CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

femto-st
sciences & technologies

UNIVERSITÉ DE FRANCHE-COMTÉ

Phase & Frequency Noise Metrology

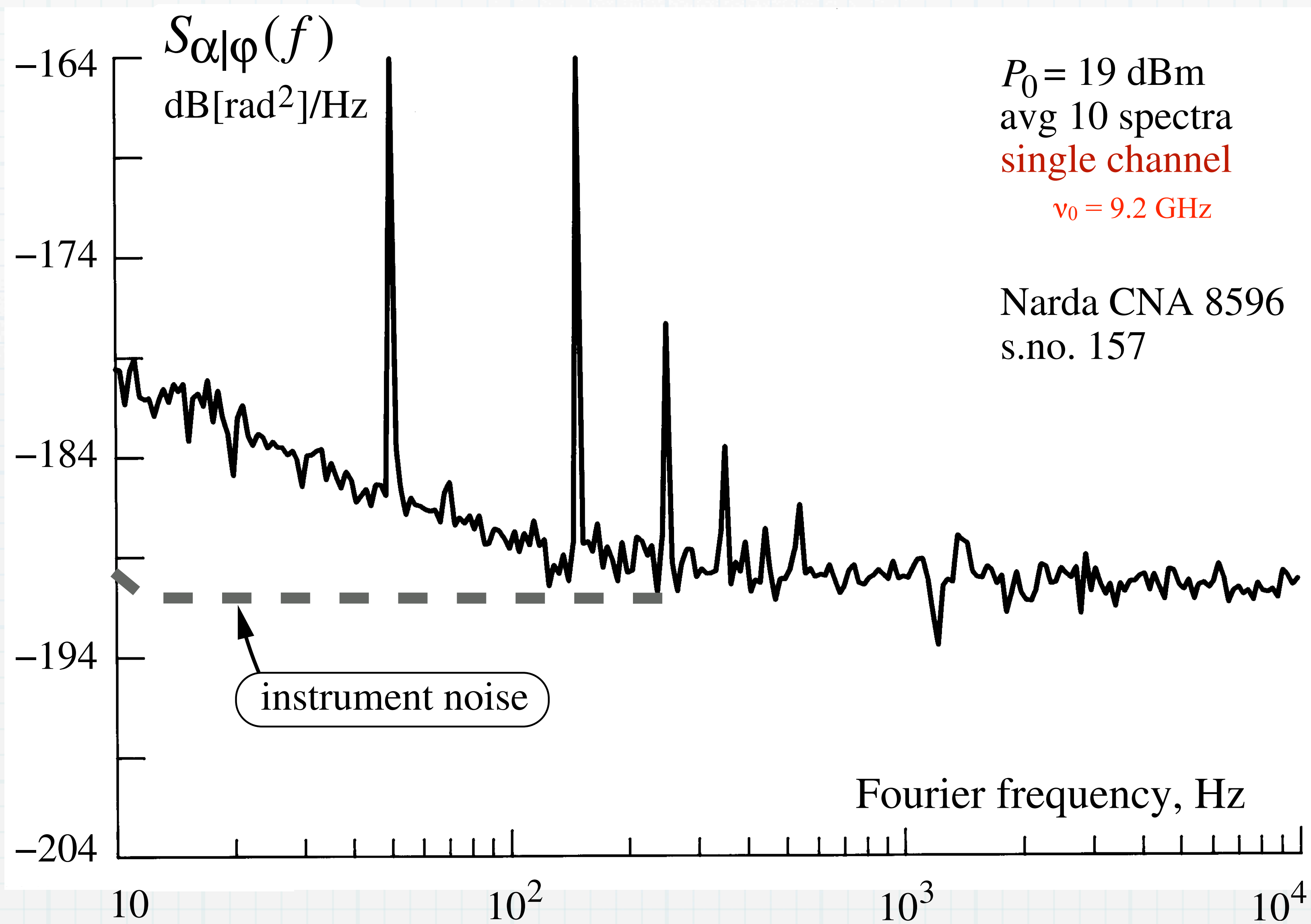
Enrico Rubiola

Outline

- Introduction
- Measurement methods
- Microwave photonics
- Electronic and optical components
- AM noise and RIN

home page http://rubiola.org

Though frequency standards are moving to optics (and beyond), RF and microwaves are inevitable



Microwave circulator

Lower phase noise is required

Phase noise & friends

$$v(t) = V_p\,[1+\alpha(t)]\cos\,[1+\varphi(t)]$$

random phase fluctuation

$$S_\varphi(f) = \text{PSD of } \varphi(t)$$

power spectral density

it is measured as

$$S_\varphi(f) = \mathbb{E}\left\{\Phi(f)\Phi^*(f)\right\} \quad \text{(expectation)}$$

$$S_\varphi(f) \approx \left\langle\Phi(f)\Phi^*(f)\right\rangle_m \quad \text{(average)}$$

$$\mathcal{L}(f) = \frac{1}{2}S_\varphi(f) \ \ \text{dBc}$$

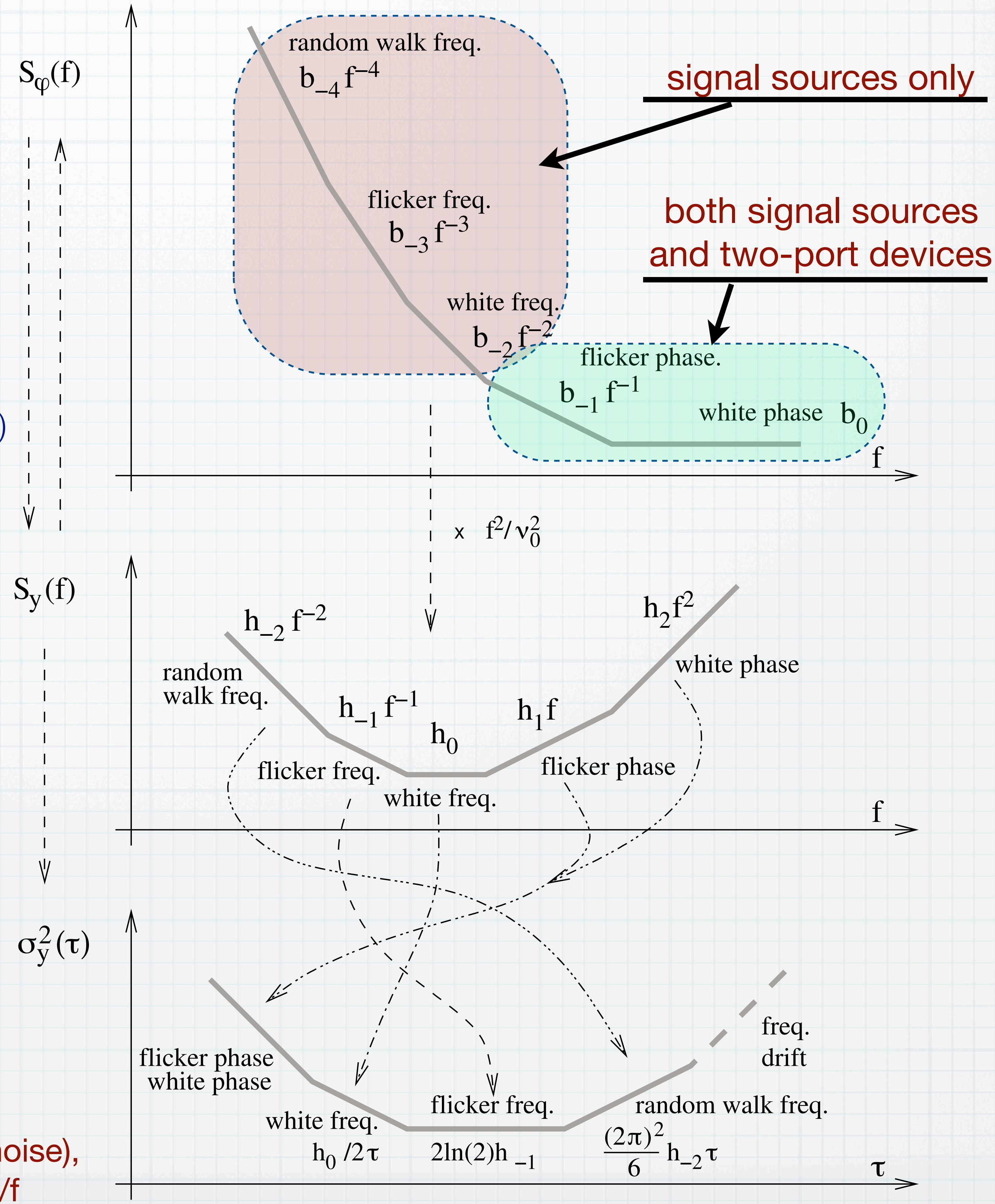
random fractional-frequency fluctuation

$$y(t) = \frac{\dot\varphi(t)}{2\pi\nu_0} \quad \Rightarrow \quad S_y = \frac{f^2}{\nu_0^2}S_\varphi(f)$$

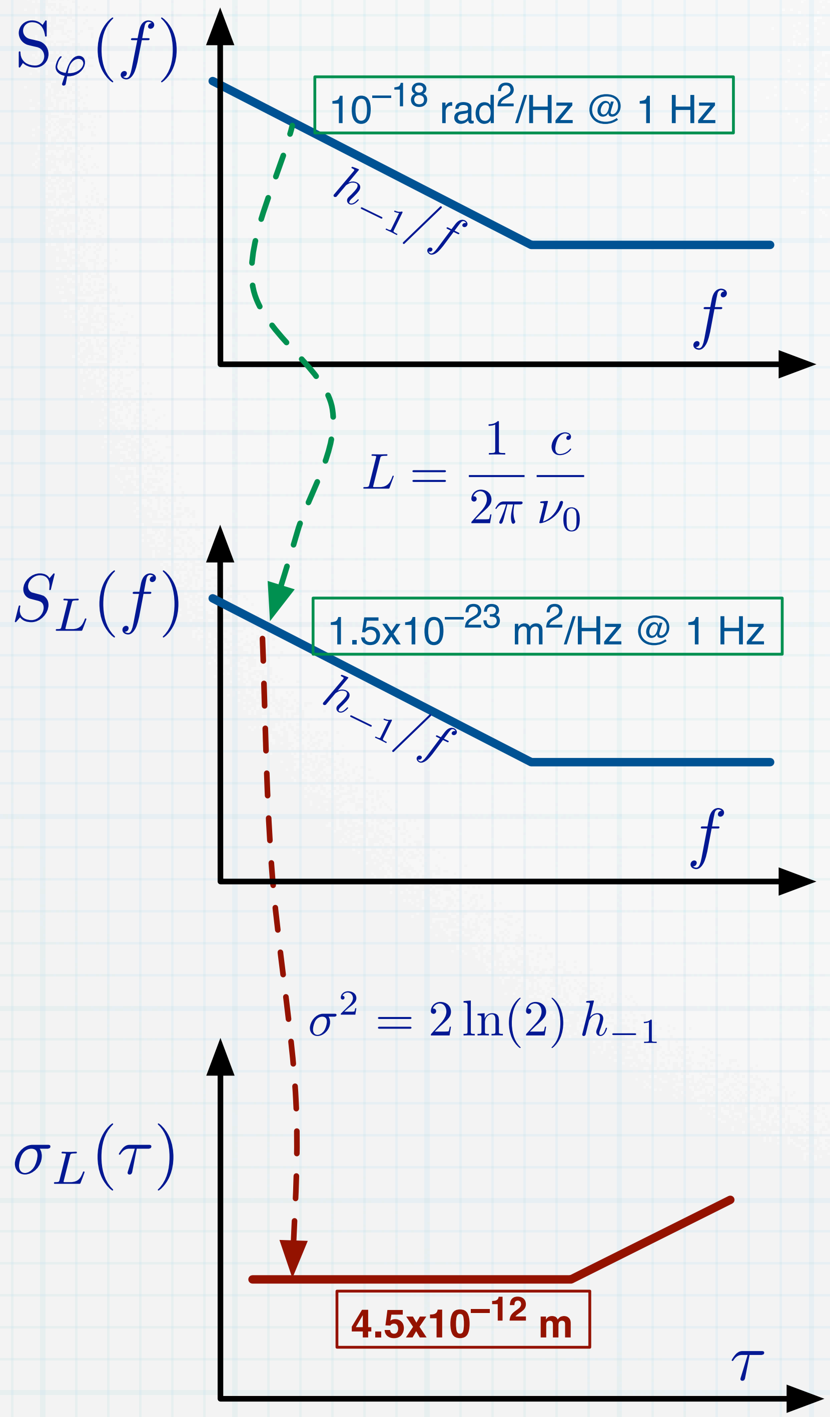
Allan variance
(two-sample wavelet-like variance)

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{y}_{k+1} - \overline{y}_k\right]^2\right\} \ .$$

approaches a half-octave bandpass filter (for white noise),
hence it converges for processes steeper than 1/f



Mechanical stability



Any phase fluctuation can be converted into length fluctuation

$$L = \frac{1}{2\pi}\frac{c}{\nu_0}$$

$b_{-1} = -180$ dBrad²/Hz and $\nu_0 = 10$ GHz is equivalent to $S_L = 1.46\times10^{-23}$ m²/Hz at f = 1 Hz

Any flicker spectrum h_{-1}/f can be converted into a flat Allan variance

$$\sigma_L^2 = 2\ln(2)\,h_{-1}$$

A residual flicker of –180 dBrad²/Hz at f = 1 Hz off the 10 GHz carrier is equivalent to

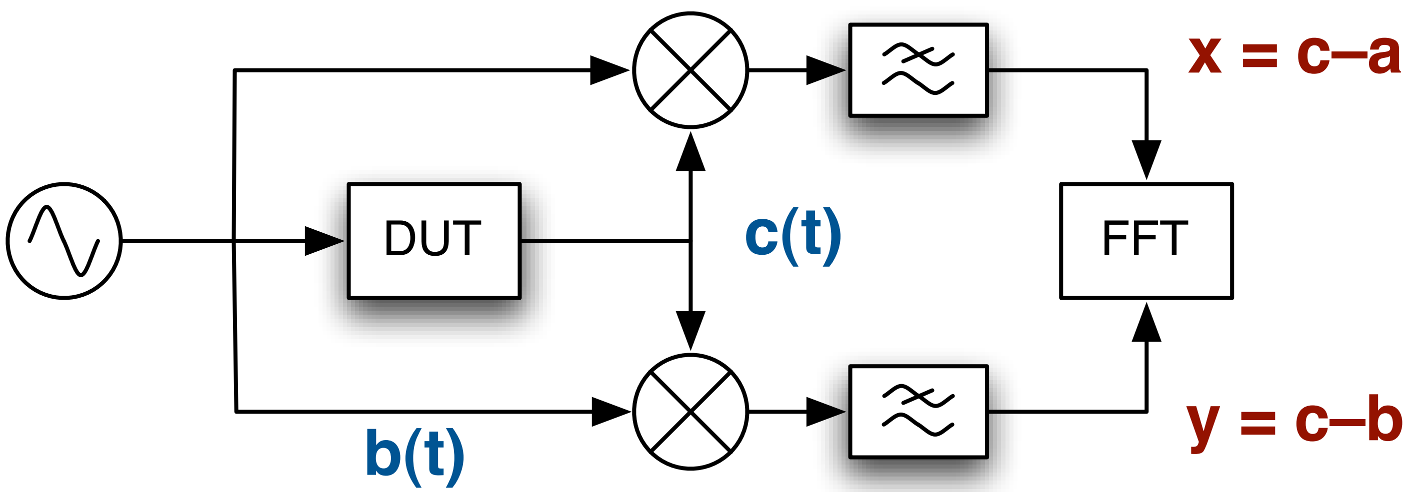
$\sigma^2 = 2\times10^{-23}$ m² thus $\sigma = 4.5\times10^{-12}$ m

for reference, the Bohr radius of the electron is R = 0.529 Å

- **Don't think "this is just engineering" !!!**
- **Learn from non-optical microscopy (bulk matter, 5×10^{-14} m)**
- **Careful DC section (capacitance and piezoelectricity)**
- **The best advice is to be *at least* paranoiac**

1 – Measurement methods

Correlation measurements



Two separate instruments measure the same DUT.
Only the DUT noise is common

a(t), b(t) –> instrument noise
c(t) –> DUT noise

phase noise measurements		
DUT noise, normal use	a, b c	instrument noise DUT noise
background, ideal case	a, b c = 0	instrument noise no DUT
background, with AM noise	a, b c ≠ 0	instrument noise AM-to-DC noise

basics of correlation

$$
\begin{aligned}
S_{yx}(f) &= \mathbb{E}\left\{Y(f)X^*(f)\right\} \\
&= \mathbb{E}\left\{(C-A)(C-B)^*\right\} \\
&= \mathbb{E}\left\{CC^* - AC^* - CB^* + AB^*\right\} \\
&= \mathbb{E}\left\{CC^*\right\}
\end{aligned}
$$

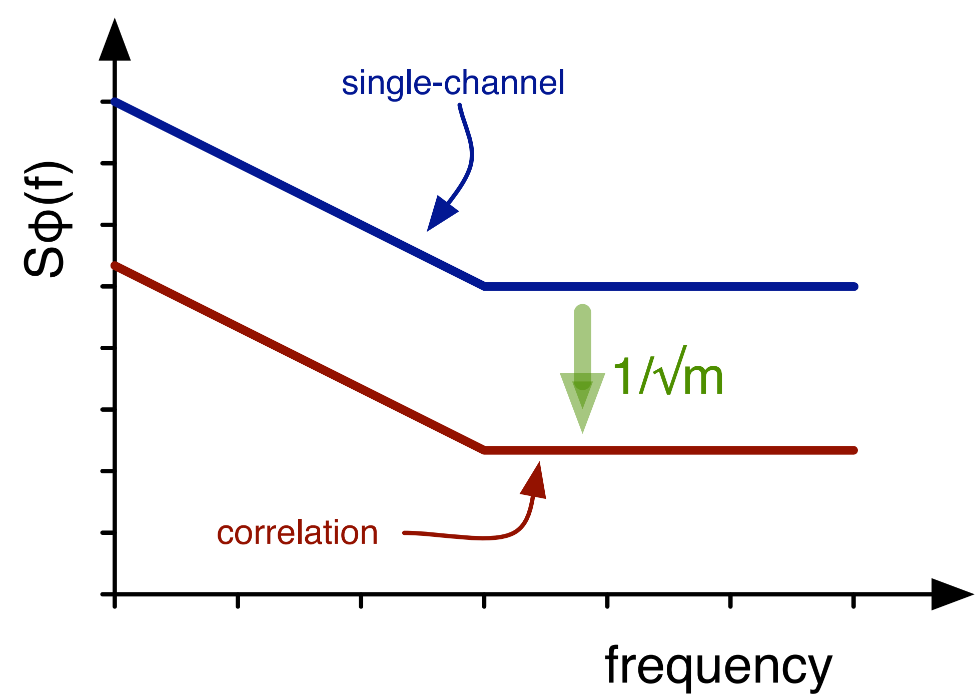
$AC^* \to 0$, $CB^* \to 0$, $AB^* \to 0$

$$S_{yx}(f) = S_{cc}(f)$$

in practice, average on m realizations

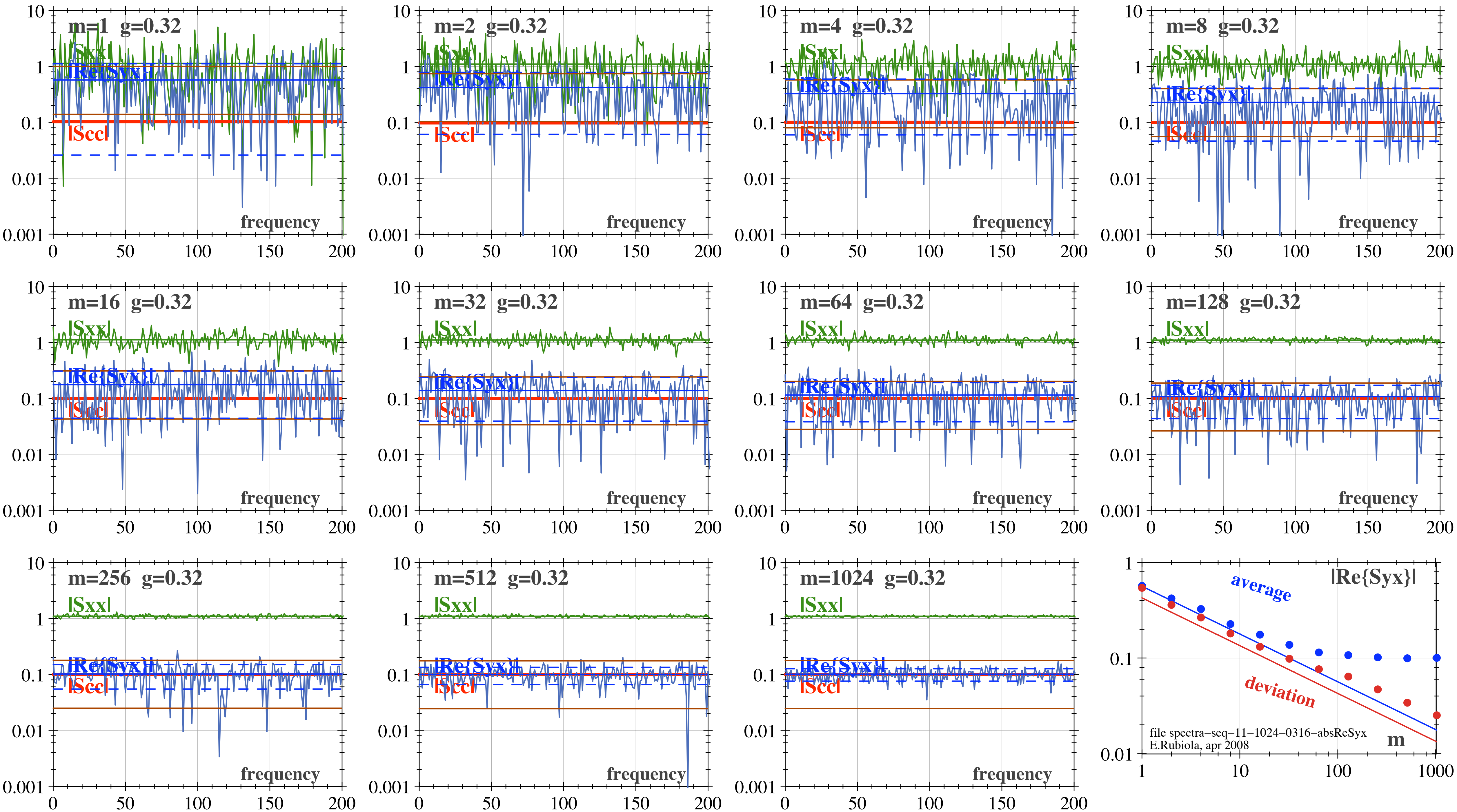
$$
\begin{aligned}
S_{yx}(f) &= \left\langle Y(f)X^*(f)\right\rangle_m \\
&= \left\langle CC^* - AC^* - CB^* + AB^*\right\rangle_m \\
&= \left\langle CC^*\right\rangle_m + O(1/m)
\end{aligned}
$$

AC^*, CB^*, AB^* → **0 as 1/√m**



Cross-spectrum, increasing m

|Re{Syx}| with C≠0,

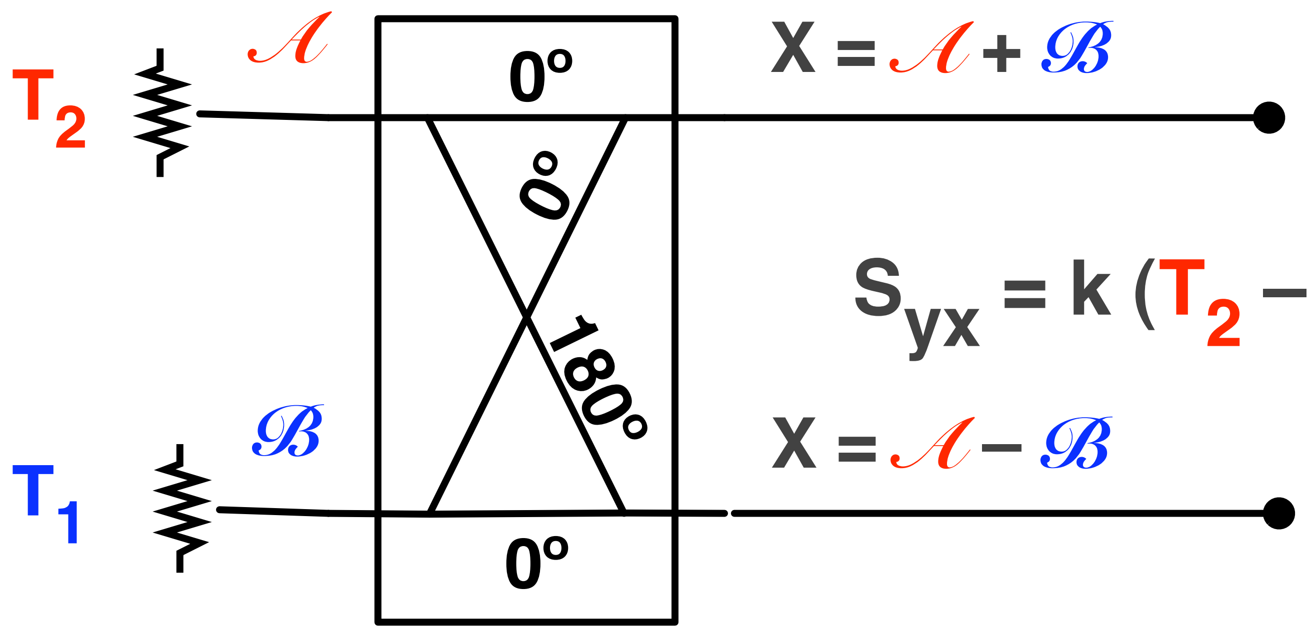


Increasing m:

first, S_{yx} decreases => single-channel noise rejection

then, S_{xx} shrinks => increased confidence level

The thermal noise is rejected as any signal. The limit $S_\varphi = P_0/kT$ does not apply



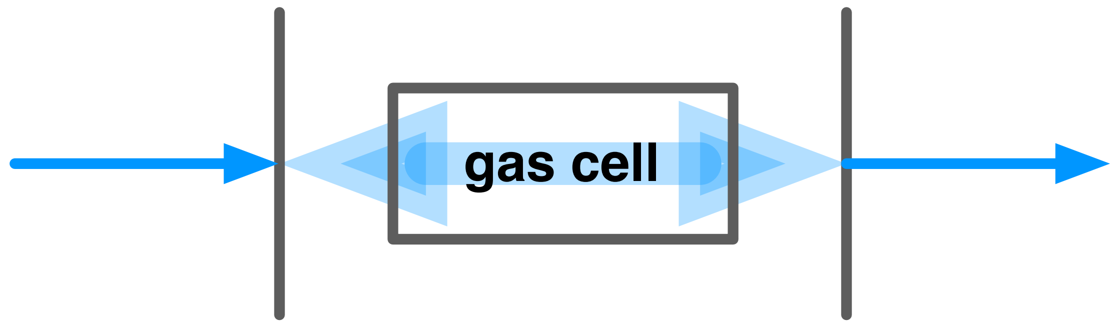
$S_{yx} = k\,(T_2 - T_1)\,/\,2$

X and Y are uncorrelated

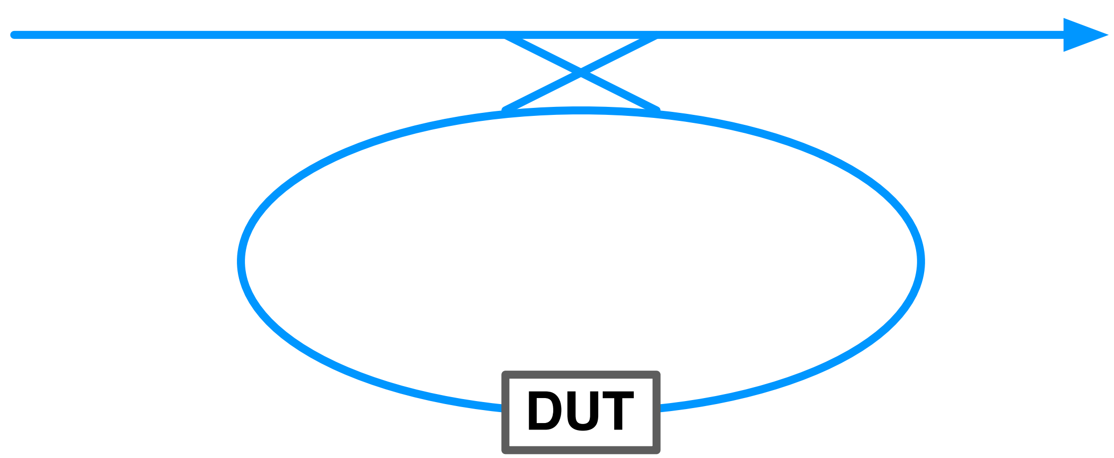
The cross spectrum is proportional to the temperature difference

C. M. Allred, A precision noise spectral density comparator, J. Res. NBS 66C no.4 p.323-330, Oct-Dec 1962
Application to AM/PM noise: E. Rubiola, V. Giordano, Rev. Sci. Instrum. 71(8) p.3085-3091, Aug 2000

Carrier recirculation

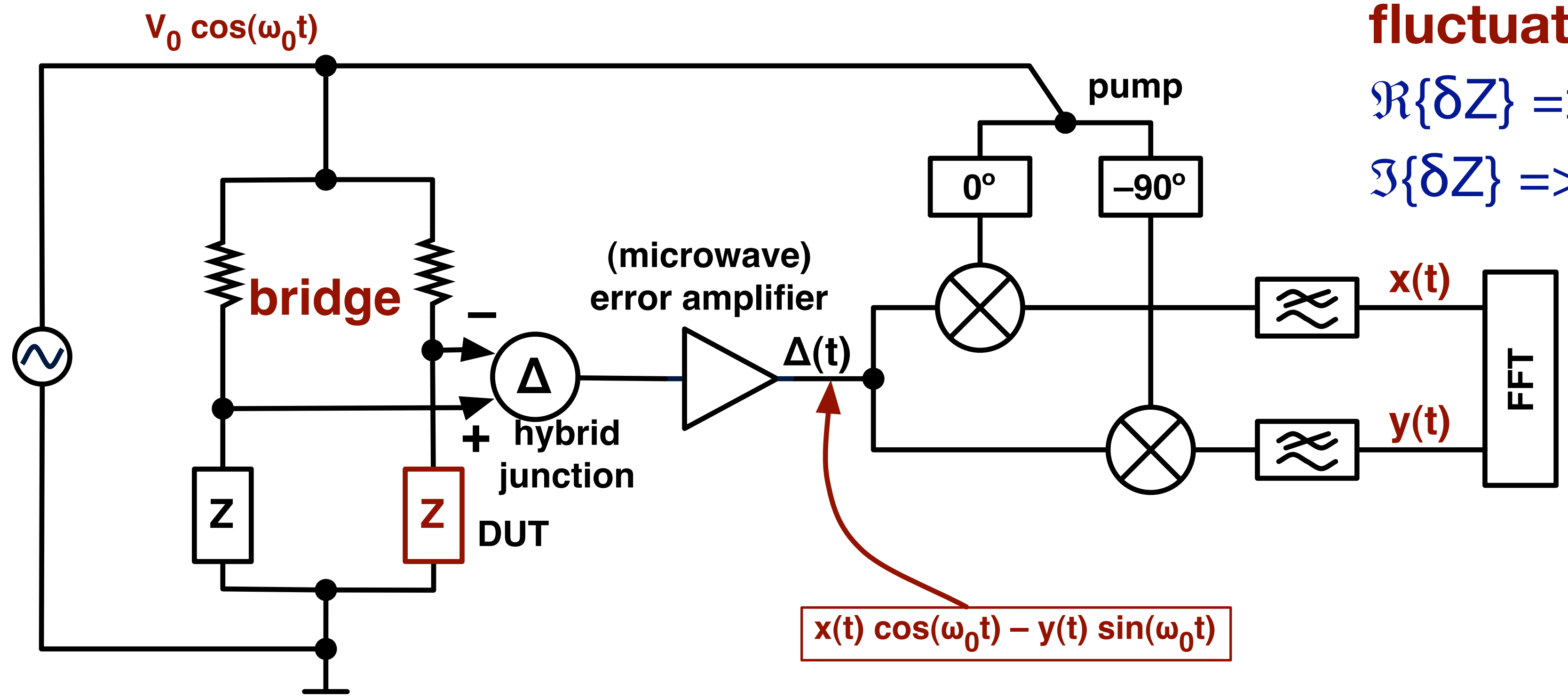


Invented by J. Hall for gas spectroscopy.
The gain is increased by the number of times the light beam circulates in the cavity



Also works with RF/microwave carrier, provided the DUT be “transparent”.
For small no. of roundtrips, gives the appearance of “real-time”

Bridge (interferometric) method



fluctuating error δZ => noise sidebands

$\Re\{\delta Z\}$ => AM noise $x(t)\cos(\omega_0 t)$

$\Im\{\delta Z\}$ => PM noise $-y(t)\sin(\omega_0 t)$

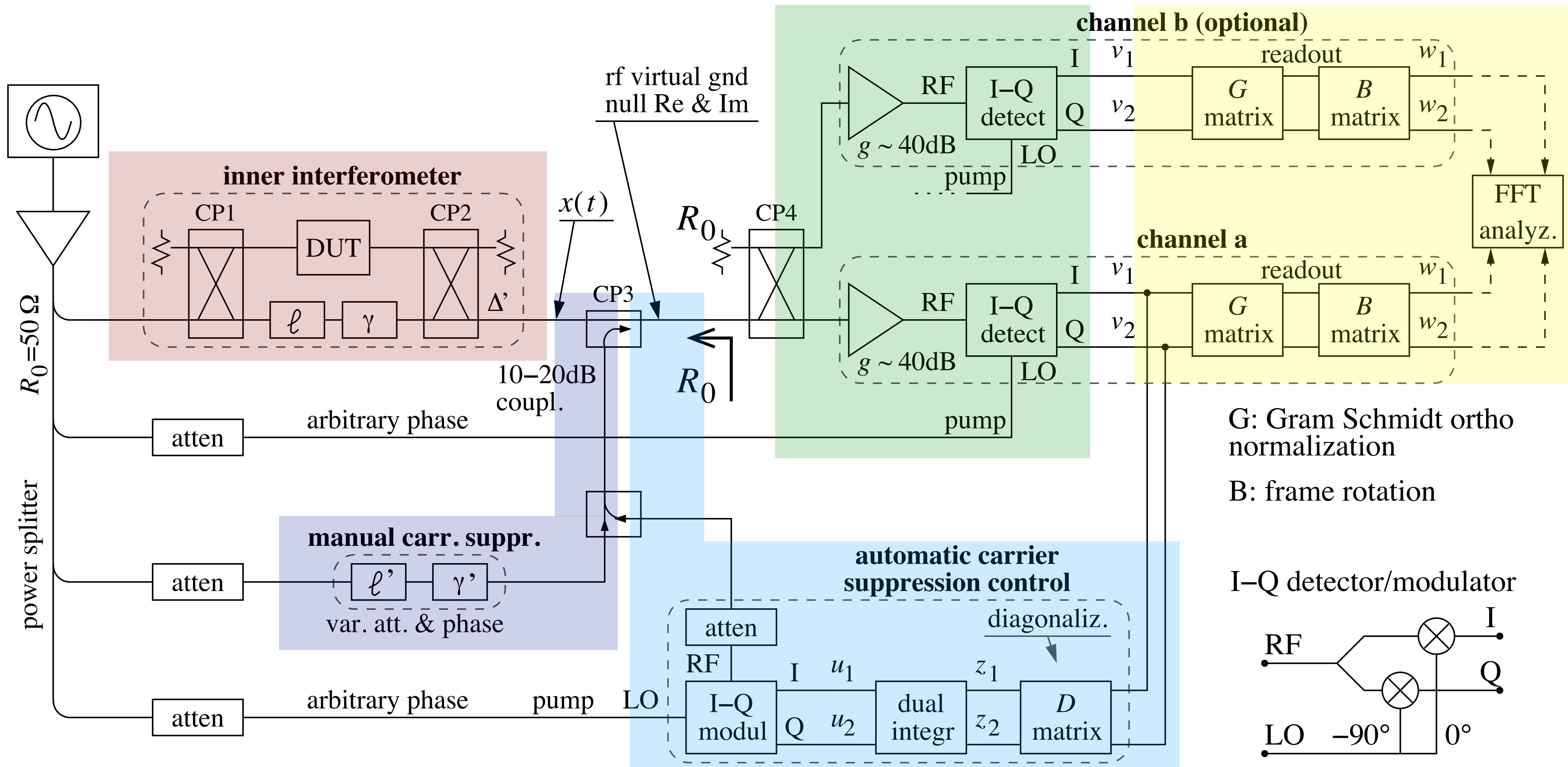
Basic ideas

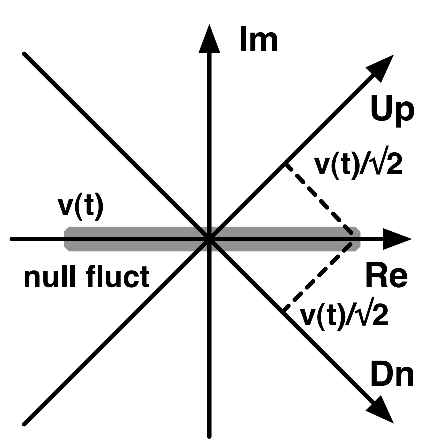
- **Carrier suppression => the error amplifier cannot flicker: it does know ω_0**
- **High gain, due to the (microwave) error amplifier**
- **Low noise floor => the noise figure of the (microwave) error amplifier**
- **High immunity to the low-frequency magnetic fields due to the microwave amplification before detecting**
- **Rejection of the master oscillator's noise**
- **Detection is a scalar product => signal-processing techniques**

Derives from H. Sann, MTT 16(9) 1968, and F. Labaar, Microwaves 21(3) 1982
Later, E. Ivanov, MTT 46(10) oct 1998, and Rubiola, RSI 70(1) jan 1999

Actual block diagram

E. Rubiola, V. Giordano, Rev. Sci. Instrum. 73(6) pp.2445-2457, June 2002





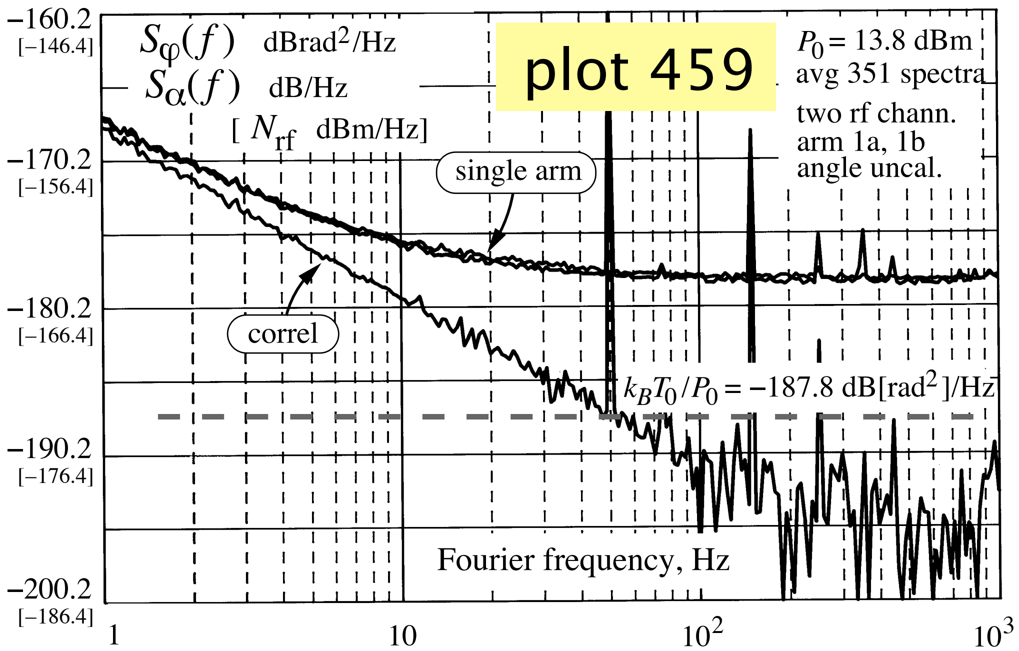
$$S_{ud}(f) = \frac{1}{2}\left[S_\alpha(f) - S_\varphi(f)\right]$$

Concepts

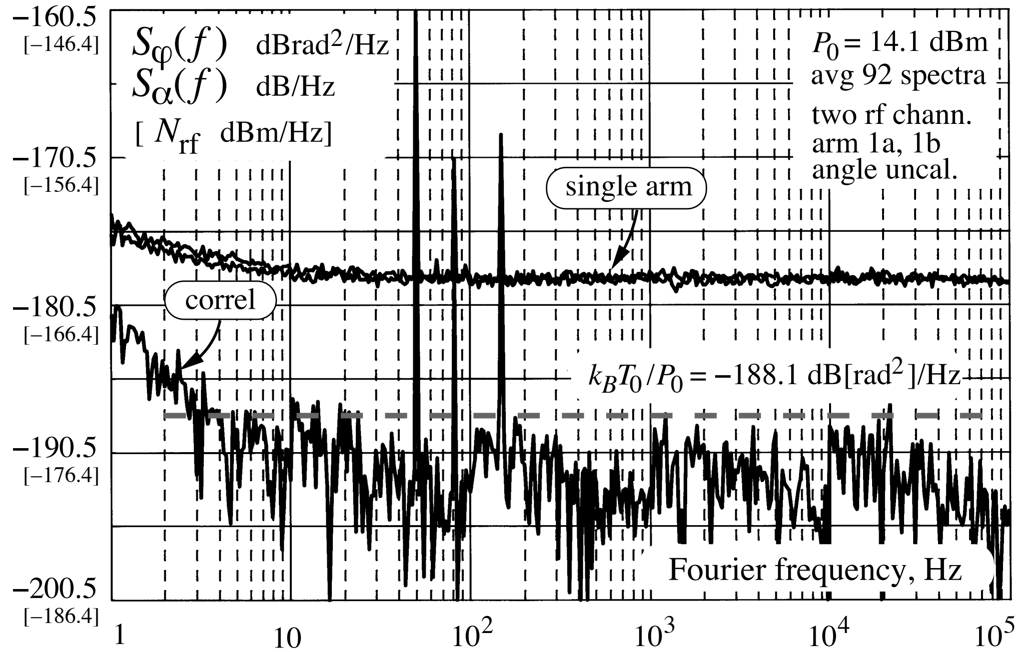
- Coarse and fine carrier suppression reduces the flicker noise
- Scalar product gives $v_1(t)$ and $v_2(t)$ in Cartesian frame. Linear algebra fixes the arbitrary phase, gain asymmetry and quadrature defect
- Closed-loop control of the carrier suppression works as a RF VGND
- Correlation is possible, using two amplifiers and two detectors
- Correlating the signals detected on two orthogonal axes (±45º) eliminates the amplifier noise. Works with a single amplifier!

Example of results

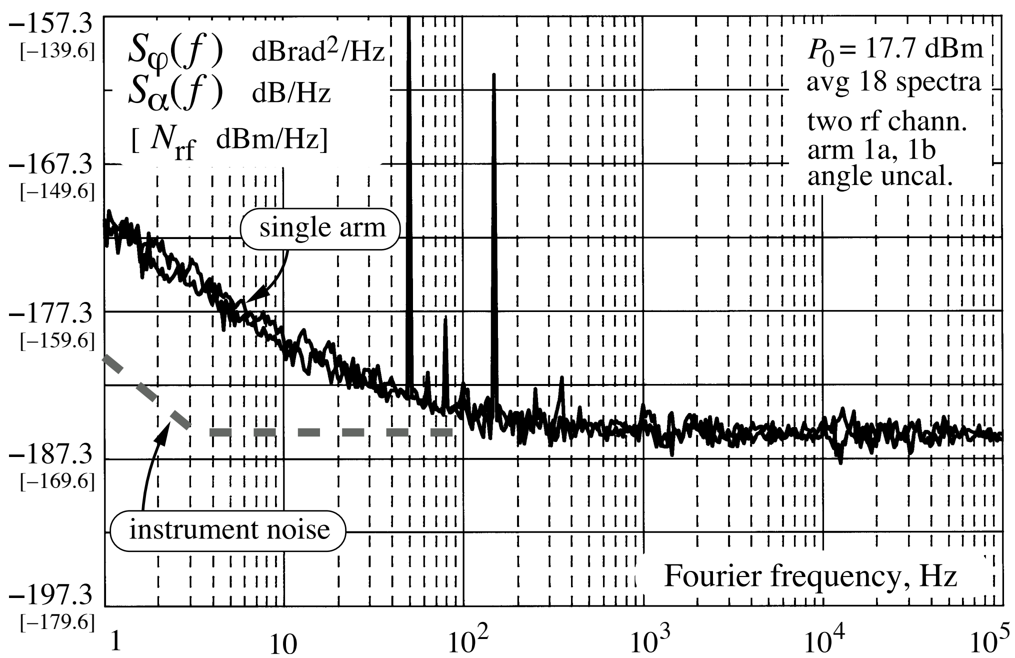
Noise of a by-step attenuator



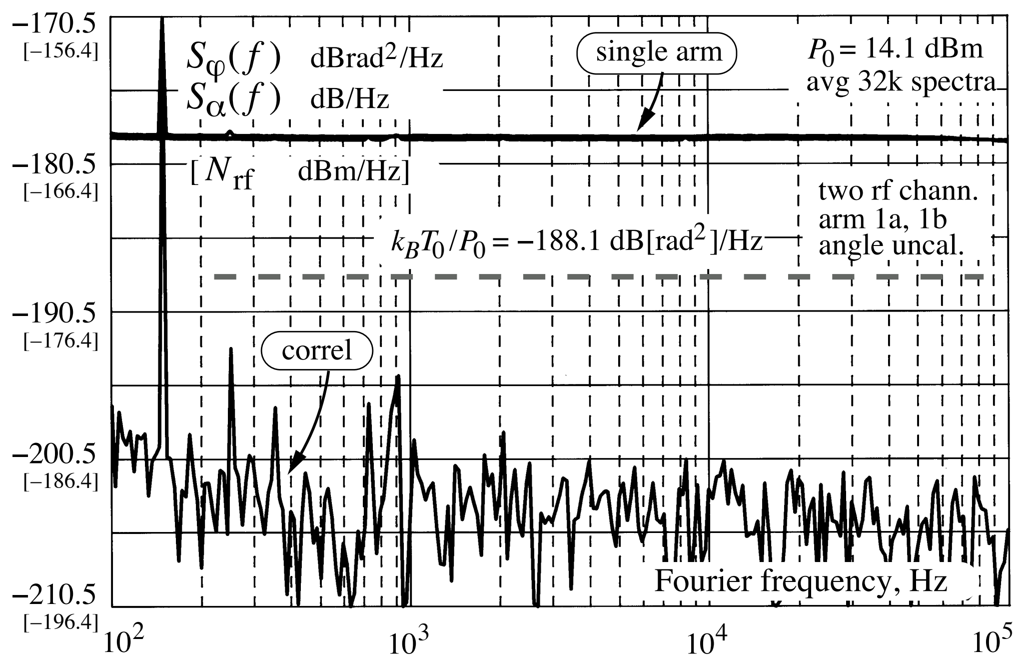
Background noise of the fixed-value bridge



Noise of a pair of HH-109 hybrid junctions



Background noise of the fixed-value bridge (larger m)

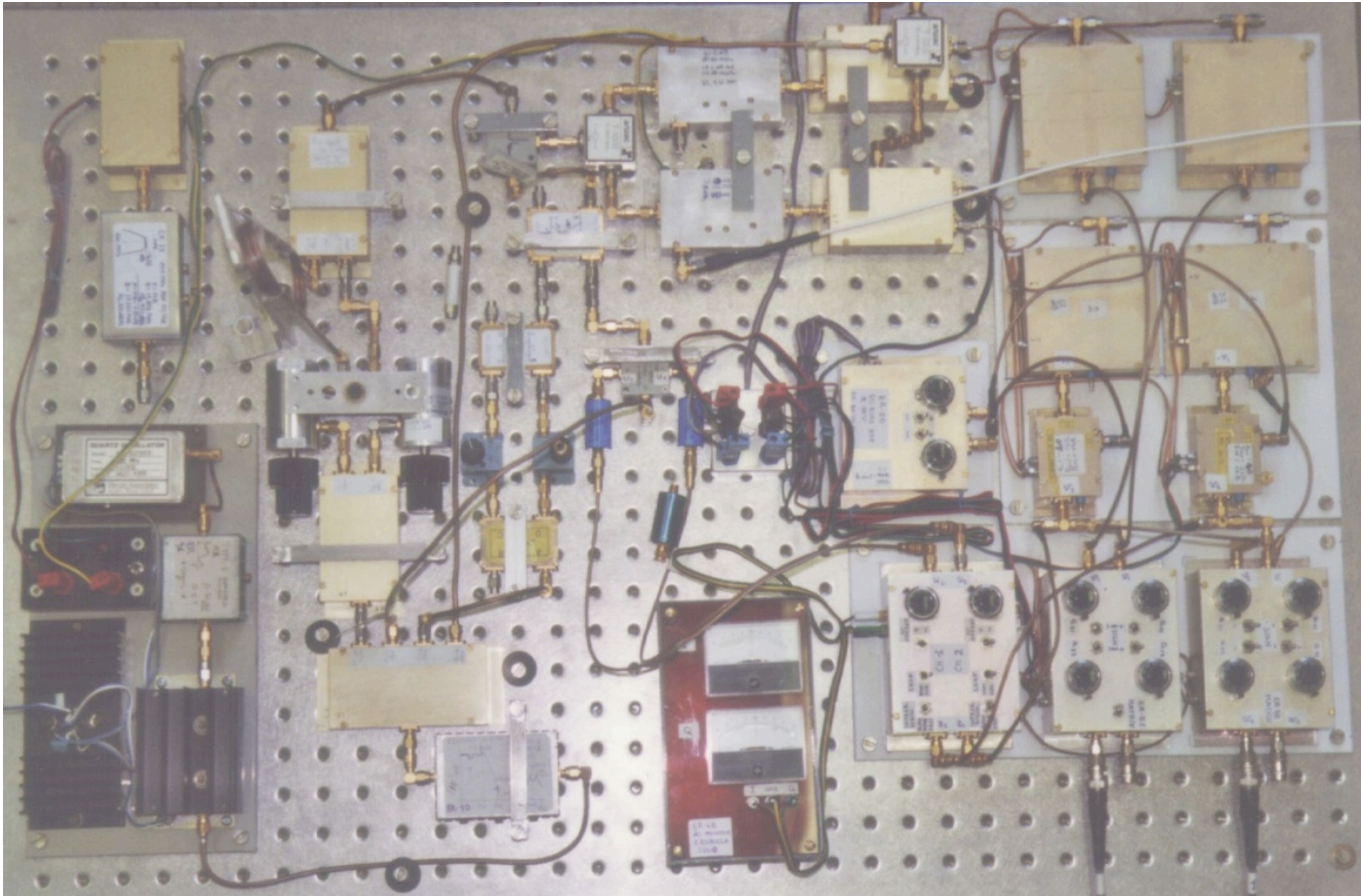


Averaged spectra must be smooth

Average on m spectra: confidence of a point improves by $1/m^{1/2}$

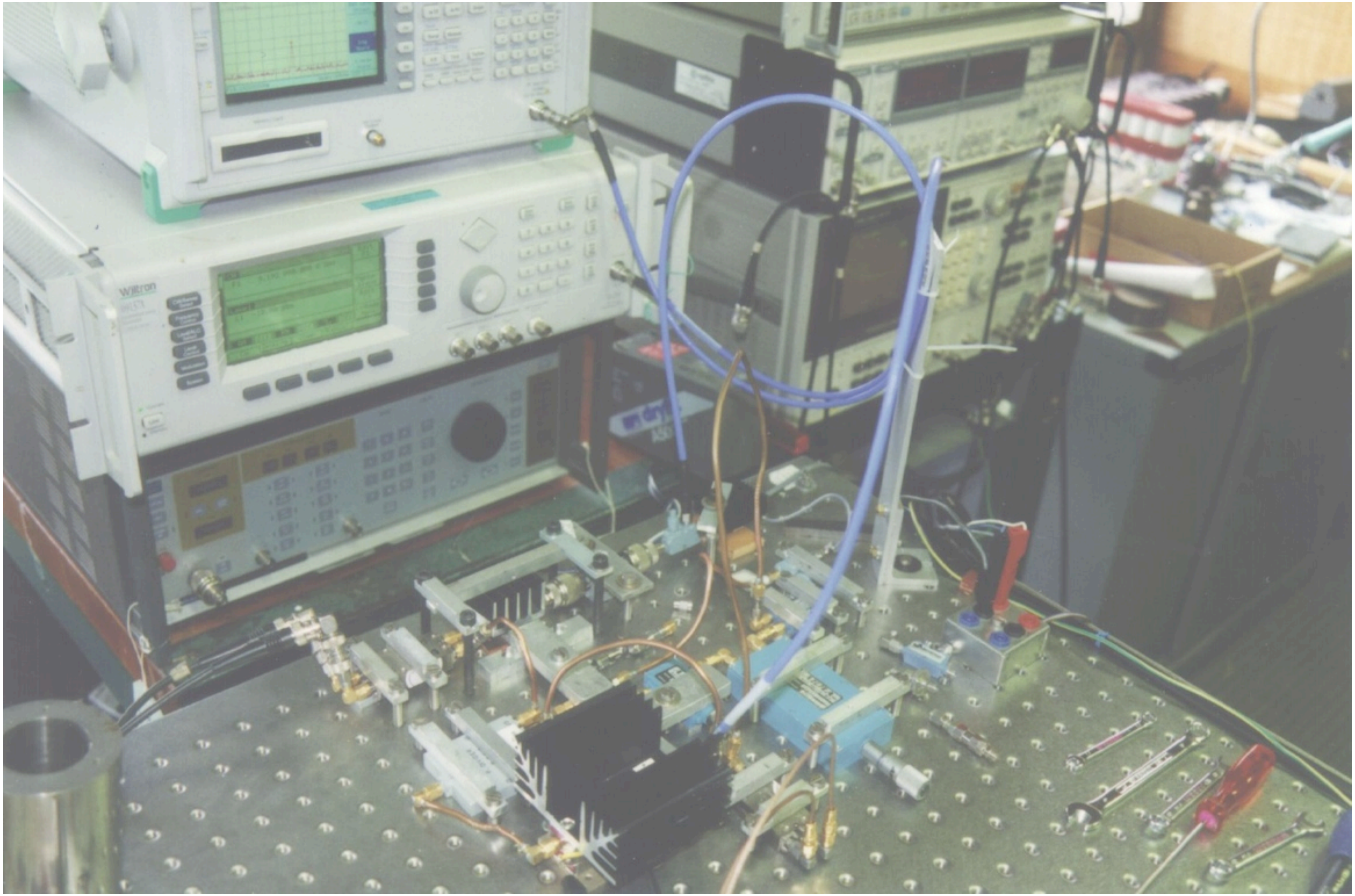
interchange ensemble with frequency: smoothness $1/m^{1/2}$

The complete machine (100 MHz)

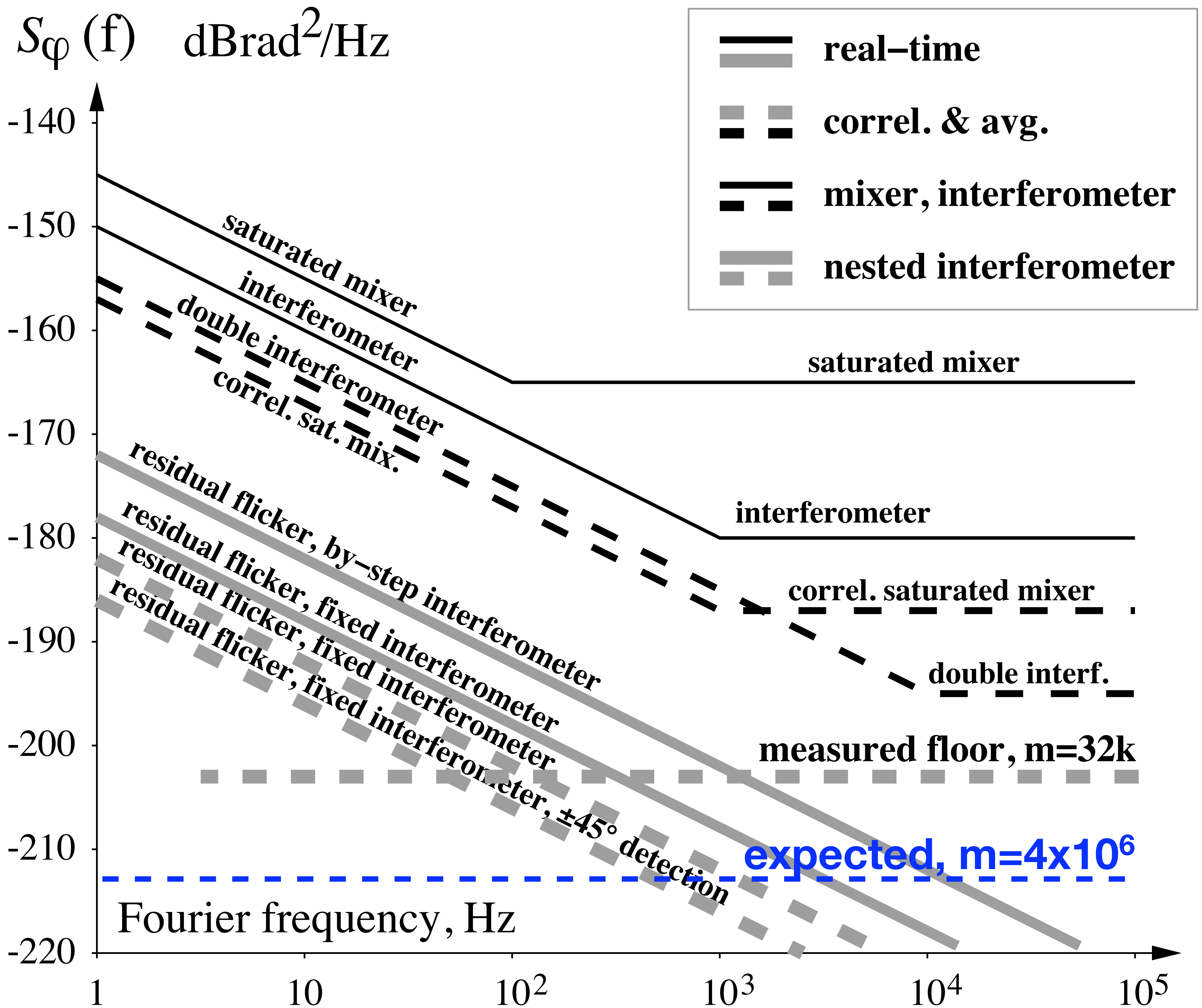


A 10 GHz experiment

(dc circuits not shown)



Comparison of the background noise



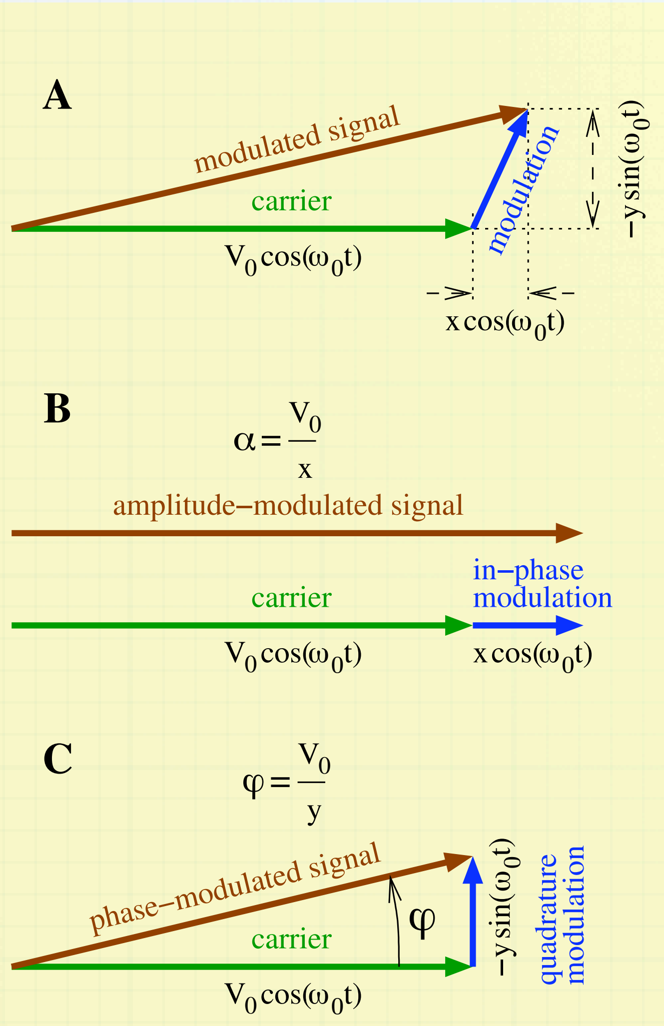
AM-PM calibration

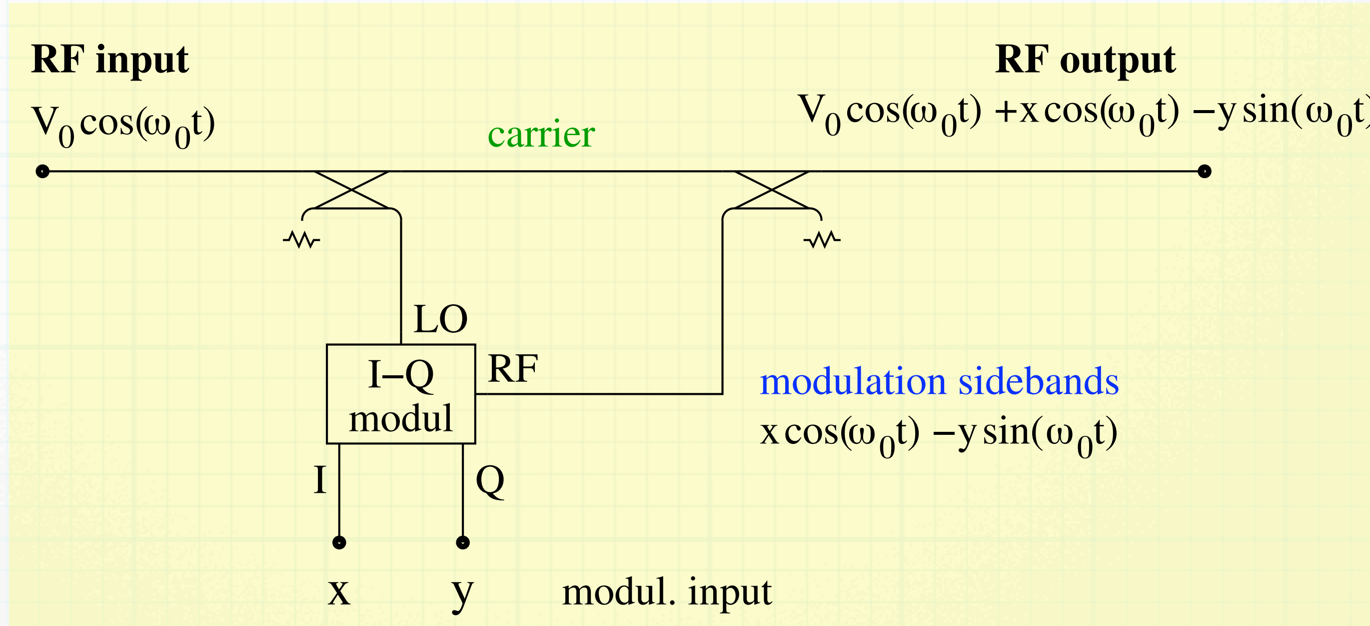
- In most cases the phase detector is a double-balanced mixer saturated at both inputs
- In a double-balanced mixer, the offset is affected by power,
- AM noise is detected as it was PM noise, which is conceptually incorrect
- k_φ/k_α can be as low as 5

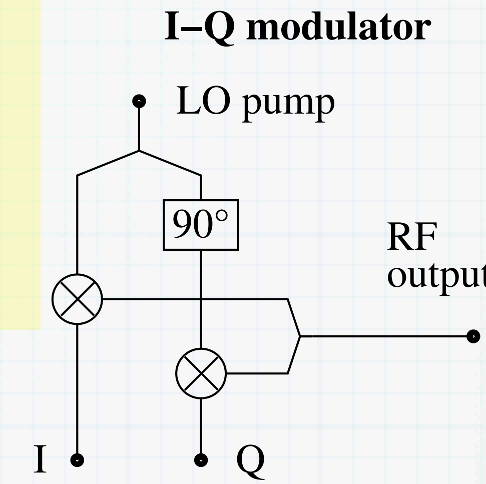
- In the bridge, the effect of AM noise is divided by the microwave gain

Primary AM-PM calibration

E. Rubiola – unpublished







AM and PM can be defined as

$$\alpha_{\text{rms}} = \sqrt{P_x/P_0}$$

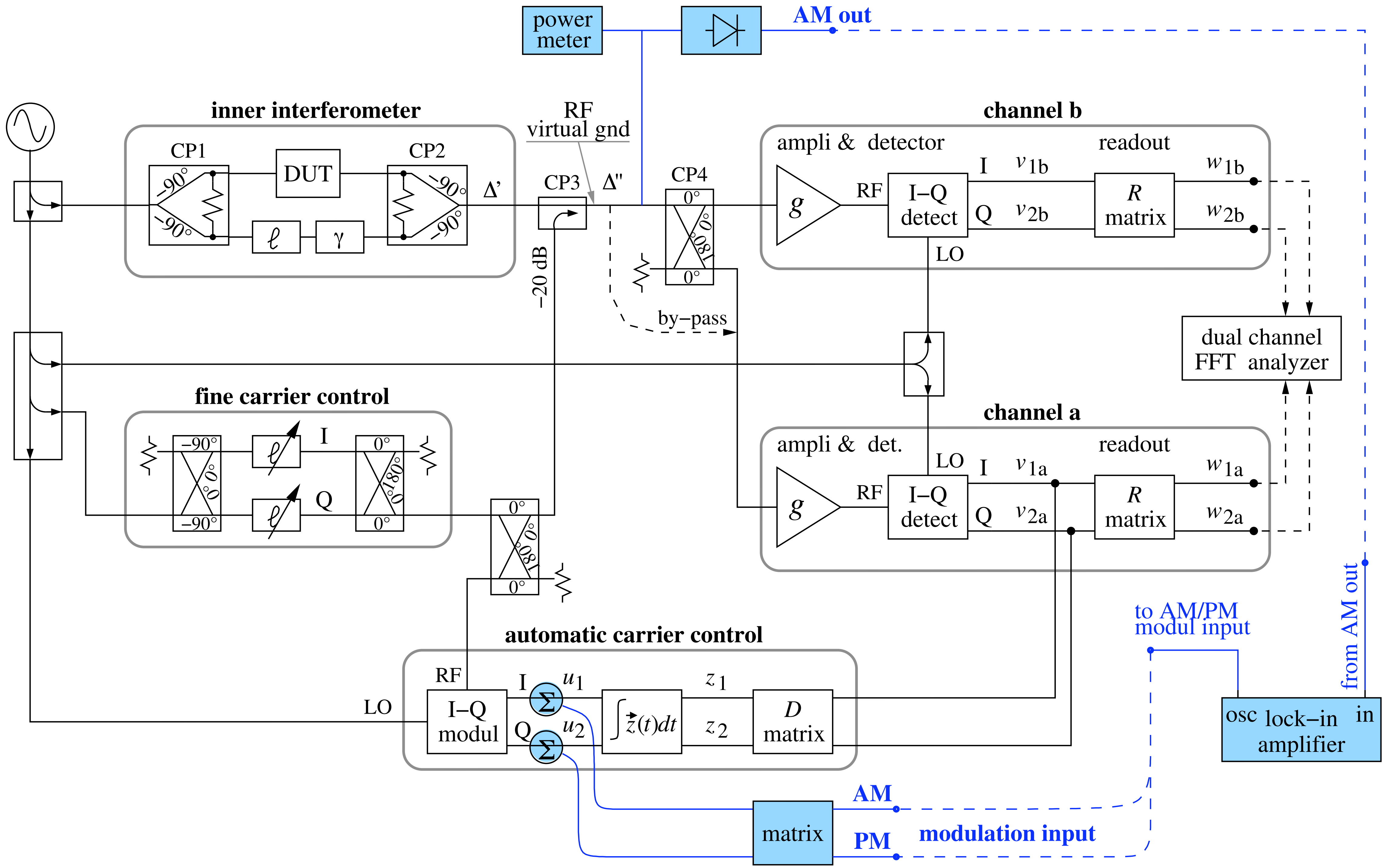
$$\varphi_{\text{rms}} = \sqrt{P_y/P_0}$$

P_0 = power of the carrier
P_x = power of the in-phase sidebands
P_y = power of the quadrature sidebands

Basic ideas

- Generate reference AM and PM by adding sidebands to a carrier
- Power detectors provide absolute reference of PM as a null of AM
- Accurate (0.02 dB) power-ratio measurement with commercial power-meters
- Correct IQ modulators and detectors with linear algebra (2x2 matrices)
- Transfer the accuracy of a LF (kHz range) lock-in amplifier to RF/microwave
- Worst-case accuracy 0.3 dB => improvement in progress

Bridge (interferometric) instrument



Light blue: work in progress

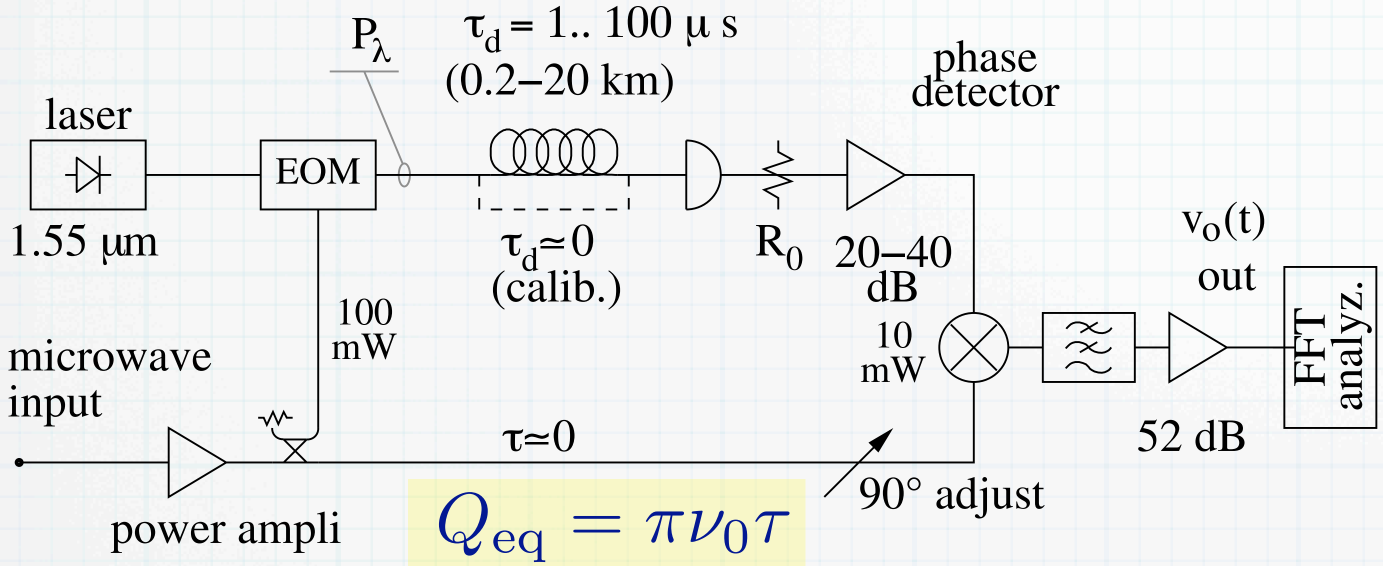
The dual-bridge contains almost all the blocks needed to calibrate the measurement

E. Rubiola, V. Giordano, Rev. Sci. Instrum 73 6 p.2445–57, jun 2002. Also arXiv:physics/0503015

2 – Microwave photonics

Opto-electronic discriminator

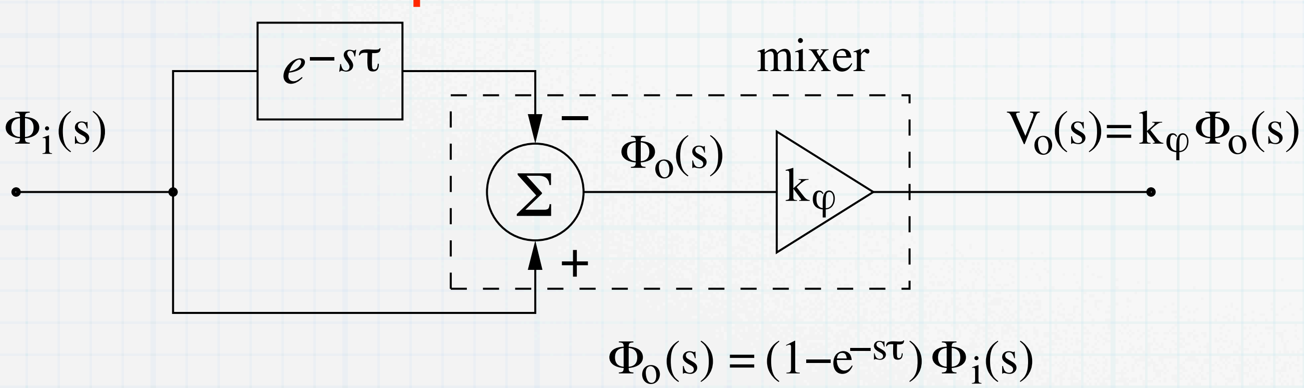
Rubiola, Salik, Huang, Yu, Maleki, JOSA-B 22(5) p.987–997 (2005)



$$Q_{eq} = \pi \nu_0 \tau$$

The short arm can be a microwave cable or a photonic channel

Laplace transforms



- delay –> frequency-to-phase conversion
- works at any frequency
- long delay (microseconds) is necessary for high sensitivity
- the delay line must be an optical fiber
  fiber: attenuation 0.2 dB/km, thermal coeff. 6.8 10^{-6}/K
  cable: attenuation 0.8 dB/m, thermal coeff. ~ 10^{-3}/K

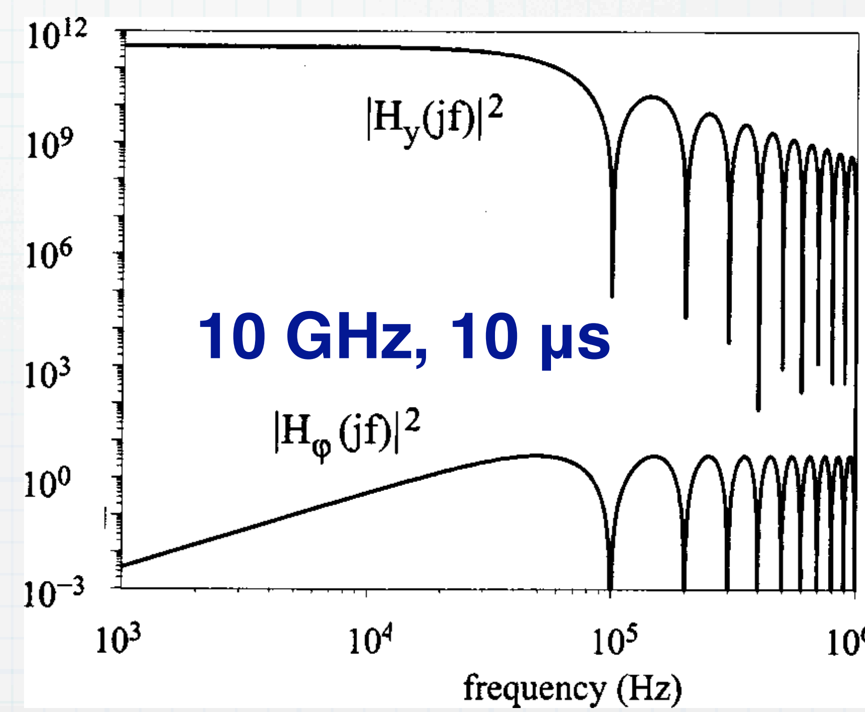
Laplace transforms

$$\Phi(s) = H_\varphi(s)\Phi_i(s)$$

$$|H_\varphi(f)|^2 = 4\sin^2(\pi f \tau)$$

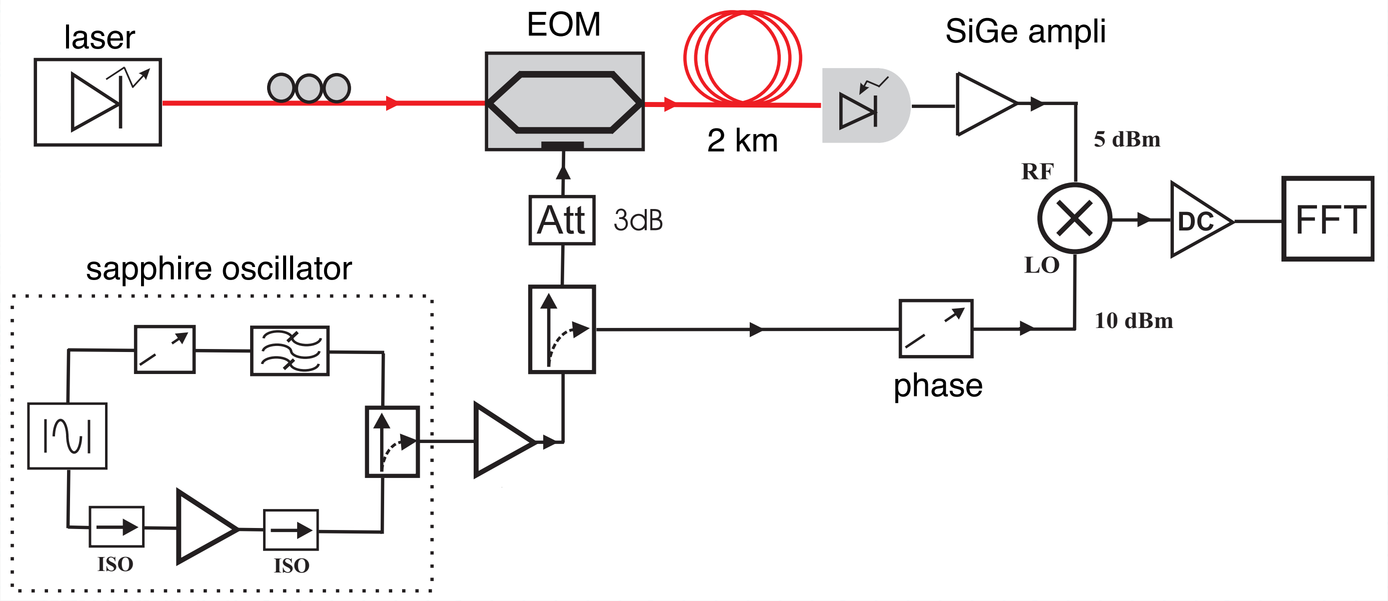
$$S_y(f) = |H_y(f)|^2\, S_{\varphi\, i}(s)$$

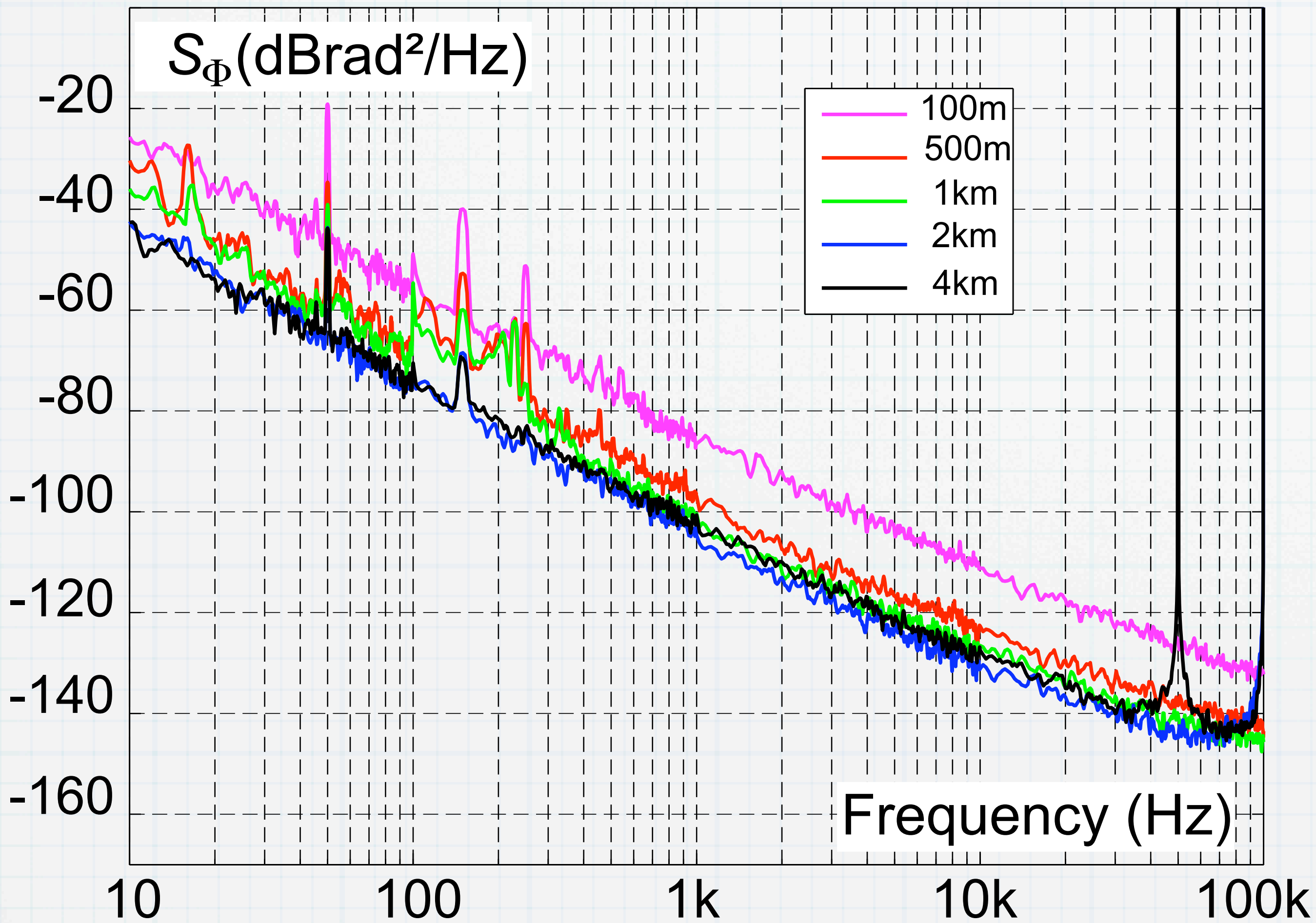
$$|H_y(f)|^2 = \frac{4\nu_0^2}{f^2}\sin^2(\pi f \tau)$$



Measurement of a sapphire oscillator

Volyanskiy & al., JOSAB (in press). Also arXiv:0807.3494v1 [physics.optics] July 2008.



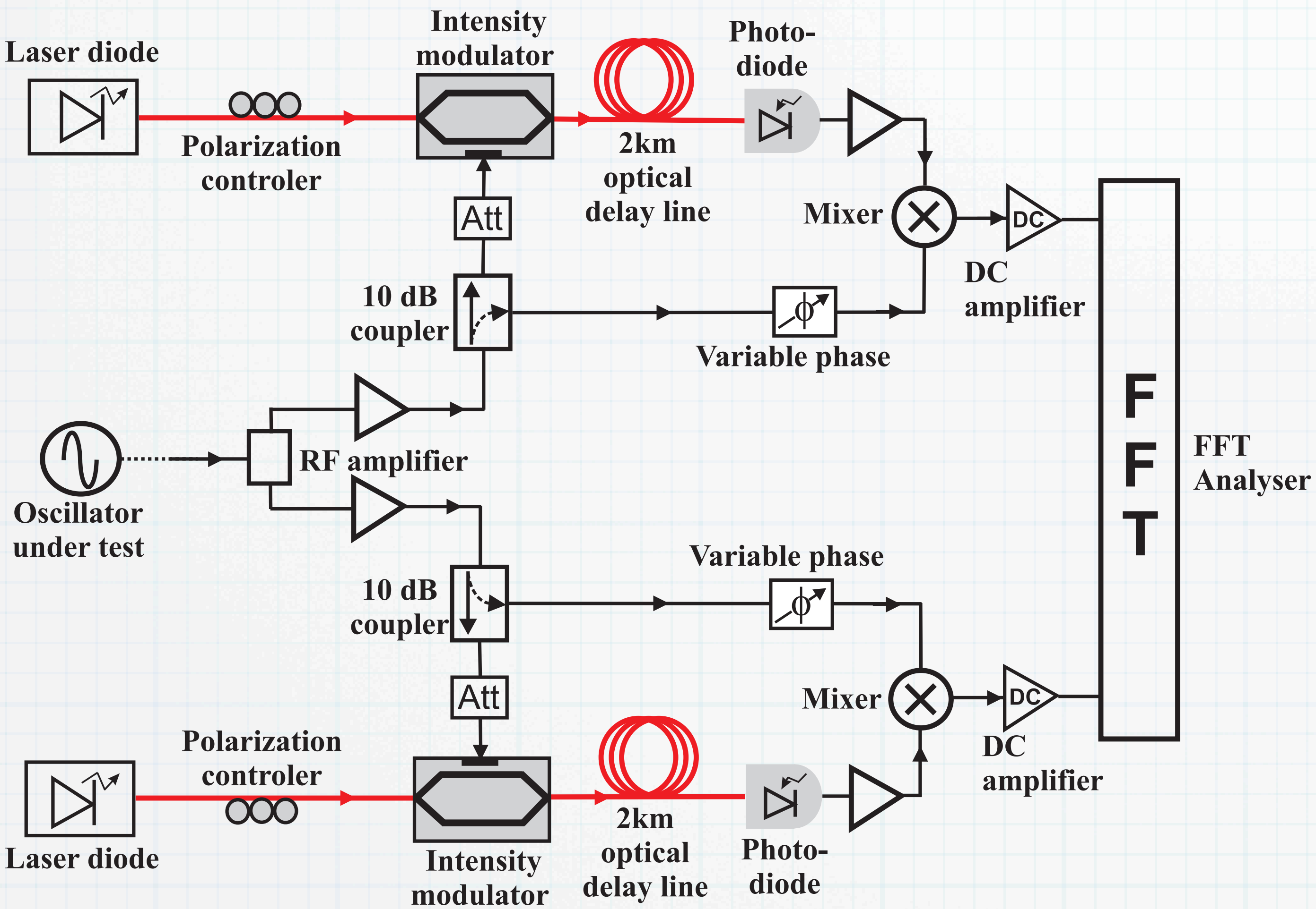


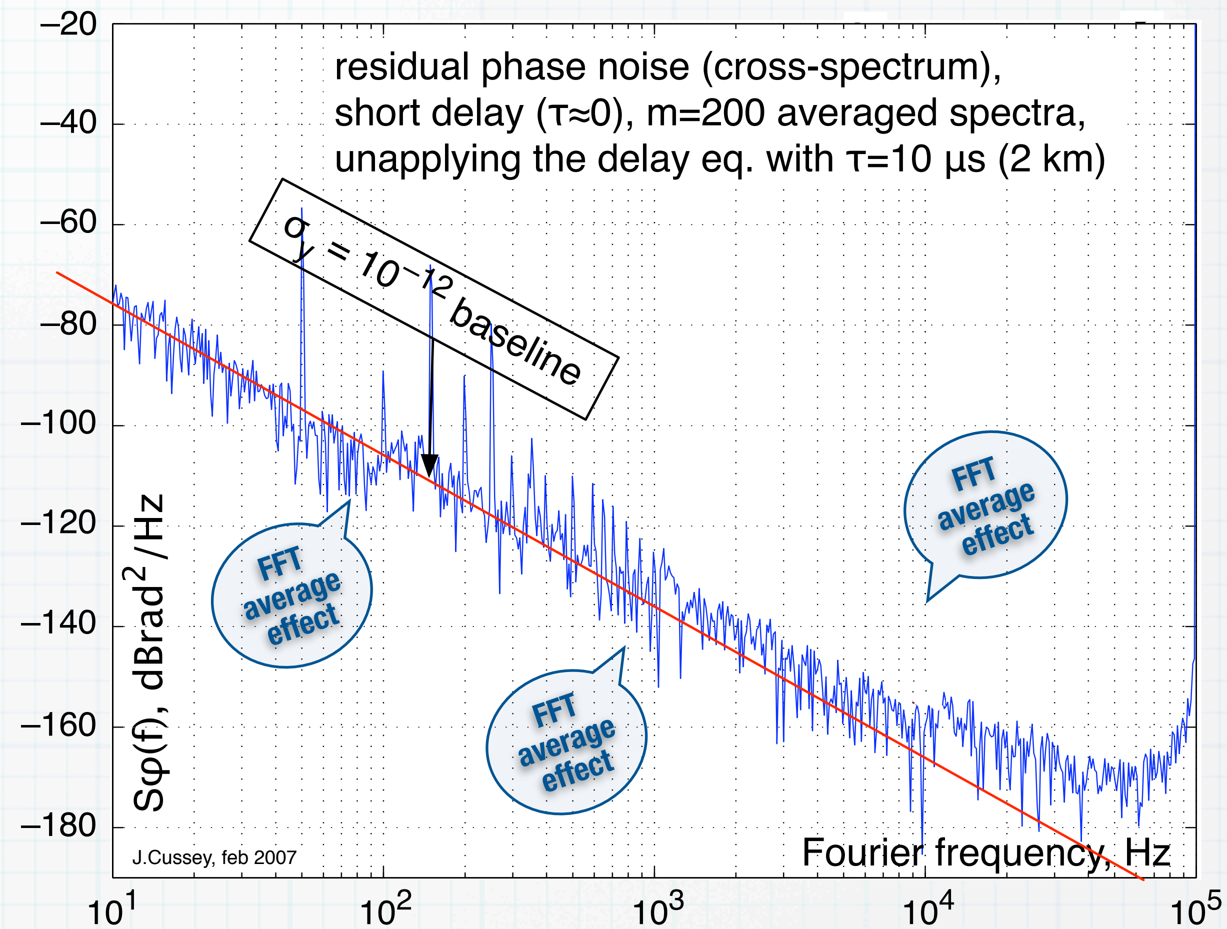
- The instrument noise scales as 1/τ, yet the blue and black plots overlap
  magenta, red, green => instrument noise
  blue, black => noise of the sapphire oscillator under test
- We can measure the $1/f^3$ phase noise (frequency flicker) of a 10 GHz sapphire oscillator (the lowest-noise microwave oscillator)
- Low AM noise of the oscillator under test is necessary

Dual-channel (correlation) measurement

Volyanskiy & al., JOSAB (in press) and arXiv:0807.3494v1 [physics.optics] July 2008.

Derives from: E. Salik, N. Yu, L. Maleki, E. Rubiola, Proc. Ultrasonics-FCS Joint Conf., Montreal, Aug 2004 p.303-306

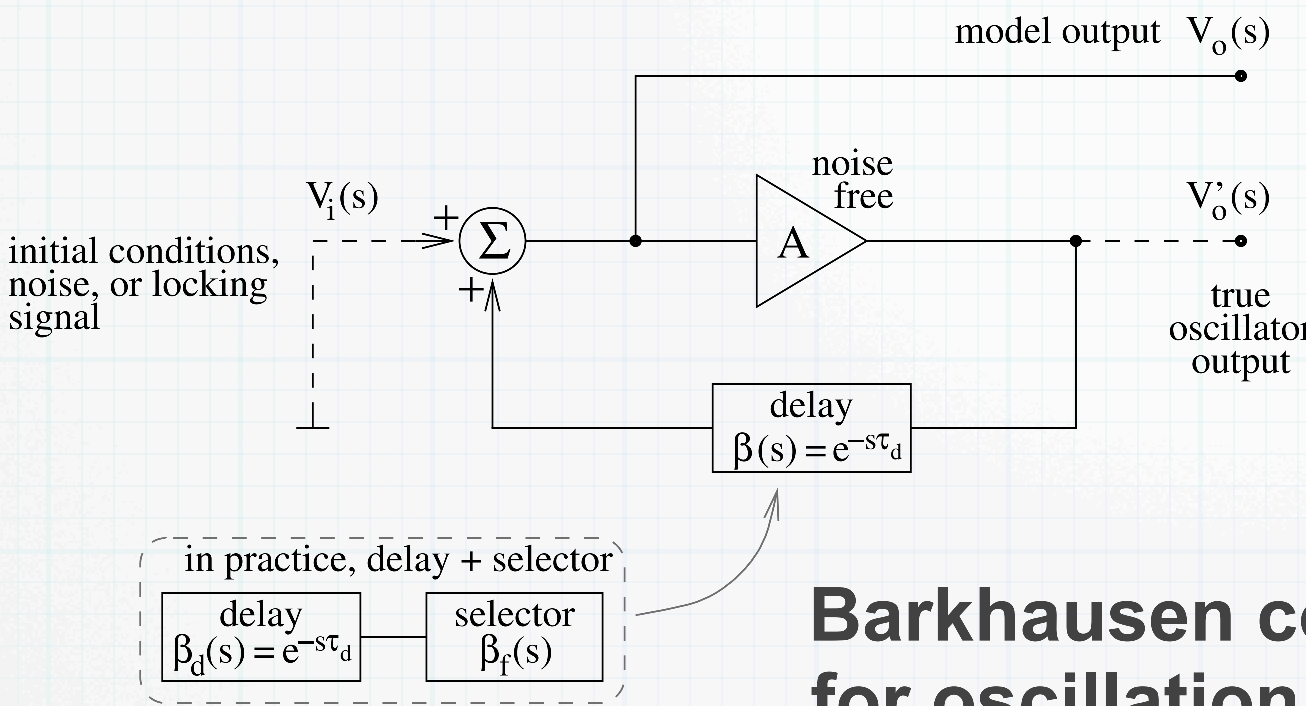


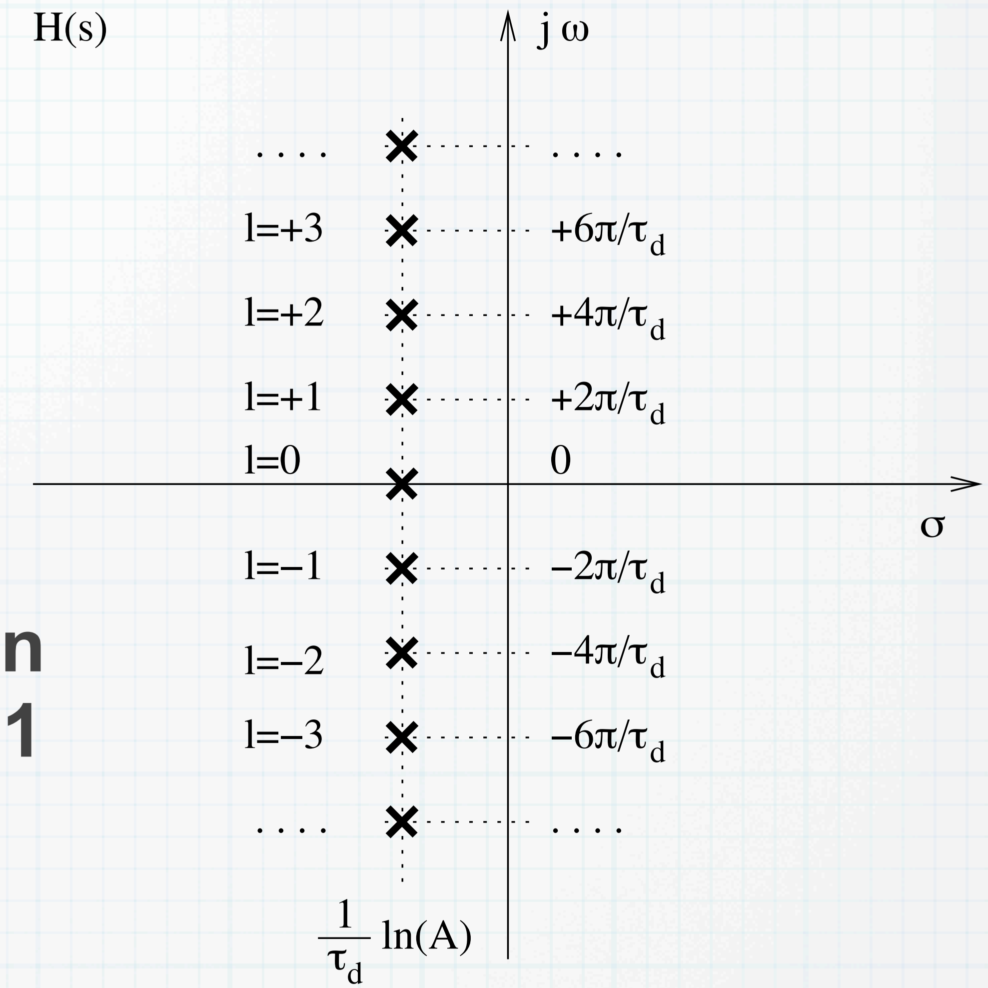


Improvements

- Understanding flicker (photodetectors and amplifiers)
- SiGe technology provides lower 1/f phase noise
- CATV laser diodes exhibit lower AM/FM noise
- Low Vπ EOMs show higher stability because of the lower RF power
- Optical fiber sub-mK temperature controlled

Delay-line oscillator – operation





Barkhausen condition for oscillation: Aß = 1

General feedback theory

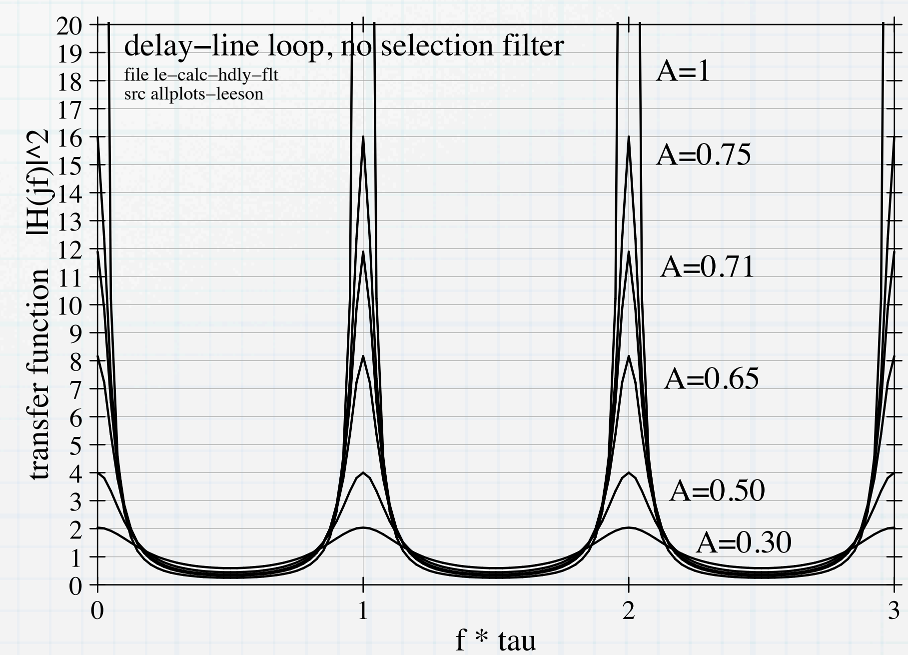
$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{1 - A\beta(s)}$$

Delay-line oscillator

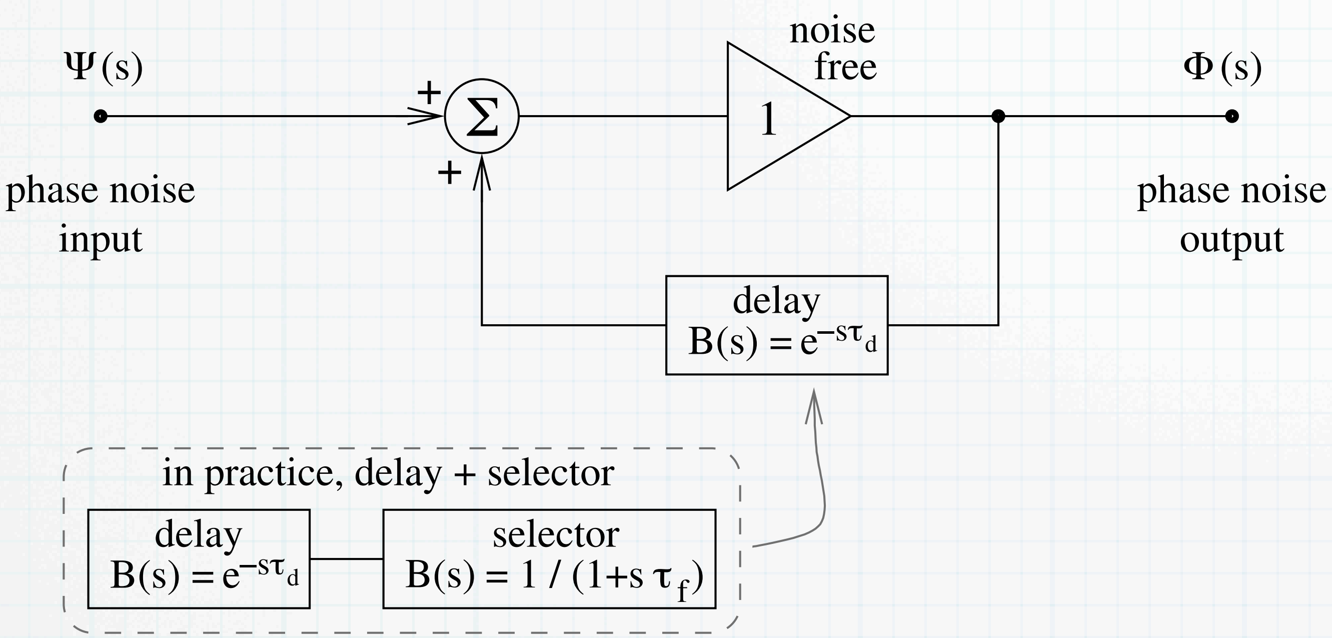
$$H(s) = \frac{1}{1 - A\, e^{-s\tau_d}}$$

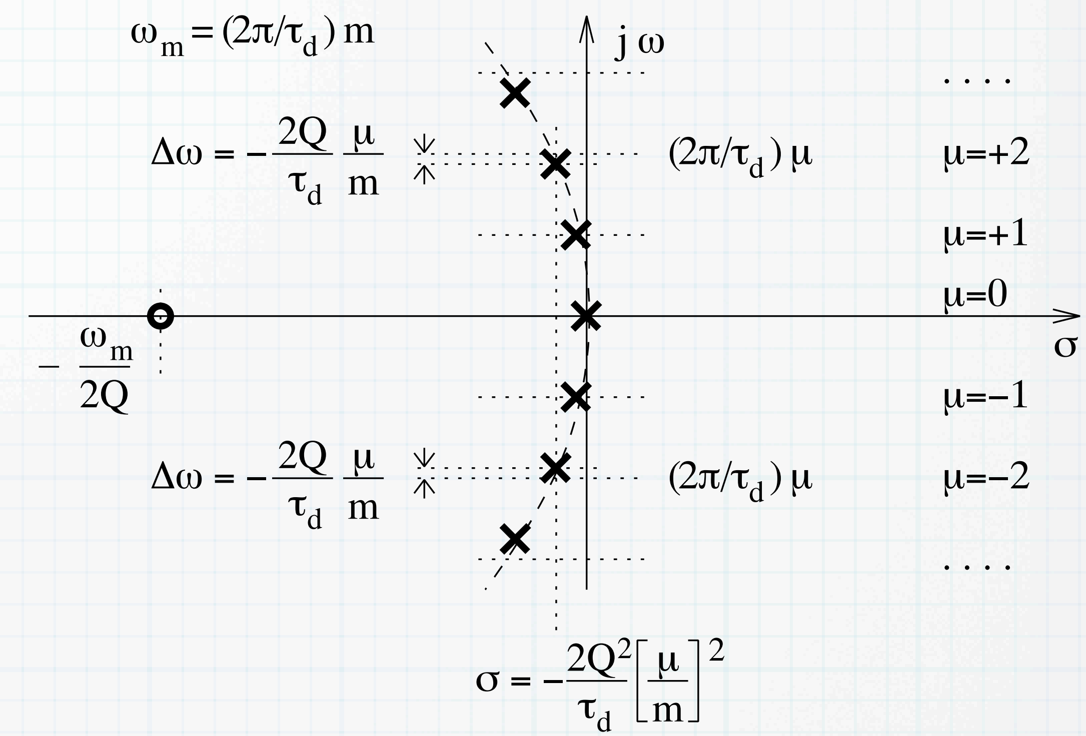
Location of the roots

$$s_l = \frac{1}{\tau_d}\ln(A) + j\,\frac{2\pi}{\tau_d}\,l \qquad \text{integer } l \in (-\infty, \infty)$$



Delay-line oscillator – phase noise





General feedback theory

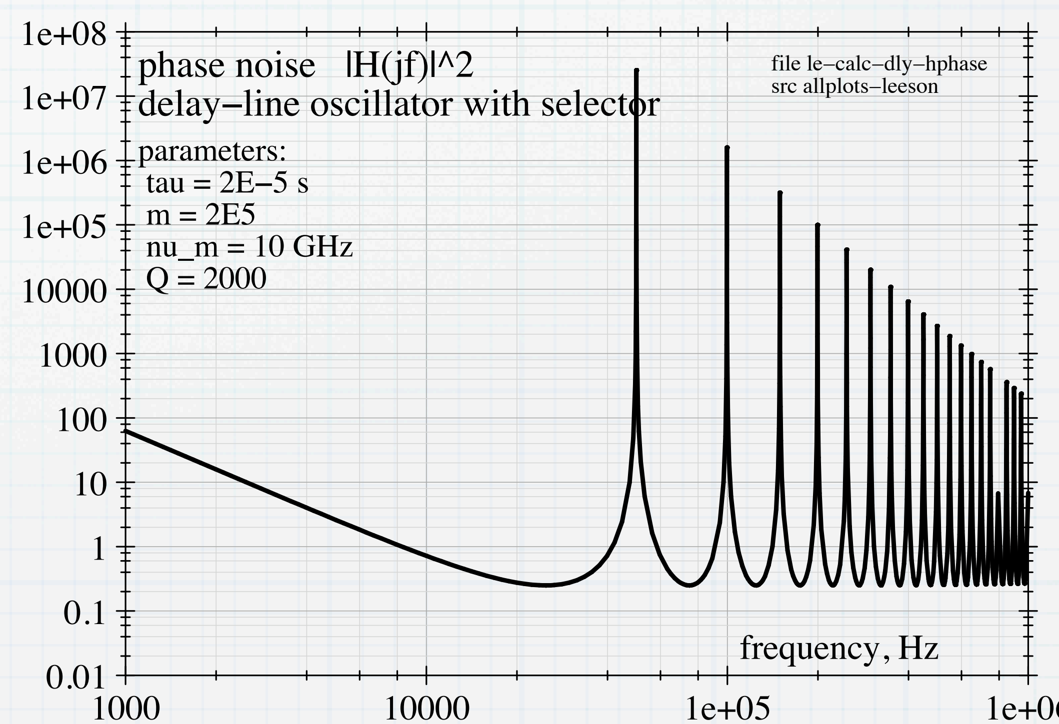
$$H(s) = \frac{\Phi(s)}{\Psi(s)} = \frac{1}{1 - B(s)}$$

Delay-line oscillator

$$H(s) = \frac{1 + s\tau_f}{1 + s\tau_f - e^{-s\tau_d}}$$

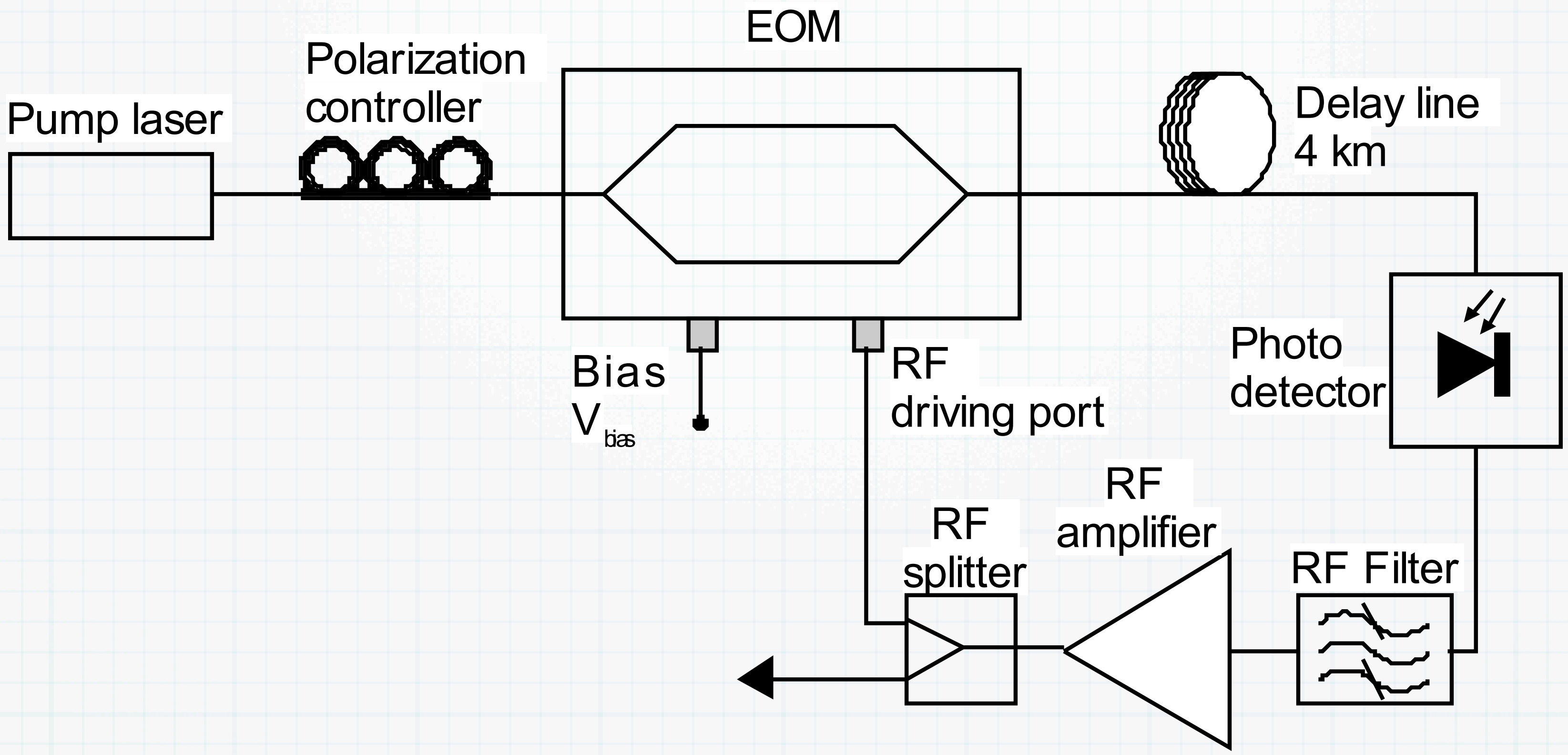
Location of the roots

$$s_\mu = -\frac{2Q^2}{\tau_d}\left(\frac{\mu}{m}\right)^2 + j\frac{2\pi}{\tau_d}\mu - \frac{2Q}{\tau_d}\,\frac{\mu}{m}$$



E. Rubiola, *Phase Noise and Frequency Stability in Oscillators*, Cambridge 2008, ISBN 978-0521-88677-2

Delay-line oscillator



$$Q_{\rm eq} = \pi \nu_0 \tau$$

$$f_L = \frac{\nu_0}{2Q}$$

$$f_L = \frac{1}{4\pi^2\tau^2}$$

$Q_{eq}=3\times10^5$ ← L=4km

f_L=8kHz

10^{-11}

Leeson formula

$$S_\varphi(f) \simeq \frac{f_L^2}{f^2}\, S_\psi(f) \quad \text{for } f \ll f_L$$

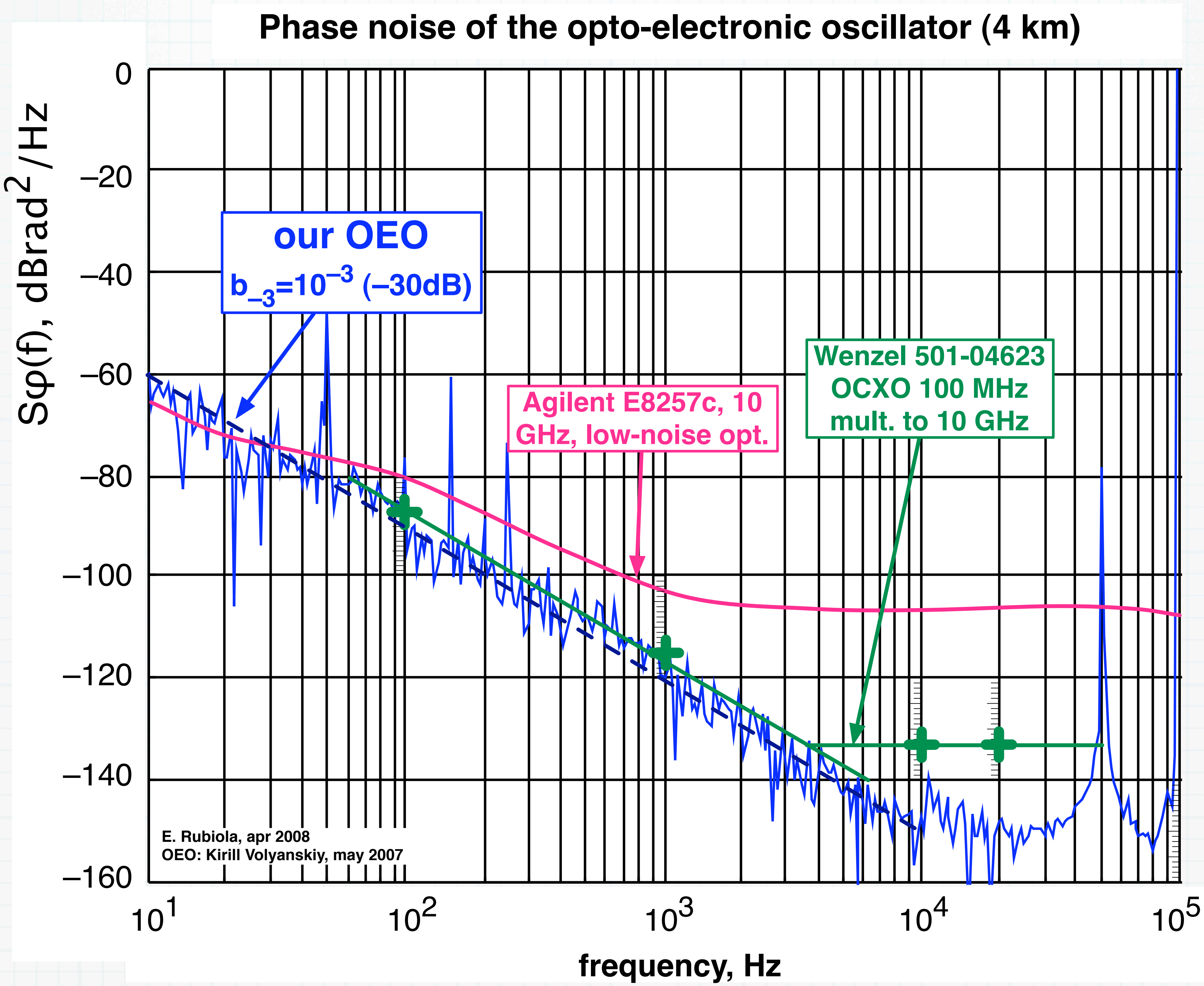
$b_{-3} = 6.3\times10^{-4}$ (–32 dB)

$$h_{-1} = b_{-3}/\nu_0^2$$ **6.3×10^{-24}**

$$\sigma_y^2 = 2\ln(2)\, h_{-1}$$ **8.8×10^{-24}**

$$\sigma_y \simeq 2.9\times10^{-12}$$ **Allan deviation**

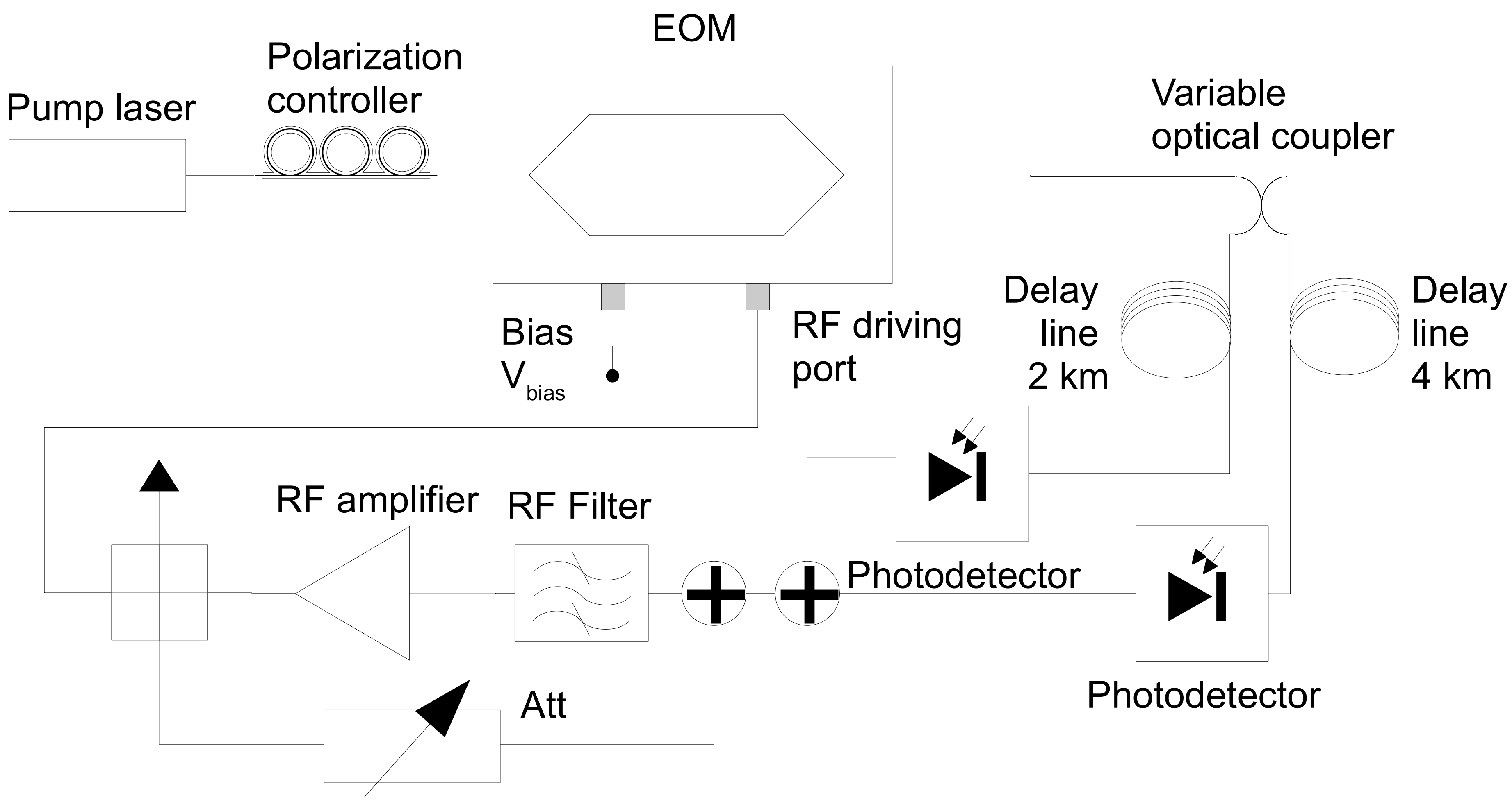
Delay-line oscillator - measured noise



- 1.310 nm DFB CATV laser
- Photodetector DSC 402 (R = 371 V/W)
- RF filter ν_0 = 10 GHz, Q = 125
- RF amplifier AML812PNB1901 (gain +22dB)

expected phase noise
$b_{-3} \approx 6.3\times10^{-4}$ (–32 dB)

Optical-fiber 10 GHz oscillator



- use positive feedback with a short cable (3-5 ns) in the feedback path to implement the mode selector filter
- the positive feedback also increase the amplifier gain
  (AML SiGe parallel amplifiers exhibits lowest flicker, but low have gain 22 dB)
- use the 2-km (10 μs) path to eliminate the 50-kHz noise peak due to the 4-km (20 μs)
- the microwave power is changed by adjusting the laser power
- high noise figure, due to the two power splitters/combiners

Kiryll Volyanskiy, jan 2008

Regenerative optical-fiber 10 GHz oscillator

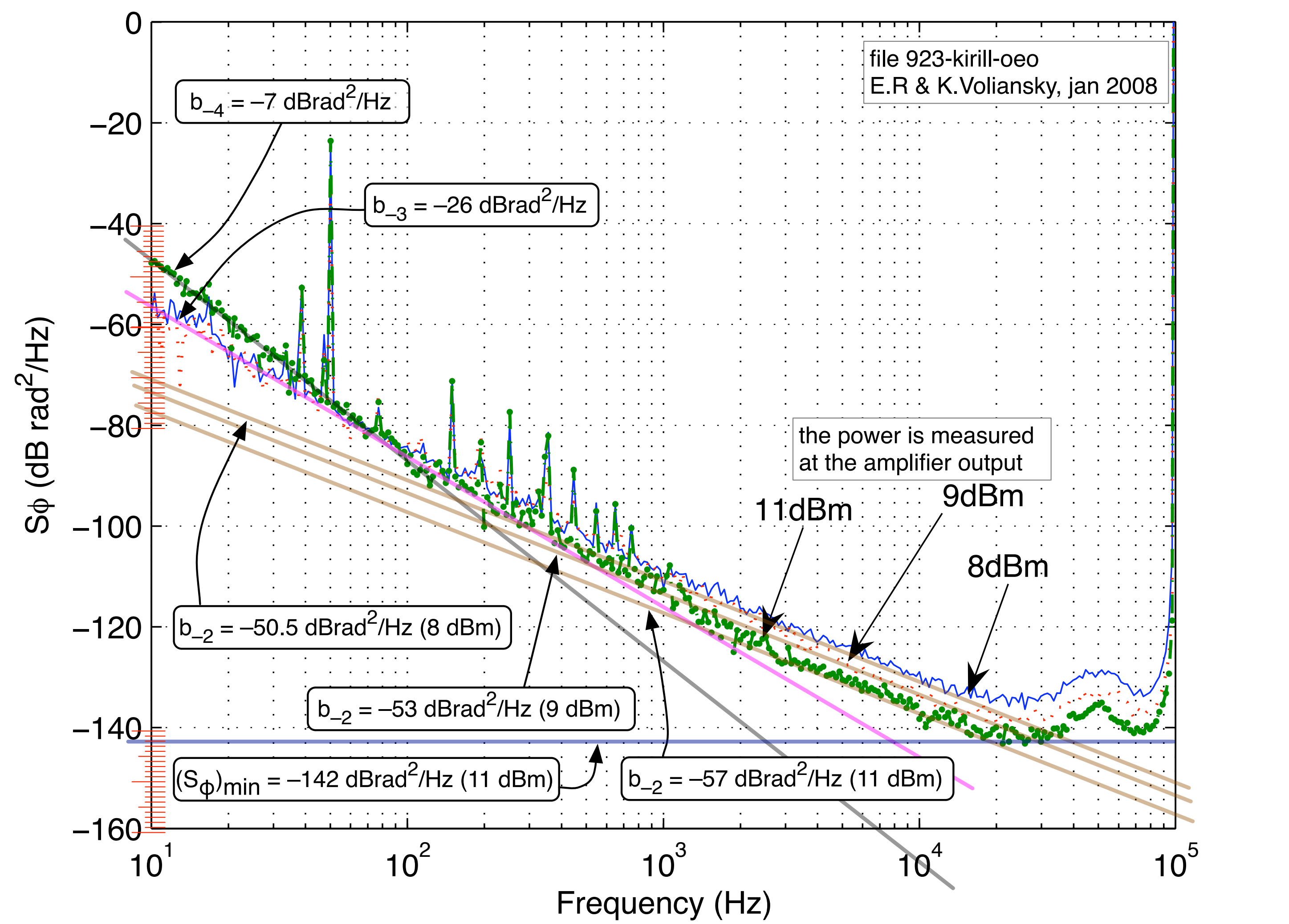
P_{rf} is given, thus $V_0 = (2RP)^{1/2}$
V_π is estimated (4.5 V at 10 GHz)

Use

$$m = 2J_1\left(\frac{\pi V_0}{V_\pi}\right)$$

Get

P, dBm	V_p, V	$\pi V_0/V_\pi$	m
11	1.122	0.860	0.783
9	0.891	0.683	0.644
8	0.794	0.6.09	0.581



The oscillator phase noise minima are 6 dB lower than $b_0=N/P_0$ (white noise)

$m = 0.725$ (P_{rf}=11 dBm)
$(S_\varphi)_{min} = -142$ dB
F = 10 dB (incl. couplers)
$\eta = 0.6$
$\nu_l = 194$ THz

Feeding the available data in the model

$$(S_\varphi)_{\min} = \frac{8}{m^2}\left\{\frac{F k_B T_0}{R_0}\left[\frac{h\nu_l}{q\eta}\right]^2 \frac{1}{\overline{P}_l^2} + 2\,\frac{h\nu_l}{\eta}\,\frac{1}{\overline{P}_l}\right\}$$

we get

$P_0 = 6.4$ µW (-22 dBm)

$P_l \approx 0.71$ mW

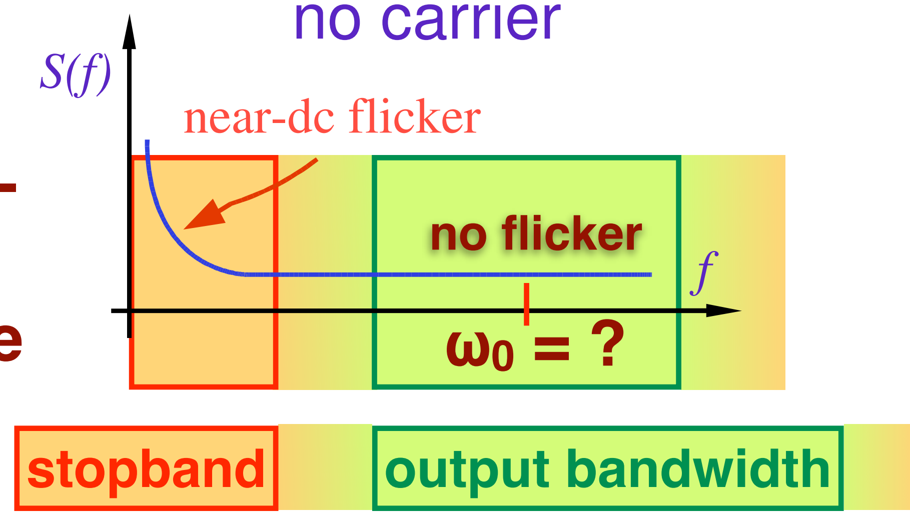
There is room for engineering

3 – Electronic and optical components

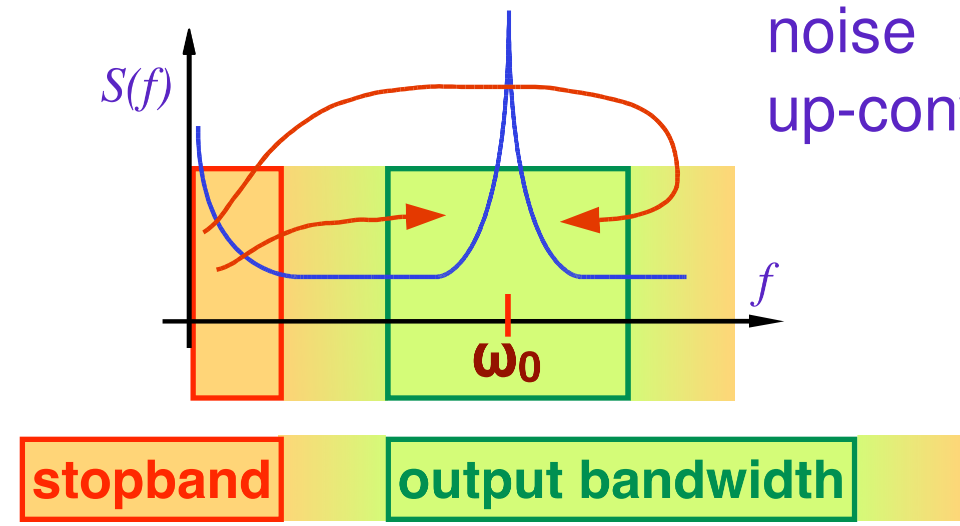
Flicker in electronic & optical components

near-dc noise

no carrier



noise up-conversion



carrier — **near-dc noise**

$$v_i(t) = V_i\, e^{j\omega_0 t} + n'(t) + jn''(t)$$

the parametric nature of 1/f noise is hidden in n' and n''

substitute
(careful, this hides the down-conversion)

$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + \ldots$$

non-linear (parametric) amplifier

expand and select the ω_0 terms

$$v_o(t) = V_i \Big\{ a_1 + 2a_2 \big[n'(t) + jn''(t) \big] \Big\} e^{j\omega_0 t}$$

The noise sidebands are proportional to the input carrier

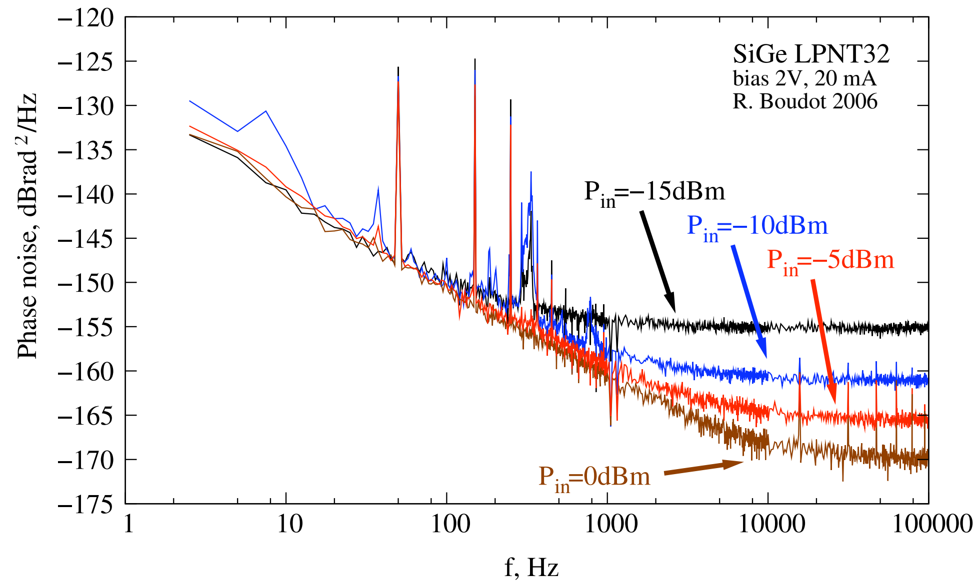
get AM and PM noise

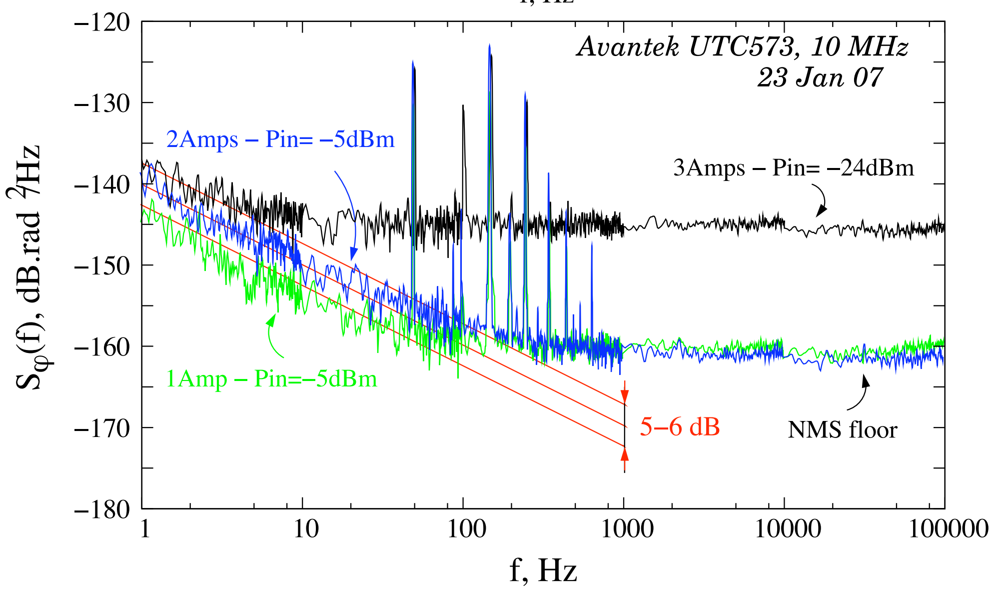
$$\alpha(t) = 2\,\frac{a_2}{a_1}\,n'(t) \qquad \varphi(t) = 2\,\frac{a_2}{a_1}\,n''(t)$$

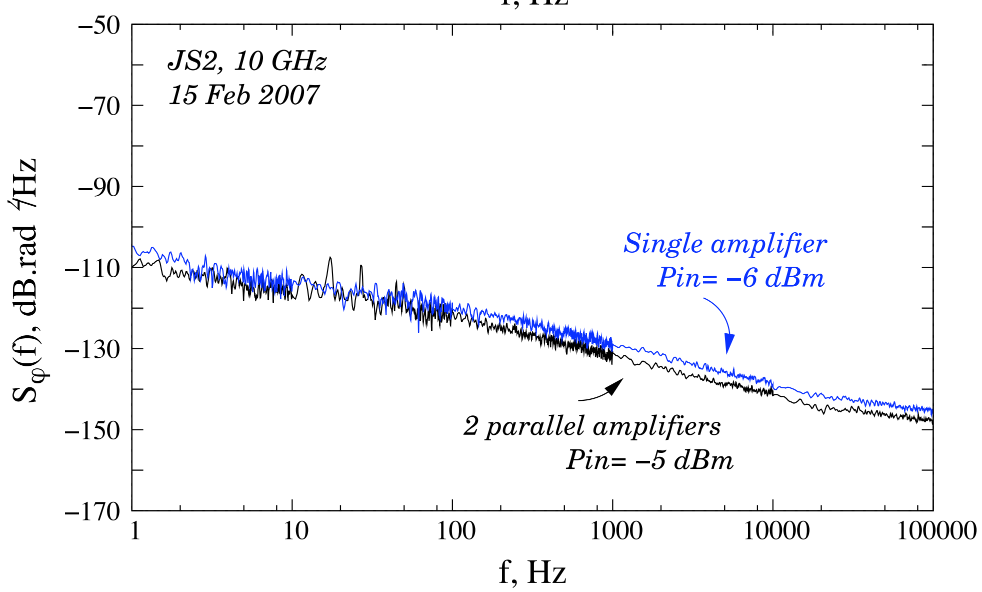
The AM and the PM noise are independent of V_i, thus of power

There is also a linear parametric model, which yields the same results

Amplifier flicker noise – experiments







Phase noise vs. power

- The 1/f phase noise b_{-1} is about independent of power
- The white noise b_0 scales up/down as $1/P_0$, i.e., the inverse of the carrier power
- Describing the 1/f noise in terms of fc is misleading because fc depends on the input power

Phase noise of cascaded amplifiers

- The expected flicker of a cascade increases by:
  3 dB, with 2 amplifiers
  5 dB, with 3 amplifiers

Regenerative amplifiers

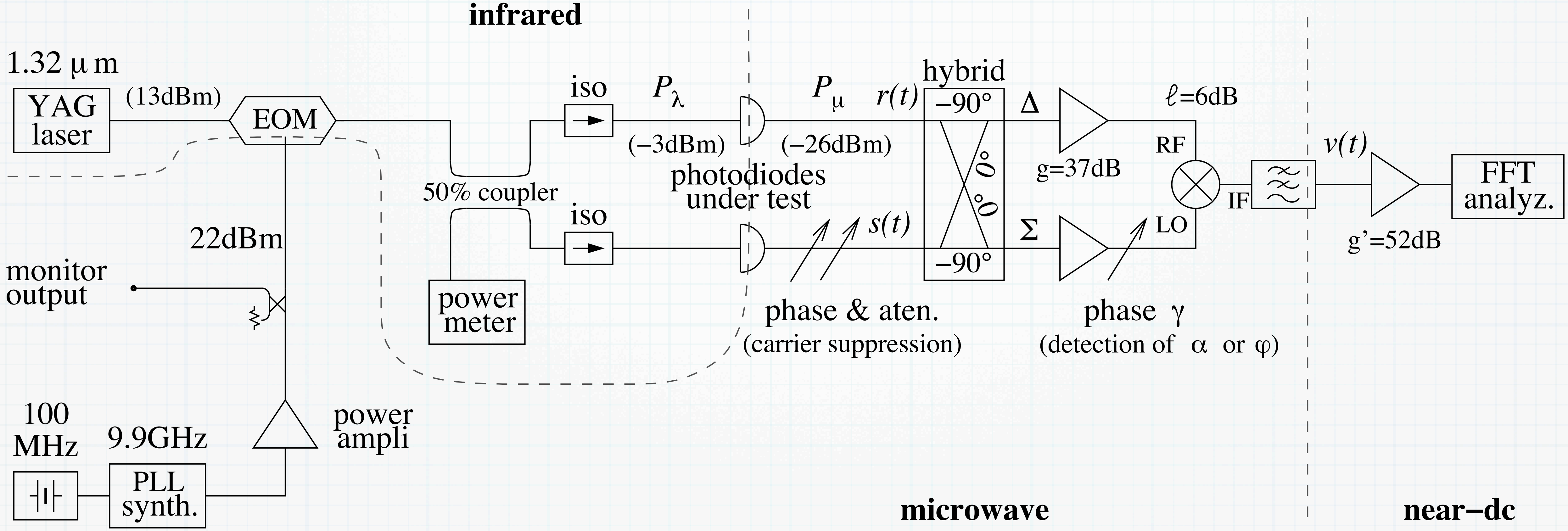
- Phase noise increase as the squared gain because the noise source at each roundtrip is correlated

Phase noise of paralleled amplifiers

- Connecting two amplifiers in parallel, the phase-noise flicker is expected to decrease by 3 dB

Photodetector 1/f noise

Rubiola, Salik, Yu, Maleki, MTT 54(2) p.816-820, Feb 2006



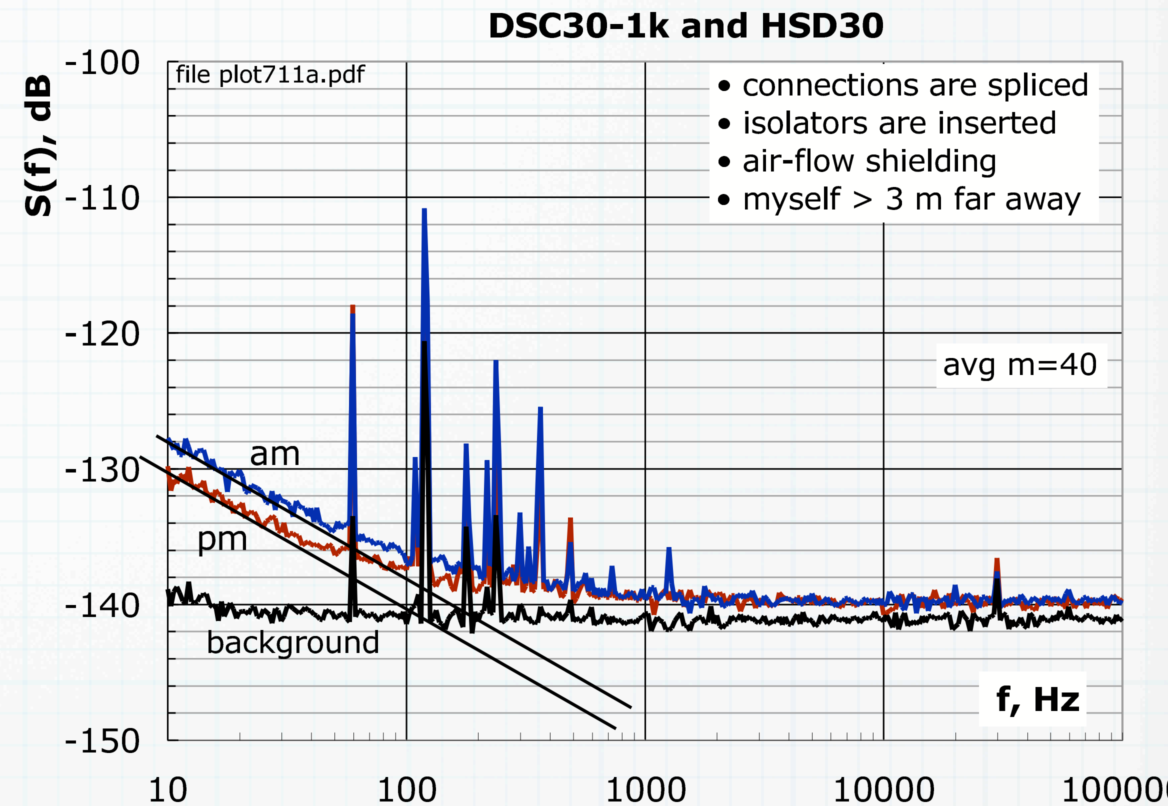
photodiode	$S_\alpha(1\,\text{Hz})$ estimate	$S_\alpha(1\,\text{Hz})$ uncertainty	$S_\varphi(1\,\text{Hz})$ estimate	$S_\varphi(1\,\text{Hz})$ uncertainty
HSD30	−122.7	−7.1 / +3.4	−127.6	−8.6 / +3.6
DSC30-1K	−119.8	−3.1 / +2.4	−120.8	−1.8 / +1.7
QDMH3	−114.3	−1.5 / +1.4	−120.2	−1.7 / +1.6
unit	dB/Hz	dB	dBrad^2/Hz	dB

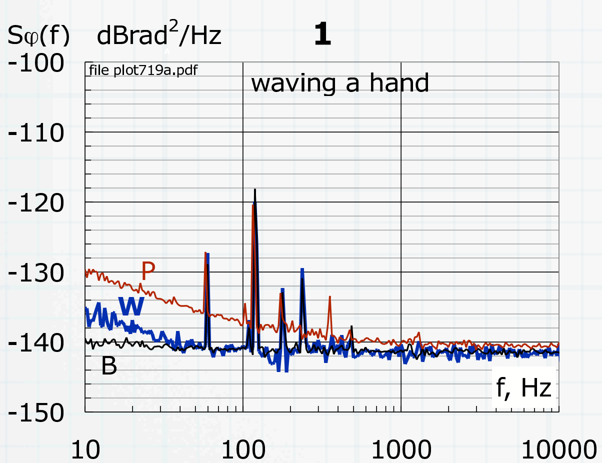
The noise of the ∑ amplifier is not detected [Rubiola, Salik, Yu, Maleki, Electron. Lett. 39(19) p.1389-1390 (2003)]

Photodetector 1/f noise

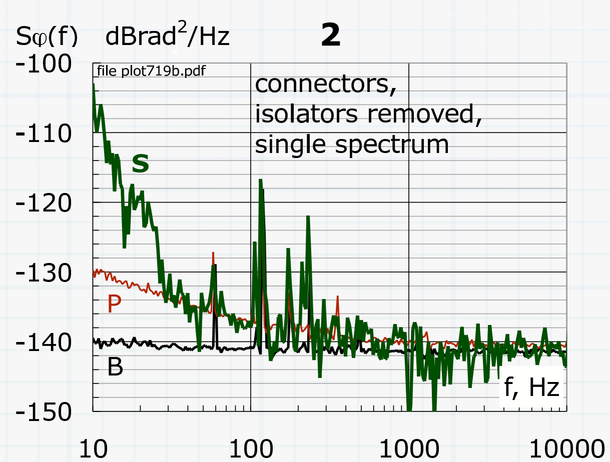
Rubiola, Salik, Yu, Maleki, MTT 54(2) p.816-820, Feb 2006

- the photodetectors we measured are similar in AM and PM 1/f noise
- the 1/f noise is about -120 dB[rad²]/Hz
- other effects are easily mistaken for the photodetector 1/f noise
- environment and packaging deserve attention in order to take the full benefit from the low noise of the junction

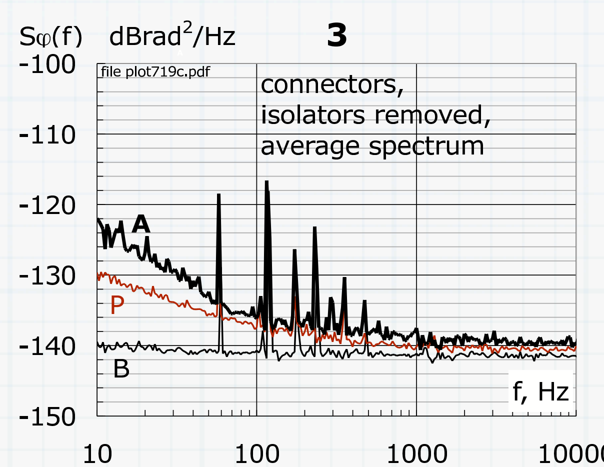




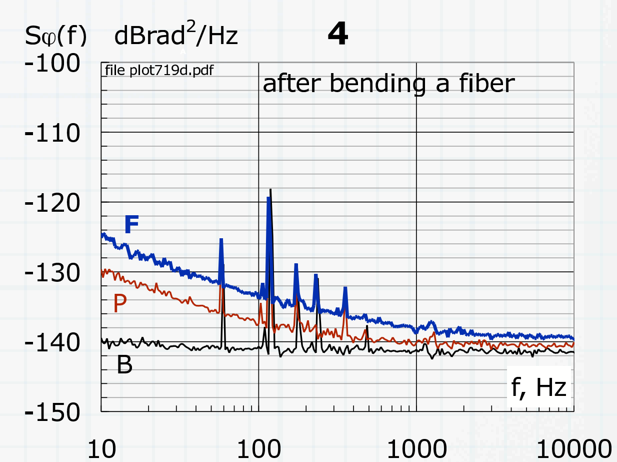
W: waving a hand 0.2 m/s, 3 m far from the system
B: background noise
P: photodiode noise



S: single spectrum, with optical connectors and no isolators
B: background noise
P: photodiode noise

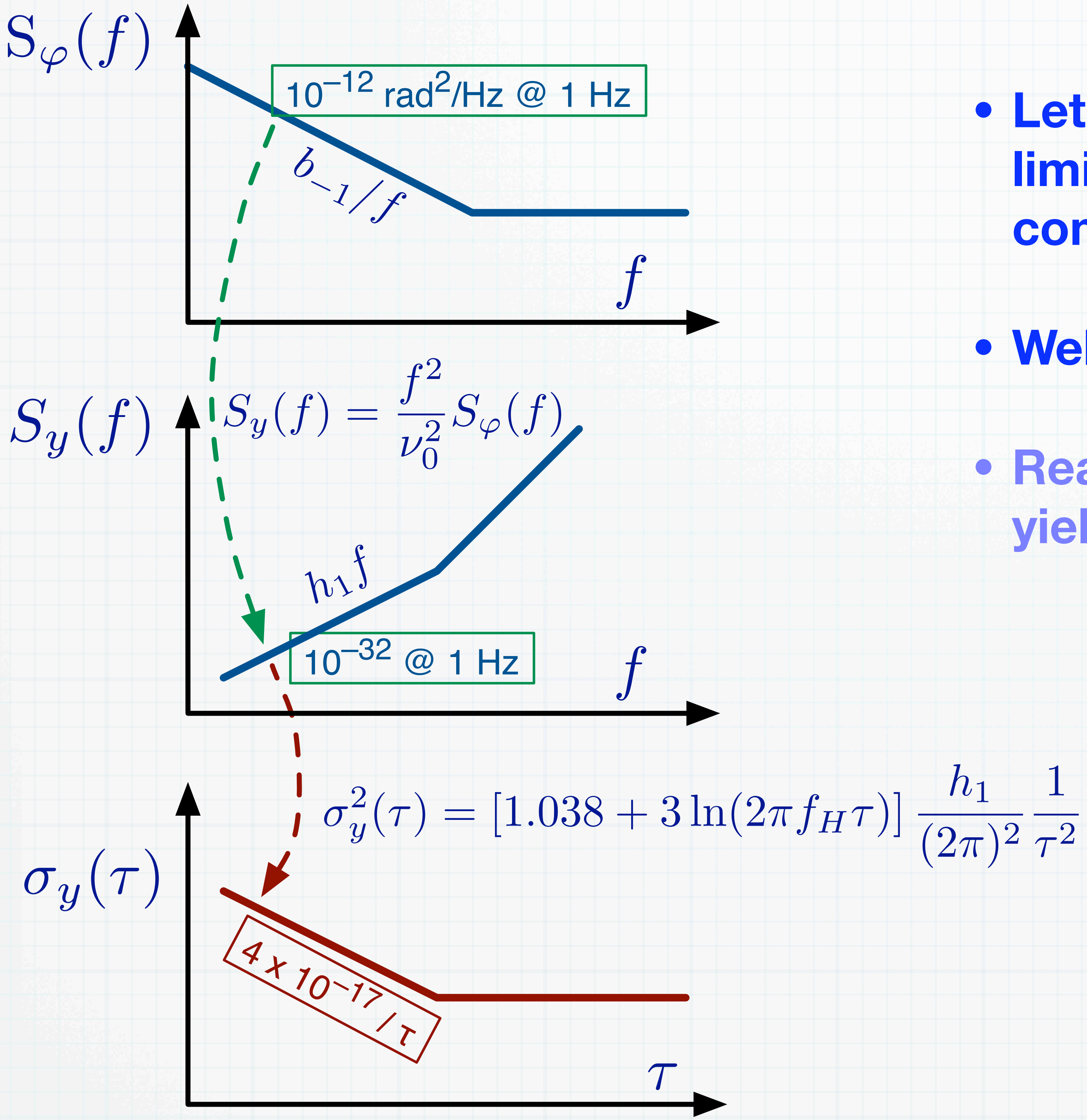


A: average spectrum, with optical connectors and no isolators
B: background noise
P: photodiode noise



F: after bending a fiber, 1/f noise can increase unpredictably
B: background noise
P: photodiode noise

Microwave optical link

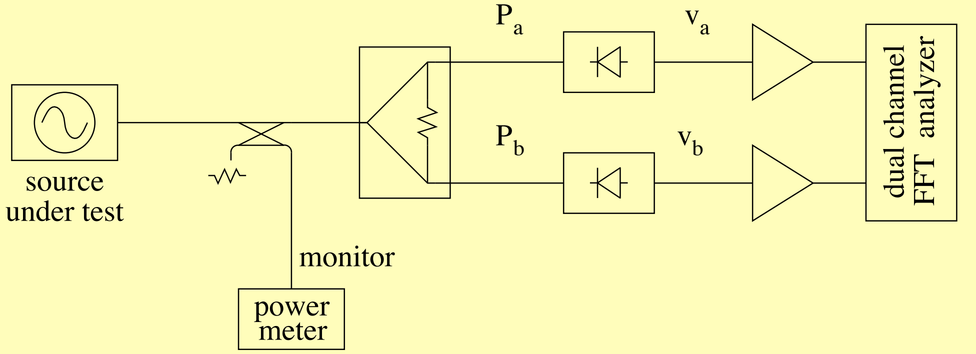


- **Let's be optimistic: a 10 GHz link is limited by the 1/f phase noise of a single component, –120 dBrad²/Hz @ f=1 Hz**
- **Well known rules give $\sigma_y(\tau) = 4\text{x}10^{-17}/\tau$**
- **Realistically, –100 dBrad²/Hz @ f=1 Hz yields $\sigma_y(\tau) = 4\text{x}10^{-16}/\tau$**

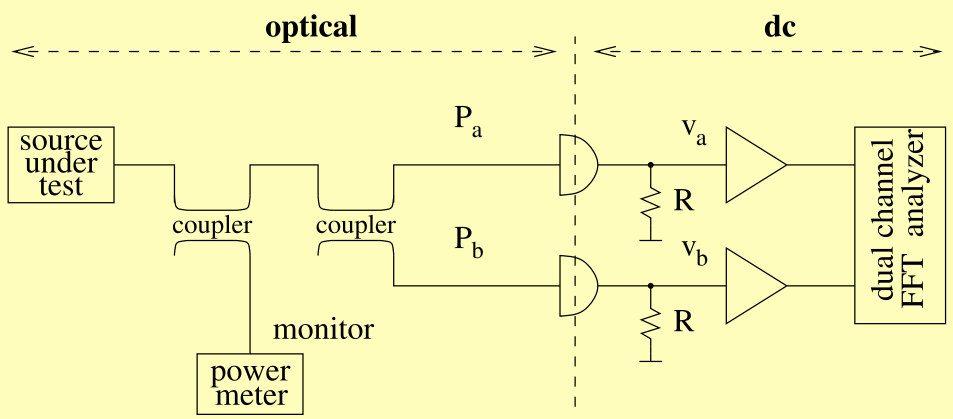
4 – AM noise and RIN

Amplitude noise & laser RIN

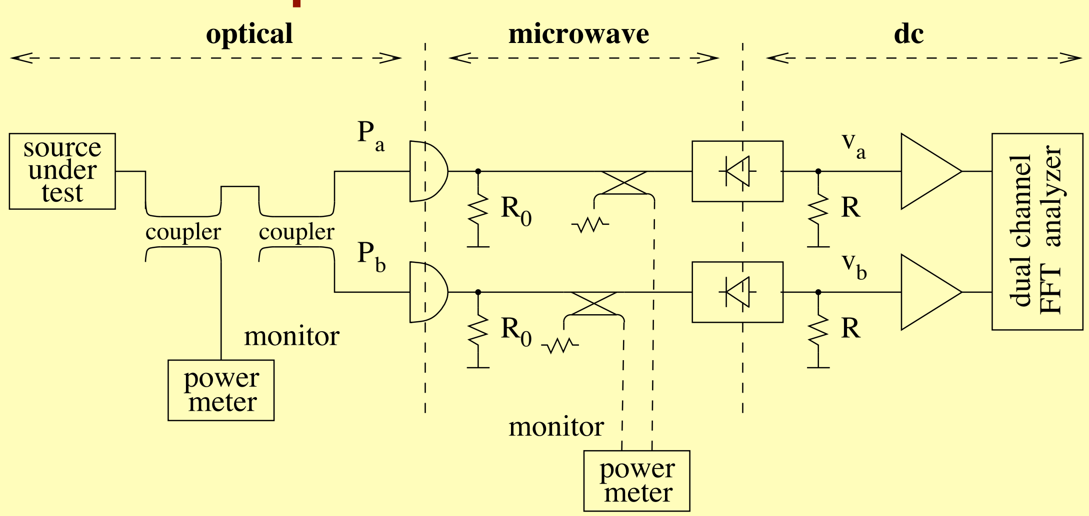
AM noise of RF/microwave sources



Laser RIN

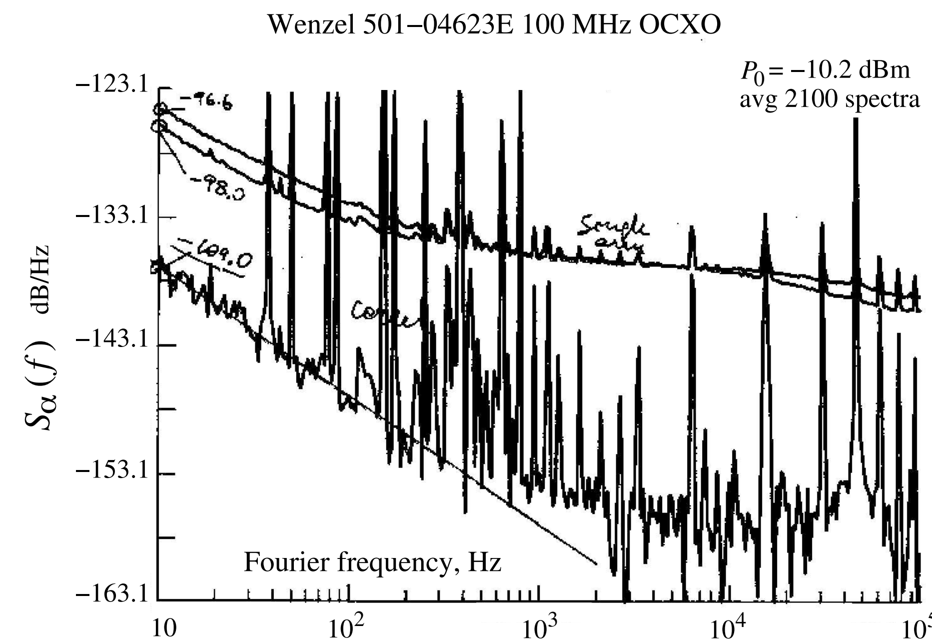


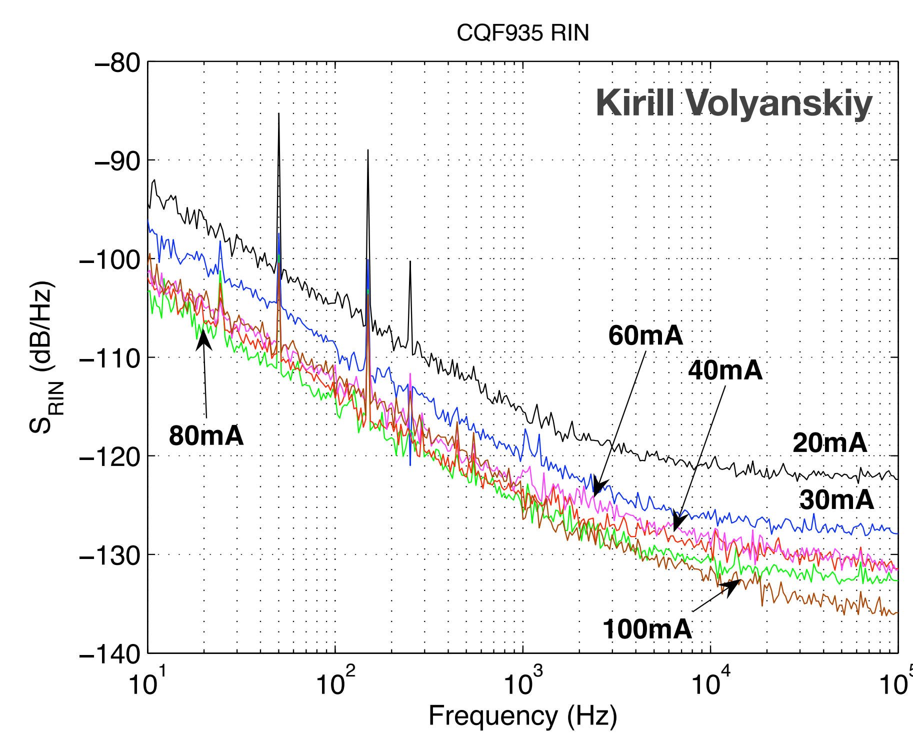
AM noise of photonic RF/microwave sources



E. Rubiola, the measurement of AM noise, dec 2005
arXiv:physics/0512082v1 [physics.ins-det]

- **In PM noise measurements, one can validate the instrument by feeding the same signal into the phase detector**
- **In AM noise this is *not possible* without a lower-noise reference**
- **Provided the crosstalk was measured otherwise, correlation enables to validate the instrument**





AM noise of some sources

source	h_{-1} (flicker)		$(\sigma_\alpha)_{\text{floor}}$
Anritsu MG3690A synthesizer (10 GHz)	2.5×10^{-11}	-106.0 dB	5.9×10^{-6}
Marconi synthesizer (5 GHz)	1.1×10^{-12}	-119.6 dB	1.2×10^{-6}
Macom PLX 32-18 0.1 $\rightarrow$ 9.9 GHz multipl.	1.0×10^{-12}	-120.0 dB	1.2×10^{-6}
Omega DRV9R192-105F 9.2 GHz DRO	8.1×10^{-11}	-100.9 dB	1.1×10^{-5}
Narda DBP-0812N733 amplifier (9.9 GHz)	2.9×10^{-11}	-105.4 dB	6.3×10^{-6}
HP 8662A no. 1 synthesizer (100 MHz)	6.8×10^{-13}	-121.7 dB	9.7×10^{-7}
HP 8662A no. 2 synthesizer (100 MHz)	1.3×10^{-12}	-118.8 dB	1.4×10^{-6}
Fluke 6160B synthesizer	1.5×10^{-12}	-118.3 dB	1.5×10^{-6}
Racal Dana 9087B synthesizer (100 MHz)	8.4×10^{-12}	-110.8 dB	3.4×10^{-6}
Wenzel 500-02789D 100 MHz OCXO	4.7×10^{-12}	-113.3 dB	2.6×10^{-6}
Wenzel 501-04623E no. 1 100 MHz OCXO	2.0×10^{-13}	-127.1 dB	5.2×10^{-7}
Wenzel 501-04623E no. 2 100 MHz OCXO	1.5×10^{-13}	-128.2 dB	4.6×10^{-7}

worst

best