RF Instrumentation

Enrico Rubiola

2019-2020

- Burden:
  21 H Lectures and work (E.R.)
  8 H labs (Pierre-Yves Bourgeois)

#1 Thursday, Sept 12, 2019

1.5 Hours

Where Are Instruments Going?

input → dedicated front-end → ADC → fast processing → slow processing → output

broadly similar iris/retina/optical-nerve/brain

Moore Law: describes exponentially growing technology

12-bit converters – from my memory (years may not be accurate)

- **1985-1990 15-20 MS/s**
- **2005 250 MS/s**
- **2018 12 GS/s**

most figures are from

ANALOG-DIGITAL CONVERSION

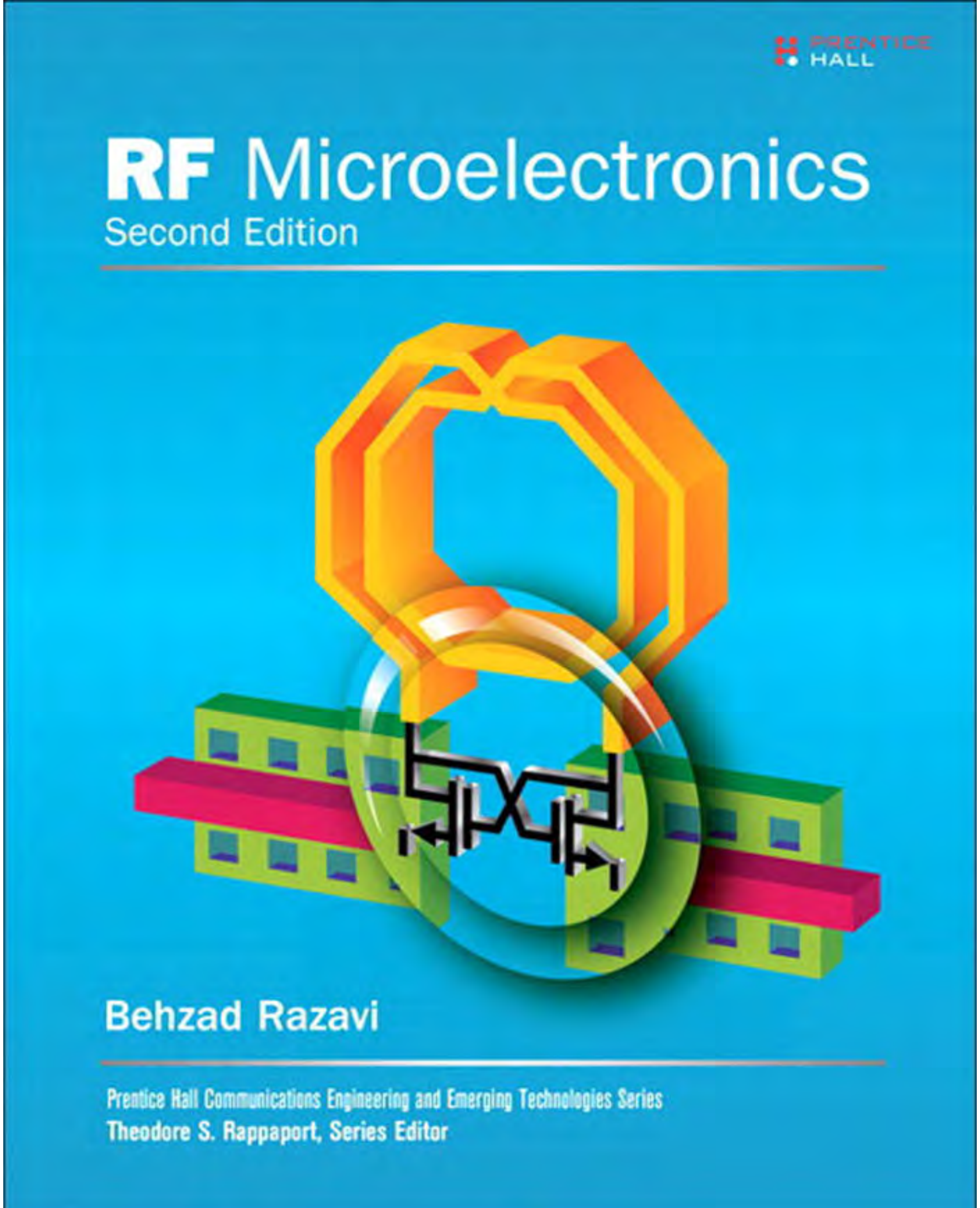
Walt Kester

Editor

ANALOG DEVICES

1138 pages
© AD, but free

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

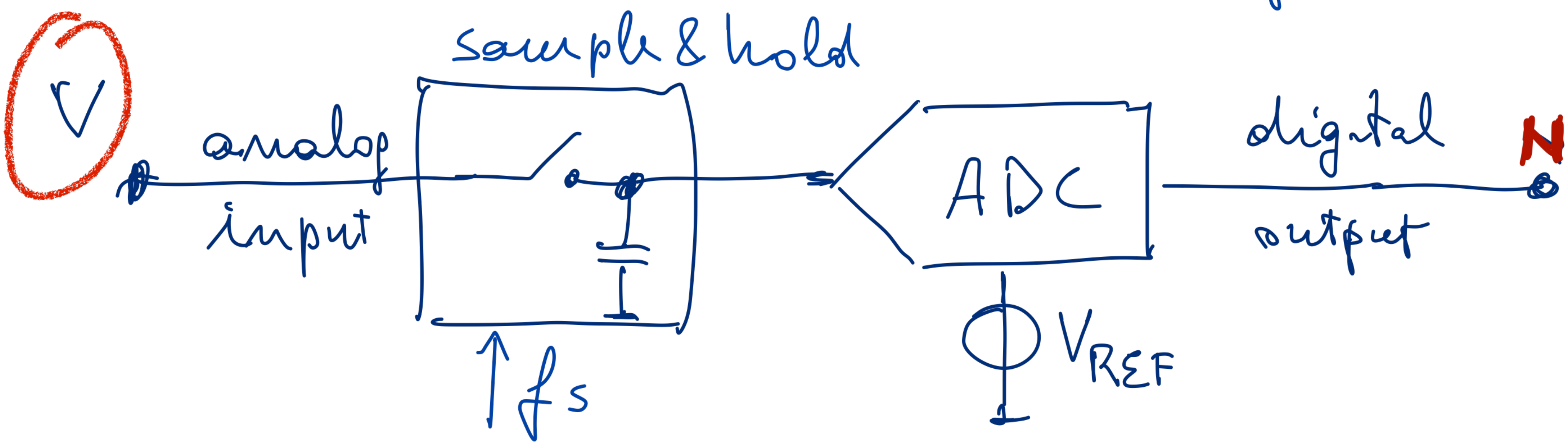


Useful textbook
(950 pages)

Conversion Basics

Analog to Digital Conversion

- Sample (time)
- Compare to a ref
- Round to integer

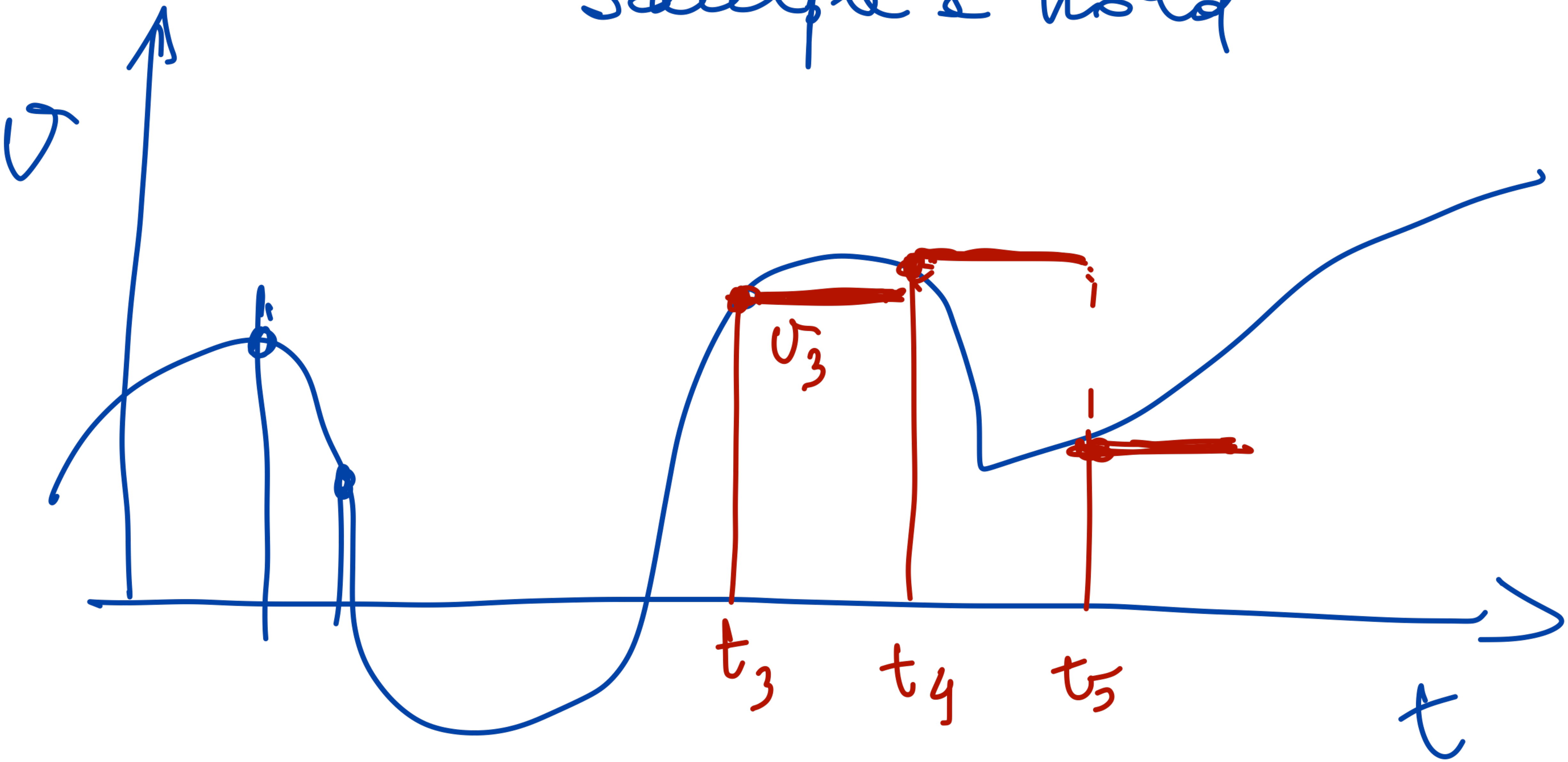


Integer

Real

$$\boxed{\mathrm{nint}(\cdot)} \quad N \sim K \frac{V}{V_{REF}}$$

Sample & hold



t_1	v_1
t_2	v_2
t_3	v_3

f_s = sampling frequency

nint = nearest integer

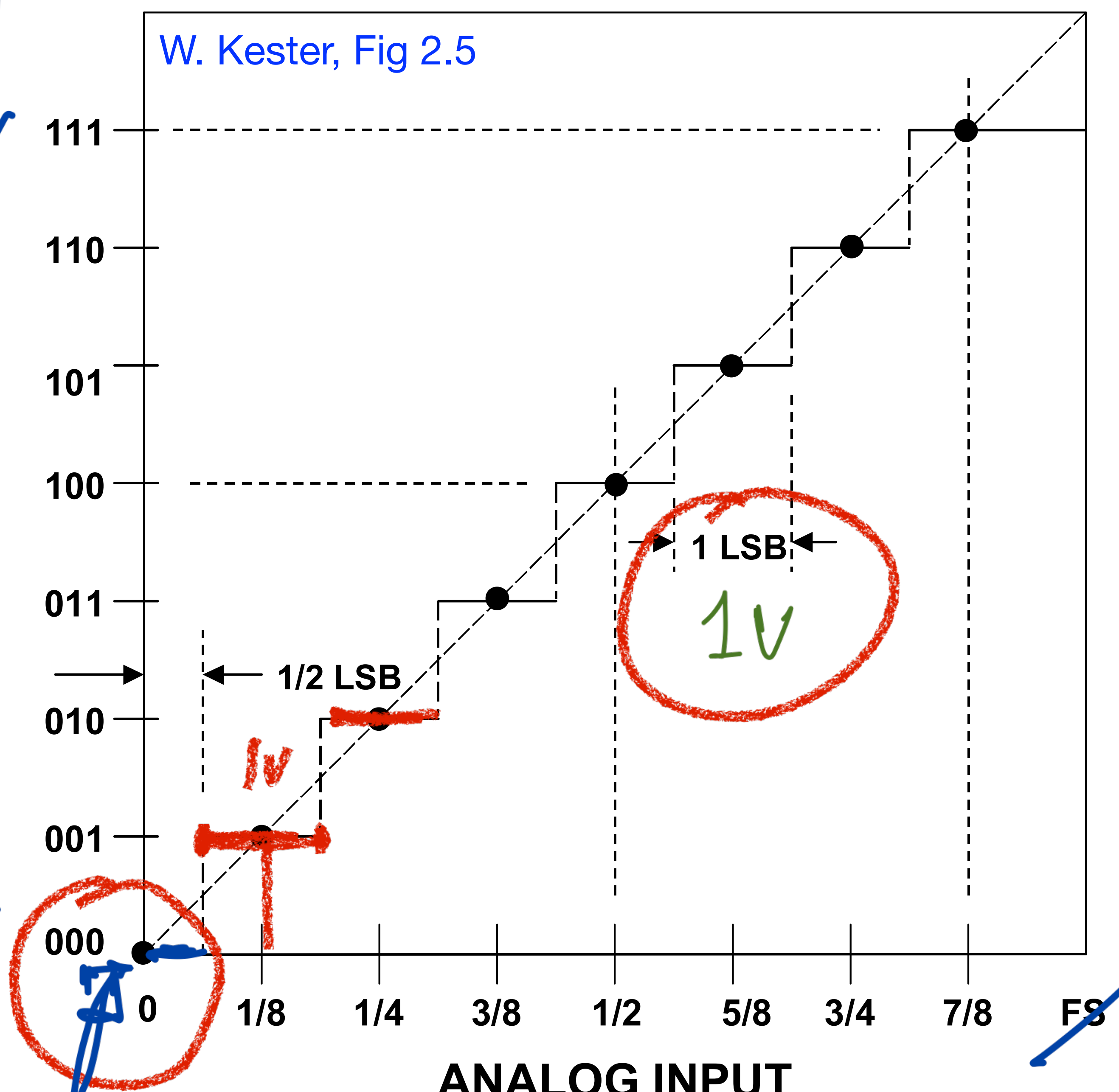
Example: 3-bit converter

N

7V

1V

0V



-0.5 .. -0 V
unused

(8V)

1 bit ⟷ 1 V

0 .. 0.5 V → 0 V

0.5 .. 1.5 V → 1 V

Least Significant Bit

V_{FSR} Full Scale Range

$$N \left\lfloor K \underbrace{\frac{V + \boxed{V_{LSB}/2}}{V_{FSR}}}_{0..1} \right\rfloor$$

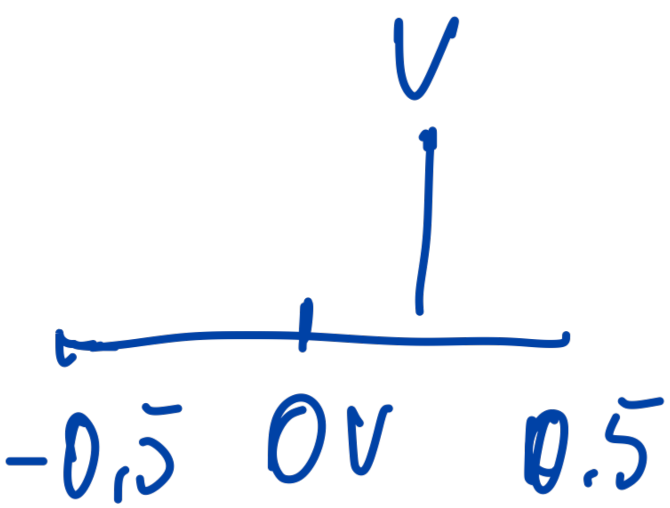
W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

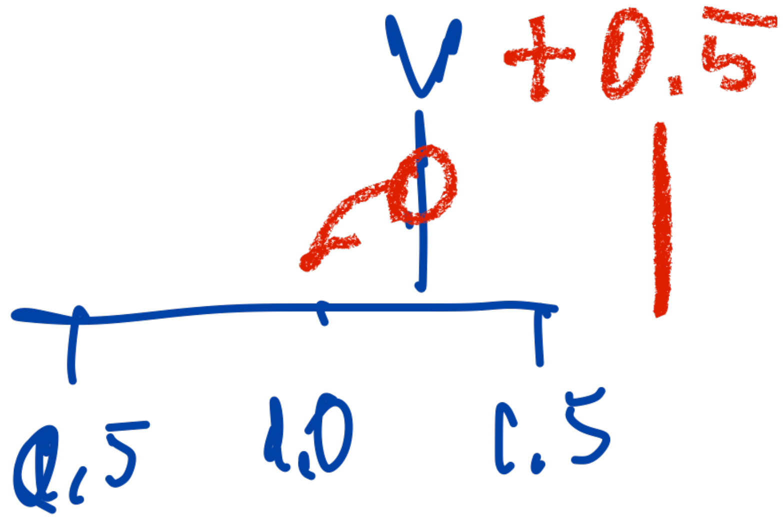
Knuth parentheses

$\lfloor \quad \rfloor$ floor
closest integer "less"

$\lceil \quad \rceil$ ceil
closest integer "more"

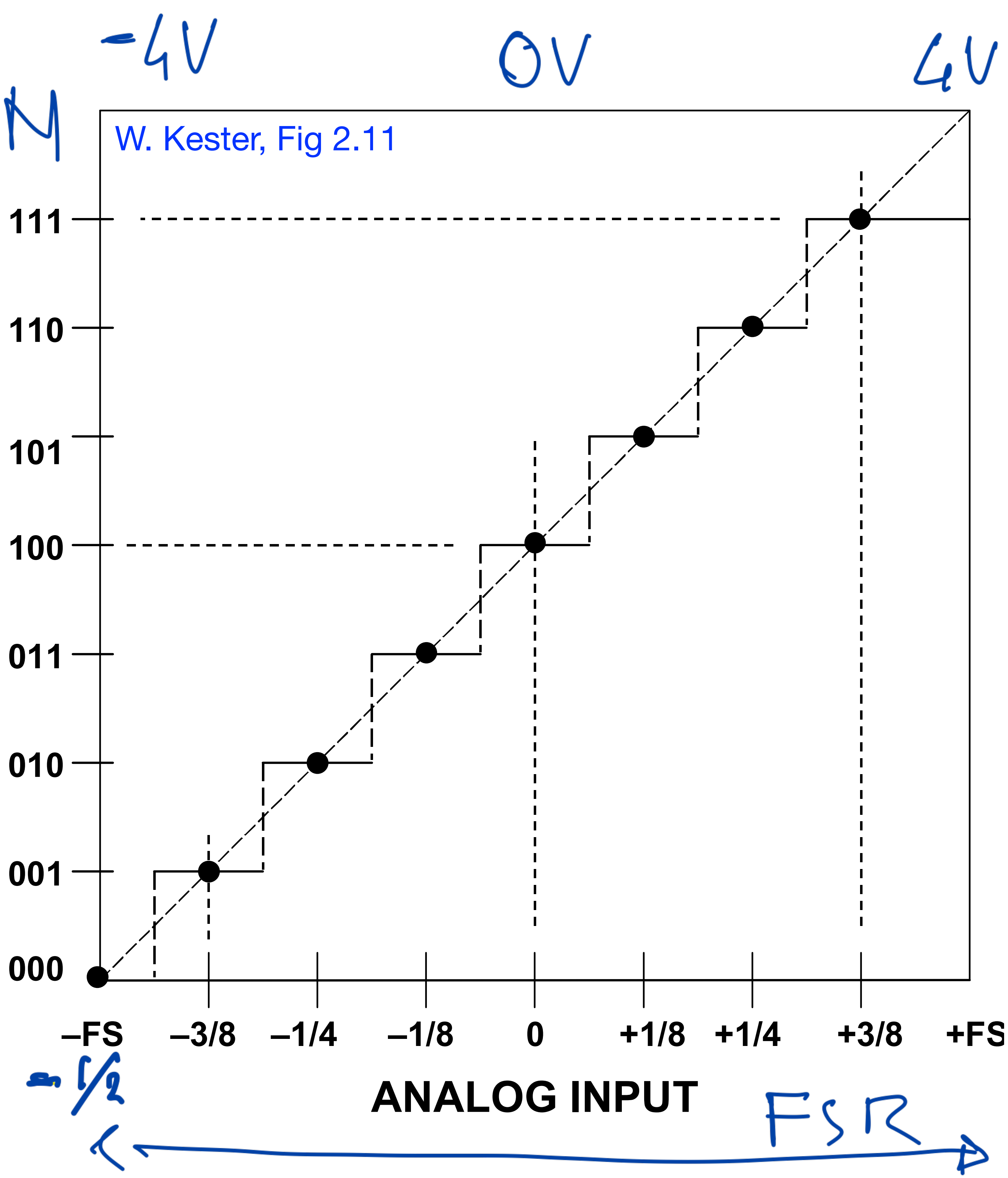
$\frac{1}{2} V_{LSB}$





$\lfloor V+0.5 \rfloor = 1$

Bipolar Input – Example



$V_{FS} = 4\,V$

$F_{FSR} = 8\,V$

$2^m = 8$ (3 bits)

<0 >0

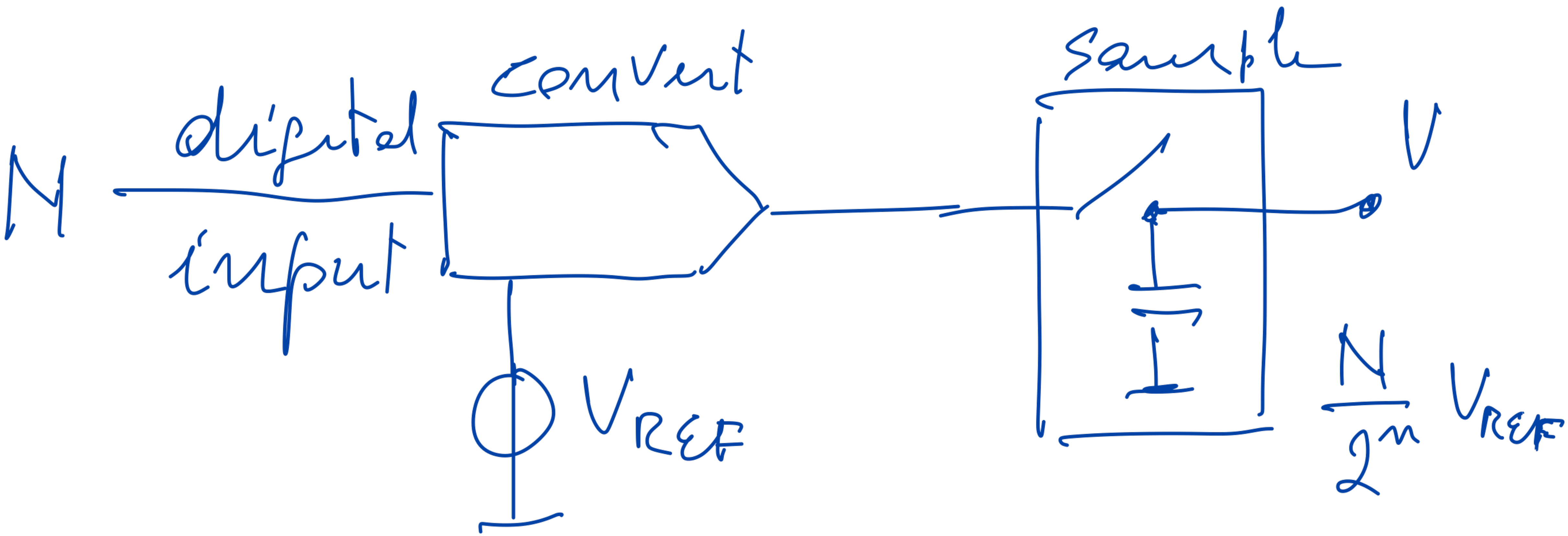
$$N \sim 2^m \underbrace{\frac{V + 1/2\, V_{FSR}}{V_{FSR}}}_{0\,..\,1}$$

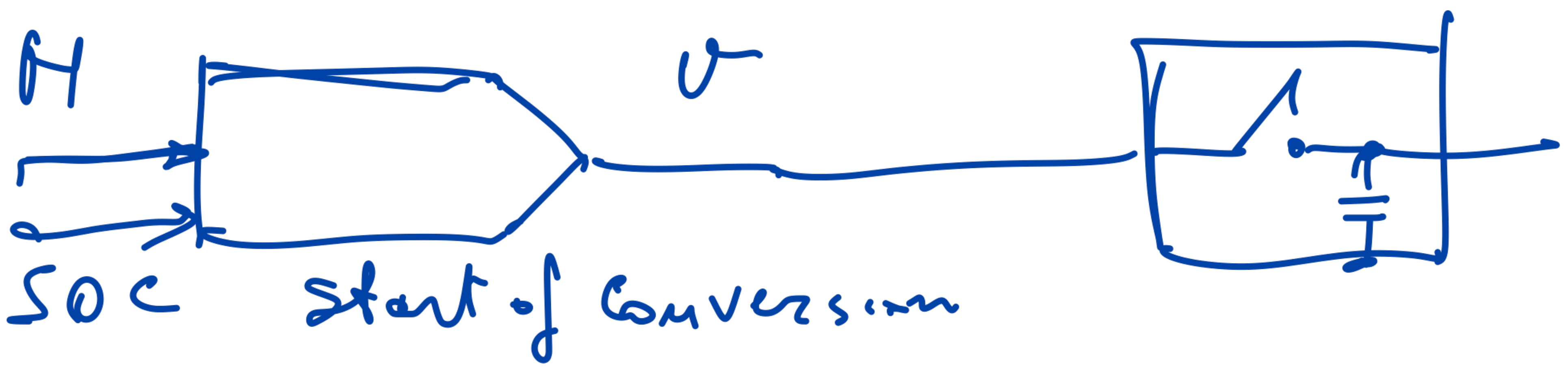
convert to the appropriate integer

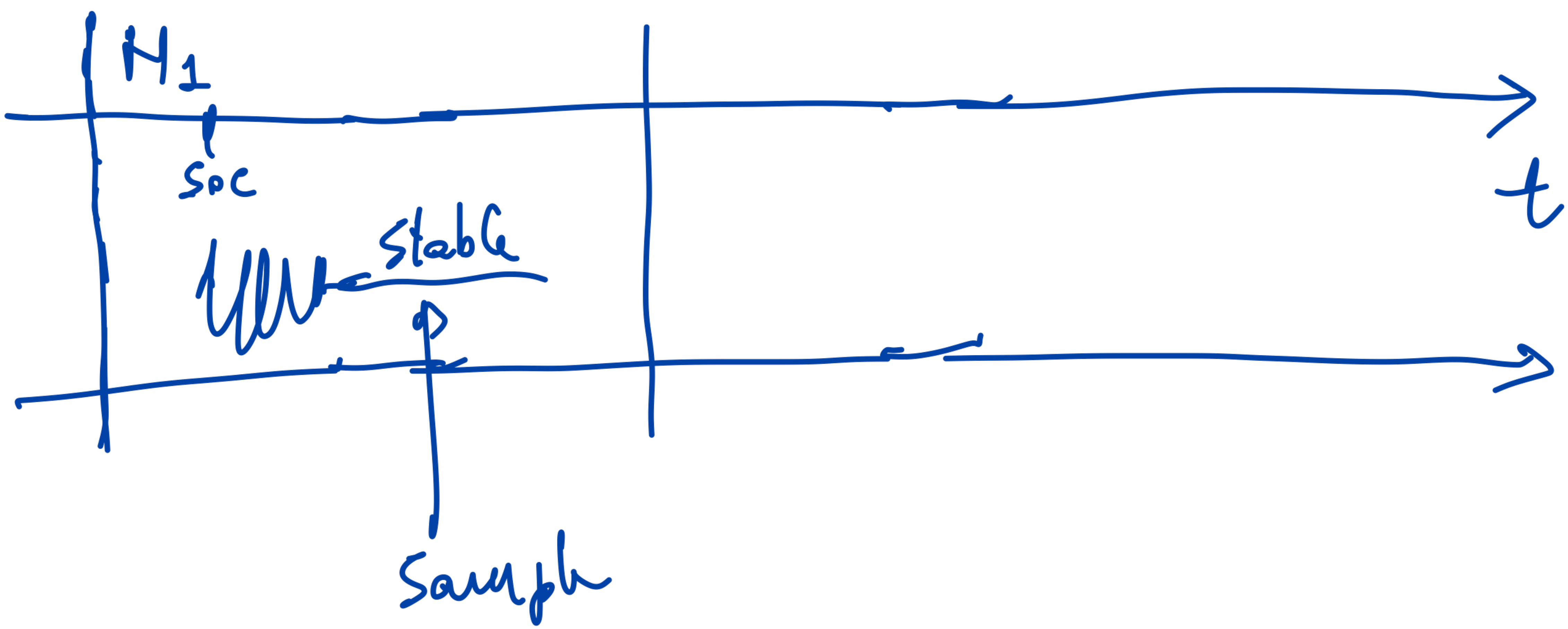
W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Digital to Analog Conversion

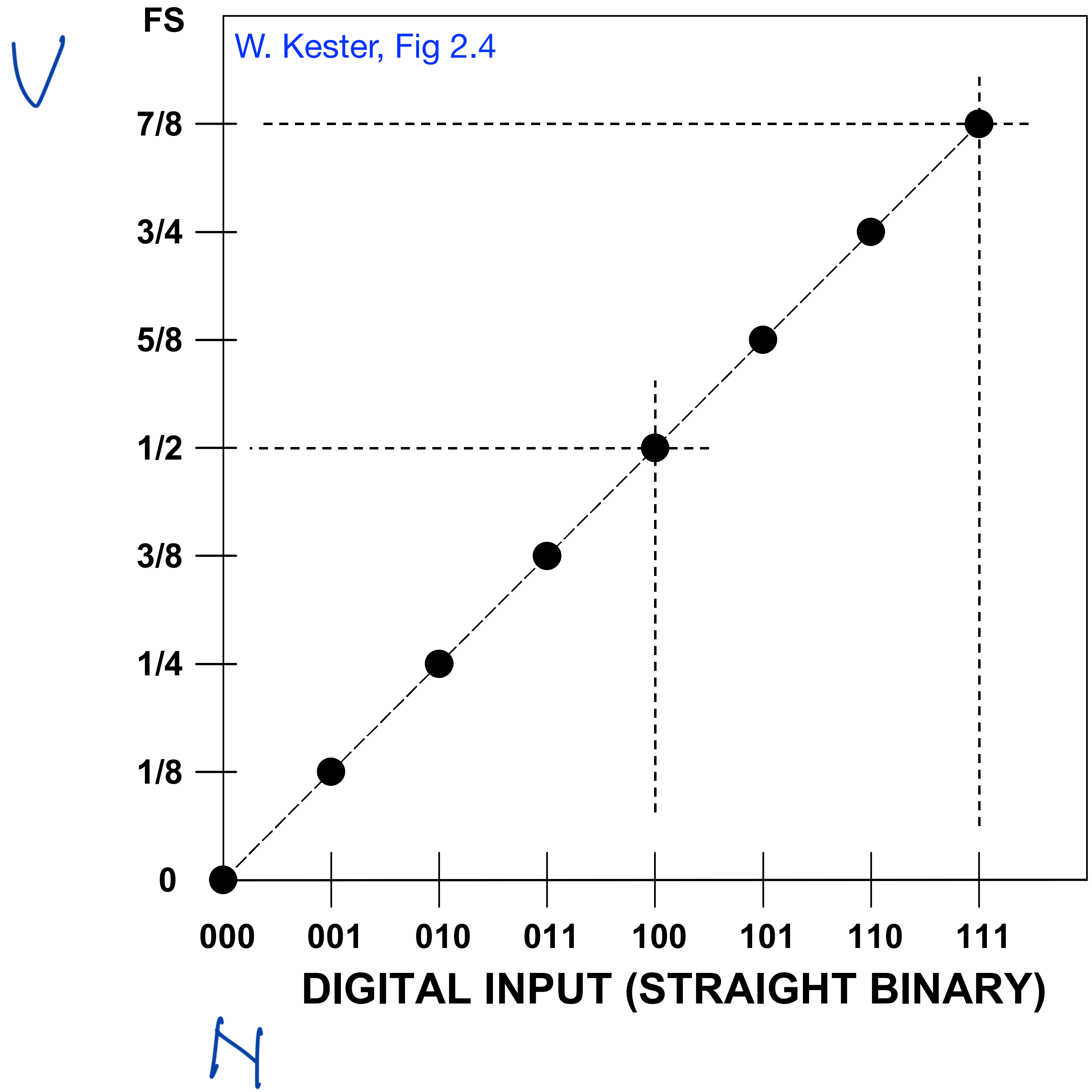
- Convert
- Sample




H
v
SOC
start of conversion


H1
SOC
stable
t
sample

Example: 3-bit Converter



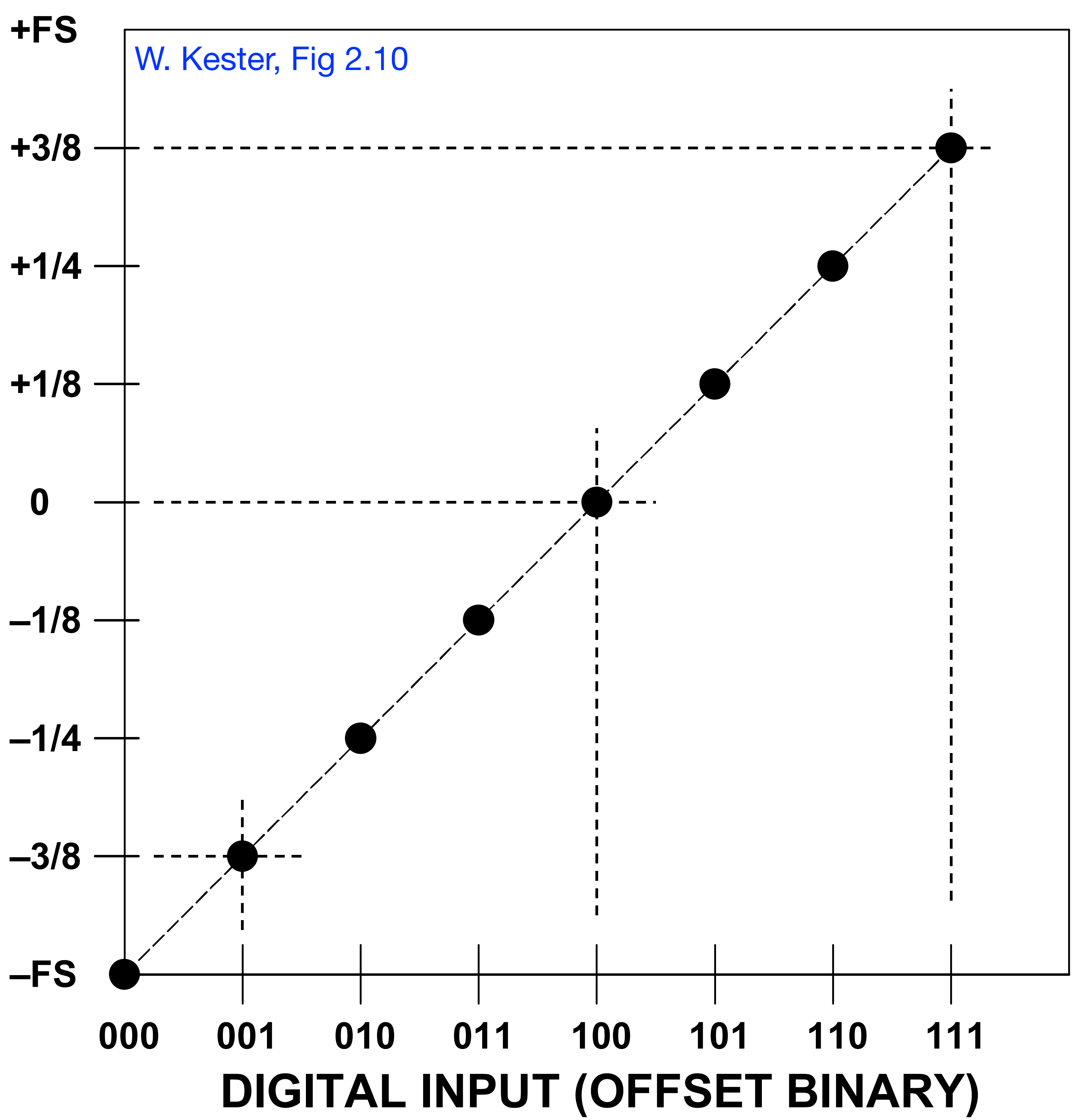
input N

$N \in 0 .. 2^n - 1$

$V = \frac{N}{2^n} V_{FSR}$

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Bipolar Conversion



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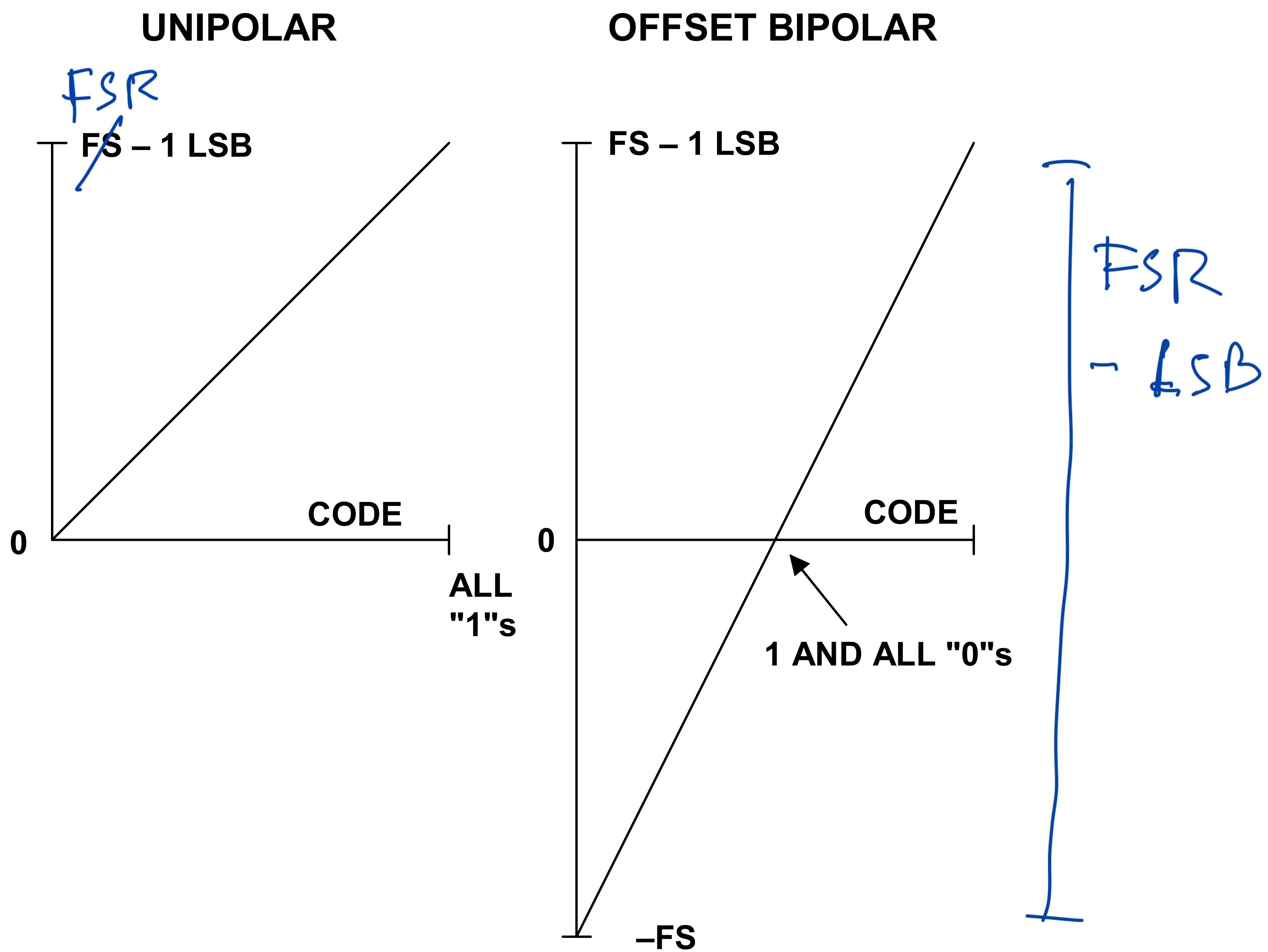
Conversion, Summary



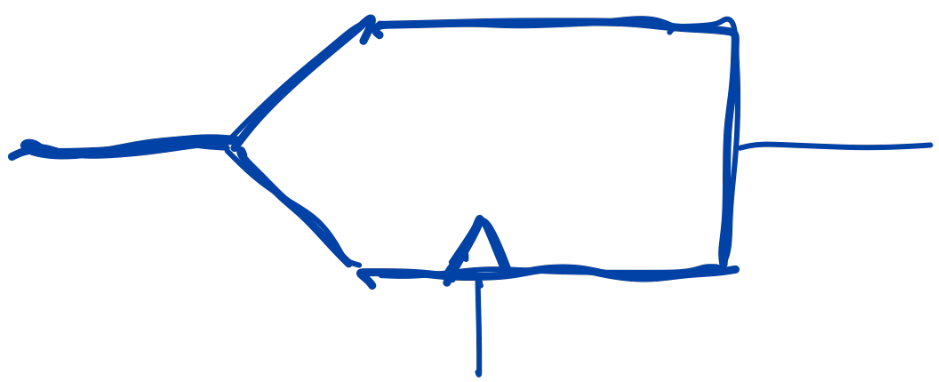
W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3 W. Kester, Fig 2.16 (excerpt)

Symbols

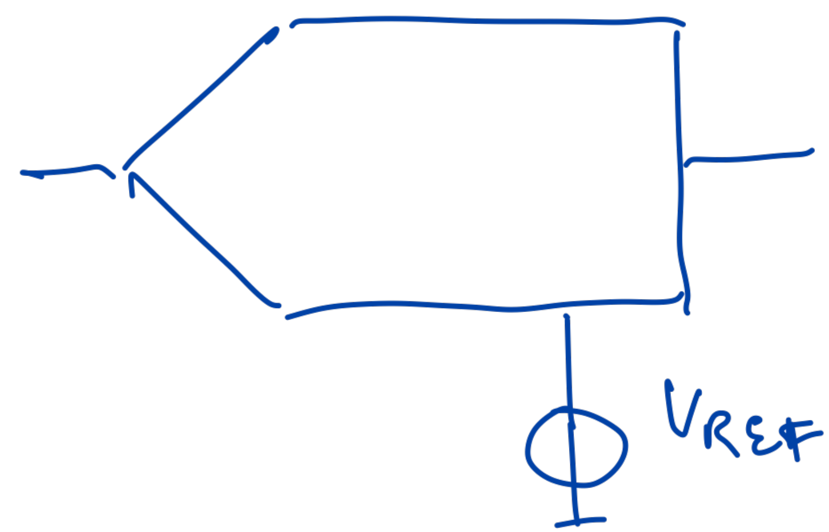
Single input

Differential input

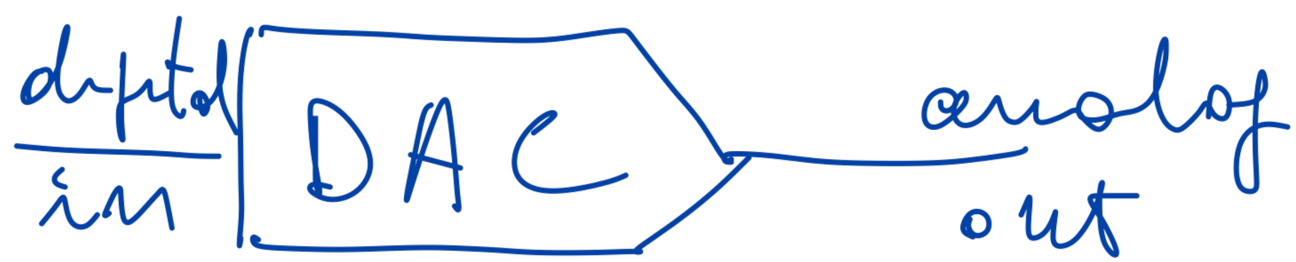
Sampling clock



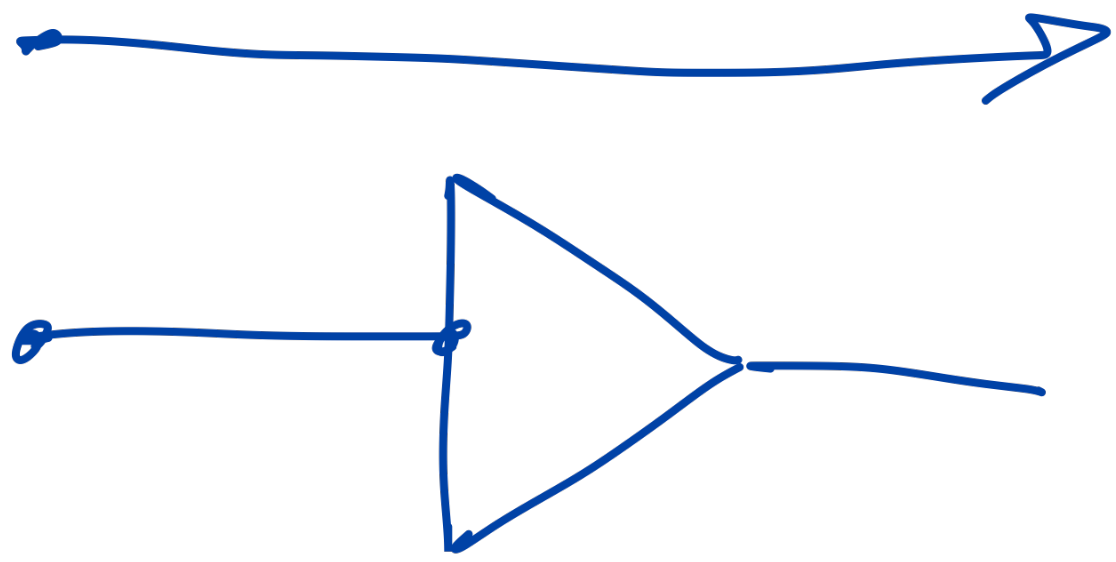
Internal/External V_{REF}

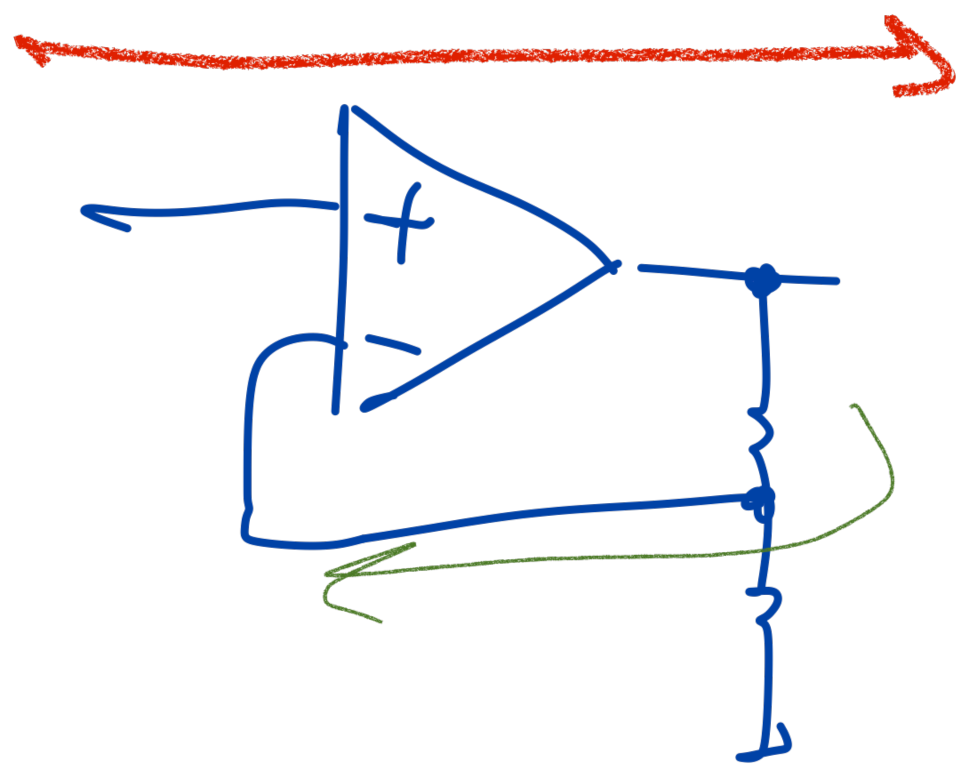


Likewise for the DACs

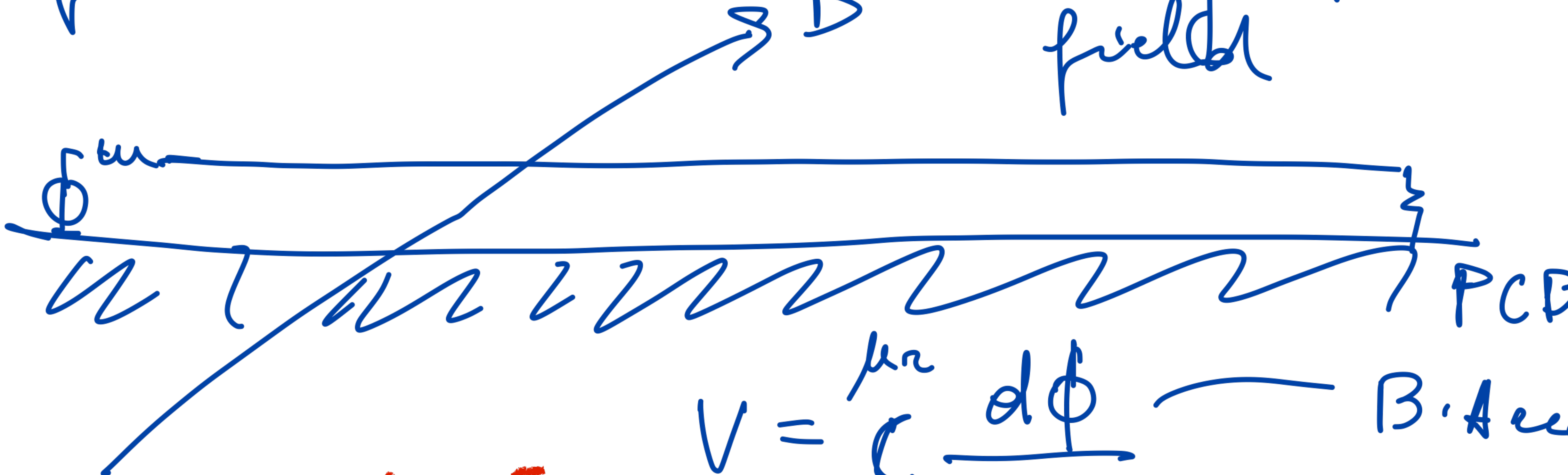


Drawing rules

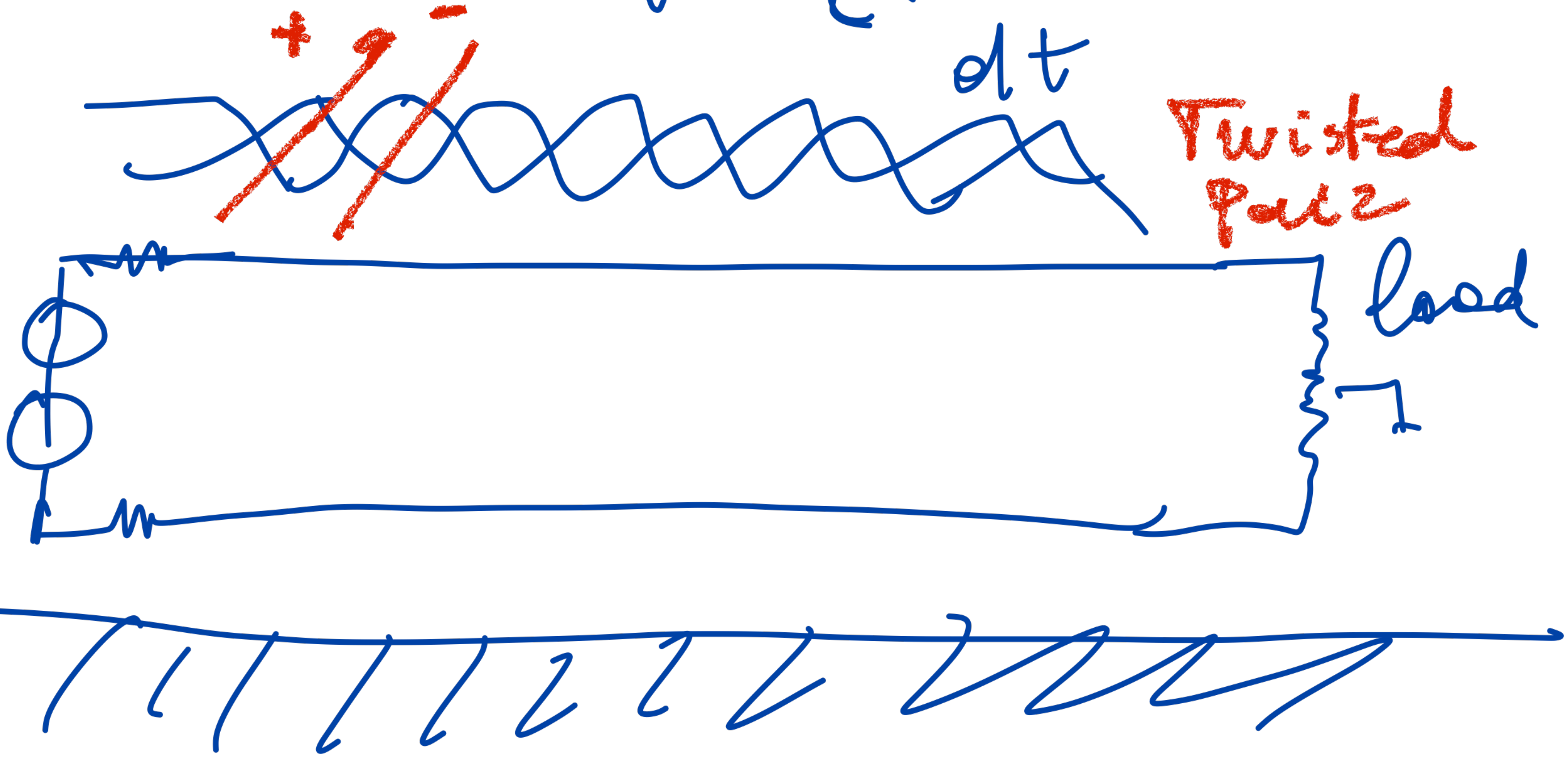




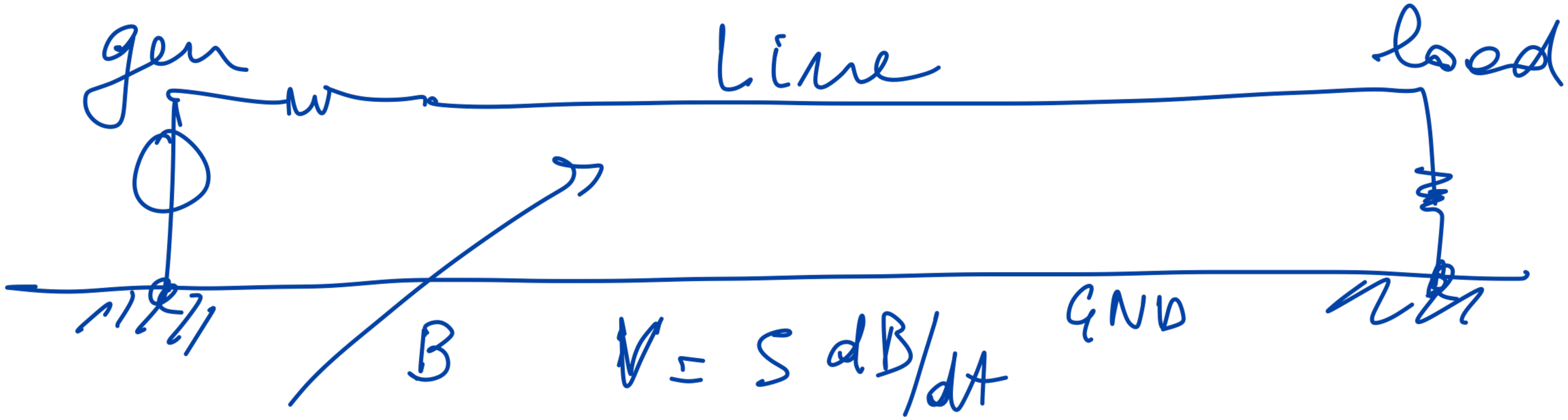
Regular

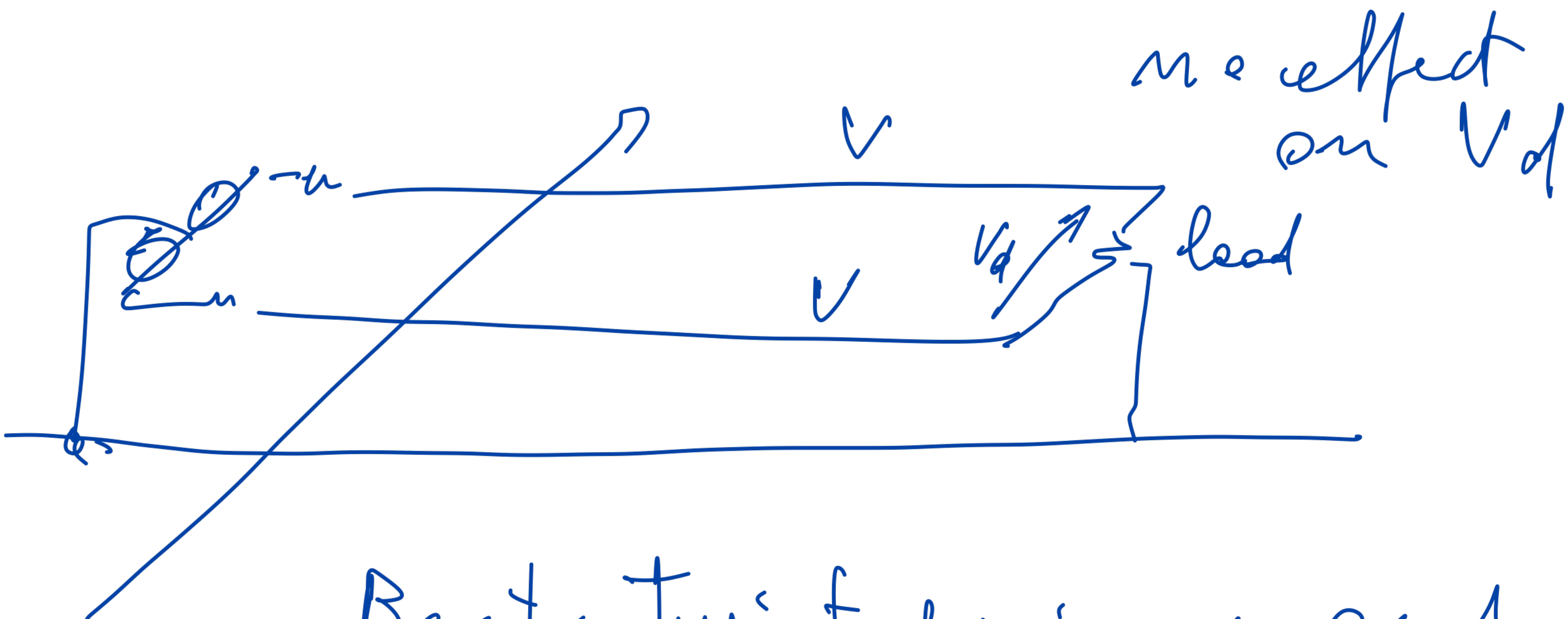


$$V = \mu_r C \frac{d\Phi}{dt} \quad B \cdot \text{Area}$$



Differential Signal & EMI





Best: Twisted pair on gnd

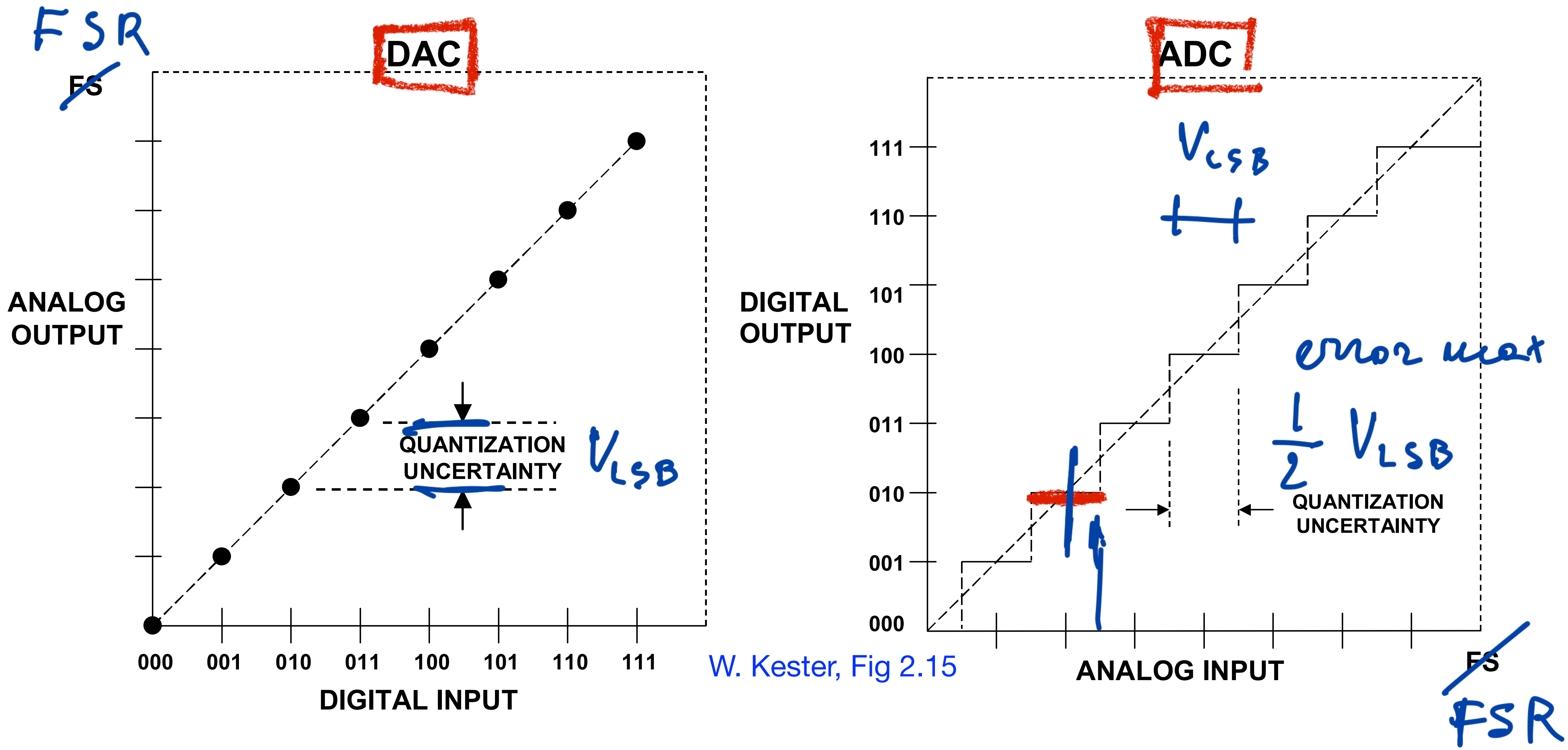
#2 Tuesday, Sept 17, 2019

1.5 Hours

Conversion Errors and Uncertainty

Quantization Uncertainty

LSB = Least Significant bit



W. Kester, Fig 2.15

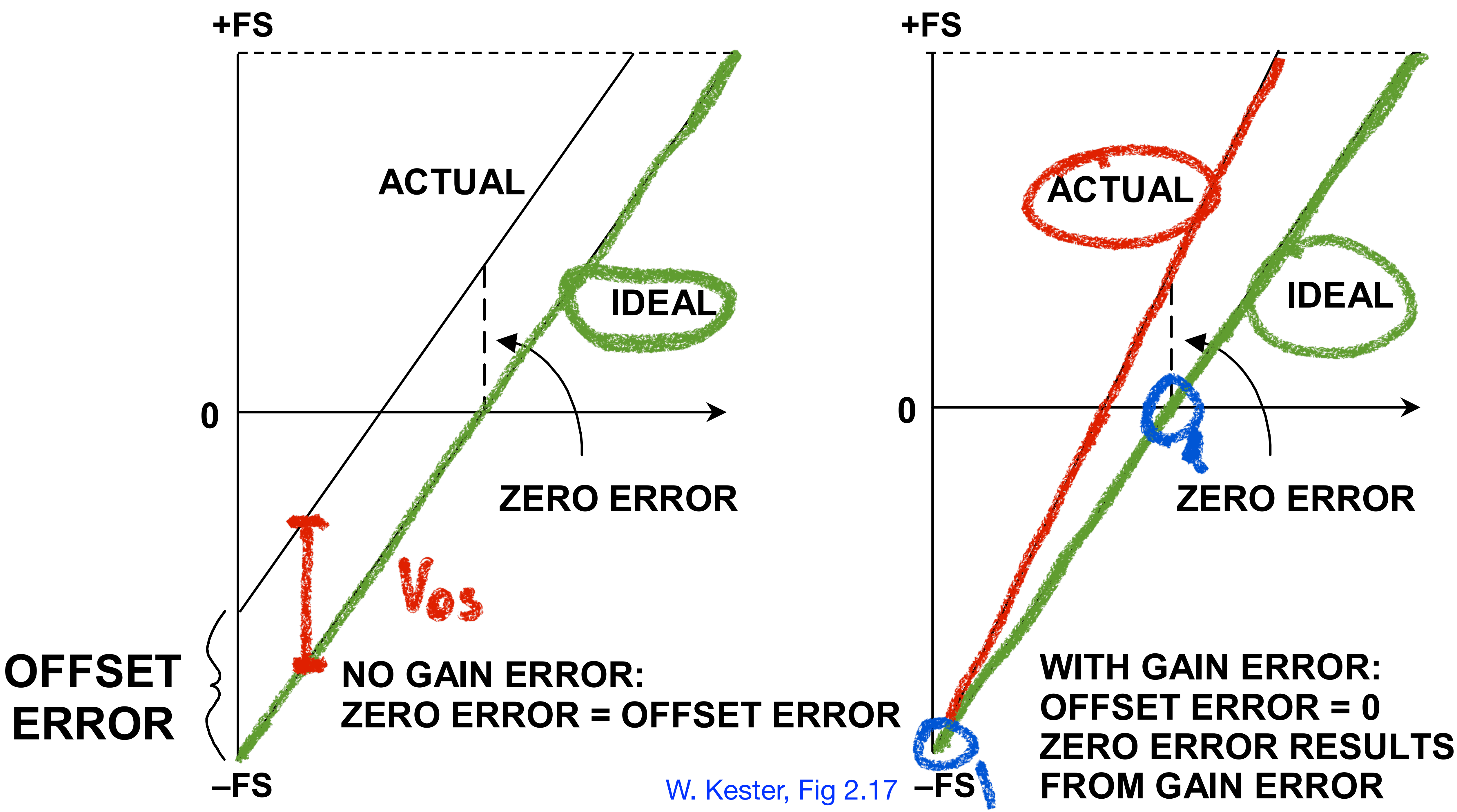
n bits

$N = 2^n$ levels

$$V_{LSB} = \frac{V_{FSR}}{2^n}$$

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Offset and Gain



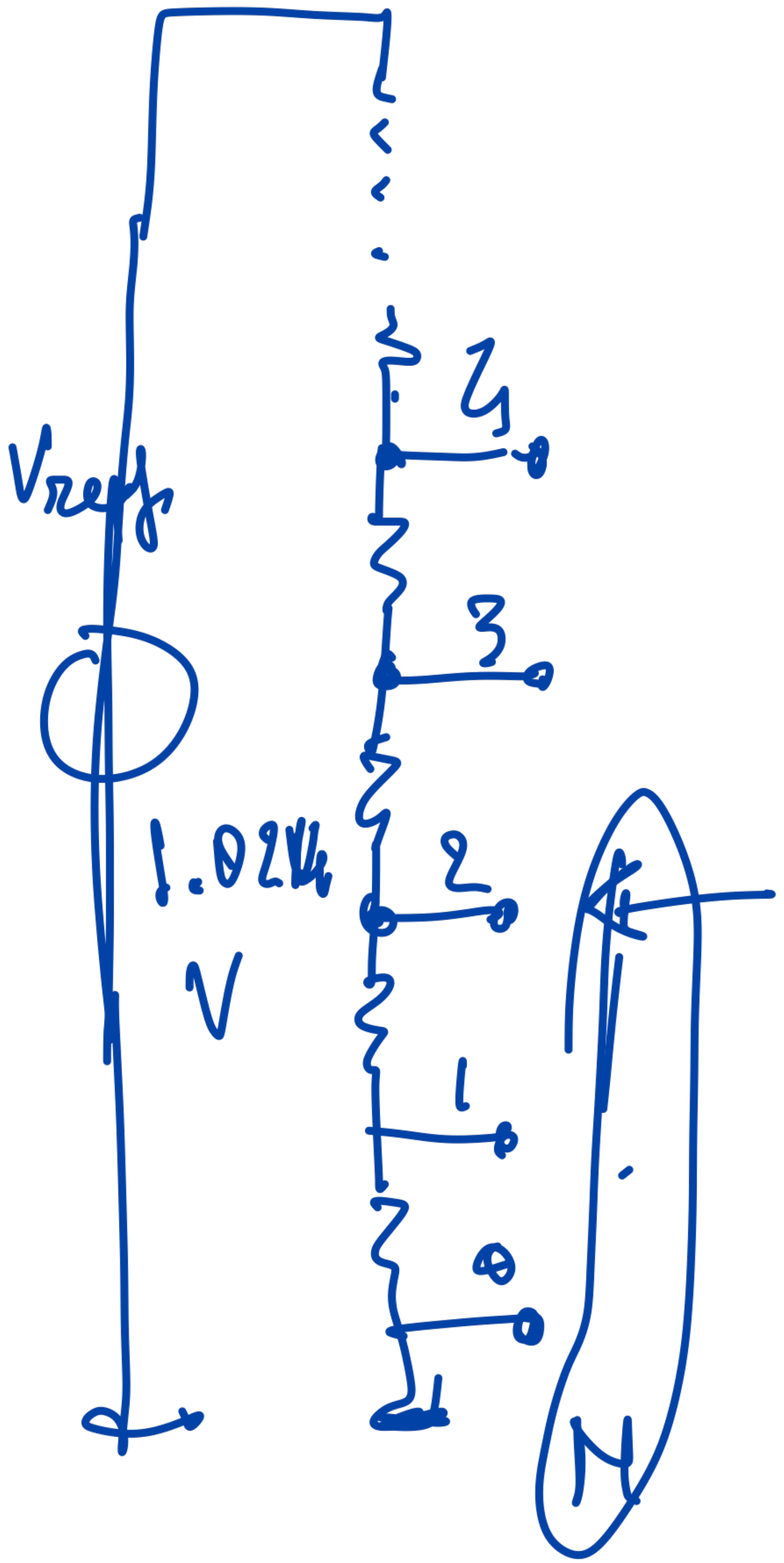
W. Kester, Fig 2.17

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

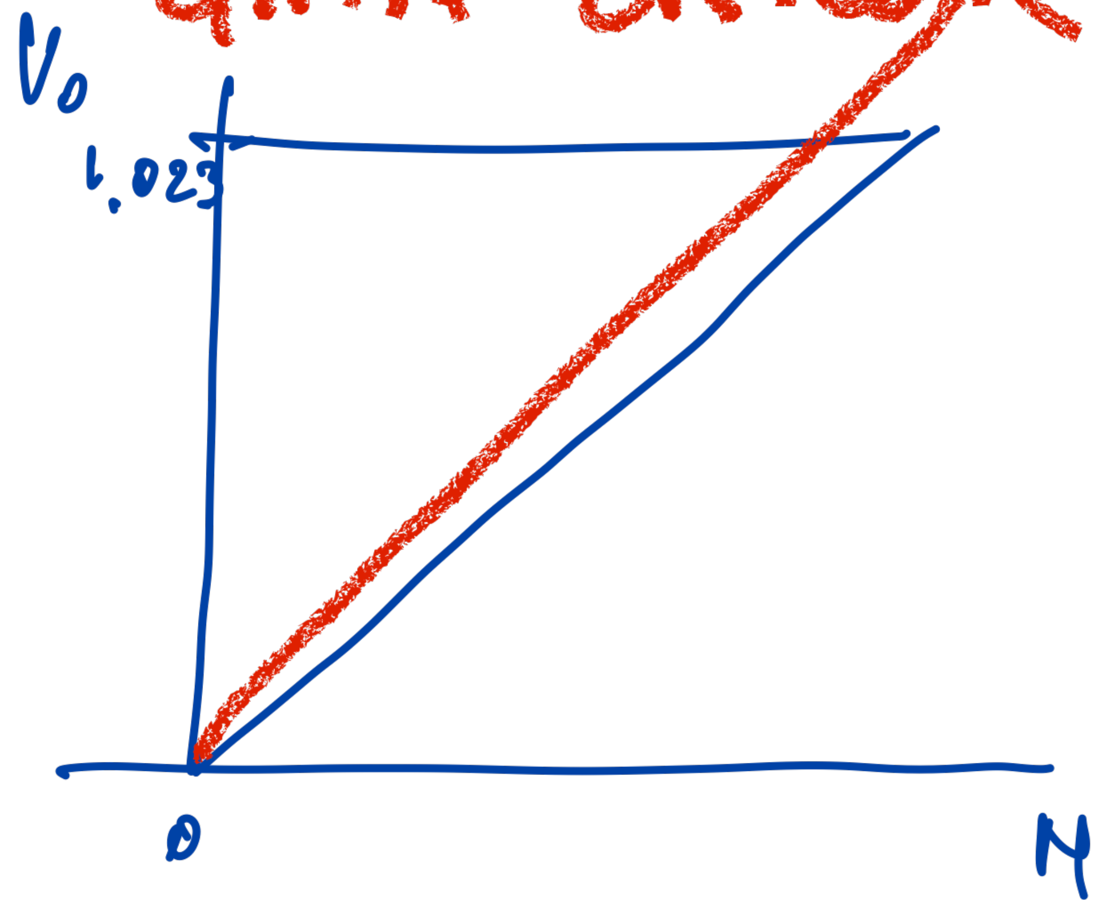
DAC

1.024 V

$V_{LSB} = 1\,mV$



GAIN ERROR



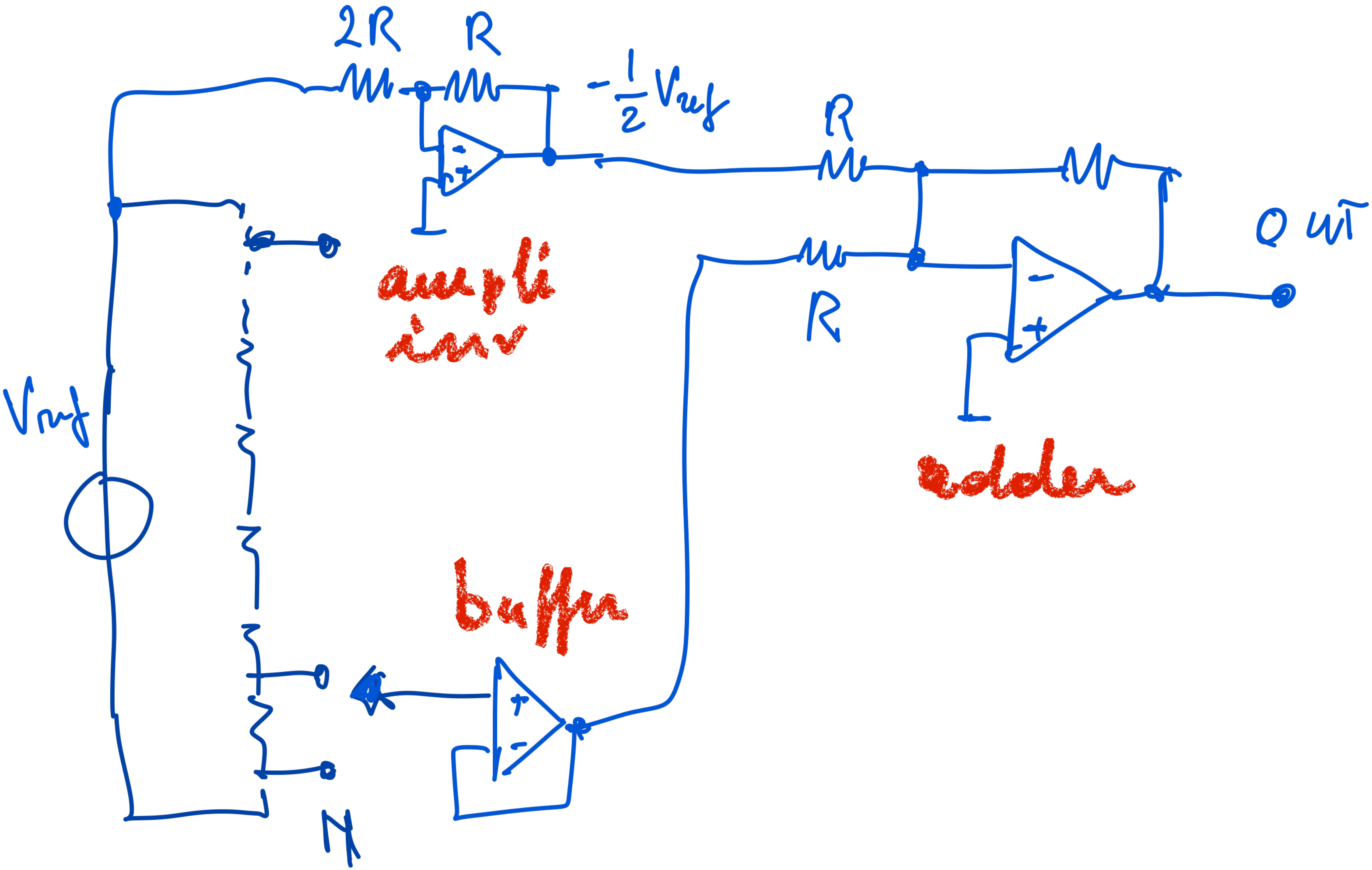
~~1.024 V~~

1.064 V

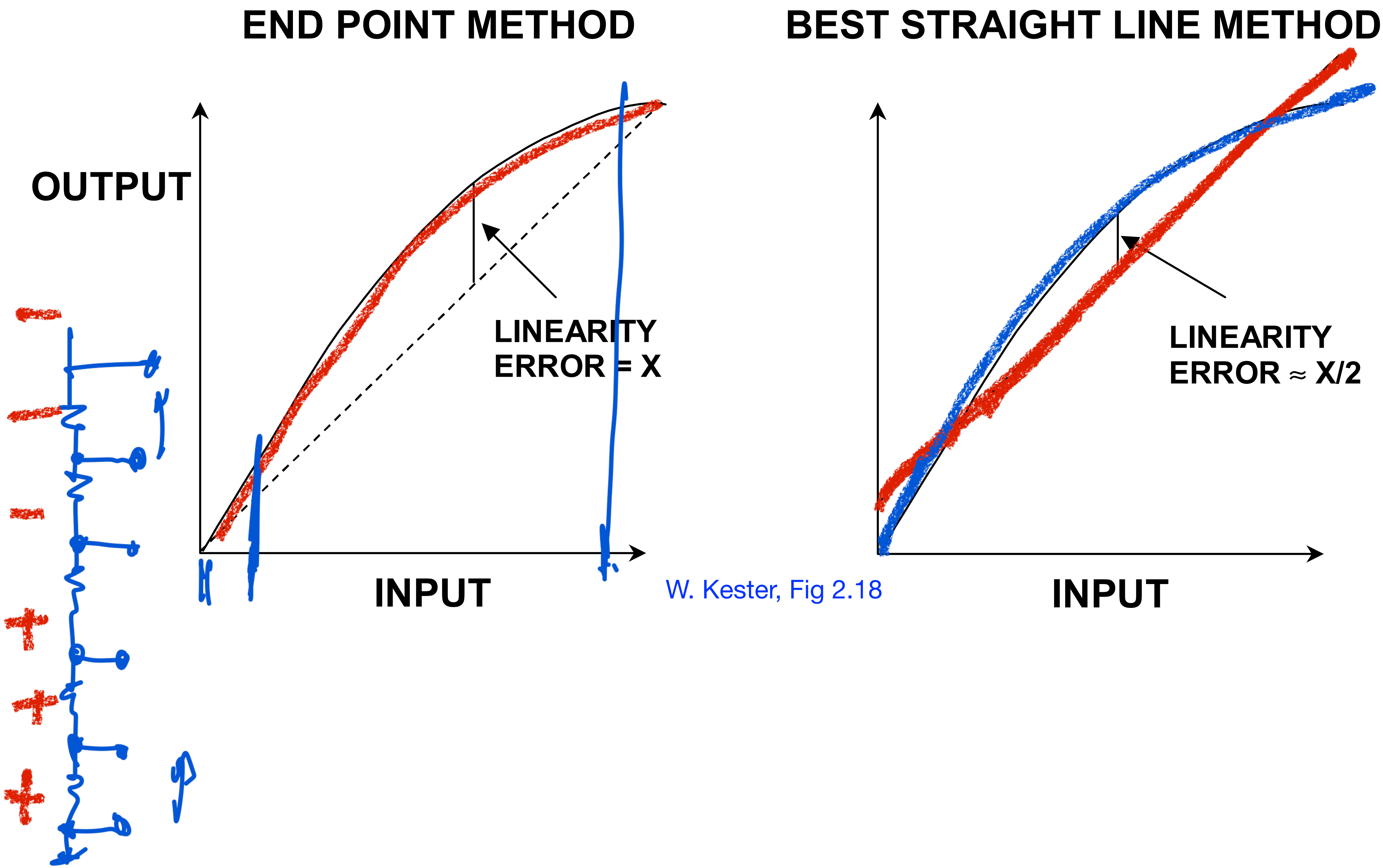
+40 mV

$$\boxed{V_o = N \frac{V_{ref}}{2^M}}$$

BIPOLAR CONVERTER



Integral Nonlinearity



W. Kester, Fig 2.18

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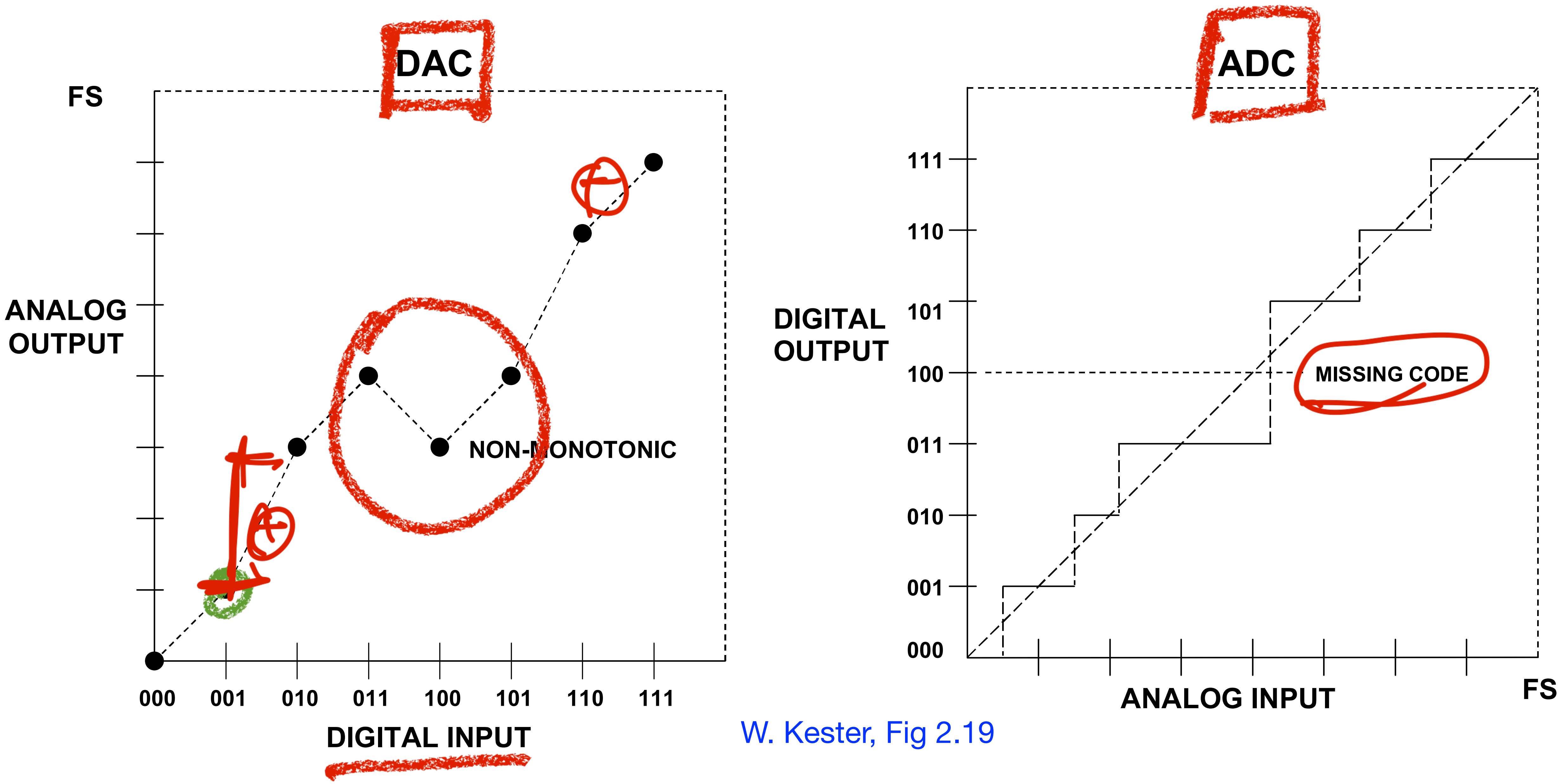
Differential Nonlinearity



W. Kester, Fig 2.19

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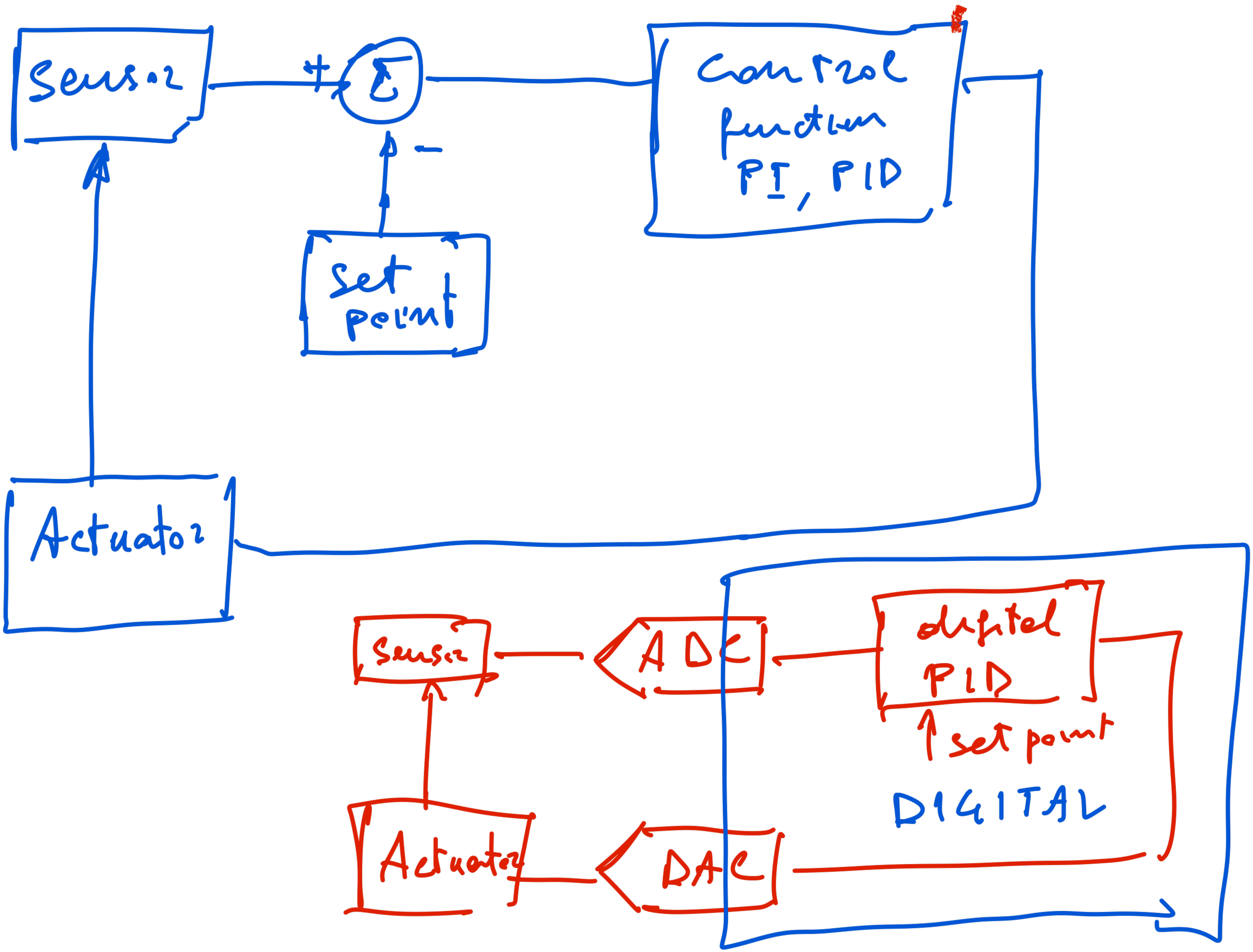
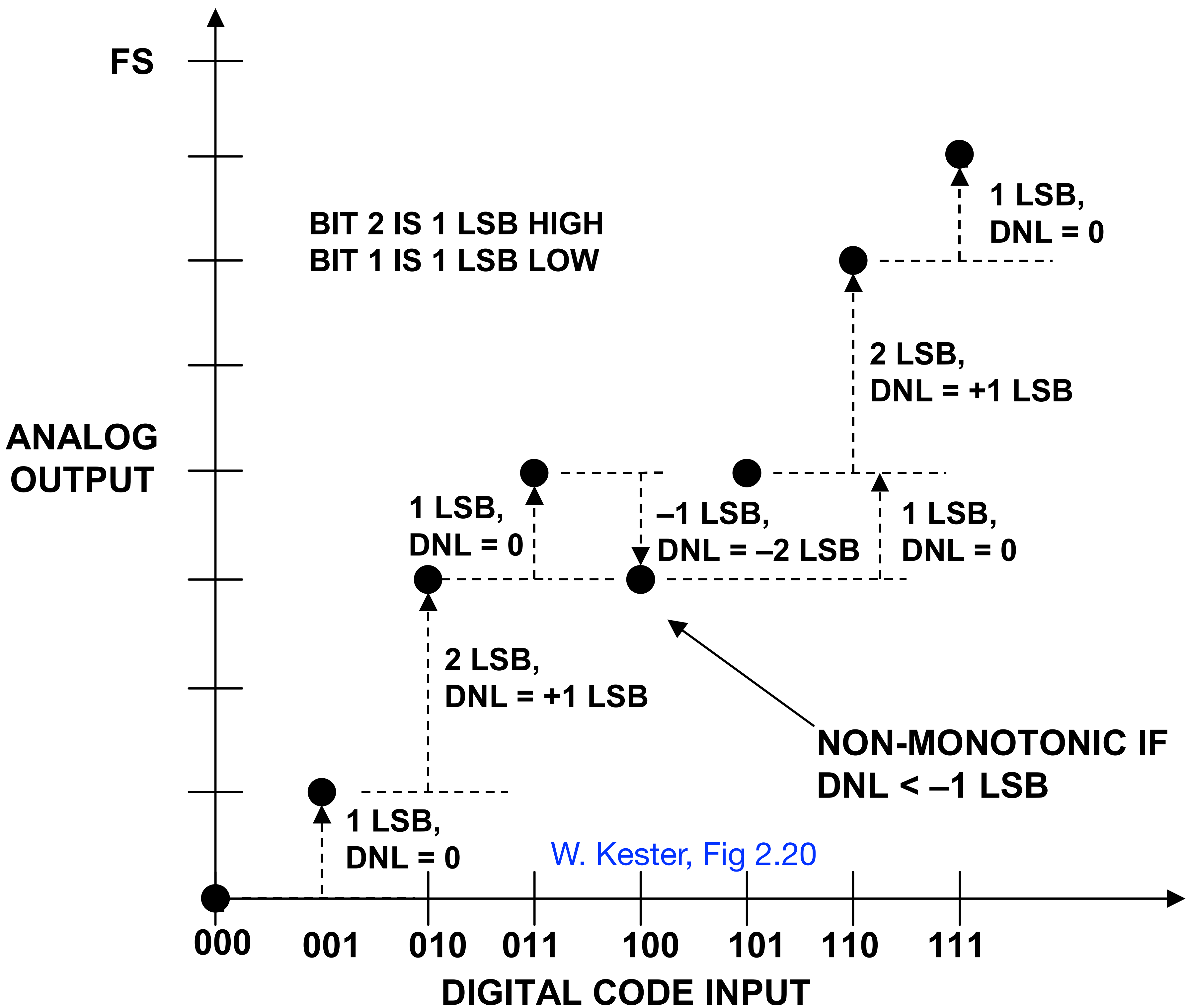
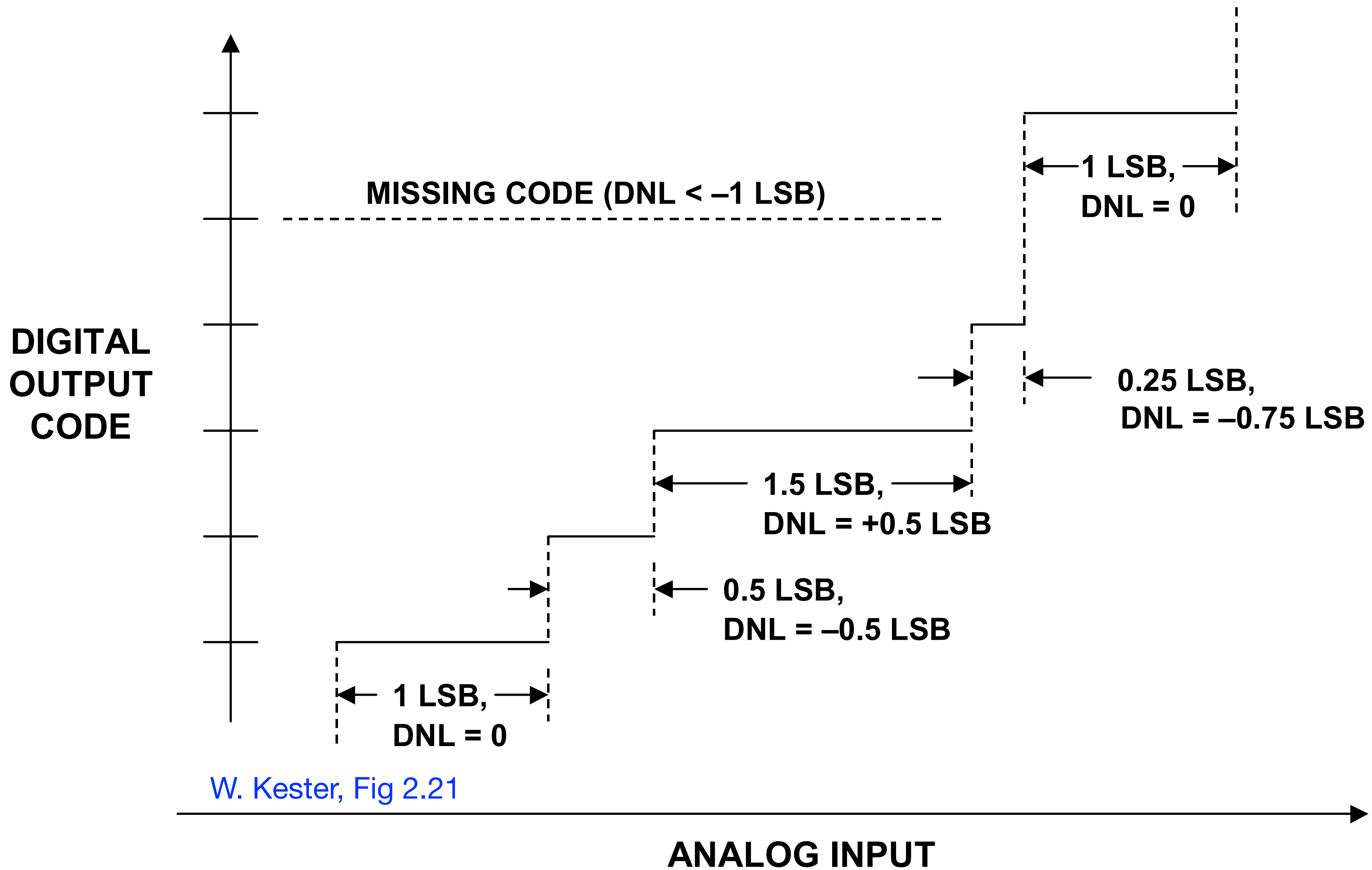

Sensor
+
Σ
−
Set point
Control function
PI, PID
Actuator
Sensor
ADC
digital PID
set point
DIGITAL
Actuator
DAC

DAC Differential Nonlinearity



W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

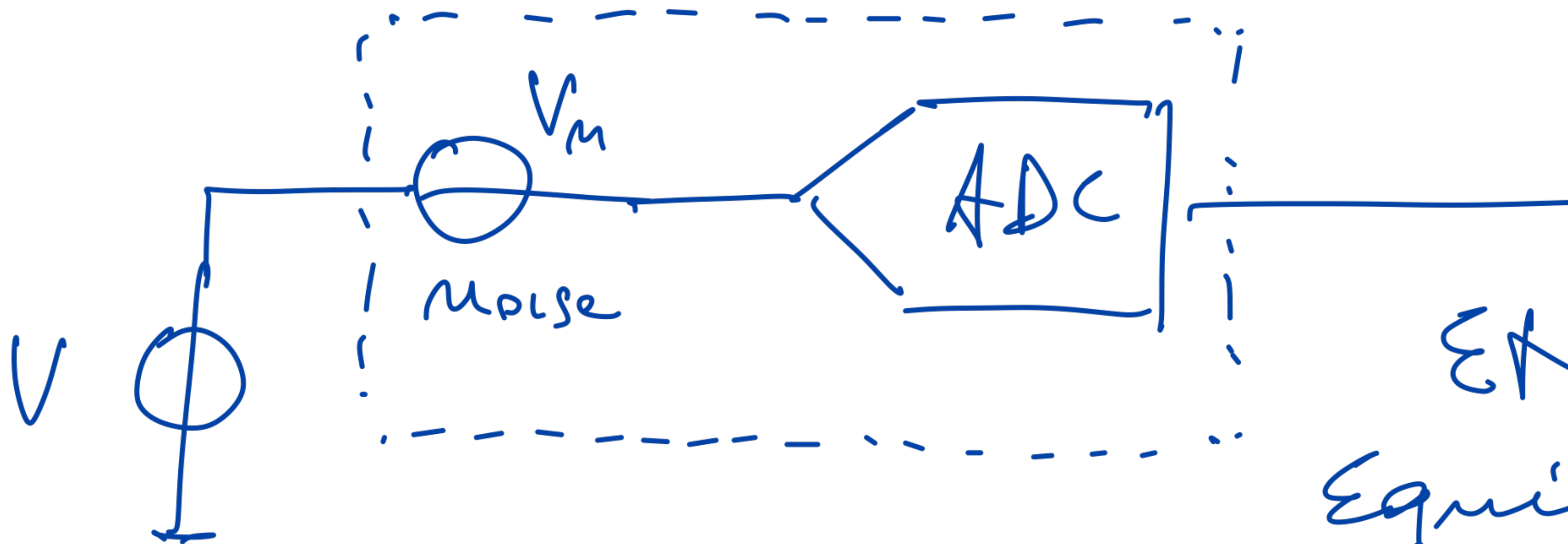
ADC Differential Nonlinearity



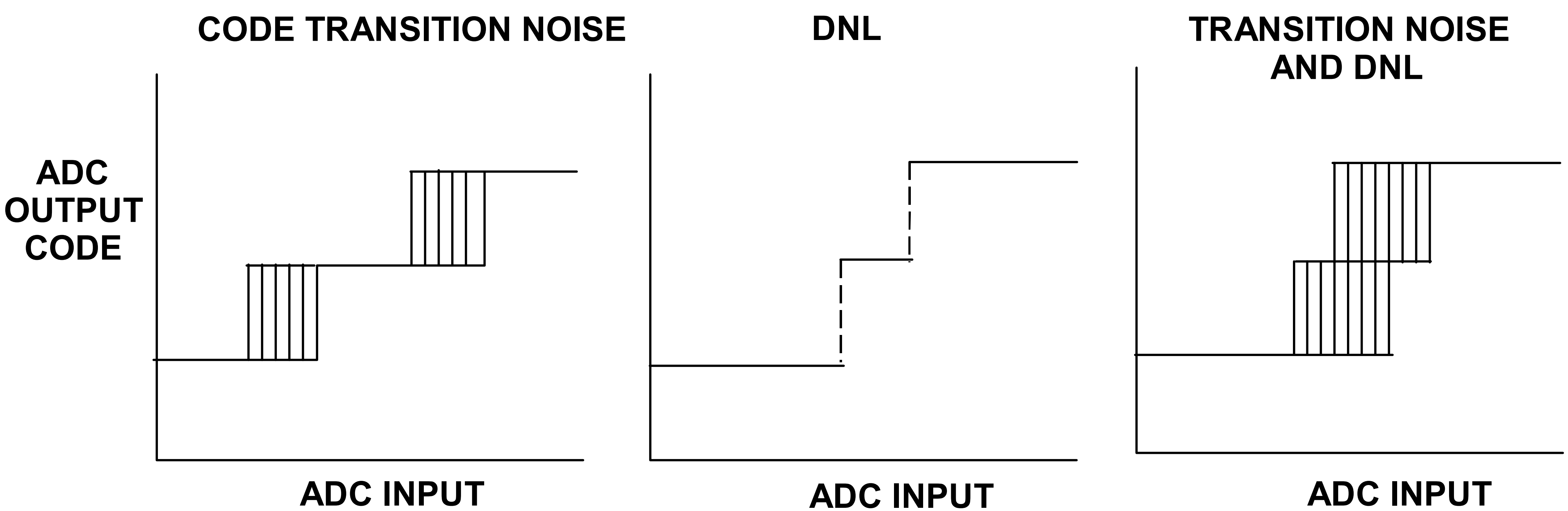
W. Kester, Fig 2.21

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Code Transition and Noise



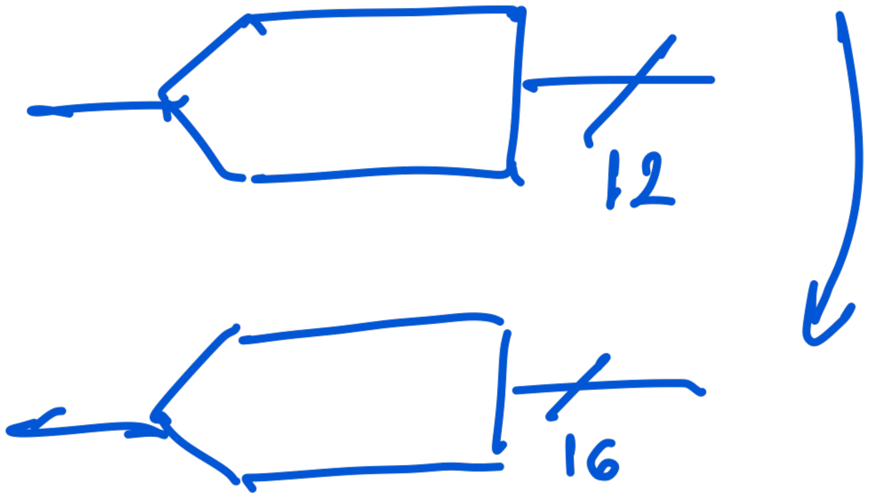
ENOB
Equivalent No
Of Bits



W. Kester, Fig 2.24

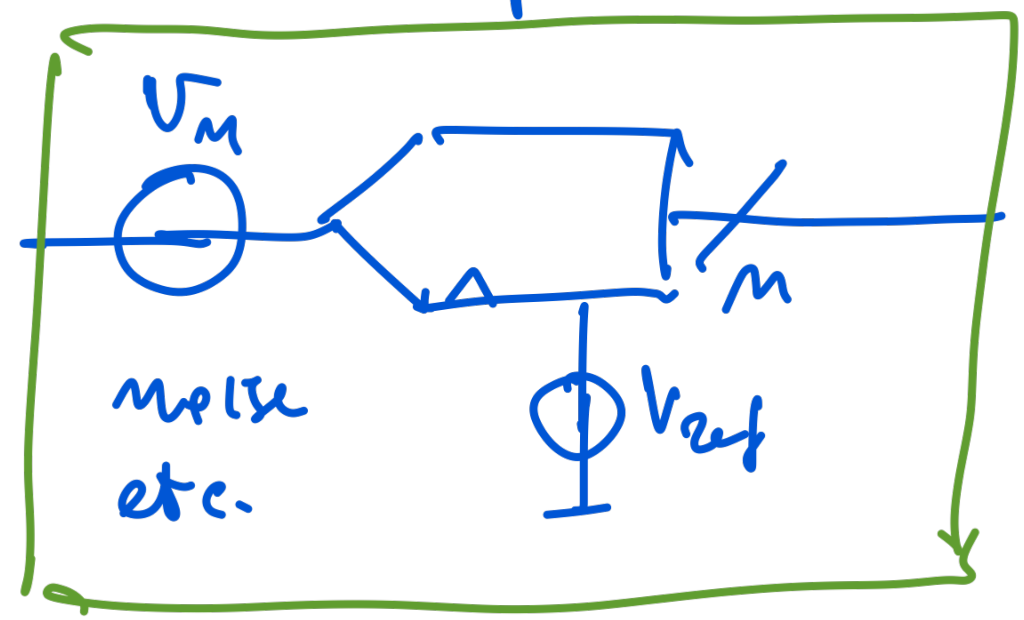
W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

BUS bits



just some
surface of Si
added

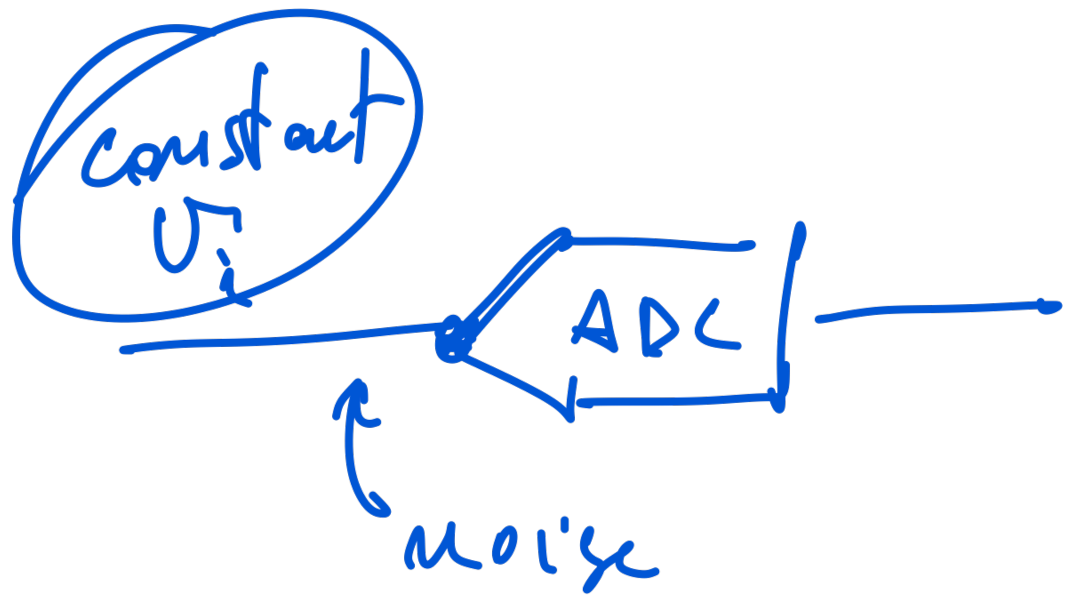
Metrological "value"



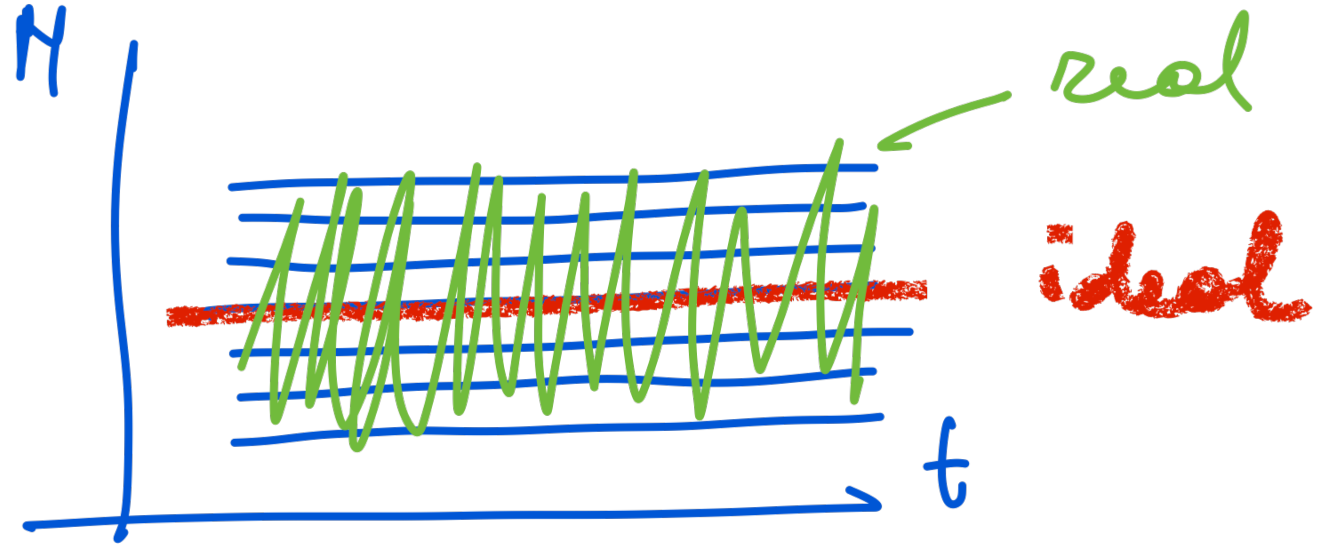
low noise
(V_{ref} accuracy)

Correct metrological approach

- Add extra bits (increase n)
  cheap
- Quantization error/noise
  gets negligible.

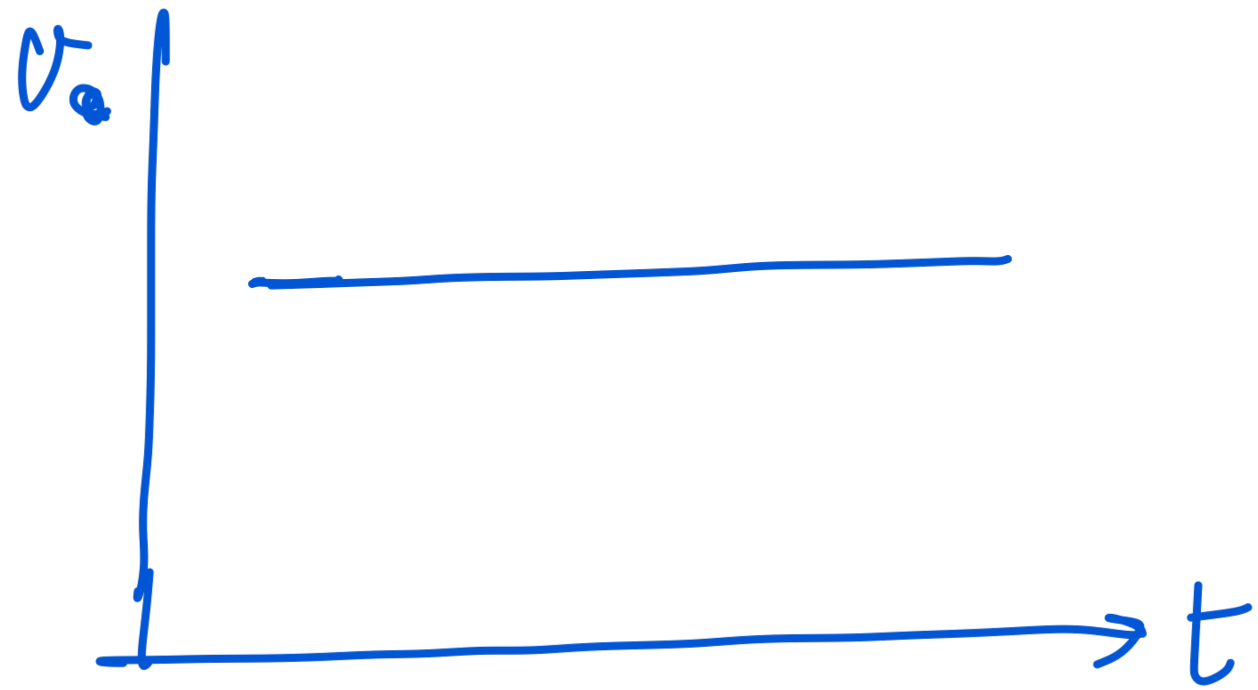


constant
v_i
ADC
noise



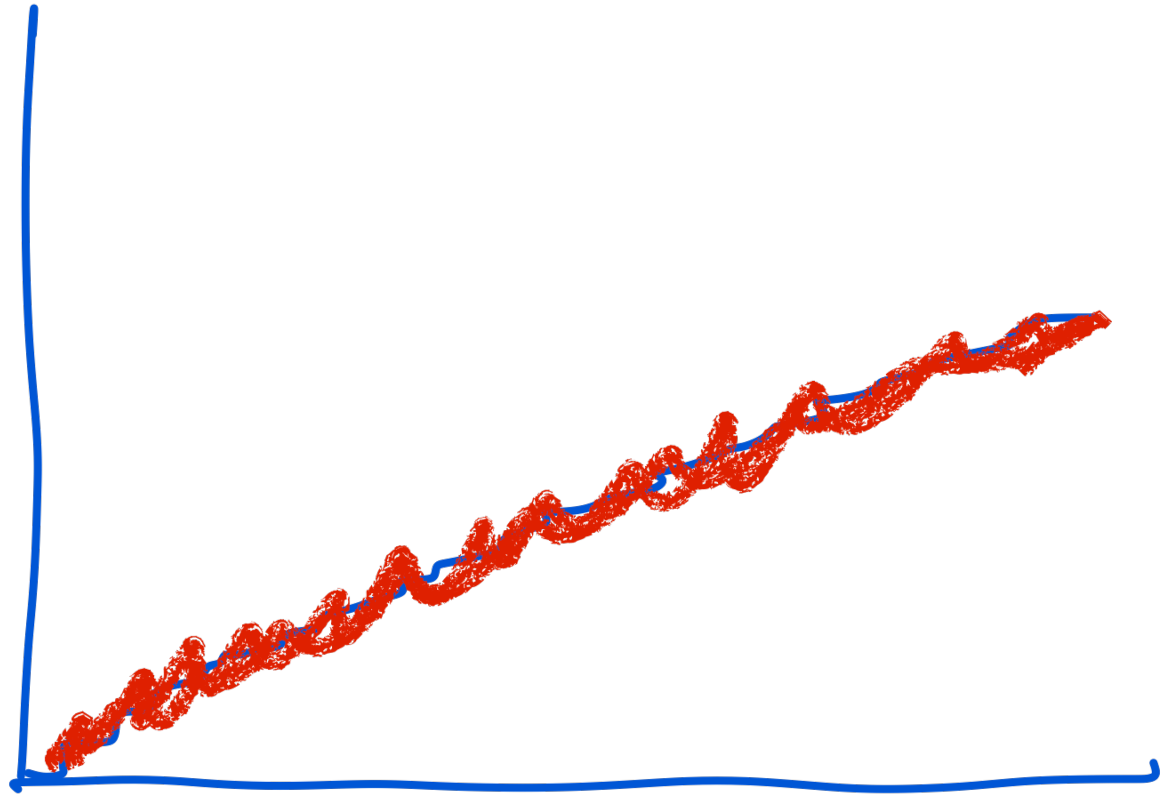
N
t
real
ideal

N
DAC
v_o



v_o
t

DA C



The DNL Is No Longer Useful

We will explain why later
First, we need sampling and noise

End of lecture 2

#3 Thursday, Sept 19, 2019

1.5 Hours

DNL & Transition Noise

↓

Sampling

↓

spectral analysis

↓

Noise

↓

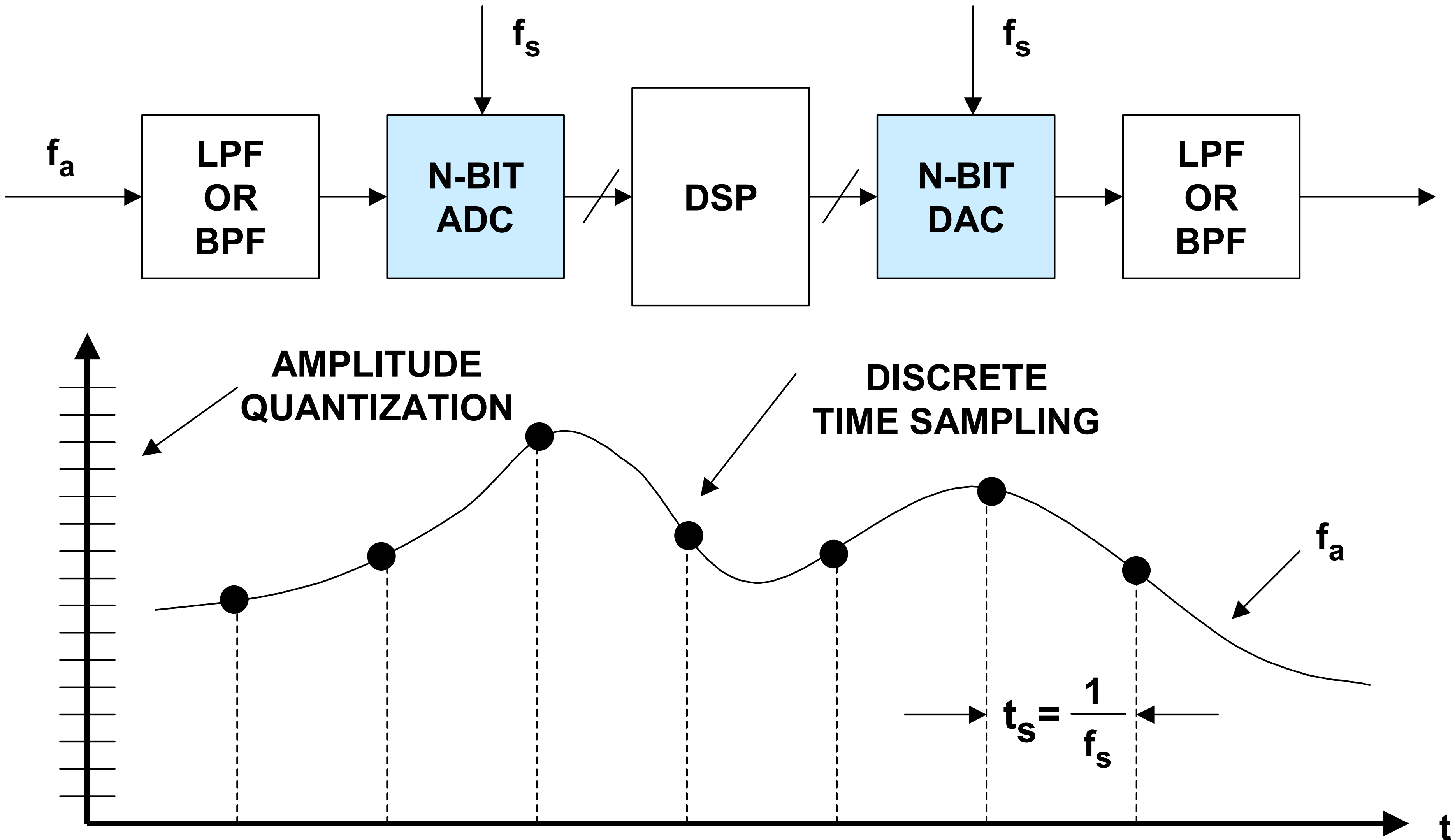
Transition Noise

Sampling

LPF = Low Pass Filter

BPF = Band Pass Filter

Basic



W. Kester, Fig 2.25

$f \lesssim f_N = \frac{1}{2} f_s$ — sampling

Nyquist

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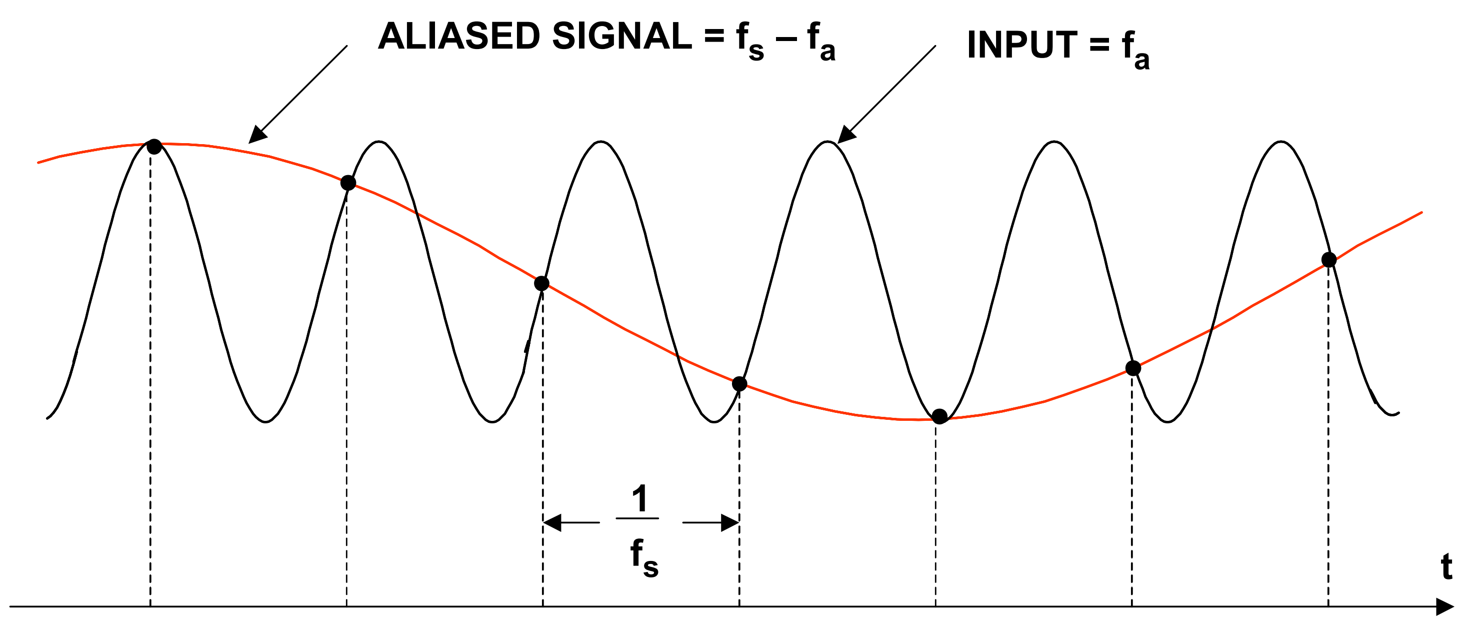
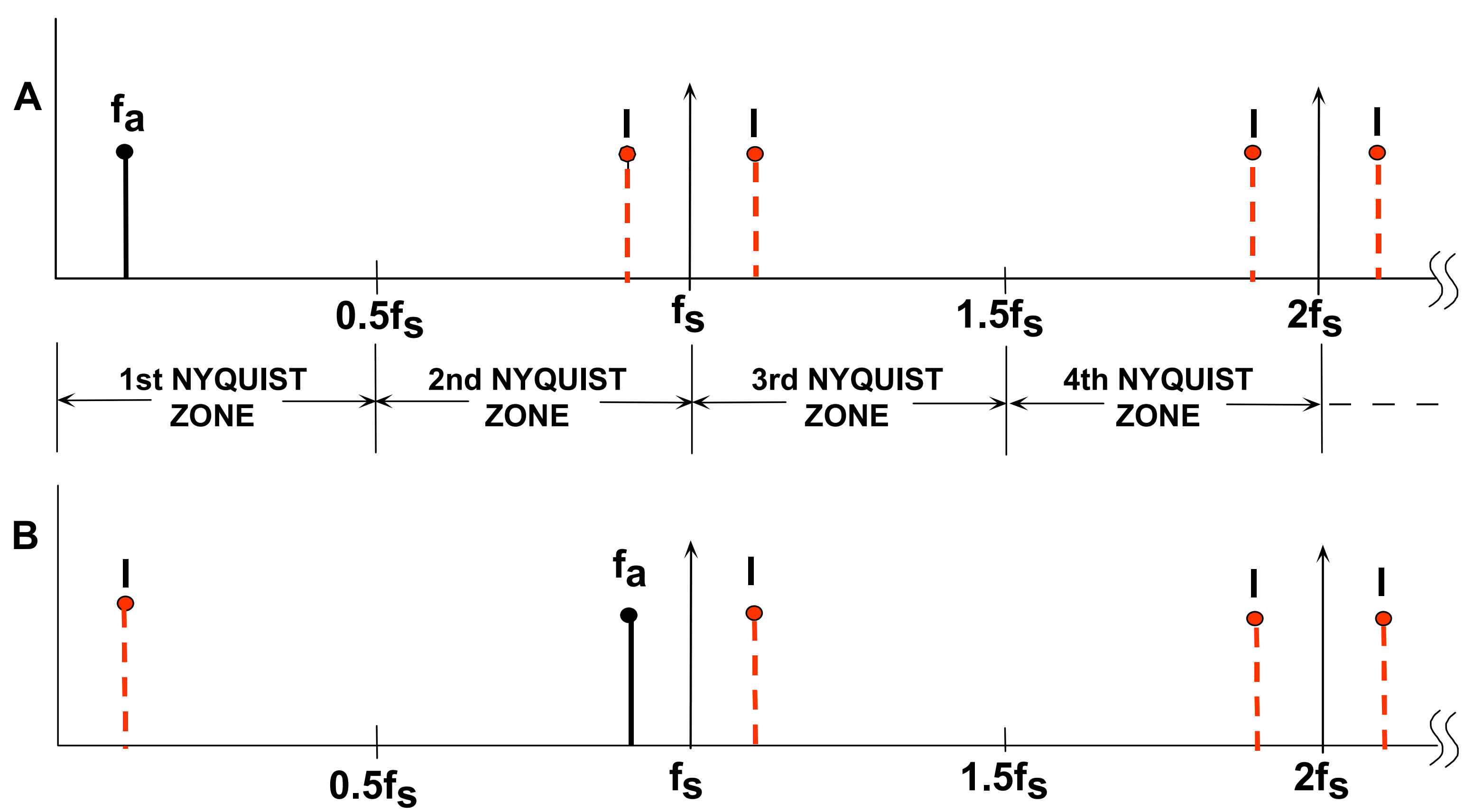
Aliasing



W. Kester, Fig 2.30

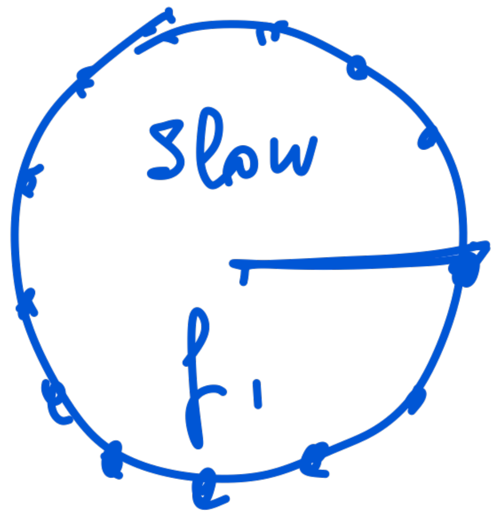


W. Kester, Fig 2.31

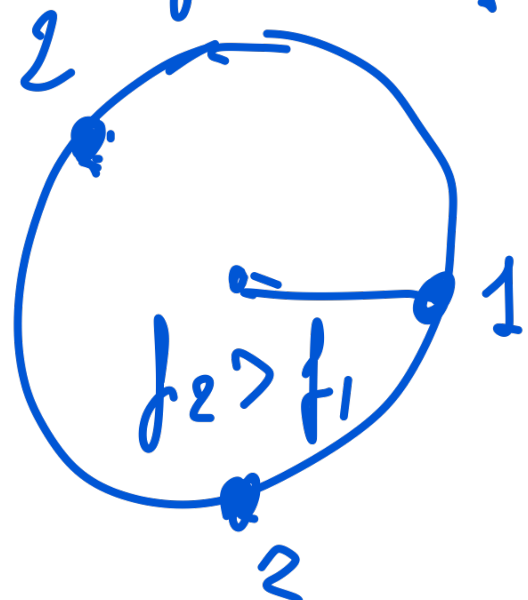
W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Aliasing

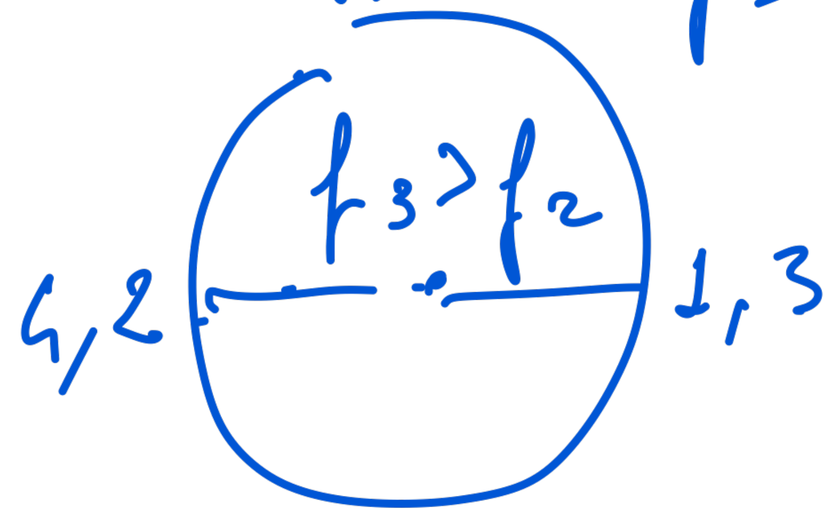
$e^{i\omega t}$



$f = 0.4 f_s$

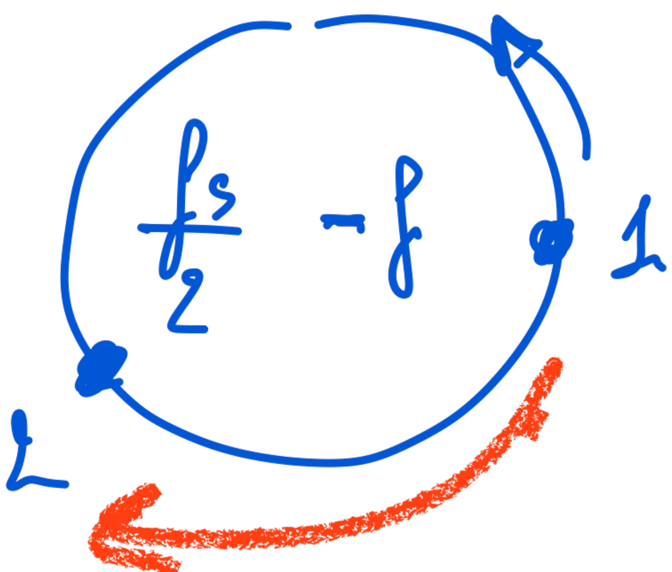


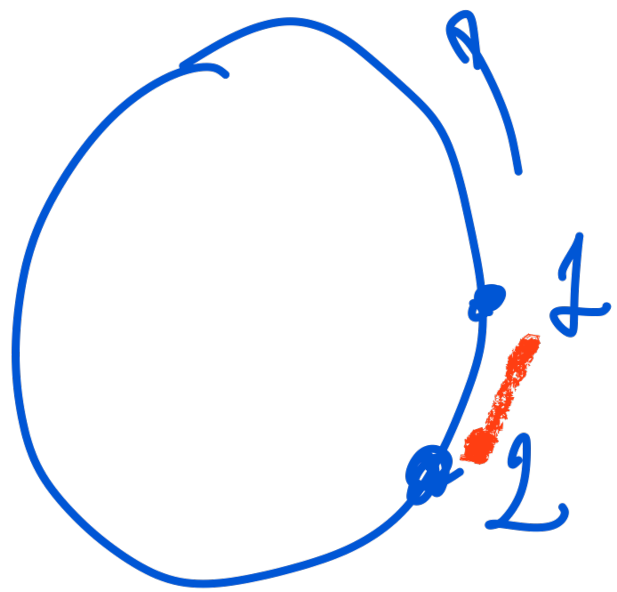
$f_3 = 0.5 f_s$

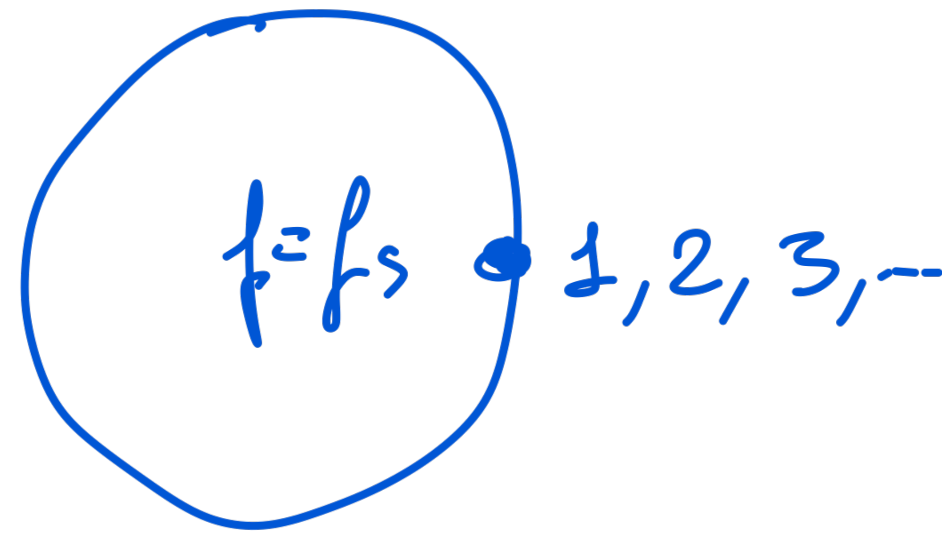


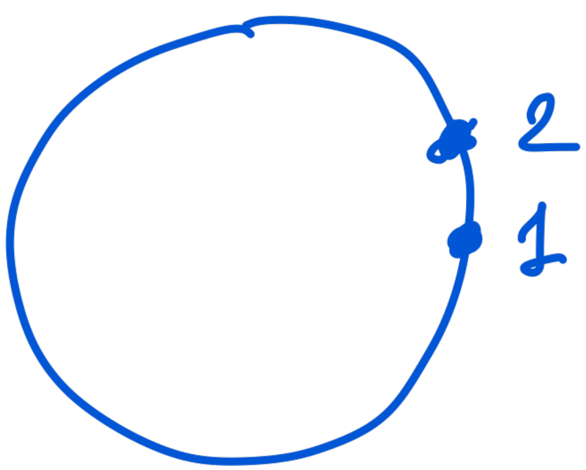
f_s = sampling frequency

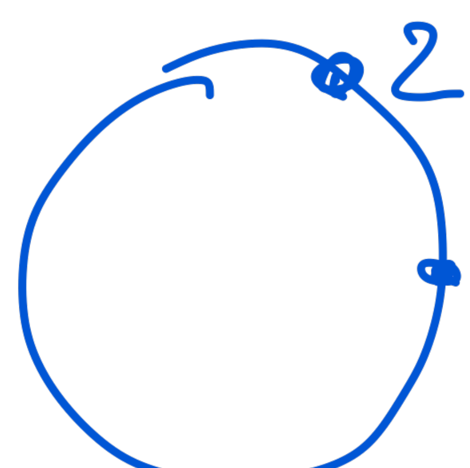
f









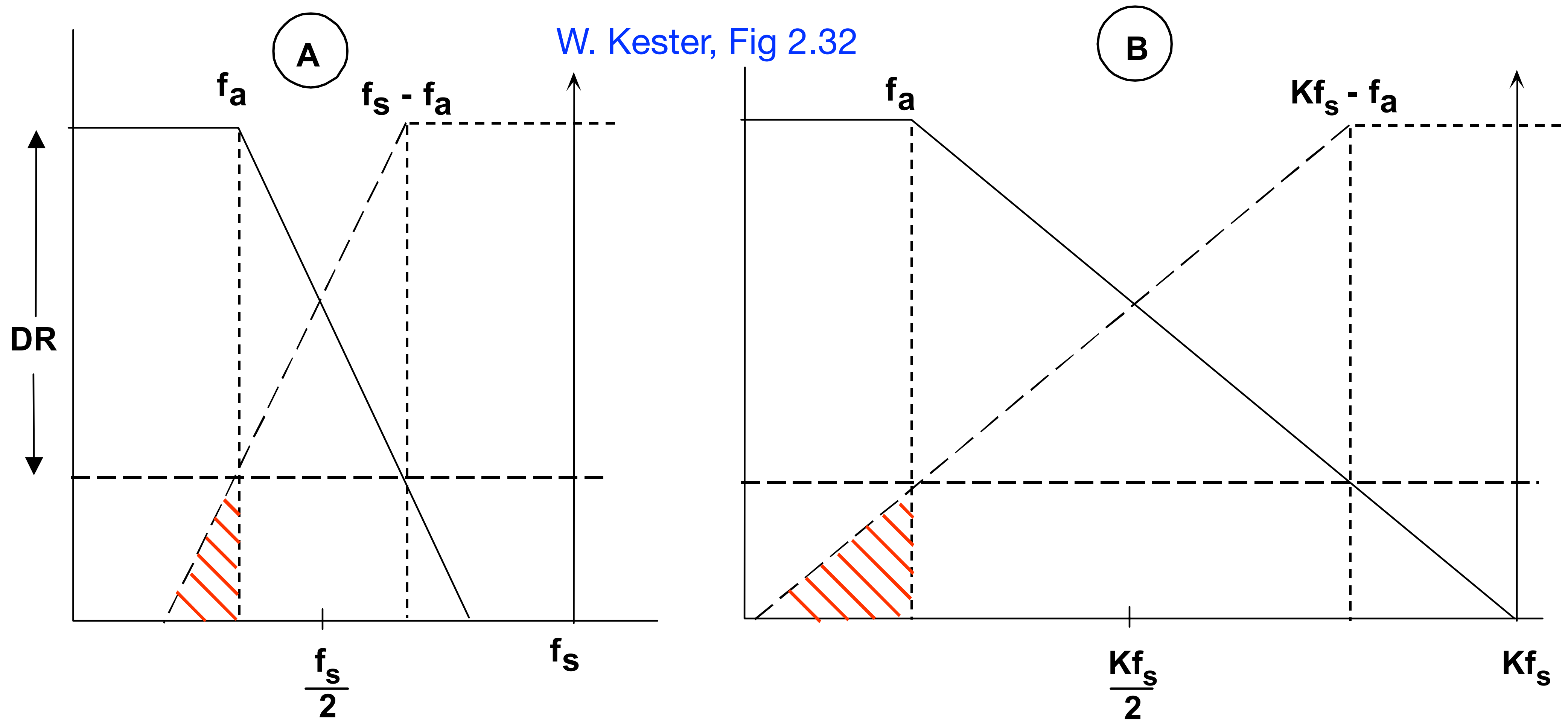


seen as

$f - f_s$

Antialiasing Filter & Oversampling

W. Kester, Fig 2.32

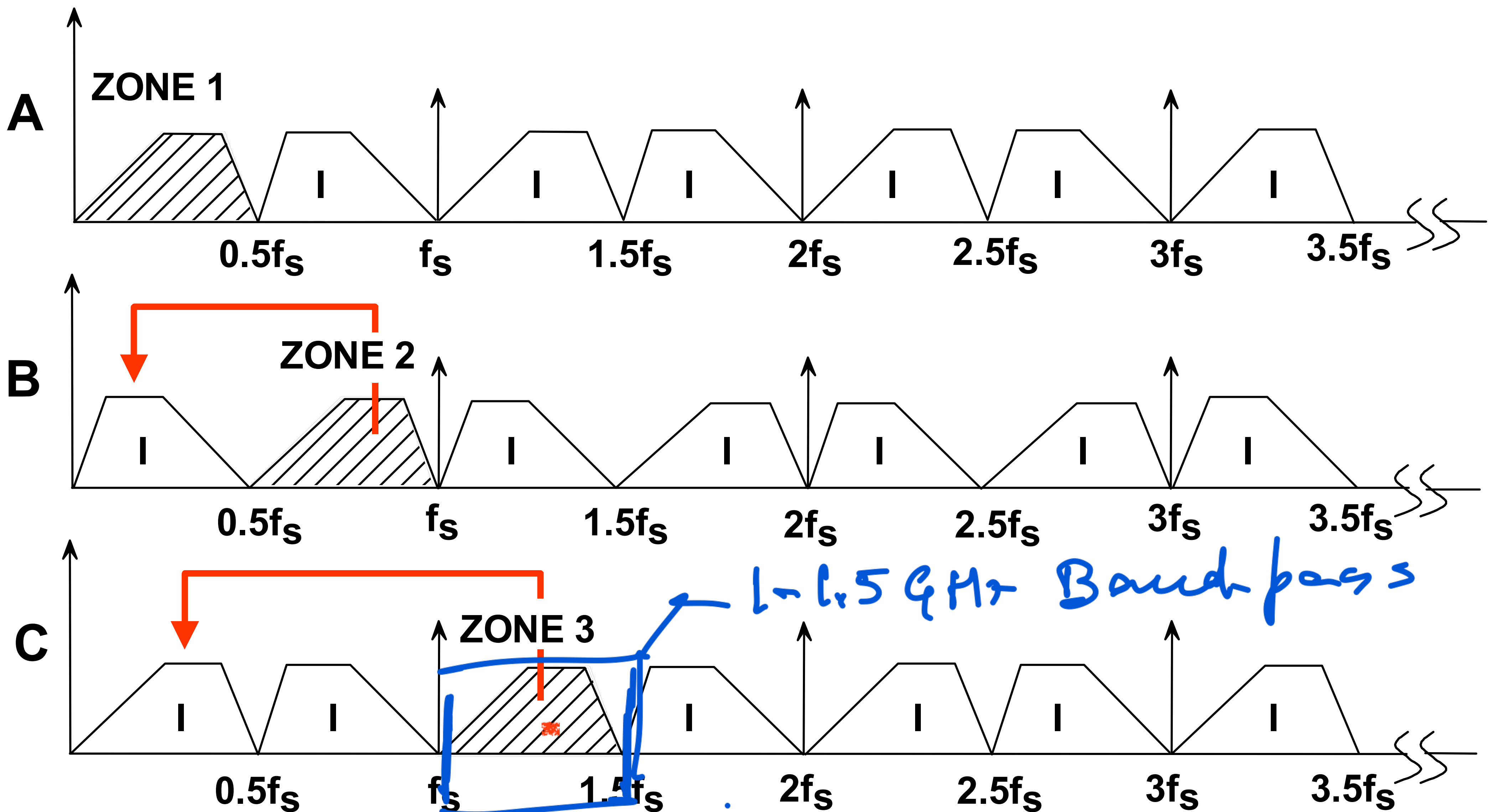


STOPBAND ATTENUATION = DR
TRANSITION BAND: f_a to f_s - f_a
CORNER FREQUENCY: f_a

STOPBAND ATTENUATION = DR
TRANSITION BAND: f_a to Kf_s - f_a
CORNER FREQUENCY: f_a

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Intentional Undersampling



W. Kester, Fig 2.34

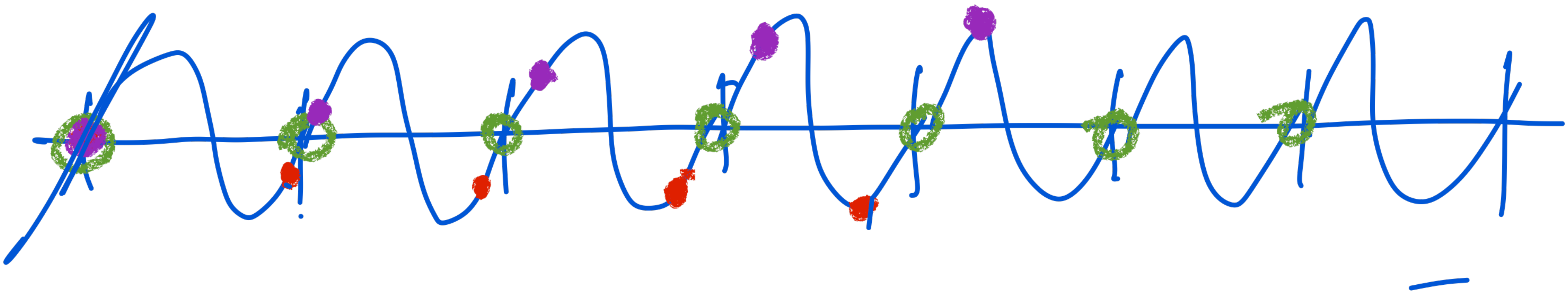
Seen at the out

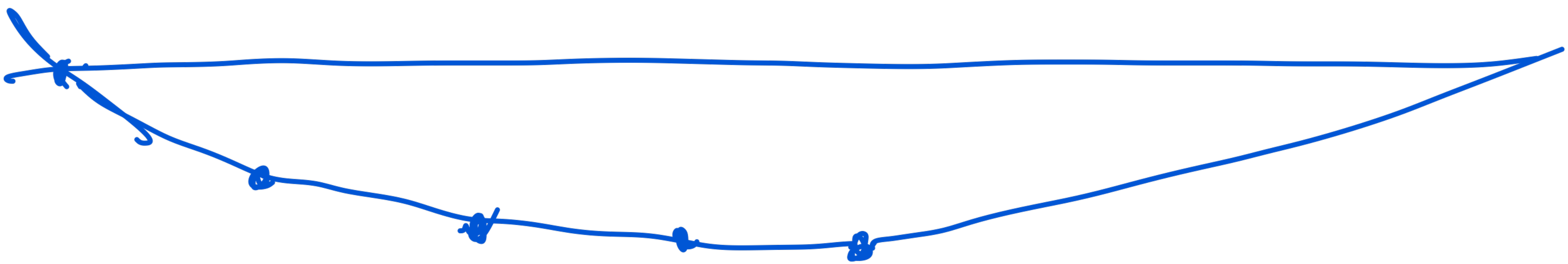
interpretation

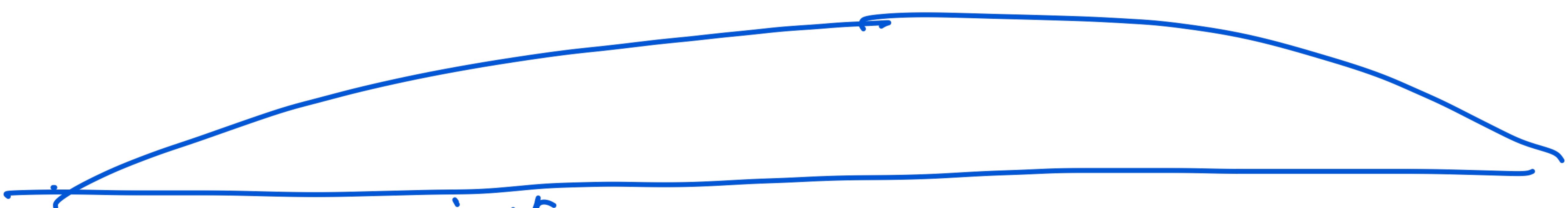
1 GS/s

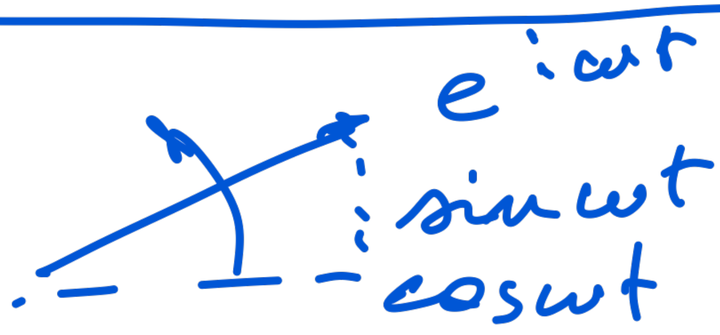
$f_N = 0.5$ GHz

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

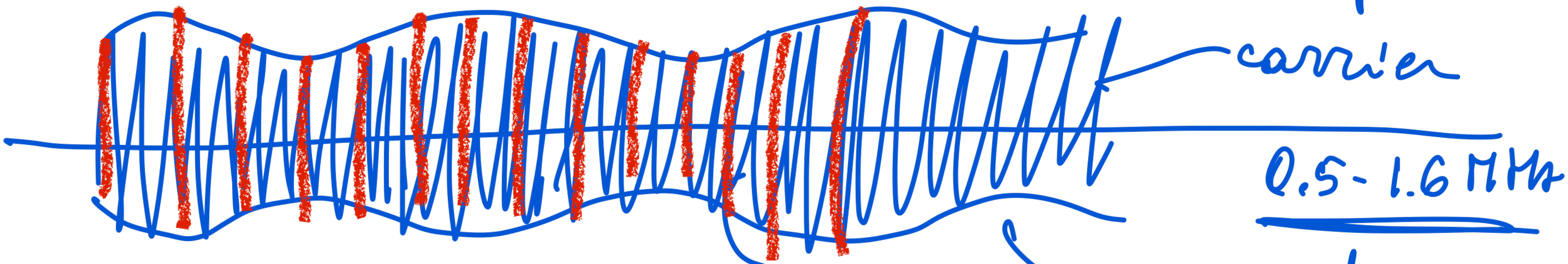







$e^{i\omega t}$
$\sin \omega t$
$\cos \omega t$

AM radio broadcasting

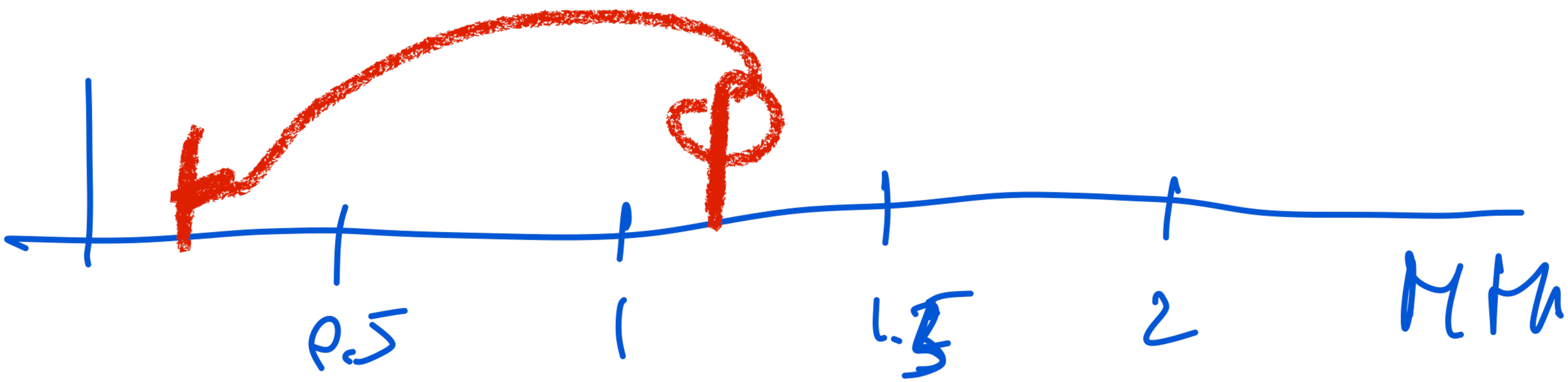


envelope

1.6 MHz → $f_s \geq 3.2$ MHz

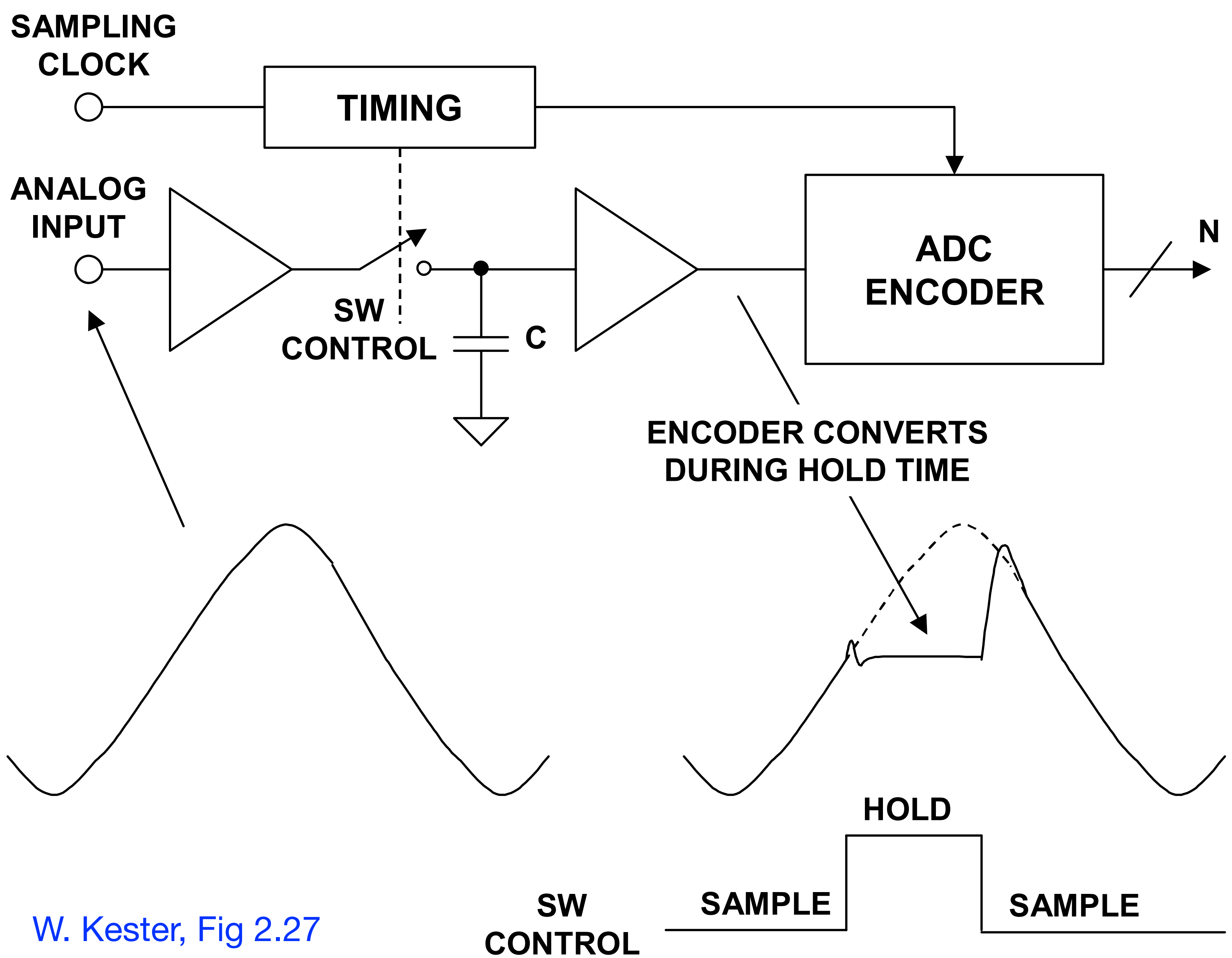
30 Hz – 3.5 kHz
(4)

1200 kHz



$f_s = 1$ MHz

Sample and Hold



W. Kester, Fig 2.27

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Power, Spectra and Probability

Recall dB, dBm, dBV, dBm/Hz, dBV^2/Hz etc.

Power $(P_2/P_1)_{dB} = 10\ \mathrm{Log}_{10}(P_2/P_1)$

Power, dBm $P_{dBm} = 10\ \mathrm{Log}_{10}(P/P_{ref})$, $P_{ref} = 1$ mW

Voltage $(V_2/V_1)_{dB} = 20\ \mathrm{Log}_{10}(V_2/V_1)$

Current $(I_2/I_1)_{dB} = 20\ \mathrm{Log}_{10}(I_2/I_1)$

Voltage, dBV $V_{dBV} = 20\ \mathrm{Log}_{10}(V/V_{ref})$, $V_{ref} = 1$ V

Obvious extension, use
10 Log for power,
20 Log for voltage and current

$V^2 \sim$ Power

$I^2 \sim$ Power

Examples

$P = 102\,kW = 102 \cdot 10^3\,W \approx 10^5\,W$

$dBm = 10\log_{10}\left(\frac{10^5}{10^{-3}}\right)$

- Car engine, 137 HP (102 kW) —> dBm

$= 10\log_{10}(10^8) = 80\,dBm$

$dBV = 20\log\left(\frac{300 \cdot 10^{-6}}{1}\right) = 20\log_{10}(3 \cdot 10^{-4})$

- Antenna signal, 300 µV —> dBV

$= 20\log_{10}(10^{3-4}) = 20(-3{,}5) = -70\,dBV$

	P_2/P_1
0 dB	1
10 dB	10
20 dB	100

etc.

---

1 dB		$\boxed{1.26}$
2	(1+1)	
3	$\frac{1}{2}\times 4$	$\boxed{2}$
4	(10−6)	2.5
5	→ $\sqrt{10}$	= 3.16
6	(3+3)	4
7	(10−3)	5
8	(5+3)	6.3
9	(3+3+3)	8

$$10 \log_{10}\left(\frac{P_2}{P_1}\right) = 1 \longrightarrow P_2/P_1 = 1.26$$

102 kW $10^5 \to +50$

W → mW $+30$

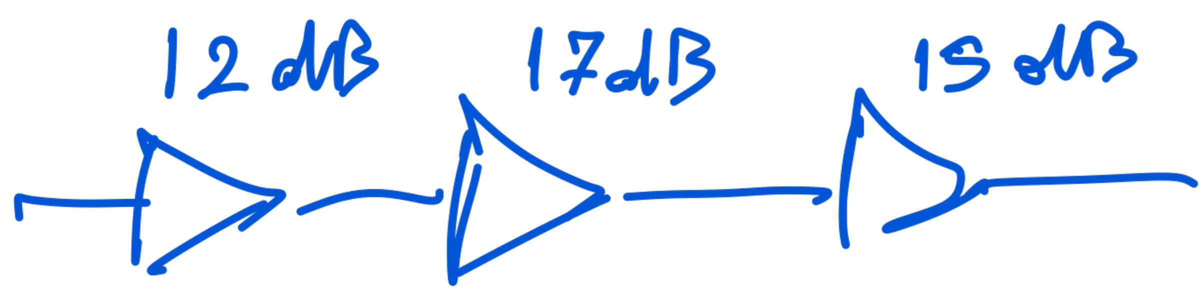
 80

---

300 μV 3×10^{-4} → −80

$+10$

−70



12+17+15
= 44

40+4   40 → 10000

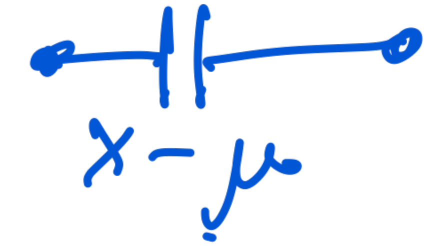
4   ×2.5

25000

Variance (signal power)

Time domain

$$\sigma^2 = \frac{1}{(R)T}\int_0^T |x(t)-\mu|^2 dt$$



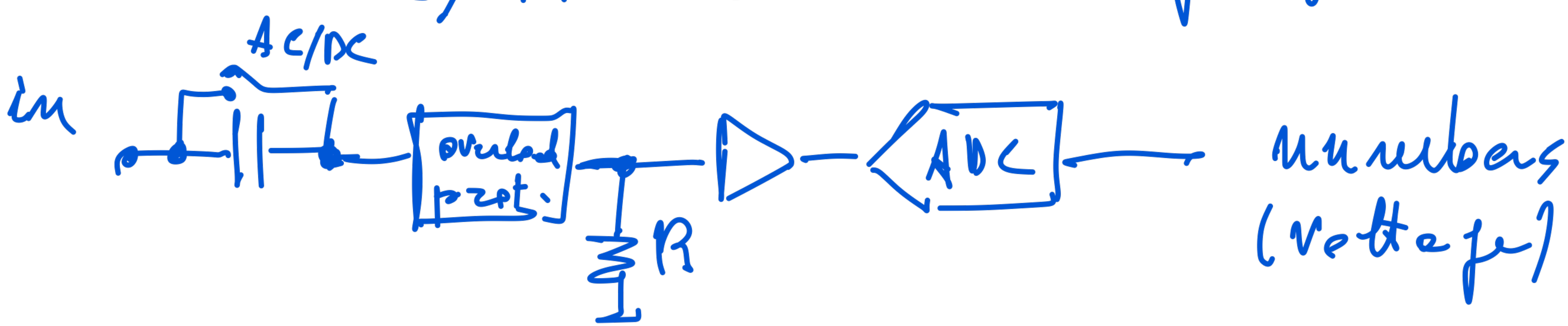
← average (dc value)

σ = deviation
rms value

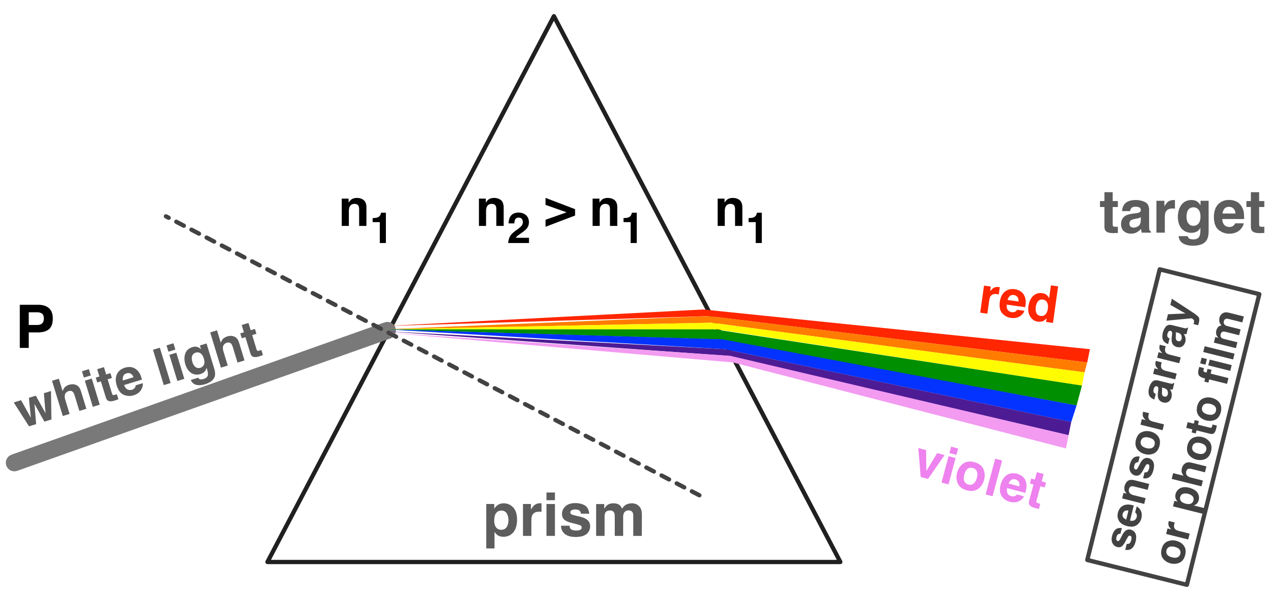
x [V] $\quad x^2$ [V^2] $\quad \frac{x^2}{R}$ [W]

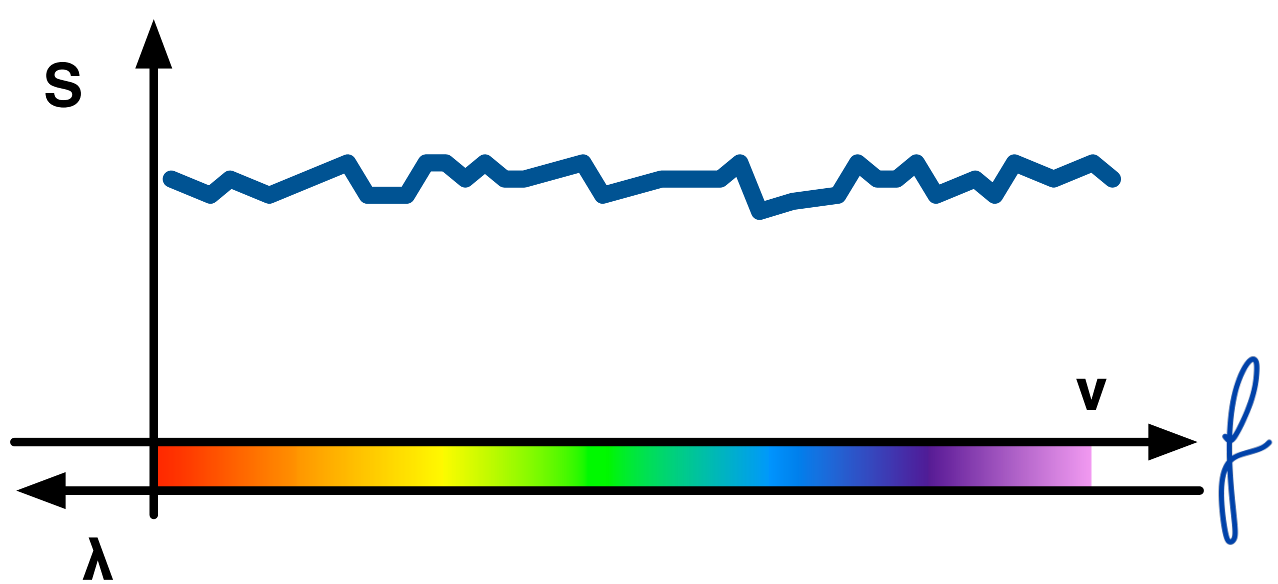
$x_1, x_2, x_3 \ldots x_N$ [V]

$\sigma^2 = [x_1^2 + x_2^2 \ldots]/RN$ [W] average power



Power Spectral Density S(f)





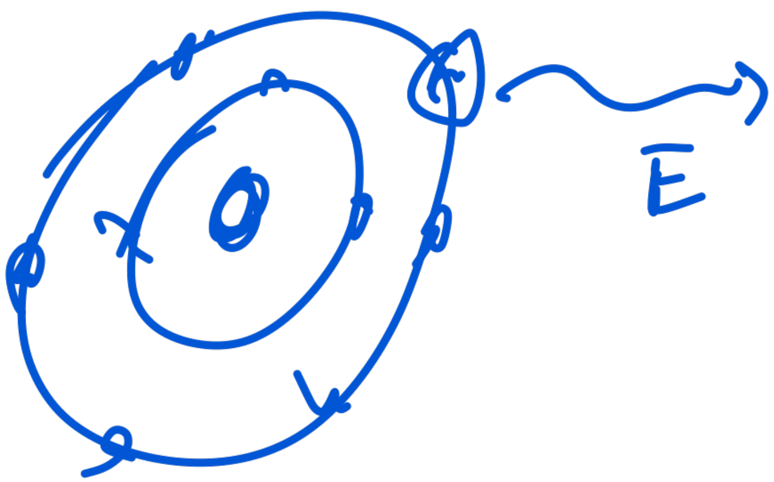
Continuous or discrete

- The PSD is the distribution of power vs. frequency (power in 1-Hz bandwidth)
- The PS is the distribution of energy vs. frequency (energy in 1-Hz bandwidth)
- Frequency can be continuous or discrete (histogram),

- In mathematics,
- the power is a square quantity
- the energy is power integrated in time

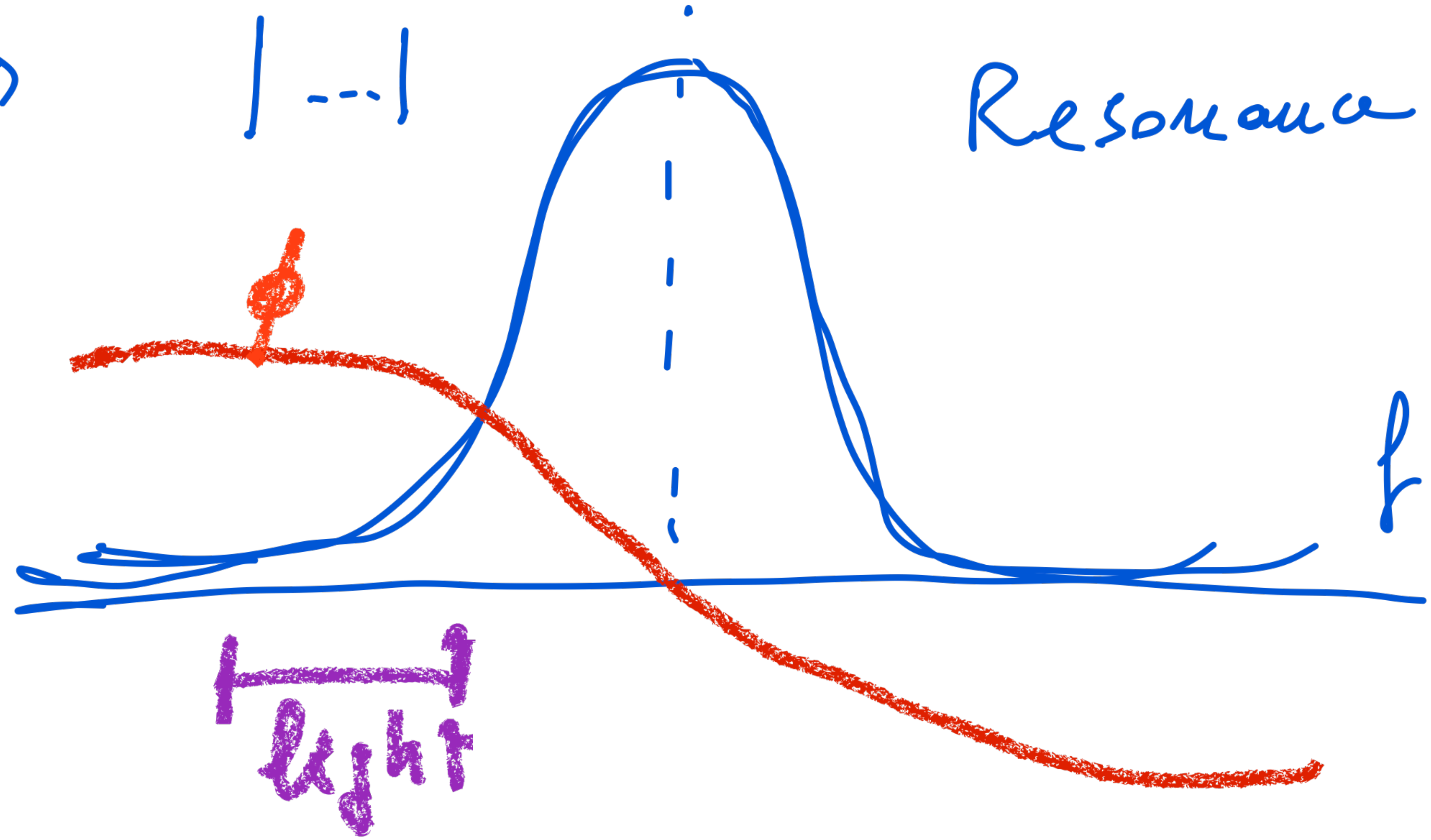
- Power (energy) in physics is a square (integrated) quantity
- PSD –> W/Hz (or V^2/Hz, A^2/Hz, etc.)
- PS –> J/Hz

Where Dispersion Comes from



group delay

$$\tau_g = \frac{d\phi}{d\omega}$$



End of lecture 3

#4 Monday, Sept 23, 2019

1.5 Hours

Power, Spectra and Probability

– continuation –

Parseval Theorem

The power P (the variance σ^2) of a signal can be evaluated in time domain or in the frequency domain, and the result is the same

$$P = \int_0^\infty S(f)\, df$$

One-sided PSD

Frequency domain

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \; dt$$

Time domain

$\int_0^T$ for deterministic signals

Fourier Transform and PSD

$$X(\omega) = \int x(t)\, e^{-i\omega t}\, dt$$

$$\left(e^{i\omega t}\right)^*$$

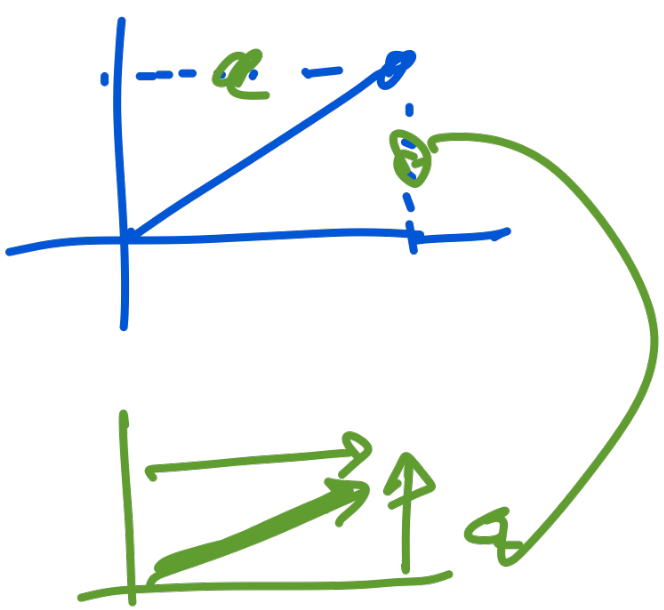
(sum)

$$x(t) = \frac{1}{2\pi} \int X(\omega)\, e^{i\omega t}\, d\omega$$

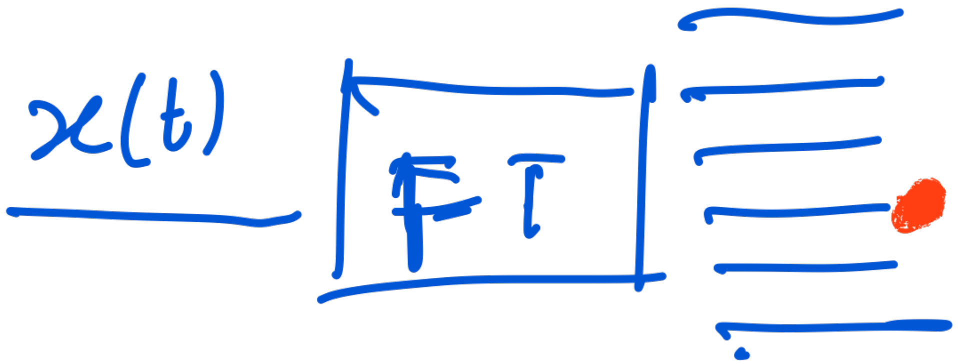
Spectral component

unity vector

Scalar product (a, b)



$$(a, b) = \int_{\mathbb{R}} a(t)\, b^*(t)\, dt$$



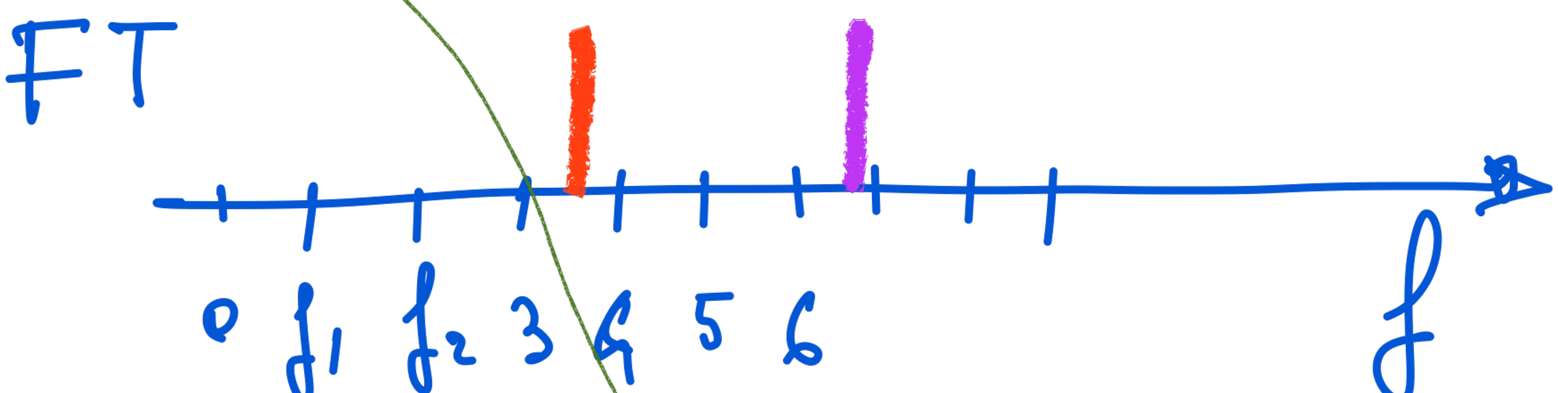
FT → DFT

discrete Fourier Transform

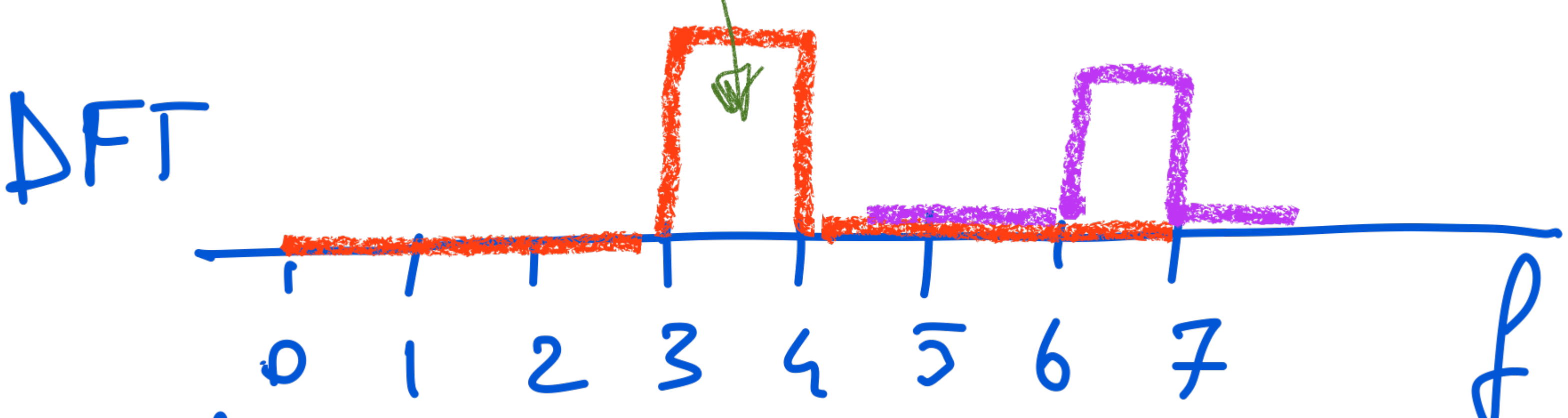
Δt

$f_3 < f < f_4$

$f_6 < f < f_7$



The DFT is equivalent to a filter bank



f_0, ----

$x(t)$ [V] → (D)FT → $v_1, v_2, \ldots$

$P = \langle v^2 \rangle$ mean square voltage

$P_i = \langle v_i^2 \rangle$

Use $\langle v^2 \rangle / R$ for true electrical power

PSD =

Careful with sinusoids

$$S^{II}(f) = \frac{1}{T} \left| X^2(f) \right|$$

TWO SIDED

ONE-SIDED → $f > 0$

T: acquisition time

$$S^{I}(f) = \frac{2}{T} \left| X^2(f) \right|$$

average definition

$$\langle a \rangle = \frac{1}{T} \int_T a(t)\,dt$$

$$\boxed{\omega = 2\pi f}$$ ← shorthand

PERIODIC SIGNALS

FT

Infinite time

Periodic $\longrightarrow$ FT contains $\delta(f)$
Dirac

$|X^2(f)|$ does not exist because of $\delta^2(f) = ?$

DFT

Finite time

No problem at all

$|X^2(f)|$ is always a valid quantity

DFT, FFT, FFTW, SFFT

The Discrete Fourier Transform (DFT) approximates the (continuous) FT

$$X\left(\frac{n}{NT_s}\right) = \sum_{k=0}^{N-1} x(kT_s)\, e^{-\imath 2\pi nk/N}$$

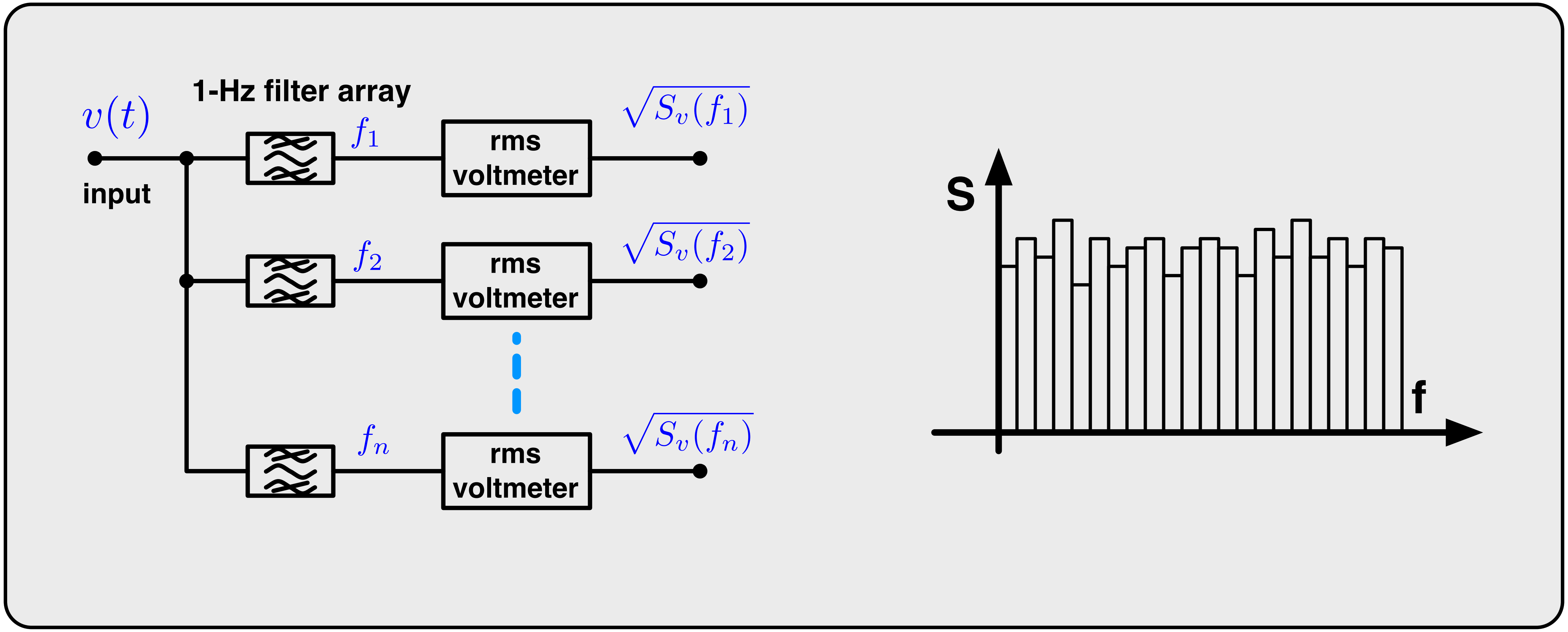
Basic DFT

T_s = sampling interval, $f_S = 1/T_s$

$n = 0 \ldots N-1$ integer frequency, $f = n/NT$

- The direct computation of the DFT takes ≈ N^2 multiplications
- The FFT is an algorithm for Fast computation of the DFT that takes ≈ N log(N) multiplications
- The FFTW, “the Fastest Fourier Transform in the West,” is an even faster. N log(N) multiplications (M. Frigo, S.G. Johnson, MIT)
  See http://fftw.org/
- SFFT “faster-than-fast” Sparse (FFT, D.Katabi, P.Indyk, MIT)
  See http://groups.csail.mit.edu/netmit/sFFT/
- For the general user (does not implement FT algorithms), the difference between DFT, FFT, and FFTW is (at most) computing time

Parallel Spectrum Analyzer

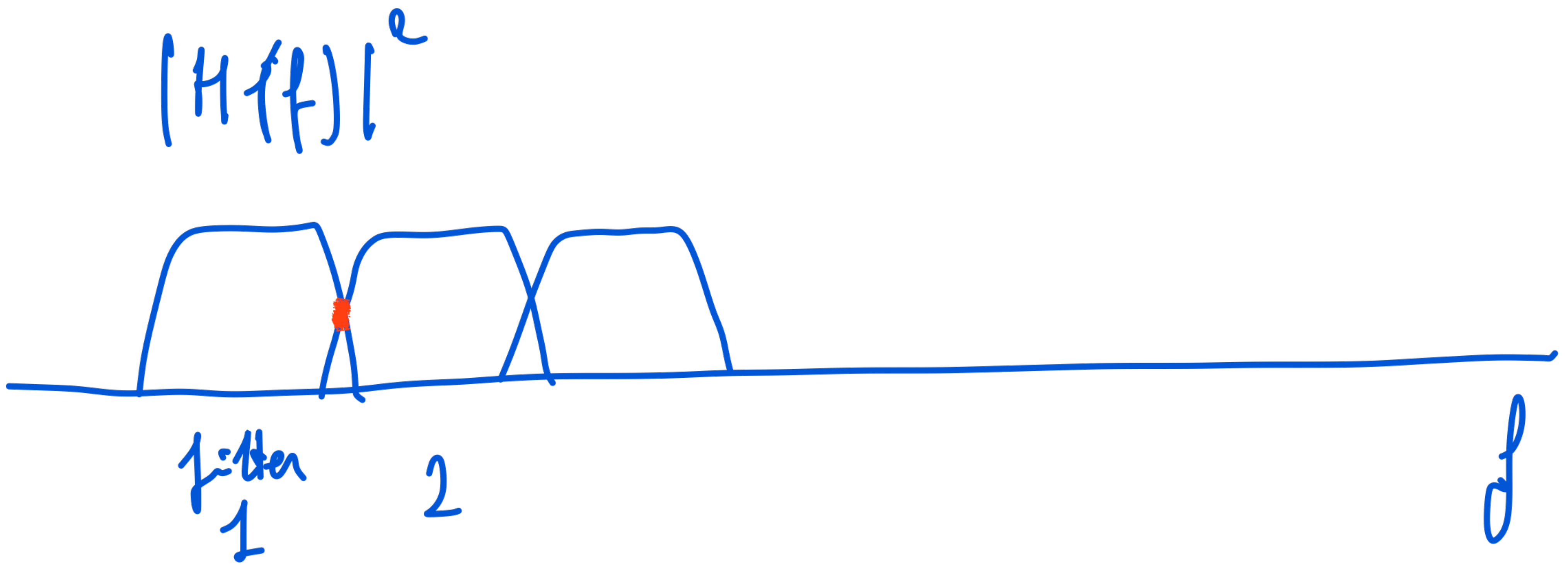


Rice representation —> Extension of the Fourier series

$$x(t) = \sum_{n=0}^{\infty} a_n(t)\cos(n\omega_0 t) - b_n(t)\sin(n\omega_0 t)$$

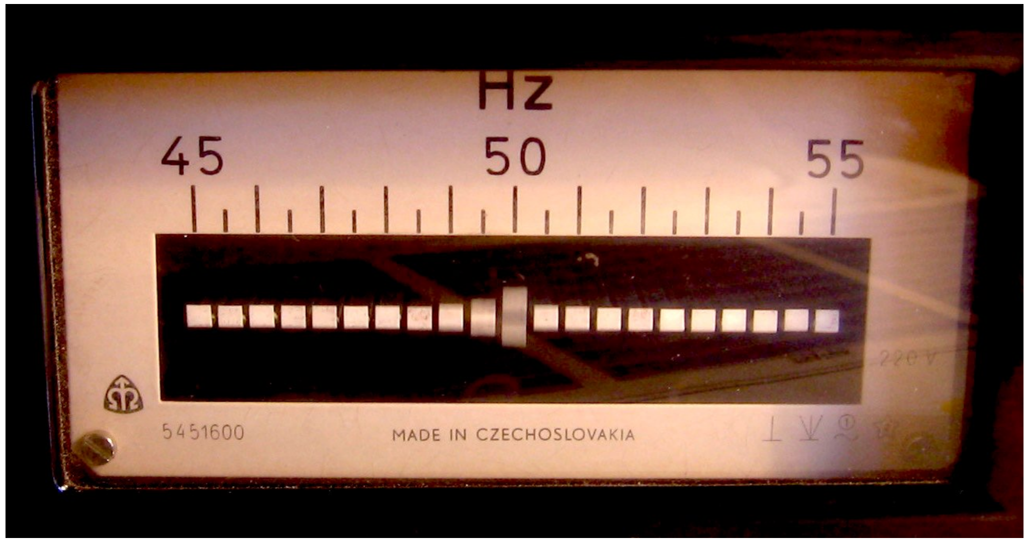
$$S_x(n\omega_0) = \left[a_n^2 + b_n^2\right]/\omega_0$$

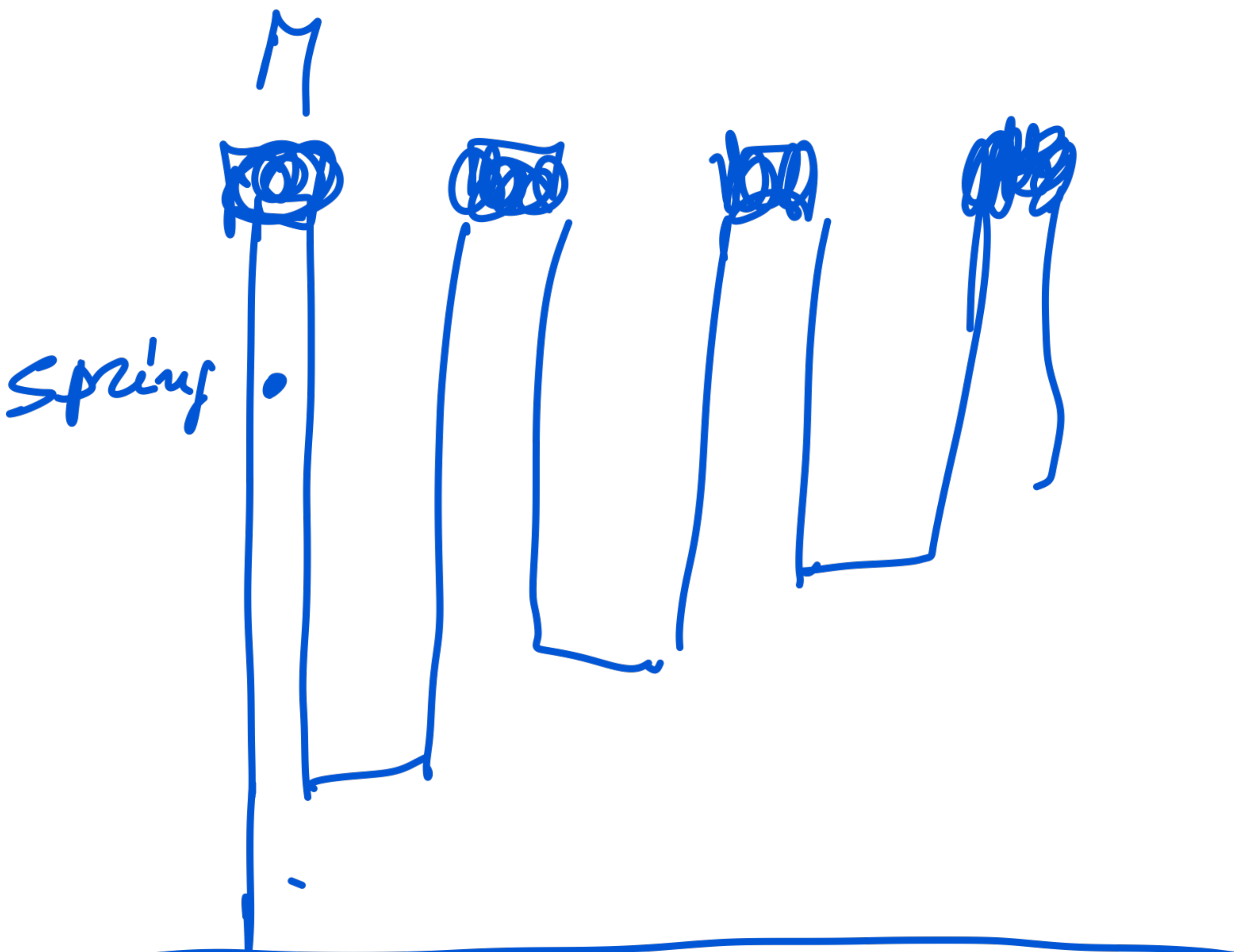
ω_0 is the analysis bandwidth



$$\sum_i H_i(f) = 1$$

Vibrating-Reed Frequency Meter



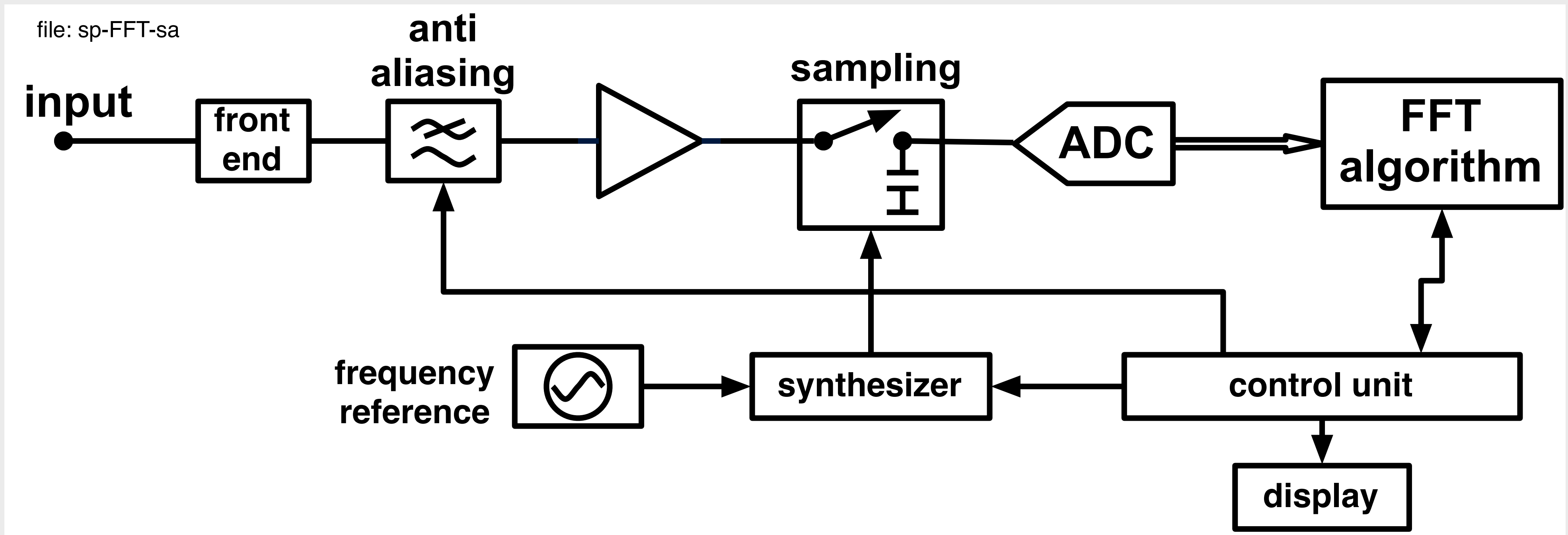


End of Lecture #4

#5 Tuesday, Sept 24, 2019

1.5 Hours

FFT Spectrum Analyzer



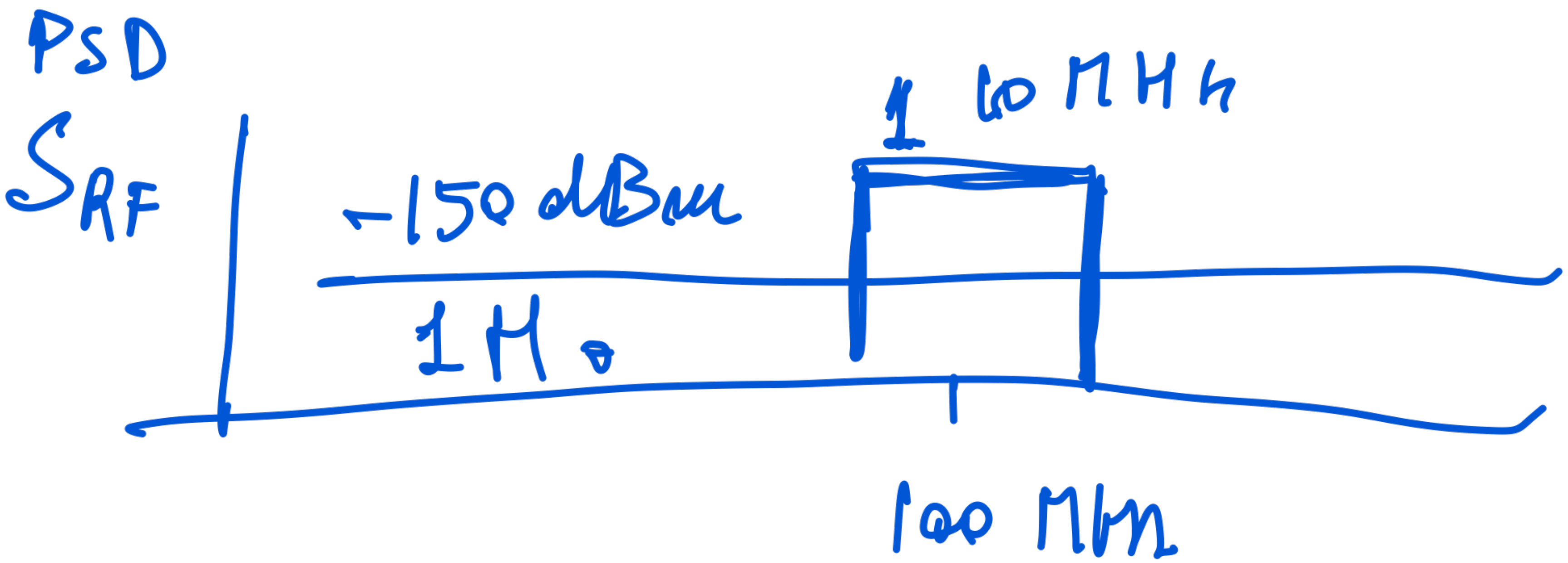
- Direct digitization of the input signal
- Fully digital process
- Limited to $f_{\text{max}} \approx 0.4 \times f_{\text{sampling}}$
- Tough tradeoff between resolution and max frequency

Examples

1. White noise, –150 dBm/Hz
   Calculate the power in B = 10 MHz centered at f_0 = 100 MHz
2. Pure 100 MHz carrier, +10 dBm power, sketch the PSD
   Assume ideally narrow bandwidth, continuous PSD
3. Pure 100 MHz carrier, +10 dBm power, sketch the PSD
   Assume discrete PSD, RBW = 5 kHz
4. Amplitude-modulated signal,
   Carrier f_0 = 100 MHz, P_0 = 10 dBm
   Sidebands f_m = 500 Hz, P_{SB} = 0 dBm
   - Assume ideally narrow bandwidth, continuous PSD
   - Assume discrete PSD, RBW = 5 kHz

Example 1

- White noise, –150 dBm/Hz
  Calculate the power in B = 10 MHz centered at f_0 = 100 MHz



$$P = \int_a^b S_{RF}(f)\,df$$

$$P = S_{RF} * B$$

1. 150 dBm = 10^{-15} mW

$$S_{RF} * B = 10^{-15} \cdot 10^{7} = 10^{-8}\ \text{mW}$$

$$10\log()\ \underline{S_{RF} \cdot B}$$

$$\downarrow$$

$$-150 + 70 = -80\ \text{dBm}$$

$P = S_{RF} \cdot B$

$10\log() \quad 10\log() + 10\log()$

$10\log 10^7$

$-150 + 70 = -80\ \text{dBm}$

dBm

$\downarrow$

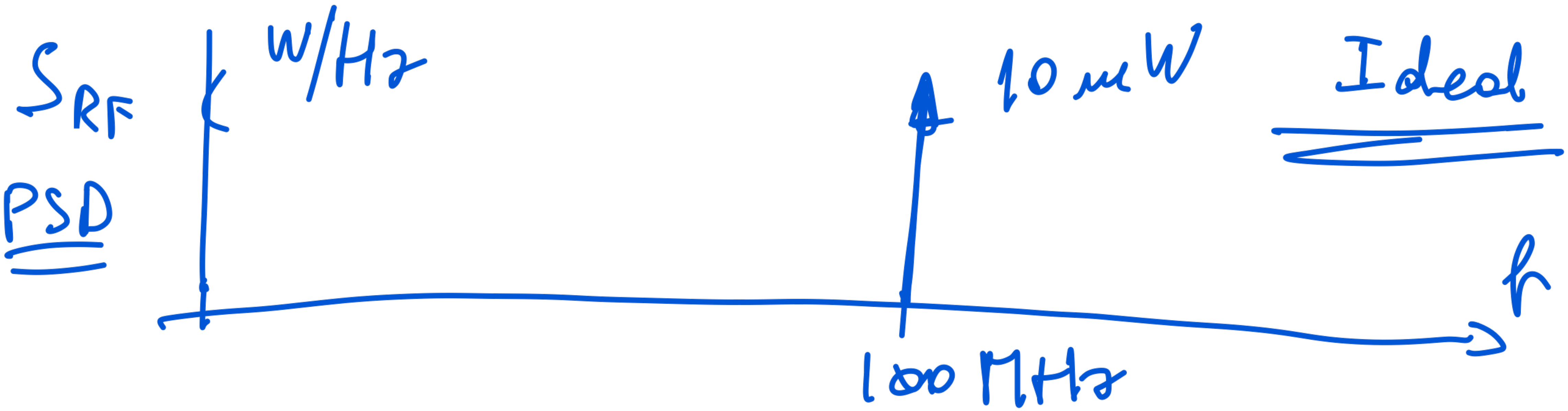
$10^{-8}\ \text{mW}$

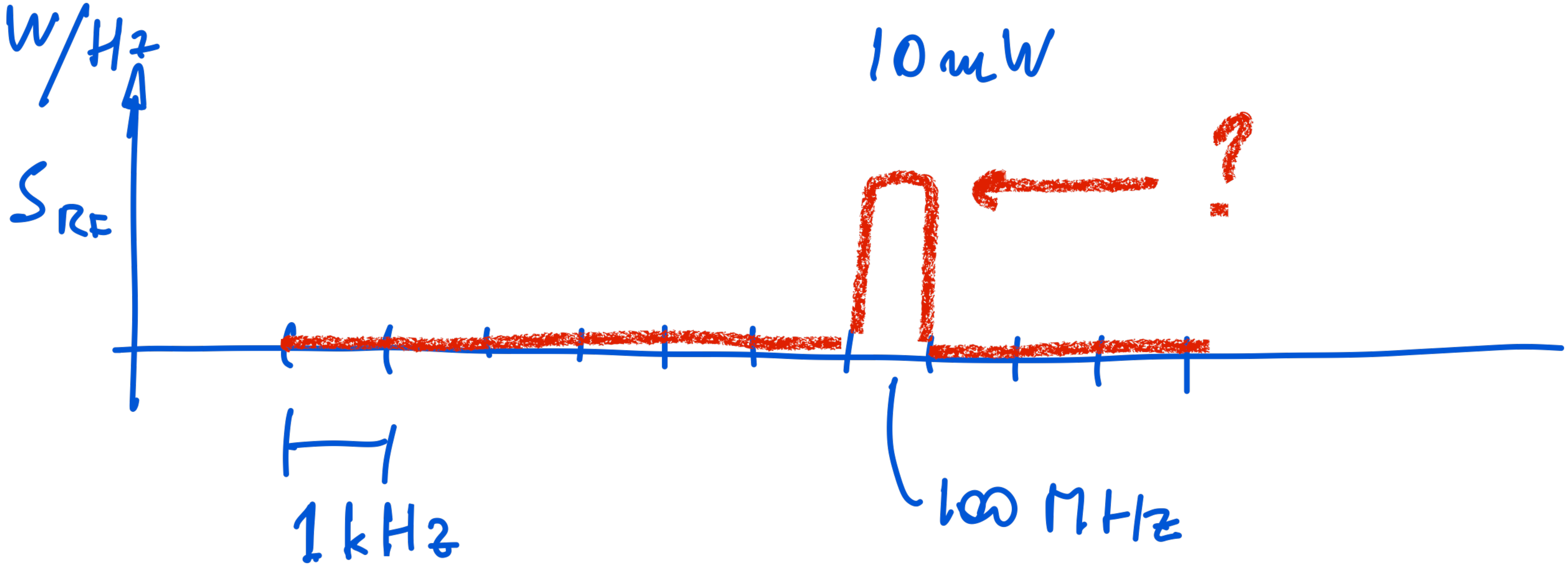
$\downarrow$

$10^{-11}\ \text{W}$

Example 2

- Pure 100 MHz carrier, +10 dBm power, sketch the PSD Assume ideally narrow bandwidth.





$P = S_{RF} \cdot B$

$$P = \int_a^b S_{RF}(f)\, df$$

$S_{RF} = \frac{P}{B}$

$10\ \text{dBm} - 10 \log B = -20\ \text{dBm}$

-30

$\frac{10\ \text{mW}}{1\ \text{kHz}} = 10^{-5}\ \text{W/Hz}$

Example 3

- Pure 100 MHz carrier, +10 dBm power, sketch the PSD
  Assume discrete PSD, RBW = 5 kHz

Already done, with 1kHz

RBW = Resolution BandWidt

Example 4

- Amplitude-modulated signal, sketch the PSD
  Carrier f_0 = 100 MHz, P_0 = 10 dBm
  Sidebands f_m = 500 Hz, P_{SB} = 0 dBm
  - Assume ideally narrow bandwidth, continuous PSD
  - Assume discrete PSD, RBW = 5 kHz

Homework

modulation index,
$\leftrightarrow$ gives the sideband power

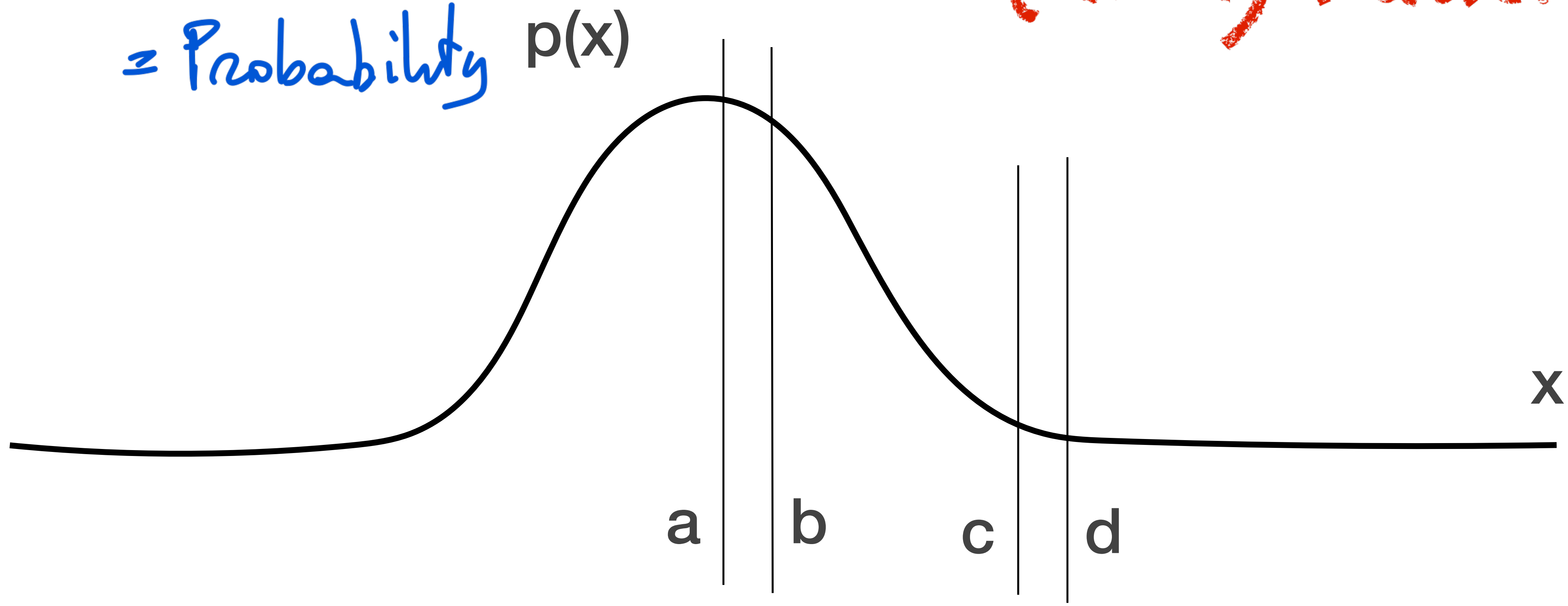
$$v = V_0 \left[1 + m\cos(\Omega t)\right]\cos(\omega t)$$

Probability Density Distribution

$p(x)$ PDF

(not a) Function

P = Probability



1

$$P_{ab} = \int_a^b p(x)\, dx$$

$P\{a < x < b\}$

2

$$\int_{-\infty}^{\infty} p(x)\, dx = 1$$

3

$$P_{ab \text{ or } cd} = P_{ab} + P_{cd}$$

iff ab and cd are separated

1) A priori (card games, roulette, etc.)

2) Experimental

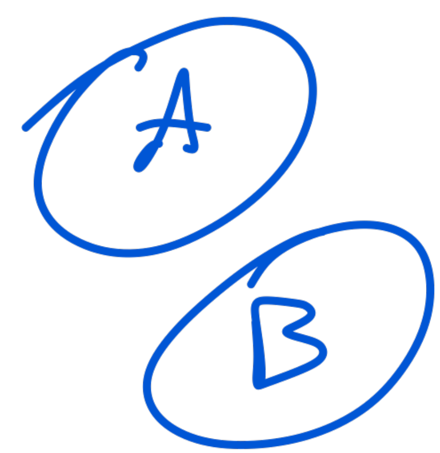
repeated trials converge to the probability

3) Kolmogorov E

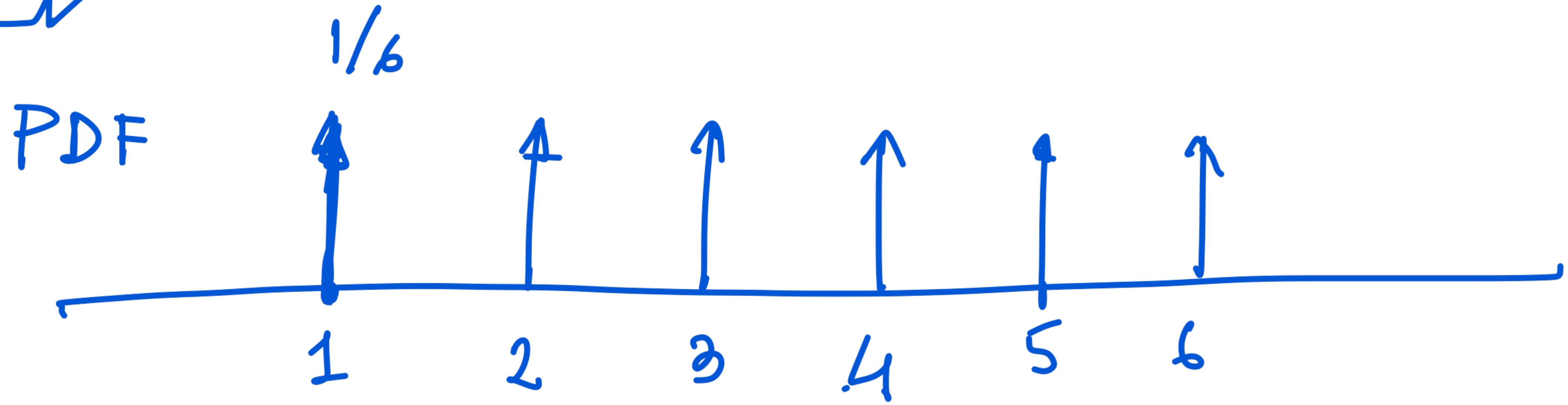
$$P \in [0, 1]$$

$$P\{E\} = 1$$

$$P\{A + B\} = P\{A\} + P\{B\}$$



$p(x)$



$$P\{a < x < b\} = \int_a^b p(x)\,dx$$

$$P\{1 \leq x \leq 6\} = 1$$

function $\quad f(x)$

distribution $\quad \delta(x)$

$$\int_{\mathbb{R}} f(x)\,\delta(x) = f(0)$$

End of lecture

\# 5

#6 Thursday, Sept 26, 2019

1.5 Hours

Variance (signal power)

Time domain

$$\sigma^2 = \frac{1}{T}\int_0^T |x(t)|^2 \, dt$$

Probability

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 p(x)\, dx$$

Spectrum

$$\sigma^2 = \int_0^{\infty} S(f)\, df$$

Parseval Theorem

Wiener Khinchin Theorem

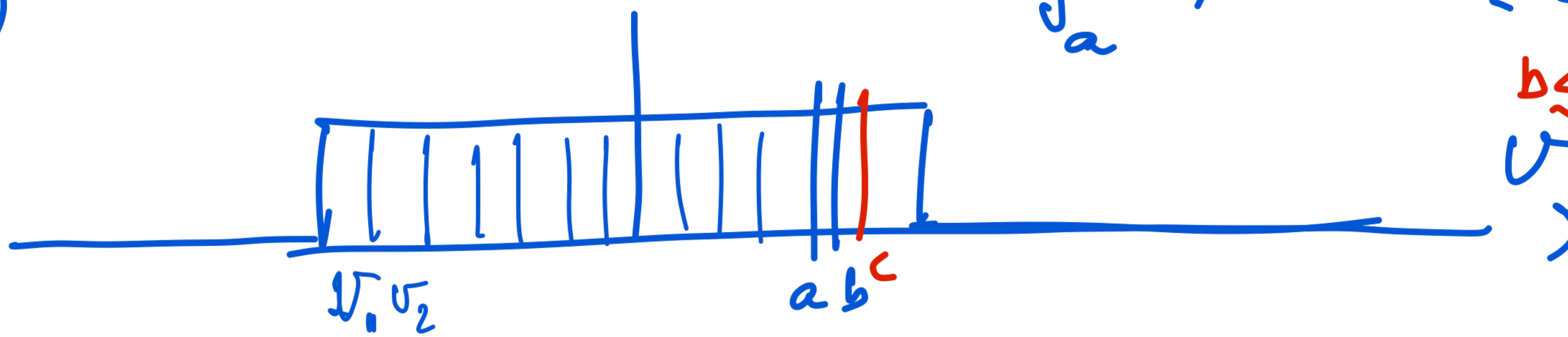
Why the Variance is Power

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 p(x)\, dx$$

$$\int_a^b p(v)\,dv = \mathbb{P}\{a \le v < b\}$$

$b \le v < c$

p(v)



probability

power

$\mathbb{P}_1 \longrightarrow P = v_1^2$ $\qquad v_1^2 \quad \mathbb{P}_1$

$\mathbb{P}_2 \longrightarrow P = v_2^2$ $\qquad v_2^2 \quad \mathbb{P}_2$

$\vdots$

$v_1^2\mathbb{P}_1 + v_2^2\mathbb{P}_2 + \cdots v_n^2\mathbb{P}_n$ = average power $\langle P \rangle$

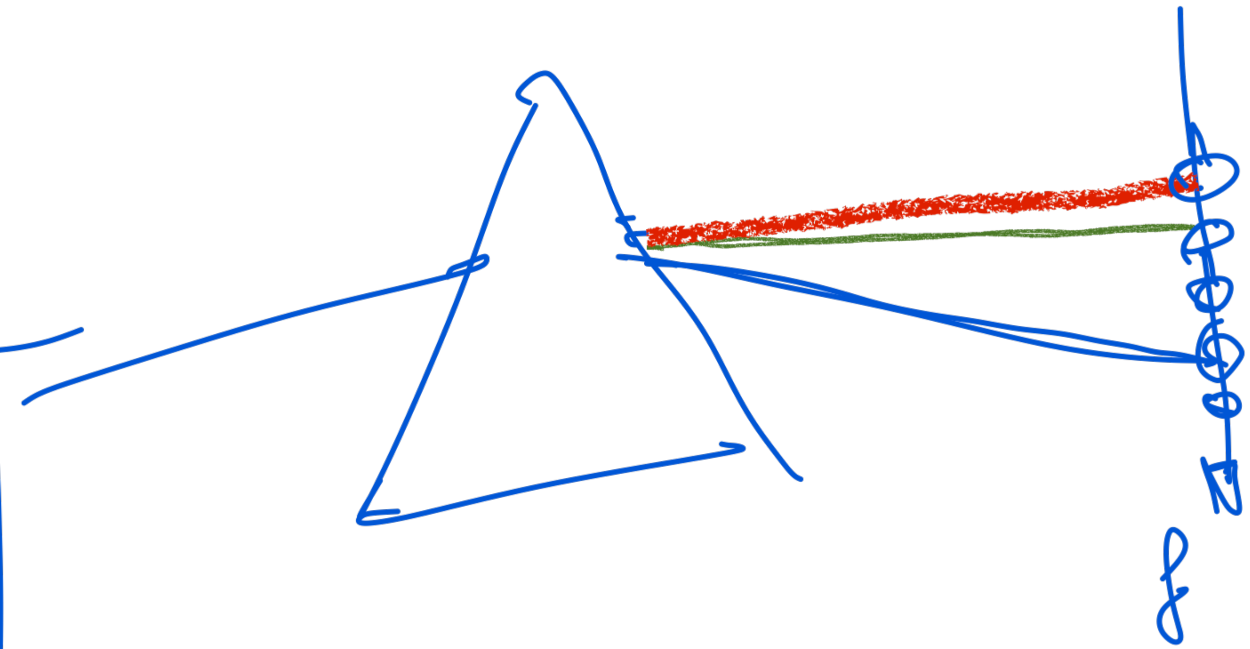
$\mathbb{E}\{P\}$

$v(t)$

instantaneous power

$$P = \frac{1}{T}\int_0^T v^2(t)\,dt$$

$$P(t) = \frac{v^2(t)}{\cancel{R}}$$

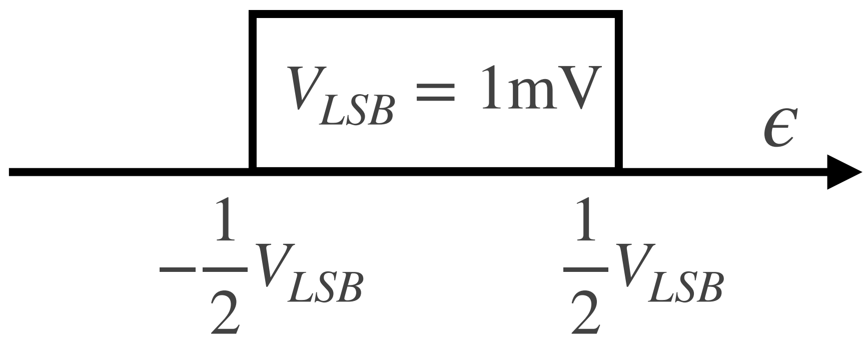


$$P = \int_0^{\infty} S(f)\,df$$

Parseval Theorem

$$P = \int_{-\infty}^{\infty} v^2\,p(v)\,dv$$

Example

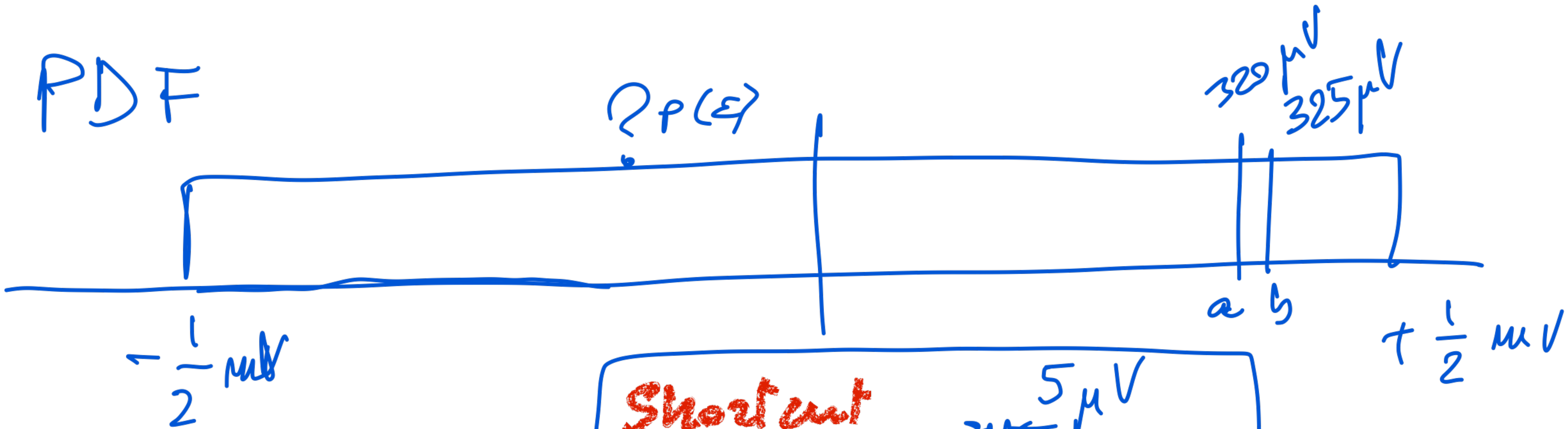


Probability

Calculate

$$P\{320\ \mu\text{V} < \epsilon < 325\ \mu\text{V}\}$$

PDF

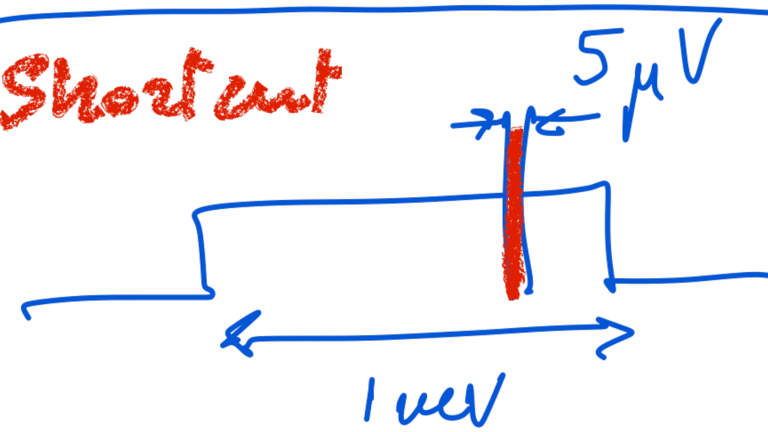


$P\{-0{,}5 \le \varepsilon \le 0{,}5\} = 1$

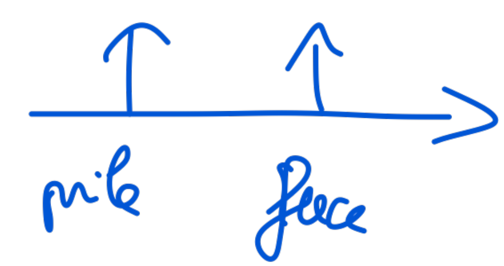
$P\{\varepsilon\} = ?$

$\int_{-5\cdot10^{-4}}^{5\cdot10^{-4}} p(\varepsilon)\, d\varepsilon = 1$

Shortcut



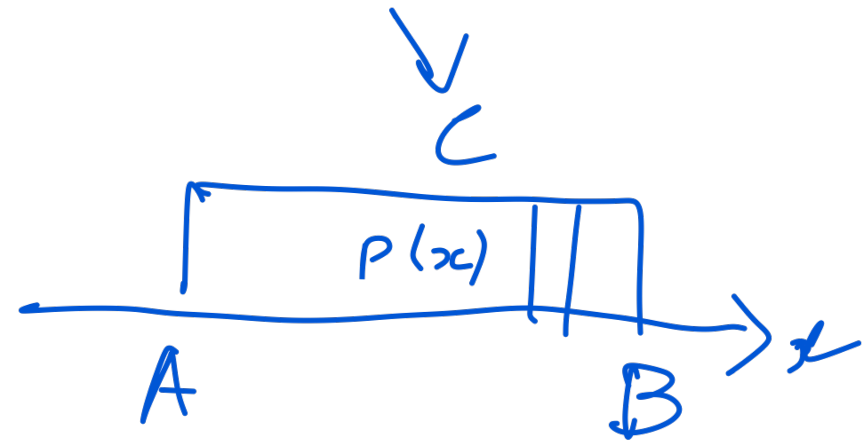
$P = \frac{5\,\mu V}{1\,mV} = 5\times10^{-3}$



2 choice

Σ choic = 1

$$\int_{-\infty}^{\infty} p(x)\,dx = 1 = C(B-A) \quad (*)$$



$$\int_{a}^{b} p(x)\,dx = P\{a < x < b\}$$

$$\begin{cases} A = -5.10^{-4} \\ B = 5.10^{-4} \end{cases}$$

$$(*) \quad 1 = C\left(5.10^{-4} - (-5.10^{-4})\right)$$

$$\Leftrightarrow 1 = C.10^{-3}$$

$$\Leftrightarrow \frac{1}{10^{-3}} = C$$

$$\Leftrightarrow \boxed{C = 10^{3}\,V^{-2}}$$

$$p(x) = \begin{cases} C \text{ si } x \in [A,B] \\ 0 \text{ sinon} \end{cases}$$

$$P\{320\,\mu V < x < 325\,\mu V\}$$

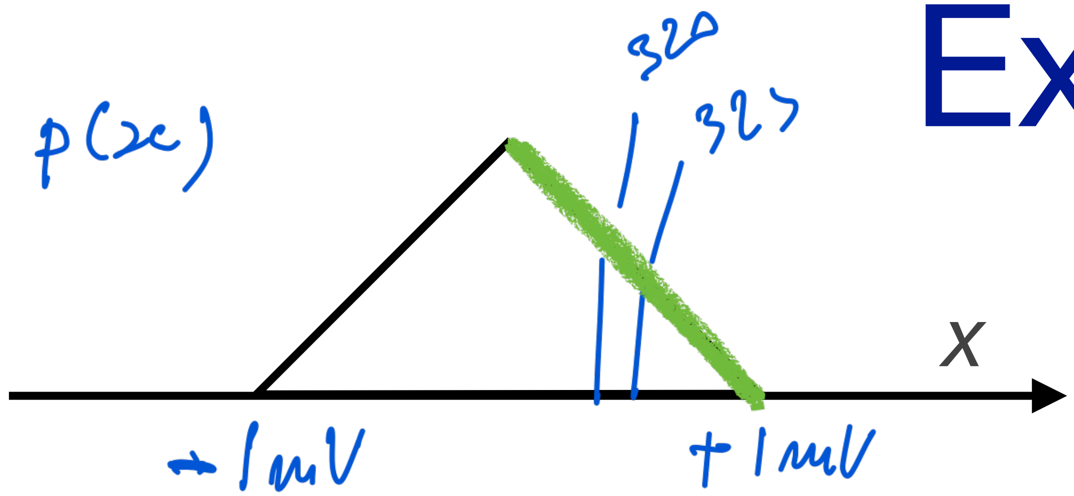
$$= \int_{320.10^{-6}}^{325.10^{-6}} C\,dx$$

$$= C \times \Delta x$$

$$= 10^{3} \times 5.10^{-6}$$

$$= \boxed{5.10^{-3}\ \varnothing}$$

Example



Calculate

$$\underbrace{P\{320\ \mu\text{V} < x < 325\ \mu\text{V}\}}_{A}$$

a b

$$p(x) = \begin{cases} 0 & x < -1\ \text{mV} \\ \text{—} & -1\ \text{mV} \\ \text{—} & +1\ \text{mV} \\ 0 & x > 1\ \text{mV} \end{cases}$$

$$\int_{-\infty}^{\infty} p(x) = 1$$

$$P\{A\} = \int_a^b p(x)$$

Homework

Gaussian (Normal) Distribution

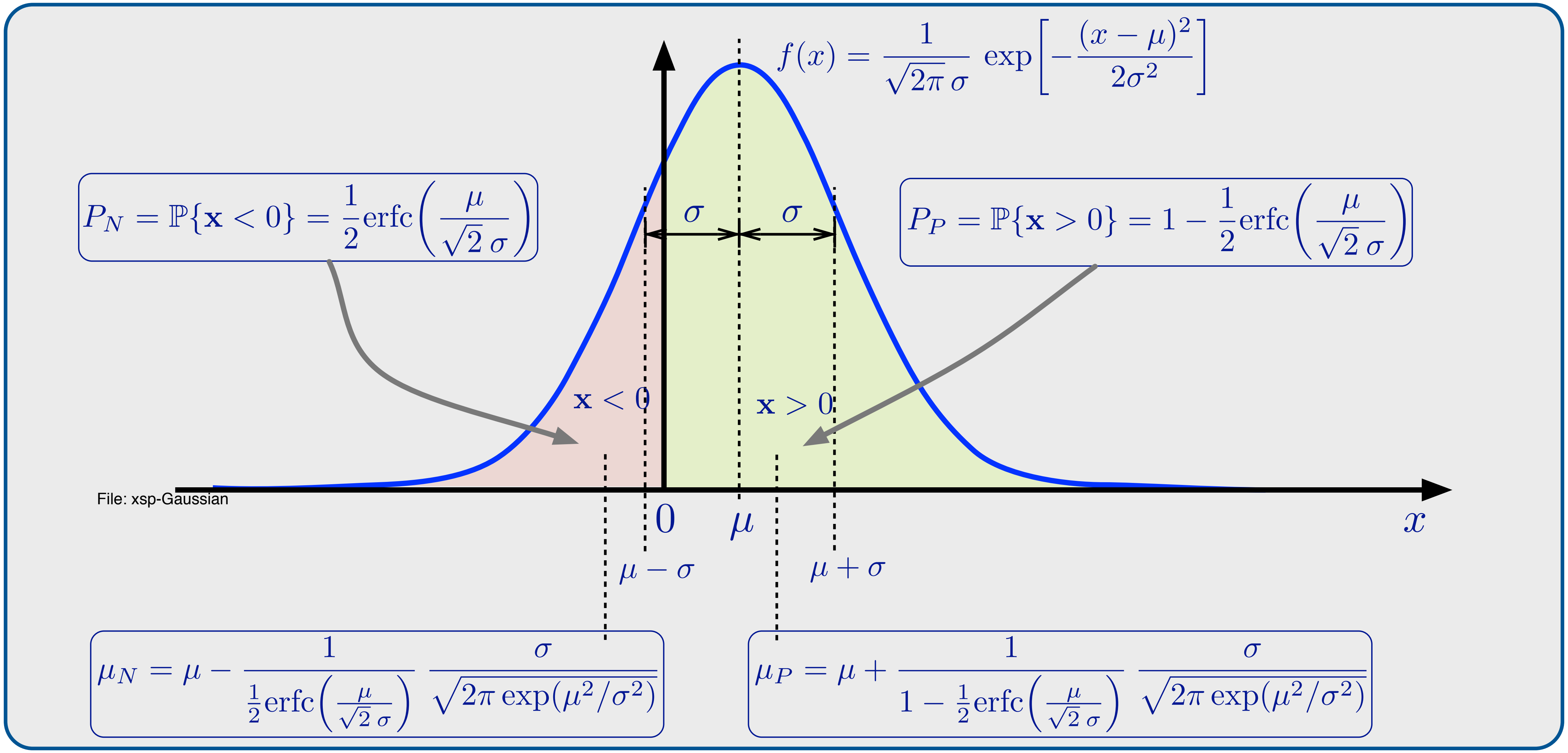
x is normal distributed with zero mean μ and variance σ²

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

$$\mathbb{E}\{f(x)\} = \mu$$

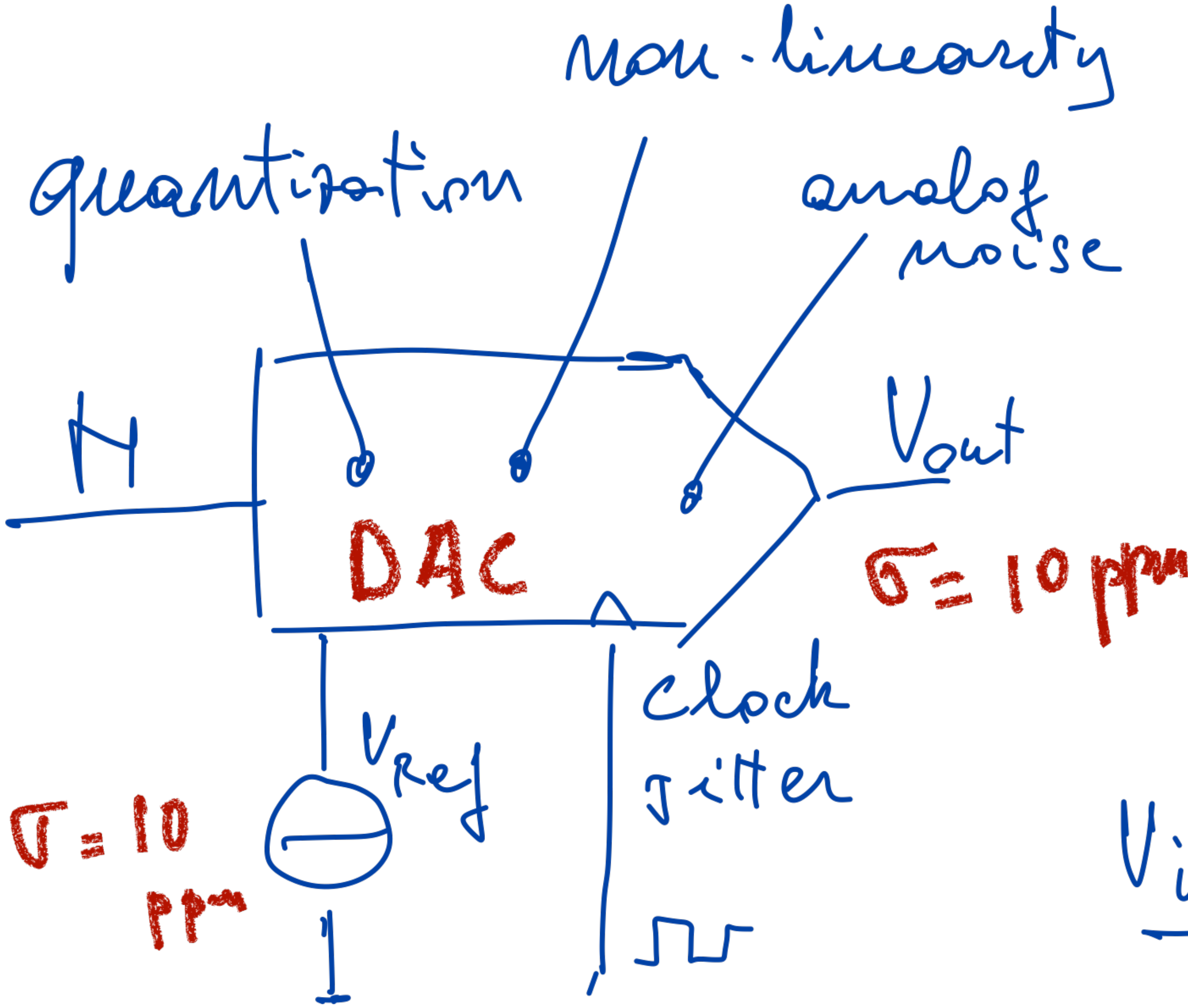
$$\mathbb{E}\{f^2(x)\} = \mu^2 + \sigma^2$$

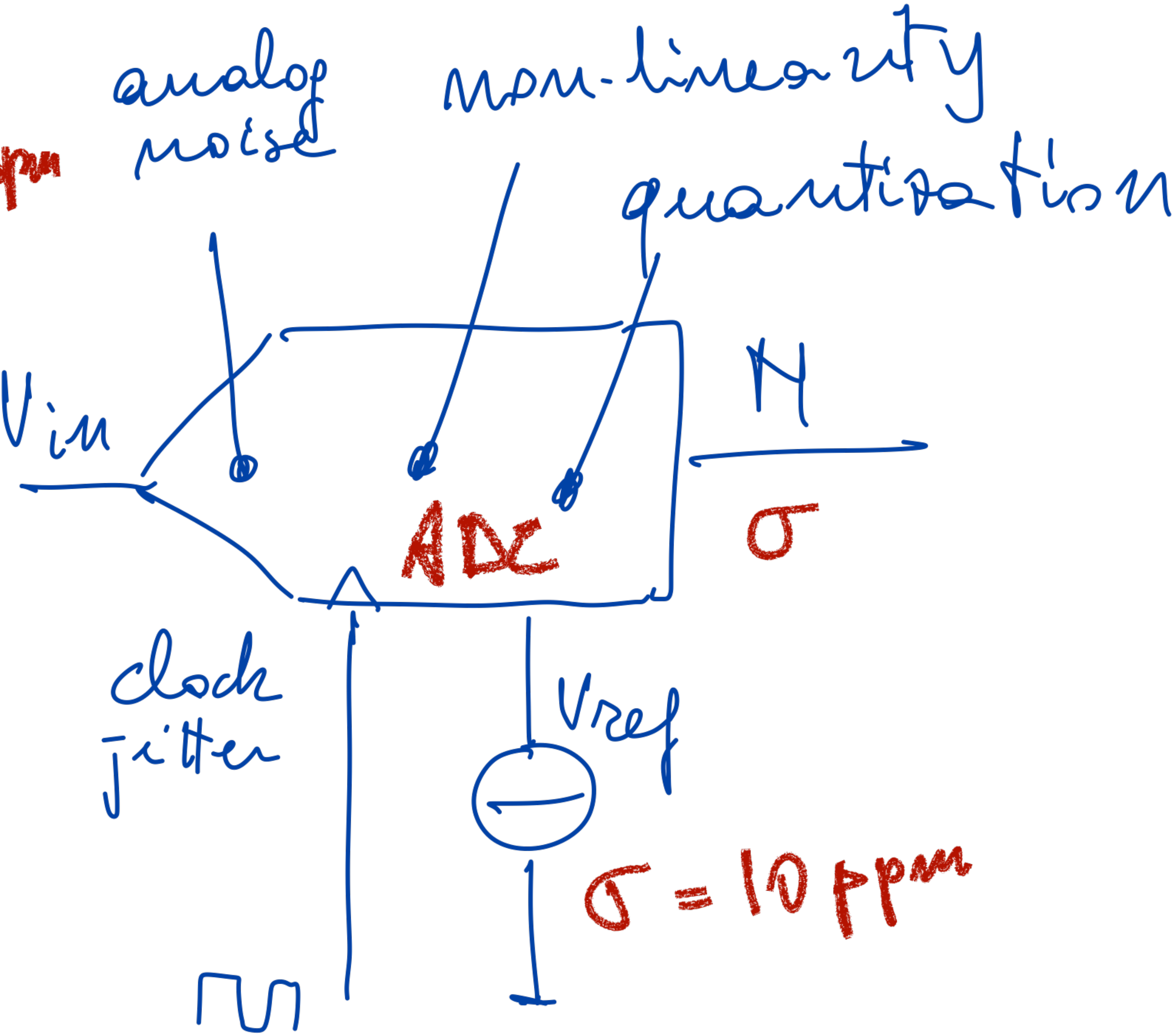
$$\mathbb{E}\{|f(x) - \mathbb{E}\{f(x)\}|^2\} = \sigma^2$$



Noise

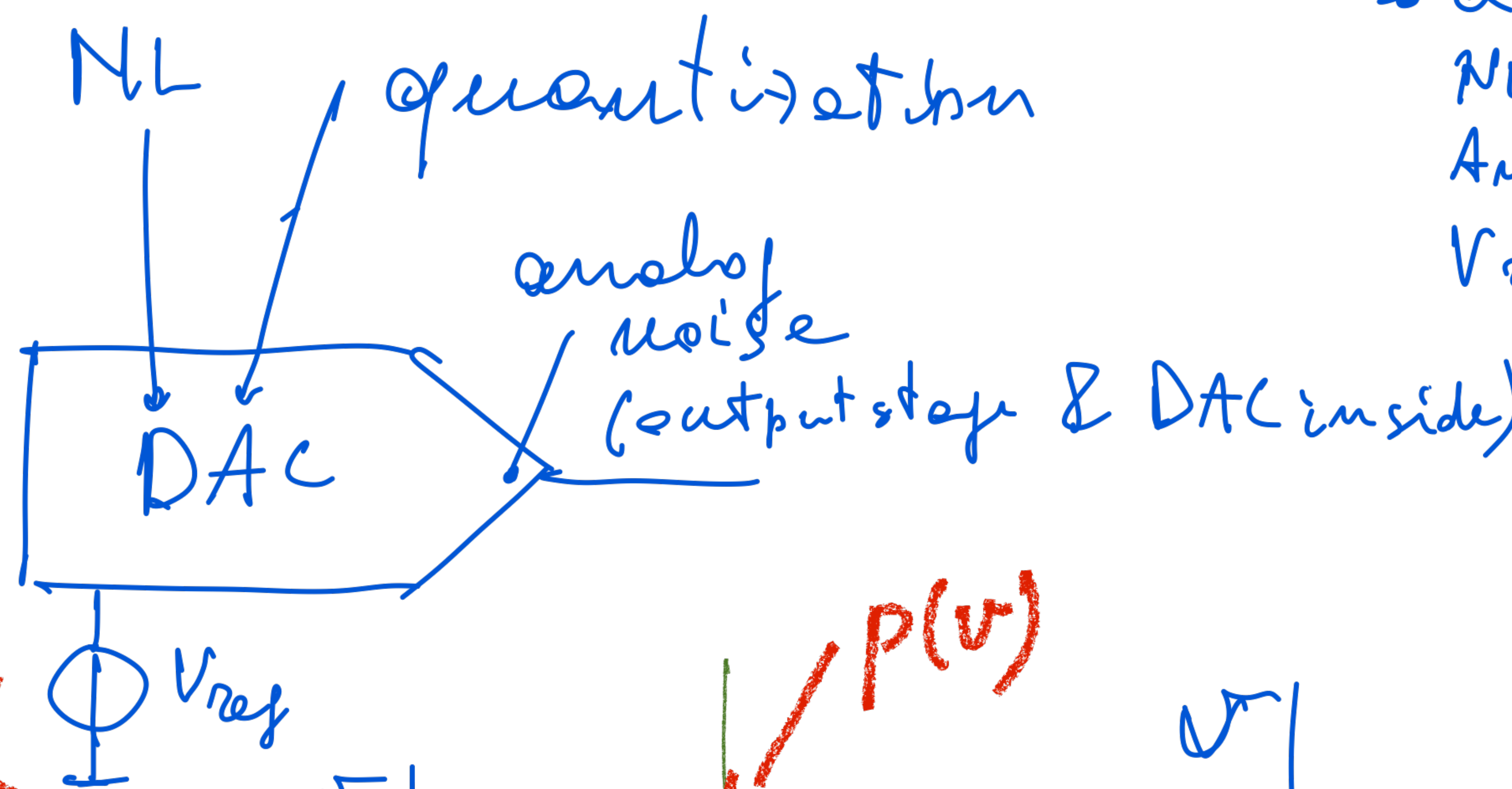
Accuracy and Noise — The Big Picture





Remember:
clock jitter affects only timing

- Q.
- NL
- Analog
- Vref

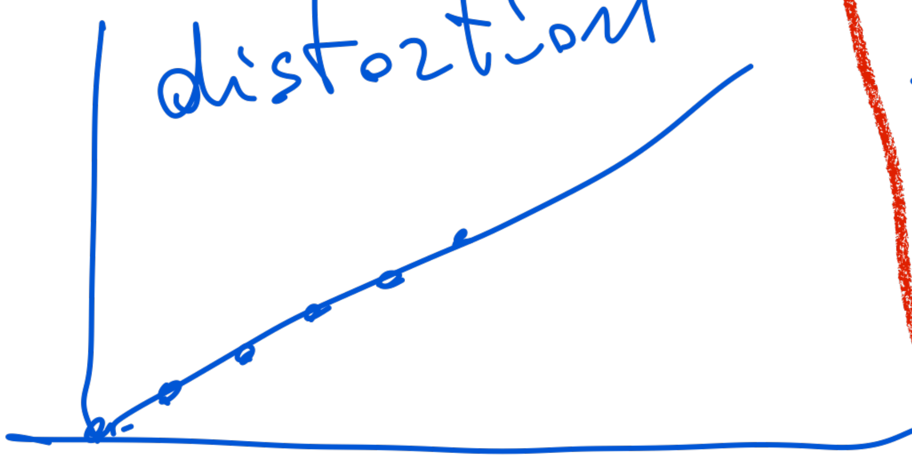


Already seen

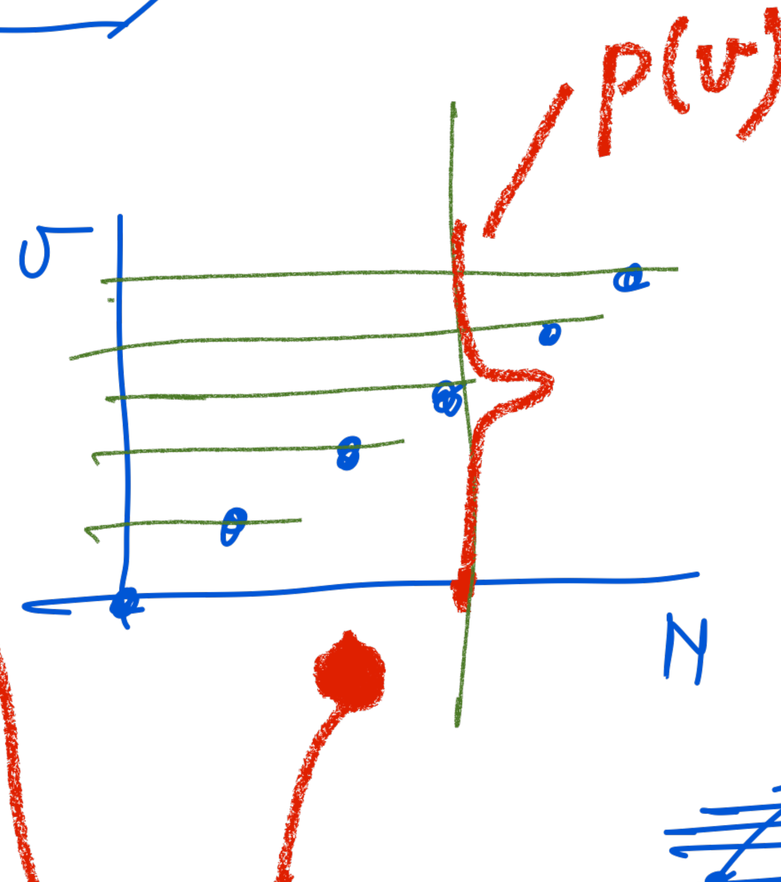
Non-linearity

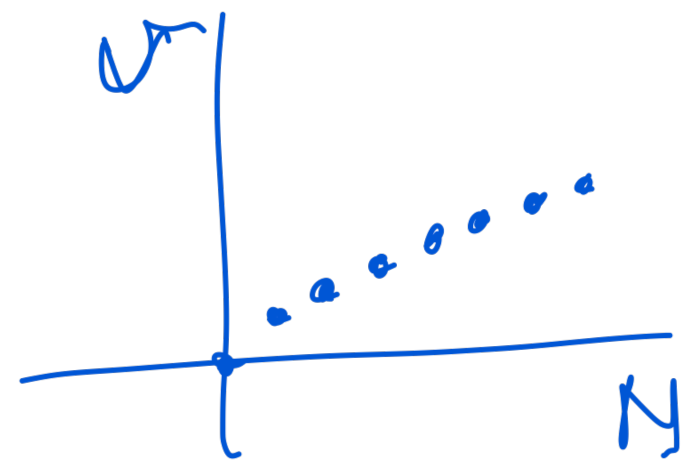
↓

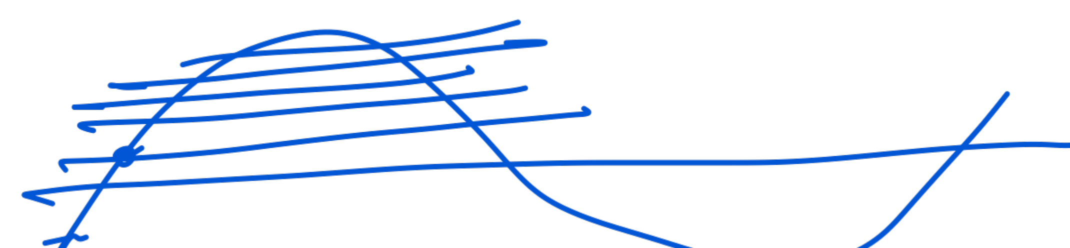
distortion



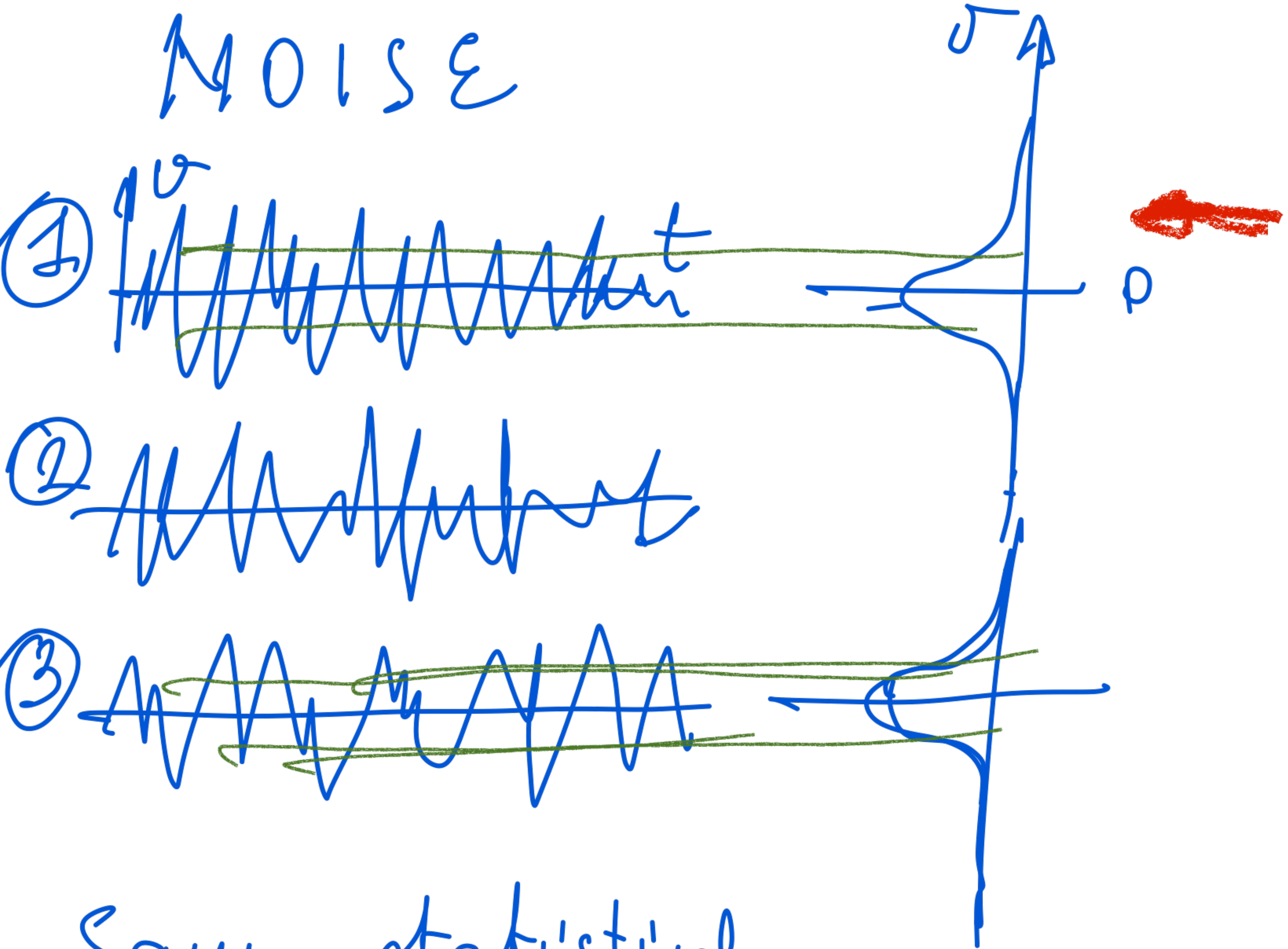
for later







NOISE



Same statistical properties

PDF

$\boxed{\text{GAUSS}}$ $\mu = 0$

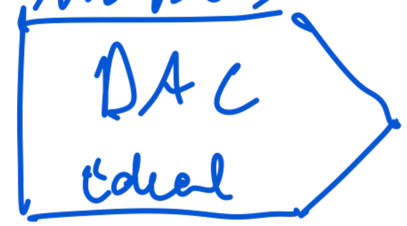
$P\{|v| < \sigma\}$ 0.68

2σ 0.95

$$P\{\mu - \sigma < v < \mu + \sigma\} \doteq 0.68$$

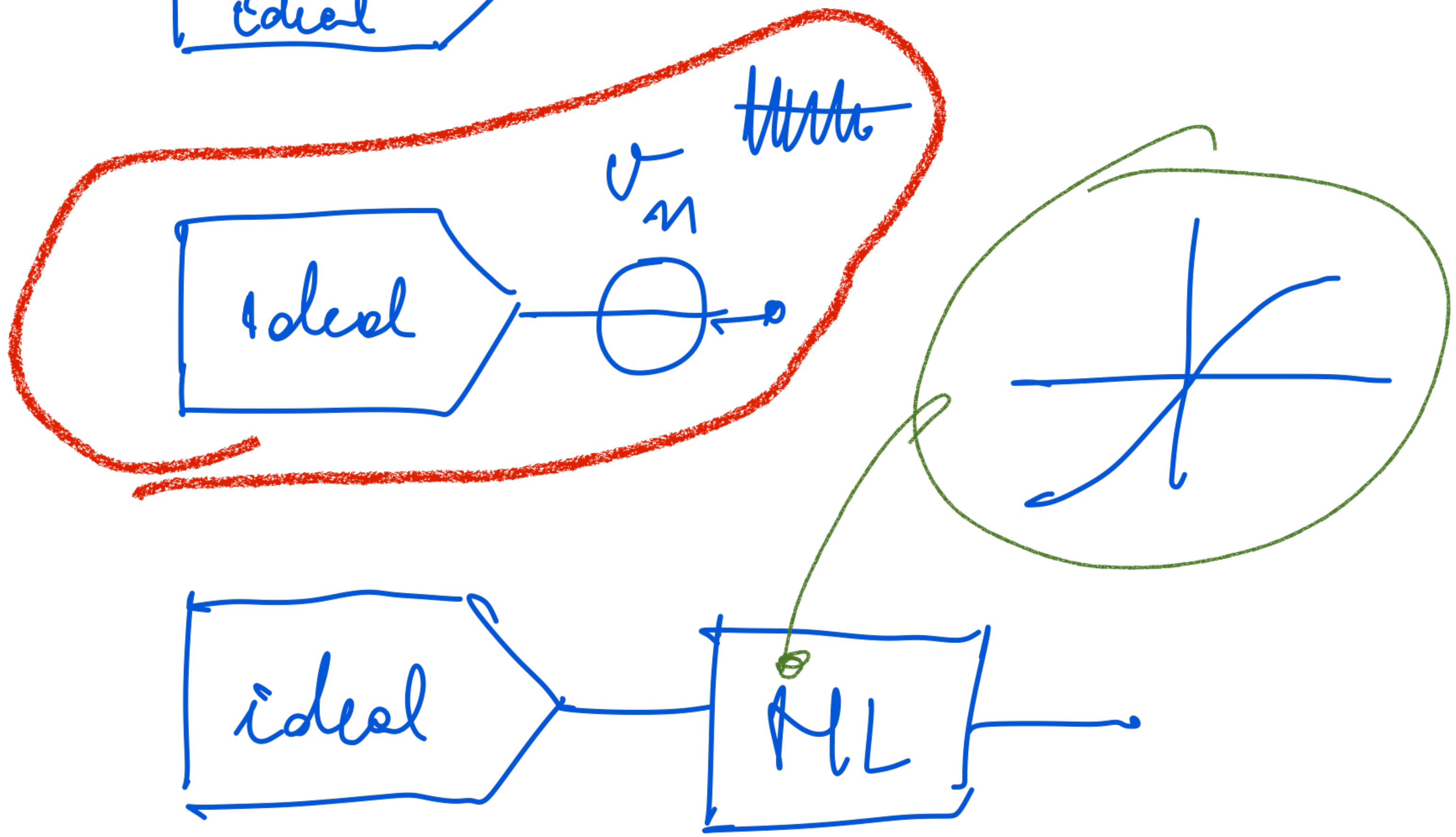
$2\sigma \to 0.95$

m bits



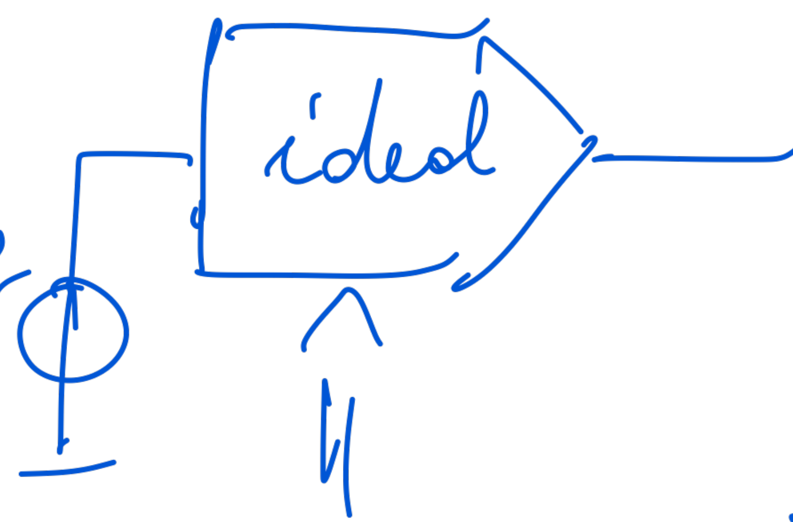
$\to$ quantization

Q
NL
Noise
Vref



unipolar

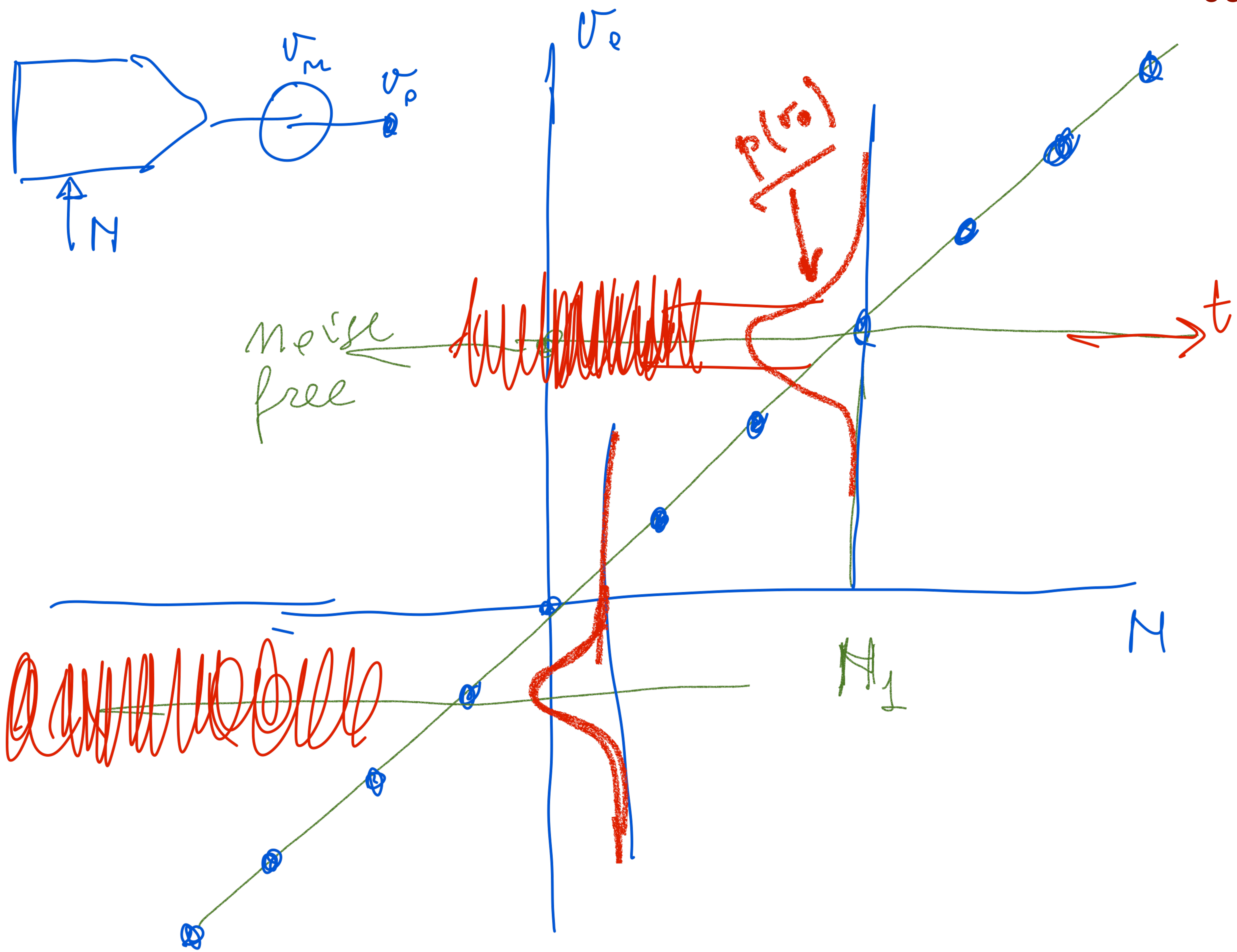
$$\frac{\Delta V_o}{V_o} = \frac{\Delta V_{ref}}{V_{ref}}$$



unipolar

$$\boxed{v_o = \frac{N}{2^m} V_{ref}}$$

$$\frac{\Delta V_{ref}}{V_{ref}} \quad \frac{(V_{ref} - V_{nom})}{V_{ref}}$$



v_m
v_p
N
v_e
$p(v_e)$
noise free
t
N
N_1

End of Lecture #6

~~End of lecture~~

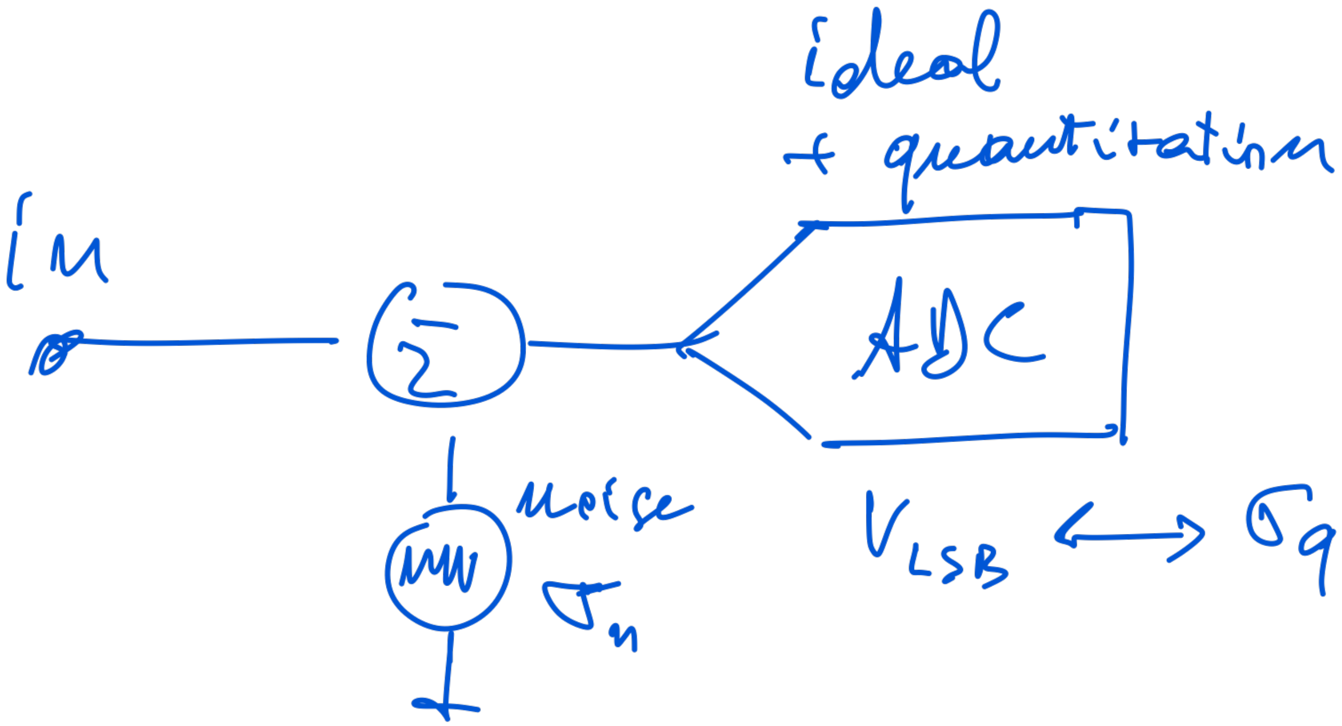
Q.

$\sigma > V_{LSB}$?

$\sigma < V_{LSB}$?

#7 Thursday, October 10, 2019

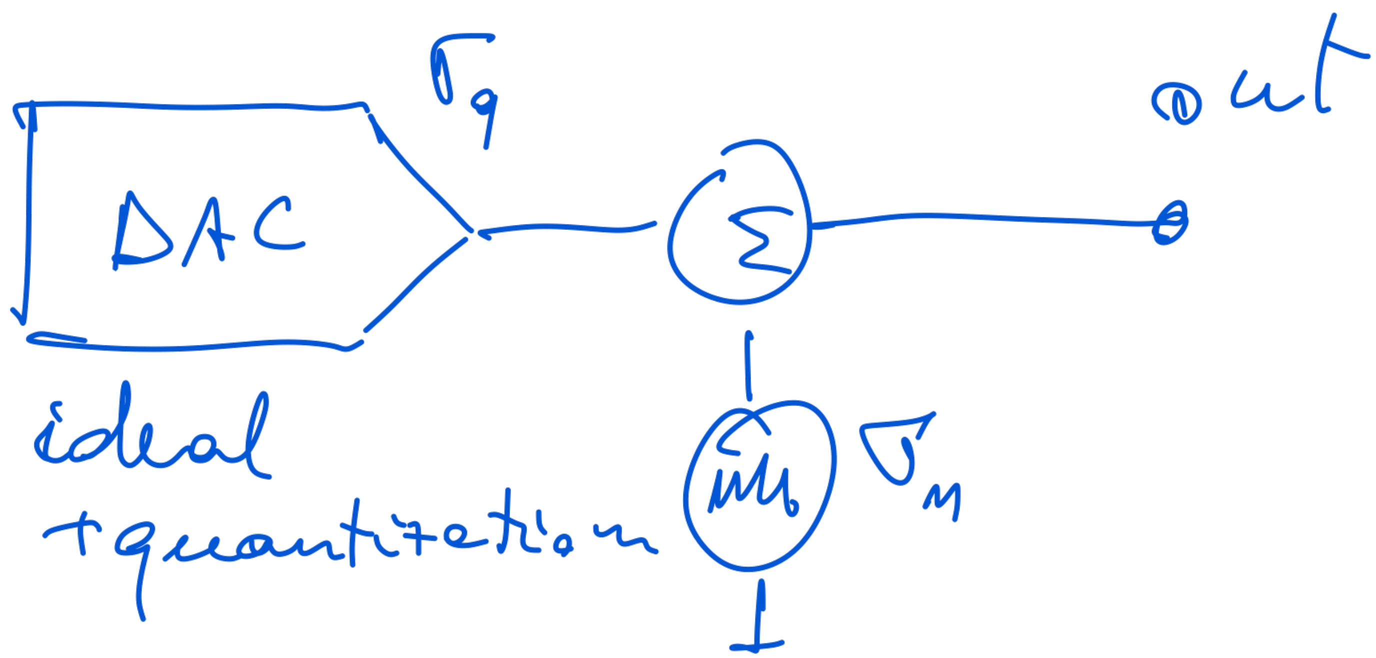
1.5 Hours



$$\sigma^2 = \cancel{\sigma_q^2} + \underline{\underline{\sigma_n^2}}$$

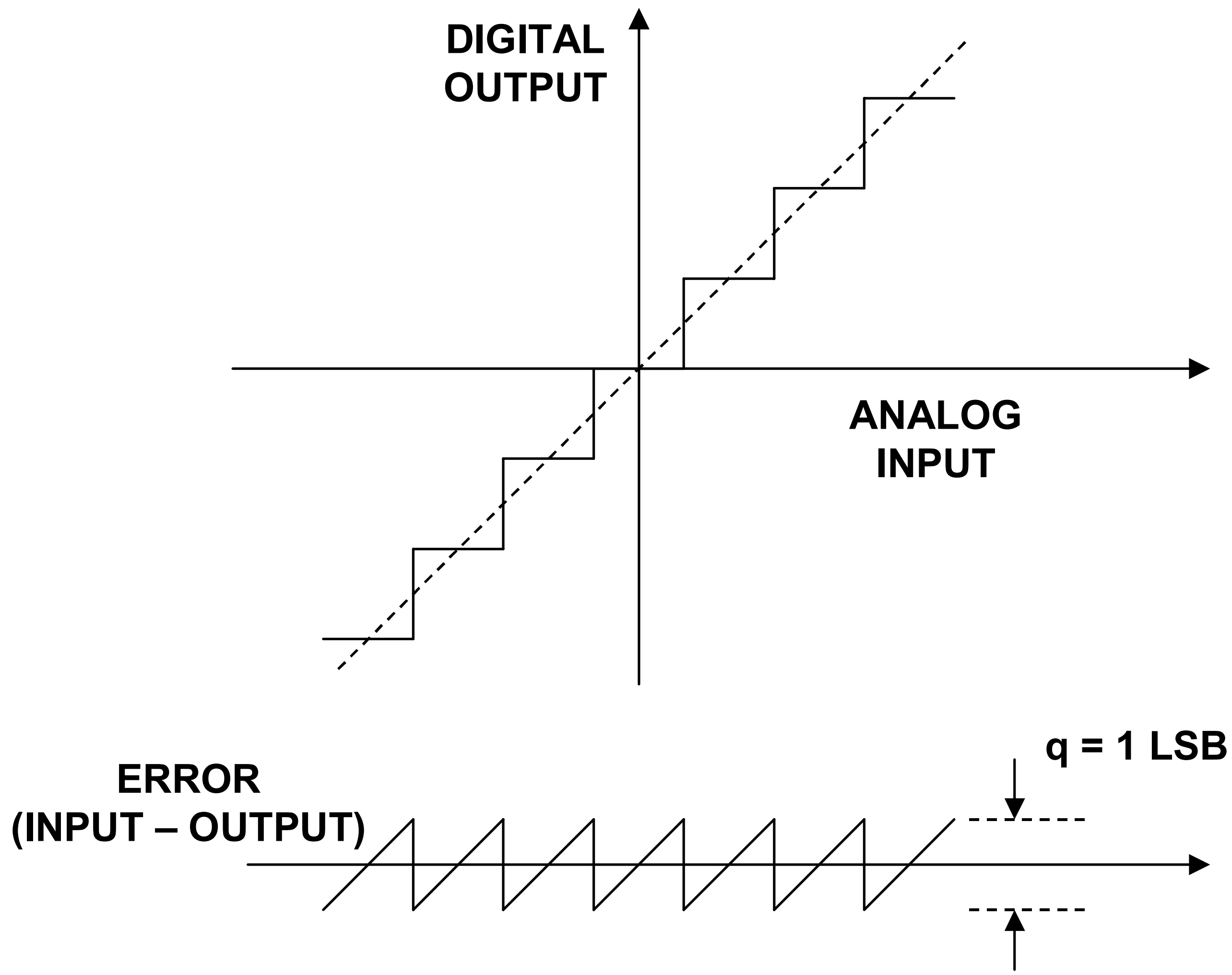
true limit

we can add some bits



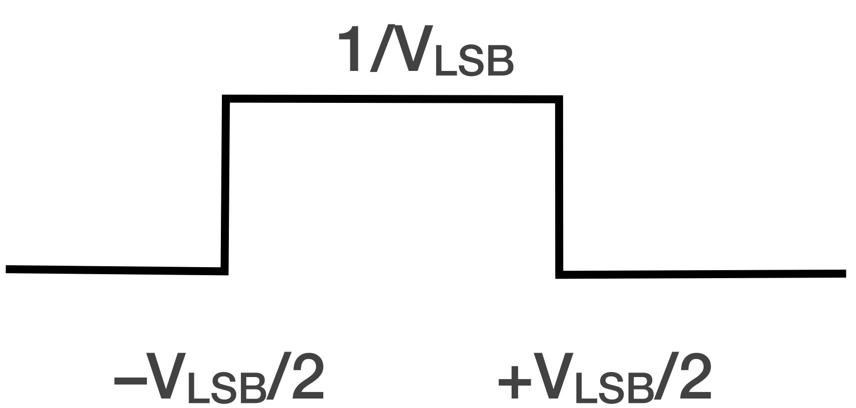
σ_n	
q	1.6×10^{-19} C
k	1.38×10^{-23} J/K
F	noise figure

Quantization



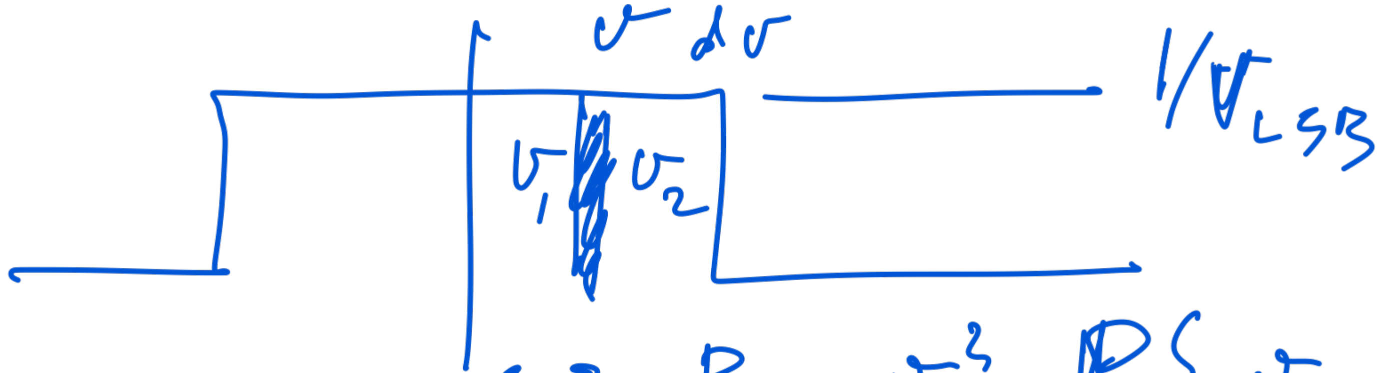
W. Kester, Fig 2.37

Probability density



$$\sigma^2 = \frac{1}{12} V_{LSB}^2$$

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

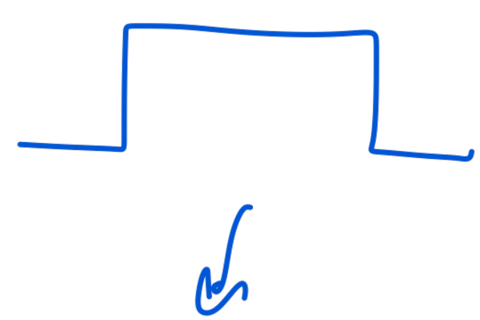


$$P = \frac{v^2\, P\{v_1 < v < v_2\}}{dv}$$

$$dP = v^2 \underbrace{\frac{1}{V_{LSB}} dv}_{\text{probability}}$$

Powr

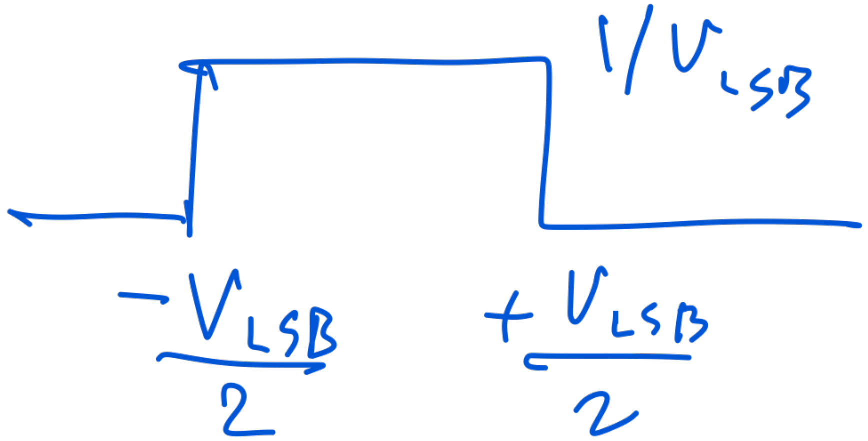
$v \to x$



total

$$P = \int_{-\infty}^{\infty} x^2\, p(x)\, dx$$

Joules



$$P = \int_{-\infty}^{+\infty} x\, p(x)\, dx$$

$$= \int_{-\frac{V_{LSB}}{2}}^{+\frac{V_{LSB}}{2}} \frac{1}{V_{LSB}} x^2\, dx$$

$$= \frac{1}{3V_{LSB}} \Big[x^3 \Big]_{-\frac{V_{LSB}}{2}}^{+\frac{V_{LSB}}{2}}$$

$$= \frac{1}{3V_{LSB}} \left[\left(\frac{V_{LSB}}{2}\right)^3 - \left(\frac{-V_{LSB}}{2}\right)^3 \right)$$

$$= \frac{1}{3\cancel{V_{LSB}}} \times \frac{1}{8} \times \left(V_{LSB}^{\cancel{3}\;2} + V_{LSB}^{\cancel{3}\;2} \right)$$

$$= \frac{2}{3 \times 8} \times V_{LSB}^2$$

$$\boxed{= \frac{1}{12} V_{LSB}^2}$$

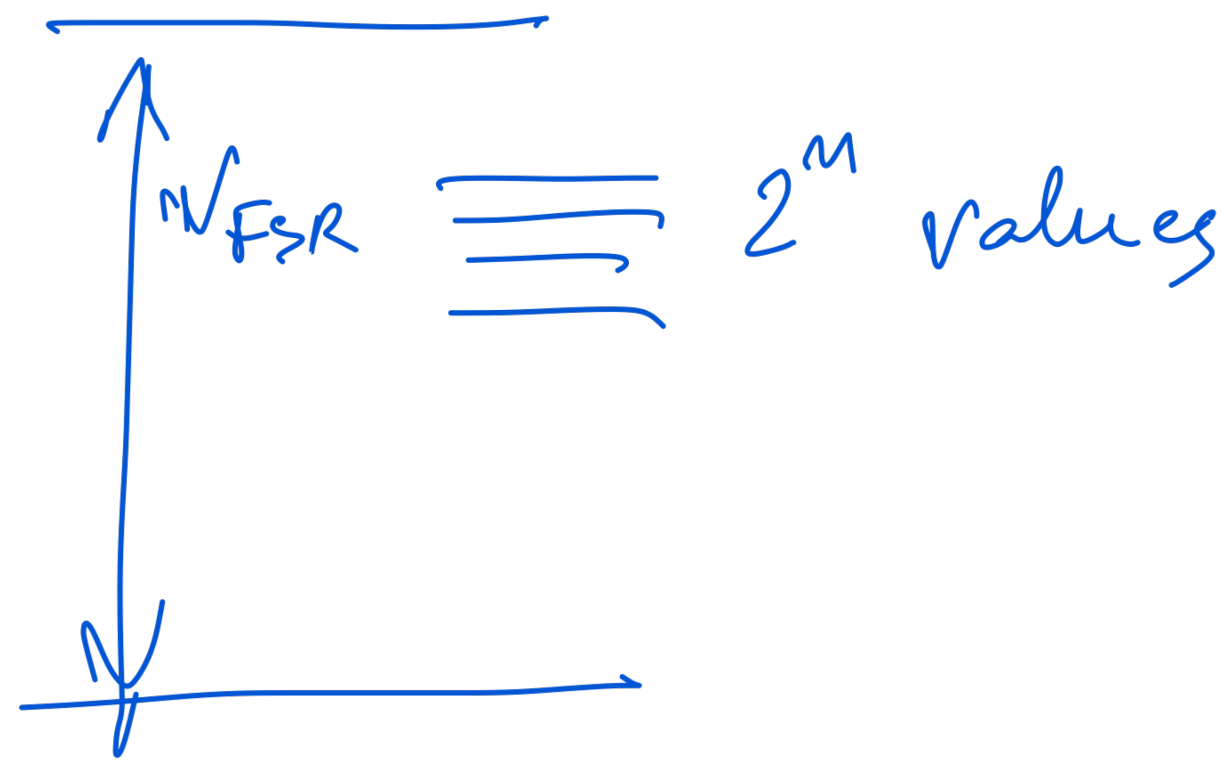
$$\sigma^2 = \frac{V_{LSB}^2}{12}$$

$$V_{LSB} = \frac{V_{FSR}}{2^n}$$

$$\boxed{n = \text{no of bits}}$$

2^n values

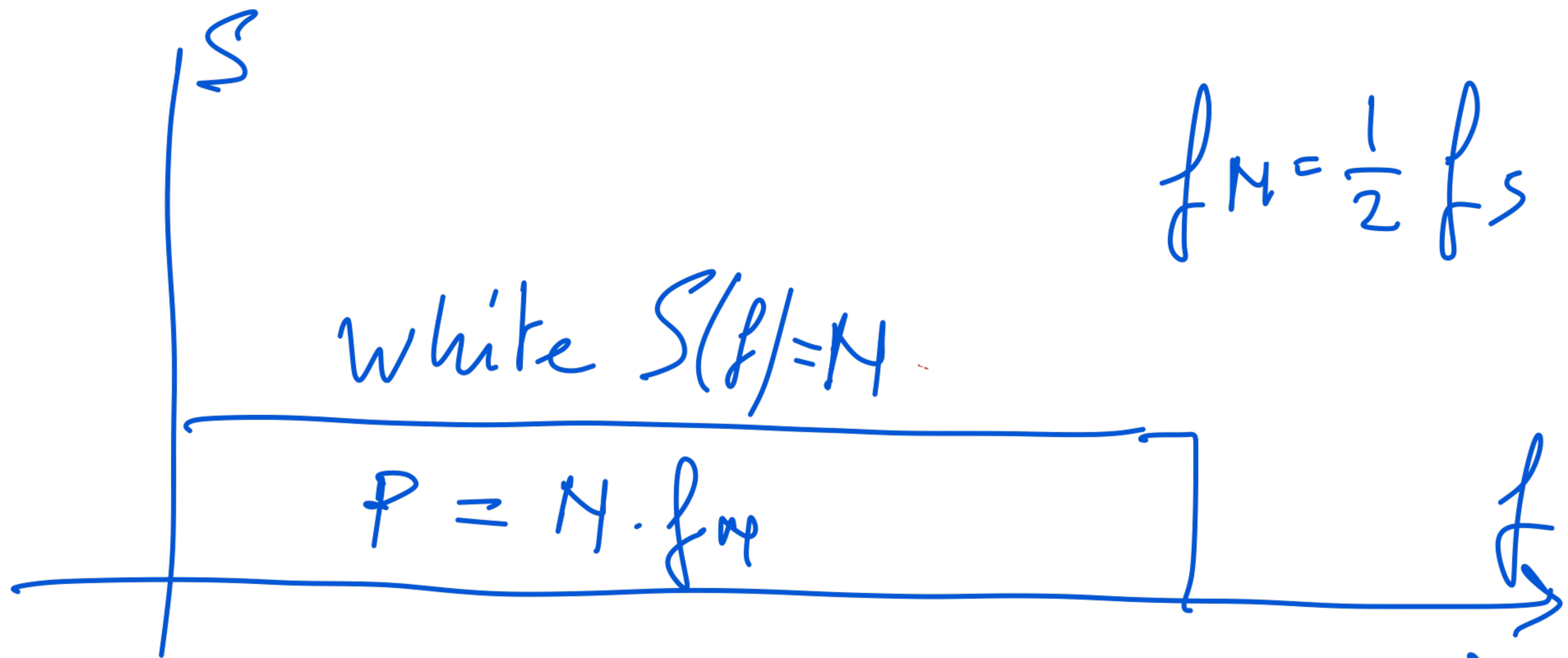
$$\sigma^2 = \frac{V_{FSR}^2}{12 \times 2^{2n}}$$



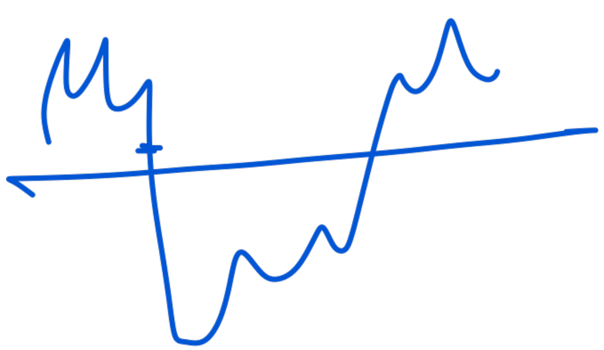
(SPECTRUM) PSD $S(f)$

$$\sigma^2 = \frac{V_{FSR}^2}{12\ 2^{2n}}$$

$$f_N = \frac{1}{2} f_s$$



random signal



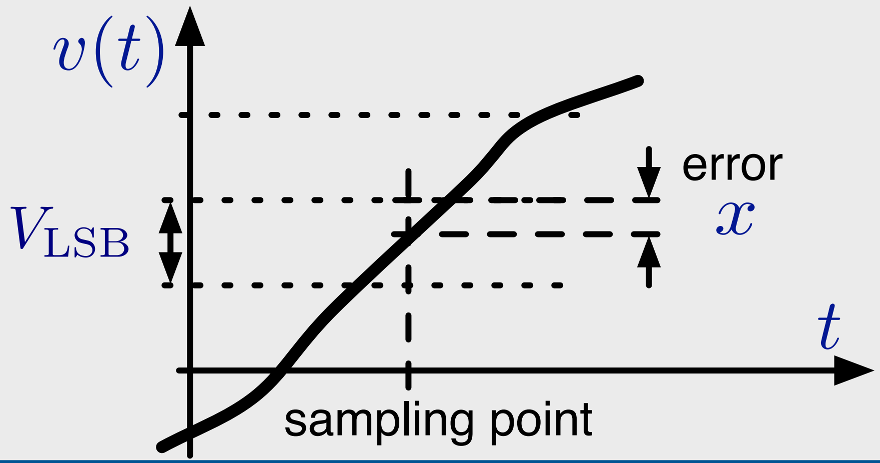
$$N = \frac{P}{f_N} \leftarrow \sigma^2 = \frac{V_{FSR}^2}{12 \cdot 2^{2n}}$$

$$N = \frac{V_{FSR}^2 \ 2}{12 \cdot 2^{2n} f_s}$$

$$\boxed{N = \frac{V_{FSR}^2}{6\ 2^{2n} f_s}}$$

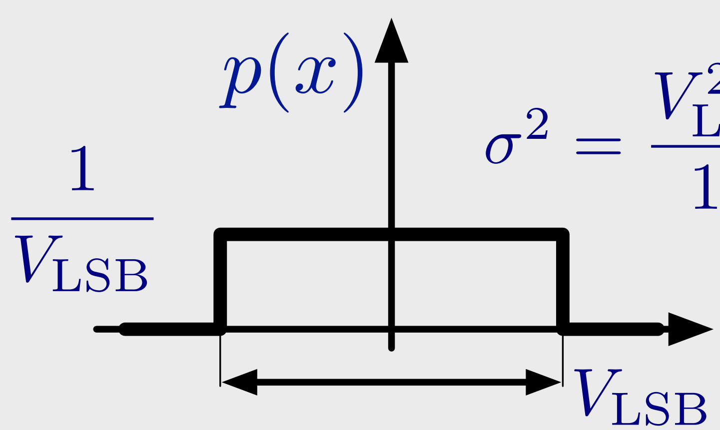
Quantization Noise

W. R. Bennett, Spectra of quantized signals, Bell System Tech J. 27(4), July 1948



Analog-to-digital conversion introduces a **quantization error** x $[-V_{\rm LSB}/2 \le x \le +V_{\rm LSB}/2]$

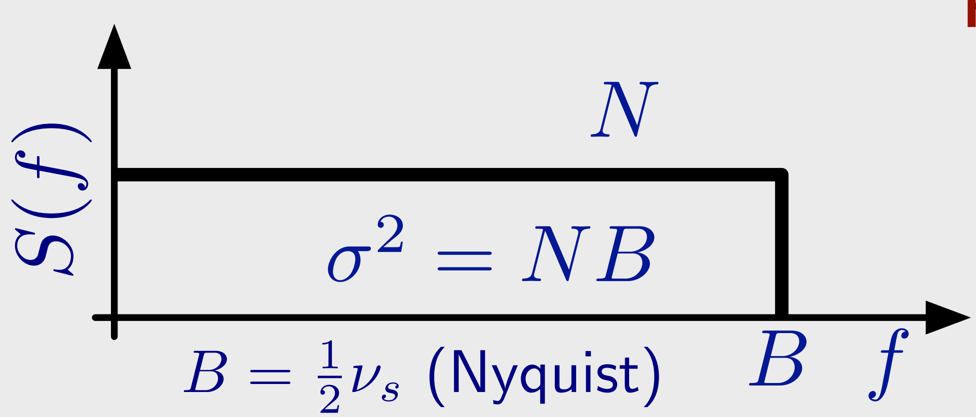
$$n\text{-bit conversion: } V_{\rm LSB} = \frac{V_{\rm FSR}}{2^n}$$



Wiener–Khintchine theorem: in ergodic systems, interchange time & ensemble
The noise can be calculated with statistics

$$\sigma^2 = \frac{V_{\rm FSR}^2}{12 \times 2^{2n}} \quad \mathrm{V}^2$$

1/12 –> –10.8 dB
2^{2n} –> 6 dB/bit



Parseval theorem: Energy (power) calculated in time and in frequency is the same

$$N = \frac{V_{\rm FSR}^2}{6 \times 2^{2n}\,\nu_s} \quad \mathrm{V}^2/\mathrm{Hz}$$

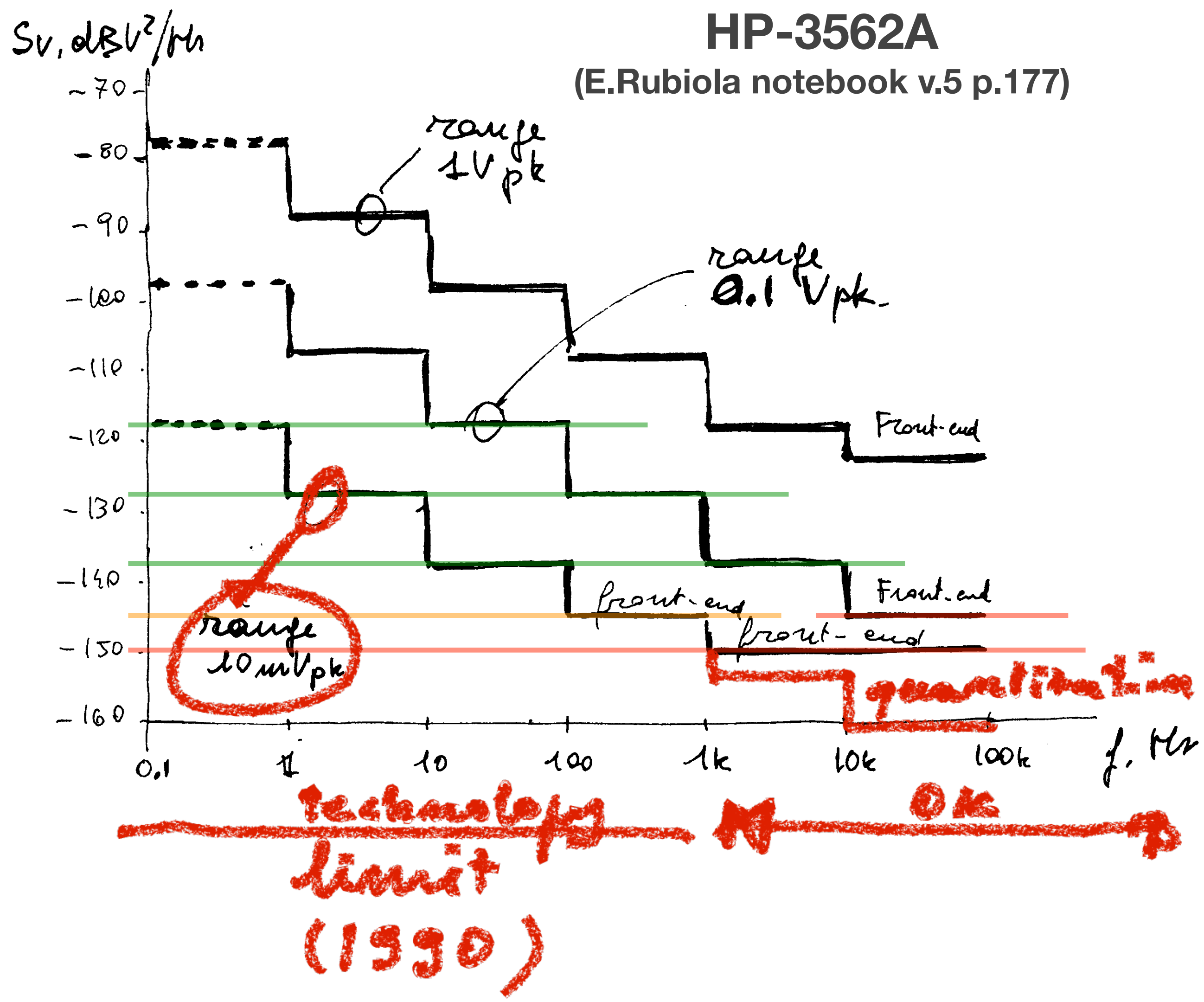
f_s

Example of FFT Analyzer Noise

Experimental observation

HP-3562A

(E.Rubiola notebook v.5 p.177)



Theoretical evaluation

DAC 12 bit resolution, including sign

range 10 mV_{peak}

V_{fsr} = 20 mV (±10 mV)

±10 mV

resolution

$V_q = V_{fsr} / 2^{12}$

= 4.88 μV

total noise

$\sigma^2 = (4.88\ \mu V)^2 / 12$

$= 2\times10^{-12}\ V^2$ (–117 dB)

quantization noise PSD

$S_v = \sigma^2 / B$

= –117 dBV^2/Hz with B = 1 Hz (etc.)

Front-end noise, evaluated from the plot

$S_v = 2\times10^{-15}\ V^2$ (–150 dB), at 10–100 kHz

or 45 $nV/Hz^{1/2}$

use Sv = 4kTR

R = 125 kΩ

or R = 100 kΩ and F = 1 dB (noise figure)

$V_{FSR} = 20\,mV$

$n = 12$ bits

$f_s = 256$ kS/s

$\sigma = ?$

$N = ?$

$$N = \frac{\sigma^2\, 2}{f_s}$$

$$2^{24} = 16 \times 2^{20}$$

$$\sigma^2 = \frac{V_{LSB}^2}{12} = \frac{V_{FSR}^2}{12 \cdot 2^{2n}}$$

$$\sigma = \frac{2 \times 10^{-2}}{\sqrt{12} \cdot 2^{12}} \quad \frac{2 \times 10^{-2}}{3{,}5 \cdot 4096}$$

$1.4\,\mu V$ RMS

$$N = \frac{V_{FSR}^2}{6 \cdot 2^{2n} f_s} = 1.6 \times 10^{-17}\ V^2/Hz$$

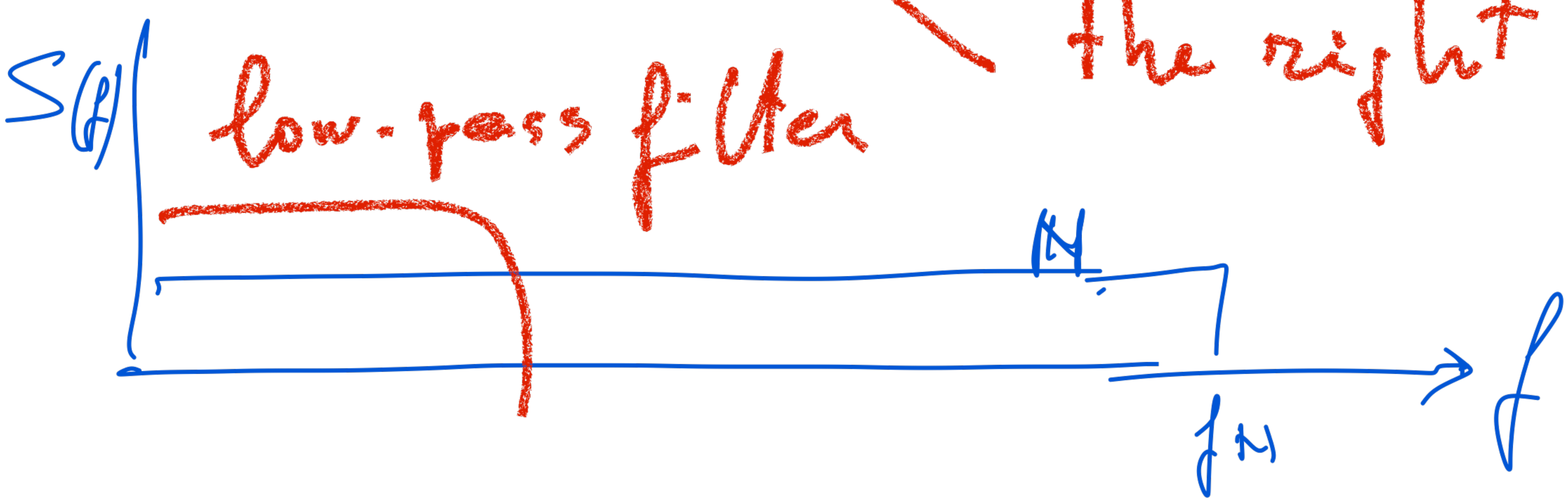
$$\sqrt{N} = 3.9\,\mu V/\sqrt{Hz}$$

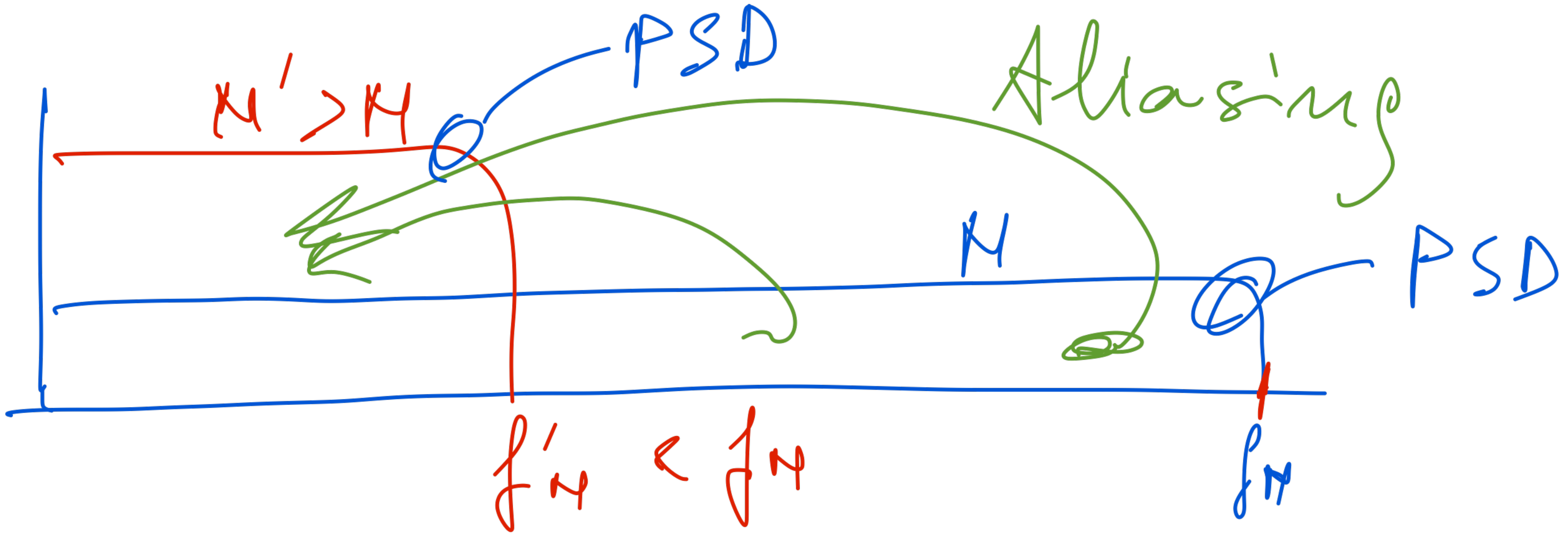
How to Get Lower f_s

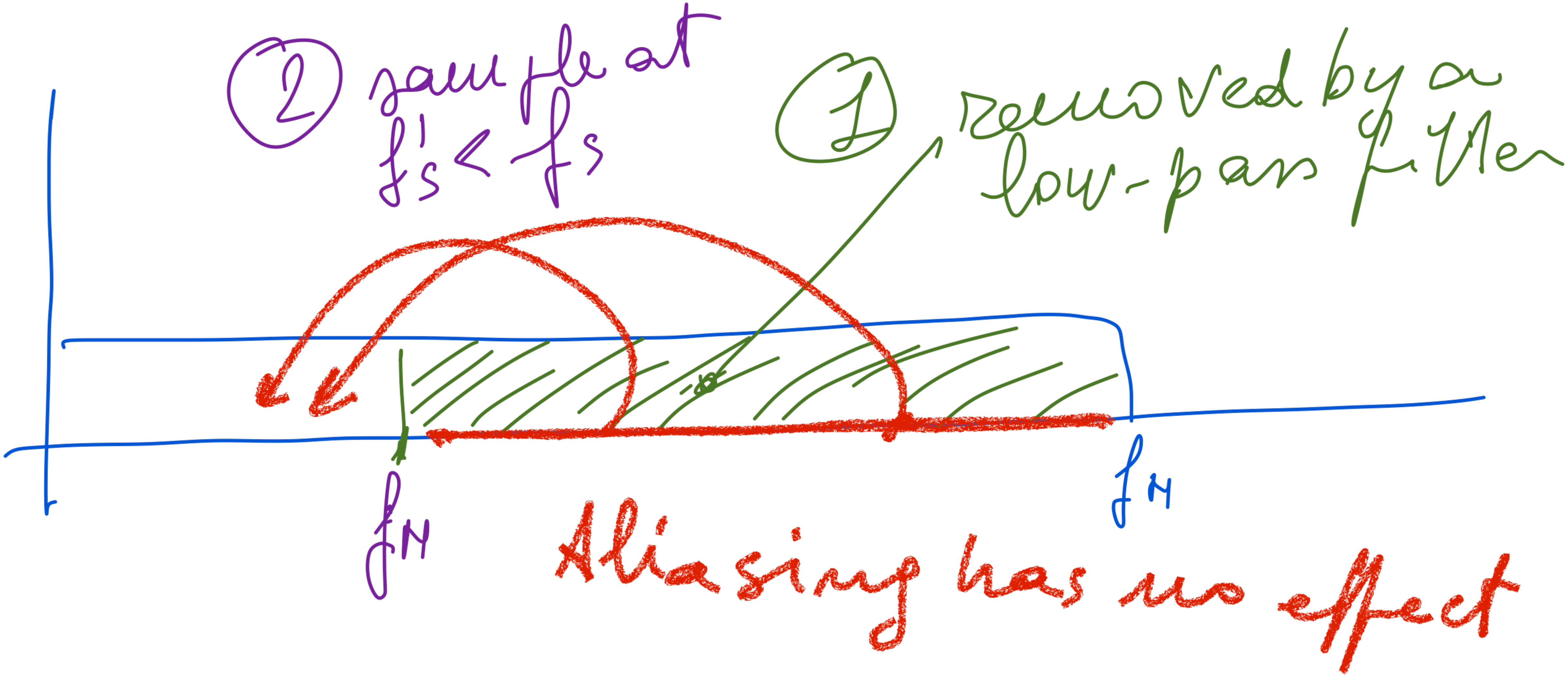
$$N = \frac{V_{FSR}^2}{6 \cdot 2^{2n} f_s}$$

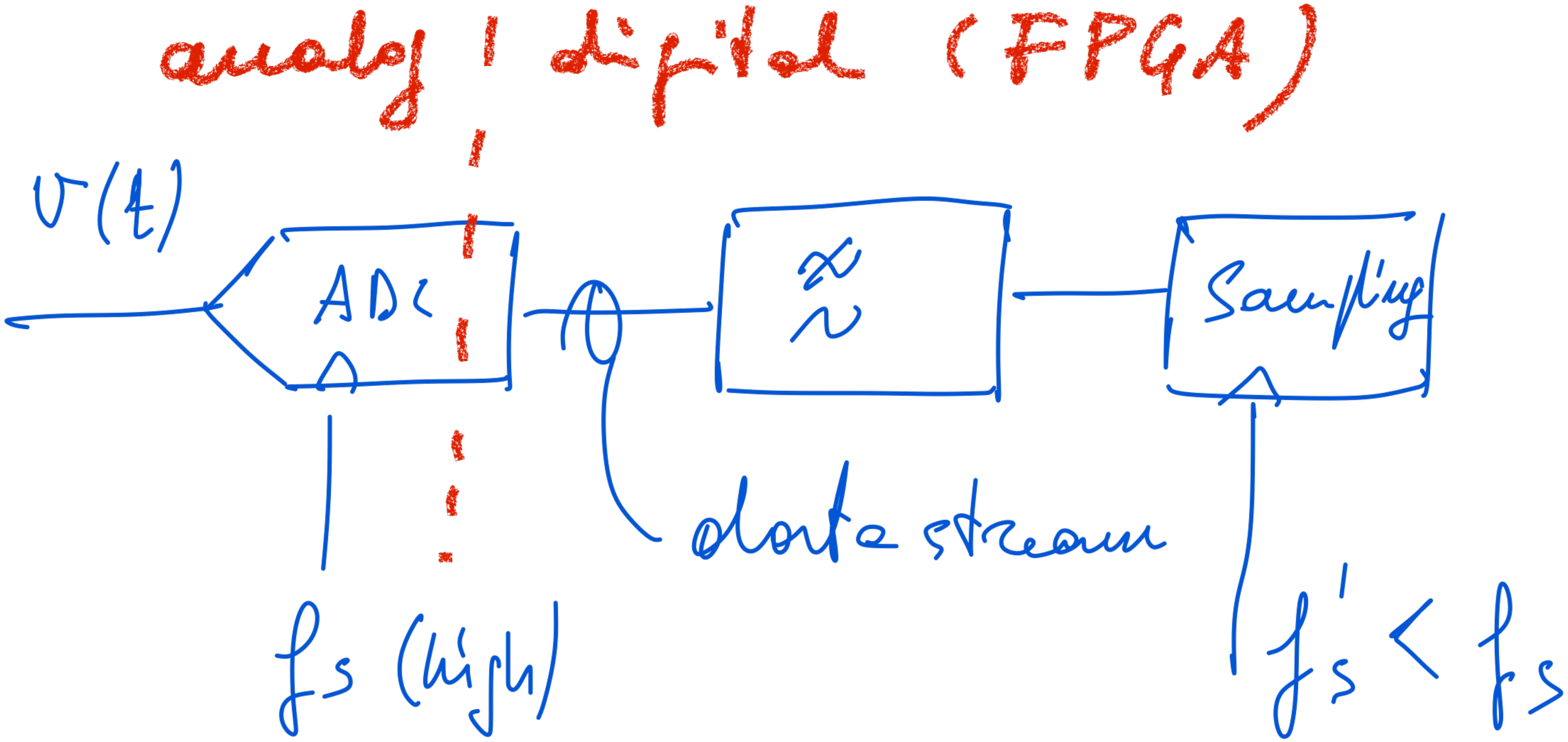
- Just reduce the clock frequency — Obsolete
- Clean decimation

the right way

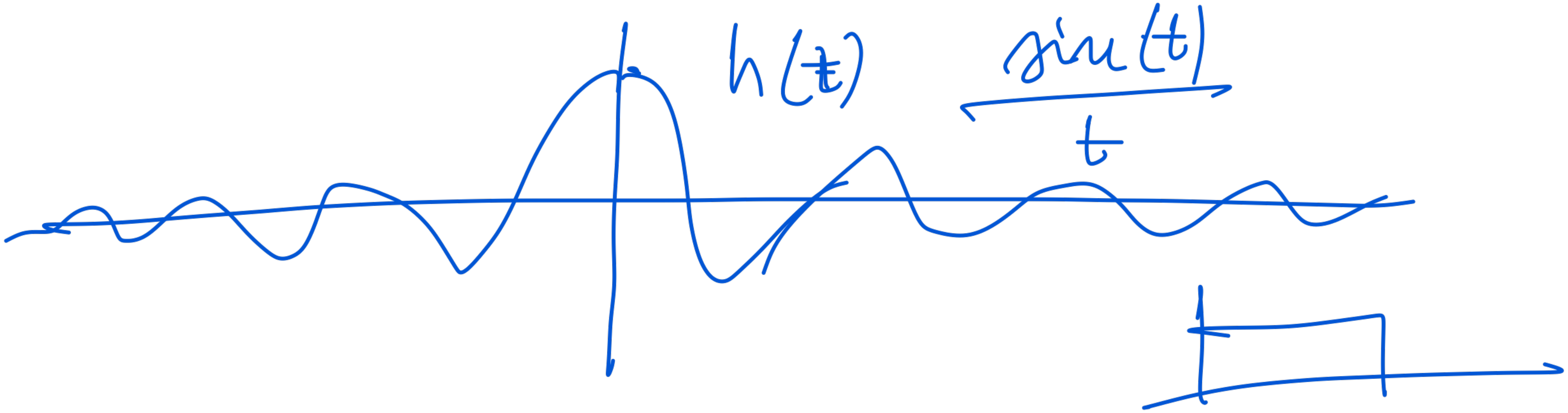



M' > M
PSD
Aliasing
M
PSD
f'N < fN
fN


2 sample at f's < fs
1 removed by a low-pass filter
fN
fN
Aliasing has no effect



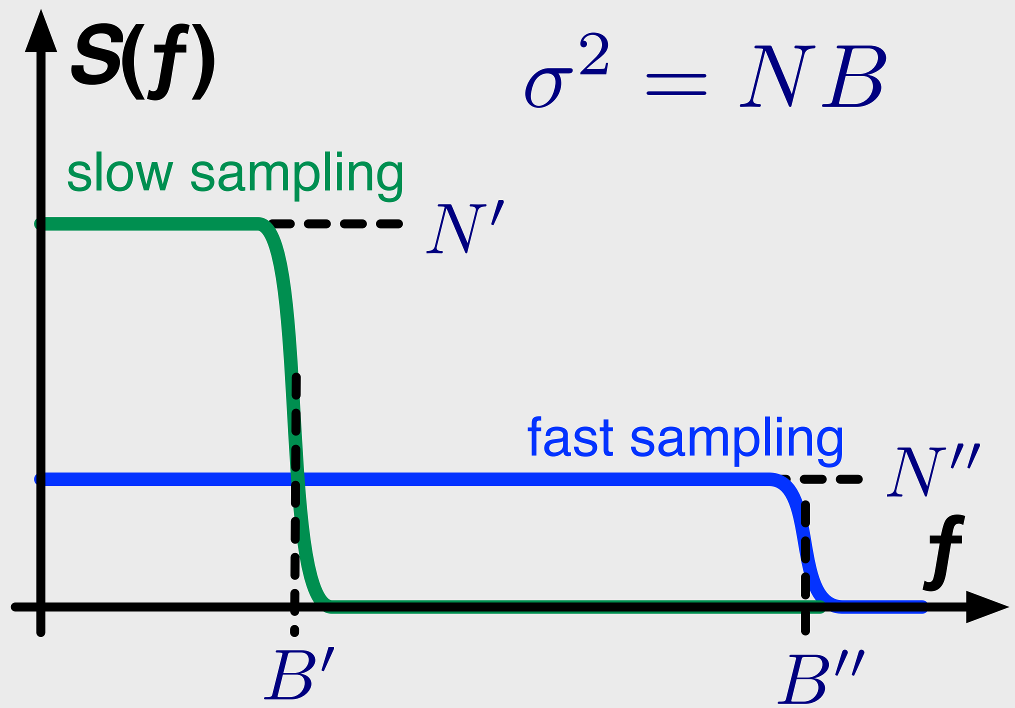
$$y(t) = x(t) * \underbrace{h(t)}$$ FIR filter



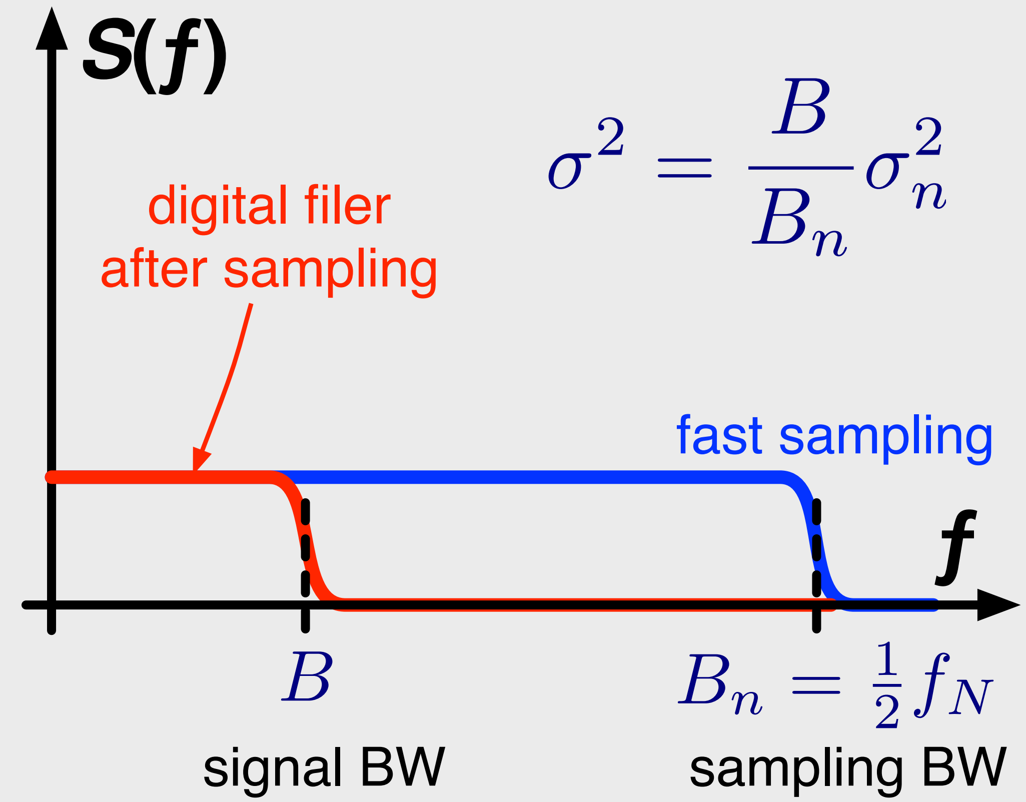
Digital Filter and Decimation

Noise, Sampling, and the Parseval theorem

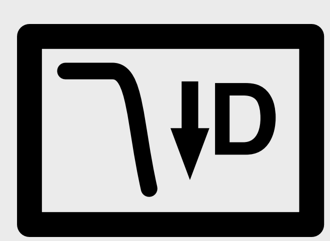
$$\sigma^2 = \frac{V_{\mathrm{FSR}}^2}{12 \times 2^{2m}}$$

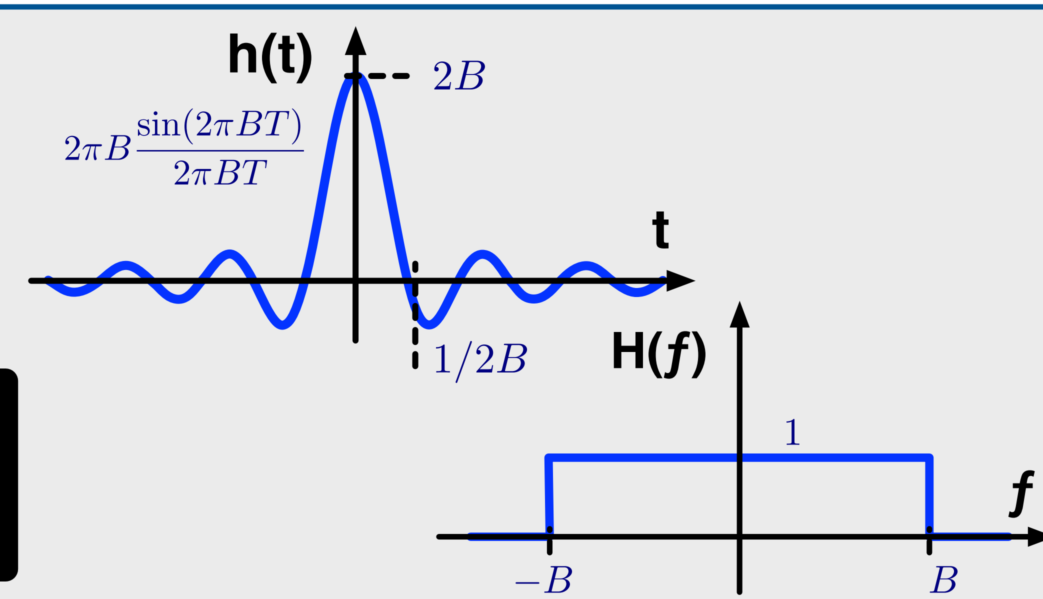


Solution: Fast sampling, and filter



- Convolution with low-pass h(t)
- 127 coeff. Blackman-Harris kernel provides 70 dB stop-band attenuation
- Future: we will use >>127 coefficients
- Need more bits





#8 Tuesday, October 15, 2019

1.5 Hours

The Formula SNR = 6.02 n + 1.76 dB

$$\mathrm{SNR} = \frac{P}{\sigma^2}$$

$P = V_{\mathrm{pp}}^2/8R$

$\sigma^2 = V_{\mathrm{LSB}}^2/12R$

Better called SQR
Signal to Quantization Ratio

$$\mathrm{SNR} = \frac{3}{2}\frac{V_{\mathrm{pp}}^2}{V_{\mathrm{LSB}}^2}$$

$V_{\mathrm{pp}} \leq V_{\mathrm{FSR}}$

$V_{\mathrm{LSB}} = V_{\mathrm{FSR}}/2^n$

Simplified

$$\mathrm{SNR} = \frac{3}{2} 2^{2n}$$

$10\log_{10}(3/2) = 1.76$

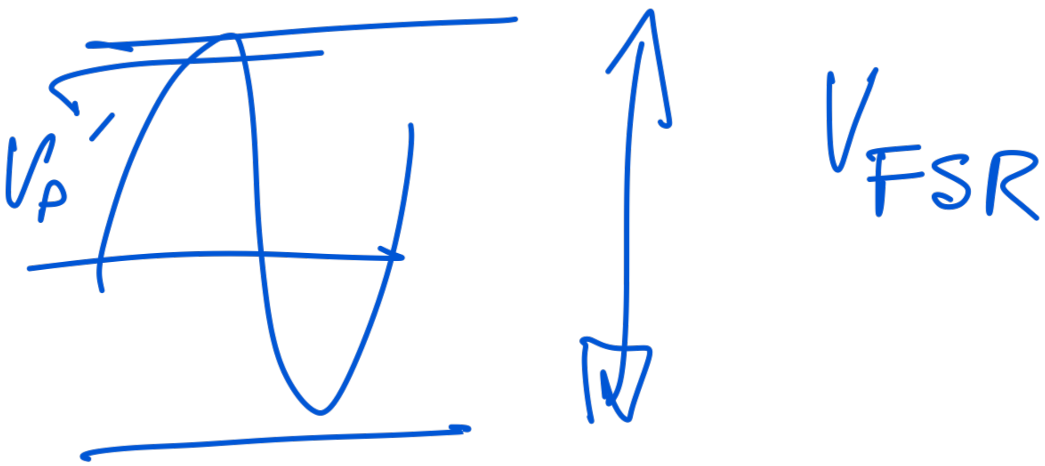
$10\log_{10}\left(2^{2n}\right) = 6.02\,n$

$$6.02\,n + 1.76 \text{ dB}$$

Full formula

$$\mathrm{SNR} = \frac{3}{2} 2^{2n} \frac{V_{\mathrm{pp}}^2}{V_{\mathrm{FSR}}^2}$$

$$6.02\,n + 1.76 + 20\log_{10}\frac{V_{\mathrm{pp}}}{V_{\mathrm{FSR}}} \text{ dB}$$



$$\boxed{V_{pp} = V_{FSR}}$$

$$V_{pp} = a\, V_{FSR}$$

$$a < 1$$

$$P = V_{FSR}^2 / 8$$

$$\boxed{SNR = \frac{P}{\sigma^2}}$$

signal power

noise power

$$\sigma^2 = \frac{V_{FSR}^2}{12 \times 2^{2n}} \qquad V^2$$

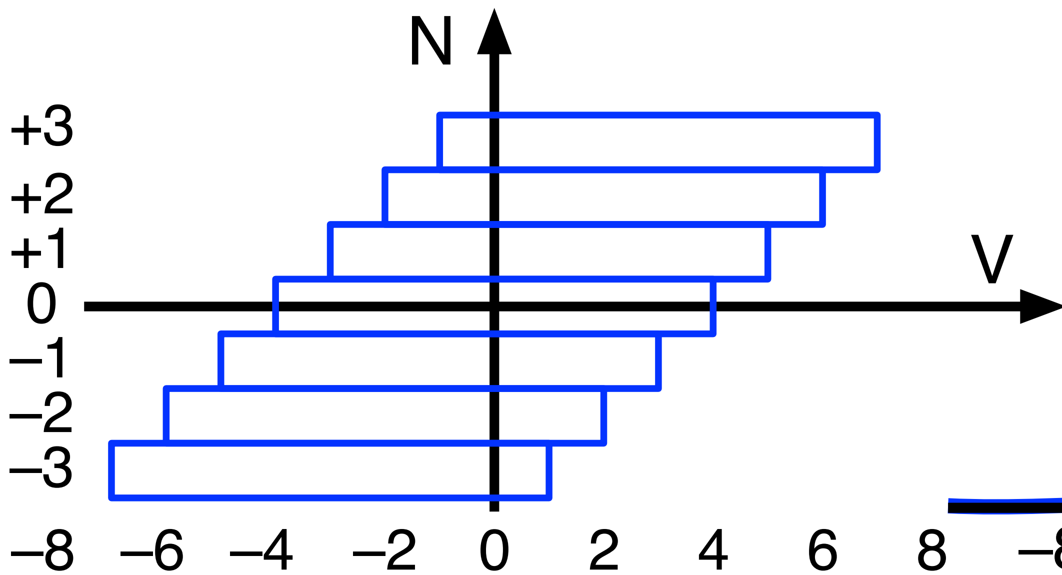
$$SNR = \frac{\cancel{V_{FSR}^2}}{\cancel{8}\,2} \cdot \frac{\cancel{12}\,3 \times 2^{2n}}{\cancel{V_{FSR}^2}}$$

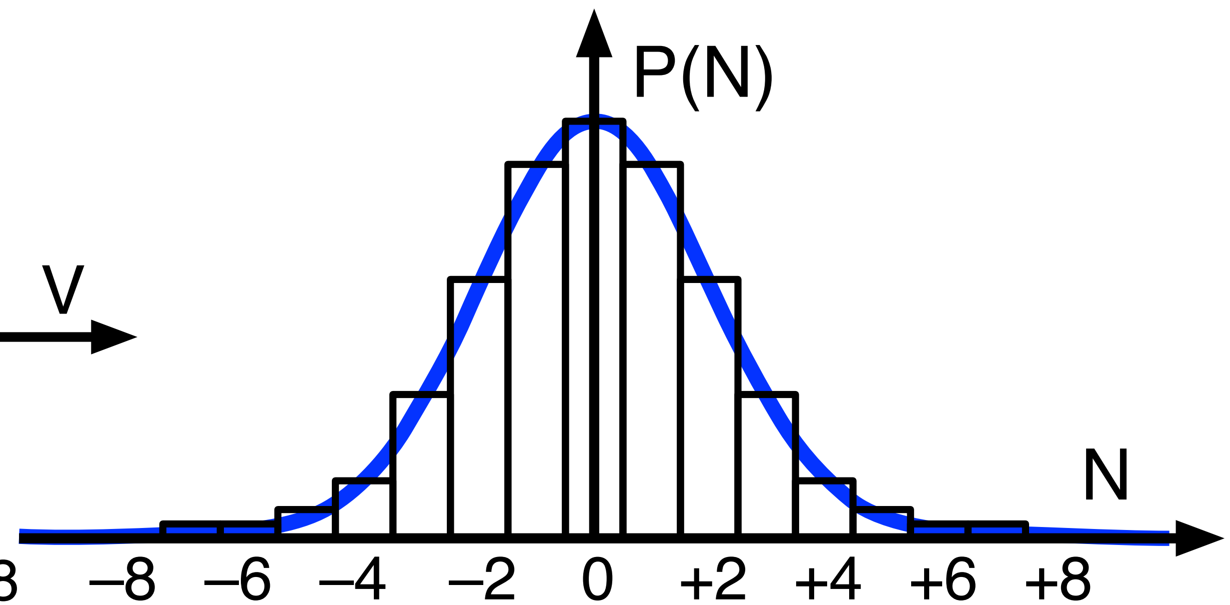
$$2^{2n} = (2^2)^n$$

$$SNR = \frac{3}{2}\, 2^{2n}$$

$$1.76 + 6.02 \times n - 20 \log_{10} a \ \text{dB}$$

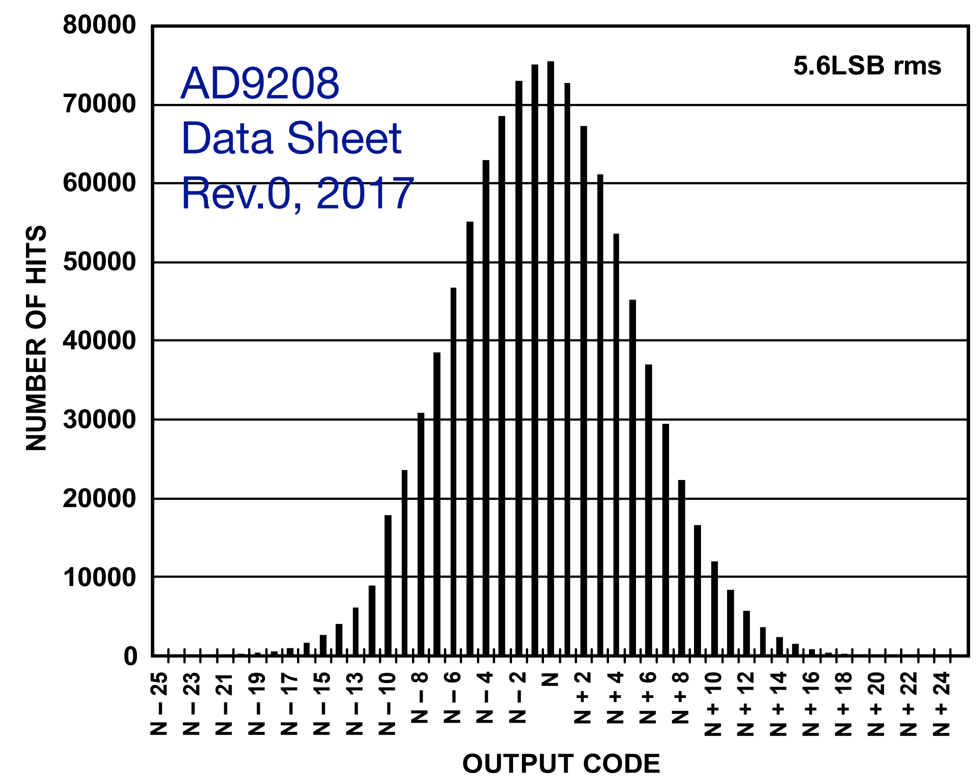
Transition Noise





- Actual noise includes quantization, analog noise, and distortion
- $\sigma^2_v = \sigma^2_q + \sigma^2_a + \sigma^2_d$
- Random distribution of output N
- Metrology suggests to make σ^2_q negligible because BUS bits are cheap

Example



ENOB = Equivalent Number of Bits

1. $\sigma_q^2 = \frac{V_{LSB}^2}{12} = \frac{V_{FSR}^2}{12 \, 2^{2N}}$ quantization

2. σ_a^2 analog noise

(3) (distorsion)

σ^2 total

$$\sigma^2 = \frac{V_{FSR}^2}{12 \, 2^{2\,ENOB}}$$

ENOB is the number that gives the actual σ_i^2 analog + quantiz

Transition Noise

High-Speed Converters

- Analog noise is higher than quantization noise
- Given a voltage V –> random distribution of output N
- This correct –> $V^2 = V^2_{analog} + V^2_{quant}$
  (don't spoil the resolution with insufficient no of bits)

Information (bits)

$$I = \sum_i -p_i \log_2(p_i)$$

Equivalent No of Bits

$$\mathrm{ENoB} = \log_2 \left[1 + \frac{V_{\mathrm{FSR}}}{\sqrt{12}\,\sigma_V}\right]$$

Transition Noise

W. Kester, Fig 2.47

- **Effective Input Noise = $e_{n\ rms}$**
- **Peak-to-Peak Input Noise = $6.6\ e_{n\ rms}$**
- **Noise-Free Code Resolution** $= \log_2\left[\dfrac{\text{Peak-to-Peak Input Range}}{\text{Peak-to-Peak Input Noise}}\right]$

$$= \log_2\left[\frac{2^N}{\text{Peak-to-Peak Input Noise (LSBs)}}\right]$$

- **"Effective Resolution"** $= \log_2\left[\dfrac{\text{Peak-to-Peak Input Range}}{\text{RMS Input Noise}}\right]$

$$= \log_2\left[\frac{2^N}{\text{RMS Input Noise (LSBs)}}\right]$$

= Noise-Free Code Resolution + 2.7 bits

$\boxed{\text{Example AD9652}}$

Similar to the HP3562A

Homework

- Specs: V_{FSR} = 2 V, ENOB = 12 bits, f_s = 250 MHz
- Calculate σ_q , noise PSD, SQR.
- What happens if f_s is lowered to 100 MHz?

SINAD, ENOB, SNR

W. Kester, Fig 2.16

- **SINAD (Signal-to-Noise-and-Distortion Ratio):**
  - **The ratio of the rms signal amplitude to the mean value of the root-sum-squares (RSS) of all other spectral components, including harmonics, but excluding DC.**
- **ENOB (Effective Number of Bits):**

$$\text{ENOB} = \frac{\text{SINAD} - 1.76\text{dB}}{6.02}$$

- **SNR (Signal-to-Noise Ratio, or Signal-to-Noise Ratio Without Harmonics:**
  - **The ratio of the rms signal amplitude to the mean value of the root-sum-squares (RSS) of all other spectral components, excluding the first 5 harmonics and DC**

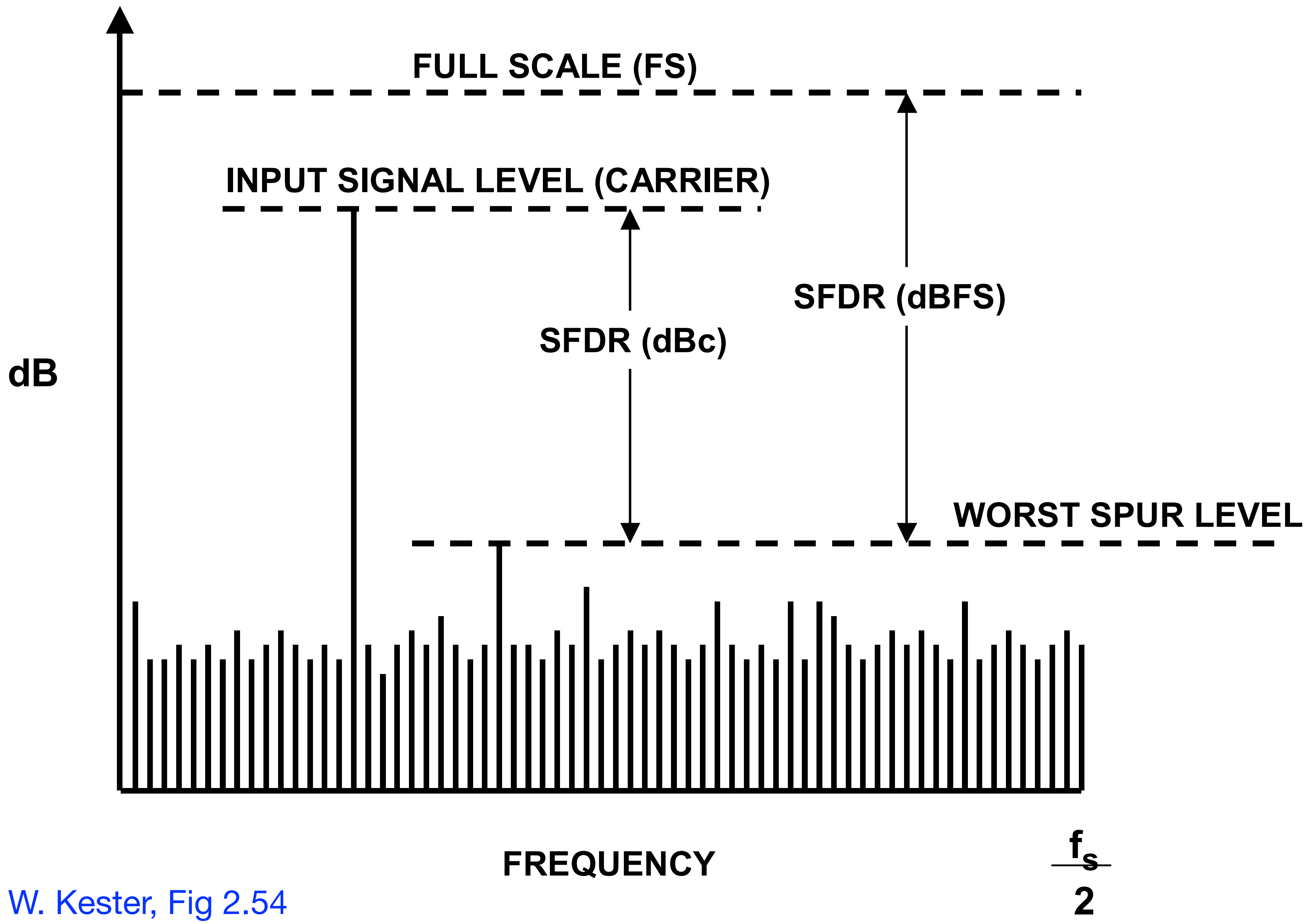
This is not a general definition!!!

$$\text{SINAD} = \frac{V_{\text{rms}}}{\sqrt{\sum V_{\text{harmon}}^2} + \sqrt{\sum V_{\text{spurs}}^2} + \sqrt{\sum \cdots}}$$

Also SQR = Signal to Quantization Ratio

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Spurious-Free Dynamic Range



W. Kester, Fig 2.54

Spurious = unwanted spectral lines, generally not understood

W. Kester ed., Analog-Digital Conversion, AD 2004, ISBN 0-916550-27-3

Information

Information (bits)

$$I = \sum_i -p_i \log_2(p_i)$$

1 bit (sign only)

$50\,\%\quad V > 0$

$50\,\%\quad V < 0$

$-0.5\log_2(0.5) = 0.5$

$\sum = 1$

3 bits

V_{FSR}

$p = 1/8$

$-(1/8)\log_2(1/8) = 3/8$

8 equal terms

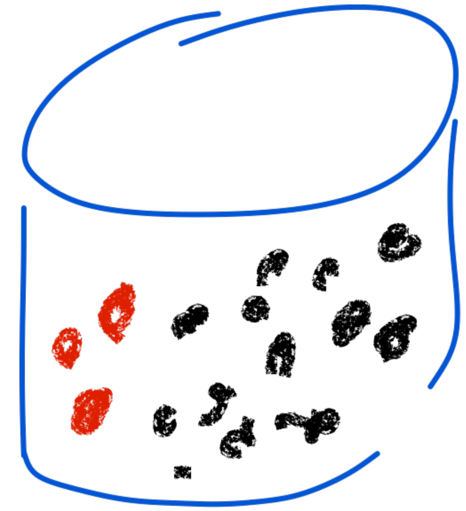
0

$$\sum_{i=1}^{8} \ldots = 3$$

Information

black = 0
red = 1

1 bit



0010000100...

1 bi

$$I = \sum_{i}^{\text{all } i} -p_i \log(p_i)$$

p_i = probability of the event i

"log" → $\ln$ →
$\log_2$ → bit
$\log_{10}$ → Hartley

EXAMPLE

comparator

$v_i > 0 \longrightarrow 1 \quad p = 0.5$

$v_i < 0 \longrightarrow 0 \quad p = 0.5$

~~$v_i = 0 \longrightarrow ? \quad p = 0$~~

$$I = \sum_i -p_i \log_2 p_i$$

$i = 0 \quad p_0 = 0.5 \qquad \log_2(0.5) = -1$

$$-0.5 \times (-1) = 0.5 \qquad I_0 = 0.5$$

$i = 1 \quad p_1 = 0.5 \qquad \text{- - -} \qquad I_1 = 0.5$

$$\sum = 1$$

comparator = one-bit ADC

EXAMPLE

3 bit converter

$i = 0 .. 7$

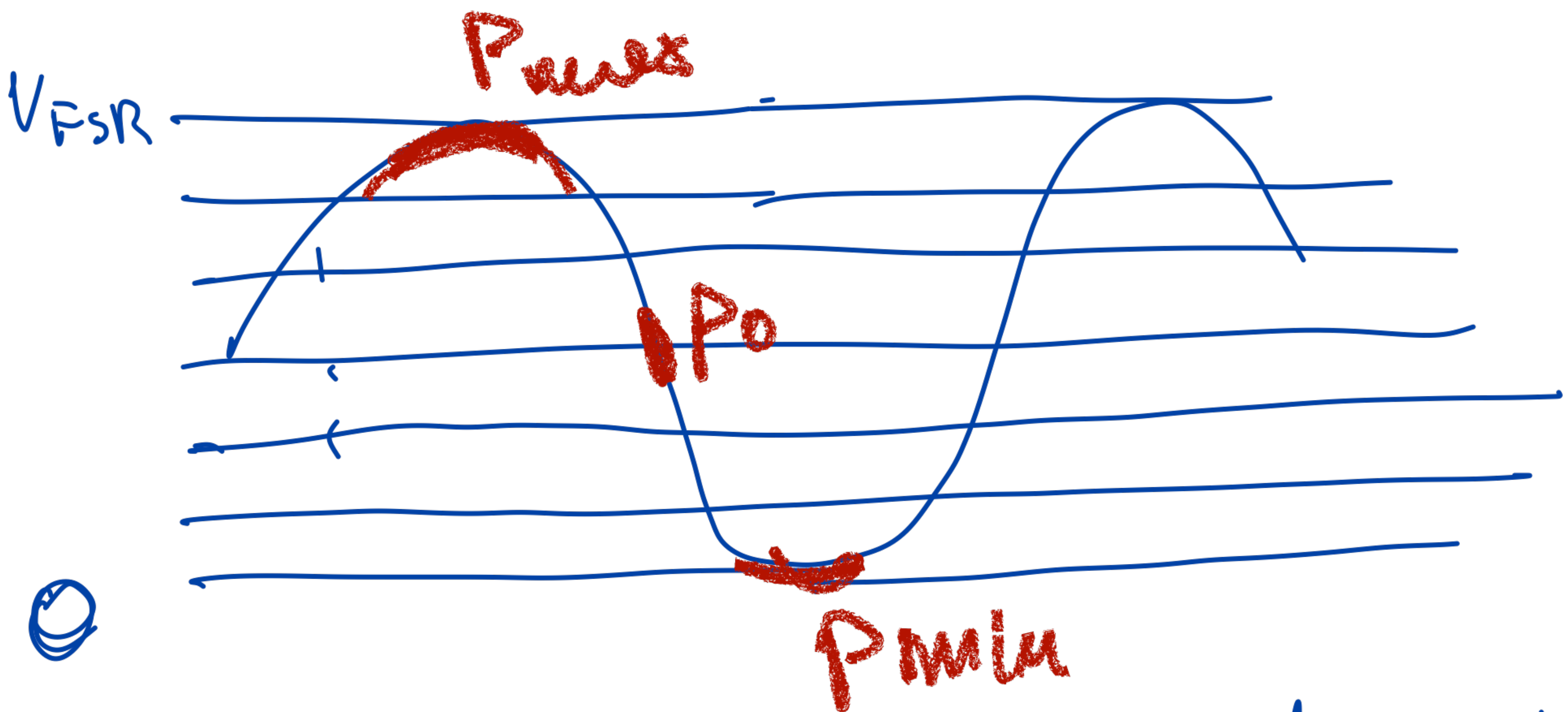
$p_i = 1/8$ (noise)

$$-\underbrace{p_i}_{1/8} \underbrace{\log_2 p_i}_{-3} \qquad \left[-\frac{1}{8} \times -3\right] 8 \text{ equal terms}$$

$-\ 1/8 \qquad -3$

3 bits

HOMEWORK

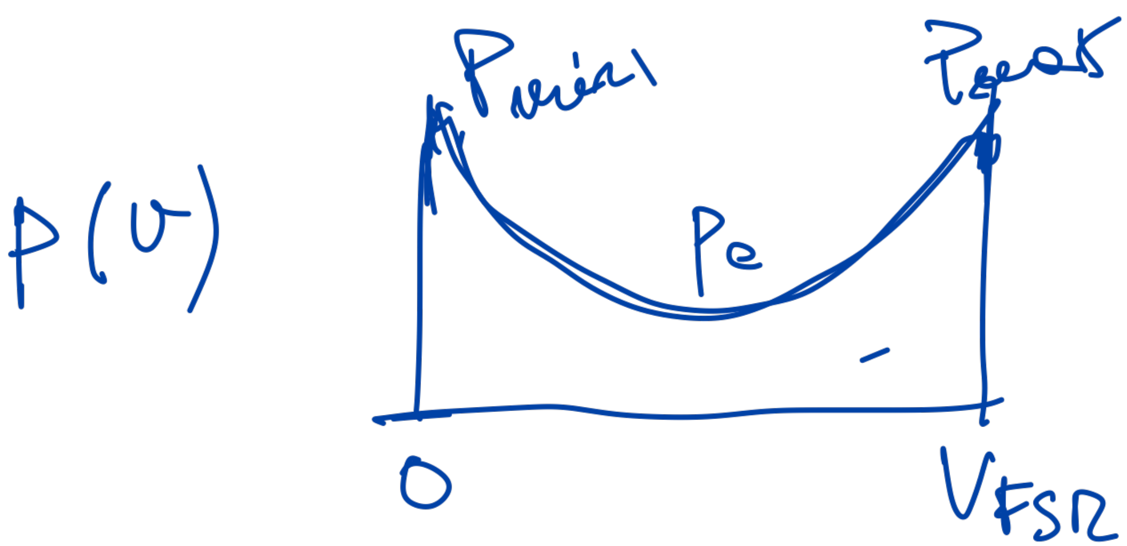


3 bit converter

8 levels

Entropy of the output stream

TIP



1000 samples in one period

V → integer

ANALOG NOISE

Thermal noise

Shot noise (electron charge, photon energy, etc)

[Noise Figure]

[Equivalent temperature]

Flicker noise

- - - - -

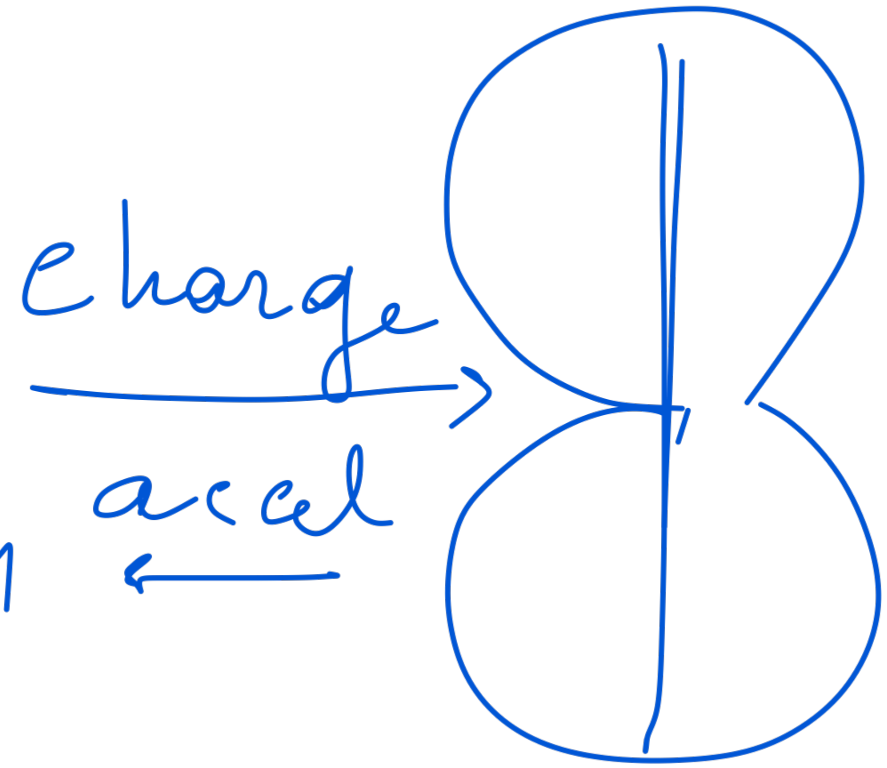
End of lecture

#9
Thursday, October 17, 2019

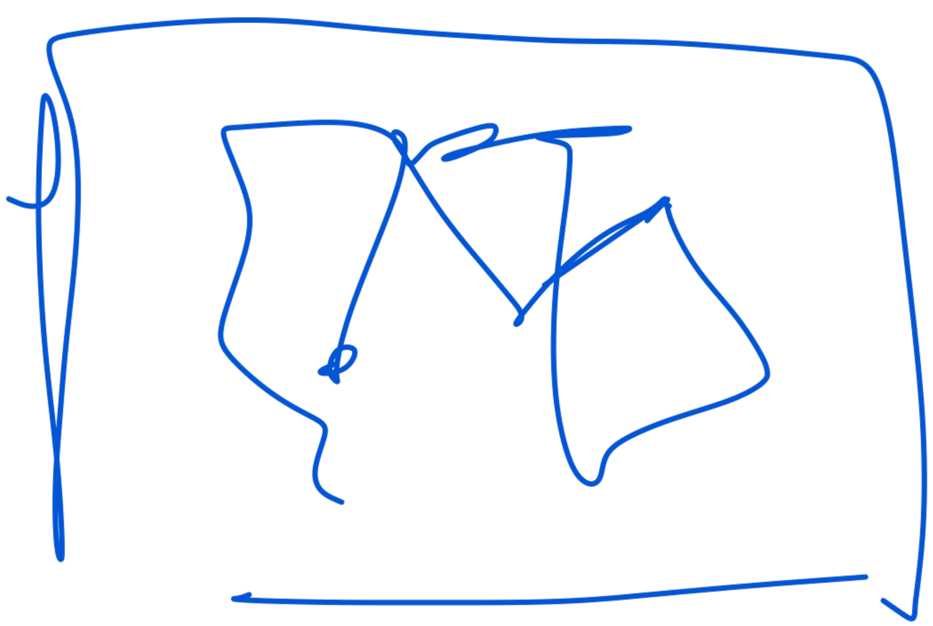
1.5 Hours

NOISE

Larmor Theorem



- synchrotron radiation
- antenna
- Led
- Blackbody radiation



gas-like "electron gas"

$\frac{1}{2} k T$ per degree of freedom

$$k = 1.380\,649 \cdot 10^{-23}\ \mathrm{J/K}$$

$$\simeq 2\ \mathrm{cal/mol}$$

blackbody radiation kT

---

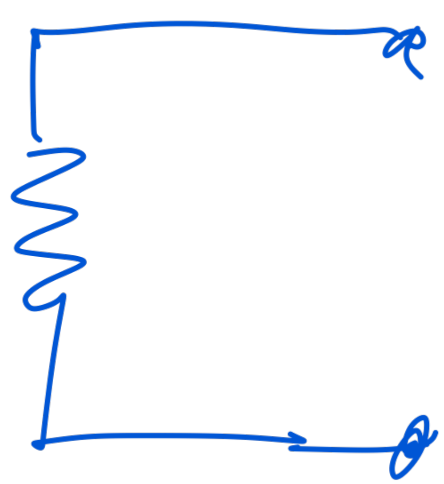
electrical current $\begin{cases} \text{particle} \\ \text{electromagnetic field} \end{cases}$

THERMAL NOISE

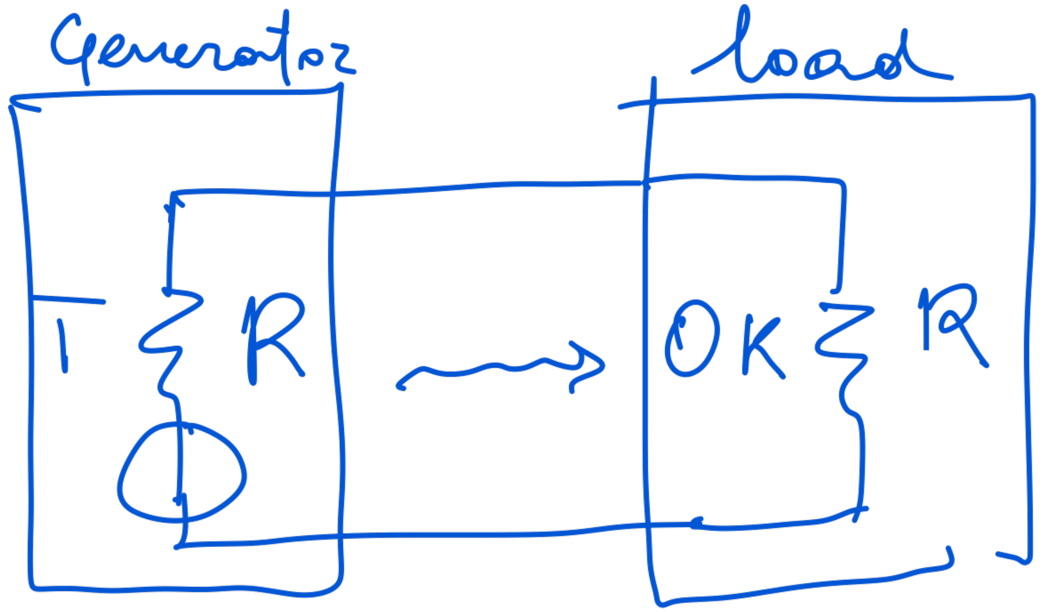
2 degrees of freedom

$\frac{1}{2}kT \rightarrow \mathcal{E}$ $\quad$ $\frac{1}{2}kT \rightarrow H$

field

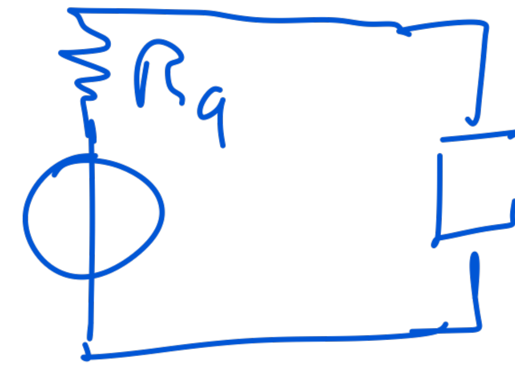


$$S = kT \quad [\mathrm{J}] \quad [\mathrm{W/Hz}]$$



$$S(f) = kT$$

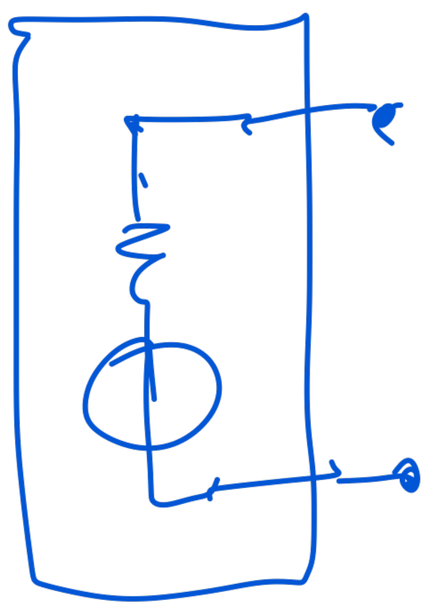
Thévenin



$R_L = \infty$

$R_L = 0$

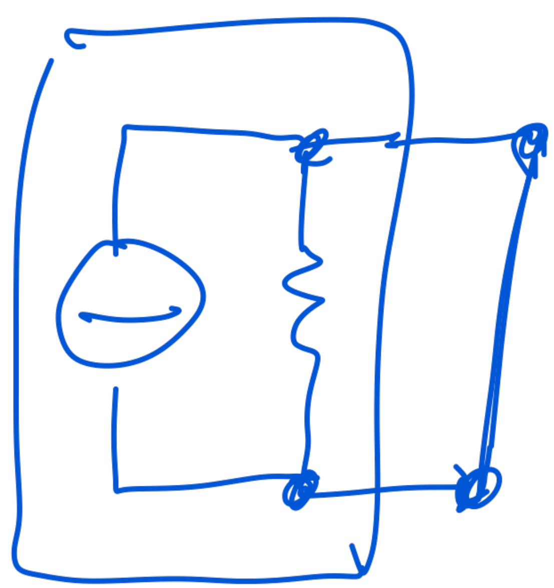
$R_L = R_g$



$$S_v(f) = \frac{4kT}{R} \quad [V^2/Hz]$$

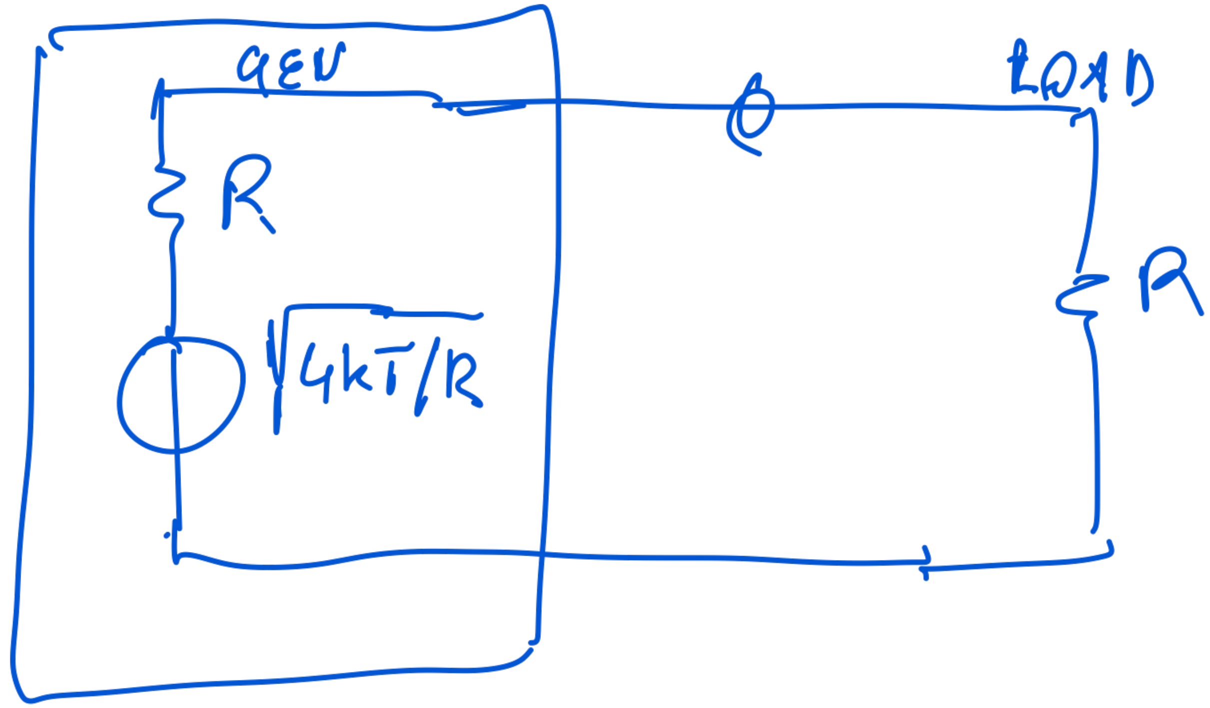
$$\sqrt{4kT/R}$$

$0.9\ nV/\sqrt{Hz}$ $\quad 50\Omega$



$$S_I(f) = 4kTR$$

$$A^2/Hz$$



$S(f)$ $[W/Hz]$

S

$\frac{W}{Hz}$ $\quad \frac{V}{\sqrt{Hz}} \cdot \frac{A}{\sqrt{Hz}}$

$$\frac{1}{2}\sqrt{\frac{4kT}{\cancel{R}}} \cdot \frac{1}{2}\sqrt{4kT\cancel{R}} = kT$$

Jargon of analog electronics

$\sqrt{S_V} \longrightarrow e_n$ $\qquad$ $nV/\sqrt{Hz}$

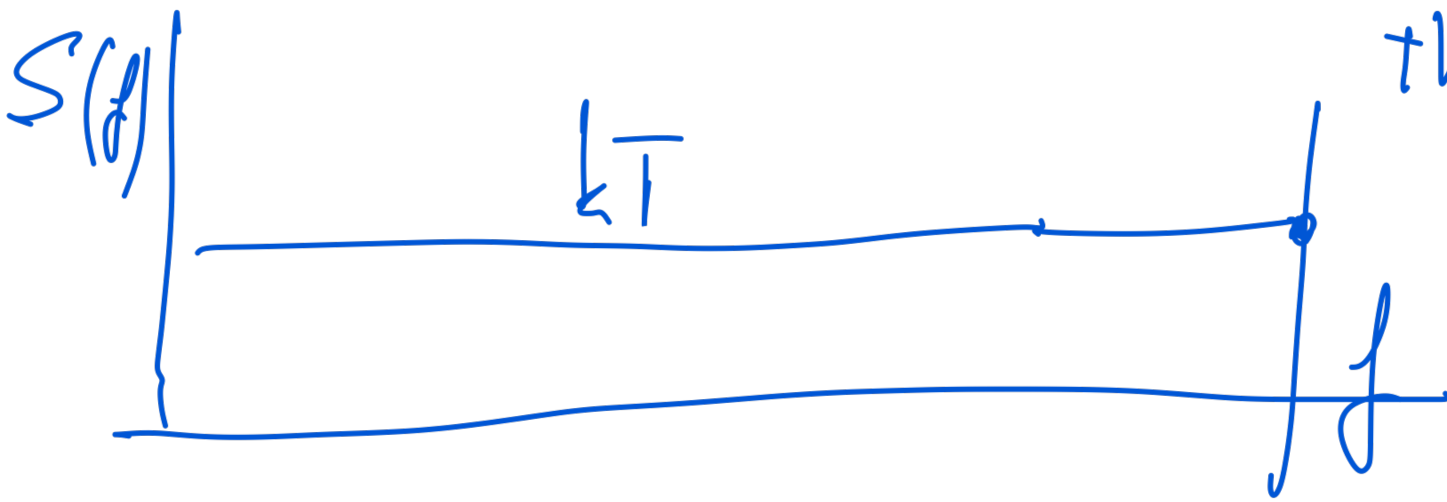
$\sqrt{S_I} \longrightarrow i_n$ $\qquad$ $pA/\sqrt{Hz}$

Cutoff Frequency

$$P = \int_a^b S(f)\,df$$

$a \to 0$

$b \to \infty$?!?



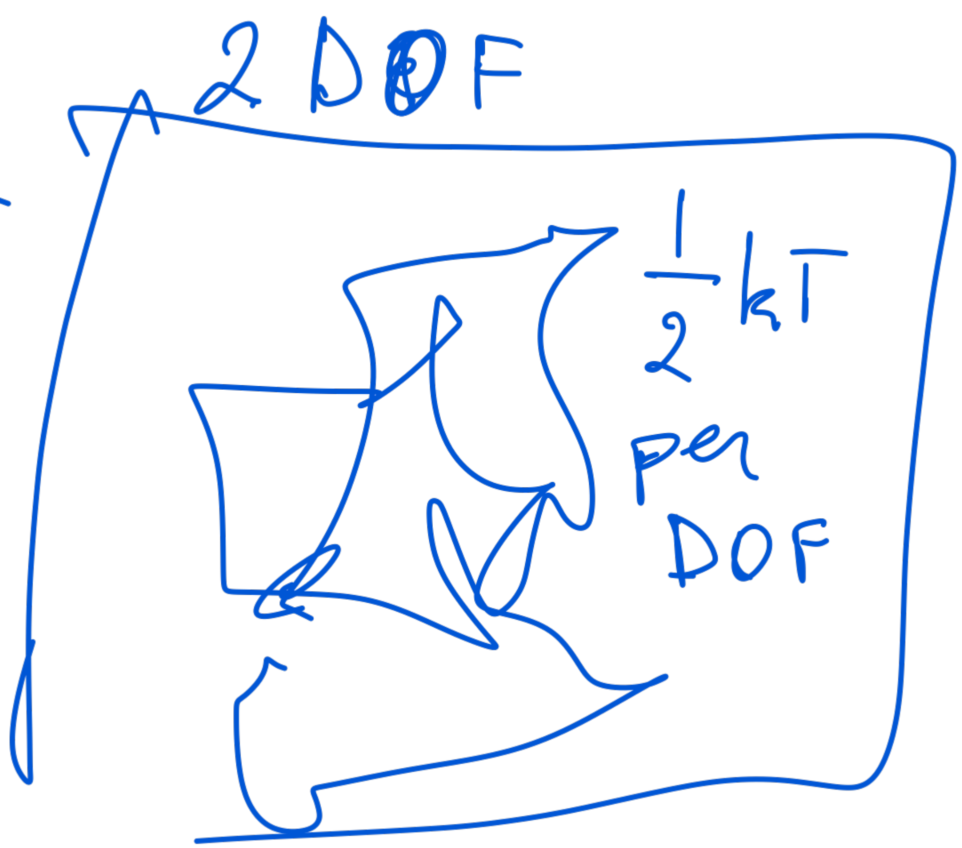
thermal energy = photon energy

$$kT = hf$$

$$f_c = \frac{kT}{h}$$

$$E = \frac{3}{2} kT$$

kT



$$\frac{\text{Jeans \& Rayleigh}}{\text{M. Planck}}$$

$$hf = kT \longrightarrow f_c = \frac{kT}{h}$$

300 K

$$\frac{1{,}38 \times 10^{-23} \cdot 300}{6{,}6 \times 10^{-34}} = 6 \times 10^{12}\ \mathrm{Hz}$$

6 THz

(50 μm)

30 THz 10.6 μm CO2 'metrology'

10.6 μm power

2 decades

0.3 THz 200 GHz electronics

End of Lecture

#10

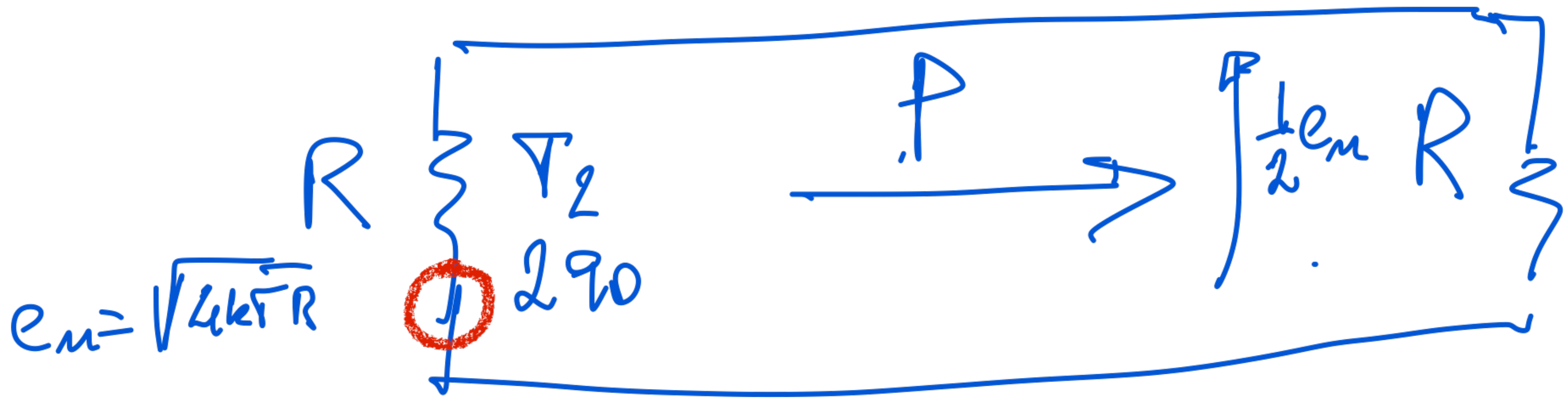
Tuesday, October 22, 2019

1.5 Hours

Examples & Exercises

1. Calculate the total power transferred from a resistor at $T_2 = 290$ K to a cold ($T_1 = 0$ K)
2. A resistor of temperature $T_R = 4.2$ K is connected to a 6-dB attenuator at the temperature $T_A = 300$ K. Which is the equivalent temperature T_E seen at the other end of the attenuator?
   Tip: Trust the 2nd principle of thermodynamics.
3. Same as (1), but the two resistors are connected by WR28 waveguide (21.1-42.2 GHz bandwidth). Assume the cable loss free.
4. Same as (3), but now $T_1 = 77$ K

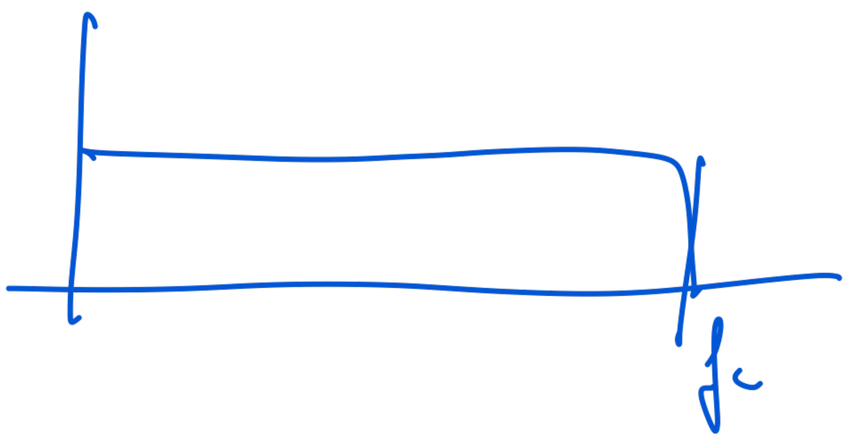
1. Calculate the total power transferred from a resistor at T_2 = 290 K to cold (T_1 = 0 K)



$T_1 = 0\ K$

$$S = \frac{1}{4} e_n^2 / R = kT$$

$$P = \int_0^\infty S(f)\, df$$



$$hf = kT$$

$$f_c = \frac{kT}{h}$$

$k = 1.38 \times 10^{-23}$

$T = 290\ K$

$kT = 4 \times 10^{-21}$

$S(f) = kT$ W/Hz

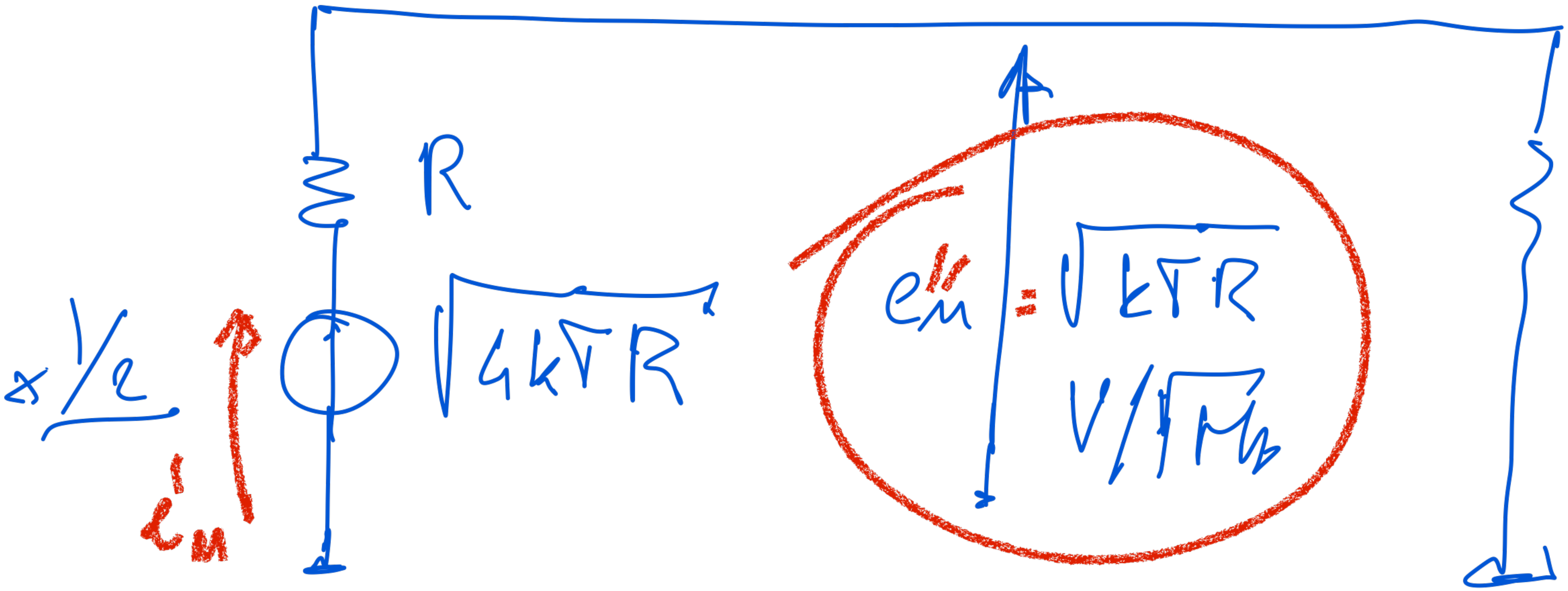
$$P = S \cdot f_c = \frac{(kT)^2}{h}$$

$$\frac{16 \times 10^{-42}}{6.6 \times 10^{-34}}$$

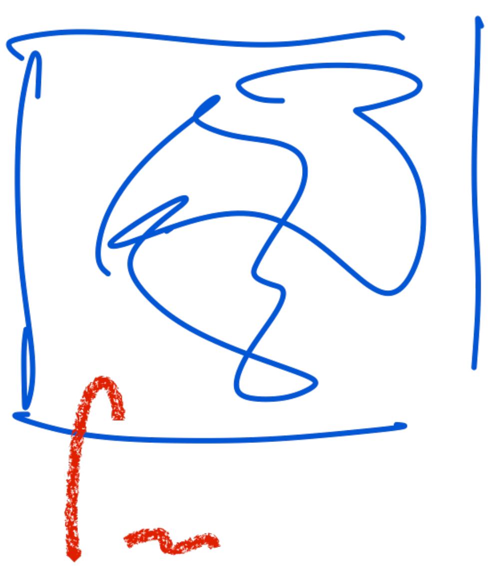
Planck constant h

-46 dBm

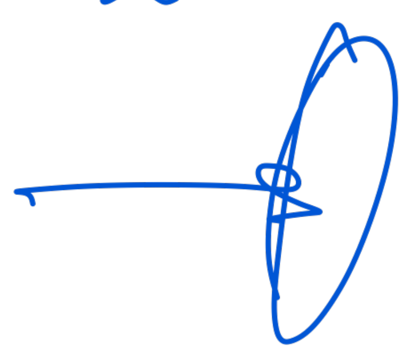
$\boxed{2.4 \times 10^{-8} \quad 24\ \text{nW}}$



$$S = \frac{e_n^2}{R} \qquad \frac{V^2}{Hz \cdot \Omega} = \frac{W}{Hz}$$

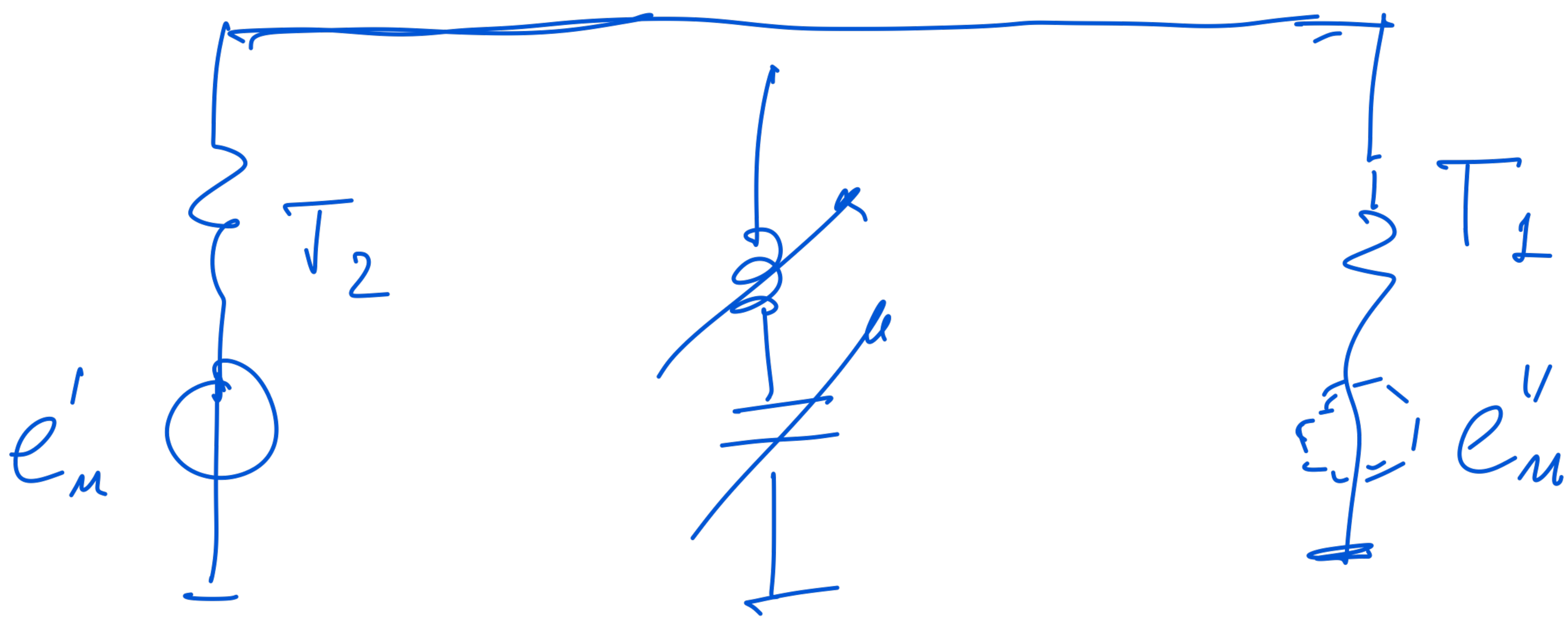


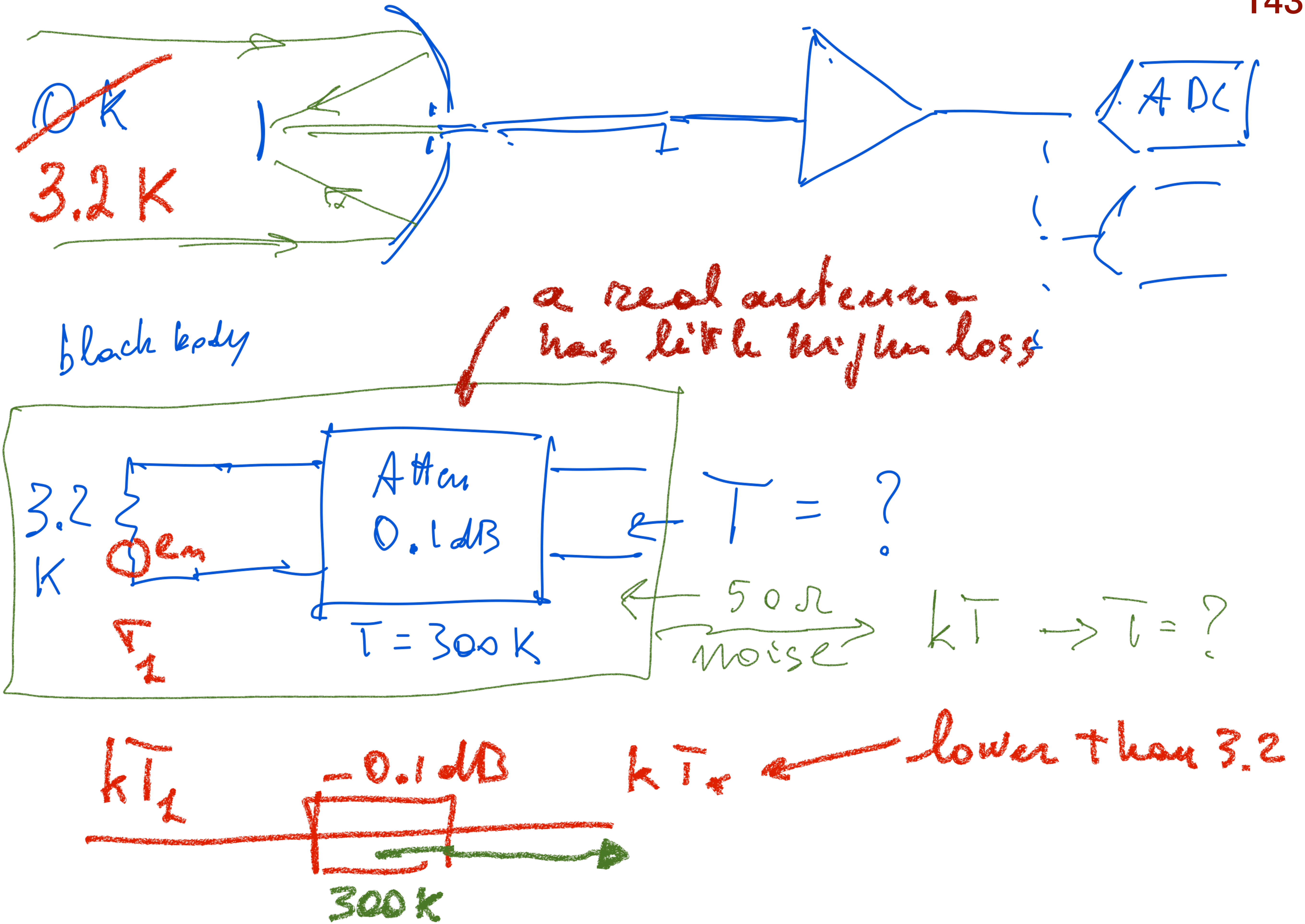
$\frac{1}{2} kT$ per degree of freedom



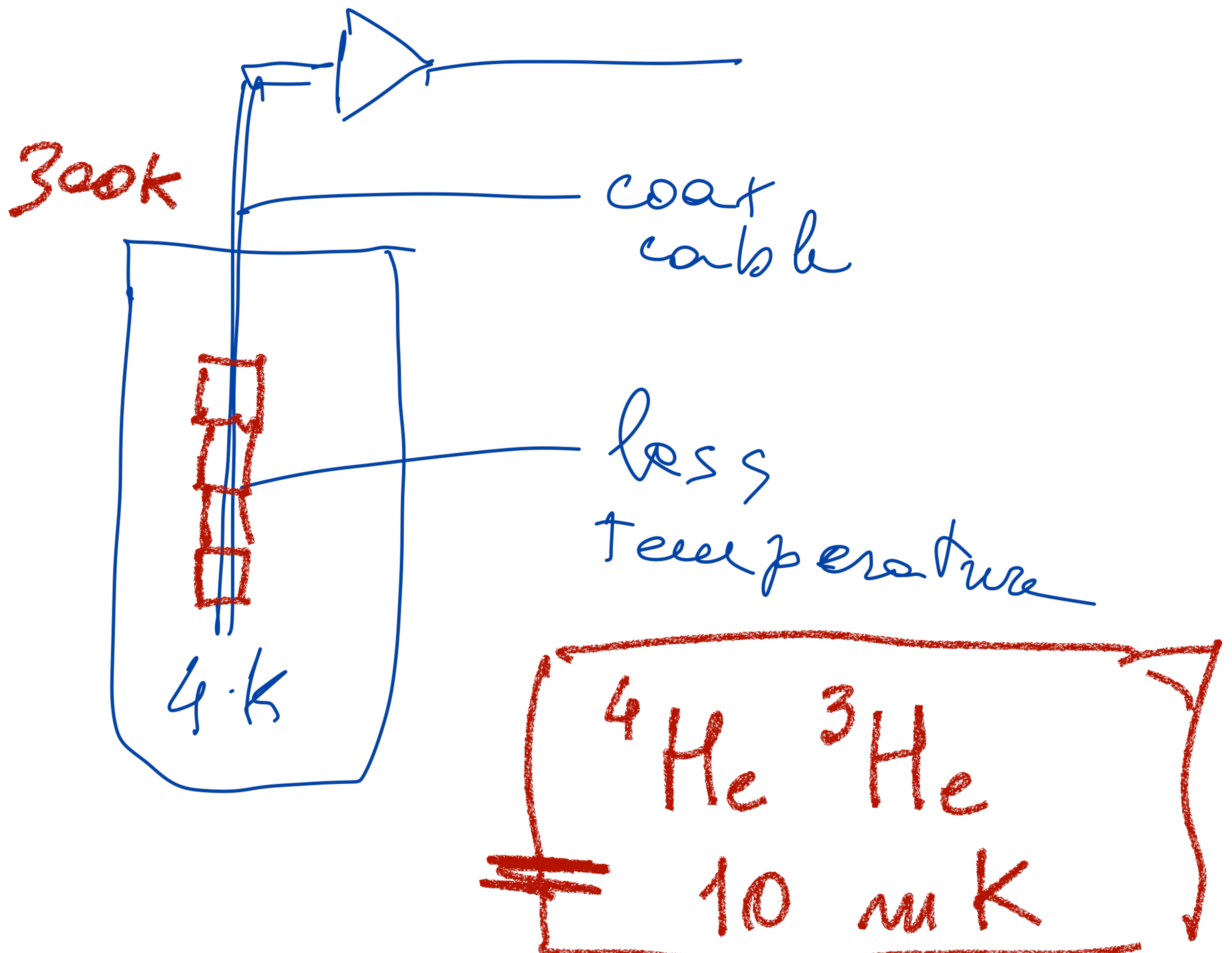
kT

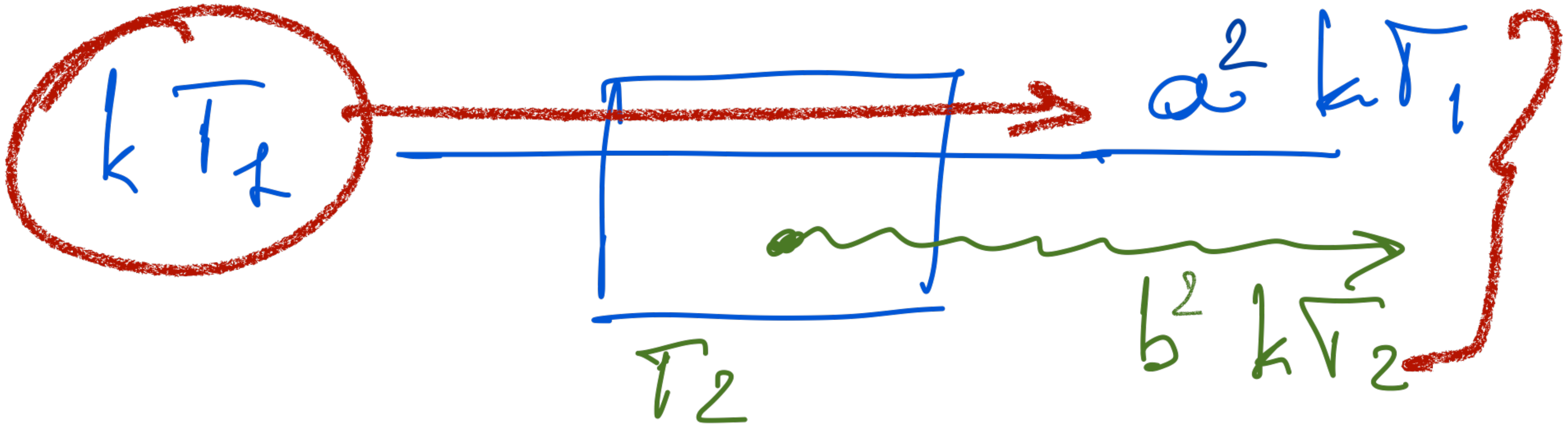
H. Nyquist ~ 1928




0 K
3.2 K
ADC
black body
a real antenna has little higher loss
3.2 K
Atten 0.1 dB
e_n
T_1
T = 300 K
T = ?
50 Ω
noise
kT → T = ?
kT_1
0.1 dB
300 K
kT_x
lower than 3.2

Liquid-He Cryostat

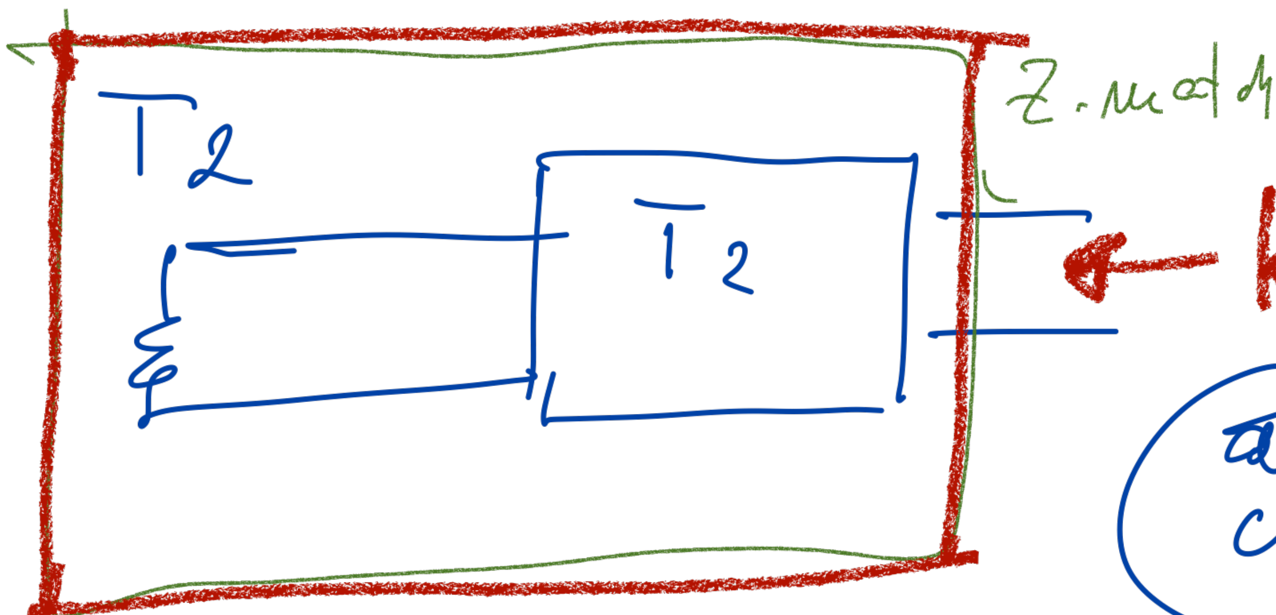




find a^2, b^2

$a^2 = 0.977$ (−0.1 dB)

Gedankenexperiment



$\leftarrow kT_2 \rightarrow a^2 kT_2$

~~atten~~ contrib $\Rightarrow$ $b^2 kT_2$

$$\frac{b^2 kT_2}{kT_2}$$

$+0.1\,\mathrm{dB} \rightarrow 1.023$

$-0.1\,\mathrm{dB} \rightarrow 0.977$

$a^2 + b^2 = 1$

$b^2 = 0.023$

$T_x = 6.9\,\mathrm{K}$

Historical article, highly educational

THERMAL AGITATION OF ELECTRIC CHARGE IN CONDUCTORS*

By H. Nyquist

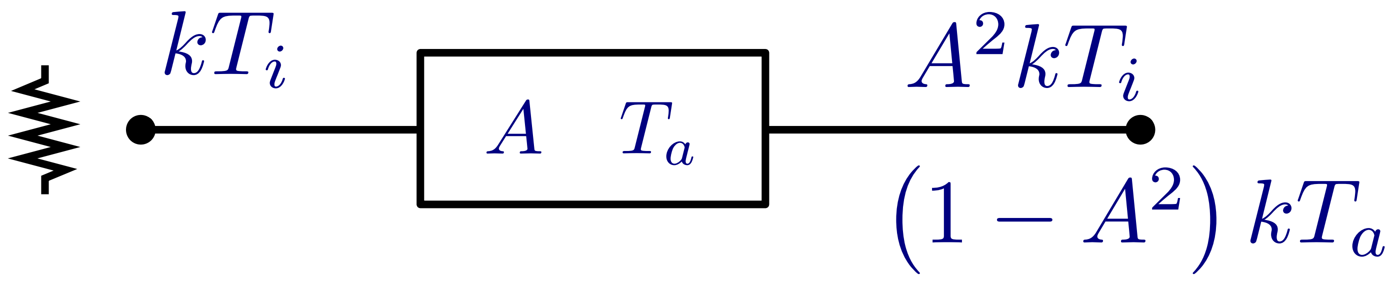
Abstract

The electromotive force due to thermal agitation in conductors is calculated by means of principles in thermodynamics and statistical mechanics. The results obtained agree with results obtained experimentally.

Dr. J. B. Johnson[1] has reported the discovery and measurement of an electromotive force in conductors which is related in a simple manner to the temperature of the conductor and which is attributed by him to the thermal agitation of the carriers of electricity in the conductors. The work to be resported in the present paper was undertaken after Johnson's results were available to the writer and consists of a theoretical deduction of the electromotive force in question from thermodynamics and statistical mechanics.[2]

Nyquist H - Thermal agitation of electric charges in conductors - Phys Rev 32(1) p110-113, July 1928

Thermal Noise of a Dissipative Device



$$S(f) = A^2 kT_i + \left(1 - A^2\right) kt_a$$

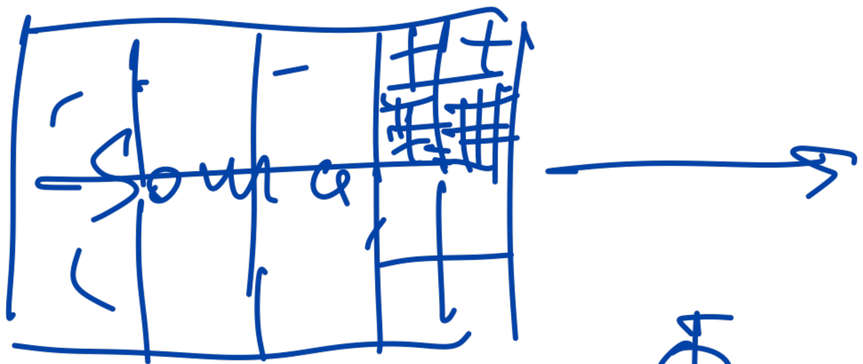
- Noise contribution of the input resistor
  - The attenuator makes no difference between “noise” and “signal”
  - The input signal is “amplified” by a factor $A^2 < 1$
- Noise contribution of the attenuator
  - At uniform temperature T the sum of the contributions must be kT
  - The input contributes A^2kT
  - The attenuator contributes the complement $(1–A^2)$kT
- The factors A^2kT and $(1–A^2)$kT do not depend on temperature

#11

Thursday, October 24, 2019

1.5 Hours

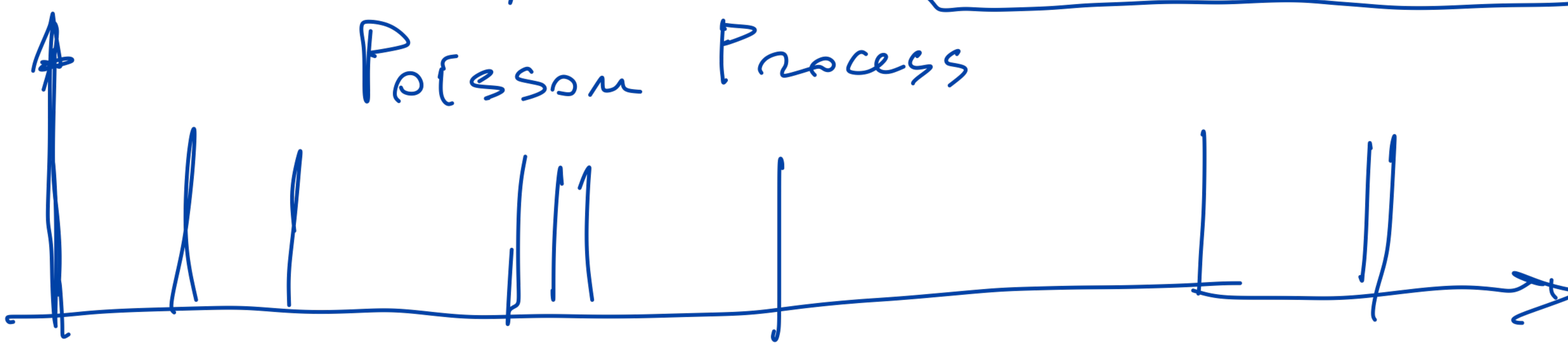
SHOT NOISE



Φ

Poisson Process
- random time
- no space correlation
- no time correlation (memory)

Poisson Process



Electrons Φ n° of electron per second

$q = 1.602 \times 10^{-19}$ J

$$\langle I \rangle = q\,\Phi$$

Photons Φ no of photons per second

$E = hf = \cancel{\frac{hc}{\lambda}}$ $\quad P = \Phi E$

Poisson Process

- emission at random time
- no space correlation
- no memory (time correlation)

Featured book

Feller, An introduction to probability theory Vol. II, J. Wiley ~ p. 15

PDF

e^{-at}

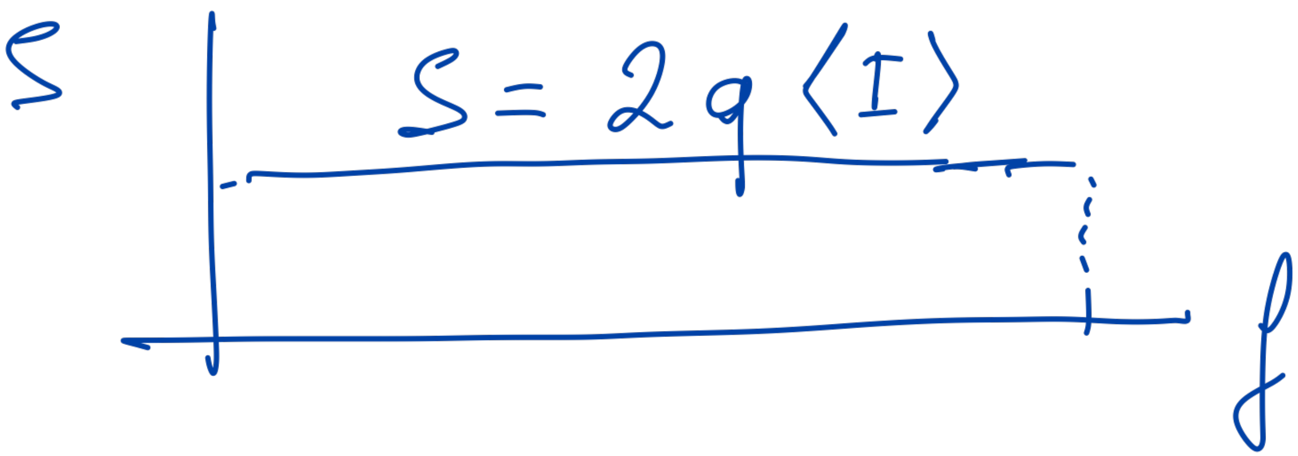
waiting time

$\mu = 2\Phi$

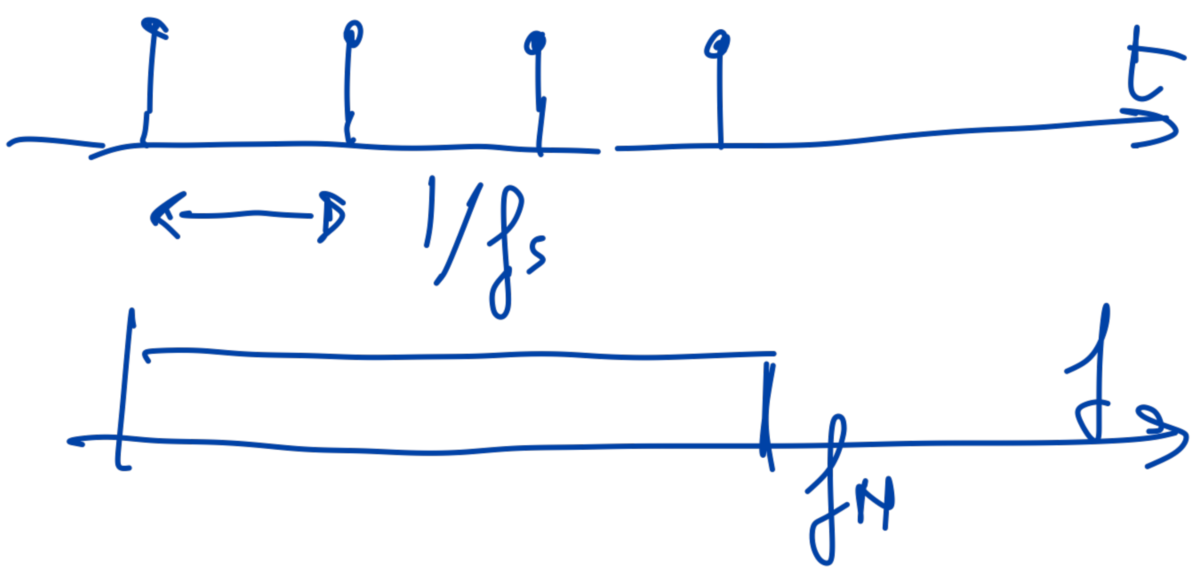
$\sigma^2 = \mu$

Feller explains that this is the maximum disorder

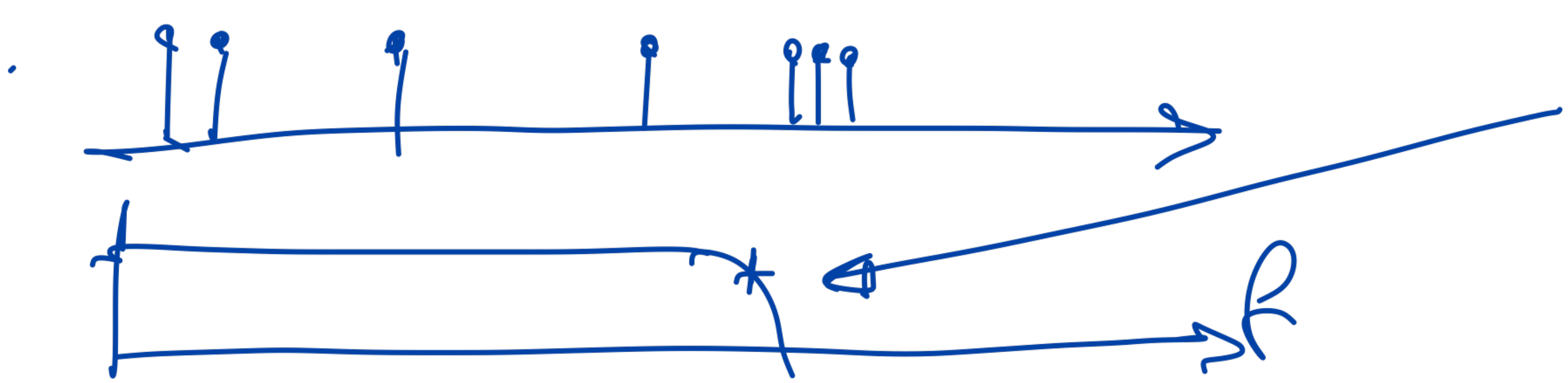
Shot noise PSD / Electrical



$$\langle I \rangle = q \langle \Phi \rangle$$



$$f_N = \frac{1}{2} f_s$$



cutoff $f_c = \bar{\Phi}/2$

Shot Noise

Example

$I = 1\,mA$

find S_I, f_c

$$\Phi = I/q = \frac{10^{-9}}{1.6\times10^{-19}}$$

$$6.25\times10^{9}\ e/s$$

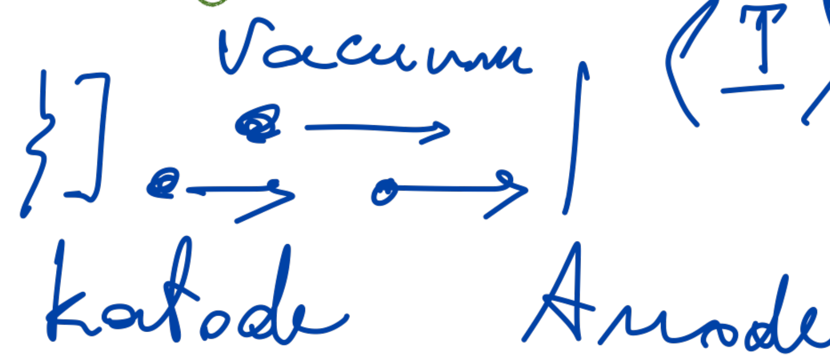
$$f_c \simeq 3\ GHz$$

$$S_I = 2\cdot 1.6\times10^{-19}\cdot 10^{-9}$$

$$= 3.2\times10^{-28}\ A^2/Hz$$

$$P = \frac{2q\langle I\rangle}{R}\,\frac{1}{2}\,\frac{\langle I\rangle}{q} = \frac{\langle I\rangle^2}{R}\quad [A^2]\ [W]$$

Cutoff $f_c = \frac{1}{2}\Phi$



$$\langle I\rangle = q\,\langle\Phi\rangle$$

$$S = 2q\langle I\rangle \quad [A^2/Hz]$$

$$\boxed{P = \int_0^{\infty} S(f)\,df}$$

$$f_c = \frac{1}{2}\Phi = \frac{1}{2}\frac{I}{q}$$

Shot noise – photodetector

$P_\lambda \longrightarrow D$ ——— $\xrightarrow{I}$ load, S_I

$\Phi_{photons} = \dfrac{P}{E}$ $\quad \eta \quad$ $\Phi_{electrons} = \eta \, \Phi_{photons}$

η = quantum efficiency

Exercise

$P = 1\,mW$ optical

$\lambda = 1550\,nm$

$\eta = 0.7$

① Find f_c, S_I, $P_{electrical} \rightarrow R = 50\,\Omega$

② $P_{optical} \rightarrow +10\,dB$

$P_{electrical} \rightarrow +\,?\,dB$

[m] [clocks]

$d = c\,t$

measured

$d = c\,t$

^{133}Cs clock

299 792 458 m/s

by def

the meter is defined this way

$\Delta\nu\, Cs$

c

e (q)

h

k

N_A

BIPM

CGPM

Weighted average of previous measures

End of lecture

#12

Tuesday, November 5, 2019

1.5 Hours

$$\Phi = \frac{P}{E} \longrightarrow \frac{\eta P}{h\nu} \longrightarrow \frac{\eta P}{hc/\lambda} \qquad \frac{0.7 \cdot 10^{-9}}{6.6\times10^{-34} \cdot 3\times10^{8}/1.55\times10^{-6}}$$

hc/λ: 1.3×10^{-19} J ~ 0.8 eV

$$\eta P = 0.7\times10^{-9}$$
$$= 7\times10^{-10}$$

$$\Phi \quad 5.4\times10^{9} \quad e/s$$

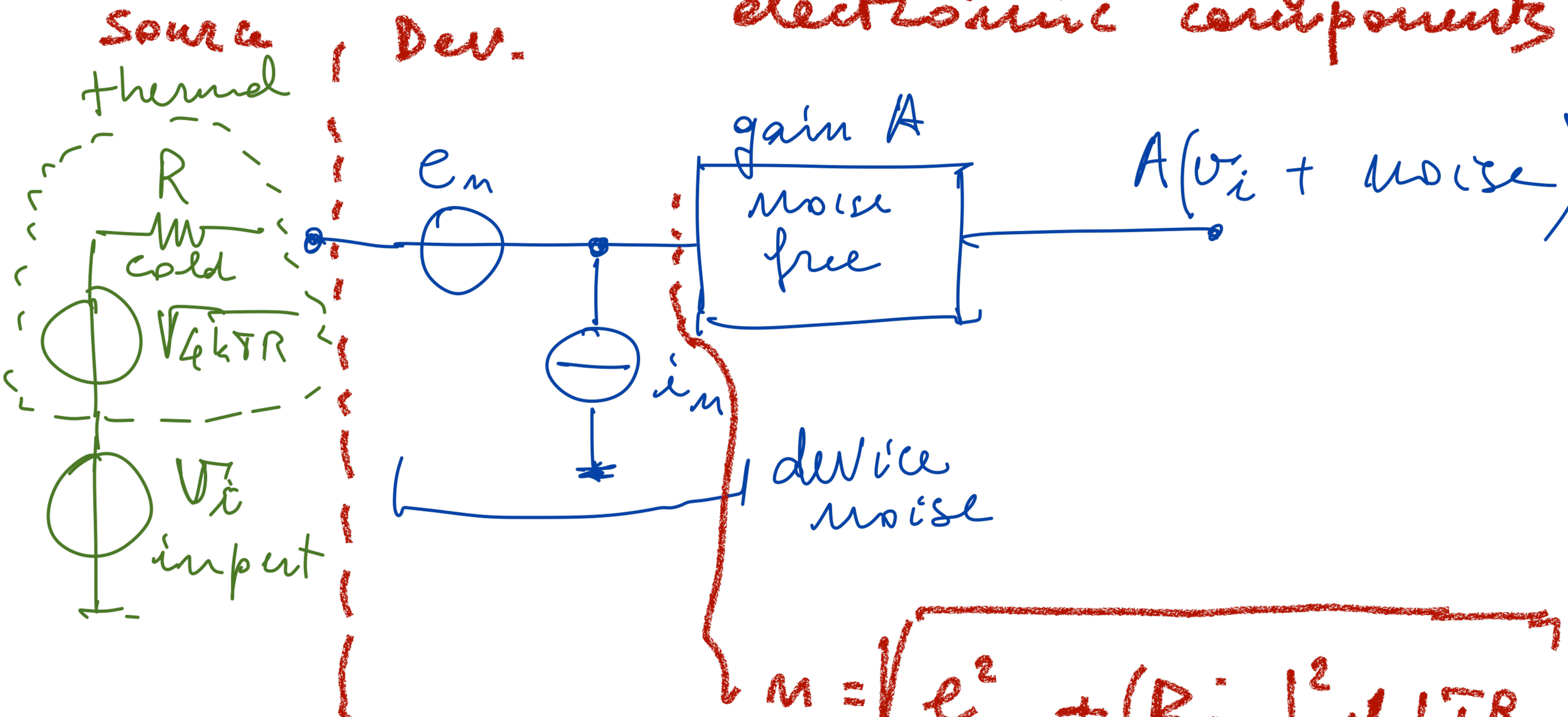
$$f_c = \frac{1}{2}\Phi = 2.7\ \text{GHz}$$

$$S_I = 2qI$$
$$= 2.8\times10^{-28}\ A^2/Hz$$
$$\longrightarrow 1.7\times10^{-14}\ A/\sqrt{Hz} \qquad 17\ fA/\sqrt{Hz}$$

$$I = q\Phi =$$
$$1.6\times10^{-19}\times 5.4\times10^{9}$$
$$8.6\times10^{-10}\ A$$

The Rothe Dahlke Model

Electronics – Analog noise of electronic components

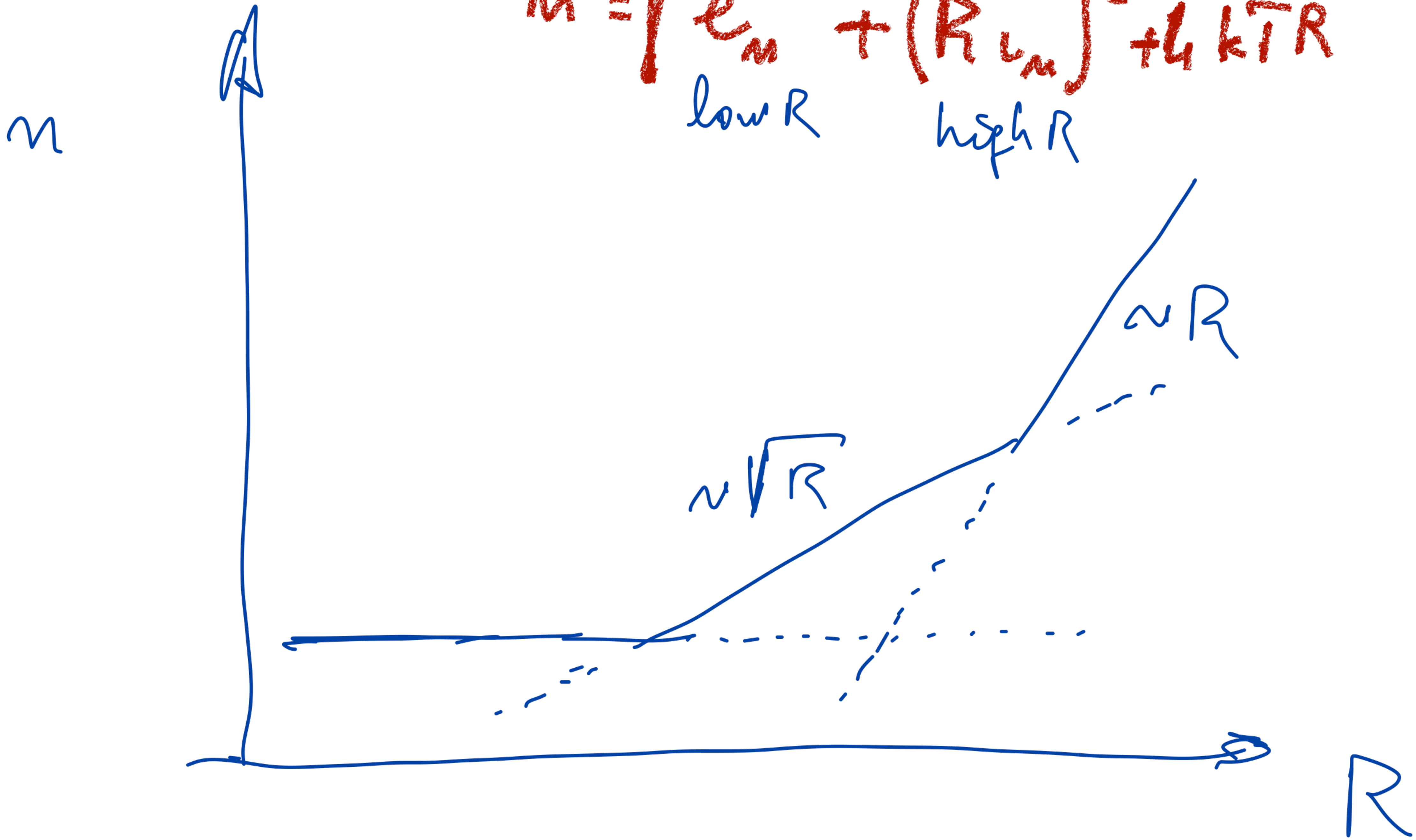


$$n = \sqrt{e_n^2 + (R\, i_n)^2 + 4kTR}$$

Rothe Dahlke 1956

$$n = \sqrt{e_n^2 + (R\, i_n)^2 + 4kTR}$$

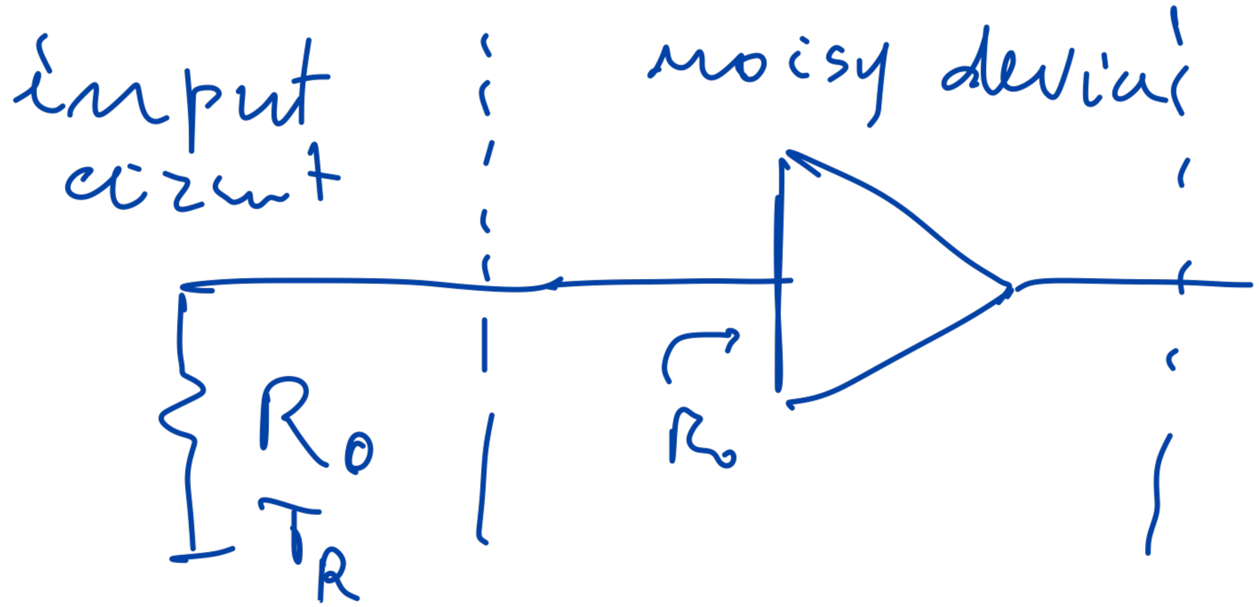
low R   high R

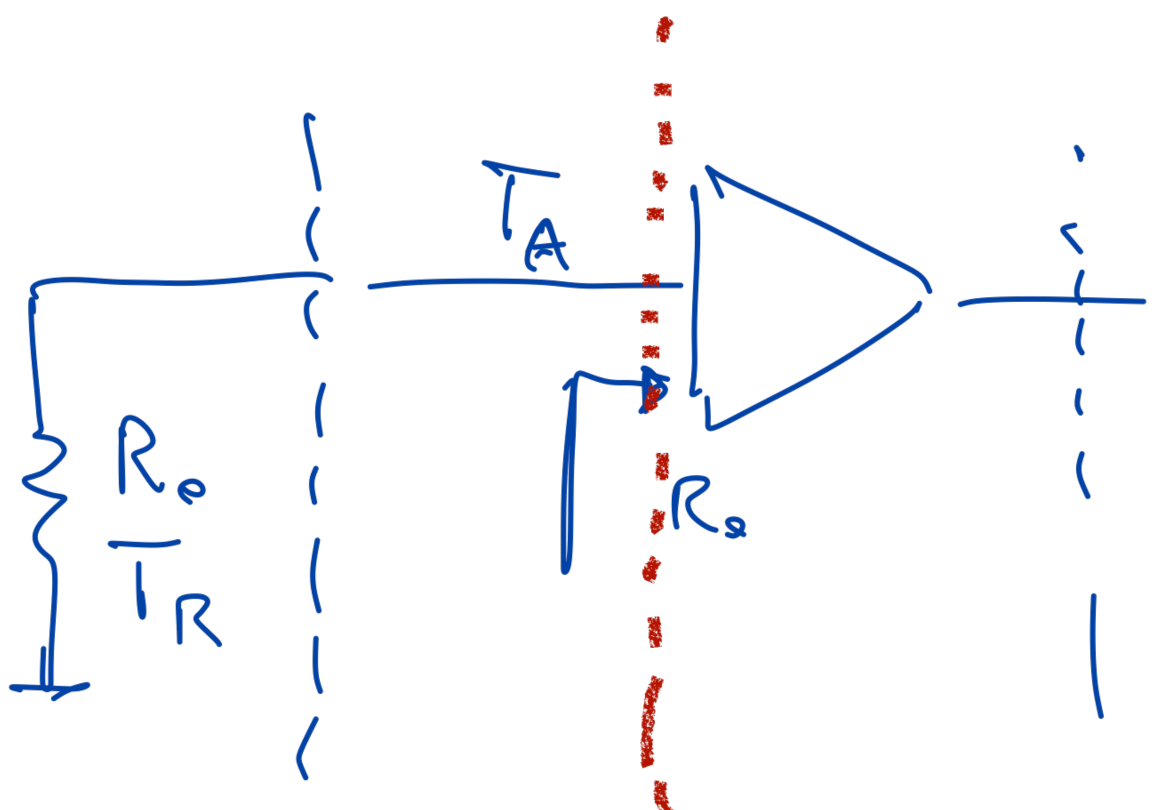


R-D model – best choice for general analog electronics

microwaves & RF → electrical lines
impedance matching

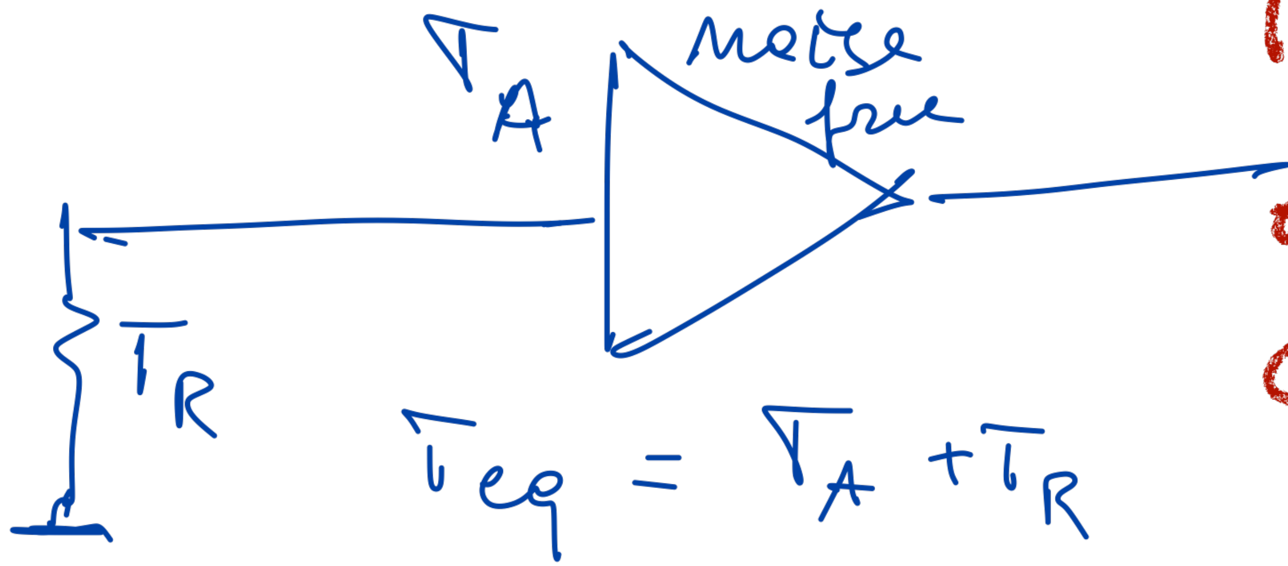
$R_0 = 50\,\Omega$





Phenomenological model only
(T_{eq} does not exist)

$kT_{eq} \longrightarrow T_{eq} = T_R + T_A$



$$T_{eq} = T_A + T_R$$

T_{eq} is a radio-engineering concept

Noise-free device

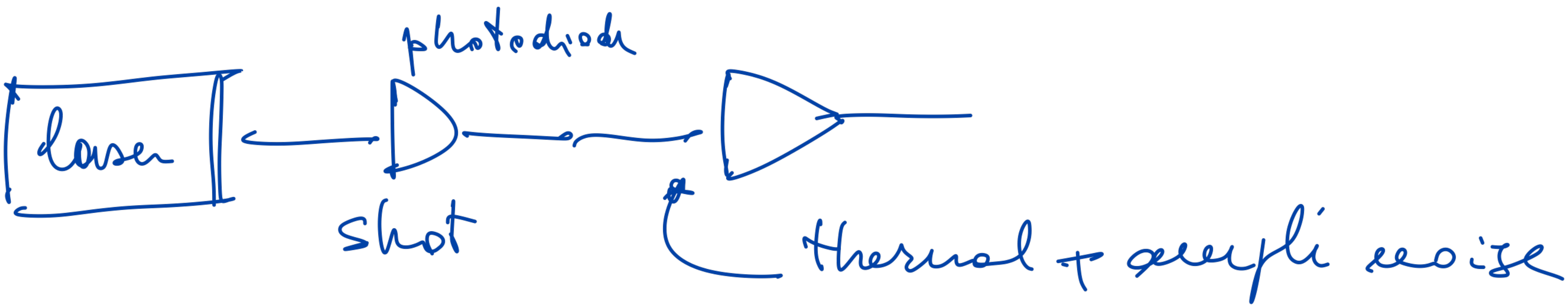
T_{eq} is the temperature of the resistor if

- the device is noise-free
- the resistor is heated to the "magic temperature" that gives the same noise...

A warning to people with main background in optics:

T_{eq} is not a true temperature

The shot noise is included in T_{eq}



$P > \sim$ a few mW

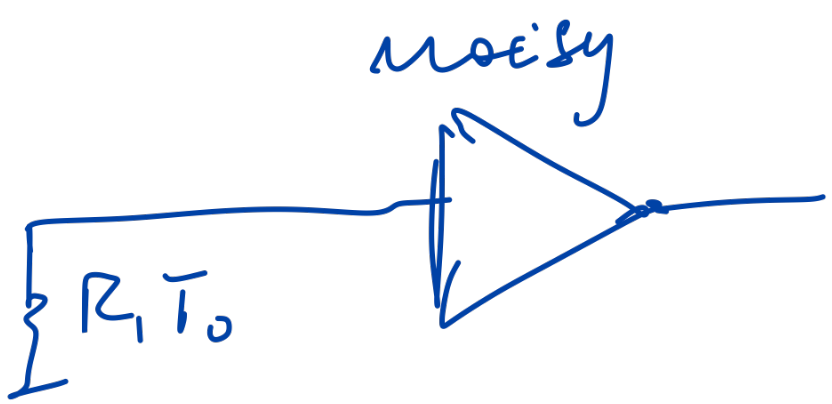
shot > thermal

Noise Factor (Noise Figure)

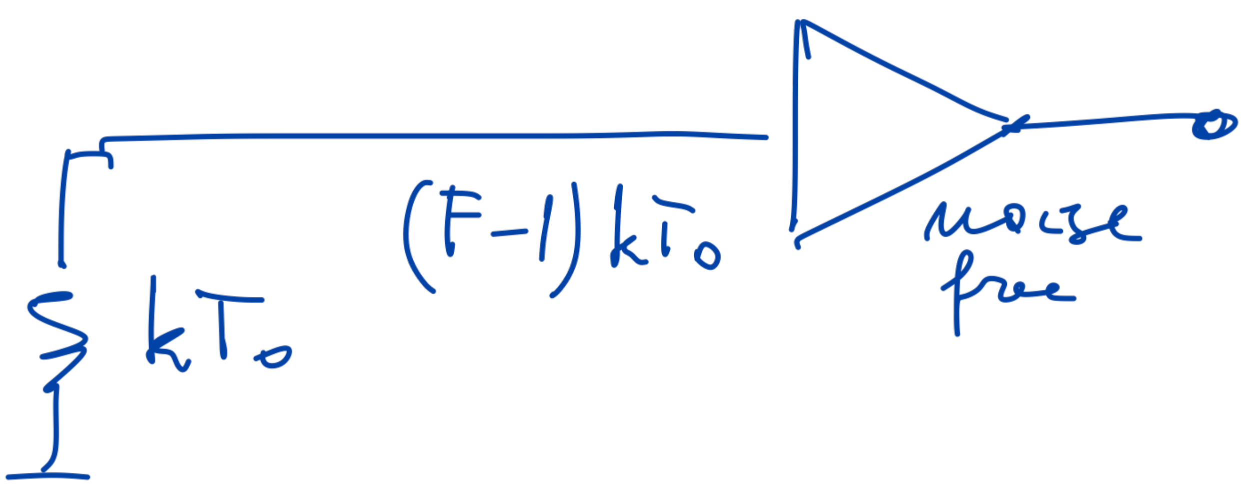
Microwaves (R_0), standard temperature

$$T_0 = 290 \text{ K} \quad (17\,^\circ\text{C})$$

$$kT_0 = 4 \times 10^{-21} \text{ J}$$



total noise FkT_0



input R	kT_0
ampli	$(F-1)kT_0$
Σ	FkT_0

Equivalent Noise Temperature

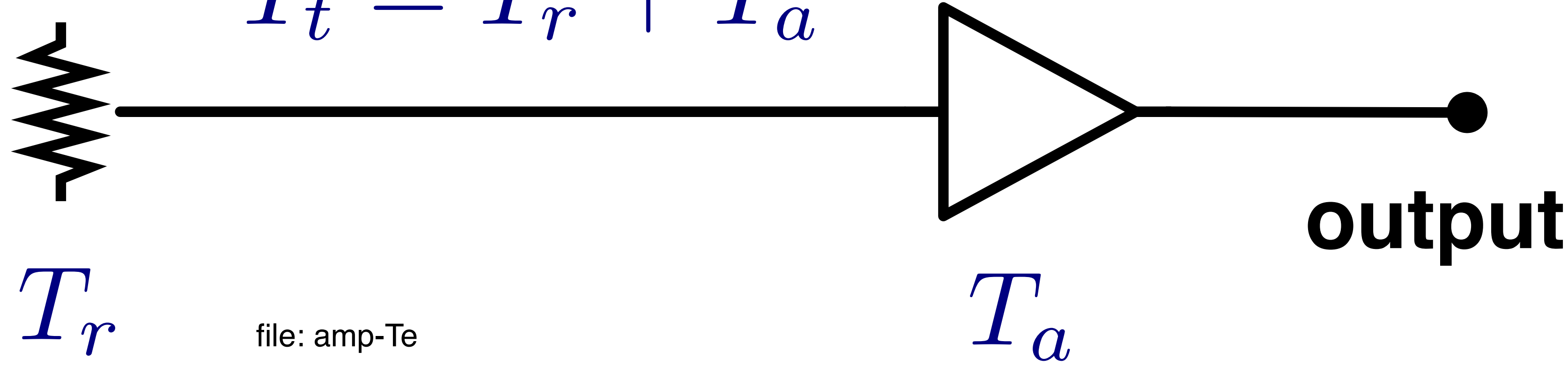
Thermal noise

$$S(\nu) = \frac{h\nu}{e^{h\nu/kT}}$$

$$S(\nu) = N_t \quad \text{constant for } h\nu \ll kT$$

$$N_t = kT$$

$$T_t = T_r + T_a$$



Ta is the equivalent noise temperature of the amplifier

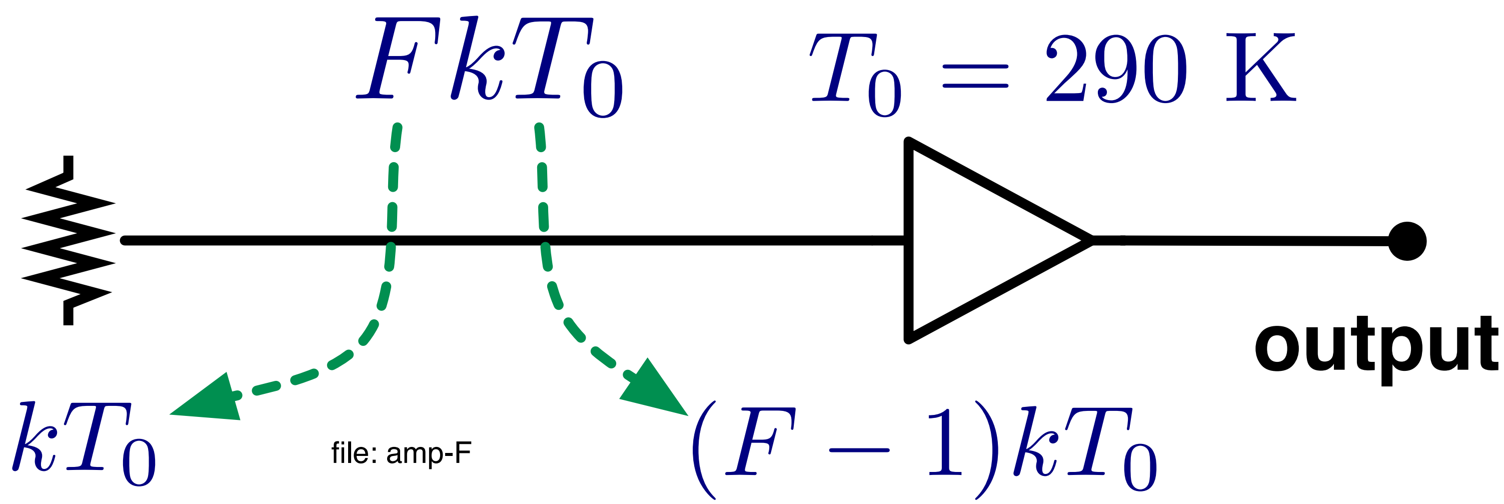
defined in specified conditions (physical temperature and input resistance)

Equivalent temperature T_a defined by $N_t = k(T_a + T_r)$

- Warning: the noise temperature a radio-engineering concept
  - The physical nature of noise does not matter
  - Often misleading in optics: the shot noise contributes to the equivalent temperature

Noise Figure

Noise figure $$F = \frac{\mathrm{SNR(out)}}{\mathrm{SNR(in)}}$$ general definition



Assume that the whole circuit is at the reference temperature T0 = 290 K (17 ºC)

The total noise referred to the amplifier input is FkT0

amplifiers and RF/μw devices

$$FkT_0 \quad = \quad kT_e \quad = \quad k(T_a + T_0) \qquad T_0 = 290\,\mathrm{K}$$

$$F = \frac{T_a + T_0}{T_0} \qquad \text{and} \qquad T_a = (F-1)T_0$$

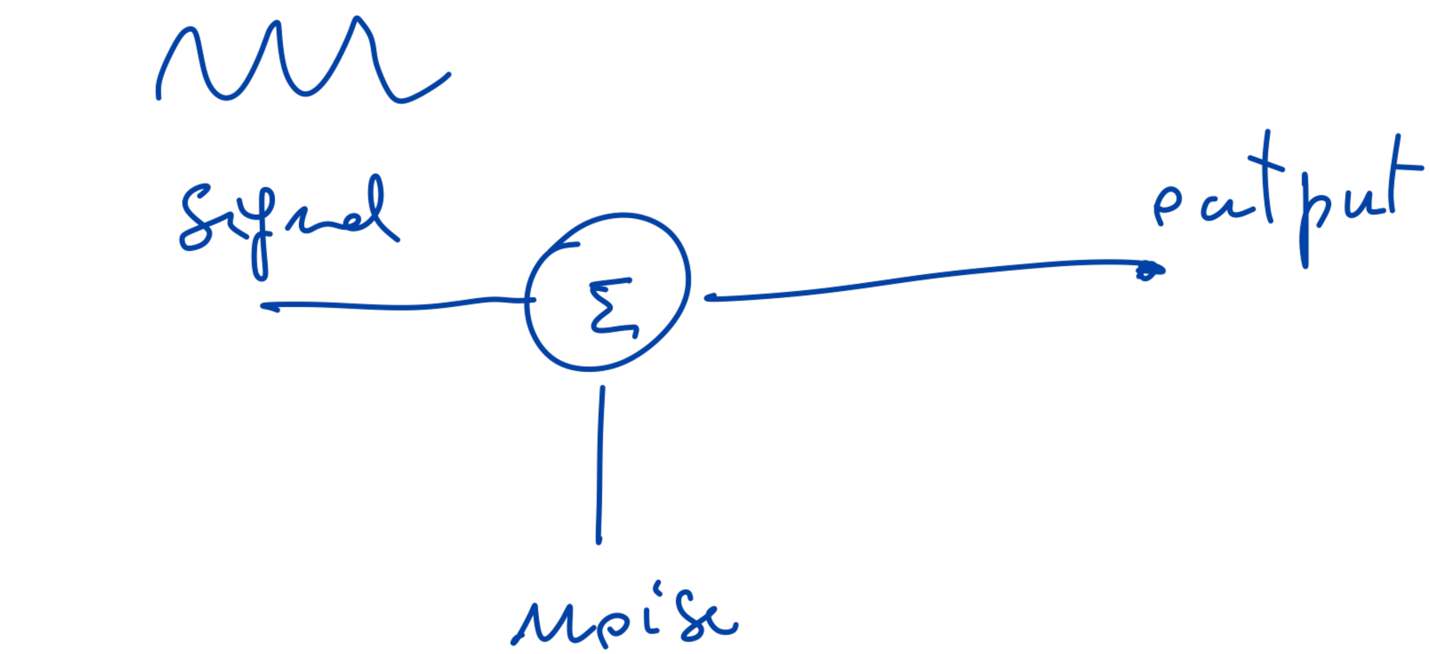
Warning: the noise figure is a radio-engineering concept, can be misleading in optics

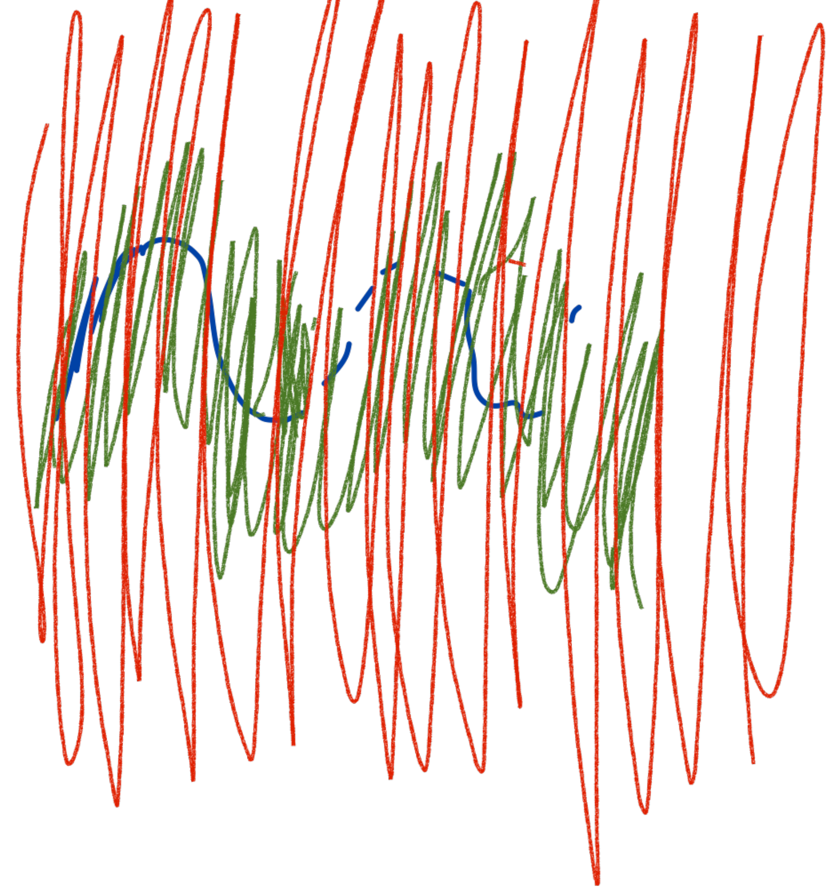
End of lecture

#13

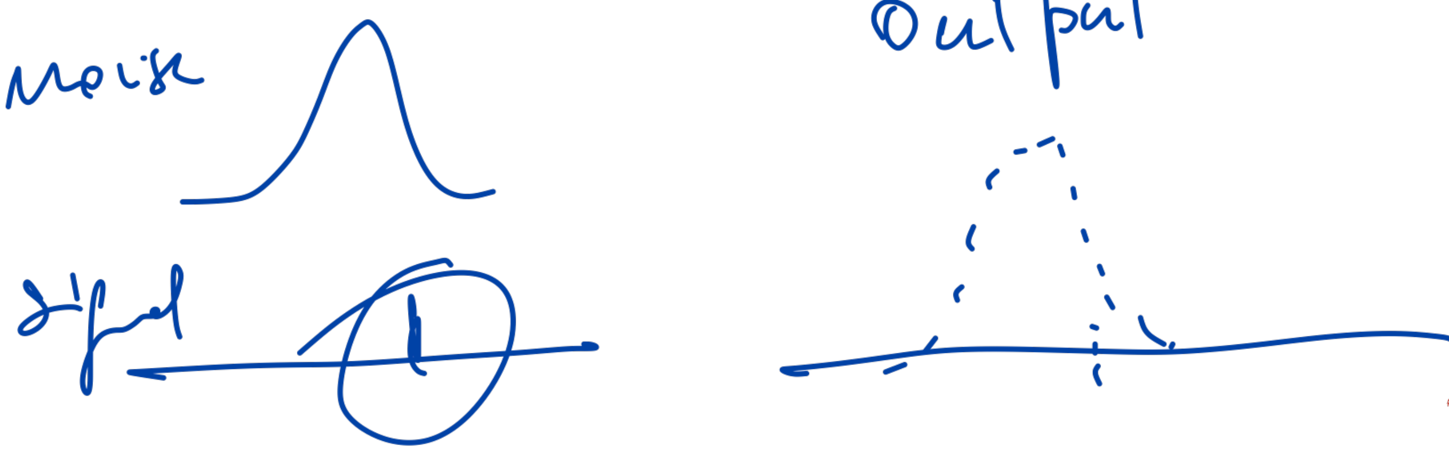
Friday, November 8, 2019

1.5 Hours





2σ 95%



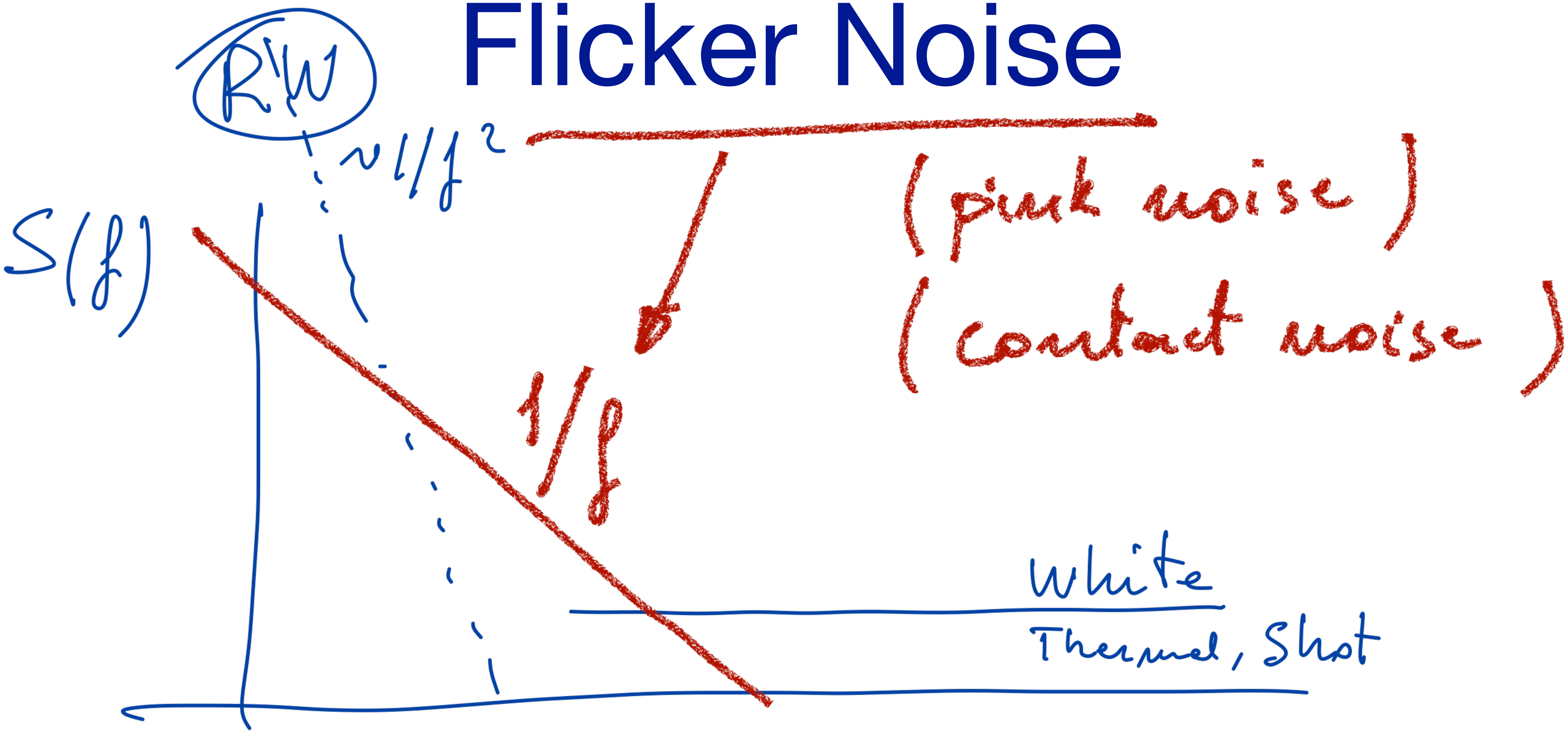
time series

Inverse Problem

Bayes statistics

$$P\{A|B\} = P\{A \& B\}/P\{B\}$$

Flicker Noise



$\int x(t)\,dt \to$ Random Walk

white

Brownian motion

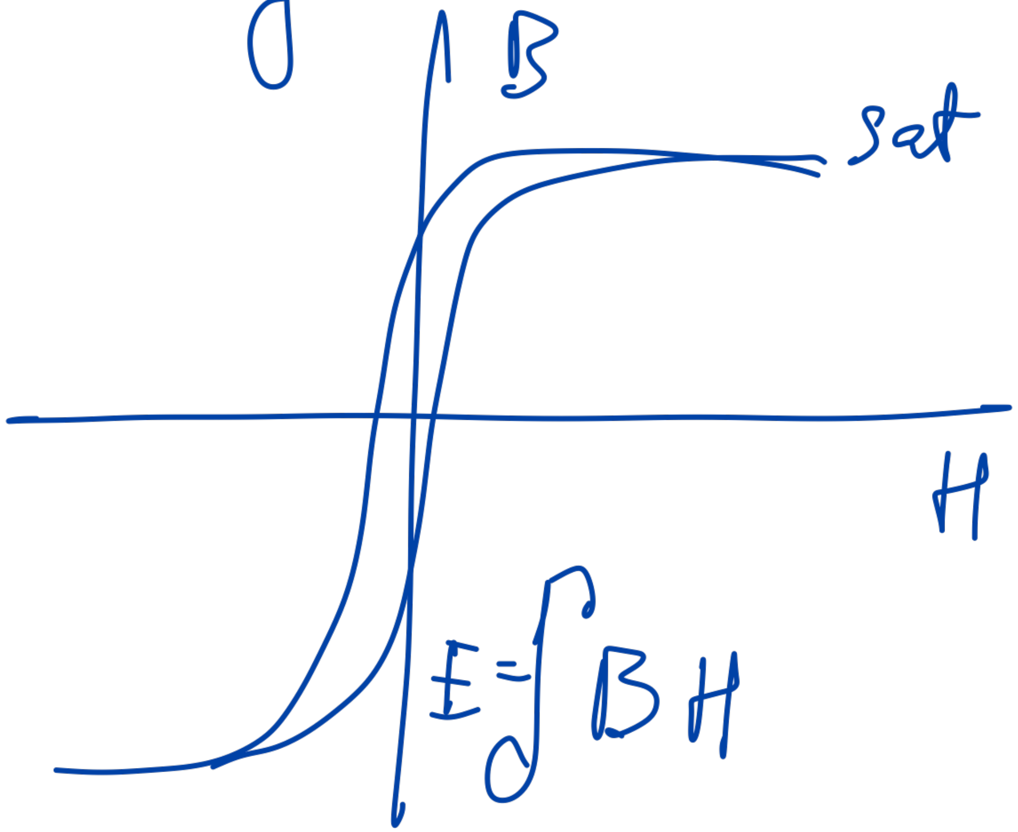
$\int \text{---}\,dt \;\Big|\; \times \frac{1}{i\omega}$

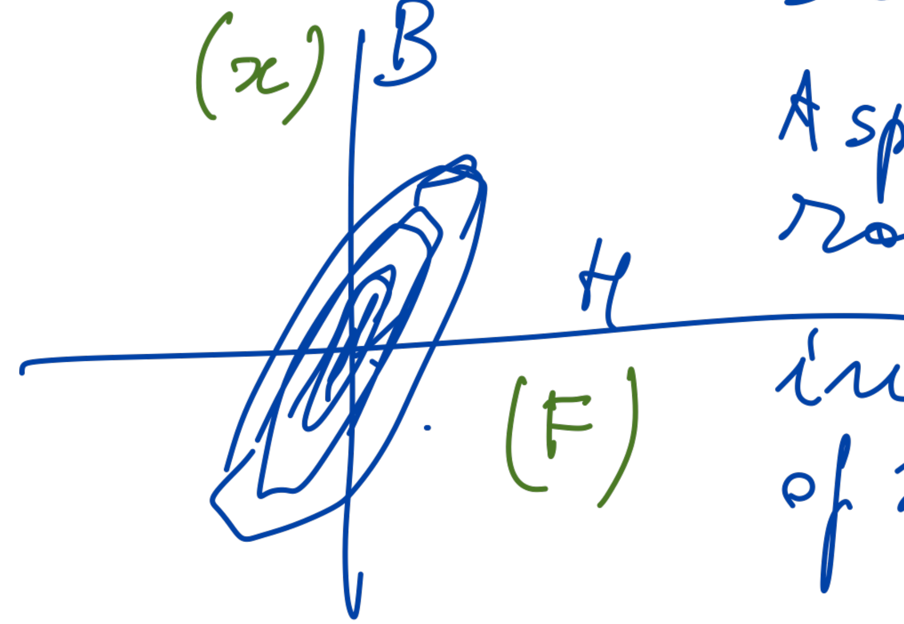
Spectrum $\sim \frac{1}{\omega^2}$

Observed in quite different phenomena

- Nile flooding
- Carbon microphones
- Semiconductors -
- Pulsar
- mechanical size (Fabry-Pérot)
- ferromagnetic materials (Barkhausen noise)
- - - - - - - -

Magnetic materials





small H

Aspect ratio

independent of size

$H_0 \cos(\omega t)$

$E \sim H_0^2$

$P \sim H_0^2\, \omega$

Thermodyn. equilibrium

1-2 T

Weiss

Langevin

$x \qquad X = \int x(t)\, e^{i\omega t}\, dt \qquad \left[\frac{V}{Hz}\right]$

$[V] \qquad [s]$

$S \qquad [V^2/Hz] \qquad [Hz^{-1}]$

$|X|^2 \qquad [V^2/Hz^2]$

$f>0$: energy conservation

$\frac{2}{T}|X|^2 \qquad [V^2/Hz]$

acquisition time

The End