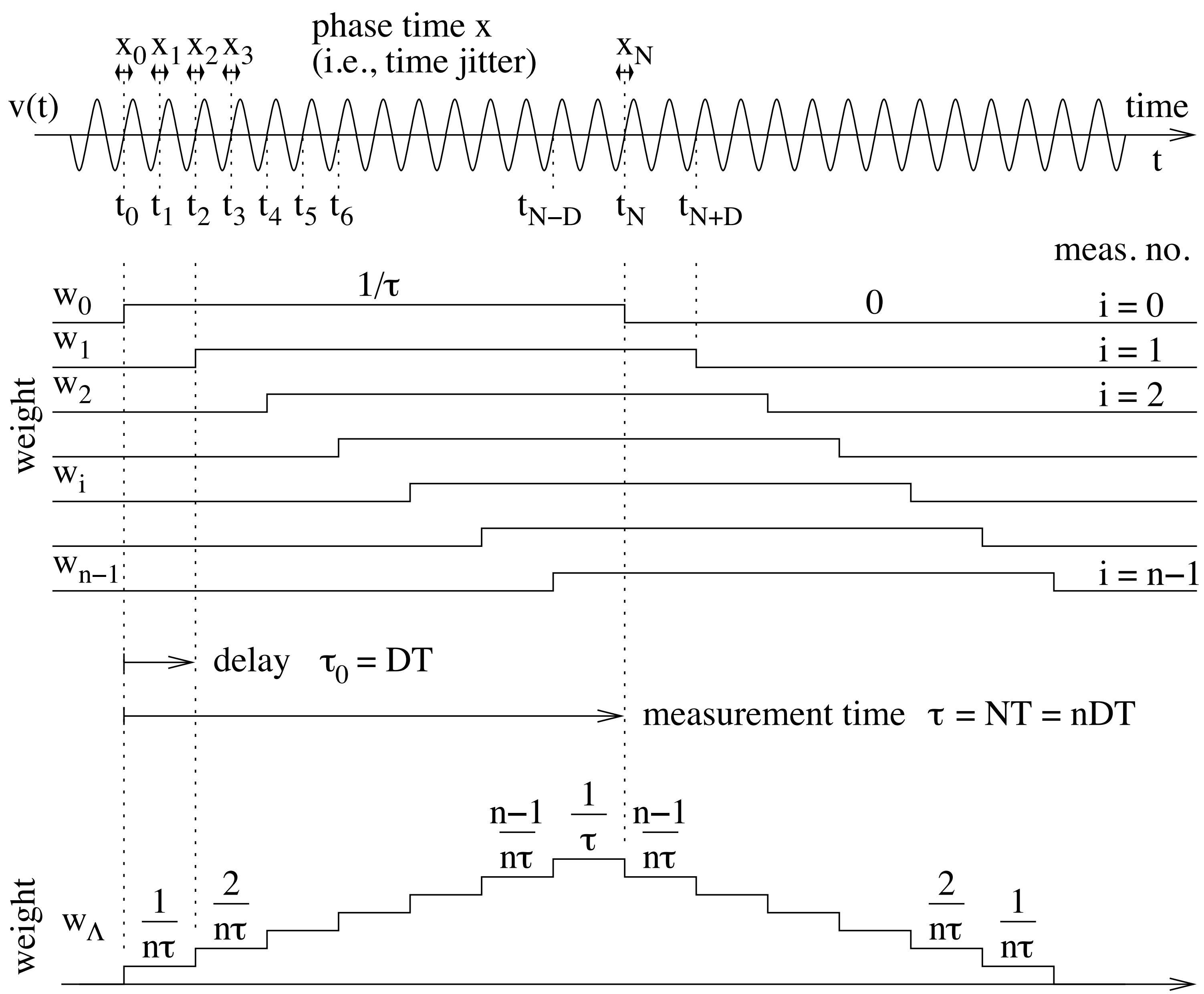
$$w_\Pi(t) = \begin{cases} 1/\tau & 0 < t < \tau \\ 0 & \text{elsewhere} \end{cases}$$ weight

$$\int_{-\infty}^{+\infty} w_\Pi(t)\, dt = 1$$ normalization

variance

$$\sigma_y^2 = \frac{2\sigma_x^2}{\tau^2}$$ classical variance

enhanced-resolution counter



$$\mathbb{E}\{\nu\} = \frac{1}{n}\sum_{i=0}^{n-1}\overline{\nu}_i \qquad \overline{\nu}_i = N/\tau_i$$

Λ estimator

$$\mathbb{E}\{\nu\} = \int_{-\infty}^{+\infty} \nu(t)\,w_\Lambda(t)\,dt$$

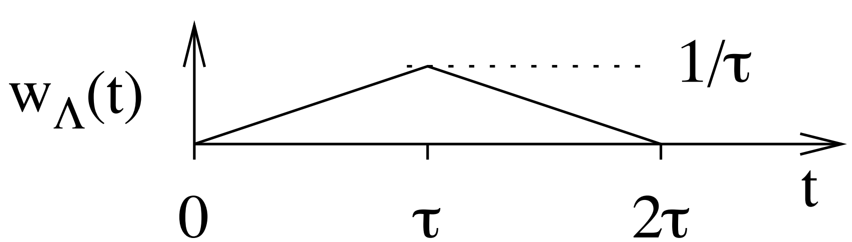
weight

$$w_\Lambda(t) = \begin{cases} t/\tau & 0 < t < \tau \\ 2 - t/\tau & \tau < t < 2\tau \\ 0 & \text{elsewhere} \end{cases}$$

normalization

$$\int_{-\infty}^{+\infty} w_\Lambda(t)\,dt = 1$$

limit τ_0 -> 0 of the weight function



white noise: the autocorrelation function is a narrow pulse, about the inverse of the bandwidth

the variance is divided by n

$$\sigma_y^2 = \frac{1}{n}\,\frac{2\sigma_x^2}{\tau^2}$$

classical variance

actual formulae look like this

$$(\Pi) \quad \sigma_y = \frac{1}{\tau}\sqrt{2(\delta t)^2_{\mathrm{trigger}} + 2(\delta t)^2_{\mathrm{interpolator}}}$$

$$(\Lambda) \quad \sigma_y = \frac{1}{\tau\sqrt{n}}\sqrt{2(\delta t)^2_{\mathrm{trigger}} + 2(\delta t)^2_{\mathrm{interpolator}}}$$

$$n = \begin{cases} \nu_0\tau & \nu_{00} \leq \nu_I \\ \nu_I\tau & \nu_{00} > \nu_I \end{cases}$$

understanding technical information

classical reciprocal counter

$$\sigma_y^2 = \frac{2\sigma_x^2}{\tau^2}$$ classical variance

enhanced-resolution counter

$$\sigma_y^2 = \frac{1}{n}\,\frac{2\sigma_x^2}{\tau^2}$$ classical variance

low frequency: full speed

$$\tau_0 = T \quad \Longrightarrow \quad n = \nu_{00}\tau$$

$$\sigma_y^2 = \frac{1}{\nu_{00}}\,\frac{2\sigma_x^2}{\tau^3}$$ classical variance

high frequency: housekeeping takes time

$$\tau_0 = DT \ \text{ with } D>1 \quad \Longrightarrow \quad n = \nu_{00}\tau$$

$$\sigma_y^2 = \frac{1}{\nu_I}\,\frac{2\sigma_x^2}{\tau^3}$$ classical variance

the slope of the classical variance tells the whole story

$$1/\tau^2 \quad \Longrightarrow \quad \Pi \text{ estimator (classical reciprocal)}$$

$$1/\tau^3 \quad \Longrightarrow \quad \Lambda \text{ estimator (enhanced-resolution)}$$

look for formulae and plots in the instruction manual

examples

Stanford SRS-620

$$\begin{bmatrix}\text{RMS}\\ \text{resolution}\\ \text{(in Hz)}\end{bmatrix} = \frac{\text{frequency}}{\text{gate time}}\sqrt{\frac{(25\text{ ps})^2 + \left[\begin{pmatrix}\text{short term}\\ \text{stability}\end{pmatrix} \times \begin{pmatrix}\text{gate}\\ \text{time}\end{pmatrix}\right]^2 + 2\times\begin{bmatrix}\text{trigger}\\ \text{jitter}\end{bmatrix}^2}{\text{N}}}$$

RMS resolution	$\sigma_\nu = \nu_{00}\sigma_y$
frequency	ν_{00}
gate time	τ

Agilent 53132A

$$\begin{bmatrix}\text{RMS}\\ \text{resolution}\end{bmatrix} = \begin{pmatrix}\text{frequency}\\ \text{or period}\end{pmatrix} \times \left[\frac{4\times\sqrt{(t_{\text{res}})^2 + 2\times(\text{trigger error})^2}}{(\text{gate time})\times\sqrt{\text{no. of samples}}} + \frac{t_{\text{jitter}}}{\text{gate time}}\right]$$

$t_{\text{res}} = 225$ ps

$t_{\text{jitter}} = 3$ ps

$$\text{number of samples} = \begin{cases}(\text{gate time})\times(\text{frequency}) & \text{for } f < 200\text{ kHz}\\ (\text{gate time})\times 2{\times}10^5 & \text{for } f \geq 200\text{ kHz}\end{cases}$$

RMS resolution	$\sigma_\nu = \nu_{00}\sigma_y$ or $\sigma_T = T_{00}\sigma_y$
frequency	ν_{00}
period	T_{00}
gate time	τ
no. of samples	$n = \begin{cases}\nu_{00}\tau & \nu_{00} < 200\text{ kHz}\\ \tau\times 2{\times}10^5 & \nu_{00} \geq 200\text{ kHz}\end{cases}$

5 – Advanced statistics

Allan variance

definition

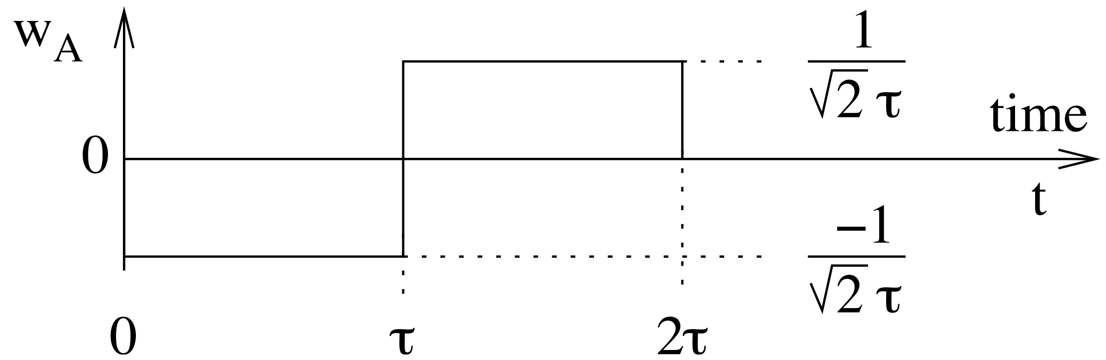
$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{y}_{k+1} - \overline{y}_k\right]^2\right\}$$

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\frac{1}{\tau}\int_{(k+1)\tau}^{(k+2)\tau} y(t)\,dt - \frac{1}{\tau}\int_{k\tau}^{(k+1)\tau} y(t)\,dt\right]^2\right\}$$

wavelet-like variance

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\left[\int_{-\infty}^{+\infty} y(t)\,w_A(t)\,dt\right]^2\right\}$$

$$w_A = \begin{cases} -\frac{1}{\sqrt{2\tau}} & 0 < t < \tau \\ \frac{1}{\sqrt{2\tau}} & \tau < t < 2\tau \\ 0 & \text{elsewhere} \end{cases}$$



energy

$$E\{w_A\} = \int_{-\infty}^{+\infty} w_A^2(t)\,dt = \frac{1}{\tau}$$

the Allan variance differs from a wavelet variance in the normalization on power, instead of on energy

modified Allan variance

definition

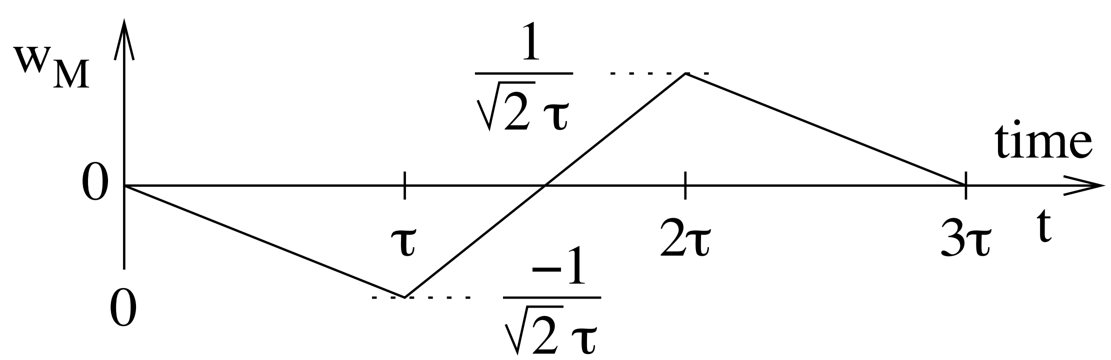
$$\mathrm{mod}\,\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\frac{1}{n}\sum_{i=0}^{n-1}\left(\frac{1}{\tau}\int_{(i+n)\tau_0}^{(i+2n)\tau_0} y(t)\,dt - \frac{1}{\tau}\int_{i\tau_0}^{(i+n)\tau_0} y(t)\,dt\right)\right]^2\right\}$$

with $\tau = n\tau_0$.

wavelet-like variance

$$\mathrm{mod}\,\sigma_y^2(\tau) = \mathbb{E}\left\{\left[\int_{-\infty}^{+\infty} y(t)\,w_M(t)\,dt\right]^2\right\}$$

$$w_M = \begin{cases} -\frac{1}{\sqrt{2}\tau^2}t & 0 < t < \tau \\ \frac{1}{\sqrt{2}\tau^2}(2t-3) & \tau < t < 2\tau \\ -\frac{1}{\sqrt{2}\tau^2}(t-3) & 2\tau < t < 3\tau \\ 0 & \text{elsewhere} \end{cases}$$



energy

$$E\{w_M\} = \int_{-\infty}^{+\infty} w_M^2(t)\,dt = \frac{1}{2\tau}$$

compare the energy

$$E\{w_M\} = \frac{1}{2}\,E\{w_A\}$$

this explains why the mod Allan variance is always lower than the Allan variance

spectra and variances

Noise Type	$S_y(f)$	Allan (σ^2_A)	Modified Allan	Triangle
White PM	$h_2 f^2$	$\frac{3\,f_H}{4\,\pi^2} h_2 \tau^{-2}$ $= \sigma^2_A(\tau)$	$\frac{3}{8\,\pi^2} h_2 \tau^{-3}$ $= \frac{1}{2\,f_H\,\tau} \sigma^2_A(\tau)$	$\frac{2}{\pi^2} h_2 \tau^{-3}$ $= \frac{8}{3\,f_H\,\tau} \sigma^2_A(\tau)$
Flicker PM	$h_1 f$	$\frac{1.038+3\,ln(2\,\pi f_H \tau)}{4\,\pi^2} h_1 \tau^{-2}$ $= \sigma^2_A(\tau)$	$\frac{3\,ln(\frac{256}{27})}{8\,\pi^2} h_1 \tau^{-2}$ $= \frac{3.37}{3.12+3\ln \pi f_H \tau} \sigma^2_A(\tau)$	$\frac{6\,ln(\frac{27}{16})}{\pi^2} h_1 \tau^{-2}$ $= \frac{12.56}{3.12+3\ln \pi f_H \tau} \sigma^2_A(\tau)$
White FM	h_0	$\frac{1}{2} h_0 \tau^{-1}$ $= \sigma^2_A(\tau)$	$\frac{1}{4} h_0 \tau^{-1}$ $= 0.50\,\sigma^2_A(\tau)$	$\frac{2}{3} h_0 \tau^{-1}$ $= 1.33\,\sigma^2_A(\tau)$
Flicker FM	$h_{-1} f^{-1}$	$2\,ln(2)\,h_{-1}$ $= \sigma^2_A(\tau)$	$2\,ln(\frac{3\,3^{11/16}}{4})\,h_{-1}$ $= 0.67\,\sigma^2_A(\tau)$	$(24\,ln(2) - \frac{27}{2}\,ln(3))\,h_{-1}$ $= 1.30\,\sigma^2_A(\tau)$
Random Walk FM	$h_{-2} f^{-2}$	$\frac{2}{3}\pi^2\,h_{-2}\,\tau$ $= \sigma^2_A(\tau)$	$\frac{11}{20}\pi^2\,h_{-2}\,\tau$ $= 0.82\,\sigma^2_A(\tau)$	$\frac{23}{30}\pi^2\,h_{-2}\,\tau$ $= 1.15\,\sigma^2_A(\tau)$
Frequency Drift ($\dot{y} = D_y$)	-	$\frac{1}{2} D_y^2 \tau^2$	$\frac{1}{2} D_y^2 \tau^2$	$\frac{1}{2} D_y^2 \tau^2$

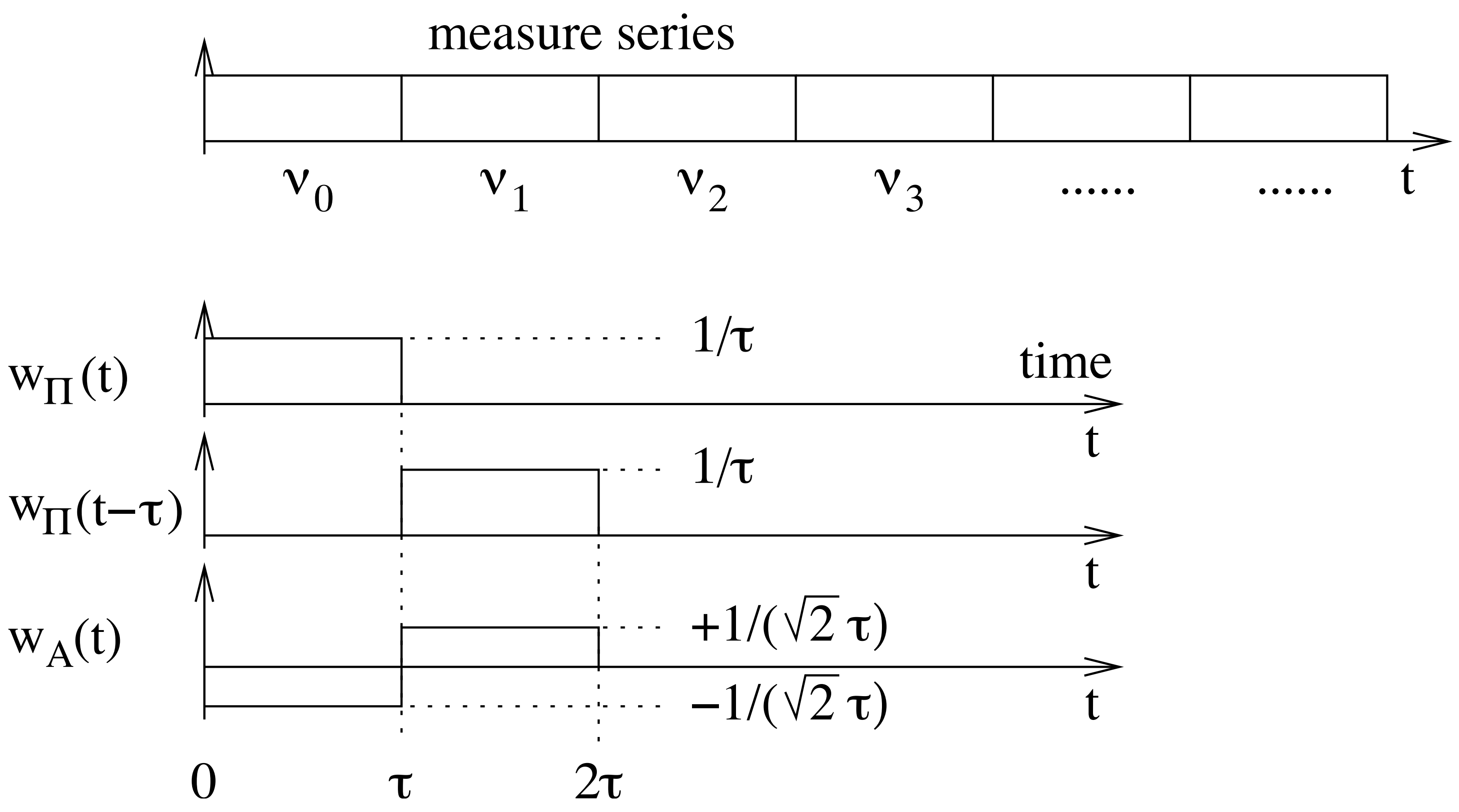
ν_{00} is replaced with ν_0 for consistency with the general literature

f_H is the high cutoff frequency, needed for the noise power to be finite

S.T. Dawkins, J.J. McFerran, A.N. Luiten, IEEE Trans. UFFC 54(5) p.918–925, May 2007

Π estimator —> Allan variance

given a series of contiguous non-overlapped measures



the Allan variance is easily evaluated

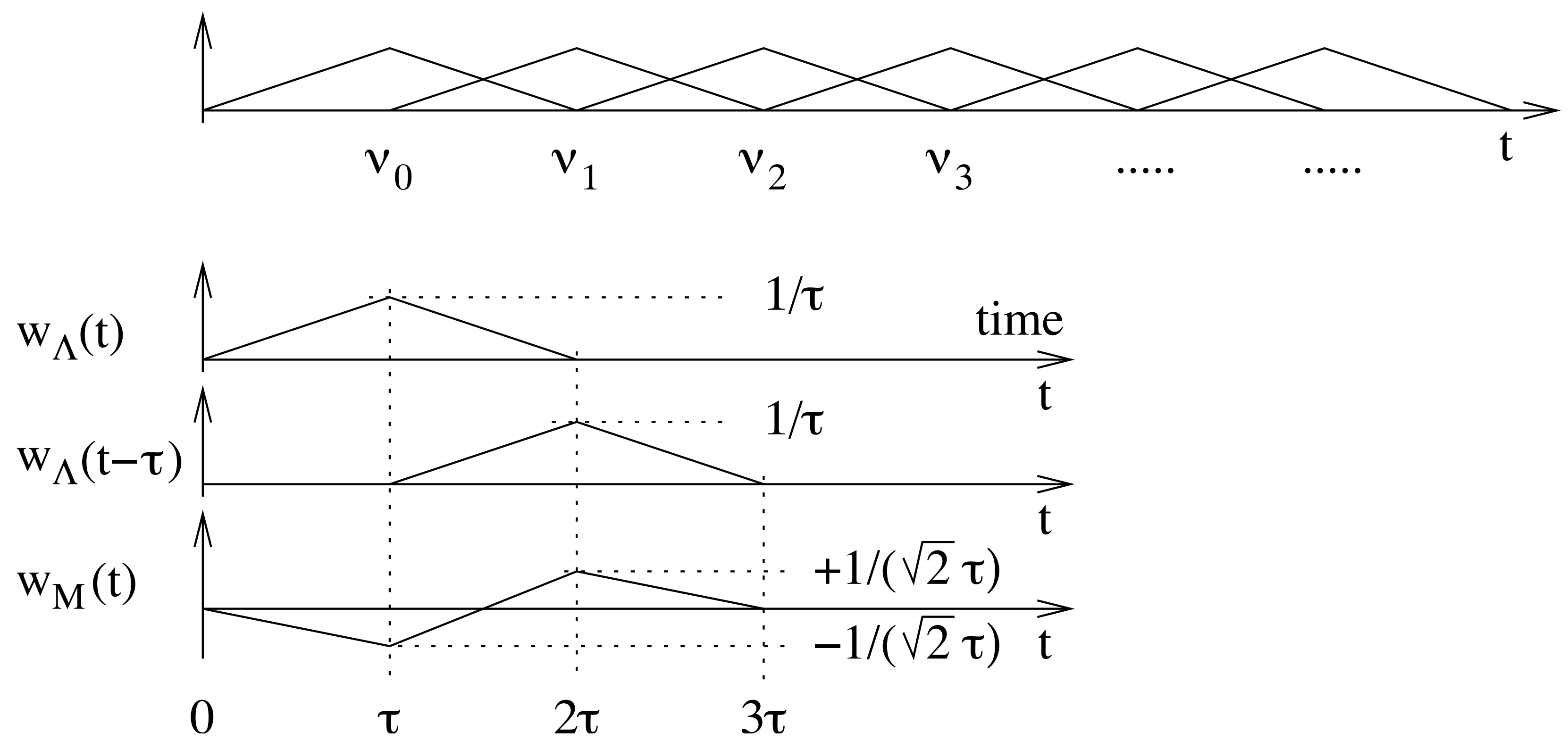
$$\sigma_y^2(\tau) = \mathbb{E}\left\{ \frac{1}{2} \left[\overline{y}_{k+1} - \overline{y}_k \right]^2 \right\}$$

overlapped Λ estimator —> MVAR

by feeding a series of Λ-estimates of frequency in the formula of the Allan variance

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{y}_{k+1} - \overline{y}_k\right]^2\right\}$$

as they were Π-estimates



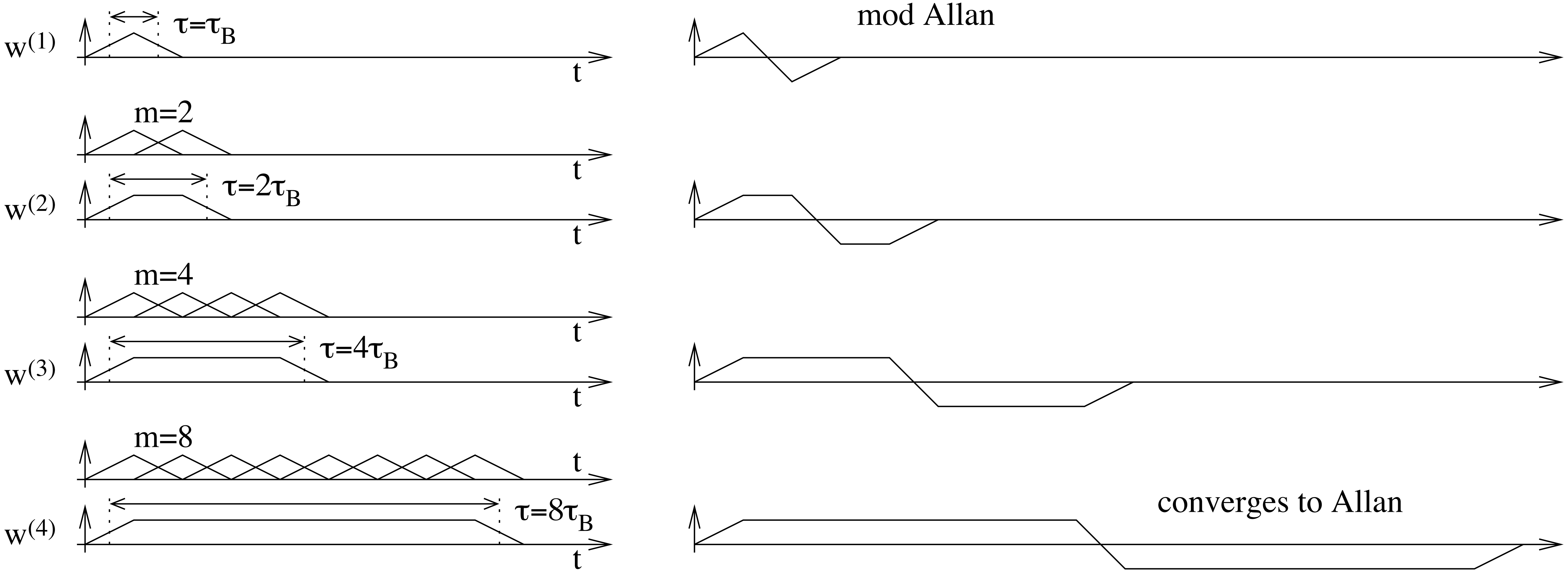
one gets exactly the modified Allan variance!

$$\operatorname{mod}\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\frac{1}{n}\sum_{i=0}^{n-1}\left(\frac{1}{\tau}\int_{(i+n)\tau_0}^{(i+2n)\tau_0} y(t)\,dt - \frac{1}{\tau}\int_{i\tau_0}^{(i+n)\tau_0} y(t)\,dt\right)\right]^2\right\}$$

with $\tau = n\tau_0$.

joining contiguous values to increase τ

graphical proof



m = 1	mod Allan
m = 2	this is not what we expected
m = 4	...
m ≥ 8	the variance converges to the (non modified) Allan variance

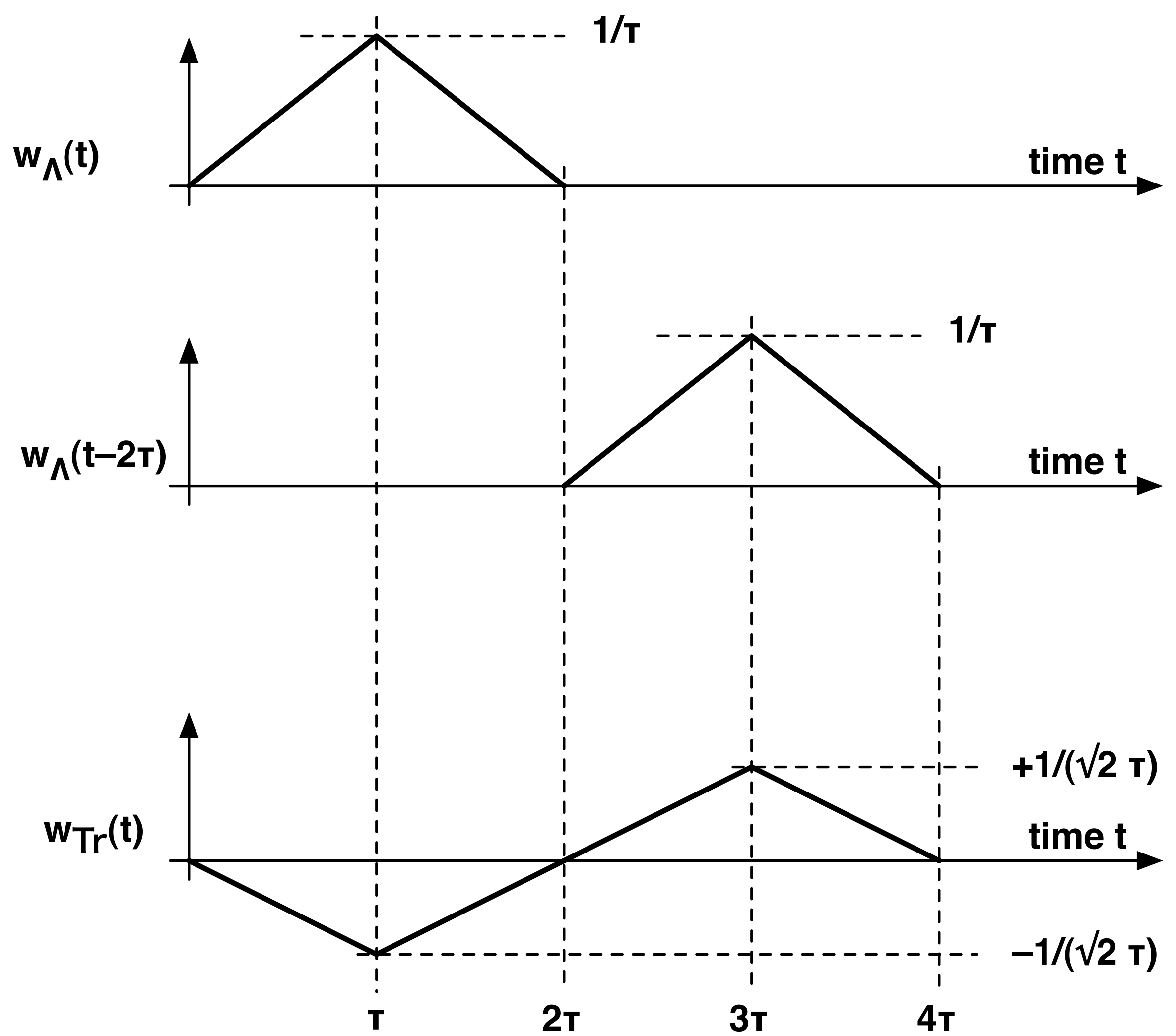
There is a mistake in one of my articles: I believed that in the case of the Agilent counters the contiguous measures were overlapped. They are not.

non-overlapped Λ estimator —> TrVAR

by feeding a series of Λ-estimates of frequency in the formula of the Allan variance

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{y}_{k+1} - \overline{y}_k\right]^2\right\}$$

as they were Π-estimates



one gets the triangular variance!

S.T. Dawkins, J.J. McFerran, A.N. Luiten, IEEE Trans. UFFC 54(5) p.918–925, May 2007

Conclusions

- The multi-tap delay-line interpolator is simple with modern FPGAs
- In frequency measurements, the Λ (triangular) estimator provides higher resolution
- The Λ estimator can not be used in single-event time-interval measurements
- Mistakes are around the corner if the counter inside is not well understood
- Some of the reported ideas are suitable to education laboratories and classroom works (I used a bicycle and milestones to demonstrate the Λ estimator)

Thanks to J. Dick (JPL), C. Greenhall (JPL), D. Howe (NIST) and M. Oxborrow (NPL) for discussions

To know more:

1 - rubiola.org, slides and articles
2 - www.arxiv.org, document arXiv:physics/0503022v1
3 - Rev. of Sci. Instrum. vol. 76 no. 5, art.no. 054703, May 2005.

home page http://rubiola.org

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IEEE Trans. Nucl. Sci 31(1) 167 (feb. 1984)

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- E.D. Peterson, R. P. Valenti, Time Interval measuring system employing digital means coarse ambiguity resolver US patent 3,218,553, nov. 16, 1965
- Hewlett Packard (now Agilent Technologies) HP-5370A operating and service manual
- Hewlett Packard - Fundamentals of electronic counters, HP AN-200, 1997
- Eldorado counter, I could not track the reference

TIME-TO-VOLTAGE CONVERTER

- Stanford SR620 operating manual.

Note: the full scheme included, is difficult to interpret. No "theory of operation" chapter is present.