Flicker Noise Measurement of HF Quartz Resonators

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***Abstract*—The frequency flicker of quartz resonators can be derived from the measurement of $S_\varphi(f)$, i.e., the power spectrum density of phase fluctuations φ. The interferometric method appears to be the best choice to measure the phase fluctuations of the quartz resonators because of its high sensitivity in the low power conditions, which is required for this type of resonator. Combining these two ideas, we built an instrument suitable to measure the frequency flicker floor of the quartz resonators, and we measured the stability of some 10-MHz high performance resonators as a function of the dissipated power. The stability limit of our instrument, described in terms of Allan deviation $\sigma_y(\tau)$, is of some 10^{-14}.**

I. Introduction

Quartz oscillators, as compared with other sources, exhibit outstanding reliability, in conjunction with an exceptionally good compromise among low noise, high stability, and a fairly low drift. For these reasons, they are the most widely used reference frequency sources in electronics and metrology; at the present time, they could hardly be replaced with other ones, such as whispering gallery oscillators [1], because of reliability or drift [2]. Yet, in some cases the short-term stability of quartz oscillators is still insufficient. This occurs, for instance, with atomic fountain frequency standards, which require ultrastable flywheel oscillators [3].

For metrological applications, frequency flicker of resonators and oscillators is our main concern. This type of noise, often referred as the flicker floor, is independent of the averaging time τ in the Allan deviation $\sigma_y(\tau)$ plot and represents the ultimate stability limit. Commercially available oscillators exhibit a flicker floor of some 10^{-13} for τ in the 0.2 to 30 s range. Selections are available with σ_y up to 1×10^{-13}, but for some special units, a floor as low as 7×10^{-14} can be expected [4]. For comparison, the drift of these outstanding oscillators can be lower than 10^{-11}/d, or a few parts in 10^{-9}/yr.

Whether the oscillator floor is due to the frequency fluctuation of the quartz resonator or whether it comes from the phase flicker noise of the amplifier converted into frequency flicker by the Leeson effect [5] depends on the particular circuit and resonator. But which is the main technological factor limiting the stability of the state-of-the-art oscillators is still matter of discussion. To answer this question, researchers have been measuring the frequency stability of quartz resonators for at least 25 yr with various techniques, most of which are based on the double balanced mixer as a phase-to-voltage converter [6]–[9]. In addition, attempts have been recently made to model the short-term stability of measuring systems for quartz resonators [10].

In this context, our attention is focused on measurements. For best stability, the typical dissipated power P_d of the quartz resonator is in the 10 to 100 μW range or even lower. In this case, resonator time-domain stability can approach parts in 10^{-14} in the flicker-of-frequency region, and the conventional resonator stability measurement apparatus does not support accurate measurements down to this level. A method is being proposed in this paper, based on frequency domain measurement of phase fluctuations, that shows improved sensitivity. A measurement system has been implemented and successfully used to measure a few high stability 10-MHz quartz resonators, listed in Table I. The flicker floor of the described prototype is close to the 10^{-14} target, depending on the resonator driving power.

II. Basics of the Proposed Method

When a quartz resonator is used in a passive phase bridge, fluctuations in the resonant frequency induce corresponding phase fluctuation in the externally generated carrier signal. Accordingly, the ultimate frequency flicker floor of a quartz oscillator caused by the resonator stability alone can be derived from the measurement of the power spectrum density $S_\varphi(f)$ of the phase fluctuation φ(t) induced on a carrier signal applied to the resonator.

All of the concepts we need to describe the frequency and phase fluctuations, as well as the relationships among the various noise representations, are well estabilished in the literature and can be found in many references, such as [11]. Details concerning the resonator and its RLC equivalent circuit, together with the measurement methods, are clearly explained in [12].

With reference to Fig. 1, let us assume that two identical quartz resonators are inserted in a bridge driven by a noise-free oscillator tuned at their series resonance frequency ν_0.

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TABLE I
Quartz resonators being tested.

Device	No.	R_s, Ω	L,H	Q_0 (unloaded)	C_v, pF	T, °C
AT	1	57.5	1.25	1.37×10^6	26.2	48
premium	2	58.2	1.25	1.35×10^6	21.5	51
AT	1	59.0	1.21	1.28×10^6	19.9	52.5
swept	2	57.8	1.20	1.31×10^6	24.8	48
QAS	1	70.0	1.37	1.23×10^6	24.9	67
	2	71.9	1.44	1.26×10^6	27.7	72
BVA	1	105	2.22	1.33×10^6	18.0	80
	2	106	2.24	1.33×10^6	19.0	80

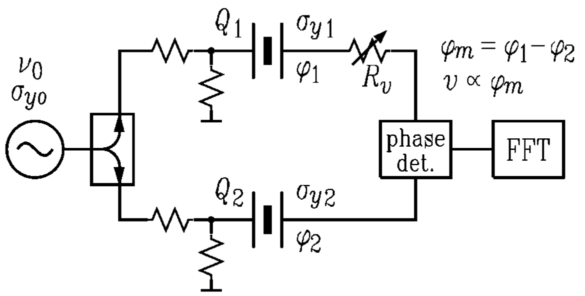

Fig. 1. Bridge quartz measurement scheme.

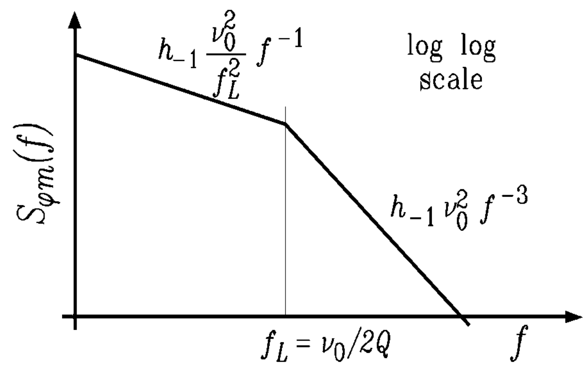

Fig. 2. Expected $S_{\varphi m}(f)$. Frequency flicker only is considered and is characterized by $S_y(f) = h_{-1}f^{-1}$.

Not to alter the usual formulas related to spectral purity, we provisionally assume that one resonator flickers and the other one is perfectly stable. Regarding the relative frequency deviation $y_q = \frac{\nu-\nu_0}{\nu_0}$ of the quartz as the input signal and the measured phase φ_m as the output, the quartz is equivalent to a low pass filter characterized by the cutoff frequency $f_L = \frac{\nu_0}{2Q}$. With the 10-MHz resonators we measured, f_L is of the order of 5 Hz, limited by the loaded merit factor Q. For reference, the unloaded merit factor Q_0 can be of the order of 1.5×10^6 at that frequency. Hence, the frequency-to-phase conversion can be rewritten as

$$S_{\varphi m}(f) = \frac{1/f_L^2}{1+f^2/f_L^2}\nu_0^2 S_{yq}(f) \tag{1}$$

where $S_{yq}(f)$ denotes the power spectrum density of the relative resonant frequency y_q and $S_{\varphi m}(f)$ is the corresponding spectrum density of the phase fluctuations induced on the carrier signal passing through the resonator. Fig. 2 reports the $S_{\varphi m}(f)$ plot thereby expected in the case of pure frequency flicker noise. The f^{-1} proportionality within the resonator bandwidth ($f < f_L$) is obvious because it comes from the quasistatic behavior of the resonator that responds with $\varphi_m = 2Q\delta\nu/\nu_0$ to a frequency fluctuation $\delta\nu$. For $f > f_L$, the quartz filters its own frequency fluctuations, yielding the f^{-3} phase slope. For unexplained reasons, this was not observed in the early experiments [6] but was reported later [7], [8].

With frequency flicker, representated as $S_y(f) = h_{-1}f^{-1}$, it holds $\sigma_y^2(\tau) = 2\ln 2\; h_{-1}$. Combining these relationships we get

$$S_y(f) = \frac{1}{2\ln 2}\frac{1}{f}\sigma_y^2(\tau). \tag{2}$$

The latter, inserted in (1), yields

$$\sigma_{yq}^2(\tau) = \frac{2\ln 2}{4Q^2}\left(1+\frac{f^2}{f_L^2}\right) f S_{\varphi m}(f). \tag{3}$$

To derive $\sigma_{yq}^2(\tau)$, we measure $S_{\varphi m}(f)$ for $f \ll f_L$; accordingly, we get

$$\sigma_{yq}^2(\tau) = \frac{2\ln 2}{4Q^2} f\, S_{\varphi m}(f). \tag{4}$$

The Allan deviation $\sigma_{yq}(\tau)$ thus obtained is the stability of the device resonant frequency, i.e., the time-domain stability of an oscillator in which that resonator is the only source of frequency instability.

There are two reasons to derive $\sigma_{yq}^2(\tau)$ from the measurement of $S_{\varphi m}(f)$ in quasistatic conditions, i.e., $f \ll f_L$. First, higher $S_{\varphi m}(f)$ relaxes noise specification for the phase detector, and, second, the slope f^{-1} yields lower spectrum analyzer uncertainty than the slope f^{-3}. Nevertheless, the f^{-3} slope, together with the corner at $f = f_L$, is a useful diagnostic tool. Obviously, only the part of $S_{\varphi m}(f)$ where the frequency flicker is the dominant process must be taken into account.

Finally, letting both resonators flicker, a first estimate of the variance of each resonator is one-half of that stated by (3) or (4) for a measured $S_{\varphi m}(f)$. At a deeper sight, the stability of two otherwise equal resonators can be different. Accordingly, the measurement of three or more resonators of the same type is required to find the actual stability of each device. This type of analysis is beyond the aim of this paper.

Resonators being tested also convert the frequency fluctuations $y_o(t)$ of the driving oscillator into phase fluctuations $\varphi_m(t)$ in the same way as they do with the fluctuations of their own resonance frequency. That oscillator, based on a quartz crystal, is usually less stable than the resonators we are testing. Yet, because of circuit symmetry, a large portion of the phase fluctuations caused by the oscillator instability, occurring within the resonator bandwidth, cancel. Accordingly, for $f \ll f_L$, the instrument noise floor is

$$\sigma^2_{y\,0}(\tau) \simeq \frac{(Q_1 - Q_2)^2}{Q_1^2 + Q_2^2}\sigma^2_{yo}(\tau) \qquad (5)$$

where Q_1 and Q_2 refer to the individual resonators. A Q mismatch within 10% warrants an oscillator noise rejection of 20 dB. If this is not sufficient with the available oscillator, the circuit must be symmetrized by means of a variable resistor that damps the higher Q resonator.

Quartz resonators show a power-induced frequency bias $\Delta\nu/\nu_0 = k_a P_d$ known as amplitude-frequency effect or anisochronism [13]. The coefficient k_a is independent of the resonator type (BVA, QAS, etc.), but it depends on mechanical and energy trapping parameters. For the high stability resonators we are interested in, the coefficient k_a is of some $10^{-9}/\mu$W [14]. Obviously, the power-induced frequency fluctuations cannot be divided from the resonator instability. Yet, the bridge scheme can make this effect negligible. In fact, the power fluctuation originates in the driving oscillator, and, therefore, it affects the latter and the two resonators being tested in the same way. But the bridge scheme is sensitive to the anisochronism mismatch of the two resonators only, and the driving oscillator is affected by the anisochronism of a single resonator. Fortunately, with equal resonators, k_a is about the same, with an expected mismatch of the same order of magnitude of the mismatch of the motional parameters; from Table I, a mismatch of parts in 10^{-2} can be inferred. Accordingly, if the flicker floor of the driving oscillator is of some 10^{-13}, and only a fraction of this is due to the power fluctuation, no anisochronism effect is expected for the 10^{-14} measurement target. Nonetheless we measured the anisochronism of one quartz pair, the BVAs, and we observed $k_a \simeq 9.8 \times 10^{-10}/\mu$W, almost equal for the two units.

Phase detection turns out to be a critical point because high sensitivity must be achieved with low power, which are mutually exclusive constraints. In fact, although details of state-of-the-art oscillators are not published, we expect that in a 10-MHz quartz crystal, the loaded Q is in the 7×10^5 to 10^6 range and the dissipated power is of the order of 10 μW. Taking $Q = 7 \times 10^5$ as a conservative value, a measurement noise floor $\sigma_{y0}(\tau) = 10^{-14}$ implies that the instrument noise specified in terms of $S_\varphi(f)$ does not exceed $S_{\varphi m0}(1\text{ Hz}) = -155.5$ dBrad^2/Hz.

A double balanced mixer used as the phase detector could offer the desired low noise, provided it is driven with sufficient power, i.e., 10 dBm or more. Hence, the small signal available at the quartz output must be amplified. Yet, according to our experience, commercially available radio frequency amplifiers do not meet the phase flicker requirement when they deliver some 10 dBm. For comparison, the best prototype built in our laboratory shows a phase noise $S_{\varphi m0}(1\text{ Hz}) = -140$ dBrad^2/Hz when the quartz dissipated power is $P_d = 50$ μW. Still under the assumption of $Q = 7 \times 10^5$ and $\nu_c = 10$ MHz, the reported noise is equivalent to a measurement limit corresponding to a stability $\sigma_{y0}(\tau) = 4 \times 10^{-14}$ referred to each quartz of a pair. This is suitable to most resonators, but still not sufficient for our purposes. The amplifier and mixer noise can be rejected by means of a correlation scheme derived from [15] in which four amplifiers and two mixers are used [9]. Yet, we opted for quite a different solution.

III. Interferometric Measurement System

The proposed measurement scheme, shown in Fig. 3, is basically an interferometric phase detector modified for quartz resonators. This kind of detector, first proposed as method for microwave measurements [16], has been ameliorated and adapted to lower frequencies [17]. In short, phase noise is regarded as a sideband pair that carries information. Hence, after adjusting the phase γ' and the attenuations ℓ_1 and ℓ_2 for best circuit symmetry, the carrier is suppressed at the input of the amplifier, but this mechanism has no effect on the noise sidebands originated by the resonator fluctuation. Thus, the amplifier amplifies the noise sidebands only. Properly setting the phase shift γ'', the mixer down converts to baseband the phase noise sidebands, rejecting amplitude noise. Consequently, the voltage $v(t)$ available at the fast Fourier transform (FFT) analyzer input is proportional to the instantaneous value of $\varphi_m(t)$.

Under the assumption of $\ell_1 = \ell_2 = 0$ dB, the overall phase detector gain is

$$K_\varphi = \frac{S_v(f)}{S_{\varphi m}(f)} = \frac{g P_c R_0}{\ell_h \ell_m} \qquad (6)$$

for one quartz. g is the amplifier gain, P_c is the carrier power at the quartz output, and R_0 is the mixer output impedance. ℓ_h is the loss of the 180° hybrid used as the power combiner in which the carrier is suppressed, not including the 3-dB intrinsic loss caused by energy conservation. ℓ_m is the mixer loss, which includes the 3-dB intrinsic loss caused by conversion into upper and lower bands. These definitions of ℓ_h and ℓ_m are those commonly used in most component databooks.

Eq. (6) is derived and discussed in [17]. Nonetheless, some commments can be useful here. A first interpretation of the interferometer operation is that the carrier suppression is a means to increase the modulation index of the phase fluctuations induced by the quartz crystal instability. This interpretation is correct in principle, but it could induce the wrong belief that a certain amount of residual

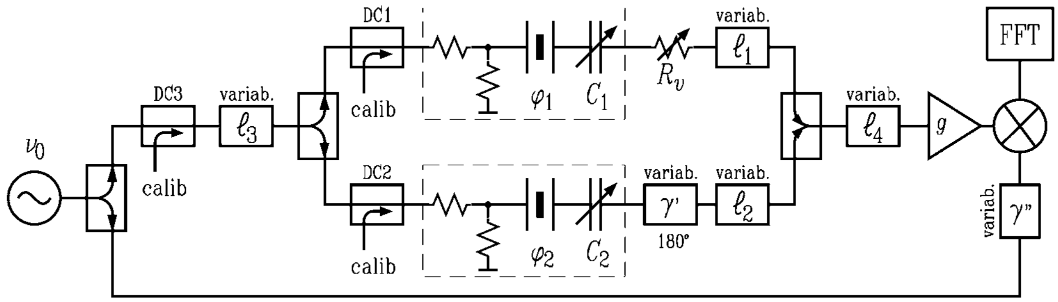

Fig. 3. Interferometric measurement system.

carrier is necessary, otherwise no modulation index could be recognized. A better interpretation is that even though the carrier is suppressed, the crystal-induced phase modulation sidebands are not, and their power is independent of the suppression ratio. Thus the noise sidebands are amplified and down converted to baseband by the mixer as they would be in a syncrodyne receiver. The mixer introduces a 3-dB gain caused by the overlapping of upper and lower sideband, which compensates for the 3-dB intrinsic loss of the 180° hybrid in which the carrier suppression takes place.

The main reason to chose the interferometric scheme is its suitability to low Fourier frequency measurements in low power conditions, where an amplifier is needed. In fact, radio frequency amplifiers flicker because their near-dc parameters flicker, and these fluctuations are up converted by the device nonlinearity [18], [19]. Yet, in the interferometric scheme, the amplifier works in small signal regime, warranted by the carrier suppression mechanism, and the effect of nonlinearity becomes negligible. Therefore, for lowest flicker, the carrier should be suppressed as much as possible. Reference [17] provides more details on the carrier suppression requirements, together with general design guidelines.

With the described prototype, the amplifier gain is $g = 43$ dB, the hybrid loss is $\ell_h \simeq 0.3$ dB, and the mixer loss is $\ell_m \simeq 6$ dB, which means $K_\varphi \simeq 4$ dBV^2/rad^2 with $P_c = 10$ μW. After proper adjustment, residual carrier does not exceed -25 dBm at the amplifier output, which is some 40 dB below the maximum deliverable power of that device, specified as the 1-dB compression point.

The white phase noise of the instrument is limited by the amplifier input noise Fk_BT_0, where F is the noise figure of the amplifier, $k_B = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant, and $T_0 = 290$ K is the reference temperature, close to room temperature. Hence, the expected phase noise is $S_{\varphi m0}(f) = 2\ell_h F k_B T_0/P_c$, ascribed to a single quartz, or

$$S_{\varphi m0}(f) = \frac{\ell_h F k_B T_0}{P_c} \tag{7}$$

for each quartz of a pair. Then, $S_{\varphi m0}(f)$ can be expressed as a function of the quartz dissipated power $P_d = \frac{R_s}{R_0}P_c$, where $R_0 = 50$ Ω is the input impedance of the power combiner in which the carrier is suppressed.

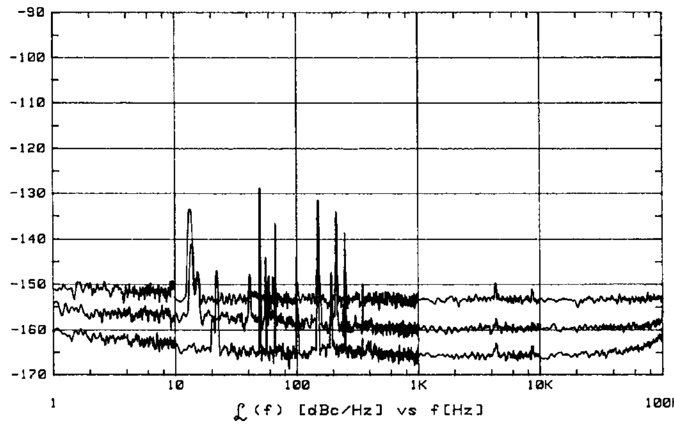

Fig. 4. Phase detector noise for some values of the dissipated power. Upper curve $P_d = 12$ μW, middle curve $P_d = 50$ μW, and lower curve $P_d = 200$ μW.

Fig. 4 shows the measured noise floor of the phase detector for some values of the power P_d. Obviously, for this type of measurement the resonator is replaced with a resistor equal to the motional resistance R_s, which is 50 Ω in this case. Because of the available software package, results are reported in terms of $\mathcal{L}(f) = \frac{1}{2}S_\varphi(f)$. The noise floor is in agreement with the value predicted by (7), within 1 dB.

To account for flicker, we replace F with $F(f)$. Inspecting Fig. 4, we observe that the 1-Hz noise is 5 dB higher than the floor level; combining this piece of information with the noise figure $F = 1.7$ dB of the available amplifier, we get $F(1\text{ Hz}) = 6.7$ dB. The noise floor thereby calculated is $\sigma_{y0}(\tau) = 6.6 \times 10^{-17}/\sqrt{P_c}$. With the high performance, 10-MHz resonators we measured on, the motional resistance is in the 50 to 110 Ω range (see Table I). Thus, the P_d/P_c ratio spans from 0 to 3.5 dB in the reported conditions. Hence, for the sake of simplicity, we account for the spread of R_s with a 3-dB margin instead of including R_s in the sensitivity model. Accordingly, we take $\sigma_{y0}(\tau) = 10^{-16}/\sqrt{P_d}$ as a conservative estimate of the measurement stability limit.

IV. Technical Aspects and Design Guidelines

Some technical problems, discussed subsequently, had to be solved before making the instrument function properly and achieve the desired stability.

The variable attenuators and phase shifters responsible for the carrier suppression are critical devices in the proposed circuit because their fluctuation contributes to the measured $S_{\varphi m}(f)$, limiting sensitivity. Selecting an attenuator requires some attempts and a pinch of good luck. This occurs because the flicker noise of the attenuators is not specified by the manufacturer. Even worse, in our experience, low flicker is not necessarily related to other good quality parameters, such resolution, accuracy, or mechanical ruggedness, nor to the cost. Hence, we advise testing all of the available units before considering other solutions. As for phase shifters, searching through many data sheets, we still have not been able to find any suitable device. When we designed the system, we decided to avoid the electrically tuned phase shifters, which would otherwise constitute the smartest solution, because of the varactor noise; nevertheless, these devices could be reconsidered after the results published in [20]. Surprisingly, even the microwave line stretchers we tested turned out not to be sufficiently stable when used at low frequencies in the HF band. As a provisional solution, we decided to build our own phase shifters, based on a low Q resonator that can be slightly detuned. Obviously, these resonators must be designed for the specific quartz frequency. The narrow dynamic range of the detuned resonator, of some $\pm 5^\circ$ for reasonably constant amplitude, is not a problem. This occurs because we use the quartz at the resonance frequency, where it is equivalent to a resistor, and, consequently, only a small phase adjustment is needed.

Most of the commercially available amplifiers and mixers suitable to the HF band show a wide bandwidth, typically of 1 to 500 MHz or so. Hence, strong harmonic distortion is present at the mixer LO input because of the saturation of the latter; because of the mixer symmetry, odd harmonics only are relevant. As a consequence, the mixer down converts the amplifier noise around $3\nu_c$, $5\nu_c$, etc. other than the desired signal. The suggested solution is the insertion of at least one bandpass filter along the amplifier chain. For best stability, a low Q filter must be selected.

Each resonator is temperature stabilized close to its turning point by means of an oven of the same type as those used for high performance oscillators. For best mechanical stability, all of the circuit is screwed on a 4-mm copper plate, which also serves as a ground plane, and put on a 120-mm thick sand layer. A type of sand, originally intended for children's games, proved to be the best choice because it is clean and shows good damping properties, probably because of the relatively large grain size.

Unfortunately, a shielded chamber was not available, while the computer network of the entire laboratory works at 10 MHz, the same frequency of our resonators. For this reason, after some attempts, the prototype was enclosed in a nearly sealed iron box that also contained the lead-acid batteries that supply all of the circuits. Only one cable connects the instrument output to the FFT spectrum analyzer, carrying a relatively high level signal. In addition, it was necessary to disconnect from the network and to switch off all of the computers present in the experiment room; only the computer used to collect and average spectrum data was on.

Finally, some of the reported measurements could be done only in the late afternoon, when most people were out.

V. Tuning and Calibration

For proper operation, the instrument first needs to be tuned. The suggested procedure is described subsequently.

• Detection Phase γ''. Both resonators are initially removed and replaced with resistors equal to the highest of the two motional resistances R_s. The variable capacitors C_1 and C_2 are removed along with the quartzes. γ'' must be tuned with arm 1 alone because the latter has no adjustable phase. Accordingly, ℓ_1 is set to 0 dB, and ℓ_2 is set to its maximum, so that arm 2 is nearly isolated. Obviously, no carrier suppression takes place with this condition.

γ'' is now adjusted for the mixer input signals to be in quadrature, so that the mixer can properly detect the phase noise. This condition can be easily recognized observing the dc voltage at the mixer output, which must be zero. For best adjustment accuracy, the attenuator ℓ_4 must be set for the mixer RF power to be some 6 to 10 dB lower than the LO power. With lower RF power, the mixer sensitivity is poor; on the other hand, higher power causes a dc offset in the mixer, because of residual diode asymmetry, and the mixer input signals are no longer in quadrature when γ'' is set at zero output dc voltage. For reference, in our prototype, the LO power is +10 dBm, and the RF power is 0 dBm, when the quartz dissipated power P_d is -10 dBm and ℓ_4 is set to 30 dB.

• Symmetry Phase γ'. The instrument must now be tuned for the mixer to detect phase noise from arm 2. To do this, the rôle of the two arms must be interchanged, setting ℓ_2 to 0 dB and ℓ_1 for isolation. Then, γ' must be tuned for the quadrature condition at the mixer inputs, which is detected observing zero dc voltage at the mixer output.

Because of the residual interaction between the two arms, reiteration of steps 1 and 2 may be necessary before going to the next step.

• Insertion of the Quartz Resonators. The resonators, together with the variable capacitors, are now inserted in the instrument. To compensate for the Q mismatch with the variable resistor R_v, the resonator with lower R_s must be put in the arm 1.

• Tuning C_1 and C_2. For frequency tunability, quartz resonators show a residual inductance at the nominal frequency to be compensated by an external capacitor of specified value; the latter is referred as C_v in Table I. C_1 is tuned first, isolating arm 2. Accordingly, ℓ_1 is set to 0 dB, and ℓ_2 is set for isolation. Acting on C_1, the overall impedance of C_1 and the quartz 1 turns into a pure resistance when the nominal value is reached. Once again, the good condition is detected from the zero dc voltage at the mixer output. Afterward, the rôle of the two arms is exchanged, and this step is repeated with C_2.

• Carrier Suppression. As a result of the previous steps, the two phases are almost equal. Hence, in principle, only amplitude asymmetry still remains. In practice, this is no longer true, but amplitude should be adjusted first, acting on ℓ_1 or ℓ_2; these attenuators are initially set to 0 dB. To suppress the carrier, the residual power at the output of the first amplifier stage must be monitored by means of a spectrum analyzer. Power monitoring requires that the amplifier works in its linear regime. For this reason, ℓ_4 must be initially set to an appropriately high value, 30 dB in our prototype, and reduced to 0 dB when the residual power is sufficiently low. The carrier power must be suppressed as much as possible, which requires some iterations of fine amplitude and phase tuning.

Provided the phases were precisely set as described in A and B, the small phase change required here has a negligible effect on the detection. In fact, an error $\delta\gamma$, where γ is either γ' or γ'', affects $S_{\varphi m}(f)$ with an error $\delta S_{\varphi m}(f)/S_{\varphi m}(f)$ not greater than $1-\cos^2(\delta\gamma)$.

Calibration is much simpler than tuning. The gain K_φ is first measured by injecting through the appropriate directional coupler a known sideband of power P_s at $f_s = 1$ Hz apart from ν_c; f_s must be lower than f_L, so that the quartz filter action is negligible. This sideband is equivalent to a sinusoidal phase modulation of rms value $\varphi = \sqrt{P_s/(2P_c)}$, which causes a voltage v_o at the mixer output. The gain thereby obtained is $K_\varphi = (v_o/\varphi)^2$. The rejection of the driving oscillator noise can be checked in the same way. In both cases a sideband is preferable to white noise because it can be measured more accurately.

The calculation of $\sigma_{yq}(\tau)$ requires the knowledge of the loaded Q in actual circuit conditions. This can be obtained from the cutoff frequency f_L, measured by injecting white noise through the directional coupler in series to each quartz and measuring the output by means of the FFT analyzer.

The driving power has a negligible effect on the phase and attenuation of the circuit, and, consequently, it is not necessary to repeat the tuning procedure each time P_d is changed. Carrier suppression only must be refined because it results from imperfect compensation of two equal quantities, and, consequently, it is impaired by small symmetry changes. Otherwise, it is negligible. In principle, the gain K_φ as a function of the driving power can be obtained from a single measurement. Nevertheless, we repeated the measurement for each value of P_d.

VI. Experimental Results and Discussion

The adjustment and tuning process just described was repeated for all of the resonators of Table I before measuring their phase noise.

Fig. 5 and 6 report an example of measurements taken with the BVA pair, each one dissipating a power $P_d = 200$ μW. Injecting white noise in series to one quartz, we ob-

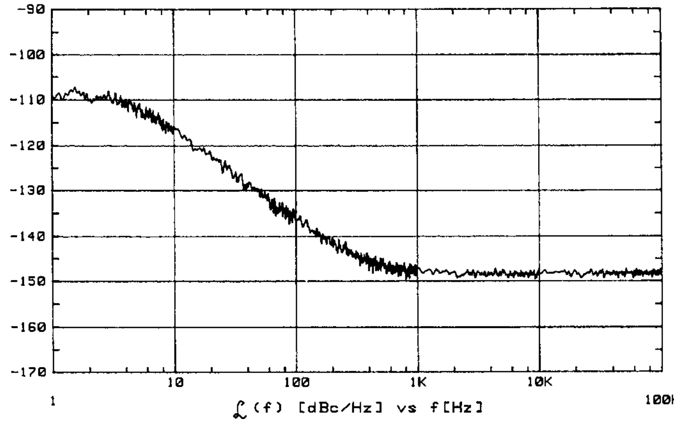

Fig. 5. Transfer function $|H(f)|$ of a BVA resonator.

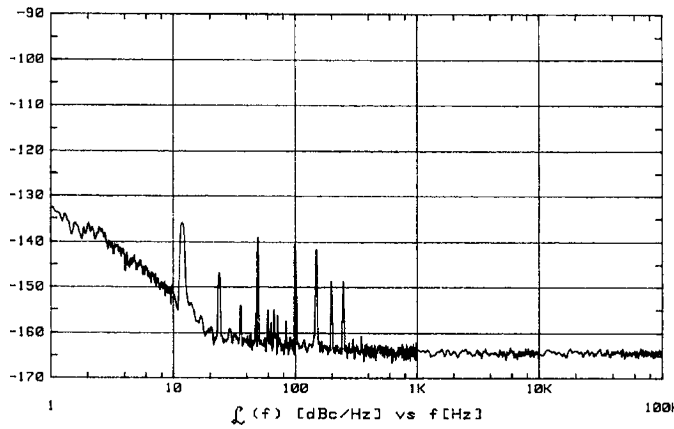

Fig. 6. Phase noise of a BVA resonator pair.

tain the frequency response of Fig. 5, from which the cutoff frequency can be individuated at $f_L = 4.5$ Hz. Accordingly, the loaded merit factor is $Q = \frac{\nu_0}{2f_L} = 1.1 \times 10^6$. From Fig. 6, we get the phase noise value $S_{\varphi m}(1\ \text{Hz}) = -131$ dBrad^2/Hz. The latter, inserted in (4), yields a flicker floor $\sigma_y(\tau) = 10^{-13}$ for each quartz.

Results of all of the stability measurements are shown in Fig. 7, together with the instrument limit. The effect of P_d on the flicker floor is evident with the QAS resonators, and it also appears with the BVA units for $P_d > 50$ μW.

The QAS resonators come from two oscillators that were explicitly disassembled for this purpose. Comparing these oscillators with one another, the flicker floor turned out to be $\sigma_y(\tau) \simeq 1.7 \times 10^{-13}$ for each unit, and the dissipated power was $P_d \simeq -13$ dBm. Comparing those data with our measurements (Fig. 7), the flicker floor of the quartz alone is 1.5 dB lower than that of the whole oscillator.

The theoretical background and the experience achieved until now can be exploited to assess the expected limitations and suitability of the proposed method to other frequencies of great interest, and particularly $\nu_0 = 5$ MHz and $\nu_0 = 100$ MHz.

Many commercially available modules (amplifiers, hy-

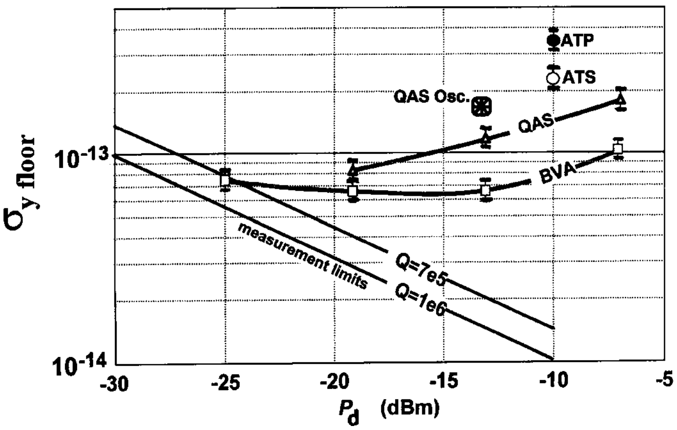

Fig. 7. Allan deviation of some quartz resonators deduced from phase noise measurements.

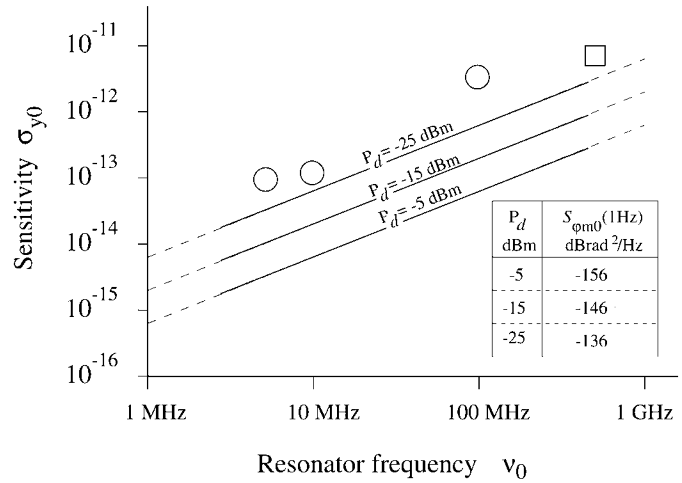

Fig. 8. Expected performances of the noise measurement system as a function of the carrier frequency. For reference, the stability of some oscillators is also reported.

brids, directional couplers, etc.) show a wide bandwidth of some 1 to 500 MHz or more; in addition, the motional resistance and the dissipated power of the resonators in the 1 to 100 MHz range are of the same order of those of the 10-MHz devices we tested. This means that the only required change concerns the filters; all other electronic circuits can be reused as they are. Moreover, the same sensitivity $S_{\varphi m0}(f)$ is expected in the same power conditions.

A widely accepted rule of thumb states that the highest achievable unloaded merit factor of quartz resonators is $Q_0\nu_0 = b$, where b is an empirical constant whose value is in the 1×10^{13} to 2×10^{13} Hz range. Because the instrument sensitivity given by (4) is related to Q instead of Q_0, we can replace $Q = (Q/Q_0)(b/\nu_0)$ in that equation; the ratio Q/Q_0, that can be typically 0.5 to 0.8, accounts for the loading effect of the measurement circuit. Thus, under the hypothesis of two equal resonators, the instrument sensitivity is

$$\sigma_{y0}(\tau) = \frac{Q_0}{Q}\frac{\sqrt{\ln 2}}{2b}\sqrt{S_{\varphi m0}(1\ \text{Hz})\nu_0} \tag{8}$$

where $S_{\varphi m0}(1\ \text{Hz}) = fS_{\varphi m0}(f)$ is the flicker noise of the instrument extrapolated to 1 Hz.

Fig. 8 reports the sensitivity derived from (8) evaluated for a reference situation, in which $b = 1.4 \times 10^{13}$ and $Q/Q_0 = 0.7$. Each line refers to a particular value of the extrapolated flicker $S_{\varphi m0}(1\ \text{Hz})$, and it is labeled with the minimum dissipated power with which that value of $S_{\varphi m0}(1\ \text{Hz})$ can be obtained according to the reported experience. The flicker floor of some high performance oscillators is also reported for reference.

As ν_0 increases, resonators exhibit lower Q, which proportionally impairs the instrument sensitivity. On the other hand, the flicker floor of the complete oscillator increases with the same law. This is not surprising because the sensitivity of the measurement instrument and the stability of the quartz oscillator depend on the loaded Q with the same law. Therefore, no additional difficulty is expected to extend the proposed instrument to the whole frequency range of actual interest, from 1 to 200 MHz or more.

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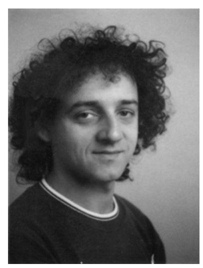

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