Flicker Noise Measurement of HF Quartz Resonators

Enrico Rubiola, Jacques Groslambert, Michel Brunet, and Vincent Giordano

Abstract—The frequency flicker of quartz resonators can be derived from the measurement of $S_\varphi(f)$, i.e., the power spectrum density of phase fluctuations $\varphi$. The interferometric method appears to be the best choice to measure the phase fluctuations of the quartz resonators because of its high sensitivity in the low power conditions, which is required for this type of resonator. Combining these two ideas, we built an instrument suitable to measure the frequency flicker floor of the quartz resonators, and we measured the stability of some 10-MHz high performance resonators as a function of the dissipated power. The stability limit of our instrument, described in terms of Allan deviation $\sigma_y(\tau)$, is of some $10^{-14}$.

I. INTRODUCTION

Quartz oscillators, as compared with other sources, exhibit outstanding reliability, in conjunction with an exceptionally good compromise among low noise, high stability, and a fairly low drift. For these reasons, they are the most widely used reference frequency sources in electronics and metrology; at the present time, they could hardly be replaced with other ones, such as whispering gallery oscillators [1], because of reliability or drift [2]. Yet, in some cases the short-term stability of quartz oscillators is still insufficient. This occurs, for instance, with atomic fountain frequency standards, which require ultrastable flywheel oscillators [3].

For metrological applications, frequency flicker of resonators and oscillators is our main concern. This type of noise, often referred as the flicker floor, is independent of the averaging time $\tau$ in the Allan deviation $\sigma_y(\tau)$ plot and represents the ultimate stability limit. Commercially available oscillators exhibit a flicker floor of some $10^{-13}$ for $\tau$ in the 0.2 to 30 s range. Selections are available with $\sigma_y$ up to $1 \times 10^{-13}$, but for some special units, a floor as low as $7 \times 10^{-14}$ can be expected [4]. For comparison, the drift of these outstanding oscillators can be lower than $10^{-11}/d$, or a few parts in $10^{-9}/\text{yr}$.

Whether the oscillator floor is due to the frequency fluctuation of the quartz resonator or whether it comes from the phase flicker noise of the amplifier converted into frequency flicker by the Leeson effect [5] depends on the particular circuit and resonator. But which is the main technological factor limiting the stability of the state-of-the-art oscillators is still matter of discussion. To answer this question, researchers have been measuring the frequency stability of quartz resonators for at least 25 yr with various techniques, most of which are based on the double balanced mixer as a phase-to-voltage converter [6]–[9]. In addition, attempts have been recently made to model the short-term stability of measuring systems for quartz resonators [10].

In this context, our attention is focused on measurements. For best stability, the typical dissipated power $P_d$ of the quartz resonator is in the 10 to 100 $\mu$W range or even lower. In this case, resonator time-domain stability can approach parts in $10^{-14}$ in the flicker-of-frequency region, and the conventional resonator stability measurement apparatus does not support accurate measurements down to this level. A method is being proposed in this paper, based on frequency domain measurement of phase fluctuations, that shows improved sensitivity. A measurement system has been implemented and successfully used to measure a few high stability 10-MHz quartz resonators, listed in Table I. The flicker floor of the described prototype is close to the $10^{-14}$ target, depending on the resonator driving power.

II. BASICS OF THE PROPOSED METHOD

When a quartz resonator is used in a passive phase bridge, fluctuations in the resonant frequency induce corresponding phase fluctuation in the externally generated carrier signal. Accordingly, the ultimate frequency flicker floor of a quartz oscillator caused by the resonator stability alone can be derived from the measurement of the power spectrum density $S_\varphi(f)$ of the phase fluctuation $\varphi(t)$ induced on a carrier signal applied to the resonator.

All of the concepts we need to describe the frequency and phase fluctuations, as well as the relationships among the various noise representations, are well established in the literature and can be found in many references, such as [11]. Details concerning the resonator and its $RLC$ equivalent circuit, together with the measurement methods, are clearly explained in [12].

With reference to Fig. 1, let us assume that two identical quartz resonators are inserted in a bridge driven by a noise-free oscillator tuned at their series resonance frequency $\nu_0$. 

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periments. Not to alter the usual formulas related to spectral purity,
we provisionally assume that one resonator flickers and the other one is perfectly stable. Regarding the relative frequency deviation \( \nu = \frac{\nu - \nu_0}{\nu_0} \) of the quartz as the input signal and the measured phase \( \varphi_m \) as the output, the quartz is equivalent to a low pass filter characterized by the cutoff frequency \( f_L = \frac{\nu_0}{2Q} \).

With the 10-MHz resonators we measured, \( f_L \) is of the order of 5 Hz, limited by the loaded merit factor \( Q \).

For reference, the unloaded merit factor \( Q_0 \) can be of the order of 1.5 \( \times 10^6 \) at that frequency. Hence, the frequency-to-phase conversion can be rewritten as

\[
S_{\varphi_m}(f) = \frac{1}{f^2} \frac{1}{1 + \frac{f^2}{f_L^2}} \nu_0^2 S_{\nu}(f) \tag{1}
\]

where \( S_{\nu}(f) \) denotes the power spectrum density of the relative resonant frequency \( \nu \) and \( S_{\varphi_m}(f) \) is the corresponding spectrum density of the phase fluctuations induced on the carrier signal passing through the resonator.

Fig. 2 reports the \( S_{\varphi_m}(f) \) plot thereby expected in the case of pure frequency flicker noise. The \( f^{-1} \) proportionality within the resonator bandwidth \( f < f_L \) is obvious because it comes from the quasistatic behavior of the resonator that responds with \( \varphi_m = 2Q\delta\nu/\nu_0 \) to a frequency fluctuation \( \delta\nu \). For \( f > f_L \), the quartz filters its own frequency fluctuations, yielding the \( f^{-3} \) phase slope. For unexplained reasons, this was not observed in the early experiments [6] but was reported later [7], [8].

With frequency flicker, represented as \( S_{\nu}(f) = h_1 f^{-1} \), it holds \( \sigma_{\nu}^2(\tau) = 2 \ln 2 h_1 \). Combining these relationships we get

\[
S_{\nu}(f) = \frac{1}{2 \ln 2} \frac{1}{f} \sigma_{\nu}^2(\tau). \tag{2}
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Resonators being tested also convert the frequency fluctuations \( y_\ell(t) \) of the driving oscillator into phase fluctuations \( \phi_m(t) \) in the same way as they do with the fluctuations of their own resonance frequency. That oscillator, based on a quartz crystal, is usually less stable than the resonators we are testing. Yet, because of circuit symmetry, a large portion of the phase fluctuations caused by the oscillator instability, occurring within the resonator bandwidth, cancel. Accordingly, for \( f \ll f_L \), the instrument noise floor is

\[
\sigma_y^2(\tau) \approx \frac{(Q_1 - Q_2)^2}{Q_1^2 + Q_2^2} \sigma_y^2(\tau)
\]

where \( Q_1 \) and \( Q_2 \) refer to the individual resonators. A \( Q \) mismatch within 10% warrants an oscillator noise rejection of 20 dB. If this is not sufficient with the available oscillator, the circuit must be symmetrized by means of a variable resistor that damps the higher \( Q \) resonator.

Quartz resonators show a power-induced frequency bias \( \Delta \nu/\nu_0 = k_\nu P_d \) known as amplitude-frequency effect or anisochronism [13]. The coefficient \( k_\nu \) is independent of the resonator type (BVA, QAS, etc.), but it depends on mechanical and energy trapping parameters. For the high stability resonators we are interested in, the coefficient \( k_\nu \) is of some \( 10^{-9}/\mu W \) [14]. Obviously, the power-induced frequency fluctuations cannot be divided from the resonator instability. Yet, the bridge scheme can make this effect negligible. In fact, the power fluctuation originates in the driving oscillator, and, therefore, it affects the latter and the two resonators being tested in the same way. But the bridge scheme is sensitive to the anisochronism mismatch of the two resonators only, and the driving oscillator is affected by the anisochronism of a single resonator. Fortunately, with equal resonators, \( k_\nu \) is about the same, with an expected mismatch of the same order of magnitude of the mismatch of the motional parameters: from Table I, a mismatch of parts in \( 10^{-2} \) can be inferred. Accordingly, if the flicker floor of the driving oscillator is of some \( 10^{-13} \), and only a fraction of this is due to the power fluctuation, no anisochronism effect is expected for the \( 10^{-14} \) measurement target. Nonetheless we measured the anisochronism of one quartz pair, the BVAs, and we observed \( k_\nu \simeq 9.8 \times 10^{-10}/\mu W \), almost equal for the two units.

Phase detection turns out to be a critical point because high sensitivity must be achieved with low power, which are mutually exclusive constraints. In fact, although details of state-of-the-art oscillators are not published, we expect that in a 10-MHz quartz crystal, the loaded \( Q \) is in the \( 7 \times 10^5 \) to \( 10^6 \) range and the dissipated power is of the order of 10 \( \mu W \). Taking \( Q = 7 \times 10^5 \) as a conservative value, a measurement noise floor \( \sigma_y^2(\tau) = 10^{-14} \) implies that the instrument noise specified in terms of \( S_\phi(f) \) does not exceed \( S_\phi(1 \text{ Hz}) = -155.5 \text{ dBm}^2/\text{Hz} \).

A double balanced mixer used as the phase detector could offer the desired low noise, provided it is driven with sufficient power, i.e., 10 dBm or more. Hence, the small signal available at the quartz output must be amplified. Yet, according to our experience, commercially available radio frequency amplifiers do not meet the phase flicker requirement when they deliver some 10 dBm. For comparison, the best prototype built in our laboratory shows a phase noise \( S_{\phi,0}(1 \text{ Hz}) = -140 \text{ dBm}^2/\text{Hz} \) when the quartz dissipates power is \( P_d = 50 \mu W \). Still under the assumption of \( Q = 7 \times 10^5 \) and \( v_c = 10 \text{ MHz} \), the reported noise is equivalent to a measurement limit corresponding to a stability \( \sigma_{y,0}(\tau) = 4 \times 10^{-14} \) referred to each quartz of a pair. This is suitable to most resonators, but still not sufficient for our purposes. The amplifier and mixer noise can be rejected by means of a correlation scheme derived from [15] in which four amplifiers and two mixers are used [9]. Yet, we opted for quite a different solution.

### III. Interferometric Measurement System

The proposed measurement scheme, shown in Fig. 3, is basically an interferometric phase detector modified for quartz resonators. This kind of detector, first proposed as a method for microwave measurements [16], has been ameliorated and adapted to lower frequencies [17]. In short, phase noise is regarded as a sideband pair that carries information. Hence, after adjusting the phase \( \gamma' \) and the attenuations \( \ell_1 \) and \( \ell_2 \) for best circuit symmetry, the carrier is suppressed at the input of the amplifier, but this mechanism has no effect on the noise sidebands originated by the resonator fluctuation. Thus, the amplifier amplifies the noise sidebands only. Properly setting the phase shift \( \gamma'' \), the mixer down converts to baseband the phase noise sidebands, rejecting amplitude noise. Consequently, the voltage \( v(t) \) available at the fast Fourier transform (FFT) analyzer input is proportional to the instantaneous value of \( \phi_m(t) \).

Under the assumption of \( \ell_1 = \ell_2 = 0 \text{ dB} \), the overall phase detector gain is

\[
K_\phi = \frac{S_\phi(f)}{S_{\phi,0}(f)} = \frac{g P_c R_0}{\ell_h \ell_m} \quad (6)
\]

for one quartz. \( g \) is the amplifier gain, \( P_c \) is the carrier power at the quartz output, and \( R_0 \) is the mixer output impedance. \( \ell_h \) is the loss of the 180° hybrid used as the power combiner in which the carrier is suppressed, not including the 3-dB intrinsic loss caused by energy conservation. \( \ell_m \) is the mixer loss, which includes the 3-dB intrinsic loss caused by conversion into upper and lower bands. These definitions of \( \ell_h \) and \( \ell_m \) are those commonly used in most component databooks.

Eq. (6) is derived and discussed in [17]. Nonetheless, some comments can be useful here. A first interpretation of the interferometer operation is that the carrier suppression is a means to increase the modulation index of the phase fluctuations induced by the quartz crystal instability. This interpretation is correct in principle, but it could induce the wrong belief that a certain amount of residual...
carrier is necessary, otherwise no modulation index could be recognized. A better interpretation is that even though the carrier is suppressed, the crystal-induced phase modulation sidebands are not, and their power is independent of the suppression ratio. Thus the noise sidebands are amplified and down converted to baseband by the mixer as they would be in a syncrodyne receiver. The mixer introduces a 3-dB gain caused by the overlapping of upper and lower sideband, which compensates for the 3-dB intrinsic loss of the 180° hybrid in which the carrier suppression takes place.

The main reason to choose the interferometric scheme is its suitability to low Fourier frequency measurements in low power conditions, where an amplifier is needed. In fact, radio frequency amplifiers flicker because their near dc parameters flicker, and these fluctuations are up converted by the device nonlinearity [18], [19]. Yet, in the interferometric scheme, the amplifier works in small signal regime, warranted by the carrier suppression mechanism, and the effect of nonlinearity becomes negligible. Therefore, for lowest flicker, the carrier should be suppressed as much as possible. Reference [17] provides more details on the carrier suppression requirements, together with general design guidelines.

With the described prototype, the amplifier gain is $g = 43 \, \text{dB}$, the hybrid loss is $\ell_h \simeq 0.3 \, \text{dB}$, and the mixer loss is $\ell_m \simeq 6 \, \text{dB}$, which means $K_\varphi \simeq 4 \, \text{dBV}^2/\text{rad}^2$ with $P_c = 10 \, \mu\text{W}$. After proper adjustment, residual carrier does not exceed $-25 \, \text{dBm}$ at the amplifier output, which is some $40 \, \text{dB}$ below the maximum deliverable power of that device, specified as the 1-dB compression point.

The white phase noise of the instrument is limited by the amplifier input noise $Fk_BT_0$, where $F$ is the noise figure of the amplifier, $k_B = 1.38 \times 10^{-21} \, \text{J/K}$ is the Boltzmann constant, and $T_0 = 290 \, \text{K}$ is the reference temperature, close to room temperature. Hence, the expected phase noise is $S_{\varphi m}(f) = 2\ell_h Fk_BT_0/P_c$, ascribed to a single quartz, or

$$S_{\varphi m}(f) = \frac{\ell_h Fk_BT_0}{P_c}$$

for each quartz of a pair. Then, $S_{\varphi m}(f)$ can be expressed as a function of the quartz dissipated power $P_d = \frac{R_0}{R_s} P_c$, where $R_0 = 50 \, \Omega$ is the input impedance of the power combiner in which the carrier is suppressed.

Fig. 3. Interferometric measurement system.

Fig. 4. Phase detector noise for some values of the dissipated power. Upper curve $P_d = 12 \, \mu\text{W}$, middle curve $P_d = 50 \, \mu\text{W}$, and lower curve $P_d = 200 \, \mu\text{W}$.

Fig. 4 shows the measured noise floor of the phase detector for some values of the power $P_d$. Obviously, for this type of measurement the resonator is replaced with a resistor equal to the motional resistance $R_s$, which is $50 \, \Omega$ in this case. Because of the available software package, results are reported in terms of $L(f) = \frac{1}{2} S_{\varphi m}(f)$. The noise floor is in agreement with the value predicted by (7), within $1 \, \text{dB}$.

To account for flicker, we replace $F$ with $F(f)$. Inspecting Fig. 4, we observe that the 1-Hz noise is $5 \, \text{dB}$ higher than the floor level; combining this piece of information with the noise figure $F = 1.7 \, \text{dB}$ of the available amplifier, we get $F(1 \, \text{Hz}) = 6.7 \, \text{dB}$. The noise floor thereby calculated is $\sigma_{\varphi m}(\tau) = 6.6 \times 10^{-17}/\sqrt{P_c}$. With the high performance, 10-MHz resonators we measured on, the motional resistance is in the 50 to 110 $\Omega$ range (see Table I). Thus, the $P_d/P_c$ ratio spans from 0 to $3.5 \, \text{dB}$ in the reported conditions. Hence, for the sake of simplicity, we account for the spread of $R_s$ with a 3-dB margin instead of including $R_s$ in the sensitivity model. Accordingly, we take $\sigma_{\varphi m}(\tau) = 10^{-16}/\sqrt{P_d}$ as a conservative estimate of the measurement stability limit.

IV. TECHNICAL ASPECTS AND DESIGN GUIDELINES

Some technical problems, discussed subsequently, had to be solved before making the instrument function properly and achieve the desired stability.
The variable attenuators and phase shifters responsible for the carrier suppression are critical devices in the proposed circuit because their fluctuation contributes to the measured $S_{2,\text{sn}}(f)$, limiting sensitivity. Selecting an attenuator requires some attempts and a pinch of good luck. This occurs because the flicker noise of the attenuators is not specified by the manufacturer. Even worse, in our experience, low flicker is not necessarily related to other good quality parameters, such resolution, accuracy, or mechanical ruggedness, nor to the cost. Hence, we advise testing all of the available units before considering other solutions. As for phase shifters, searching through many data sheets, we still have not been able to find any suitable device. When we designed the system, we decided to avoid the electrically tuned phase shifters, which would otherwise constitute the smartest solution, because of the varactor noise; nevertheless, these devices could be reconsidered after the results published in [20]. Surprisingly, even the microwave line stretchers we tested turned out not to be sufficiently stable when used at low frequencies in the HF band. As a provisional solution, we decided to build our own phase shifters, based on a low $Q$ resonator that can be slightly detuned. Obviously, these resonators must be designed for the specific quartz frequency. The narrow dynamic range of the detuned resonator, of some $\pm 5^\circ$ for reasonably constant amplitude, is not a problem. This occurs because we use the quartz at the resonance frequency, where it is equivalent to a resistor, and, consequently, only a small phase adjustment is needed.

Most of the commercially available amplifiers and mixers suitable to the HF band show a wide bandwidth, typically of 1 to 500 MHz or so. Hence, strong harmonic distortion is present at the mixer LO input because of the saturation of the latter; because of the mixer symmetry, odd harmonics only are relevant. As a consequence, the mixer down converts the amplifier noise around $\nu_0$, $5\nu_0$, etc. other than the desired signal. The suggested solution is the insertion of at least one bandpass filter along the amplifier chain. For best stability, a low $Q$ filter must be selected.

Each resonator is temperature stabilized close to its turning point by means of an oven of the same type as those used for high performance oscillators. For best mechanical stability, all of the circuit is screwed on a 4-mm copper plate, which also serves as a ground plane, and put on a 120-mm thick sand layer. A type of sand, originally intended for children’s games, proved to be the best choice because it is clean and shows good damping properties, probably because of the relatively large grain size.

Unfortunately, a shielded chamber was not available, while the computer network of the entire laboratory works at 10 MHz, the same frequency of our resonators. For this reason, after some attempts, the prototype was enclosed in a nearly sealed iron box that also contained the lead-acid batteries that supply all of the circuits. Only one cable connects the instrument output to the FFT spectrum analyzer, carrying a relatively high level signal. In addition, it was necessary to disconnect from the network and to switch off all of the computers present in the experiment room; only the computer used to collect and average spectrum data was on.

Finally, some of the reported measurements could be done only in the late afternoon, when most people were out.

V. TUNING AND CALIBRATION

For proper operation, the instrument first needs to be tuned. The suggested procedure is described subsequently.

- **Detection Phase $\gamma''$.** Both resonators are initially removed and replaced with resistors equal to the highest of the two motional resistances $R_s$. The variable capacitors $C_1$ and $C_2$ are removed along with the quartzes. $\gamma''$ must be tuned with arm 1 alone because the latter has no adjustable phase. Accordingly, $\ell_1$ is set to 0 dB, and $\ell_2$ is set to its maximum, so that arm 2 is nearly isolated. Obviously, no carrier suppression takes place with this condition.

$\gamma''$ is now adjusted for the mixer input signals to be in quadrature, so that the mixer can properly detect the phase noise. This condition can be easily recognized observing the dc voltage at the mixer output, which must be zero. For best adjustment accuracy, the attenuator $\ell_1$ must be set for the mixer RF power to be some 6 to 10 dB lower than the LO power. With lower RF power, the mixer sensitivity is poor; on the other hand, higher power causes a dc offset in the mixer, because of residual diode asymmetry, and the mixer input signals are no longer in quadrature when $\gamma''$ is set at zero output dc voltage. For reference, in our prototype, the LO power is $+10$ dBm, and the RF power is 0 dBm, when the quartz dissipated power $P_d$ is $-10$ dBm and $\ell_1$ is set to 30 dB.

- **Symmetry Phase $\gamma'$.** The instrument must now be tuned for the mixer to detect phase noise from arm 2. To do this, the role of the two arms must be interchanged, setting $\ell_2$ to 0 dB and $\ell_1$ for isolation. Then, $\gamma'$ must be tuned for the quadrature condition at the mixer inputs, which is detected observing zero dc voltage at the mixer output.

Because of the residual interaction between the two arms, reiteration of steps 1 and 2 may be necessary before going to the next step.

- **Insertion of the Quartz Resonators.** The resonators, together with the variable capacitors, are now inserted in the instrument. To compensate for the $Q$ mismatch with the variable resistor $R_v$, the resonator with lower $R_v$ must be put in the arm 1.

- **Tuning $C_1$ and $C_2$.** For frequency tunability, quartz resonators show a residual inductance at the nominal frequency to be compensated by an external capacitor of specified value; the latter is referred as $C_v$ in Table I. $C_1$ is tuned first, isolating arm 2. Accordingly, $\ell_1$ is set to 0 dB, and $\ell_2$ is set for isolation. Acting on $C_1$, the overall impedance of $C_1$ and the quartz 1 turns into a pure resistance when the nominal value is reached. Once again,
the good condition is detected from the zero dc voltage at the mixer output. Afterward, the rôle of the two arms is exchanged, and this step is repeated with \( C_2 \).

- Carrier Suppression. As a result of the previous steps, the two phases are almost equal. Hence, in principle, only amplitude asymmetry still remains. In practice, this is no longer true, but amplitude should be adjusted first, acting on \( \ell_1 \) or \( \ell_2 \); these attenuators are initially set to 0 dB. To suppress the carrier, the residual power at the output of the first amplifier stage must be monitored by means of a spectrum analyzer. Power monitoring requires that the amplifier works in its linear regime. For this reason, \( \ell_4 \) must be initially set to an appropriately high value, 30 dB in our prototype, and reduced to 0 dB when the residual power is sufficiently low. The carrier power must be suppressed as much as possible, which requires some iterations of fine amplitude and phase tuning.

Provided the phases were precisely set as described in \( A \) and \( B \), the small phase change required here has a negligible effect on the detection. In fact, an error \( \delta \gamma \), where \( \gamma \) is either \( \gamma' \) or \( \gamma'' \), affects \( S_{\varphi m}(f) \) with an error \( \delta S_{\varphi m}(f)/S_{\varphi m}(f) \) not greater than \( 1 - \cos^2(\delta \gamma) \).

Calibration is much simpler than tuning. The gain \( K_\varphi \) is first measured by injecting through the appropriate directional coupler a known sideband of power \( P_s \) at \( f_s = 1 \) Hz apart from \( \nu_q \); \( f_s \) must be lower than \( f_L \), so that the quartz filter action is negligible. This sideband is equivalent to a sinusoidal phase modulation of rms value \( \varphi = \sqrt{P_s/(2P_s)} \), which causes a voltage \( v_o \) at the mixer output. The gain thereby obtained is \( K_\varphi = (v_o/\varphi)^2 \). The rejection of the driving oscillator noise can be checked in the same way. In both cases a sideband is preferable to white noise because it can be measured more accurately.

The calculation of \( \sigma_{\varphi q}(\tau) \) requires the knowledge of the loaded \( Q \) in actual circuit conditions. This can be obtained from the cutoff frequency \( f_L \), measured by injecting white noise through the directional coupler in series to each quartz and measuring the output by means of the FFT analyzer.

The driving power has a negligible effect on the phase and attenuation of the circuit, and, consequently, it is not necessary to repeat the tuning procedure each time \( P_d \) is changed. Carrier suppression only must be refined because it results from imperfect compensation of two equal quantities, and, consequently, it is impaired by small symmetry changes. Otherwise, it is negligible. In principle, the gain \( K_\varphi \) as a function of the driving power can be obtained from a single measurement. Nevertheless, we repeated the measurement for each value of \( P_d \).

**VI. Experimental Results and Discussion**

The adjustment and tuning process just described was repeated for all of the resonators of Table I before measuring their phase noise.

Fig. 5 and 6 report an example of measurements taken with the BVA pair, each one dissipating a power \( P_d = 200 \) \( \mu \)W. Injecting white noise in series to one quartz, we obtain the frequency response of Fig. 5, from which the cutoff frequency can be individuated at \( f_L = 4.5 \) Hz. Accordingly, the loaded merit factor is \( Q = 2f_L/\gamma = 1.1 \times 10^6 \). From Fig. 6, we get the phase noise value \( S_{\varphi m}(1 \) Hz) = \(-131 \) dBrad\(^2/\)Hz. The latter, inserted in (4), yields a flicker floor \( \sigma_{\varphi}(\tau) = 10^{-13} \) for each quartz.

Results of all of the stability measurements are shown in Fig. 7, together with the instrument limit. The effect of \( P_d \) on the flicker floor is evident with the QAS resonators, and it also appears with the BVA units for \( P_d > 50 \) \( \mu \)W.

The QAS resonators come from two oscillators that were explicitly disassembled for this purpose. Comparing these oscillators with one another, the flicker floor turned out to be \( \sigma_{\varphi}(\tau) \approx 1.7 \times 10^{-13} \) for each unit, and the dissipated power was \( P_d \approx -13 \) dBm. Comparing those data with our measurements (Fig. 7), the flicker floor of the quartz alone is 1.5 dB lower than that of the whole oscillator.

The theoretical background and the experience achieved until now can be exploited to assess the expected limitations and suitability of the proposed method to other frequencies of great interest, and particularly \( \nu_0 = 5 \) MHz and \( \nu_0 = 100 \) MHz.

Many commercially available modules (amplifiers, hy-
bridges, directional couplers, etc.) show a wide bandwidth of some 1 to 500 MHz or more; in addition, the motional resistance and the dissipated power of the resonators in the 1 to 100 MHz range are of the same order of those of the 10-MHz devices we tested. This means that the only required change concerns the filters; all other electronic circuits can be reused as they are. Moreover, the same sensitivity \( S_{\phi m}(f) \) is expected in the same power conditions.

A widely accepted rule of thumb states that the highest achievable unloaded merit factor of quartz resonators is \( Q_0 b/Q_0 = b \), where \( b \) is an empirical constant whose value is in the \( 1 \times 10^{13} \) to \( 2 \times 10^{13} \) Hz range. Because the instrument sensitivity given by (4) is related to \( Q \) instead of \( Q_0 \), we can replace \( Q = (Q/Q_0)(b/\nu_0) \) in that equation; the ratio \( Q/Q_0 \), that can be typically 0.5 to 0.8, accounts for the loading effect of the measurement circuit. Thus, under the hypothesis of two equal resonators, the instrument sensitivity is

\[
\sigma_{\phi 0}(\tau) = \frac{Q_0}{Q} \frac{\sqrt{\ln 2}}{2b} \sqrt{S_{\phi m}(1 \text{ Hz})} \nu_0
\]

where \( S_{\phi m}(1 \text{ Hz}) = f S_{\phi m}(f) \) is the flicker noise of the instrument extrapolated to 1 Hz.

Fig. 8 reports the sensitivity derived from (8) evaluated for a reference situation, in which \( b = 1.4 \times 10^{13} \) and \( Q/Q_0 = 0.7 \). Each line refers to a particular value of the extrapolated flicker \( S_{\phi m}(1 \text{ Hz}) \), and it is labeled with the minimum dissipated power with which that value of \( S_{\phi m}(1 \text{ Hz}) \) can be obtained according to the reported experience. The flicker floor of some high performance oscillators is also reported for reference.

As \( \nu_0 \) increases, resonators exhibit lower \( Q \), which proportionally impairs the instrument sensitivity. On the other hand, the flicker floor of the complete oscillator increases with the same law. This is not surprising because the sensitivity of the measurement instrument and the stability of the quartz oscillator depend on the loaded \( Q \) with the same law. Therefore, no additional difficulty is expected to extend the proposed instrument to the whole frequency range of actual interest, from 1 to 200 MHz or more.

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**REFERENCES**


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