Abstract—In this paper, we report the characterization results of our cryogenic sapphire oscillators. The oscillator incorporates a sapphire resonator cooled down at the liquid helium temperature in a cryocooled ultra-low vibration cryostat. The phase noise of a single CSO is $-104 \text{ dBc/Hz}$ at 1 Hz offset of the 10 GHz carrier. The frequency fluctuations were measured with three frequency counters using different statistical algorithms. The best result at 1 s integration time is $\sigma_v(1\text{ s}) = 6.5 \times 10^{-19}$. For the integration time above 10 s, the Allan deviation of a single CSOs computed from each data sets reaches a floor around $3.2 \times 10^{-16}$ at 100 s integration time for a daily frequency stability of $3.5 \times 10^{-15}$.

I. INTRODUCTION

The preliminary measurement of two identical Cryogenic Sapphire Oscillators was realized in the frame of the ULISS project and presented during the joint meeting UFFC/EFTF in Prague in 2013 [1]. Since, the two instruments have been improved, leading today to an unprecedented frequency stability better than $1 \times 10^{-15}$ between 1 s and 10 000 s integration times (one unit floor: $\sigma_v(100\text{ s}) = 3.2 \times 10^{-16}$) with a frequency stability of $3.5 \times 10^{-15}$ per day. The frequency synthesis provided from the CSO signal the useful frequencies (10 GHz, 1 GHz, 100 MHz) has been also improved and completely characterized. The feedbacks from the ULISS Odyssey experience, where the transportable CSO was successively tested in few laboratories around Europe, allowed us to understand the main limitations in the oscillator performances. Thus, the thermal configuration of the cryogenic resonator was modified to increase the rejection of the temperature modulation arising form the Pulse-Tube Cryocooler. We conducted careful measurements to better understand the power sensibility of the cryogenic resonator [2]. The measurement campaign of these ultra-stable oscillators also allowed us to compare different commercial frequency counters.

II. TECHNOLOGY

A. Cryogenic sapphire oscillator

The cryogenic sapphire oscillator is composed of a cylindrical sapphire crystal resonator cooled down at the liquid helium temperature in a cryocooled low-vibration cryostat. The loaded Q-factors shown by the sapphire modes reach 1 billion at 4K when excited on whispering gallery modes at X-band frequencies. The resonator is temperature controlled at its frequency-temperature turnover point (around 6 K) and integrated into a Pound-Galani oscillator loop. The oscillator resonance is locked on the ultra-narrow resonance of the sapphire resonator. The power injected into the resonator is also controlled at the frequency-power turnover point [2].
B. Measurement setup

Two equivalent CSOs, called Uliss and Marmotte, are compared (figure 2). However, the loaded Q-factors of the two sapphire resonators differ. The loaded Q-factor of the sapphire Marmotte is about 1 billion compared to 350 million for Uliss. The coupling of the resonator Uliss is not optimized which justifies the difference. The beatnote frequency resulting from the mixing of the two output signals is around 7 MHz. This signal output is connected to a frequency distribution amplifier manufactured by Timetech [6]. The isolation of each outputs is better than 120 dB and makes negligible the cross-talks between the different instruments. The frequency reference is a BVA quartz manufactured by Oscilloquartz [7]. The reference signal is also distributed by the same frequency distribution amplifier. The beatnote frequency is counted by three different instruments: the Agilent 53230A, the Symmetricom 5125A and the K+K FXE SCR.

b) Agilent 53230A: is a $\Lambda$-counter [8]. In order to improve the instrument resolution, the counter involves multiple averaging within the gate time $\tau$ approximated by a $\Lambda$-estimator. In fact, the standard deviation $\sigma_\Lambda(\tau)$ slightly differs from the “true” Allan deviation $\sigma_\alpha(\tau)$. Dawkins [9] estimates the relation between $\sigma_\Lambda$ and $\sigma_\alpha(\tau)$ for each kind of noises. The 53230A is the new version of the commonly used Agilent 53132A. The improvement is the reducing of the residual input jitter to 20 ps which increases the frequency measurement resolution of one more digit than the previous model. Moreover, a new acquisition mode allows a measurement without dead time.

c) Symmetricom 5125A: is a phase noise and Allan deviation test set [10]. The phase noise measurement does not require that the device under test (DUT) and the frequency reference are phase-locked. The reference and the DUT outputs are directly converted to digital after anti-aliasing filtering. The down-conversion and the phase detection are digitally made which allows to measure both phase noise and frequency stability simultaneously. Moreover, the instrument uses cross-correlation to enable the test set noise to be below the noise floor of a single channel.

d) K+K FXE SCR: is a multi-channel phase recorder [11]. The instrument use the picket fence measurement method [12]. The time intervals between the picket fence derived from the frequency reference and the next front of the input signal after the strobe is measured. Counting all the pulses of the input signal, it is possible to determine with an high resolution the frequency of the device under test without the error produced by a synchronisation delay.

III. RESULTS

A. Phase noise spectrum

The phase noise spectrum of two CSOs is traced on figure 3.

![Two CSOs phase noise spectrum measured with the Symmetricom 5125A test set](image)

The spurious spikes present in the decade 1 Hz-10 Hz are generated by the residual mechanical vibrations of the cryocooler transfer to the sapphire. Other spurious spikes are mainly due to the electrical network 50 Hz and its harmonics. Moreover, we notice few humps on the spectrum at low Fourier frequencies generated by the temperature control of Uliss and by the temperature modulation due to the air conditioning system. The results for one CSO are in table I.

<table>
<thead>
<tr>
<th>Noise type</th>
<th>Asymptotic value</th>
<th>calculated $\sigma_\alpha(\tau)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>white phase</td>
<td>$-145$ dBc/Hz</td>
<td>not calculated</td>
</tr>
<tr>
<td>white frequency</td>
<td>$-104$ dBc/Hz</td>
<td>$6.3 \times 10^{-16}$</td>
</tr>
<tr>
<td>flicker frequency</td>
<td>$-115$ dBc/Hz</td>
<td>$2.1 \times 10^{-16}$</td>
</tr>
</tbody>
</table>

TABLE I. PHASE NOISE RESULTS OF ONE CSO

The results are comparable with the state-of-the-art microwave oscillator from the University of Western Australia [13] and the 10 GHz microwave signal generated from a femto comb from the National Institute of Standards and Technology [14].

B. Frequency stability

The Allan deviation of two CSOs calculated from the data of one week acquisition of the three instruments are traced on figure 4. Any post-processings were applied to the data sets.

We notice that the Allan deviation computed from the three instrument data sets are equals for integration time above 10 s. One CSO frequency stability reaches a floor around $3.2 \times 10^{-16}$ at 100 s integration time which differs from the phase noise conversion reported in table I. The increase of the floor is
The results given by the Symmetricom 5125A differ from the results obtained with the two other instruments. The 5125A allows to reduce the measurement bandwidth at 500 mHz. In this condition, the power density of noise is greatly reduced and the Allan deviation at 1 s integration time is about $\sigma_\alpha(1 \text{ s}) = 6.5 \times 10^{-16}$ for one CSO which is consistent with the value determined from the phase noise measurement (cf. table I). However, the humps generated by the temperature controller degrade the frequency stability which does not follow a $\tau^{-1/2}$ slope for other short integration times. For the two other instruments, we notice that the standard deviation values computed from the Agilent counter differ to a factor 1.2 from the K+K ones due to a $\Lambda$-estimator. This factor is compatible with the factor calculated in [9].

### IV. Conclusion

In this paper, we reported an Allan deviation better than $1 \times 10^{-15}$ from 1 s to 10 000 s integration times measured with three frequency counters. The instruments based on different kind of measurement scheme were used to record the frequency fluctuations of our CSOs: the Agilent 53230A, the Symmetricom 5125A and the K+K FXE SCR. We have shown that the Allan deviation computed from the three data sets differ at 1 s to 10 s integration times but are equivalents for higher $\tau$. The Symmetricom 5125A gave the best results by reducing the bandwidth analysis at 500 mHz. The K+K FXE SCR gives equivalent frequency stability than the 5125A taking into account the wider input noise bandwidth. Finally, the Agilent 53230A data are the most biased. Due to the approximation by a $\Lambda$-estimator, the standard deviation $\sigma_\alpha(\tau)$ results are slightly degraded compared to the $\sigma_\alpha(\tau)$ obtained with the data set of other counters. However the factor found between the result values is coherent with the factor calculated by Dawkins. In presence of modulations, the factor ratio is not applicable and makes the data analysis more difficult. In conclusion, the counter 53230A is the cheapest and the most usefull frequency counter with its clear user/instrument interface but its measurement algorithm makes the result interpretation quite difficult and tendencious. The K+K FXE SCR and Symmetricom 5125A, by recording the phase fluctuations, give a true image of the CSO performances.

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**REFERENCES**


