Phase noise measurements of low power signals

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Measuring the phase fluctuation between a pair of low-power microwave signals, the signals must be amplified before detection. In such cases the phase noise of the amplifier pair is the main cause of $1/f$ residual noise. A new scheme is proposed that makes amplification possible while rejecting the $1/f$ (flicker) noise of the two amplifiers in real time. The first prototype, compared to a traditional saturated mixer system under the same conditions, shows a 24 dB noise reduction in the $1/f$ region.

Introduction: Phase noise is conventionally described in terms of the power spectral density $S_\varphi(f)$, which refers to the representation $x(t) = V_0[1 + \alpha(t)]\cos[\omega_0 t + \varphi(t)]$. $\varphi(t)$ and $\alpha(t)$ are the phase and amplitude fluctuations, ω_0 is the carrier angular frequency, and f is the Fourier frequency. It is a common practice to measure $\varphi(t)$ with a Schottky diode mixer as a phase-to-voltage converter, comparing the signal to a reference. Yet, the mixer needs power to saturate, and amplification becomes necessary if the signals are smaller than 0–5 dBm. In the case of signals distributed over optical fibres, for example, the output power of a photodetector can be -20 dBm or less, requiring further amplification before they are fed to the mixer. Of course amplifiers flicker, which turns out to be the main measurement limit at low f. This limit is even more severe if both the signal and the reference must be amplified. We observed that the $1/f$ noise of both amplifiers can be eliminated if a hybrid junction, which generates the sum and the difference of the two input signals, is inserted in the circuit. Fig. 1 shows the traditional and new configurations.

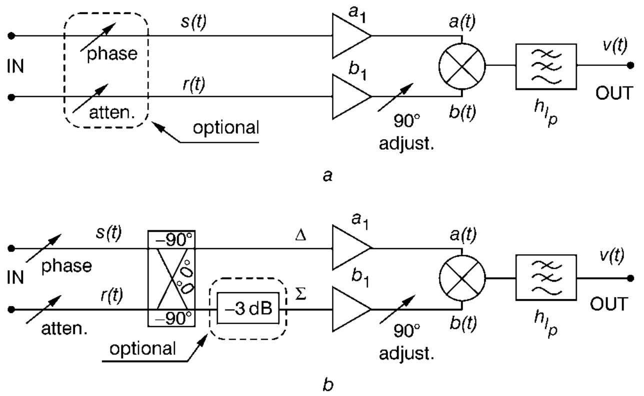

Fig. 1 *Mixer and interferometer*

Components in dotted ovals serve only to compare two configurations in equal conditions, and would otherwise be omitted
a Saturated mixer (traditional)
b Low-power interferometer (new)

Close-in flicker noise of amplifiers originates from the near-DC flickering that modulates some internal parameters, and in turn the carrier. Amplifying a pure sinusoid of amplitude V_0, the output signal is of the form $y(t) = a_1 V_0[\cos\omega_0 t + m'n'(t)\cos\omega_0 t - m''n''(t)\sin\omega_0 t]$. Up to a few dB below $P_{1\text{dB}}$ (the 1 dB compression power), the amplifier is linear, described by the gain a_1. n' and n'' are near-DC flicker, and m' and m'' the corresponding modulation factors. Below $P_{1\text{dB}}$, m'' (and probably also m') tends to be constant in a wide range of V_0, which means that phase noise is independent of power. This has been observed in a variety of amplifiers, bipolar and field-effect, from a few MHz to microwaves [1–3]. The $1/f$ coefficient of actual microwave amplifiers is in the range from -100 to -110 dBrad^2/Hz.

Phase detector: In the conventional scheme (Fig. 1*a*), the residual noise is due to the amplifiers, which are independent; the $1/f$ noise of the mixer is much lower than that of the amplifiers. Letting $r = V_0\cos\omega_0 t$ and $s = V_0\cos(\omega_0 t + \varphi)$, and taking a perfect multiplier as the mixer, the signal-to-noise ratio is

$$\text{SNR}_{mixer} = \frac{\overline{\varphi^2}}{\overline{(m''_a n''_a)^2} + \overline{(m''_b n''_b)^2}} \tag{1}$$

The subscripts a and b identify the circuit branches. Actual mixers must be saturated, which has minor implications discussed later.

Low-power interferometer: Inserting a hybrid coupler and adjusting the input amplitude and phase, the instrument works as an interferometer (Fig. 1*b*), the details of which are reported in [4]. Yet, the presence of two amplifiers makes the circuit operation unique. Ideally, all the carrier power goes to Σ, while Δ only contains the signal (noise) sidebands. The sidebands are amplified and down-converted. Let $s = V_0\cos(\omega_0 t + \varphi)$ and $r = V_0[\cos\omega_0 t + \varepsilon_\alpha\cos\omega_0 t - \varepsilon_\varphi\sin\omega_0 t]$ the input signals, where ε_α and ε_φ form the residual carrier due to imperfect adjustment, and $\varepsilon^2 = \varepsilon_\alpha^2 + \varepsilon_\varphi^2$; of course, $\varepsilon^2 \ll 1$. In the absence of amplifier noise ($n' = 0$ and $n'' = 0$), the output signal is $v = (1/2)a_1 b_1 V_0^2\varphi$. The background noise is calculated by re-introducing the amplifier noise, and discarding the signal ($\varphi = 0$). Letting $V_\delta = (1/2)V_0\varepsilon$ and $V_\sigma = \sqrt{2}V_0$ be the carrier voltages at the hybrid outputs, the mixer input signals are

$$a = a_1 V_\delta[\cos\omega_0 t + m'_a n'_a\cos\omega_0 t - m''_a n''_a\sin\omega_0 t] \tag{2}$$

$$b = b_1 V_\sigma[-\sin\omega_0 t - m'_b n'_b\sin\omega_0 t + m''_b n''_b\cos\omega_0 t] \tag{3}$$

Thus, the detected signal is $v = (1/2)a_1 b_1 V_\delta V_\sigma[m''_a n''_a(t) + m''_b n''_b(t)]$. Hence, the signal-to-noise ratio is

$$\text{SNR}_{interferom.} = \frac{\overline{\varphi^2}}{4\varepsilon^2\left[\overline{(m''_a n''_a)^2} + \overline{(m''_b n''_b)^2}\right]} \tag{4}$$

Comparing (1) to (4), the amplifier noise is rejected by a factor $4\varepsilon^2$, which is the carrier rejection parameter. The physical interpretation of this result is simple. The amplifier in the arm a can not flicker because there is no carrier at its input (eqn. (2)); this is known [4]. The amplifier in the arm b flickers (eqn. (3)), yet its noise does not contribute because there is no carrier at the opposite input of the mixer. Interestingly, this noise rejection mechanism is effective in real time. However, it naturally does not eliminate the additive random noise.

The assumption has been implicitly made in (2) that the phase of the residual carrier is 0; letting this phase be arbitrary makes the formalism more complicated, but does not change the results. Actual mixers must be saturated at the LO port (b in Fig. 1*b*), thus the conversion gain is lower than that of a perfect multiplier. A similar limitation applies to the phase detector, which is saturated at both inputs. Under these assumptions, the improvement in $1/f$ noise would still be of the order of ε^2. Nonetheless, whereas low ε values (10^{-3} or less) are easily obtained, the actual noise reduction is limited by second order factors, the most likely of which is the amplitude noise of the pump taken in via the mixer asymmetry.

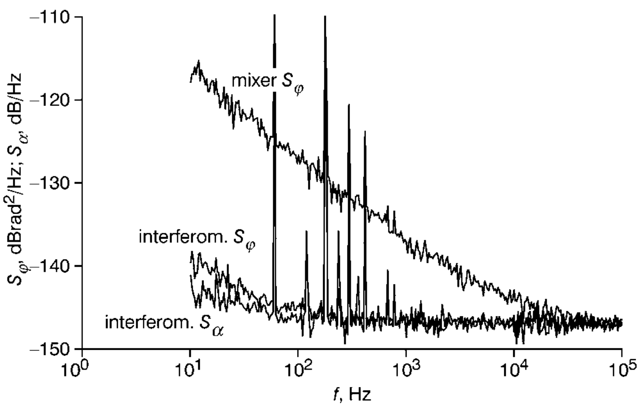

Fig. 2 *Background noise of two configurations*

Experimental results: The instrument is driven by two signals of frequency $\omega_0/2\pi = 9.9$ GHz and of power of -20 dBm obtained from a common synthesiser and directional couplers. The couplers are virtually noise free, hence the measured noise is the residual noise of the instrument. The amplifiers each produce 32 dB of gain with a 3 dB output attenuator that improves impedance matching and protects the mixer, and have a noise figure of 3 dB. We compare the two configurations of Fig. 1 under the same conditions, inserting and removing the hybrid coupler and the 3 dB attenuator; the interferometer is adjusted for ε lower than some 10^{-3}. The background noise is shown in Fig. 2. The white noise is -147 dBrad^2/Hz. This is due to the additive white noise of the amplifier, which is the same in all cases. The saturated-mixer scheme shows a residual flicker of -106 dBrad^2/Hz at $f = 1$ Hz (extrapolated), which is consistent with the $1/f$ noise of the amplifiers alone. The amplitude noise of the interferometer, hardly visible, is of some -135 dB/Hz at $f = 1$ Hz. The phase noise is -130 dBrad^2/Hz at $f = 1$ Hz (extrapolated), which improves by 24 dB as compared with the saturated mixer scheme. As expected, the full benefit of a factor $4\varepsilon^2$ could not be obtained.

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