The Origin and the Measurement of Phase Noise in Oscillators

Enrico Rubiola
FEMTO-ST Institute, Besancon, France, and INRiM, Torino, Italy
Home page http://rubiola.org

The scientific and technological interest for lowest noise oscillators and lasers is ever growing because phase noise relates to the time fluctuation. In this lecture, we go through the origin of phase noise in oscillators and in resonators, and the measurement techniques.

Letting $V_0[1+\alpha(t)]\cos[2\pi\nu t+\varphi(t)]$ the clock signal, $\alpha(t)$ is the random fractional-amplitude fluctuation, and $\varphi(t)$ is the random phase. Such noise is properly described in terms of the power spectral $S_\alpha(f)$ and $S_\varphi(f)$, while industries prefer $L(f) = (1/2)\, S_\varphi(f)$. The Allan deviation $\sigma_{\mathrm{y}}(\tau)$ is also often used, for slower fluctuations and for timekeeping.

The Leeson effect is a system-oriented heuristic approach which explains how the noise of the electronics turns into the AM and PM noise of the oscillator, and how the AM noise – often neglected – impacts on PM noise. Being broader than just a model, the Leeson effect applies also to lasers and to microwave-photonic oscillators. The analysis relies on a low-frequency space, where cyclostationary noise of parametric nature is easily represented as additive noise. The resonator is seen a low-pass filter, or a delay line in the case of lasers and photonic oscillators. The same framework helps to understand the Pound and the PDH frequency control used to stabilize lasers and microwave oscillators to a reference cavity, ending in a stability of 10^{-15} ... 10^{-16}.

However, the Leeson effect fails in a number of practical cases, where the frequency flicker ($1/f^3$ PM noise) is set by the flicker ($1/f$) fluctuation of the natural frequency. Flicker noise in the resonators is understood only in some cases. A theory starts from the fact that mechanical dissipation in low-loss condensed matter is of structural nature, as opposed to viscous nature. The dissipation shows up as a hysteresis cycle which keeps its aspect ratio from microscopic scale to microscopic scale. Combining structural dissipation with the fluctuation dissipation theorem predicts successfully the stability of Fabry-Perot etalons made of ULE glass or monocrystalline Si, of the order of 10^{-15} ... 10^{-16}. Unfortunately, the same theory fails to explain the stability of quartz resonators (10^{-12} ... 10^{-13}). A heuristic theory, suggesting that flicker is proportional to the reciprocal of the volume, applies to the resonator's frequency fluctuations and to the phase fluctuations of two-port devices.

Virtually all modern phase-noise analyzers exploit two separate channels, each consisting of a detector and a frequency reference, that measure simultaneously the oscillator under test. Under the hypothesis that the two channels are independent, the cross spectrum converges to the oscillator's $S_\varphi(f)$, or $S_\alpha(f)$. The large memory and computing power of nowadays digital electronics give the false impression of high rejection of the single-channel noise, while the actual limit is set by the systematic (B-Type) uncertainty, generally not documented. Gross errors and nonsensical results are around the corner.