Phase Noise

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Summary

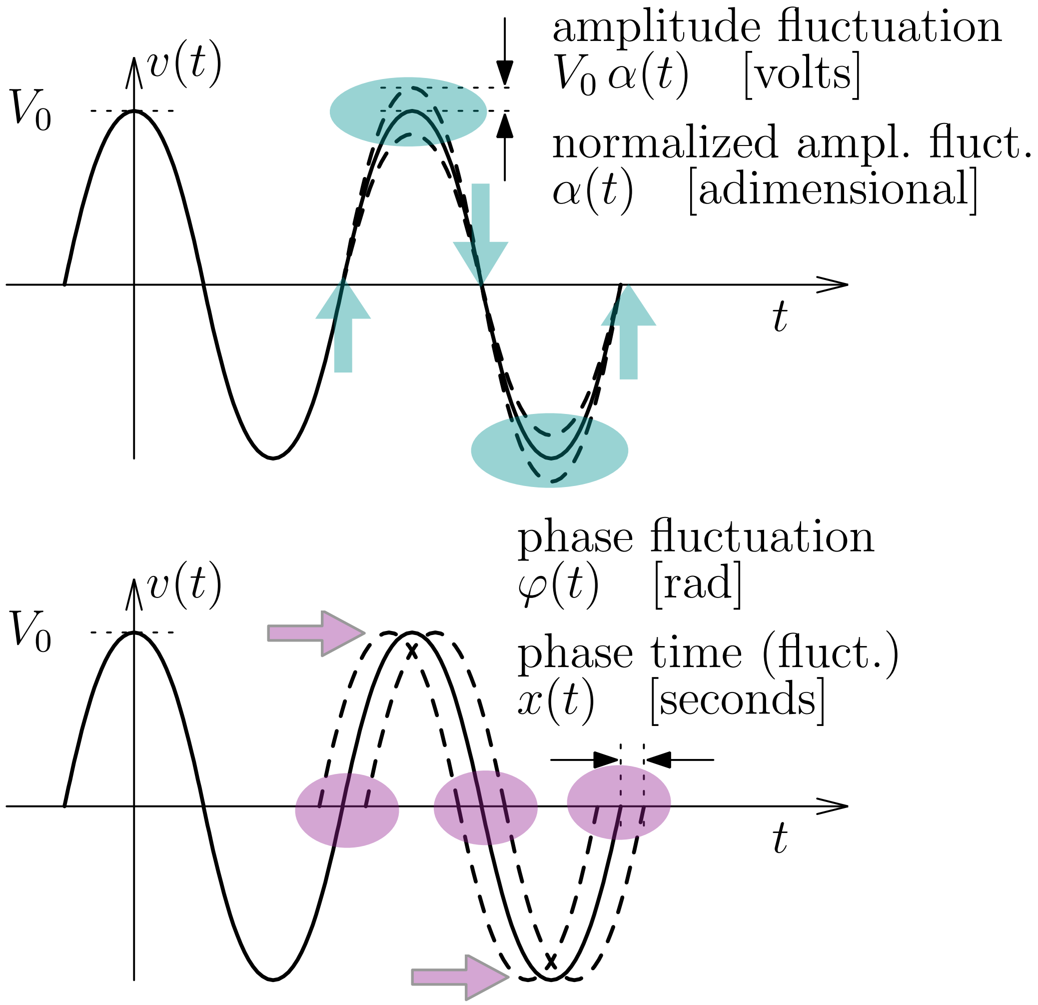
www.rubiola.org

you can download *this presentation*, an *e-book* on the Leeson effect, and some other documents on noise (amplitude and phase) and on precision electronics from my web page

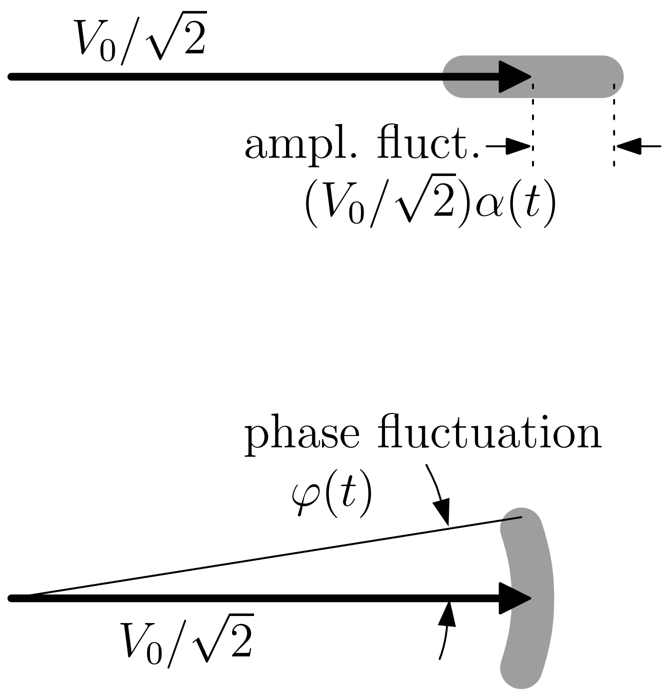
1 – Introduction

Representations of a sinusoid with noise

Time Domain



Phasor Representation



polar coordinates $\quad v(t) = V_0\,[1+\alpha(t)]\cos[\omega_0 t + \varphi(t)]$

Cartesian coordinates $\quad v(t) = V_0 \cos\omega_0 t + n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t$

under low noise approximation

$$|n_c(t)| \ll V_0 \quad \text{and} \quad |n_s(t)| \ll V_0$$

It holds that

$$\alpha(t) = \frac{n_c(t)}{V_0} \quad \text{and} \quad \varphi(t) = \frac{n_s(t)}{V_0}$$

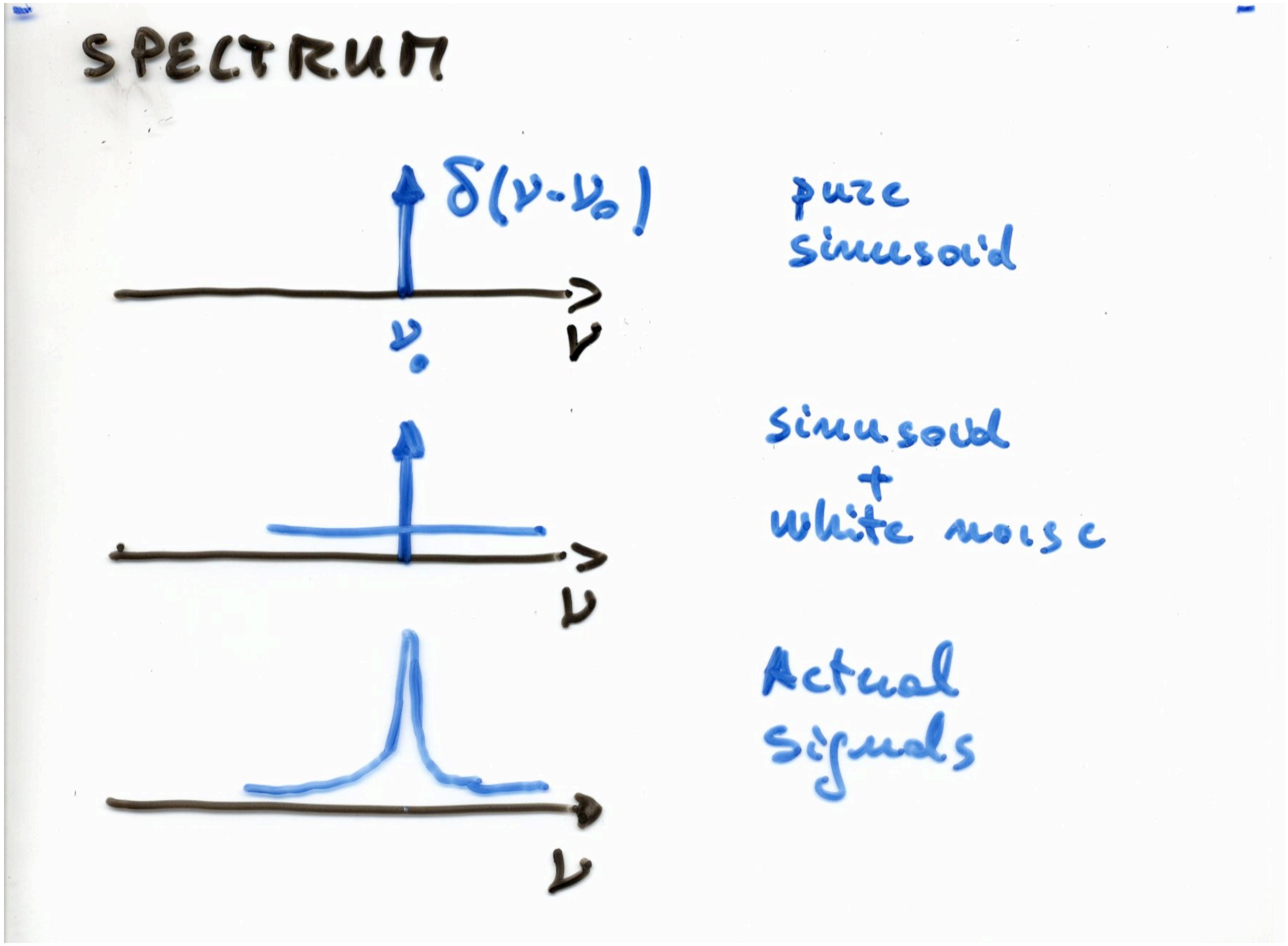
Noise broadens the spectrum

$$v(t) = V_0\,[1 + \alpha(t)] \cos\,[\omega_0 t + \varphi(t)]$$

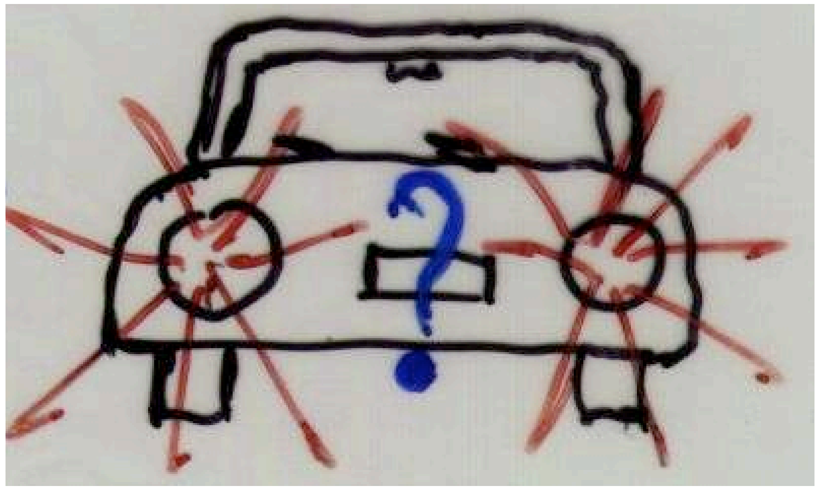
$$v(t) = V_0 \cos\omega_0 t + n_c(t) \cos\omega_0 t - n_s(t) \sin\omega_0 t$$

$$\omega = 2\pi\nu$$

$$\nu = \frac{\omega}{2\pi}$$



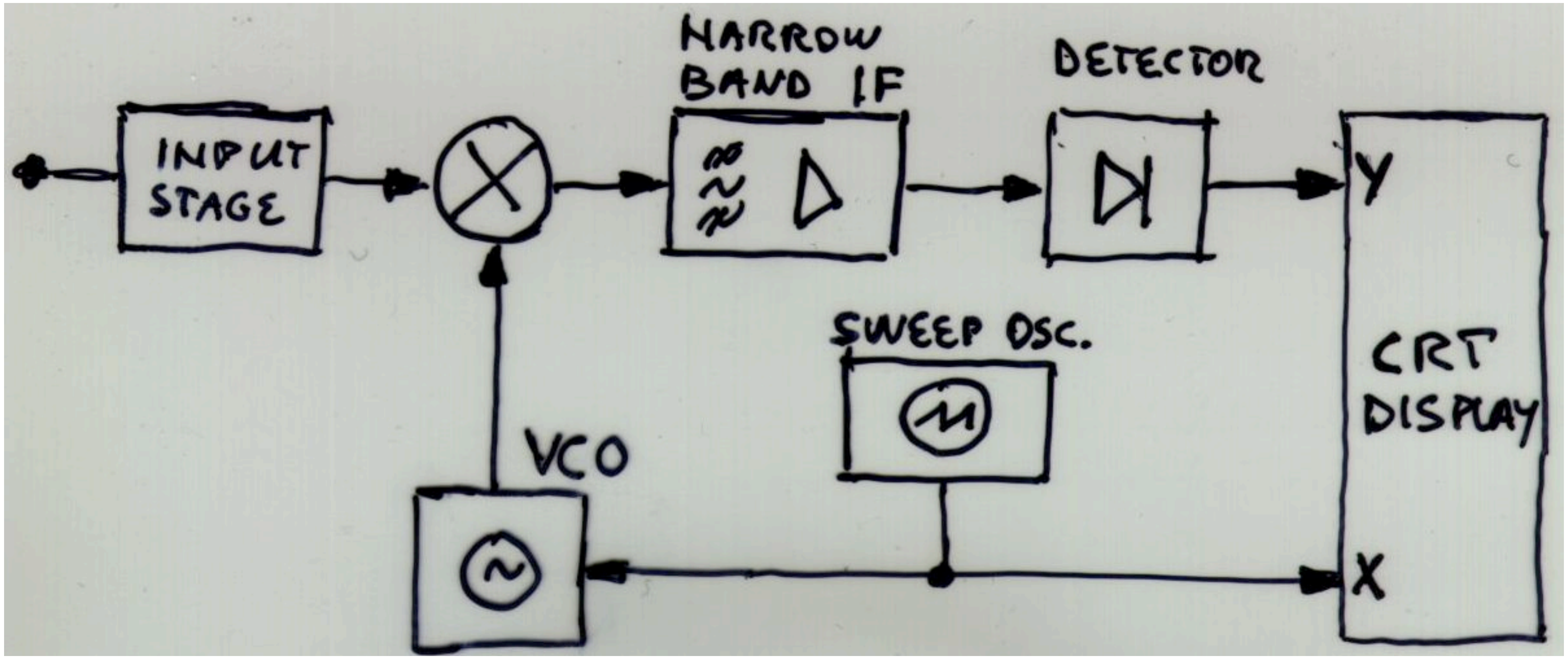
Basic problem: how can we measure a low random signal (noise sidebands) close to a strong dazzling carrier?



solution(s): suppress the carrier and measure the noise

convolution (low-pass)	$s(t) * h_{lp}(t)$	distorsiometer, audio-frequency instruments
time-domain product	$s(t) \times r(t - T/4)$	traditional instruments for phase-noise measurement (saturated mixer)
vector difference	$s(t) - r(t)$	bridge (interferometric) instruments

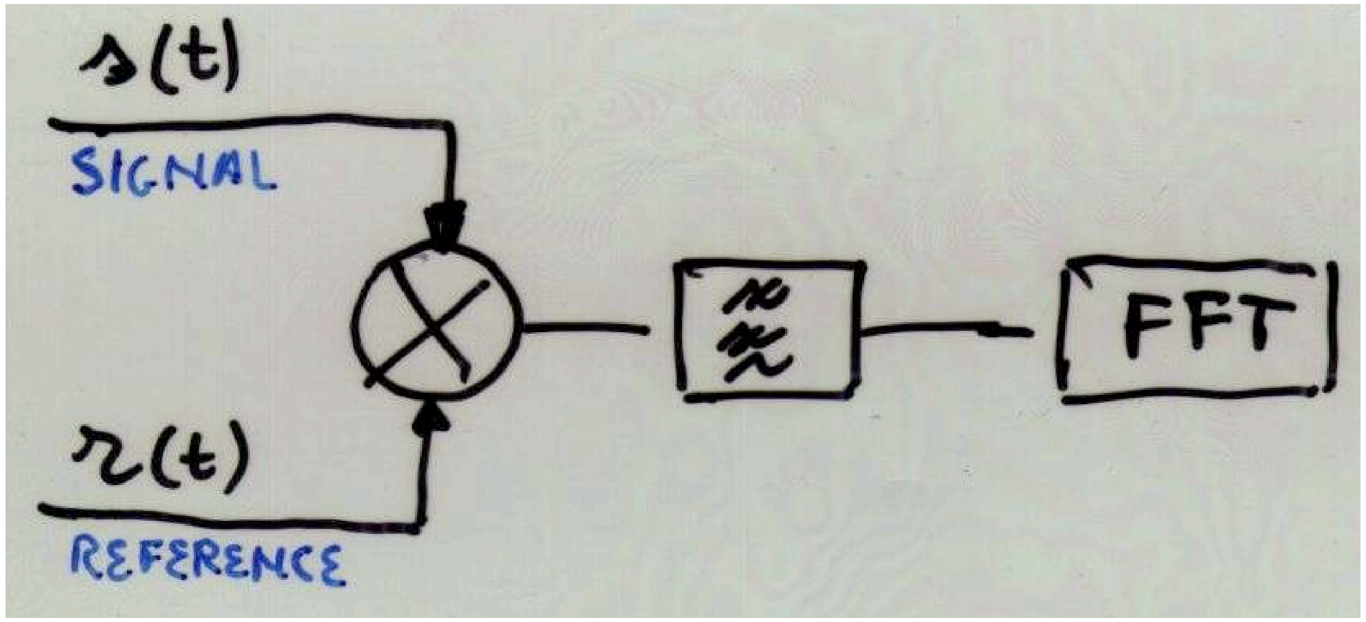
Why a spectrum analyzer does not work?



1. too wide IF bandwidth
2. noise and instability of the conversion oscillator (VCO)
3. detects both AM and PM noise
4. insufficient dynamic range

Some commercial analyzers provide phase noise measurements, yet limited (at least) by the oscillator stability

The Schottky-diode double-balanced mixer saturated at both inputs is the most used phase detector



signal $$s(t) = \sqrt{2R_0P_0}\,\cos\left[2\pi\nu_0 t + \varphi(t)\right]$$

reference $$r(t) = \sqrt{2R_0P_0}\,\cos\left[2\pi\nu_0 t + \pi/2\right]$$

filtered out

product $$r(t)s(t) = k_\varphi\varphi(t) + \text{“}2\nu_0\text{” terms}$$

The AM noise is rejected by saturation

Saturation also account for the phase-to-voltage gain k_φ

2 – Spectra

Power spectrum density

In general, the power spectrum density $S_v(f)$ of a random process $v(t)$ is defined as

$$S_v(f) = \mathbb{E}\Big\{\mathcal{F}\{\mathcal{R}_v(t_1,t_2)\}\Big\}$$

$\mathbb{E}\{\,\cdot\,\}$	statistical expectation
$\mathcal{F}\{\,\cdot\,\}$	Fourier transform
$\mathcal{R}_v(t_1,t_2)$	autocorrelation function

In practice, we measure $S_v(f)$ as

$$S_v(f) = \big|\mathcal{F}\{v(t)\}\big|^2$$

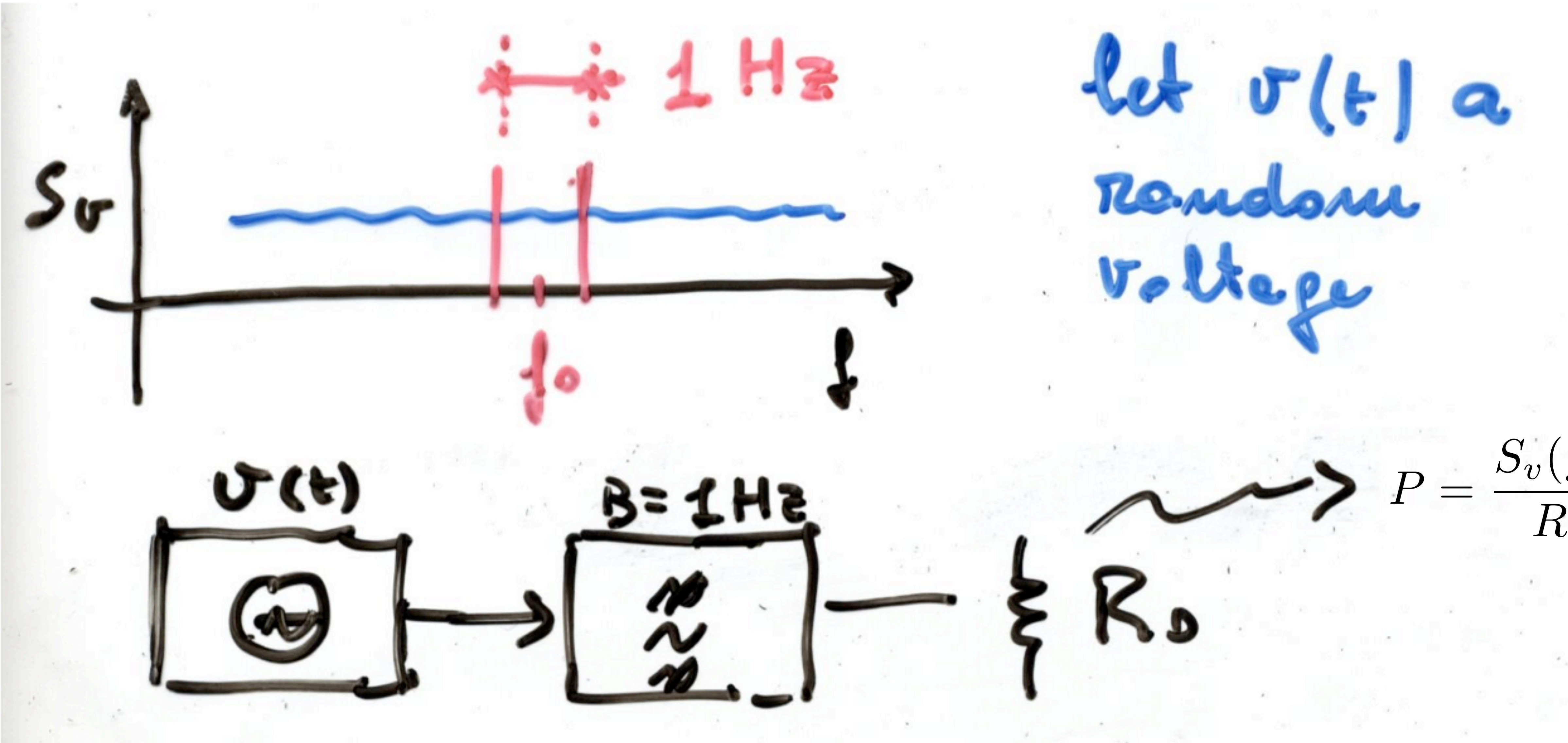
This is possible (Wiener-Khinchin theorem) with ergodic processes

In many real-life cases, processes are ergodic and stationary

Ergodicity: ensemble and time-domain statistics can be interchanged. This is the formalization of the reproducibility of an experiment

Stationarity: the statistics is independent of the origin of time. This is the formalization of the repeatability of an experiment

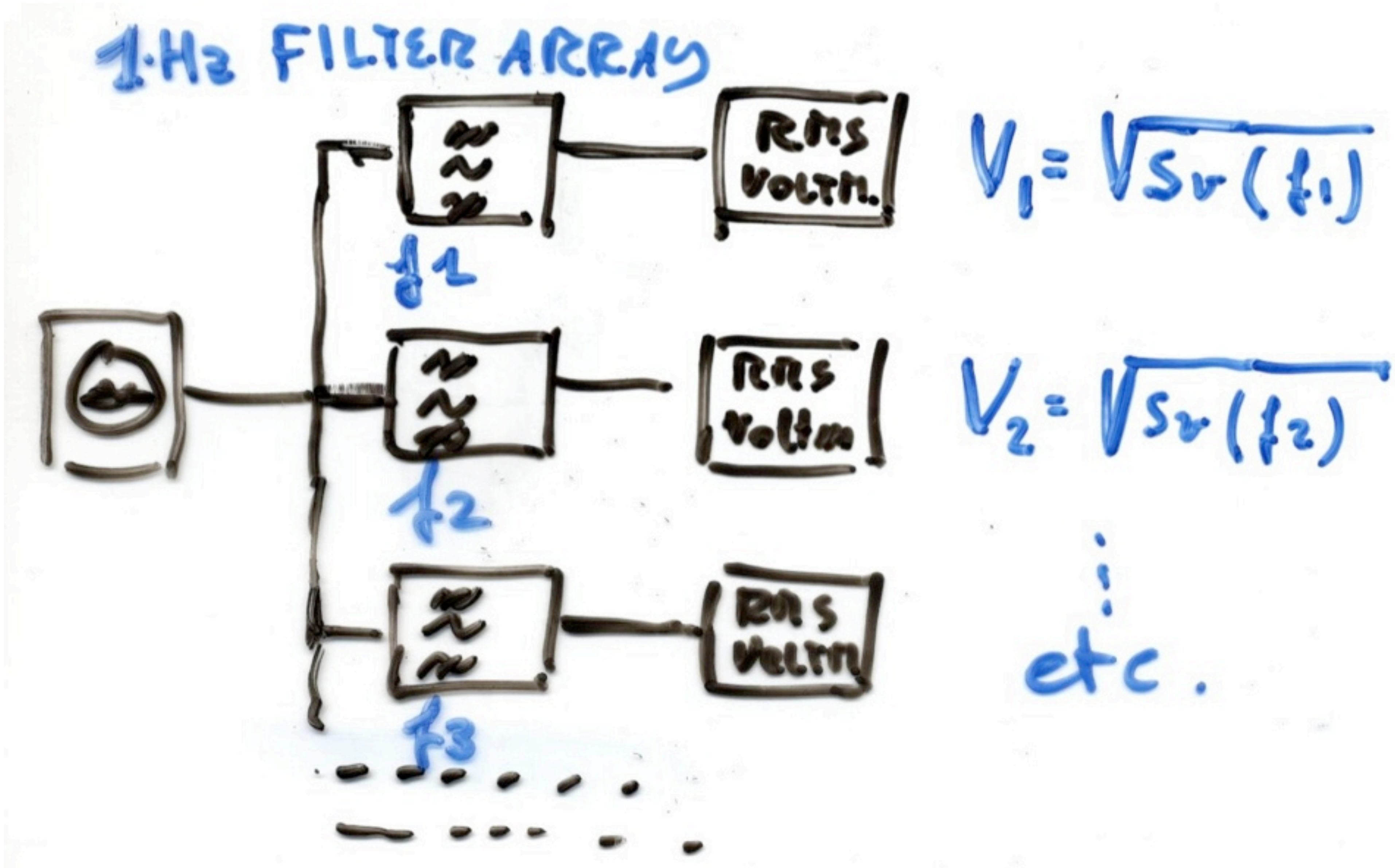
Physical meaning of the power spectrum density



$$P = \frac{S_v(f_0)}{R_0}$$

$\dfrac{S_v(f_0)}{R_0}$ power in 1 Hz bandwidth dissipated by R_0

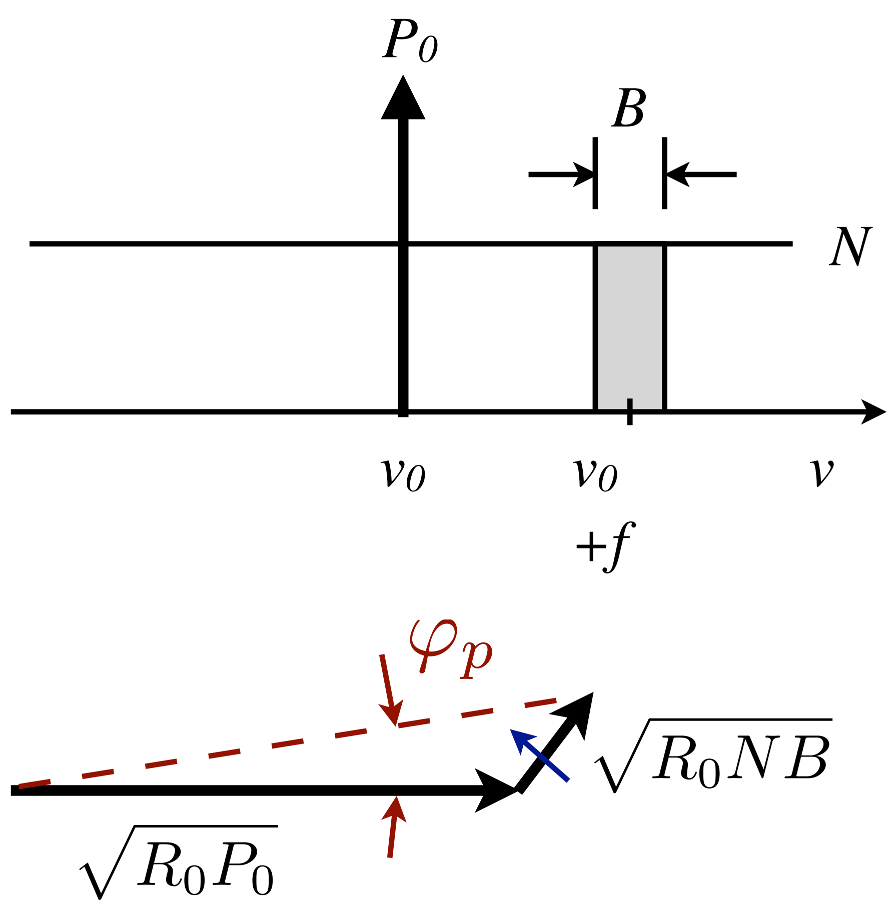
Physical meaning of the power spectrum density



The power spectrum density extends the concept of root-mean-square value to the frequency domain

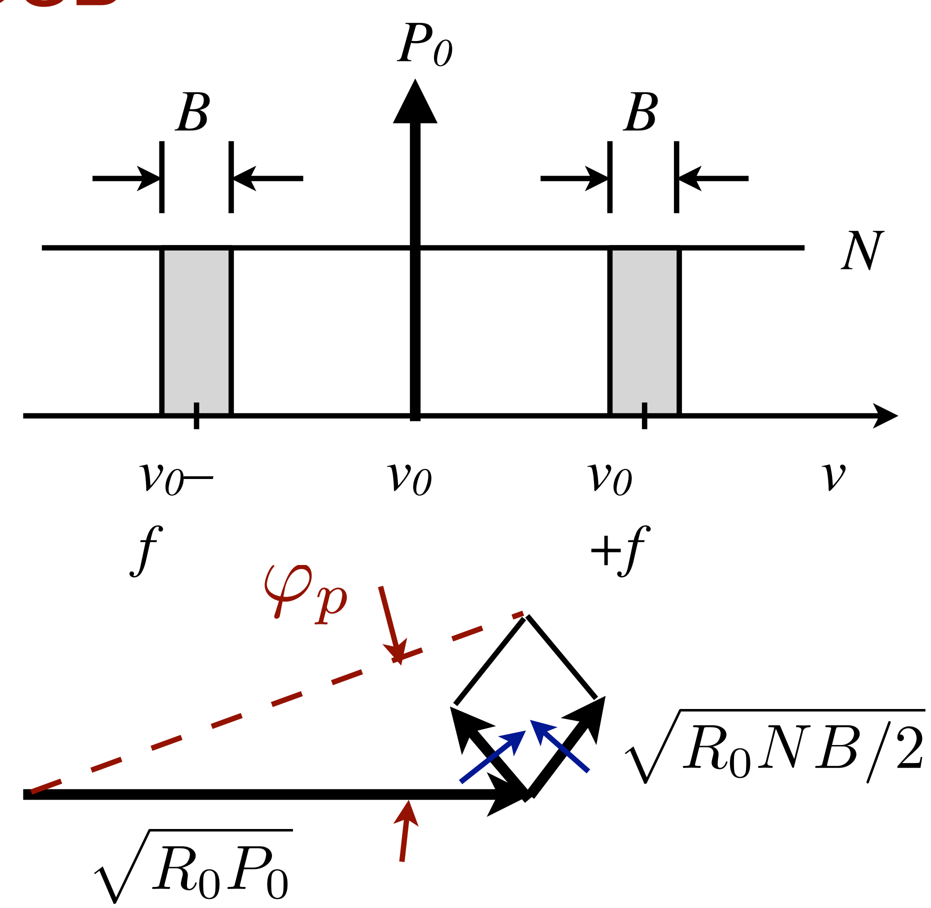
S_φ(f) and $\mathscr{L}$(f) in the presence of (white) noise

SSB



$$\varphi_{\text{rms}} = \sqrt{\frac{1}{2}}\,\varphi_p = \sqrt{\frac{NB}{2P_0}}$$

DSB



$$\varphi_{\text{rms}} = \sqrt{\frac{1}{2}}\,\varphi_p = \sqrt{\frac{NB}{P_0}}$$

dBc/Hz $\quad \mathcal{L} = \dfrac{N}{2P_0}$

3 dB

$S_\varphi = \dfrac{N}{P_0}$ $\quad$ dBrad²/Hz

$\mathscr{L}(f)$ (re)defined

The first definition of $\mathscr{L}(f)$ was

$\mathscr{L}(f)$ = (SSB power in 1Hz bandwidth) / (carrier power)

The problem with this definition is that it does not divide AM noise from PM noise, which yields to ambiguous results

Engineers (manufacturers even more) like $\mathscr{L}(f)$

The IEEE Std 1139-1999 redefines $\mathscr{L}(f)$ as

$$\mathscr{L}(f) = (1/2) \times S_\varphi(f)$$

Useful quantities

phase time

$$x(t) = \frac{1}{2\pi\nu_0}\,\varphi(t)$$

$x(t)$ is the phase noise converted into time fluctuation
physical dimension: time (seconds)

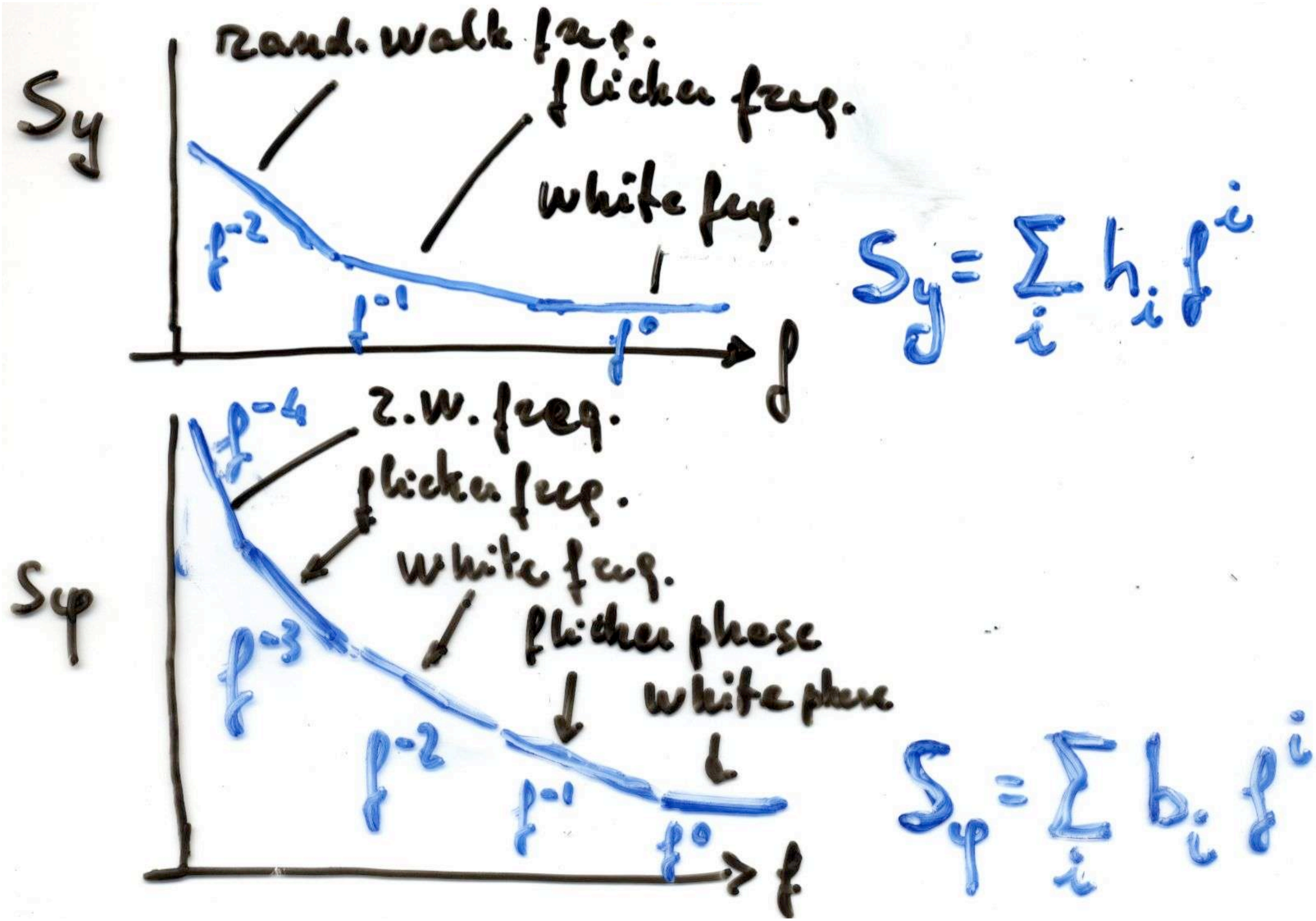
fractional frequency fluctuation

$$y(t) = \frac{1}{2\pi\nu_0}\,\dot{\varphi}(t) = \dot{x}(t)$$

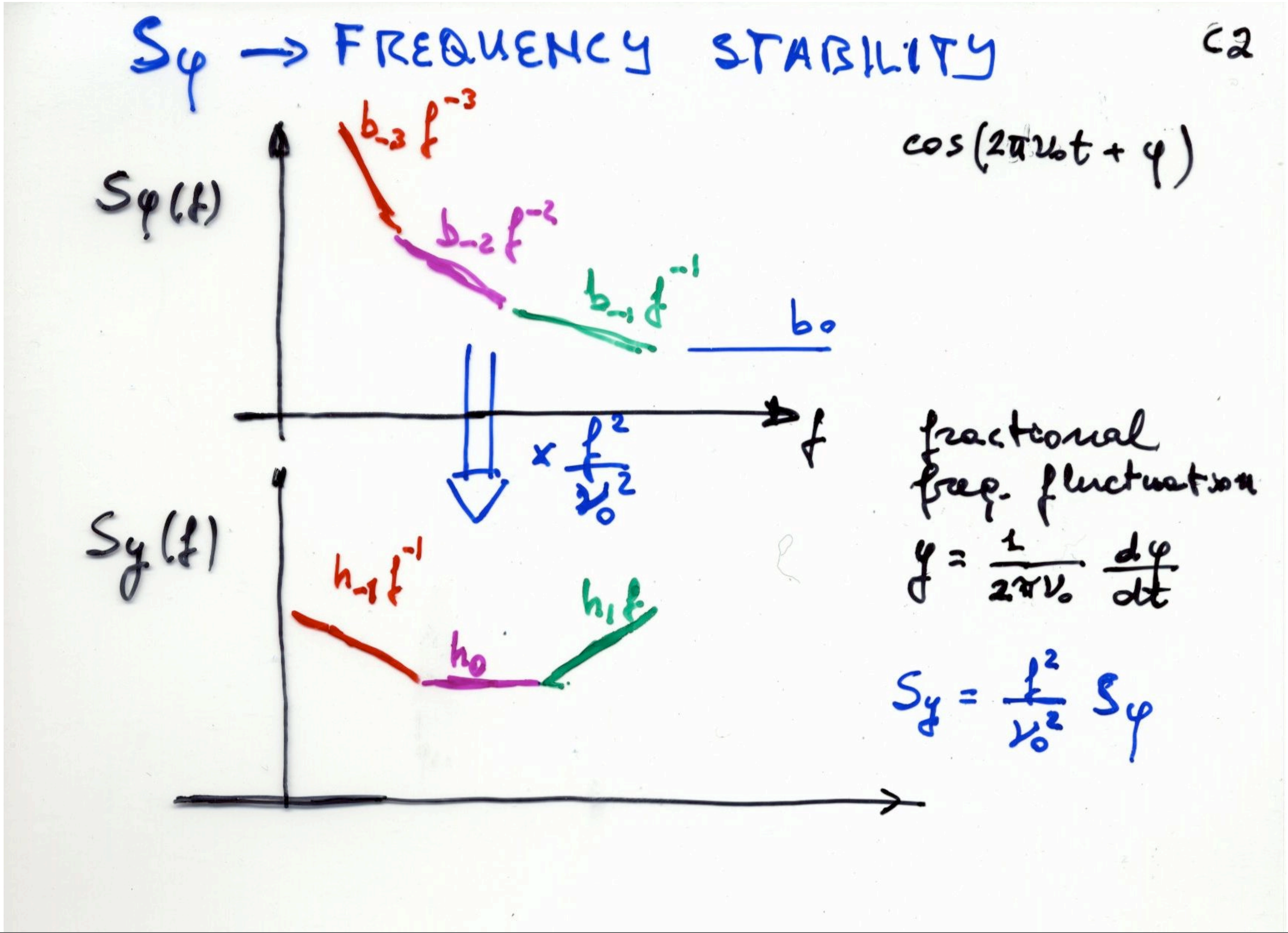
$y(t)$ is the fractional frequency fluctuation $\nu-\nu_0$ normalized to the nominal frequency ν_0 (dimensionless)

$$y(t) = \frac{\nu - \nu_0}{\nu_0}$$

Power-law and noise processes in oscillators



Relationships between $S_\varphi(f)$ and $S_y(f)$



Jitter

The phase fluctuation can be described in terms of a single parameter, either phase jitter or time jitter

The phase noise must be integrated over the bandwidth B of the system (which may be difficult to identify)

phase jitter

$$\varphi_{rms} = \sqrt{\int_B S_\varphi(f)\, df}$$

radians

time jitter

phase jitter converted into time

$$x_{rms} = \frac{1}{2\pi\nu_0}\sqrt{\int_B S_\varphi(f)\, df}$$

seconds

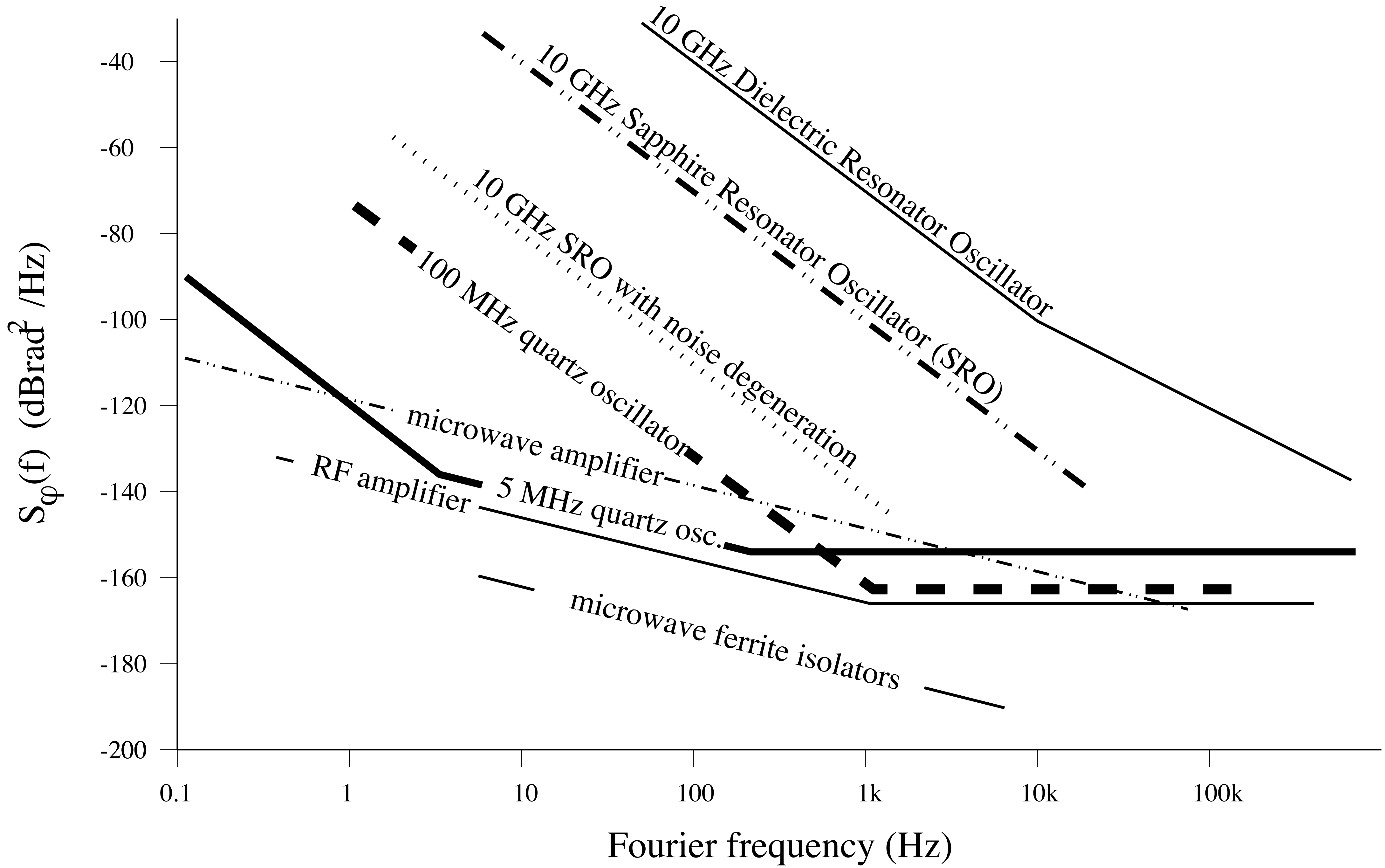
The jitter is useful in digital circuits because the bandwidth *B* is known

– lower limit: the inverse propagation time through the system this excludes the low-frequency divergent processes)

– upper limit: ~ the inverse switching speed

Victor Reinhardt (invited), A Review of Time Jitter and Digital Systems, Proc. 2005 FCS-PTTI joint meeting

Typical phase noise of some devices and oscillators



3 – Variances

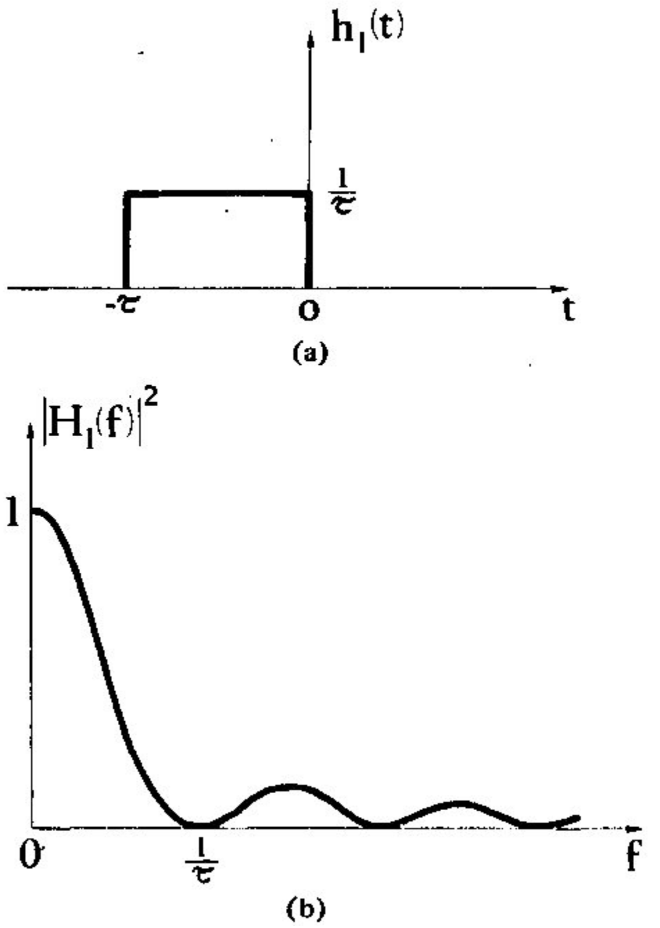
Classical variance

$$\bar{y} = \frac{1}{\nu_0}\frac{1}{\tau}\int_{\tau} \nu(t)\,dt$$

normalized reading of a counter that measures (averages) over a time *T*

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(\bar{y}_i - \underbrace{\frac{1}{N}\sum_{j=1}^{N}\bar{y}_j}_{\text{average of the } N \text{ readings}}\right)^2$$

classical variance,
file of *N* counter readings



For a given process, the classical variance depends of N

Even worse, if the spectrum is f^{-1} or steeper, the classical variance diverges

The filter associated to the measure takes in the dc component

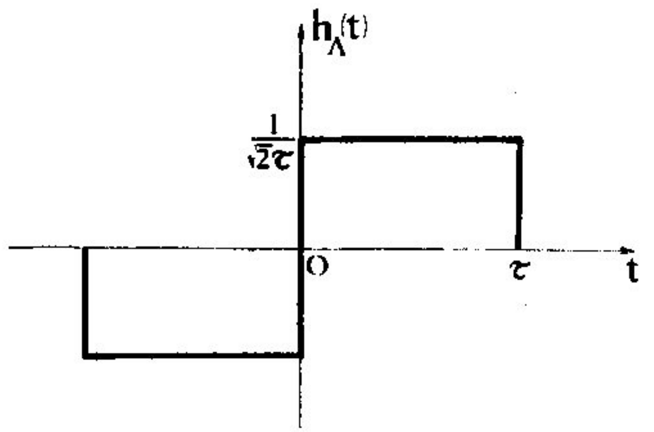
Zero dead-time two-sample variance (Allan variance)

$$\sigma_y^2 = \frac{1}{2}\left\langle\left(y_2 - y_1\right)^2\right\rangle$$

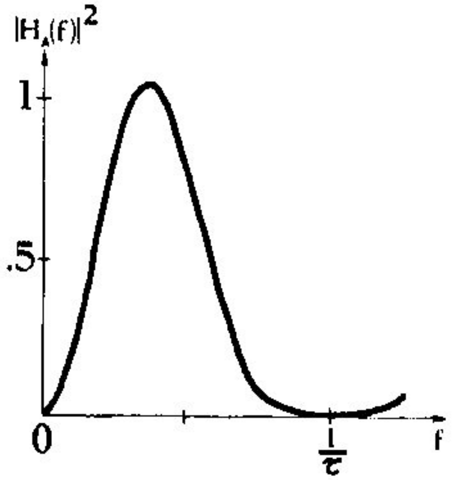
Definition
(Let N = 2, and average)

$$\sigma_y^2 = \frac{1}{2(m-1)}\sum_{i=1}^{m-1}\left(\overline{y}_2 - \overline{y}_1\right)^2$$

Estimated Allan variance,
file of m counter readings



(a)

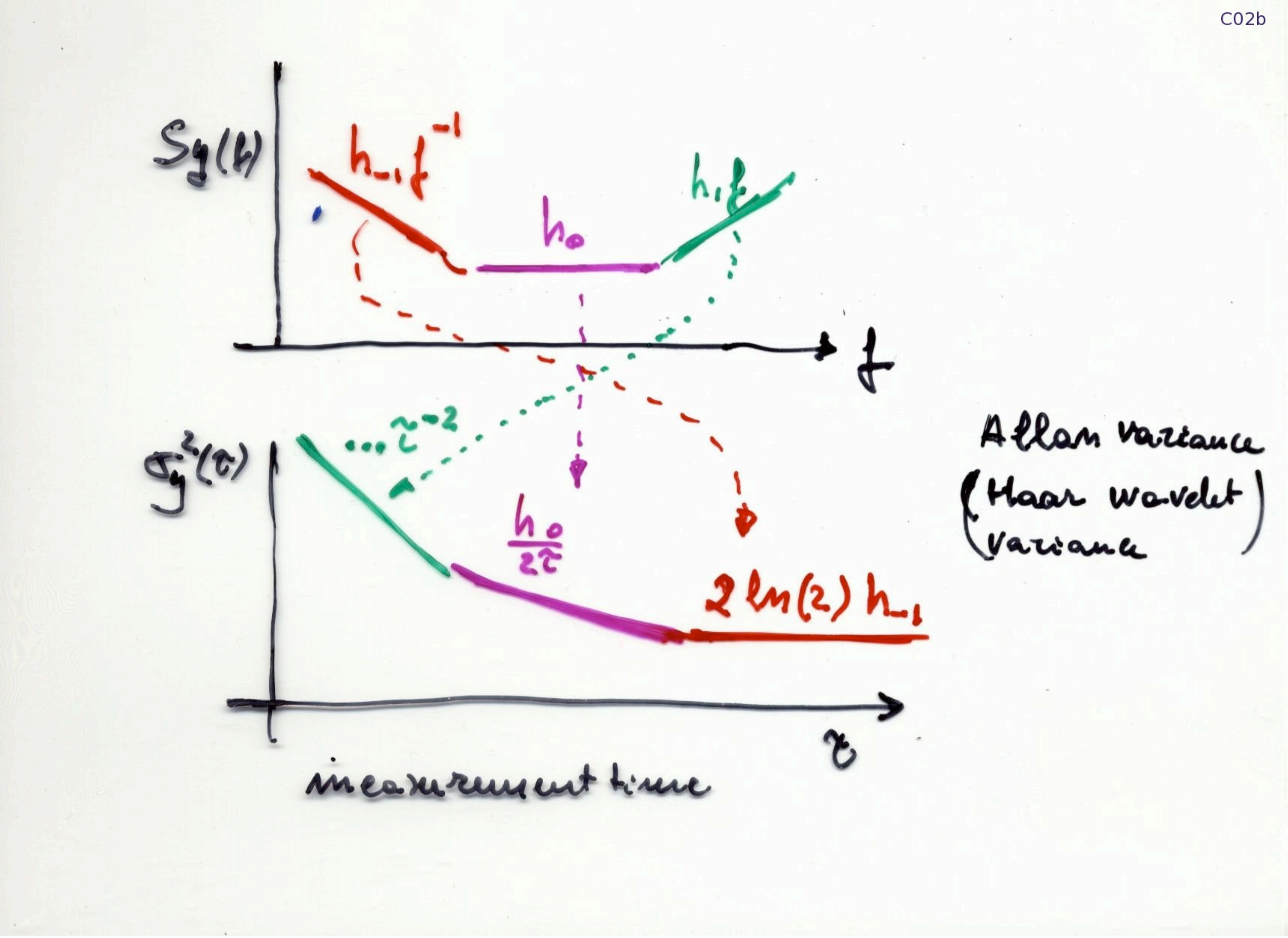


(b)

The filter associated to the difference
of two contiguous measures is a band-pass

The estimate converges to the variance

The Allan variance is related to the spectrum $S_y(f)$



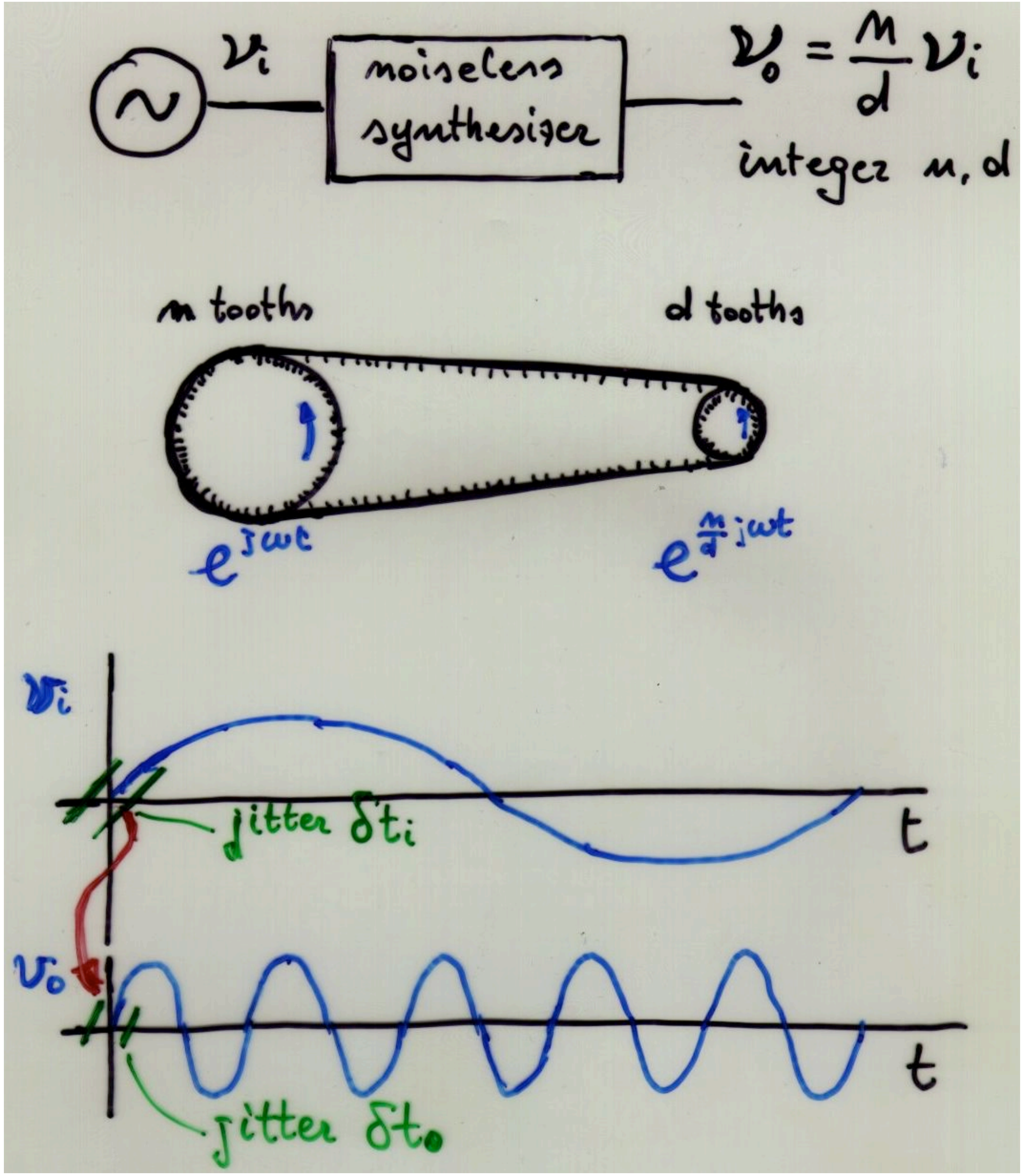
Convert S_φ and S_y into Allan variance

noise type	$S_\varphi(f)$	$S_y(f)$	$S_\varphi \leftrightarrow S_y$	$\sigma_y^2(\tau)$	$\mathrm{mod}\,\sigma_y^2(\tau)$
white PM	b_0	$h_2 f^2$	$h_2 = \dfrac{b_0}{\nu_0^2}$	$\dfrac{3 f_H h_2}{(2\pi)^2}\tau^{-2}$ $2\pi\tau f_H \gg 1$	$\dfrac{3 f_H \tau_0 h_2}{(2\pi)^2}\,\tau^{-3}$
flicker PM	$b_{-1} f^{-1}$	$h_1 f$	$h_1 = \dfrac{b_{-1}}{\nu_0^2}$	$[1.038 + 3\ln(2\pi f_H \tau)]\,\dfrac{h_1}{(2\pi)^2}\,\tau^{-2}$	$0.084\,h_1\,\tau^{-2}$ $n \gg 1$
white FM	$b_{-2} f^{-2}$	h_0	$h_0 = \dfrac{b_{-2}}{\nu_0^2}$	$\dfrac{1}{2} h_0\,\tau^{-1}$	$\dfrac{1}{4} h_0\,\tau^{-1}$
flicker FM	$b_{-3} f^{-3}$	$h_{-1} f^{-1}$	$h_{-1} = \dfrac{b_{-3}}{\nu_0^2}$	$2\ln(2)\;h_{-1}$	$\dfrac{27}{20}\ln(2)\;h_{-1}$
random walk FM	$b_{-4} f^{-4}$	$h_{-2} f^{-2}$	$h_{-2} = \dfrac{b_{-4}}{\nu_0^2}$	$\dfrac{(2\pi)^2}{6} h_{-2}\tau$	$0.824\,\dfrac{(2\pi)^2}{6} h_{-2}\,\tau$
frequency drift $\dot{y} = D_y$				$\dfrac{1}{2}\,D_y^2\,\tau^2$	$\dfrac{1}{2}\,D_y^2\,\tau^2$

f_H is the high cutoff frequency, needed for the noise power to be finite.

4 – Properties of phase noise

Frequency synthesis



Ideal synthesizer
- noise-free
- zero delay time

time translation:
output jitter = input jitter
phase time $x_o = x_i$

linearity of the integral and the derivative operators:

$\varphi_o = (n/d)\varphi_i \quad \Rightarrow \quad \nu_o = (n/d)\nu_i$

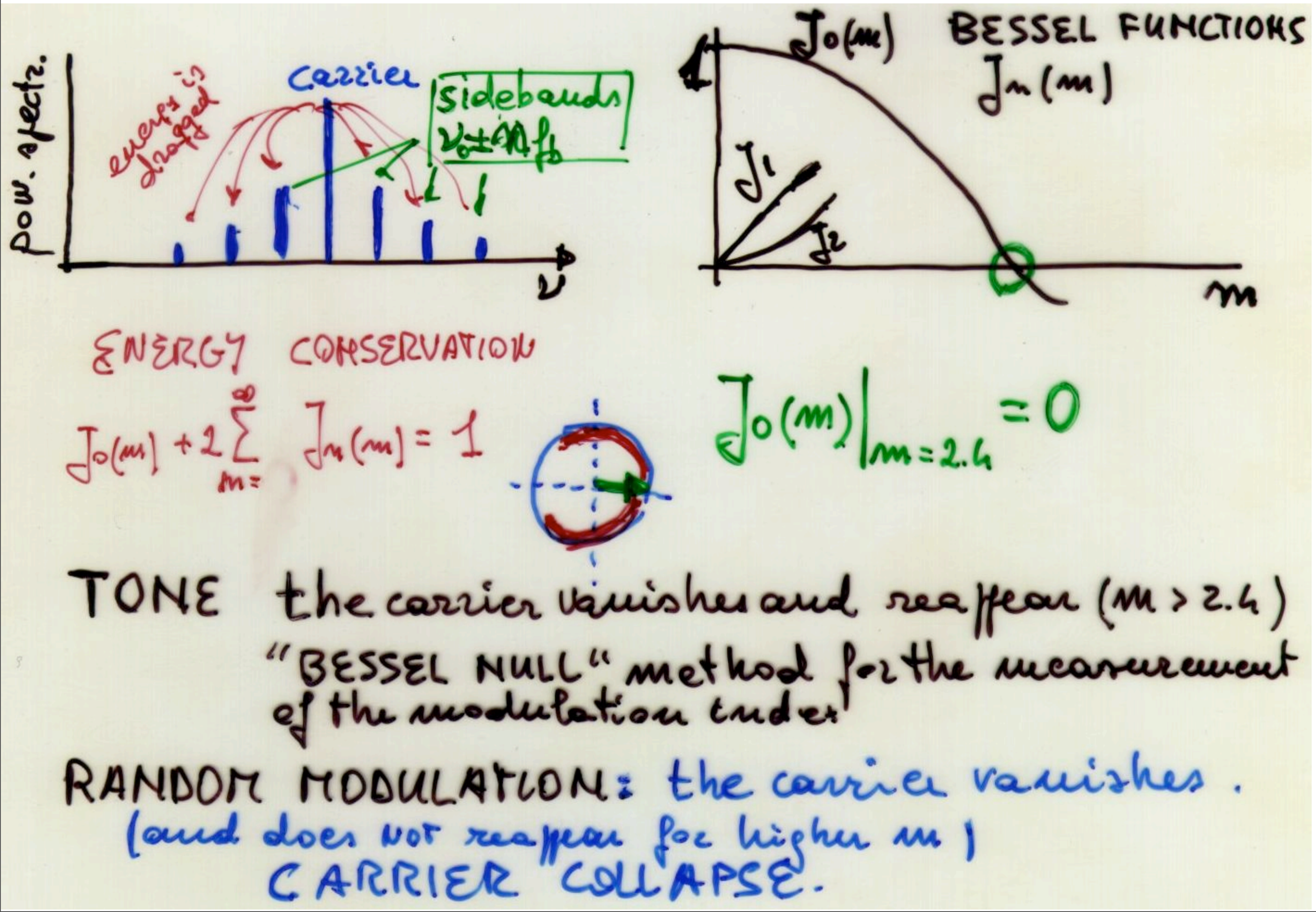
spectra

$$S_{\varphi o}(f) = \left(\frac{n}{d}\right)^2 S_{\varphi i}(f)$$

Carrier collapse

Simple physical meaning, complex mathematics. Easy to understand in the case of sinusoidal phase modulation

$$v(t) = V_0 \cos\left[2\pi \nu_0 t + m \cos(2\pi \nu_m t)\right]$$



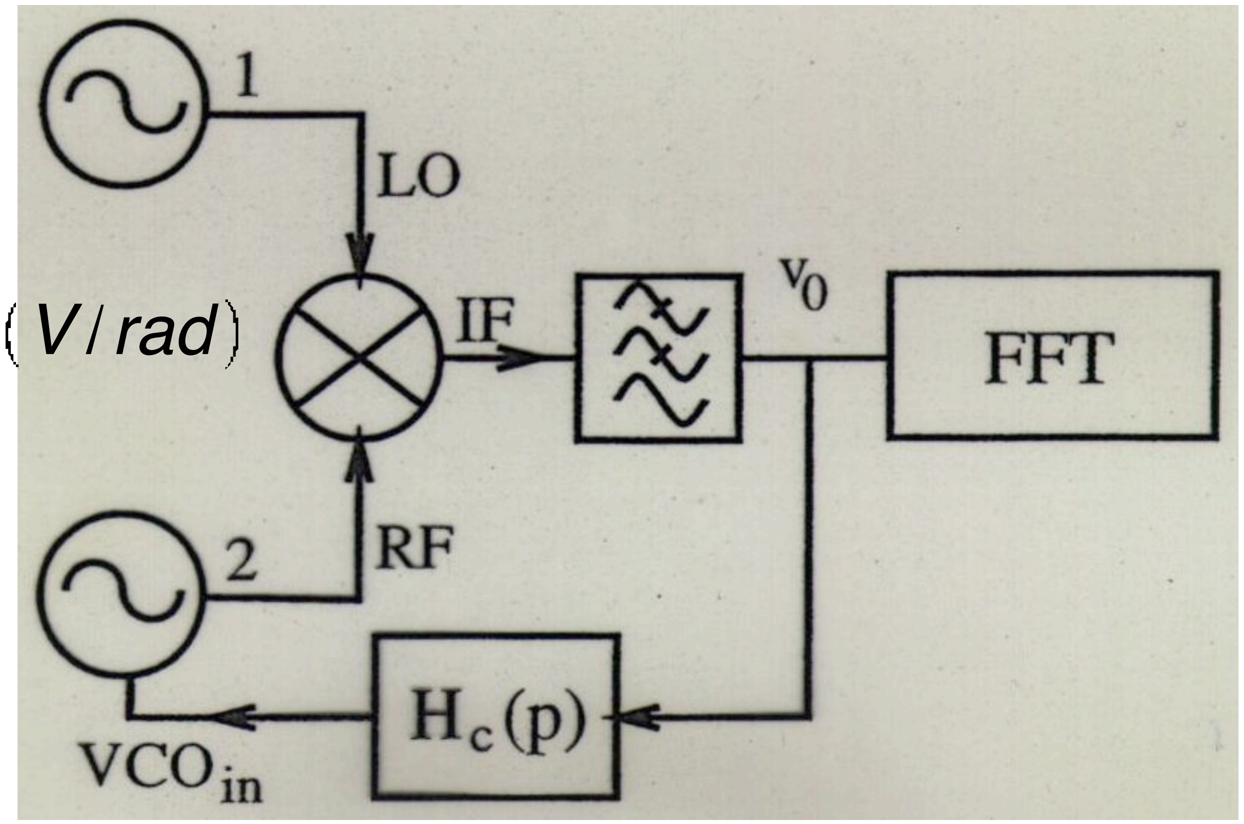
random noise => phase fluctuation

$$\overline{\varphi^2} = \int_B S_\varphi(f)\, df$$

Filtering <=> Phase Locked Loop (PLL)

$k_\varphi \ (V/rad)$

$k_o \ (rad/s/V)$



The signal "2" tracks "1"

The FFT analyzer (not needed here) can be used to measure $S_\varphi(f)$

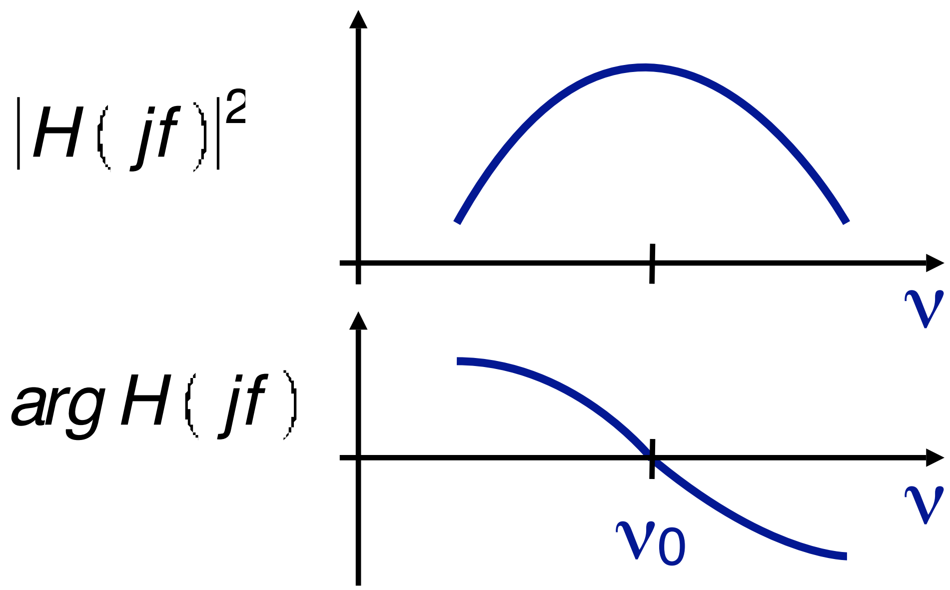
The PLL low-pass filters the phase

$$\frac{S_{\varphi 2}(f)}{S_{\varphi 1}(f)} = \frac{\left|k_o k_\varphi H_c(f)\right|^2}{4\pi^2 f^2 + \left|k_o k_\varphi H_c(f)\right|^2}$$

Output voltage: the PLL is a high-pass filter

$$\frac{S_{vo}(f)}{S_{\varphi 1}(f)} = \frac{4\pi f^2 k_\varphi^2}{4\pi^2 f^2 + \left|k_o k_\varphi H_c(f)\right|^2}$$

Frequency discriminator



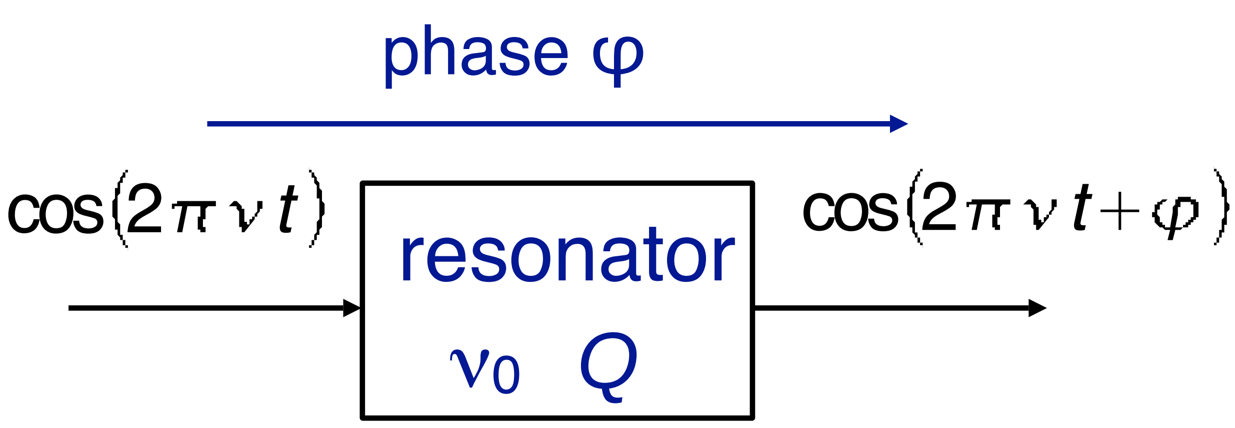
A resonator turns a slow frequency fluctuation $\Delta\nu$ into a phase fluctuation

$$\varphi = \frac{1}{2Q}\,\frac{\Delta\nu}{\nu_0}$$

Parameters

ν_0 resonant frequency

Q merit factor

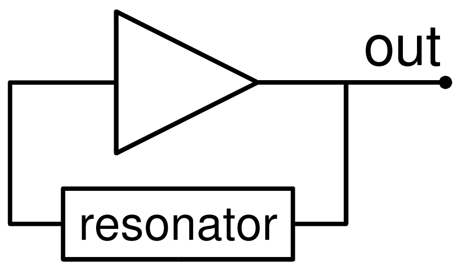


For slow frequency fluctuations, a delay-line t is equivalent to a resonator of merit factor

$$Q = \pi\tau\nu_0$$

The Leeson effect: phase-to-frequency noise conversion in oscillators

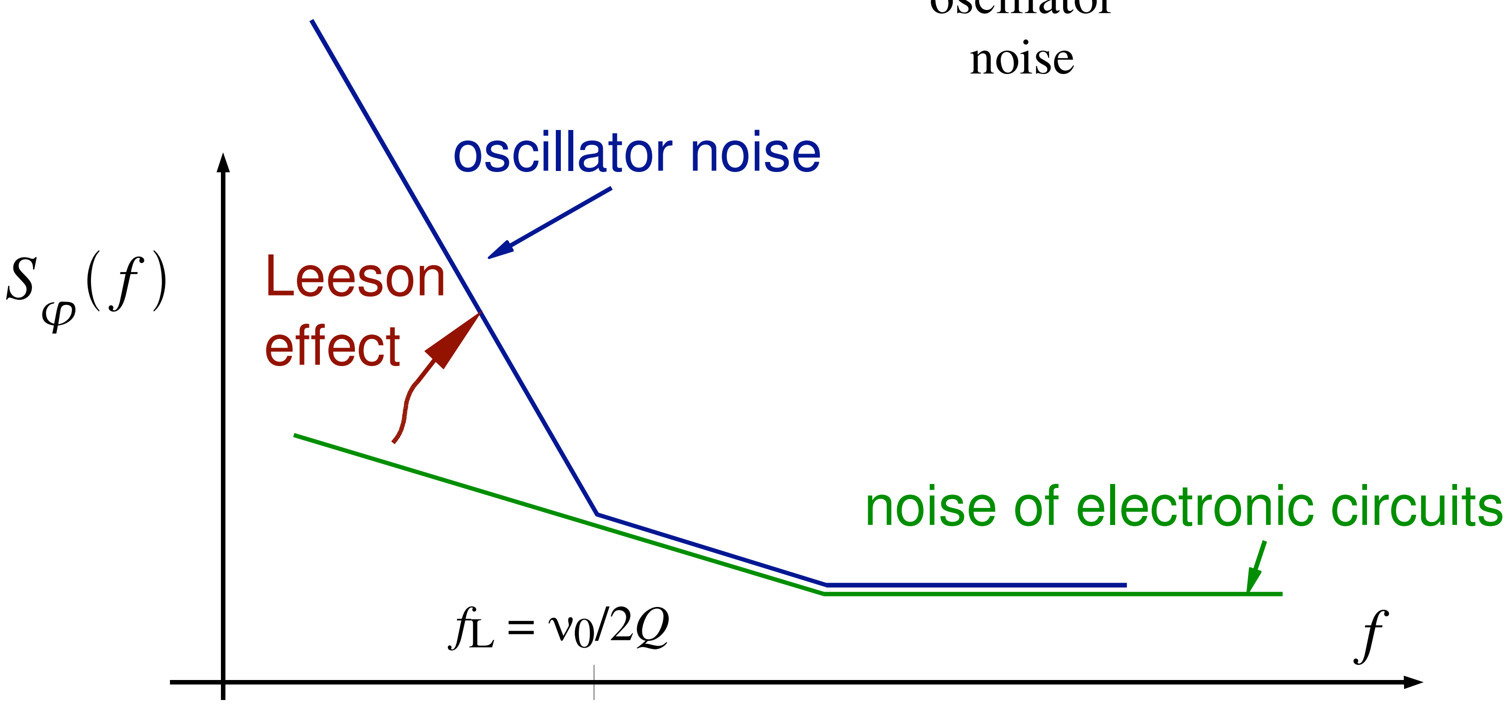
D. B. Leeson, A simple model for feed back oscillator noise, Proc. IEEE 54(2):329 (Feb 1966)



$$S_\varphi(f) = \left[1 + \left(\frac{\nu_0}{2Q}\right)^2 \frac{1}{f^2}\right] S_\psi(f)$$

oscillator noise

ampli noise

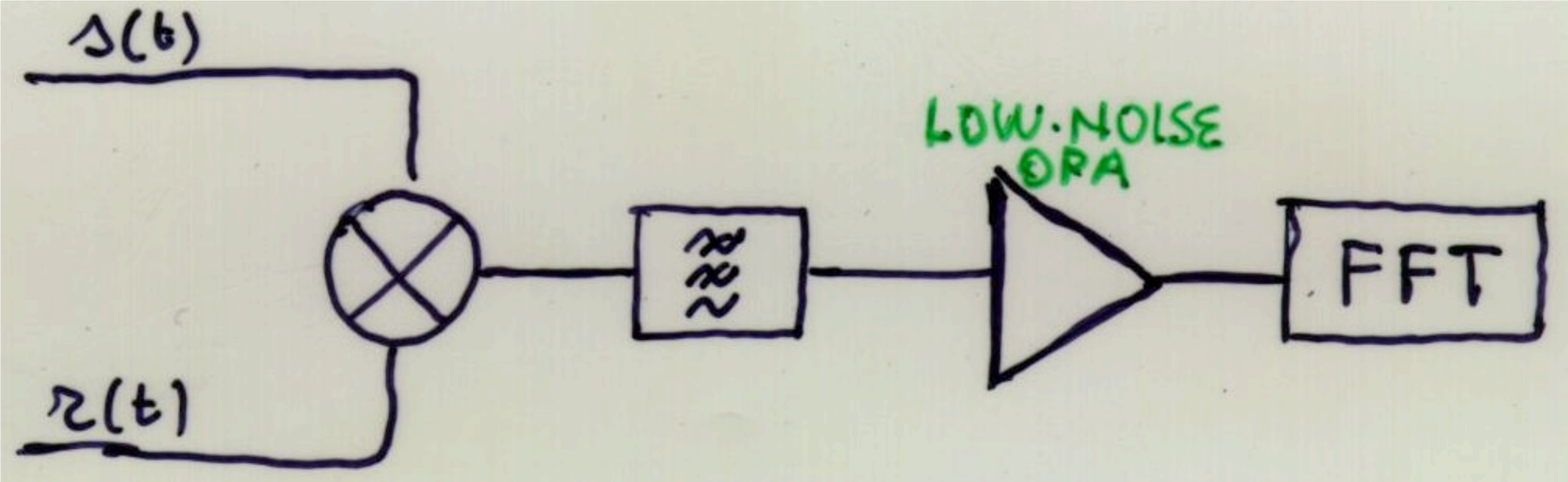


E. Rubiola, The Leeson effect, Tutorial 2A, Proc. 2005 FCS-PTTI (tutorials)

E. Rubiola, The Leeson effect, e-book, (http://arxiv.org/abs/physics/0502143 or rubiola.org)

5 – Laboratory practice

Practical limitations of the double-balanced mixer



1 – Power

narrow power range: ±5 dB around P_{nom} = 5-10 dBm

r(t) and s(t) should have (about) the same power

2 – Flicker noise due to the mixer internal diodes

typical $S_\varphi = -140$ dBrad²/Hz at 1 Hz in average-good conditions

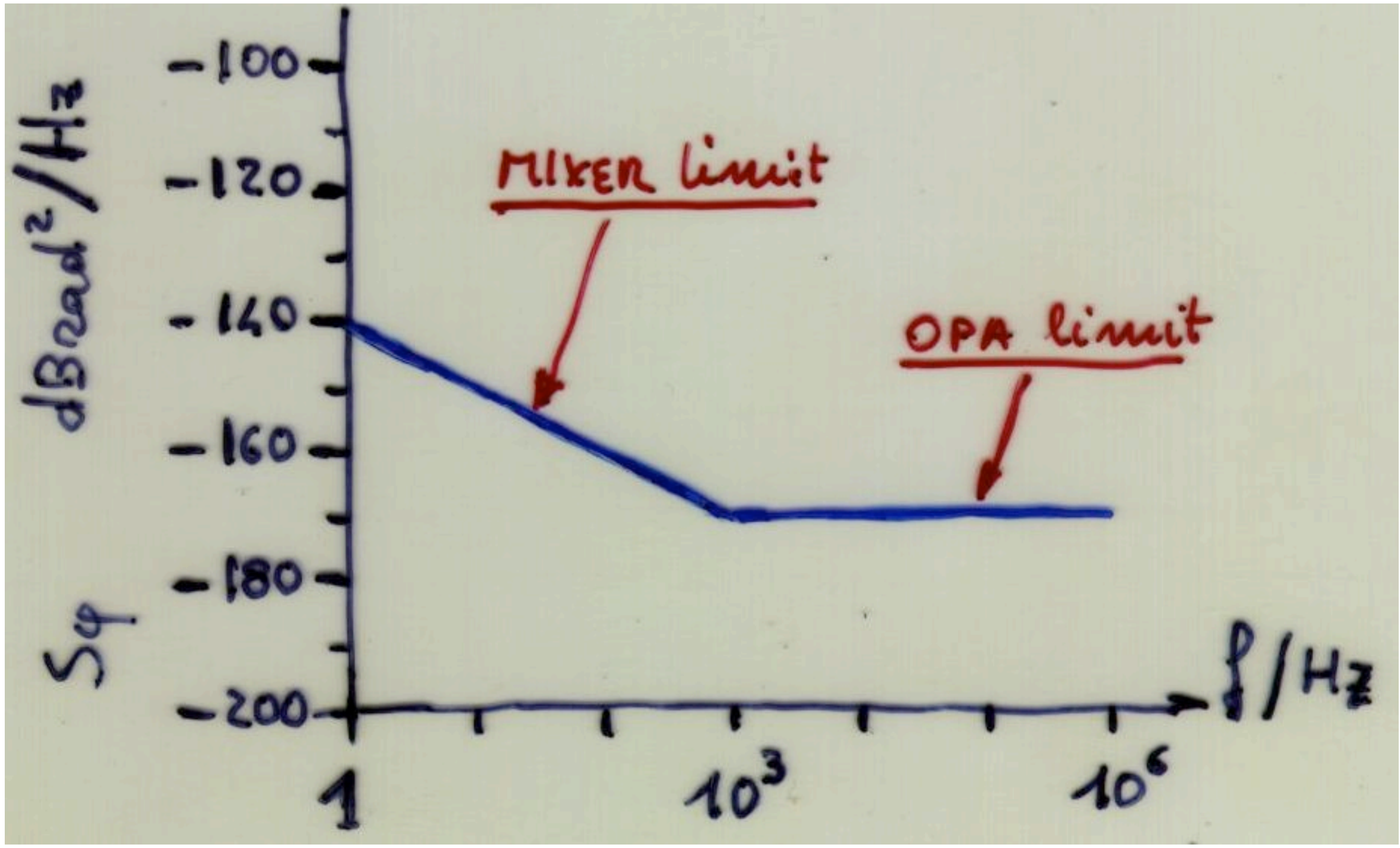
3 – Low gain

$k_\varphi \sim -10$ to -14 dBV/rad typical (0.2-0.3 V/rad)

4 – White noise

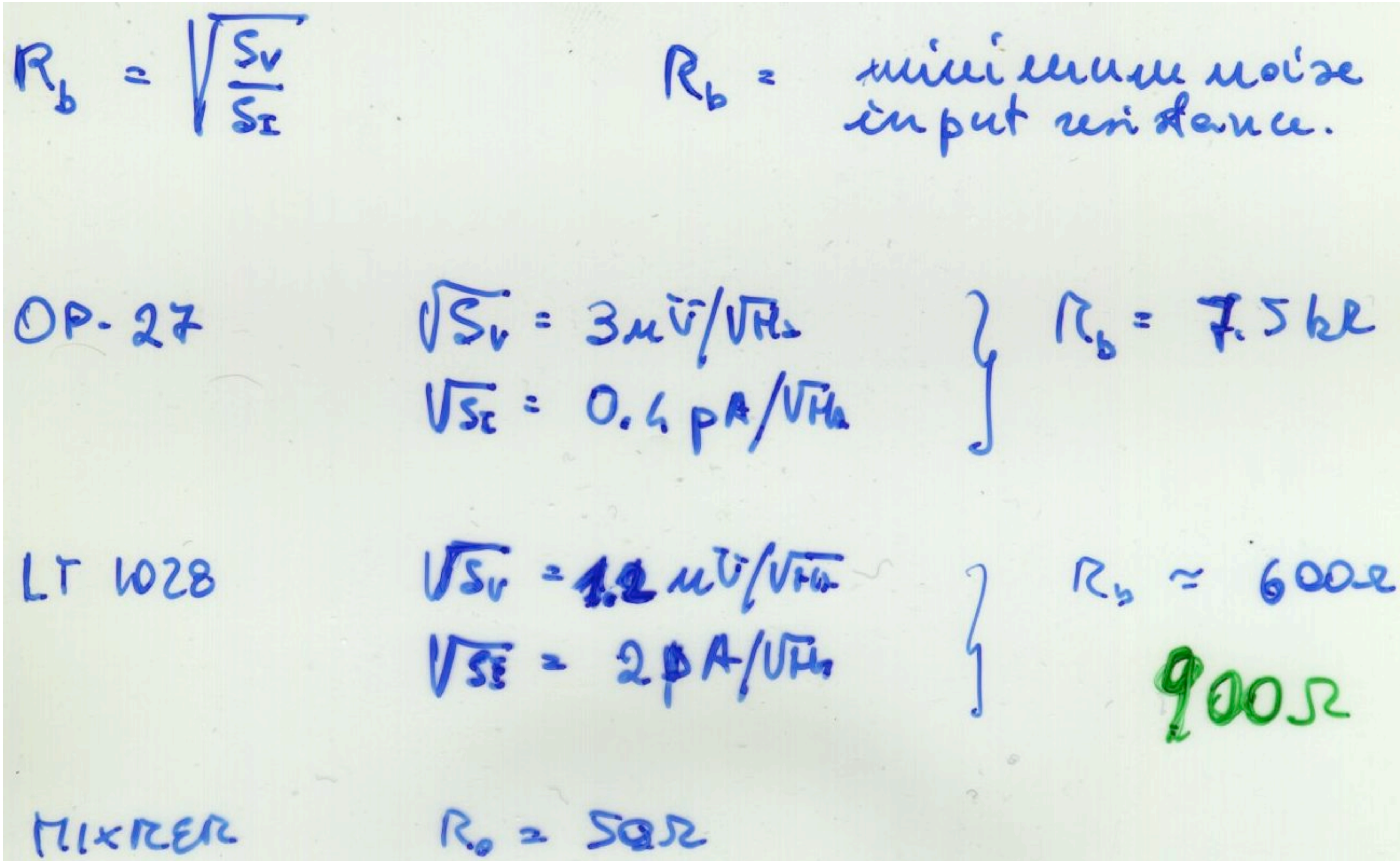
due to the operational amplifier

Typical background noise



RF mixer (5-10) MHz
Good operating conditions (10 dBm each input)
Low-noise preamplifier (1 nV/√Hz)

The operational amplifier is often misused



Warning: if only one arm of the power supply is disconnected, the LT1028 may delivers a current from the input (I killed a $2k mixer in this way!)

You may duplicate the low-noise amplifier designed at the FEMTO-ST
Rubiola, Lardet-Vieudrin, Rev. Scientific Instruments 75(5) pp. 1323-1326, May 2004

A proper mechanical assembly is vital

$$\ell = \frac{\lambda_{cable}}{2\pi}\,\varphi$$

length fluct.

phase fluct.

-180 dB rad $\rightarrow$ 10^{-9} rad

$\hookrightarrow$ 4×10^{-12} m @ 10 GHz

4×10^{-10} m @ 100 MHz

$$\sigma_\ell^2 = 2\ln(2)\,h_{-1}$$

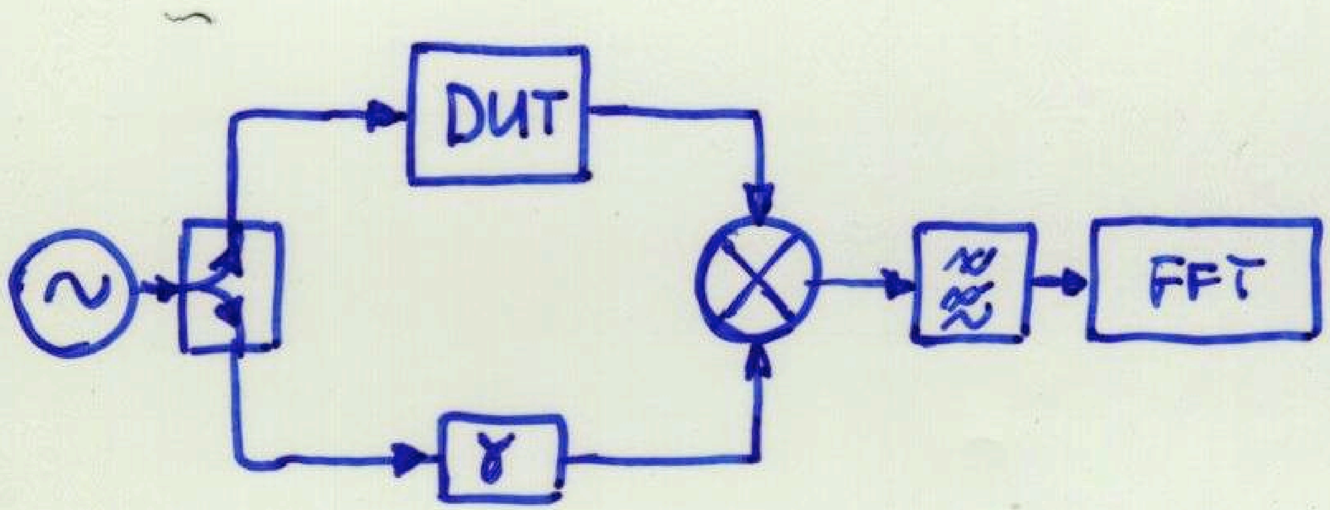
Allan variance

flicker coeff. of S_ℓ

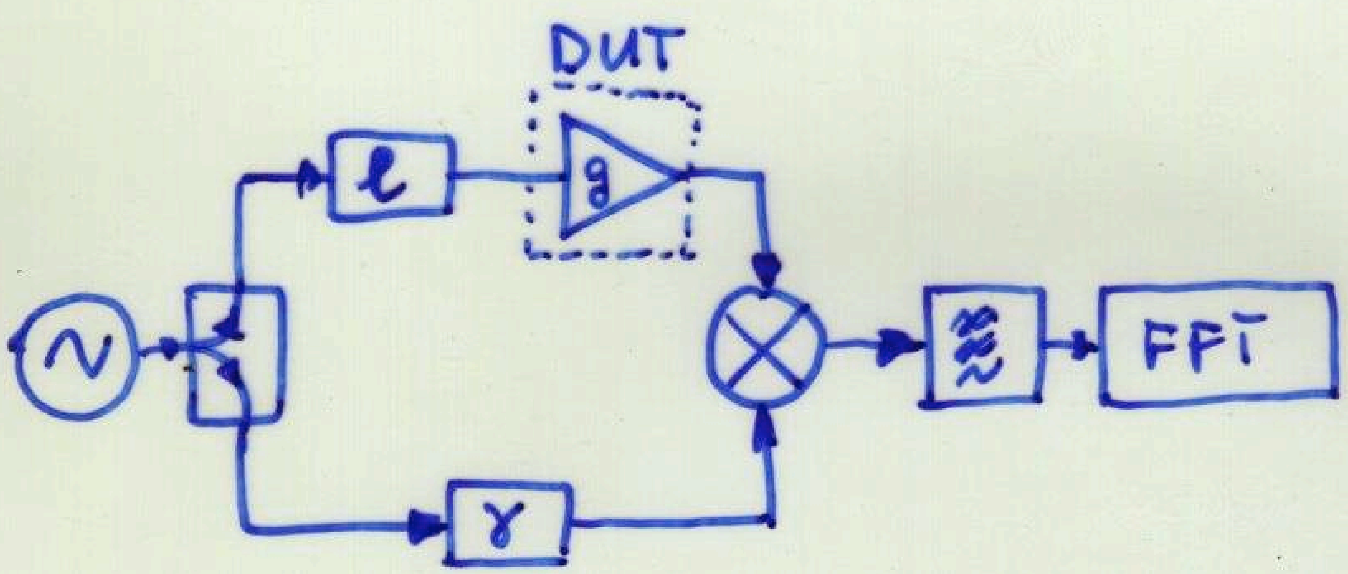
10^{-9} rad $\rightarrow$ $\sigma_\ell \approx 4.8$ pm @ 10 GHz

4.8 Å @ 100 MHz

Two-port device under test (DUT)



- The phase shifter γ ensures the 90° condition at the mixer inputs
- The oscillator noise is rejected

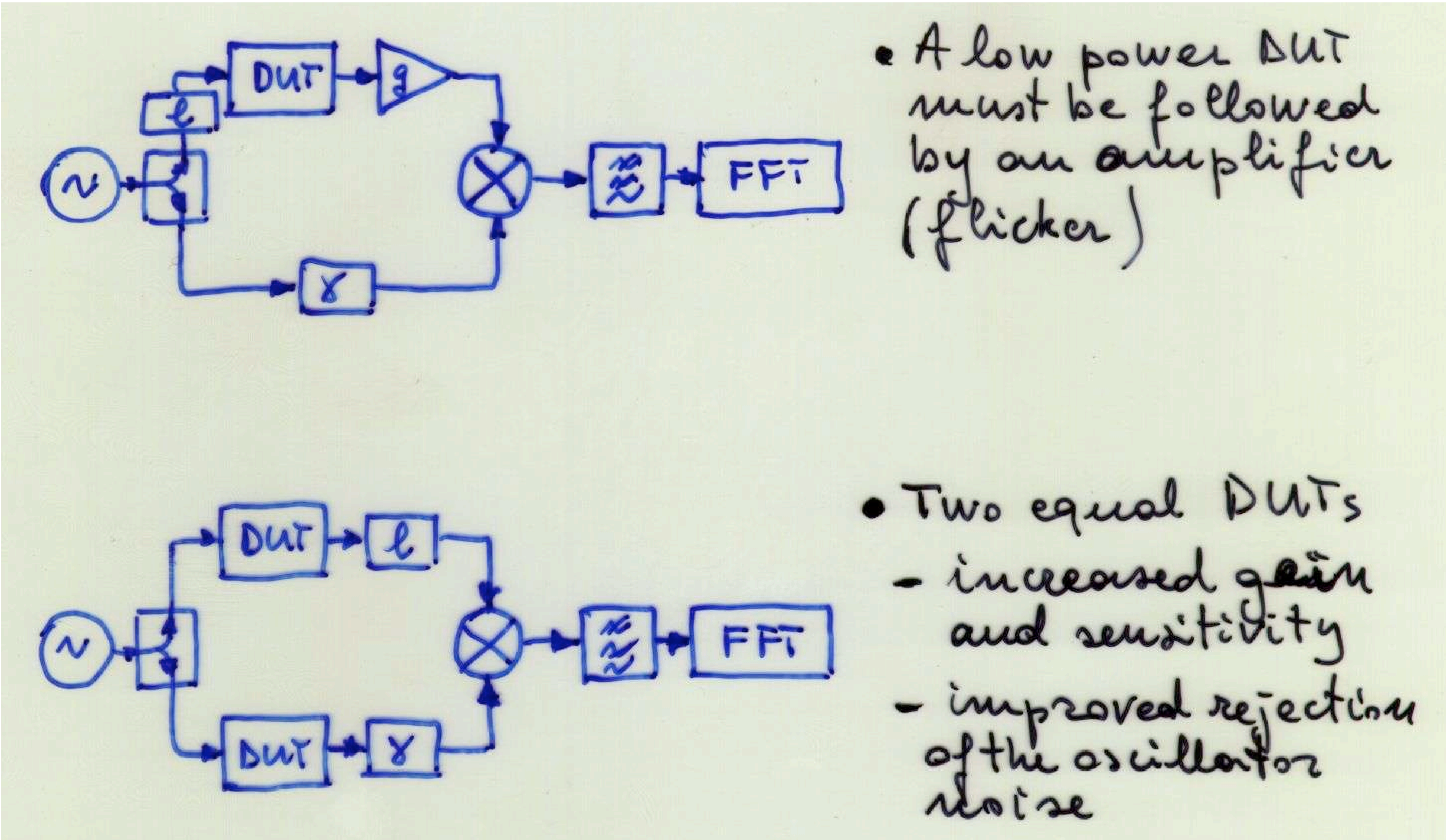


- Testing an amplifier, it must be preceded by an attenuator

F increases

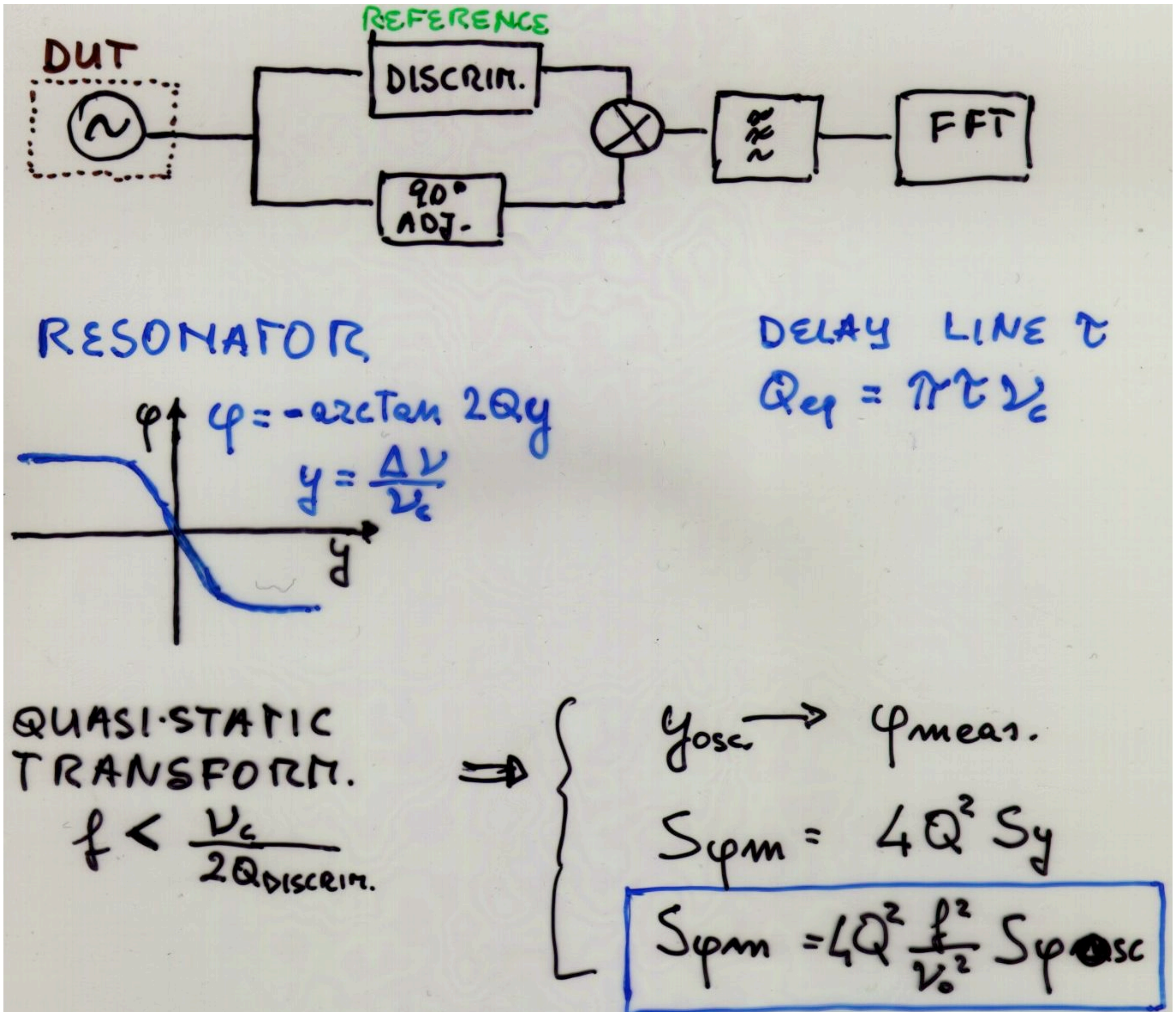
- Higher power ampli may be followed by an attenuator

Two-port device under test (DUT)



other configurations are possible

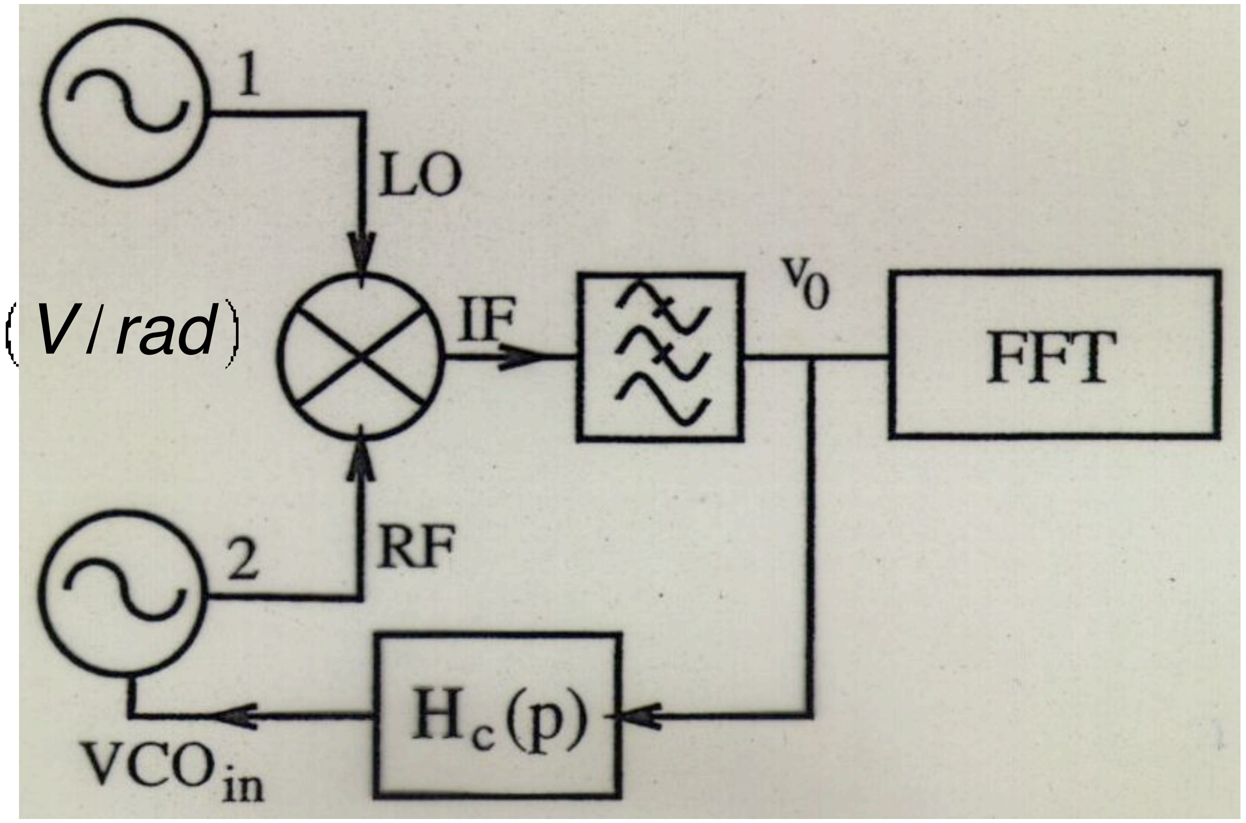
A frequency discriminator can be used to measure the phase noise of an oscillator



Phase Locked Loop (PLL)

$k_\varphi \;(V/rad)$

$k_o \;(rad/s/V)$



Phase: the PLL
is a low-pass filter

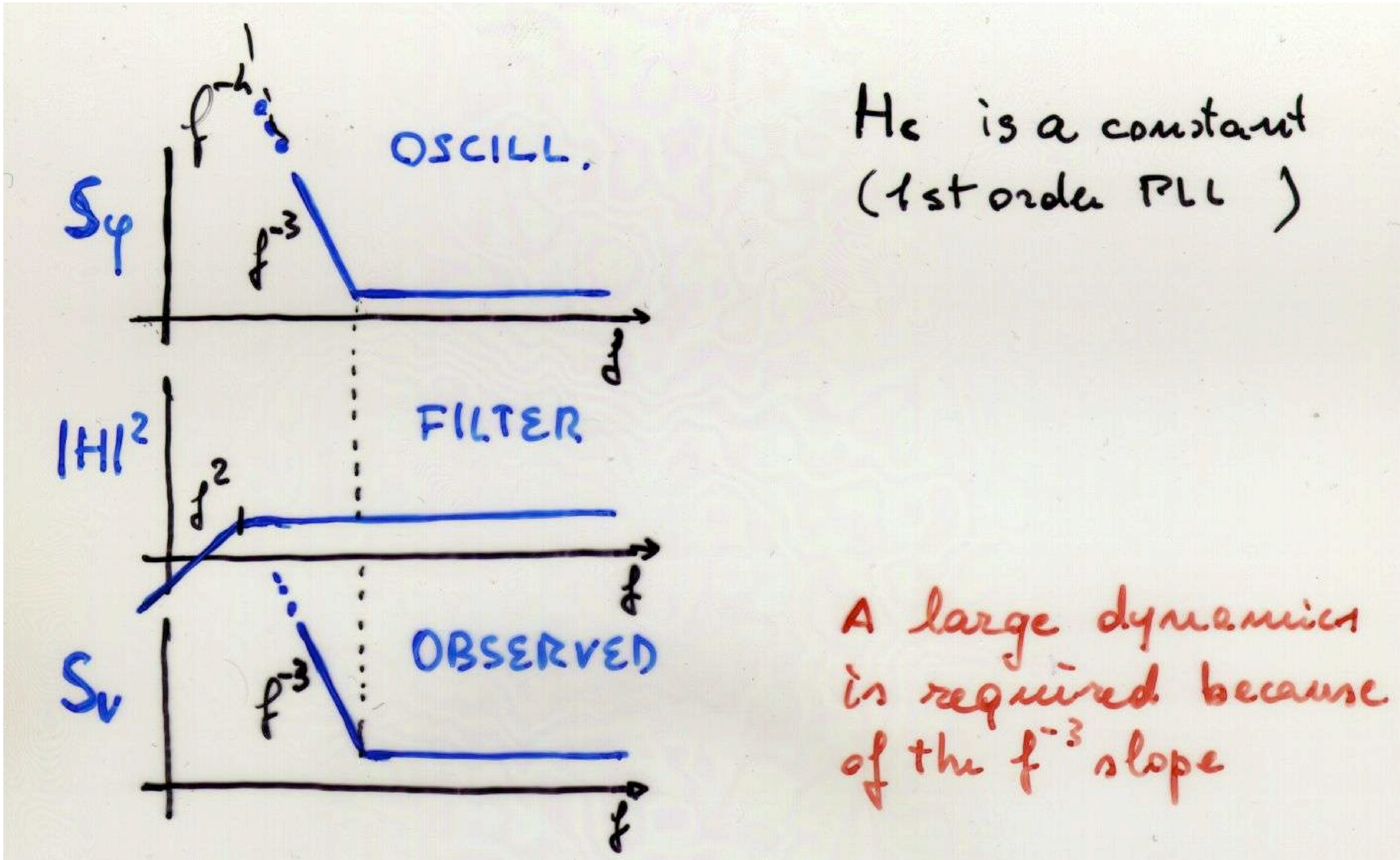
$$\frac{S_{\varphi 2}(f)}{S_{\varphi 1}(f)} = \frac{\left|k_o k_\varphi H_c(f)\right|^2}{4\pi^2 f^2 + \left|k_o k_\varphi H_c(f)\right|^2}$$

Output voltage: the PLL
is a high-pass filter

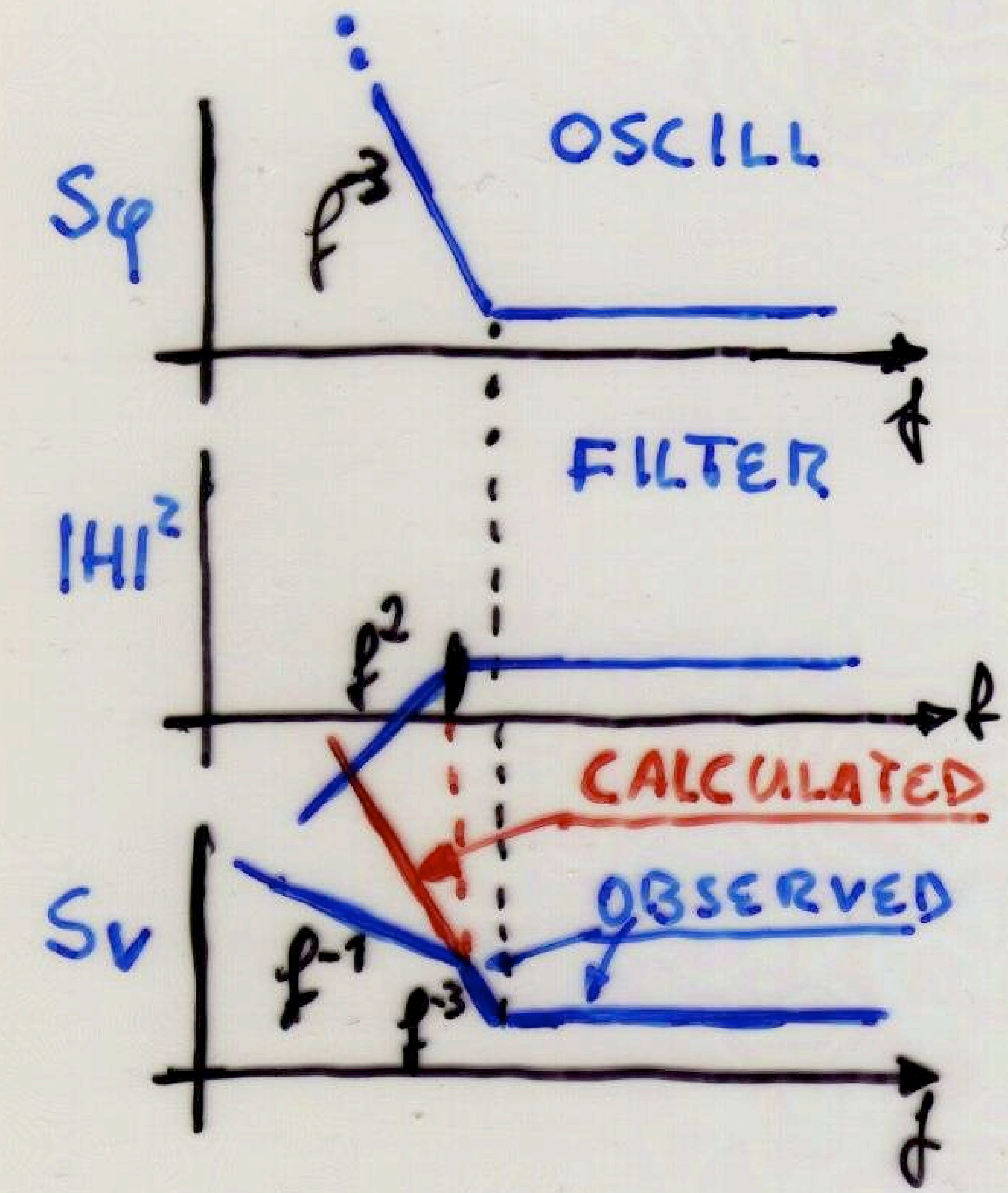
$$\frac{S_{vo}(f)}{S_{\varphi 1}(f)} = \frac{4\pi f^2 k_\varphi^2}{4\pi^2 f^2 + \left|k_o k_\varphi H_c(f)\right|^2}$$

compare an oscillator under test to a reference low-noise oscillator
– or –
compare two equal oscillators and divide the spectrum by 2 (take away 3 dB)

Phase Locked Loop (PLL)



A tight PLL shows many advantages



Still 1st order PLL

increased gain

a narrower dynamics is sufficient
slope: $f^{-3} \to f^{-1}$

but you have to correct the spectrum for the PLL transfer function

Practical measurement of $S_\varphi(f)$ with a PLL

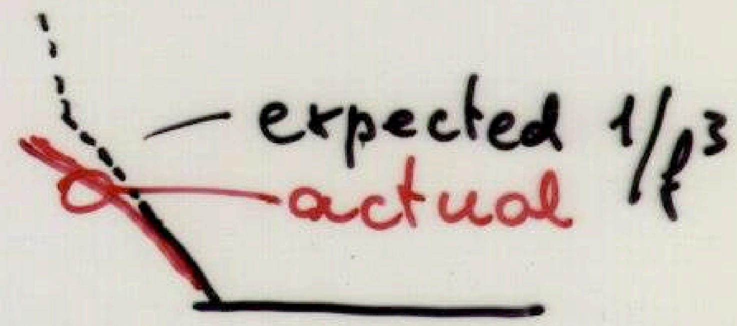
1. Set the circuit for proper electrical operation
   a. power level
   b. lock condition (there is no beat note at the mixer out)
   c. zero dc error at the mixer output (a small V can be tolerated)
2. Choose the appropriate time constant
3. Measure the oscillator noise
4. At end, measure the background noise

Warning: a PLL may not be what it seems

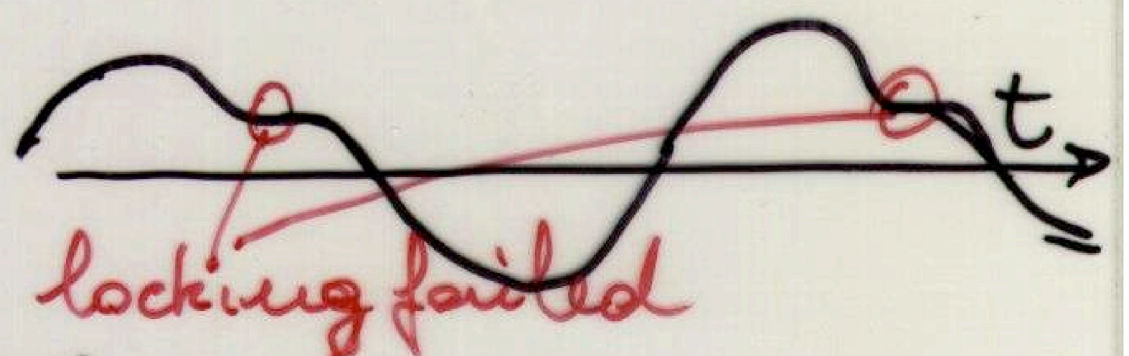
Parasitic locking or coupling of the oscillators may impair the result

BAD SYMPTOMS:

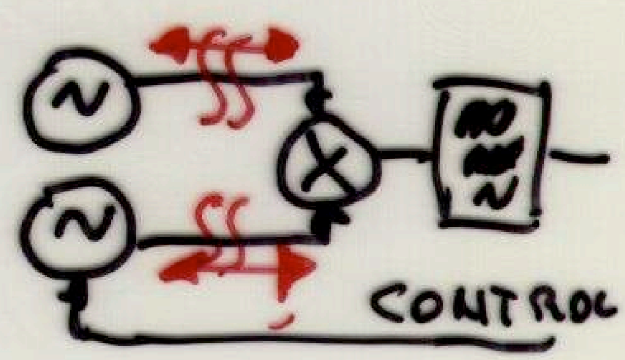
- odd slope S_φ



- open-loop waveforms IFout

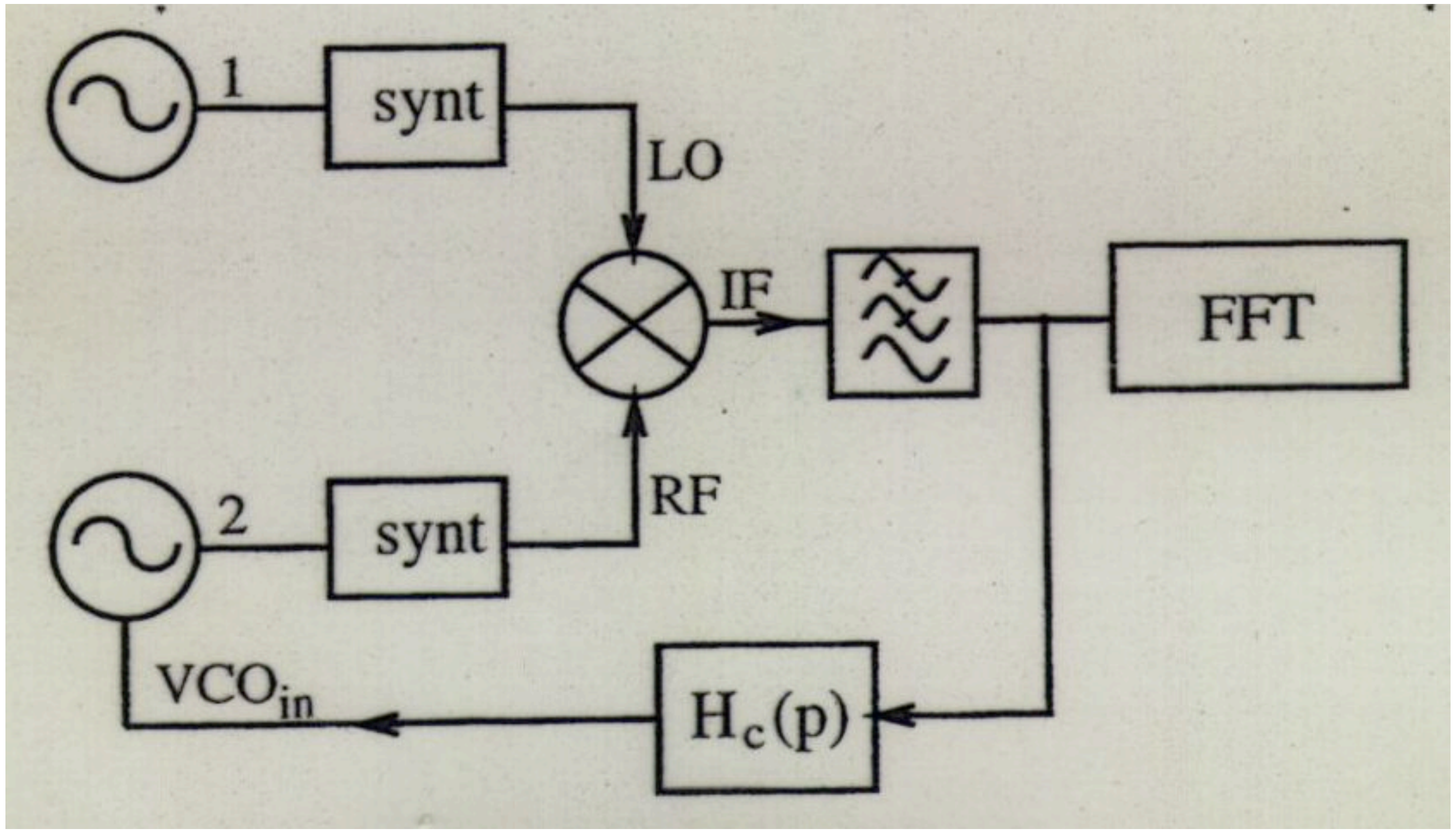


- results (S_φ) depend on the cable length



PLL – two frequencies

The output frequency of the two oscillators is not the same.
A synthesizer (or two synth.) is necessary to match the frequencies

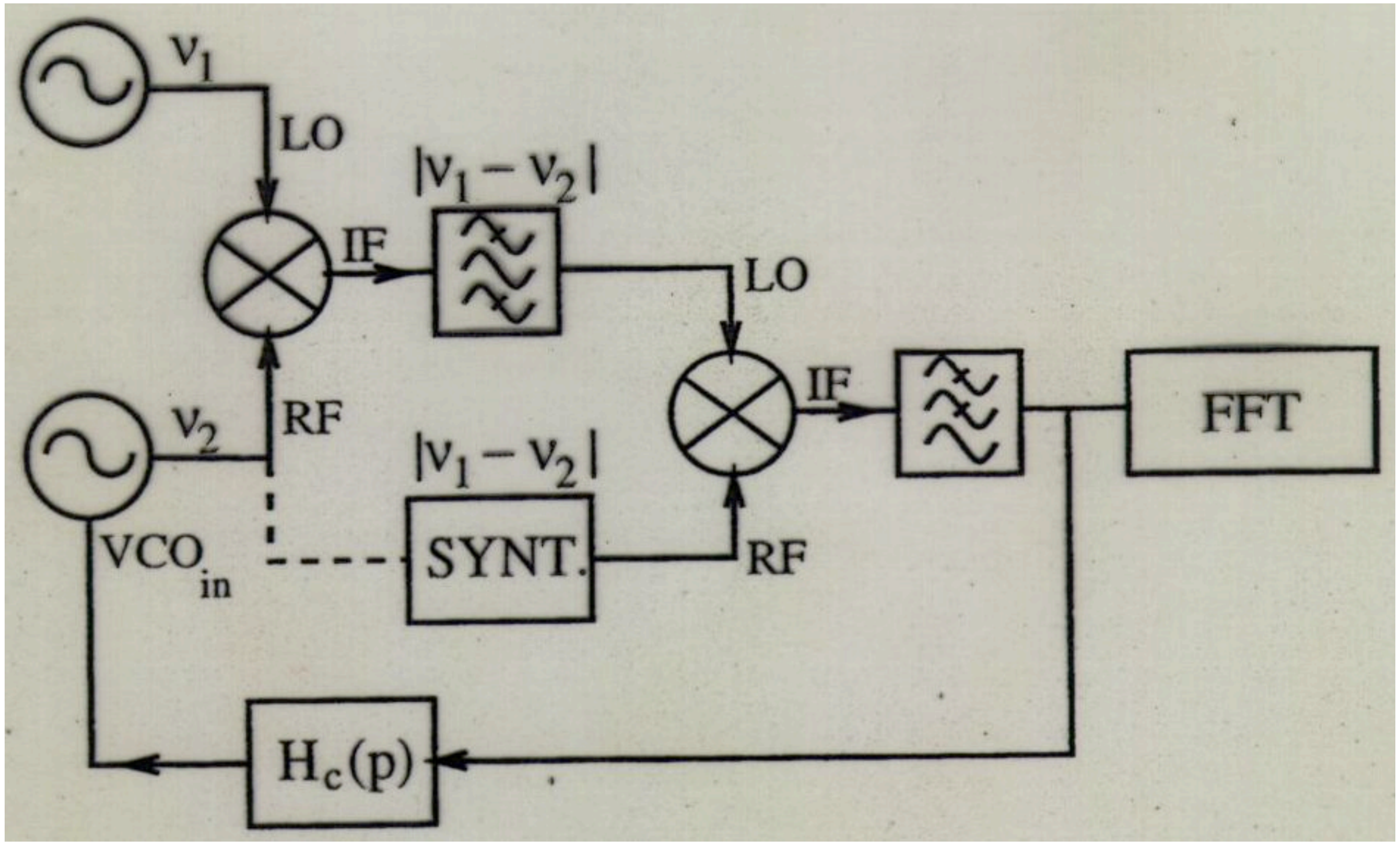


At low Fourier frequencies, the synthesizer noise is lower than the oscillator noise

At higher Fourier frequencies, the white and flicker of phase of the synthesizer may dominate

PLL – low noise microwave oscillators

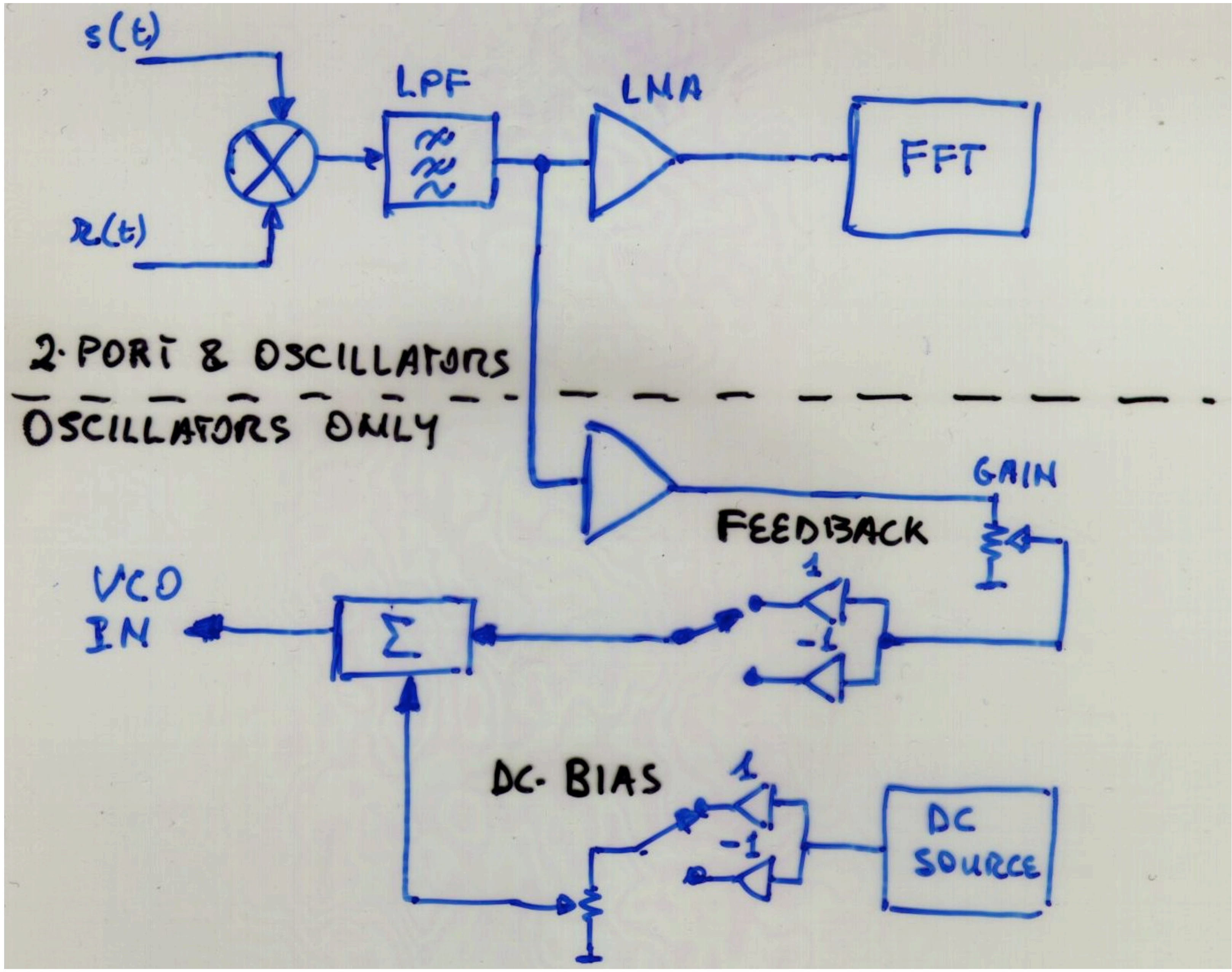
With low-noise microwave oscillators (like whispering gallery) the noise of a microwave synthesizer at the oscillator output can not be tolerated.



Due to the lower carrier frequency, the noise of a VHF synthesizer is lower than the noise of a microwave synthesizer.

This scheme is useful
- with narrow tuning-range oscillator, which can not work at the same freq.
- to prevent injection locking due to microwave leakage

Designing your own instrument is simple



Standard commercial parts:

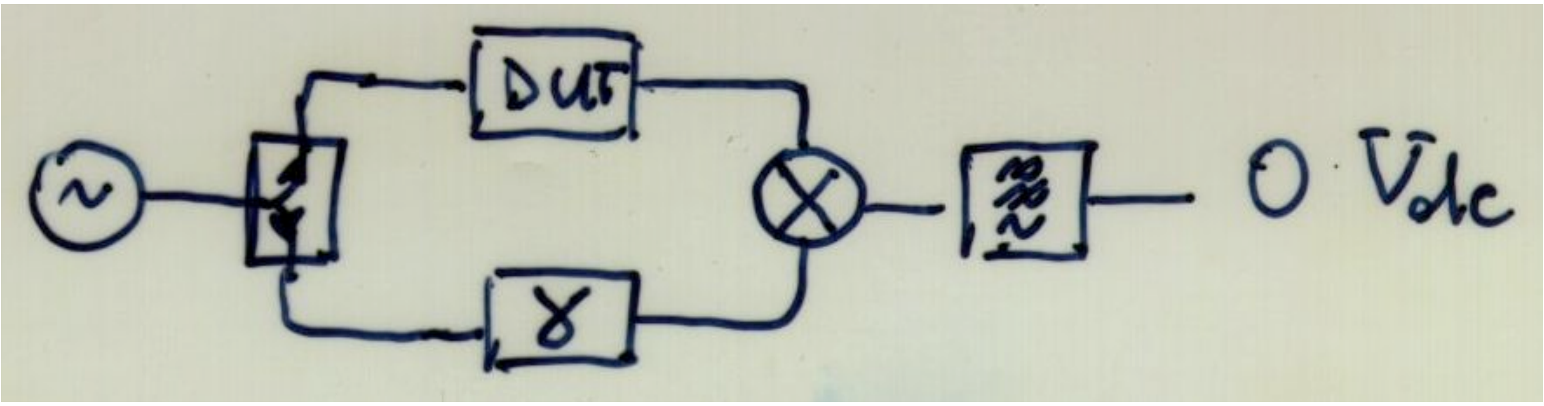
- double balanced mixer
- low-noise op-amp
- standard low-noise dc components in the feedback path
- commercial FFT analyzer

Afterwards, you will appreciate more the commercial instruments:
– assembly
– instruction manual
– computer interface and software

6 – Calibration

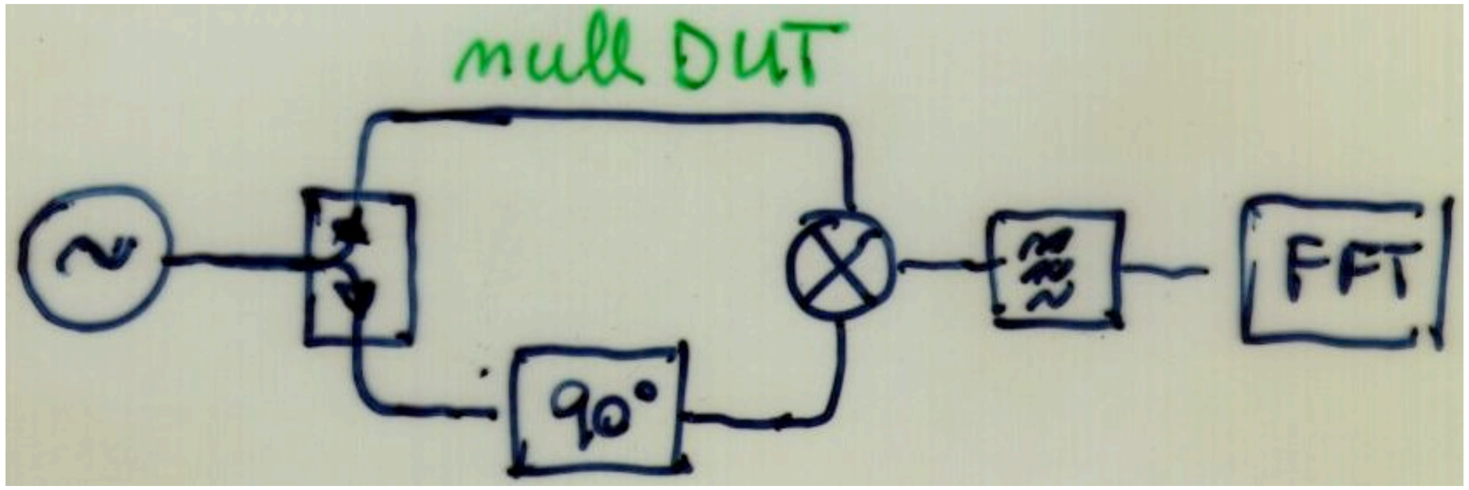
Calibration – general procedure

1 – adjust for proper operation: driving power and quadrature



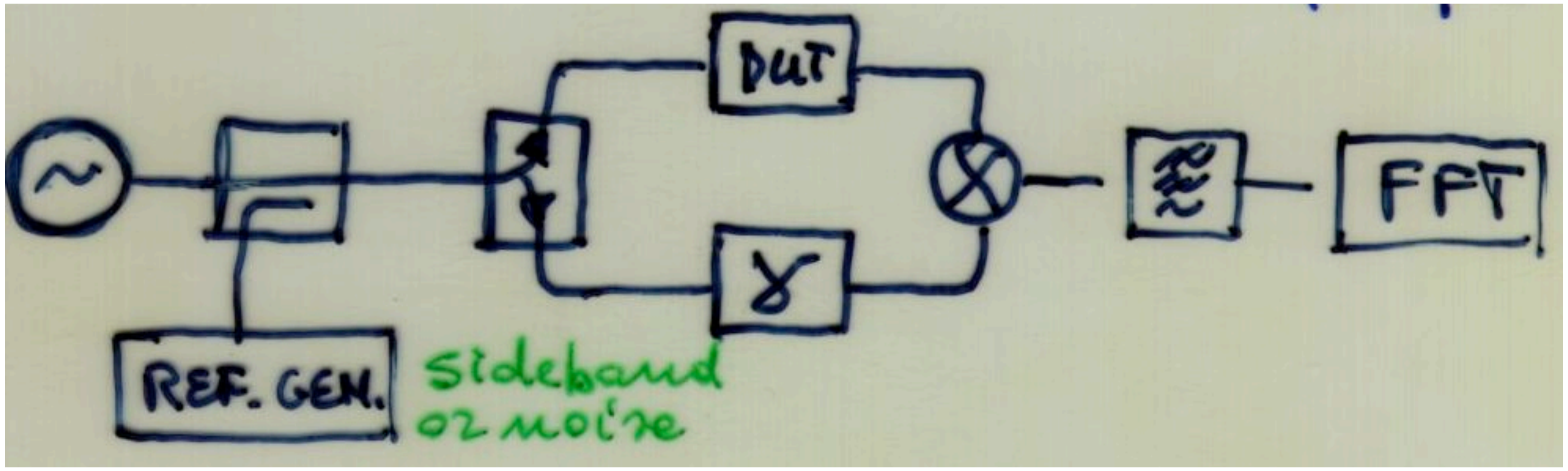
2 – measure the mixer gain k_φ (volts/rad) —> next

3 – measure the residual noise of the instrument



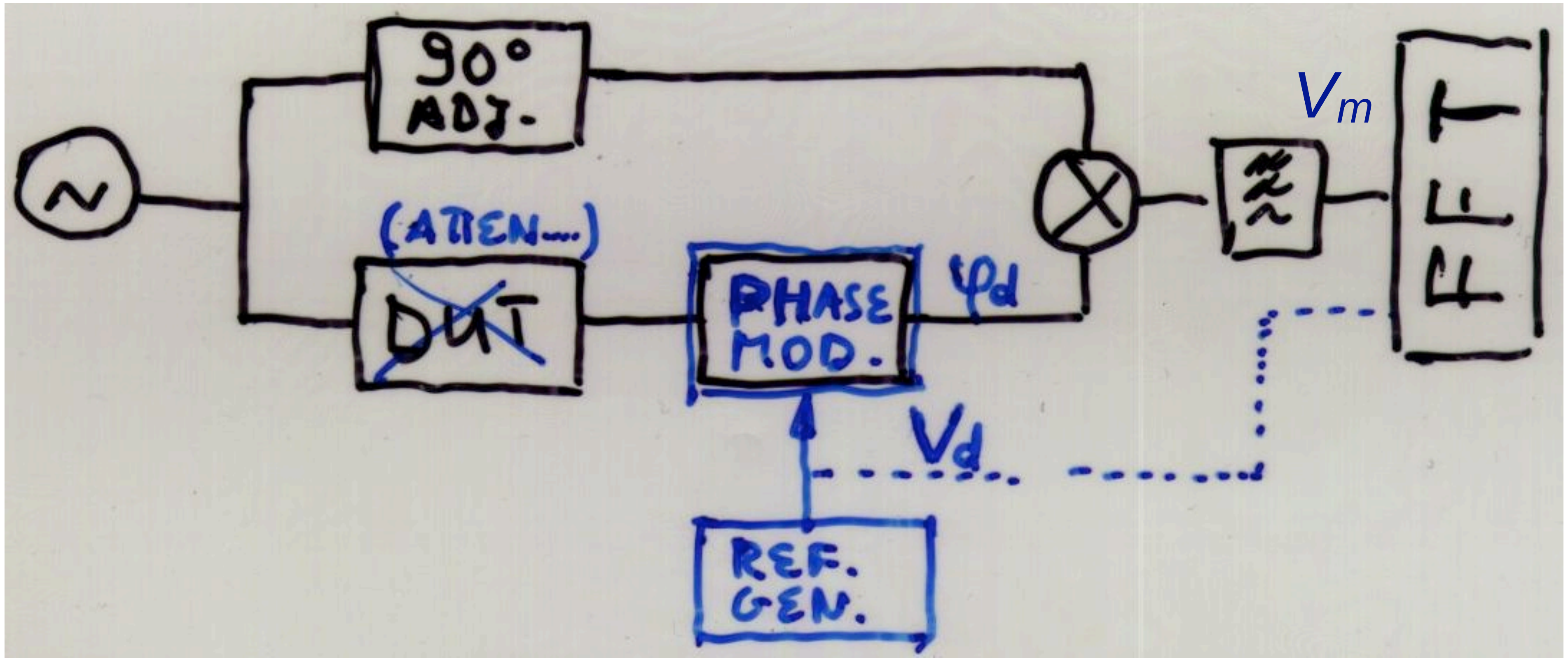
Calibration – general procedure

4 – measure the rejection of the oscillator noise



Make sure that the power and the quadrature are the same during all the calibration process

Calibration – measurement of k_φ (phase mod.)



The reference signal can be a

tone:

detect with the FFT,
with a dual-channel FFT, or
with a lock-in

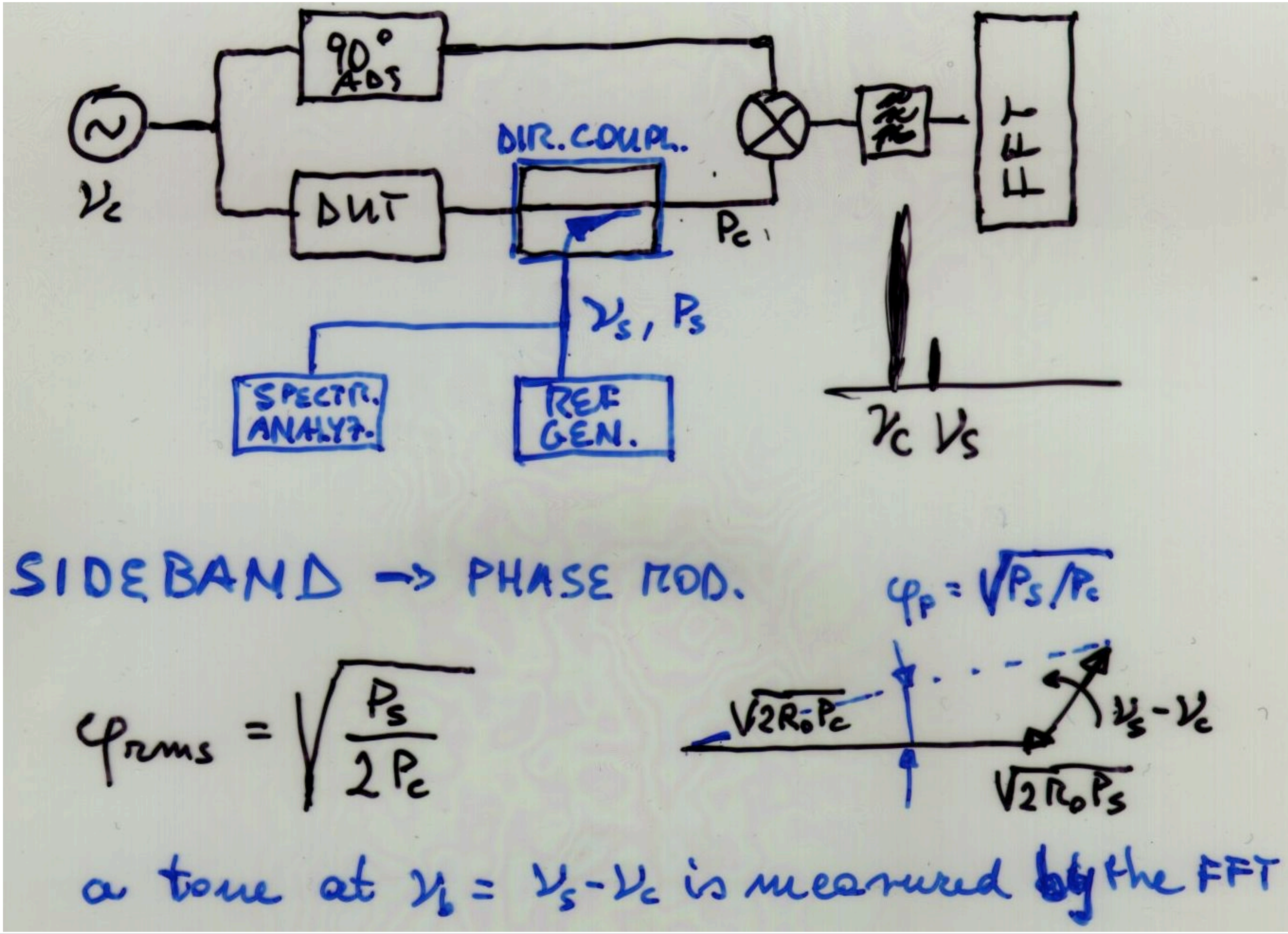
(pseudo-)random white noise

tone: $$k_\varphi = \frac{V_m}{k_m V_d}$$

white noise $$k_\varphi^2 = \frac{S_{Vm}}{k_m^2 S_{Vd}}$$

Some FFTs have a white noise output
Dual-channel FFTs calculate the transfer function $|H(f)|^2 = S_{Vm}/S_{Vd}$

Calibration – measurement of k_φ (rf signal)

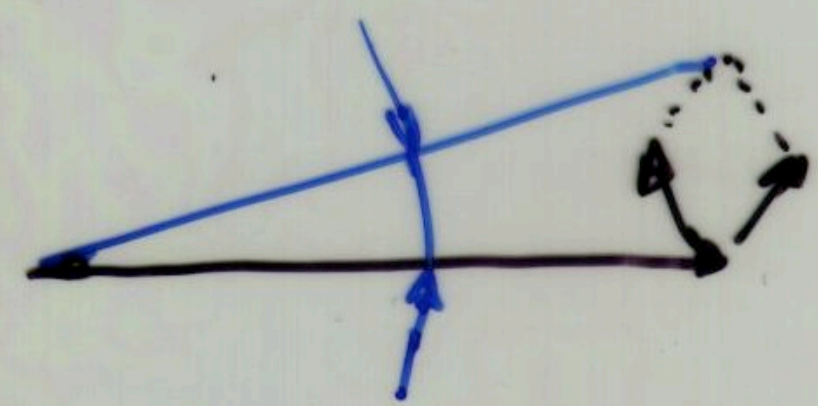


Calibration – measurement of k_φ (rf noise)

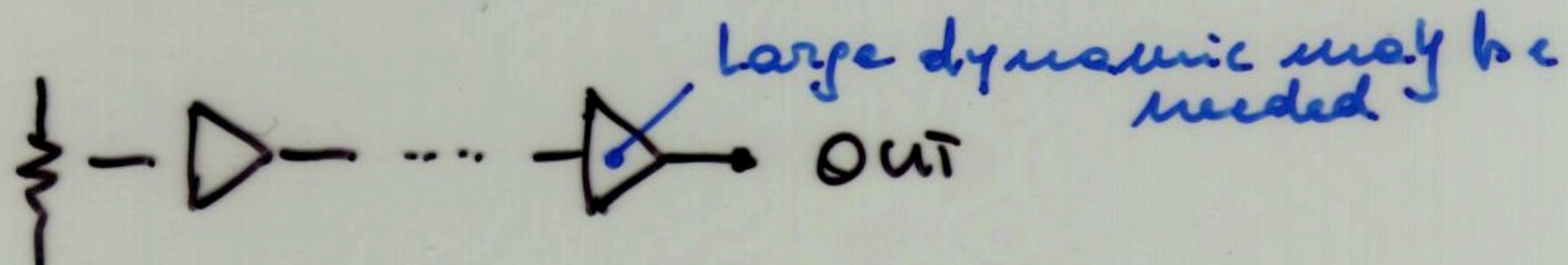
A reference rf noise is injected in the DUT path
through a directional coupler

WHITE NOISE

$$S_\varphi = \frac{N}{P_c}$$



WHITE NOISE GENERATOR

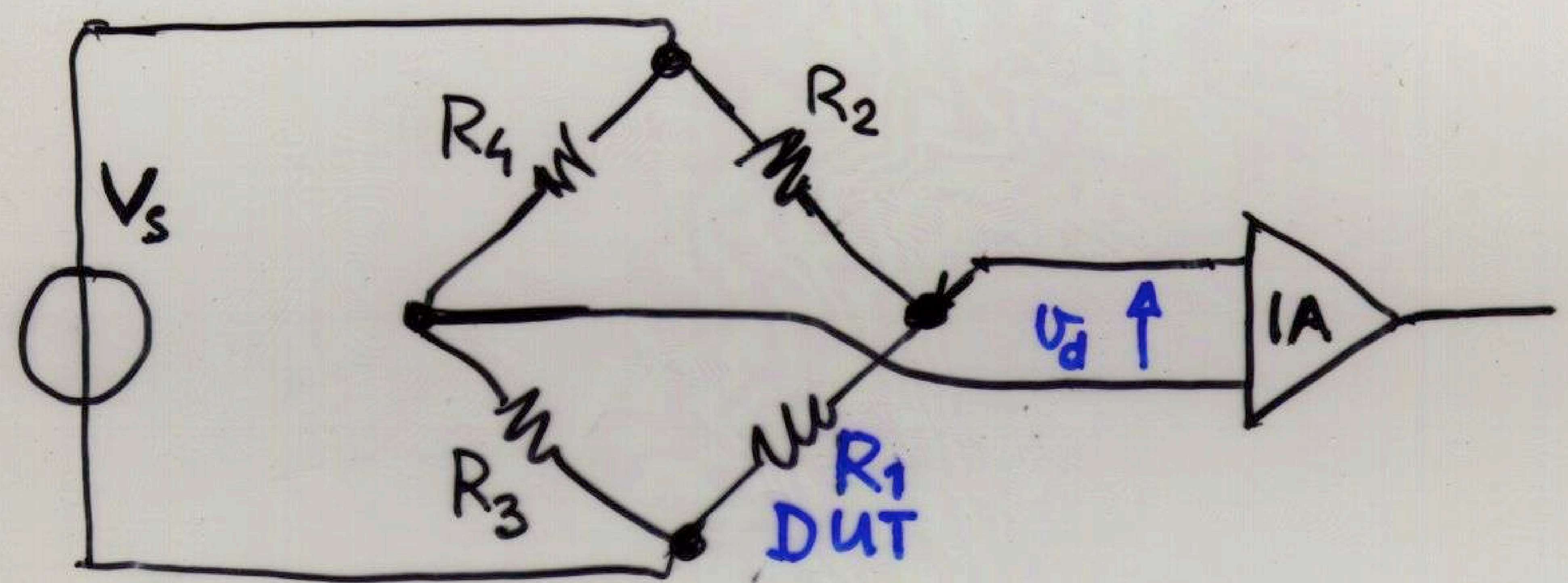


$$P_{1dB} \gg P_{peak} \approx P_{avg} + 20\ \mathrm{dB}$$

$$N_{OUT} \cdot B = F k_B T g B$$

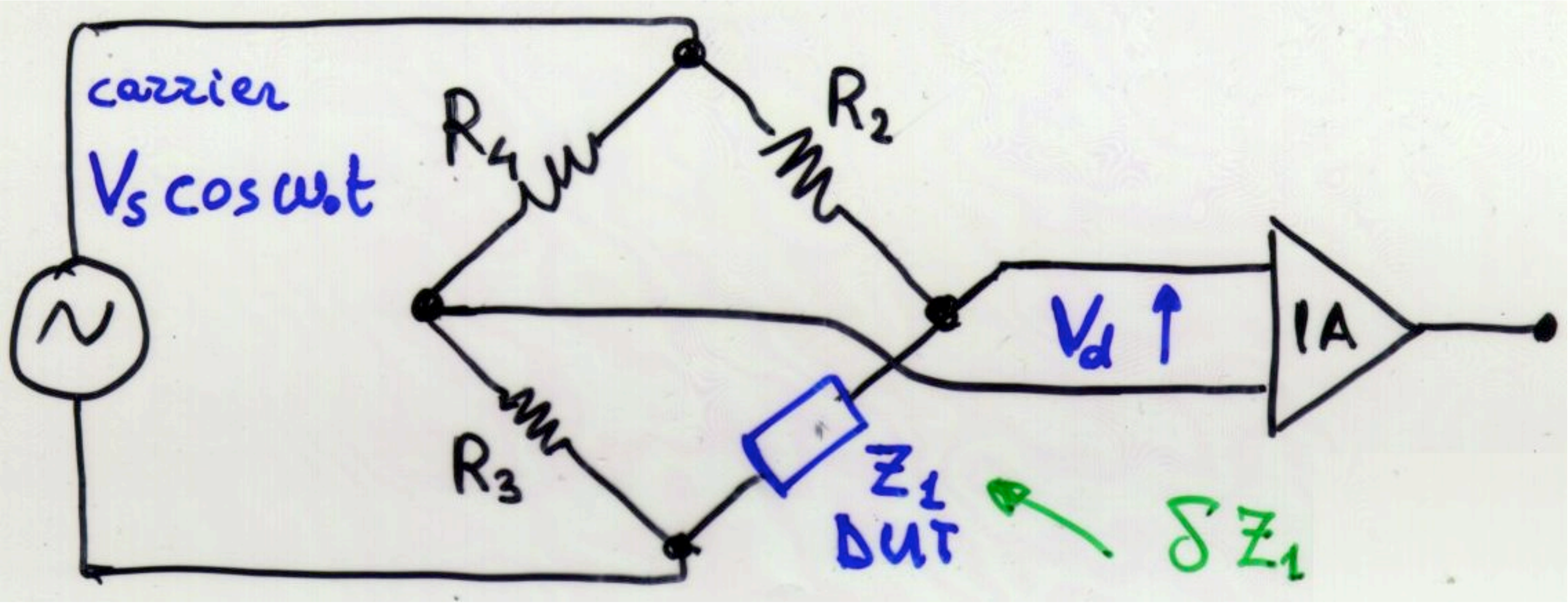
7 – Bridge (interferometric) measurements

Wheatstone bridge



- Rejection of δV_s ⟶ amplitude
  phase, if the group delay... is short.
- A narrow dynamic (v_d) is sufficient

Wheatstone bridge – ac version

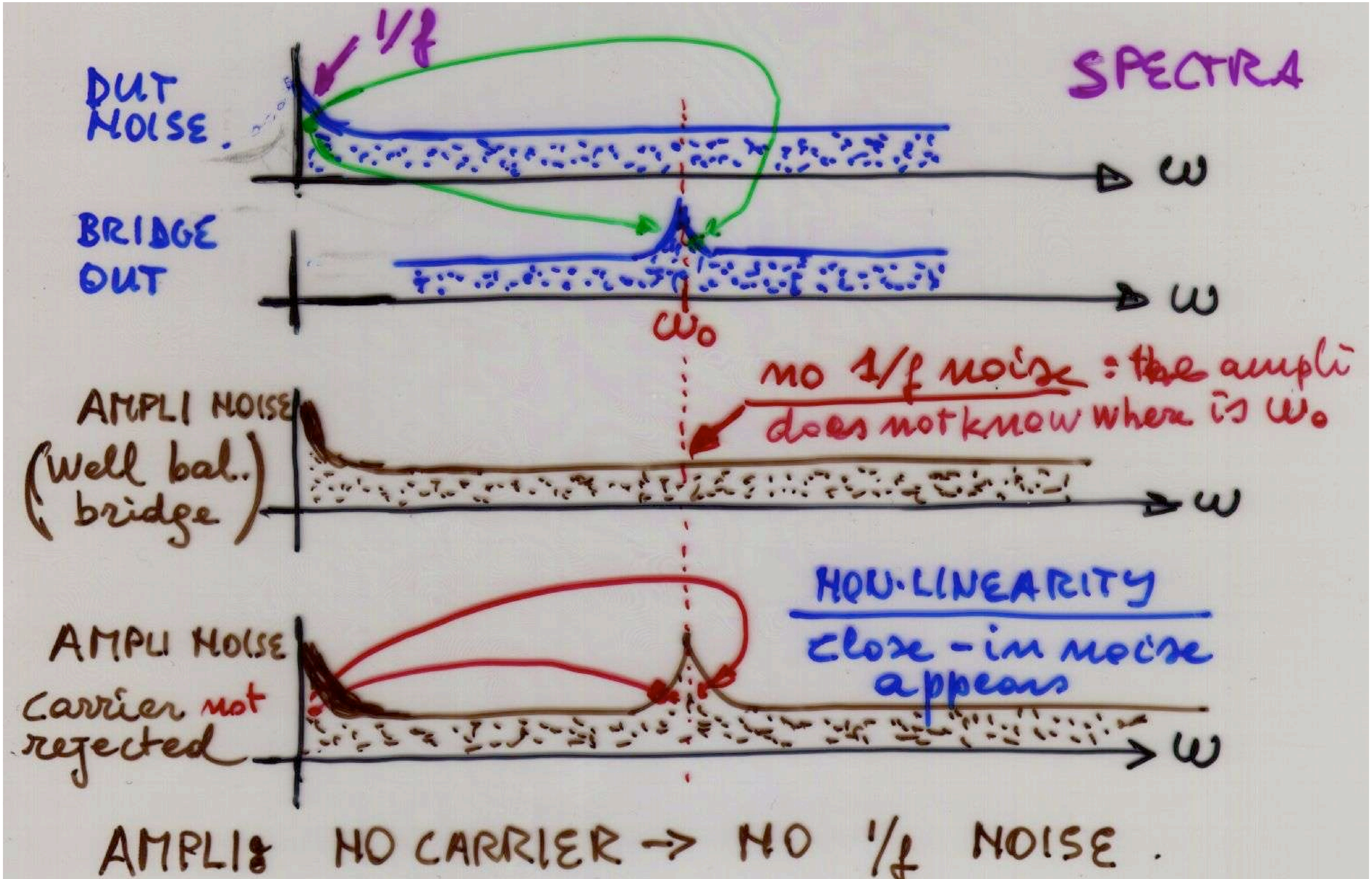


equilibrium: $V_d = 0$ –> carrier suppression

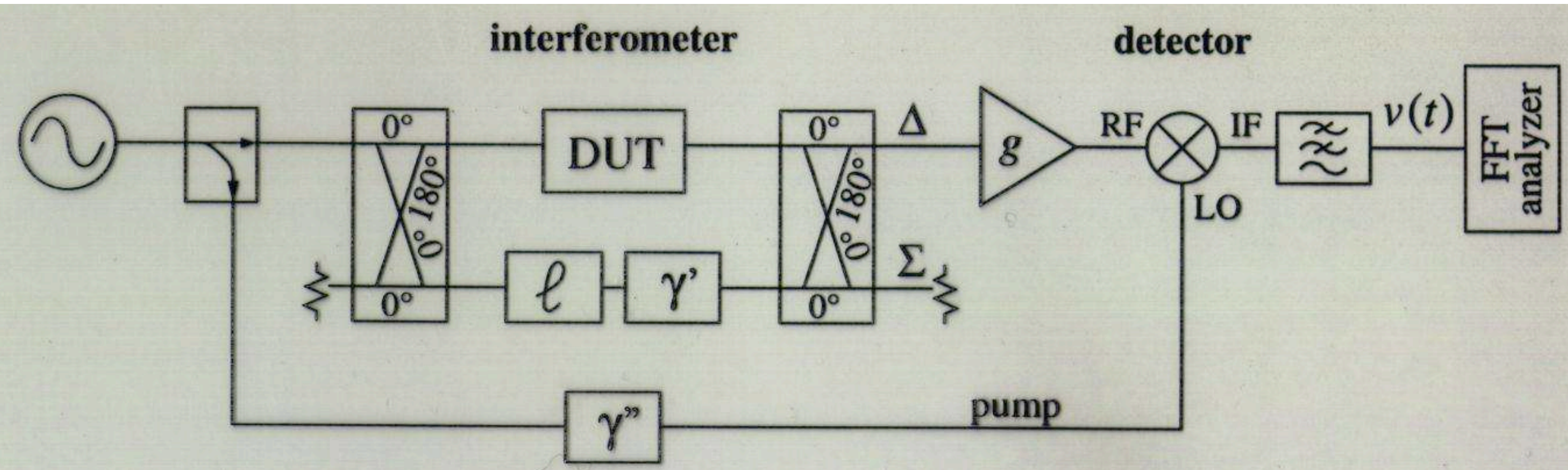
static error δZ_1 –> some residual carrier
real δZ_1 => in-phase residual carrier $V_{re} \cos(\omega_0 t)$
imaginary δZ_1 => quadrature residual carrier $V_{im} \sin(\omega_0 t)$

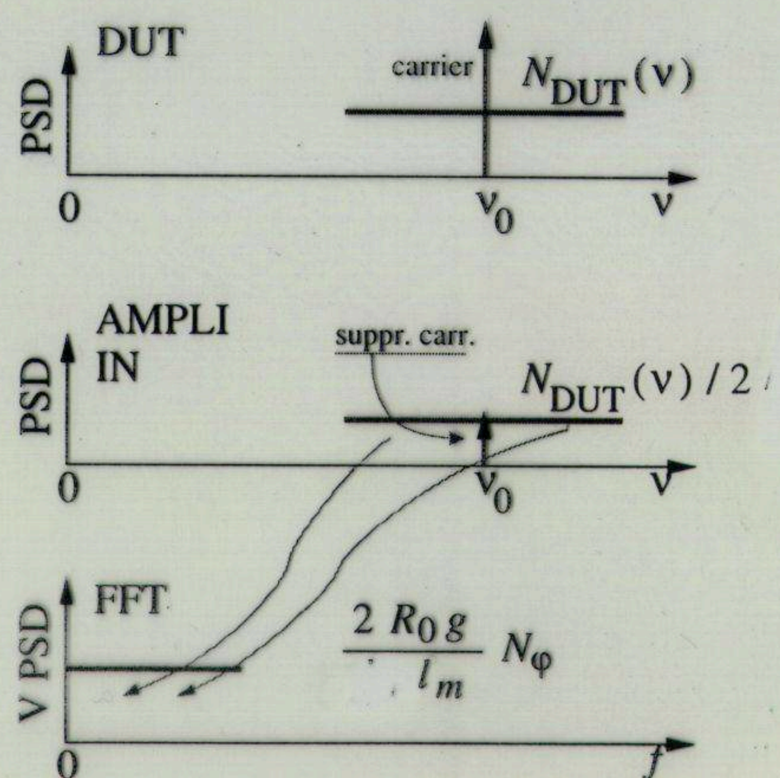
fluctuating error δZ_1 => noise sidebands
real δZ_1 => AM noise $n_c(t) \cos(\omega_0 t)$
imaginary δZ_1 => PM noise $-n_s(t) \sin(\omega_0 t)$

Wheatstone bridge – ac version



Bridge (interferometric) phase-noise and amplitude-noise measurement





High carrier suppression:

no carrier ⇒ the amplifier can't flicker

High gain:

$$k_\varphi = \frac{v(t)}{\varphi(t)} = \sqrt{\frac{R_0 g P_0}{l_m}} - \textit{dissip. losses}$$

Low Noise Floor:

$$S_{\varphi 0} = \frac{2 F k_B T_0}{P_0} + \textit{dissip. losses}$$

High immunity to 50 Hz B-fields

Improved, from RSI **70** 1 pp. 220-225, Jan 1999

Synchronous detection

$$s(t) = V_0\,[1+\alpha(t)]\cos[2\pi\nu_0 t + \varphi(t)]$$

for small α and φ, is equivalent to

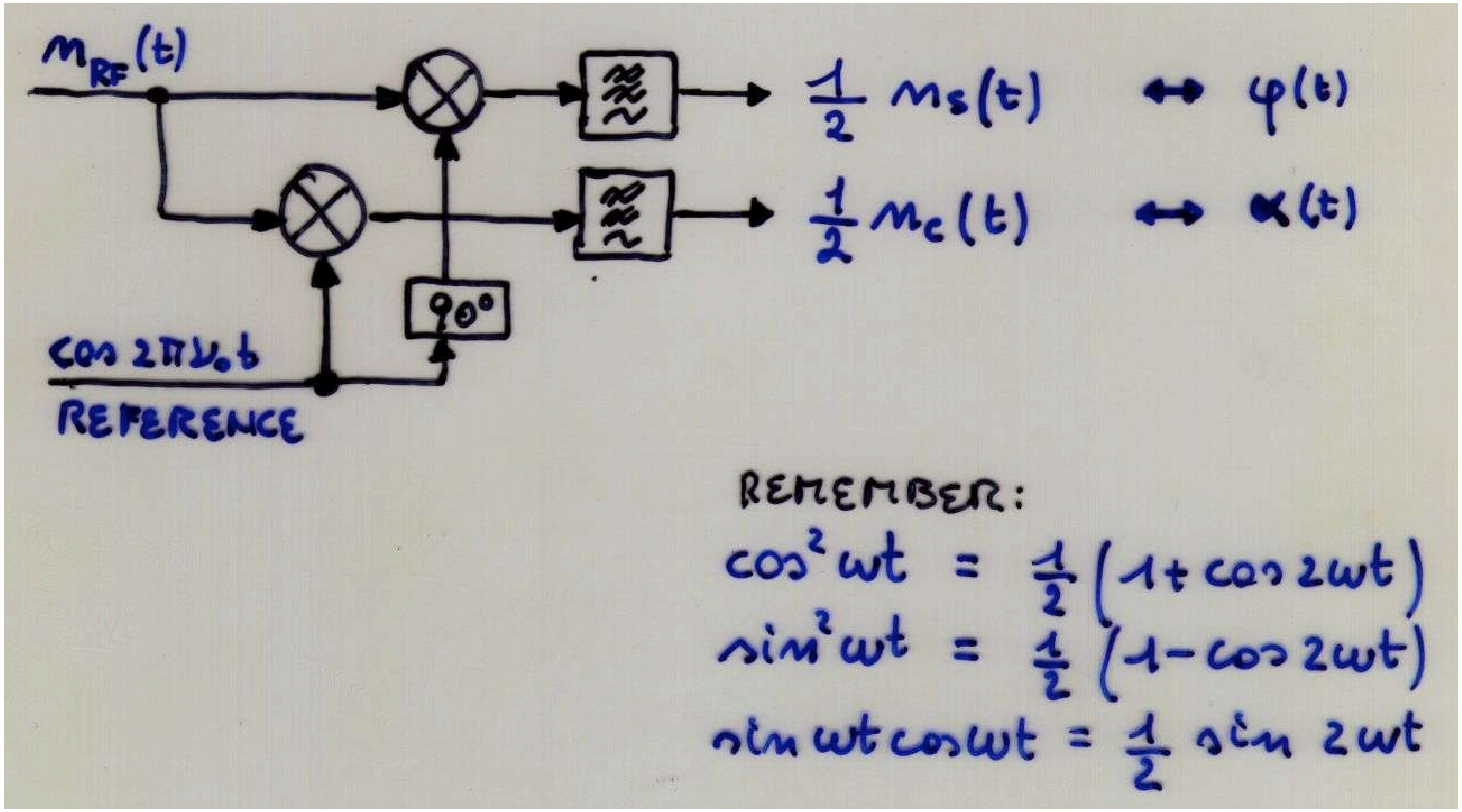
$$s(t) = n_{RF}(t) + V_0 \cos[2\pi\nu_0 t]$$

$$\underbrace{n_{RF}(t)}_{\text{close to the carrier}} = \underbrace{n_c(t)}_{\text{near dc}}\cos 2\pi\nu_0 t - \underbrace{n_s(t)}_{\text{near dc}}\sin 2\pi\nu_0 t$$

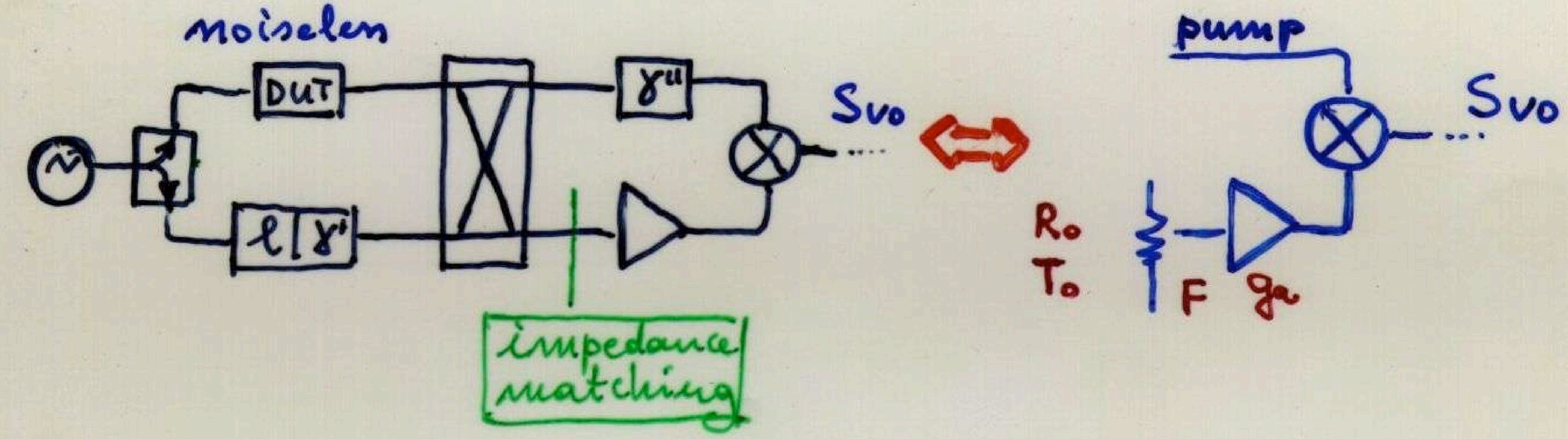
$$\alpha(t) = \frac{n_c(t)}{V_0}$$

$$\varphi(t) = \frac{n_s(t)}{V_0}$$

Synchronous in-phase and quadrature detection



White noise floor



$$S_{vo} = 2\,\frac{R_0\, g_a\, F\, k_B T_0}{\ell_m}$$

divide by $k_\varphi^2 = R_0\, g_a\, P_0 / \ell_h\, \ell_m$

$$S_{\varphi 0} = 2\,\ell_h\,\frac{F\, k_B T_0}{P_0}$$

thermal noise

$k_B = 1.38 \times 10^{-23}$ J/K

$T_0 \simeq 290$ K

$k_B T_0 = -174$ dBm/Hz

White noise floor – example

$k_B T_0 = -174$ dBm/Hz

$F = 2$ dB

$\ell_h = 1$ dB

$P_0 = 10$ dBm

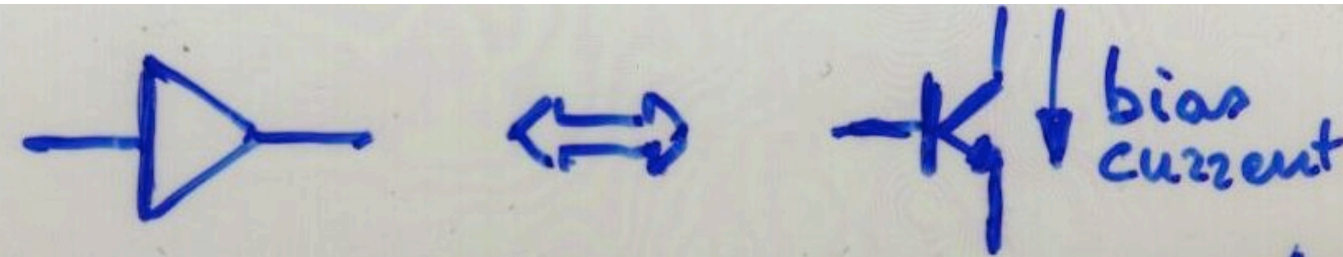
$S_{\varphi 0} = -178$ dBrad^2/Hz

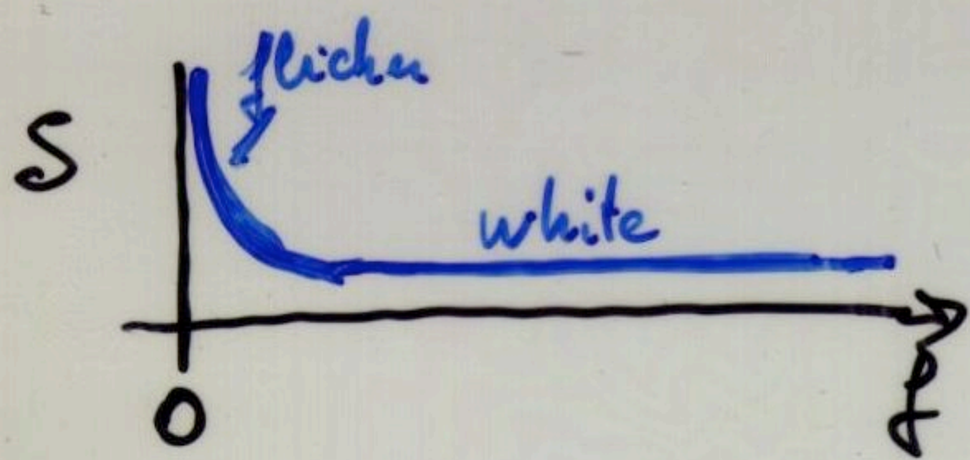
In the same conditions, changing P_0 to 32 dBm yields

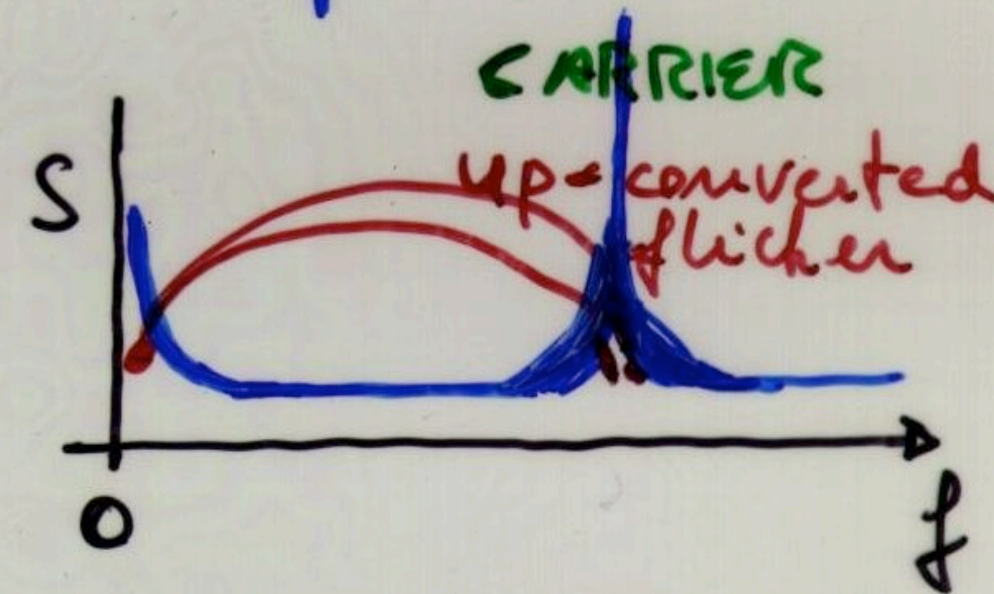
$S_{\varphi 0} = -200$ dBrad^2/Hz

What really matters (1)

1





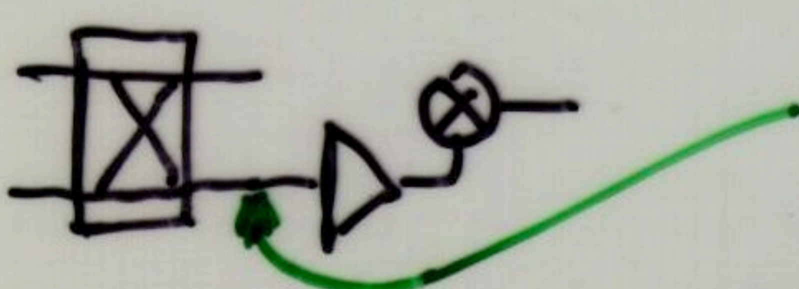


close-to-the-carrier flicker comes from near-dc flicker up-converted.

lowest carrier → best linearity → lowest flicker

WIDEBAND NOISE AND HARMONICS MAY ALSO CONTRIBUTE.

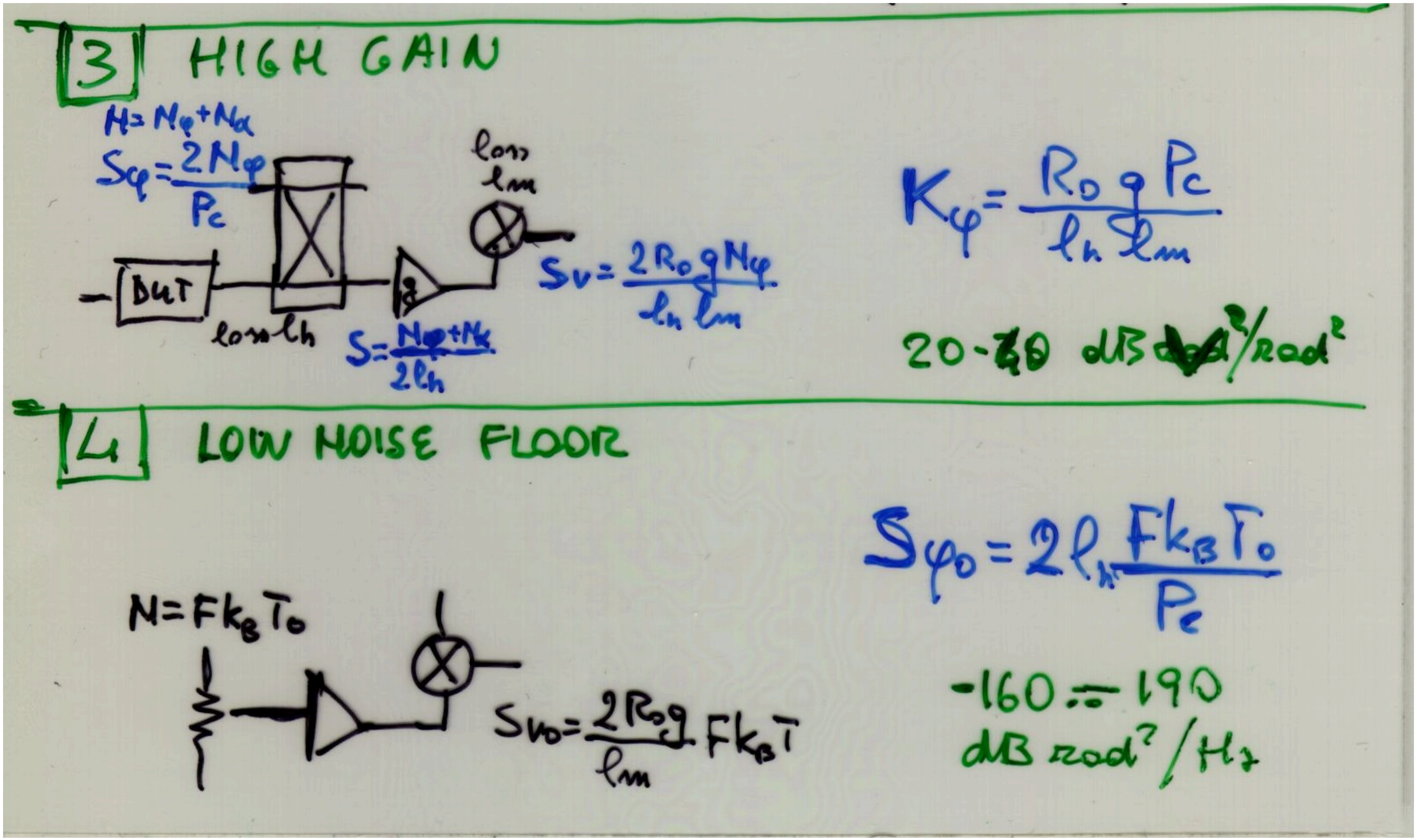
2



The smallest signal is a RF signal

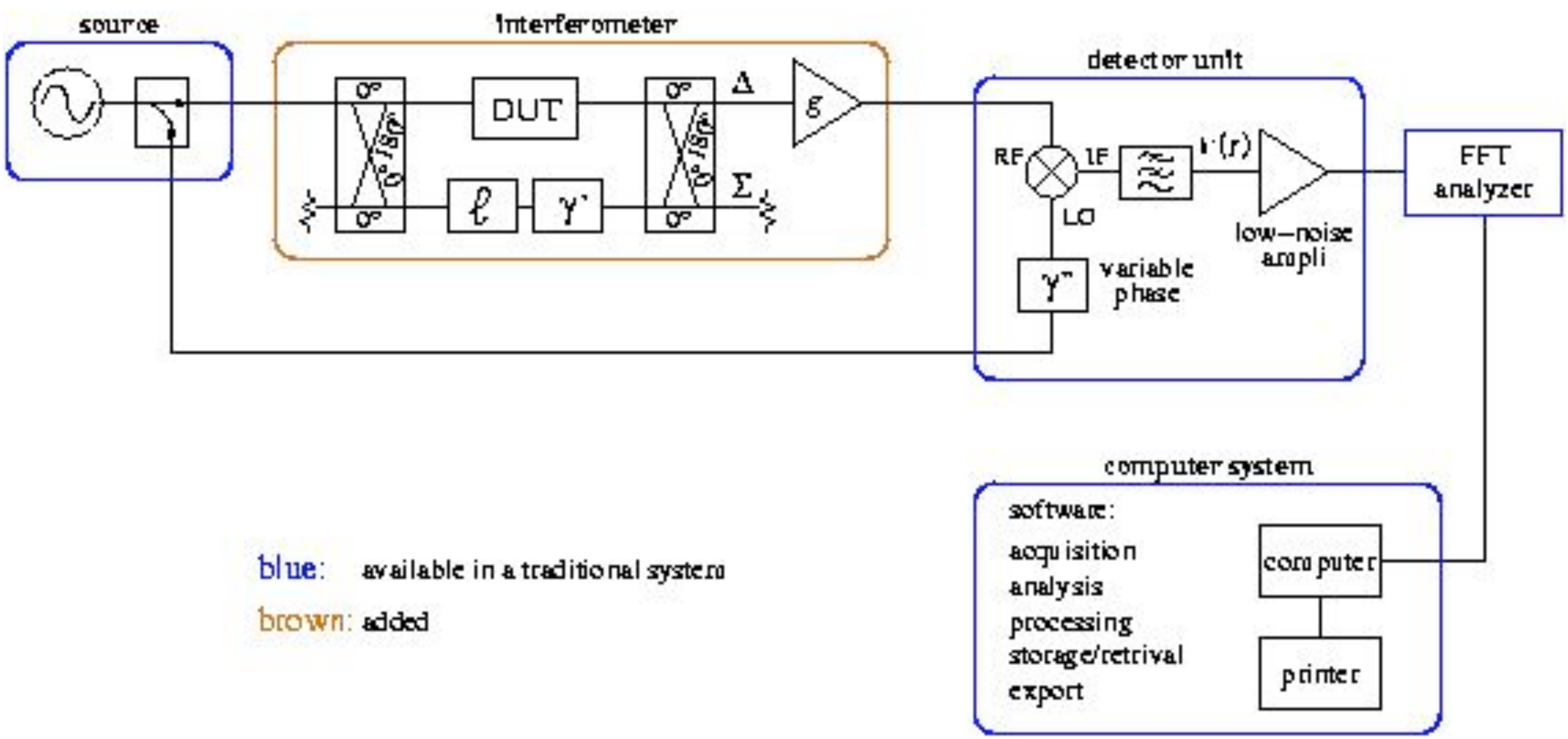
⇒ easy to shield from low freq. $\underline{B}$ fields

What really matters (2)



A bridge (interferometric) instrument can be built around a commercial instrument

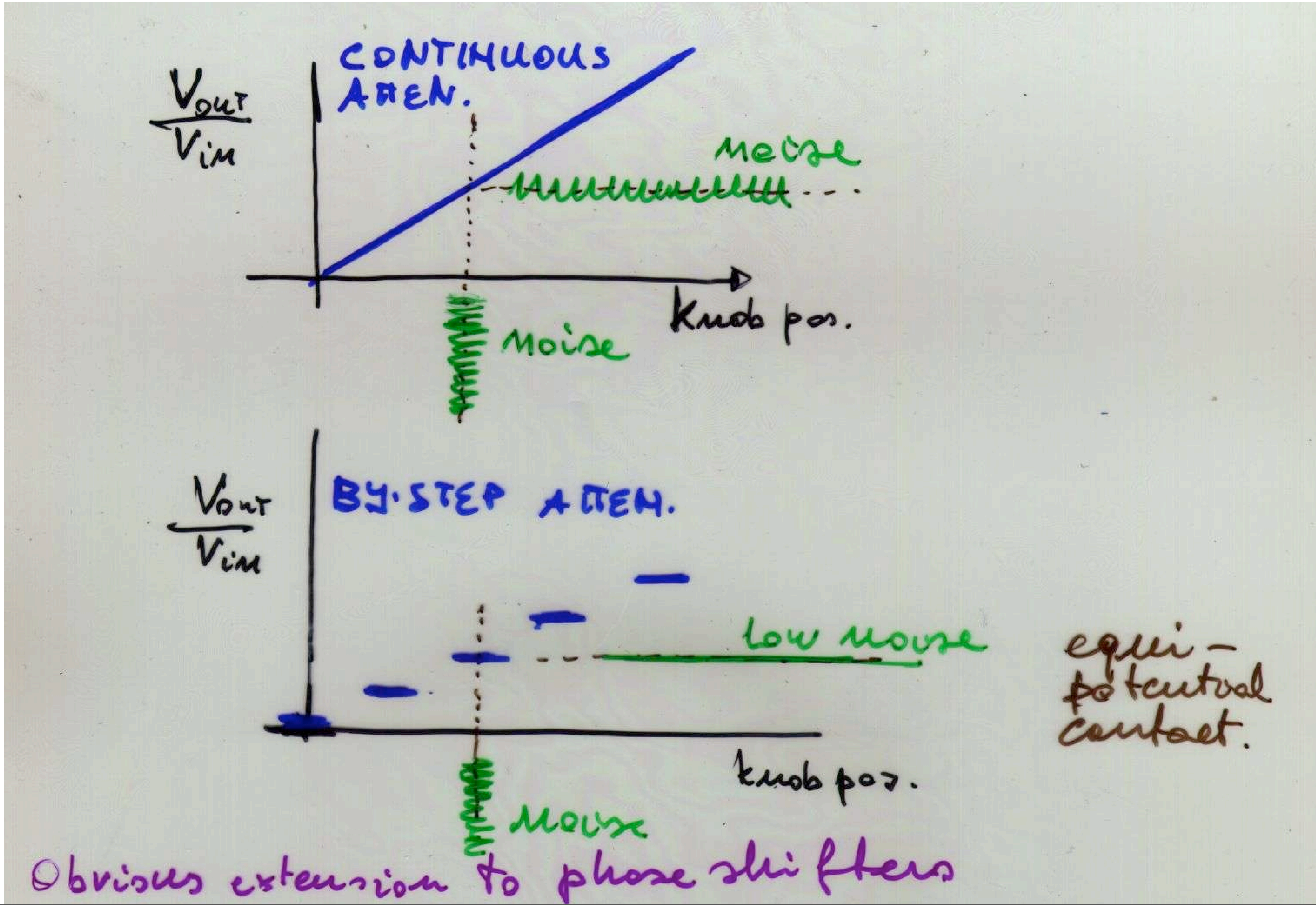
How to build an interferometer around a commercial instrument



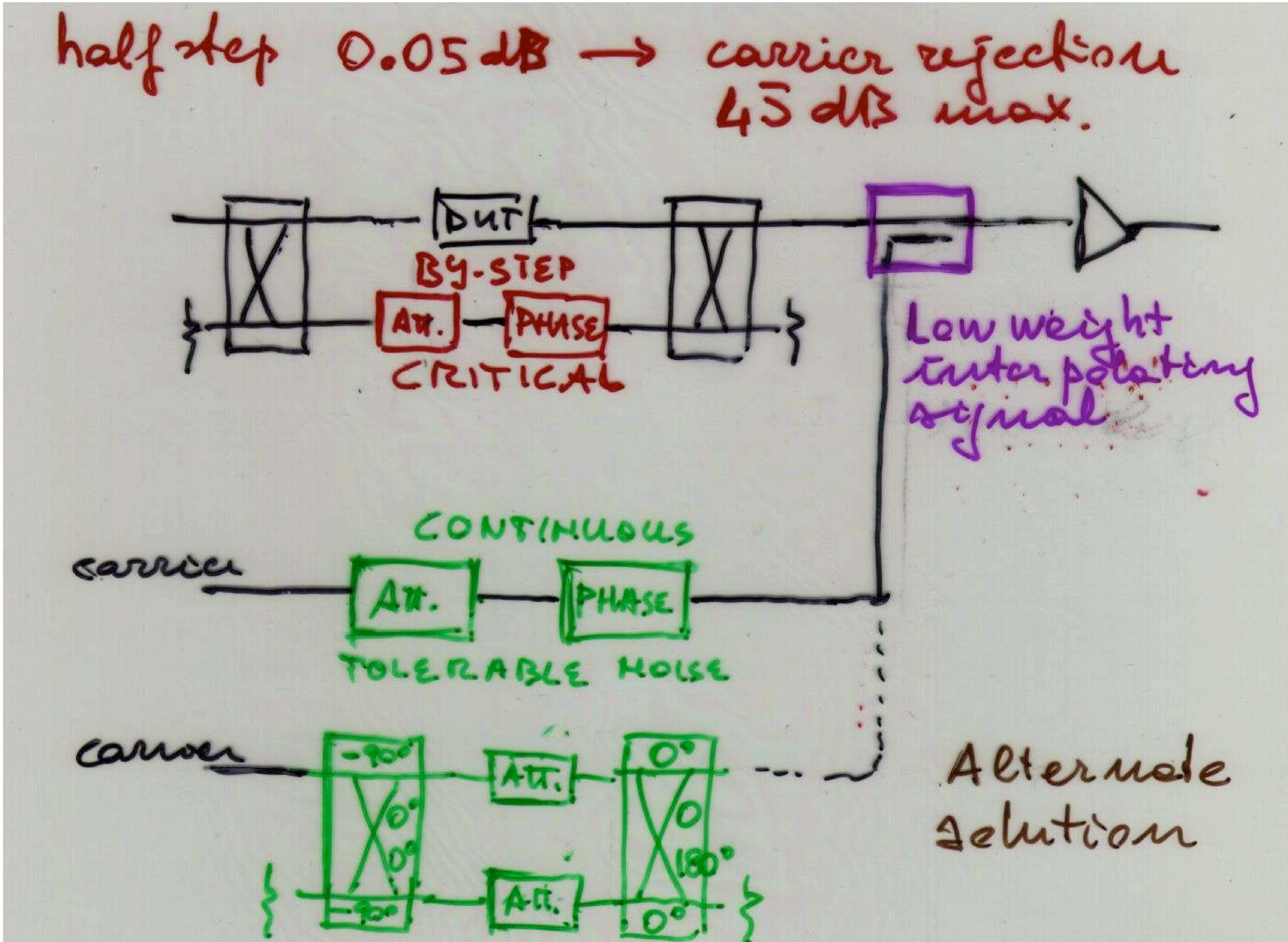
You will appreciate the computer interface and the software ready for use

8 – Advanced Techniques

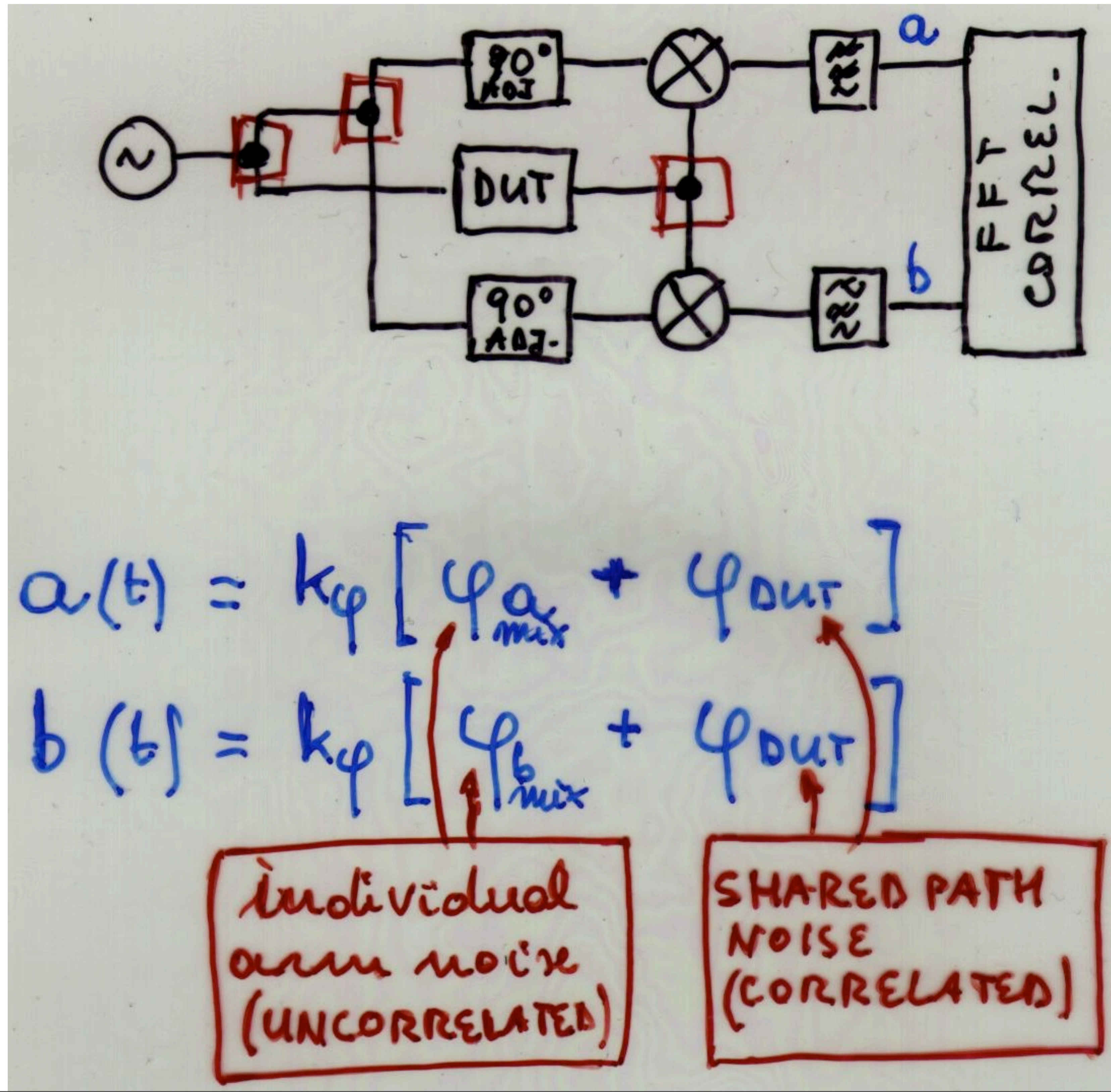
Low-flicker scheme



Interpolation is necessary



Correlation can be used to reject the mixer noise



Correlation – how it works

$$S_{ab} = k_\varphi^2 \, \underline{S_{\varphi\,\mathrm{DUT}}} + \text{residual} \ldots$$

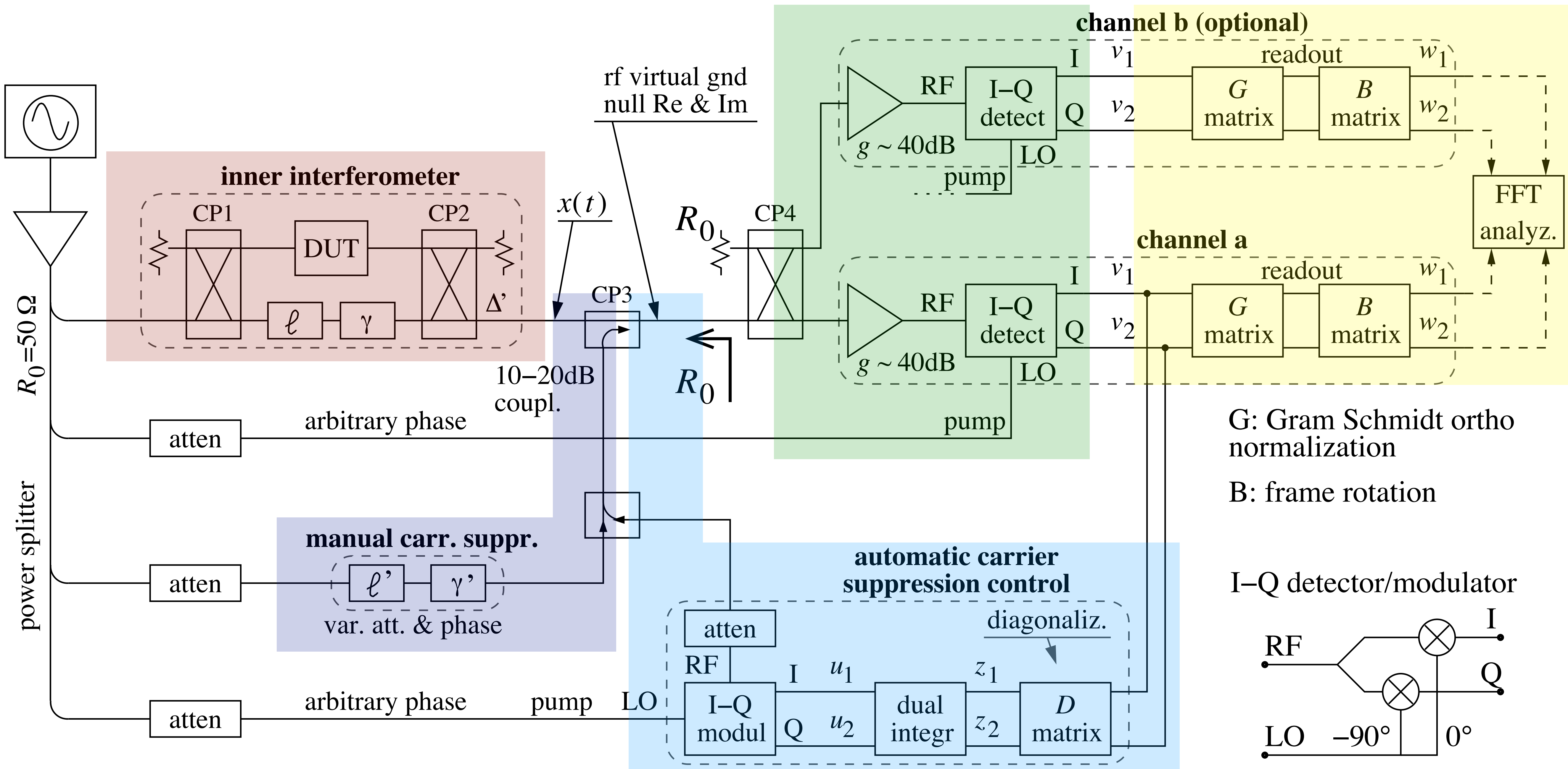
long averaging to remove

$$S_{\varphi\,\mathrm{resid}} \propto \frac{1}{\sqrt{2m}}$$

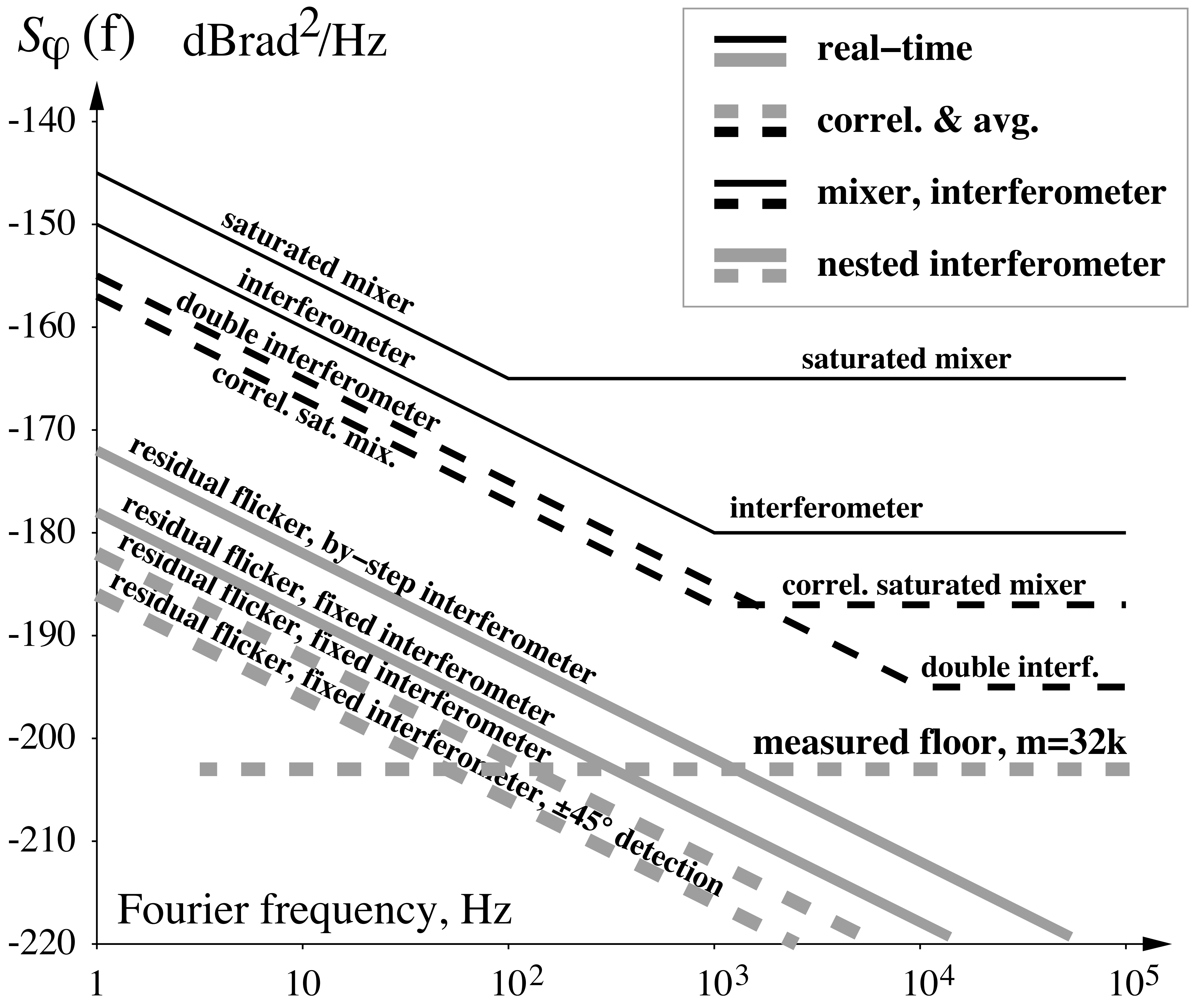
$$S_{ab} = A(f) \cdot B^*(f)$$

$$= k_\varphi^2 \left[\Phi_{a\,\mathrm{mix}} + \underline{\Phi_{\mathrm{DUT}}} \right] \left[\Phi^*_{b\,\mathrm{mix}} + \underline{\Phi^*_{\mathrm{DUT}}} \right]$$

Flicker reduction, correlation, and closed-loop carrier suppression can be combined



Comparison of the background noise



9 – References

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E. Rubiola, V. Giordano, Advanced Interferometric Phase and Amplitude Noise Measurements, Rev. of Scientific Instruments vol.73 no.6 pp.2445-2457, June 2002.
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BOOKS

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Oran E. Brigham, The Fast Fourier Transform and its Applications, Prentice-Hall 1988.
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W. D. Davenport, Jr., W. L."Root, An Introduction to Random Signals and Noise, McGraw Hill 1958.
Reprinted by the IEEE Press, 1987.
One of the best references on electrical noise in general and on its mathematical properties.

E. Rubiola, The Leeson effect (e-book, 117 pages, 50 figures) arxiv.org, document arXiv:physics/0502143

E. Rubiola, Phase Noise Metrology, book in preparation

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