Scientific Instruments

— and —

Phase Noise and Frequency Stability in Oscillators

Lectures for PhD Students and Young Scientists

Updated
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Part 1: General

Part 2: Phase noise and oscillators

Part 3: The International System of Units SI

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Lecture 11

Scientific Instruments & Oscillators

Lectures for PhD Students and Young Scientists

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Contents

- Uncertainty & related concepts
- The new SI

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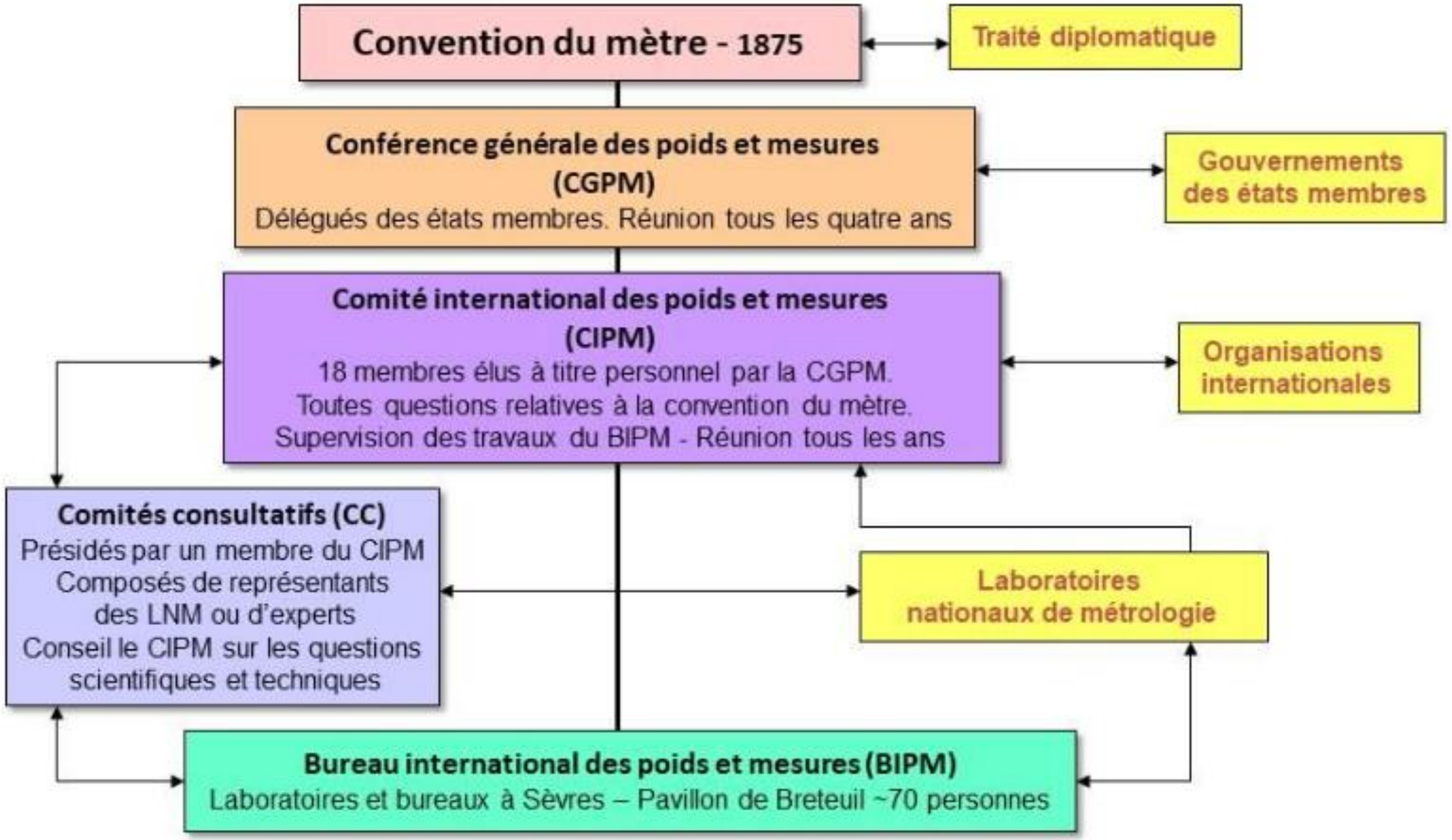
home page http://rubiola.org

International coordination of metrology



https://metrologie-francaise.lne.fr/fr/metrologie/les-organisations-internationales

Uncertainty

Reference documents

“VIM” JCGM, International Vocabulary of Metrology, 4th edition (draft), January 2021

https://www.bipm.org/documents/20126/54295284/VIM4_CD_210111c.pdf

“GUM” JCGM, Guide to the expression of uncertainty in measurement (a few parts)

https://www.bipm.org/en/committees/jc/jcgm/publications

“17025” ISO/IEC 17025:2017, General requirements for the competence of testing and calibration laboratories

https://www.iso.org/standard/66912.html

Measured quantity and orthography

$$Q = \{Q\}[\mathrm{Q}]$$

Numerical value   Unit

Write the result (general)

$$\mathrm{Q} = \mathrm{L}^{\alpha}\mathrm{M}^{\beta}\mathrm{T}^{\gamma}\mathrm{I}^{\delta}\Theta^{\epsilon}\mathrm{N}^{\zeta}\mathrm{J}^{\eta}$$

The 7 base quantities

length

mass

time

current

Temperature

mole

Illumin. intensity

Good examples

$m_e = 9.109 \times 10^{-31}\ \mathrm{kg}$

$c = 299792458\ \mathrm{ms}^{-1}$

$G = 6.674 \times 10^{-11}\ \mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$

$T = 373.15\ \mathrm{K}$

$\theta = 3.21 \times 20^{-2}\ \mathrm{rad}$

Bad examples

$m_e = 9.109 \cdot 10^{-31}\ \mathrm{kg}$

$m_e = 9.109 \times 10^{-31}\ kg$

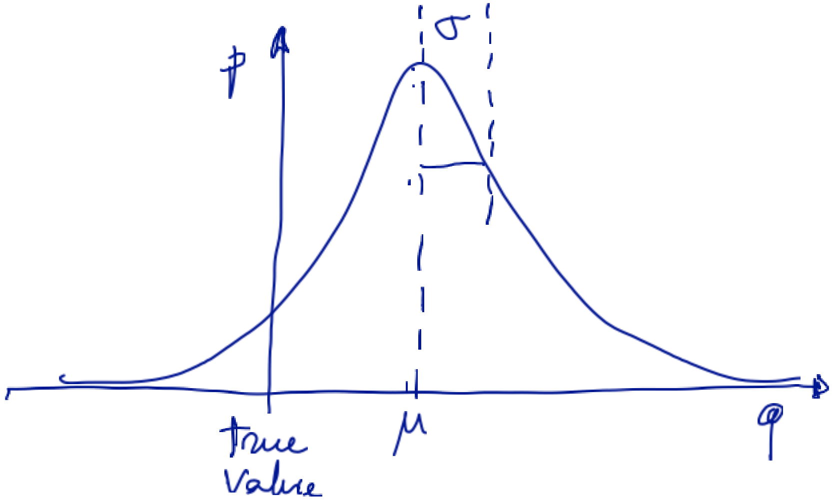
$c = 299{,}792{,}458\ \mathrm{m/s}$

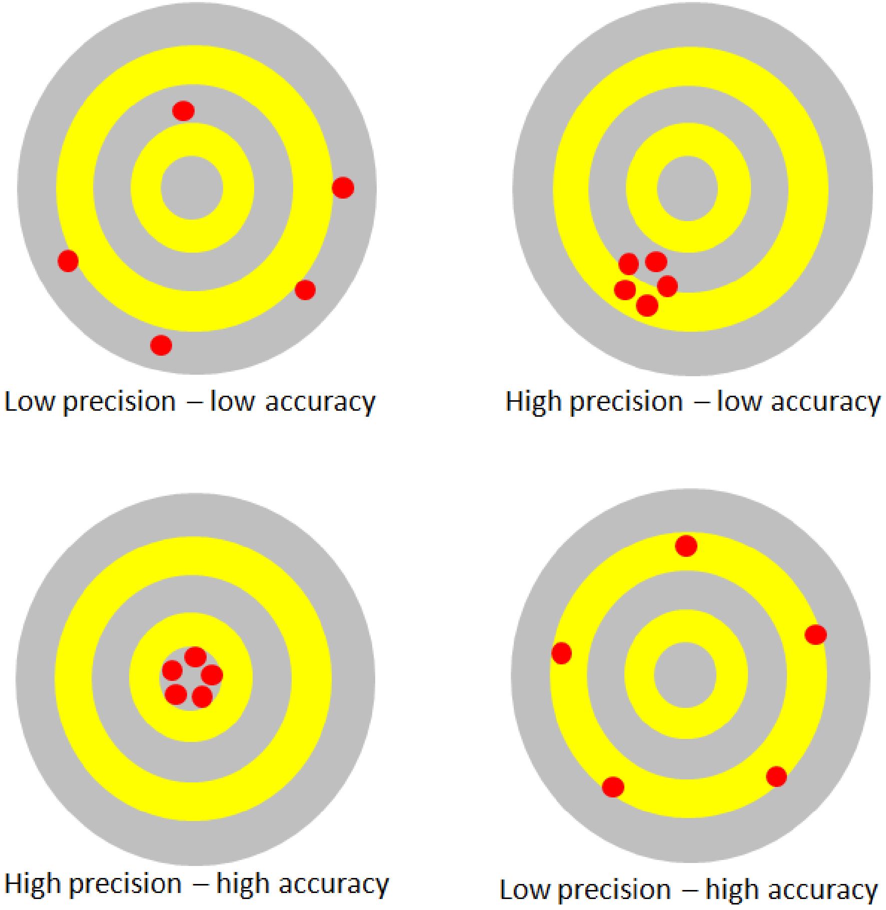
$T = 373.15\ {}^{\circ}\mathrm{K}$

Read “Rules and style conventions”, § 5.4 of the SI Brochure
BIPM, *The International System of Units*, 9th ed., 2019

Precision vs accuracy

- Let us start with old concepts
- True value Q_T
- Measured value Q_M
- Error $E = Q_M - Q_T$
- Averaging repeated measures improves the estimate



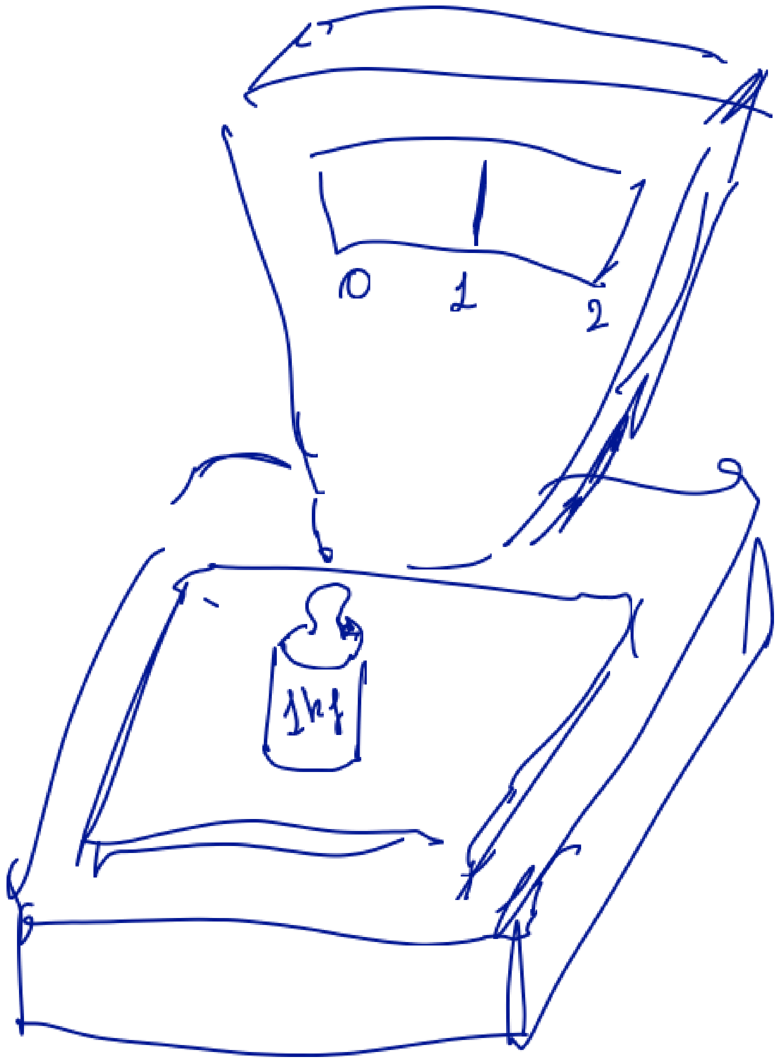


Expressing the uncertainty

Value	$m_e = 9.1093837015 \times 10^{-31}\ \mathrm{kg}$	
Standard uncertainty (1σ if not otherwise stated)	$u(m_e) = 0.0000000028 \times 10^{-31}\ \mathrm{kg}$	
Standard relative uncertainty	$u(m_e)/m_e = 3 \times 10^{-10}$	If Q has sign, use
“r” —> “relative” (fractional)	$u_r(m_e) = 3 \times 10^{-10}$	$u(Q)/\lvert Q\rvert$
Full expression	$m_e = (9.1093837015 \pm 0.0000000028) \times 10^{-31}\ \mathrm{kg}$	
Most used Condensed expression	$m_e = 9.1093837015(28) \times 10^{-31}\ \mathrm{kg}$	
Expanded uncertainty	$\mathfrak{U}(m_e) = ku$	eg., 95%
Non standard expression	$U(Q)$	uncertainty (absolute)
	$u(Q) = U(Q)/\lvert Q\rvert$	relative uncertainty

Statistics and uncertainty

- Something goes wrong even in everyday life.
- Example: The needle shows
$$m = 1.02\ \text{kg}$$
- Repeated measurements give the same value. Statistics says
$$\mu = 1.02\ \text{kg}$$
$$\sigma = 0$$
- Perfect measure?
- Nope, the time series does not tell us the whole story

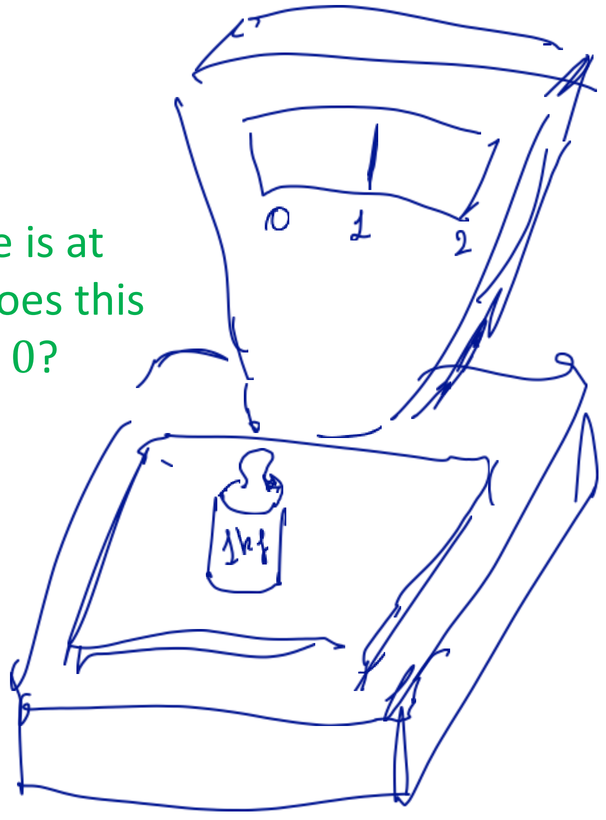


A-type uncertainty

2.28 – Type A evaluation of measurement uncertainty
Type A evaluation. Evaluation of a component of measurement uncertainty by a statistical analysis of measured quantity values obtained under defined measurement conditions

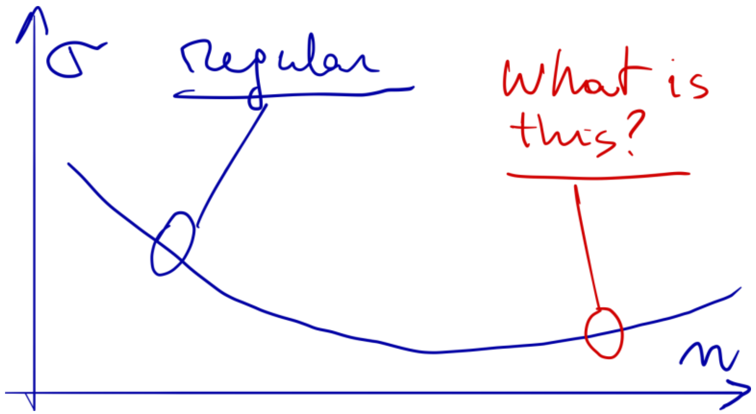
The definition is copied verbatim from the VIM, © JCGM 2012

The needle is at the rest, does this mean $\sigma = 0$?



Things may not be that simple

Increasing the number of measures may not reduce the uncertainty



B-type uncertainty

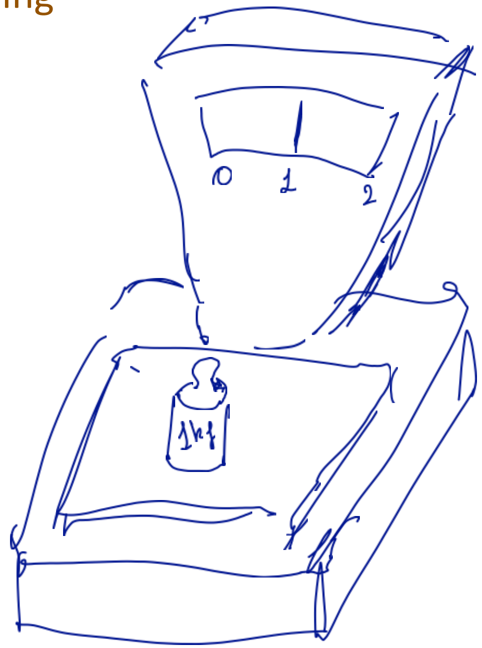
2.29 – Type B evaluation of measurement uncertainty
Type B evaluation. Evaluation of a component of measurement uncertainty determined by means other than a Type A evaluation of measurement uncertainty

Rules

- Understand the system in depth
- Identify/measure all corrections and influence quantities
- Ensemble statistics may help

…and good luck

The needle is at the rest, but a



Model, and correct for errors
Uncertainty is what remains afterwards

Combined uncertainty

$$u_C^2 = u_A^2 + u_B^2$$

$$u_B^2 = \sum_j u_{B,j}^2$$

$$u_A^2 = \sum_i u_{A,i}^2$$

Be careful with correlations

Example – PTB CSF2, x10^{-15}

Frequency shift	Correction	Uncertainty
Quadratic Zeeman shift	−99.85	0.06
Blackbody radiation shift	16.60	0.06
Gravity + relativistic Doppler effect	−8.567	0.006
Collisional shift	0.32	0.62
Cavity phase shift	0.0	0.15
Light shift	0.0	0.001
Majorana transitions	0.0	0.0001
Rabi pulling	0.0	0.0002
Ramsey pulling	0.0	0.001
Electronics	0.0	0.20
Microwave leakage	0.0	0.10
Microwave power dependence	0.0	0.40
Background pressure	0.0	0.05
Total	−91.50	0.80

Table 1, from V. Gerginov et al., "Uncertainty evaluation of the caesium fountain clock PTB-CSF2," Metrologia 47 008 p.65-79, 2008. ©IOP

Note: replace this table with a CC BY example

Compatibility

Measurement of the Planck constant

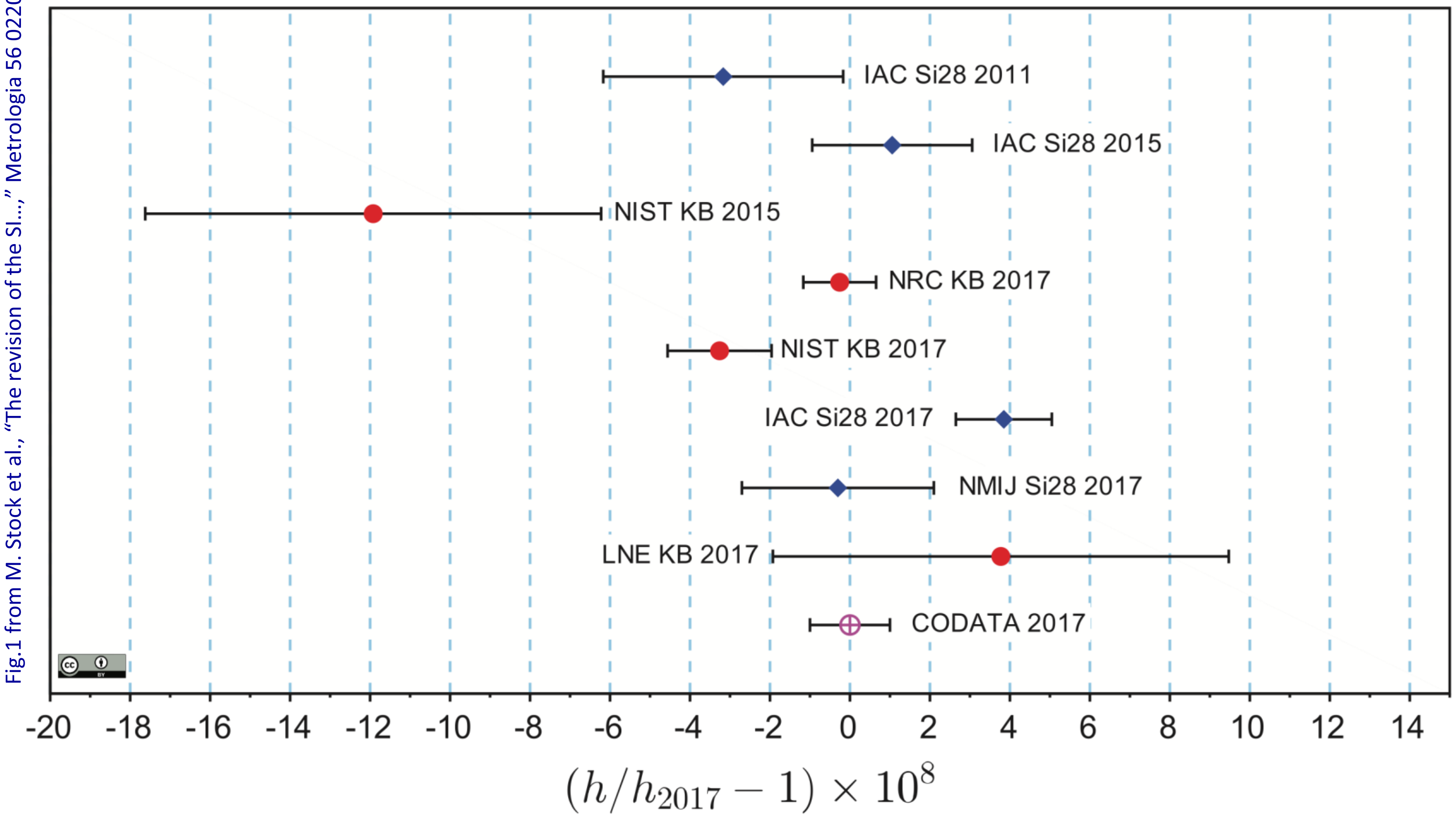


Fig.1 from M. Stock et al., "The revision of the SI…," Metrologia 56 022001, 2019

- Measures are said compatible when the uncertainty regions overlap
- This is not always the case

Beware, measuring h is no longer possible

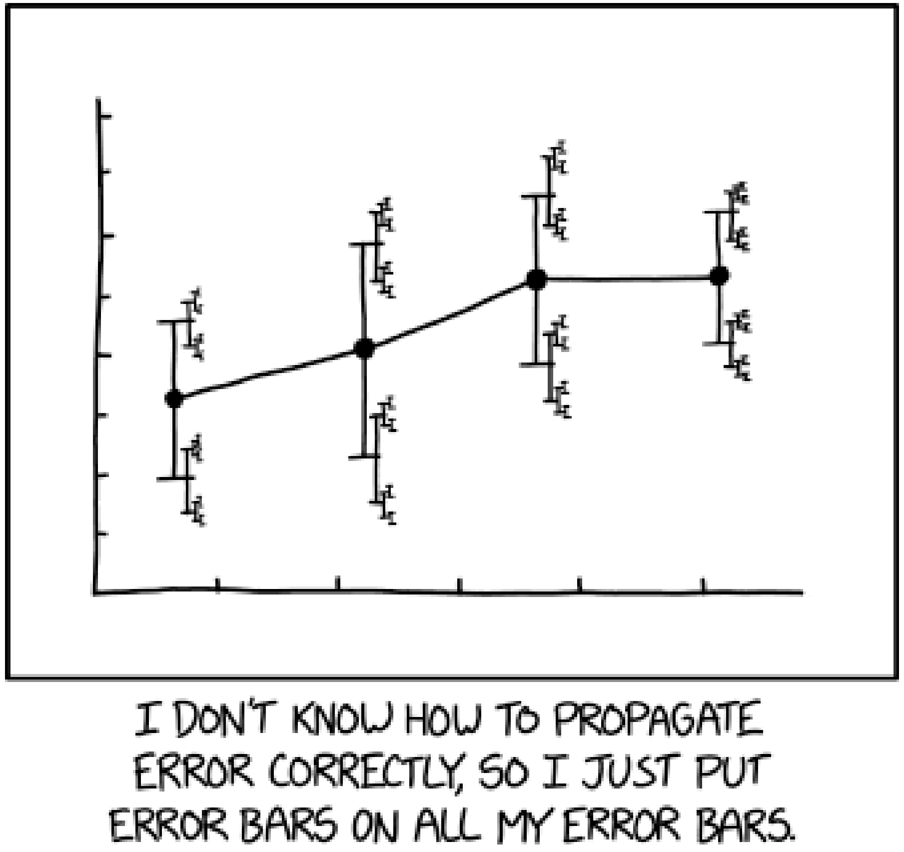
Uncertainty of uncertainty?

This is what I asked the first time I heard about uncertainty in a class

- Uncertainty is an experimental concept
  - It includes everything
  - No recursive uncertainty of uncertainty
- Moments are mathematical concepts
  - σ, the square root of the central 2nd moment
  - Higher moments are allowed, at will
- If something goes wrong
  - The typical wrong outcome is that measures are incompatible

Don't do this

You end up with a fractal



Zero (null) uncertainty

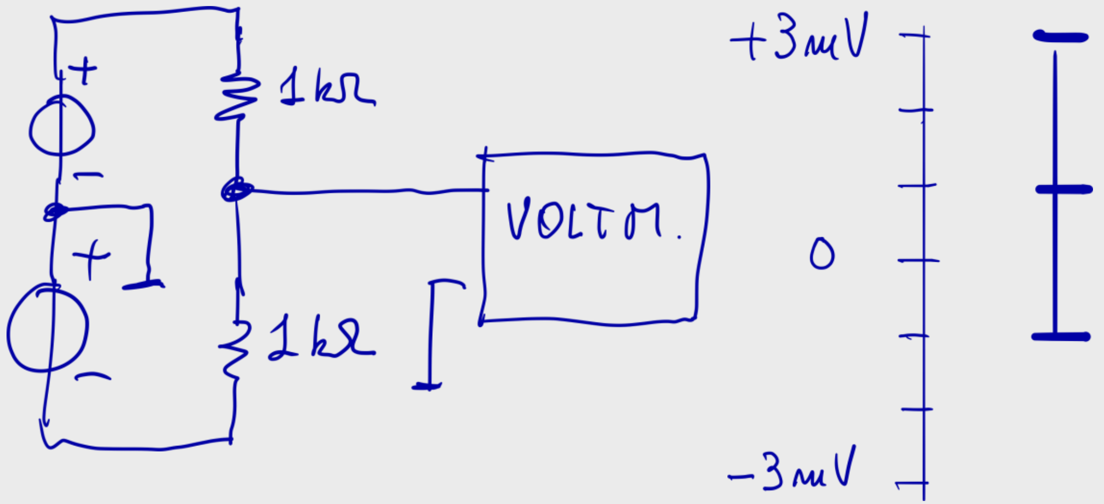
4.29 – null measurement uncertainty
measurement uncertainty where the specified measured quantity value is zero

Legal metrology

- Pollution from Hg in water
- Calculus says
  (320 ± 400) µg/m^3
- What can you say?

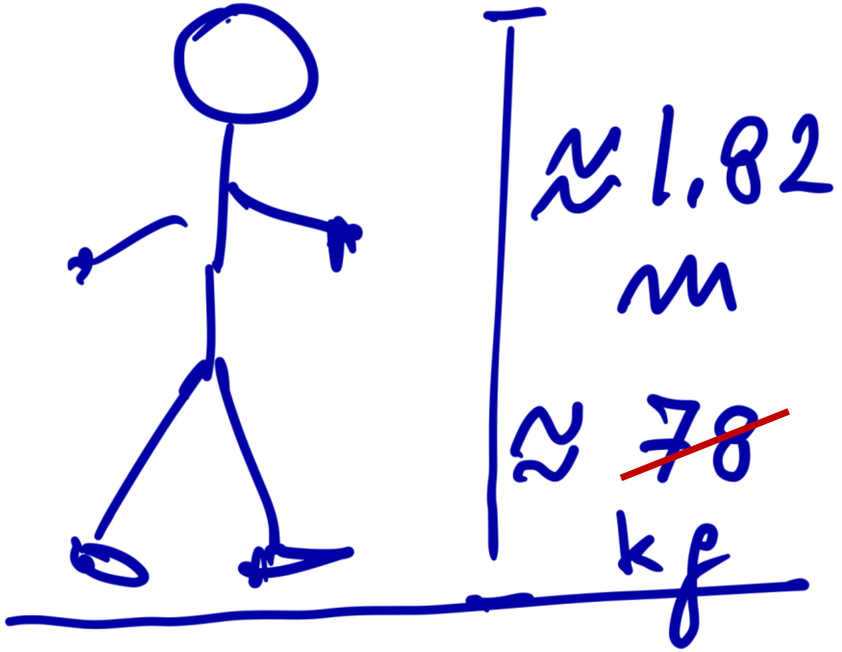
Funny example – Is the result valid?

Answer separately for DC and in AC measurements?



Intrinsic uncertainty of the measurand

2.27 – definitional uncertainty.
Component of measurement uncertainty resulting from the finite amount of detail in the definition of a measurand



81 kg ☹

Meaning

- You cannot measure better than what the measurand allows

Examples

- Height of human body (1-2 cm morning-to-evening)
- Mass of human body (≈1 kg morning-to-evening)
- Thickness of a sleeve

Influence quantity

2.52 influence quantity
quantity that, in a direct measurement, does not affect the quantity that is actually measured, but affects the relation between the indication and the measurement result

IQs affect the measurement chain, not the measurand

As our skill progresses, IQs —> corrections

Examples

- Frequency, in the measurement of V_{AC}
- Voltage measurement of a well-shielded Weston cell
  - Temperature of the room, thermal gradients, air flow —> thermocouples, voltmeter
  - Magnetic fields —> voltage fluctuations

Counterexamples

- Cs frequency standard
  - Blackbody radiation
  - Magnetic field
  - Blackbody radiation
- Probe capacitance in the measurement of a rise time

Repeatability

2.21 (3.6)
measurement repeatability
measurement precision under a set of repeatability conditions of measurement

2.20 (3.6, Notes 1 and 2)
repeatability condition of measurement repeatability condition
condition of measurement, out of a set of conditions that includes the same measurement procedure, same operators, same measuring system, same operating conditions and same location, and replicate measurements on the same or similar objects over a short period of time

NOTE 1 A condition of measurement is a repeatability condition only with respect to a specified set of repeatability conditions.

NOTE 2 In chemistry, the term “intra-serial precision condition of measurement” is sometimes used to designate this concept.

Repeat means same experiment: same equipment, conditions and operator

Repeatability is broadly related to the mathematical concept of *Stationarity*

- The result and its statistical properties do not depend on the origin of time
- Stationarity should go with specific parameters (typically, average and variance)

Caveat: repeatability and stationarity belong to different branches of science

Reproducibility

2.25 (3.7)

measurement reproducibility

measurement precision under reproducibility conditions of measurement

NOTE Relevant statistical terms are given in ISO 5725-1:1994 and ISO 5725-2:1994.

2.24 (3.7, Note 2)

reproducibility condition of measurement reproducibility condition

condition of measurement, out of a set of conditions that includes different locations, operators, measuring systems, and replicate measurements on the same or similar objects

NOTE1 The different measuring systems may use different measurement procedures.

NOTE2 A specification should give the conditions changed and unchanged, to the extent practical.

Reproduce means make a nominally equal experiment.

Equipment, conditions and operator may be different

Reproducibility is broadly related to the mathematical concept of *Ergodicity*

- The result and its statistical properties do not depend on the sample in the statistical ensemble
- Ergodicity should go with specific parameters (typically, average and variance
- Ergodicity makes sense only with stationary processes

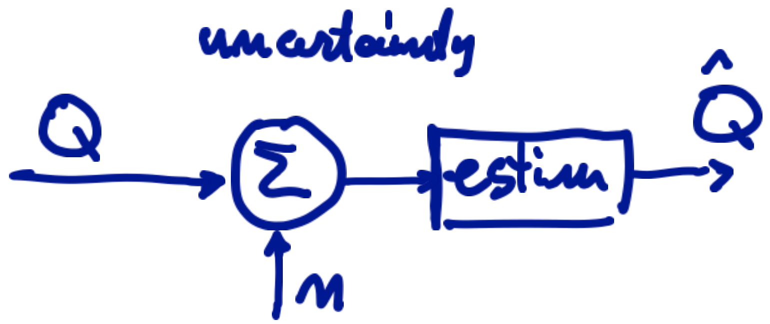
Read also section 2 of the VIM

You may not need to learn all of this
At least, be aware that these terms have a precise technical meaning

- Principle is not the same as Method
- True value, and Conventional value
- Accuracy, Trueness, and Precision
- Error, Systematic error, Bias, Random error,
- Calibration, Calibration hierarchy,
- Traceability, Traceability chain

International Vocabulary of Metrology, https://jcgm.bipm.org/vim/en/index.html

Estimation



Estimator: algorithm/process used to estimate the value of *Q*

Examples

- One value from an instrument
- The average of m values
- The length of the shadow to estimate the height of a tower
- The ß decay to estimate the amount of ^{137}Cs

Expectation $\mathbb{E}\{Q\}$ what the ideal world should be

In many cases the finite average is a good estimator

$$\hat{Q} = \langle Q \rangle_m$$

…but other estimators are possible

Unbiased estimator
$\mathbb{E}\{\hat{Q}\} = \mathbb{E}\{Q\}$

Biased estimator
$\mathbb{E}\{\hat{Q}\} \neq \mathbb{E}\{Q\}$

Best estimator: the best choice, according to some criteria

Uncertainty and noise are
- In the measurand
- In the measurement process

Example – The AC voltmeter

Definition

$$V_{\rm rms} = \sqrt{\mathbb{E}\left\{V^2(t)\right\}}$$

In the case of

$$V(t) = V_0 \cos(\omega t)$$

it holds that

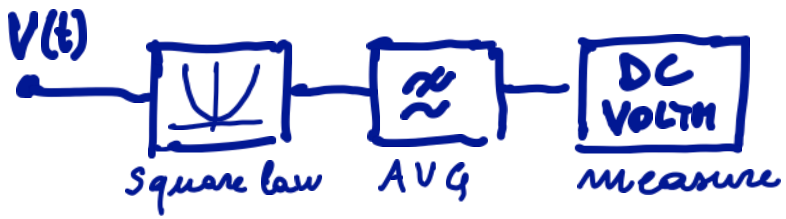
$$V_{\rm rms} = V_0/\sqrt{2}$$

rms = root mean square

"True RMS" voltmeter

$$\widehat{V_{\rm rms}} = \sqrt{\langle V^2(t)\rangle_\tau}$$

Implementation



Ergodicity, replace

$$\mathbb{E}\{\ \} \to \langle\ \rangle_\tau$$

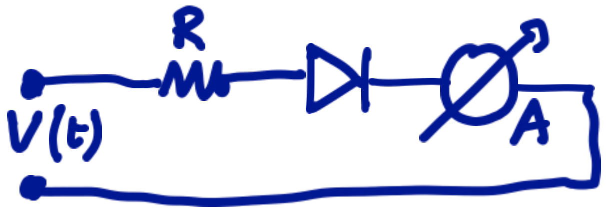
Unbiased estimator for

$\tau \gg 2\pi/\omega$ or $\tau = n2\pi/\omega$

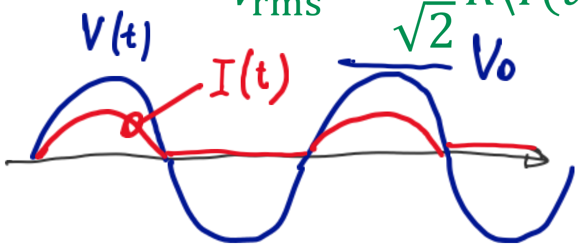
"Mean-Value" voltmeter

$$\widehat{V_{\rm rms}} = \frac{\pi}{\sqrt{2}}\langle |V(t)|\rangle_\tau$$

Implementation



$$\hat{V}_{\rm rms} = \frac{V_0}{\sqrt{2}} R\langle I(t)\rangle_\tau$$



- Biased estimator if the signal is not sinusoidal
- Suffers from distortion and noise
- Most used in low-cost and hand-held instruments

Uncertainty

At the top of the metrology chain, we do not have a standard for calibration

Questions

- What do accuracy and precision mean?
- Where do the units come from
- Can we have friendly standard and units?
  - Universal, in time and space
  - Improve accuracy as physics progresses
  - Little or no need to transport artifacts
- Which is the ultimate limit?
- What is the metrology chain, from fundamental science to layman

Reference documents

- VIM
- GUM
- Articles (eg. Possolo-Iyer, RSI 2017)
- BIPM web site
- National labs: INRiM, LNE, METAS, NPL, NIST, PTB, etc
- Put aside your domain and think big:
- Astrometry, biology, chemistry, electricity, environment, geodesy, magnetism, mechanics, nuclear, photometry, radiometry, etc.

In force May 20, 2019

The New SI

Motivations for these lectures

"Metrology is truly the mother of science"
John Lewis "Jan" Hall, Nobel Prize in Physics 2005

- Standard university education gives little insight in
  - Fundamental constants and measurement units
  - How we make sure that the units are the same everywhere
  - What is accuracy/precision, and how to express it
- The new International system of units is in force since May 20, 2019
- Electrical methods are ubiquitous
- Understanding electrical instruments is a must for the successful experimentalist

Some changes in electrical units

1990-1 —> may 2019

Unit	Symbol	$\Delta Q/Q$, x10^{-9}
volt	V	–107
ohm	Ω	–18
ampere	A	–89
coulomb	C	–89
watt	W	–196
farad	F	+18
henry	H	–18

Nothing relevant to the layman – Example

Enrico uses 3.6 MWH electrical energy in one year (≈10 kWH/day)

The annual bill is of 540 € (0.15 €/kWH)

The new Watt is 1.96x10^{-7} smaller.

Thus, the new annual bill for the same amount of energy is 105.8 μ€ higher

Questions

Which is the physical quantity given in MWH/y?

Convert 3.6 MWH into the appropriate SI unit

Bibliography

The International System of Units

- CGPM, *The International System of Units*, 9th ed., 2019
- M. Stock, R. Davis, E. de Mirandes, M. J. T. Milton. “The revision of the SI – The result of three decades of progress in metrology,” Metrologia 56 022001 14pp, 2019 [open access]
- T. Quinn, *From Artefacts to Atoms*, Oxford 2012, ISBN 978-0-19-530786-3
- E. O. Göbel, U. Siegner, *The New international System of Units (SI)*, ISBN 978-3-527-34459-8, Wiley-VCH 2019
- [obsolete] E. O. Göbel, U. Siegner, *Quantum Metrology: Foundation of Units and Measurements*, Wiley-VCH 2015, ISBN 978-3-527-41265-5
- W. Nawrocki, Introduction to Quantum Metrology, Springer 2019, ISBN 978-3-319-15668-2
- CODATA Internationally recommended (2018) values of the Fundamental Physical Constants https://physics.nist.gov/cuu/Constants/index.html

Basic material

JCGM, International vocabulary of metrology – Basic and general concepts and associated terms (VIM), 4th edition (committed draft), January 2021 ~~3rd ed., 2012~~

JCGM, Evaluation of measurement data

- An introduction to the “Guide to the expression of uncertainty in measurement” and related documents
- Guide to the expression of uncertainty in measurement
- Supplement 1 to the “Guide to the expression of uncertainty in measurement” — Propagation of distributions using a Monte Carlo method
- Supplement 2 to the “Guide to the expression of uncertainty in measurement” – Extension to any number of output quantities
- The role of measurement uncertainty in conformity assessment

Antonio Possolo (NIST)

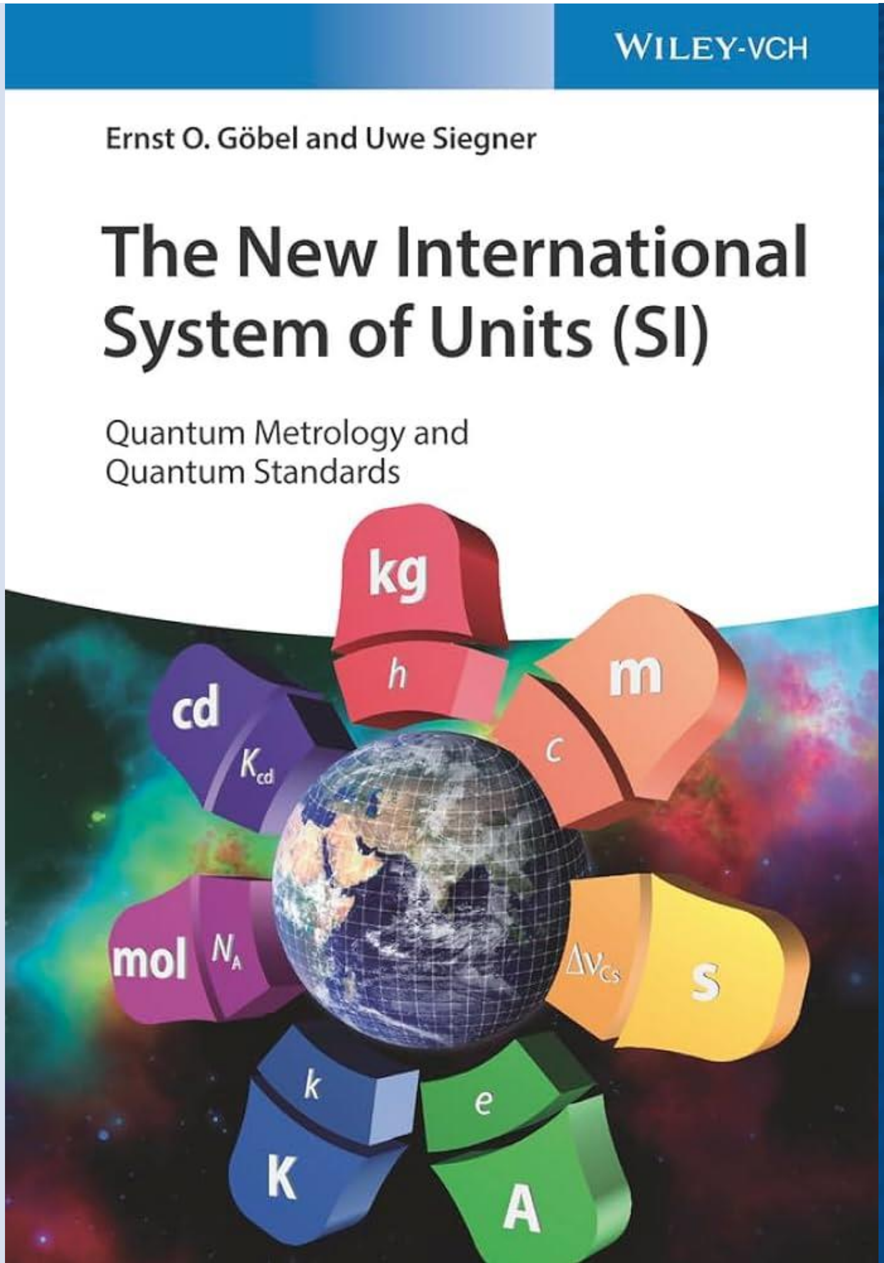
- Simple Guide for Evaluating and Expressing the Uncertainty of NIST Measurement Results, NIST Technical Note 1900, October 2015
- Introducing a Simple Guide for the evaluation and expression of the uncertainty of NIST measurement results, Metrologia 53 S17-S24, 2016
- Possolo A, Iyer HK - Concepts and tools for the evaluation of measurement uncertainty - RSI 88011301, January 2017

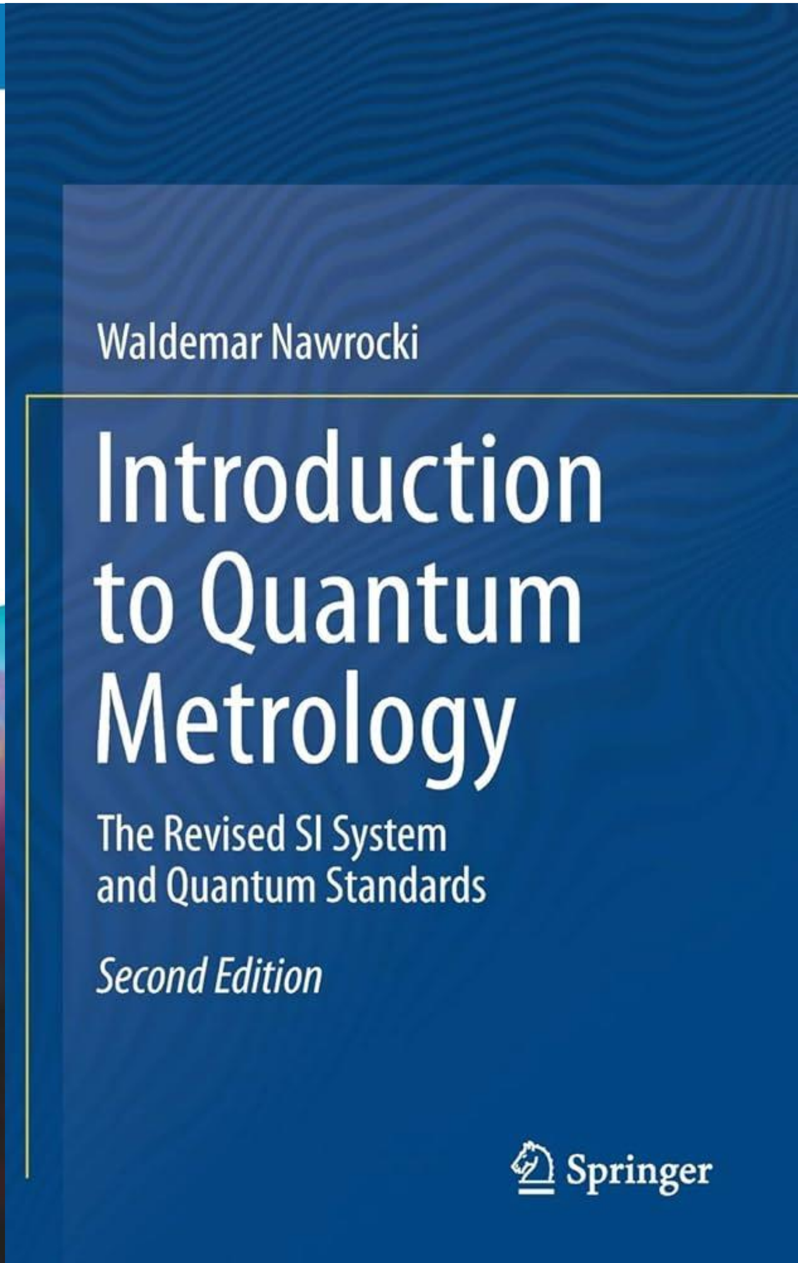
NIST Technical Note 1297
https://www.nist.gov/pml/nist-technical-note-1297

BIPM/JCGM documents are available free of charge from

Where to learn

Open access





Let us agree for consistent measurements

Political

- Originally driven by economy and trading
- Politicians often do not understand science

Scientific

- A plethora of system and units

Technical/operational

- Compare, reproduce, duplicate, …
- Cooperation among engineers, researchers, scientists…
- Make sure that the whole system is consistent

Featured reading: T. Quinn, From Artefacts to Atoms, Oxford 2012, ISBN 978-0-19-530786-3

Convention du mètre

Int’l treatise,
May 20, 1875

BIPM International Bureau of Weights and Measures

- Int’l organization, Sèvres, small int’l land in Paris area
- Executive / helper role
- No authority on Countries!

CGPM General Conference on Weights and Measures

- One representative per Member State (in 2019, 61 + 41 associates)
- Meeting every 4 years
- Supreme authority of BIPM

CIPM International Committee for Weights and Measures

- Elected by the CGPM (18 members) + BIPM Director
- Discuss / recommend / annual report
- Major reports, and BIPM Brochure
- Two meetings per year (was one)

CCs Consultative Committees (10)

- Advise the CIPM on specific fields

Mutual recognition arrangement (MRA)

CIPM Int’l equivalence of measurements

Signed by the representatives of 106 institutes

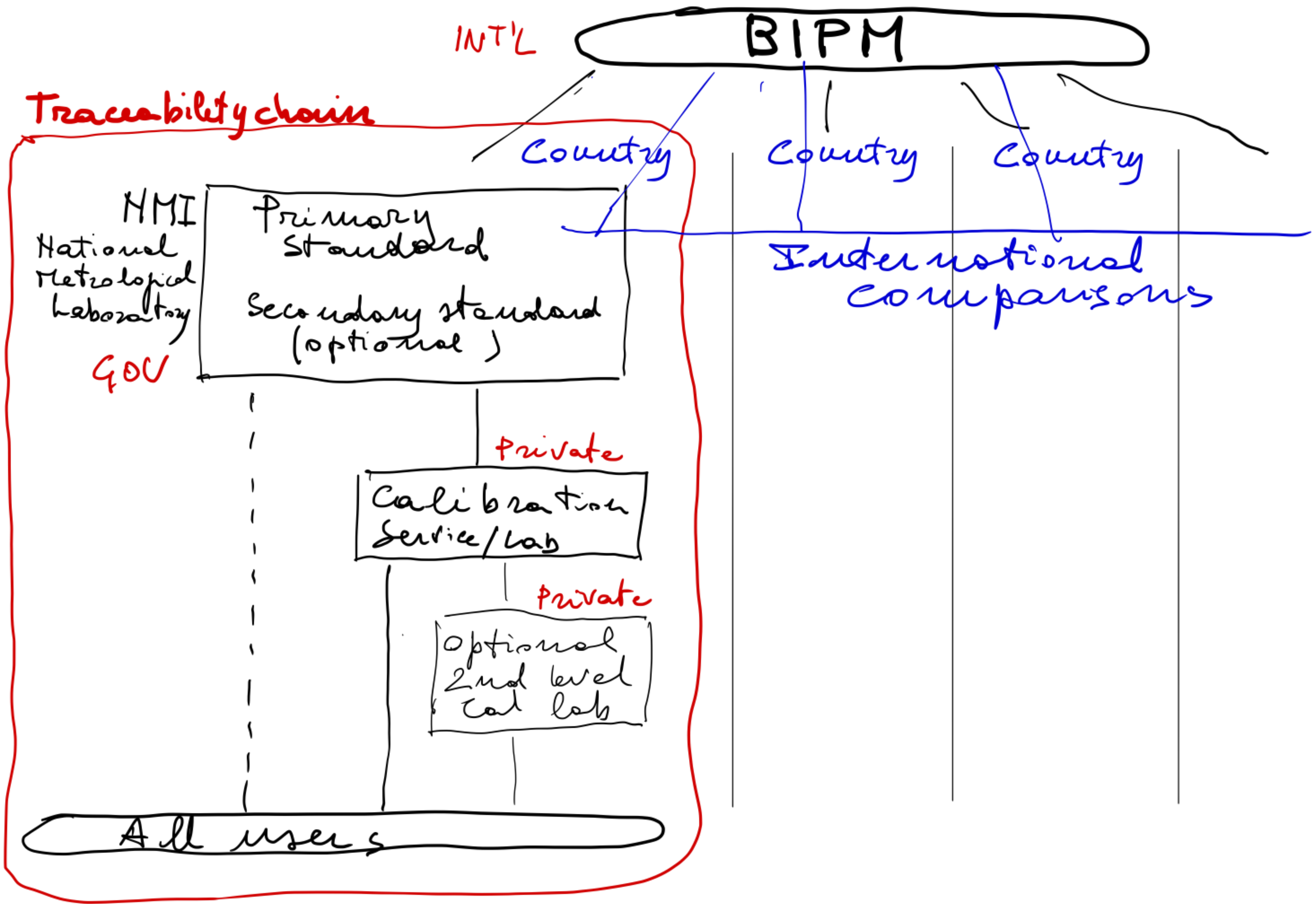
- 61 Member States,
- 41 Associates of the CGPM, and
- 4 int’l,
- Covers a further 157 institutes designated by the signatory bodies.

check on updates

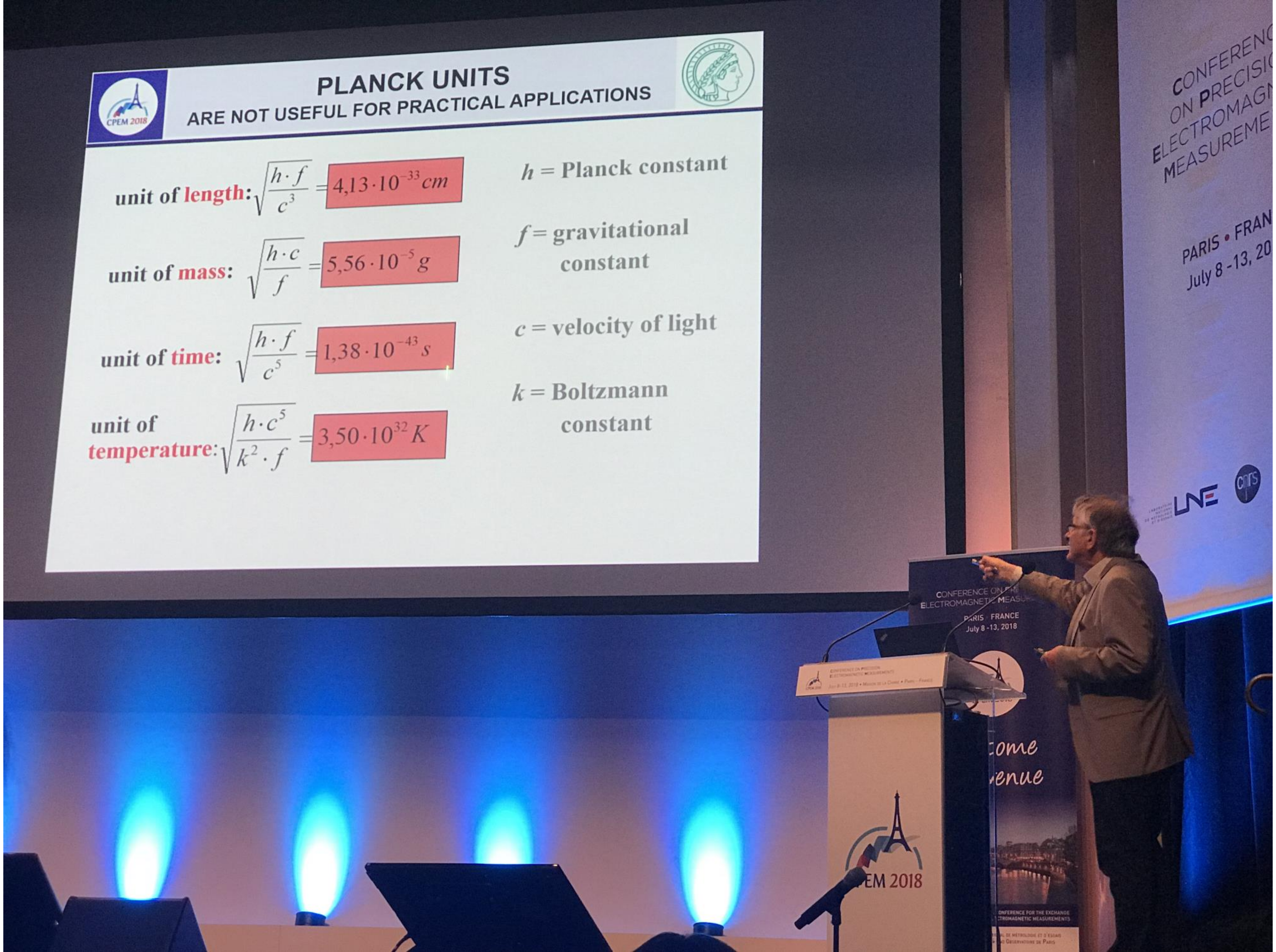
Traceability

Driven by Economy & Law

- make sure that all measurements in a Country are consistent
- Examples
- Some examples look ridiculous, they are not
  - Cooking timers
  - Hand-held voltmeters
  - Oscilloscopes



also electrical charge



Klaus von Klitzing, Nobel Prize in Physics 1985

Planck natural (and impractical) units

$$\ell_P = \sqrt{\frac{hG}{c^3}} = 4.33 \times 10^{-35}\ \mathrm{m}$$

$$m_P = \sqrt{\frac{hc}{G}} = 5.56 \times 10^{-8}\ \mathrm{kg}$$

$$t_P = \sqrt{\frac{hG}{c^5}} = 1.38 \times 10^{-43}\ \mathrm{s}$$

$$q_P = \sqrt{4\pi hc\epsilon_0} = 4.7 \times 10^{-18}\ \mathrm{C}$$

$$T_P = \sqrt{\frac{hc^5}{Gk^2}} = 3.5 \times 10^{32}\ \mathrm{K}$$

These are *cosmological* units

However, Planck is most known for *quantum* physics

Motivations

The new SI

In force May 20, 2019

(More) Motivations

- Quantum physics provides the most reproducible, precise and future-proof measurements
- Energy concepts
- Artifact standards (the kg) are a problem
- Fundamental constants are
  - Universally accessible
  - Supposed not to age

Must be backward compatible (keep accuracy)

Picture ©BIPM

Mechanical vs electrical units

< 1988

mechanical units
kg

electrical units
V, A, Ω

uncertainty of electrical units limited by mechanical measurements

1988 – 2019

mechanical units
kg

electrical units
V, A, Ω

inconsistent

quantum electrical units
V, A, Ω

May 20, 2019

quantum electrical units
V, A, Ω

electrical units
V, A, Ω

adjustment of all units

Mechanical vs quantum electrical units

Josephson effect

Brian D. Josephson, Nobel 1973

$$K_J = \frac{2e}{h}$$

$K_J \simeq 484\ \mathrm{THz/V}$

Quantum Hall effect

Klaus von Klitzing, Nobel 1985

$$R_K = \frac{h}{e^2}$$

$R_K \simeq 25.8\ \mathrm{k\Omega}$

Newton law

$$F = ma$$

Quantum Electrical

$$P = VI$$

$P = V^2/R$

new →

← old

Mechanical

$$P = Fv$$

$G = 6.67430(15) \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$

Energy and force

No Energy and Force standard

Mechanical

$$E = \int F(x)dx$$

Electrical

$$E = \int V(t)I(t)dt$$

Force between currents (or charges)

Thermodynamic

$$E = kT$$

Gas law

$$pV = NkT$$

Quantum

$$E = h\nu$$

momentum

$$p = h\nu/c$$

End of lecture 11

Lecture 12

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Lectures for PhD Students and Young Scientists

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home page http://rubiola.org

The New SI

Definitions

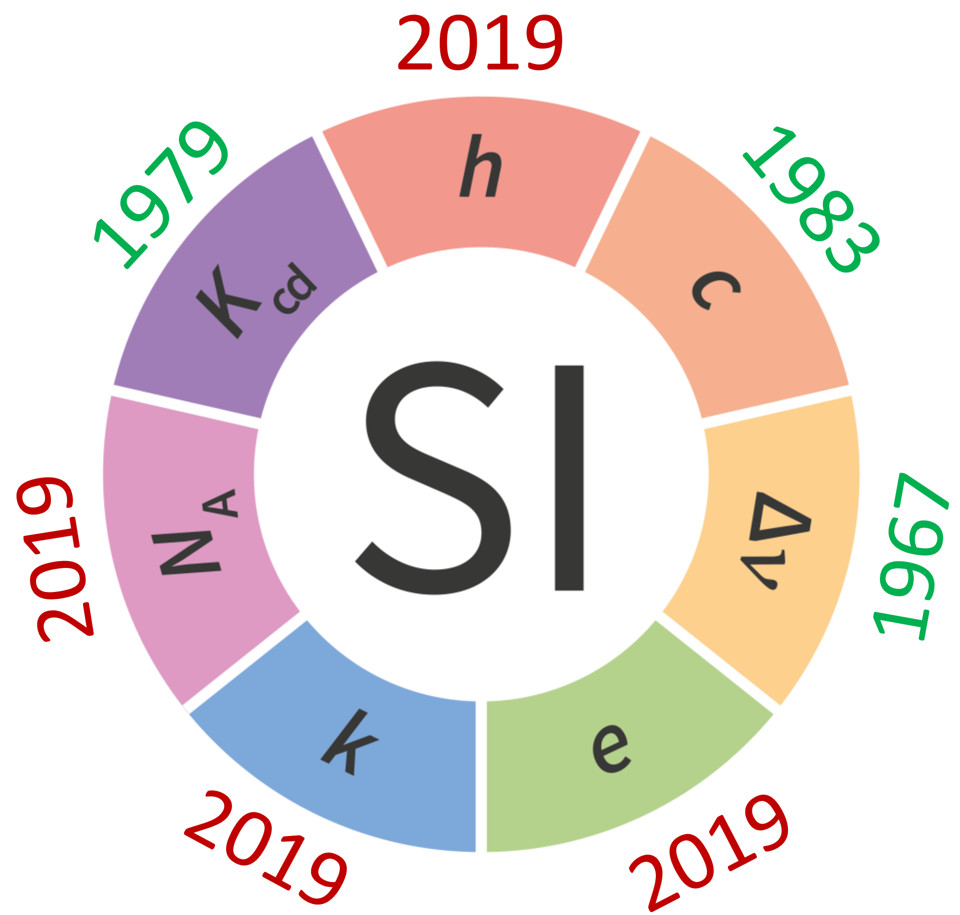
Official documents

Everything is on the BIPM web site
https://www.bipm.org/en/publications/si-brochure/

- The complete brochure, bilingual (EN/FR), 218 pages
  - 27 pp text + appendices 1,3,4, front matter and back matter
- Appendix 2: *Mise en pratique* (practical realization) of the 7 base units is available only in electronic format
  https://www.bipm.org/en/publications/mises-en-pratique/
  - second
  - metre
  - kilogram
  - ampere
  - kelvin
  - mole
  - candela

Most of these documents go with recommendations and guidance

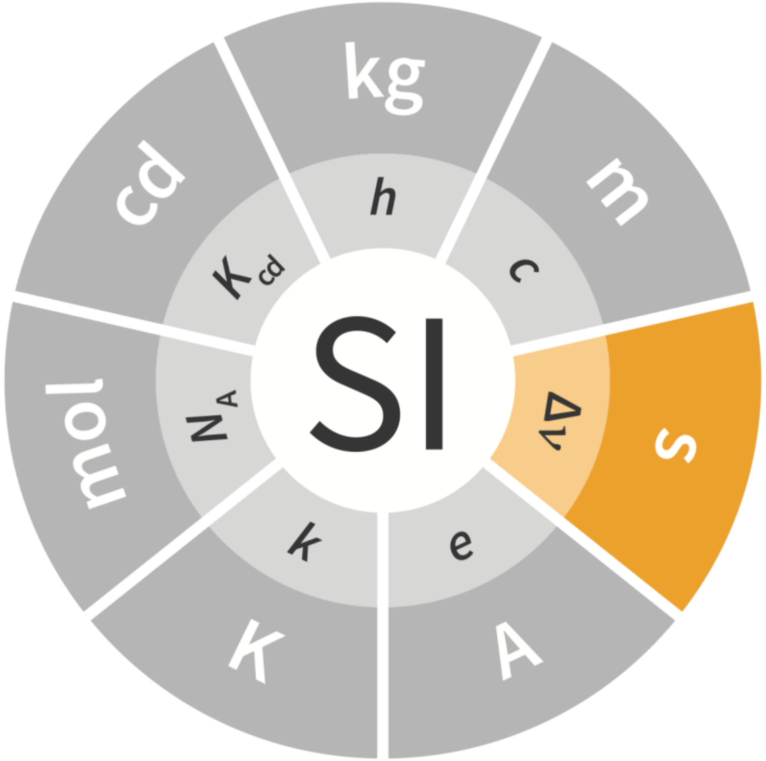
The SI and the fundamental constants



- The unperturbed ground state hyperfine transition frequency of the ^{133}Cs atom $\Delta\nu$Cs is 9 192 631 770 Hz
- The speed of light in vacuum *c* is 299 792 458 m/s
- The Planck constant *h* is 6.626 070 15 × 10^{-34} J s
- The elementary charge *e* is 1.602 176 634 × 10^{-19} C
- The Boltzmann constant *k* is 1.380 649 × 10^{-23} J/K
- The Avogadro constant N_A is 6.022 140 76 × 1023 mol^{-1}
- The luminous efficacy of monochromatic radiation of frequency 540 × 1012 Hz, K_{cd}, is 683 lm/W

Red: the new definitions
Green: definitions already in force (rephrased version)

Second



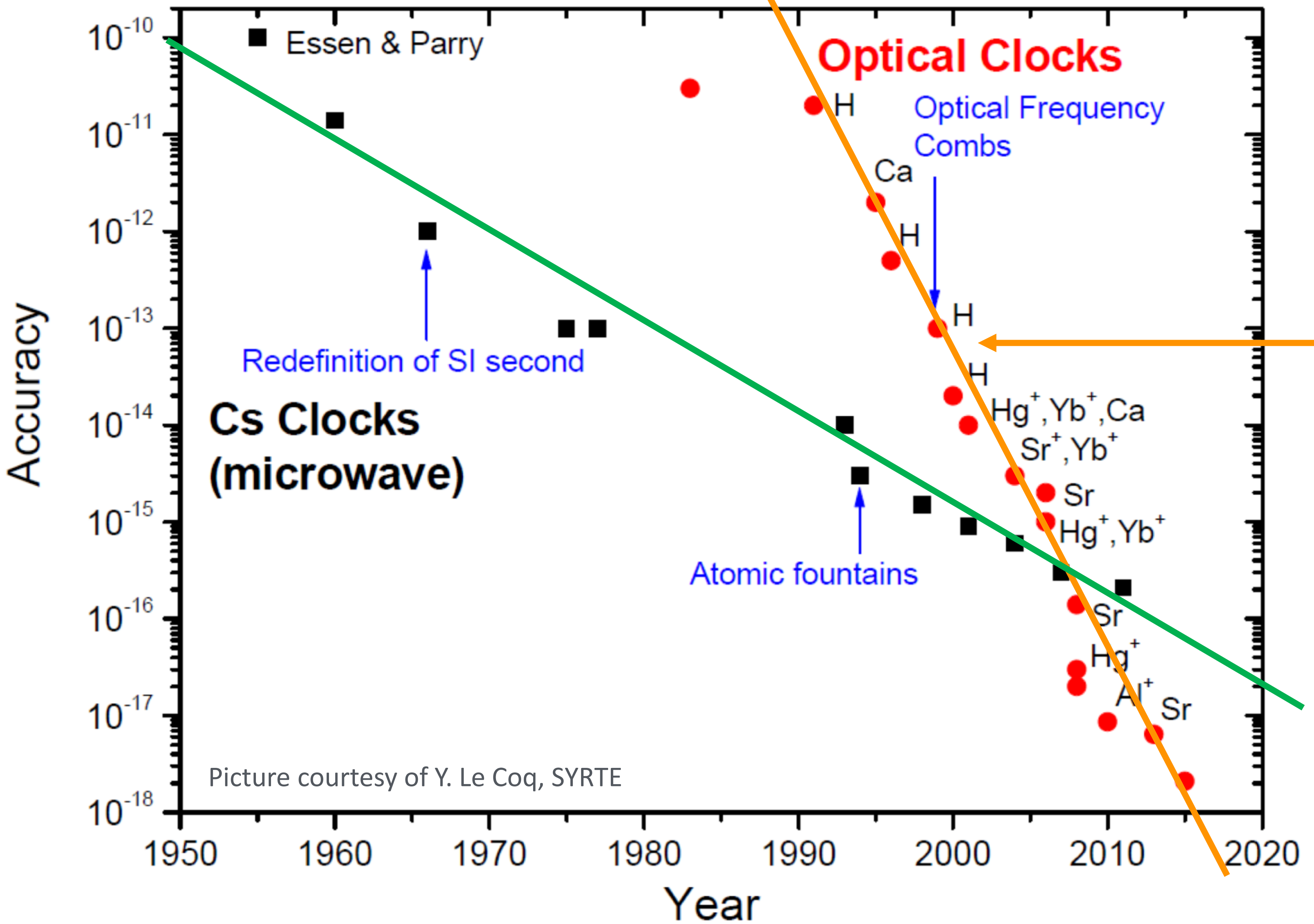
The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{Cs}$, the unperturbed ground-state hyperfine transition frequency of the caesium 133 atom, to be 9192631770 when expressed in the unit Hz, which is equal to s^{-1}.

A rephrased version of the 1967 definition

The formulation defines the *time interval* through the frequency $\Delta\nu_{Cs}$
A *time scale* is required to associate the time to an event. Time scales are a separate topic

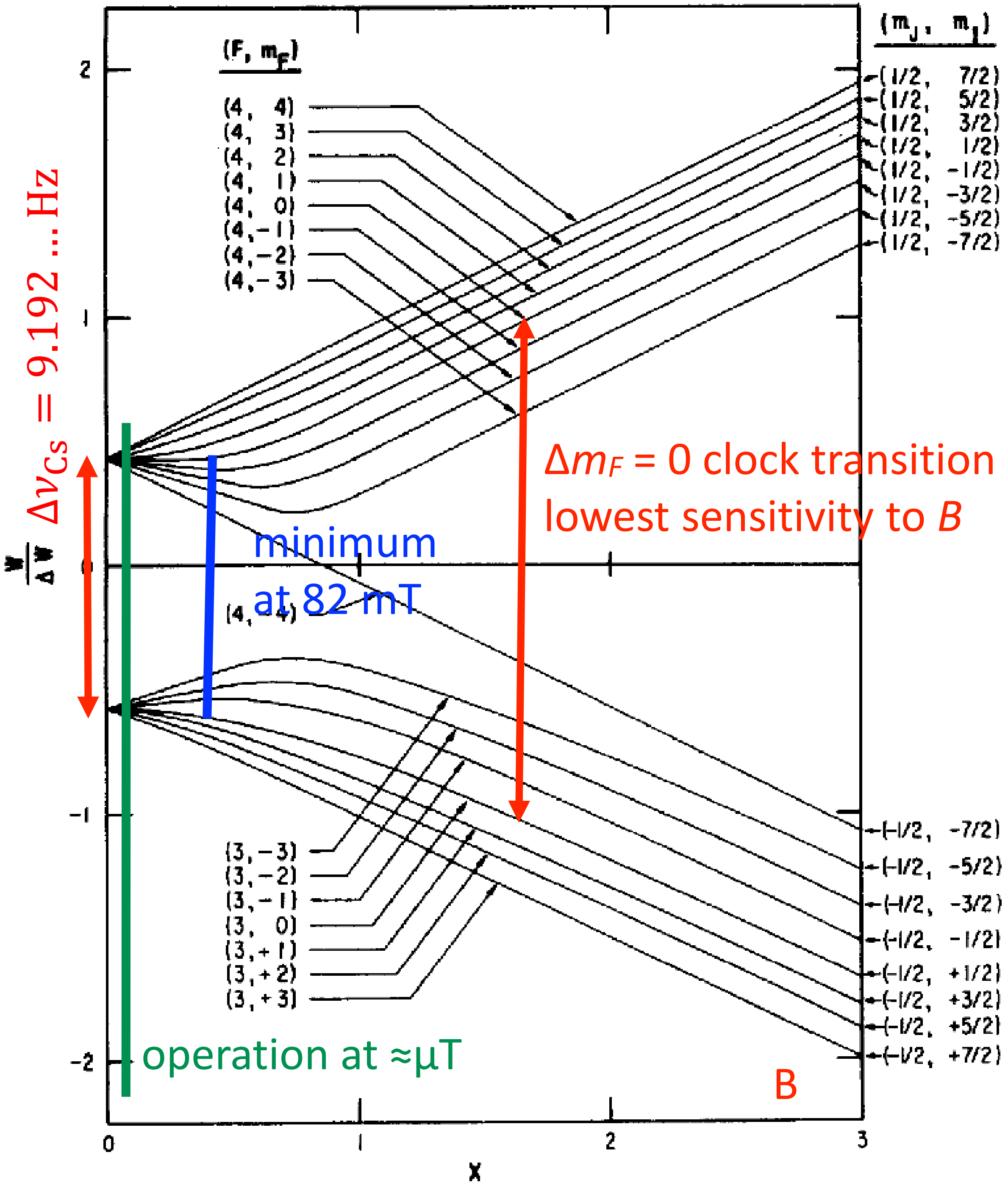
POI: the International Union of Applied and Pure Chemistry (IUPAC) spelling is Caesium, but the common American English word is Cesium

Frequency is the most accurate quantity



This is reproducibility rather than accuracy, optical clock do not rely on the definition of the second

Hyperfine structure of Cs



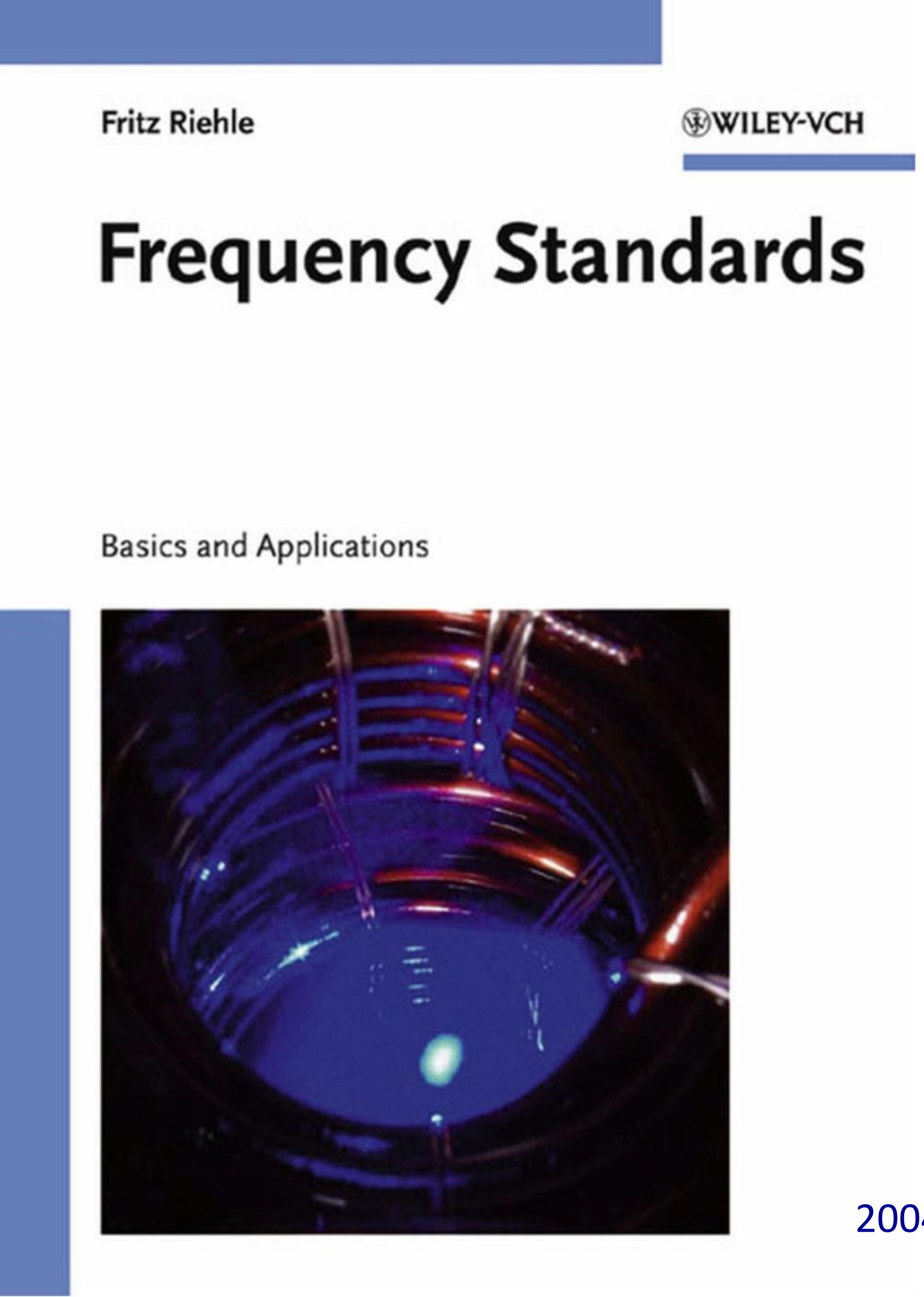
hyperfine structure, in the presence of a magnetic field (C field)

$$6^2S_{1/2}$$

magnetic field (C field) is necessary for the wave/atom interaction

≈1 Hz bias (10^{-10})

Picture from Richard C. Mockler, Atomic Beam Frequency Standards,
In B. E. Blair, A. H. Morgan (ed), NBS (now NIST) Special Publication 300 vol.5, June 1972

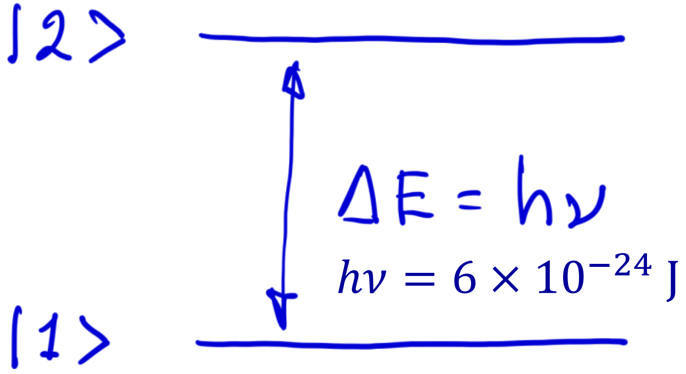


2004

[Link to Wiley-VCH]

General principle

Atomic quantum states



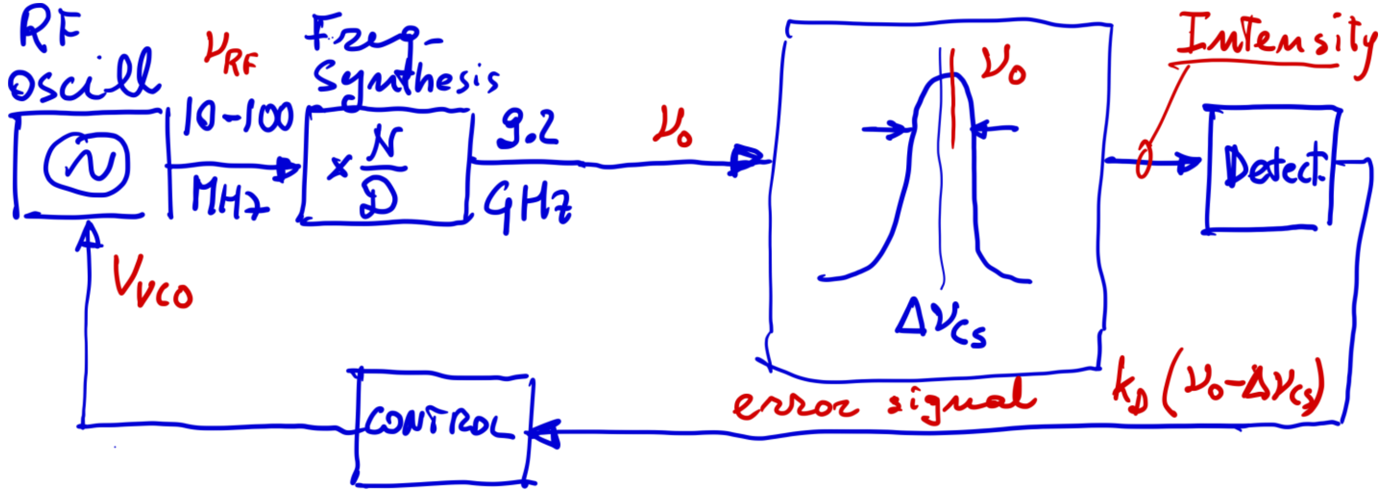
$\nu \approx 9.2\,GHz$ $\quad \lambda \approx 3.3\,cm$

Select (or prepare) the atoms

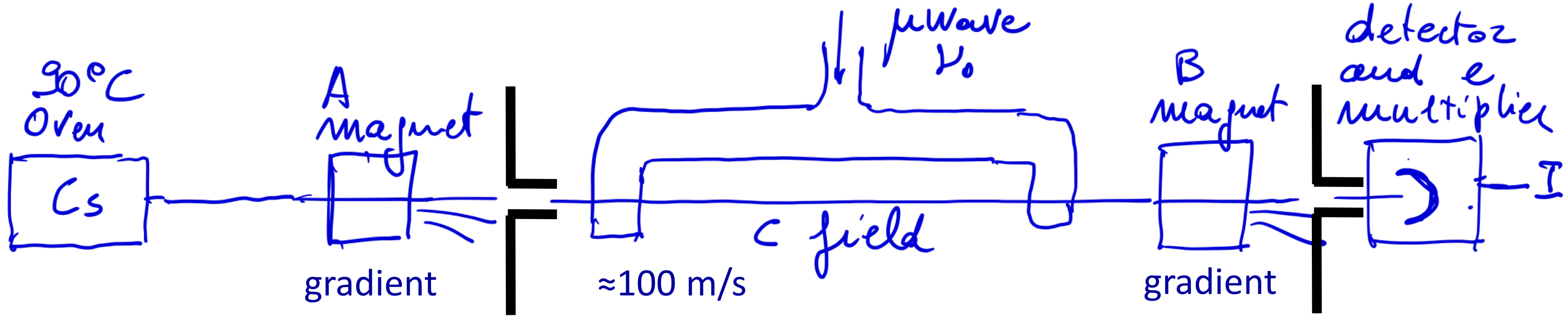
Interrogation

Detection

Tune the μwave frequency

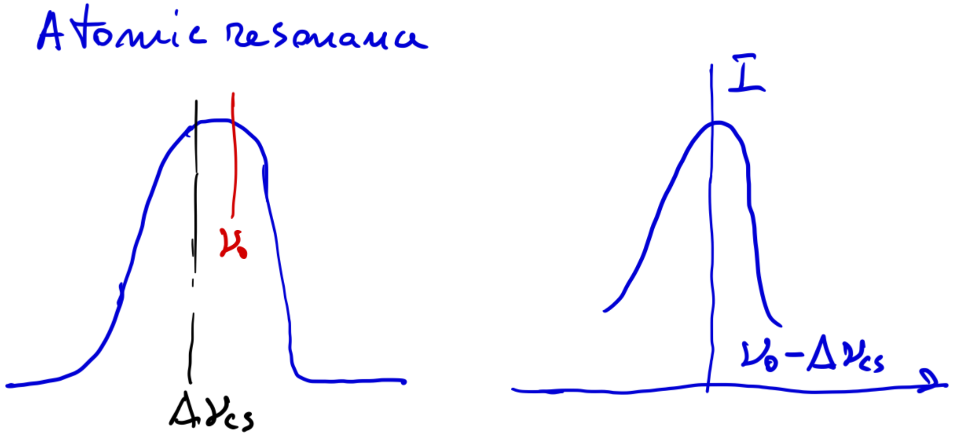


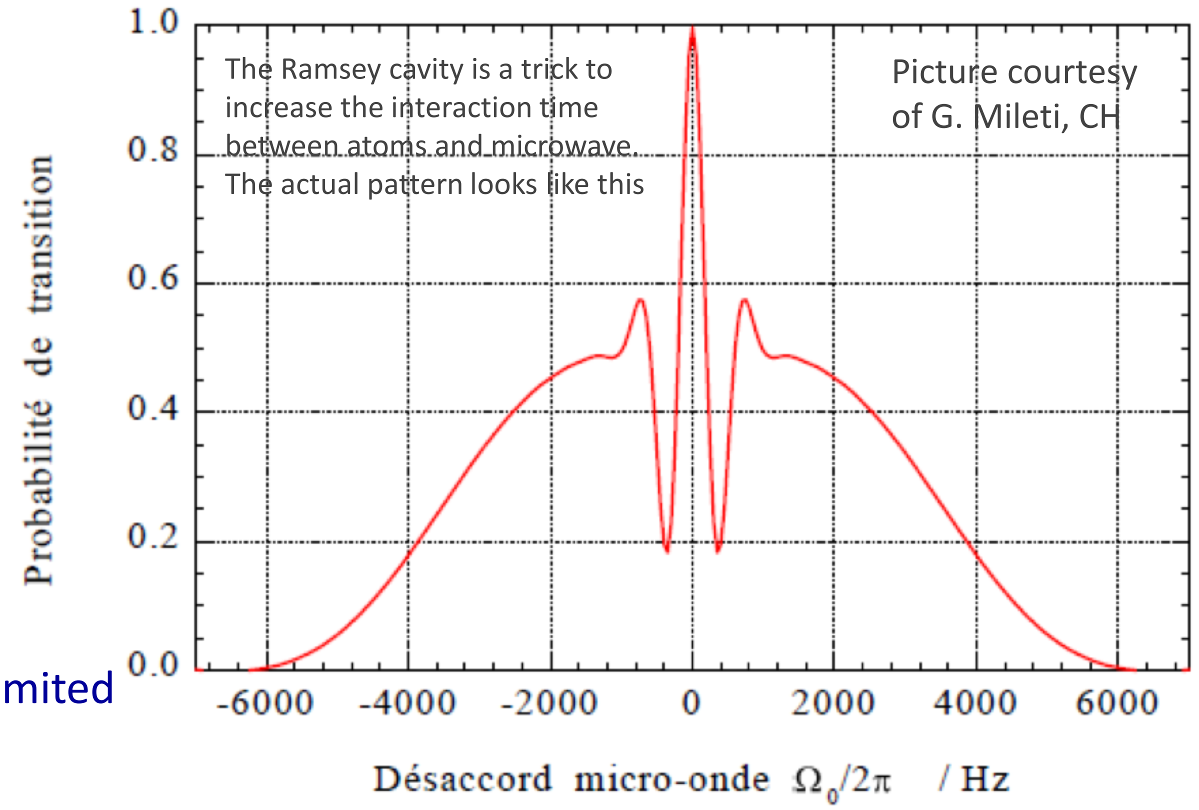
The old good Cs-beam standard



Major contribution from Norman Foster Ramsey Jr., Nobel in Physics in 1944

Still used in commercial Cs standards

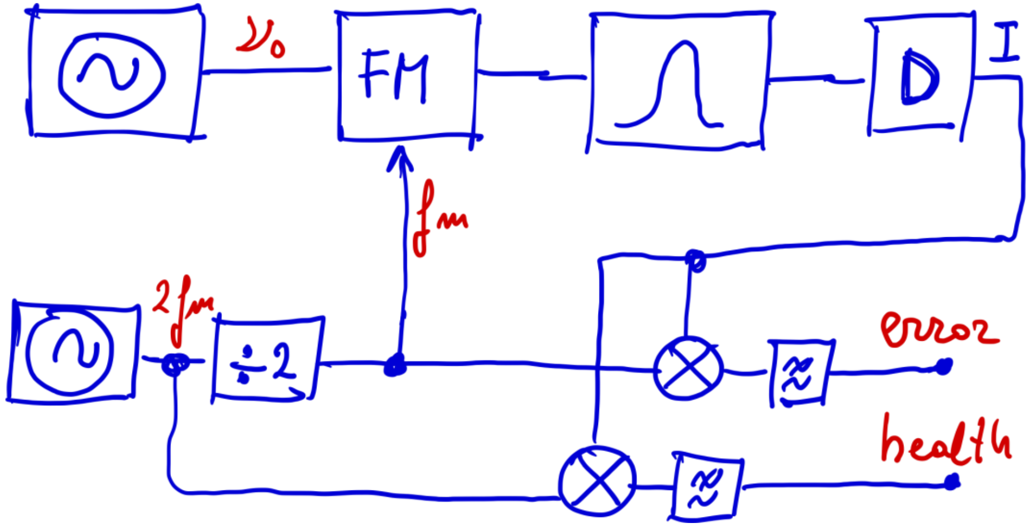


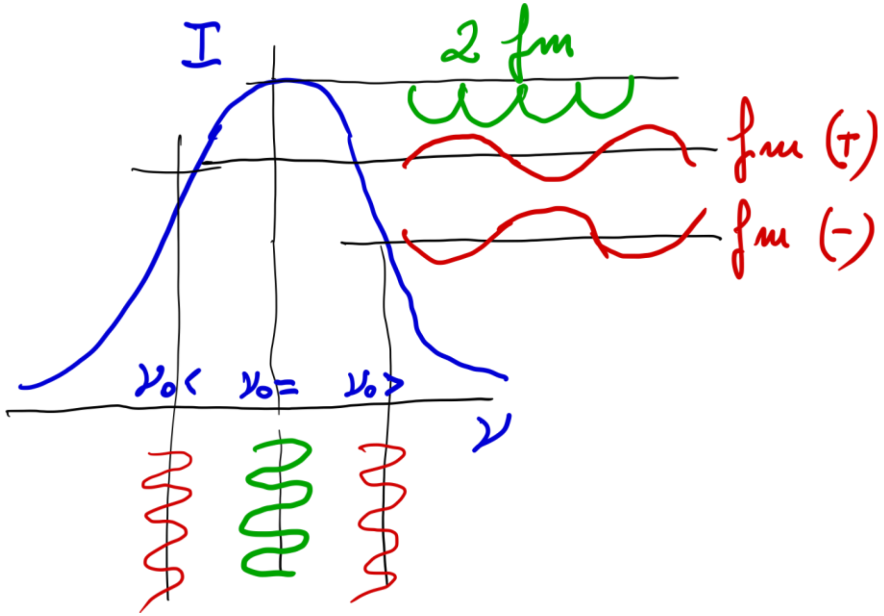


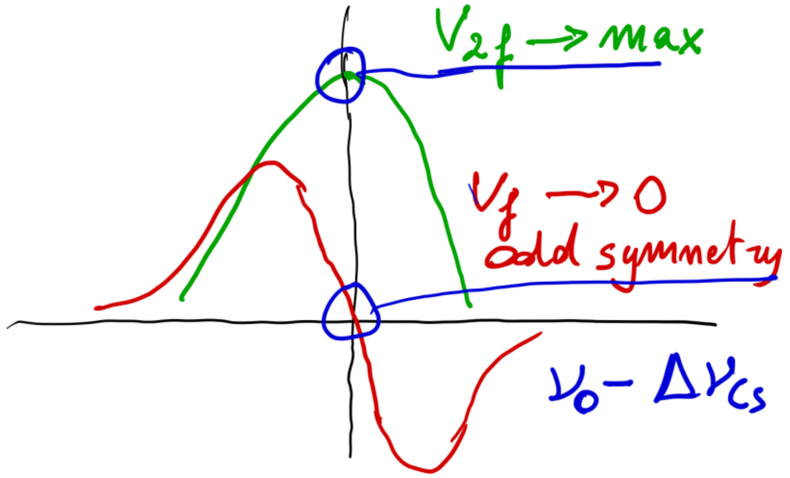
Features

- No long-term drift
- 5-7 y expected life, limited by the Cs reservoir

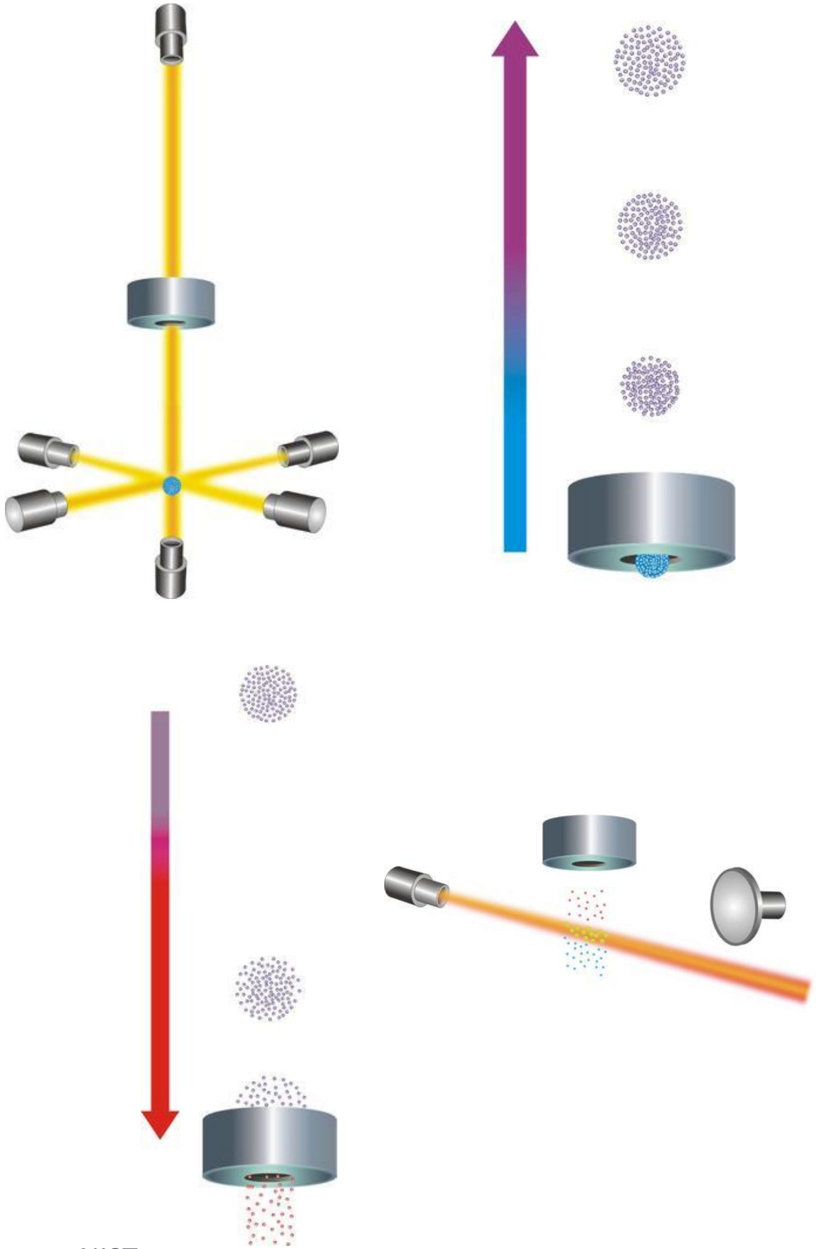
The f-2f detector







The Cs fountain



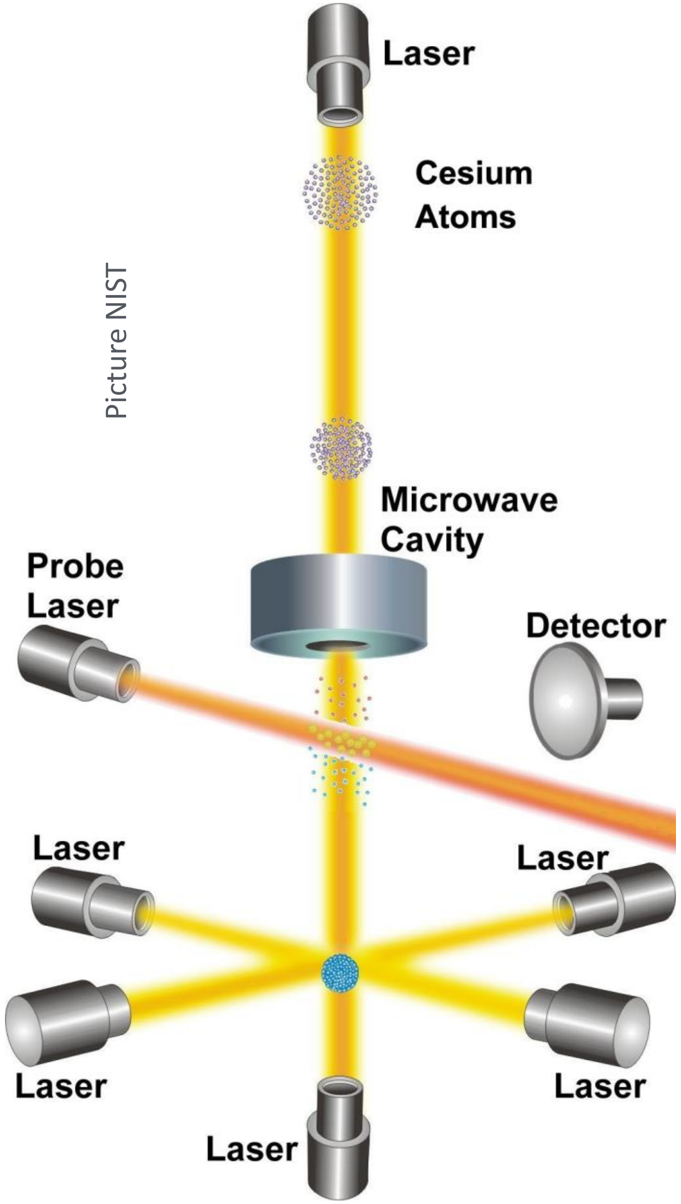
Atom cooling technology

- Gather a cluster of cold atoms (10^4)
- Launch the cluster upward
- Highly predictable vertical trajectory (v_0, g)
  - Cooling —> little/no horizontal spread
- Interrogation time set by the up/down time
- Optical detection with fluorescence
- Ultimate accuracy, ≈10^{-16}

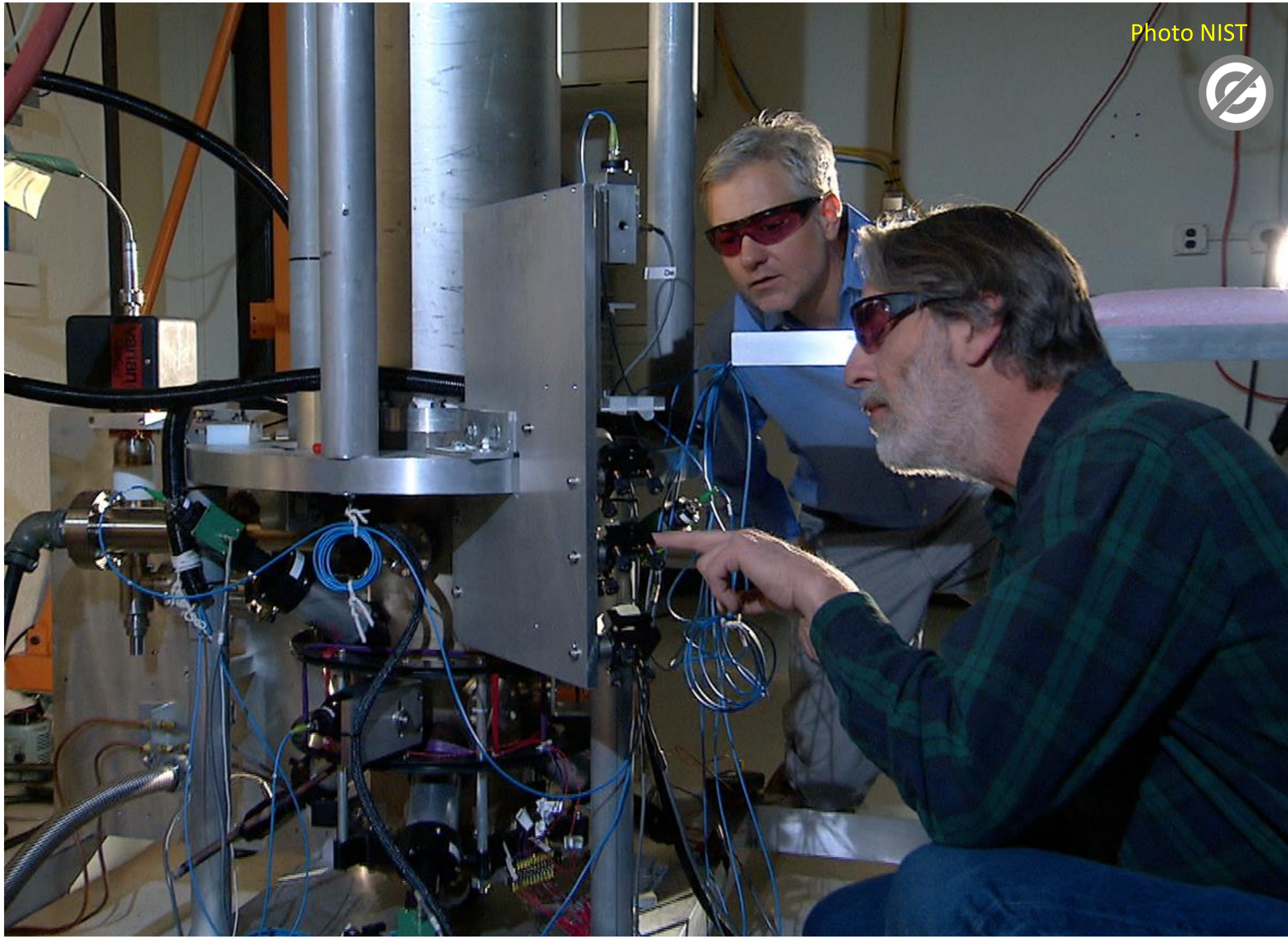
Atom cooling and trapping with laser light
Nobel Prize in Physics in 1997
Steven Chu, Claude Cohen-Tannoudji, and William “Bill” D. Phillips

Pictures NIST

The Cs fountain



Picture NIST



Thomas P. “Tom” Heavner (left), and Steven R. “Steve” Jefferts (right)

Why the Cs atom was chosen

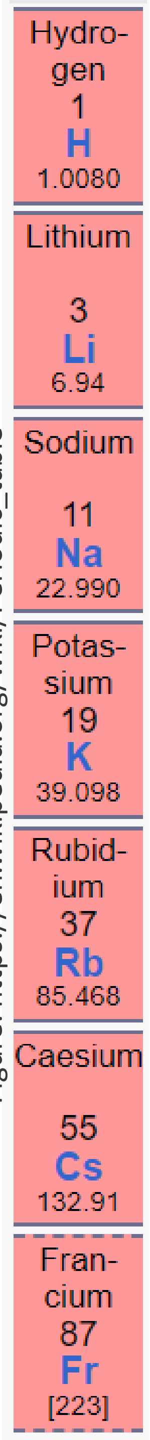


Figure: https://en.wikipedia.org/wiki/Periodic_table

Masers

Magnetometers

Widely used, secondary

Widely used, primary/secondary

Nope, radioactive, 22 m half life!

- Heavy atom —> small Doppler
- Alkali —> simple Hydrogen-like spectrum
- High vapor pressure —> gas easy to obtain and remove

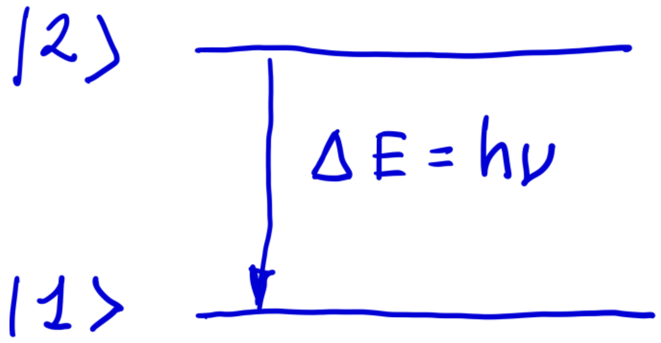
≈1 Hz bias (10^{-10})

Frequency shift $\times 10^{-15}$	Correction	Uncertainty
Quadratic Zeeman shift	−99.85	0.06
Blackbody radiation shift	16.60	0.06
Gravity + relativistic Doppler effect	−8.567	0.006
Collisional shift	0.32	0.62
Cavity phase shift	0.0	0.15
Light shift	0.0	0.001
Majorana transitions	0.0	0.0001
Rabi pulling	0.0	0.0002
Ramsey pulling	0.0	0.001
Electronics	0.0	0.20
Microwave leakage	0.0	0.10
Microwave power dependence	0.0	0.40
Background pressure	0.0	0.05
Total	−91.50	0.80

Table 1, from V. Gerginov et al., "Uncertainty evaluation of the caesium fountain clock PTB-CSF2," Metrologia 47 008 p.65-79, 2008. ©IOP

The H maser

The most reliable “serious” atomic frequency standard, 15 y expected life



Select the atoms

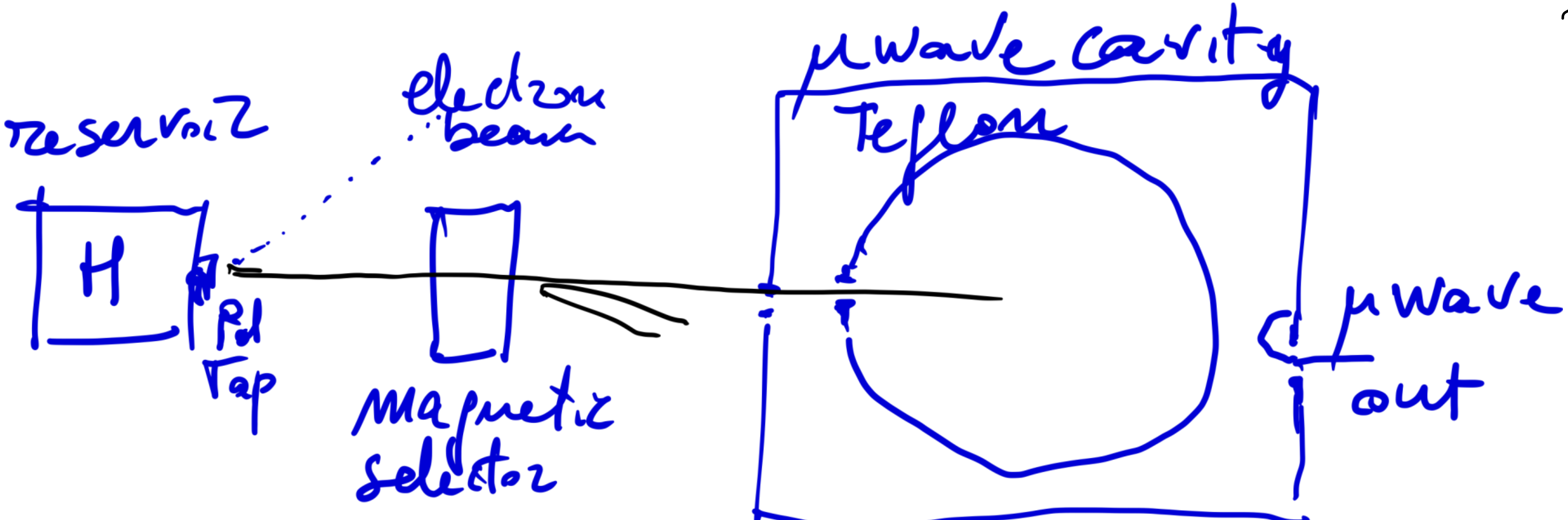
Stimulated emission

Frequency (or phase) lock a receiver

T4Science white paper

Key Features

Stability (Allan Deviation)	
1s	9E-14
10s	2E-14
100s	5E-15
1'000s	2E-15
10'000s & floor	2E-15



Pd film releases atomic H, with no H_2

High ∇B needed because H atoms are light & fast

Commercial products

- Microchip
- T4Science
- Vremya

Vendor	Product	ADEV ($\tau = 1$ s)	$\mathscr{L}$ (dBc/Hz, 10 Hz offset)	Drift (monthly)	Retrace ($\pm$)	T range (°C)	Tempco	Size (cm^3)	Weight (kg)	Power (W)
Microchip	SA45.s CSAC	3×10^{-10}	-70	9×10^{-10}	5×10^{-10}	-10 to 70	1×10^{-9}	17	0.035	0.12
Microchip	SA35.m MAC	3×10^{-11}	-87	1×10^{-10}	5×10^{-11}	0 to 75	7×10^{-11}	50	0.086	5
Microchip	SA22.c	3×10^{-11}	-90	4×10^{-11}	2×10^{-11}	-10 to 75	1×10^{-10}	208	0.43	10
Microchip	5071A	5×10^{-12}	-130			0 to 55		29700	30	50
Microchip	CsIII 4310B	1.2×10^{-11}	-130			0 to 50		16544	13.5	30
Microchip	MHM	8×10^{-14}	-138	6×10^{-14}				371000	246	75
Accubeat	NAC	2×10^{-10}	-86	3×10^{-10}		-20 to 65	2×10^{-9}	32	0.075	1.2
Accubeat	AR133A	5×10^{-12}	-116	1×10^{-11}	5×10^{-11}	-20 to 65	1×10^{-10}	146	0.295	8.25
FEI	FE-5669	6×10^{-12}	-140	1×10^{-11}	2×10^{-11}	-20 to 60	5×10^{-11}	669	1.69	20
FEI	FEI RAFS	6×10^{-13}	-138		5×10^{-12}	-4 to 25		4902	7.5	39
Spectratime	LP Rb	1×10^{-11}	-100	3×10^{-11}	5×10^{-11}	-25 to 55	2×10^{-10}	216	0.29	10
Spectratime	iSpace RAFS	3×10^{-12}	-120			-5 to 10		3224	3.4	35
Spectratime	miniRAFS	1×10^{-11}	-84			-15 to 55		388	0.45	10
T4Science	iMaser-3000	6×10^{-14}	-136	6×10^{-15}				436800	100	100
T4Science	pHMaser	5×10^{-13}	-130					49820	33	90
SRS	PRS10	2×10^{-11}	-130	5×10^{-11}	5×10^{-11}	-20 to 65	2×10^{-10}	155	0.6	14.4
Excelitas	RAFS	2×10^{-12}	-105	3×10^{-12}	5×10^{-12}	-20 to 45		1645	6.35	39
IQD	IQRB-1	8×10^{-11}	-95	5×10^{-11}	2×10^{-11}	0 to 50	5×10^{-10}	66	0.105	6
IQD	IQRB-2	2×10^{-12}	-138	4×10^{-11}	2×10^{-11}			230	0.22	6
Vremya	VCH-1003M	6×10^{-14}	-135	9×10^{-15}				305525	100	100
Chengdu Spaceon	XHTF1031 Rb	5×10^{-11}	-95	5×10^{-11}		-30 to 65	2×10^{-10}	65	0.2	6
Chengdu Spaceon	XHTF1021 Rb	3×10^{-11}	-100	5×10^{-11}	2×10^{-11}	-20 to 60	3×10^{-10}	189	0.27	7.8
Chengdu Spaceon	TA1000 OPC	1.2×10^{-11}	-125					48266	40	100
Chengdu Spaceon	CPT	2×10^{-10}	-90		5×10^{-11}	-45 to 70	5×10^{-10}	24	0.045	1.6
Teledyne	TCSAC	3×10^{-10}	-85	3×10^{-10}	3×10^{-10}	-10 to 60	1×10^{-9}	23	0.042	0.18
Muquans	MuClock	3×10^{-13}	-151					682000	135	200
Spectradynamics	cRb	5×10^{-13}	-138					39806	30.5	75

IEEE Trans UFFC 68(6), 2021

IEEE TRANSACTIONS ON ULTRASONICS, FERROELECTRICS, AND FREQUENCY CONTROL, VOL. 68, NO. 6, JUNE 2021 2007

A Review of Commercial and Emerging Atomic Frequency Standards

Bonnie L. Schmittberger Marlow and David R. Scherer, Senior Member, IEEE

(Invited Paper)

arXiv manuscript

A Review of Contemporary Atomic Frequency Standards

Bonnie L. Schmittberger and David R. Scherer

arXiv:2004.09987v1 [physics.atom-ph] 21 Apr 2020

Relevant literature

Huge interest in tiny atomic oscillators

APPLIED PHYSICS REVIEWS 5, 031302 (2018)

APPLIED PHYSICS REVIEWS

Chip-scale atomic devices

John Kitching
Time and Frequency Division, National Institute of Standards and Technology, Boulder, Colorado 80305, USA

(Received 16 February 2018; accepted 1 May 2018; published online 14 August 2018)

Chip-scale atomic devices combine elements of precision atomic spectroscopy, silicon micromachining, and advanced diode laser technology to create compact, low-power, and manufacturable instruments with high precision and stability. Microfabricated alkali vapor cells are at the heart of most of these technologies, and the fabrication of these cells is discussed in detail. We review the design, fabrication, and performance of chip-scale atomic clocks, magnetometers, and gyroscopes and discuss many applications in which these novel instruments are being used. Finally, we present prospects for future generations of miniaturized devices, such as photonically integrated systems and manufacturable devices, which may enable embedded absolute measurement of a broad range of physical quantities. https://doi.org/10.1063/1.5026238

Link to APR

The physics of atomic oscillators finds application in wonderful sensors

IEEE SENSORS JOURNAL, VOL. 11, NO. 9, SEPTEMBER 2011 1749

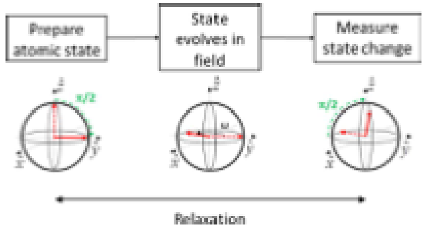
Atomic Sensors – A Review

John Kitching, Svenja Knappe, and Elizabeth A. Donley

(Invited Paper)

***Abstract*—We discuss the basic physics and instrumentation issues related to high-performance physical and inertial sensors based on atomic spectroscopy. Recent work on atomic magnetometers, NMR gyroscopes, and atom interferometers is reviewed, with a focus on precision sensing of electromagnetic and gravitational fields and inertial motion. Atomic sensors have growing relevance to many facets of modern science and technology, from understanding the human brain to enabling precision navigation of moving platforms.**

***Index Terms*—Atomic spectroscopy, inertial sensor, magnetometry.**



Link to IEEE Sensors J

Anticipated by R. A. Heinlein *Starship Trooper*

Your "eyes" and your "ears" are rigged to help you without cluttering up your attention, too. Say you have three audio circuits, common in a marauder suit. The frequency control to maintain tactical security is very complex, at least two frequencies for each circuit both of which are necessary for any signal at all and each of which wobbles under the control of a cesium clock timed to a micromicrosecond with the other end — but all this is no problem of yours. You want circuit A to your squad leader, you bite down once — for circuit B, bite down twice — and so on.

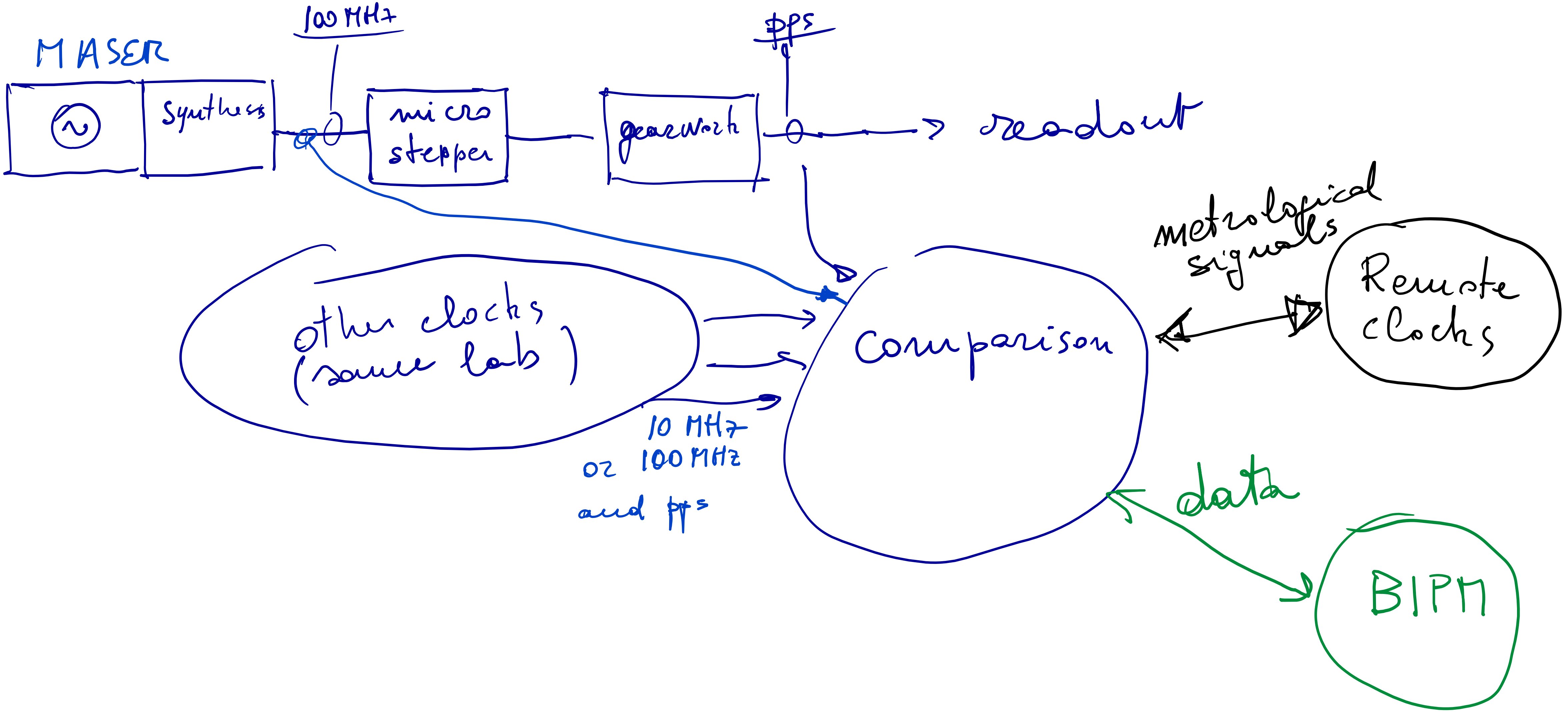
Magnetic fields (Zeeman effect)
Gyroscope (Larmor precession)
Gravity (atom interferometry)
Temperature (Doppler broadening)
Etc.

What time is it? —> Time scales

- Atomic "clock" —> frequency
  - Not a time standard!
  - Jargon: the atomic oscillator is called clock if it contributes to TAI continuously for months/years
- JD (Julian Date), origin Jan 1, 4713 BC at noon
  - Old-style astronomers needed not to change date nighttime
- Modified JD, MJD = JD – 2400000.5
  - Origin November 17, 1858
  - Date changes at midnight
  - Often used
  - 60000 MJD = Feb 25, 2023
- Temps Atomique International (TAI)
  - Set by an ensemble of atomic clocks
  - Coordinated by the BIPM
  - The Earth slows down, TAI does not
- Universal Time Coordinated (UTC)
  - TAI – UTC = n seconds (today, 37 s)
  - Synchronized to the rotation of the Earth
  - Leap second when needed
    - Clock counts 58, 59, 60, 00, 01
- Leap second is a nightmare of Internet
  - Synchronize thousands of servers
  - Google adjusts the clocks over 1 day

Today, March 26, 2024, is 60395 MJD

What does a time scale look like?

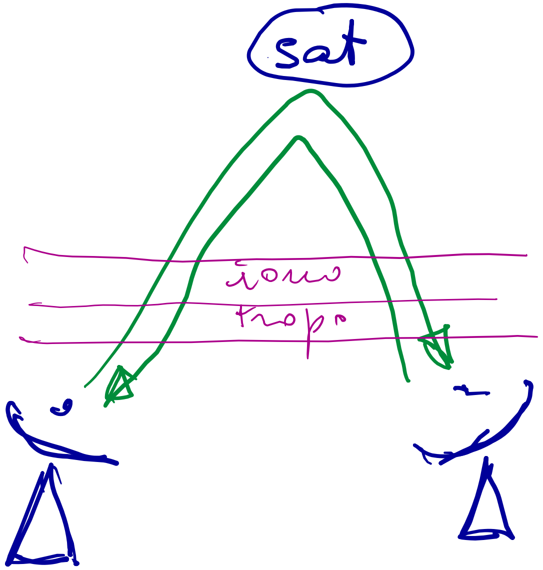


Clock comparisons

$10^{-16} \longrightarrow$ 10 ps / 3 mm in a day

New methods based on optical fibers

Two Way

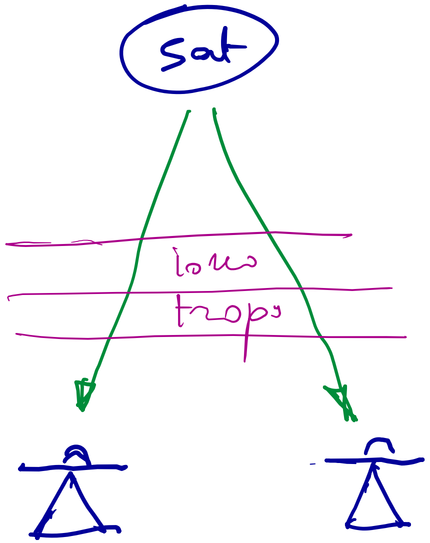


use the roundtrip time to calibrate the propagation

assume channel symmetry

TWSTFT
- Geostat. sat

10 / 50 ps
precision / accuracy



Same signal received by the two labs

Independent of emission time

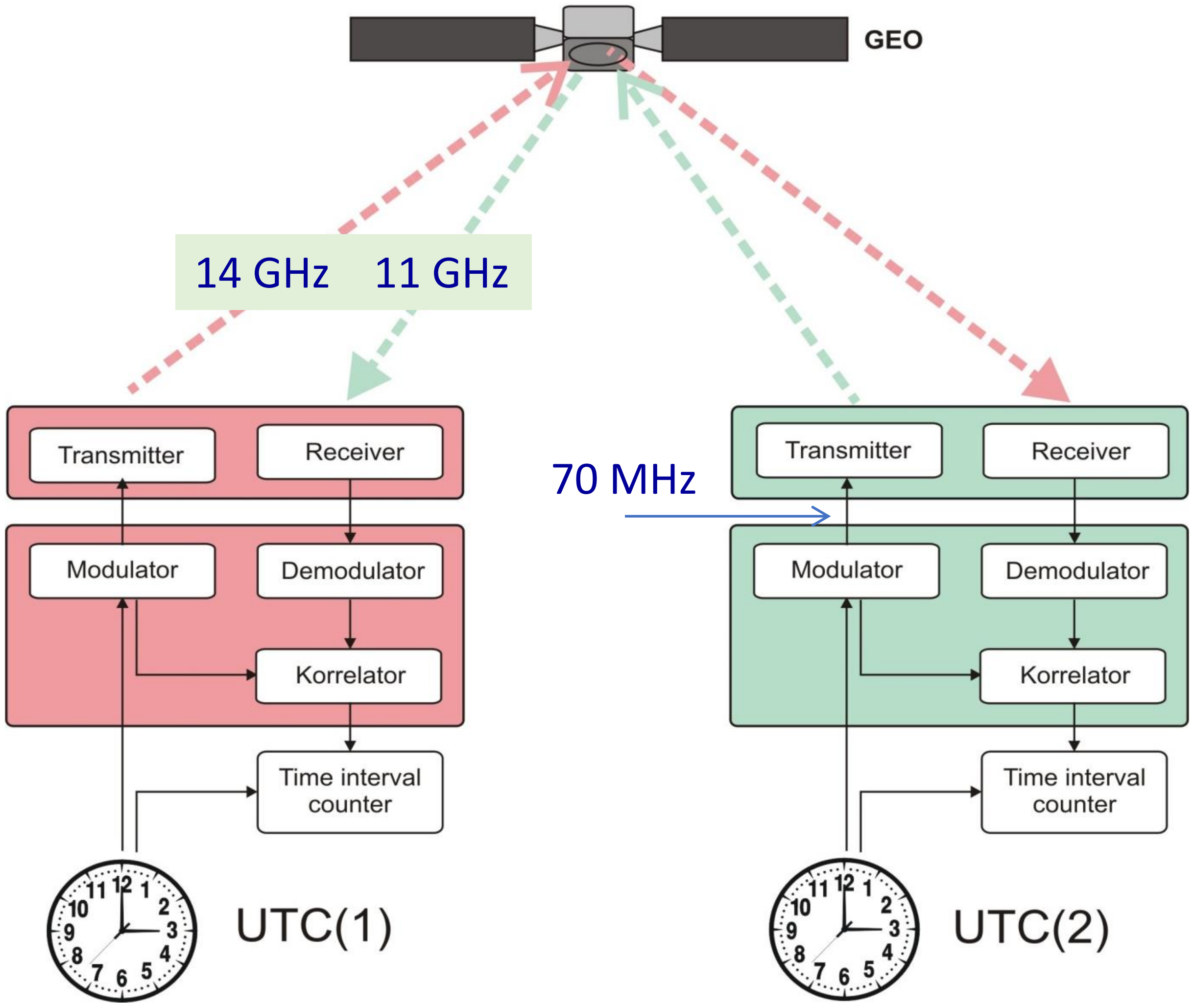
estimation of propagation time and perturbations

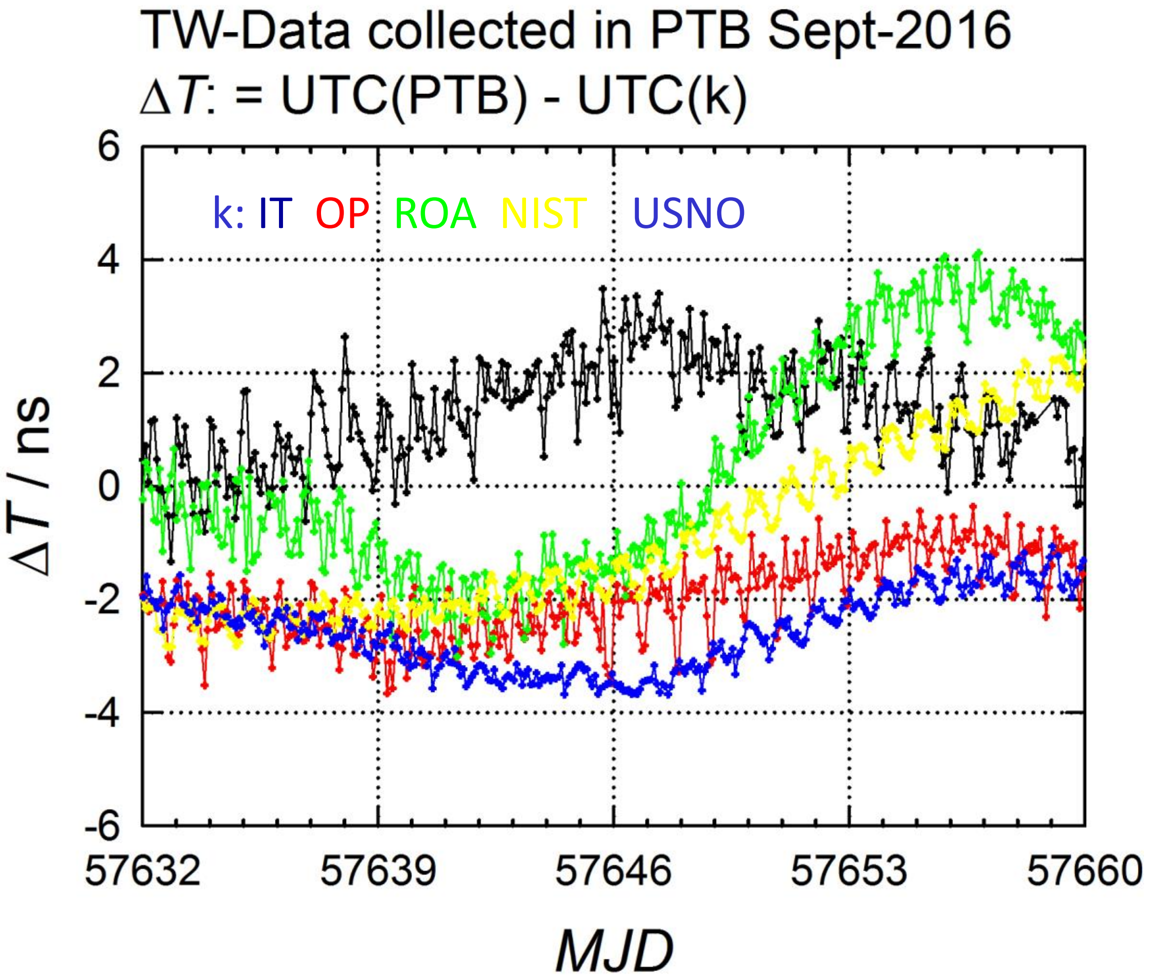
GPS/GNSS

< 1 ns

Clock comparisons – TWSTFT

Two Way Satellite Time & Frequency Transfer





All figures Courtesy of Andreas Bauch, PTB, DE

GNSS – Global Navigation Satellite Systems

Mobile phones "GPS" is actually GNSS,
and "GPS coordinates" are actually geodetic coordinates

GPS, USA
Galileo, Europe
GLONASS, Russia
BeiDou, China

Regional systems
QZSS, Japan
IRNSS/NaviC, India

- All satellites are synchronized
- One-way communication
  - Solve x, y, z, t
  - Requires 4 satellites
- Mobile equipment may use height
  - A "satellite" at the center of the Earth
- Warning
  - Low level signals
  - Fragile to jamming and spoofing

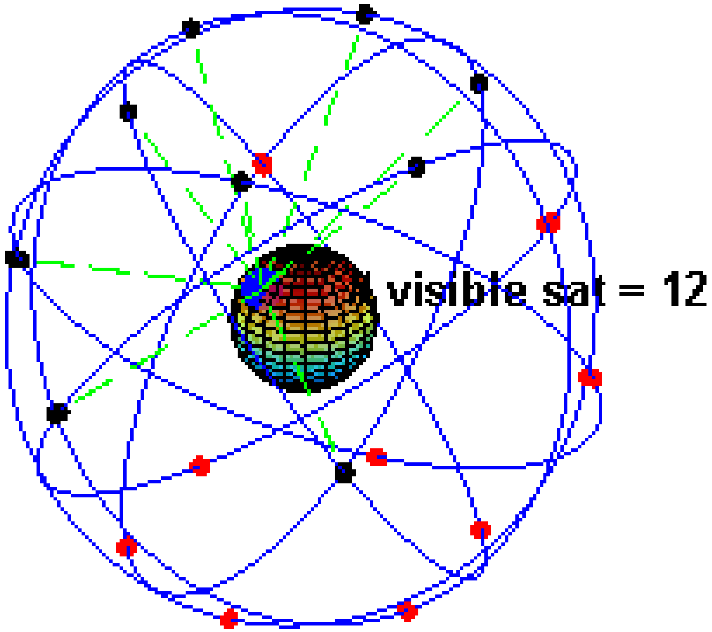
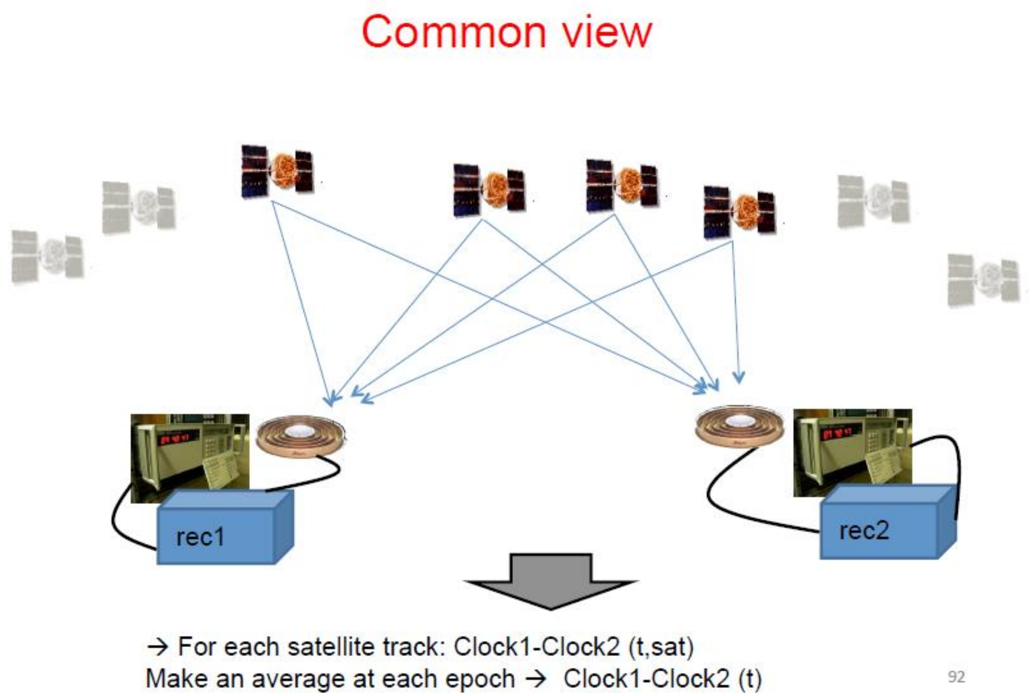
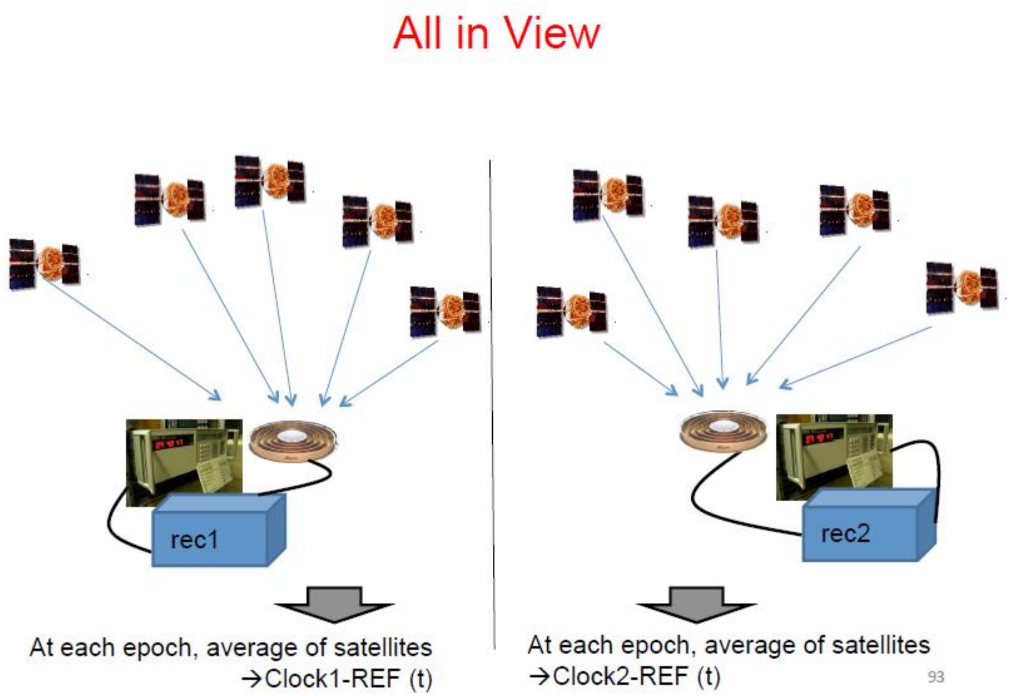


Image El pak, Wikimedia Commons

GNSS for clock comparison

POI: The satellite time may be independent of UTC
But common view is independent of the satellite time

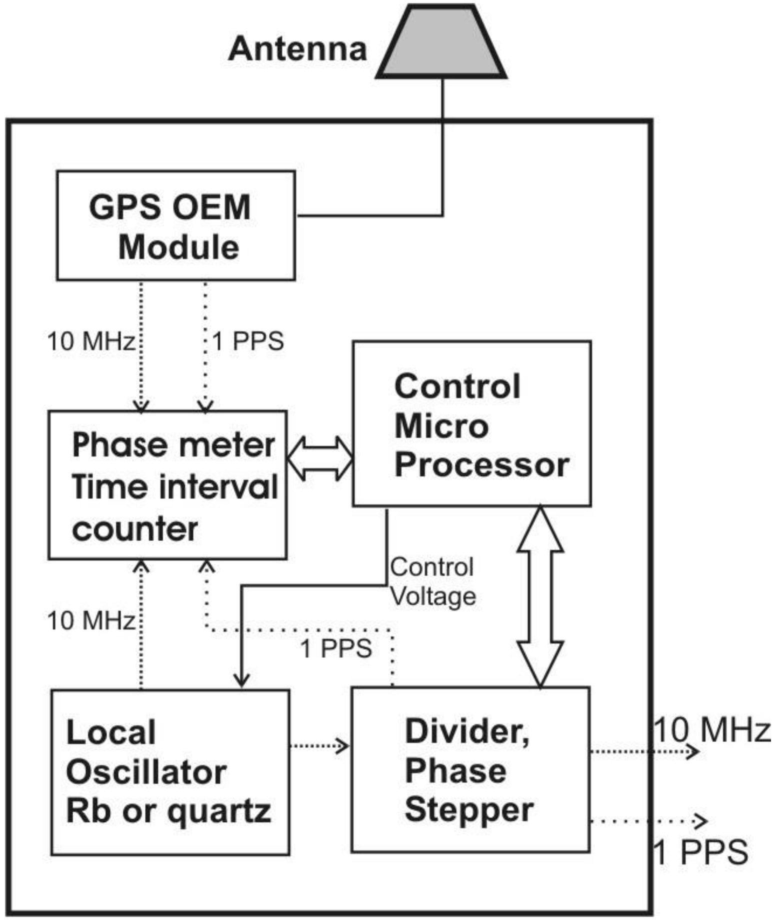




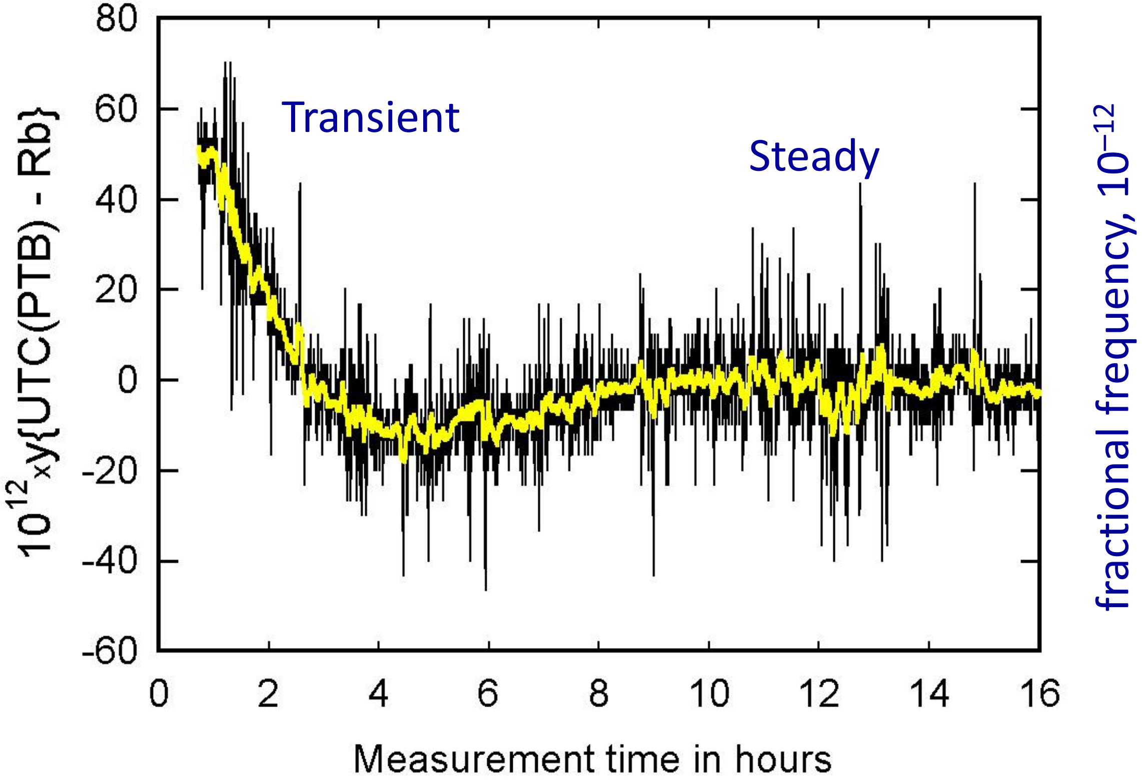
All figures Courtesy of Andreas Bauch, PTB, DE

GNSS disciplined oscillator

High accuracy for cheap
Radio Amateur solutions exist – Also used unit for $300 on eBay



black: hourly average
yellow: 6-h sliding avg



All figures Courtesy of Andreas Bauch, PTB, DE

What’s next?

- The ^{133}Cs survived the 2019 revolution
  - but it will capitulate soon
    - because optical clocks are sieging it
- Nowadays nobody relies on the angular position of the Earth
  - Leap should retire, as it takes > 1 k year before TAI-UTC is significant
    - GLONASS does not support TAI-UTC > 1 s
      (is this a problem?)
- Central European Time (CET)
  - Too wide longitude span in one time zone –> health problem
- Summer Time
  - No real reason
  - Solar noon at 2 pm in France, 3 pm in Spain
- Historical note, in 1582 there was a 10 day hole in the calendar

Funny, yet serious reading: Abner Shimony, Tibaldo and the Hole in the Calendar

What's next?

OPEN ACCESS
IOP Publishing | Bureau International des Poids et Mesures Metrologia

Metrologia **61** (2024) 012001 (19pp) https://doi.org/10.1088/1681-7575/ad17d2

Review

Roadmap towards the redefinition of the second

N Dimarcq[1], M Gertsvolf[2], G Mileti[3], S Bize[4], C W Oates[5], E Peik[6], D Calonico[7], T Ido[8], P Tavella[9,*], F Meynadier[9], G Petit[9], G Panfilo[9], J Bartholomew[10], P Defraigne[11], E A Donley[5], P O Hedekvist[12], I Sesia[7], M Wouters[13], P Dubé[2], F Fang[14], F Levi[7], J Lodewyck[4], H S Margolis[15], D Newell[5], S Slyusarev[16], S Weyers[6], J-P Uzan[17], M Yasuda[18], D-H Yu[19], C Rieck[12], H Schnatz[6], Y Hanado[8], M Fujieda[8,21], P-E Pottie[4], J Hanssen[20], A Malimon[16] and N Ashby[5]

https://iopscience.iop.org/article/10.1088/1681-7575/ad17d2

End of lecture 12

Lecture 13

Scientific Instruments & Oscillators

Lectures for PhD Students and Young Scientists

Enrico Rubiola

CNRS FEMTO-ST Institute, Besancon, France

INRiM, Torino, Italy

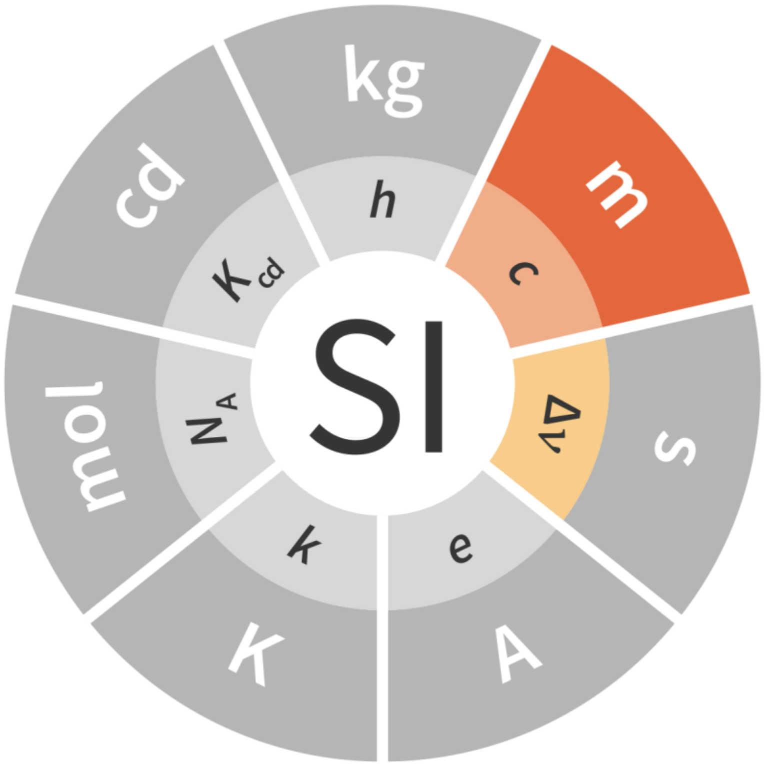
Contents

-

ORCID 0000-0002-5364-1835

home page http://rubiola.org

Metre



The metre, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299792458 when expressed in the unit m s^{-1}, where the second is defined in terms of the caesium frequency $\Delta\nu_{Cs}$.

A rephrased version of the 1983 definition

POI: the CGPM uses French spelling metre (with no accent), but the common American English spell is meter

The historical metre

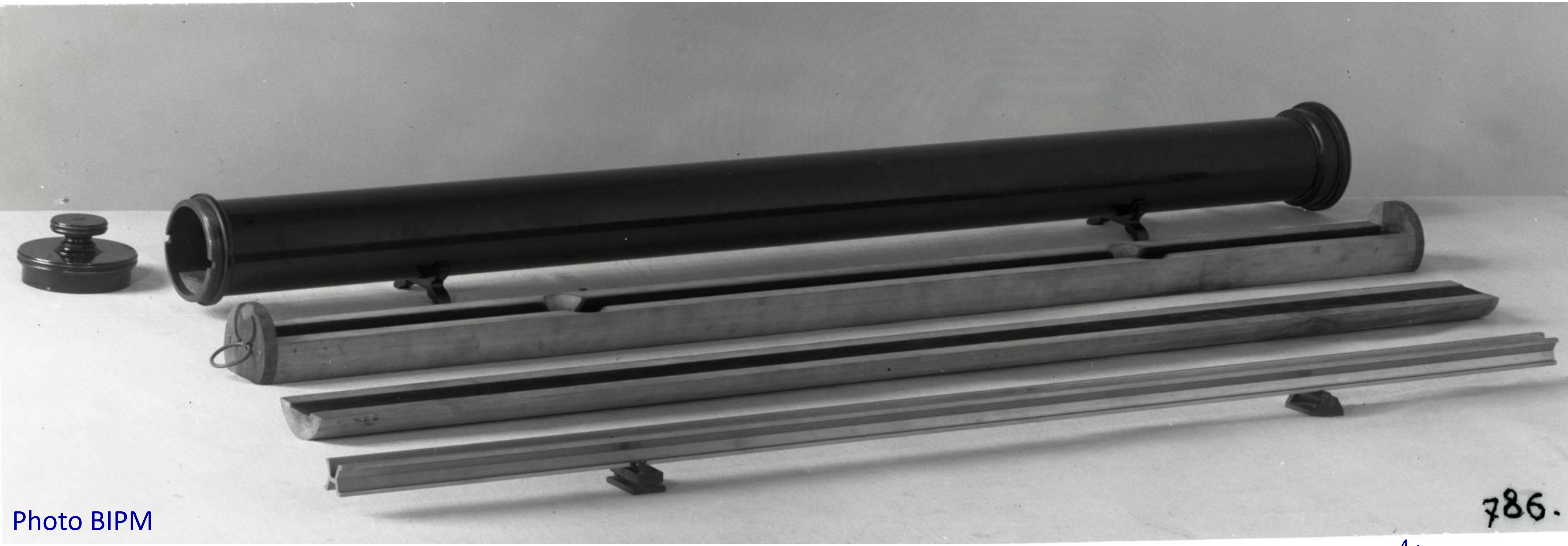
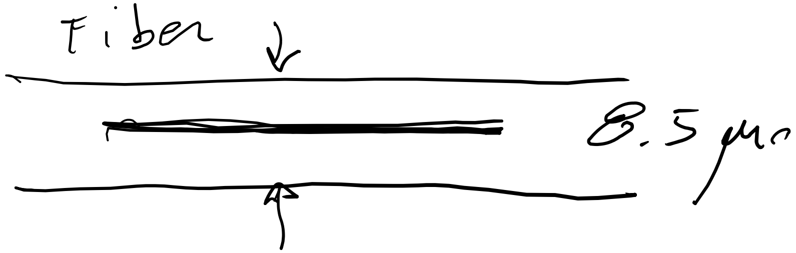
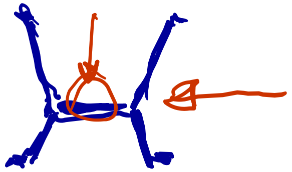
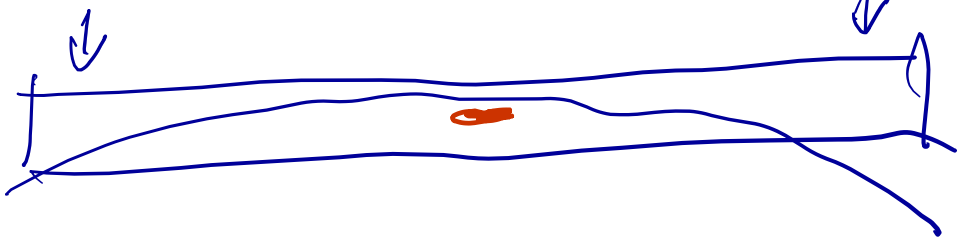


Photo BIPM





Neutral axis



Time delay

$$\ell = c\Delta t$$

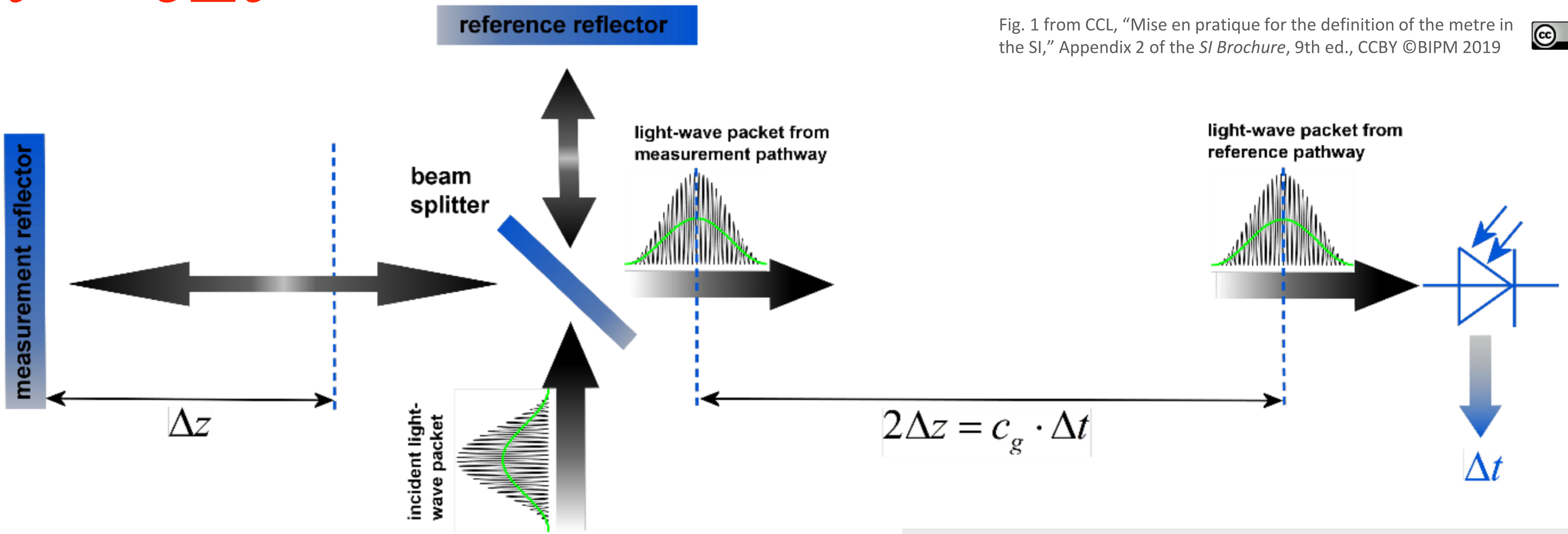


Fig. 1 from CCL, “Mise en pratique for the definition of the metre in the SI,” Appendix 2 of the *SI Brochure*, 9th ed., CCBY ©BIPM 2019

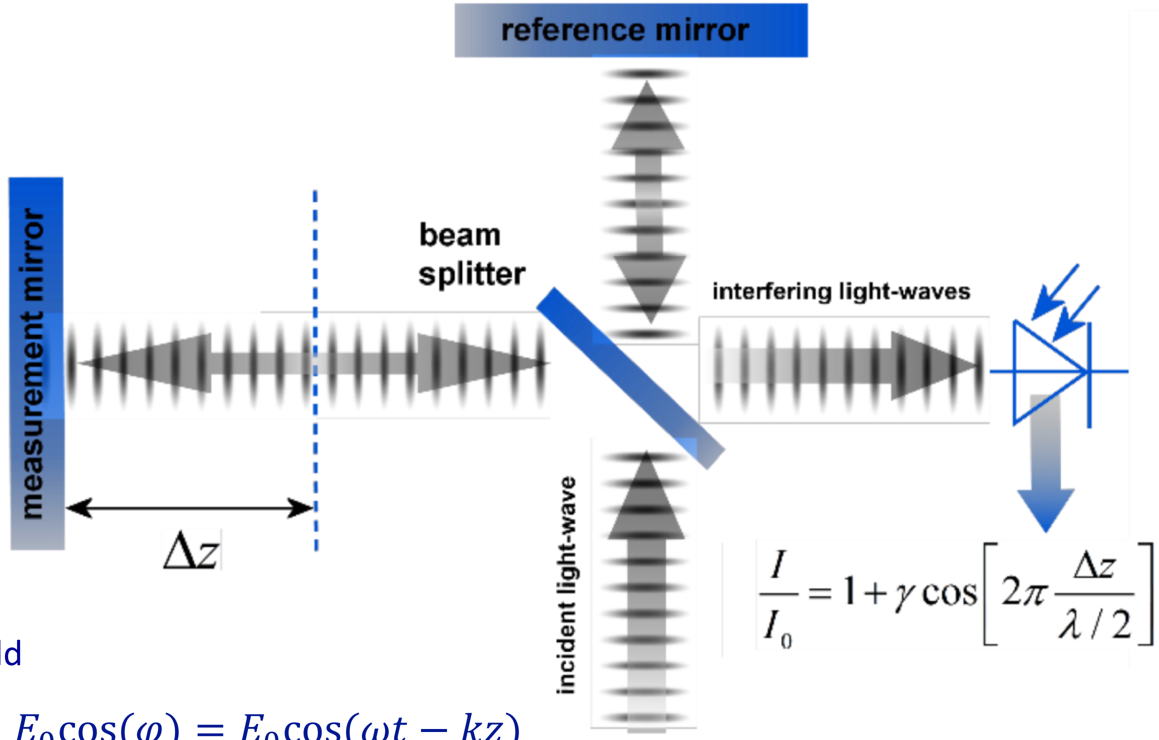
$$\ell = \Delta z = \frac{1}{2} c_g \Delta t$$

$$c_g = c/n_g \qquad n_g(\lambda) = n(\lambda) - \lambda \frac{dn}{d\lambda}$$

group velocity and refraction index

- Resolution of Δt limited by the electronics
  - Single shot 10 ps —> 3 mm roundtrip (1.5 mm resolution)
- Suitable for long distances
- Pulse train
- Pseudo random code

Optical interferometry



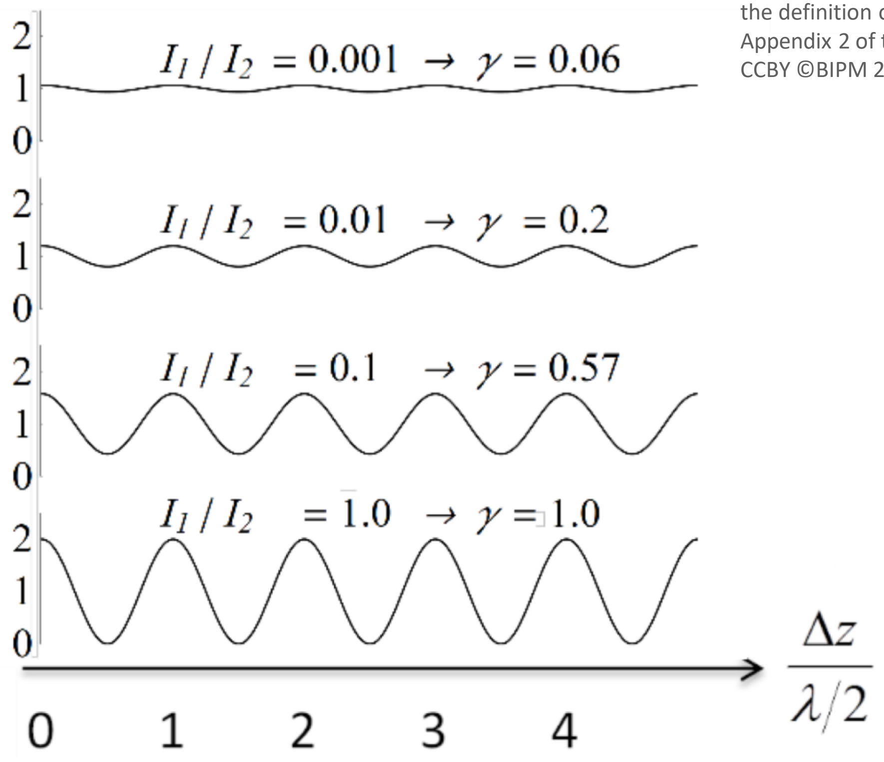


Fig. 2 from CCL, “Mise en pratique for the definition of the metre in the SI,” Appendix 2 of the *SI Brochure*, 9th ed., CCBY ©BIPM 2019

Electric field

$E(z,t) = E_0\cos(\varphi) = E_0\cos(\omega t - kz)$

$k = 2\pi/\lambda$

Intensity

$$I = I_0[1 + \gamma\cos(\Delta\varphi)]$$

γ contrast, $\gamma \leq 1$

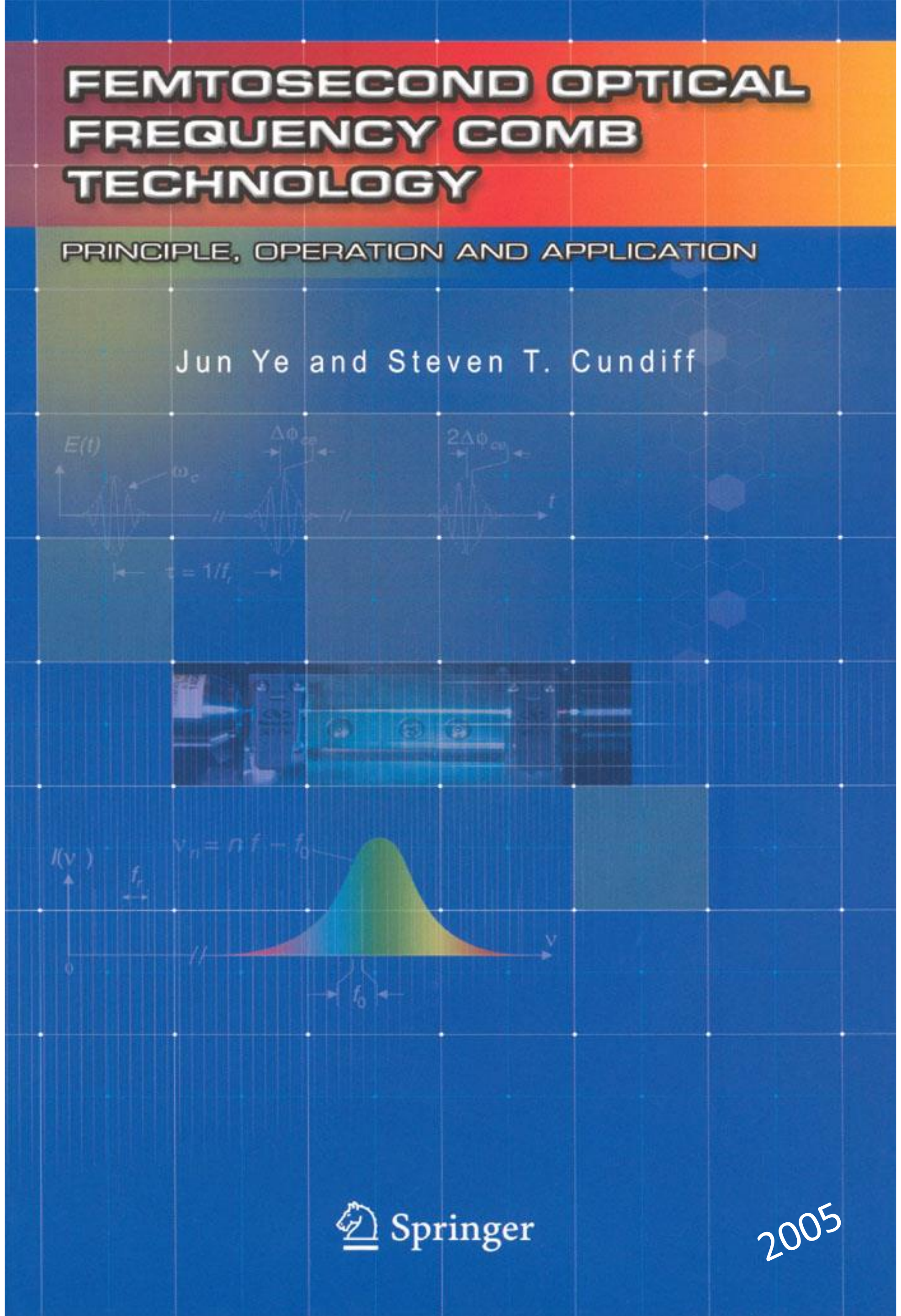
$$\ell = \Delta z = \frac{\lambda}{2}\frac{\Delta\varphi}{2\pi}$$

- High resolution (λ ≈ µm)
- Relies on frequency accuracy (λ=c/ν) and fringes counting
- Suitable for short length

The Femtosecond Comb

John Lewis “Jan” Hall and Theodor Wolfgang Hänsch, Nobel prize in Physics in 2005
(together with Roy Jay Glauber)

Bring the Definition of the Second to Optics



2005

REVIEWS OF MODERN PHYSICS, VOLUME 75, JANUARY 2003

Colloquium: Femtosecond optical frequency combs

Steven T. Cundiff* and Jun Ye†

JILA, University of Colorado and National Institute of Standards and Technology, Boulder, Colorado 80309-0440

(Published 10 March 2003)

Recently there has been a remarkable synergy between the technologies of precision laser stabilization and mode-locked ultrafast lasers. This has resulted in control of the frequency spectrum produced by mode-locked lasers, which consists of a regular comb of sharp lines. Thus such a controlled mode-locked laser is a “femtosecond optical frequency comb generator.” For a sufficiently broad comb, it is possible to determine the absolute frequencies of all of the comb lines. This ability has revolutionized optical frequency metrology and synthesis. It has also served as the basis for the recent demonstrations of atomic clocks that utilize an optical frequency transition. In addition, it is having an impact on time-domain applications, including synthesis of a single pulse from two independent lasers. In this Colloquium, we first review the frequency-domain description of a mode-locked laser and the connection between the pulse phase and the frequency spectrum in order to provide a basis for understanding how the absolute frequencies can be determined and controlled. Using this understanding, applications in optical frequency metrology and synthesis and optical atomic clocks are discussed. This is followed by a brief overview of how the comb technology is affecting and will affect time-domain experiments.

CONTENTS

I. INTRODUCTION

Mode-locked lasers generate ultrashort optical pulses by establishing a fixed phase relationship across a broad spectrum of frequencies. Progress in the technology of mode-locked lasers has resulted in the generation of optical pulses that are only 5 fs in duration (Morgner *et al.*, 1999; Sutter *et al.*, 1999), which corresponds to less than two cycles of the laser light. Although mode locking is a frequency-domain concept, mode-locked lasers and their applications are typically discussed in the time domain. Recently, a paradigm shift in the field of ultrafast optics has been brought about by switching to a frequency-domain treatment of the lasers and the pulse trains that they generate. Understanding mode-locked lasers in the frequency domain has allowed the extensive tools of frequency-domain laser stabilization to be employed, with dramatic results.

The central concept to these advances is that the pulse train generated by a mode-locked laser has a frequency spectrum that consists of a discrete, regularly spaced series of sharp lines, known as a frequency comb. As described below, if the comb spectrum is sufficiently broad, it is possible to directly measure the two radio frequencies (rf) that describe the comb. This fact has had immediate impact in the field of optical frequency metrology/synthesis[1] and has been enabling for the recent demonstration of optical atomic clocks (Diddams *et al.*, 2001; Ye *et al.*, 2001). Because the comb spectrum can be related to phase evolution in the pulse train (Apolonski *et al.*, 2000; Jones *et al.*, 2000), these results also promise important advances in ultrafast science, specifically extreme nonlinear optics (Brabec and Krausz, 2000) and coherent control. In addition, the union of time and frequency-domain techniques has yielded remarkable results in pulse synthesis (Shelton *et al.*, 2001).

The idea that a regularly spaced train of pulses corresponds to a comb in the frequency domain and can

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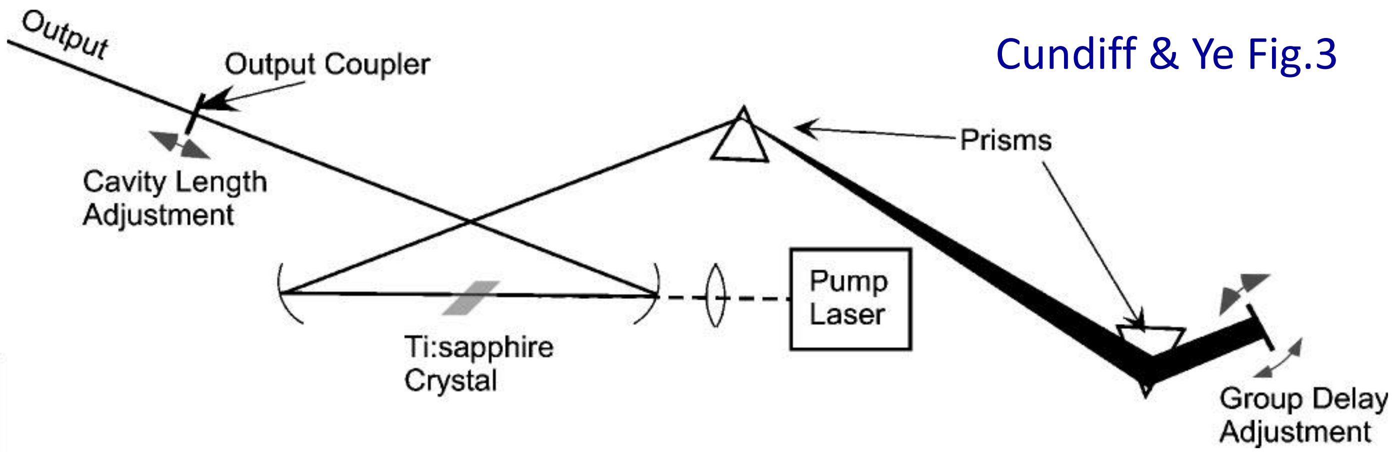
[1]There have been a large number of results in the last two years. A few of the more notable ones are reported by Diddams, Jones, Ye, *et al.* (2000), Holzwarth *et al.* (2000), Jones *et al.* (2000), Stenger, Binnewies, *et al.* (2001), and Udem *et al.* (2001). For a more extensive compilation, see Cundiff *et al.* (2001).

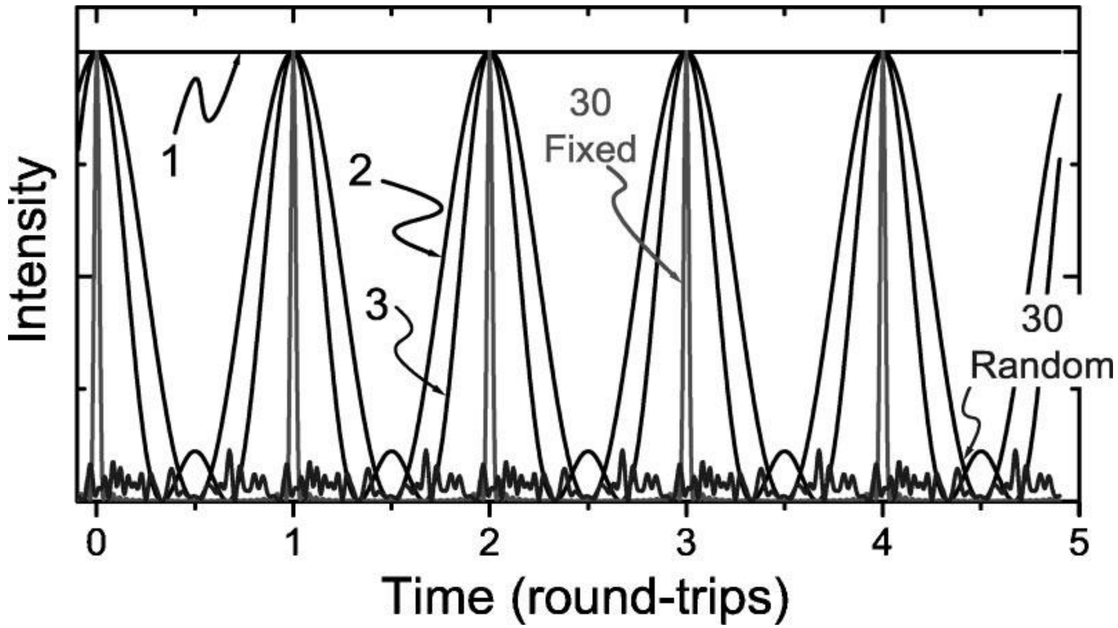
Step 1: the mode locked laser

Mode locking results from fixed phase relationships between laser longitudinal modes



Cundiff & Ye Fig.3

Kerr-lens mode-locked Ti:sapphire laser



Cundiff & Ye Fig.2

S. T. Cundiff, J. Ye, Colloquium, Femtosecond optical frequency combs, Rev. Mod. Phys. 75(1), January 2003

Theodore W. Hänsch and John L. Hall, Nobel in Physics in 2005 (with Roy J. Glauber)

Step 2: the frequency comb

one-step microwave-to optics frequency synthesis

Fig.1 and Fig.3 from Th. Udem, R. Holzwarth, T. W. Hänsch, Optical frequency metrology, Nature 416(6877) p.233-237, 14 March 2002

Sharp laser pulses

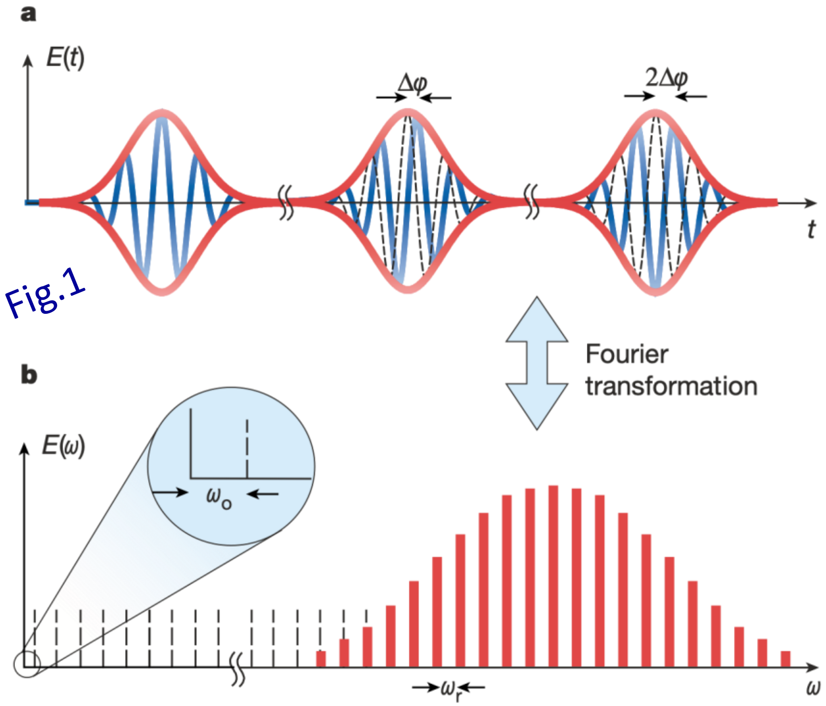


Fig.1

The main idea: f-2f locking

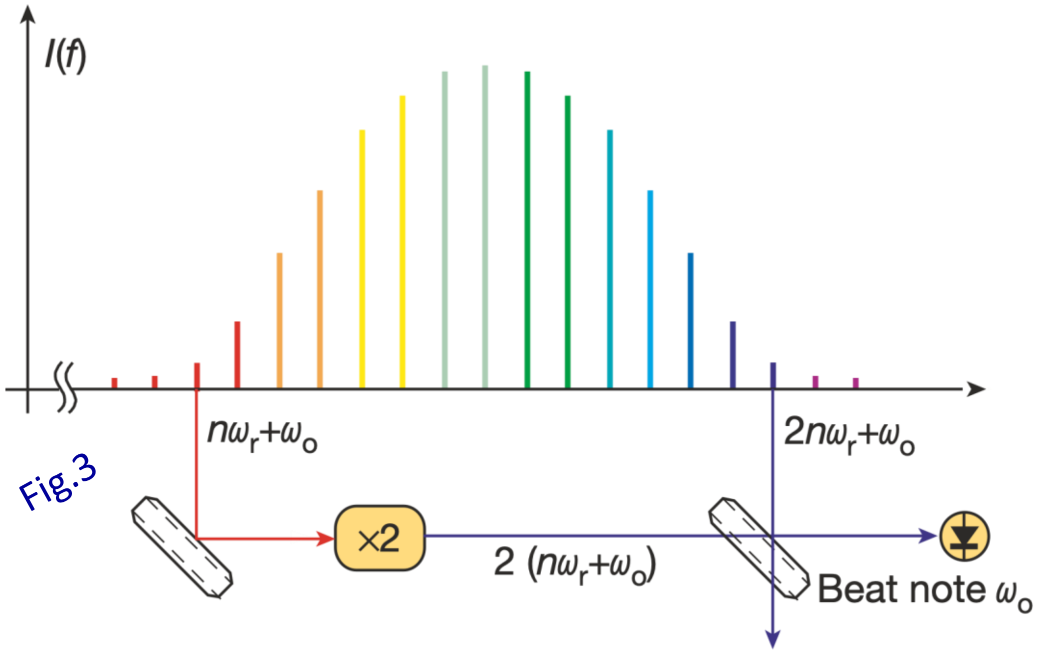


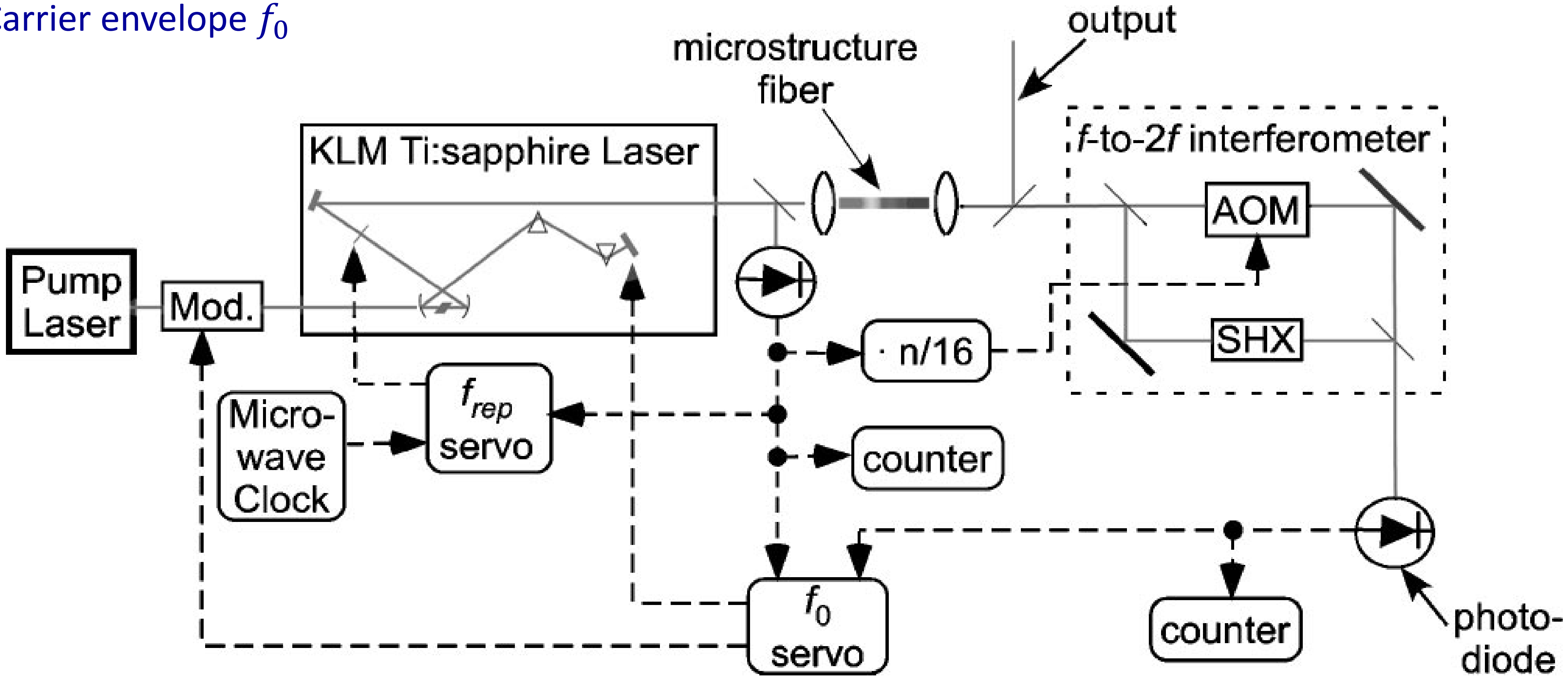
Fig.3

The carrier travels at the phase velocity, the pulse travels at the group velocity.
There results a shift $\omega_0 = \omega_r \Delta\varphi_{CE}$, where $\Delta\varphi_{CE} = (1/v_g + 1/v_p) l_c \omega_c$ is the carrier-envelope phase shift, l_c the roundtrip length, and ω_c the carrier angular frequency

Femtosecond comb generator

Key parameters
- Repetition f_r
- Carrier envelope f_0

Cundiff & Ye Fig.7



S. T. Cundiff, J. Ye, Colloquium, Femtosecond optical frequency combs, Rev. Mod. Phys. 75(1), January 2003

Secondary methods for nanometrology

Neither Time-of-flight nor optical interferometry
are suitable to nanometrology

Outcome of the Avogadro experiment (the Silicon sphere)

- CODATA (2018 revision of the SI): The value of the natural Si {220} lattice spacing at 22.5 °C in vacuum is

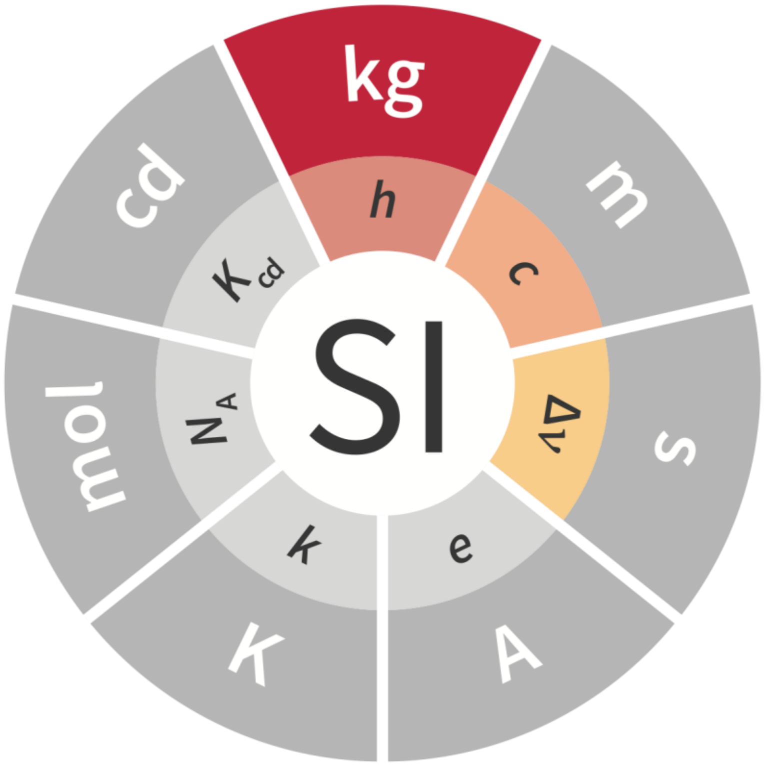
$$d_{220} = 192.0155714(32) \times 10^{-12}\ \mathrm{m}$$
$$u(d_{220})/d_{220} = 1.67 \times 10^{-8}$$

- Recommendations of CCL/WG-N on: Realization of SI metre using silicon lattice and Transmission Electron Microscopy for Dimensional Nanometrology

Bibliography

- CCL, “Mise en pratique for the definition of the metre in the SI,” Appendix 2 of the SI Brochure, 9th ed., 2019, CCBY ©BIPM.
- Editorial - Documents Concerning the New Definition of the Metre, Metrologia 19(4) p.163-178, 1984
- F. Riehle, P. Gill, F. Arias, L. Robertsson, The CIPM list of recommended frequency standard values: guidelines and procedures, *Metrologia* 55 p.188-200, February 2018
- J. M. Rüeger, Electronic Distance Measurement 4th ed., *Springer* 1996, ISBN 978-3-540-61159-2.
- S. T. Cundiff, J. Ye, Colloquium, Femtosecond optical frequency combs, Rev. Mod. Phys. 75(1), January 2003
- J. Ye, S. T. Cundiff ST, Femtosecond optical frequency comb Technology, ISBN 0-387-23790-9, Springer 2005
- Th. Udem, R. Holzwarth, T. W. Hänsch, Optical frequency metrology, Nature 416(6877) p.233-237, 14 March 2002
- D. Flack D, J. Hannaford, Fundamental Good Practice in Dimensional Metrology, Good Practice Guide No. 80, NPL, UK, October 2012

Kilogram



The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be 6.62607015×10^{-34} when expressed in the unit J s, which is equal to kg m^2 s^{-1}, where the metre and the second are defined in terms of c and $\Delta\nu_{Cs}$.

New in 2019

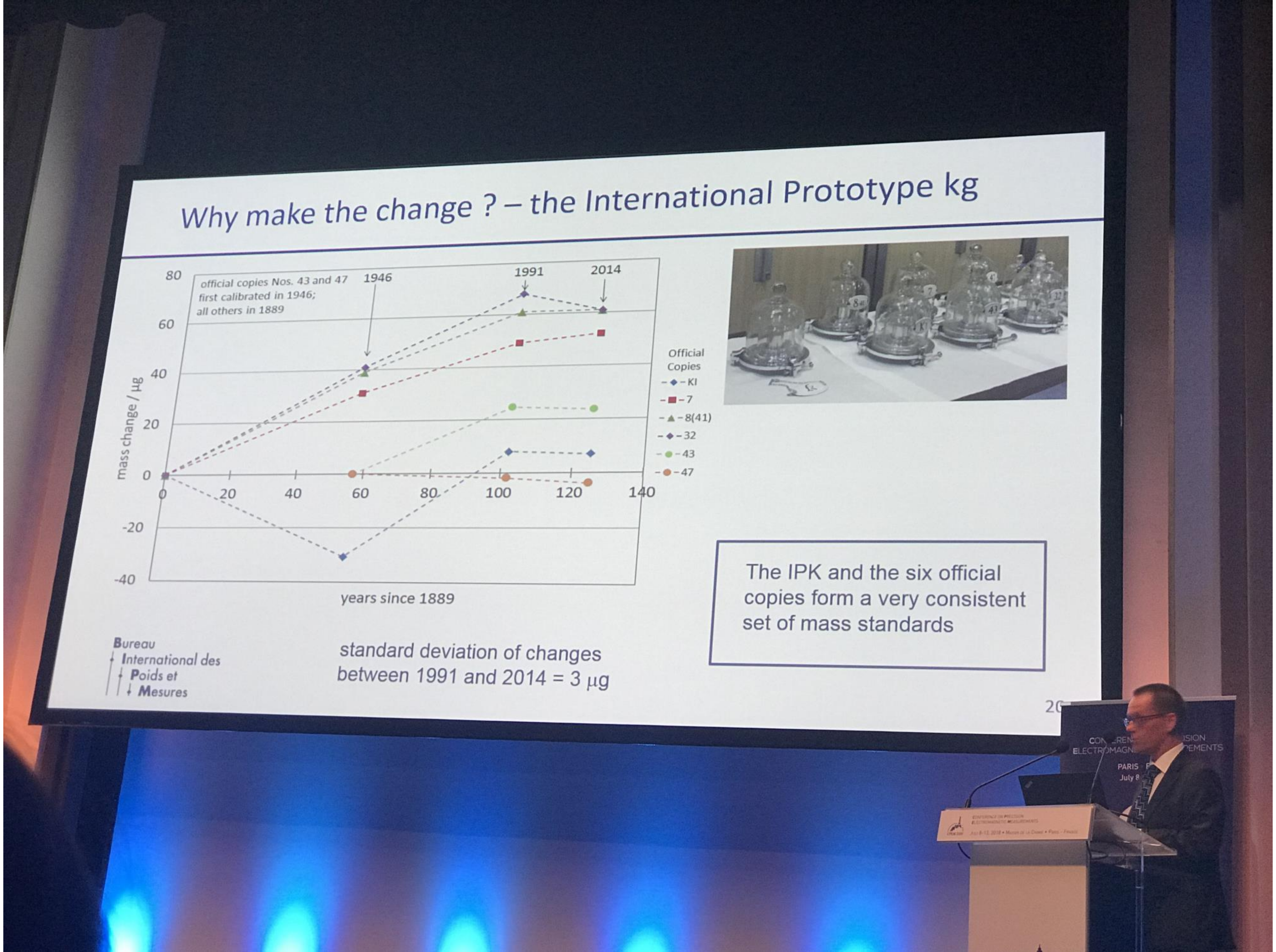
POI: "k" is a part of the name of the unit, but is removed in multiples (blind uses of rules gives "mkg" for g)

h is a good choice to define the kg because

dim h = kg m^2 s^{-1}

the units s and m are measured accurately

Picture ©BIPM.



Martin Milton, Director of BIPM

BIPM Open Session on the New SI
Nov.18, 2018. Photo Enrico Rubiola

Most is about the kilogram

The one and only artifact in the former SI

Most known
1900's equations

General relativity

$$E = mc^2$$

Quantum mechanics

$$E = h\nu$$

Photon mass

$$m = \frac{h\nu}{c^2}$$

The photon experiences and generates gravity

The mass can be defined in terms of

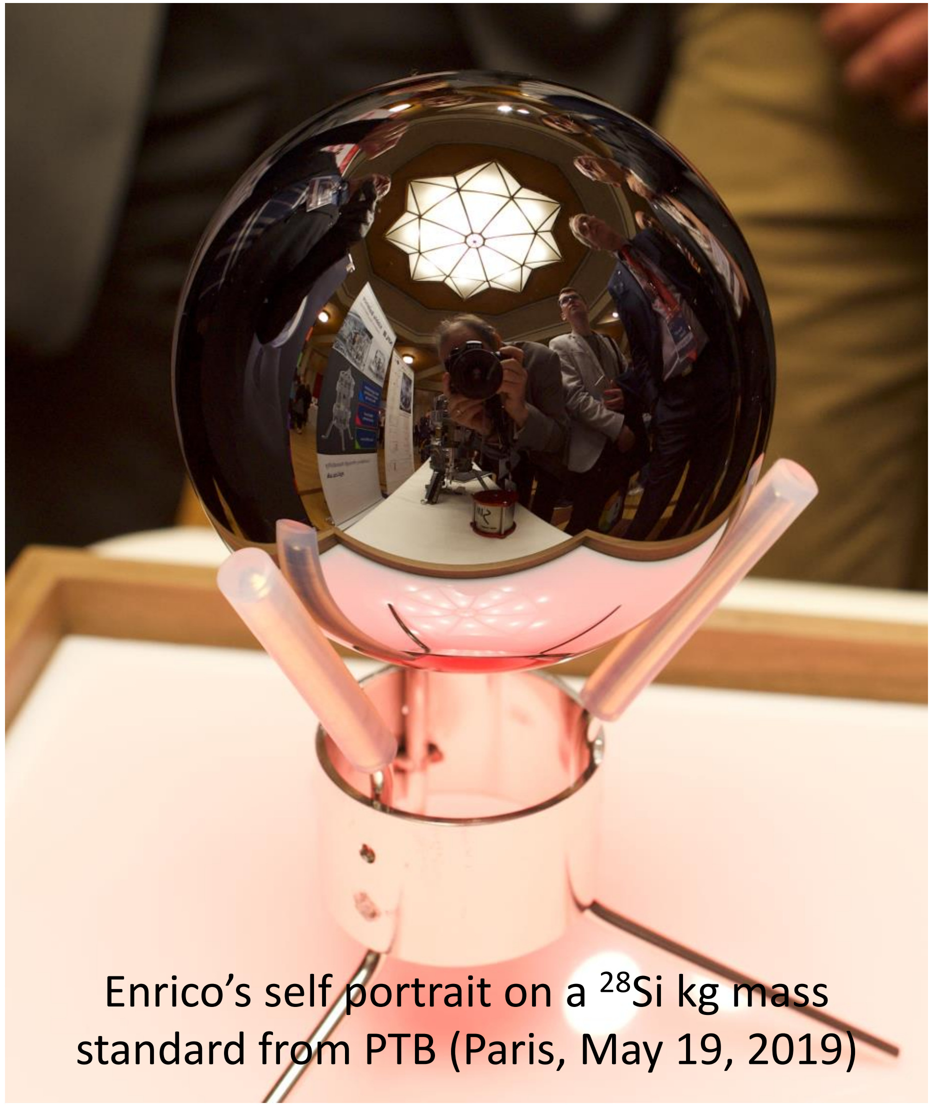
$$h, c, \Delta\nu_{\mathrm{Cs}}$$

POI: How "real" is the mass associated to energy?

- Mercury orbit, late 1800
- Gravitational lens
- Proton $\mathrm{p} = \mathrm{uud}$, neutron $\mathrm{n} = \mathrm{udd}$
  <1% of the mass is in the quarks

Two technologies in competition

The Avogadro Experiment



Enrico’s self portrait on a ^{28}Si kg mass standard from PTB (Paris, May 19, 2019)

Photo Enrico Rubiola

Do you know the lithography *Hand with reflecting sphere*, Maurits Cornelius Escher 1935?

The Kibble (Watt) Balance

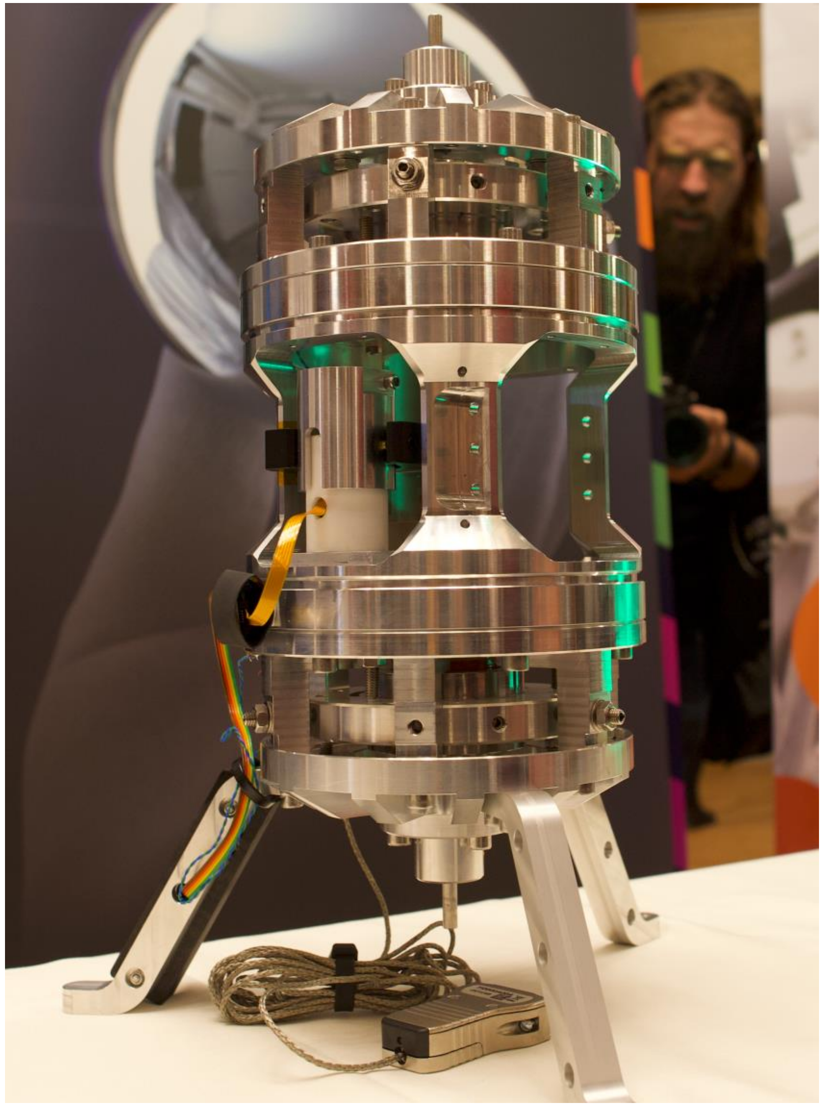
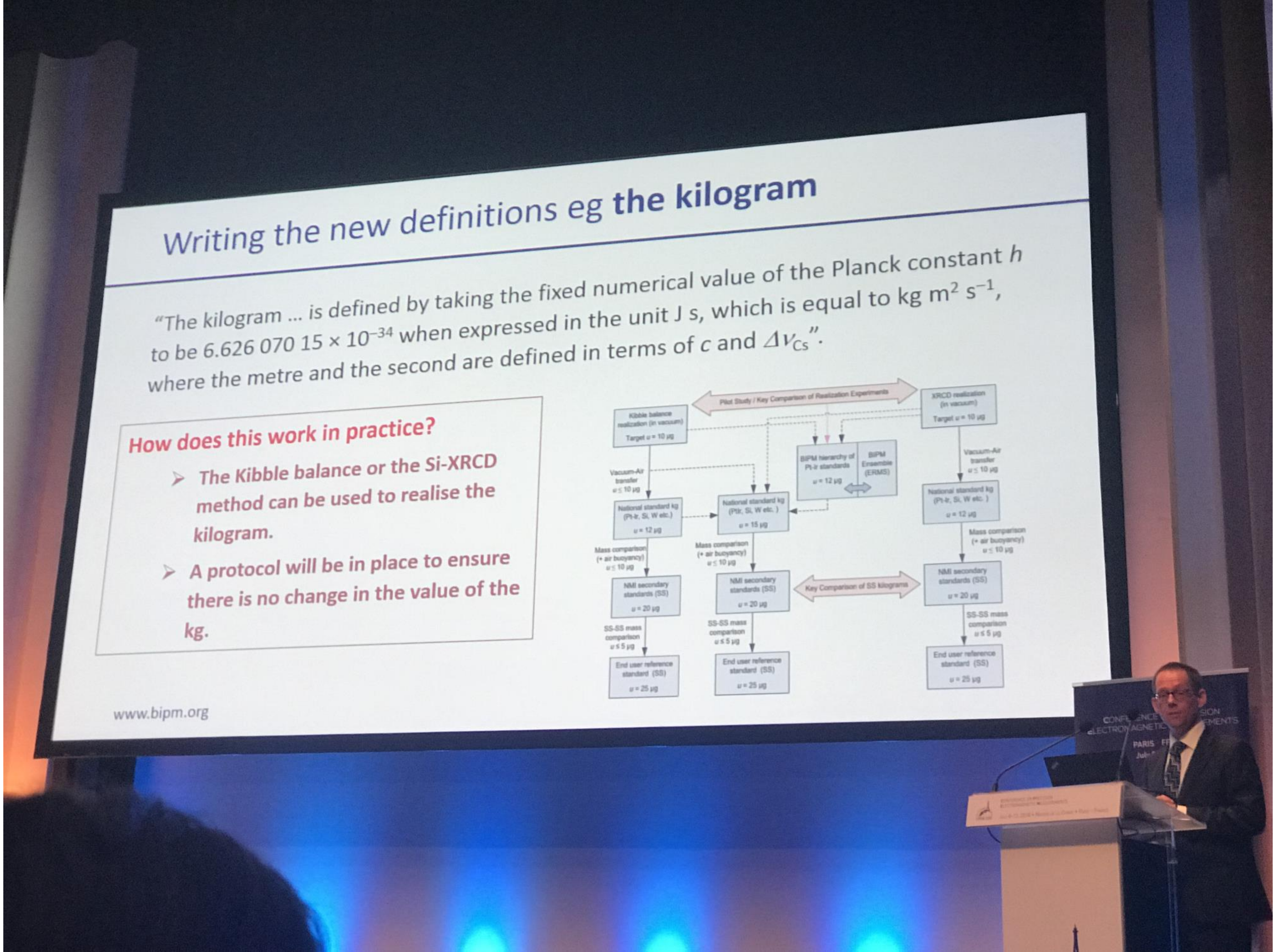


Photo Enrico Rubiola



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Measurement consistency

OPEN ACCESS
IOP Publishing | Bureau International des Poids et Mesures — Metrologia
Metrologia 53 (2016) A1–A5 — doi:10.1088/0026-1394/53/5/A1

CrossMark

Foreword

Realization, maintenance and dissemination of the kilogram in the revised SI

Guest Editors

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A substantial change to the International System of Units (SI) is currently under discussion and might become effective in 2019. The General Conference on Weights and Measures (CGPM) is scheduled to have its 26th meeting in 2018 and will likely vote on a proposal put forward by the International Committee for Weights and Measures (CIPM) to revise the SI. According to this proposal, the structure of the SI will change fundamentally. The present SI is built upon seven base units: the metre, the second, the kilogram, the ampere, the kelvin, the mole, and the candela. Each of the seven base units has its own definition, which sometimes draws on other base units. The unit of mass, the kilogram, is defined via an artefact, the International Prototype of the Kilogram (IPK) that is kept at the International Bureau of Weights and Measures (BIPM).

The foundation of the revised SI is based on seven defining constants. In addition to the three existing defining constants of the hyperfine transition frequency $\Delta\nu_{Cs}$ of caesium (Cs), the speed of light in vacuum c, and the luminous efficacy K_{cd}, four additional defining constants are introduced: the Planck constant h, the elementary charge e, the Boltzmann constant k, and the Avogadro constant N_A, see figure 1. Unlike an artefact based definition that allows the realization of the unit only at the location of the artefact, a system based on defining constants allows the realization of the units everywhere. Hence, the revision of the SI will completely implement an idea conceived in the middle of the 18th century by Charles-Marie de La Condamine [1] to establish a universal system of units 'for all time, for all people' as a phrase of later coinage succinctly put it.

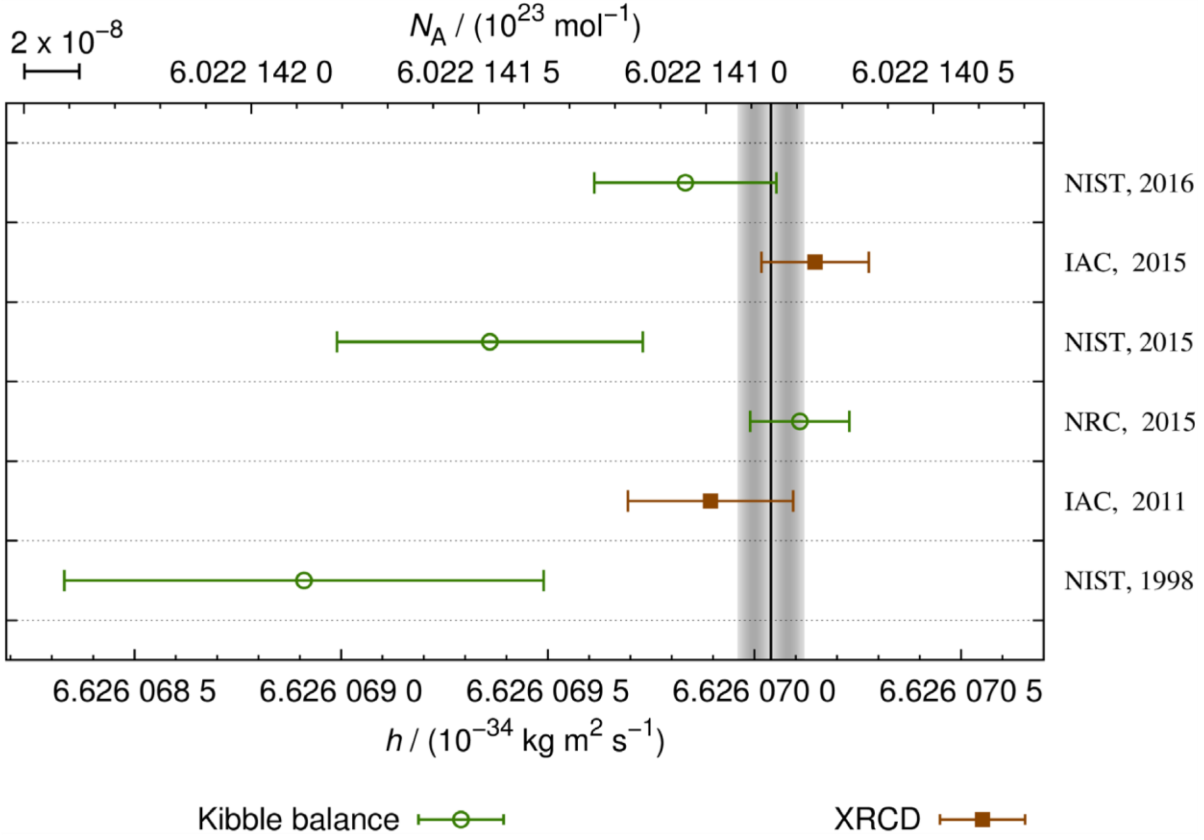
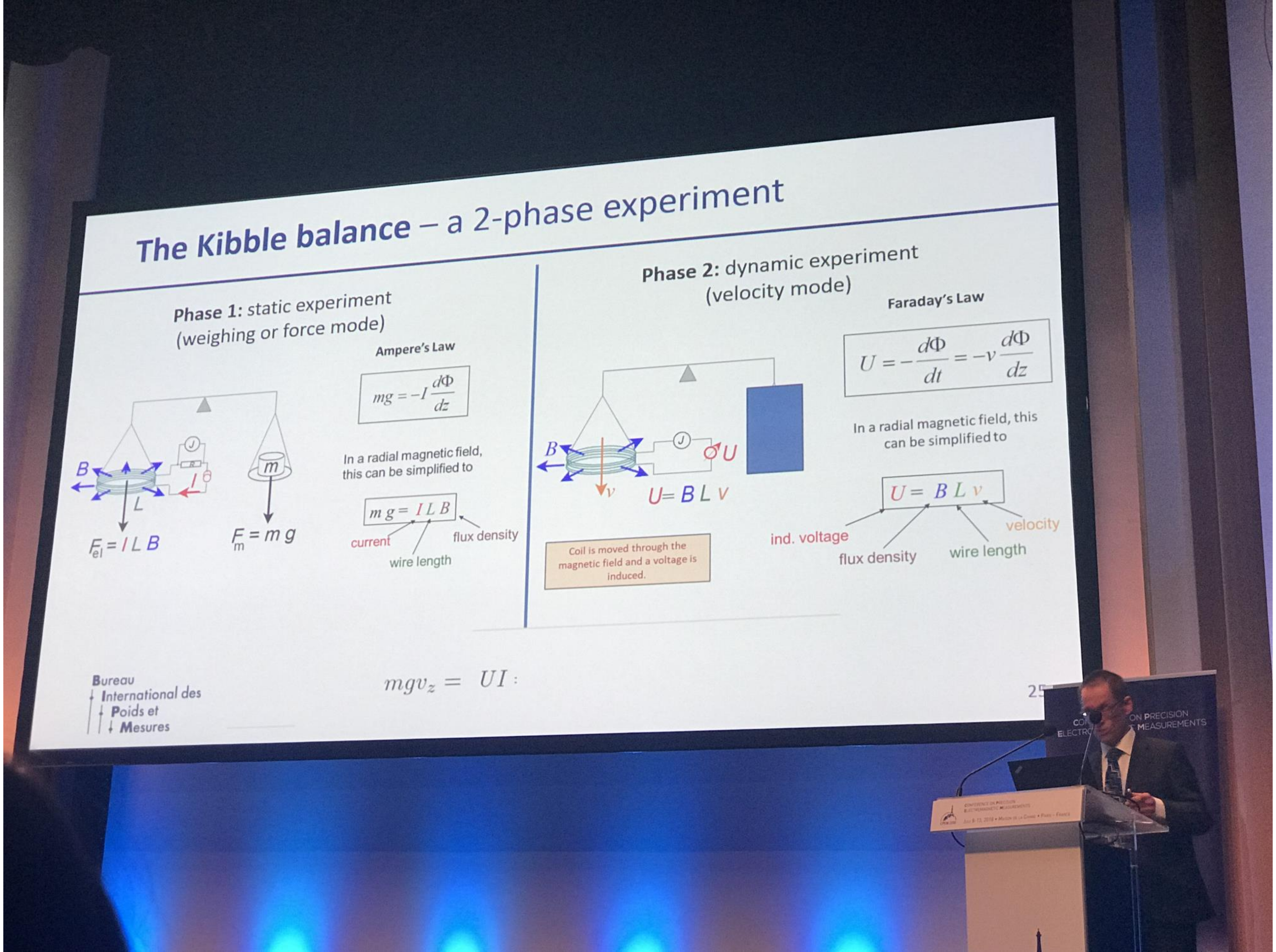


Figure 3. Determinations of the Planck constant or the Avogadro constant with the smallest uncertainties. The data shown as open circles were obtained using Kibble balances. The data shown as solid squares were obtained using the x-ray crystal density (XRCD) method by the International Avogadro Coordination (IAC). The error bars denote the standard uncertainty reported by the experiment. The vertical black line denotes the recommended value based on the 2014 adjustment of fundamental constants carried out by the Task Group on Fundamental Constants by the Committee on Data for Science and Technology (CODATA). The gray band surrounding the black line gives the standard uncertainty associated with the recommended value. The results shown in the figure can be found in [11–13, 18–20].



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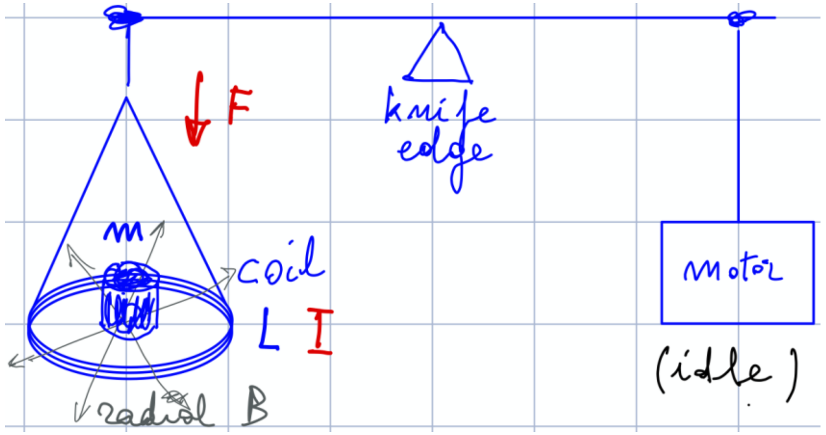
Nov.18, 2028. Photo Enrico Rubiola

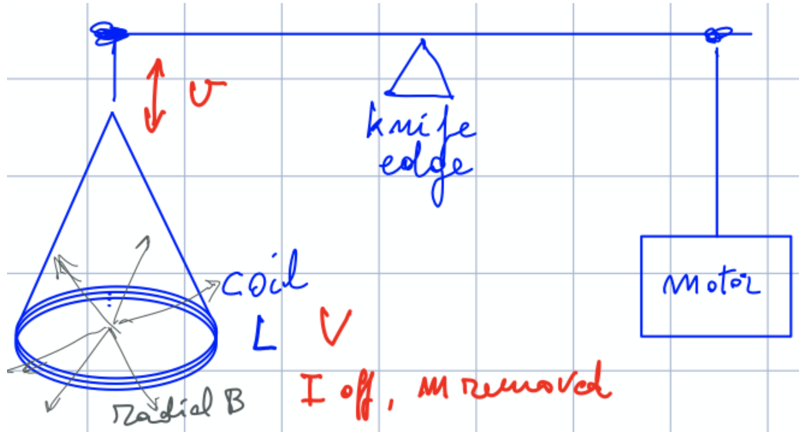
Martin Milton, Director of the BIPM

The Kibble (Watt) balance

Static operation



Dynamic operation



$$mgv = VI$$

mechanical power

electrical power

external gravimeter required

$u_r(g) \approx 10^{-9}$

Lorentz force

$\vec{F} = q\vec{v} \times \vec{B}$

Ampere's force law

$F = -I \dfrac{d\Phi}{dz}$

inductance L in a radial field B

$F = ILB$

Strong field

$B \approx 0.5 \ldots 1\ \mathrm{T}$

equilibrium

$mg = ILB$

Faraday's law

$V = -\dfrac{d\Phi}{dt}$

$\dfrac{d\Phi}{dt} \to \dfrac{d\Phi}{dz}\dfrac{dz}{dt} = v\dfrac{d\Phi}{dz}$

inductance L in a radial field B

$V = LBv$

v = speed

Featured reading: I.A. Robinson, S. Schlamminger, The watt or Kibble balance: a technique for implementing the new SI definition of the unit of mass, Metrologia 2016, doi:10.1088/0026-1394/53/5/A46

Ian A. Robinson, Stephan Schlamminger, The watt or Kibble balance: a technique for implementing the new SI definition of the unit of mass, Metrologia 2016, doi:10.1088/0026-1394/53/5/A46

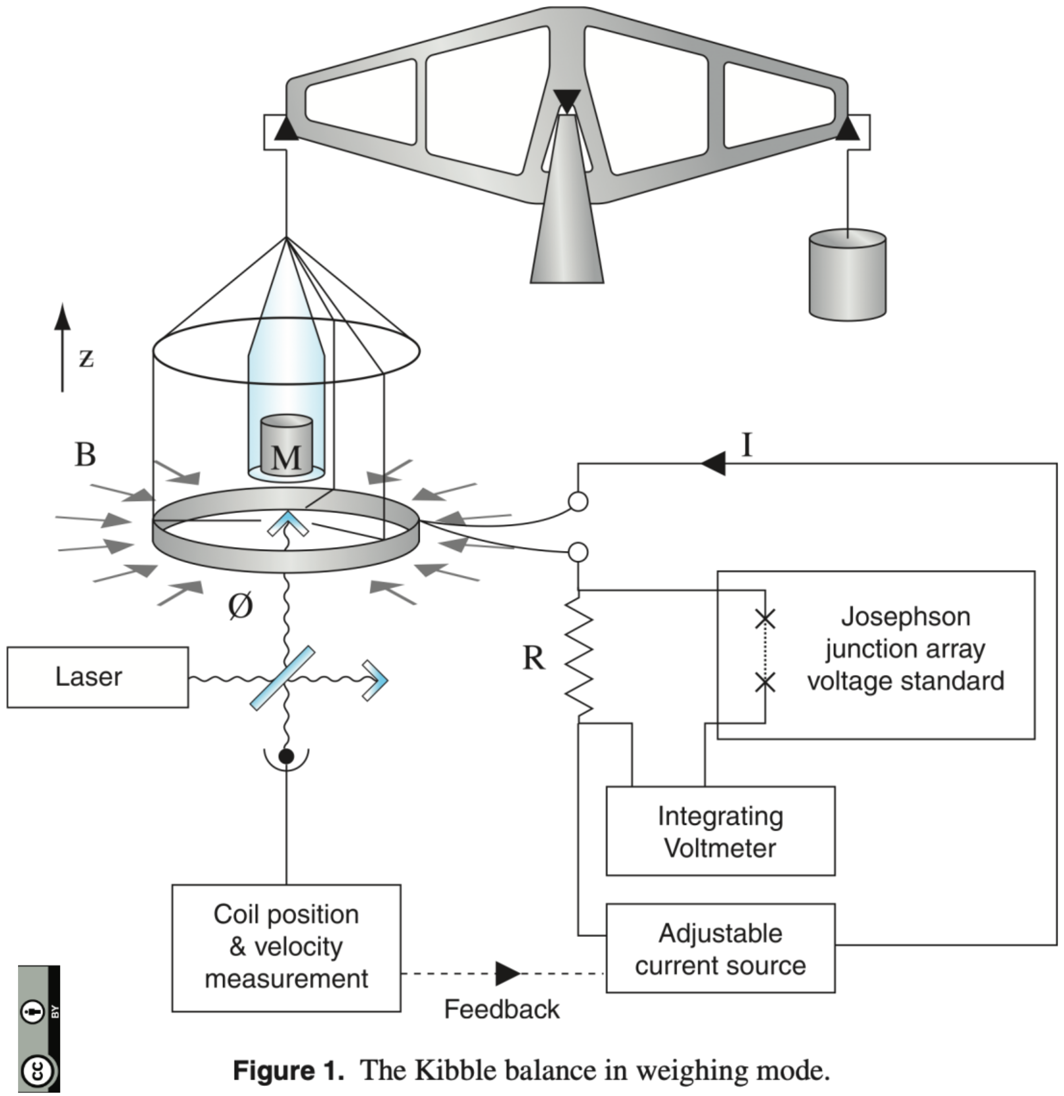


Figure 1. The Kibble balance in weighing mode.

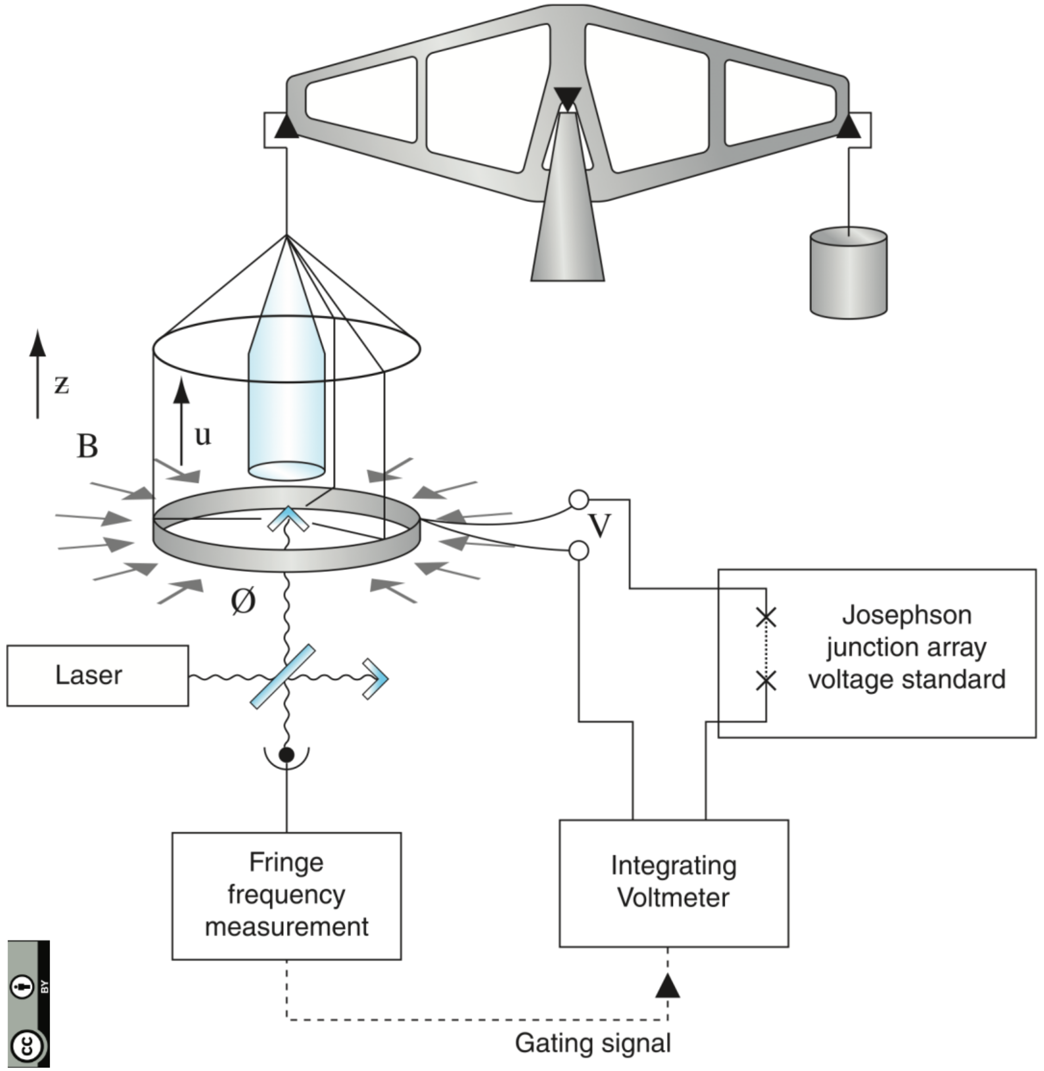
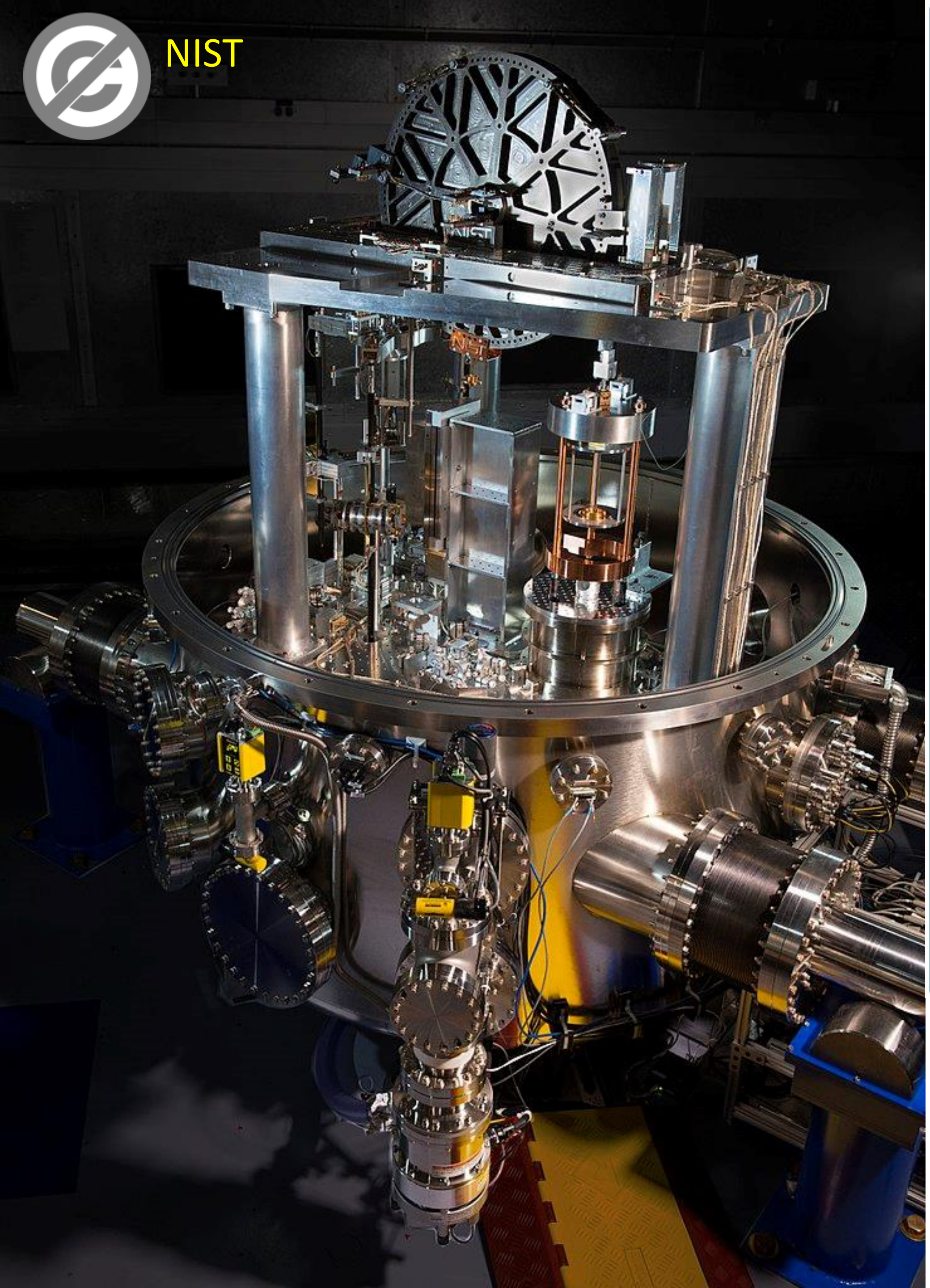
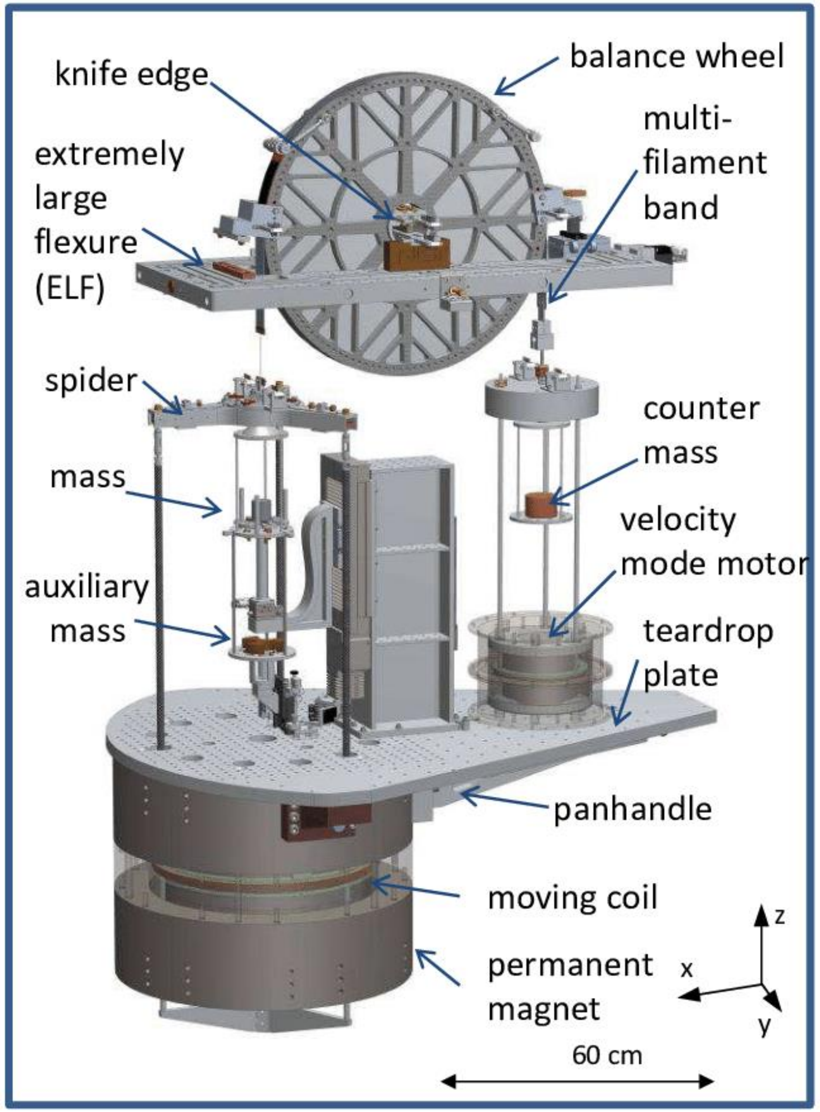
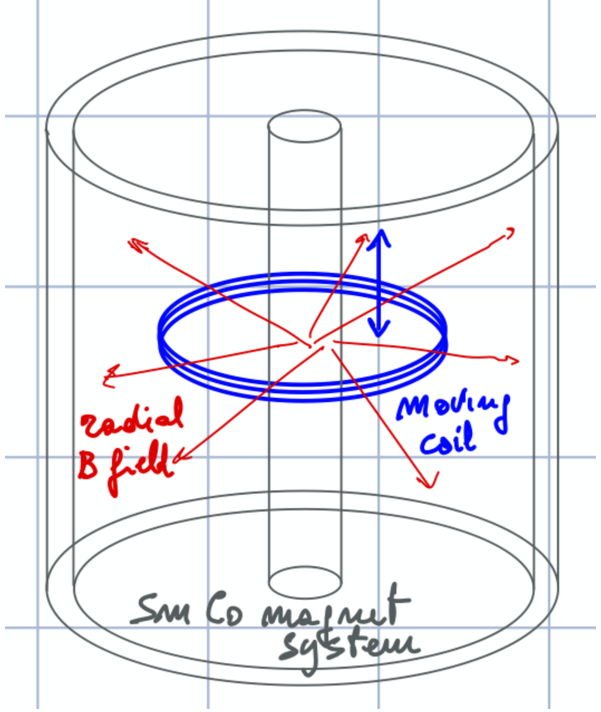


Figure 2. The Kibble balance in moving mode.







arXiv:1503.04656v1 [physics.ins-det] 16 Mar 2015

Darine El Haddad, Frank C. Seifert, Leon S. Chao, Yusi A. Cao, Georgio A. Sineriz, Jon R. Pratt, David B. Newell, Stephan Schlamminger, First measurements of the flux integral with the new NIST-4 watt balance, IEEE Transact. on IM 64(6) p.1642-1649, 31 March 2015.

Historical masterpieces

Photos/figures from B. P. Kibble, A Realization of the SI Watt by the NPL Moving-Coil Balance, Metrologia 27(4) p.173-192, 1990, © IOP

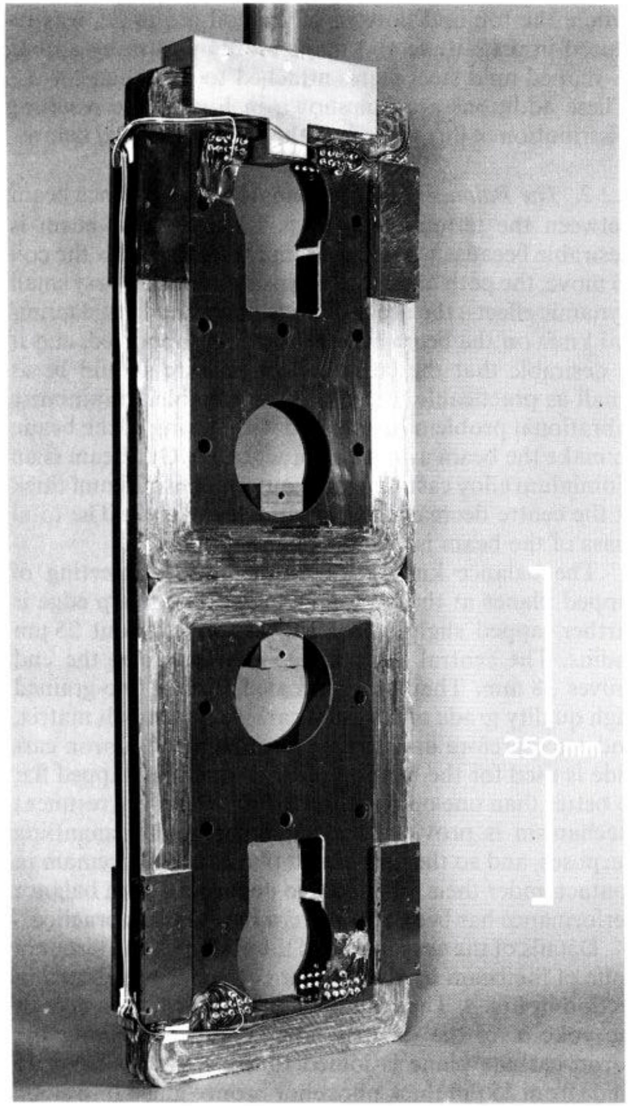
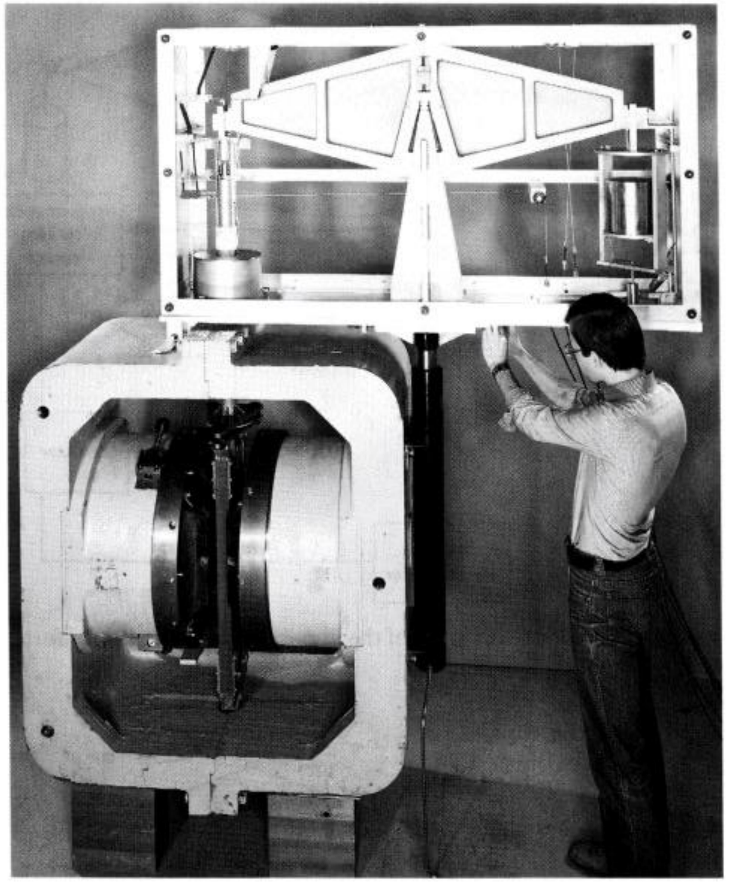


Fig.1. The coil

Fig.2. Overview



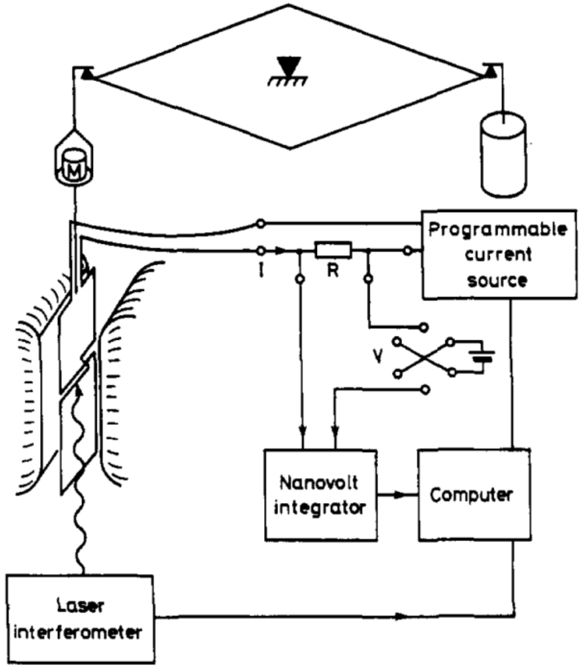


Fig.3a. Force measurement

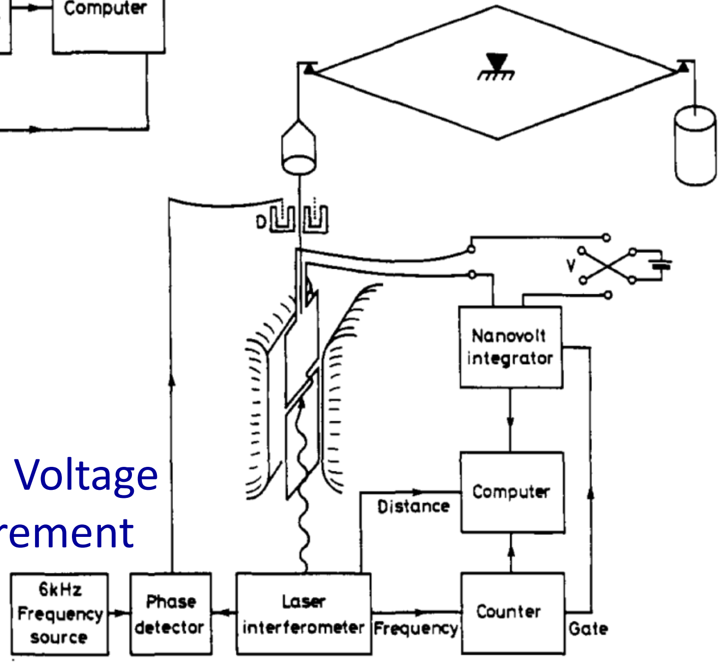
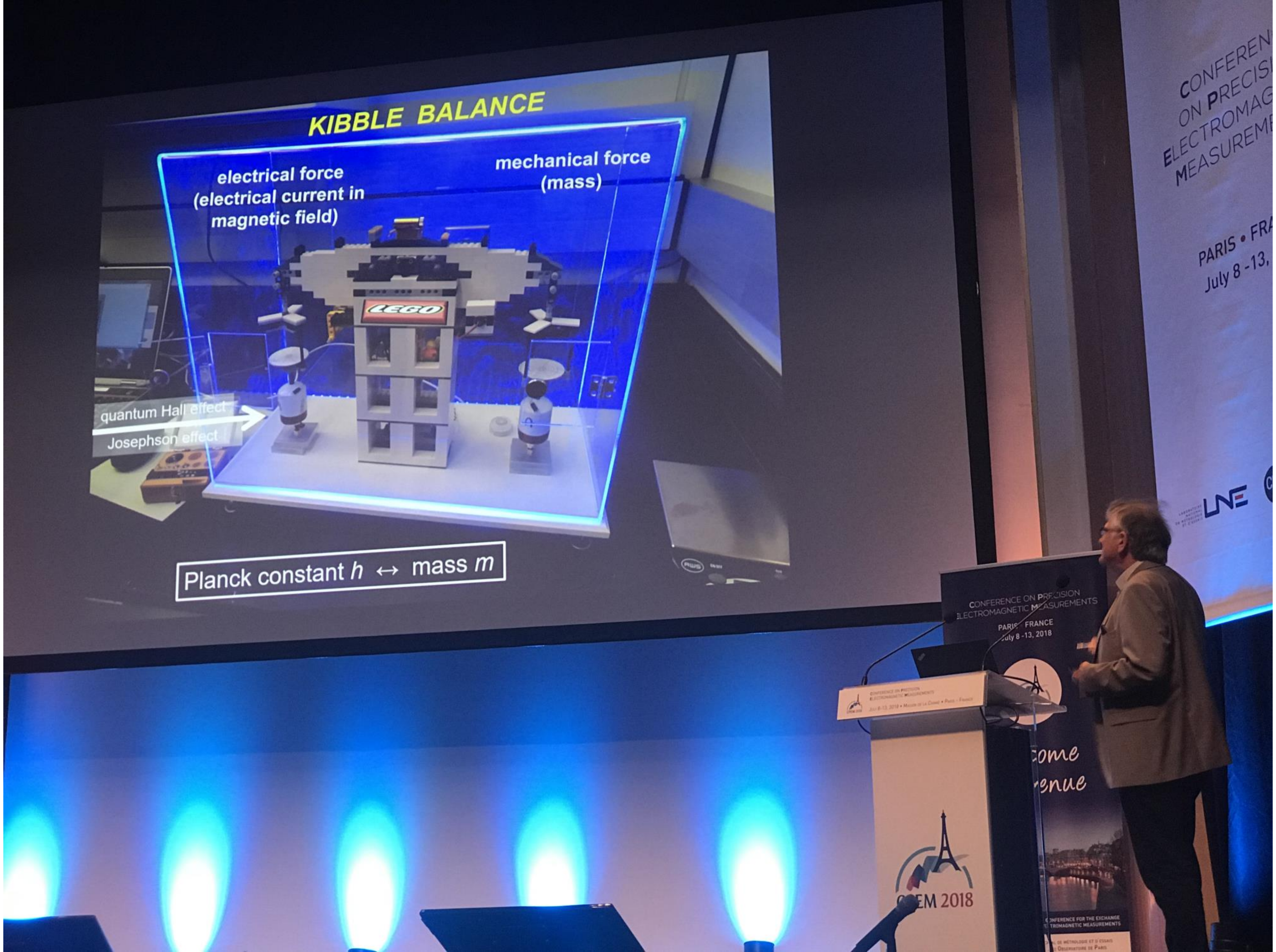


Fig.3b. Voltage measurement



Klaus von Klitzing (1943 –), Nobel Prize in Physics 1985

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Why not an electrostatic balance?

The Electrostatic Balance would be similar to the magnetic balance.

- Technically difficult or impossible for kg-sized bodies.
- Good for small masses
  - G. A. Shaw, Milligram mass metrology using an electrostatic force balance, Metrologia 53, 2016, doi:10.1088/0026-1394/53/5/A86

The Avogadro (XRCD) experiment

XCRD = X-Ray Computed Densitometry

$$h = \frac{\alpha^2 c}{2R_\infty}\frac{A_r(e)}{A_r(Si)}\frac{M_{\mathrm{sp}} a_0^3}{8V}$$

Relies on
- understanding of crystal structure
- theory of Hydrogen atom

α fine structure constant

$$\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c^2} \approx 1/137$$

c = speed of light

R_∞ = Rydberg constant

$$R_\infty = \frac{m_e \alpha^2 c}{2h}$$

10973731.568160(21) m^{-1}

M_{sp} = Mass of the sphere

a_0 = lattice parameter, the edge of the cube

V = Volume of the core cell, it contains 8 atoms

$$A_r() = \frac{m()}{\frac{1}{12}m(^{12}\mathrm{C})}$$

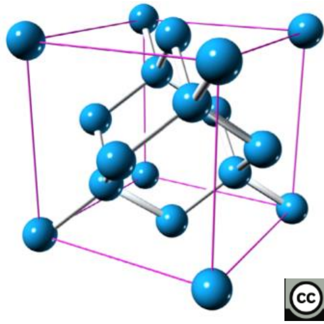

Fig.1 from K. Fuji et al, Metrologia 2016 doi:10.1088/0026-1394/53/5/A19

Takes benefit of the low uncertainty of R_∞ and α

$u_r\,(R_\infty) = 5.9 \times 10^{-12}$, and $u_r(\alpha) = 2.3 \times 10^{-10}$

Technology!

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IOP Publishing | Bureau International des Poids et Mesures
Made open access 6 January 2017
Metrologia
Metrologia 53 (2016) A19–A45
doi:10.1088/0026-1394/53/5/A19

Realization of the kilogram by the XRCD method

Kenichi Fujii[1], Horst Bettin[2], Peter Becker[2], Enrico Massa[3], Olaf Rienitz[2], Axel Pramann[2], Arnold Nicolaus[2], Naoki Kuramoto[1], Ingo Busch[2] and Michael Borys[2]

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[2] Physikalisch-Technische Bundesanstalt (PTB), Bundesallee 100, 38116 Braunschweig, Germany
[3] Istituto Nazionale di Ricerca Metrologica (INRIM), Str. delle Cacce 91, 10135 Torino, Italy

Natural F is $\approx$100% ^{19}F

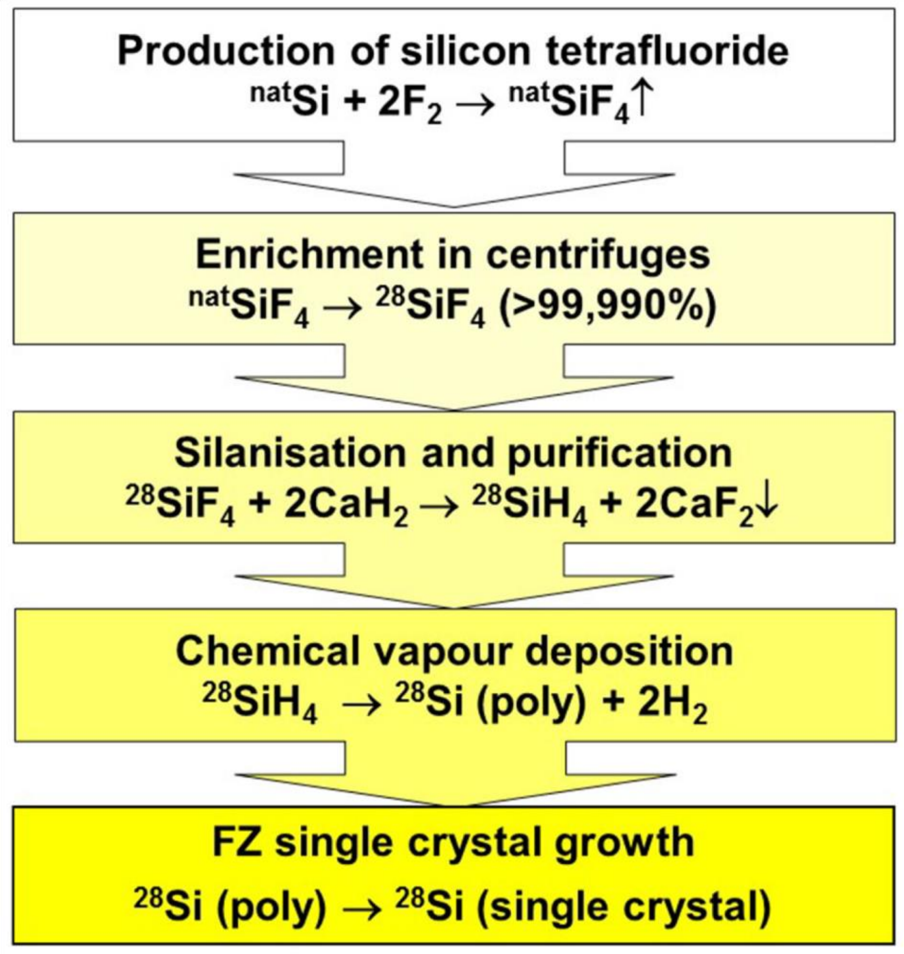

Figure 3. Flow chart showing the production of a ^{28}Si-enriched silicon single crystal (^{nat}Si: silicon with natural isotopic composition, poly: polycrystal).

Si isotopes	
Mass no Z	Abundance
28	92.2 %
29	4.7 %
30	3.1 %

XCRD = X-Ray Computed Densitometry

The technology of ^{28}Si kilogram

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Metrologia 54 (2017) 693–715 https://doi.org/10.1088/1681-7575/aa7820

A new ^{28}Si single crystal: counting the atoms for the new kilogram definition

G Bartl[1], P Becker[1], B Beckhoff[2], H Bettin[1], E Beyer[1], M Borys[1], I Busch[1], L Cibik[2], G D'Agostino[3], E Darlatt[2], M Di Luzio[3,4], K Fujii[5], H Fujimoto[5], K Fujita[5], M Kolbe[2], M Krumrey[2], N Kuramoto[5], E Massa[3], M Mecke[1], S Mizushima[5], M Müller[2], T Narukawa[5], A Nicolaus[1], A Pramann[1], D Rauch[1], O Rienitz[1], C P Sasso[3], A Stopic[6], R Stosch[1], A Waseda[5], S Wundrack[1], L Zhang[5] and X W Zhang[7]

[1] Physikalisch-Technische Bundesanstalt PTB, Bundesallee 100, 38116 Braunschweig, Germany
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[6] Australian Nuclear Science and Technology Organisation ANSTO, New Illawarra Road, Lucas Heights, New South Wales 2234, Australia
[7] Institute of High Energy Physics IHEP, Chinese Academy of Science, 19B Yuquan Road, Shijingshan District, Beijing, People's Republic of China

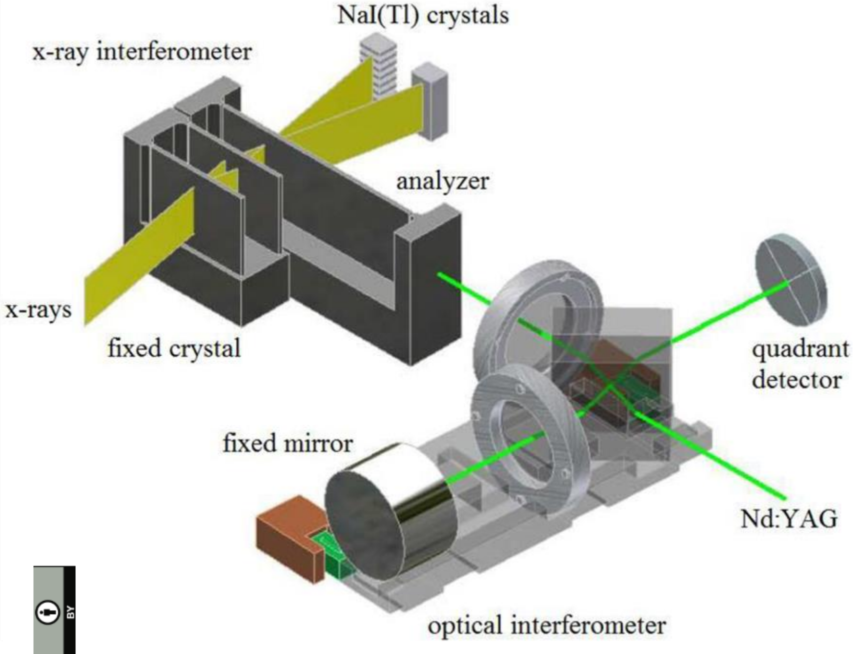

Figure 2. Sketch of a combined x-ray and optical interferometer.

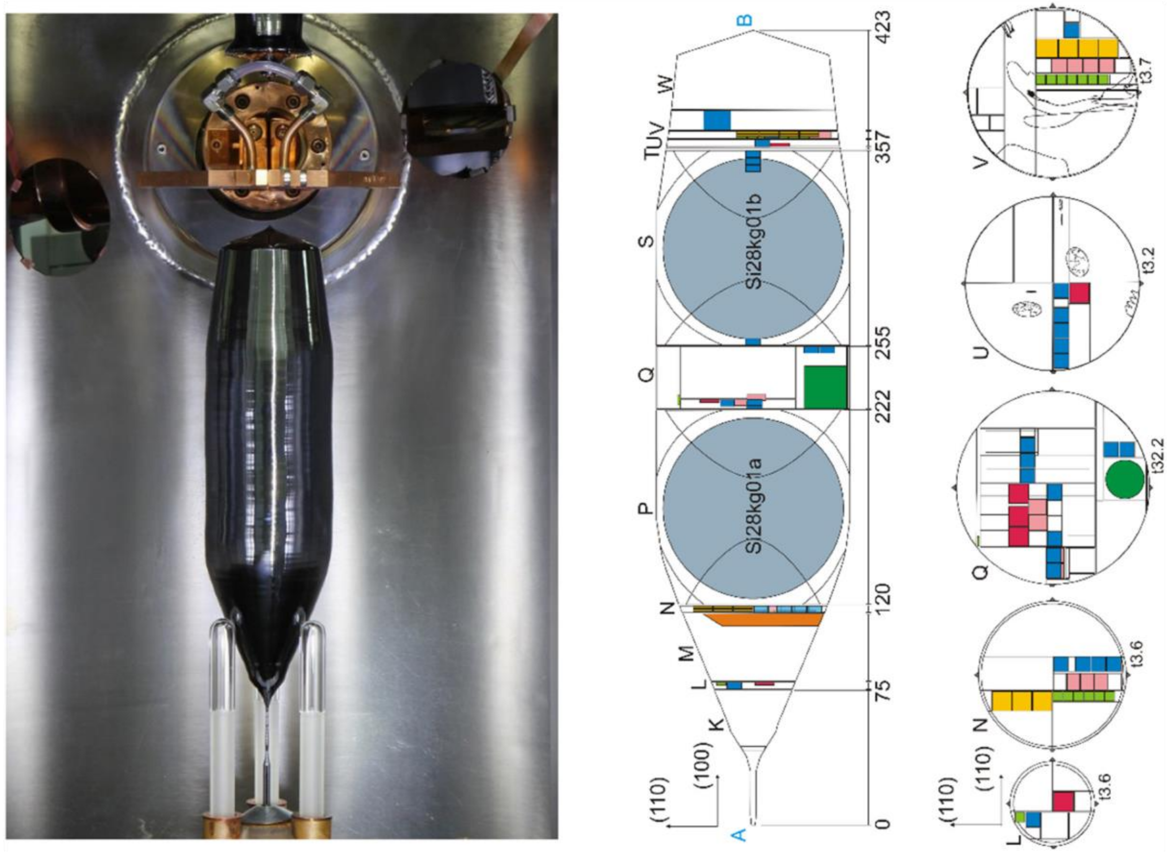

Figure 1. The new float-zone ^{28}Si crystal Si28-23Pr11 and its cutting scheme. The isotopic enrichment is about 0.999 985. To determine density, two spheres (Si28kg01a and Si28kg01b) were manufactured from parts P and S. From the parts L, N, Q, U, and V small samples were prepared for the determination of other crystal properties. The parts U and V contain some cracks (thin irregular lines). No cracks were detected in the parts K to T.

Density measurement

OPEN ACCESS
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Metrologia
Metrologia 57 (2020) 025006 (11pp)
https://doi.org/10.1088/1681-7575/ab2d30

Absolute measurement of the density of silicon spheres to improve the primary density standard of NMIJ

Naoki Kuramoto, Lulu Zhang, Shigeki Mizushima, Atsushi Waseda, Sho Okubo, Hajime Inaba, Akira Kurokawa and Kenichi Fujii

National Metrology Institute of Japan, National Institute of Advanced Industrial Science and Technology, 1-1-1 Umezono, Tsukuba, Ibaraki 305-8563, Japan

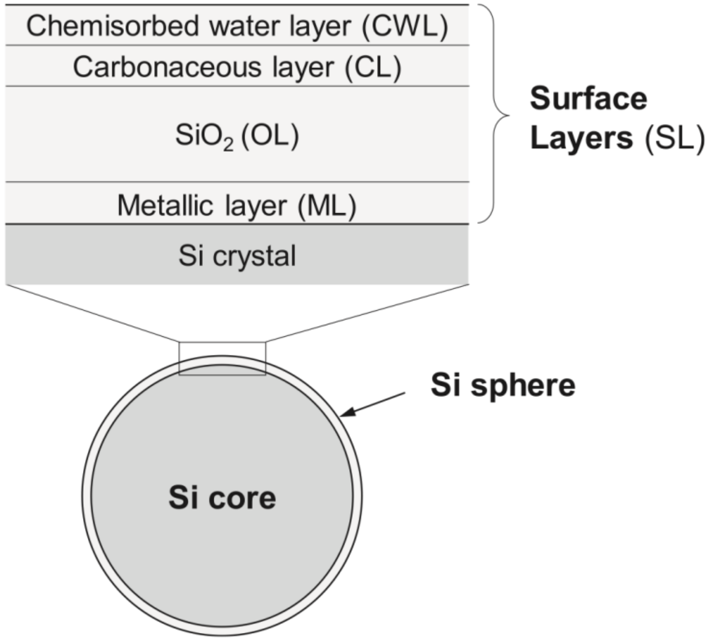


Figure 2. Model of the SL on silicon spheres in vacuum.

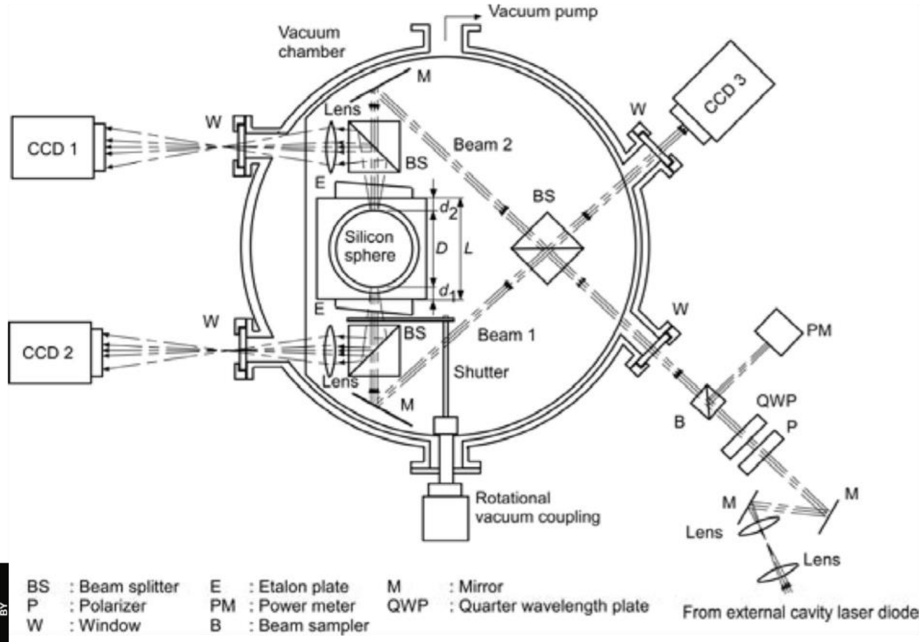


Figure 3. Optical interferometer used to measure diameters of silicon spheres.

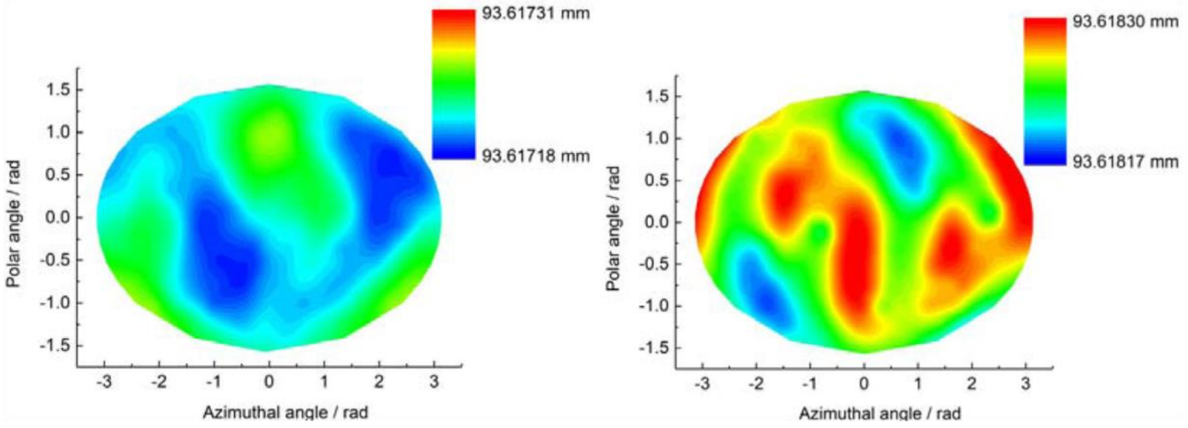


Figure 4. Mollweide map projection of the distribution of the diameter based on 145 directions for NMIJ-S4 (left) and NMIJ-S5 (right). The difference between the minimum and maximum diameters is 79 nm for NMIJ-S4 and 124 nm for NMIJ-S5.

Comparing the kg prototypes

is a tough exercise

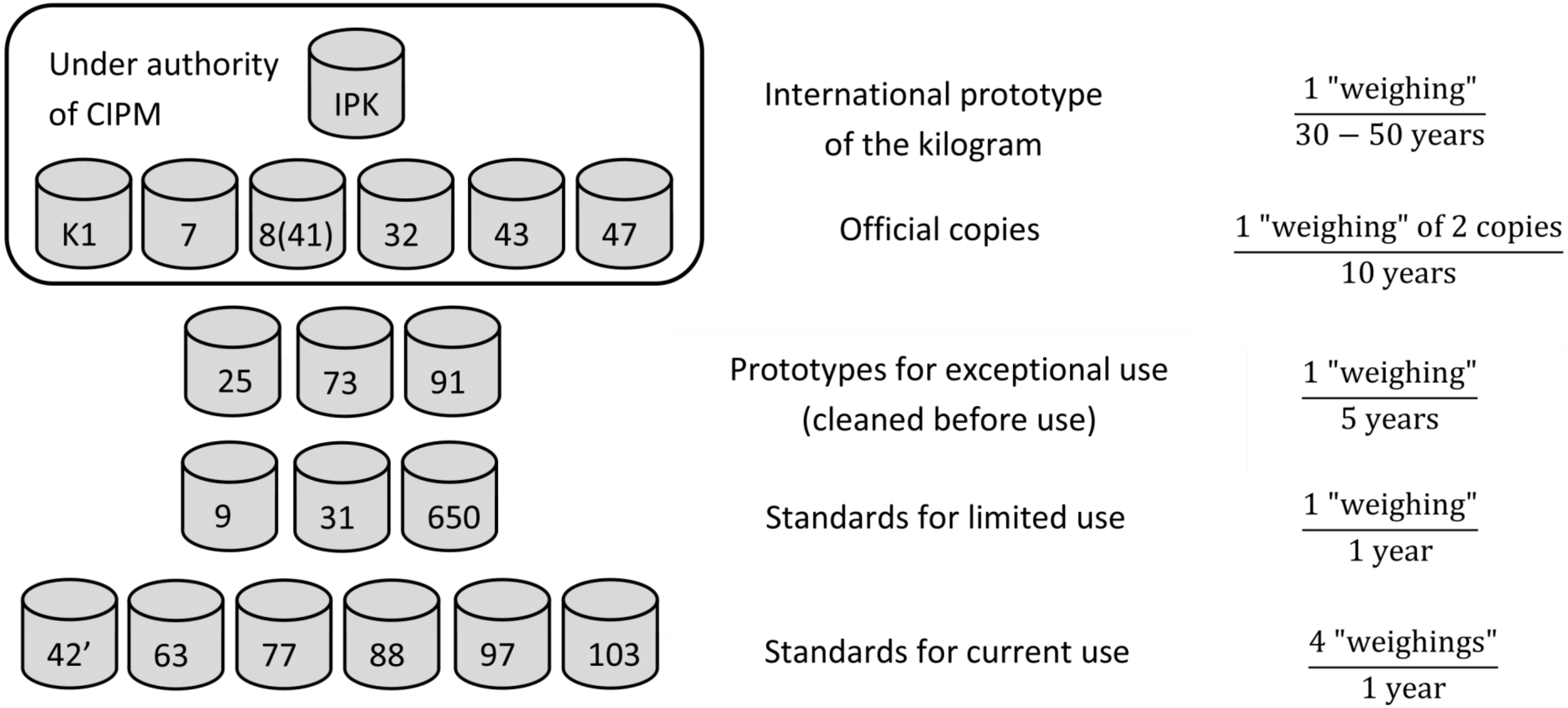


Figure 1. Hierarchy of BIPM Pt–Ir working standards, established after the Extraordinary Calibration campaign in 2014. The standards for current use serve to calibrate national prototypes during two calibration campaigns per year. Their stability is verified once per year by comparison with the standards for limited use. The latter are compared every 5 years with the prototypes for exceptional use. The IPK and the official copies will lose their special status after the redefinition.

M Stock et al., Maintaining and disseminating the kilogram following its redefinition, Metrologia 54 (2017) S99–SS107

Traceability chain

before may 2029

after may 2029, proposed

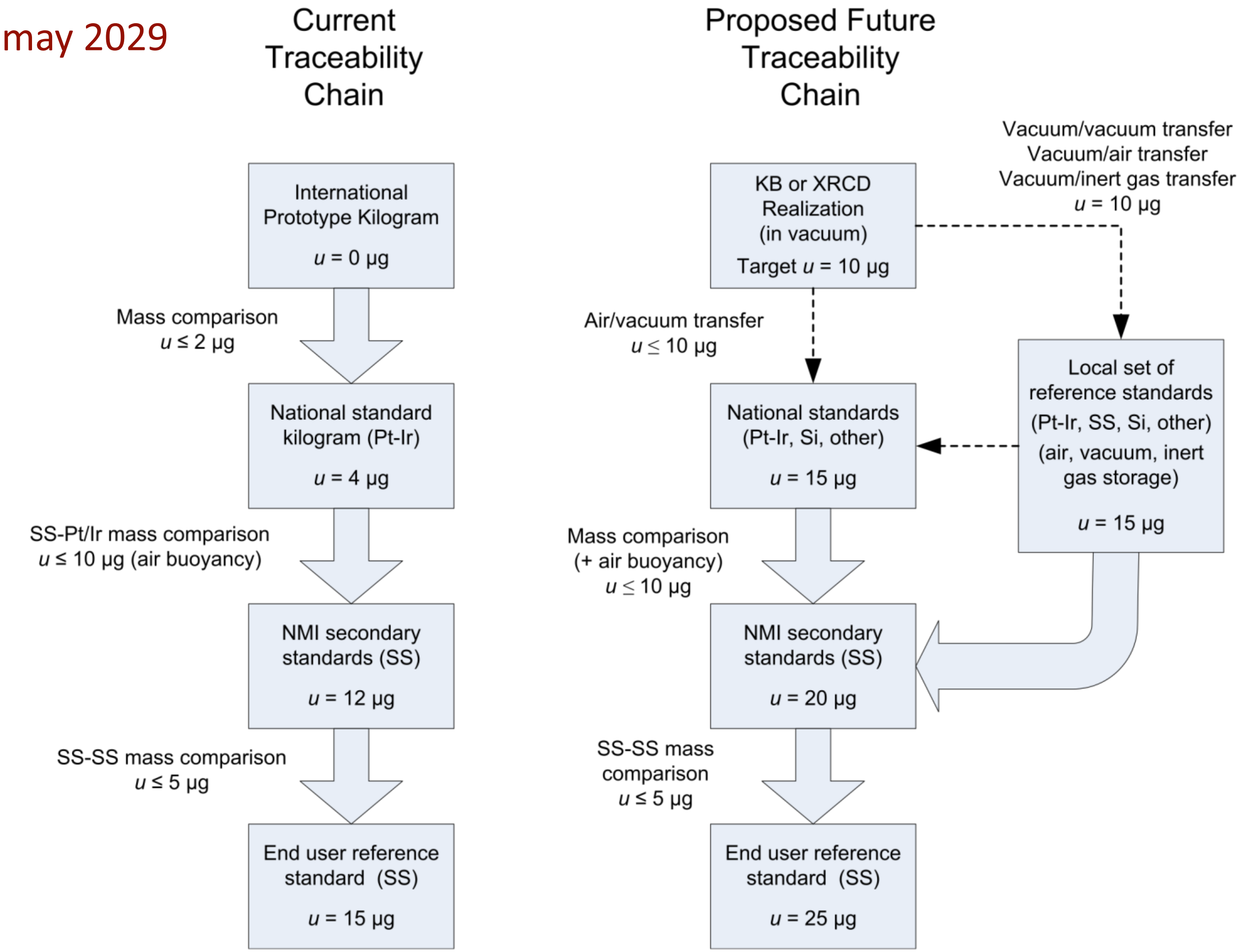


Figure 2. Current traceability route to the IPK and indicative future traceability route to primary methods after the redefinition. All uncertainties are given at the $k = 1$ level. They have not been rigorously evaluated due to the variation in or lack of data but are meant to give an idea of the potential increase in uncertainty after redefinition. (Pt–Ir: platinum–iridium, SS: stainless steel, Si: silicon.)

M. Stock et al., Maintaining and disseminating the kilogram following its redefinition, Metrologia 54 (2017) S99–SS107

The bottom line

Kibble balance and ^{28}Si sphere realize the kg, and have similar uncertainty, $\approx 10^{-10}$

Kibble balance

- Relies on the accuracy of
  - Mechanical design
  - Magnetic field
  - Temperature control, etc.
- Training takes months
- Fragile
- Not transportable
  - International comparisons rely on artifacts

^{28}Si sphere

- Relies on
  - The accuracy of atomic constants
  - A blend of technologies
  - Complex international collaboration
- Handling training takes just hours
- Rock solid
  - Accurate after “dirty hands” and cleaning
  - Just don’t drop it on the floor!!!
- Transportable (flies in a cabin bag)

Radically different technologies and know-how —> Safe consistency

Transferring 10^{-10} accuracy from electrical units to mechanical units requires a complex mechanical experiment

Alternate ideas

Define the kg based on N_A and the mass of a chosen atom (likely, ^{28}Si)

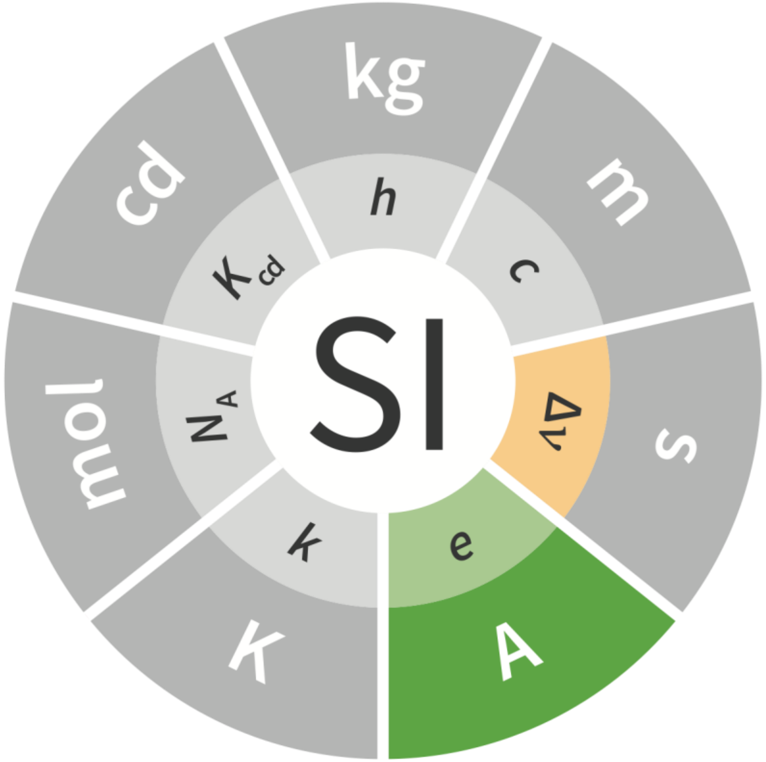
This idea has been debated for long before opting for the Planck constant

- Way simpler to explain to the layperson (and to engineers)
- Same accuracy, and same easy handling of the SI sphere, as used now
- The benefit to the other branches of metrology (electrical metrology) is lost

Bibliography

- B. P. Kibble, A Realization of the SI Watt by the NPL Moving-Coil Balance, Metrologia 27(4) p.173-192, 1990
- Terry J. Quinn, From Artefacts to Atoms, Oxford 2012
- H. Bettin, S. Schlamminger, Realization, maintenance and dissemination of the kilogram in the revised SI, Metrologia 53, A1–A5, 2016. DOI: 10.1088/0026-1394/53/5/A1
- Darine El Haddad, Frank C. Seifert, Leon S. Chao, Yusi A. Cao, Georgio A. Sineriz, Jon R. Pratt, David B. Newell, Stephan Schlamminger, First measurements of the flux integral with the new NIST-4 watt balance, IEEE Transact. on IM 64(6) p.1642-1649, 31 March 2015.
- I. A. Robinson, S. Schlamminger, The watt or Kibble balance: a technique for implementing the new SI definition of the unit of mass, Metrologia 2016. DOI: 10.1088/0026-1394/53/5/A46
- P. Becker P, History and progress in the accurate determination of the Avogadro constant, Rep. Progr. Phys. 64(12) p.1945-2008, 15 Nov. 2001
- K. Fujii et al., …
- Bartl et al., …
- Kuramoto et al., …
- M. Stock et al., Maintaining and disseminating the kilogram following its redefinition, Metrologia 54 (2017) S99–SS107

Ampere



The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge *e* to be $1.602176634 \times 10^{-19}$ when expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu_{Cs}$.

New in 2019

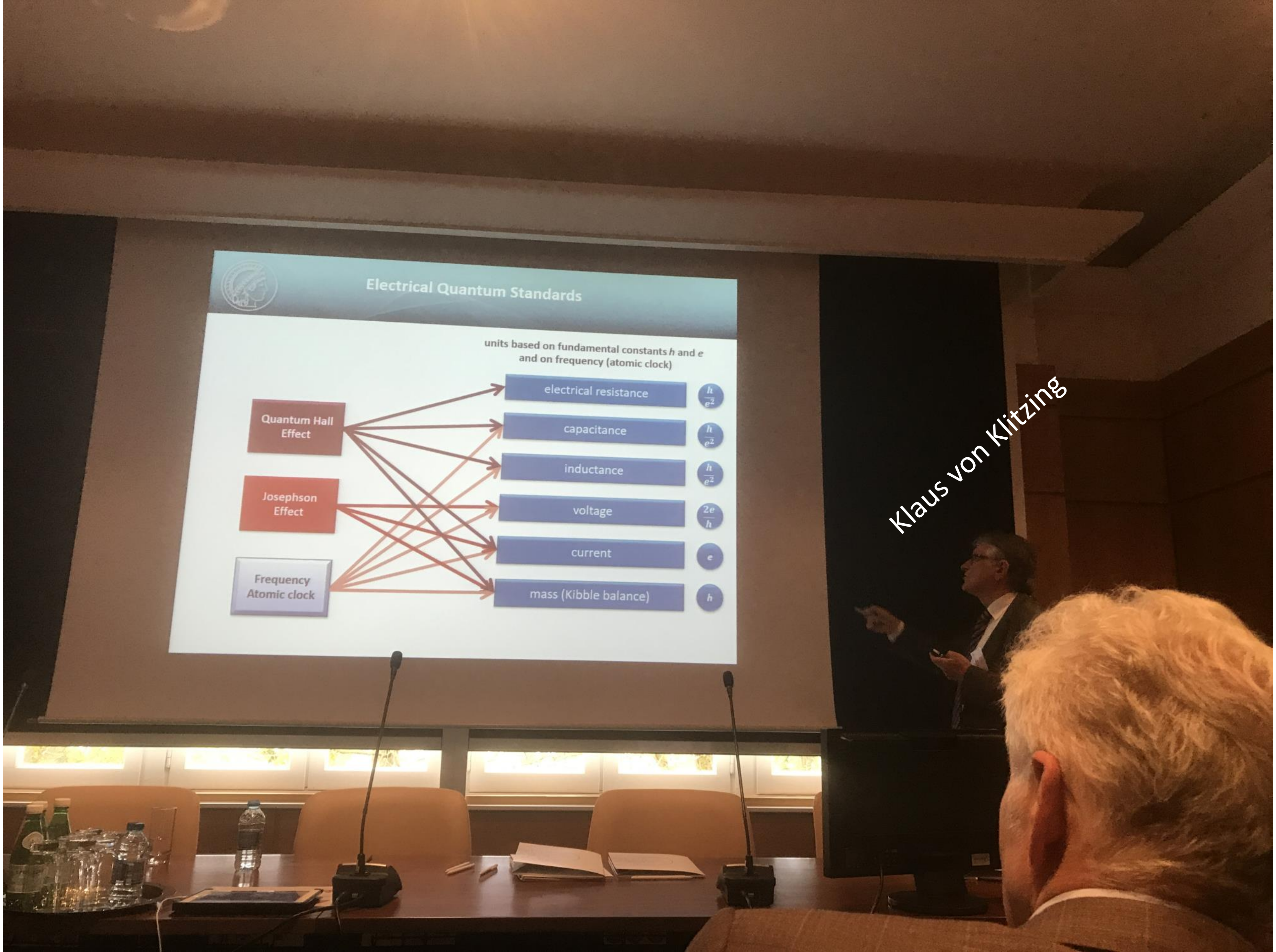


Photo Enrico Rubiola

The quantum metrology triangle

photon-assisted tunneling

V
Josephson effect

$$V = n\frac{h\nu}{2e}$$

$n =$ Shapiro no

A
single-electron transport

$$I = me\nu$$

$m =$ no of charges/event

Ω
Quantum Hall effect

$$R_K = \frac{h}{e^2}$$

Also $R_K = \frac{\mu_0 c}{2\alpha}$

They are not free because

$$P = VI$$
$$R = V/I$$

One definition sets all the electrical unit

Alessandro Volta, 1745-1827

André-Marie Ampère, 1775-1836

George Simon Ohm, 1789-1854

Superconductivity

Unfinished list

Heike Kamerlingh Onnes (1853-1926)

- Studied at U Heidelberg under Robert Bunsen and Gustav Kirchoff
- Liquid Helium, 1908
- Superconductivity, 1911
- Nobel Prize in Physics in 2013 for
  "his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium"

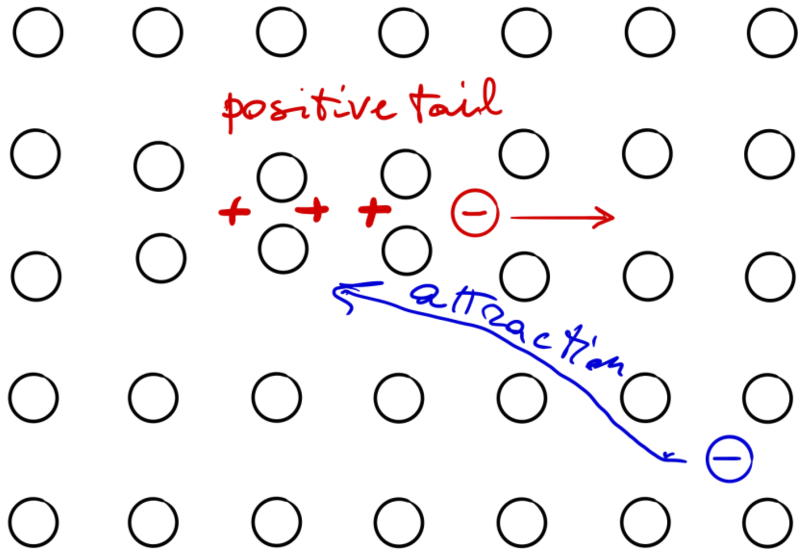
Ginzburg and Landau

- GL theory, 1950
- Lev Davidovich Landau (1908-1968), Nobel Prize in Physics in 1962 for a mathematical theory of superfluidity that accounts…
- Vitaly Ginzburg (1916-2009), Nobel Prize in Physics in 2003, together with Alexei Alexeyevich Abrikosov and Anthony James Leggett for their "pioneering contributions to the theory of superconductors and superfluids"

John Bardeen, Leon Cooper, and John Robert Schrieffer

- BCS theory, 1957
- Nobel Prize in Physics in 1972 (Bardeen also Nobel Prize in Physics in 1956)

Superconductivity



Required temperature ≈1–10 K

1957, Bardeen-Cooper-Schrieffer (BCS) theory
1972 Nobel prize

Good slideshow on
http://hep.ph.liv.ac.uk/~hock/Teaching/2010-2011/7-cooper-pair.pdf

Conductor

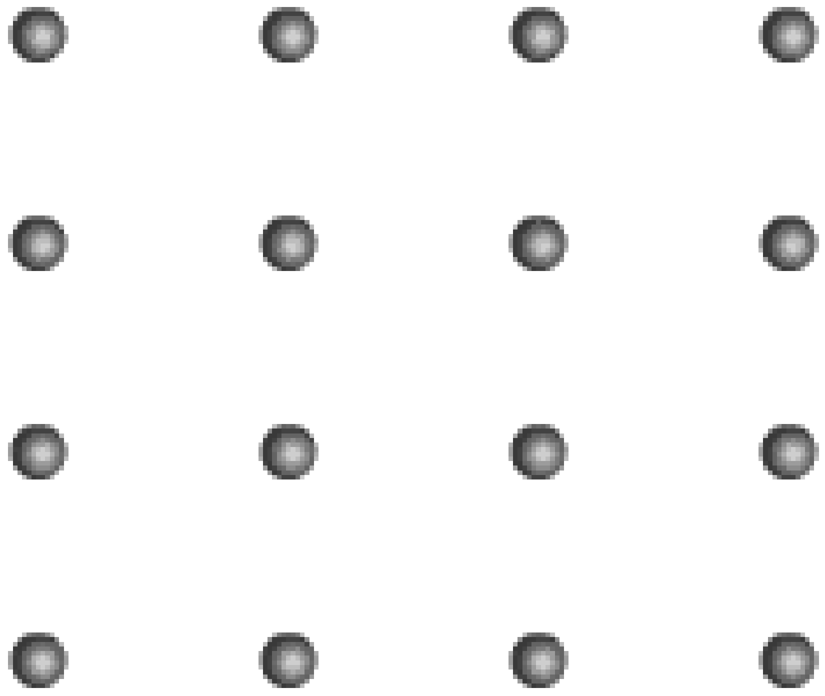
- Electrons-lattice collisions/scattering
- Electrostatic repulsion between electrons, e^-

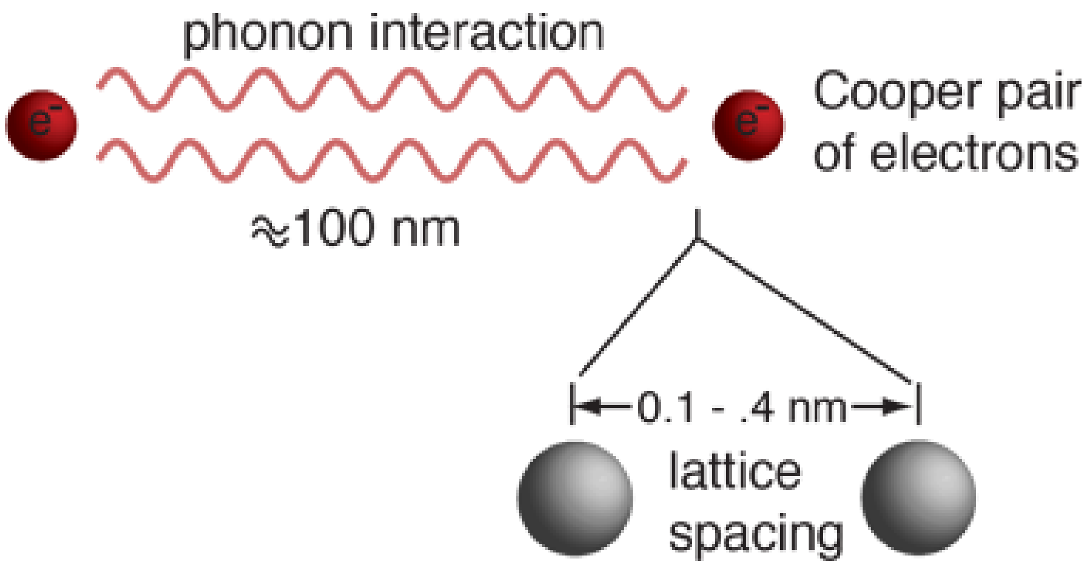
Superconductor

- Positive tail attracts electrons (described as phonon interaction)
- Electron spin s = 1/2 (fermion) —> Pauli exclusion
- At low T, weak spin coupling BCS pairs, spin s = 0 (boson!) —> the crystal is transparent
  Distance order of ≈ 100 nm (lattice 3 A)
- Bose condensation of Cooper pairs in a coherent quantum state

Featured readings: Tinkham M - Introduction to superconductivity 2ed - McGraw Hill 1996, ISBN 0-07-064878-6
Weisskopf VF - The formation of cooper pairs and the nature of superconducting currents - Contemporary Physics 22(4) p.375-395, 1981,

Superconductivity made simple





Downloaded from http://hyperphysics.phy-astr.gsu.edu/hbase/Solids/bcs2.html#c1
R. Nave, Hyperphysics

Downloaded from http://www.superconductors.org/oxtheory.htm

Conductor vs superconductor
Also fluid vs superfluid

Picture from A. Gurevich, General aspects of superconductivity, Tutorial presented at the Superconducting Radio Frequency Workshop, 2007 (CERN archive)

End of lecture 13

Lecture 14

Scientific Instruments & Oscillators

Lectures for PhD Students and Young Scientists

Enrico Rubiola

CNRS FEMTO-ST Institute, Besancon, France

INRiM, Torino, Italy

Contents

- Electrical units, cont.

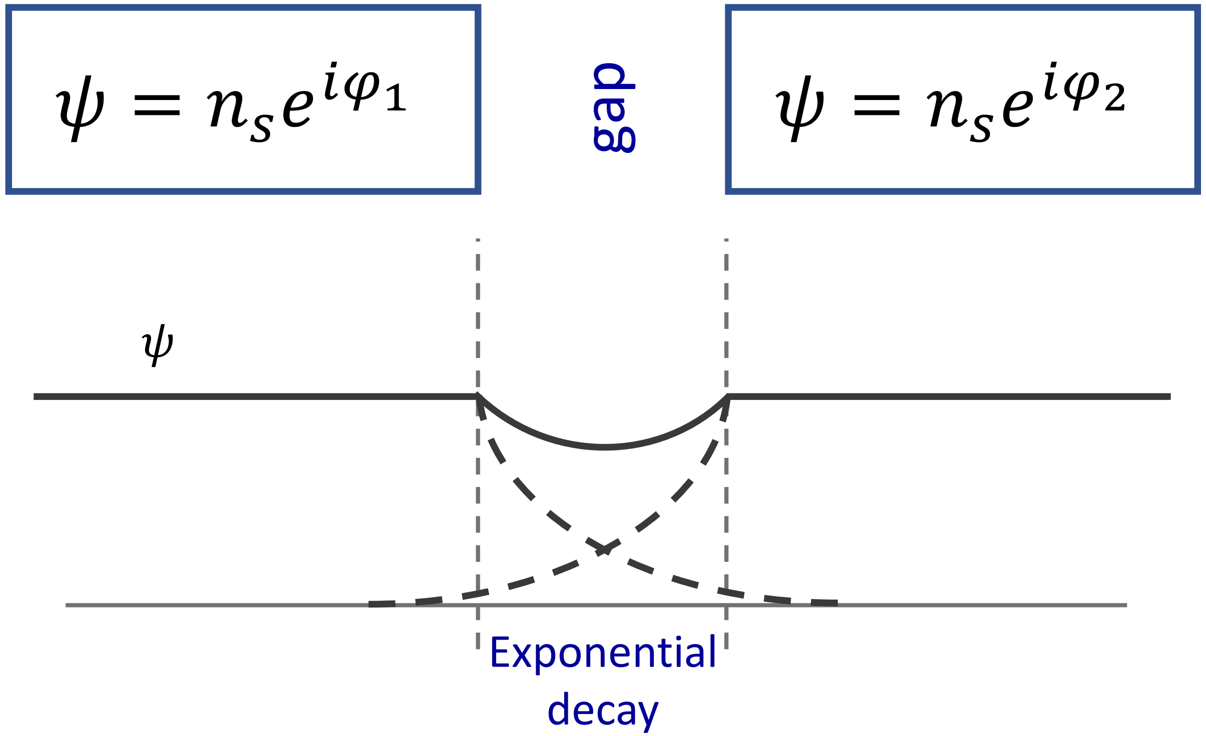
ORCID 0000-0002-5364-1835

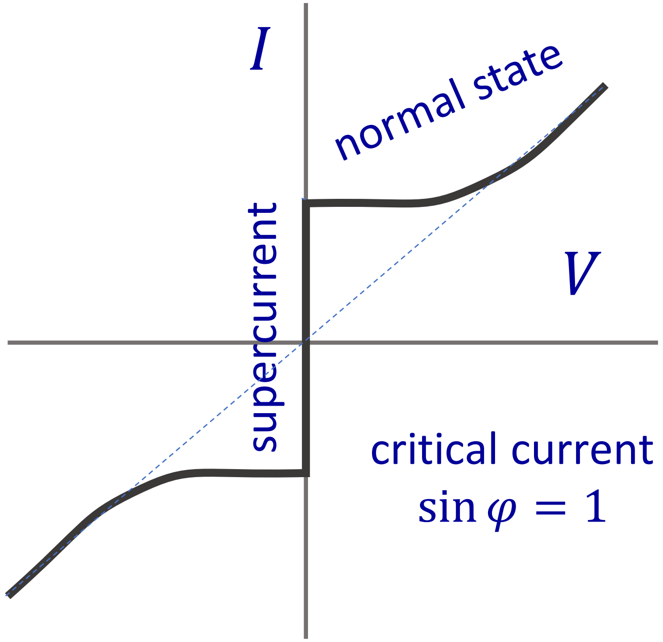
home page http://rubiola.org

The Quantum Electrical Standards

Josephson junction

BEC = all the Cooper pairs are in a single quantum state





1st Josephson equation

$$I_s = I_c \sin\varphi$$

$$\varphi = \varphi_2 - \varphi_1$$

2nd Josephson equation

$$2eV = \hbar \frac{\partial \varphi}{\partial t}$$

voltage impacts on φ

voltage results in oscillation
(RF has been observed)

Josephson junction and Shapiro steps

Constant DC voltage V results in

$$I_s(t) = I_{\max}\sin\left[\frac{2e}{\hbar}Vt + \varphi_0\right]$$

Instead of *detecting* AC, *apply* AC (together with DC voltage)

$$V(t) = V_{\mathrm{DC}} + V_{\mathrm{AC}}\cos(\omega_{\mathrm{AC}}t)$$

results in a series

$$I_s(t) = I_{\max}\sum_{n=-\infty}^{+\infty}(-1)^n J_n\left(\frac{2eV_{\mathrm{AC}}}{\hbar\omega_{\mathrm{AC}}}\right)\sin[(\omega_J - n\omega_{\mathrm{AC}})t + \varphi_0]$$

The condition $\omega_J - n\omega_{\mathrm{AC}} = 0$ results in

$$V_n = n\frac{h}{2e}\nu_{\mathrm{AC}}$$

This is a quantum voltage standard

3x10[5] junctions at 16-20 GHz —> 10 V

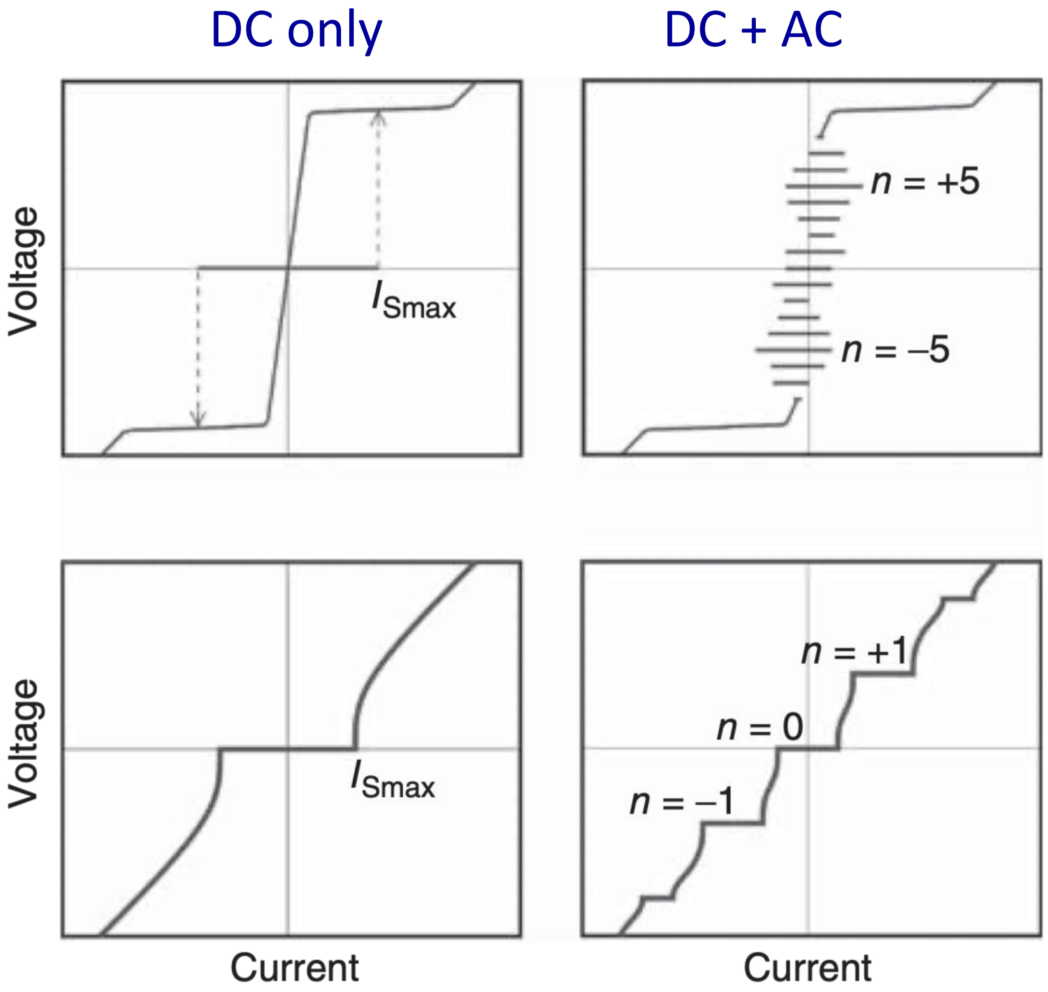
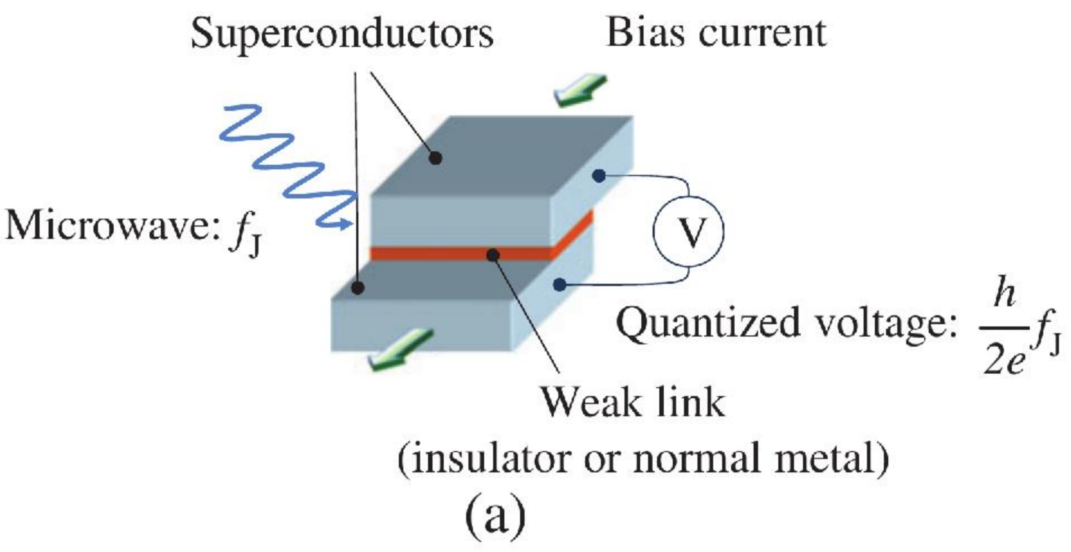
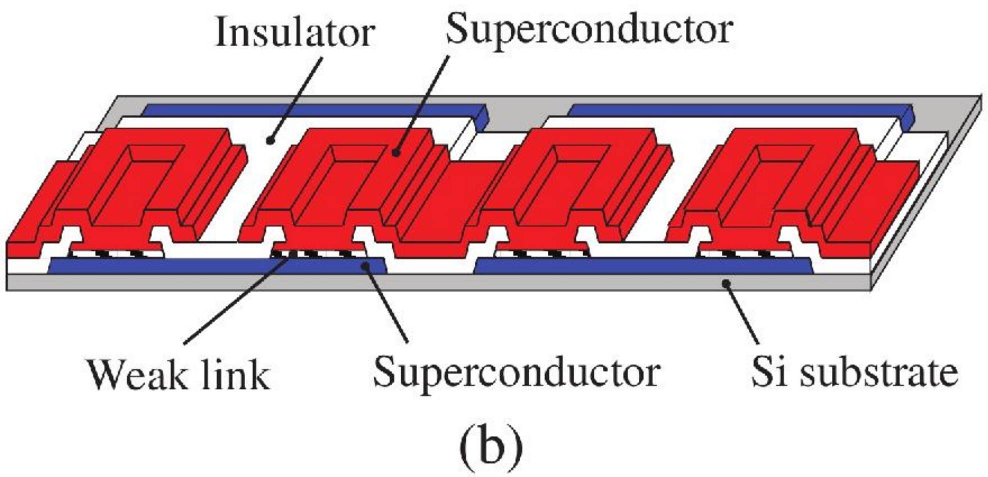


Fig. 4.5 from E. O. Göbel, U. Siegner, The New international System of Units (SI), Wiley-VCH 2019, ISBN 978-3-527-34459-8

What a Josephson junction looks like





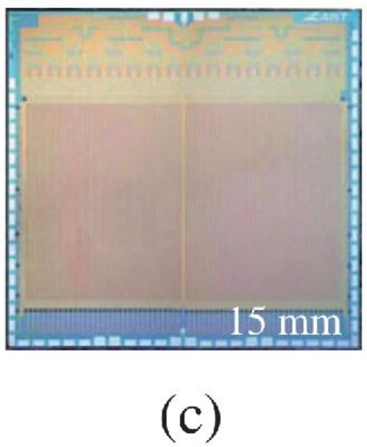


Fig. 3. Schematic of a part of a JJA and a picture of the PJVS fabricated at AIST.

(a) Schematic of a single JJ,
(b) schematic of a JJA, and
(c) photograph of the AIST PJVS device: a double-stack NbN/TiNx/NbN/TiNx /NbN device (524288 JJs, 17 V at the irradiating frequency of 16 GHz, working on a liquid-He-free compact cryocooler (Fig. 6). (Courtesy of Clean Room for Analog & digital superconductiVITY (CRAVITY), M. Maruyama and H. Yamamori, AIST)

Figure and caption from:
Kaneko NH - Review of Quantum Electrical Standards and Benefits and Effects of the Implementation of the Revised SI IEEJ T

What a Josephson standard looks like

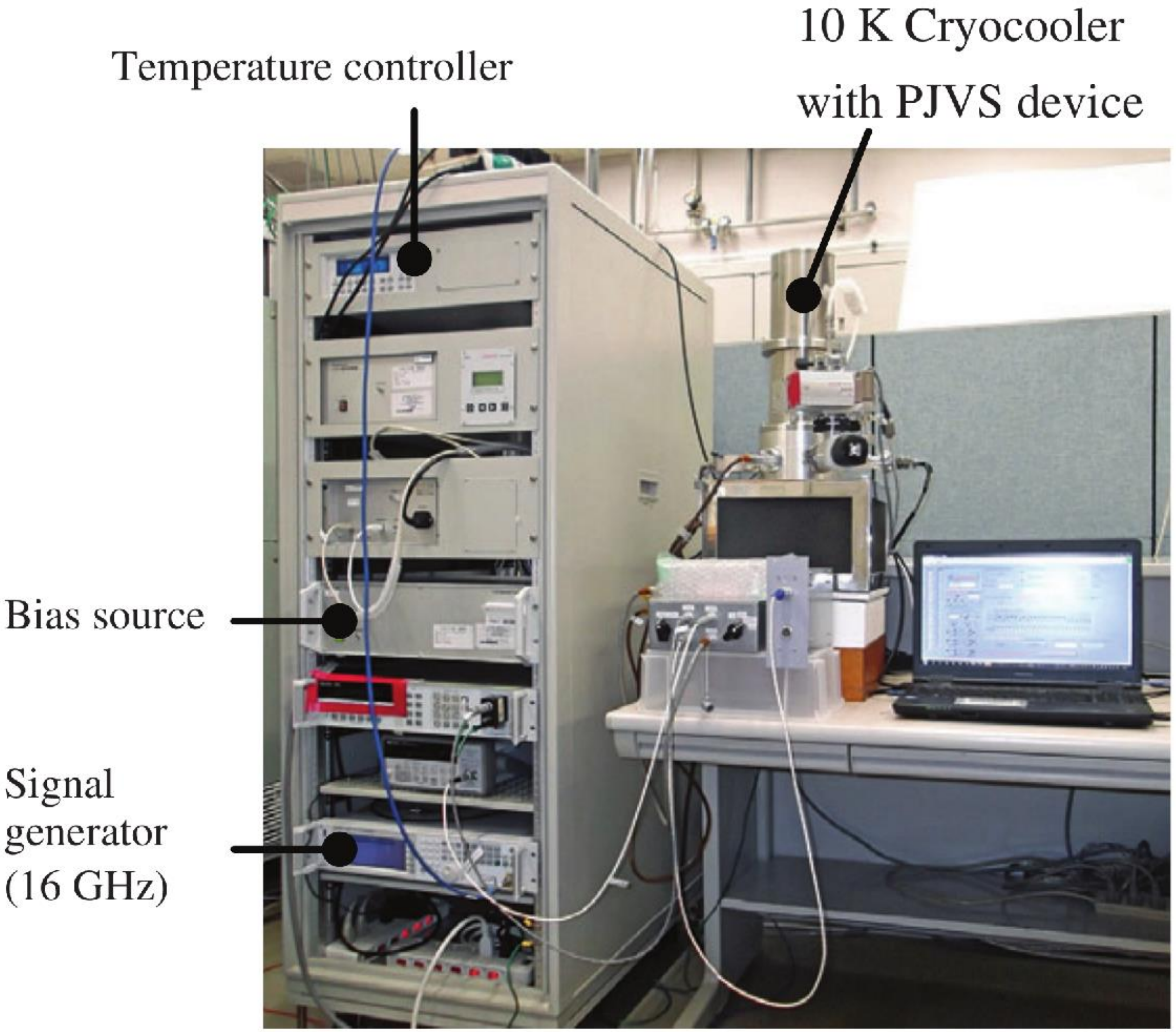


Fig. 6. Photograph of the NMIJ/AIST PJVS system (the national measurement standard of DC voltage equipped with a PJVS device (Fig. 3(c)) developed at the NMIJ/AIST [63]

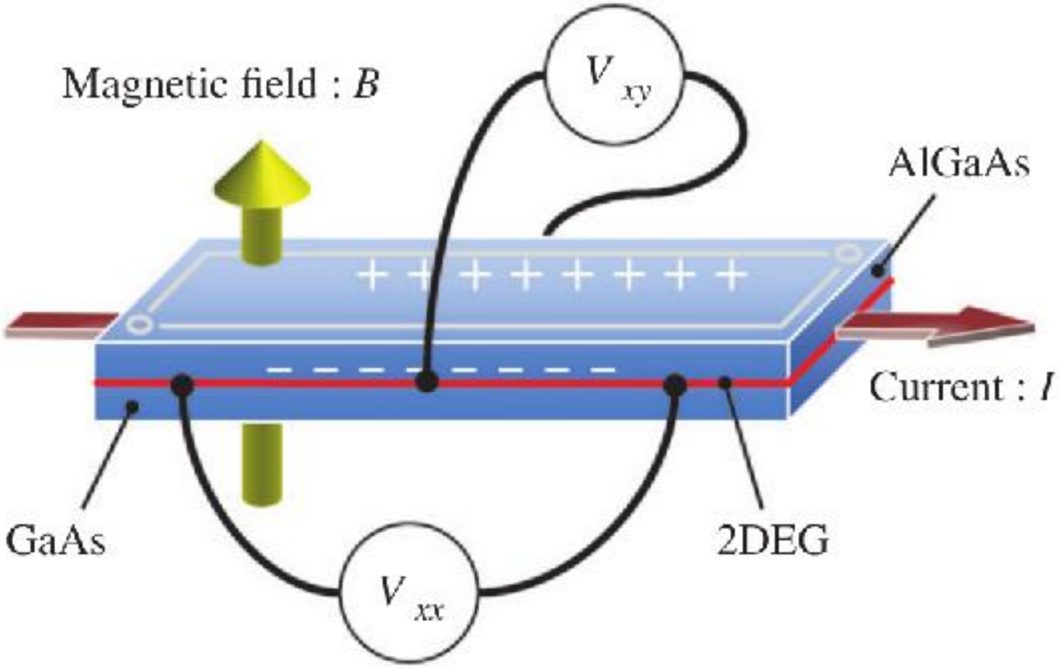
Seeback effect

$$V = k_S(T_2 - T_1)$$

$$k_S \approx 30\ \mu\text{V/K}$$

Figure and caption from:
Kaneko NH - Review of Quantum Electrical Standards and Benefits and Effects of the Implementation of the Revised SI IEEJ T

Classical Hall effect



Lorentz force

$$\vec{F} = e\,\vec{v} \times \vec{B}$$

A traverse voltage appears, proportional to B

$$V = K\,I\,B_{\perp}$$

Thus

$$R_{xy} = V_{xy}/I$$

Used as a magnetic sensor

BUT

- At large B, electron trajectories become circles
- At low T, quantization shows up

Figure 7(a) from:
Kaneko NH - Review of Quantum Electrical Standards and Benefits and Effects of the Implementation of the Revised SI IEEJ T

Quantum Hall effect

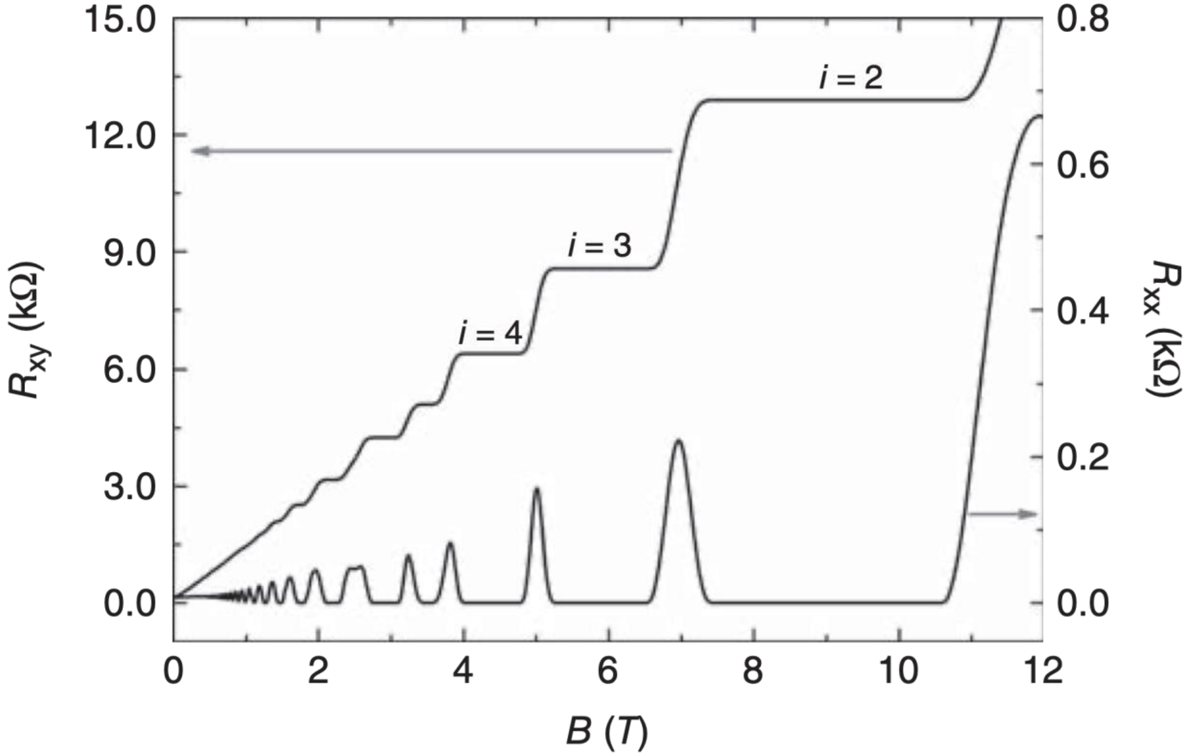


Fig. 5.7 from E. O. Göbel, U. Siegner, The New international System of Units (SI), Wiley-VCH 2019, ISBN 978-3-527-34459-8

Quantized Hall resistance

$$R_{xy} = \frac{h}{e^2}\frac{1}{i} \quad \text{integer } i$$

Von Klitzing constant

$$R_K = \frac{h}{e^2} \simeq 25.8\ \text{k}\Omega$$

Very low T
Very high B (10 T)
GaAs/AlGaAs heterostructure
Mostly $i = 2$ or $i = 4$

Klaus von Klitzing, Nobel prize in Physics in 1985

Feature reading:
Yennie DR, Integral quantum Hall effect for nonspecialists, Rev Modern Phys 59(3-1) p.781-824, July 1987

Implications of R_K on the impedance

$$R_K = \frac{h}{e^2} \simeq 25.8\ \mathrm{k\Omega}$$

$$\boxed{\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c} \qquad \alpha = \frac{\mu_0}{4\pi}\frac{e^2 c}{\hbar}}$$

$$\epsilon_0 = \frac{1}{2\alpha c} \qquad \epsilon_0 \neq \frac{10^7}{4\pi c^2} \qquad C = 4\pi\epsilon_0 r$$

Farad

Wave propagation

$$c^2 = \frac{1}{\epsilon_0\mu_0}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \mu_0 c = \frac{1}{\epsilon_0 c}$$

$$\mu_0 = \frac{2}{c\alpha}\frac{h}{e^2}$$

$$\mu_0 \neq 4\pi 10^{-7}$$

$$L = \mu_0 l$$

Henry

$$R_K = \frac{h}{e^2}$$

$$Z_0 = \frac{2h}{\alpha e^2} \neq 4\pi 10^{-7} c$$

Ohm

What ate QH resistance standard looks like

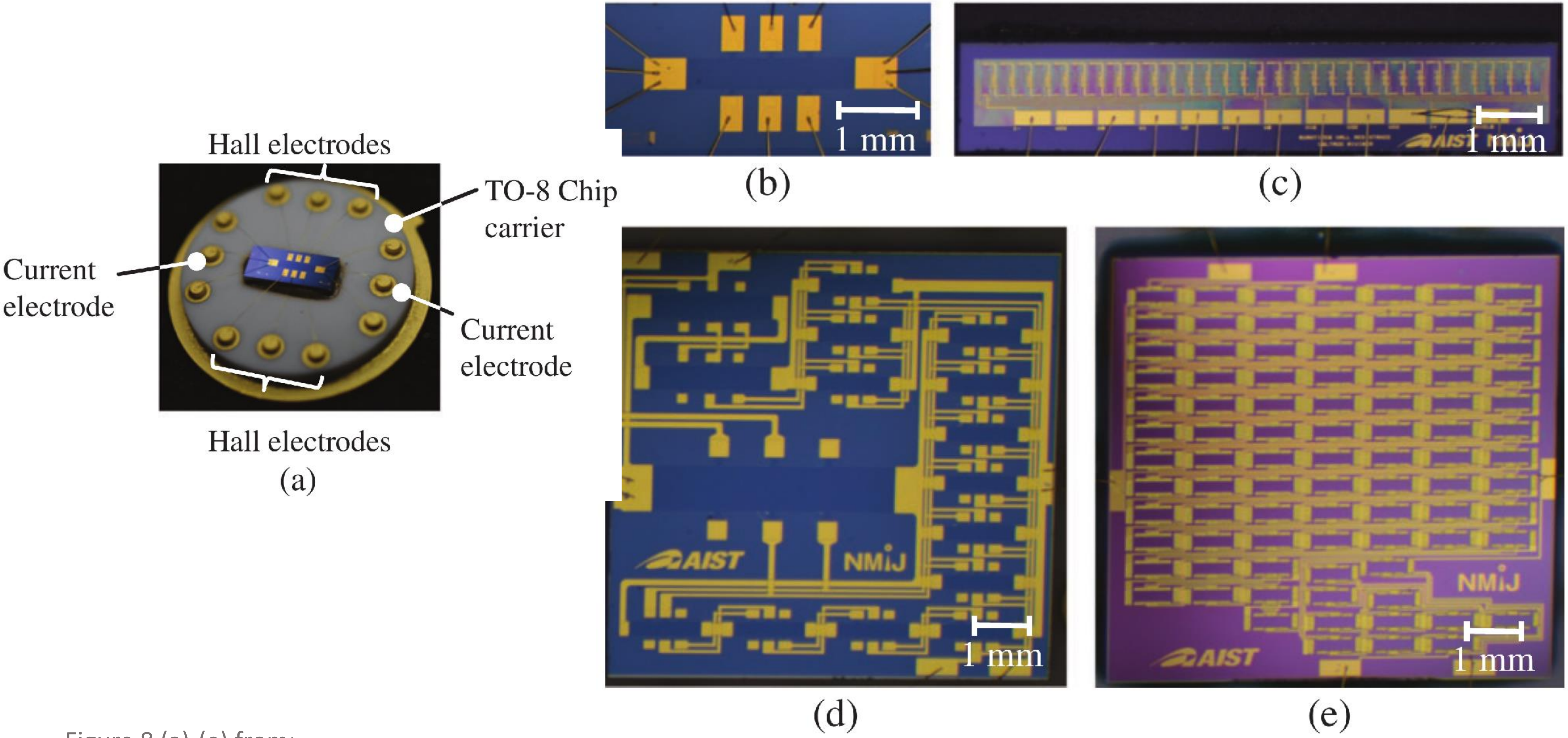


Figure 8 (a)-(e) from:
Kaneko NH - Review of Quantum Electrical Standards and Benefits and Effects of the Implementation of the Revised SI IEEJ T

What a QH resistance standard looks like

Gas handling system

Temperature and magnet controllers

Cryogenic current comparator

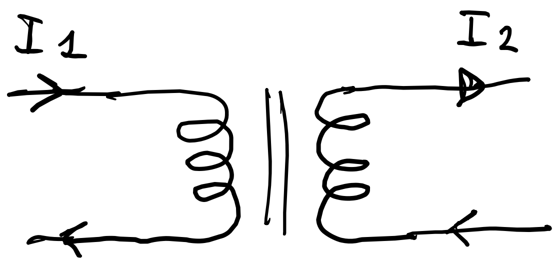
Top-loading ^{3}He refrigerator (500 mK) and superconducting magnet (15 T/17 T) with QHR device

Fig. 10. Photograph of the NMIJ/AIST QHRS (the national measurement standard of DC resistance) developed and maintained by NMIJ/AIST. All the components of the system are hand-made or custom-made. The liquid ^{4}He Dewar for the ^{3}He refrigerator (cryostat) is inserted in the pit on the floor

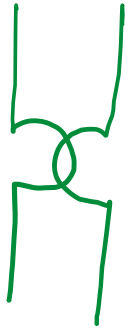
Figure 10 from:
Kaneko NH - Review of Quantum Electrical Standards and Benefits and Effects of the Implementation of the Revised SI IEEJ T

The cryogenic current comparator

TRANSFORMER



$$N_1 I_1 = N_2 I_2$$



A superconducting transformer
- Works in dc !
- does not need iron core

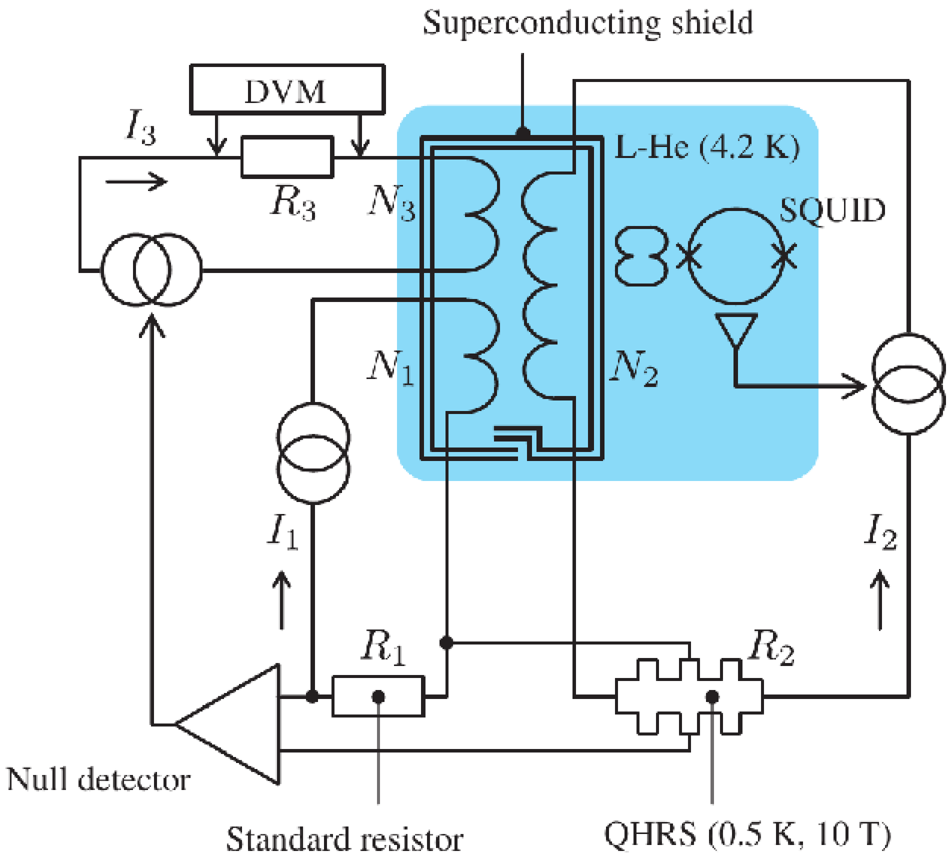


Figure 9 from:
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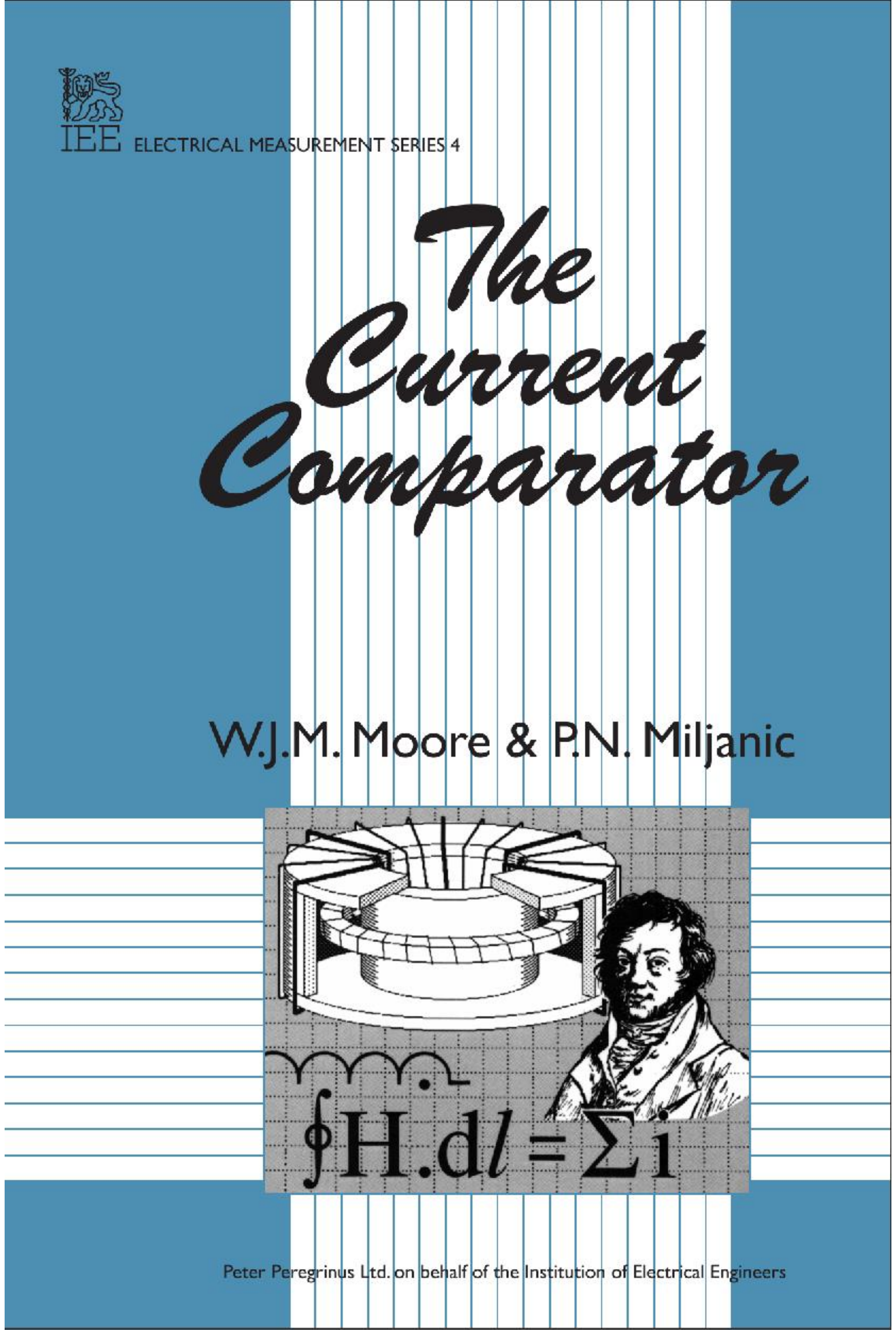
Moore, W. J. M.
The current comparator—(IEE electrical measurement series; 4).
1. Electronic transformers
I. Title II. Miljanic, P. N.
III. Institution of Electrical Engineers
IV. Series
621.3815'1 TK7872.T7

ISBN 0 86341 112 6

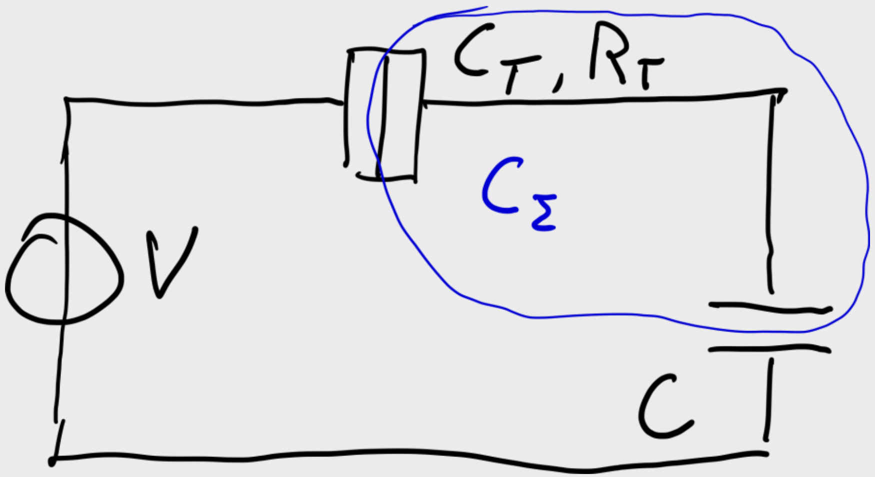
The room-temperature current comparator is more complex

Printed in England by Short Run Press Ltd., Exeter



Single-Electron Transport

$$I = \nu e$$



Actual implementation takes DC bias and AC pump.
Single-electron transistors

Energy Conditions

Adding 1 electron, $E_{1e} \gg kT$

$$E_{1e} = \frac{1}{2}\frac{e^2}{C_\Sigma} \gg kT \qquad \frac{1}{2}\frac{q^2}{C} \quad q \to e,\ C \to C_\Sigma$$

Quantum fluctuation

$$E_{1e} \gg E_{fluct}$$

$$\frac{1}{2}\frac{e^2}{C_\Sigma} \qquad E = \frac{\hbar}{\tau} \qquad \tau = R_T C_\Sigma$$

$$R_T \gg \frac{1}{\pi}\frac{h}{e^2} \simeq 8\,k\Omega$$

Practical realization of the electrical units

ampere

- Josephson and Quantum Hall

$$I = V/R$$

- Single-electron transport

$$I = e\nu$$

- Capacitance

$$I = C\,dV/dt$$

volt

- Josephson

ohm / siemens

- Quantum Hall
- Compare a resistor to a capacitor

coulomb

- Same as ampere

$$q = \int I(t)\,dt$$

farad

- Compare the impedance with QH
- Calculated capacitor

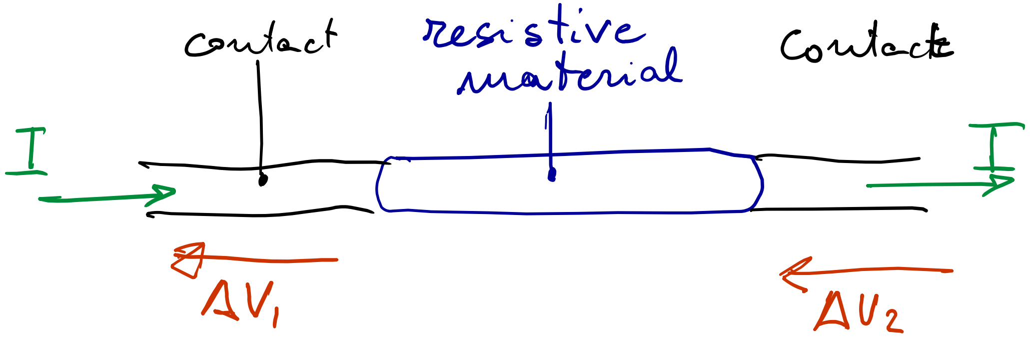
$$\epsilon_0 = \frac{1}{\mu_0\,c^2}$$

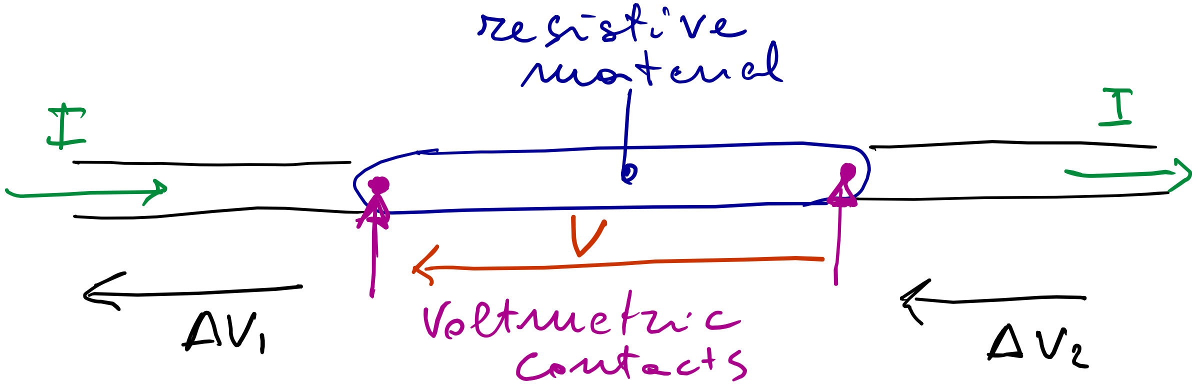
Also: henry, watt, tesla, weber

Reference document: “Mise en pratique for the definition of the ampere and other electric units” in the SI, Appendix 2 of the SI Brochure

Secondary Standard & Working Standards

Two wire and four wire connections





The resistance of contacts is not well defined, not stable.

error/uncertainty ΔV

The resistance between the two V contacts is well defined

$$R = \frac{V}{I}$$

across V contact

flowing through the A contacts

Zener/Avalanche diodes

Zener effect

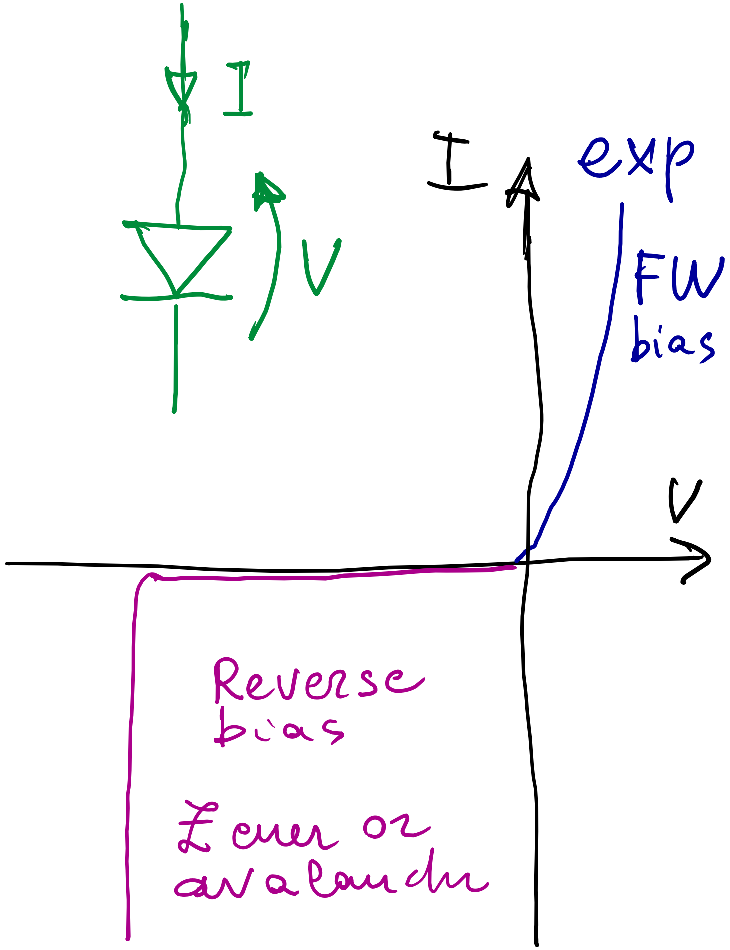
- reverse-biased junction
- breakdown
- electrons tunnel from valence band to conduction band
- sudden increase of minority carriers
- Positive TC

PN junction 30 MV/m
moderately doped P
heavily doped N
narrow depletion region

Avalanche effect

- reverse-biased junction
- breakdown
- minority carriers accelerated to an energy level sufficient to create e-h pairs by collision
- sudden increase of carriers
- Negative TC

Highly doped PN junction
wide depletion region

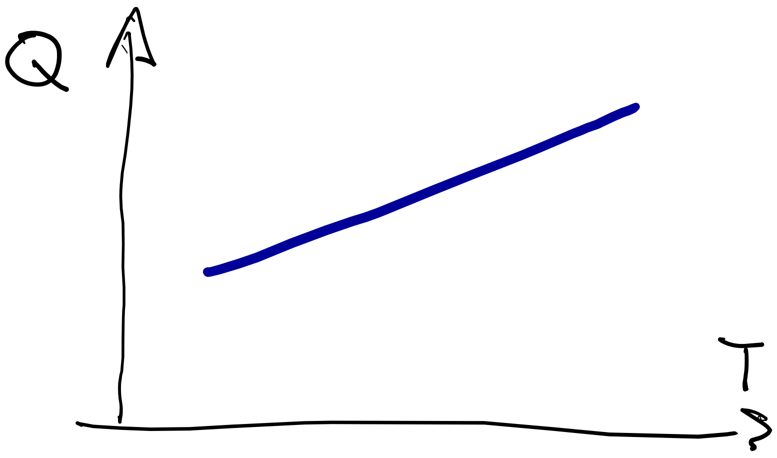


Avalanche has hysteresis → noise

Clarence Melvin Zener,

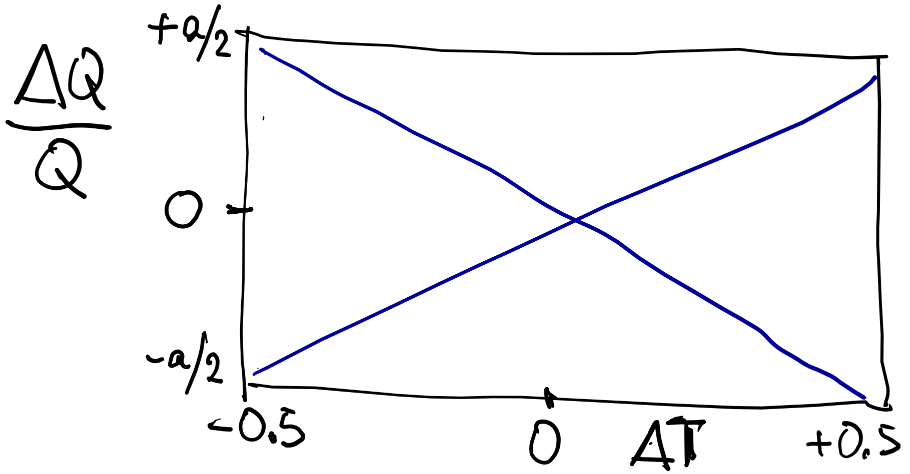
Thermal coefficient

(True) TC



- Well known effect
- Can be used to measure T
  - Volume of Hg
  - Pt resistor

TC of Electronic Components

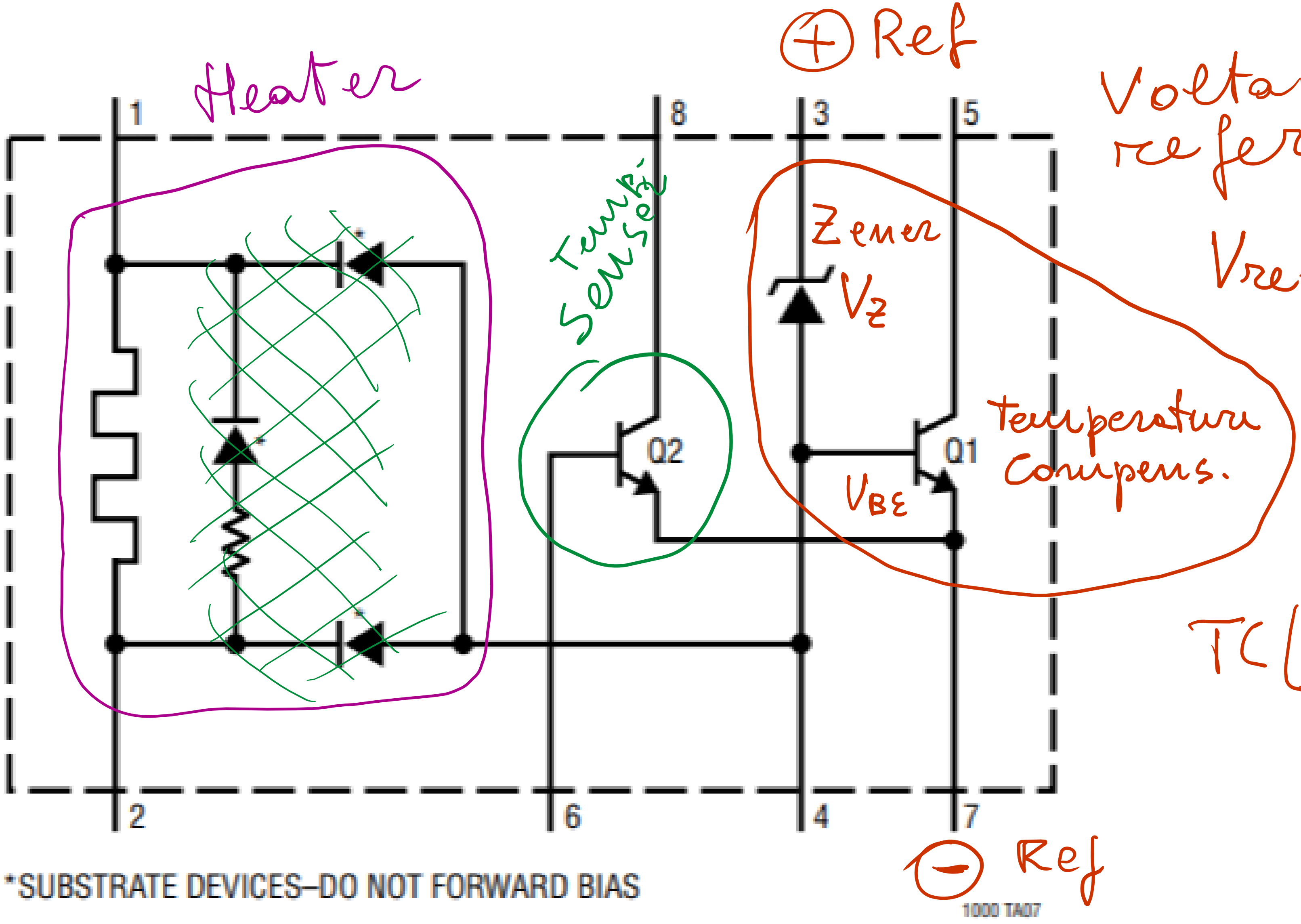


Meaning: max $|(\Delta Q/Q)/\Delta T|$

This is B-Type uncertainty

Usually given in %/K, ppm/K etc.
Improperly, also %/°C, ppm/°C ....

LTZ 1000 / LTZ 1000A voltage reference



Voltage reference

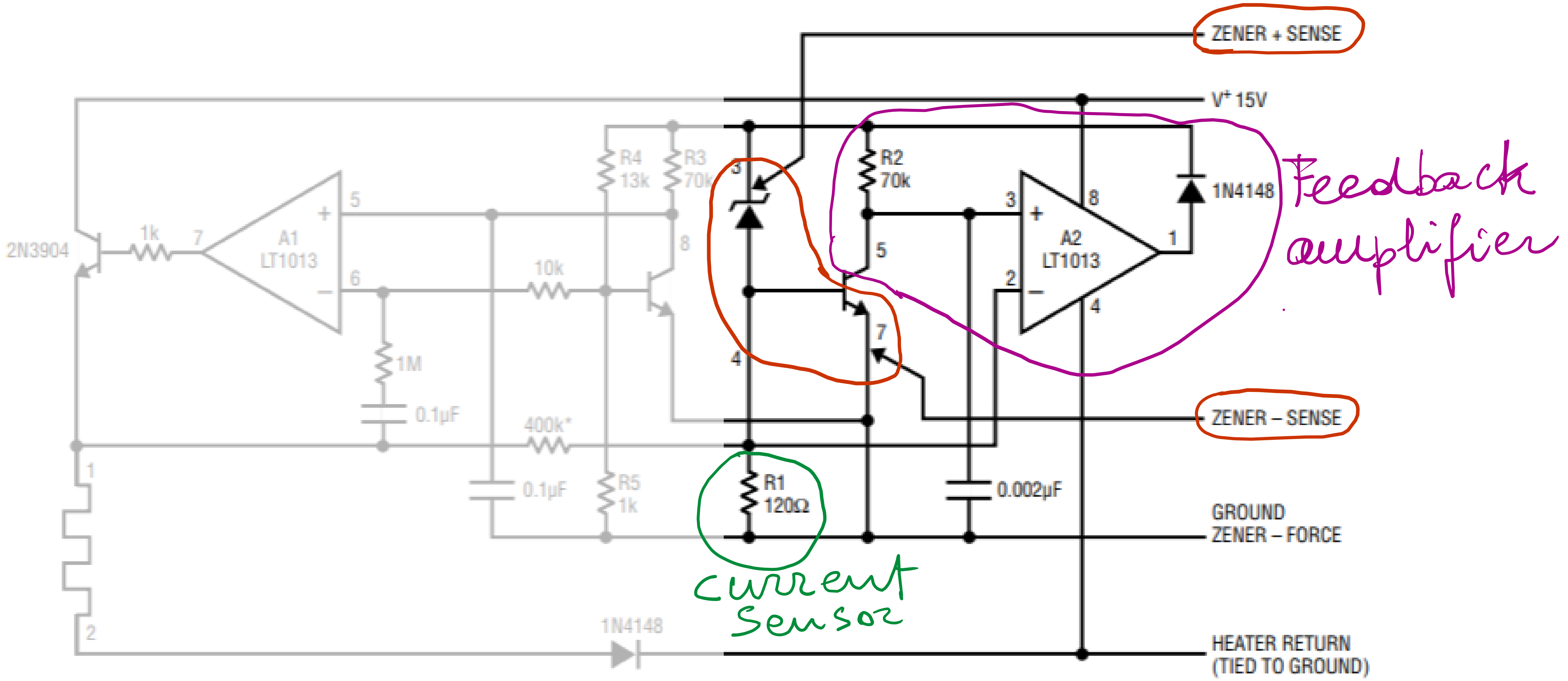
$$V_{ref} = V_Z + V_{BE}$$

$$\frac{\Delta V_{BE}}{\Delta T} \approx -2.5 \frac{mV}{K}$$

$$TC(V_Z) + TC(V_{BE}) = 0$$

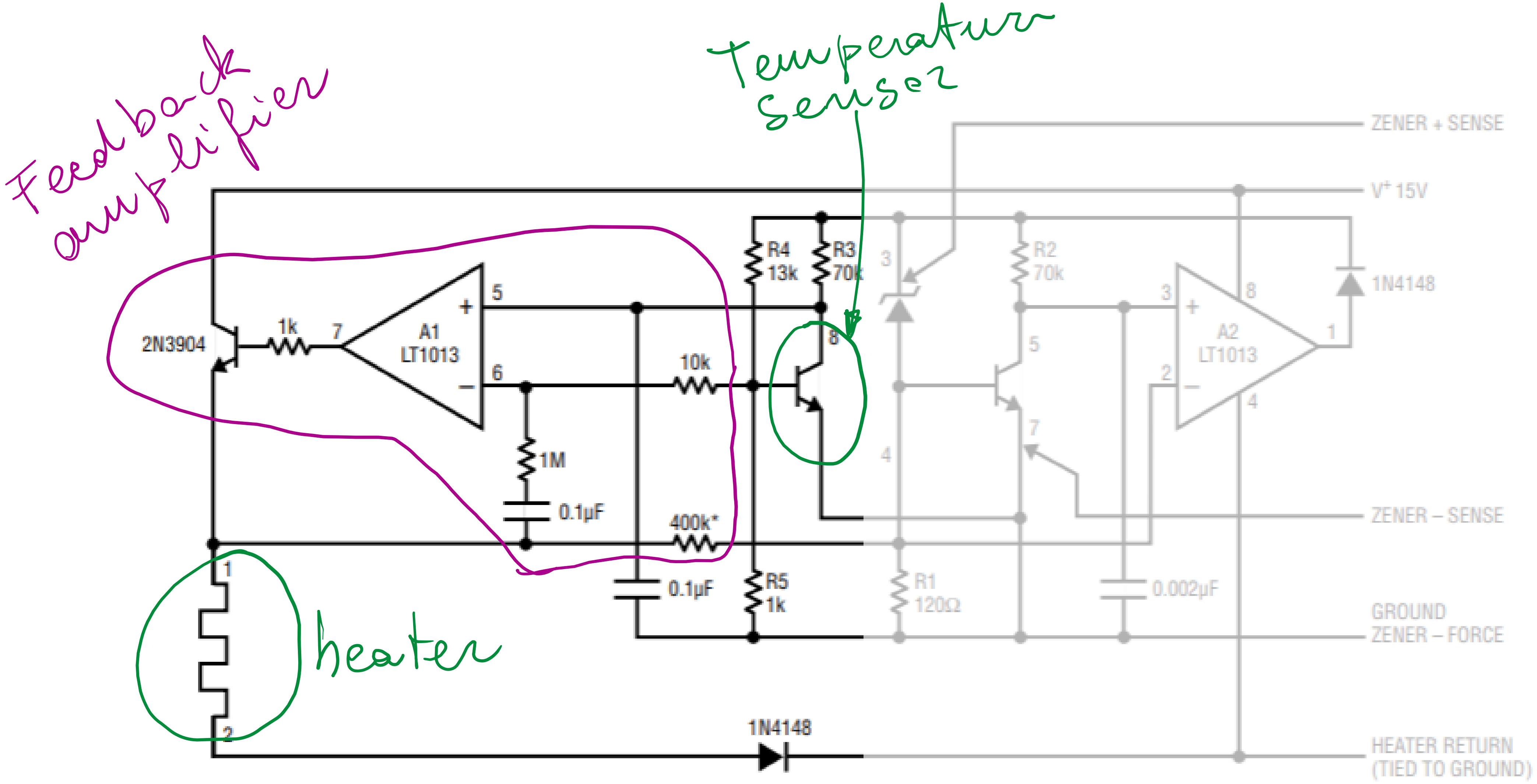
Picture from the LTZ1000/LTZ1000A data sheet, © Analog Devices

LTZ 1000 / LTZ 1000A Voltage Reference



Picture from the LTZ1000/LTZ1000A data sheet, © Analog Devices

LTZ 1000 / LTZ 1000A voltage reference

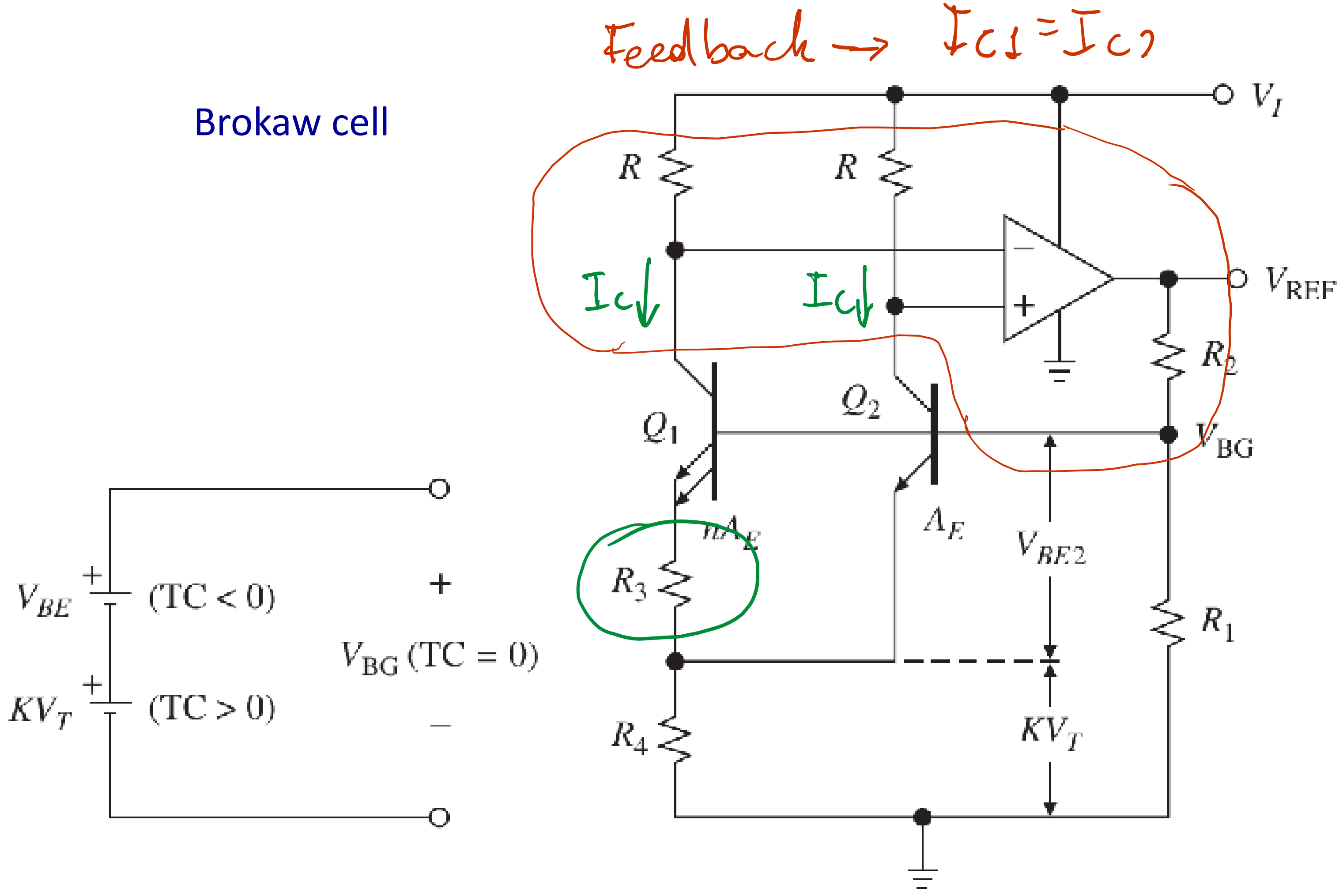


Picture from the LTZ1000/LTZ1000A data sheet, © Analog Devices

Bandgap voltage reference

Brokaw cell

S. Franco Fig.11.7 p.545



$I = I_s \left[e^{V_{BE}/V_T} - \cancel{1} \right]$

$V_T = kT/e \simeq 25 mV$

emitter area A_E

$I_{S1} = n I_{S2}$

(I_0) $(n I_0)$

$n I_0 e^{V_{BE1}/V_T} = I_0 e^{V_{BE2}/V_T}$

solve:

$V_{BE1} - V_{BE2} = \ln(n) V_T$

Sergio Franco, Design with operational amplifiers and analog integrated circuits, 4th ed, McGraw Hill 2015, ISBN 978-0-07-802816-8

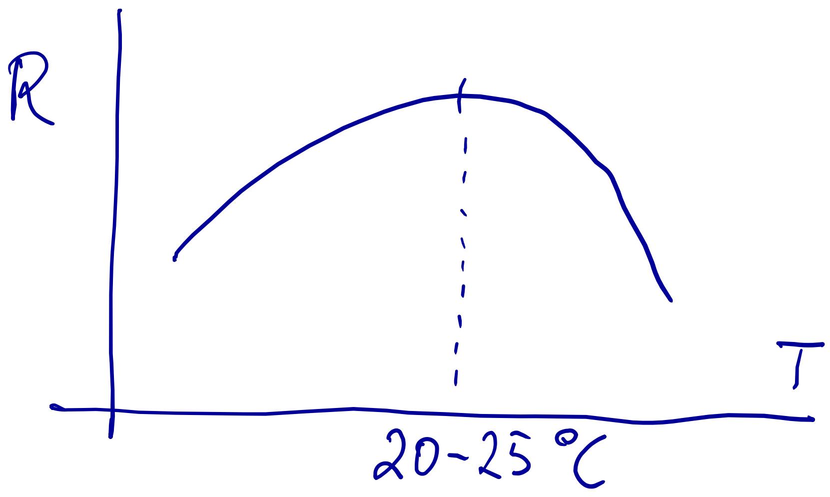
Manganin & Zeranin

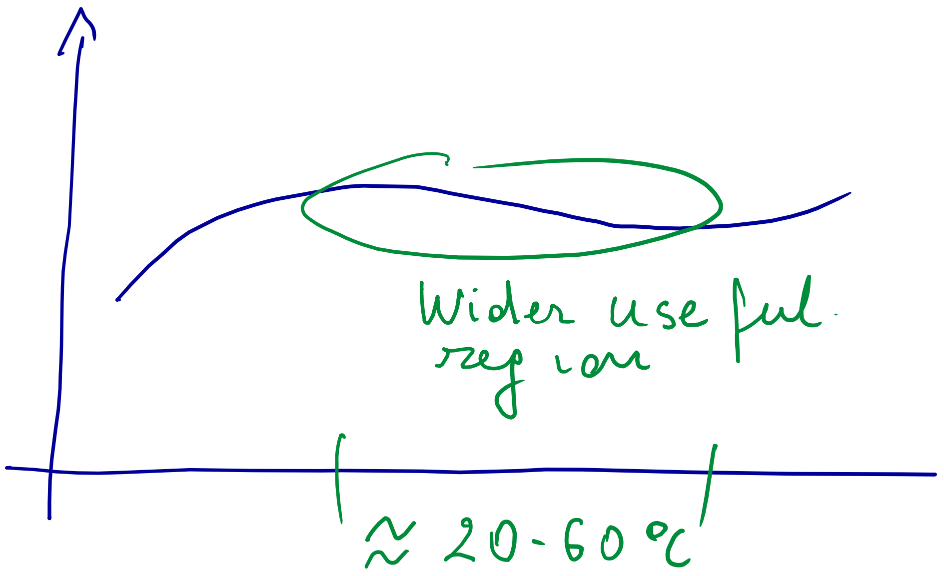
Manganin

84.2% Cu, 12.1% Mg, 3.7% Ni

Zeranin

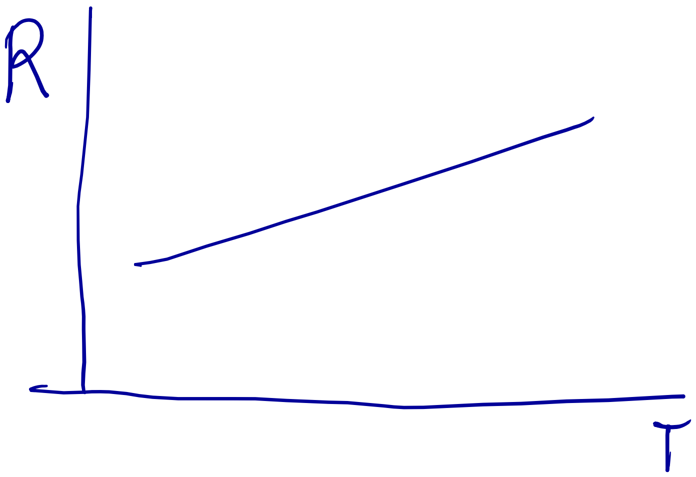
... Cu, 7% Mn, 2.3% Sn





Isabellenhütte, Germany

Vishay resistors



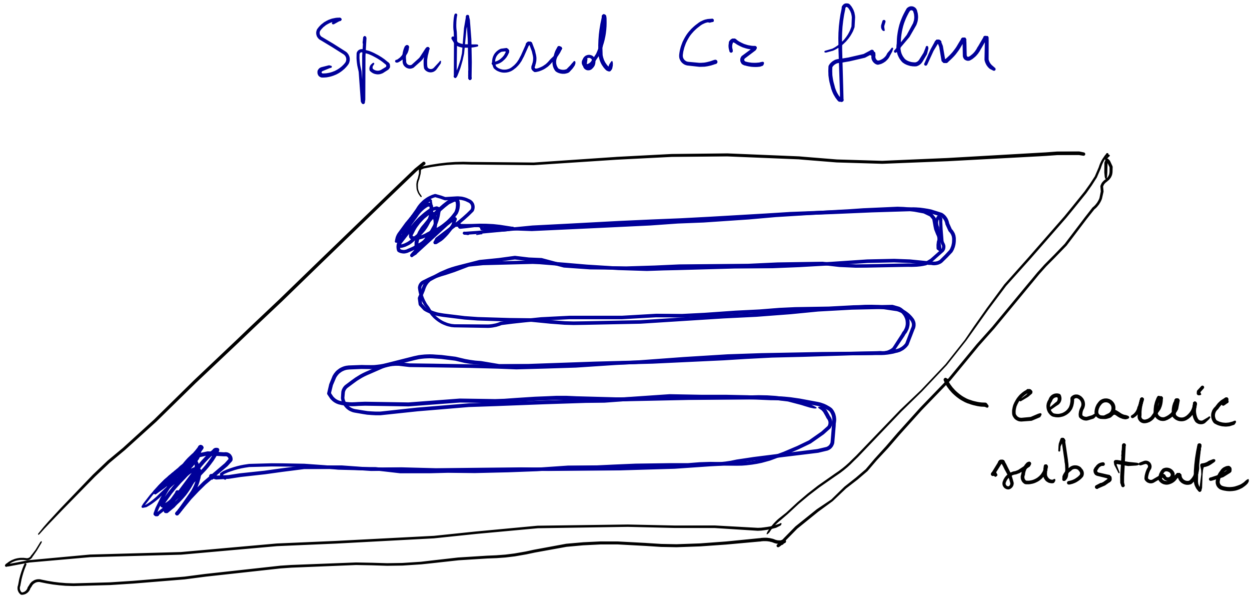
Cu $\sim 3.9\times10^{-3}/K$

Cr

Include thermal expansion

Cu 17 ppm/K

Cr 4.9 ppm/K



- Add a strain effect to TC(R)
- Chemistry/physics of ceramic
- Get TC(R) $\approx$ 0 in a range at RT

University of Montpellier

Vishay Inc

Vishay resistors

In practice

- < 1 ppm TC
- Laser trimmed to 10^{-4} accuracy
- Matched pairs

RESISTOR THEORY AND TECHNOLOGY

Deep inside today's exciting new electronics devices, from smart appliances to cutting-edge personal data assistants, lies some form of resistor.

Resistor Theory and Technology hands engineers, scientists, technicians, and business people in the electronics industries a much-needed roadmap to understanding resistor design, fabrication, and use. The authors give an overview of resistor theory and technology, showing how it arises from scientific theories of resistive properties of materials. They explain selection and use of virtually every discrete resistor type, including high-performance precision resistors, sophisticated anistropic magnetoresistance for hard disk drives and recording devices, potentiometers for automated control systems, the increasingly popular nonlinear resistors, and many others.

Each chapter of the book stands alone, making it perfect as a resource for a variety of resistor-related subjects, including:

- Types of resistor materials
- Limitations of resistive circuits
- Modifying solid and thin-film metals and alloys
- Conductive materials used in resistors
- Dealing with heat transfer issues
- And much more!

Much of the practical information on resistors comes from documentation developed by Vishay Intertechnology, Inc., and other manufacturers and is unavailable in any other single source. With Resistor Theory and Technology as a handy reference, design engineers and technicians will be able to optimize circuitry with exactly the right resistor choices.

ABOUT THE AUTHORS

Felix Zandman holds a Doctorate in physics from the Sorbonne, Paris, France, and a degree in mechanical engineering. He is the founder of Vishay Intertechnology, Inc., and is its Chairman, Scientific Director, and Chief Executive Officer, as well as the author of three textbooks and numerous scientific papers. Vishay Intertechnology, Inc., is the largest U.S. and European manufacturer of passive electronic components and a major manufacturer of discrete semiconductors and selected integrated circuits.

Paul-René Simon holds a Ph.D. in physics from the University of Toulouse, France. He is currently Scientific Consultant for the CEE and the French ministry of research and development in electronic components and related fields. He was formerly Director of Research of Sfernice Group, a subsidiary of Vishay Intertechnology, Inc.

Joseph Szwarc is Chief Engineer for Foil Resistors at Vishay, Israel. He studied engineering in Poland and France, and holds diplomas in mechanical and electrical engineering.

ISBN: 1-891121-12-X

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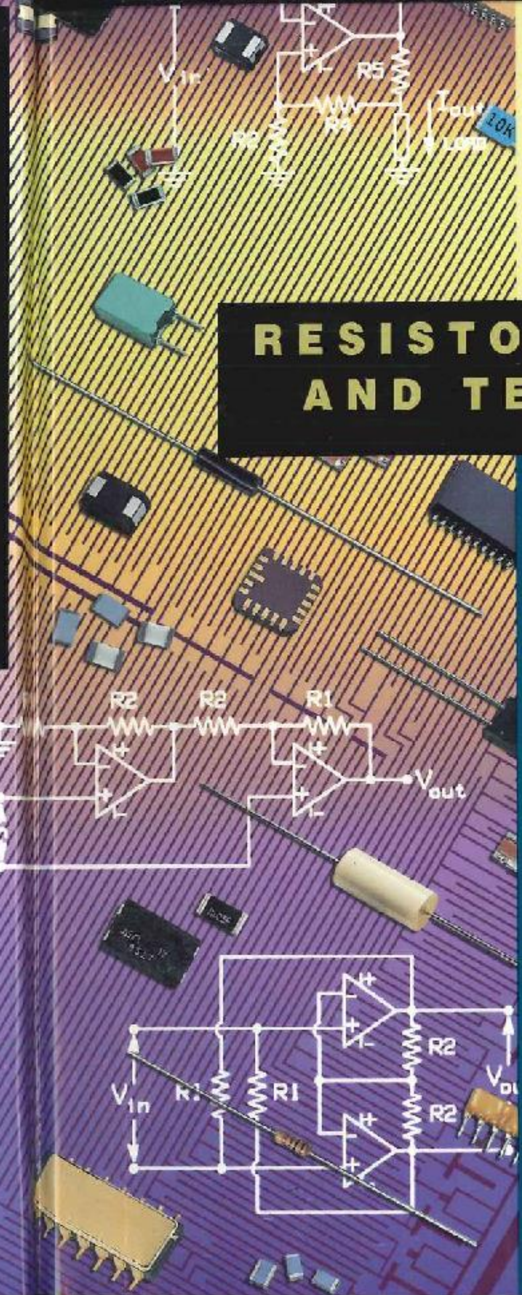
VISHAY

RESISTOR THEORY AND TECHNOLOGY

By Felix Zandman, Paul-René Simon, Joseph Szwarc

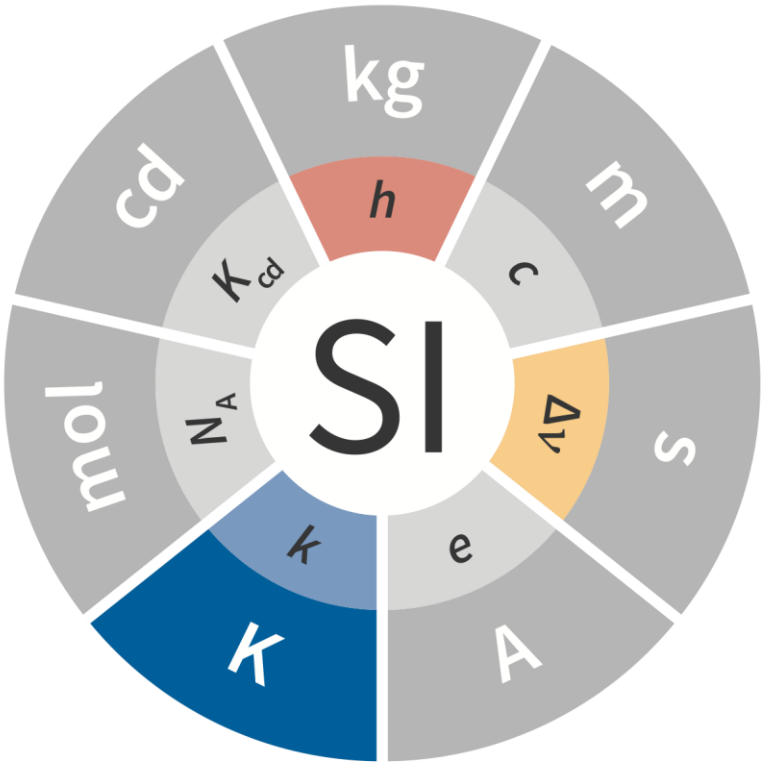


RESISTOR THEORY AND TECHNOLOGY

By
Felix Zandman
Paul-René Simon
Joseph Szwarc

Bibliography

Kelvin



The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k to be 1.380649×10^{-23} when expressed in the unit $\mathrm{J\,K^{-1}}$, which is equal to $\mathrm{kg\,m^2\,s^{-2}\,K^{-1}}$, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{\mathrm{Cs}}$.

New in 2019

William Thomson, 1st Baron Kelvin, 1824–1907
Ludwig Eduard Boltzmann, 1844–1906 (suicide)

Note that c is *not colored*. The kelvin depends on the kg, the m and the s. Although the kg and the m depend both *individually* on c, this is not the case for the kelvin, because c *drops out* from its definition

Temperature metrology is extremely complex

Often, hostile environment
Temperature uniformity
Heath conduction
Radiation

POI: spelling
Temperature (Latin)
Thermometer, Thermal, etc. (Greek)

POI: Greek to modern Latin alphabet
τ –> t   θ –> th

Acoustic thermometry in gas (He, Ar)

Good at room temperature

$$v_0 = \sqrt{\frac{\gamma R T}{M}}$$

v_0 = speed of sound

$\gamma = C_P / C_V$

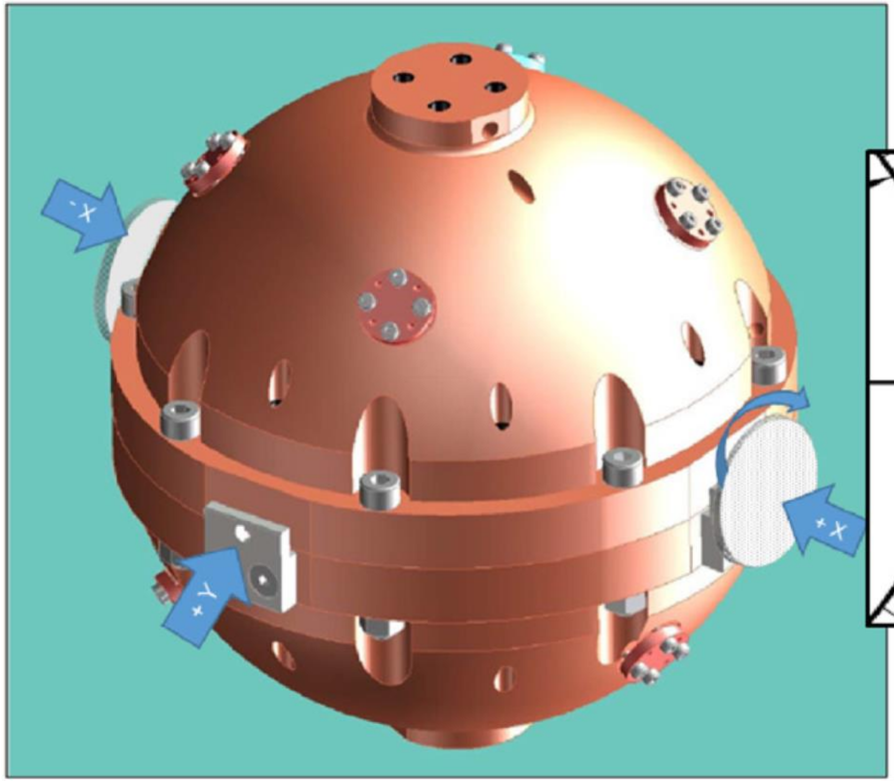
monoatomic –> $\gamma \approx 5/3$

M = molar mass

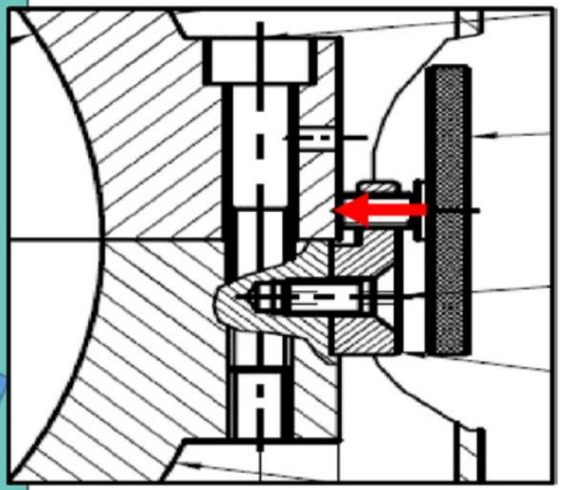
R = molar gas constant

Measure v_0 from acoustic resonance

Speed of sound
natural He –> 972 m/s (mainly ^{4}He)
Natural Ar –> 323 m/s (99.6% ^{40}Ar)



Gavioso et al, Fig.4



The diameter can be calculated from the microwave resonance

Gavioso et al., A determination of the molar gas constant R by acoustic thermometry in helium, Metrologia 52 S274, 2015

End of lecture 14

Lecture 15

Scientific Instruments & Oscillators

Lectures for PhD Students and Young Scientists

Enrico Rubiola

CNRS FEMTO-ST Institute, Besancon, France

INRiM, Torino, Italy

Contents

- Kelvin and temperature metrology
- (skip the mole)
- Candela, just a minimalist introduction

ORCID 0000-0002-5364-1835

home page http://rubiola.org

Dielectric-constant gas thermometry (DCGT)

Cryogenic and room temperature

$$p = kT\frac{\epsilon - \epsilon_0}{\alpha_0}$$

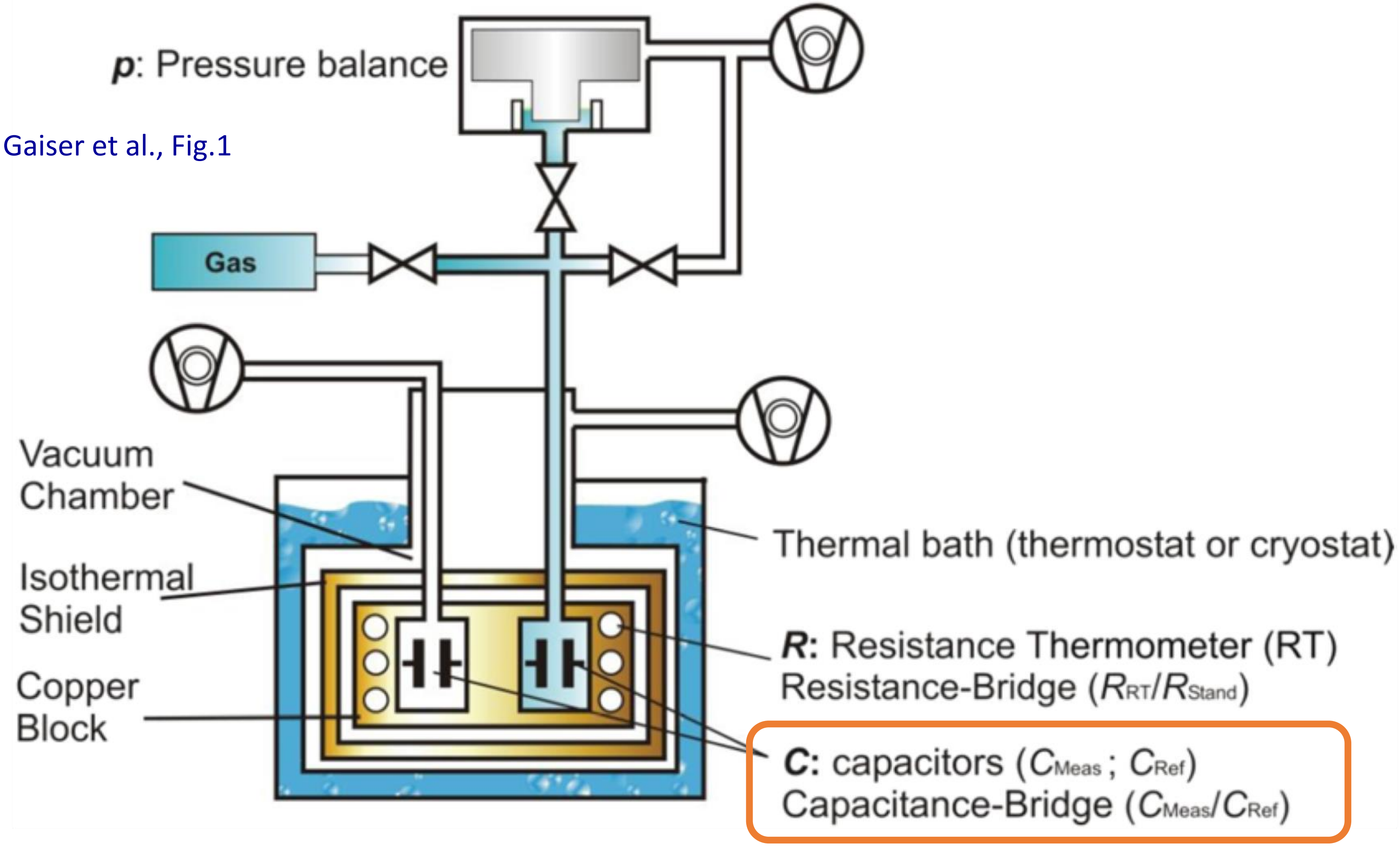
p = pressure

ϵ, ϵ_0 = dielectric const

α_0 = static polarizability

Measure $\epsilon - \epsilon_0$ with a capacitive bridge

Gaiser et al., Fig.1



Gaiser C, Zandt T, Fellmuth B, Dielectric-constant gas thermometry, Metrologia 52 S217, 2015

Refractive-index gas thermometry (RIGT)

Similar to DCGT, but usually exploits the measurement the resonance of the microwave cavity filled with gas
Large cavity —> large volume-to-surface-area ratio

$$p = kT\frac{(n^2-1)\epsilon_0}{\alpha_0}$$

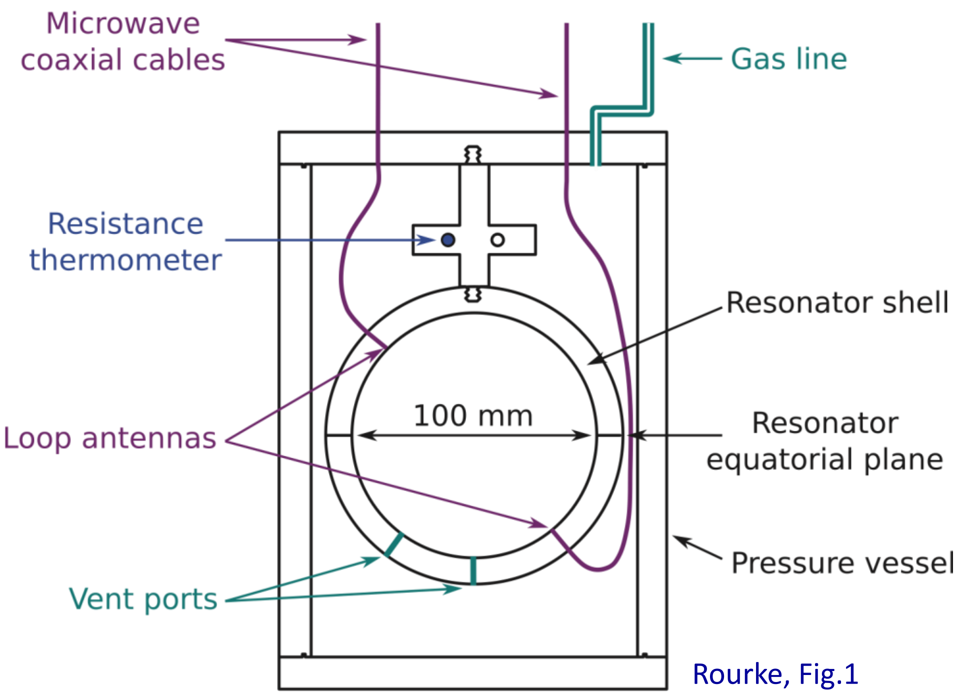
n = refractive index

α_0 = static polarizability

Measure n with µwave resonance

Smart use of Acoustic and Refractive-Index thermometry in a single experiment

He, Ne, Ar preferred



Rourke, Fig.1

Rourke, Tab.3

Table 3. Absolute primary optical RIGT uncertainty budget from a measurement of the Boltzmann constant k at $T = 293.1529$ K by Egan *et al* (2017), using helium as the working gas. All uncertainty components are listed as standard uncertainties.

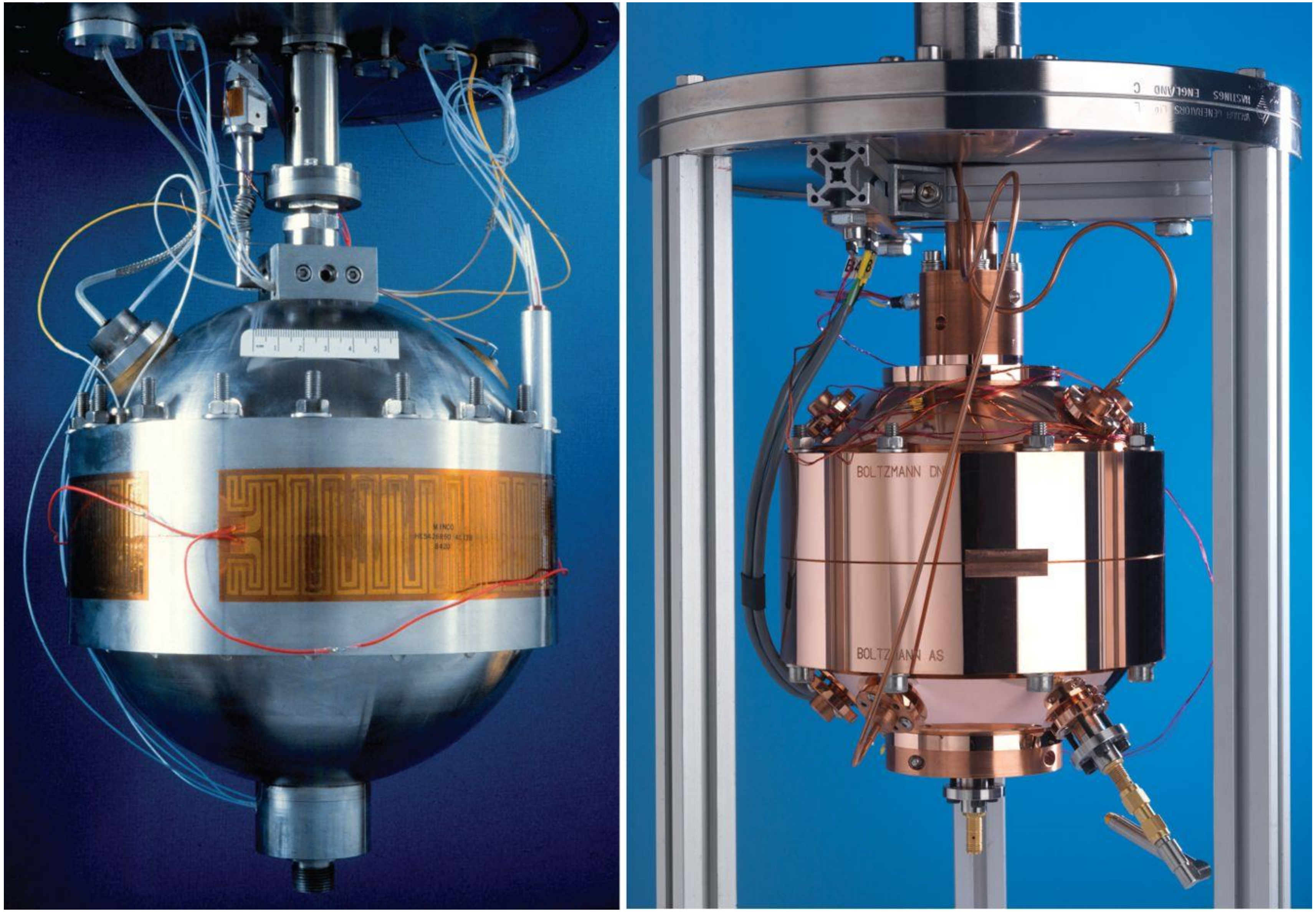
Uncertainty component	$u_r(k) \times 10^6$	$u_r(k) \times 293.1529$ K
Optical window distortion and stress, d_w	9.8	2.9 mK
Thermodynamic temperature, T	4.5	1.3 mK
Gas pressure, p	4.3	1.3 mK
Gas impurities	4.0	1.2 mK
Interferometric phase change, $\Delta\Phi$	2.2	0.64 mK
Relative cell length, ΔL	0.6	0.2 mK
Refractivity first virial coefficient, A_R	0.12	0.035 mK
k combined standard uncertainty	12.5	3.7 mK

Review article: P. M. C. Rourke et al., Refractive-index gas thermometry, Metrologia 56 032001, 2019

Combined microwave and acoustic resonator

Moldover et al, Fig.1

Left:
NIST cavity
USA



Right:
NPL cavity
UK

M. R. Moldover, W. L. Tew, H. W. Yoon, Advances in thermometry, Nature Phys 12 3618 pp.7-11, January 2016

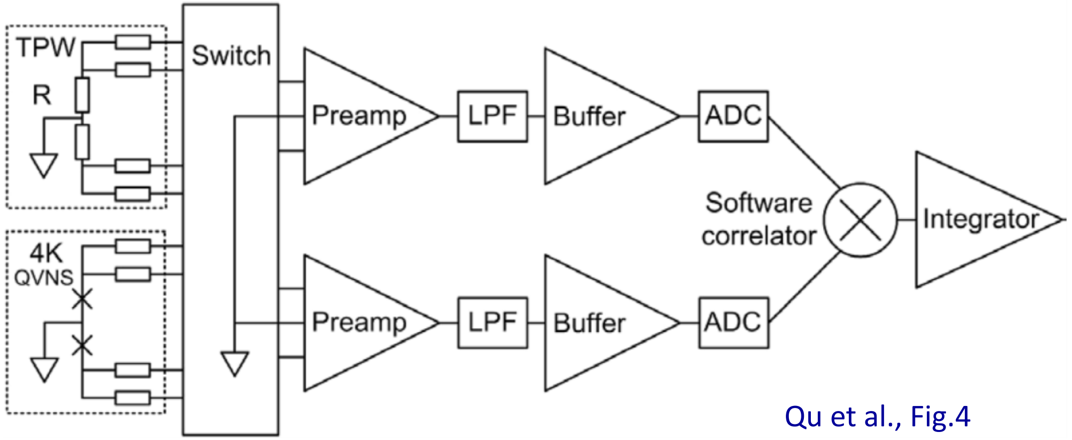
Johnson thermometry

Qu et al., Tab.1

Component	Term	Relative uncertainty
Ratio of the power spectral densities, S_R/S_Q	Statistical	3.2 ppm
	Model Ambiguity	1.8
	Aberrations / Dielectric losses	1.0
	Non-linearity	0.1
	EMI	0.4
	Total $u_r(S_R/S_Q)$	3.8
QVNS waveform S_Q	Frequency reference	< 0.001
	Quantization effects	0.1
	Total(S_Q)	0.11
TPW temperature T	Reference standard TPW cell	0.29 (0.08 mK)
	Temperature measurement	0.04 (0.01 mK)
	Hydrostatic pressure correction	0.08 (0.02 mK)
	Immersion effects	0.18 (0.05 mK)
	Total $u_r(T_W)$	0.35
Resistance R	Ratio measurement	0.05
	Transfer Standard	0.1
	AC–dc difference	0.1
	Relaxation effect	0.5
	Thermoelectric effect	0.1
	Total $u_r(R)$	0.53
	TOTAL (k_B)	3.9

$$\langle V^2 \rangle = 4kTR\Delta f$$

R = resistance



Qu et al., Fig.4

Measure ΔT from thermal noise

J. Qu et al., Improved electronic measurement of the Boltzmann constant by Johnson noise thermometry, Metrologia 52 S242, 2015

Doppler broadening thermometry

$$\Delta f_D = \sqrt{\frac{2kT}{mc^2}}\,\nu_0$$

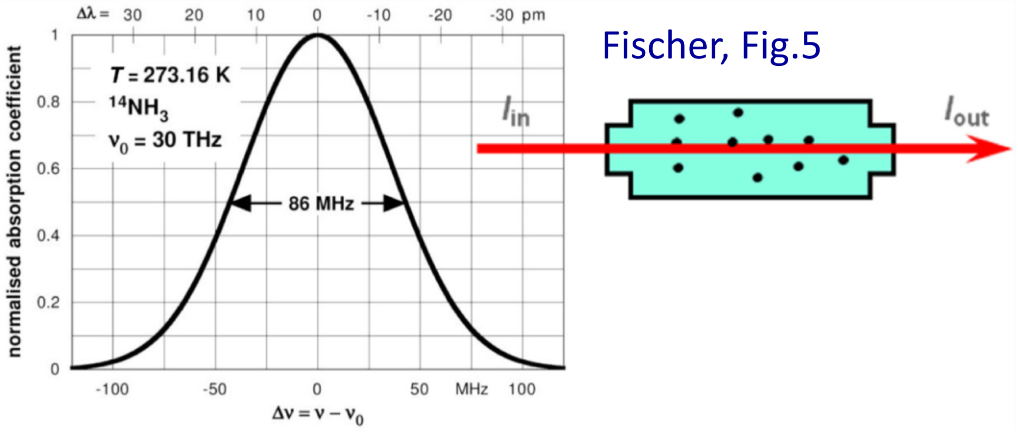
ν_0 = absorption frequency

Δf_D = Doppler broadening

m = atomic mass

J. Fischer, Progress towards a new definition of the kelvin, Metrologia 52 (2015) S364–S375

Fischer, Fig.5



Best results with alkali vapors (Cs, Rb)
Technology derives from the vapor-cell (secondary) frequency standards

Truong et al, Tab.6

Table 6. Experimental error budget for the determination of the Boltzmann constant at 296 K with a 2 h measurement campaign.

Source	Current $u_r(k_B)$ (ppm)	Upgrade $u_r(k_B)$ (ppm)
Statistical	5.8	0.1
Lorentz Width (Γ_L^{fit})	65	1.5
Laser Gaussian noise	16	0.001
Optical pumping	15	1
Etalons (misidentification)	15	0
Etalons (unresolved)	3	0.1
Spontaneous Emission	3.6	0.2
Temperature	1.9	0.2
Temp. Gradient	1.2	0.2
PD Linearity	1	1
Zeeman Splitting	<0.1	<0.1
Atomic recoil	<0.1	<0.1
Total (fit Γ_L)	71	2.0

G. W. Truong et al., Atomic spectroscopy for primary thermometry, Metrologia 52 S324, 2015

Review of laws and experiments

Constant-Volume Gas Thermometry (CVGT) Outdated, too high uncertainty	$pV = nRT$	R = molar gas constant
Acoustic Thermometry in Gas (AGT) He, Ar Gavioso et al., A determination of the molar gas constant R by acoustic thermometry in helium, Metrologia 52 S274, 2015	$v_0 = \sqrt{\frac{\gamma RT}{M}}$	$\gamma = C_p/C_V$ R = molar gas constant M = molar mass
Dielectric-Constant Gas Thermometry (DCGT) C. Gaiser C, T. Zandt, B. Fellmuth, Dielectric-constant gas thermometry, Metrologia 52 S217, 2015	$p = kT\frac{\epsilon - \epsilon_0}{\alpha_0}$	p = pressure ϵ, ϵ_0 = dielectric const α_0 = static polarizability
Refractive-Index Gas Thermometry (RIGT) P. M. C. Rourke et al., Refractive-index gas thermometry, Metrologia 56 032001, 2019	$p = kT\frac{(n^2 - 1)\epsilon_0}{\alpha_0}$	n = refractive index α_0 = static polarizability
Johnson Noise Thermometry Qu J et al., Improved electronic measurement of the Boltzmann constant by Johnson noise thermometry - Metrologia 52 S242, 2015	$\langle V^2 \rangle = 4kTR\Delta f$	R = resistance
Doppler-Broadening Thermometry G. W. Truong et al., Atomic spectroscopy for primary thermometry, Metrologia 52 S324, 2015	$\Delta f_D = \sqrt{\frac{2kT}{mc^2}}\nu_0$	Δf_D = Doppler broadening m = atomic mass ν_0 = absorption frequency
Blackbody Radiation One and only option for high temperature / Hand-held thermometers. P. Saunders, Uncertainty propagation through integrated quantities for radiation thermometry - Metrologia 55 863, 2018	$L_B(\lambda, T) = \frac{c_1}{n^2\lambda^5\left[\exp\left(\frac{c_2}{n\lambda T}\right) - 1\right]}$	Planck law c_1, c_2 = radiation constants n = refractive index

Featured reading: R. White, J. Fischer (guest editors), Focus on the Boltzmann Constant, Metrologia 52 (special issue) p.S213-S216, 2015.
J. Fischer, Progress towards a new definition of the kelvin, Metrologia 52 (2015) S364–S375.

Practical Temperature Measurements

Beware of cryogenic temperatures

- Matter approaches the ground state
- Low thermal conductance
- Low thermal capacitance
- Often, surprisingly long thermal constants
- Extreme rigidity, mechanical fragility
- Structures made of mixed materials may break down during cooling
- Thermal contacts are extremely poor
- Superconductors and superfluids are no friends
- Measurements and temperature controls are difficult
- Experimental difficulties everywhere

Suggested readings

- Franck Pobell, *Matter and Methods at Low Temperatures*, 3rd ed., Springer 2007
- T. H. K. Barron and G. K. White, *Heat Capacity and Thermal Expansion at Low Temperatures*, Springer 1999
- Steven W. Van Sciver, Helium Cryogenics, 2nd ed., Springer 2012
- P. Duthil P, Material Properties at Low Temperature, arXiv 1501.07100, 2015

Triple point of water

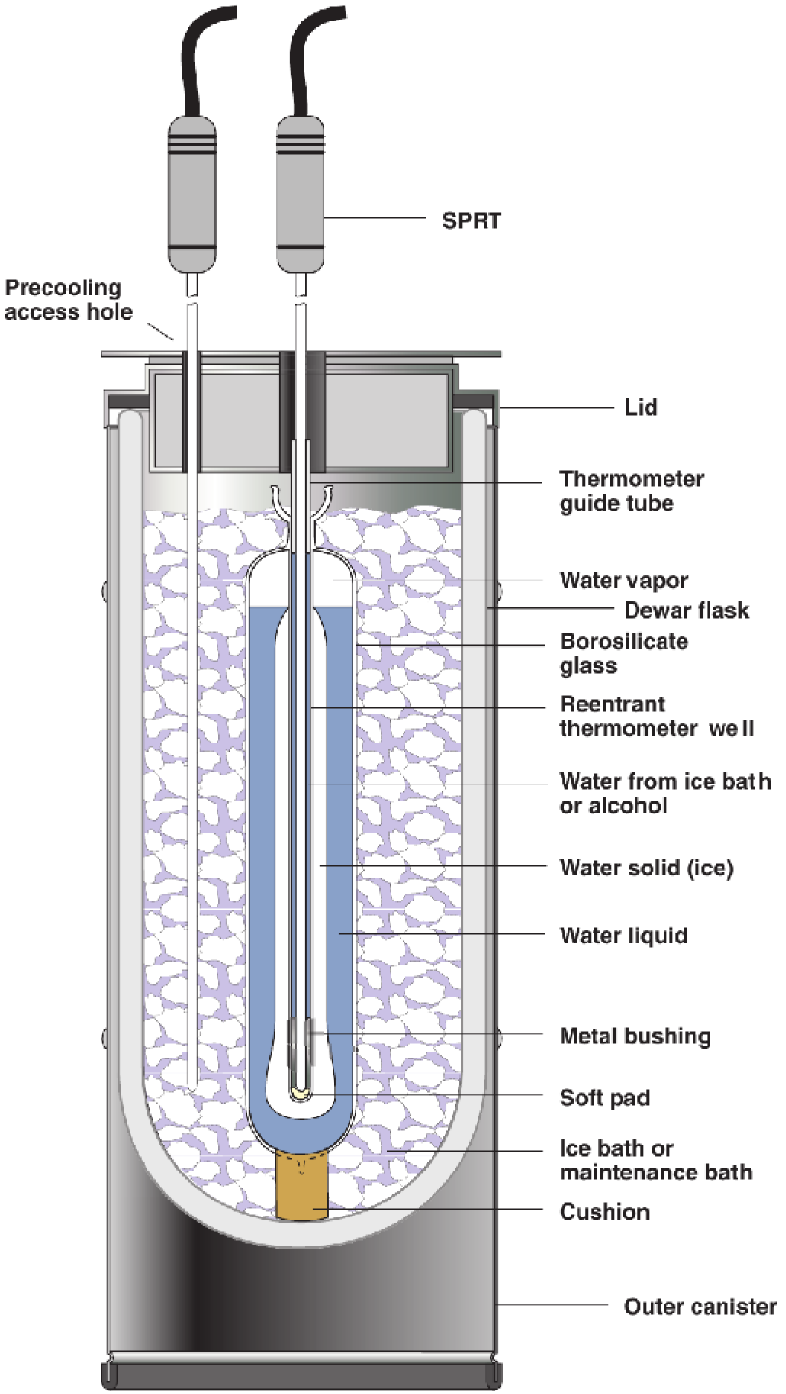
Water, vapor and ice in equilibrium

T = 373 K = 0.01 °C,

P = 620 Pa

Slowly-boiling water with pieces of ice in the bath!

Atmospheric pressure ≈10^5 Pa



Picture from "Why use a triple point of water?" Fluke Calibration application note, 2013

International Temperature Scale ITS90

Tab.1 of Guide to the Realization of the ITS-90, Introduction, BIPM

T_{90} / K	t_{90} / °C	Substance[1]	State[2]	$W_r(T_{90})$[3]	$u(T_{90})$ / mK	$T-T_{90}$ / mK	$u(T-T_{90})$ / mK
3 to 5	–270 to –268	He	vp	–	0.2 (0.03)	0	0.1
13.8033	–259.3467	e-H_2	tp	0.00119007	0.03 (0.05)	0.44	0.14
≈ 17.035	≈ –256.115	e-H_2 or He	vp or gp	0.00229646	0.2 (0.03)	0.51	0.16
≈ 20.27	≈ –252.88	e-H_2 or He	vp or gp	0.00423536	0.2 (0.03)	0.32	0.17
24.5561	–248.5939	Ne	tp	0.00844974	0.09 (0.05)	–0.23	0.20
54.3584	–218.7916	O_2	tp	0.09171804	0.06 (0.02)	–1.06	1.6
83.8058	–189.3442	Ar	tp	0.21585975	0.06 (0.02)	–4.38	1.3
234.3156	–38.8344	Hg	tp	0.84414211	0.2 (0.1)	–3.25	1.0
273.16	0.01	H_2O	tp	1.00000000	0.05 (0.03)	0	0
302.9146	29.7646	Ga	mp	1.11813889	0.2 (0.03)	4.38	0.4
429.7485	156.5985	In	fp	1.60980185	0.8 (0.2)	10.1	0.8
505.078	231.928	Sn	fp	1.89279768	0.6 (0.2)	11.5	1.3
692.677	419.527	Zn	fp	2.56891730	0.8 (0.4)	13.8	6.9
933.473	660.323	Al	fp	3.37600860	2 (0.5)	28.7	6.6
1234.93	961.78	Ag	fp	4.28642053	4 (0.6)	46.2	14
1337.33	1064.18	Au	fp	–	25 (8)	39.9	20
1357.77	1084.62	Cu	fp	–	25 (8)	52.1	20

Absolute measurement is too tough.
Practical scale published by the CIPM/BIPM CCT.
A set of calibration points 0.65 K to 1357.77 K

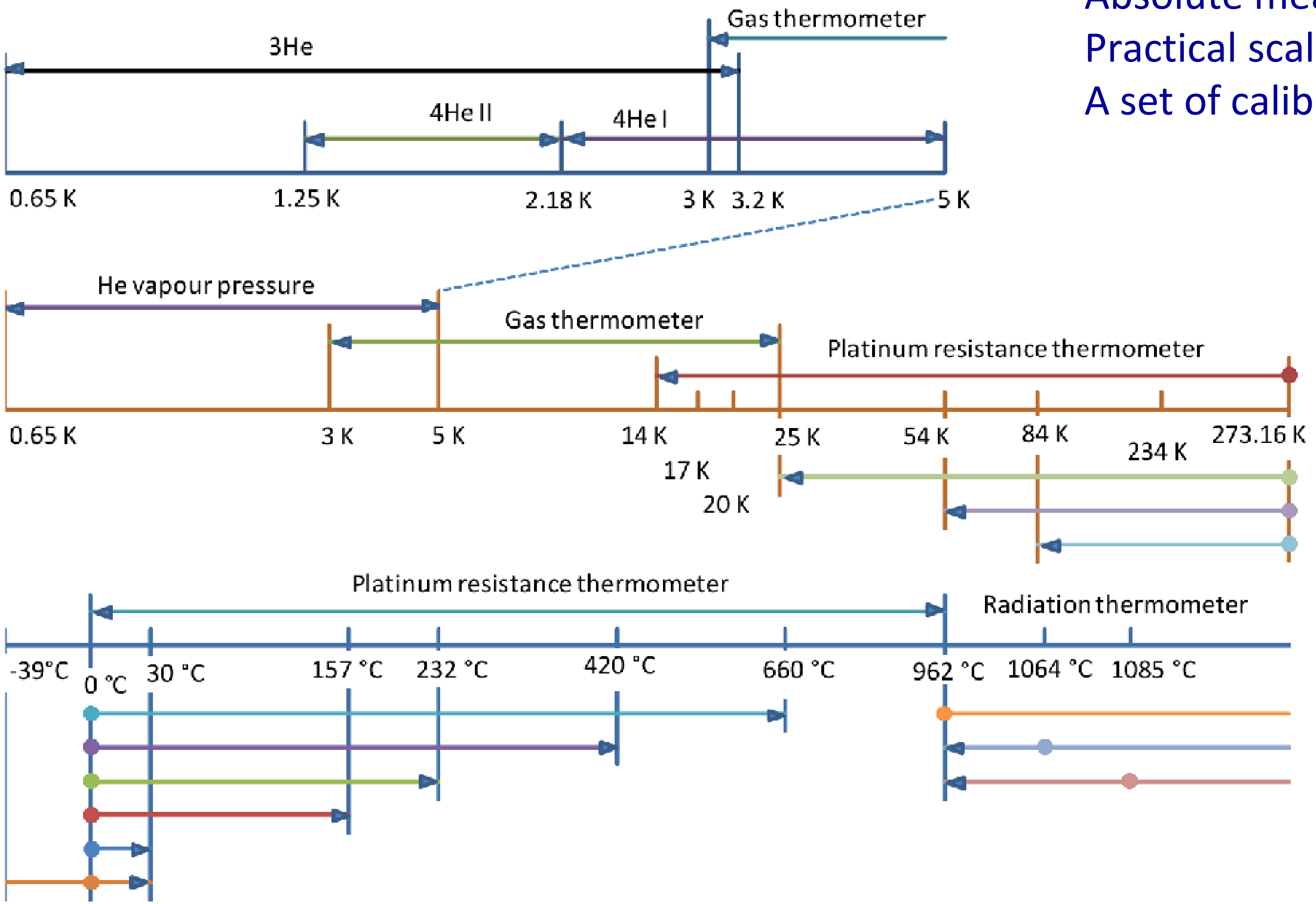
Feature reading

H. Preston-Thomas, The International Temperature Scale of 1990 (ITS-90), Metrologia 27 002, 1990

Reference document: Guide to the Realization of the ITS-90, BIPM

International Temperature Scale ITS90

Fig.1 of Guide to the Realization of the ITS-90, Introduction, BIPM



Absolute measurement is too tough.
Practical scale published by the CIPM/BIPM CCT.
A set of calibration points 0.65 K to 1357.77 K

Calibration points, see Tab.1 of the Guide to the Realization of the ITS-90

H. Preston-Thomas, The International Temperature Scale of 1990 (ITS-90), Metrologia 27 002, 1990

Platinum resistance thermometry

SPRT = Standard Platinum Resistance Thermometer

Properties of Pt

- Chemically stable
- High melting point (2041 K)
- Resistivity 105 nΩ m at room temperature
- Temperature Coefficient TC(R) ~ 3.9 10^{-3}/K at room T

SPRT in ITS-90

- Widest range of temperature 13.8033 … 1234.93 K (962 Celsius)
- The "Guide to the Realization of the ITS-90" has a 71-page section on the Pt resistance thermometry

For reference, Cu has resistivity 16.8 nΩ m at room temperature, and similar TC

Sealed Pt resistance thermometer

Fig. 1, 25 Ω SPRT

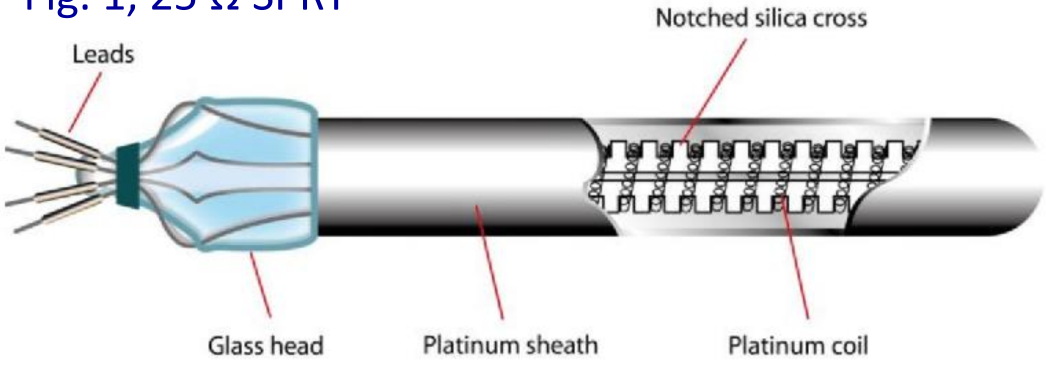


Fig. 2 Long-stem 25 Ω SPRT

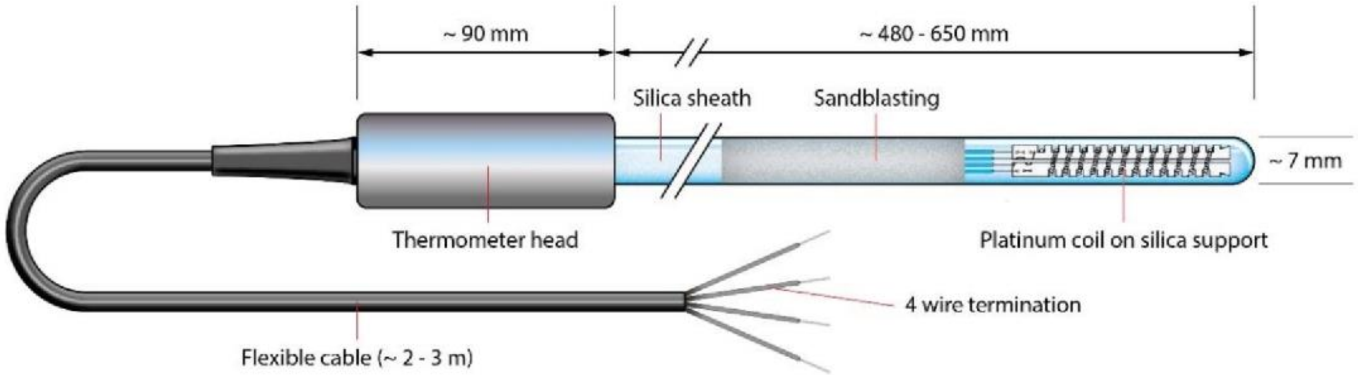
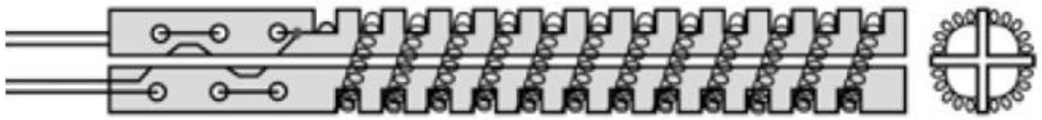


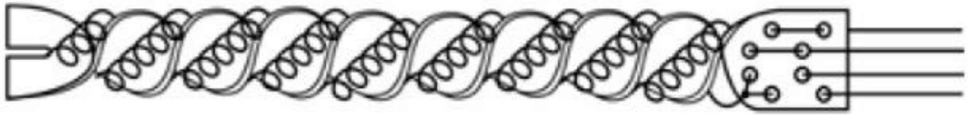
Fig. 3

High-temperature 0.25 Ω SPRT

Room-temperature 25 Ω SPRT



Room-temperature 25 Ω SPRT



Figures from the Guide to the Realization of the ITS-90, Platinum Resistance Thermometry

Platinum thermometer

- Best “simple” method
  - Most accurate
  - Most reproducible
- Metrological resistors
  - 0.25 Ω for high T
  - 25 Ω resistors for “regular” T
- General purpose resistors 100 Ω and 1 kΩ at room T
- Low TC, $3.9\text{x}10^{-3}$/K at room T
  - Small signal
  - Poor for temperature control if fast operation is required
  - four-wire measurement is mandatory

Negative Temperature Coefficient (NTC) resistors

- Semiconducting material such as sintered metal oxides, Fe_2O_3 with Ti doping (n-type), or NiO with Li doping (p-type)
- Negative TC achieved via generation of carriers
- Compared to Pt resistors
  - Narrow range, −55 °C to +150 °C typical
  - Higher R, 1-100 kΩ
  - TC ten-fold higher
  - Overall, suitable to temperature controls
  - Poorer reproducibility, 0.1-0.3 K (or worse) if replaced
- YSI brand provides 0.1 K replaceability, and 0.01 K ageing after 100 months

The Steinhart Hart equation

Better than 10 mK accuracy in the temperature range

$$\frac{1}{T} = a + b\ \ln R + c(\ln R)^3$$

T = absolute temperature
a, b, c = Steinhart-Hart Parameters
R = resistance

$$\ln R = \frac{b}{3cx^{1/3}} - x^{1/3}$$

$$y = \frac{1}{2c}\left(a - \frac{1}{T}\right)$$

$$x = y + \sqrt{\left(\frac{b}{3c}\right)^3 + y^2}$$

Example, 3 kΩ NTC
$a = 1.4 \times 10^{-3}$
$b = 2.37 \times 10^{-4}$
$c = 9.9 \times 10^{-8}$

300 K, $R = 2829.4\ \Omega$
301 K, $R = 2709.4\ \Omega$

at 300 K, $\Delta R/R\Delta T = 4.33 \times 10^{-2}/\mathrm{K}$

(compare to $3.9 \times 10^{-3}/\mathrm{K}$ of Pt_{100})

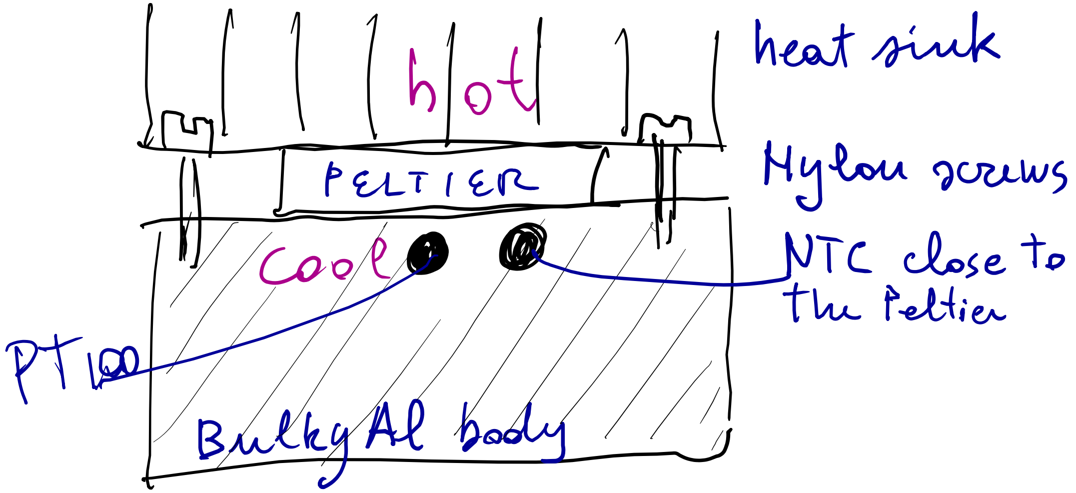
Steinhart JS, Hart SR - Calibration curves for thermistors - Deep-Sea Research 15, 1968

Practical issues

- Self heating, $P = RI^2 = V^2/R$
  - Choose optimal I as a tread off
    - High I, higher gain
    - Low I, low self-heating
- Heat conduction through wires
  - Thermalize the wires
- Thermocouples
  - O_2 absorbed in Cu wires results in 100-200 nV/K unpredictable Seeback effect

Experimental tricks for temperature control

beware of the Peltier
Thermal conductance



NTC for gain & SNR
Pt100 for absolute meas

What if the NTC is broken?

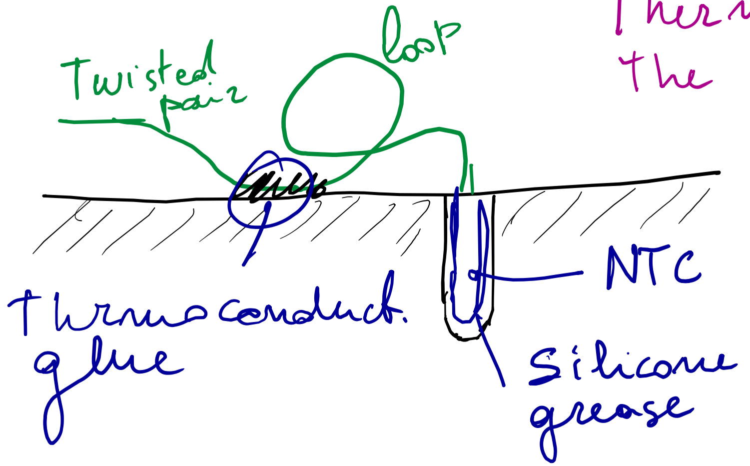
Thermal conductivity

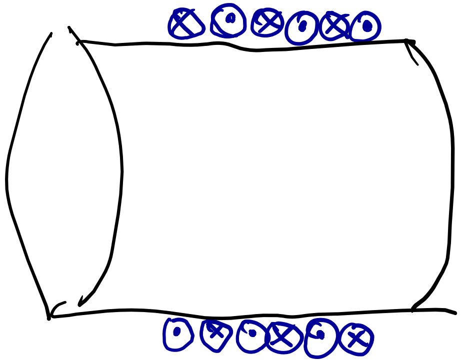
Al	220	W/m K	→ easy machining
Cu	390	W/m K	→ machining nightmare

$^{63.5}$Cu, $\rho = 8.96\ \mathrm{g/cm^3}$ ^{27}Al, $\rho = 2.7\ \mathrm{g/cm^3}$

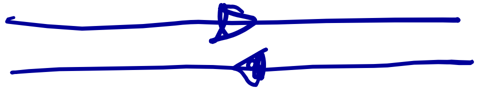
Whenever possible, avoid cooling

thermalize the wire





heating with twin wires minimizes magnetic field



PI and PID temperature control – Ziegler-Nichols

In the PID control, the power P needed to correct the temperature error Θ is

$$P = k\left(\Theta + T_d \frac{d\Theta}{dt} + \frac{1}{T_i}\int \Theta\, dt\right)$$

In the PI control, the derivative term is suppressed.

Tuning the PI/PID control means finding the most appropriate

gain k and

time constants T_d and T_i.

The Zigler-Nichols method is a simple, well established (dates to the '40s), rock solid method to tune the control. One can chose between:

1A. suppress the derivative and integral terms, thus $P = k\Theta$

2A. measure the step response

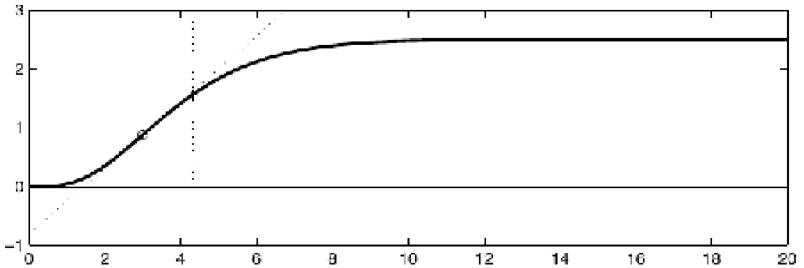
1B. suppress the derivative and integral terms, thus $P = k\Theta$

2B. measure gain and frequency at the small-oscillation critical point

The ZN method is the workhorse in the art of (simple) process control.

Ziegler-Nichols' Step Response Method

- Switch controller to manual.
- Make a step in the control variable.
- Log process output. Normalize the curve so that it corresponds to a unit step.
- Determine intercepts of tangent with steepest slope i.e. parameters a and L. The controller parameters are obtained from a table.



Ziegler-Nichols' Step Response Method

Data: apparent time delay L and intercept a. Controller parameters are given by

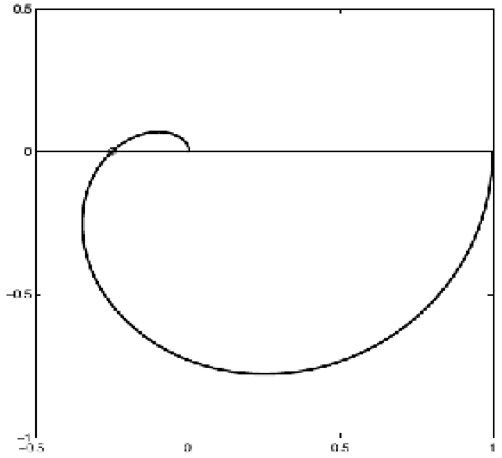
Controller	k	T_i	T_d	T_p
P	$1/a$			$4L$
PI	$0.9/a$	$3L$		$5.7L$
PID	$1.2/a$	$2L$	$L/2$	$3.4L$

Parameter T_p is an estimate of the response time of the closed loop system.

Preferred

Ziegler-Nichols' Frequency Response Method

- Switch the controller to pure proportional.
- Adjust the gain so that the closed loop system is at the stability boundary.
- Determine the gain k_u (the ultimate gain) and the period T_u (the ultimate period) of the oscillation.
- Suitable controller parameters are obtained from a table.



Ziegler-Nichols' Frequency Response Method

Data: ultimate gain k_u and ultimate period T_u. Controller parameters given by.

Reg.	k	T_i	T_d	T_p
P	$0.5k_u$			T_u
PI	$0.4k_u$	$0.8T_u$		$1.4T_u$
PID	$0.6k_u$	$0.5T_u$	$0.125T_u$	$0.85T_u$

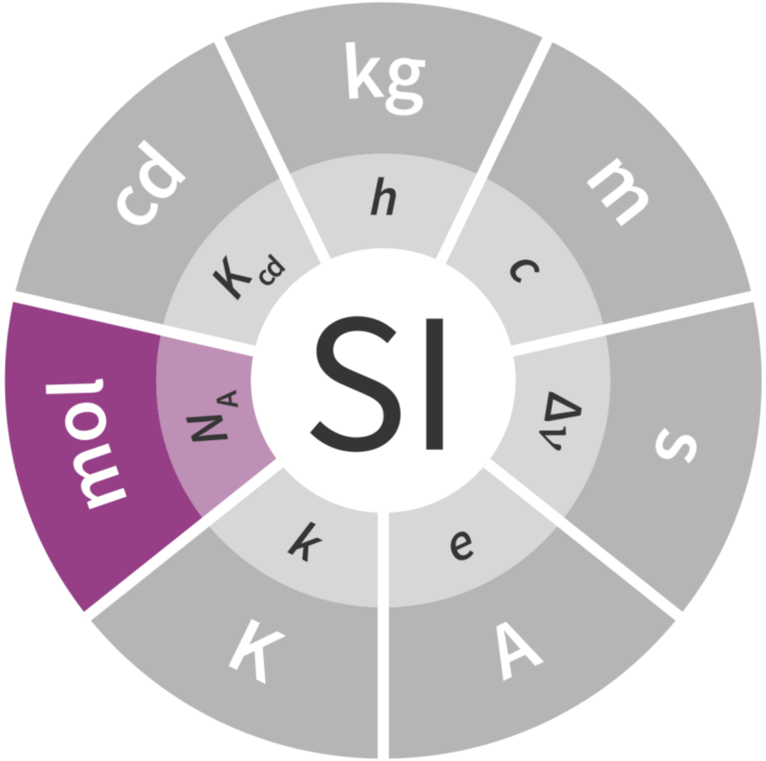
Parameter T_p is an estimate of the response time of the closed loop system.

Bibliography

Classical references

- H. S. Carslaw, J. C. Jaeger, *Conduction of heat in solids*, 2nd ed., Oxford 1959
- Terry J. Quinn, *Temperature*, Academic press 1983

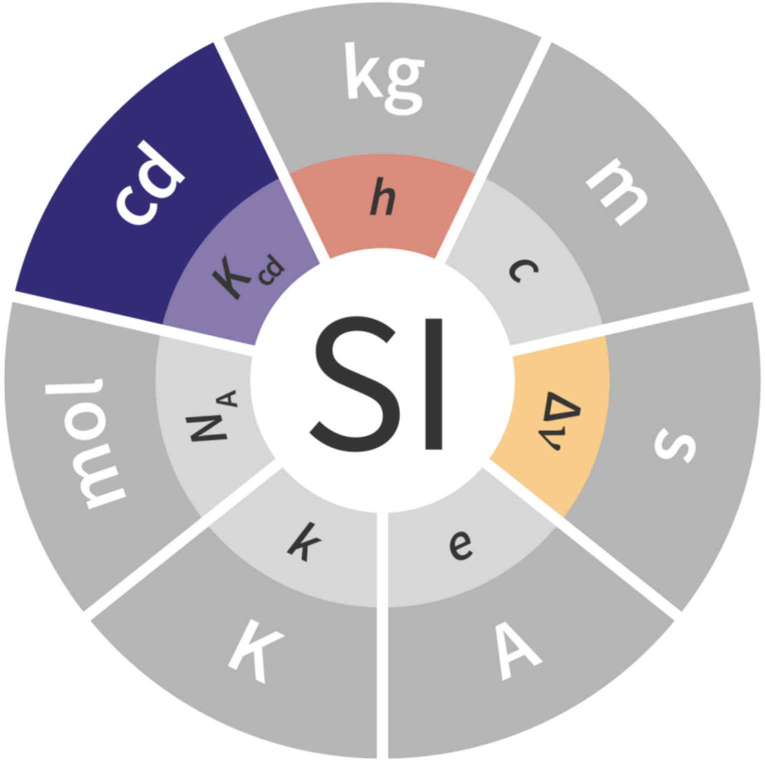
Mole



The mole, symbol mol, is the SI unit of amount of substance. One mole contains exactly 6.02214076×10^{23} elementary entities. This number is the fixed numerical value of the Avogadro constant, N_A, when expressed in the unit mol^{-1} and is called the Avogadro number. The amount of substance, symbol n, of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.

New in 2019

Candela



The candela, symbol cd, is the SI unit of luminous intensity in a given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, K_{cd}, to be 683 when expressed in the unit of lm W^{-1}, which is equal to cd sr W^{-1}, or cd sr $kg^{-1}\ m^{-2}\ s^{3}$, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{Cs}$.

Same as in 1979

Note that c is *not colored*. Although both kg and m depend *individually* on c, the candela does not because c *drops out* from its definition

Radiometric and photometric quantities

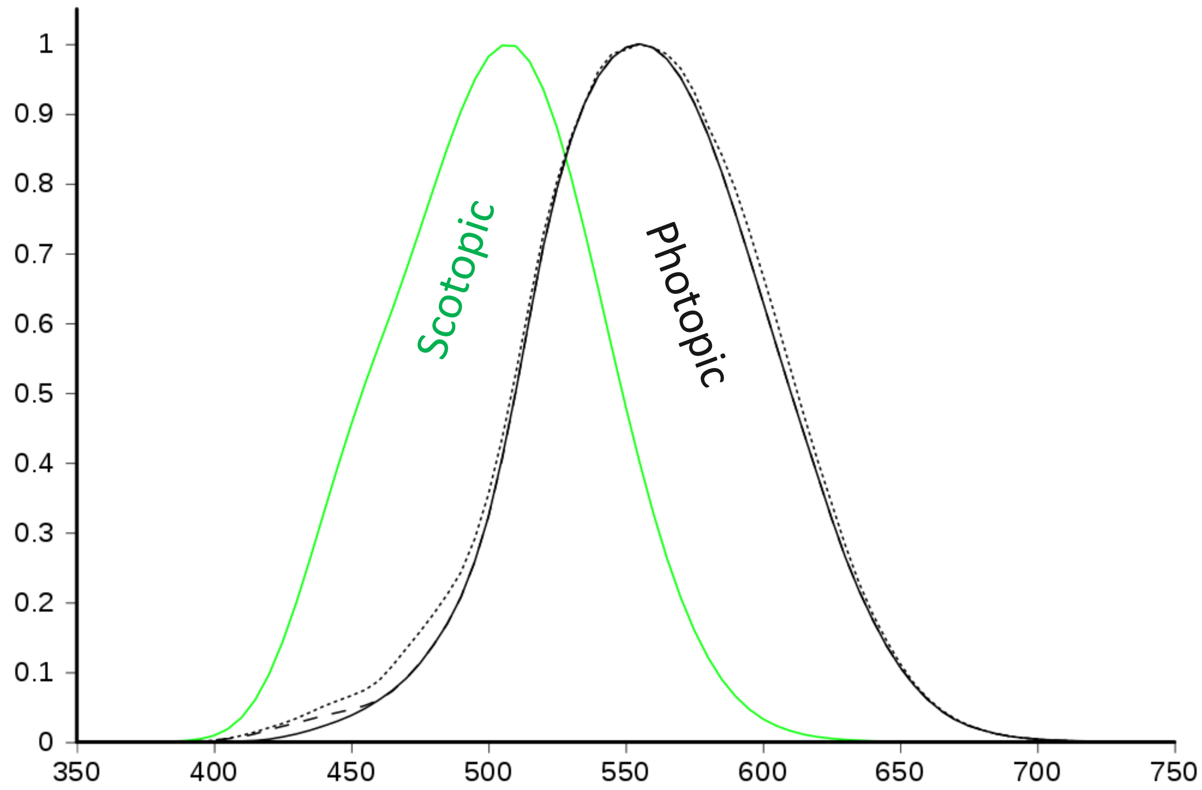
Photometry differs from radiometry in that all quantities are weighted for the response of the human eye

Radiometry			Photometriy		
Quantity	Symbol	Units	Quantity	Symbol	Units
radiant Power	Φ_e	W	Luminous flux	Φ_v	lumen (lm)
Radiant intensity	I_e	W/sr	Luminous intensity	I_v	candela (cd)
Radiance	L_e	W m^{-2} sr^{-1}	Luminance	L_v	cd/m^2 (nit)
Irradiance	E_e	W/m^2	Illuminance	E_v	lux (lx)

- High efficiency (led and fluorescent) consumer lamps are confusing
  - People think in “watts of the equivalent tungsten lamp”
  - and do not understand the luminous flux
- The “brightness” of monitors and mobile phones is actually luminance
- 500 nit is comfortable outdoor vision
- Mobile phones 500-1500 nits (record 2000, Apple and Samsung)
- Monitors 400-500 nits, up to 1000 nits for pro photography

Vision Types

Vision type		Luminance	Sensors	Colors
Photopic	sunny to twilight	3 to 10^6 cd/m^2	mainly cones	full colors
Scotopic	dark to full moon	10^{-6} to $3\text{x}10^{-3}$ cd/m^2	mainly rods	black and white
Mesopic	in between	in between	rods and cones	degraded colors



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