

# Improved Interferometric Method to Measure Near-Carrier AM and PM Noise

Enrico Rubiola, Vincent Giordano, and Jacques Gros Lambert

**Abstract**—An improved version of the interferometric method to measure near-carrier AM and PM noise is being presented. The main feature of this scheme is the capability to reduce the instrument noise by correlating the output of two equal interferometers built around the same device to be tested, thus enhancing the sensitivity. Two double interferometers are described, operating in the microwave and VHF bands. The latter shows a noise floor of  $-194$  dB rad<sup>2</sup>/Hz when the signal power is  $+8$  dBm. The sensitivity of both the instruments turns out to be significantly higher than the ratio of the thermal noise divided by the carrier power.

**Index Terms**—AM noise, correlation, phase noise, random noise.

## I. INTRODUCTION

IT HAS been recognized recently that the interferometric method, although based on old ideas [1], allows real-time near-carrier noise measurement in the microwave region with higher sensitivity compared to the traditional method based on a saturated mixer. In fact, it has been demonstrated that for the X-band a PM noise measurement system (NMS), based on this method, shows a sensitivity  $S_{\varphi 0}(f)$  close to  $-190$  dB rad<sup>2</sup>/Hz (white noise) if a power level as high as 20 dBm can be tolerated [2]. Besides, it is known that the sensitivity of the saturated-mixer NMS can be improved by some 15 dB by means of correlation of the output of two equal NMS's that measure the same device [3], [11]. So, in search of the most sensitive instrument we combined the two techniques, correlating the output of two interferometers measuring the same DUT (device under test). After having proposed some preliminary results [4], [5], full details and experiments are being presented. Two NMS's were built, operating at 7.3 GHz and 100 MHz, with sensitivity values of  $-192$  dB rad<sup>2</sup>/Hz and  $-194$  dB rad<sup>2</sup>/Hz with carrier power of 8 and 15 dBm, respectively.

As the noise theory of the proposed NMS is still under development, we will spend most of the following sections describing the instrument and the experiments.

## II. BASICS OF THE DOUBLE INTERFEROMETER

The scheme of the double interferometer is shown in Fig. 1. Let us first analyze one of the two interferometers, for instance that shown in the upper half of the figure. The 90° hybrid

coupler makes the vector addition, i.e., the *interference*, of its input signals. Thus, setting  $\ell'$  and  $\gamma'$  equal to the DUT attenuation and phase shift, respectively, all the carrier power goes to the  $\Sigma$  output, and the carrier is suppressed at the  $\Delta$  output. The DUT noise sidebands, which are *not* suppressed by the interference mechanism, are amplified by the low noise amplifier (LNA) and down-converted to baseband by the mixer. The latter down converts PM or AM noise, depending on the detection phase  $\gamma''$ . Setting  $\gamma'' = 90^\circ$ , the voltage  $x(t)$  at the output of the mixer is proportional to the instant value  $\varphi(t)$  of the phase noise. The power gain  $K_\varphi$  of the NMS, defined as  $K_\varphi = x^2(t)/\varphi^2(t)$ , is given by

$$K_\varphi = \frac{R_0 g_a P_o}{2 \ell_s \ell_a \ell_h \ell_m} \quad (1)$$

where

- $R_0$  output impedance of the mixer;
- $g_a$  power gain of the amplifier;
- $P_o$  power available at the DUT output;
- $\ell_s$  loss of the power splitter;
- $\ell_a$  additional loss needed to compensate the residual loss of the variable attenuator and phase shifter;
- $\ell_h$  loss of the hybrid;
- $\ell_m$  loss of the mixer.

According to the usual definitions,  $\ell_s$  and  $\ell_h$  do not include the 3 dB intrinsic loss due to power splitting;  $\ell_m$ , by contrast, includes the 3 dB intrinsic loss due to the fact that the mixer makes the sum and the difference of its input frequencies, and consequently, it splits the input power into two bands.

Replacing the DUT with a short cable, the residual PSD (power spectrum density)  $S_{\varphi 0}(f)$  of the white noise represents the sensitivity of the interferometer, given by

$$S_{\varphi 0}(f) = \frac{4F_a k_B T_0 \ell_s \ell_h \ell_a}{P_o} \quad (2)$$

where  $F_a$  is the noise figure of the amplifier and  $k_B = 1.38 \times 10^{-23}$  J/K is the Boltzmann constant; the temperature of the interferometer is assumed to be close to the reference temperature  $T_0 = 290$  K at which  $F_a$  is defined. It is worth pointing out that if the DUT noise is independent of the carrier power  $P_o$ , as it happens for the additive noise,  $S_{\varphi 0}$  is decreased by increasing  $P_o$ .

The microwave amplifier is also a potential source of flicker noise because of nonlinear phenomena that up-convert the dc bias flicker, as shown in [6] and [7]. Hence, linearity must be ensured by keeping the residual carrier power at the amplifier input as low as possible; a carrier suppression of 60–80 dB

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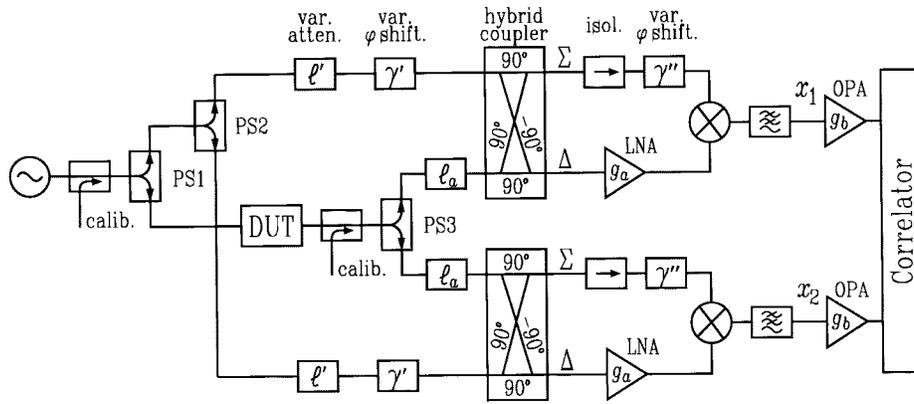


Fig. 1. Block diagram of the double interferometer.

can be necessary, depending on the implementation and  $P_o$ . Moreover, the phase shifters  $\gamma'$  and the attenuators  $\ell'$  can generate additional white and flicker PM noise.

Setting  $\gamma'' = 0^\circ$ ,  $x(t)$  is proportional to the instantaneous value  $\alpha(t)$  of the relative amplitude noise. The above formulae (1) and (2) still hold, replacing  $K_\varphi$  and  $S_{\varphi 0}$  with  $K_\alpha$  and  $S_{\alpha 0}$ , whose meanings are obvious.

We can now consider the whole scheme of Fig. 1, which allows the DUT noise to be measured through the cross PSD  $S_{x_1 x_2}(f)$  of the two interferometer outputs. Let us define  $a$ ,  $b$ , and  $c$  the noise processes taking place in the upper arm only, in the lower arm only, and in the common path, respectively. Accordingly, the output PSD's are  $S_{x_1} = S_a + S_c$  and  $S_{x_2} = S_b + S_c$ . Provided that the detection phases  $\gamma''$  are set equal, only the contribution of  $c$  remains in the average product  $\langle S_{x_1} S_{x_2} \rangle$ , while all the cross terms are expected to approach zero as  $1/\sqrt{n}$ , where  $n$  is the number of measures, under the assumption that  $a$ ,  $b$ , and  $c$  are independent. So, we can measure  $S_c$  by means of a digital correlator, and consequently, get  $S_\varphi$  or  $S_\alpha$ , depending on  $\gamma''$ .

To sum up, all the sources of noise that are independent in the two arms are rejected by correlation. This applies to the most relevant ones, i.e., those located in the amplifiers, in the variable attenuators, and in the variable phase shifters. So, comparing our scheme to the single interferometer, a lower noise floor is expected or more noisy components can be adopted. The latter makes feasible an automatically tuned version of the double interferometer, which would require electrically controlled attenuators and phase shifters, generally more noisy than the manually adjustable ones.

### III. IMPLEMENTATION AND RESULTS

We have built two NMS prototypes, one for the microwave band and one for VHF, described below.

The microwave NMS, shown in Fig. 2, operates at the carrier frequency  $\nu_c = 7.3$  GHz, chosen because of the availability of all the components. The amplifier consists of two cascaded modules, showing an overall gain of about 45 dB, a noise figure  $F_n = 2$  dB, a maximum power  $P_m = 18$  dBm (1 dB compression), and a bandwidth  $B = 250$  MHz. This relatively narrow band is needed to ensure that the noise

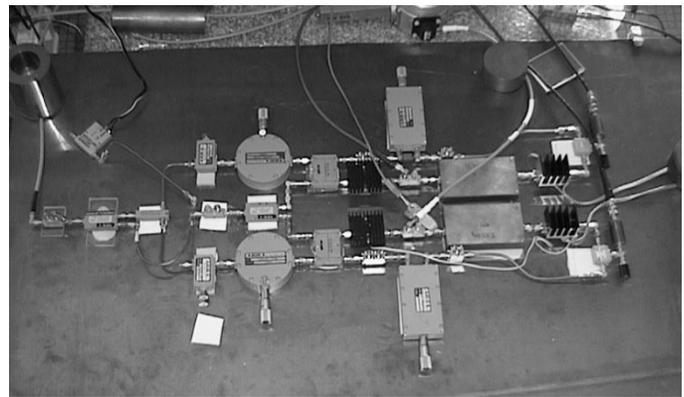


Fig. 2. Microwave implementation of the double interferometer.

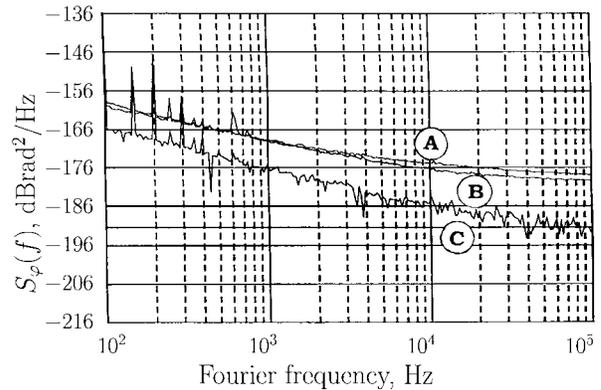


Fig. 3. Instrument noise of the microwave double interferometer. Curves A and B: single arm noise, and curve C: cross correlation.

power is sufficiently low to keep the amplifier in its fully linear region, thus preventing it to flicker. Operating with a DUT output power  $P_o = 15$  dBm,  $K_\varphi$  turns out to be  $36 \text{ dB V}^2/\text{rad}^2$ ; the overall gain is  $56 \text{ dB V}^2/\text{rad}^2$ , including the amplifiers inserted between the mixers and the correlator ( $g_b = 20 \text{ dB}$ ). A directional coupler, inserted between the two stages, allows the monitoring of the residual carrier. Some ferrite isolators are inserted at the  $\Sigma$  output of the hybrids to prevent oscillation or measurement alteration due the unwanted feedback through the hybrid, which shows poor isolation, of the order of 20 dB.

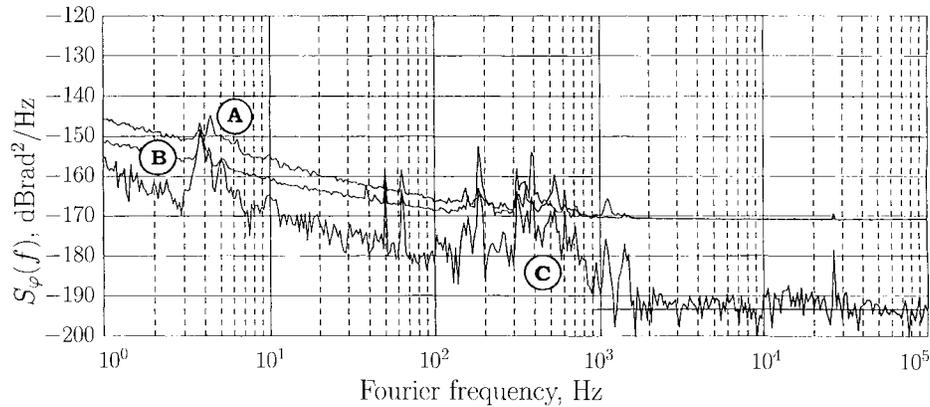


Fig. 4. Instrument noise of the VHF double interferometer. Curves A and B: single arm noise, and curve C: cross correlation.

A carrier suppression of  $-80$  to  $-90$  dB, stable for 30 min, can be achieved. After observing that the source is critical because of its noise, we adopted the best available one, i.e., a cryogenic sapphire whispering gallery oscillator [8], which exhibits a white noise of  $-180$  dB  $\text{rad}^2/\text{Hz}$  and an estimated flicker noise of  $-94$  dB  $\text{rad}^2/\text{Hz}$  at 100 Hz.

The VHF prototype, which operates at  $\nu_c = 100$  MHz, is similar to the microwave one, but it was able to be constructed after solving certain additional problems, mainly due to the variable phase shifters and to harmonic distortion. The latter turns into a relevant difficulty because most VHF components show a bandwidth of (2–3) decades, while the carrier suppression mechanism is effective only at  $\nu = \nu_c$  and the unwanted harmonics could be stronger than the residual fundamental. The carrier suppression of this NMS is about (65–70) dB, limited by the variable attenuators, which are based on a potentiometer. In fact, these attenuators seem to be less stable than those based on a movable absorbing surface; the latter type, unfortunately, is not available for the VHF. On the other hand, because the driving oscillator is less critical than in the other band, we adopted a commercial device [9], which shows a white noise floor of  $-157$  dB  $\text{rad}^2/\text{Hz}$  and a flicker noise of  $-127$  dB  $\text{rad}^2/\text{Hz}$  at 100 Hz. Operating with a DUT output power  $P_o = 8$  dBm,  $K_\varphi$  turns out to be 24 dB  $\text{V}^2/\text{rad}^2$ , not including the amplifiers ( $g_b = 20$  dB) at the output of the mixers. As design strategies and implementations are close to those of the single interferometers operating in the same bands, further details can be found in [10]. Because we are mainly interested in the sensitivity of our NMS's, we did all the noise measurements with a short cable replacing the DUT.

The microwave NMS was calibrated by means of a directional coupler inserted in the DUT path. After injecting an amplitude modulation through this coupler, we set  $\gamma_1''$  and  $\gamma_2''$  to null  $x_1$  and  $x_2$ , the latter being monitored by a lock-in amplifier; in this condition, both the arms of the NMS detect the PM noise. Then, the gain  $K_\varphi$  was measured by injecting a sideband of known amplitude in the coupler and measuring the amplitude of  $x_1$  and  $x_2$ . After calibrating, the coupler input port was connected to a matched termination. The VHF NMS was calibrated by means of a varactor phase modulator,

explicitly built for this purpose, inserted in the DUT path.  $\gamma_1''$  and  $\gamma_2''$  were set for the null of  $x_1$  and  $x_2$ , monitored with the lock-in. Then, by inspecting with a network analyzer,  $\gamma_1''$  and  $\gamma_2''$  were increased by  $90^\circ$ . Finally, the varactor of the phase modulator was replaced by a less noisy variable capacitor, set to the same value.

The results of noise measurement for the microwave NMS are shown in Fig. 3. Curves A and B are the  $S_{\varphi_1}(f)$  and  $S_{\varphi_2}(f)$  of the two individual interferometers, while curve C is the cross PSD, which represents the sensitivity of the NMS. Unfortunately, the asymptotic values in the rightmost part of the figure can only be inferred because the flicker is not negligible even at  $f = 100$  kHz, which is the maximum input frequency for the available correlator. Curves A and B approach values close to  $-180$  dB  $\text{rad}^2/\text{Hz}$ , which is the noise floor predicted by (2) evaluated for the described implementation. At the lowest frequencies, below 10 kHz, the NMS noise (curve C) is of the flicker type, showing a value of  $-167$  dB  $\text{rad}^2/\text{Hz}$  at  $f = 100$  Hz. Despite the use of the cryogenic oscillator, this flicker is probably due to the noise of the source instead of the NMS limit. In fact, we measured a source noise rejection of about 75 dB, and the measured flicker is close to the oscillator noise attenuated by this rejection ratio. The white noise of the instrument, which can only be inferred in the rightmost part of Fig. 3, is approximately  $-192$  dB  $\text{rad}^2/\text{Hz}$ .

Similar measurements, made with the VHF NMS, yield the results shown in Fig. 4, where curves A, B, and C have the same meaning as before. The white noise of the individual arms (curves A and B) is  $-172$  dB  $\text{rad}^2/\text{Hz}$ , close to the value predicted by (2). At the lowest frequencies,  $f < 1$  kHz, the NMS noise (curve C) is of the flicker type, showing a value of  $-171$  dB  $\text{rad}^2/\text{Hz}$  at  $f = 10$  Hz. In addition, the effect of acoustic vibrations is dominant in the region between 200 Hz and 2 kHz. The white noise floor is  $-194$  dB  $\text{rad}^2/\text{Hz}$ .

We wish to stress that no signal processing was done to mask any disturbance—like those due to acoustic vibrations, electromagnetic pollution, or the residual of the mains—before reporting the results shown in Figs. 3 and 4. The noise measurements were done placing the NMS's on an antivibrating table, of the same type of those commonly used for optics,

without using a shielded chamber. The above results were able to be obtained after a refinement of the mechanical assembly and a careful choice of the path of cables, ground, and power supply.

#### IV. DISCUSSION OF THE RESULTS AND CONCLUSION

Basically, the double interferometer noise floor is determined by the noise sources shared by the two arms. The most relevant of these sources are the power splitter internal resistors, whose noise  $k_B T_0$  is equally divided into the two outputs. In fact, the power splitters PS1–PS3, which are of the reactive type, are actually four-port 3 dB hybrid couplers with one input connected to a resistive termination ( $R_1$ ,  $R_2$ , and  $R_3$ , respectively). Let us first consider  $R_1$ . Because of the phase relationships shown in Fig. 1, the noise  $k_B T_0$  is equally divided into the two LNA inputs. Taking into account the losses, noise contributions  $k_B T_0 / (2\ell_s^2 \ell_a \ell_h)$  are expected at the two LNA inputs, fully correlated because they come from a single source. For similar reasons,  $R_2$  and  $R_3$  yield correlated noise. Summing the noise power due to  $R_1$ ,  $R_2$ , and  $R_3$  at the LNA inputs, we get a fully correlated noise  $k_B T_0 / (\ell_s^2 \ell_a \ell_h)$ ; the complement to  $k_B T_0$ , due to the individual arm losses, is uncorrelated. Taking into account the NMS gain (1), the expected noise floor of the double interferometer is

$$S_{\varphi_{\text{th di}}} \geq \frac{4k_B T_0}{P_o} \quad (3)$$

where the equality holds in the case of minimum losses. It should be observed that the lower bound given by (3) is 6 dB higher than the thermal noise limit  $S_{\varphi_{\text{th 0}}} = k_B T_0 / P_o$ . This is due to the power splitter PS3 and the hybrids, which limit to 1/4 the fraction of the DUT output noise that reaches the input of each LNA.

Because the transmission coefficients from the power splitter internal resistors ( $R_1$ ,  $R_2$ , and  $R_3$ ) to the LNA inputs are considered relevant to the comprehension of the noise floor, we used externally terminated 4-port hybrids as power splitters in the VHF NMS, and we measured these coefficients by means of a network analyzer. The transmission coefficients turn out to be  $-6$  dB from the  $R_1$  port to the LNA inputs, and  $-8.5$  dB from the  $R_2$  and  $R_3$  ports to the LNA inputs, in a close agreement to the expected values. The latter are based on the nominal losses  $\ell_s = 0.8$  dB,  $\ell_a = 1$  dB, and  $\ell_h = 0.8$  dB of the available devices.

Although the resistances  $R_1$ ,  $R_2$ , and  $R_3$  seem to give correlated contribution to the two arms of the NMS, the measured noise floor  $S_{\varphi_0}$  is lower than that predicted by (3). In fact, the microwave NMS shows a noise floor of  $-192$  dB  $\text{rad}^2/\text{Hz}$ , while (3) yields  $-183$  dB  $\text{rad}^2/\text{Hz}$  for  $P_o = 15$  dBm. Moreover, the VHF prototype shows a noise floor of  $-194$  dB  $\text{rad}^2/\text{Hz}$ , which is 18 dB lower than the value predicted by (3) for  $P = 8$  dBm. A theoretical explanation for the above result is still under development. Anyway, our results should be compared to those reported in [11], where another correlation-based NMS is described, showing an exceptionally low noise floor, of the order of  $-200$  dB  $\text{rad}^2/\text{Hz}$  at  $\nu_c = 5$  MHz. That scheme is different from ours, mainly because of the absence of the carrier suppression mechanism and because

the mixers are used as phase-to-voltage converters, saturated at both inputs. Although only a brief system description is given in [11], and the presence of thermal noise sources common to the two arms due to the power splitter internal resistors is not mentioned, the reported noise floor is some 10 dB lower than  $k_B T_0 / P_o$ .

To conclude, the double interferometer allows noise measurements with a lower instrument noise floor as compared to single interferometer systems. Our prototypes, although not optimized, prove the feasibility and the benefits of the proposed scheme in a wide frequency range. Further work is in progress, to give an adequate theoretical explanation for the reported results.

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