Phase Noise in the Regenerative Frequency Dividers

Enrico Rubiola, Marcel Olivier, and Jacques Groslambert

Abstract—**The aim of this paper is twofold: to give a theoretical analysis of the phase noise in regenerative dividers, and to provide new design rules for the best spectral unity. The spectral purity is, in fact, the reason why regenerative dividers may be preferred to simpler schemes whenever that characteristic is a fundamental requirement. Moreover, this class of dividers is suitable for high frequencies, out of reach to other techniques. The nucleus of our theory is the description of the mixer, driven by coherent signals, in terms of differential *phase gain*, so as to relate noise at the divider output to the noise generated inside the divider itself. After introducing a model of the mixer and some related measurement techniques, we show that the most favorable noise condition for the divider is reached by tuning it off the maximum output amplitude. Methods for approximating this condition are then outlined. Our experiments prove the feasibility of the proposed approach; reported results show a phase noise reduction of 10 dB with respect to the maximum amplitude condition.**

I. Introduction

FREQUENCY dividers seem to have recently become once again an interesting field of research. In frequency metrology, as research on frequency sources moves toward higher frequencies, low-noise dividers are of growing importance, especially if phase-locking techniques are used. However, the *noise* behavior of dividers is less well known than most of the other aspects of frequency synthesis.

Analog dividers show some advantages, compared with digital dividers, due to their higher limits to the input frequency and their lower noise. Data on digital dividers can be found in [4]. The regenerative divider seems to be a very low-noise scheme, very likely the best one.

In recent years monolithic implementation of the regenerative divider has been shown to be feasible, by resorting to various technologies [1]–[3], thus making this scheme more attractive.

Since the regenerative scheme first appeared in the literature a long time ago [5], [6], it has been extended to very low frequencies [7], and better explained in more recent papers [8], [9]. However, a few experimental results dealing with phase noise have been published, such as [10], [11], but no theory or heuristic explanation of the phenomenon has been found until now.

An overview of the frequency divider's operation is given in Section II, where analytical concepts are introduced. In Section III a mixer model is developed, based on experimental parameters and suitable for dealing with noise in dividers. The measurement techniques, discussed in Section IV, make this model a useful design tool for the phase noise theory developed in Section V.

It turns out that the contribution of the divider to the overall output phase noise can be significantly reduced with a new design approach, proposed in Section VI. This has been confirmed by experimental results (Section VII), obtained with a divider by two. A phase noise reduction of 10 dB from the classical to the new design approach has been observed.

Manuscript received June 20, 1991; revised December 11, 1991.
The authors are with the Laboratoire de Physique et Métrolgie des Oscillateurs (LPMO) CNRS, F-25000 Besançon, France.
IEEE Log Number 9107615.

II. Frequency Divider Operation

The block diagram of the regenerative frequency divider dividing by $N + 1$ is shown in Fig. 1, where most of the symbols used in this paper are also defined. The mixer is a nonlinear three-port device. The combined action of mixer and filter is such that at the input of the amplifier the frequency is:

$$f_o = f_R - f_L. \tag{1}$$

Because of the multiplier in the feedback path ($f_L = Nf_o$) the LO signal regenerates the IF one. Consequently, a synchronous mode of operation can be established with $f_o = f_R/(N + 1)$.

The signals at the mixer ports, *radio frequency* (RF) input, *local oscillator* (LO) input, and *intermediate frequency* (IF) output are the following:

$$\text{RF:} \quad x_R = A_R \cos(2\pi f_R t) \tag{2}$$

$$\text{LO:} \quad x_L(t) = A_L \cos(2\pi N f_o t + \Theta) \tag{3}$$

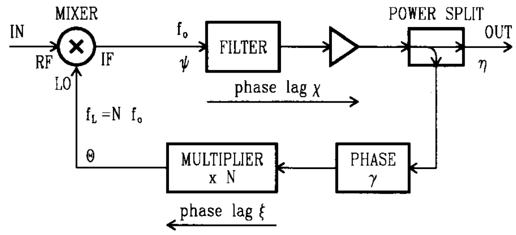
$$\text{IF:} \quad x_o(t) = A_o \cos(2\pi f_o t + \psi) \tag{4}$$

where A_o is a function of A_R and A_L. Since these signals are coherent, it make sense to speak about phase relationships between signals at different frequencies. Taking the RF signal as the zero phase reference, at the output of an ideal mixer (i.e., a perfect analog multiplier) the phase is:

$$\psi = -\Theta. \tag{5}$$

The closed-loop stability constraints, unity gain and zero phase ($+2k\pi$, with integer k) are easily met. The desired gain must be ensured by saturation either in the amplifier, or in the mixer, or in the frequency multiplier. The parameter Θ is a degree of freedom of the system, and it will satisfy the phase condition if jumps of multiples of 2π can be avoided.

0018-9456/92$03.00



LIST OF SYMBOLS

- ψ phase at the IF mixer output
- Θ phase at the LO mixer input
- η phase at the divider output
- ξ phase lag of the frequency multiplier, defined at the output frequency Nf_o
- χ phase lag of amplifier and filter
- γ includes all other phase lags, such as cables etc.

Fig. 1. Basic scheme of the regenerative frequency divider.

Regeneration usually starts from noise. The small-signal loop gain must be greater than one. This can be a critical constraint because some frequency multipliers have an input level threshold.

In the case of division by two ($N = 1$) especially, asynchronous modes of operation can arise, originating self-consistent oscillations incoherent with the input signal. A chaotic behavior is also possible, either alone or in conjunction with the proper dividing operation. These problems are discussed in a qualitative but clear way in [12].

An accurate analysis, based on *actual* mixers, shows that nonlinearity in these devices produces many harmonics of the input signals. Some combinations of harmonics contribute to the signal $x_o(t)$. As a consequence, (5) is no longer valid. The amplitude A_o and the phase ψ are also functions of Θ, affecting both the stability and the noise of the divider.

III. Mixers and Frequency Dividers

A comprehensive mixer model, including all schemes and technologies, can hardly be given in closed form. Therefore, we have chosen an experimental description tailored for the phase noise problem in the dividers.

When two signals of frequencies f_R and f_L are supplied at the inputs of a mixer, the output spectrum contains many signals, each one with its own amplitude, whose frequencies are:

$$f_{n,m} = nf_L + mf_R \tag{6}$$

where n and m are integers. Since in the divider we need a frequency difference, we choose n and m with opposite signs.

The closed-loop condition of the divider, $f_L = Nf_o$, imposes a selection on m and n. The output frequency f_o comes from all the *mixing modes* n, m satisfying the following relations:

$$n = N + h(N + 1) \tag{7}$$

$$m = 1 - N(h + 1) \tag{8}$$

where h is an integer. A further selection on n and m may arise from the mixer symmetry, if any. In the double-balanced devices used in the experimental part of this paper, the even harmonics are strongly attenuated by the symmetry of the inputs.

The IF signal, at the frequency f_o, is given by the following sum of harmonics:

$$x_o(t) = \sum_{n,m} I_{n,m} \cos(2\pi f_o + \beta_{n,m}) \tag{9}$$

where n and m are subject to the above-described selection rules; and I and β are amplitude and phase of the n, m mixing mode. In *actual* mixers, $I_{n,m}$ decreases rapidly with $|n|$ and $|m|$, with a law which depends on the particular mixer and on its working conditions. The strongest mixing mode is $n = -1$, $m = 1$.

Let us now focus on the phases $\beta_{n,m}$. Each term of the sum (9) comes from (6). This means that the LO frequency is multiplied by n, and, consequently, any phase shift of the LO signal is multiplied by n also. This can be formulated as:

$$\beta_{n,m} = n\Theta + \alpha_{n,m} \tag{10}$$

where $\alpha_{n,m}$ is the *phase offset* of each mixing mode.

In order to evaluate the phase ψ, and its nonlinear dependence on Θ, we introduce the concept of *significance* ($c_{n,m}$) associated with each mixing mode:

$$c_{n,m} = ni_{n,m}$$

referring to the normalized amplitudes $i_{n,m} = I_{n,m}/I_{-1,1}$.

Returning to (9), $x_o(t)$ is determined by a series of signals at the same frequency, to be dealt with as phasors: the main one ($n = -1$, $m = 1$), the first-order perturbing one ($n = n_p \neq -1$, and $m = m_p \neq 1$, with the highest $|c|$), and many second-order perturbing signals.

A phasor representation of the main signal and the first-order perturbing one on the complex plane gives an overview of what happens within the mixer. Equation (10) states that since these two phasors have different values of n, they rotate by different angles when a change is imposed on Θ. Two values of Θ are peculiar: where the phasors are parallel and antiparallel, Θ_A and Θ_B, respectively. Since the output signal is the sum of these phasors, the output amplitude has its maximum and minimum there respectively. Considering the special case of the divider dividing by two, in which the first-order perturbing signal is almost always $n_p = 3$, $m_p = -1$, Fig. 2 gives a plot of these two extreme situations. A small phase shift $d\Theta$ is added to Θ, and the corresponding shift $d\psi$ is observed at the output. In case A the signals are added:

$$d\psi = \frac{c_{-1,1} + c_{3,-1}}{1 + i_{3,-1}} d\Theta \tag{11}$$

while in case B the signals are opposite in phase:

$$d\psi = \frac{c_{-1,1} - c_{3,-1}}{1 - i_{3,-1}} d\Theta. \tag{12}$$

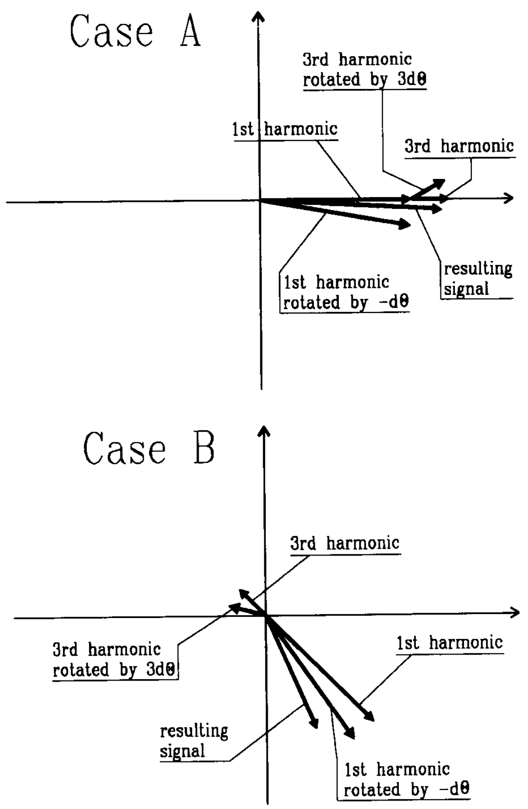


Fig. 2. Phasor representation of the two main signals contributing to the output frequency f_o in a divider by two. The effects of a perturbation $d\Theta$ imposed to the LO signal are considered for the two extreme situations: A) signals are in phase, the output amplitude is maximum, and the resulting perturbation is smaller than $d\Theta$; B) signals are 180° out of phase, the output amplitude is minimum, and the resulting perturbation is stronger than $d\Theta$.

In the general case, with other values of N but considering only the above-mentioned two phasors, the structure of formulas (11) and (12) is exactly the same except for the subscripts of coefficients, which indicate a different selection of the first-order perturbing phasor. Let us now add the second-order perturbing phasors, whose amplitudes decrease rapidly as $|n|$ and $|m|$ increase. As a consequence of a perturbation $d\Theta$, each of them rotates by $n\, d\Theta$ and gives its contribution to $d\psi$.

The derivative $G = d\psi/d\Theta$ can be interpreted as the differential *phase gain* of the mixer, defined for small perturbations. Thus, a linearization of ψ can be introduced as:

$$\psi(\Theta + d\Theta) = \psi(\Theta) + G\, d\Theta.$$

Equations (11) and (12), extended to other n and m, give the meaning of the dimensionless parameters $c_{n,m}$. The maximum capability of the n, m mixing mode for influencing G is proportional to $|c_{n,m}|$. Although G is usually a negative quantity, in some circumstances it can assume positive values. With reference to the example reported in Fig. 2, it happens that $G > 0$ in case A if the perturbing mixing mode is *more significant* than the main one; that is, if $|c_{3,-1}| > |c_{-1,1}|$. In an ideal mixer, without any contribution of higher-order harmonics, $\psi = -\Theta$, and it is always $G = -1$.

In the two-phasor model G is maximum at $\Theta = \Theta_A$, when the output amplitude is also at maximum. In the opposite situation, $\Theta = \Theta_B$, both G and the output amplitude A_o are at a minimum. In the general case, in which all the phasors are taken into consideration, the phase gain G *tends* to be maximum or minimum *near* the above-described conditions.

In some cases the hypothesis can be made that the terms $\alpha_{n,m}$ defined in (10) are zero. This means that all the mixing modes are in phase for $\Theta = 0$. If this happens, the knowledge of the amplitudes $I_{n,m}$ is sufficient for a complete characterization of the device.

In principle, there is no difference in the resulting behavior whether the harmonics actually originate inside the mixer, or come from the outside. Therefore, we can take into account distortions either present in the input signal or originating in the amplifier and the multiplier.

IV. Measurement Techniques for Mixers

Mixers are described in terms of phase gain G. In this section, we discuss three experimental techniques for characterizing these devices, as well as some experimental results. The methods proposed are equivalent. The first has some advantages of simplicity, while the other two are suitable for a wider frequency range.

At the mixer output there are many signals at frequencies other than f_o representing about half the total power. The strongest of these signals is at the frequency $f_R + f_L = (2N + 1) f_o$. When these signals are reflected back to the mixer, the nonlinearity is enhanced, thus affecting the phase gain. The common practice of matching the impedance for the main signal only can therefore lead to experimental errors. Performing measurements under the same conditions in which the mixer will operate is a safe rule.

A. First Method

The mixer is driven by two synthesizers whose frequency ratio is $(N + 1)/N$, both phase locked to the same reference (Fig. 3). In this scheme the mixer is used in the same way as in the divider, since LO and RF signals are synchronous, with the proper frequency ratio, and the IF output gives the frequency f_o. The two dividers, by N and by $N + 1$, permit phase measurements at the same frequency f_o.

Taking the phase of the RF signal as a reference, vector voltmeters provide A_R, A_L, Θ, A_o, and ψ. The phase Θ of the LO signal is shifted manually during the experiment, by adjusting one synthesizer (bottom left of Fig. 3). This permits the measurement of A_o and ψ as functions of Θ, and, consequently, G.

B. Second Method

The signal $x_o(t)$ can be found from (9) if the amplitudes $I_{n,m}$ are known. Using the same equation (9), A_o, ψ, and

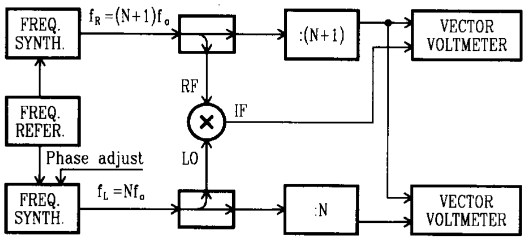


Fig. 3. Basic measurement method for mixers characterization (first method).

G can be easily calculated. Following this method, we assume that all mixing modes contributing to f_o are in phase for $\Theta = 0$.

If a small relative frequency offset ϵ is imposed on the LO signal ($f_L \rightarrow f_L(1 + \epsilon)$, where $\epsilon << 1$), the mixing modes are split around f_o. Each one row appears as a separate spectral line whose power is proportional to $I^2_{n,m}$ and shifted by $n\epsilon f_o$ from f_o. This happens because the n, m mode implies a frequency multiplication by n.

In this method the mixer is driven by two synthesizers whose frequencies are $(N + 1)f_o$ and $Nf_o(1 + \epsilon)$. The frequency ratio is *almost* $(N + 1)/N$. A spectrum analyzer measuring the IF output provides the amplitude $I_{n,m}$ of each mode.

This method relies on the hypothesis that the mixer has a sufficient bandwidth, and that therefore the amplitudes $I_{n,m}$ are not affected by the small relative frequency offset ϵ.

C. Comparison Between the Methods

A double-balanced mixer TFM-10514 has been tested, simulating a frequency divider by two. All the parameters (A_o, ψ, and G) have been measured with the first method, and evaluated with the second, taking into account all mixing modes down to -50 dB below the main one, in the following conditions:

LO input 235 MHz +10 dBm

(add 20 kHz for the second method)

RF input 470 MHz +14 dBm.

From the results, reported in Fig. 4, the evaluated average amplitude is 0.2 dB higher than the measured amplitude. Subtracting this bias, the amplitude difference is within ± 0.3 dB.

Comparing the two plots of ψ, there is a systematic difference of 13.5°, with variations of $\pm 3°$ (peak).

Measured and calculated phase gains G differ by about 0.2, excluding the narrow regions around the minima, but these regions are not meaningful for our purposes because the amplitude A_o has its minimum as well. This extreme condition should be avoided in the divider because keeping a stable operation is difficult. In this specific condition the two methods are equivalent.

D. Third Method

The mixer is driven by two synthesizers whose frequencies are $(N + 1)f_o$ and Nf_o, as shown in Fig. 5. A small auxiliary signal at the frequency $Nf_o(1 + \epsilon)$ (where $\epsilon << 1$) is injected at the LO input with a power ratio $\rho << 1$. The resulting signal can be interpreted as a sum of two components of equal power which can be considered as amplitude and phase modulations. The latter has a modulation index given, in radians, by $M_0 = \sqrt{(1/2)\rho}$. Assuming that the mixer input is saturated, or that an amplitude limiter is inserted, only the phase modulation is present at the output, with a modulation index M. Since the spectrum analyzer gives no hint concerning the sign, it is necessary to know *a priori* that $G < 0$ before concluding that $G = -M/M_0$. A quick inspection with the second method is therefore helpful.

V. Phase Noise Theory

A common practice is to describe a sinusoidal signal with *weak* narrow-band noise as:

$$v(t) = V_0[1 + \epsilon(t)] \sin [2\pi\nu_0 t + \phi(t)]$$

where $\epsilon(t)$ and $\phi(t)$ represent the amplitude and phase noise. A description of the phase fluctuations of $v(t)$ will be given by means of $S_\phi(f)$, defined as the unilateral power spectral density of $\phi(t)$.

In an ideal divider by $N + 1$, the phase noise power spectral density at the output is the same as that of the input signal, divided by $(N + 1)^2$. This is equivalent to stating that the output time jitter is the same as for the input signal. While it is impossible to overcome this limit, an actual divider adds its own noise contribution to the signal.

This noise is generated by the electronic circuits contained in the divider. We take into consideration especially the amplifier and the frequency multiplier because, in our experience, they are the most significant sources of phase noise. Other noise sources can be included in the model of one of them.

Being interested in the phase noise close to the carrier, the measurement bandwidth of the phase noise will be assumed to be small compared to the divider bandwidth and to the carrier. Because of this hypothesis, a static analysis is valid. In fact, the period of the highest Fourier frequency f is long, in comparison with the signal transit time around the loop.

Using the symbols introduced in Fig. 1, we start the analysis of the divider by considering the phase noise at the amplifier input as a small perturbation $d\chi$ to the amplifier phase lag χ. Similarly, a perturbation $d\xi$ affects the phase lag ξ of the frequency multiplier. Both statistically independent perturbations affect the output phase η.

The closed-loop phase condition of the divider (see Fig. 1) is:

$$N(\psi + \chi + \gamma) + \xi = \Theta \tag{13}$$

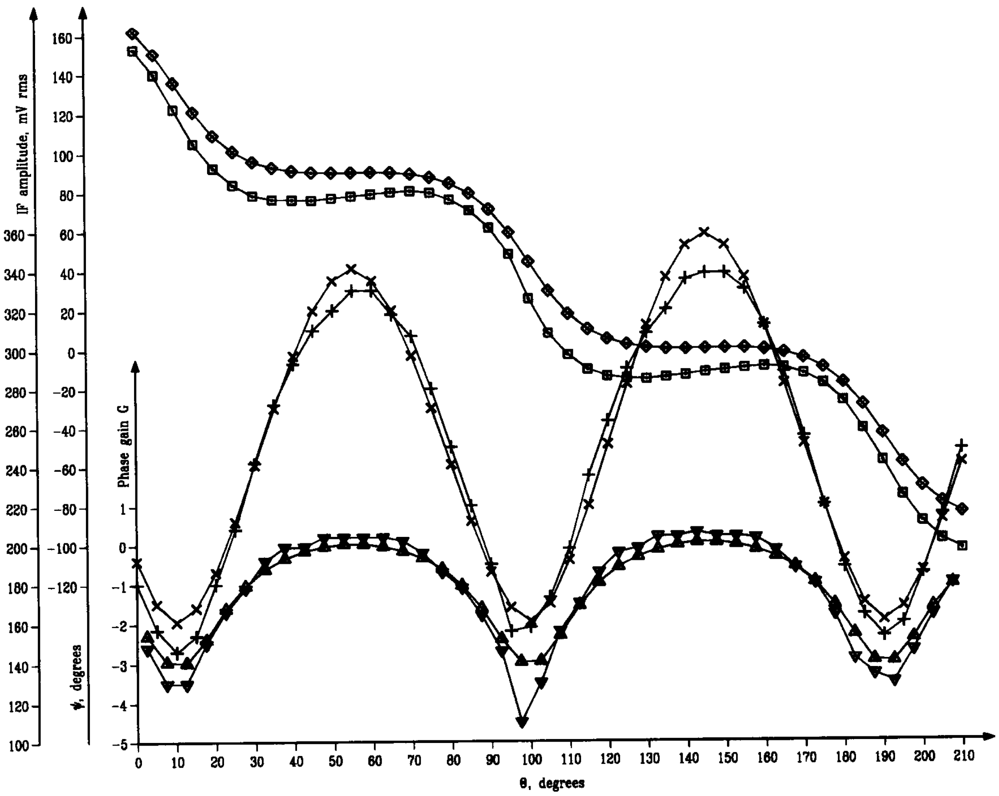


Fig. 4. Comparison between measured (first method) and evaluated (second method) parameters of a double balanced mixer. × calculated amplitude, + measured amplitude, ◇ calculated phase, □ measured phase, ∇ calculated phase gain, Δ measured phase gain.

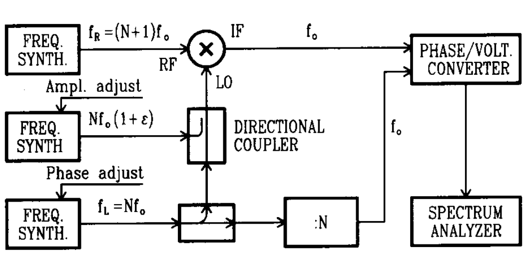


Fig. 5. Direct phase gain measurement scheme (third method).

and the output phase η can be written in two ways:

$$\eta = \frac{\Theta - \xi}{N} - \gamma \tag{14}$$

$$\eta = \psi + \chi. \tag{15}$$

The contribution of the *amplifier* can be found by differentiating (13) and assuming: (i) $d\gamma = 0$, because γ is a constant, and (ii) $d\xi = 0$, because the multiplier is here assumed noiseless. It is then:

$$N\,d\psi + N\,d\chi = d\Theta. \tag{16}$$

Differentiating (14), with the same hypothesis, we get:

$$d\eta = \frac{1}{N}\,d\Theta. \tag{17}$$

Combining (16) and (17), and remembering that $d\psi = G\,d\Theta$, we find:

$$\frac{d\eta}{d\chi} = \frac{1}{1 - NG}. \tag{18}$$

The contribution of the *frequency multiplier* can be found in a similar way, assuming $d\gamma = 0$ and $d\chi = 0$. Thus, differentiating (13) and (15) we obtain:

$$N\,d\psi + d\xi = d\Theta \tag{19}$$

$$d\eta = d\psi \tag{20}$$

and combining these results, we obtain:

$$\frac{d\eta}{d\xi} = \frac{G}{1 - NG}. \tag{21}$$

Equations (18) and (21) lead to the final result:

$$S_\eta(f) = \left[\frac{1}{1 - NG}\right]^2 S_\chi(f) + \left[\frac{G}{1 - NG}\right]^2 S_\xi(f) \tag{22}$$

where $S_\eta[f]$, $S_\chi(f)$, and $S_\xi(f)$ are the above-defined phase noise power spectral densities for η, χ, and ψ. $S_\chi(f)$ and $S_\xi(f)$ come from the device characterization, while $S_\eta(f)$ represents the phase noise generated by the divider.

We obtained the result (22) by taking into account the instantaneous phase noise value. There is no reason, in principle, to restrict its validity to some particular types of noise, such as the flicker or the white floor.

VI. Low-Noise Design

If the mixer can be considered an ideal device, as follows from (22), the divider attenuates $S_\chi(f)$ and $S_\xi(f)$ by the factor $(N + 1)^2$. When these power spectra are converted into time jitter, the latter turns out to be divided by $N + 1$. This is an advantageous property of the divider loop.

Now we consider actual mixers, whose behavior is perturbed by harmonics.

The classical approach is to tune the loop phase lag (γ) of the divider for maximum output amplitude. This choice *seems* to be the best one, since it ensures that: (i) the largest operating bandwidth is obtained, (ii) stability and startup conditions are easily met, and (iii) a chaotic behavior is rare, etc. However, remembering the results given in Section 4, when the amplitude A_o reaches its maximum, G is also close to its maximum. According to (22), this is the *worst* case for spectral purity. Phase noise is enhanced.

Instead, an optimized version of the divider requires minimum G. This occurs when the output amplitude is minimum. Obviously, all of the previously described advantages would be lost in this case. We can expect that the best divider will be a compromise taking advantage of a reduction in G.

A complete characterization of the mixer is therefore needed in order to know $A_o(\Theta)$ and $G(\Theta)$ for various values of A_R and A_L. The result of this first step will be a series of plots similar to those shown in Fig. 4.

Two alternative design criteria can be adopted in order to choose the mixer working point:

1. ensure $G \simeq -1$ for any Θ, or
2. try to adjust the divider for $G < -1$.

The first approach is simpler, and therefore recommended for low-noise, wide-band applications. It implies $|c_{n,m}| \ll 1$ for all mixing modes except the main one. A helpful, but not sufficient, condition is that the mixer work below the saturation level. In some cases, filter and amplifier (Fig. 1) can be interchanged in order to improve impedance matching, thus keeping the mixer working point more constant in the operating bandwidth.

For extremely low-noise applications, the second approach is the most attractive. A careful design is needed around a strongly saturated mixer. The condition $G < -1$ can be obtained in a subrange of the possible values of Θ, in partial conflict with other design rules. Unless a cumbersome filter is used, a compromise will be achieved only in a narrow band.

Any configuration in which the mixer is strongly perturbed by harmonics, and no consideration is given to G, should be definitively avoided. A rough, but quick, estimate of the range in which G falls can be obtained with the method described in Section IV.B, by taking into account only two mixing modes, the main and the first-order perturbing ones. This quick evaluation is recommended in any case.

In the next part of this section we will consider the second approach.

A. Parameter Adjustment

The main technique of acting on G is by adjusting γ. This *detunes* the divider out of the maximum amplitude working point.

The noise of the amplifier should be analyzed, since it generates phase noise. In many cases, the amplifier noise turns out to be independent of signal power. In this condition, the amplifier phase noise will decrease when the signal power is increased. This is not necessarily true if the amplifier is pushed into saturation, because it tends to keep its output amplitude constant. Similar considerations hold for the frequency multiplier.

The range of G limits the improvement that can be achieved. A proper design of the filter at the mixer output could enhance this range, as was shown in Section IV.

B. Analytical Approach

The analytical approach is based on the complete characterization of all the devices in the divider. The principal tool is the plot of $G(\Theta)$ and $A_o(\Theta)$.

The divider is initially set for its maximum output amplitude on the G and A_o plots, as in the classical design. Then it is detuned by small steps towards the minimum G, evaluating the phase noise at each step. As the divider is progressively detuned, its behavior is as follows: (i) A_o decreases more rapidly than G (see Fig. 4), but the saturation of the circuits keeps the divider output amplitude about constant; (ii) the output amplitude is slightly reduced (a few dB), but G decreases significantly, thus reducing the phase noise, which is the compromise desired; (iii) the output power is significantly reduced and the phase noise increases again; and (iv) the divider stops working.

C. Experimental Approach

From the operational point of view, the most important result is that the divider must be detuned in order to improve its noise performance.

Phase noise measurements usually require the comparison between two equal devices driven by the same source. A purely experimental procedure in which two equal prototypes are adjusted by steps towards the lowest-noise working point appears cumbersome. However, remembering that the phase noise and modulation are equivalent under our hypotheses, it is still possible to adjust and evaluate a *single* divider.

This method is based on the comparison between the divider under test, in which a phase modulation is injected, and a reference divider. We measure the rejection of *phase modulation* at the divider output *instead of* phase noise.

This technique should be considered with some reservations, since it is based on the phase gain, without allowing for possible noise variations with signal level in the various devices. Despite the above-mentioned limitation, this approach proved to be a useful tool in the experimental part of this work.

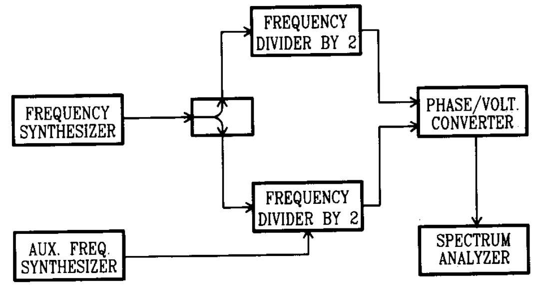


Fig. 6. Phase noise measurement technique. A phase modulation injected at the amplifier input is used to demonstrate the divider's capability for attenuating a phase perturbation, and consequently the noise.

VII. Experimental Results

In this section the experimental proof of the phase noise theory will be discussed, following two different approaches.

We have carried out some experiments with two 470-MHz dividers by two. The significant components in our prototypes are the following:

mixer	TFM-10514	Mini Circuits
amplifier	AMP-77	Mini Circuits
filter	bw $\simeq$ 10 MHz	house built.

The goal of this section is *not* a minimum noise divider, but the experimental verification of the noise reduction that can be obtained by exploiting the conclusions of the previous section.

Two situations are compared in which the divider is:

- tuned for the maximum output amplitude,
- detuned adding to the phase lag γ of delay equivalent to 50°.

The detuning phase shift has been found after several attempts, searching for the best compromise between stability and minimum noise.

A. First Method

In this experiment, according to the suggestions given in Section IV.C, we measure the divider's capability for rejecting a phase modulation inserted in the loop.

This is done, as shown in Fig. 6, by adding a small auxiliary signal at the amplifier input with a frequency offset of 15 kHz, which induces a phase modulation, plus an amplitude modulation rejected by the saturated amplifier, as explained in Section IV.D. The phase modulation at the divider output is measured with a phase-to-voltage converter and a spectrum analyzer, and it appears as a spectral line at 15 kHz.

Comparing the results of the two above-mentioned situations, in which the divider is tuned or detuned, we measure a phase excursion smaller by 8.5 dB when the divider is detuned. Thus, we expect a phase noise reduction of the same order as a consequence of the detuning operation.

B. Second Method

A more conclusive experiment consists of the measurement of the phase noise spectral density $S_\eta(f)$ at the divider output in the two cases, with the divider tuned and detuned. This experiment is complicated by the unavailability of a noiseless source or a noiseless divider to be used as reference. Thus, we compare two equal dividers with the same setup of Fig. 6, except for the presence of the modulating signal generated by the auxiliary synthesizer.

Since in our dividers most of the phase noise originates in the amplifiers, we have selected two equally noisy devices, verifying that their phase noise spectral densities $S_\chi(f)$ remain the same, in spite of the amplitude variation due to the detuning procedure. We have chosen as the noise test source the phase flicker of the amplifiers (-125 dB rad^2/Hz f^{-1}), because it is the only type of noise significantly higher than the limit of the phase noise measurement equipment (-139 dB rad^2/Hz f^{-1}).

The results are shown in Fig. 7(a) (tuned for the maximum amplitude) and (b) (detuned), together with the test equipment limit (c). Spurious lines, mainly odd harmonics of the 50-Hz mains, are present in these spectra.

Comparing 7(a) and (b), a reduction of the flicker noise is evident. Taking into account the contribution of the test equipment, which raises by 0.3 dB the flicker of (a) and by 1.8 dB the flicker of (b), the actual improvement is 10 dB. This result can be considered to be in good agreement with the previous one.

As for the white phase noise, we expect a similar result. An improvement is evident, but a quantitative evaluation is not possible. Since, for white noise, Fig. 7(b) overlaps exactly the test equipment limit (c), one can infer that the noise generated by the dividers is at most 5–6 dB below the equipment limit. If this hypothesis is true, the improvement measured for the flicker is also confirmed for the white noise.

The noise of one divider, tuned and detuned, can be obtained by subtracting 3 dB from 7(a) and (b), because both circuits have the same scheme, and components have been selected to be equally noisy. In Table I our results are compared with similar ones found in the literature.

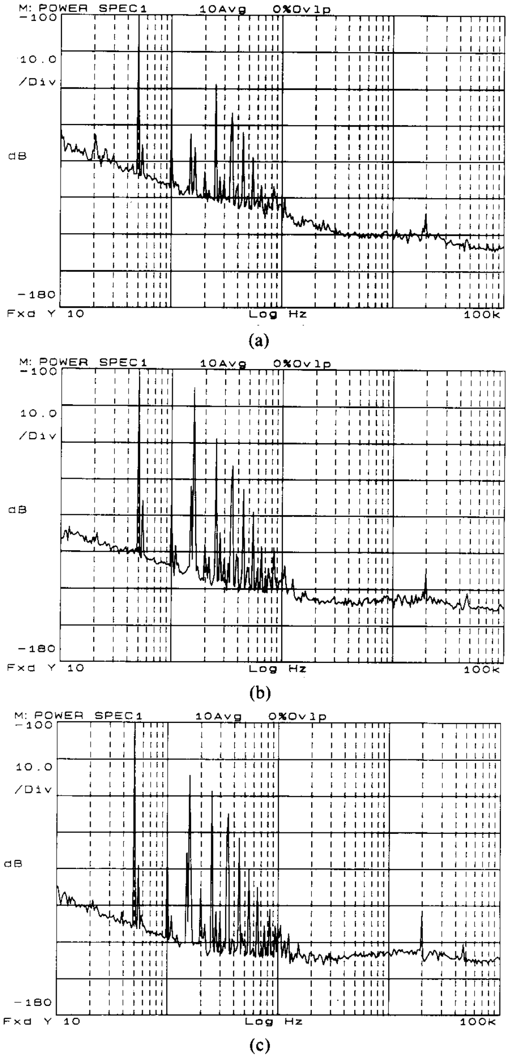


Fig. 7. Phase noise of two equal dividers. All values are given in dB below 1 rad^2/Hz. Top: (a) the dividers are tuned for the maximum output amplitude (usual approach). Center: (b) the dividers are detuned, improving their noise performances (new method). Bottom: (c) measurement equipment limit.

TABLE I
COMPARISON OF THE PHASE NOISE FOR ONE REGENERATIVE FREQUENCY DIVIDER. ALL VALUES ARE IN dB BELOW 1 rad^2/Hz

Reference	flicker	white
[11]	−128 dB f^{-1} ¶	−165 dB f^0 ¶
our divider (tuned)	−130 dB f^{-1}	−165 dB f^0
our divider (detuned)	−140 dB f^{-1}	−175 dB f^0 §
¶equivalent noise at $f_o = 235$ MHz		§inferred

The two measurement methods, under our hypotheses, can be considered basically equivalent from both a theoretical and an experimental point of view. The latter is a *true* noise measurement, and, consequently, it gives a conclusive result. However the former, as pointed out in Section IV.C, is far more useful for searching for the best working point of the divider.

VIII. CONCLUSIONS

In low-noise radio frequency applications it is convenient to work with high-level signals, of the order of 10 dBm. In this condition, mixers produce a strong distortion responsible for the phase gain phenomenon. The mixer can amplify or attenuate a phase perturbation due to noise. The regenerative frequency divider is, in a certain sense, similar to a linear phase control system, in which the noise is attenuated by negative feedback. With few restrictions on bandwidth and on Fourier frequency range, a static analysis is sufficient.

From our point of view, the best divider tuning condition is in partial disagreement with the usual design criteria, which favor the output amplitude. In some cases, by adjusting the divider off the maximum output amplitude working point, it is possible to reduce phase noise. This is predicted by the theory developed here and has been verified experimentally. In our experiments a careful choice of time delays inside the loop allows, in fact, an improvement of 10 dB in the phase noise of the divider.

Mixers are very complex devices, not sufficiently well understood in the special case of high-level synchronous input signals, as is the case in regenerative dividers. Further analysis is encouraged.

Our study, and some of the experimental techniques suggested here, seem suitable at any frequency for amplifiers and mixers, up to optical frequencies.

REFERENCES

[1] K. Kanazawa, M. Hagio, M. Kazamura, and G. Kano, "A 15-GHz single-stage GaAs dual-gate FET monolithic analog frequency divider with reduced input threshold," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 1908–1912, Dec. 1988.
[2] R. H. Derksen, H. M. Rein, and K. Wörner, "Monolithic integration of a 5.3 GHz regenerative frequency divider using a standard bipolar technology," *Electron. Lett.*, vol. 21, pp. 1037–1039, Oct. 24, 1985.
[3] T. Ohira, K. Araki, T. Tanaka, and H. Kato, "14-GHz band GaAs monolithic analogue frequency divider," *Electron. Lett.*, vol. no. 22, pp. 1057–1058, Oct. 24, 1985.
[4] W. F. Egan, "Modelling phase noise in frequency dividers," *IEEE Trans. Ultrason. Ferroelectrics and Frequency Control*, vol. 37, pp. 307–315, July 1990.
[5] R. C. Miller, "Fractional-frequency generators utilizing regenerative modulation," *Proc. IRE*, vol. 27, pp. 446–457, July 1939.
[6] R. L. Fortescue, "Quasi stable frequency dividing circuits," *J. Inst. Elec. Eng. (U.K.)*, vol. 84, pp. 693–698, 1939.
[7] E. Gatti, "Demoltiplicatore quasi aperiodico per basse frequenze acustiche," in *Publicazioni dell'Istituto Elettrotecnico Nazionale Galileo Ferrais (I.E.N.)*, no. 208, pp. 295–301, Torino, Italy, 1948. Reprinted from *Atti del Congresso Marconiano*, Roma, Italy, Sept. 28–Oct. 5, 1947.
[8] G. Immovilli and G. Mantovani, "Analysis of the Miller Frequency divider by two in view of applications to wideband FM signals," *Alta Frequenza*, vol. 17, no. 11, pp. 313–323, Nov. 1973.
[9] R. C. Harrison, "Theory of regenerative frequency dividers using double balanced mixers," *IEEE Trans. Microwave Theory Tech.*, vol. 1, pp. 459–462, June 1989.
[10] M. M. Driscoll, "Phase noise performance of analog frequency dividers," *Proc. 43th Annual Frequency Control Symposium*, 1989, pp. 342–348.
[11] M. M. Driscoll, "Phase noise performance of analog frequency dividers," *IEEE Trans. Ultrason., Ferroelectrics and Frequency Control*, vol. 37, pp. 295–301, July 1990.
[12] S. V. Ahamed, J. C. Irving, and H. Seidel, "Study and fabrication of a frequency divider-multiplier scheme for high efficiency microwave power," *IEEE Trans. Commun.*, vol. COM-24, pp. 243–249, Feb. 1976.