Summary

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2. Spectra
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4. Properties of phase noise
5. Laboratory practice
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1 – Introduction
Representations of a sinusoid with noise

**Time Domain**

- Amplitude fluctuation: $V_0 \alpha(t)$ [volts]
- Normalized amplitude fluctuation: $\alpha(t)$ [adimensional]
- Phase fluctuation: $\varphi(t)$ [rad]
- Phase time (fluctuation): $x(t)$ [seconds]

**Phasor Representation**

- Amplitude fluctuation: $(V_0/\sqrt{2})\alpha(t)$
- Phase fluctuation: $\varphi(t)$

**Polar Coordinates**

- $v(t) = V_0 [1 + \alpha(t)] \cos[\omega_0 t + \varphi(t)]$

**Cartesian Coordinates**

- $v(t) = V_0 \cos \omega_0 t + n_c(t) \cos \omega_0 t - n_s(t) \sin \omega_0 t$

**Under Low Noise Approximation**

- $|n_c(t)| \ll V_0$ and $|n_s(t)| \ll V_0$

**It Holds That**

- $\alpha(t) = \frac{n_c(t)}{V_0}$ and $\varphi(t) = \frac{n_s(t)}{V_0}$

**Introduction – Noisy Sinusoid**

Enrico Rubiola – Phase Noise – 3
Noise broadens the spectrum

\[ v(t) = V_0 \left[ 1 + \alpha(t) \right] \cos [\omega_0 t + \varphi(t)] \]

\[ v(t) = V_0 \cos \omega_0 t + n_c(t) \cos \omega_0 t - n_s(t) \sin \omega_0 t \]

\[ \omega = 2\pi \nu \]

\[ \nu = \frac{\omega}{2\pi} \]

[Diagram showing the spectrum of a pure sinusoid, sinusoid + white noise, and actual signals.]
**Basic problem:** how can we measure a low random signal (noise sidebands) close to a strong dazzling carrier?

**Solution(s):** suppress the carrier and measure the noise

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<th>Convolution (low-pass)</th>
<th>( s(t) \times h_{lp}(t) )</th>
<th>Distorsiometer, audio-frequency instruments</th>
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<td>Time-domain product</td>
<td>( s(t) \times r(t - T/4) )</td>
<td>Traditional instruments for phase-noise measurement (saturated mixer)</td>
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<td>Vector difference</td>
<td>( s(t) - r(t) )</td>
<td>Bridge (interferometric) instruments</td>
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</table>
Why a spectrum analyzer does not work?

1. too wide IF bandwidth
2. noise and instability of the conversion oscillator (VCO)
3. detects both AM and PM noise
4. insufficient dynamic range

Some commercial analyzers provide phase noise measurements, yet limited (at least) by the oscillator stability
The Schottky-diode double-balanced mixer saturated at both inputs is the most used phase detector

\[
\begin{align*}
s(t) &= \sqrt{2R_0P_0} \cos [2\pi\nu_0t + \varphi(t)] \\
r(t) &= \sqrt{2R_0P_0} \cos [2\pi\nu_0t + \pi/2] \\
r(t)s(t) &= k_{\varphi} \varphi(t) + "2\nu_0" \text{ terms}
\end{align*}
\]

The AM noise is rejected by saturation
Saturation also account for the phase-to-voltage gain \( k_{\varphi} \)
2 – Spectra
Power spectrum density

In general, the power spectrum density $S_v(f)$ of a random process $v(t)$ is defined as

$$S_v(f) = \mathbb{E}\left\{ \mathcal{F}\{ \mathcal{R}_v(t_1, t_2) \} \right\}$$

$\mathbb{E}\{ \cdot \}$ statistical expectation

$\mathcal{F}\{ \cdot \}$ Fourier transform

$\mathcal{R}_v(t_1, t_2)$ autocorrelation function

In practice, we measure $S_v(f)$ as

$$S_v(f) = |\mathcal{F}\{v(t)\}|^2$$

This is possible (Wiener-Khinchin theorem) with ergodic processes.

In many real-life cases, processes are ergodic and stationary.

**Ergodicity:** ensemble and time-domain statistics can be interchanged. This is the formalization of the reproducibility of an experiment.

**Stationarity:** the statistics is independent of the origin of time. This is the formalization of the repeatability of an experiment.
Physical meaning of the power spectrum density

\[ P = \frac{S_v(f_0)}{R_0} \]

\( S_v(f_0) \) power in 1 Hz bandwidth dissipated by \( R_0 \)
Physical meaning of the power spectrum density

The power spectrum density extends the concept of root-mean-square value to the frequency domain.
Sφ(f) and ℒ(f) in the presence of (white) noise

SSB

DSB

\[ \varphi_{\text{rms}} = \sqrt{\frac{1}{2}} \varphi_p = \sqrt{\frac{NB}{2P_0}} \]

\[ \varphi_{\text{rms}} = \sqrt{\frac{1}{2}} \varphi_p = \sqrt{\frac{NB}{P_0}} \]

dBc/Hz \[ \mathcal{L} = \frac{N}{2P_0} \] 3 dBdBrad\(^2\)/Hz \[ S\varphi = \frac{N}{P_0} \]
The first definition of $\mathcal{L}(f)$ was

$$\mathcal{L}(f) = \frac{\text{SSB power in 1Hz bandwidth}}{\text{carrier power}}$$

The problem with this definition is that it does not divide AM noise from PM noise, which yields to ambiguous results.

Engineers (manufacturers even more) like $\mathcal{L}(f)$.

The IEEE Std 1139-1999 redefines $\mathcal{L}(f)$ as

$$\mathcal{L}(f) = \frac{1}{2} \times S_\phi(f)$$
**Useful quantities**

phase time

\[ x(t) = \frac{1}{2\pi \nu_0} \phi(t) \]

\( x(t) \) is the phase noise converted into time fluctuation

physical dimension: time (seconds)

fractional frequency fluctuation

\[ y(t) = \frac{1}{2\pi \nu_0} \dot{\phi}(t) = \dot{x}(t) \]

\( y(t) \) is the fractional frequency fluctuation \( \nu - \nu_0 \) normalized to the nominal frequency \( \nu_0 \)

(dimensionless)

\[ y(t) = \frac{\nu - \nu_0}{\nu_0} \]
Power-law and noise processes in oscillators

\[ S_y = \sum_i h_i f_i \]

\[ S_p = \sum_i b_i f_i \]
Relationships between $S_{\varphi}(f)$ and $S_y(f)$

\[ S_{\varphi} \rightarrow \text{FREQUENCY STABILITY} \]

\[ \cos (2\pi \nu t + \varphi) \]

\[ S_{\varphi}(f) \]

\[ h_3 f^{-3} \]

\[ h_2 f^{-2} \]

\[ h_1 f^{-1} \]

\[ h_0 \]

\[ \frac{\nu^2}{2} \frac{\partial^2 \varphi}{\partial t^2} \]

\[ S_y = \frac{f^2}{h_0^2} S_{\varphi} \]
Jitter

The phase fluctuation can be described in terms of a single parameter, either phase jitter or time jitter.

The phase noise must be integrated over the bandwidth $B$ of the system (which may be difficult to identify).

$$\varphi_{\text{rms}} = \sqrt{\int_B S_\varphi(f) \, df} \quad \text{radians}$$

$$x_{\text{rms}} = \frac{1}{2\pi\nu_0} \sqrt{\int_B S_\varphi(f) \, df} \quad \text{seconds}$$

The jitter is useful in digital circuits because the bandwidth $B$ is known:
- lower limit: the inverse propagation time through the system (this excludes the low-frequency divergent processes);
- upper limit: $\sim$ the inverse switching speed.
Typical phase noise of some devices and oscillators

Spectra – examples

The one-side spectrum is preferred because this is what spectrum analyzers display. Complying to the usual terminology, we use the symbol \( \nu \) for the frequency and \( f \) for the Fourier frequency, i.e. the frequency of the detected signal when the sidebands around \( \nu \) are down-converted to baseband.

The power-law model is most frequently used for describing phase noise. It assumes that \( S_\phi(x) f^y \) is equal to the sum of terms, each of which varies as an integer power of frequency. Thus each term, that corresponds to a noise process, is completely specified by two parameters, namely the exponent and the value at \( f = 1 \) Hz.

Five power-law processes, listed underneath, are common with electronics.

- White phase noise
- flicker phase noise
- white frequency noise
- flicker frequency noise
- random-walk frequency noise

All these noise types are generally present at the output of oscillators, while two-port devices show white phase and flicker phase noise only.

For reference, Fig. 1 reports the typical phase noise of some oscillators and devices.
3 – Variances
Classical variance

\[
\bar{y} = \frac{1}{\nu} \frac{1}{\tau} \int \nu(t) \, dt
\]

normalized reading of a counter that measures (averages) over a time \( T \)

\[
\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} \left( \bar{y}_i - \frac{1}{N} \sum_{j=1}^{N} \bar{y}_j \right)^2
\]

classical variance, file of \( N \) counter readings

average of the \( N \) readings

For a given process, the classical variance depends of \( N \)

Even worse, if the spectrum is \( f^{-1} \) or steeper, the classical variance diverges

The filter associated to the measure takes in the dc component
Zero dead-time two-sample variance
(Allan variance)

\[ \sigma_y^2 = \frac{1}{2} \left\langle \left( y_2 - y_1 \right)^2 \right\rangle \]

Definition
(Let \( N = 2 \), and average)

\[ \sigma_y^2 = \frac{1}{2(m-1)} \sum_{i=1}^{m-1} \left( \bar{y}_2 - \bar{y}_1 \right)^2 \]

Estimated Allan variance, file of \( m \) counter readings

The filter associated to the difference of two contiguous measures is a band-pass

The estimate converges to the variance
The Allan variance is related to the spectrum $S_y(f)$.
# Convert $S_\varphi$ and $S_y$ into Allan variance

<table>
<thead>
<tr>
<th>noise type</th>
<th>$S_\varphi(f)$</th>
<th>$S_y(f)$</th>
<th>$S_\varphi \leftrightarrow S_y$</th>
<th>$\sigma_y^2(\tau)$</th>
<th>mod $\sigma_y^2(\tau)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>white PM</td>
<td>$b_0$</td>
<td>$h_2 f^2$</td>
<td>$h_2 = \frac{b_0}{\nu_0^2}$</td>
<td>$\frac{3 f_H h_2}{(2\pi)^2} \tau^{-2}$</td>
<td>$\frac{3 f_H \tau_0 h_2}{(2\pi)^2} \tau^{-3}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$2\pi \tau f_H \gg 1$</td>
<td></td>
</tr>
<tr>
<td>flicker PM</td>
<td>$b_{-1} f^{-1}$</td>
<td>$h_1 f$</td>
<td>$h_1 = \frac{b_{-1}}{\nu_0^2}$</td>
<td>$[1.038 + 3 \ln(2\pi f_H \tau)] \frac{h_1}{(2\pi)^2} \tau^{-2}$</td>
<td>$0.084 h_1 \tau^{-2}$</td>
</tr>
<tr>
<td>FM</td>
<td>$b_{-2} f^{-2}$</td>
<td>$h_0$</td>
<td>$h_0 = \frac{b_{-2}}{\nu_0^2}$</td>
<td>$\frac{1}{2} h_0 \tau^{-1}$</td>
<td>$\frac{1}{4} h_0 \tau^{-1}$</td>
</tr>
<tr>
<td>flicker FM</td>
<td>$b_{-3} f^{-3}$</td>
<td>$h_{-1} f^{-1}$</td>
<td>$h_{-1} = \frac{b_{-3}}{\nu_0^2}$</td>
<td>$2 \ln(2) h_{-1}$</td>
<td>$\frac{27}{20} \ln(2) h_{-1}$</td>
</tr>
<tr>
<td>random</td>
<td>$b_{-4} f^{-4}$</td>
<td>$h_{-2} f^{-2}$</td>
<td>$h_{-2} = \frac{b_{-4}}{\nu_0^2}$</td>
<td>$\frac{(2\pi)^2}{6} h_{-2} \tau$</td>
<td>$0.824 \frac{(2\pi)^2}{6} h_{-2} \tau$</td>
</tr>
<tr>
<td>walk FM</td>
<td>$b_{-4} f^{-4}$</td>
<td>$h_{-2} f^{-2}$</td>
<td>$h_{-2} = \frac{b_{-4}}{\nu_0^2}$</td>
<td>$\frac{(2\pi)^2}{6} h_{-2} \tau$</td>
<td>$0.824 \frac{(2\pi)^2}{6} h_{-2} \tau$</td>
</tr>
<tr>
<td>frequency drift $\dot{y} = D_y$</td>
<td></td>
<td></td>
<td></td>
<td>$\frac{1}{2} D_y^2 \tau^2$</td>
<td>$\frac{1}{2} D_y^2 \tau^2$</td>
</tr>
</tbody>
</table>

$f_H$ is the high cutoff frequency, needed for the noise power to be finite.
4 – Properties of phase noise
Frequency synthesis

Ideal synthesizer
- noise-free
- zero delay time

time translation:
output jitter = input jitter
phase time $x_o = x_i$

linearity of the integral and the derivative operators:
$$\varphi_o = \left(\frac{n}{d}\right) \varphi_i \Rightarrow \nu_o = \left(\frac{n}{d}\right) \nu_i$$

spectra
$$S_{\varphi_o}(f) = \left(\frac{n}{d}\right)^2 S_{\varphi_i}(f)$$
Carrier collapse

Simple physical meaning, complex mathematics. Easy to understand in the case of sinusoidal phase modulation

\[ v(t) = V_0 \cos \left[ 2\pi \nu_0 t + m \cos (2\pi \nu_m t) \right] \]

random noise => phase fluctuation

\[ \varphi^2 = \int_B S_\varphi(f) \, df \]
Filtering $\iff$ Phase Locked Loop (PLL)

The PLL low-pass filters the phase

Output voltage: the PLL is a high-pass filter

The signal "2" tracks "1"

The FFT analyzer (not needed here) can be used to measure $S_{\phi}(f)$

\[
\frac{S_{\phi_2}(f)}{S_{\phi_1}(f)} = \frac{|k_o k_\varphi H_c(f)|^2}{4\pi^2 f^2 + |k_o k_\varphi H_c(f)|^2}
\]

\[
\frac{S_{v_0}(f)}{S_{\phi_1}(f)} = \frac{4\pi f^2 k_\varphi^2}{4\pi^2 f^2 + |k_o k_\varphi H_c(f)|^2}
\]
Frequency discriminator

For slow frequency fluctuations, a delay-line $t$ is equivalent to a resonator of merit factor

$$Q = \pi \tau \nu_0$$

A resonator turns a slow frequency fluctuation $\Delta \nu$ into a phase fluctuation $\phi$

$$\phi = \frac{1}{2Q} \frac{\Delta \nu}{\nu_0}$$

Parameters
- $\nu_0$: resonant frequency
- $Q$: merit factor

Properties of phase noise – discriminator
The Leeson effect: phase-to-frequency noise conversion in oscillators


\[ S_\varphi(f) = \left[ 1 + \left( \frac{v_0}{2Q} \right)^2 \frac{1}{f^2} \right] S_\psi(f) \]

5 – Laboratory practice
Practical limitations of the double-balanced mixer

1 – Power
narrow power range: ±5 dB around $P_{nom} = 5\text{--}10 \text{ dBm}$
r(t) and s(t) should have (about) the same power

2 – Flicker noise due to the mixer internal diodes
typical $S_\phi = -140 \text{ dB} \text{rad}^2/\text{Hz}$ at 1 Hz in average-good conditions

3 – Low gain $k_\phi \sim -10 \text{ to } -14 \text{ dBV}/\text{rad}$ typical (0.2-0.3 V/\text{rad})

4 – White noise due to the operational amplifier
Typical background noise

RF mixer (5-10) MHz
Good operating conditions (10 dBm each input)
Low-noise preamplifier (1 nV/√Hz)
The operational amplifier is often misused

\[ R_b = \sqrt{\frac{5V}{5k}} \]

**Warning:** if only one arm of the power supply is disconnected, the LT1028 may deliver a current from the input (I killed a $2k$ mixer in this way!)

You may duplicate the low-noise amplifier designed at the FEMTO-ST Rubiola, Lardet-Vieudrin, Rev. Scientific Instruments 75(5) pp. 1323-1326, May 2004
A proper mechanical assembly is vital

\[ l = \frac{\lambda_{\text{cable}} \varphi}{2\pi} \]

- 180 dB rad \rightarrow 10^{-9} \text{ rad}

- \rightarrow 4 \times 10^{-12} \text{ m} \quad @ \quad 10 \text{ GHz}
- 4 \times 10^{-10} \text{ m} \quad @ \quad 100 \text{ MHz}

\[ \sigma_e^2 = 2 \ln(2) h_b \]

Allow Variance

10^{-9} \text{ rad} \rightarrow \sigma_e \approx 4.8 \text{ pm} \quad @ \quad 10 \text{ GHz}

4.8 \mu \text{m} \quad @ \quad 100 \text{ MHz}
Two-port device under test (DUT)

- The phase shifter $\varphi$ ensures the 90° condition at the mixer inputs.
- The oscillator noise is rejected.

- Testing an amplifier, it must be preceded by an attenuator. 
  Frequency increases.
- Higher power amplifiers may be followed by an attenuator.
Two-port device under test (DUT)

- A low power DUT must be followed by an amplifier (flicker)

- Two equal DUTs
  - increased gain and sensitivity
  - improved rejection of the oscillator noise

other configurations are possible
A frequency discriminator can be used to measure the phase noise of an oscillator.
Phase Locked Loop (PLL)

\[ S_{\phi_2}(f) \frac{S_{\phi_1}(f)}{S_{\phi_1}(f)} = \frac{|k_o k_\phi H_c(f)|^2}{4\pi^2 f^2 + |k_o k_\phi H_c(f)|^2} \]

\[ S_{v_0}(f) \frac{S_{\phi_1}(f)}{S_{\phi_1}(f)} = \frac{4\pi f^2 k_\phi^2}{4\pi^2 f^2 + |k_o k_\phi H_c(f)|^2} \]

Phase: the PLL is a low-pass filter

Output voltage: the PLL is a high-pass filter

compare an oscillator under test to a reference low-noise oscillator

– or –

compare two equal oscillators and divide the spectrum by 2 (take away 3 dB)
Phase Locked Loop (PLL)

He is a constant (1st order PLL)

A large dynamics is required because of the $f^{-3}$ slope
A tight PLL shows many advantages

but you have to correct the spectrum for the PLL transfer function
Practical measurement of $S_{\phi}(f)$ with a PLL

1. Set the circuit for proper electrical operation
   a. power level
   b. lock condition (there is no beat note at the mixer out)
   c. zero dc error at the mixer output (a small V can be tolerated)

2. Choose the appropriate time constant

3. Measure the oscillator noise

4. At end, measure the background noise
Warning: a PLL may not be what it seems

Parasitic locking or coupling of the oscillators may impair the result

BAD SYMPTOMS:
- odd slope $S_p$
- open-loop waveform $I_{Fout}$
- results $(S_p)$ depend on the cable length

Diagram:
- Expected $1/f^2$
- Actual $1/f$
- Locking failed
- Control circuit
PLL – two frequencies

The output frequency of the two oscillators is not the same. A synthesizer (or two synth.) is necessary to match the frequencies.

At low Fourier frequencies, the synthesizer noise is lower than the oscillator noise.

At higher Fourier frequencies, the white and flicker of phase of the synthesizer may dominate.
PLL – low noise microwave oscillators

With low-noise microwave oscillators (like whispering gallery) the noise of a microwave synthesizer at the oscillator output can not be tolerated.

Due to the lower carrier frequency, the noise of a VHF synthesizer is lower than the noise of a microwave synthesizer.

This scheme is useful
• with narrow tuning-range oscillator, which can not work at the same freq.
• to prevent injection locking due to microwave leakage
Designing your own instrument is simple

Standard commercial parts:
• double balanced mixer
• low-noise op-amp
• standard low-noise dc components in the feedback path
• commercial FFT analyzer

Afterwards, you will appreciate more the commercial instruments:
– assembly
– instruction manual
– computer interface and software
6 – Calibration
Calibration – general procedure

1 – adjust for proper operation: driving power and quadrature

2 – measure the mixer gain \( k_\phi \) (volts/rad) \( \rightarrow \) next

3 – measure the residual noise of the instrument
Calibration – general procedure

4 – measure the rejection of the oscillator noise

Make sure that the power and the quadrature are the same during all the calibration process
Calibration – measurement of $k_\phi$ (phase mod.)

The reference signal can be a

tone:
- detect with the FFT,
- with a dual-channel FFT, or
- with a lock-in

(pseudo-)random white noise

tone: $k_\phi = \frac{V_m}{k_m V_d}$

white noise $k^2_\phi = \frac{S_{V_m}}{k^2_m S_{V_d}}$

Some FFTs have a white noise output
Dual-channel FFTs calculate the transfer function $|H(f)|^2 = S_{V_m}/S_{V_d}$
Calibration – measurement of $k_\phi$ (rf signal)

SIDEBAND $\rightarrow$ PHASE MOD.

\[ q_{rms} = \sqrt{\frac{P_s}{2P_c}} \]

A tone at $f_s = f_s - f_c$ is measured by the FFT.
Calibration – measurement of $k_\phi$ (rf noise)

A reference rf noise is injected in the DUT path through a directional coupler.

\[ S_\phi = \frac{N}{P_e} \]

WHITE NOISE GENERATOR

Output power relationships:

\[ P_{\text{dB}} \gg P_{\text{peak}} \approx P_{\text{avg}} + 20 \text{ dB} \]

\[ N_{\text{out}} B = F k T g B \]
7 – Bridge (interferometric) measurements
Wheatstone bridge

- Rejection of $dV_s$ $\rightarrow$ amplitude phase, if the group delay... is short.
- A narrow dynamic $(dA)$ is sufficient.
Wheatstone bridge – ac version

**equilibrium:** $V_d = 0 \implies$ carrier suppression

**static error** $\delta Z_1 \implies$ some residual carrier
- real $\delta Z_1 \implies$ in-phase residual carrier $V_{re} \cos(\omega_0 t)$
- imaginary $\delta Z_1 \implies$ quadrature residual carrier $V_{im} \sin(\omega_0 t)$

**fluctuating error** $\delta Z_1 \implies$ noise sidebands
- real $\delta Z_1 \implies$ AM noise $n_c(t) \cos(\omega_0 t)$
- imaginary $\delta Z_1 \implies$ PM noise $-n_s(t) \sin(\omega_0 t)$
Wheatstone bridge – ac version

- **DUT noise**
- **Bridge out**
- **Ampli noise**
  - Well-bal. bridge

**SPECTRA**

- **$w_0$**
- **$w$**

**Ampli noise**
- Carrier not rejected

**NO CARRIER → NO $1/f$ NOISE**

**NO 1/f noise**: the ampli does not know where is $w_0$.

**HOU-LINEARITY**
- Close-in noise appears
Bridge (interferometric) phase-noise and amplitude-noise measurement

High carrier suppression:
no carrier ⇒ the amplifier can’t flicker

High gain:
\[ k_\varphi = \frac{v(t)}{\varphi(t)} = \sqrt{\frac{R_0 g P_0}{l_m}} - \text{dissip. losses} \]

Low Noise Floor:
\[ S_{\varphi_0} = \frac{2Fk_B T_0}{P_0} + \text{dissip. losses} \]

Improved, from RSI 70 1 pp. 220-225, Jan 1999

High immunity to 50 Hz B-fields
Synchronous detection

\[ s(t) = V_0 \left[ 1 + \alpha(t) \right] \cos \left[ 2\pi f_0 t + \varphi(t) \right] \]

For small \( \Delta f \) and \( \varphi \), is equivalent to

\[ s(t) = m_{RF}(t) + V_0 \cos \left[ 2\pi f_0 t \right] \]

\[ m_{RF}(t) = m_c(t) \cos 2\pi f_0 t - m_s(t) \sin 2\pi f_0 t \]

(close to the carrier)

(near dc)

\[ \alpha(t) = \frac{m_c(t)}{V_0} \]

\[ \varphi(t) = \frac{m_s(t)}{V_0} \]
Synchronous in-phase and quadrature detection

[Diagram showing the synchronization process]

\[ m_{RF}(t) \]

\[ m_s(t) = \frac{1}{2} m_{RF}(t) \]

\[ m_c(t) = \frac{1}{2} m_{RF}(t) \]

\[ \cos 2\pi f_0 t \]

\[ 90^\circ \]

\[ \text{REFERENCE} \]

**Remember:**

\[ \cos^2 \omega t = \frac{1}{2} (1 + \cos 2\omega t) \]

\[ \sin^2 \omega t = \frac{1}{2} (1 - \cos 2\omega t) \]

\[ \sin \omega t \cos \omega t = \frac{1}{2} \sin 2\omega t \]
White noise floor

\[ S_{\text{vo}} = 2 \frac{R_o g_a F k_b T_0}{l_n l_m} \]

thermal noise
\[ k_b = 1.38 \times 10^{-23} \text{ J/K} \]
\[ T_0 = 290 \text{ K} \]
\[ k_b T_0 = -174 \text{ dBm/Hz} \]
White noise floor – example

\[ k_B T_0 = -174 \text{ dBm/Hz} \]
\[ F = 2 \text{ dB} \]
\[ l_n = 1 \text{ dB} \]
\[ P_0 = 10 \text{ dBm} \]

\[ S_{\phi_0} = -178 \text{ dB} \text{rad}^2/\text{Hz} \]

In the same conditions, changing \( P_0 \) to 32 dBm yields

\[ S_{\phi_0} = -200 \text{ dB} \text{rad}^2/\text{Hz} \]
What really matters (1)

Close-to-the-carrier flicker comes from near-DC flicker up-converted.

Lowest carrier $\rightarrow$ best linearity $\rightarrow$ lowest flicker

WidEBAND NOISE AND HARMONICS MAY ALSO CONTRIBUTE.

The smallest signal is a RF signal $\rightarrow$ easy to shield from low freq. $B$ fields
What really matters (2)

3 HIGH GAIN

$H = N_q + N_f$

$S_{q0} = \frac{2N_q}{P_c}$

$S_v = \frac{2R_0gN_q}{\ln P_m}$

$K_q = \frac{R_0gP_c}{\ln P_m}$

20-260 dB dB/rd^2

4 LOW NOISE FLOOR

$N = Fk_B T_0$

$S_{v0} = \frac{2R_0gFk_B T}{P_c}$

$S = \frac{N_q + N_f}{2P_m}$

-160 - 190 dB dB/rd^2/Hz
A bridge (interferometric) instrument can be built around a commercial instrument.

How to build an interferometer around a commercial instrument:

- Source
- Interferometer
- Detector unit
- Computer system
  - Software:
    - Acquisition
    - Analysis
    - Processing
    - Storage/retrieval
    - Export

You will appreciate the computer interface and the software ready for use.
8 – Advanced Techniques
Low-flicker scheme

Obvious extension to phase shifters
Interpolation is necessary

half step 0.05 dB → carrier rejection 45 dB max.

Low weight interpolating signal

Continuous tolerable noise

Alternate solution
Correlation can be used to reject the mixer noise

\[ a(t) = k \phi \left[ \phi_{\text{mix}} + \phi_{\text{out}} \right] \]

\[ b(t) = k \phi \left[ \phi_{\text{mix}} + \phi_{\text{out}} \right] \]

**Individual arm noise (uncorrelated)**

**Shared path noise (correlated)**
Correlation – how it works

\[ S_{ab} = k_\phi S_{\phi \text{out}} + \text{residual} \]

\[ S_{\phi \text{read}} \propto \frac{1}{\sqrt{2m}} \]

\[ S_{ab} = A(t) \cdot B(t) \]

\[ = k_\phi [\phi_{\text{in}} + \phi_{\text{out}}] [\phi^*_{\text{in}} + \phi^*_{\text{out}}] \]
Flicker reduction, correlation, and closed-loop carrier suppression can be combined

Comparison of the background noise

\[ S_{\phi}(f) = \text{dBrad}^2/\text{Hz} \]

- real-time
- correl. & avg.
- mixer, interferometer
- nested interferometer

- saturated mixer
- interferometer
- residual flicker, by-step interferometer
- residual flicker, fixed interferometer
- residual flicker, fixed interferometer, ±45° detection
- measured floor, m=32k

Fourier frequency, Hz
9 – References

STANDARDS

ARTICLES
Interferometers, low-flicker methods, correlation, coordinate transformation, calibration strategies, advanced experimental techniques

BOOKS
Good and simple reference, although dated
Specific on phase noise, but dated. Unusual notation, sometimes difficult to read.
A must on the subject, most PM noise measurements make use of the FFT
One of the best references on electrical noise in general and on its mathematical properties.

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