

# Phase Noise in RF and Microwave Amplifiers

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This article is a short introduction to the main results

The full manuscript [1] is available on <http://arxiv.org/abs/1001.2047>

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The conference slides can be downloaded from <http://rubiola.org>

*Abstract*—The amplifier phase noise is a critical issue in numerous fields of engineering and physics, like oscillators, frequency synthesis, telecommunications, radars, spectroscopy, in the emerging domain of microwave photonics, and in more exotic domains like radio astronomy, particle accelerators, etc.

We analyze the two main types of phase noise in amplifiers, white and flicker. Using the polynomial model, the phase-noise power spectral density is  $S_\varphi(f) = b_0 + b_{-1}/f$ . Phase noise results from adding white noise to the RF spectrum around the carrier. For a given amount of RF noise added,  $b_0$  is proportional to the inverse of the carrier power  $P_0$ . By contrast,  $b_{-1}$  is a constant parameter of the amplifier, in a wide range of carrier power. Accordingly, the flicker phase noise  $b_{-1}/f$  is independent of  $P_0$ . This fact has amazing consequences on different amplifier topologies. Connecting  $m$  equal amplifiers in parallel,  $b_{-1}$  is  $1/m$  times that of one device. Cascading  $m$  equal amplifiers,  $b_{-1}$  is  $m$  times that of one amplifier. Recirculating the signal in an amplifier so that the gain increases by a power of  $m$  (a factor of  $m$  in dB) due to positive feedback (regeneration), which for integer  $m$  is similar to the case of  $m$  amplifiers, we find that  $b_{-1}$  is  $m^2$  times that of the amplifier alone.

The simplest model for the  $1/f$  phase noise is that the near-dc  $1/f$  noise phase-modulates the carrier through some parametric effect in the semiconductor. This model predicts the behavior of the (simple) amplifier and of the different amplifier topologies. Numerous measurements on amplifiers from different technologies and frequencies (HF to microwaves), also including some obsolete amplifiers, validate the theory. This model also applies to other devices, including passive components.

## I. AMPLIFIER PHASE NOISE MECHANISMS

The phase noise in amplifiers originates from two basic mechanisms, additive and parametric.

Additive random noise in wide-band amplifiers has white spectrum because the noise has no preference for a specific frequency. Electromagnetic pollution is not considered here. White noise tuns into a random phase modulation whose spectrum is white:  $S_\varphi(f) = b_0$ . This noise derives by adding to the carrier a random noise of power spectral density  $N = FkT_0$ , where  $k$  is the Boltzmann constant, and  $F$  is the amplifier noise figure defined at the reference temperature  $T_0 = 290$  K (17 °C). In modern low-noise amplifiers  $F$  is

typically of 1–2 dB. It may depend on bandwidth through the loss of the input impedance-matching network, and on technology. In the presence of a carrier of power  $P_0$ , the noise  $N$  results in random modulation of power spectral density

$$b_0 = \frac{FkT_0}{P_0} . \quad (1)$$

Energy equipartition suggests that white phase noise goes with an equal amount of white amplitude noise.

Letting aside the trivial cases of non-uniform gain and of additive pollution, non-white random noise is generated by parametric modulation from a near-dc phenomenon. In this case, one can expect that the near-dc noise also affects the amplitude, so that there is some correlation between AM and PM noise.

We can divide the parametric noise into two basic classes, *environmental* and *microscopic*.

However important, the environmentally-originated noise is not our scope. The main fact is that a near-dc random signal external to the amplifier modulates a parameter of the amplifier, and in turn the phase. This process is associated to a conversion gain (or efficiency) and to a time constant or to a more complex propagation mechanism.

Flicker noise has spectrum of  $1/f$  law. Near-dc flicker noise takes place at the microscopic scale, for little or no correlation is expected between different region of the device. This is supported by the fact that the probability density function is normal, which originates from the central-limit theorem in the presence of a large population of independent phenomena. So, phase flickering can only originate from up-conversion of the near-dc  $1/f$  noise

$$S_\varphi(f) = \frac{b_{-1}}{f} . \quad (2)$$

This mechanism can be described either as a nonlinear process or as a traditional modulation, with same results. Thus, at first approximation the amplifier can be seen as a linear phase

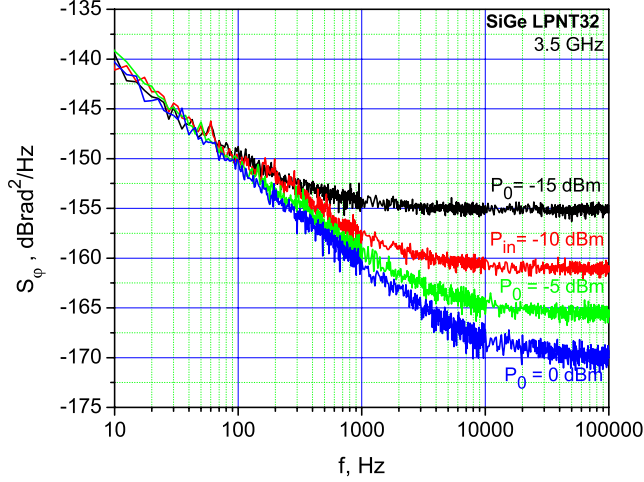


Fig. 1. Phase noise of a SiGe amplifier at different input power, measured at LAAS (Toulouse, France). Reprinted from [2]

modulator driven by a near-dc process, hence

$$b_{-1} = C \quad (\text{independent of power}). \quad (3)$$

Equations (1) and (3) yield a phase noise spectrum like the example shown in Fig. 1, where the corner frequency between flicker and white noise is

$$f_c = \frac{b_{-1}P_0}{FkT}. \quad (4)$$

Our experiments, detailed in in the main text [1] confirm that this behavioral model occurs in virtually all cases.

## II. PHASE NOISE IN AMPLIFIER NETWORKS

### A. Cascaded Amplifiers

When amplifiers are cascaded, the noise figure is governed by the Friis formula [3], which states that the noise contribution of each stage referred to the input is divided by the power gain of the preceding stage. In the case of phase noise, the Friis formula writes

$$b_0 = \left[ F_1 + \frac{F_2 - 1}{A_1^2} + \frac{F_3 - 1}{A_1^2 A_2^2} + \frac{F_4 - 1}{A_1^2 A_2^2 A_3^2} + \dots \right] \frac{kT_0}{P_0}. \quad (5)$$

where  $A_i^2$  is the power gain of the  $i$ -th stage.

The flicker phase noise is radically different. Since the amplifier  $1/f$  phase noises are statistically independent and independent of the carrier power, the  $1/f$  noise of a chain of  $m$  amplifiers is

$$b_{-1} = \sum_{i=1}^m (b_{-1})_i, \quad (6)$$

independently of the order of the amplifier in the chain.

### B. Parallel Amplifiers

Often, when amplifiers are connected in parallel this is done for the trivial purpose to increase the power because the power needed cannot be obtained with a single stage. Amplifiers can be paralleled in different ways. A  $180^\circ$  junction coupling two outputs helps in reducing the third-order distortion. Otherwise, connecting two amplifiers in parallel with  $90^\circ$  junctions at the input and at the output improves the impedance matching.

White noise cannot be reduced by paralleling amplifiers for the same reason why the noise figure cannot be improved in this way. In reality, the loss of the input junction results in a (small) degradation of the noise figure.

Conversely, flicker noise is improved by connecting  $m$  amplifiers (cells) in parallel according to

$$(b_{-1})_{\text{parallel ampli}} = \frac{1}{m} (b_{-1})_{\text{cell}}. \quad (7)$$

This happens because  $b_{-1}$  is independent of the carrier power. So when the input signal is split into  $m$  cells, each cell produces the same  $1/f$  phase noise as if it received the total carrier power. In the output junction, the carrier adds up coherently, while the flicker adds up statistically because the cells are independent. This mechanism was first explained in [4].

### C. Regenerative Amplifiers

Regeneration, i.e., positive feedback, is used since the early time of electronics to increase the gain of an amplifier and to narrow the bandwidth of a filter by recirculating at the input a fraction of the output signal. However obvious, it must be pointed out that the roundtrip gain must be less than one, so that the loop does not to oscillate. Denoting with  $A_0$  the voltage gain of the simple amplifier, in the special case where the frequency response of the feedback path is flat, the gain of the regenerative amplifier can be written as

$$A = A_0^m. \quad (8)$$

Accordingly, the regenerative amplifier can be seen as a cascade of  $m$  amplifiers of the same type of the simple amplifier inside. Of course there is no reason for  $m$  to be integer, which makes the ‘cascade’ an abstraction.

For our purposes, the most interesting and *unfortunate* fact about the regenerative amplifier is that the flicker noise is higher than that of a cascade of separate equal amplifiers having the same gain of the regenerative amplifier

$$(b_{-1})_{\text{regen ampli}} = m(b_{-1})_{\text{chain}}. \quad (9)$$

This happens because the roundtrip is short as compared to the time range at which the flicker fluctuations show up. Hence, the instantaneous value of the random phase is a constant in the decaying time of the recirculating signal. The conclusion is that the phase noise adds up coherently during the decaying time of the recirculating signal, while in a cascade the noise contributions of the  $m$  cells are independent.

TABLE I  
EXAMPLES OF FLICKER IN RF AND MICROWAVE AMPLIFIERS. REPRINTED FROM [1].

Amplifier	Frequency (GHz)	Gain (dB)	$P_1$ dB (dBm)	$F$ (dB)	DC bias	$b_{-1}$ (meas.) (dBrad <sup>2</sup> /Hz)
AML812PNB1901	8 – 12	22	17	7	15 V, 425 mA	–122
AML412L2001	4 – 12	20	10	2.5	15 V, 100 mA	–112.5
AML612L2201	6 – 12	22	10	2	15 V, 100 mA	–115.5
AML812PNB2401	8 – 12	24	26	7	15 V, 1.1A	–119
AFS6	8 – 12	44	16	1.2	15 V, 171 mA	–105
JS2	8 – 12	17.5	13.5	1.3	15 V, 92 mA	–106
SiGe LPNT32	3.5	13	11	1	2 V, 10 mA	–130
Avantek UC573	0.01 – 0.5	14.5	13	3.5	15 V, 100 mA	–141.5

#### D. Noise-Degeneration Amplifiers

A side effect of Eq. (3) is that the amplifier flicker-noise sidebands are proportional to the carrier. Hence, an error amplifier that receives the null signal of a bridge is virtually free from AM and PM flicker. This fact can be exploited in the feedforward amplifier and in the baseband-feedback amplifier.

#### E. Feedforward Amplifier

In this type of amplifier [5], input and output of the power amplifier (PA) are subtracted with appropriate weights and phases, so that the difference is the distortion and the noise of the PA. This difference is amplified by the error amplifier (EA) and added at the main output with weight and phase that null the PA noise.

#### F. Baseband-Feedback Amplifier

Comparing the input-output phase and the input-output amplitude of an amplifier, we get a pair of error signals that are used to modulate the input amplitude and the phase in closed loop, so that the amplifier AM-PM noise is canceled. This idea comes from the linearization of the SSB transmitters [6], [7, Chapter 4], and was later used to reduce the noise in microwave oscillators [8]. The noise is limited by the background of the detector, for the detector most suitable to the reduction of flicker is the bridge (interferometric) detector extensively described in [9].

### III. RESULTS

The full manuscript [1] describe in detail the measurement methods and reports a bunch of measured spectra, with interpretation and comments. Figure I provide some examples of flicker in RF and microwave amplifiers.

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