throughout nature, as its surface has low energy and high symmetry. Examples of biologically self-assembled structures include membranes in mitochondria[6] and butterfly wings[7]. In chemistry, the gyroid surface can facilitate catalysis[8], and the triple action of hydrophobic, hydrophilic and amphiphilic molecules often sets up the conditions for gyroids[9], for example, in soap and ketchup. For the photonic crystals considered here, the lack of inversion symmetry ($g(r) \neq g(-r)$) of a single gyroid suggests that the addition of a second gyroid structure is a better starting point (Fig. 1b).

Gyroid and double-gyroid geometries for engineered structures have previously been considered for both charge[10] and light[11] transport. The advance of Lu *et al.* was to introduce symmetry-breaking defects to isolate the Weyl points and lines, thus allowing their unique properties to be tapped while they remain robust to perturbations[1]. Lu *et al.* also characterized the Weyl point behaviour as these defects were introduced. This is often difficult to achieve, as image points with opposite chirality must appear to keep the overall system neutral. Tuning the parameters therefore creates a phase space of singular states with (topological) phase transitions between the different transport regimes.

By itself, the double-gyroid geometry has a three-fold degeneracy between bands three, four and five. This triple degeneracy can be partially lifted by introducing defects (for example, air holes) into the photonic structure. For the correct placement of holes, a line node is created inside an otherwise complete bandgap; the resulting dispersion curve is very flat, which can be useful if the structure is capable of emitting light from its surfaces. In particular, truncation of the crystal can control whether the surface modes are radiative or non-radiative.

A second way of breaking degeneracy is to violate time invariance. As implied by the spin matrices in the Hamiltonian, this can be accomplished by applying a d.c. magnetic field. However, preserving parity invariance limits the possible consequences of the magnetic field. To overcome this, Lu *et al.* removed one of the air holes, thereby breaking both parity and time symmetries[1]. This greatly extended the possibilities because it allowed much more of the 4 × 4 dimensional phase space $[(t, \mathbf{x}) \times (\omega, \mathbf{k})]$ to be explored. At the same time, many propagation modes, such as backscattering[12,13], disappear when parity–time symmetry is violated. By characterizing the lowest-order dynamics, Lu *et al.* showed how pairs of ±1 Weyl points can be created and annihilated depending on the orientation of the magnetic field with respect to the inversion asymmetry. These, in turn, give different classes of radiation and transport states.

As is typical for engineered photonic crystals, the first implementation of Weyl geometries will probably occur for longer wavelengths, primarily because of the availability of gyromagnetic materials at microwave frequences[12]. However, fabrication at optical frequencies should also be possible, for example, using germanium. Although direct construction methods could be used, the possibility of self-assembled structures (as in nature) is particularly exciting.

Equally promising are the potential directions that this research could take as the geometry and materials are engineered further. In terms of basic science, slight modifications to the Dirac/Weyl points give rise to interesting physics: translations in momentum space correspond to gauge fields (vector potentials), whereas changing the angle of band intersection gives rise to effective gravity[3]. In applied science, these mappings give new photonic tools for controlling the confinement, scattering and radiation of light. Beyond this, it will be attractive to exploit the special surface geometry of gyroids across combined disciplines, such as chemistry and optics for enhanced photocatalysis or mechanics and optics for topologically protected strain and rotation sensors. These applications, and many others, will follow as the topology of field theory gets translated to the photonic domain. ❐

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FREQUENCY COMBS

The purest microwave oscillations

A new femtosecond frequency comb scheme is capable of generating microwave signals at a noise level below the shot noise of light.

Enrico Rubiola and Giorgio Santarelli

Despite the fact that time (and equivalently frequency) is the physical quantity that we can measure most accurately, the demand for more precise and spectrally purer electrical oscillators is continually growing. Although optical techniques have a significant part to play in the race towards the ideal oscillator, they have some particularly challenging problems.

The oscillator we consider here is a mode-locked femtosecond laser whose optical signal is converted to a microwave signal by photodiode detection. The spectral purity of a photonic oscillator is determined by its signal-to-noise ratio, which is the ratio of the thermal and shot noise to the microwave power. This measure is proportional to the optical power P in the thermal region, and to $\sqrt{P}$ at high optical power for which the shot noise exceeds the thermal noise of the photodiode output load. An important question is whether it is possible to overcome these noise limits.

Reporting in *Nature Photonics*, Franklyn Quinlan and colleagues have now demonstrated that Poisson statistics does not apply to the sharp light pulses generated by a femtosecond frequency

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comb[1]. Consequently, at high optical power, the jitter of the photodetected microwave oscillation can be lower than a generally agreed limit due to shot noise. Circumventing the classical shot noise therefore results in a net improvement in the microwave spectral purity. Using this principle, Quinlan *et al.* have realized a noise floor for photodetected pulse train timing of 25 zs $Hz^{-1/2}$, which corresponds to a phase noise of –179 dBc Hz^{-1} for a 10-GHz carrier[1]; this is ~5 dB below that predicted by the accepted stationary (time-invariant) shot-noise behaviour.

It is insightful to consider the origin of the terms 'jitter' and 'spectral purity'. For historical reasons, the oscillator spectral purity is described by $L(f) = S_\varphi(f)/2$ and is expressed in decibels relative to the carrier (dBc). The quantity $S_\varphi(f)$ is the power spectral density of the random phase φ as a function of the Fourier (offset) frequency f. Alternatively, we can use the phase-time fluctuation $x = \varphi/2\pi\nu$, where ν is the carrier frequency. The associated power spectral density is given by $S_x(f) = (1/2\pi\nu)^2 S_\varphi(f)$. For example, –180 dBc at 10 GHz carrier gives $\sqrt{S_x} = 2.25 \times 10^{-20}$ s $Hz^{-1/2}$ (Fig. 1).

High-spectral-purity oscillators have a wealth of potential applications. Among them, modern radar is one of the most demanding. The typical carrier frequency for radar is in the 10-GHz band, and the measurement time is determined by the target range and speed. The ability to detect small targets in cluttered and hostile environments depends on the oscillator phase noise. Some particle accelerators require a very low jitter of a few femtoseconds. High spectral purity increases with increasing carrier frequency because the phase-time fluctuation x is proportional to $1/\nu$.

A classical rule states that the additive phase noise is given by $S_\varphi(f) = N/P_\mu$, where P_μ is the microwave carrier power and N is the noise power spectral density, which accounts for shot effect and thermal energy, and includes the noise figure when appropriate. By using high-power photodiodes, Quinlan *et al.* have overcome this limitation by operating their photodiodes in the pulsed shot-noise regime. The underlying principle of their approach is both conceptually simple and technically difficult.

The shot noise is generally treated as a Poisson process; that is, a stream of memory-free statistically independent events. The associated power spectral density is $N_{sh} = 2qIR$, where q is the electron charge, I is the average photocurrent and R is the load resistance (usually 50 Ω). The implicit assumption that the noise is stationary is not valid in this case; rather, the femtosecond comb is a relatively extreme cyclostationary process. The statistics depends strongly on the origin of time, but repeats with the repetition rate ν_R. Instead of having a uniform distribution, the shot noise is clustered around the light peaks. There is no noise during the dead time, when the light is off. The sharp periodic pulses introduce a periodic structure in the noise spectrum, which Quinlan *et al.* have exploited to reduce the phase noise[1].

The pulsed shot-noise regime is based on the wavefunction $|\Psi|^2$ and on its semiclassical interpretation as the detection probability in either space or time. If a photon's time of arrival is affected by an uncertainty σ, which is equal to the pulse half-width (standard deviation), the timing of an m-photon pulse will be a factor of $\sqrt{m}$ more precise. Consider, for example, a stream of 4×10^{17} photons s^{-1} (~8 mW, depending on the wavelength) in 1-ps pulses with a repetition rate of 2 GHz. Because each pulse contains 2×10^8 photons, the total jitter will be 7×10^{-16} s, uniformly distributed from zero to the Nyquist frequency (1 GHz). However, this trivial reasoning hides the need to master a series of cutting-edge technologies (such as an ultrapure Ti:sapphire laser, pulse interleaving and high-power photodiodes), not to mention numerous experimental subtleties.

The generation of very pure microwave signals from optics has been explored by several groups[2–5] using Ti:sapphire femtosecond lasers and erbium-doped fibre femtosecond lasers. Hopefully, in the near future, micrometre-scale resonators will be used to implement a miniature frequency comb[6]. The optoelectronic oscillator[7] is an alternative path for which the classical shot noise applies; this scheme has never achieved a floor lower than approximately –160 dBc.

The femtosecond laser is very stable at $f > 100$ kHz (ref. 8), but its noise increases rapidly with decreasing f. High frequency stabilization in a high-finesse Fabry–Pérot cavity overcomes this limitation[9], and is needed in most practical cases. A state-of-the-art Fabry–Pérot cavity shows a fractional frequency stability approaching 10^{-16} for measurement times in the range 0.1–1 s (refs 10,11). The reported data indicate that the cavity stability is certainly better than –170 dBc at an offset of 1 kHz (10 GHz carrier).

Among microwave oscillators, the sapphire whispering-gallery oscillator offers the highest spectral purity and it is based on reliable technology. At room temperature,

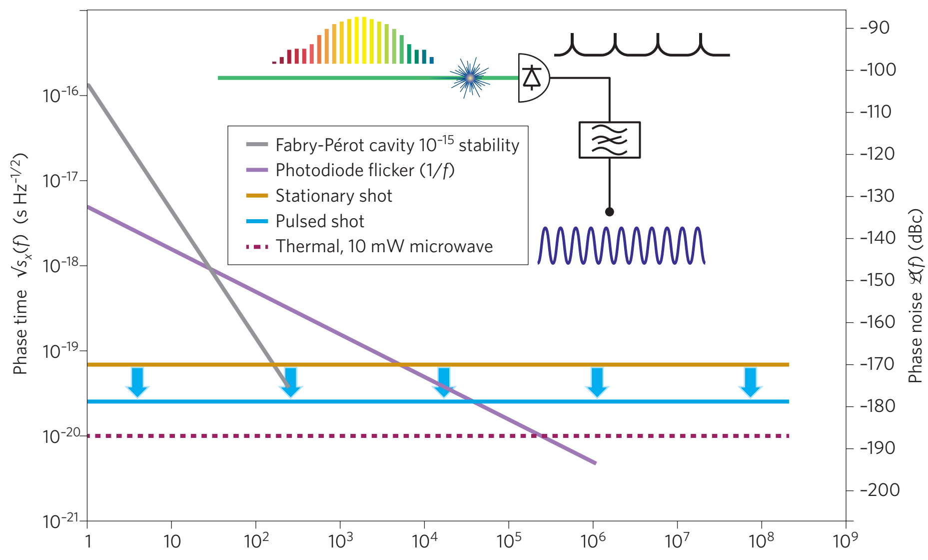

Figure 1 | Power spectral density of random phase for a 10-GHz femtosecond frequency comb detected with a high-power photodiode and expressed in terms of different units. The brown and light-blue lines are the shot noise given by Quinlan *et al.*[1] in the stationary and pulsed shot-noise regimes, respectively. By further reducing the shot noise, the pulsed shot-noise floor could reach the thermal noise (dashed line), unless the microwave power increases proportionally. Flicker (1/*f*) noise is inherent in the detectors; the lowest flicker noise is observed at −133 dBc at 1 Hz (solid purple line). Owing to the photodiode flicker, the full advantages of the pulsed-shot regime are attained only at 100 kHz and beyond. At the lowest frequencies, the flicker-of-frequency noise of the Fabry–Pérot cavity (grey line with slope $1/f^3$) may limit the spectral purity.

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a 10-GHz oscillator has a phase noise of −160 dBc (2.2×10^{-19} s $Hz^{-1/2}$) at 1 kHz offset (ref. 12). The liquid-helium version[13] is of great interest because it has a narrower resonator linewidth (<100 kHz at 300 K to a few hertz at 5 K). Both the room-temperature and cryogenic devices are mature, fully engineered and reliable oscillators[14]. The pulsed shot-noise regime realized by Quinlan *et al.* challenges sapphire-based technology.

In principle, the narrow optical pulses and wide optical bandwidth required by the femtosecond mechanism will give an extremely low phase noise that is much lower than any practical measurement capability. On the other hand, the duration of the microwave pulses is determined by the photodiode and the microwave bandwidth. Such electrical pulses are several orders of magnitude longer than the optical pulses and are subject to phase fluctuations. Moreover, experience indicates that the amplitude noise is difficult to control and affects the phase noise.

The optical comb enables a phase noise that is significantly lower than the experimental spectra. The floor given by the ratio of the thermal noise to the microwave power gives a lowest bound for the phase noise. At sufficiently high optical powers, the shot noise in the detector will exceed the thermal noise. The experimental discovery of the pulsed shot-noise regime by Quinlan *et al.*[1] provides insights into the detection mechanism and opens the way to improve the spectral purity of oscillators. On the other hand, this regime is difficult to manage and requires photodiodes capable of operating in the linear regime at high peak powers of over 100 mW. Further progress depends on advances in semiconductor technology. Finally, the best available photodiodes are affected by $1/f$ phase noise, which is about −133 dBc at an offset of 1 Hz. This is quite a difficult technical problem. Microwave amplifiers and other active devices that can be used at the photodiode output are affected by an even higher $1/f$ phase noise. Thus, the $1/f$ noise represents a technical problem that can be overcome only by efforts in the fields of optics, semiconductor technology and microwave engineering. ❐

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ANDERSON LOCALIZATION

The role of quantum symmetries

Experiments with a quantum simulator made from an integrated optical circuit reveal that bosons and fermions react to disorder in different ways, experiencing different strengths of Anderson localization.

Andrea Fratalocchi

Fifty years after the discovery of Anderson localization, the influence of disorder continues to surprise. In 1958, the US physicist P. W. Anderson discovered the phenomenon of disorder-induced localization while pursuing a general theory of electron transport[1]. Anderson predicted for the first time that disorder can arrest the transport of energy in random systems through the generation of spatially localized wavefunctions that trap electrons. As is sometimes the case after a great discovery, very few people were fully convinced of the initial prediction and its consequences, including Anderson himself[2]. It took about 20 years for the scientific community to recognize the universal character of Anderson's work; he was jointly awarded the Nobel Prize in physics in 1977. The importance of Anderson localization extends beyond quantum mechanics as it is a ubiquitous state for waves, which also manifests itself in classical media[3–7].

Anderson localization of light is being intensively investigated in the field of photonics. Besides the many implications at the level of fundamental understanding[3], Anderson localization has excited considerable interest as it has the potential to confine energy in space and time, which could act as a precursor to important phenomena such as random lasing effects[4]. Of the various challenges initiated by Anderson, the role of quantum symmetries in inducing localized states represents an important, open problem. Physicists have always been fascinated with symmetries, and its study has fostered some great leaps in science.

Writing in *Nature Photonics*, Paolo Mataloni and co-workers report a series of new significant discoveries in the field made using an ingenious quantum simulator for studying this problem[8]. Figure 1a shows a schematic of the simulator. It consists of an integrated optical circuit made by a cascade of beamsplitters arranged in a network of Mach–Zender interferometers that have controllable phase delays. The simulator is designed to mimic the dynamics of a quantum random walk for non-interacting particles with different bosonic/fermionic symmetries. A quantum random walk generalizes the concept of a classical random walk, in which the path of motion is broken into a series of steps and the direction taken at each step is determined by the flip of a coin. The quantum variant includes interference phenomena that occur at the quantum level. Such interference yields a non-classical behaviour, which can be exploited to implement different operations, ranging from quantum computing to exponentially fast search algorithms[9].

In the integrated system realized by Mataloni *et al.*, photons perform a quantum walk by moving in space in different arms of the various Mach–Zender interferometers, and by moving in time by

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