Phase Noise in RF and Microwave Amplifiers

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***Abstract*—Understanding amplifier phase noise is a critical issue in many fields of engineering and physics, such as oscillators, frequency synthesis, telecommunication, radar, and spectroscopy; in the emerging domain of microwave photonics; and in exotic fields, such as radio astronomy, particle accelerators, etc.**

Focusing on the two main types of base noise in amplifiers, white and flicker, the power spectral density of the random phase $\varphi(t)$ is $S_\varphi(f) = b_0 + b_{-1}/f$. White phase noise results from adding white noise to the RF spectrum in the carrier region. For a given RF noise level, b_0 is proportional to the reciprocal of the carrier power P_0. By contrast, flicker results from a near-dc $1/f$ noise—present in all electronic devices—which modulates the carrier through some parametric effect in the semiconductor. Thus, b_{-1} is a parameter of the amplifier, constant in a wide range of P_0. The consequences are the following: Connecting m equal amplifiers in parallel, b_{-1} is $1/m$ times that of one device. Cascading m equal amplifiers, b_{-1} is m times that of one amplifier. Recirculating the signal in an amplifier so that the gain increases by a power of m (a factor of m in decibels) as a result of positive feedback (regeneration), we find that b_{-1} is m^2 times that of the amplifier alone. The feedforward amplifier exhibits extremely low b_{-1} because the carrier is ideally nulled at the input of its internal error amplifier.

Starting with an extensive review of the literature, this article introduces a system-oriented model which describes the phase flickering. Several amplifier architectures (cascaded, parallel, etc.) are analyzed systematically, deriving the phase noise from the general model. There follow numerous measurements of amplifiers using different technologies, including some old samples, and in a wide frequency range (HF to microwaves), which validate the theory. In turn, theory and results provide design guidelines and give suggestions for CAD and simulation.

To conclude, this article is intended as a tutorial, a review, and a systematic treatise on the subject, supported by extensive experiments.

I. Introduction

Low-phase-noise amplification is crucial in a variety of applications. In the oscillator, the phase noise of the sustaining amplifier is converted into frequency noise via the Leeson effect [1]–[5]. Hence the oscillator phase fluctuation, which is the integral of frequency, diverges in the long run. The impact of the amplifier $1/f$ noise on the oscillator stability is investigated in numerous articles, mostly from the experimental standpoint. See, for example, [6]–[13]. In turn, the oscillator noise affects the bit error rate and security in communications [12], [14], [15], and radar [16], [17]. Doppler and chirp radars require ultra-low phase noise to avoid having the oscillator noise sidebands exceed the echo signal. Low-phase-noise amplification is also important in precise synchronization systems because phase represents time. Finally, the books [18] and [19] provide useful overview, although they are not up to date.

Near-dc $1/f$ noise, discovered in the 1930s [20], is clearly a ubiquitous phenomenon [21], [22]. However, no generally-agreed unification is available. Most models for electronic components resort to two original articles, [23] and [24] (see also [25]). Phase flickering can only originate from near-dc $1/f$ noise brought to the vicinity of the carrier. This occurs because in the absence of a carrier, the noise at the amplifier output is nearly white. Because the near-dc flicker is generally stationary, $1/f$ phase noise is cyclostationary.

The problem with nonlinear noise modeling is that the models rely on the identification of the near-dc noise sources, which can, in turn, be nonlinear or associated with a nonlinear circuit element [13], [26], [27]. Because the conversion of near-dc noise into phase noise is generally not implemented in CAD programs, the simulation may require dedicated software. Although these models are not a perfect representation of the device physics, some of them provide results in quite reasonable agreement with the measured phase noise [27]–[29]. Some theoretical models, supported by experiments, provide useful information about amplifier $1/f$ phase noise for several technologies [7]–[9], [12], [30]–[42] and specific schemes [43]–[47]. Conversely, more accurate semiconductor-physics approaches, such as [48] and the related microscopic models, are complex and difficult to use. Additionally, some valuable measurements of commercial amplifiers are available: for example, [12], [35], [49], [50].

To conclude, the amplifier phase noise is more or less understood, but the information is scattered in many articles. By contrast, little information is available about the consequences of these mechanisms, or about more complex amplifier architectures. This article is intended to fill this gap, providing systematic treatment, insight, practical knowledge, design rules, and extensive experimental confirmation.

II. Phase Noise Mechanisms

Fig. 1 presents a rather general overview of noise in amplifiers, suggested by experience and physical insight. In this article, we restrict our attention to white and flicker noise because, among the noise types originating from *inside* the amplifier, white and flicker are those responsible

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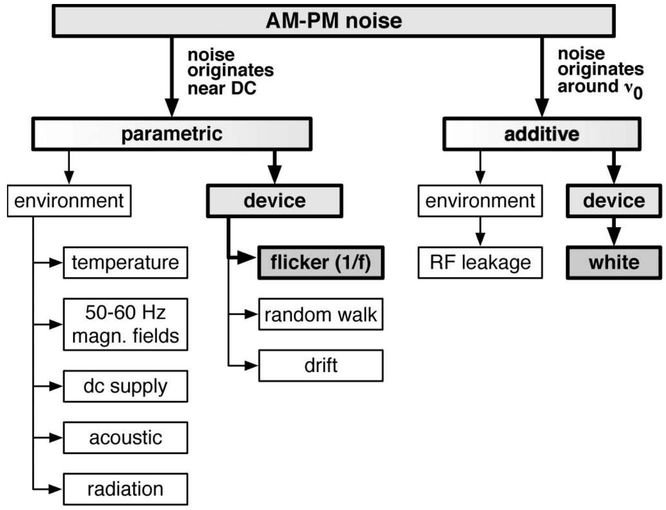

Fig. 1. Amplifier phase noise mechanisms.

for short-term phase noise. Therefore, the phase noise spectrum is completely described by the first two terms of the polynomial law

$$S_\varphi(f) = b_0 + \frac{b_{-1}}{f} \quad [\mathrm{rad}^2/\mathrm{Hz}]. \tag{1}$$

The white phase noise b_0 is derived by adding to the carrier a random noise of power spectral density $N = FkT_0$, where k is the Boltzmann constant and F is the amplifier noise figure defined at the reference temperature $T_0 =$ 290K (17°C). It is useful to have on hand the following numerical value:

$$kT_0 = 4 \times 10^{-21}\ \mathrm{J} \quad (-174\ \mathrm{dBm/Hz}).$$

In modern low-noise amplifiers, F is typically of 0.5 to 2 dB. It may depend on bandwidth, on the loss of the input impedance-matching network, and on technology. If the actual temperature is not close enough to T_0, the quantity F is meaningless. In this case, the noise is described by $N = kT_\mathrm{e}$, where T_e is the equivalent noise temperature, which includes the amplifier and its input termination. We assume that N is independent of frequency in a wide range around the carrier frequency ν_0, as is true in most practical cases.

Adding N to a carrier of power P_0 results in random phase modulation of power spectral density

$$b_0 = \frac{FkT_0}{P_0}. \tag{2}$$

Eq. (2) holds true in the linear region of the amplifier. If the amplifier is operated in the large-signal regime, where it is nonlinear or saturated, F may increase [51], [52].

At low frequencies, the amplifier phase noise is of the $1/f$ type, which currently referred to as flicker. Near-dc flicker noise takes place at the microscopic scale [23], [24], and little or no correlation is expected between different regions of the device. This is supported by the fact that the probability density function is normal [53]. Such a distribution originates from the central-limit theorem in the presence of a large population of independent phenomena.

Understanding phase flickering in amplifiers starts from the simple fact that noise is white in the absence of a carrier. Besides the experimental evidence, the heuristic proof given by Nyquist [54] for thermal noise is also convincing after introducing the noise figure F, which is not necessarily a thermal phenomenon. Close-in noise shows up only when the carrier is sent at the input. This means that phase flickering can only originate from up-conversion of the near-dc $1/f$ noise, as shown in Fig. 2(a). The noise up-conversion can be described as follows. We denote with $u(t) = U_0 e^{j2\pi\nu_0 t} + n'(t) + jn''(t)$ the input signal, where $U_0 e^{j2\pi\nu_0 t}$ is the true (accessible) input and $n = n' + jn''$ is the near-dc equivalent noise at the amplifier input; and with $v(t) = a_1 u(t) + a_2 u^2(t) +$ noise as the output signal. The near-dc noise $n(t)$ is *not* the random signal that would ideally be measured with an oscilloscope. Instead, it is an abstract quantity with spectrum proportional to $1/f$ that accounts for the parametric nature of flicker. The amplifier is described as a (smooth) nonlinear function truncated at the second order, where the coefficient a_1 is the (usual) voltage gain denoted A elsewhere in this article. Expanding $v(t)$ and selecting only the $2\pi\nu_0$ terms, we get

$$v(t) = a_1 U_0 e^{j2\pi\nu_0 t} + 2a_2[n' + jn'']U_0 e^{j2\pi\nu_0 t}, \tag{3}$$

from which

$$\alpha(t) = 2\frac{a_2}{a_1} n'(t) S_\alpha(f) = 4\frac{a_2^2}{a_1^2} S_{n'}(f) \tag{4}$$

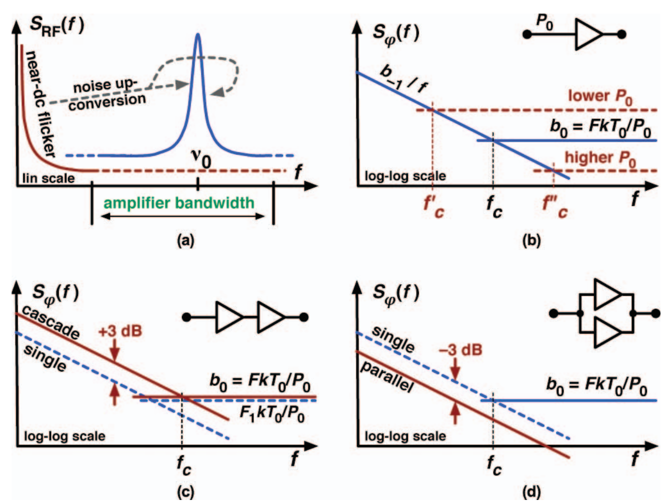

Fig. 2. Phase noise rules for several amplifier topologies. (a) Noise up-conversion from near-dc the carrier frequency, which originates $1/f$ phase noise; (b) single amplifier; (c) cascaded amplifiers; (d) parallel amplifiers. Note that (a) is a radio-frequency/microwave power spectral density, as seen by a classical spectrum analyzer, whereas (b)–(d) are the power spectral densities of the random phase fluctuation $\varphi(t)$.

$$\varphi(t) = 2\frac{a_2}{a_1}n''(t)S_\varphi(f) = 4\frac{a_2^2}{a_1^2}S_{n''}(f). \tag{5}$$

Eqs. (3), (4), and (5) express the simple fact that the noise sidebands are proportional to the carrier amplitude and, therefore, AM and PM noise are independent of the carrier amplitude or power. In this representation, we use the nonlinearity, present in virtually all devices, to transpose the random signal $n(t)$. Of course, a fully parametric model yields the same results, at a cost of heavier formalism.

Experiments show that b_{-1} is almost independent of the carrier power [29], [30], [55], [56] if the amplifier operates in the linear regime or in mild compression. The quasi-static perturbation technique provides fairly good agreement between simulated and experimental $1/f$ phase noise data in silicon and SiGe amplifiers [28]. Other investigations describe the $1/f$ phase noise as a modulation from the near-dc $1/f$ current fluctuation in microwave HBT amplifiers [32] and in InGaP/GaAs HBTs [57]. The analysis of the literature cited indicates that, regardless of the theoretical approach and of the amplifier technology, the amplifier *behavior* is that of a linear phase modulator driven by a near-dc process

$$b_{-1} = C \quad \text{(constant, independent of } P_0\text{)}. \tag{6}$$

Neither the near-dc noise nor the modulation efficiency is affected by the carrier power, unless the amplifier is pushed in strong compression. If this happens, the dc bias changes. In turn, small changes of b_{-1} are expected in an unpredictable way. Our experiments, detailed in Section IV confirm this behavioral model.

III. Analysis and Design Rules

A. Single Amplifier

The typical phase-noise pattern found in amplifiers is shown in Fig. 2(b). An amazing fact comes immediately from (2) and (6): the corner frequency is given by

$$f_c = \frac{b_{-1}}{FkT_0}P_0. \tag{7}$$

This fact has been successfully used to reverse-engineer the oscillators from their noise, identifying some relevant parameters, such as the resonator Q and driving power [2, ch. 6], [58].

The flicker corner frequency f_c sometimes found in the amplifier specifications is misleading because it is presented as a parameter of the amplifier, as it was rather constant, at least in the normal operating range. In SPICE and in some other CAD programs, the flicker is described by f_c, introduced as a fixed parameter in the device model. This is an unfortunate choice for the same reason. Replacing the parameter f_c with (7) would result in improved usability.

B. Cascaded Amplifiers

When several amplifiers are cascaded [Fig. 2(c)], the noise figure of the chain is given by the Friis formula [59]:

$$F = F_1 + \frac{F_2 - 1}{A_1^2} + \frac{F_3 - 1}{A_1^2A_2^2} + \frac{F_4 - 1}{A_1^2A_2^2A_3^2} + \cdots, \tag{8}$$

where A is the voltage gain. The Friis formula expresses the fact that the noise of the first stage is F_1kT_0, including the input termination, and the noise $(F_i - 1)kT_0$ of the ith stage ($i \geq 2$) is referred to the input after dividing by the power gain of the $i-1$ preceding stages. By virtue of (2), the obvious extension of the Friis formula to phase noise is

$$b_0 = \left[F_1 + \frac{F_2 - 1}{A_1^2} + \frac{F_3 - 1}{A_1^2A_2^2} + \frac{F_4 - 1}{A_1^2A_2^2A_3^2} + \cdots\right]\frac{kT_0}{P_0}. \tag{9}$$

In most practical cases, the noise of the chain is chiefly determined by the noise of the first stage. This applies to the RF spectrum, and also to the phase noise spectrum.

By contrast, the flicker phase noise is ruled by (6). Because the amplifier $1/f$ phase noise processes in different devices are statistically independent and also independent of the carrier power, the $1/f$ noise of a chain of m amplifiers is

$$b_{-1} = \sum_{i=1}^{m}(b_{-1})_i. \tag{10}$$

Cascading two (three) equal amplifiers, the phase flicker is 3 dB (4.8 dB) higher than that of the single amplifier.

Combining white noise (9) and flicker noise (10), we find the spectrum shown in Fig. 2(c).

C. Parallel Amplifiers

A *parallel amplifier* (PA) as an amplifier network in which m amplifier cells of the same gain equally share the burden of delivering the desired output power. Several configurations are possible. The push-pull configuration uses 180° junctions, which suppresses the even-order harmonic distortion, appreciated in audio applications. The balanced amplifier [60] uses 90° junctions to improve input and output impedance matching. The distributed amplifier [60], preferred when a wide frequency range is to be achieved at any cost, uses a series of taps in a delay line to put the cells to work.

For the sake of analysis simplification, we assume that

- the cells are equal, and have voltage gain A, input and output impedance R_0, and noise figure F, and
- the input power-splitter and the output power-combiner are loss-free[1] and impedance matched to R_0.

[1] In the case of the distributed amplifiers, it is conceptually impossible that all cells handle the same power. However, this hypothesis helps to understand the analysis.

Accordingly, the gain is equal to the gain A of a cell, and the compression power is m times the compression power of one cell.

Denoting with P_0 the power at the PA input, the input power of each cell is P_0/m. Consequently, the white phase noise is

$$(b_0)_{\text{cell}} = \frac{FkT_0}{P_0/m}$$

at the output of each cell, and

$$b_0 = \frac{FkT_0}{P_0} \tag{11}$$

at the output of the parallel amplifier, after adding m independent signals of equal power and the same statistical properties. This also means that the noise figure of the parallel amplifier is equal to the noise figure F of one cell.

The flicker noise of one cell can be derived from the $1/f$ component of (5):

$$(b_{-1})_{\text{cell}} = 4\frac{a_2^2}{a_1^2}[S_{n''}(f)]_{\text{flicker}}.$$

Therefore, combining m statistically-independent signals of equal power and same statistical properties gives

$$b_{-1} = \frac{1}{m}(b_{-1})_{\text{cell}}. \tag{12}$$

The important conclusion is that the parallel configuration features a flicker-noise reduction of a factor m, or $\log_2(m) \times 3$ dB, assuming perfect symmetry and no dissipative losses in the splitter/combiner networks. This is shown in Fig. 2(d). In practice, a noise reduction of 2.5 dB per factor-of-two is expected. Similar architectures have already been employed to reduce flicker phase noise of photodiodes by connecting several units in parallel [61], [62]. Conversely, general theory states that white noise cannot be improved in this way. In practice, the loss of the input power-splitter increases the noise figure, and thus b_0.

D. The Regenerative Amplifier

The regenerative amplifier (RA) is an amplifier in which positive feedback (regeneration) is used to increase the gain, as shown in Fig. 3. Interestingly, this technique is much known better in optics than in radio engineering. A sub-threshold laser is a common example of an optical regenerative amplifier.

Denoting with A_0 the voltage gain of the simple amplifier and with β the gain of the feedback path, elementary feedback theory suggests that the regenerative-amplifier gain is

$$A = \frac{A_0}{1 - A_0\beta}. \tag{13}$$

As an analogy with m cascaded amplifiers, we can also write

$$A_0^m \quad \text{with} \quad \beta = \frac{A_0^{m-1} - 1}{A_0^m}. \tag{14}$$

Of course, there is no reason to restrict this representation to integer m.

On closer examination, one should introduce coupling coefficients κ_{i} and κ_{o} of the input and output couplers, and also the dissipative losses. The effect of the coefficient κ is an intrinsic power loss $1 - \kappa^2$ if the coupler, for the regenerative-amplifier gain, is reduced by a factor $\sqrt{(1-\kappa_{\text{i}}^2)(1-\kappa_{\text{o}}^2)}$. The small effect of the coupler losses will be neglected in the rest of this section.

It is wise to adjust the phase for the roundtrip gain $A_0\beta$ to be real, hence G is real. This ensures that $0 < A_0\beta < 1$. The condition $A_0\beta > 0$ means that the feedback is positive, whereas $A_0\beta < 1$ is necessary to keep the loop gain below the oscillation threshold.

The equivalent noise temperature is the noise temperature of the internal amplifier referred to the RA input. This is the temperature of the internal amplifier increased by the loss of the input coupler. The detailed analytical proof given in [63] for the Q-multiplier (which is an application of the regenerative amplifier in which a resonator is inserted in the feedback) holds true for the regenerative amplifier in the general case. The consequence is that the regenerative-amplifier white noise is

$$b_0 = \frac{FkT_0}{P_0} + \text{losses}. \tag{15}$$

The flicker noise is best understood by replacing the gain A_0 with $A_0e^{j\psi}$, where $\psi(t)$ is the instantaneous value of the internal-amplifier noise. In practical design, the flicker of phase shows up at low frequencies—at least a factor of 10^2 lower than the inverse of the roundtrip time. In these conditions, the signal circulating in the loop sees a quasi-static phase ψ and, hence, the gain can be written as

$$A = \frac{A_0e^{j\psi}}{1 - A_0\beta e^{j\psi}} \tag{16}$$

and expanded using $e^x = 1 + x$ for low noise:

$$A = \frac{A_0}{1 - A_0\beta}\left[1 + j\frac{1}{1 - A_0\beta}\psi\right]. \tag{17}$$

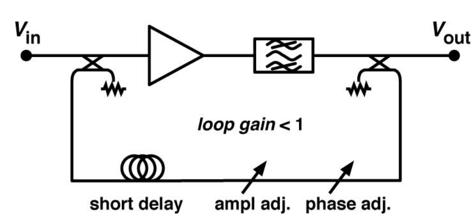

Fig. 3. Regenerative (positive-feedback) amplifier.

Accordingly, the RA phase noise is

$$\varphi(t) = \frac{1}{1 - A_0\beta}\,\psi(t) \tag{18}$$

$$(b_{-1})_{\mathrm{RA}} = \left[\frac{1}{1 - A_0\beta}\right]^2 (b_{-1})_{\mathrm{ampli}}, \tag{19}$$

which, after (14), is equivalent to

$$(b_{-1})_{\mathrm{RA}} = m^2 (b_{-1})_{\mathrm{ampli}}. \tag{20}$$

It is instructive to compare the $1/f$ of a cascade of m amplifiers to that of an RA. The comparison makes sense only if the two configurations use the same type of amplifier and have the same gain. The latter condition sets the value of β. It follows from (10) that the flicker of the cascade is

$$(b_{-1})_{\mathrm{chain}} = m (b_{-1})_{\mathrm{ampli}}, \tag{21}$$

thus

$$(b_{-1})_{\mathrm{RA}} = m (b_{-1})_{\mathrm{chain}}. \tag{22}$$

However counterintuitive, this conclusion not a surprise because the carrier is phase-shifted by m independent random processes in the cascade, whereas in the RA, it is shifted m times by the same slow process.

E. The Virtues of the Error Amplifier

A side effect of (6) is that the amplifier noise sidebands are proportional to the carrier. Because the amplifier $1/f$ noise sidebands are proportional to the carrier, an error amplifier that receives the null signal of a bridge is virtually free from close-in flicker.

The feedforward amplifier (Fig. 4, and [64]) is based on the idea that a low distortion is achieved by introducing an error amplifier that processes only the error of the power amplifier, which is a small signal. For the same reasons, the feedforward amplifier also exhibits low $1/f$ phase noise (see [65], and [66] for a review).

Our noise-measurement system of Fig. 5(b)—further described in Section IV—exploits the fact that the error amplifier (the microwave amplifier at the mixer input) cannot up-convert the near-dc $1/f$ noise if the carrier is suppressed at its input. This is the main point in [67], although at that time the relevance was, unfortunately, not made sufficiently clear. More precisely, the contribution of the error amplifier to the background b_{-1} is divided by the carrier rejection ratio, that is approximately the DUT power divided by the residual carrier at the input of the error amplifier. This ratio can be 60 to 100 dB.

The same idea can be used for the reduction of the oscillator $1/f$ frequency noise [68], [69]. In this case, the device under test (DUT) is an amplifier shared by the noise-measurement system and by an external circuit, and the DUT carrier is nulled in a closed loop. The $1/f$ noise

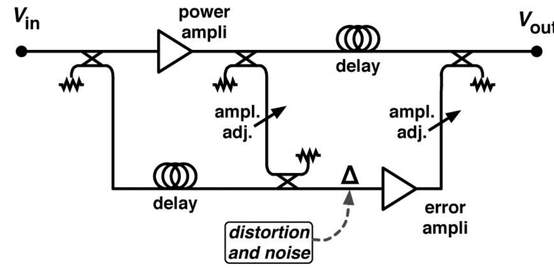

Fig. 4. Feedforward amplifier.

is limited by the background of the noise-measurement system.

F. The Effect of Physical Size

Physical insight suggests that the flicker coefficient b_{-1} is proportional to the inverse of the volume of the amplifier active region. This can be seen through a gedankenexperiment in which we set up an m-cell parallel amplifier, whose flicker is $b_{-1} = 1/m(b_{-1})_{\mathrm{cell}}$ [see (12)]. Then we join the m cells forming a single large device, trusting the fact that flicker is of microscopic origin and that the elementary volumes are uncorrelated. This assumption is supported by the fact that the sum of a large number of independent processes by virtue of the central-limit theorem yields a Gaussian distribution, which is generally observed. Moreover, the variety of flicker models for specific cases share the fact that flicker is of microscopic origin.

Our inverse-volume law must be used with prudence. First, for a given volume, flicker depends on technology. Second, the volume law certainly breaks down at the nanoscale, where the size is smaller than the coherence length of the flicker phenomenon and the elementary volumes are no longer independent; and likely also at large scale. Nonetheless, the inverse-volume law is a useful design guideline.

IV. Experimental Proof

A. Measurement Method

Two different schemes, shown in Fig. 5, have been used to measure the amplifier phase noise, depending on needs.

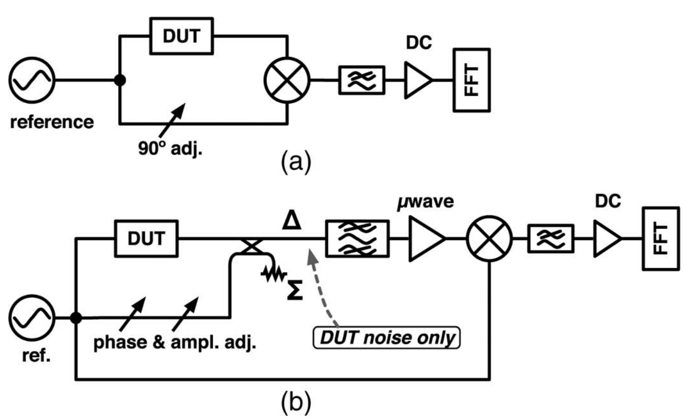

Fig. 5. Phase noise measurement methods: (a) saturated mixer and (b) low-flicker carrier-suppression scheme.

The scheme A is that of commercial phase-noise measurement systems. A Schottky-diode double-balanced mixer saturated at both inputs with 7 to 10 dBm driving power is used as the phase detector. The two inputs are to be in quadrature. In this condition, the mixer converts the phase difference φ into a voltage $V = k_\mathrm{d}\varphi$ with a typical conversion factor of 100 to 500 mV/rad. The mixer output is low-pass filtered, amplified, and sent to the fast Fourier transform (FFT) analyzer. The background $1/f$ noise is chiefly due to the mixer. Typical values are of -140 dB·rad^2/Hz for RF mixers and -120 dB·rad^2/Hz for microwave mixers. The white part of the background noise is generally due to the dc-amplifier (1.5 nV/$\sqrt{\mathrm{Hz}}$) referred to the mixer input. Values of -155 to -170 dB·rad^2/Hz are common in average or good experimental conditions. At the mixer output, we used the amplifier described in [71]. The key feature of this amplifier is that it is designed to have the lowest flicker when connected to a 50-Ω source, so it helps to keep the $1/f$ background noise low.

The detector shown in Fig. 5(b) exhibits the lowest background noise. This is typically needed for the measurement of SiGe amplifiers, whose low flicker can be similar or lower than that of Fig. 5(a). This detector, well known in the literature [67], [72]–[74], works as a Wheatstone bridge followed by a microwave amplifier and a synchronous detector. Because all of the DUT noise is contained in the sidebands, low $1/f$ background is achieved by suppressing the carrier at the input of the microwave amplifier. The latter amplifies only the DUT noise sidebands, which are low-power signals, so that virtually no flicker up-conversion takes place. Microwave amplification before down-conversion to baseband has the additional advantages of low white-noise background, and of reduced 50 to 60 Hz spurs. This happens because the dc amplifiers take in low-frequency magnetic fields, whereas microwave amplifiers do not. Neglecting dissipative losses, the white-noise background is

$$(b_0)_\mathrm{bg} = \frac{2FkT_0}{P_\mathrm{hyb}}, \tag{23}$$

where F is the noise figure of the microwave amplifier, P_hyb is the microwave power at the inputs of the hybrid junction, and the factor 2 is the junction intrinsic loss. The value of -185 dB·rad^2/Hz is easily achieved at 15 dBm power level. The $1/f$ background is not limited by necessary and known factors. We obtained $(b_{-1})_\mathrm{bg} = -150$ dB·rad^2/Hz in the very first experiments [73], and $(b_{-1})_\mathrm{bg} = -180$ dB·rad^2/Hz with a series of tricks [74]. The phase-to-voltage gain can be 40 dB higher than that of the saturated mixer. Interestingly, the scheme of Fig. 5(b) can be built around a commercial instrument [Fig. 5(a)], reusing the mixer, dc amplifier, FFT, and data acquisition system. The only problem with Fig. 5(b) is that the carrier suppression must be adjusted manually, which may take patience and experimental skill. Often, some parts must be replaced when the carrier frequency is changed.

B. Experimental Results

We measured the amplifiers listed in Table I. All are commercial products but the LPNT32, which was designed and implemented at the Laboratoire d'Analyse et d'Architecture des Systèmes (LAAS), Toulouse [28]. We believe that the AML812PNB1901 and the AML812PNB2401, claimed to be ultra-low noise units by Microsemi-RFIS (formerly AML, Camarillo, CA), are actually parallel amplifiers; there is a series of 5 AML amplifiers with dc bias current in powers of 2, from 0.1 to 1.6 A, and output power proportional to the dc bias. Interestingly, b_{-1} scales down by almost 3 dB per factor-of-two increase in the dc bias [2, ch. 2]. Our measurements are intended to determine the coefficient b_{-1}, and to experimentally confirm the behavioral rules stated in Section III. The results are given as a series of spectra discussed subsequently. Additionally, b_{-1} is reported in Table I.

White phase noise, though understood in the literature, is a necessary complement to this work and a sanity check for the results.

1) Phase Noise of a Single Amplifier:

The first experiment is the simple measurement of the phase noise of several microwave amplifiers at different values of input power (Fig. 6). It is clearly seen on all spectra that b_{-1} is independent of power. The fact that b_{-1} is constant versus power holds for different technologies, and in the moderate compression regime. This confirms the parametric nature of flicker and validates the main point of the behavioral model.

TABLE I. RF and Microwave Amplifiers Tested.

Amplifier	Frequency (GHz)	Gain (dB)	P_1dB (dBm)	F (dB)	DC bias	b_{-1} (meas.) (dB·rad^2/Hz)
AML812PNB1901	8–12	22	17	7	15 V, 425 mA	−122
AML412L2001	4–12	20	10	2.5	15 V, 100 mA	−112.5
AML612L2201	6–12	22	10	2	15 V, 100 mA	−115.5
AML812PNB2401	8–12	24	26	7	15 V, 1.1A	−119
AFS6	8–12	44	16	1.2	15 V, 171 mA	−105
JS2	8–12	17.5	13.5	1.3	15 V, 92 mA	−106
SiGe LPNT32	3.5	13	11	1	2 V, 10 mA	−130
Avantek UC573	0.01–0.5	14.5	13	3.5	15 V, 100 mA	−141.5

In Figs. 6(a) and 6(b), the white noise b_0 follows exactly the $1/P_0$ law (2). The white phase noise cannot be observed in Figs. 6(c) and 6(d) because the frequency span of our FFT analyzer is insufficient. In Fig. 6(e), the white noise b_0 follows exactly the $1/P_0$ law up to -30 dBm input power. At -25 dBm (dark green curve in the online version), we observe that between 100 Hz and 10 kHz, the noise is higher than the flicker we expected from the general rules stated. This is likely the consequence of saturation in an intermediate stage.

The AML812PNB1901 and LPNT32 amplifiers [Figs. 6(a) and 6(b)] are intended for low-phase-noise applications and for high-spectral-purity oscillators [28], [13], [70]. These amplifiers exhibit $b_{-1} < 120$ dB·rad^2/Hz. The white noise shown, though remarkably low, is the noise predicted by (2).

The power efficiency (output power divided by dc-bias power) is 50% for the LPNT32 (LAAS laboratory design [28]), and 0.5% to 2.5% for the commercial amplifiers. This indicates that low-flicker design is not incompatible with efficiency.

Our experience indicates that the flicker of a given amplifier does not change significantly in the frequency range of interest. Because this fact is observed all the time, we

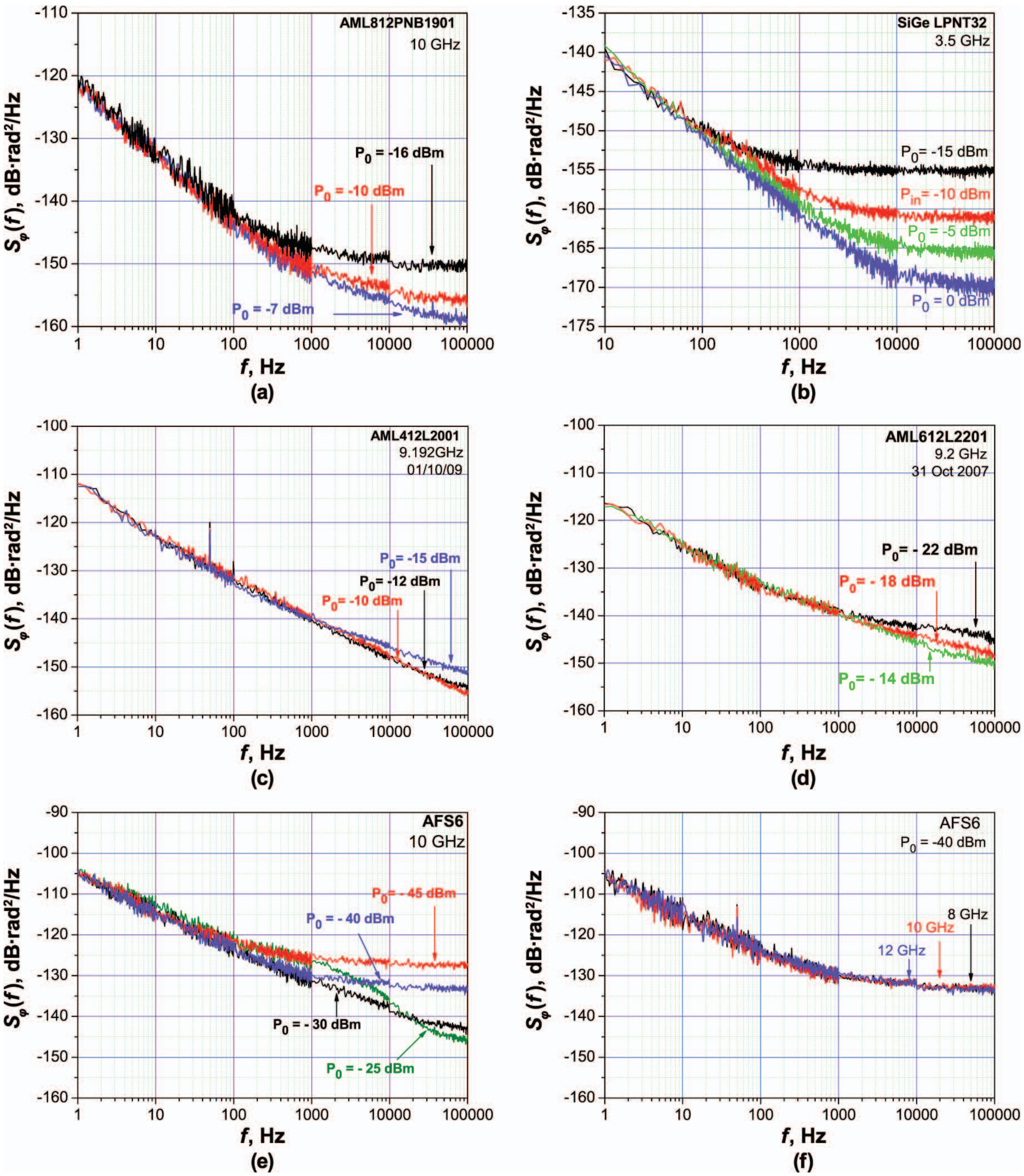

Fig. 6. Phase noise of some amplifiers, measured at different input power and frequency. The plot in (b) was measured at the Laboratoire d'Analyse et d'Architecture des Systèmes (LAAS, Toulouse, France) using the system described in [51], and first made available in [70, Fig. 3.16].

did not repeat the test systematically and we show only one case in Fig. 6(f).

2) Cascaded Amplifiers:

In a second experiment, we checked on the rule of cascaded amplifiers versus (10) by connecting 2 to 3 equal units. We did not insert attenuators in the chain. The consequence is that the input power must be scaled down proportionally to the total gain for the output to be kept in the linear or moderate-compression region. Still, impedance matching is improved with microwave isolators. The noise spectra are shown in Fig. 7.

Fig. 7(a) shows the phase noise of a chain consisting of 1 to 3 UTC573 amplifiers operated at 10 MHz. The flicker fits almost exactly the model, which predicts an increase of 3 dB for 2 cascaded units, and an increase of 4.8 dB for 3 units. The small discrepancy is ascribed to the difference between the amplifiers. The reference (one amplifier) is the noise of a single device instead of the average of the 2 to 3 amplifiers. For the single amplifier measured at −3 dBm input power, the white noise hits the background of the instrument. Otherwise it follows (9). The same result is obtained with two AML812PNB1901 tested at 10 GHz, as seen in Fig. 7(b).

Fig. 7(c) shows the phase noise of two cascaded AML812PNB2401 amplifiers at 10 GHz, measured at low input power and compared with the single amplifier. The flicker coefficient is $b_{-1} = -119$ dB·rad^2/Hz for one amplifier, and −116.5 dB·rad^2/Hz for the two amplifiers, independent of power. The reason for careful noise measurement in low-power conditions relates to frequency synthesis for fundamental metrology [75]–[77], where the typical microwave power after detecting a femtosecond comb is of the order of −30 dBm.

3) Parallel Amplifiers:

In a third experiment, we measured the phase noise of a pair of amplifiers (AFS6 or JS2) connected in parallel. We used Wilkinson power splitters/combiners at the input and at the output instead of 90° couplers for the trivial reason that layout and trimming are simpler. The demonstration of our ideas is independent of the impedance-matching benefit of the 90° couplers. The power P_0 refers to the main input, before splitting the signal. Measuring the AFS6, we had to adapt the power to experimental needs, whereas the JS2 could be measured at about the same level for the single amplifier and for the parallel configuration. The spectra are shown in Fig. 8.

In both cases, we observe that the flicker of the pair is 2.5 dB lower than the noise of the single amplifier, whereas the model predicts 3 dB. This is ascribed to the gain asymmetry and to the asymmetry of the power splitter and combiner.

In Fig. 8(a), below 100 Hz, we observe a significant discrepancy with respect to the power law (1). A slope of −7 dB/decade shows up in the left-hand side of the

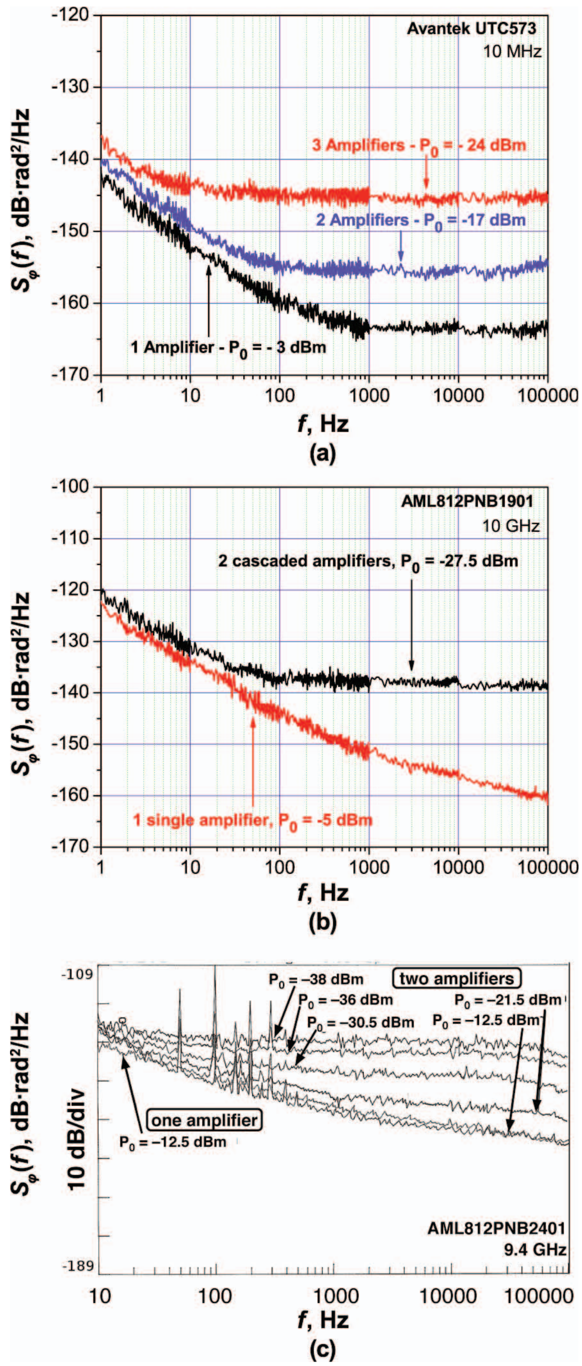

Fig. 7. Phase noise of cascaded amplifiers, compared with the noise of a single amplifier.

spectrum, up to 10 to 30 Hz, followed by a small bump. A careful check indicates that there is no damage, and the result is reproducible. Having no explanation for this anomalous behavior, we report the spectrum as a counter example, as yet, the only one found.

In Fig. 8(b), the white noise is consistent with the carrier power in the two experimental conditions.

4) Regenerative Amplifier:

The fourth experiment is the indirect measurement of the noise of a regenerative amplifier used as the sustain-

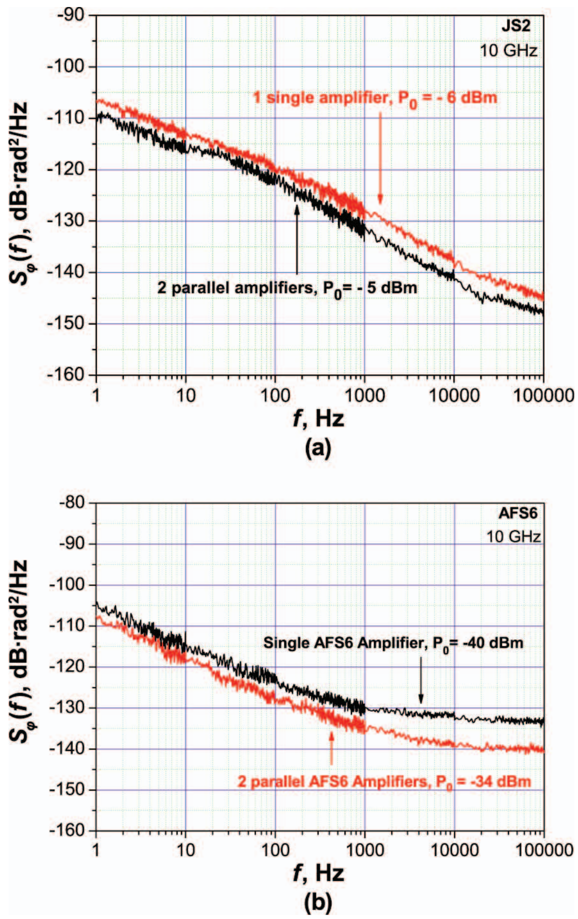

Fig. 8. Phase noise of parallel amplifiers.

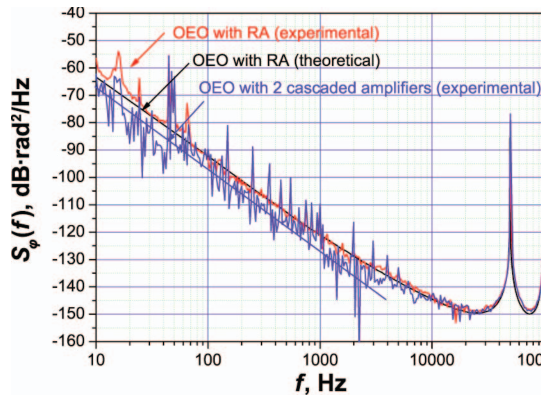

Fig. 9. Phase noise of an opto-electronic oscillator (OEO) set at 10 GHz carrier, from [78].

ing amplifier in a photonic microwave oscillator. Rather than an odd measurement choice, this experiment is a fortunate outcome of a separate research program on that topic. In this case, the oscillation frequency is an integer multiple of $1/\tau$, where τ is the group delay of an optical fiber (20 µs in our case) [78]–[80]. A colleague used regeneration to increase the gain of the sustaining amplifier, as a replacement for a second amplifier that was temporarily unavailable.

The oscillator phase noise is governed by the Leeson effect [2, ch. 4], [80], [81]. In the case of the delay-line oscillator, for $f < 1/\tau$, the flicker noise is given by

$$[b_{-3}]_{\text{osc}} = \frac{1}{4\pi^2\tau^2}[b_{-1}]_{\text{ampli}}. \quad (24)$$

This states that the oscillator integrates the phase noise of the sustaining amplifier, turning the phase flicker into frequency flicker, whose phase spectrum is $S_\varphi(f) = b_{-3}/f^3$.

Fig. 9 shows the oscillator spectrum in two configurations, with a RA used to obtain 44 dB gain from one 22-dB AML amplifier, and the other with two cascaded amplifiers of the same type—of course, with no regeneration. Knowing the $1/f$ noise of the AML812PNB1901 amplifier, we calculated the oscillator $1/f^3$ noise for the two cases. In the $1/f^3$ region (10^1 to 10^3 Hz), the noise is 3 dB higher when the regenerative amplifier is used instead of the two cascaded amplifiers, as predicted by (19) and (21). This fact validates the model in full.

V. Final Remarks

This work derives from a long-term research program on high-end oscillators and on frequency synthesis mainly for metrology and for military and space applications. The measurements reported here were done in different contexts, over more than five years. In the domain of oscillators, people are interested only in PM noise, whereas AM noise is considered a scientific curiosity and mentioned only for completeness. Amplitude noise is sometimes measured carefully [82], yet for quite different purposes, or is investigated because of its detrimental effect on phase-noise measurements [83].

It was only at the time of writing that it became clear that parametric AM and PM noise processes are partially correlated, and therefore that the amplifier noise is best modeled as in Fig. 10. This model is implied in several articles focusing on the $1/f$ noise up-conversion at the component level [6]–[8], [31], [37]–[39], [47], but not made explicit as an inherent property of the amplifier as a system building block. Such correlation is justified by the physics of the most popular amplifier devices. In a bipolar transistor, the fluctuation of the carriers in the base region acts on the base thickness, thus on the gain and on the capacitance of the reverse-biased base-collector junction. Of course, a fluctuating capacitance impacts on phase noise. In a field-effect transistor, the fluctuation of the carriers in the channel acts on the drain-source current, thus on the gate-channel capacitance via the channel thickness. In a vacuum tube, the fluctuation of the space charge affects gain and phase. In a laser amplifier, the fluctuation of the pump power acts on the density of the excited atoms, and in turn on gain, maximum power, and refraction index. In all of these examples, AM and PM fluctuations are correlated because both originate from a single near-dc random process.

Because the experiments are now terminated, we can only support the model with simulations. In the simula-

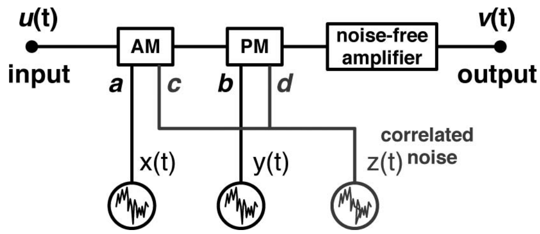

Fig. 10. Generalized model for the AM-PM noise in amplifiers.

tions shown in Fig. 11, we normalize on the carrier power, we linearize for low noise, and we set $a^2 + b^2 + c^2 + d^2 = 1$ so that the noise power is equal to one. The simulated noise is shown as it would be measured by the two-channel version of the noise-measurement system shown in Fig. 5(b), where we simultaneously detect the real and the imaginary parts with an in-phase and quadrature (I-Q) mixer [74].

In simplest form, the noise is a Gaussian process of power equally split into the real and imaginary parts. This is the symmetric two-dimensional Gaussian distribution of Fig. 11(a). If the noise is not equally split between AM and PM, for example, as happens when the amplifier is in the power compression region, there results an asymmetric Gaussian distribution [Fig. 11(c)]. The perfectly saturated amplifier has no AM noise, so it would be represented as a vertical line in a scatter plot.

Fig. 11(b) shows the case of flicker noise of an amplifier operated in the compression region. The amount of AM and PM is not the same, but there is some correlation between AM and PM noise. For comparison, the plot of Fig. 11(d) represents an (unrealistic) amplifier in which AM and PM noise originates from a single random process with the same modulation efficiency.

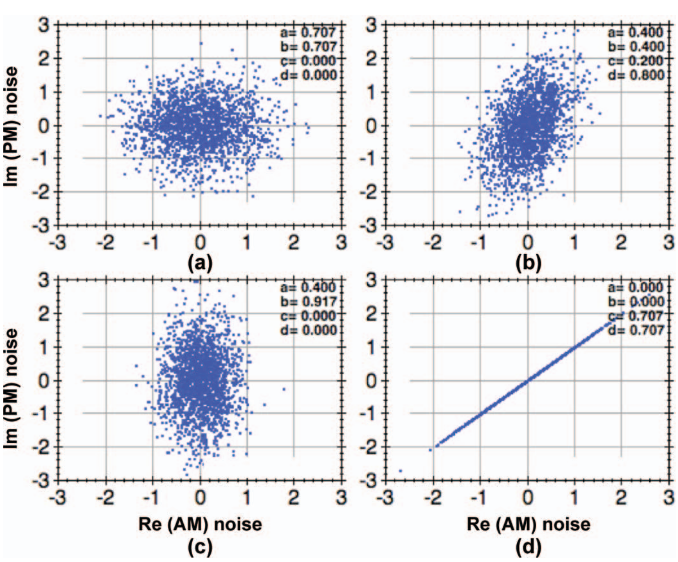

Fig. 11. Simulated parametric noise, real part (AM noise) and imaginary part (PM noise). The coefficient a, b, c, d are defined in Fig. 10.

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