Experimental methods
for the measurement of phase noise and frequency stability

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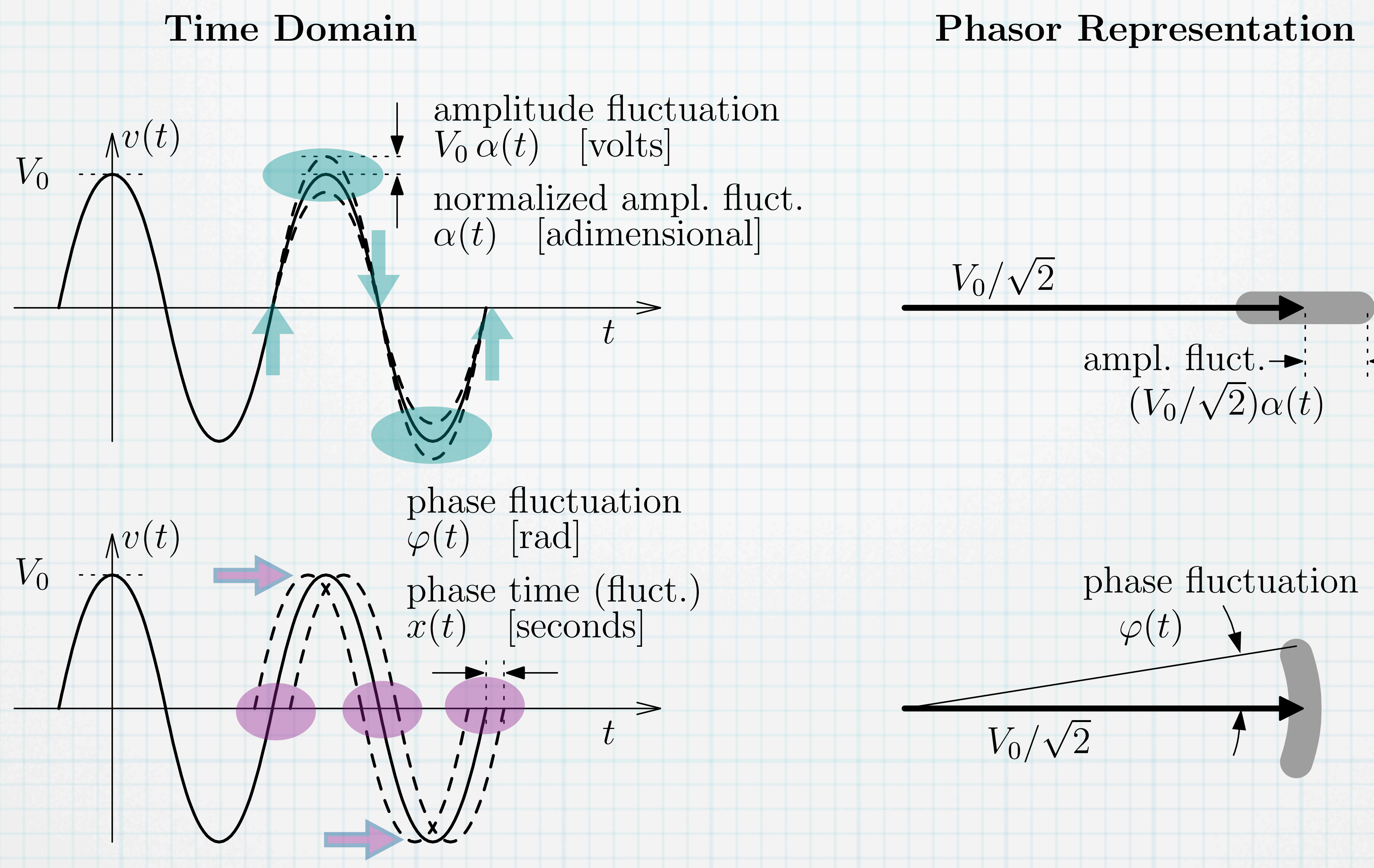
* Phase noise & friends
* Saturated mixer
* Correlation (dual-channel) measurements
* Oscillator phase noise
* Calibration methods
* Bridge techniques
* AM noise
* Noise in systems
* Time-domain methods

home page http://rubiola.org

Phase noise & friends

Clock signal affected by noise



polar coordinates $$v(t) = V_0\,[1+\alpha(t)]\cos\left[\omega_0 t + \varphi(t)\right]$$

Cartesian coordinates $$v(t) = V_0 \cos\omega_0 t + n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t$$

under low noise approximation

$$|n_c(t)| \ll V_0 \quad \text{and} \quad |n_s(t)| \ll V_0$$

It holds that

$$\alpha(t) = \frac{n_c(t)}{V_0} \quad \text{and} \quad \varphi(t) = \frac{n_s(t)}{V_0}$$

Phase noise & friends

random phase fluctuation

$$S_\varphi(f) = \text{PSD of } \varphi(t)$$

power spectral density

it is measured as

$$S_\varphi(f) = \mathbb{E}\left\{\Phi(f)\Phi^*(f)\right\} \quad \text{(expectation)}$$

$$S_\varphi(f) \approx \left\langle \Phi(f)\Phi^*(f)\right\rangle_m \quad \text{(average)}$$

$$\mathcal{L}(f) = \frac{1}{2} S_\varphi(f) \;\; \text{dBc}$$

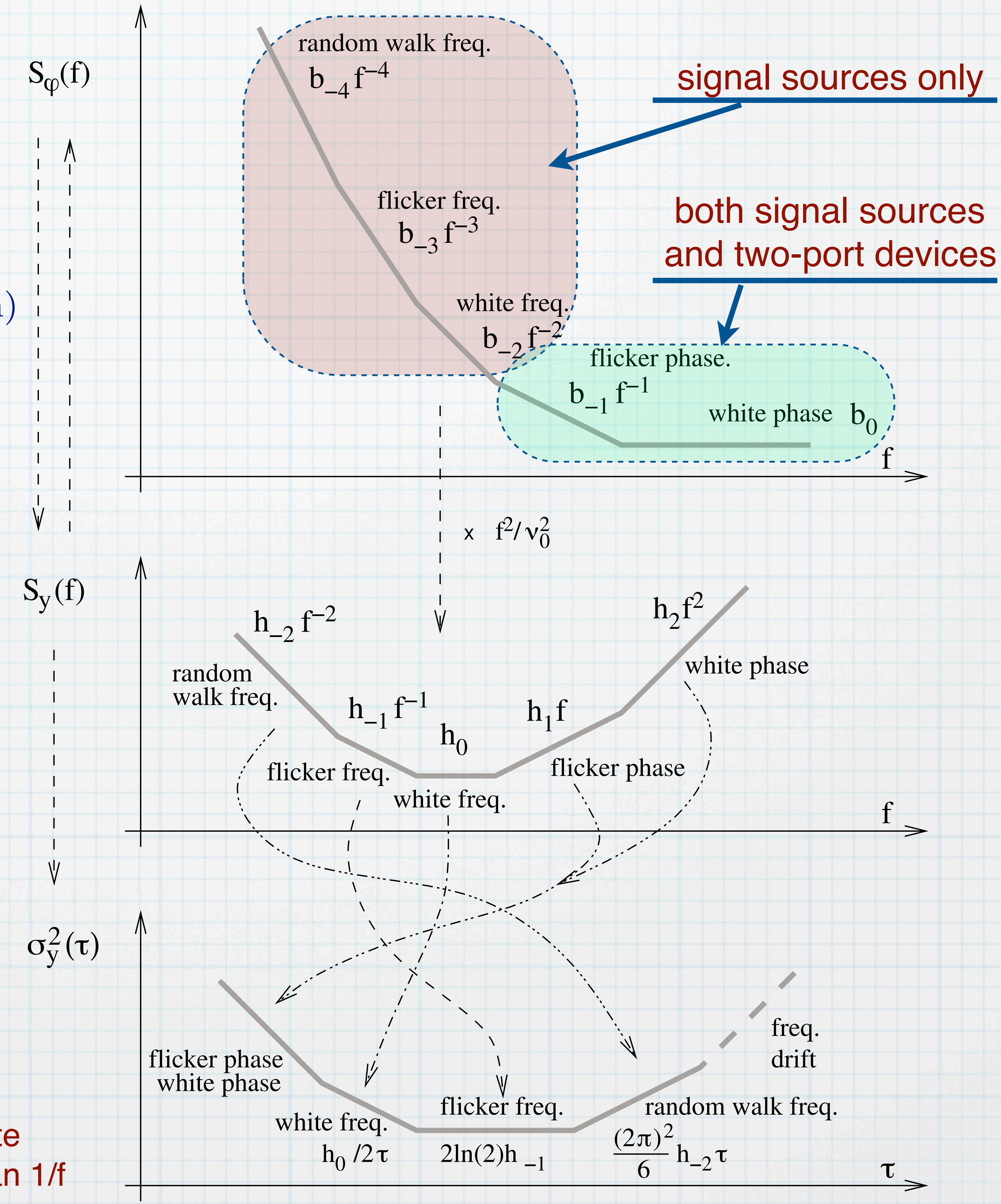
random fractional-frequency fluctuation

$$y(t) = \frac{\dot\varphi(t)}{2\pi\nu_0} \quad \Rightarrow \quad S_y = \frac{f^2}{\nu_0^2} S_\varphi(f)$$

Allan variance
(two-sample wavelet-like variance)

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{y}_{k+1} - \overline{y}_k\right]^2\right\} .$$

approaches a half-octave bandpass filter (for white noise), hence it converges for processes steeper than 1/f



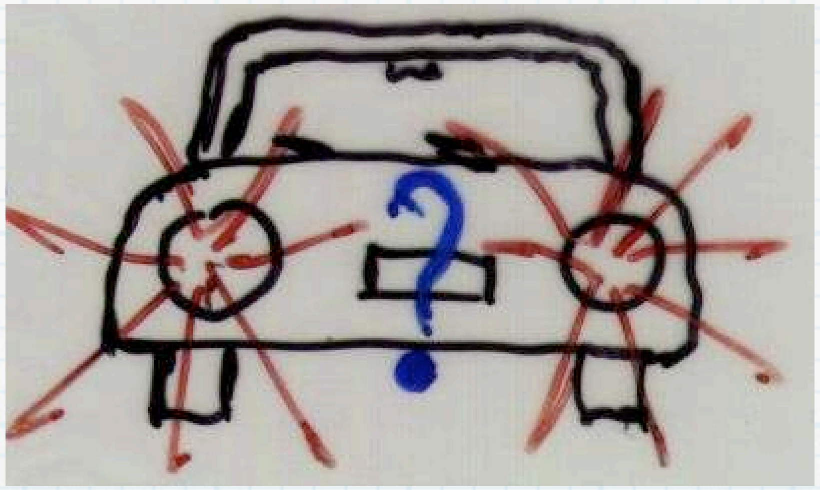
E. Rubiola, *The Leeson Effect* Chap.1, arXiv:physics/0502143

Relationships between spectra and variances

noise type	$S_\varphi(f)$	$S_y(f)$	$S_\varphi \leftrightarrow S_y$	$\sigma_y^2(\tau)$	mod $\sigma_y^2(\tau)$
white PM	b_0	$h_2 f^2$	$h_2 = \frac{b_0}{\nu_0^2}$	$\frac{3 f_H h_2}{(2\pi)^2}\tau^{-2}$ $2\pi\tau f_H \gg 1$	$\frac{3 f_H \tau_0 h_2}{(2\pi)^2}\,\tau^{-3}$
flicker PM	$b_{-1} f^{-1}$	$h_1 f$	$h_1 = \frac{b_{-1}}{\nu_0^2}$	$[1.038 + 3\ln(2\pi f_H \tau)]\,\frac{h_1}{(2\pi)^2}\,\tau^{-2}$	$0.084\, h_1 \tau^{-2}$ $n \gg 1$
white FM	$b_{-2} f^{-2}$	h_0	$h_0 = \frac{b_{-2}}{\nu_0^2}$	$\frac{1}{2} h_0\, \tau^{-1}$	$\frac{1}{4} h_0\, \tau^{-1}$
flicker FM	$b_{-3} f^{-3}$	$h_{-1} f^{-1}$	$h_{-1} = \frac{b_{-3}}{\nu_0^2}$	$2\ln(2)\; h_{-1}$	$\frac{27}{20}\ln(2)\; h_{-1}$
random walk FM	$b_{-4} f^{-4}$	$h_{-2} f^{-2}$	$h_{-2} = \frac{b_{-4}}{\nu_0^2}$	$\frac{(2\pi)^2}{6} h_{-2}\tau$	$0.824\,\frac{(2\pi)^2}{6} h_{-2}\,\tau$
linear frequency drift $\dot{y}$				$\frac{1}{2}\,(\dot{y})^2\,\tau^2$	$\frac{1}{2}\,(\dot{y})^2\,\tau^2$

f_H is the high cutoff frequency, needed for the noise power to be finite.

Basic problem: how can we measure a low random signal (noise sidebands) close to a strong dazzling carrier?



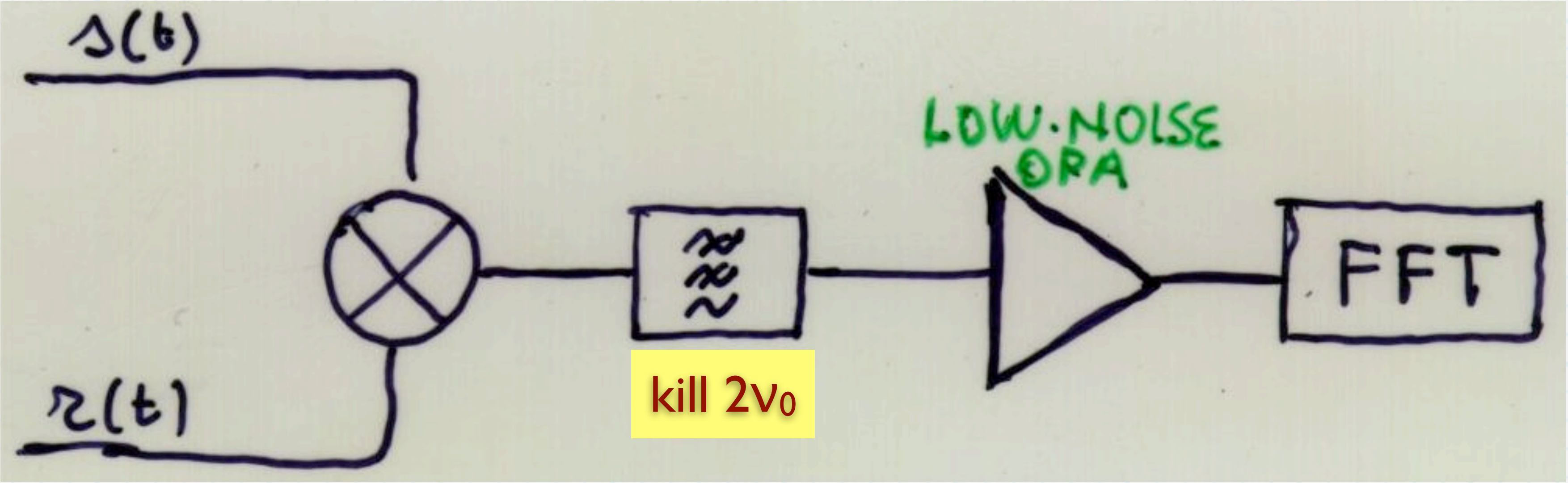
solution(s): suppress the carrier and measure the noise

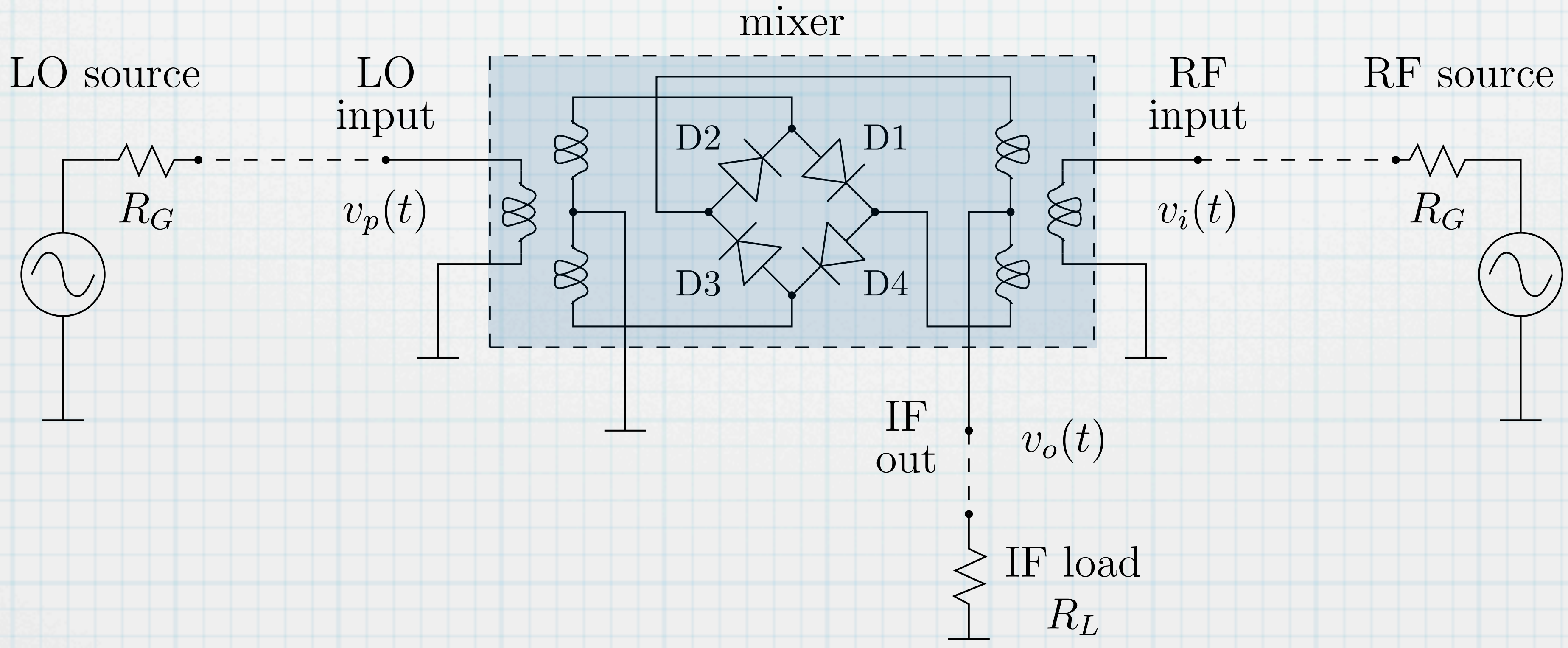
convolution (low-pass)	$s(t) * h_{lp}(t)$	distorsiometer, audio-frequency instruments
time-domain product	$s(t) \times r(t - T/4)$	traditional instruments for phase-noise measurement (saturated mixer)
vector difference	$s(t) - r(t)$	bridge (interferometric) instruments

Saturated mixer

Double-balanced mixer

saturated multiplier => phase-to-voltage detector $v_o(t) = k_\phi\, \phi(t)$

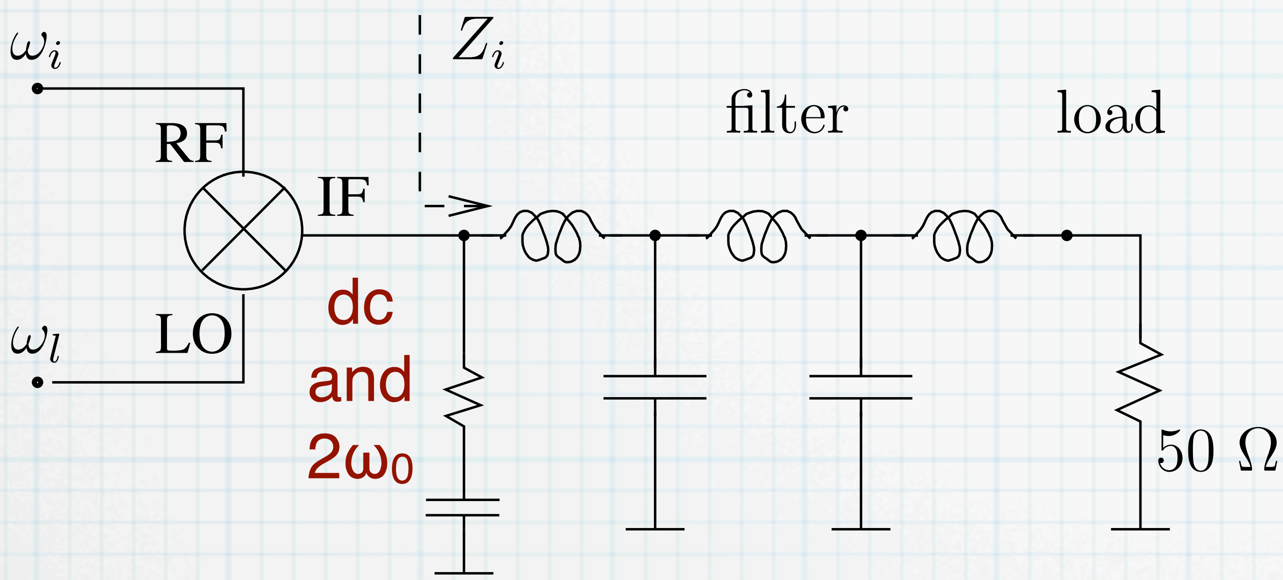


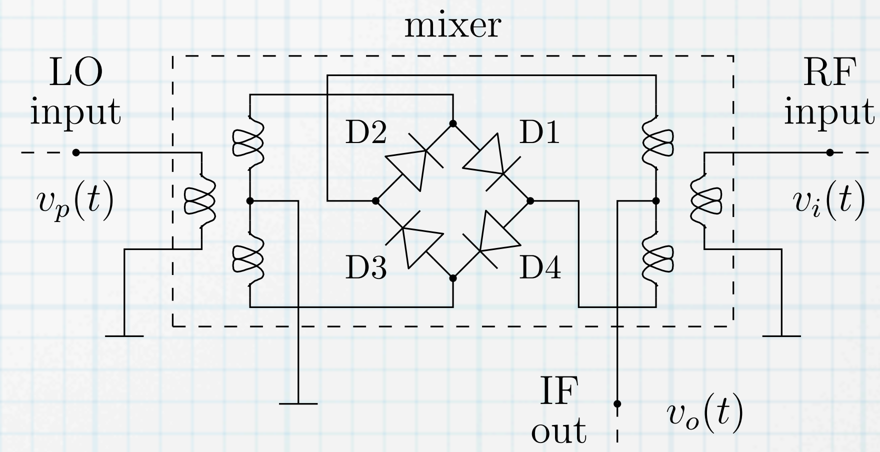


E. Rubiola, *Tutorial on the double-balanced mixer*, arXiv/physics/0608211,

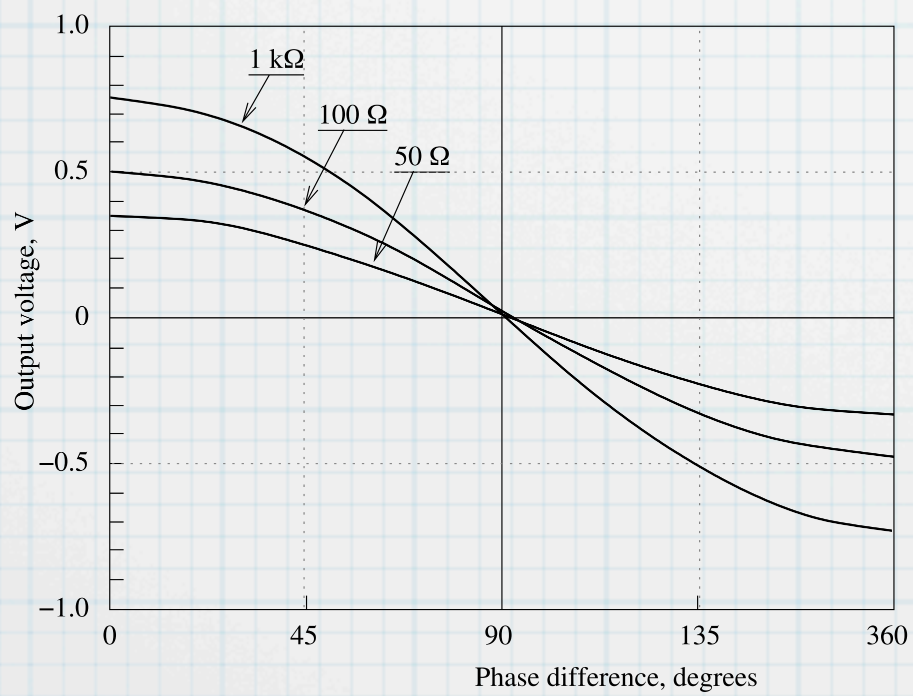
Practical issues

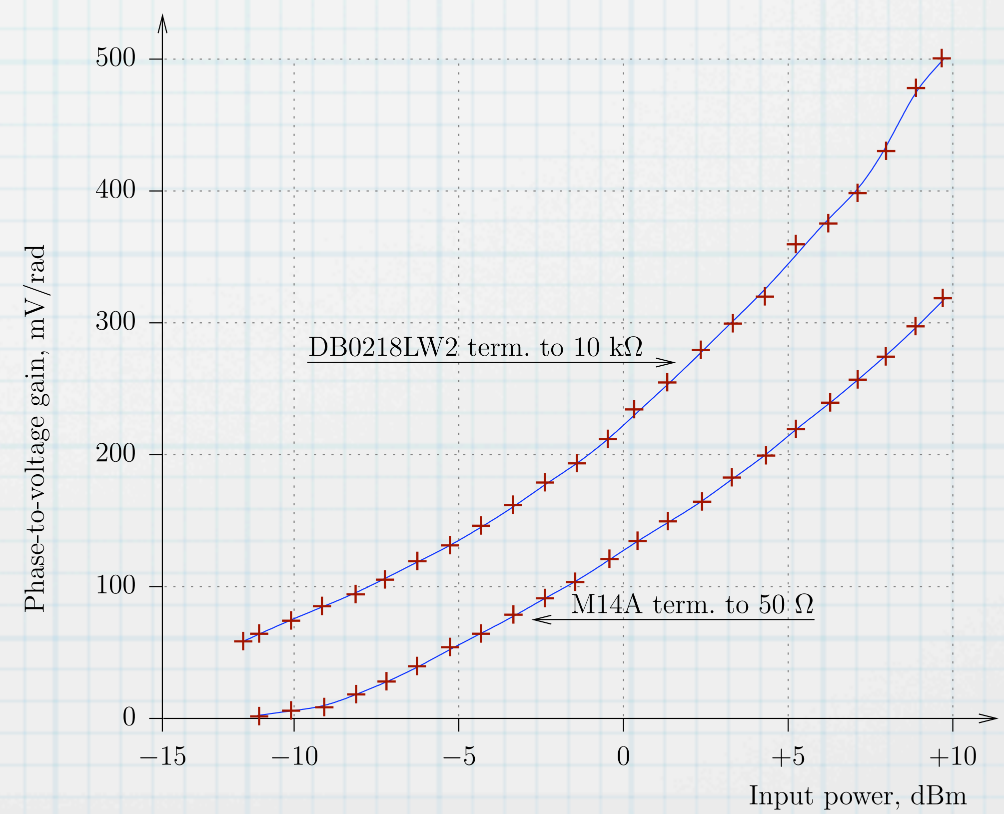
needs a capacitive-input filter to recirculate the $2\omega_0$ output signal





actual phase-to-voltage conversion

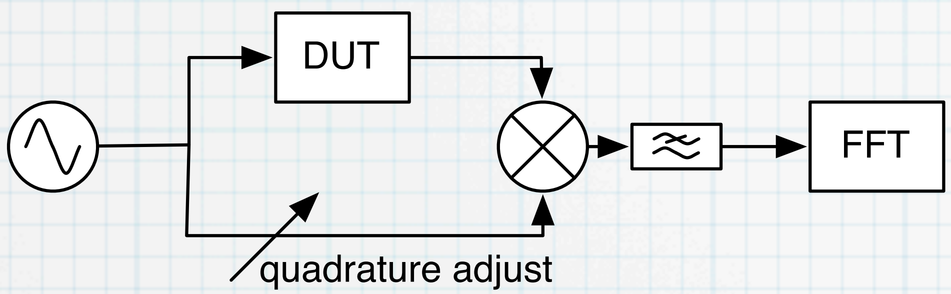




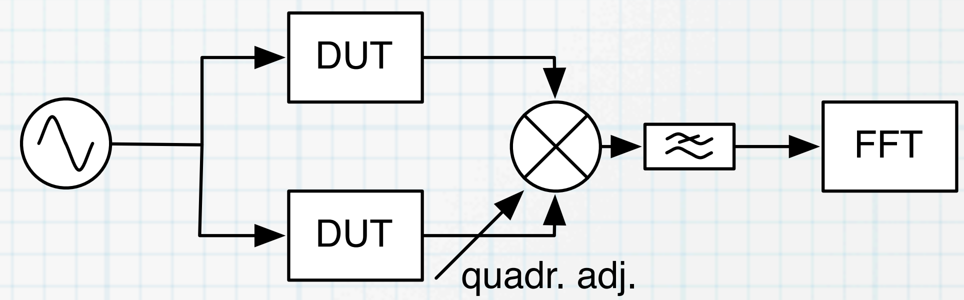
E. Rubiola, *Tutorial on the double-balanced mixer*, arXiv/physics/0608211,

Useful schemes

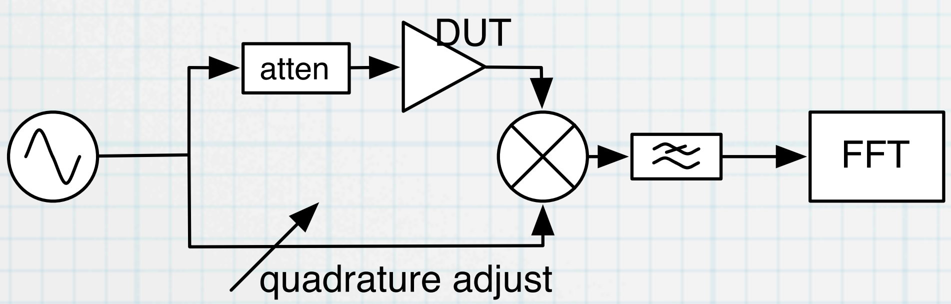
two-port device under test



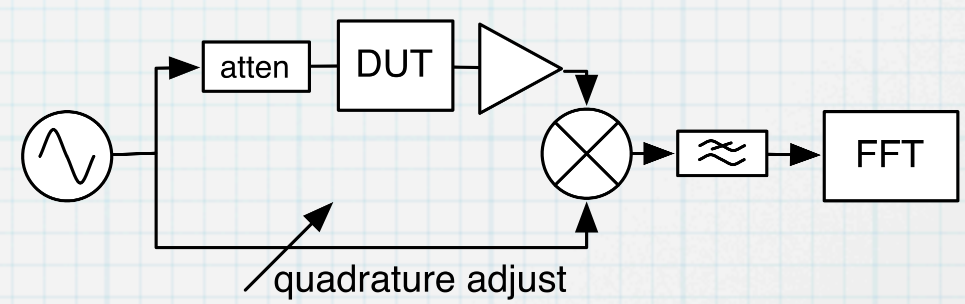
a pair of two-port devices
3 dB improved sensitivity



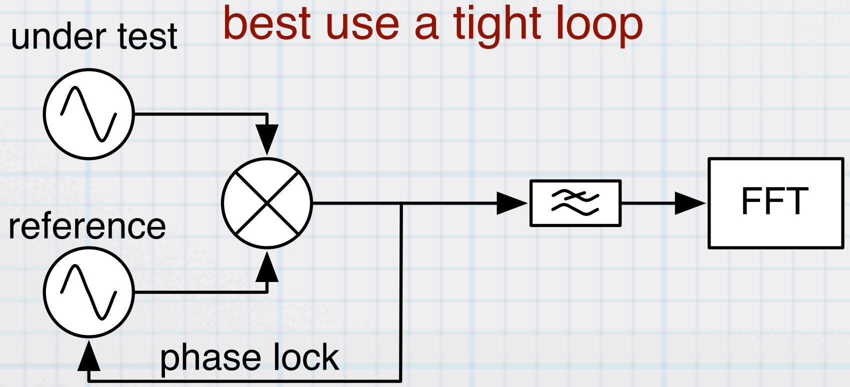
the measurement of an amplifier
needs an attenuator



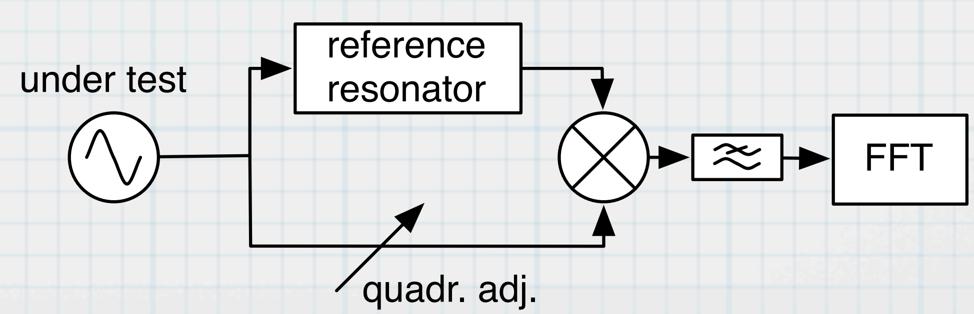
the measurement of a low-power DUT
needs an amplifier, which flickers



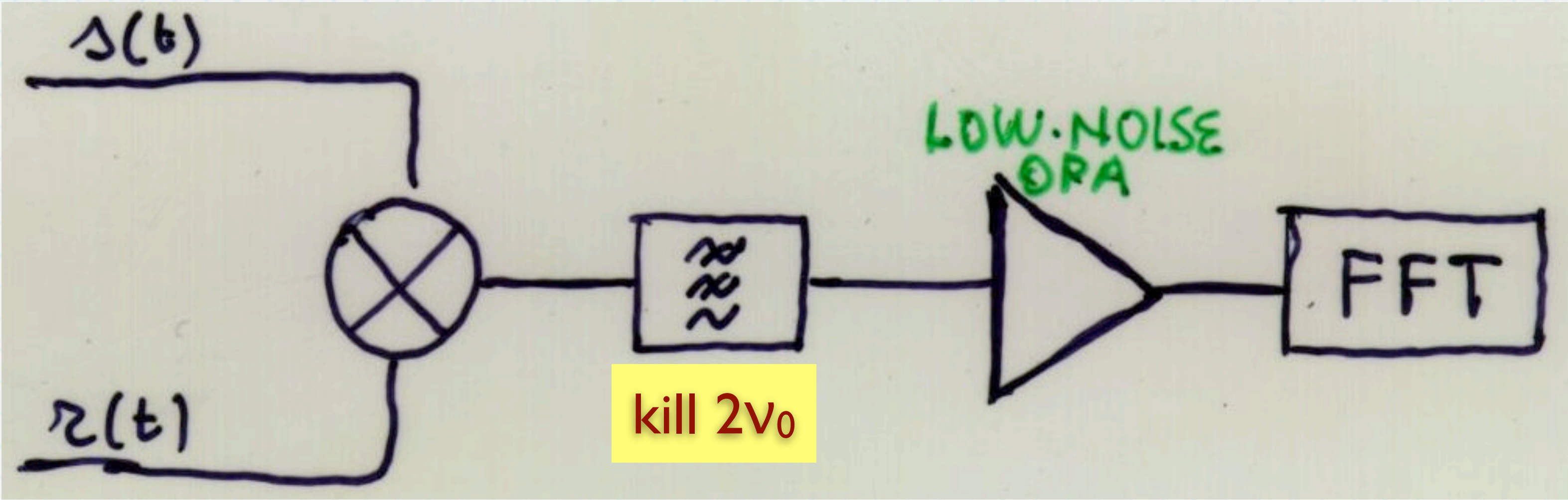
measure two oscillators
best use a tight loop



measure an oscillator vs. a resonator



Mixer limitations



1 – Power

narrow power range:
±5 dB around P_{nom} = 7–13 dBm
r(t) and s(t) should have ~ same P

2 – Flicker noise

due to the mixer internal diodes
typical S_φ = –140 dBrad²/Hz at 1 Hz
in average-good conditions

3 – Low gain

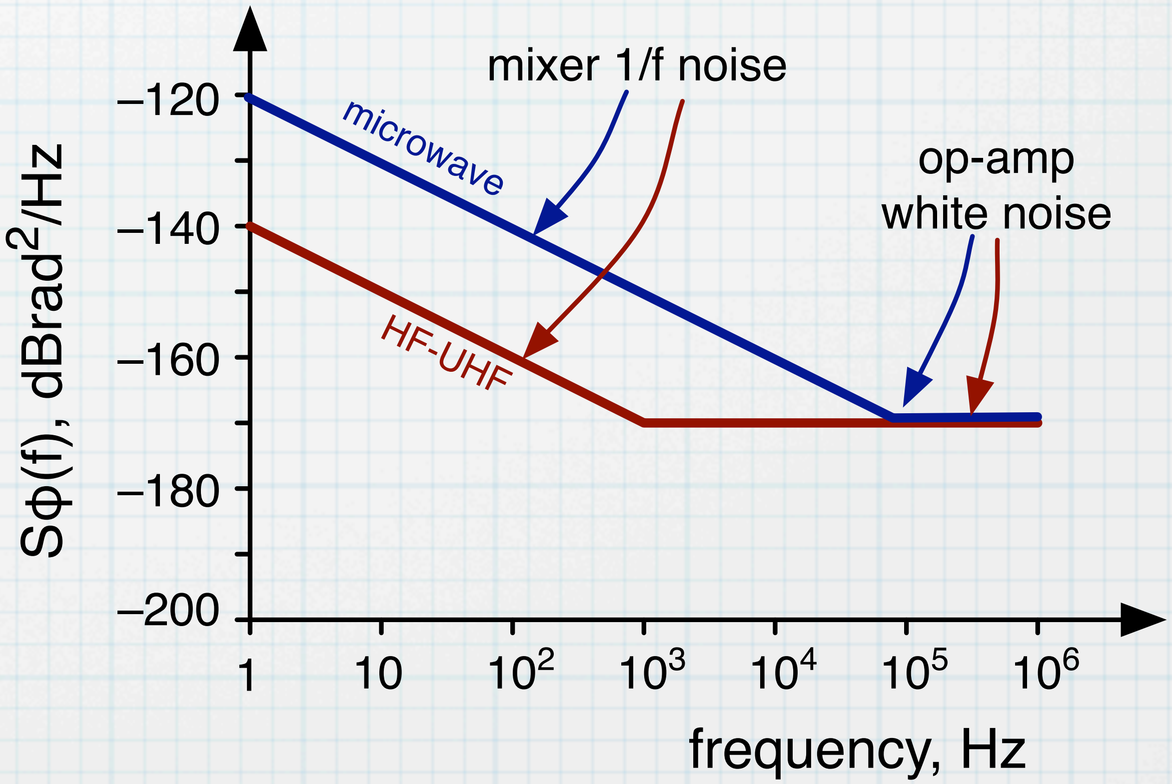
k_φ ~ 0.2–0.3 V/rad typ.
–10 to –14 dBV/rad

4 – White noise

due to the operational amplifier

5 – Takes in AM noise

due to the residual power-to-offset conversion



E. Rubiola, *Tutorial on the double-balanced mixer*, arXiv/physics/0608211,

The operational amplifier is often misused

$R_b = \sqrt{\frac{S_v}{S_i}}$ R_b = minimum noise input resistance.

OP-27 $\sqrt{S_v} = 3\,nV/\sqrt{Hz}$
$\sqrt{S_i} = 0.4\,pA/\sqrt{Hz}$ $\Big\}$ $R_b = 7.5\,k\Omega$

LT 1028 $\sqrt{S_v} = 1.2\,nV/\sqrt{Hz}$
$\sqrt{S_i} = 2\,pA/\sqrt{Hz}$ $\Big\}$ $R_b \simeq 600\,\Omega$

900Ω

MIXER $R_0 = 50\,\Omega$

OP27: [3.2 nV/Hz$^{1/2}$] / [0.2 V/rad] = 16 nrad/Hz$^{1/2}$ (–156 dBrad^2/Hz)

LT1028: [1.2 nV/Hz$^{1/2}$] / [0.2 V/rad] = 2.4 nrad/Hz$^{1/2}$ (–164 dBrad^2/Hz)

Warning: if only one arm of the power supply is disconnected, the LT1028 may delivers a current from the input (I killed a \$2k mixer in this way!)

You may duplicate the low-noise amplifier designed at the FEMTO-ST
Rubiola, Lardet-Vieudrin, Rev. Scientific Instruments 75(5) pp. 1323-1326, May 2004

Mechanical stability

any flicker spectrum $S(f) = h_{-1}/f$ can be transformed

into the Allan variance $\sigma^2 = 2\ln(2)\,h_{-1}$

(roughly speaking, the integral over one octave)

a phase fluctuation is equivalent to a length fluctuation

$$L \;=\; \frac{\varphi}{2\pi}\lambda \;=\; \frac{\varphi}{2\pi}\,\frac{c}{\nu_0} \qquad S_L(f) = \frac{1}{4\pi^2}\,\frac{c^2}{\nu_0^2}\,S_\varphi(f)$$

-180 dBrad^2/Hz at $f = 1$ Hz and $\nu_0 = 9.2$ GHz $(c = 0.8\,c_0)$ is equivalent to

$S_L = 1.73{\times}10^{-23}$ m^2/Hz $\qquad$ $(\sqrt{S_L} = 4.16{\times}10^{-12}$ m/$\sqrt{\mathrm{H}z})$

a residual flicker of -180 dBrad^2/Hz at $f = 1$ Hz off the $\nu_0 = 9.2$ GHz carrier

$(h_{-1} = 1.73{\times}10^{-23})$ is equivalent to a mechanical stability

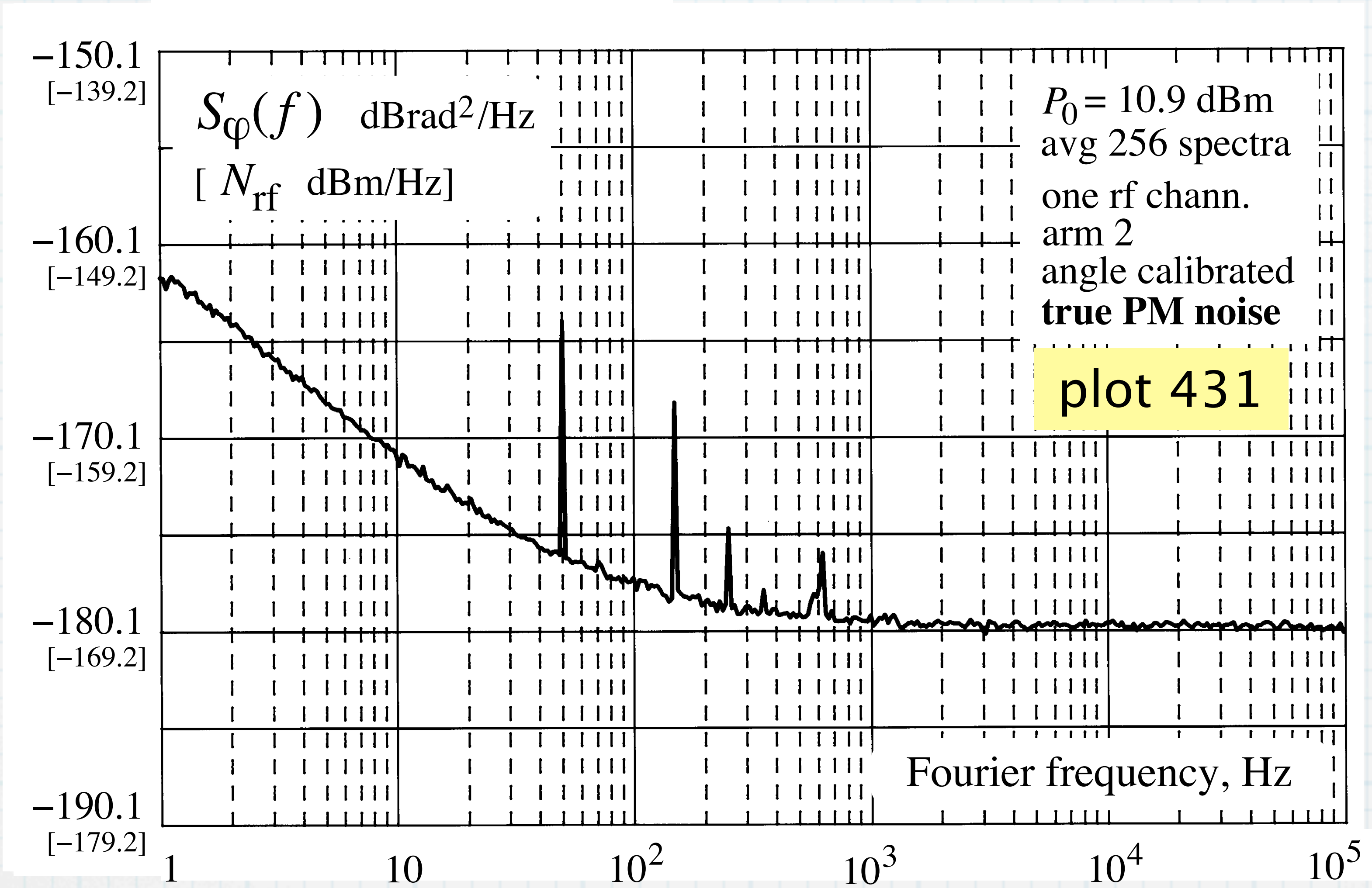
$$\sigma_L \;=\; \sqrt{1.38 \times 1.73{\times}10^{-23}} \;=\; 4.9{\times}10^{-12} \quad \mathrm{m}$$

 # don’t think “that’s only engineering” !!!

I learned a lot from non-optical microscopy

bulk solid matter is that stable

Averaged spectra must be smooth



Rice representation
$$v(t) = \sum_{n=0}^{\infty} a_n(t)\cos(n\omega_0 t) - b_n(t)\sin(n\omega_0 t)$$
$$S_v(n\omega_0) = \left[a_n^2 + b_n^2\right]/\omega_0$$

$a_n(t)$ and $b_n(t)$ contain the noise in the $\omega_0/2$ band centered at ω_0

stationary & ergodic process (means repeatable and reproducible): the statistics of all $a_n(t)$ and $b_n(t)$ is the same

average on m spectra: confidence of a point improves by $1/m^{1/2}$

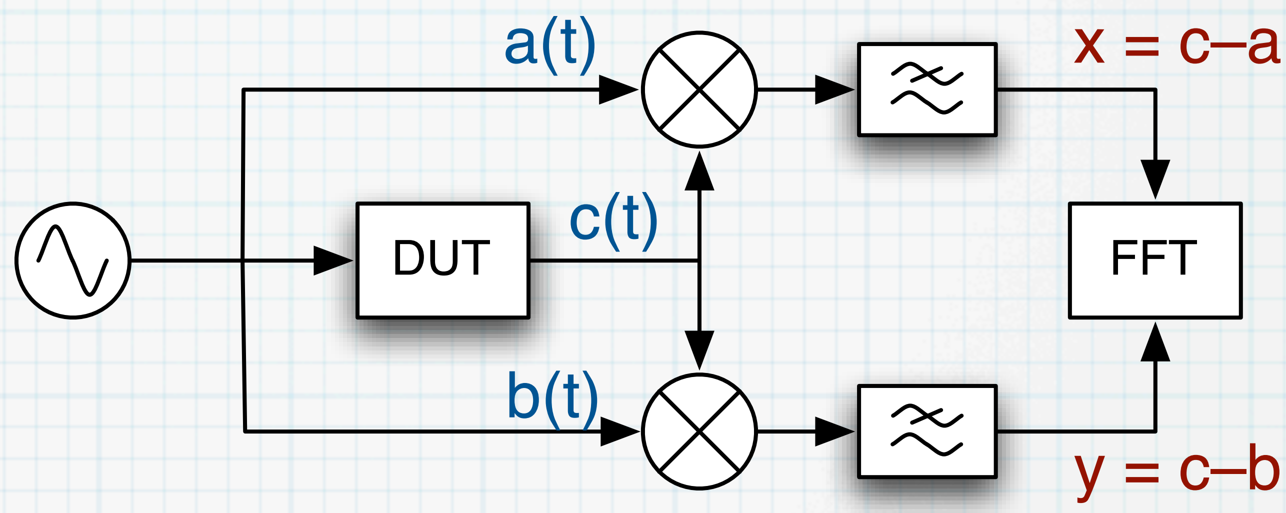
interchange ensemble with frequency: smoothness $1/m^{1/2}$

Correlation (dual-channel) measurements

Correlation measurements

Two separate mixers measure the same DUT. Only the DUT noise is common

a(t), b(t) –> mixer noise
c(t) –> DUT noise



basics of correlation

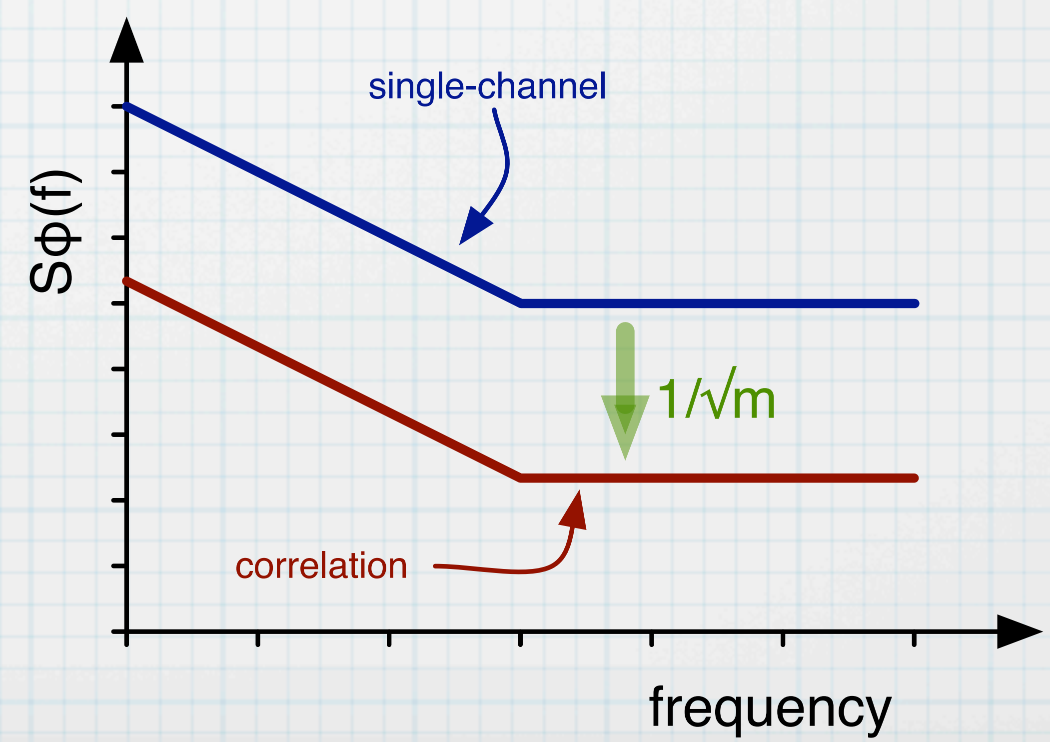
$$
\begin{aligned}
S_{yx}(f) &= \mathbb{E}\left\{Y(f)X^*(f)\right\} \\
&= \mathbb{E}\left\{(C-A)(C-B)^*\right\} \\
&= \mathbb{E}\left\{CC^* - AC^* - CB^* + AB^*\right\} \\
&= \mathbb{E}\left\{CC^*\right\} \qquad 0 \qquad 0 \qquad 0
\end{aligned}
$$

$$S_{yx}(f) = S_{cc}(f)$$

in practice, average on m realizations

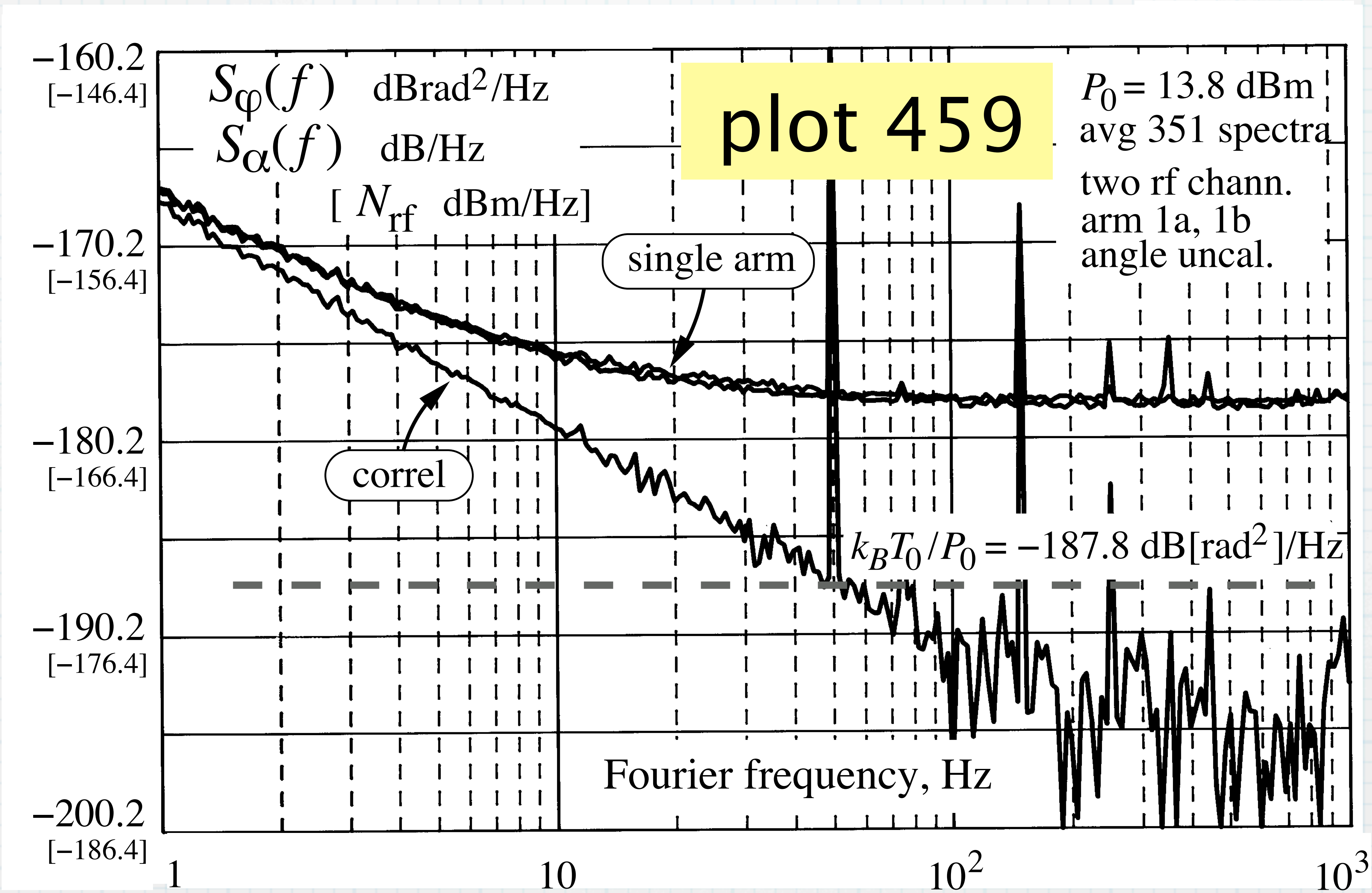
$$
\begin{aligned}
S_{yx}(f) &= \left\langle Y(f)X^*(f)\right\rangle_m \\
&= \left\langle CC^* - AC^* - CB^* + AB^*\right\rangle_m \\
&= \left\langle CC^*\right\rangle_m + O(1/m)
\end{aligned}
$$

0 as 1/√m



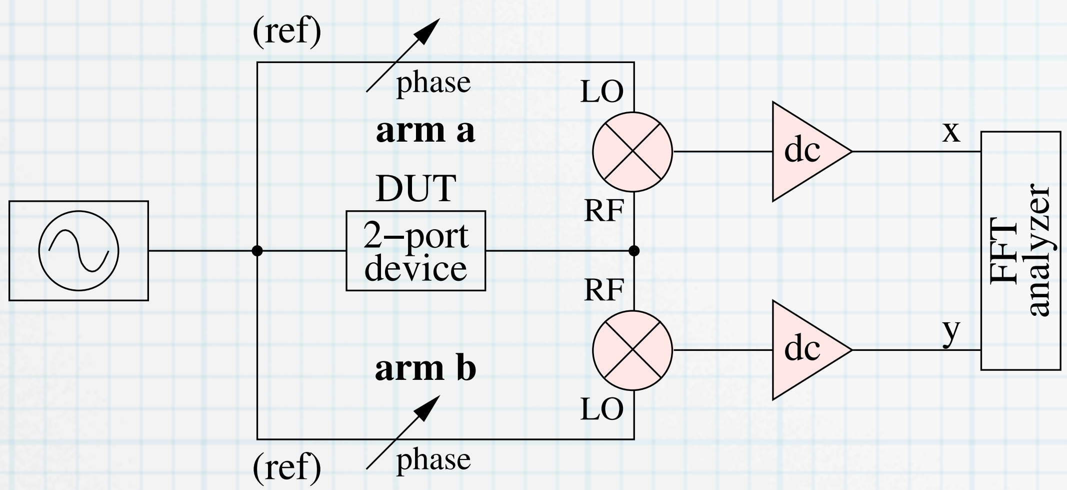
Example of correlation measurement

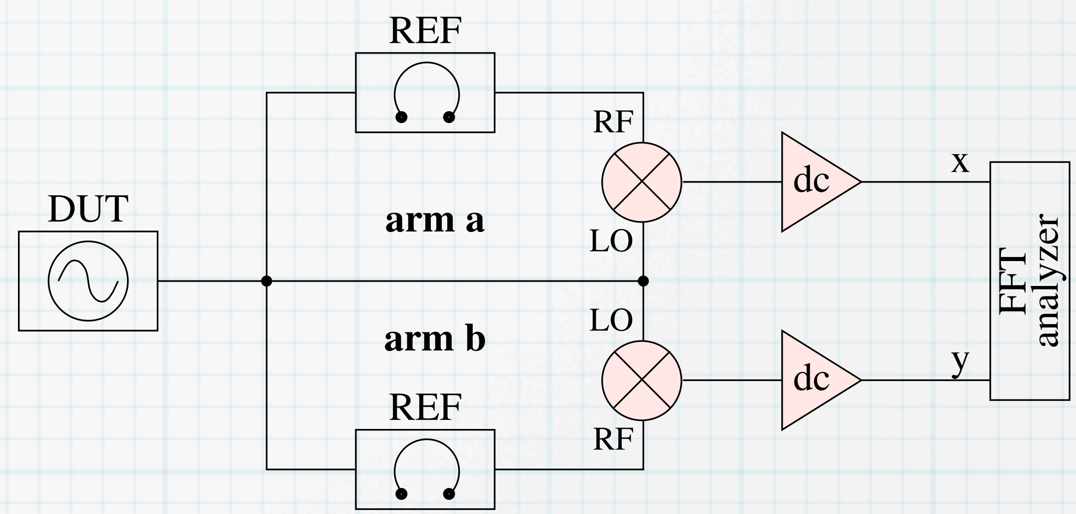
100 MHz carrier

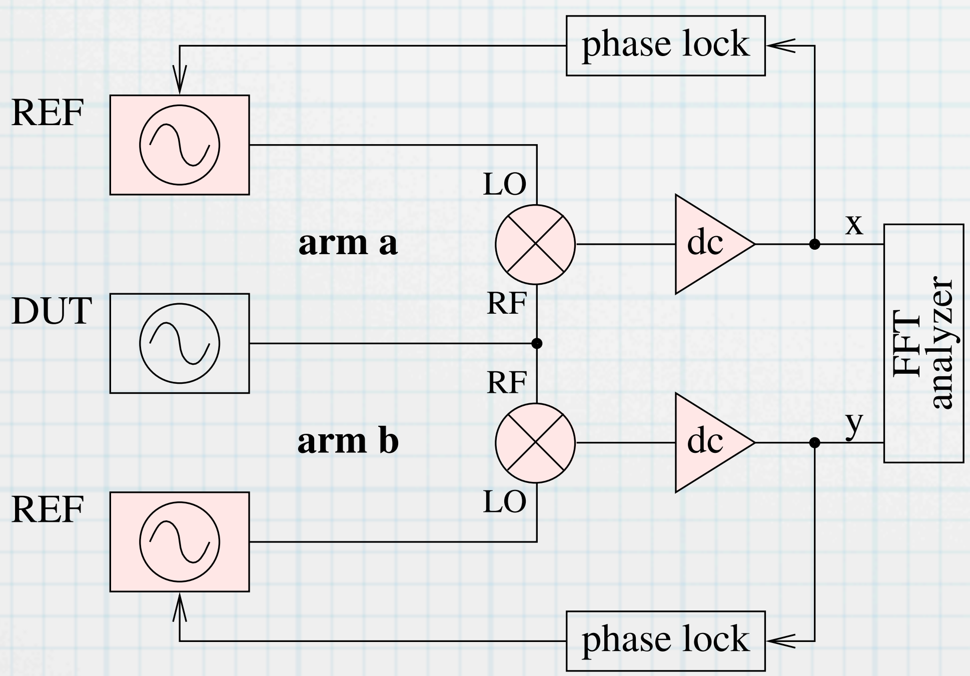


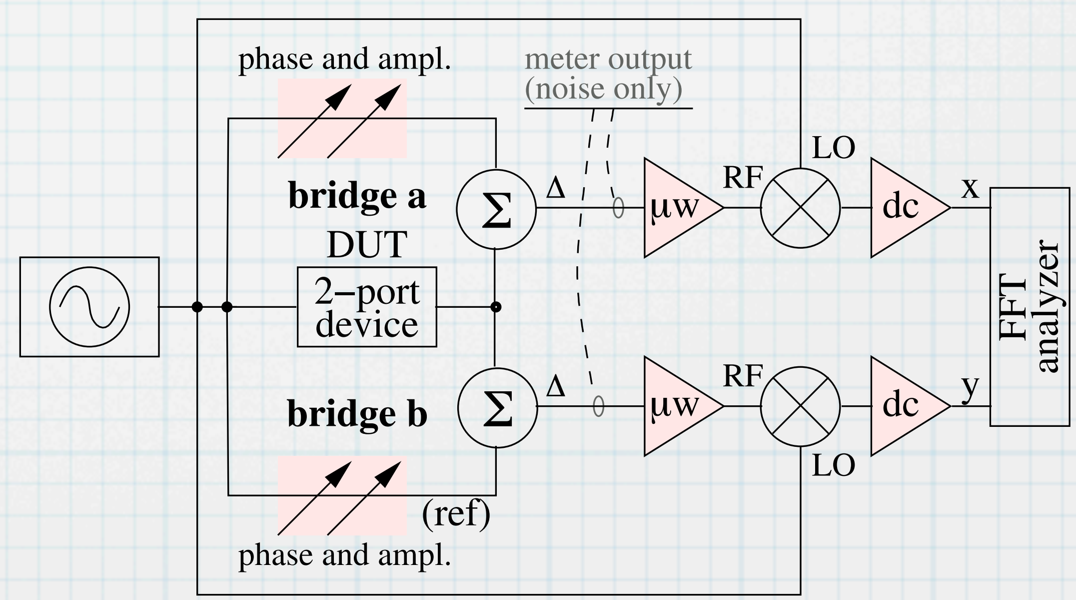
Noise of a by-step attenuator, measured at 100 MHz by correlation. The mixer is replaced with a bridge.

Useful schemes

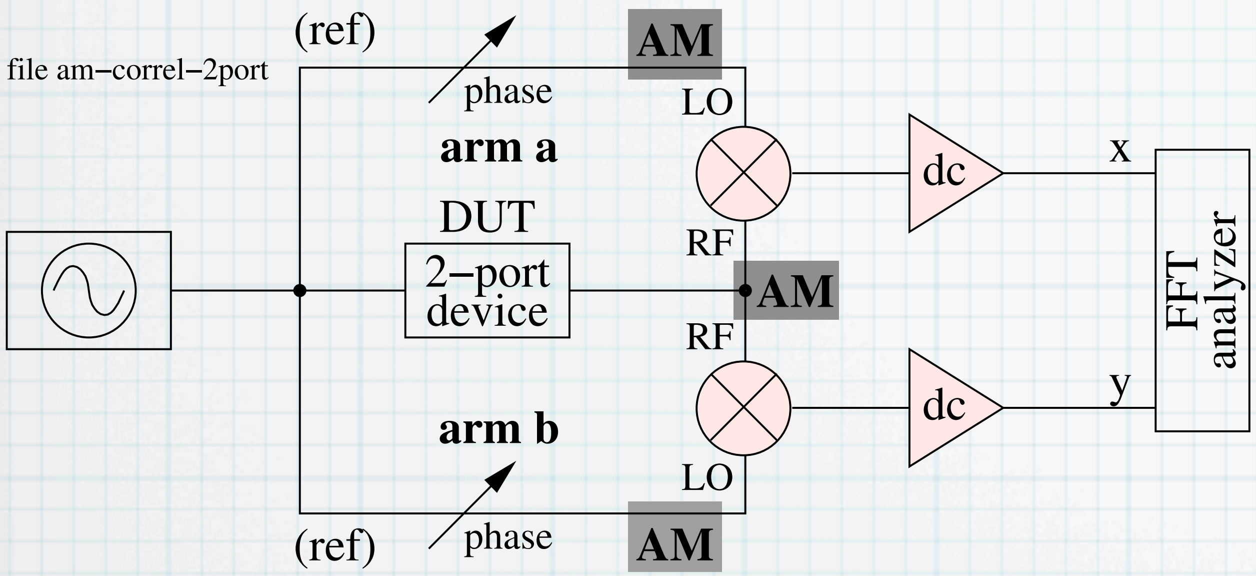








Pollution from AM noise

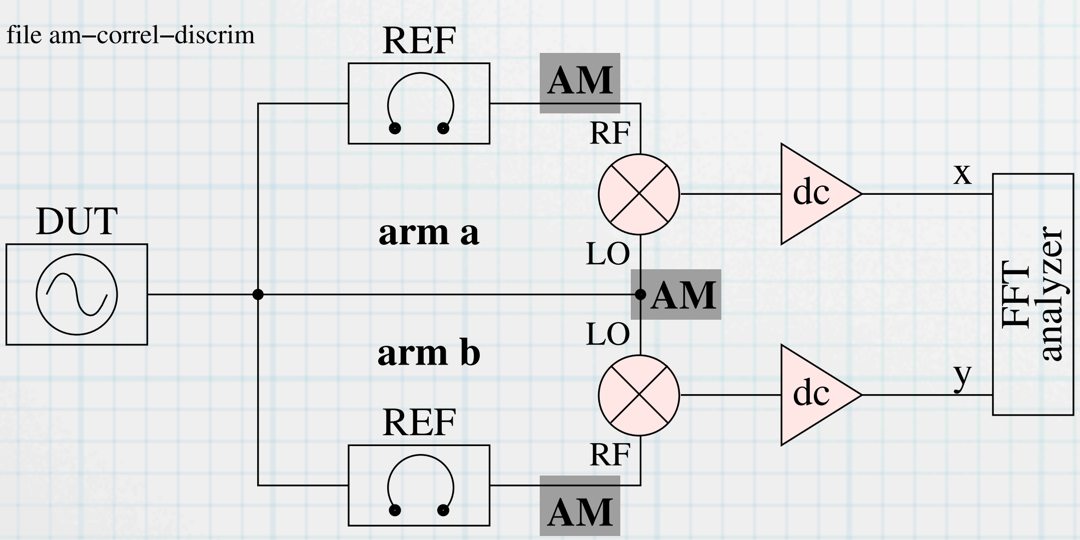


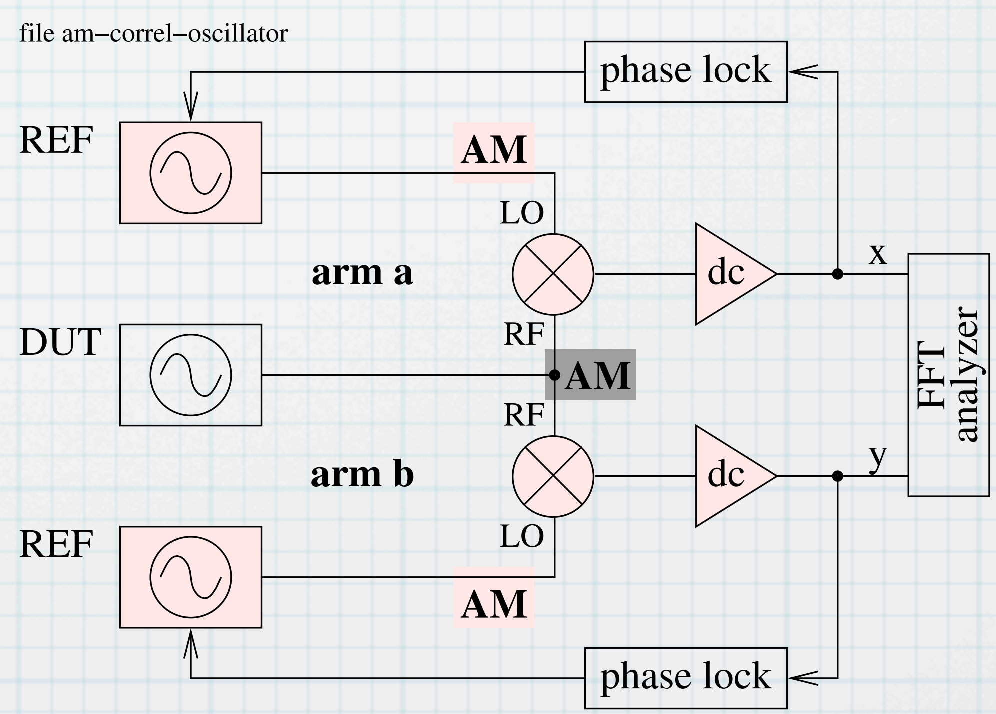
The mixer converts power into dc-offset,
thus AM noise into dc-noise,
which is mistaken for PM noise

$$v(t) = k_\phi\, \phi(t) + k_{LO}\, \alpha_{LO} + k_{RF}\, \alpha_{RF}$$

rejected by correlation and avg

not rejected by correlation and avg

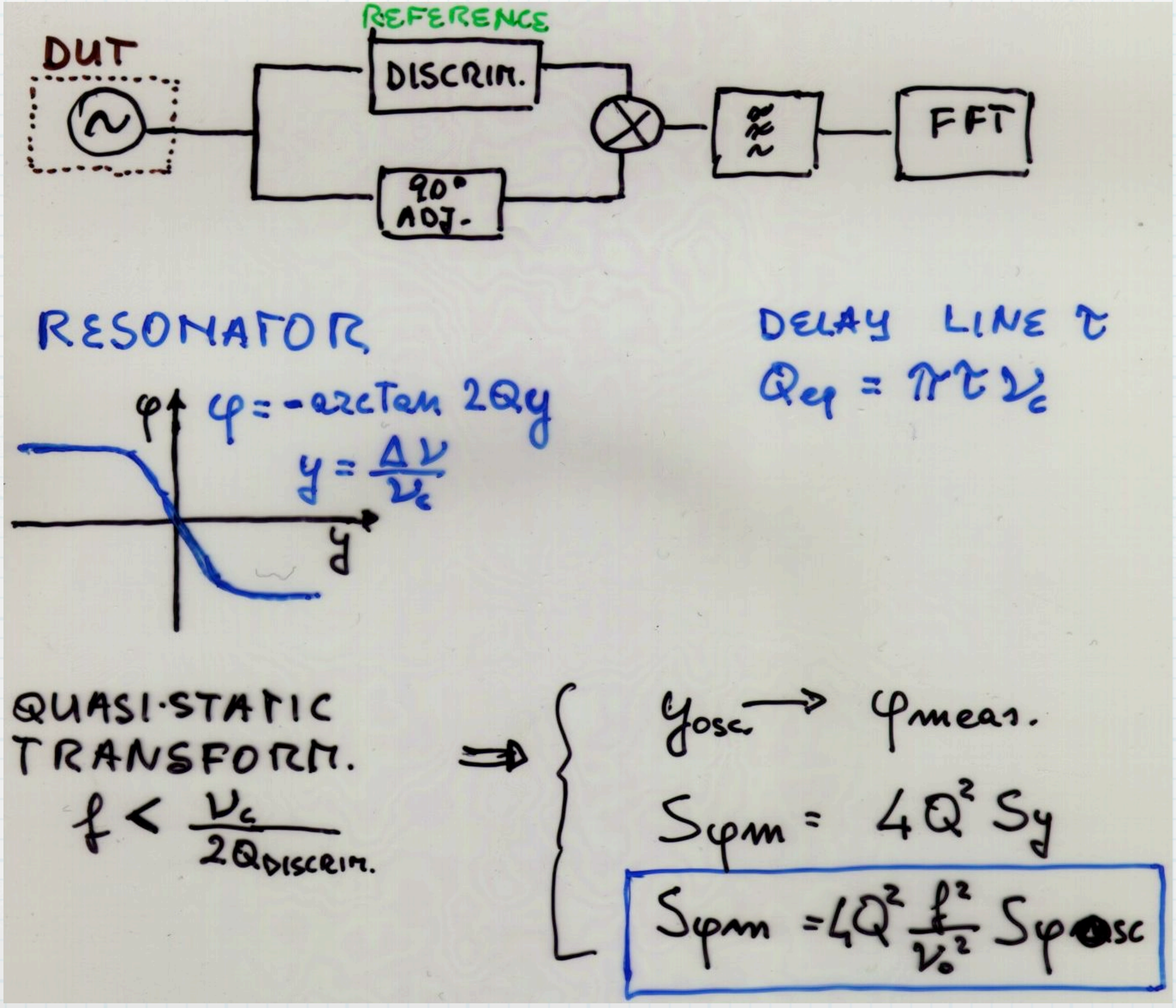




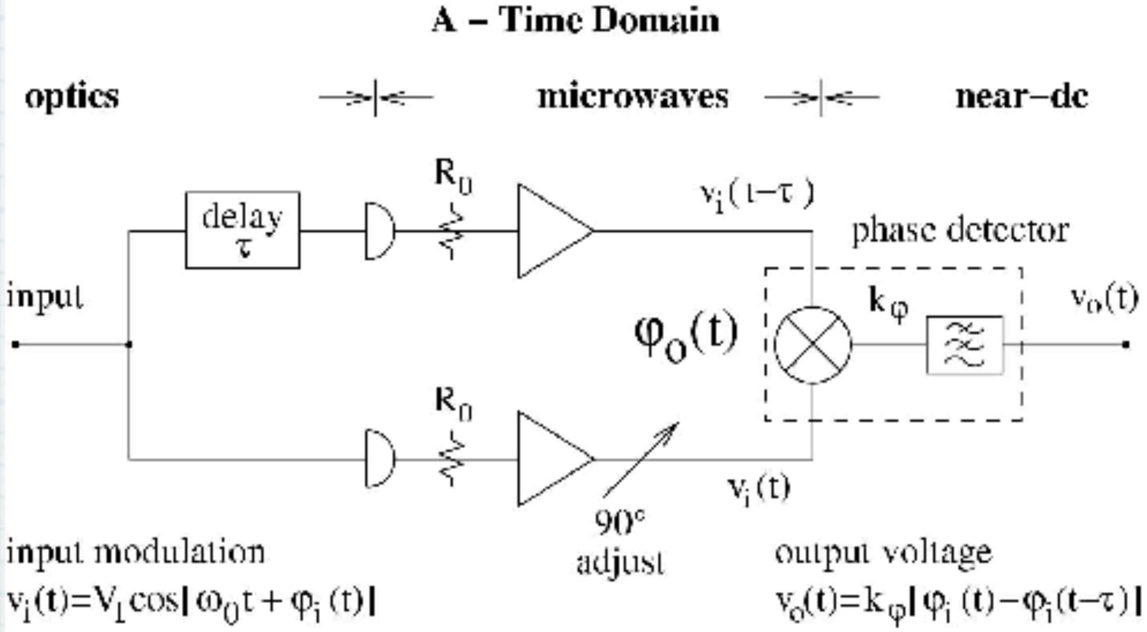
E. Rubiola, R. Boudot, *The effect of AM noise on correlation phase noise measurements*, arXiv/physics/0609147

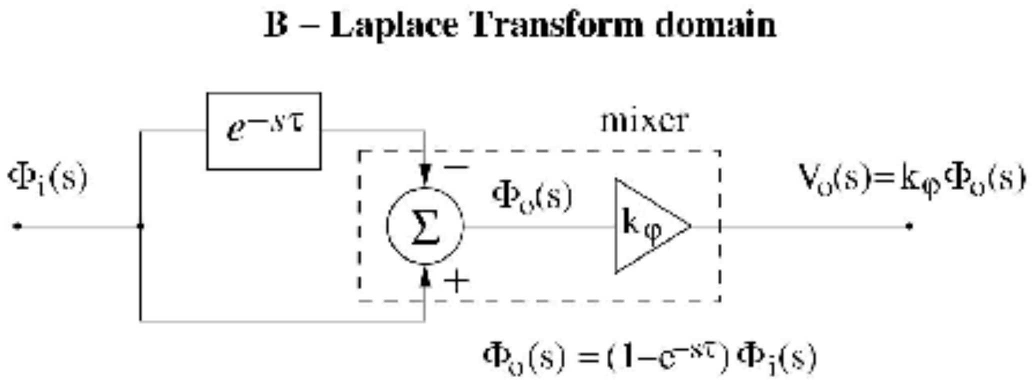
Oscillator phase noise

A frequency discriminator can be used to measure the phase noise of an oscillator



Photonic delay line method





$$\Phi_o(s) = H_\varphi(s)\Phi_i(s)$$ defines H_ϕ

$$S_{\varphi o}(f) = |H_\varphi(jf)|^2 S_{\varphi i}(f)$$ Laplace to spectra

$$S_y(f) = |H_y(jf)|^2 S_{\varphi i}(f)$$ defines H_y

$y = (\nu - \nu_0)/\nu_0\,,\qquad \nu_0$ *is the carrier frequency*

$$S_y(f) = \frac{f^2}{\nu_0^2} S_{\varphi i}(f)$$ property of the derivative

$$H_\varphi(s) = 1 - \exp(-s\tau)$$ by inspection

$$|H_\varphi(s)|^2 = 4\sin^2(\pi f \tau)$$

$$|H_y(s)|^2 = \frac{4\nu_0^2}{f^2}\sin^2(\pi f \tau)$$

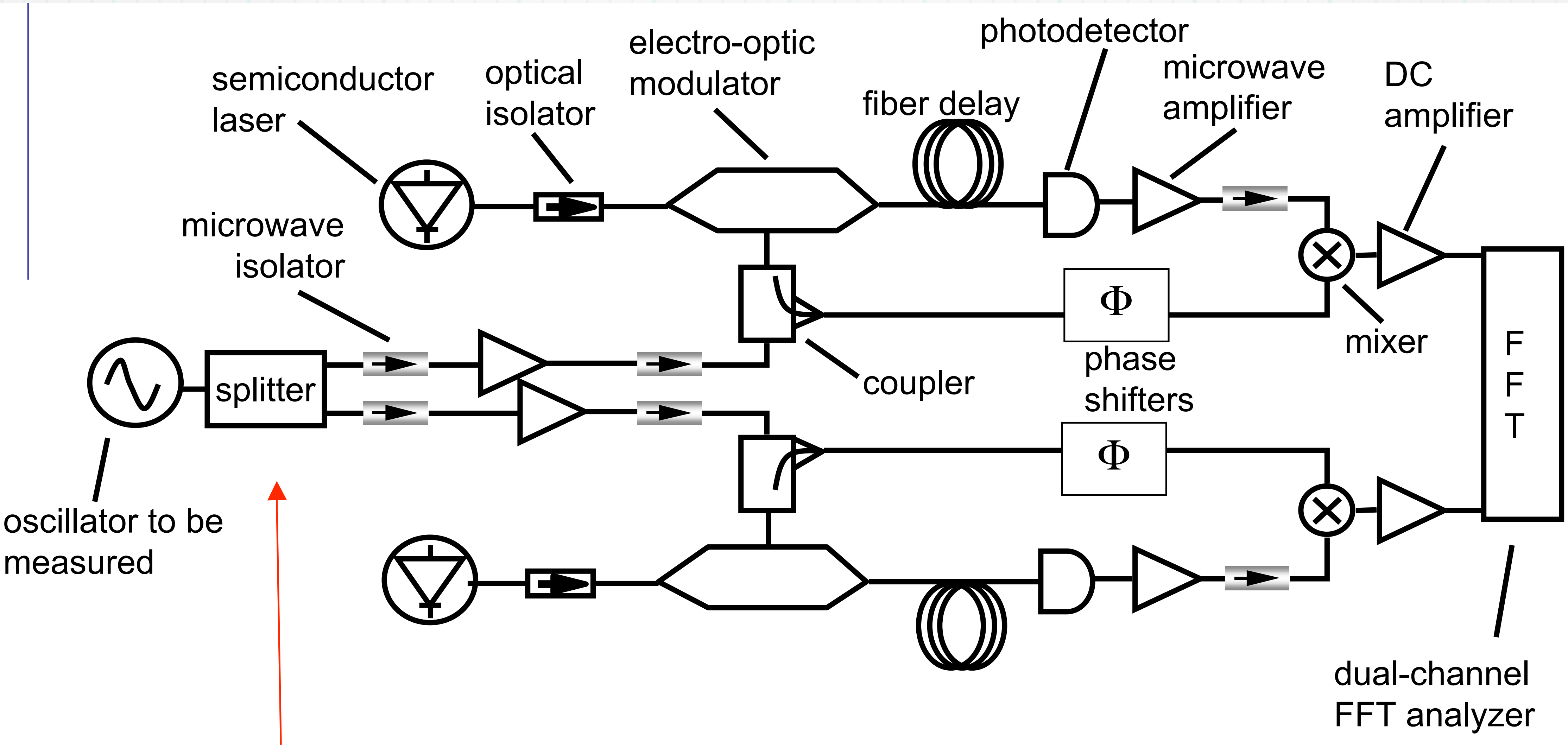
transfer functions

1 – The delay line turns the oscillator frequency noise into phase noise, which is measured by the mixer.

2 – The oscillator noise is calculated by unapplying the equation of the delay line

3 – Photonic delay: the optical fiber exhibits low loss (0.2 dB/km)

Correlation dual-delay-line method



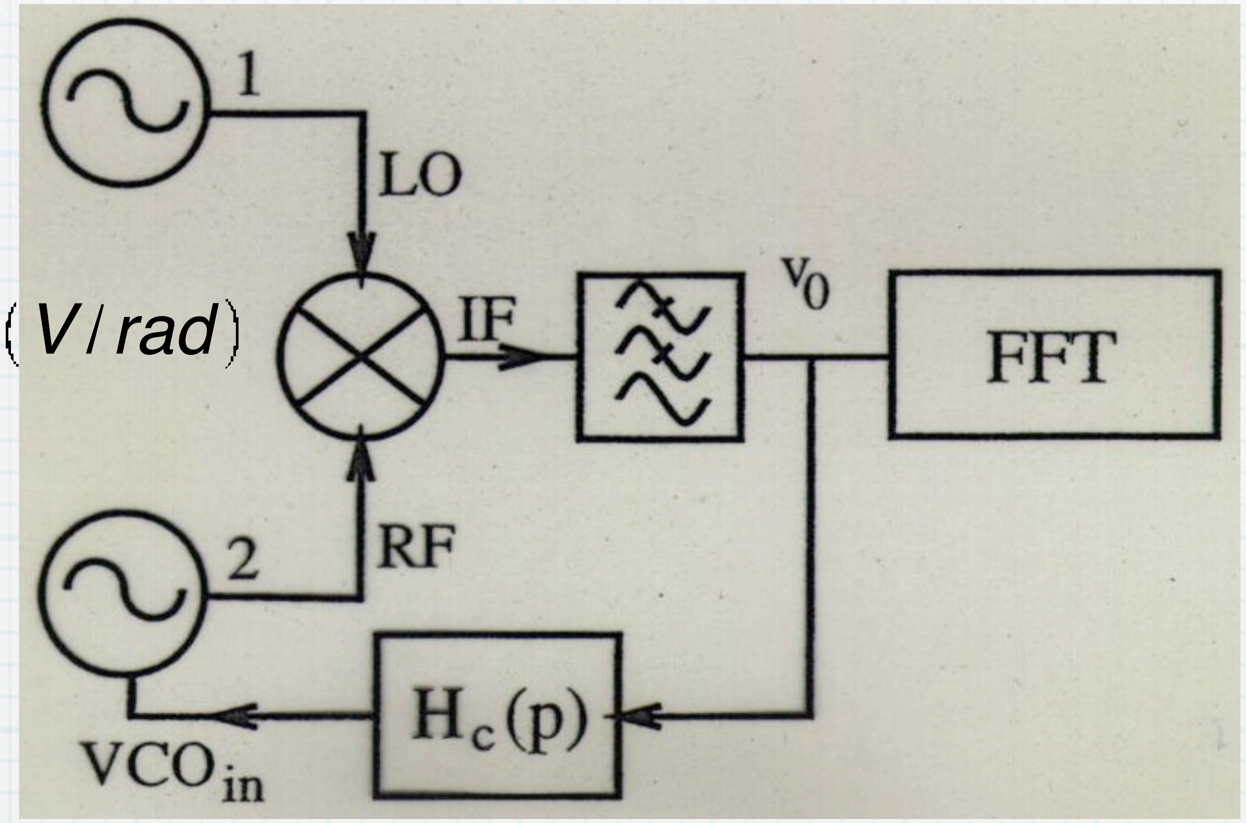
The only common part of the setup is the power splitter.

Two completely separate systems measure the same oscillator under test

Phase Locked Loop (PLL)

k_{φ} (V/rad)

k_o $(rad/s/V$



Phase: the PLL is a low-pass filter

$$\frac{S_{\varphi 2}(f)}{S_{\varphi 1}(f)} = \frac{\left|k_o k_{\varphi} H_c(f)\right|^2}{4\pi^2 f^2 + \left|k_o k_{\varphi} H_c(f)\right|^2}$$

Output voltage: the PLL is a high-pass filter

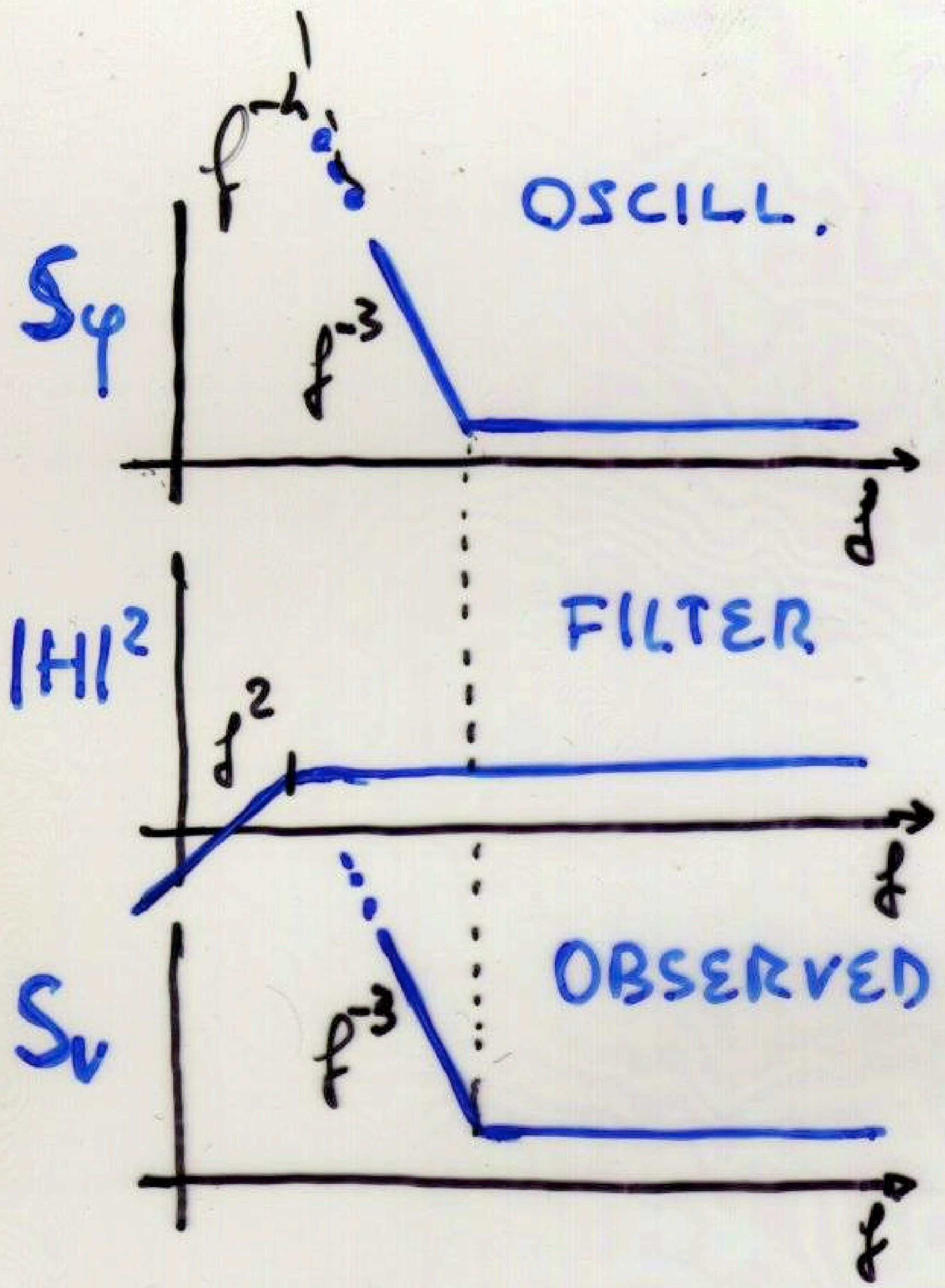
$$\frac{S_{vo}(f)}{S_{\varphi 1}(f)} = \frac{4\pi f^2 k_{\varphi}^2}{4\pi^2 f^2 + \left|k_o k_{\varphi} H_c(f)\right|^2}$$

compare an oscillator under test to a reference low-noise oscillator

– or –

compare two equal oscillators and divide the spectrum by 2 (take away 3 dB)

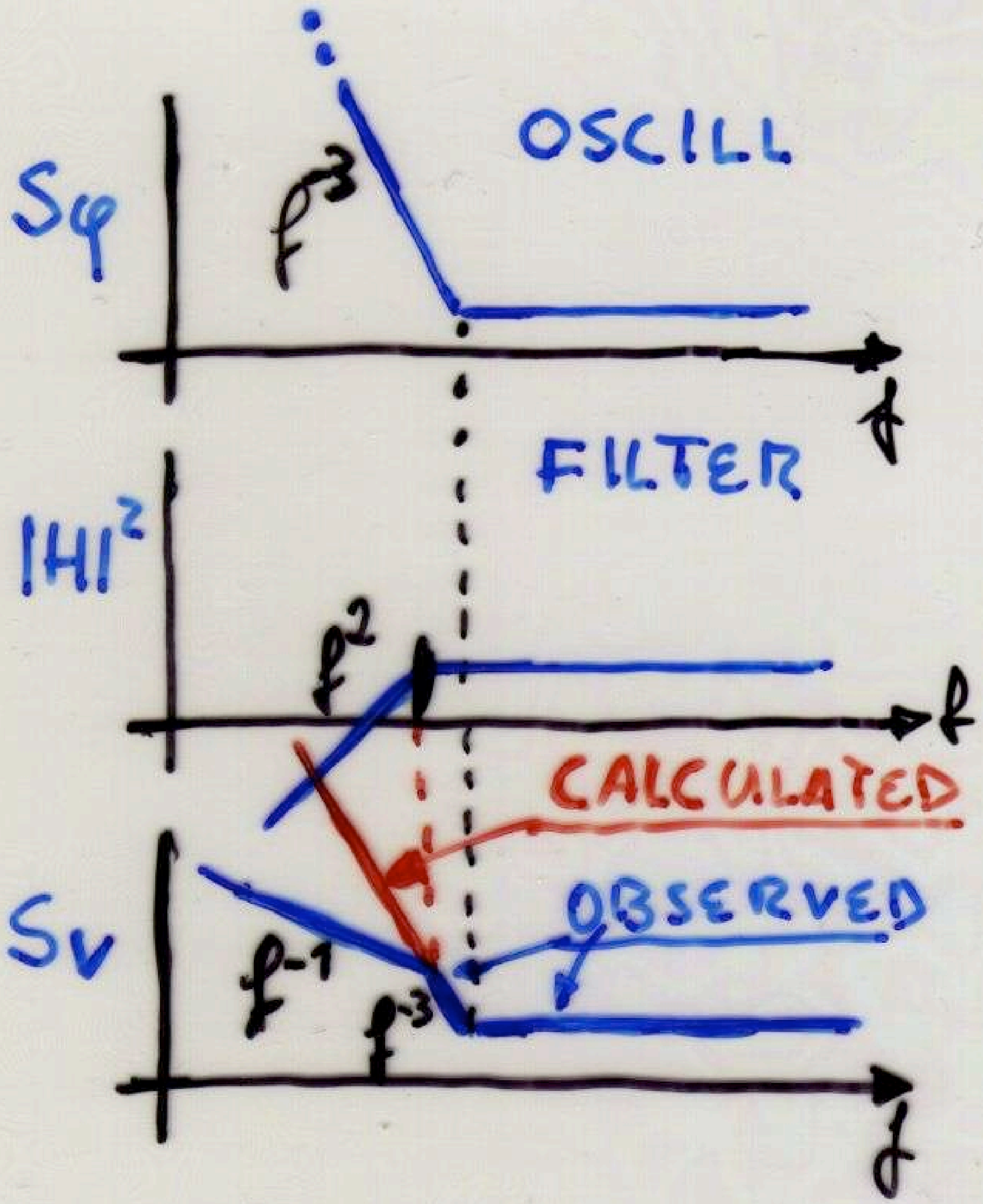
Phase Locked Loop (PLL)



H_c is a constant
(1st order PLL)

A large dynamics is required because of the f^{-3} slope

A tight PLL shows many advantages



Still 1st order PLL

increased gain

a narrower dynamics is sufficient
slope: $f^{-3} \rightarrow f^{-1}$

but you have to correct the spectrum for the PLL transfer function

Practical measurement of $S_\varphi(f)$ with a PLL

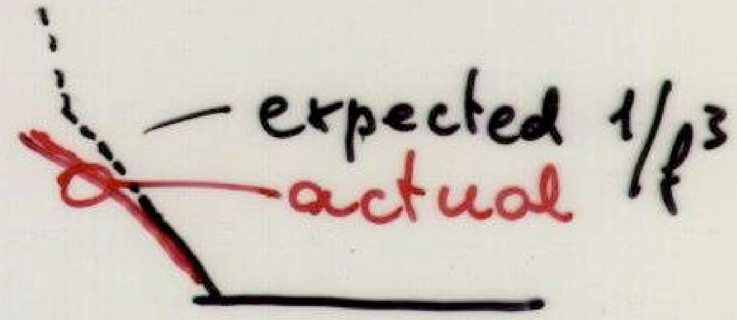
1. **Set the circuit for proper electrical operation**
    a. power level
    b. lock condition (there is no beat note at the mixer out)
    c. zero dc error at the mixer output (a small V can be tolerated)
2. **Choose the appropriate time constant**
3. **Measure the oscillator noise**
4. **At end, measure the background noise**

Warning: a PLL may not be what it seems

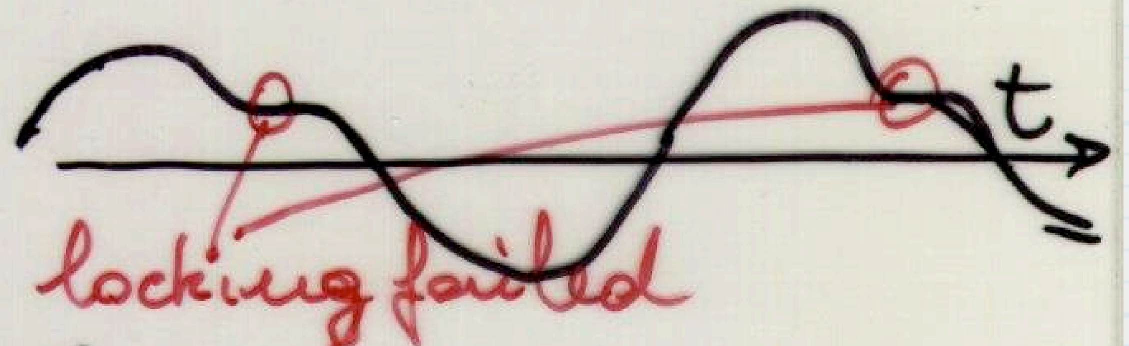
Parasitic locking or coupling of the oscillators may impair the result

BAD SYMPTOMS:

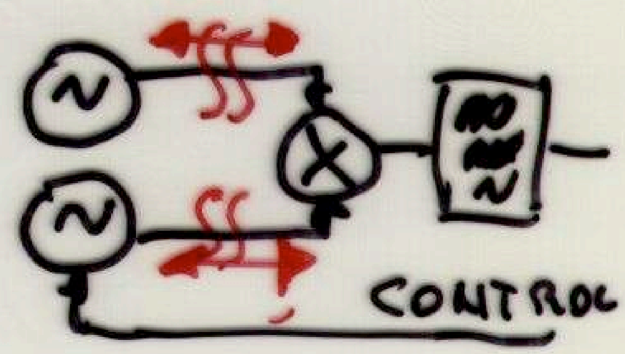
- odd slope S_φ



- open-loop waveforms IFout

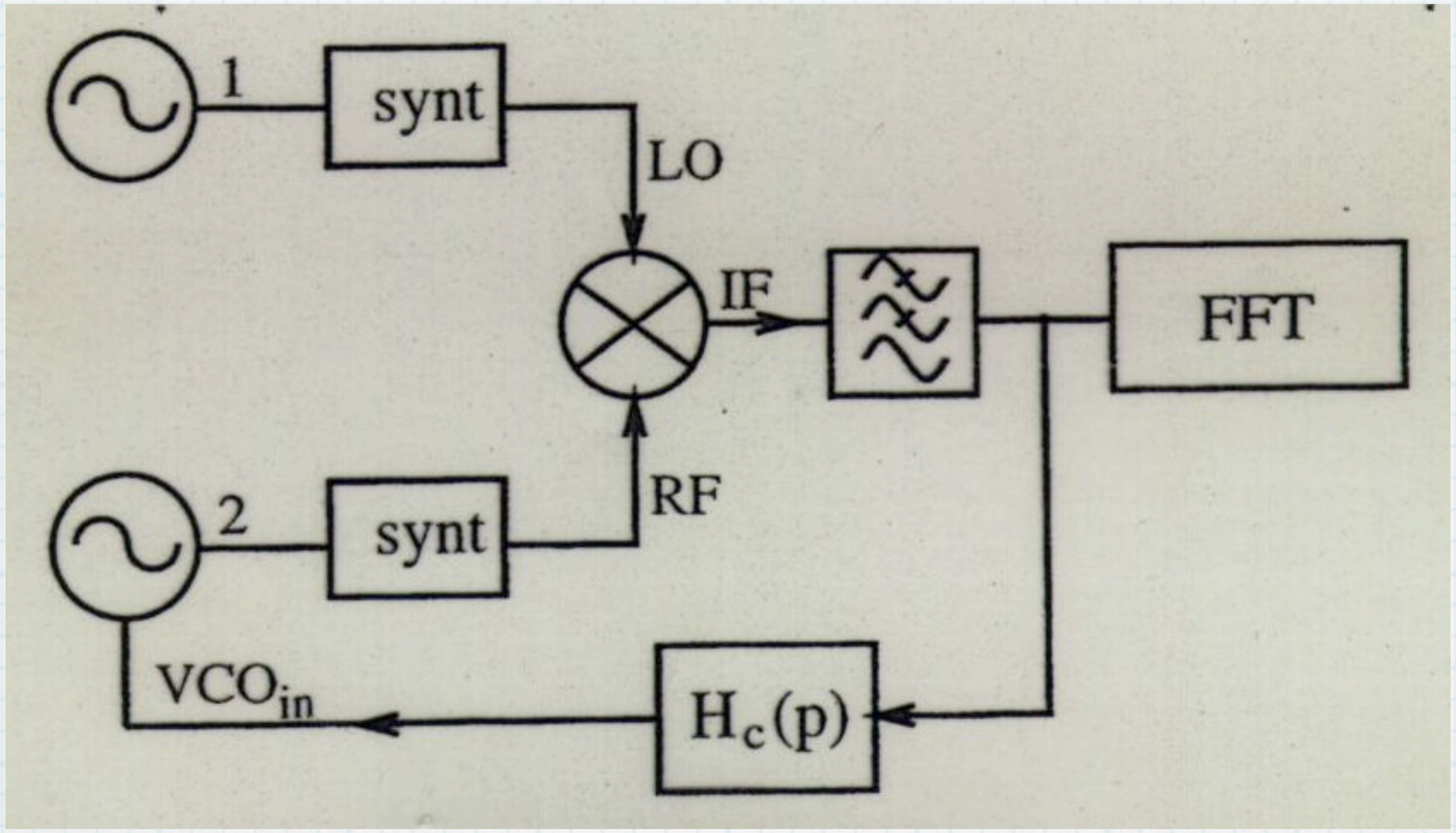


- results (S_φ) depend on the cable length



PLL – two frequencies

The output frequency of the two oscillators is not the same.
A synthesizer (or two synth.) is necessary to match the frequencies

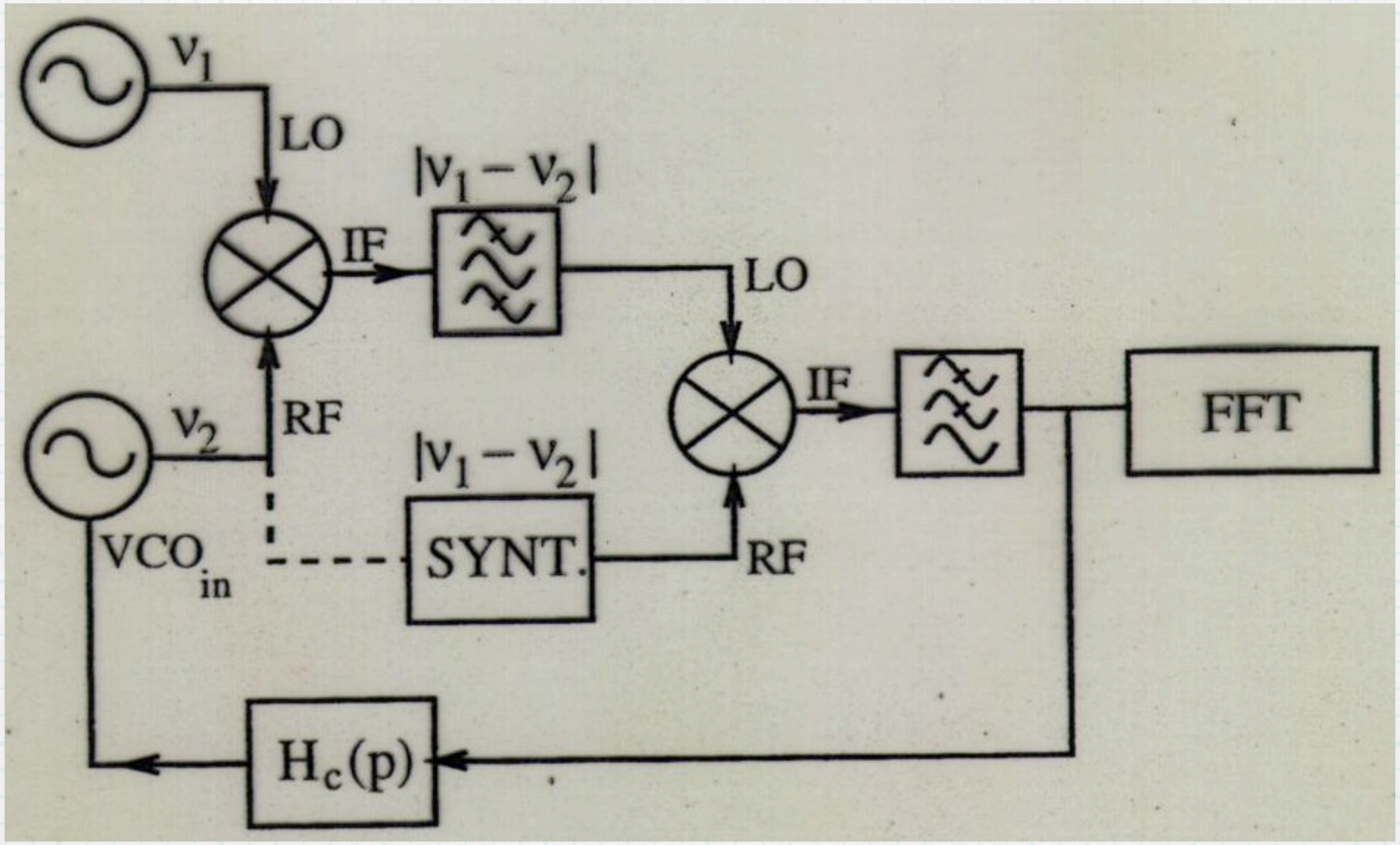


At low Fourier frequencies, the synthesizer noise is lower than the oscillator noise

At higher Fourier frequencies, the white and flicker of phase of the synthesizer may dominate

PLL – low noise microwave oscillators

With low-noise microwave oscillators (like whispering gallery) the noise of a microwave synthesizer at the oscillator output can not be tolerated.

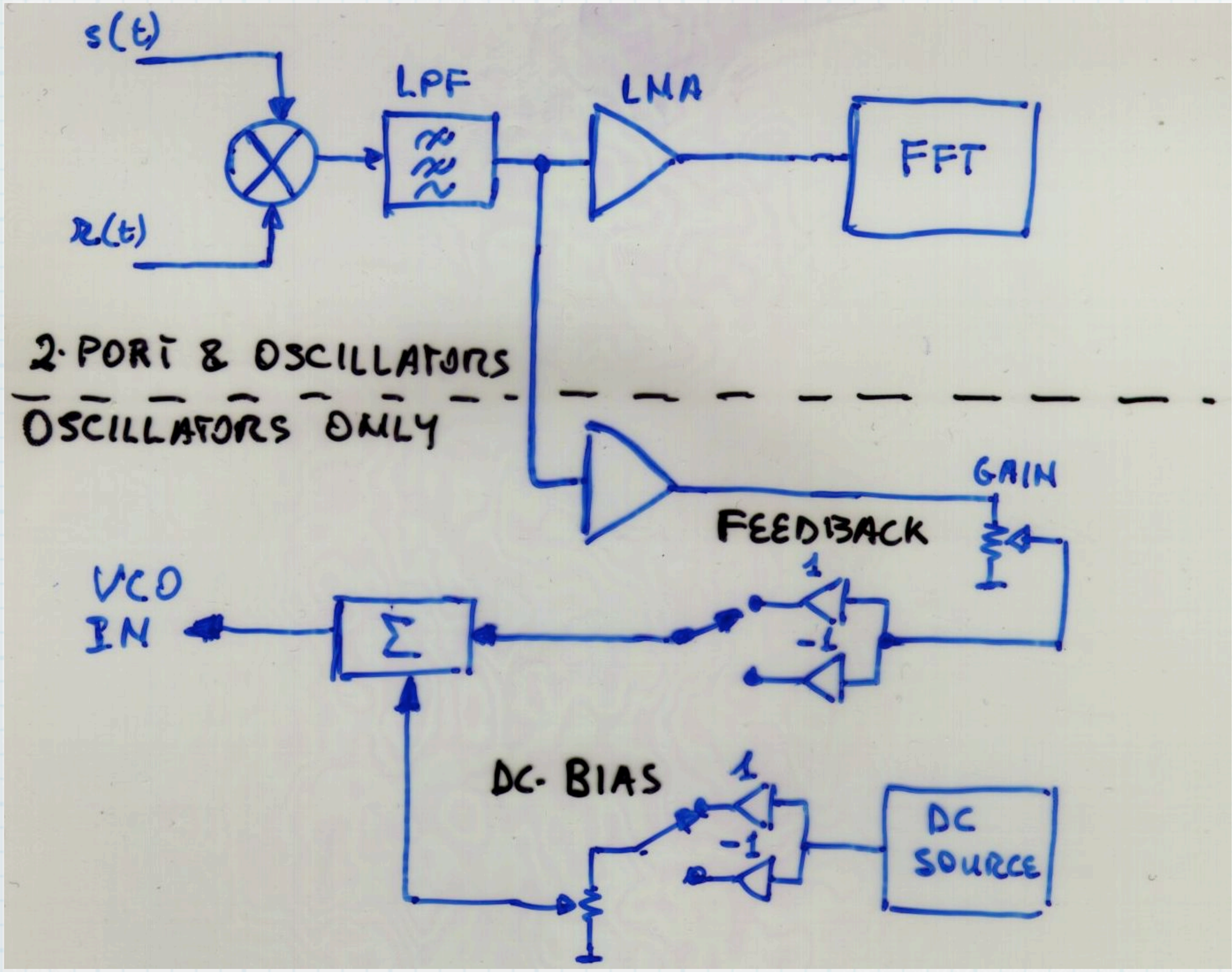


Due to the lower carrier frequency, the noise of a VHF synthesizer is lower than the noise of a microwave synthesizer.

This scheme is useful

- with narrow tuning-range oscillator, which can not work at the same freq.
- to prevent injection locking due to microwave leakage

Designing your own instrument is simple



Standard commercial parts:

- double balanced mixer
- low-noise op-amp
- standard low-noise dc components in the feedback path
- commercial FFT analyzer

Afterwards, you will appreciate more the commercial instruments:

– assembly

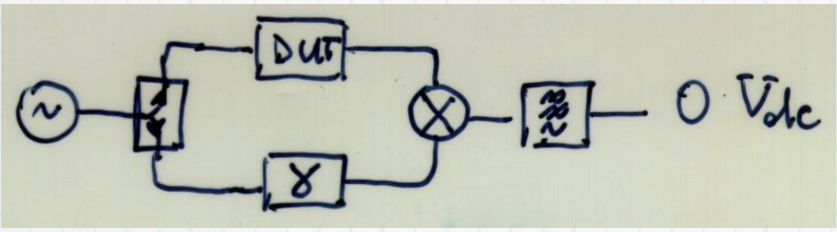
– instruction manual

– computer interface and software

Calibration methods

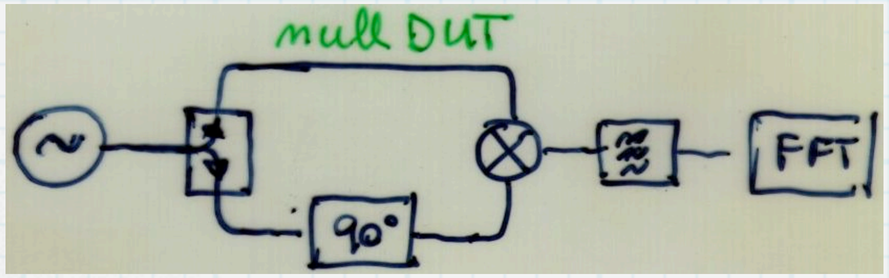
Calibration – general procedure

1 – adjust for proper operation: driving power and quadrature

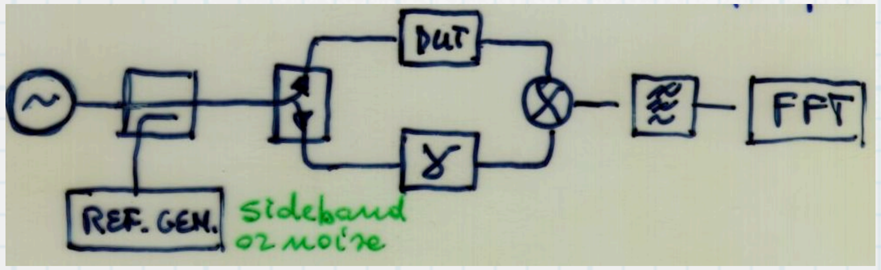


2 – measure the mixer gain k_φ (volts/rad) —> next

3 – measure the residual noise of the instrument

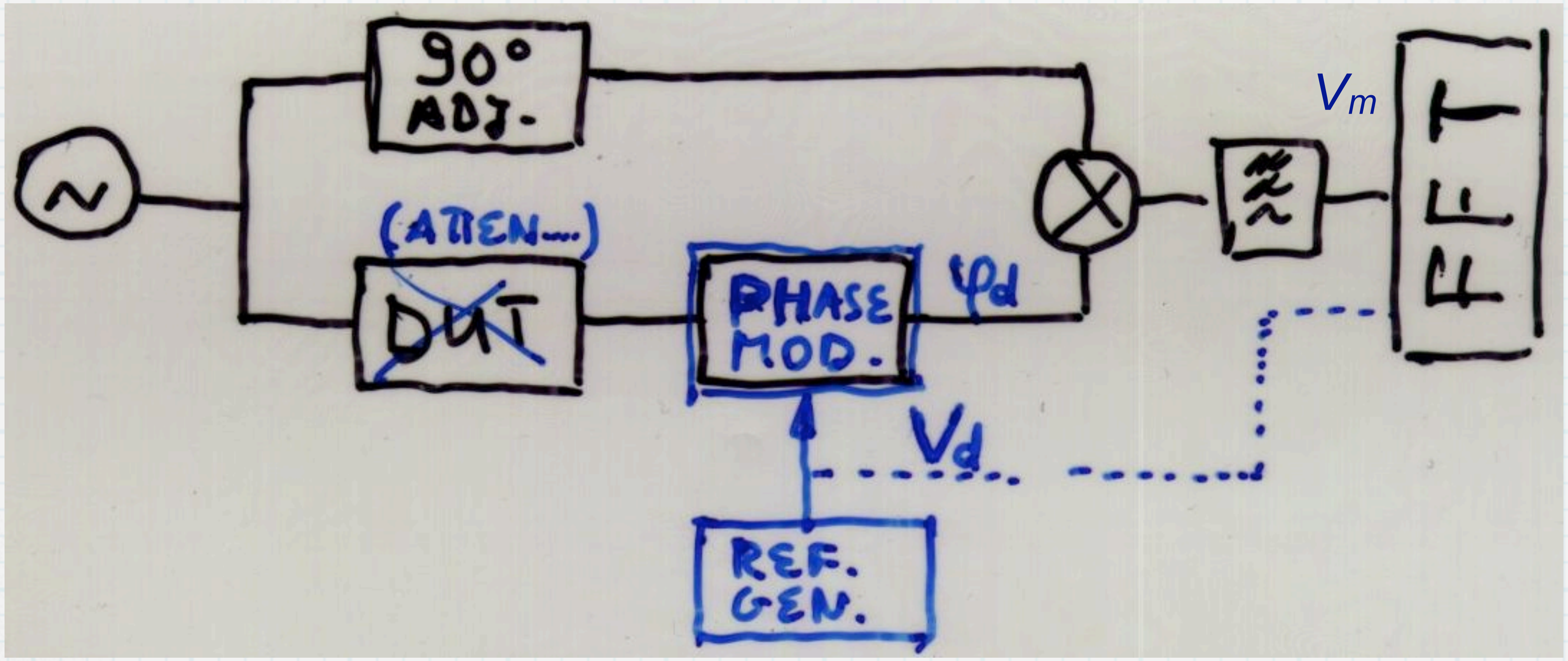


4 – measure the rejection of the oscillator noise



Make sure that the power and the quadrature
are the same during all the calibration process

Calibration – measurement of k_φ (phase mod.)



The reference signal can be:

a) a tone:

detect with the FFT,
with a dual-channel FFT, or
with a lock-in

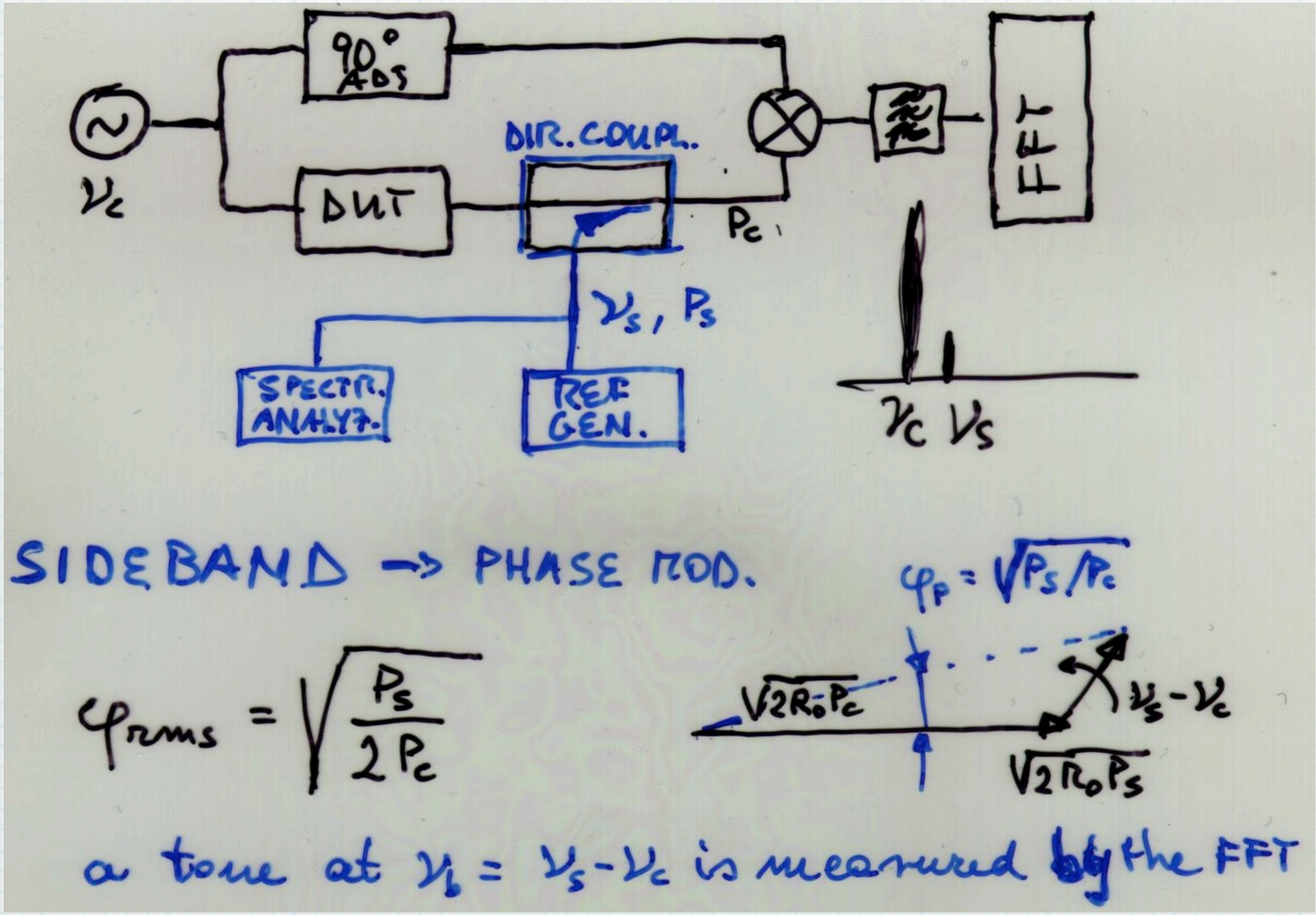
b) random or pseudo-random
white noise

tone: $$k_\varphi = \frac{V_m}{k_m V_d}$$

white noise $$k_\varphi^2 = \frac{S_{Vm}}{k_m^2 S_{Vd}}$$

Some FFTs have a white noise output
Dual-channel FFTs calculate the transfer function $|H(f)|^2 = S_{Vm}/S_{Vd}$

Calibration – measurement of k_φ (rf signal)



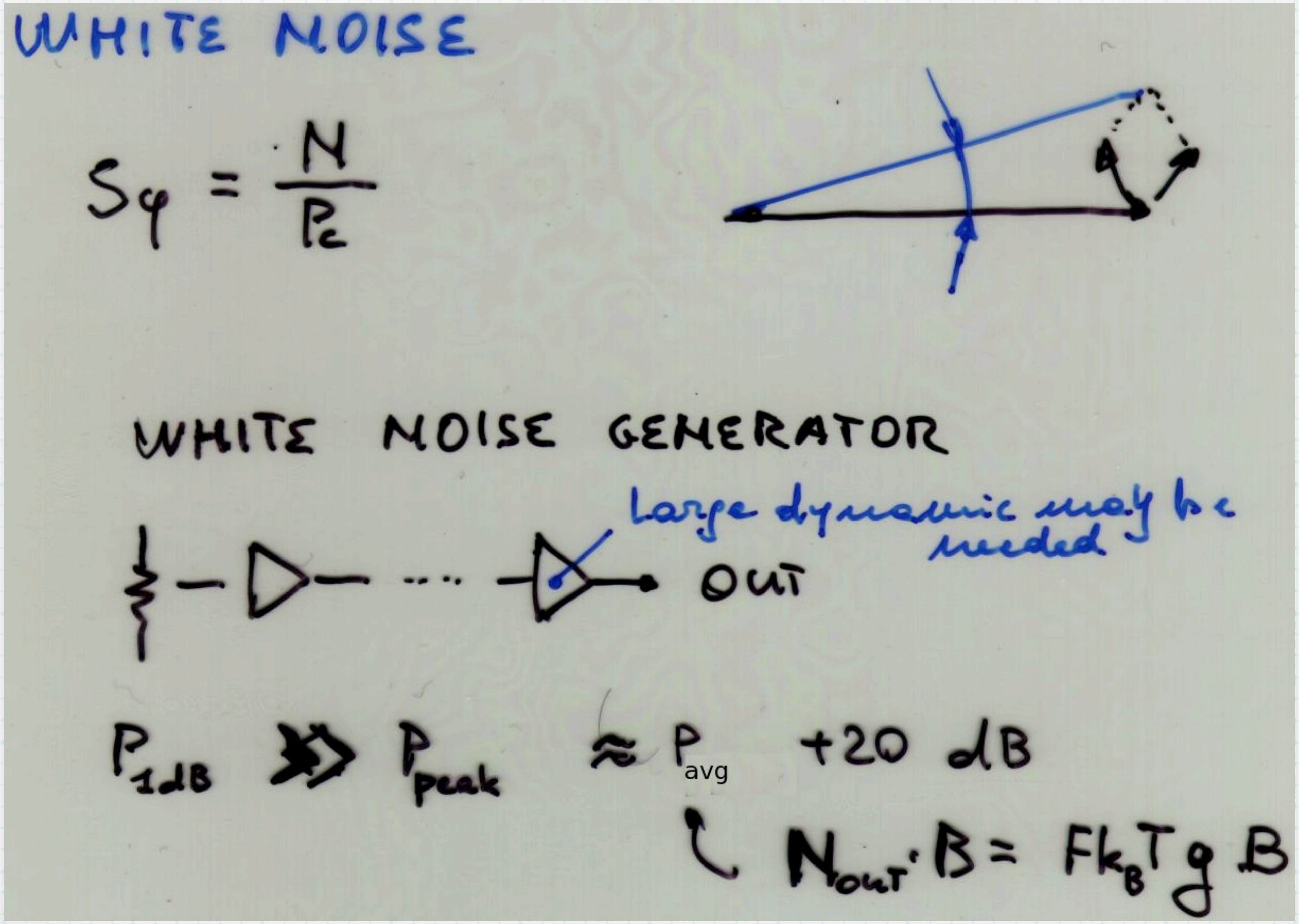
However often used, even in major laboratories this method is incorrect because:

1 – the calibration signal yields AM and PM of equal depth,

2 – the mixer shows a residual sensitivity to AM noise

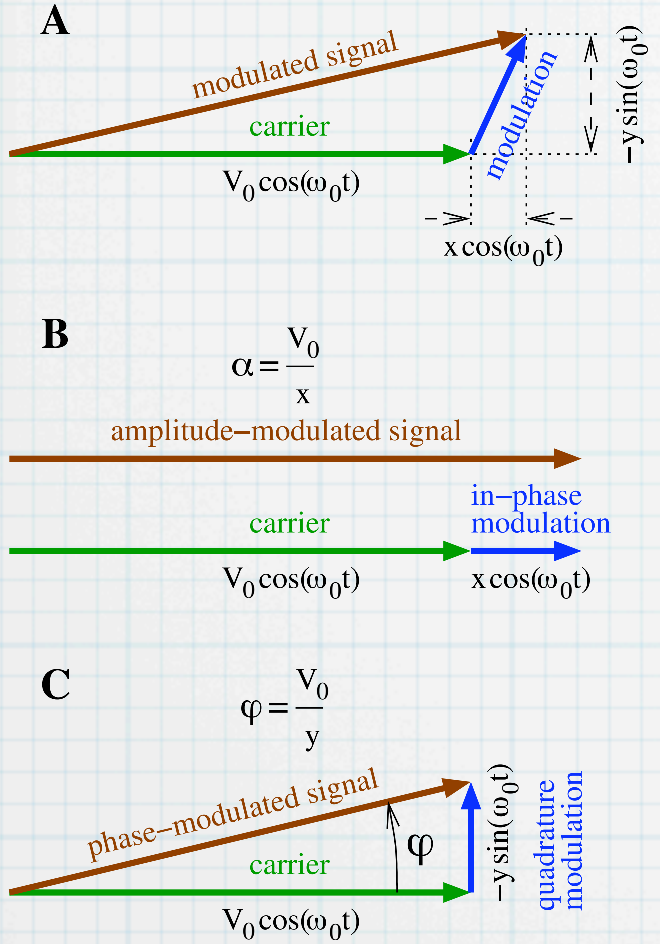
Calibration – measurement of k_φ (rf signal)

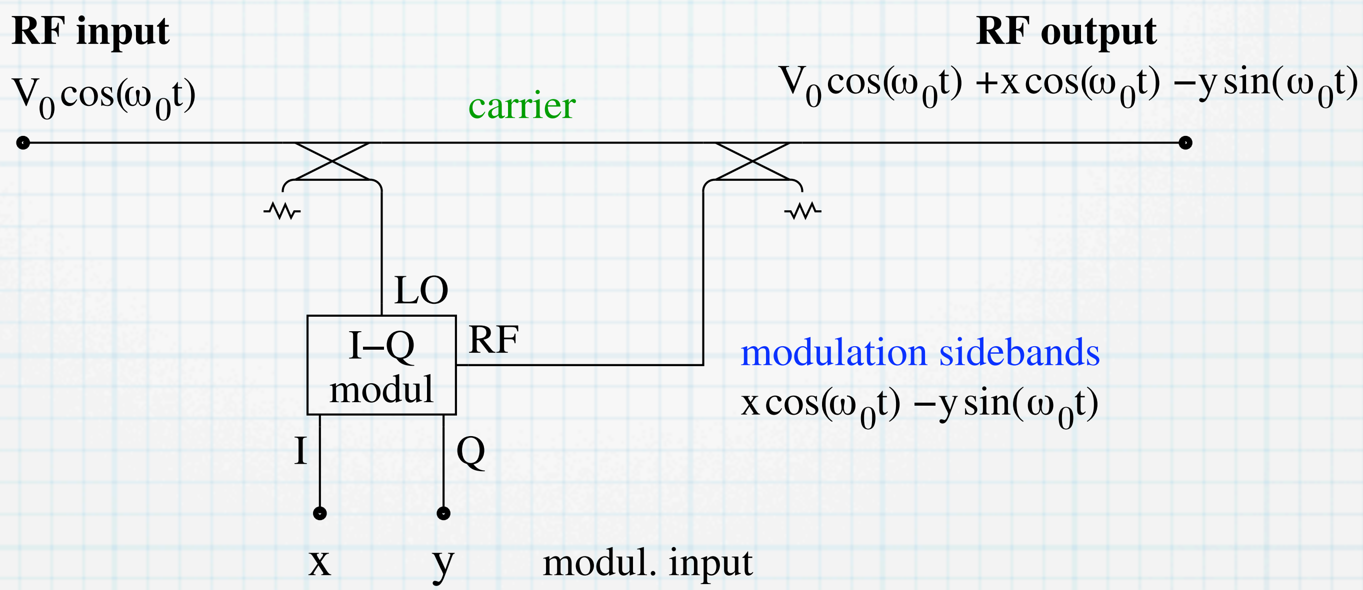
A reference rf noise is injected in the DUT path
through a directional coupler

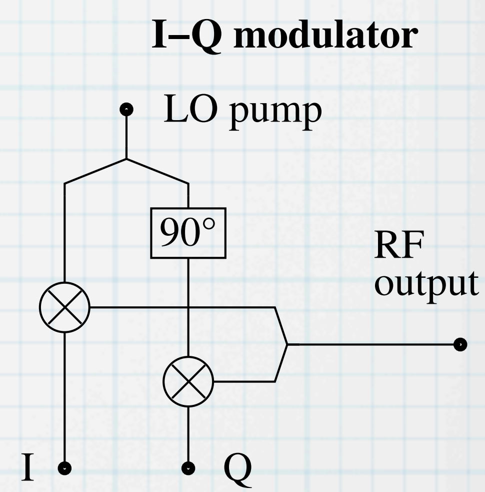


However often used, this method is incorrect – see previous slide

Primary calibration





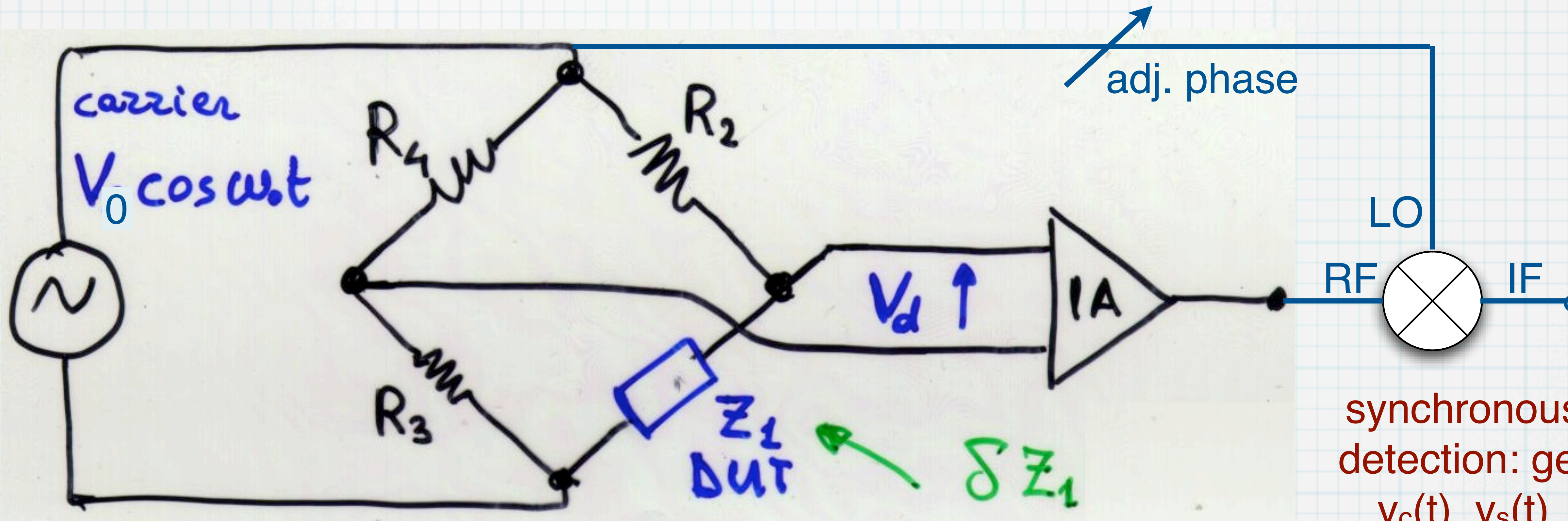


$$\alpha_{\mathrm{rms}} = \sqrt{\frac{P_x}{P_0}} \qquad \text{and} \qquad \varphi_{\mathrm{rms}} = \sqrt{\frac{P_y}{P_0}}$$

P_0 = power of the carrier
P_x = power of the in-phase sidebands
P_y = power of the quadrature sidebands

Bridge techniques

Wheatstone bridge



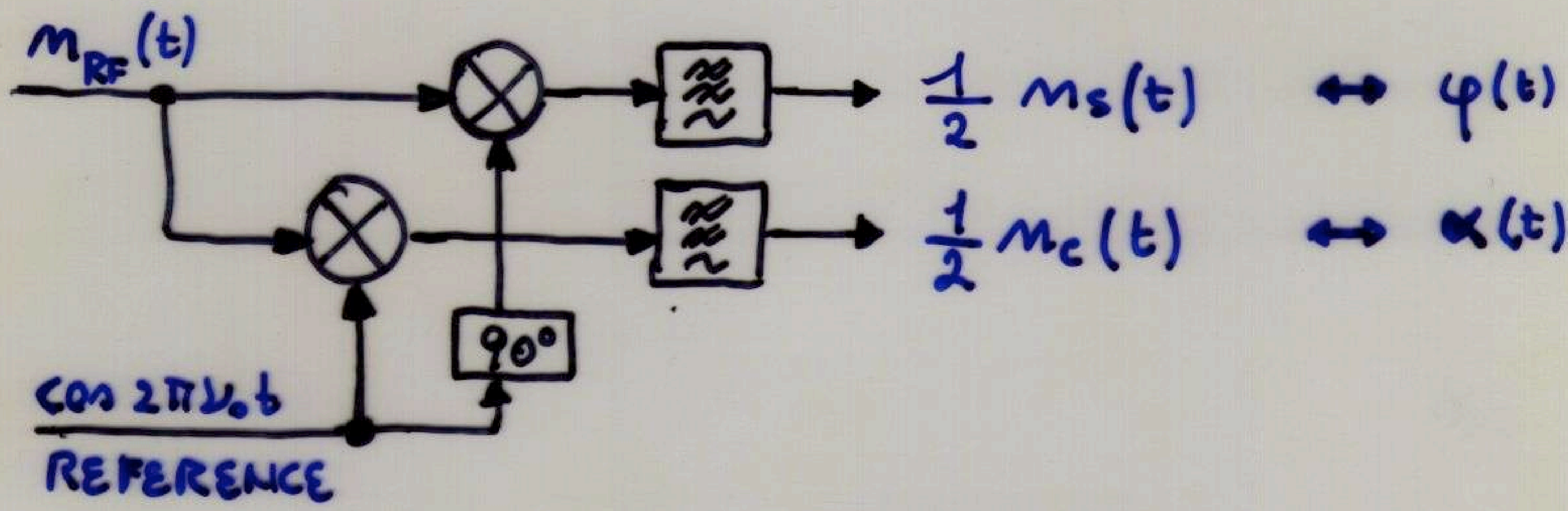
synchronous detection: get $v_c(t)$ $v_s(t)$ (AM or PM noise)

equilibrium: $V_d = 0$ –> carrier suppression

static error δZ_1 –> some residual carrier
real δZ_1 => in-phase residual carrier $V_{re} \cos(\omega_0 t)$
imaginary δZ_1 => quadrature residual carrier $V_{im} \sin(\omega_0 t)$

fluctuating error δZ_1 => noise sidebands
real δZ_1 => AM noise $v_c(t) \cos(\omega_0 t)$
imaginary δZ_1 => PM noise $-v_s(t) \sin(\omega_0 t)$

Synchronous detection



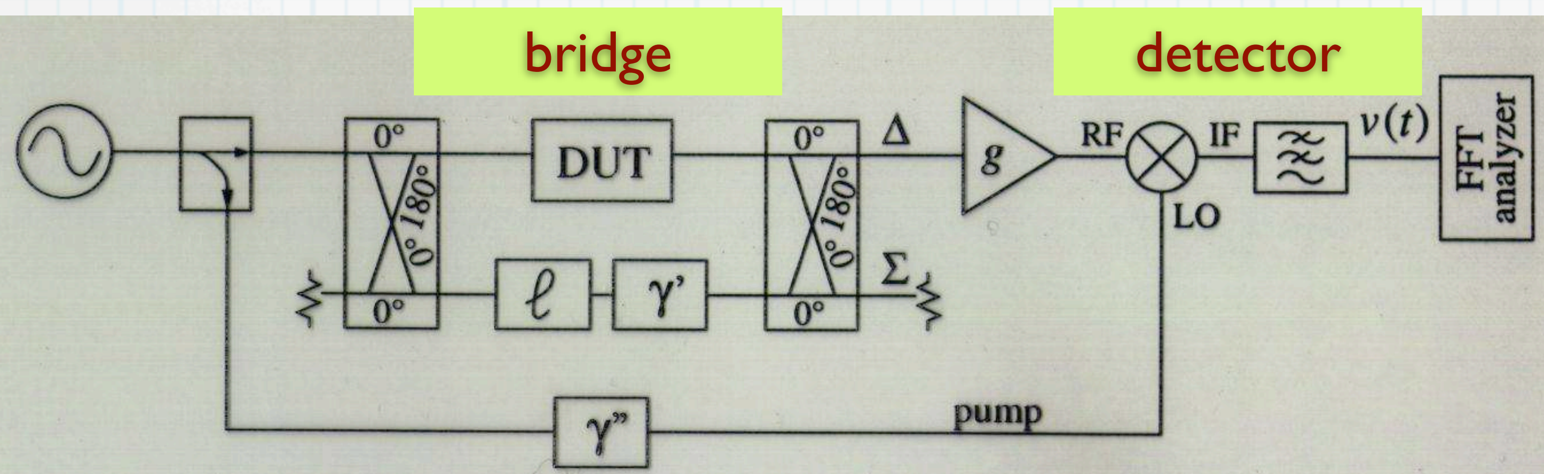
REMEMBER:

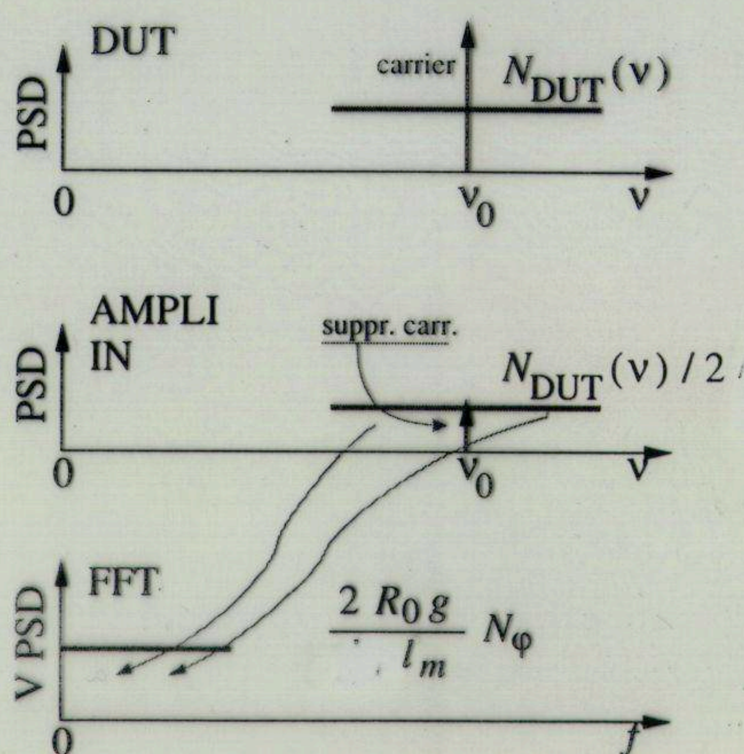
$$\cos^2 \omega t = \frac{1}{2}(1 + \cos 2\omega t)$$

$$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t)$$

$$\sin \omega t \cos \omega t = \frac{1}{2} \sin 2\omega t$$

Bridge (interferometric) PM and AM noise measurement





High carrier suppression:
no carrier ⇒ the amplifier can't flicker

High gain:

$$k_\varphi = \frac{v(t)}{\varphi(t)} = \sqrt{\frac{R_0 g P_0}{l_m}} - \text{dissip. losses}$$

Low Noise Floor:

$$S_{\varphi 0} = \frac{2 F k_B T_0}{P_0} + \text{dissip. losses}$$

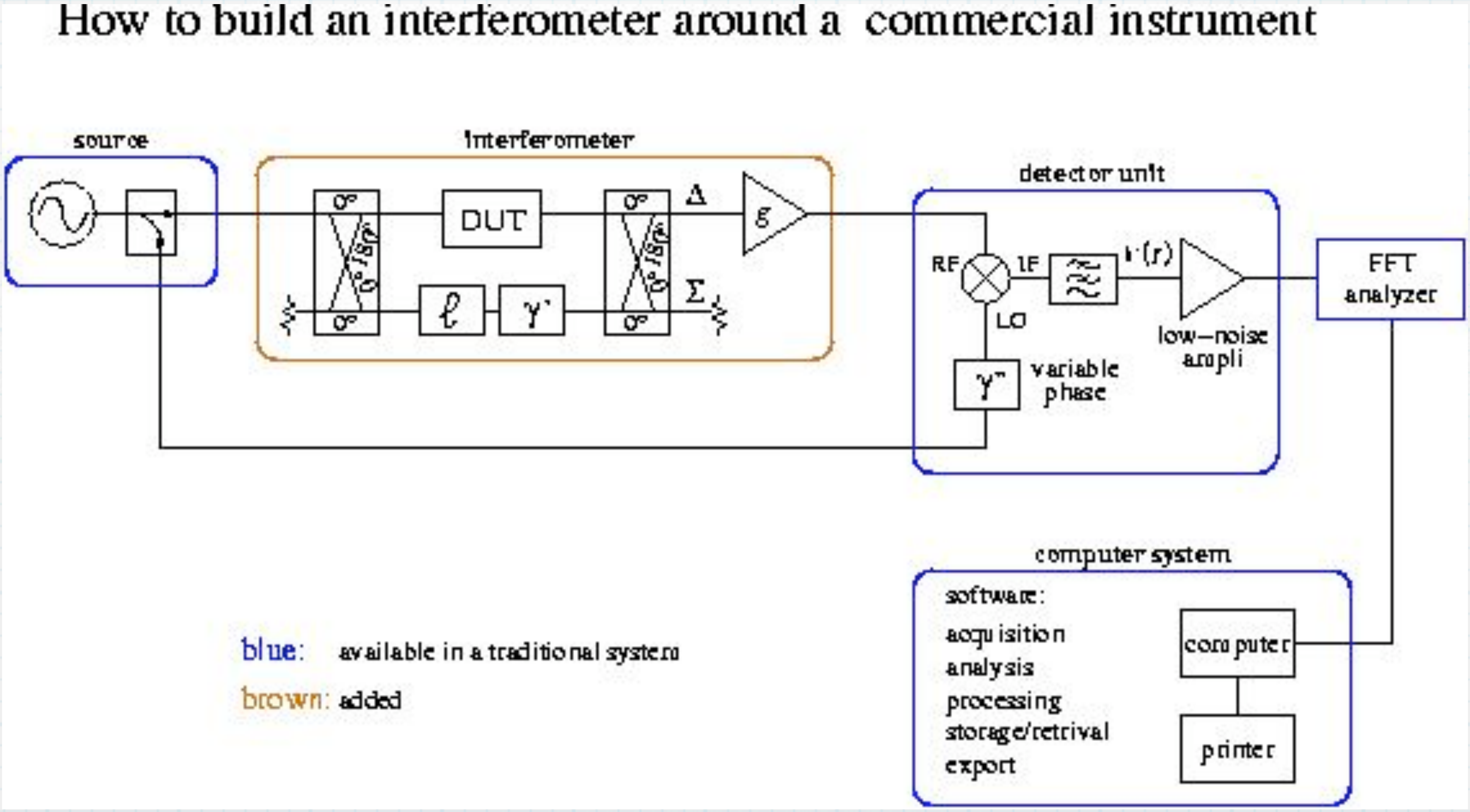
High immunity to 50 Hz B fields

Improved, from RSI **70** 1 pp. 220-225, Jan 1999

and rejection of the master-oscillator noise

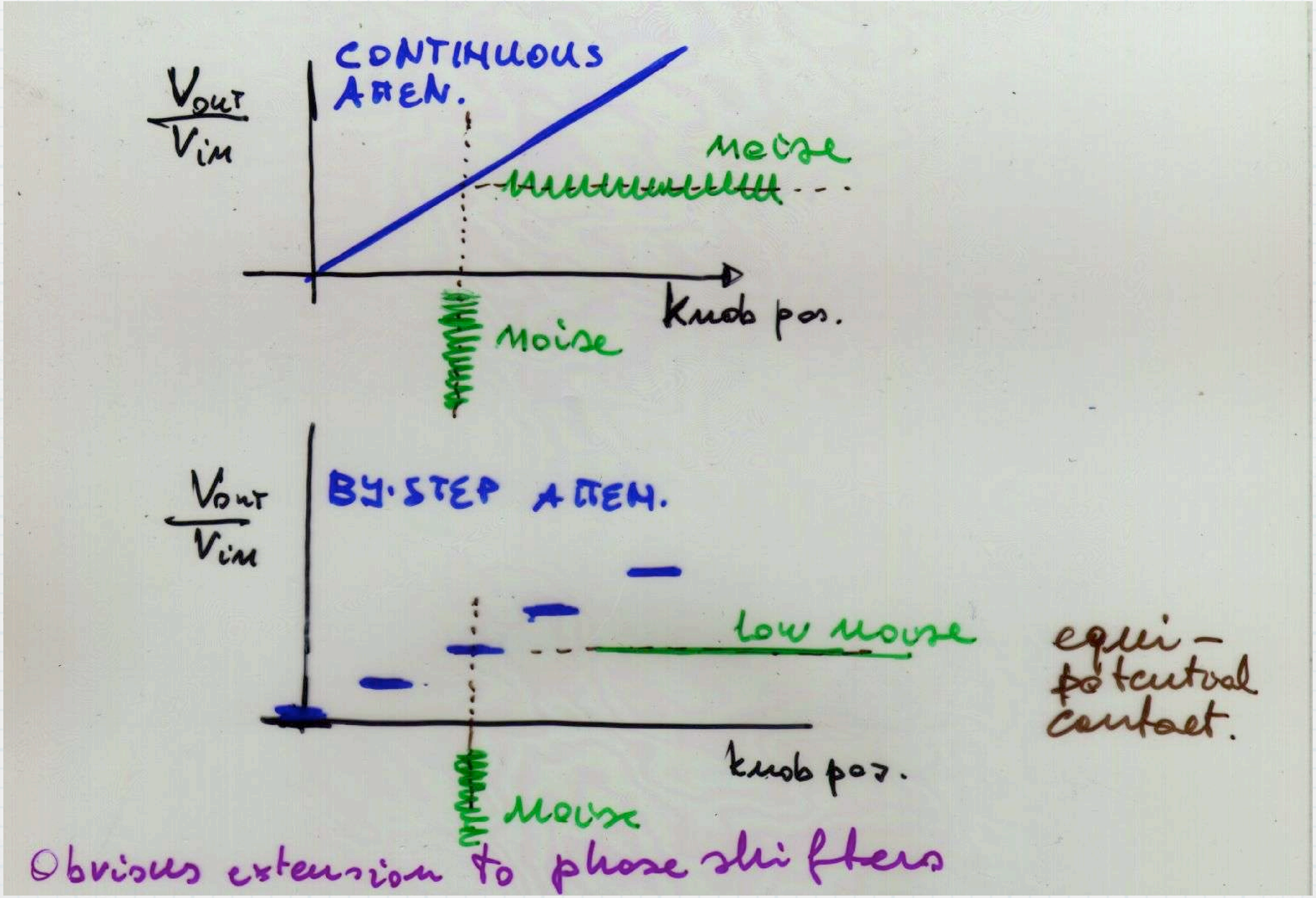
yet, difficult for the measurement of oscillators

A bridge (interferometric) instrument can be built around a commercial instrument



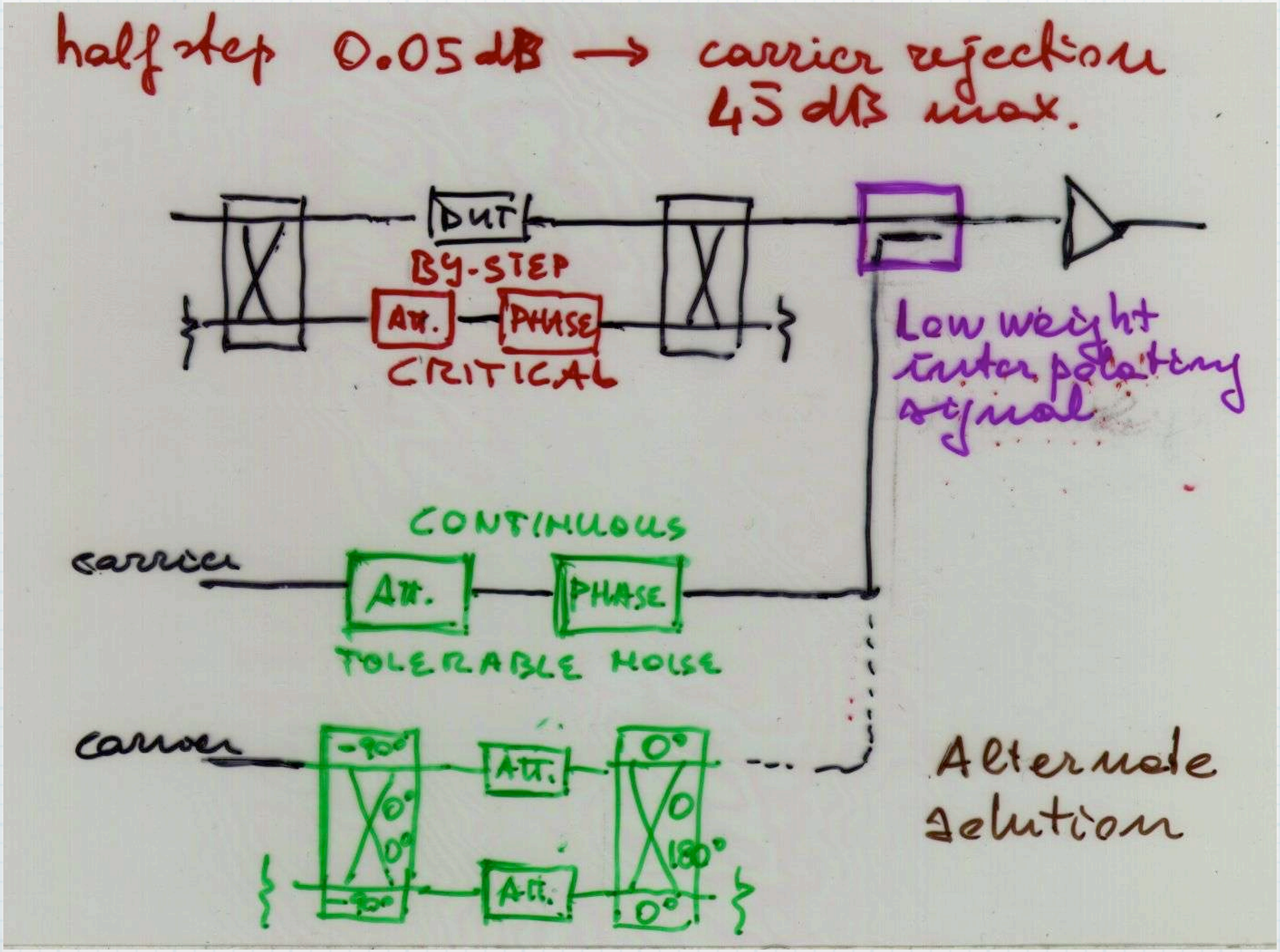
You will appreciate the computer interface and the software ready for use

Origin of flicker in the bridge

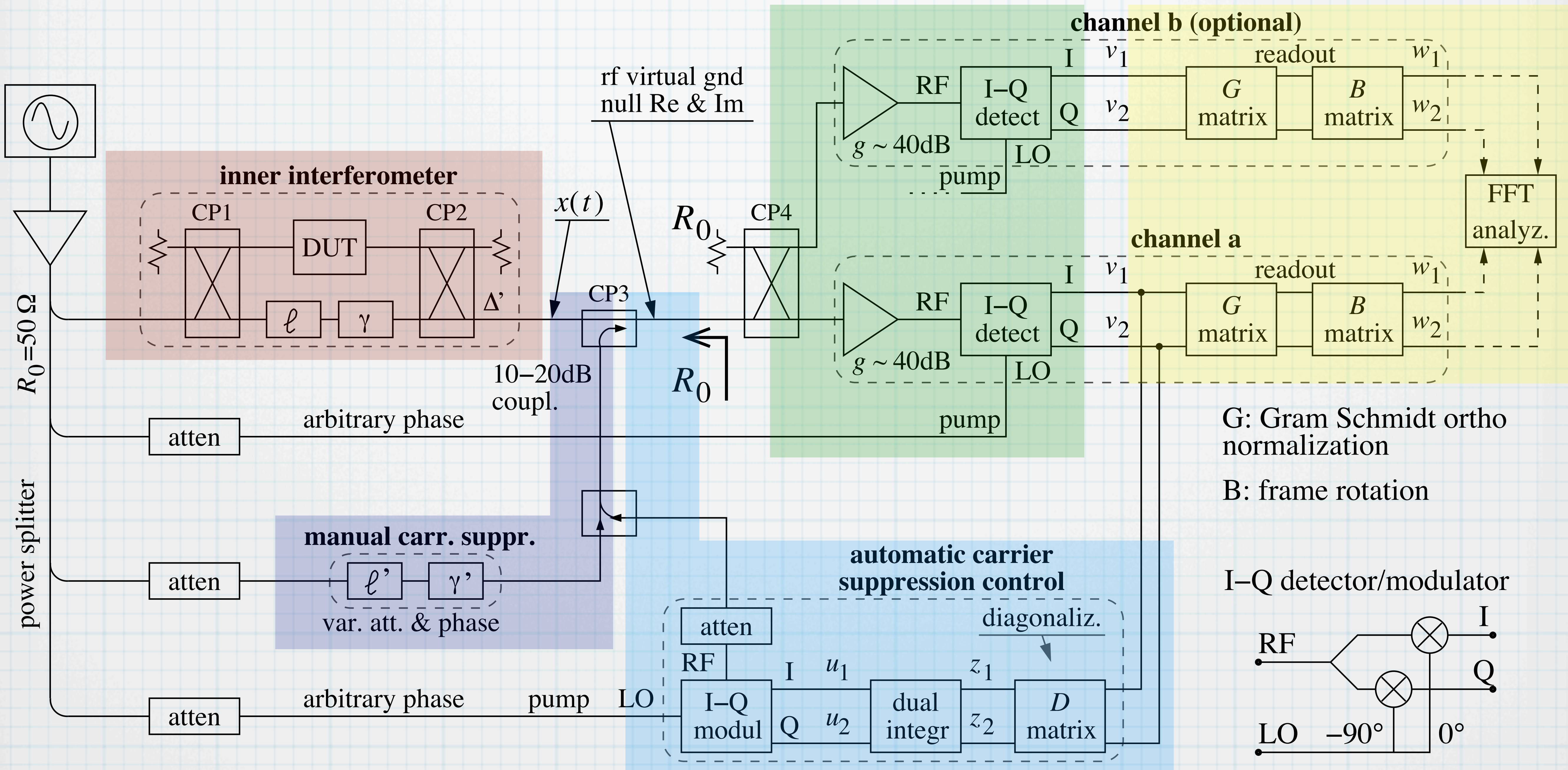


In the early time of electronics, flicker was called “contact noise”

Coarse and fine adjustment of the bridge null are necessary

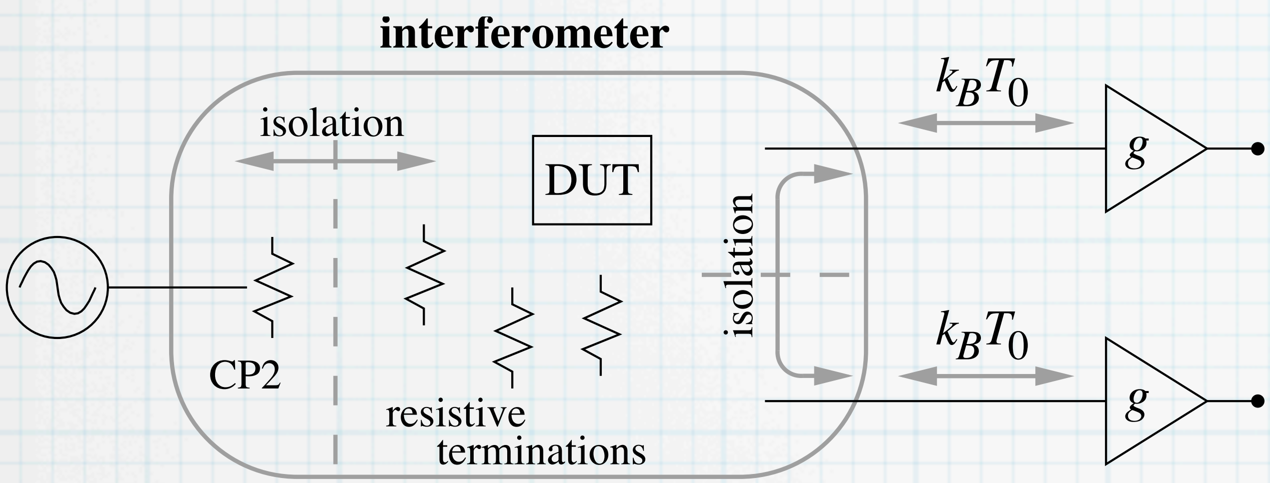


Flicker reduction, correlation, and closed-loop carrier suppression can be combined

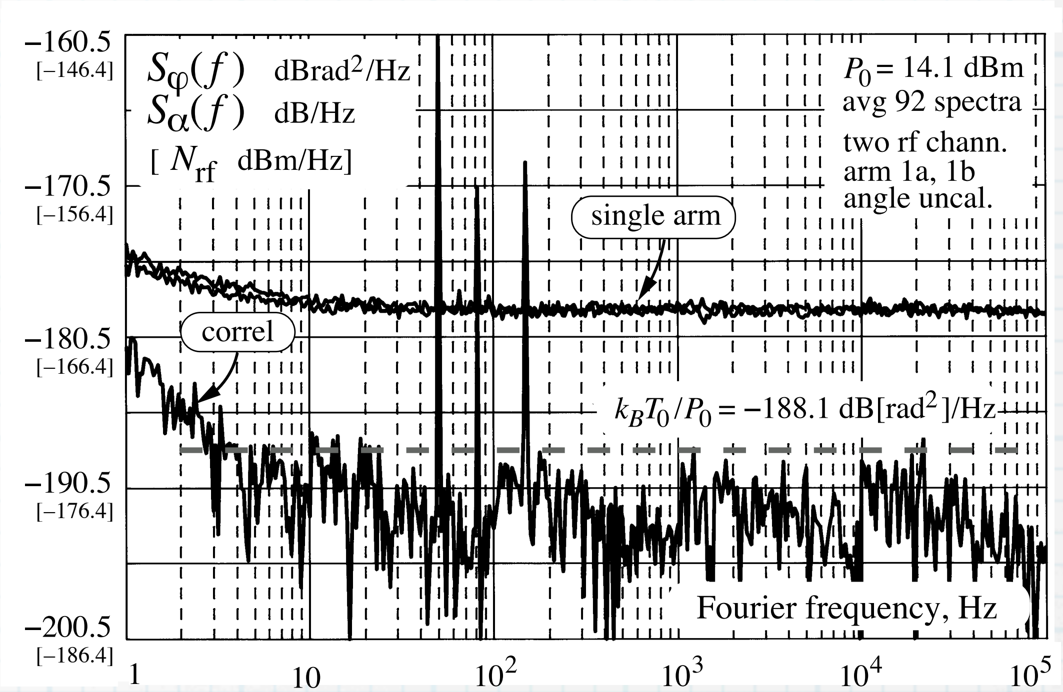


E. Rubiola, V. Giordano, Rev. Scientific Instruments 73(6) pp.2445-2457, June 2002

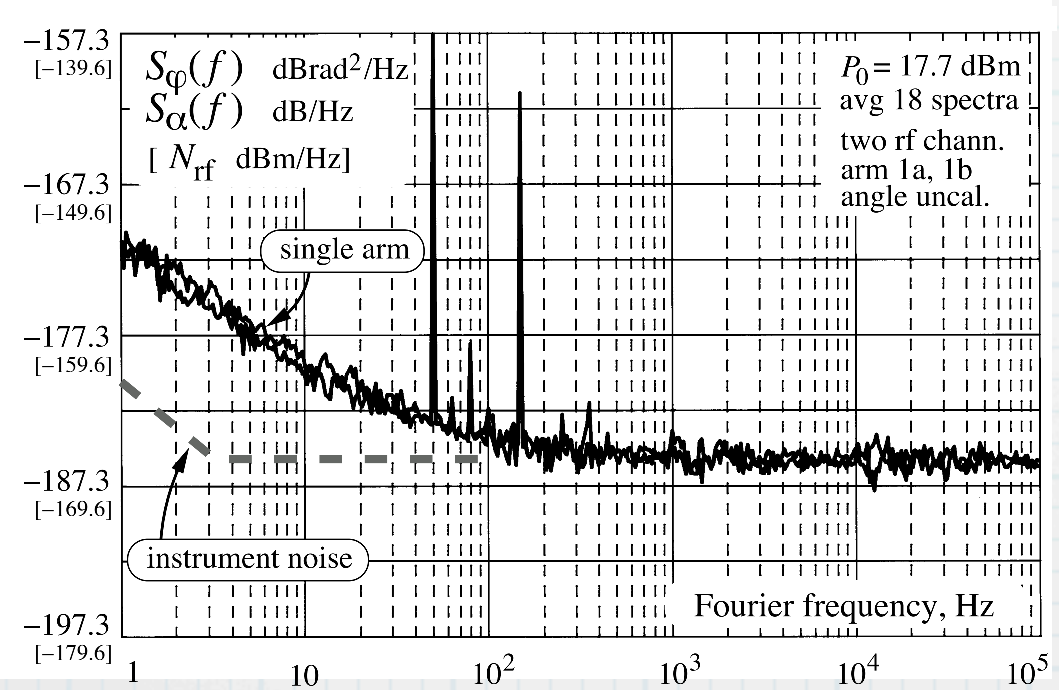
Example of results



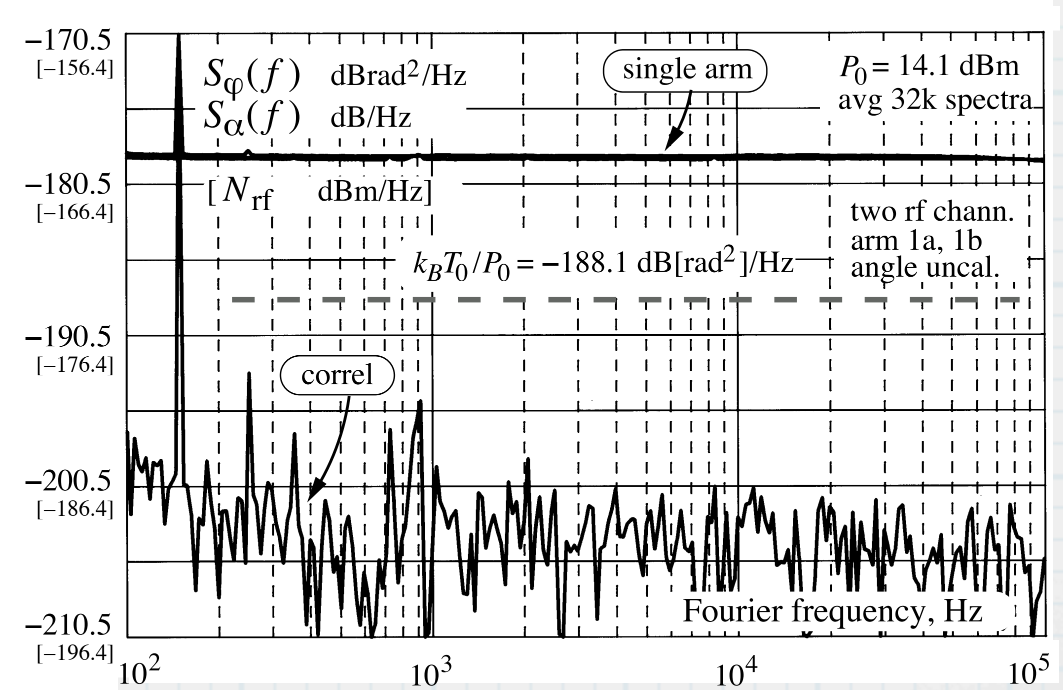
Correlation-and-averaging rejects the thermal noise



Residual noise of the fixed-value bridge, in the absence of the DUT



Noise of a pair of HH-109 hybrid couplers measured at 100 MHz



Residual noise of the fixed-value bridge. Same as above, but larger m

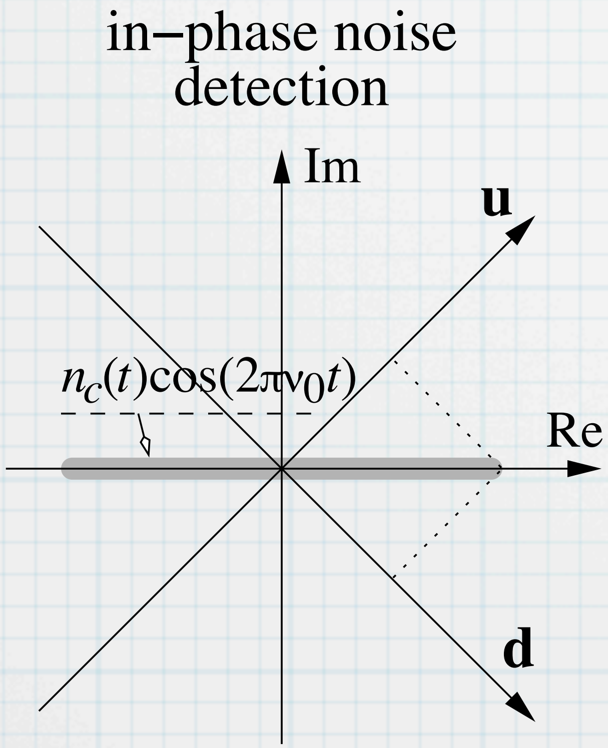
±45º detection

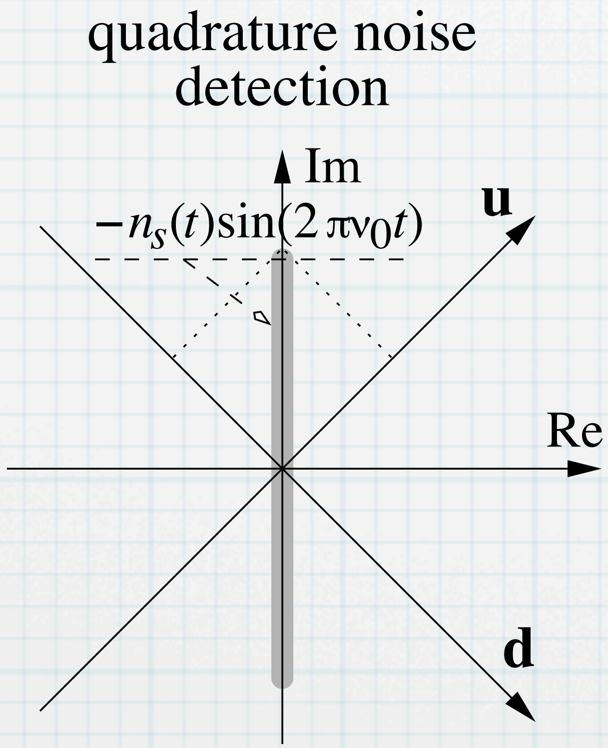
DUT noise without carrier $n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t$

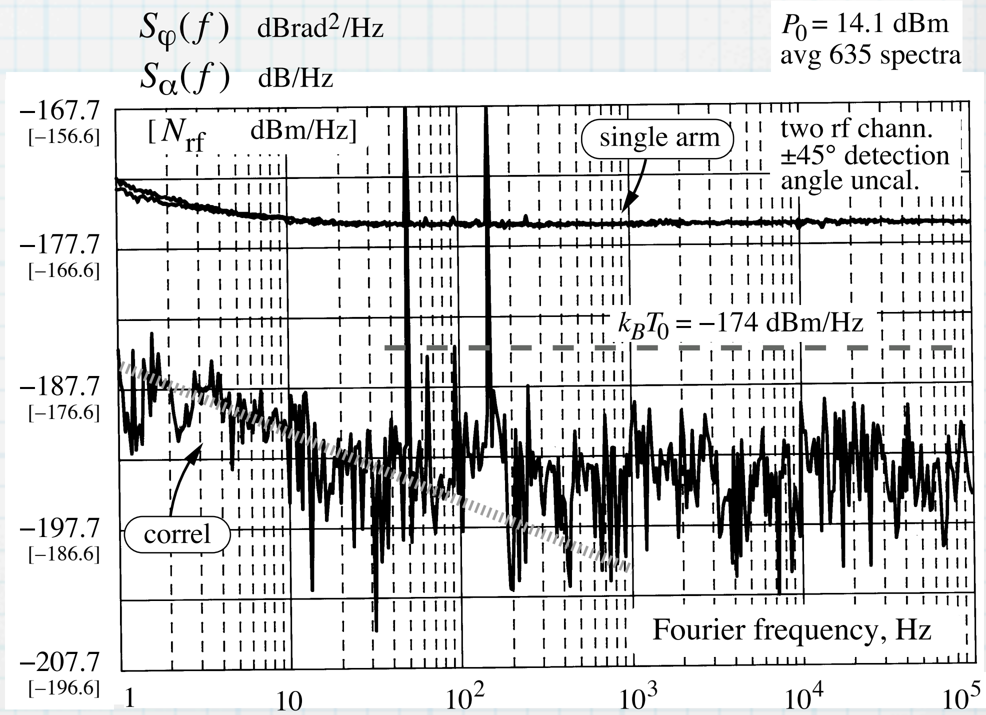
UP reference $u(t) = V_P\cos(\omega_0 t - \pi/4)$

DOWN reference $d(t) = V_P\cos(\omega_0 t + \pi/4)$

cross spectral density $S_{ud}(f) = \frac{1}{2}\Big[S_\alpha(f) - S_\varphi(f)\Big]$



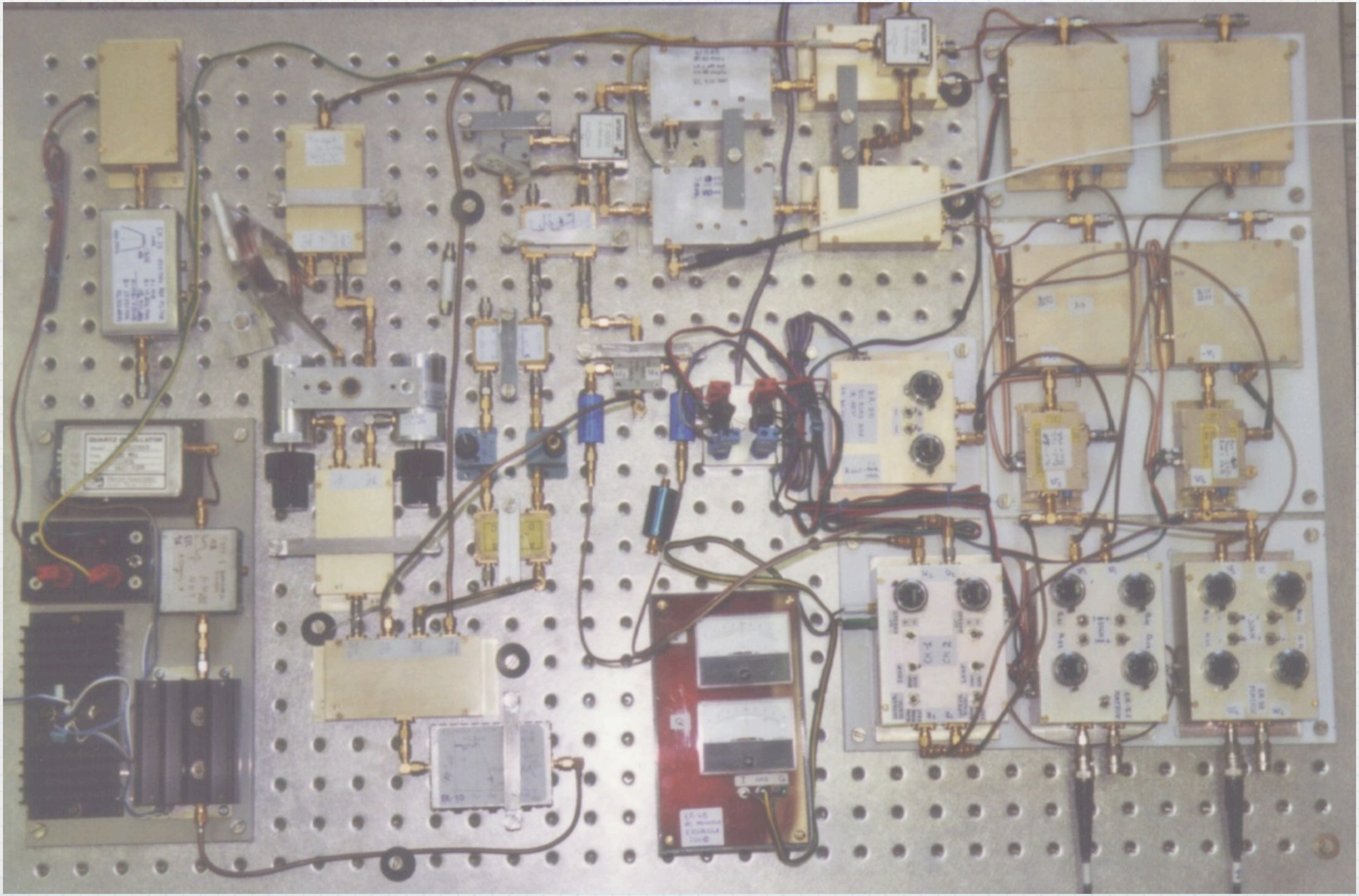




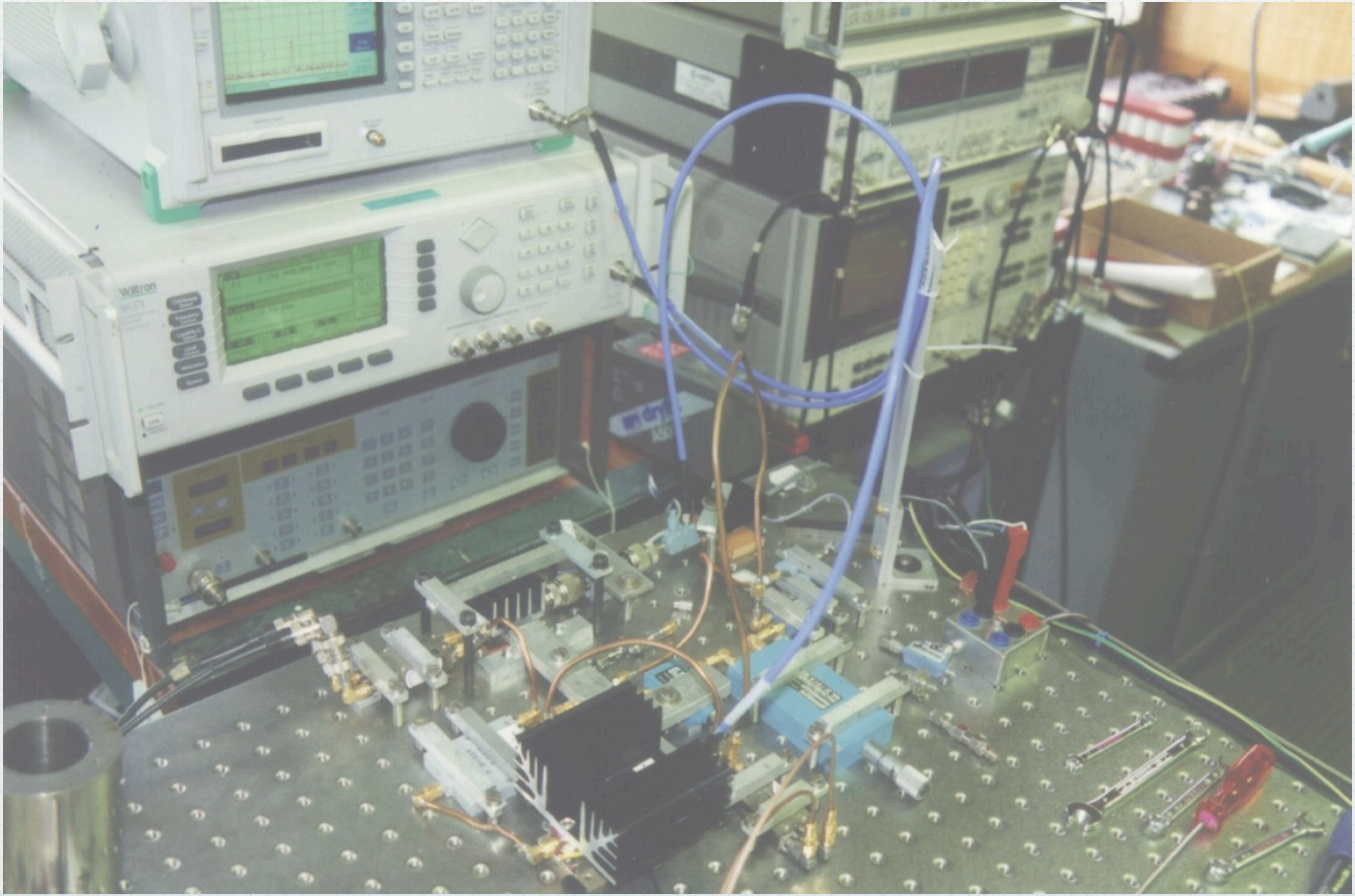
Residual noise, in the absence of the DUT

Smart and nerdy, yet of scarce practical usefulness
First used at 2 kHz to measure electromigration on metals (H. Stoll, MPI)

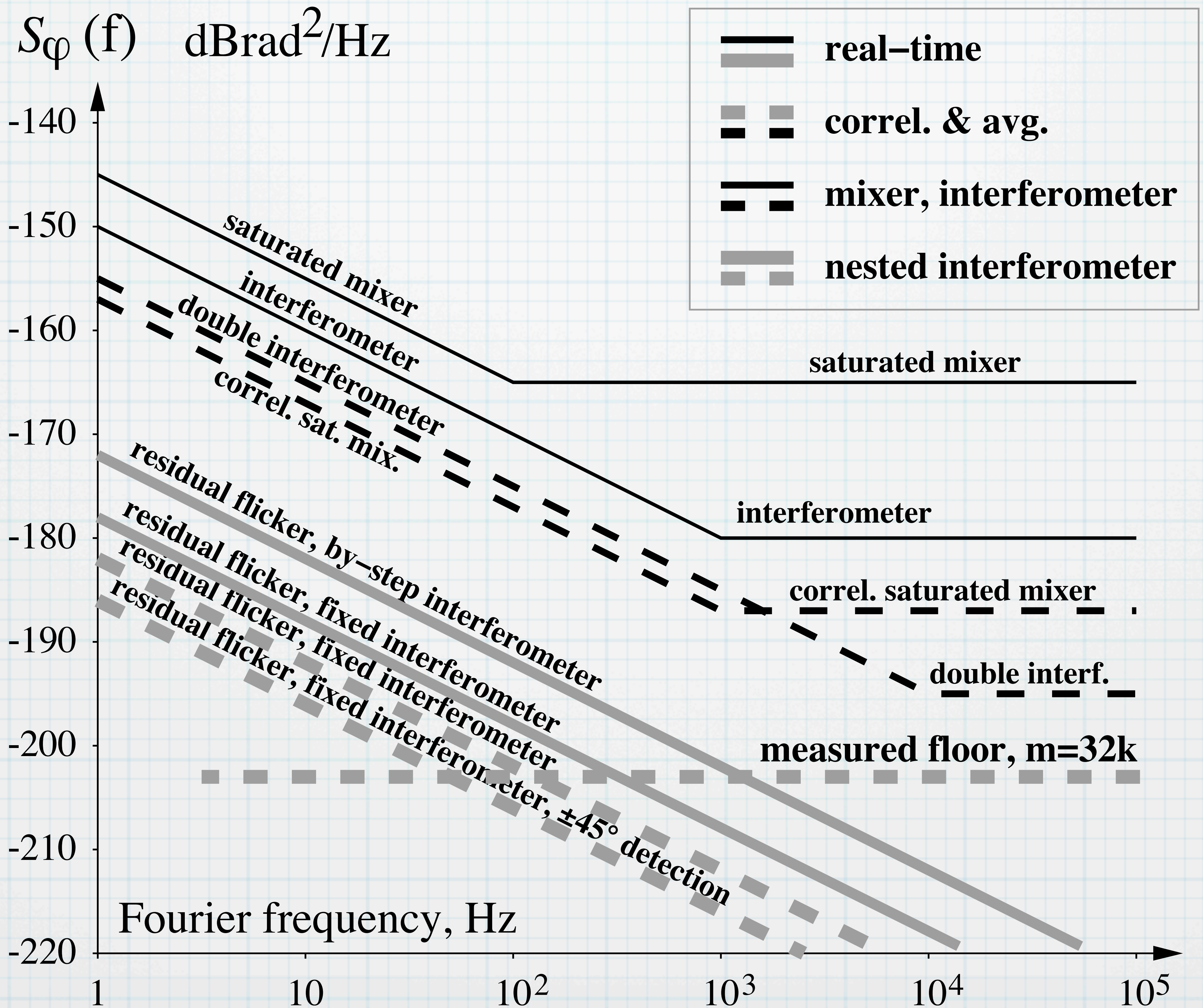
The complete machine (100 MHz)



A 9 GHz experiment (dc circuits not shown)



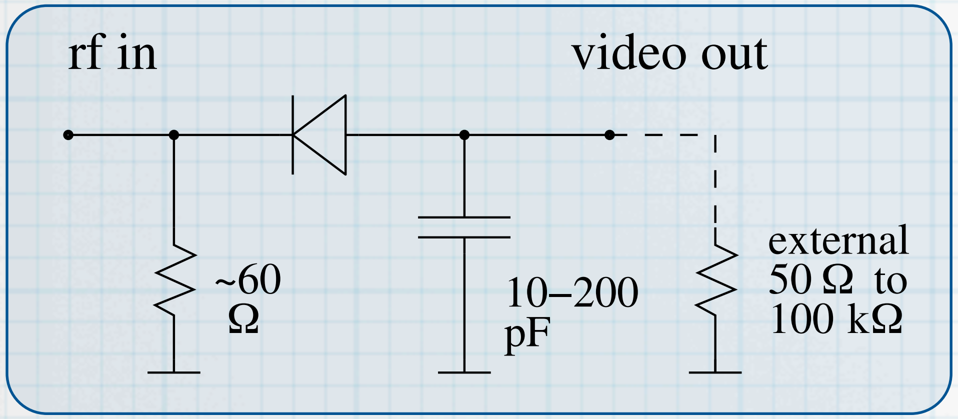
Comparison of the background noise



AM noise

Tunnel and Schottky power detectors

law: $v = k_d P$

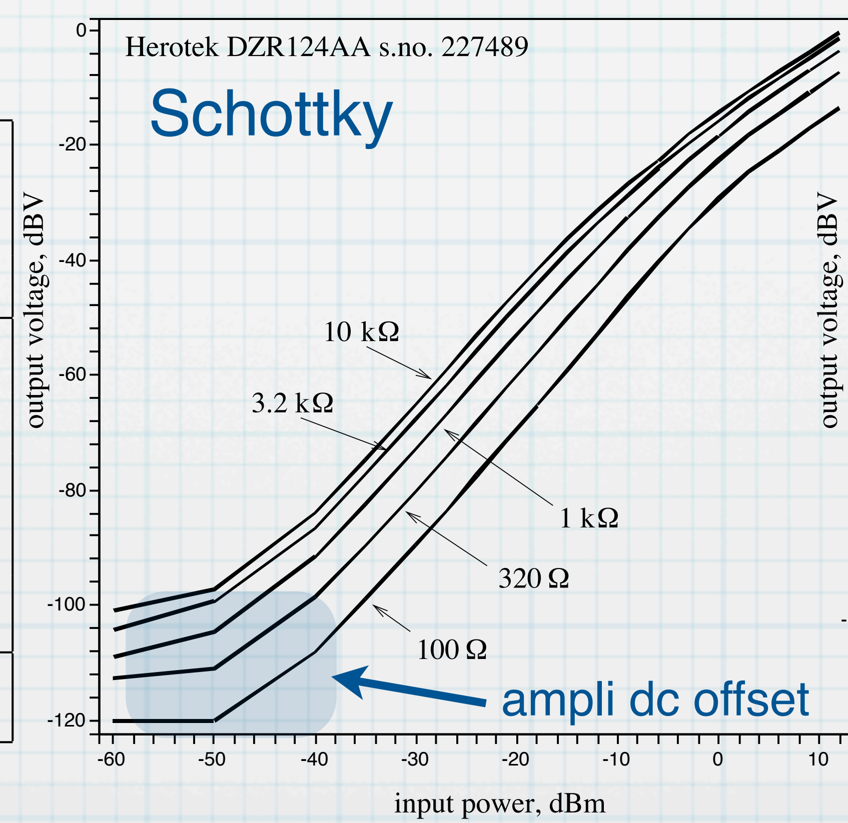


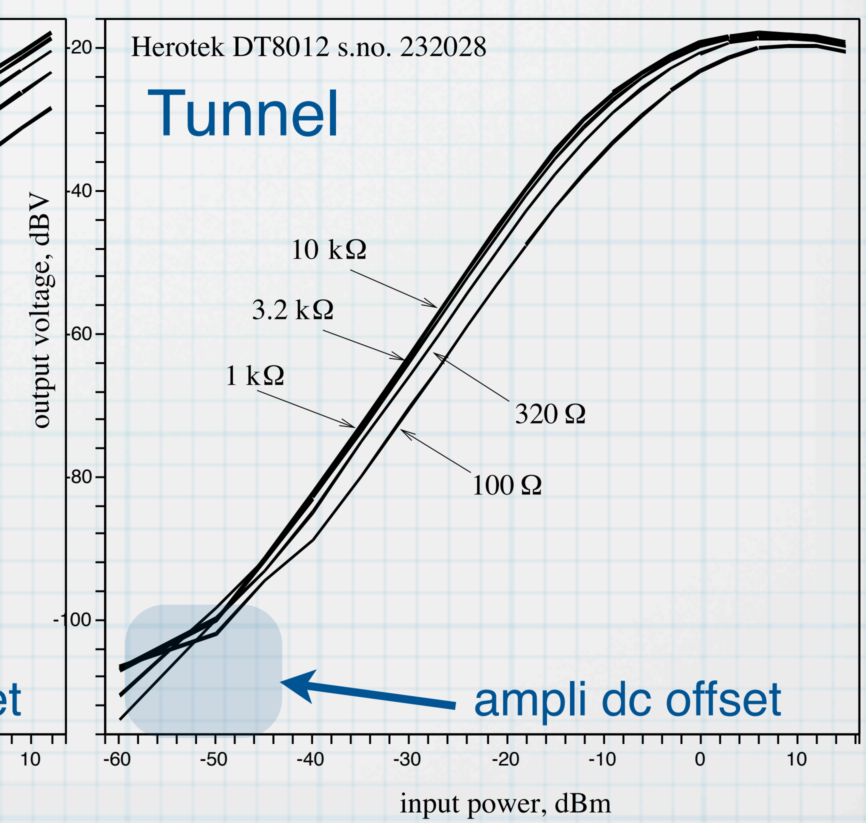
The “tunnel” diode is actually a backward diode. The negative resistance region is absent.

parameter	Schottky	tunnel
input bandwidth	up to 4 decades 10 MHz to 20 GHz	1–3 octaves up to 40 GHz
VSVR max.	1.5:1	3.5:1
max. input power (spec.)	−15 dBm	−15 dBm
absolute max. input power	20 dBm or more	20 dBm
output resistance	1–10 kΩ	50–200 Ω
output capacitance	20–200 pF	10–50 pF
gain	300 V/W	1000 V/W
cryogenic temperature	no	yes
electrically fragile	no	yes

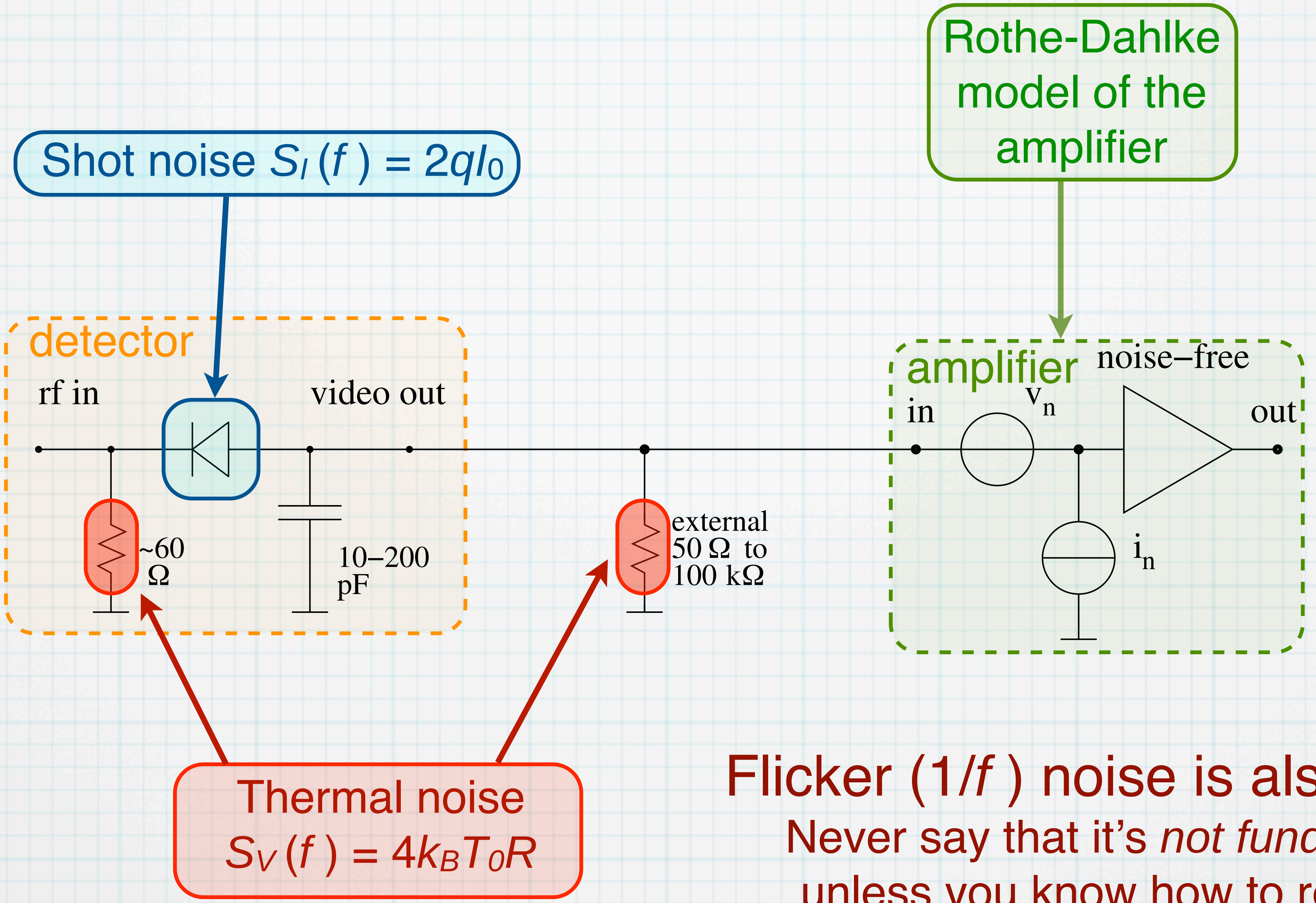
Measured

load resistance, Ω	detector gain, A^{-1} DZR124AA (Schottky)	detector gain, A^{-1} DT8012 (tunnel)
1×10^2	35	292
3.2×10^2	98	505
1×10^3	217	652
3.2×10^3	374	724
1×10^4	494	750
conditions: power −50 to −20 dBm		





Noise mechanisms



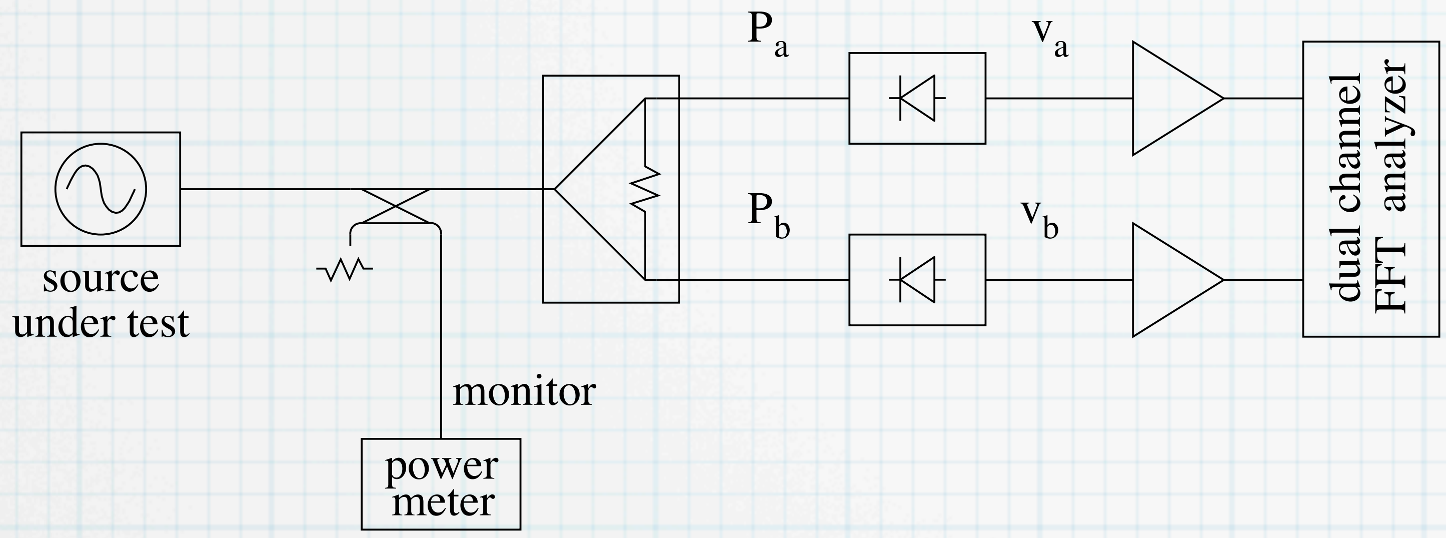
Flicker (1/*f*) noise is also present

Never say that it's *not fundamental*, unless you know how to remove it

In practice

the amplifier white noise turns out to be higher than the detector noise and the amplifier flicker noise is even higher

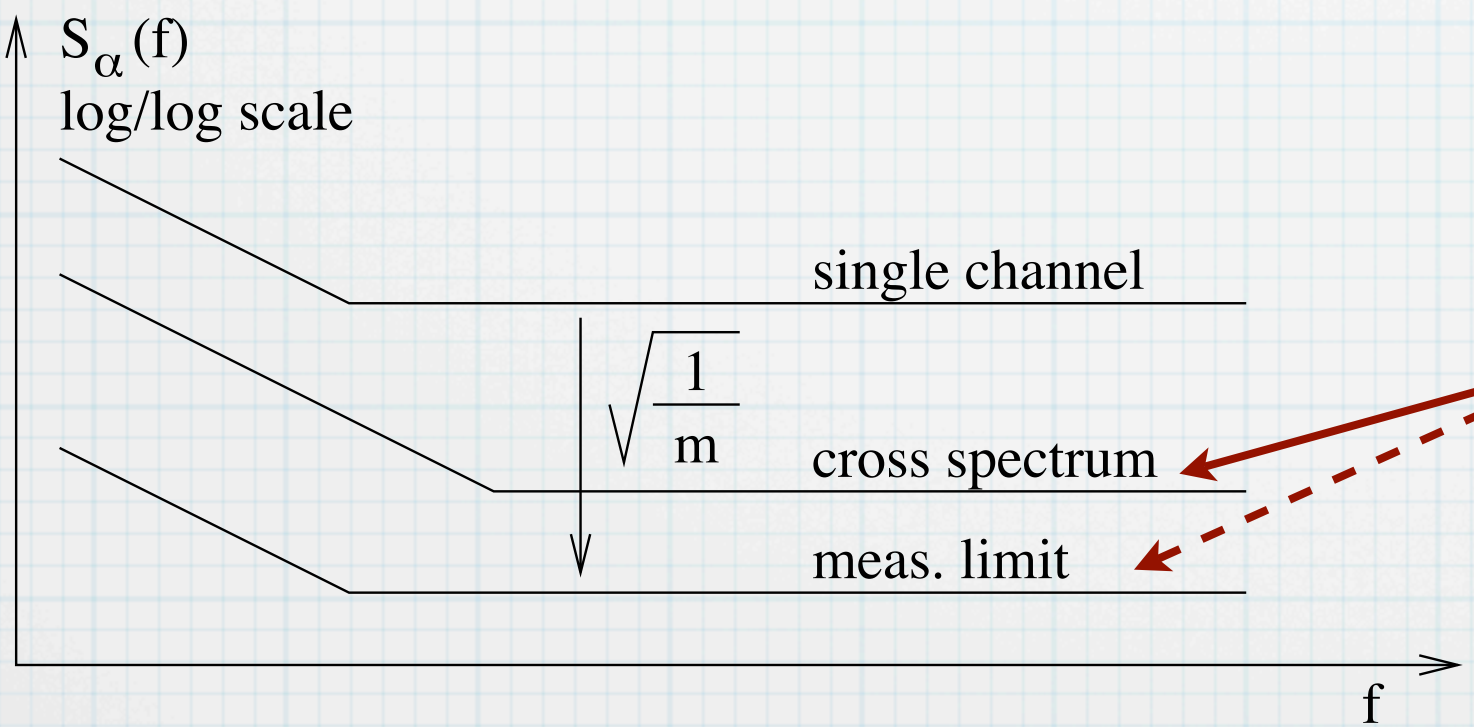
Cross-spectrum method



$$v_a(t) = 2k_a P_a \alpha(t) + \text{noise}$$
$$v_b(t) = 2k_a P_b \alpha(t) + \text{noise}$$

The cross spectrum $S_{ba}(f)$ rejects the single-channel noise because the two channels are independent.

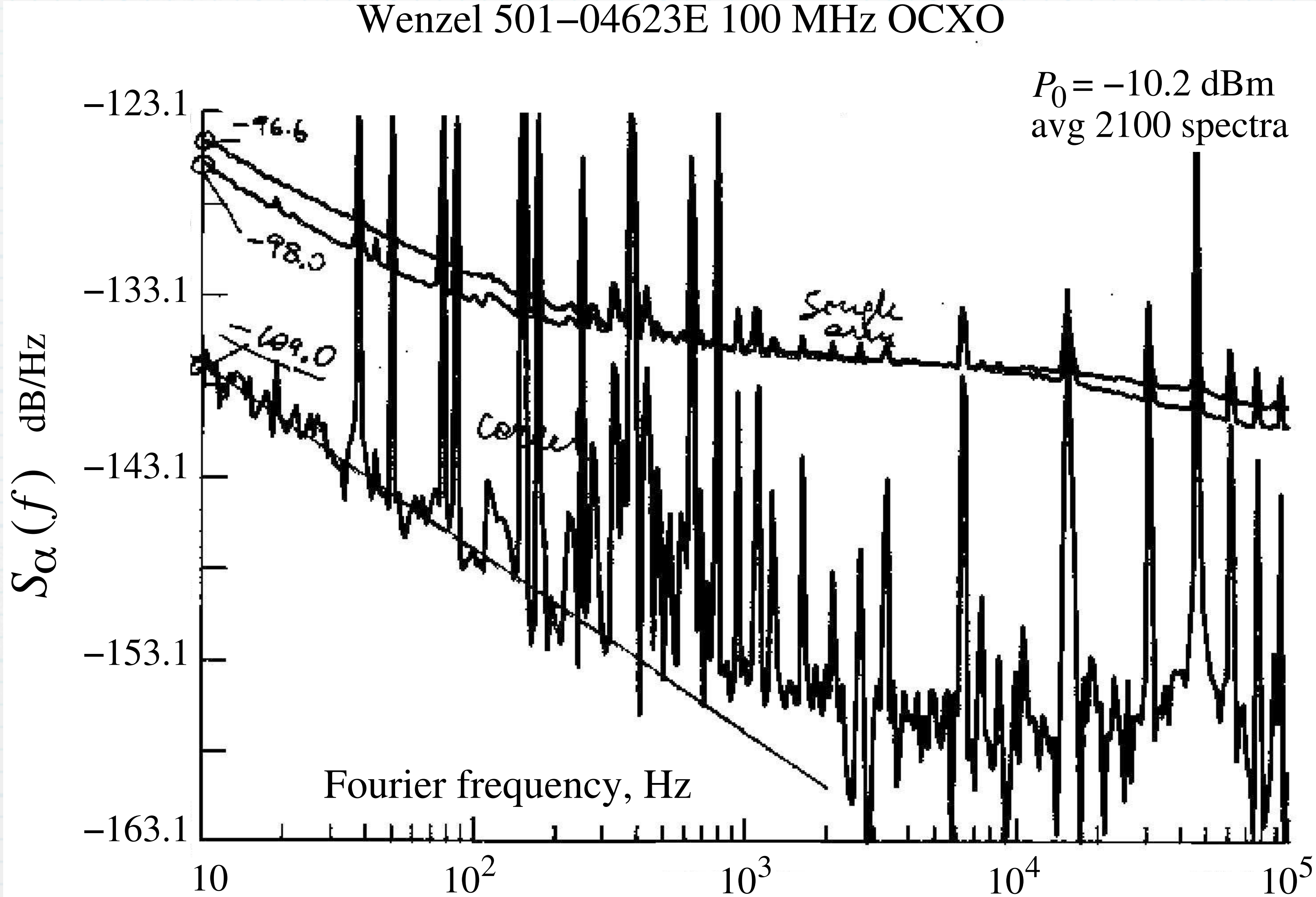
$$S_{ba}(f) = \frac{1}{4k_a k_b P_a P_b} S_\alpha(f)$$



$$\sqrt{\frac{1}{m}}$$

- Averaging on m spectra, the single-channel noise is rejected by √1/2m
- A cross-spectrum higher than the averaging limit validates the measure
- The knowledge of the single-channel noise is not necessary

Example of AM noise spectrum



flicker: $h_{-1} = 1.5{\times}10^{-13}\ \mathrm{Hz}^{-1}\ (-128.2\ \mathrm{dB}) \quad \Rightarrow \quad \sigma_\alpha = 4.6{\times}10^{-7}$

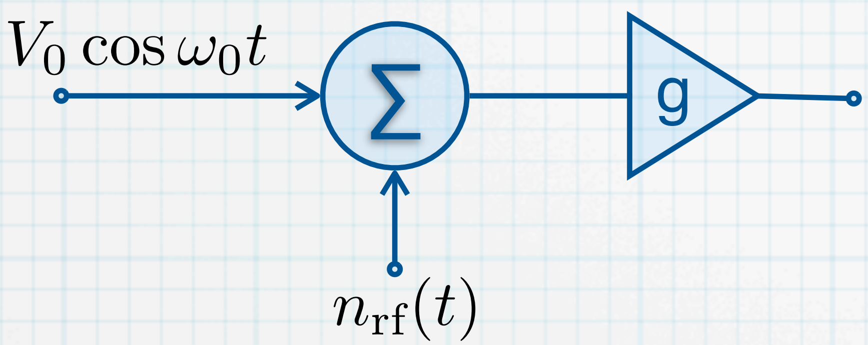
Single-arm 1/f noise is that of the dc amplifier
(the amplifier is still not optimized)

Noise in systems

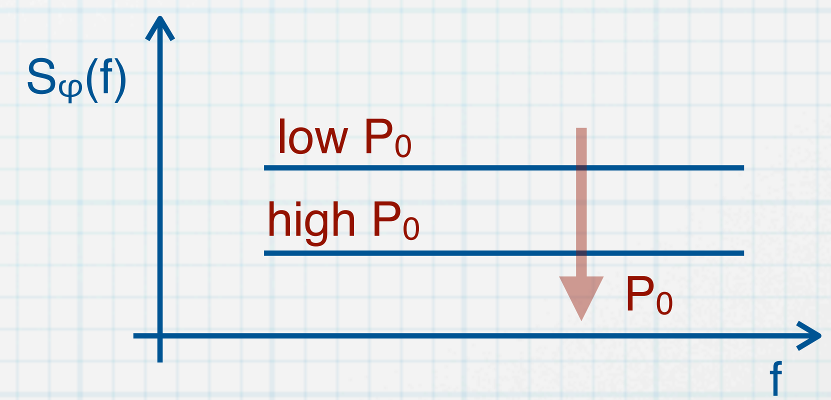
Additive (white) noise in amplifiers etc.

Noise figure F
Input power P_0

power law $$S_\varphi = \sum_{i=-4}^{0} b_i f^i$$



white phase noise $$b_0 = \frac{FkT_0}{P_0}$$



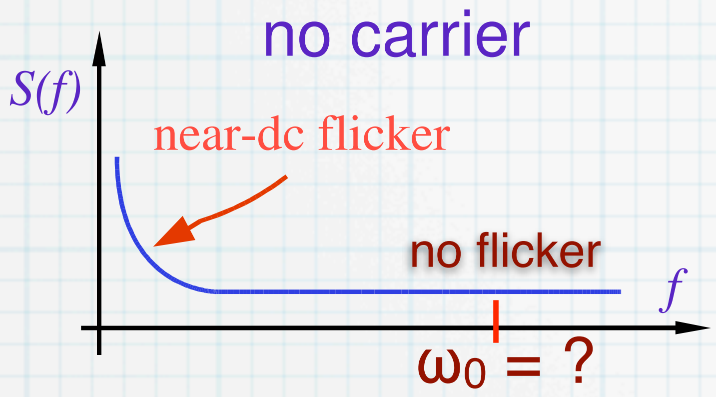
Cascaded amplifiers (Friis formula)

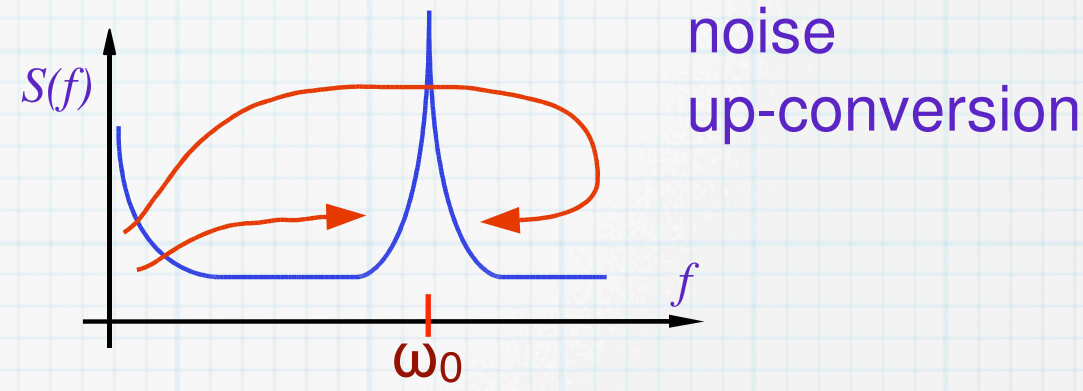
$$N = F_1 kT_0 + \frac{(F_2 - 1)kT_0}{g_1^2} + \dots$$

As a consequence, (phase) noise is chiefly that of the 1st stage

Parametric (flicker) noise in amplifiers etc.

parametric up-conversion of the near-dc noise





carrier + near-dc noise

$$v_i(t) = V_i\, e^{j\omega_0 t} + n'(t) + jn''(t)$$

the parametric nature of 1/f noise is hidden in n' and n''

substitute

(careful, this hides the down-conversion)

$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + \dots$$

non-linear amplifier

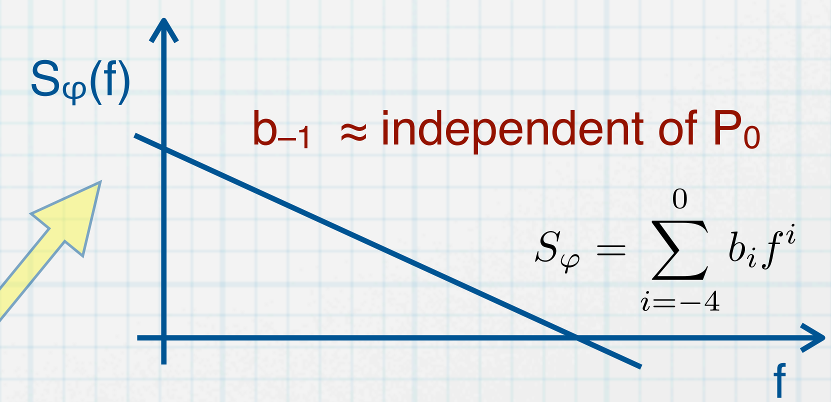
expand and select the ω_0 terms

$$v_o(t) = V_i \Big\{ a_1 + 2a_2 \big[n'(t) + jn''(t) \big] \Big\} e^{j\omega_0 t}$$

get AM and PM noise

$$\alpha(t) = 2\,\frac{a_2}{a_1}\,n'(t) \qquad \varphi(t) = 2\,\frac{a_2}{a_1}\,n''(t)$$

independent of V_i (!)



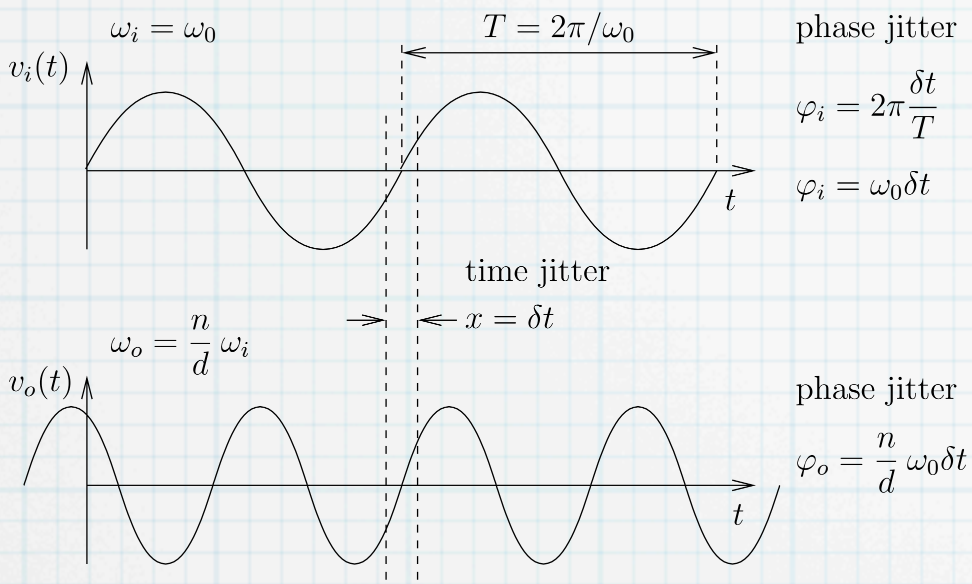
m cascaded amplifiers

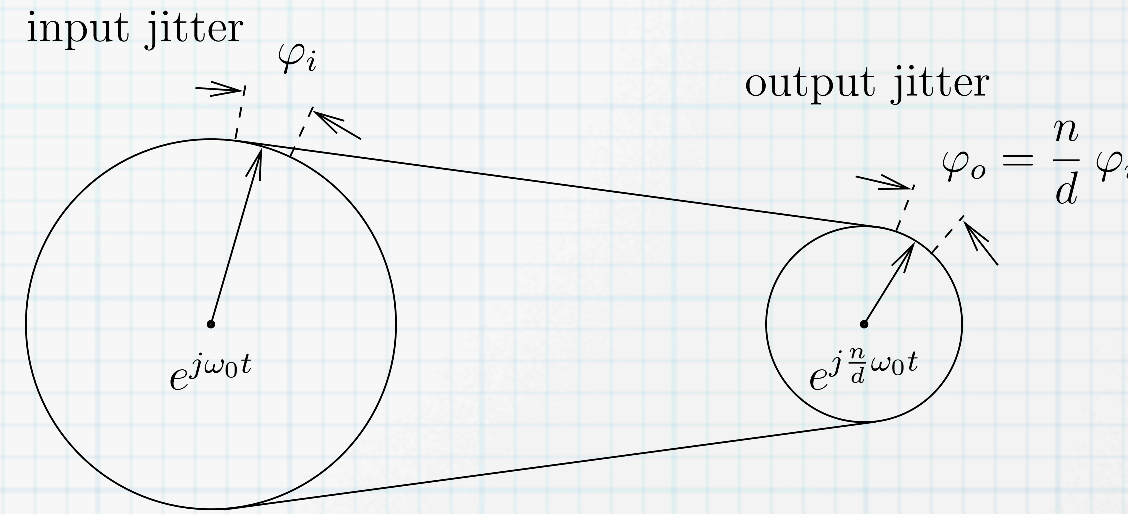
$$(b_{-1})_{\text{cascade}} = \sum_{i=1}^{m} (b_{-1})_i$$

In practice, each stage contributes ≈ equally

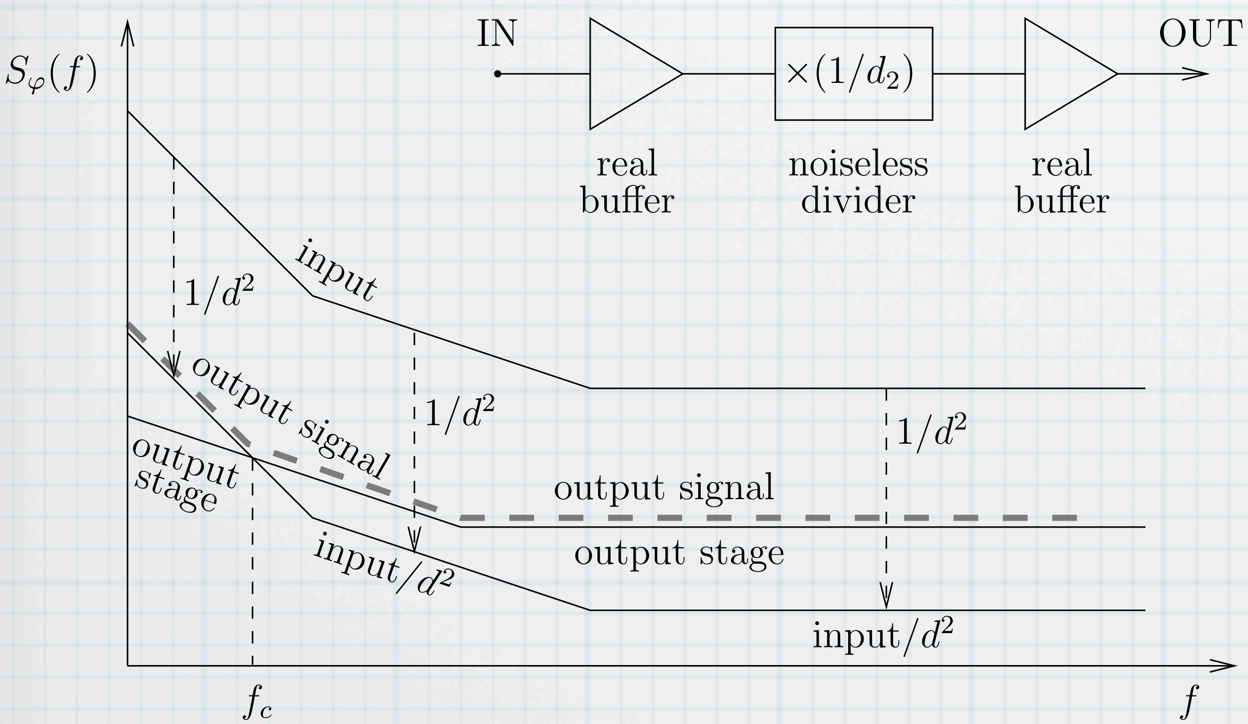
Frequency synthesis

The ideal noise-free frequency synthesizer repeats the input time jitter

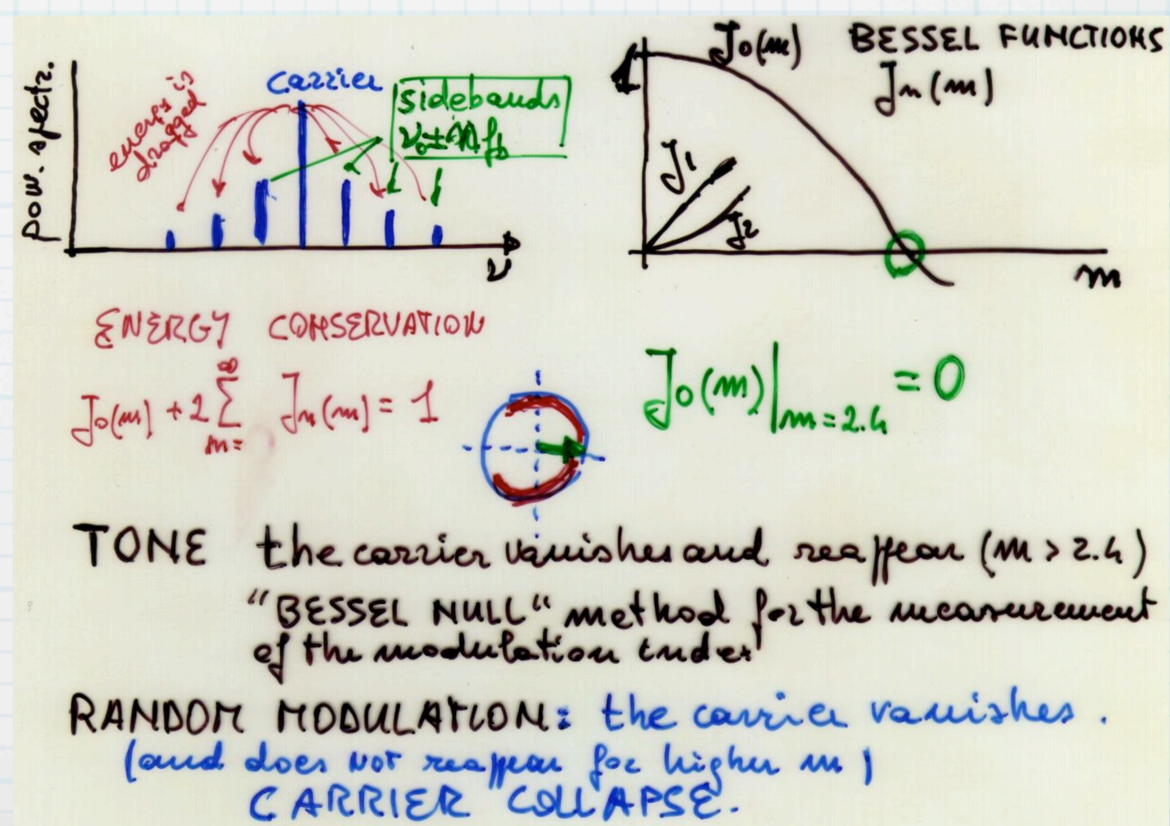




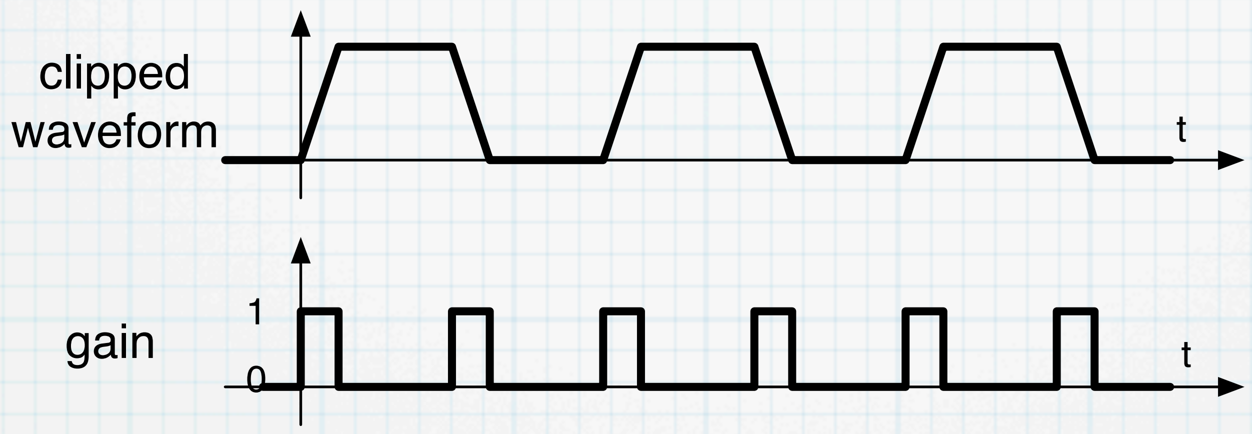
After division, the noise of the output buffer may be larger than the input-noise scaled down



After multiplication, the scaled-up phase noise sinks energy from the carrier. At m ≈ 2.4, the carrier vanishes



Saturation and sampling



Saturation is equivalent to reducing the gain

Digital circuits, for example, amplify (linearly) only during the transitions

Photodiode white noise

intensity modulation $P(t) = \overline{P}(1 + m \cos \omega_\mu t)$

photocurrent $i(t) = \frac{q\eta}{h\nu}\,\overline{P}(1 + m \cos \omega_\mu t)$

microwave power $\overline{P}_\mu = \frac{1}{2}\, m^2 R_0 \left(\frac{q\eta}{h\nu}\right)^2 P^2$

shot noise $N_s = 2\frac{q^2\eta}{h\nu}\,\overline{P} R_0$

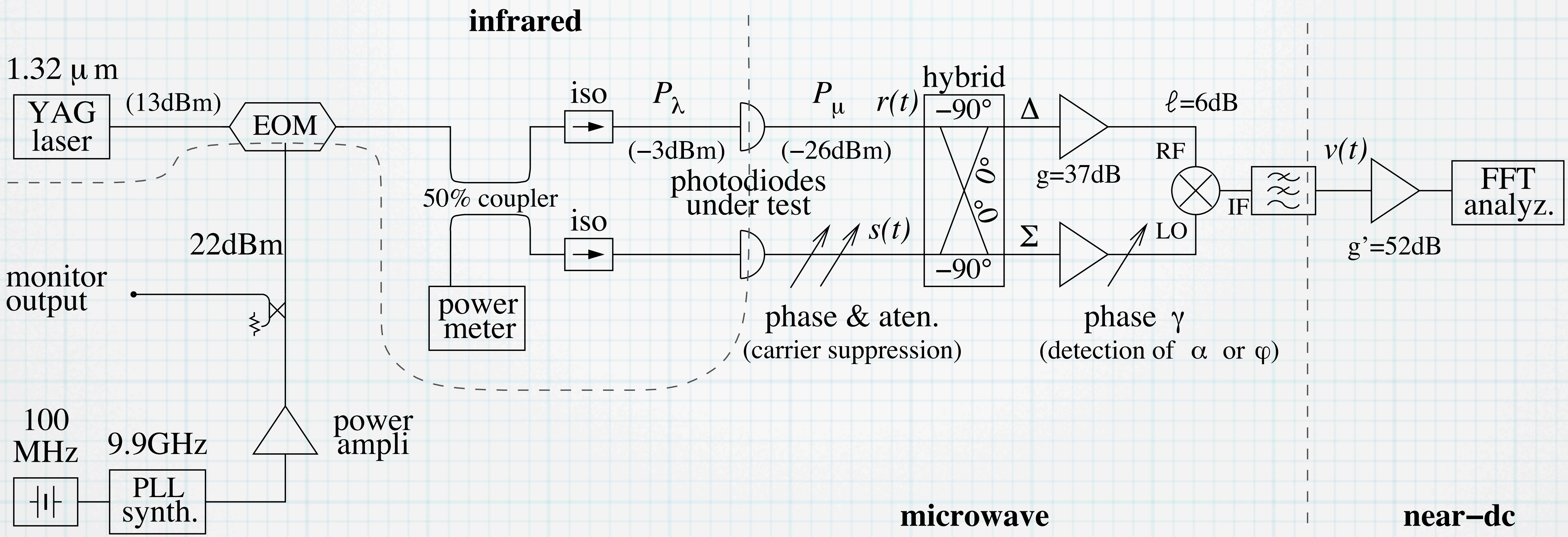
thermal noise $N_t = FkT_0$

total white noise (one detector)

$$S_{\varphi 0} = \frac{2}{m^2}\left[2\frac{h\nu_\lambda}{\eta}\,\frac{1}{\overline{P}} + \frac{FkT_0}{R_0}\left(\frac{h\nu_\lambda}{q\eta}\right)^2\left(\frac{1}{\overline{P}}\right)^2\right]$$

Threshold power ≈ 0.5–1 mW

Photodetector noise

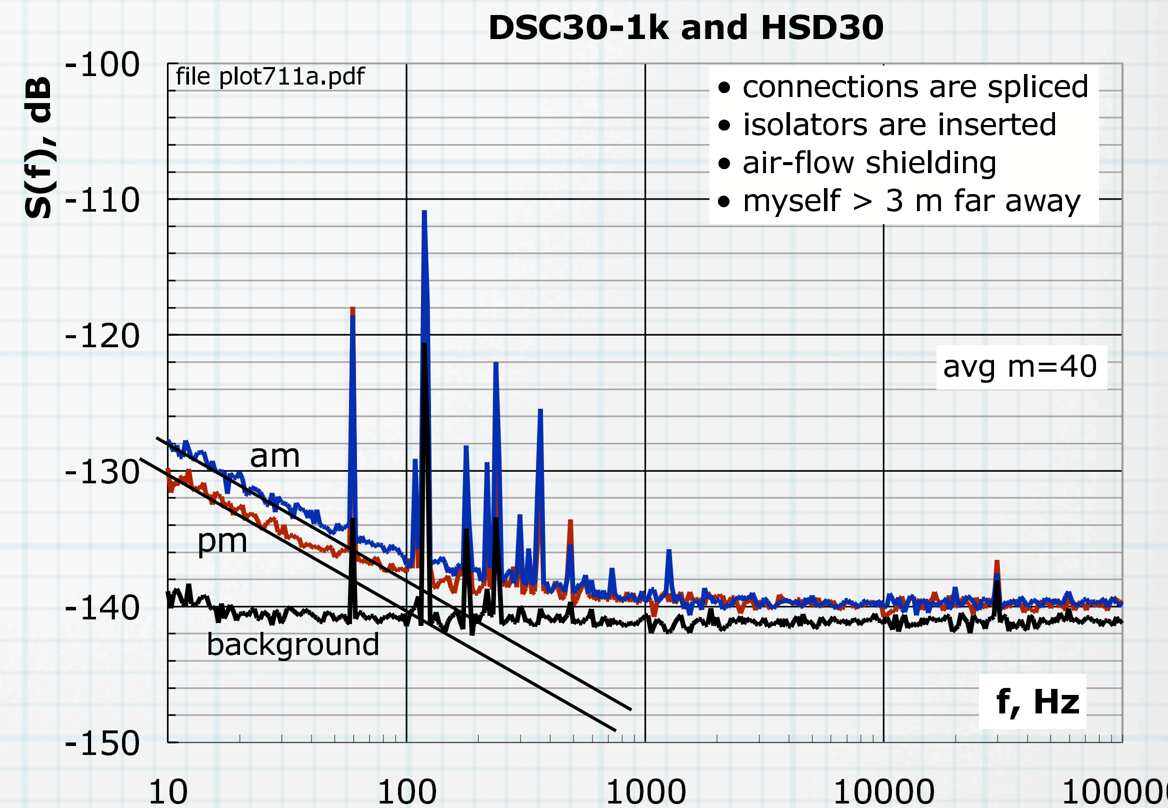


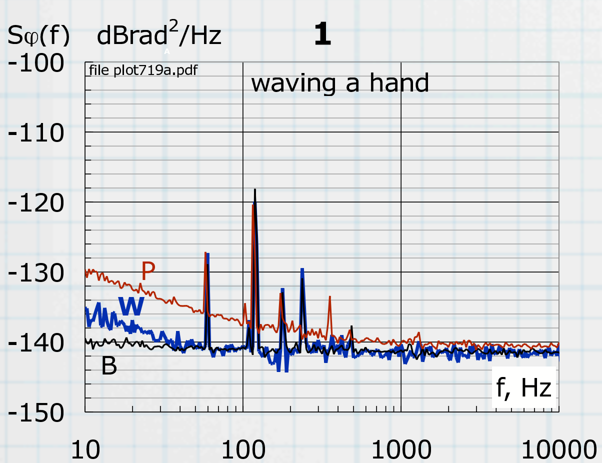
photodiode	$S_\alpha(1\,\mathrm{Hz})$ estimate	$S_\alpha(1\,\mathrm{Hz})$ uncertainty	$S_\varphi(1\,\mathrm{Hz})$ estimate	$S_\varphi(1\,\mathrm{Hz})$ uncertainty
HSD30	−122.7	−7.1 +3.4	−127.6	−8.6 +3.6
DSC30-1K	−119.8	−3.1 +2.4	−120.8	−1.8 +1.7
QDMH3	−114.3	−1.5 +1.4	−120.2	−1.7 +1.6
unit	dB/Hz	dB	dBrad^2/Hz	dB

The noise of the Σ amplifier is not detected Electron. Lett. **39** 19 p. 1389 (2003)

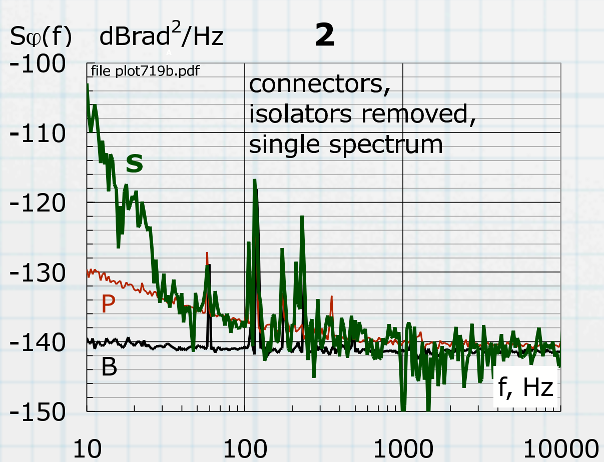
Photodetector noise

- the photodetectors we measured are similar in AM and PM 1/f noise
- the 1/f noise is about -120 dB[rad²]/Hz
- other effects are easily mistaken for the photodetector 1/f noise
- environment and packaging deserve attention in order to take the full benefit from the low noise of the junction

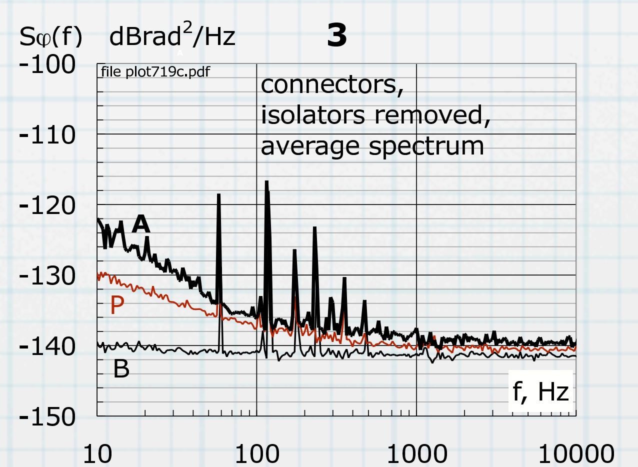




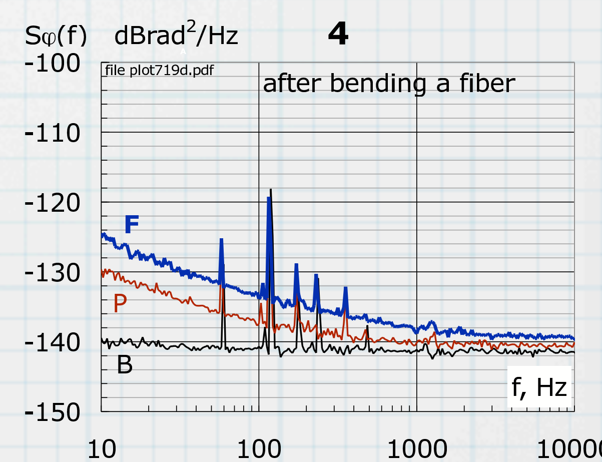
W: waving a hand 0.2 m/s, 3 m far from the system
B: background noise
P: photodiode noise



S: single spectrum, with optical connectors and no isolators
B: background noise
P: photodiode noise



A: average spectrum, with optical connectors and no isolators
B: background noise
P: photodiode noise



F: after bending a fiber, 1/f noise can increase unpredictably
B: background noise
P: photodiode noise

Physical phenomena in optical fibers

Birefringence. Common optical fibers are made of amorphous Ge-doped silica, for an ideal fiber is not expected to be birefringent. Nonetheless, actual fibers show birefringent behavior due to a variety of reasons, namely: core ellipticity, internal defects and forces, external forces (bending, twisting, tension, kinks), external electric and magnetic fields. The overall effect is that light propagates through the fiber core in a non-degenerate, orthogonal pair of axes at different speed. Polarization effects are strongly reduced in polarization maintaining (PM) fibers. In this case, the cladding structure stresses the core in order to increase the difference in refraction index between the two modes.

Rayleigh scattering. This is random scattering due to molecules in a disordered medium, by which light looses direction and polarization. A small fraction of the light intensity is thereby back-scattered one or more times, for it reaches the fiber end after a stochastic to-and-fro path, which originates phase noise. In the early fibers it contributed 0.1 dB/km to the optical loss.

Bragg scattering. In the presence of monocromatic light (usually X-rays), the periodic structure of a crystal turns the randomness of scattering into an interference pattern. This is a weak phenomenon at micron wavelengths because the inter-atom distance is of the order of 0.3--0.5 nm. Bragg scattering is not present in amorphous materials.

Brillouin scattering. In solids, the photon-atom collision involves the emission or the absorption of an acoustic phonon, hence the scattered photons have a wavelength slightly different from incoming photons. An exotic form of Brillouin scattering has been reported in optical fibers, due to a transverse mechanical resonance in the cladding, which stresses the core and originates a noise bump on the region of 200--400 MHz.

Raman scattering. This phenomenon is somewhat similar to Rayleigh scattering, but the emission or the absorption of an optical phonon.

Kerr effect. This effect states that an electric field changes the refraction index. So, the electric field of light modulate the refraction index, which originates the 2nd-order nonlinearity.

Discontinuities. Discontinuities cause the wave to be reflected and/or to change polarization. As the pulse can be split into a pulse train depending on wavelength, this effect can turn into noise.

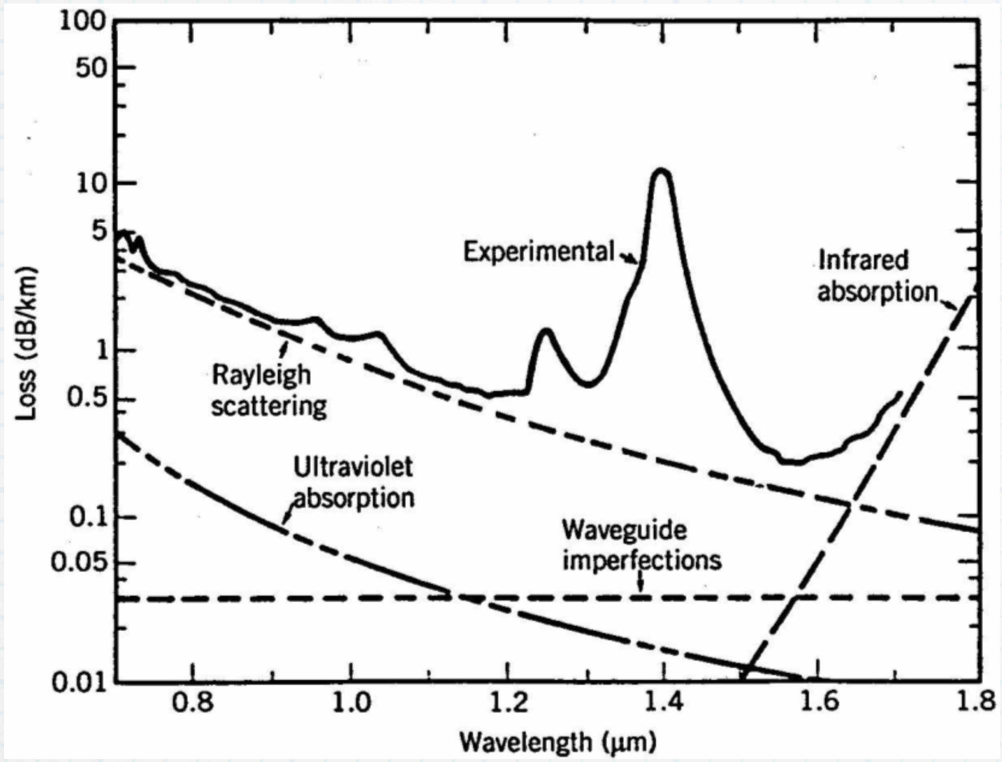
Group delay dispersion (GVD). There exist dispersion-shifted fibers, that have a minimum GVD at 1550 nm. GVD compensators are also available.

Polarization mode dispersion (PMD). This effect rises from the random birefringence of the optical fiber. The optical pulse can choose many different paths, for it broadens into a bell-like shape bounded by the propagation times determined by the highest and the lowest refraction index. Polarization vanishes exponentially along the light path. It is to be understood that PMD results from the vector sum over multiple forward paths, for it yields a well-shaped dispersion pattern.

PMD-Kerr compensation. In principle, it is possible that PMD and Kerr effect null one another. This requires to launch the appropriate power into each polarization mode, for two power controllers are needed. Of course, this is incompatible with PM fibers.

Which is the most important effect? In the community of optical communications, PMD is considered the most significant effect. Yet, this is related to the fact that excessive PMD increases the error rate and destroys the eye pattern of a channel. In the case of the photonic oscillator, the signal is a pure sinusoid, with no symbol randomness. My feeling is that Rayleigh scattering is the most relevant stochastic phenomenon.

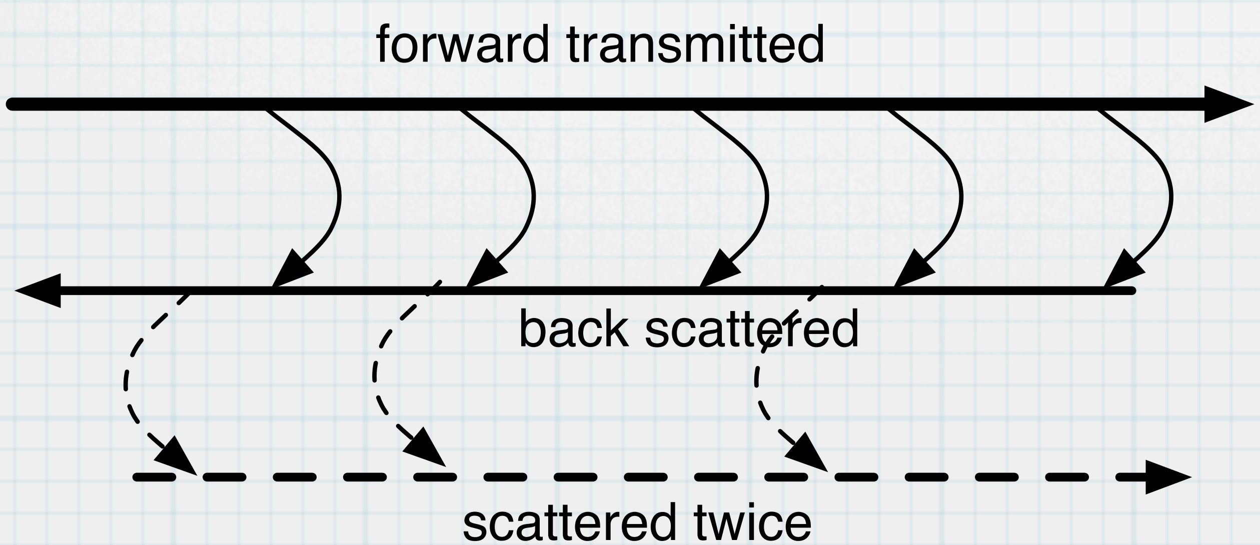
Rayleigh scattering



Rayleigh scattering contributes some 0.1 dB/km to the loss

G. Agrawal, *Fiber-optic communications systems*, Wiley 1997

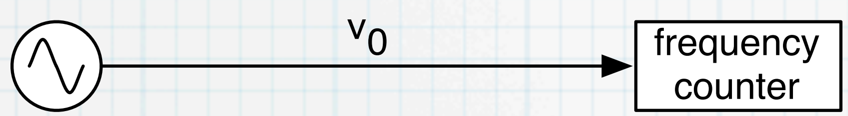
Stochastic scattering



Time-domain methods

Time-domain measurements

direct measurement

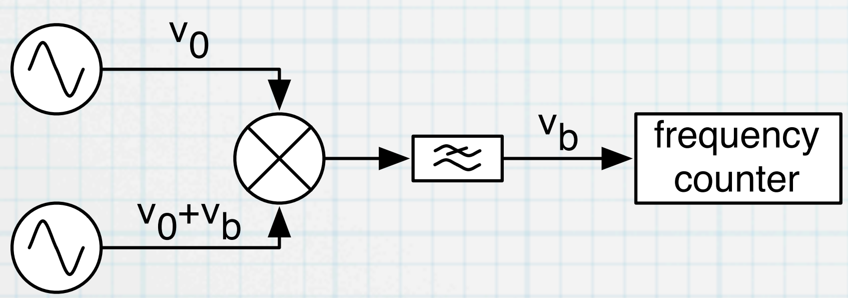


Resolution limited by the counter quantization
Example: resolution 100 ps => $\sigma_y(1s) = 10^{-10}$

Beat methods

The beat mechanism keeps ϕ, for it amplifies the phase-time x by factor ν_0/ν_b

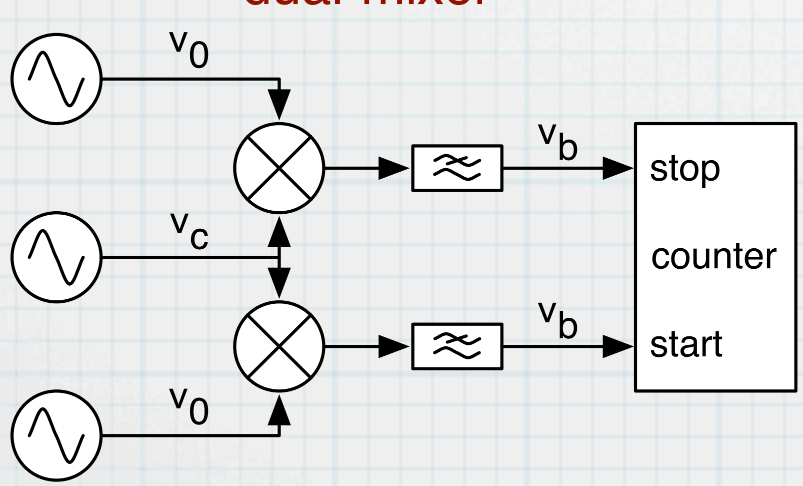
beat-note



Counter resolution improved by a factor ν_0/ν_b
Example: counter 1ns ν_0=10MHz, ν_b=10Hz
$\sigma_y(1s) = 10^{-9}/10^{6} = 10^{-15}$ [plus trigger noise]

Need a frequency offset ν_b

dual-mixer



Counter resolution improved by a factor ν_0/ν_b
Does not need the frequency offset ν_b
The noise of the common oscillator cancels