Measurement of the laser relative intensity noise

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Abstract— This article describes the measurement of the RIN power spectral density with the cross-spectrum method. Averaging the cross spectrum of two equal and independent channels that measure the same beam, the instrument background is not limited by the single-channel noise. Additionally, the measure can be validated on the basis of simple mathematical properties. This removes the need of a low-noise beam to validate the instrument background, which is a relevant experimental limitation of the (traditional) single-channel method.

I. INTRODUCTION AND STATE OF THE ART

This article is about the measurement of the power spectral density of the relative intensity noise (RIN) $\delta I/I$ in optical-fiber systems. Due to the guided propagation over a distance of at least 10^4 wavelengths, we assume that the beam shape is determined by the propagation law rather than by the lasing mechanism, thus the relative power fluctuation $\delta P/P$ is equal to the RIN. In electrical engineering the fractional amplitude fluctuation $\alpha = \delta V/V$ is generally preferred, where V the electric voltage or field. As $P \sim V^2$, it holds that $\alpha = \frac{1}{2}$RIN for small α.

The RIN has been a concern since the early beginning of laser optics [1]. Generally, the RIN is measured as the DC fluctuation of the current at the output of a photodetector [2], which is proportional to power. The main problem of this method is the difficulty of validating the measure with the knowledge of the detector noise. This occurs for the following reasons. First, the source can not be removed, otherwise there is no signal at the detector output. Second, a reference source more stable than the detector is not available in the general case. Third, flicker and other excess or parametric noises are not governed by simple and well understood physical laws. Hence, the instrument is to be validated in the same conditions of the measurement.

II. THE CROSS-SPECTRUM METHOD

In physical experiments the power spectral density (PSD) of a process is usually measured as the square absolute value of the Fourier transform

$$\left\langle S_x(f) \right\rangle_m = \left\langle |X(f)|^2 \right\rangle_m = \left\langle X(f)X(f)^* \right\rangle_m , \quad (1)$$

averaged over a suitable number m of spectra samples. This relies on the Wiener-Khinchine theorem for ergodic stationary processes, and ultimately on the repeatability and
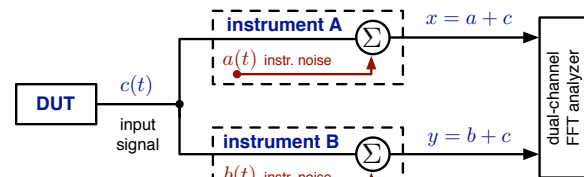

Fig. 1. Basic cross-spectrum measurement.

reproducibility of the experiment. Similarly, the cross spectrum is measured as

$$\left\langle S_{yx}(f) \right\rangle_m = \left\langle Y(f)X(f)^* \right\rangle_m . \quad (2)$$

Notice that in this section we use the word 'power' as a mathematical concept, the square of a quantity (the RIN, which is dimensionless), while later it will be the optical power. Though, we believe that the context is clear enough.

Let us assume that the physical quantity $c(t)$ is measured with two separate instruments, as shown in Fig. 1. The instruments noise contributions are $a(t)$ and $b(t)$. Of course, $a(t)$, $b(t)$ and $c(t)$ are statistically independent. Hence, the instrument outputs are

$$x(t) = c(t) + a(t) \quad (3)$$
$$y(t) = c(t) + b(t) , \quad (4)$$

Using the uppercase for the Fourier transform and expanding (2), we get

$$S_{yx}(f) = \langle CC^* \rangle_m + \langle CB^* \rangle_m + \langle AC^* \rangle_m + \langle AB^* \rangle_m$$
$$= S_c(f) + O(\sqrt{1/m}) , \quad (5)$$

where $O()$ means 'order of.' Owing to statistical independence of A, B and C, the cross terms decrease as $\sqrt{1/m}$.

The measurement sensitivity is limited by two parameters. The first is m, which must be large enough for the term $O(\sqrt{1/m})$ to be negligible, so that $S_{yx}(f) = S_c(f)$. The second is the correlated noise due to the hardware. It can be due to crosstalk, to the fluctuations induced by the environment, and to any other correlated phenomena. Of course these two issues are to be analyzed separately.

A. Statistical limit

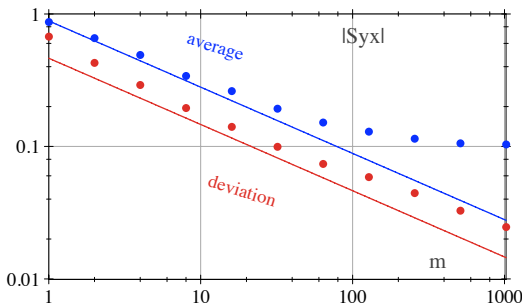We analyze the convergence to the DUT spectrum assuming that the instrument are fully independent, so that there is no

Fig. 2. Convergence of $\langle|S_{yx}|\rangle_m$ to the DUT noise. In this example the single-channel background noise is normalized to one, and the DUT noise is 10 dB lower.

Fig. 3. Basic RIN measurement method.

Fig. 4. RIN floor as a function of optical power.

correlated noise other than the DUT. Additionally, we assume that phase difference between the two channels is zero, or that it is known and taken away.

Under the above hypotheses $S_c(f)$ is real, while $\Im\{S_{yx}(f)\}$ contains only the instrument noise. The best estimator is the average real part $\langle\Re\{S_{yx}(f)\}\rangle$, and the residual is $1/\sqrt{2m}$. This estimator is uncomfortable in laboratory practice because it takes positive and negative values before m is sufficient to reject the single-channel noise, hence the spectrum cannot be displayed in logarithmic scale (dB). Hence we opt for $\langle|S_{yx}(f)|\rangle$, which is an always-positive estimator, albeit it is inevitably biased. This estimator is available in the front-panel menu of commercial FFT analyzers.

A relevant feature of the cross spectrum is that the smoothness can be used to validate the measurement on the ground of simple statistics. Let us temporarily assume that $c = 0$, so that $S_c = 0$ and only the term $O(\sqrt{1/m})$ remains. Thanks to the central limit theorem, most noises we deal with are Gaussian in time domain, hence the Fourier transform is Gaussian. Taking one frequency and normalizing the process for the variance of X and Y to be equal one, the quantity $\langle|S_{yx}(f)|\rangle_m$ has Rayleigh distribution, average

$$\mathrm{avg} = \mathbb{E}\{|S|\} \simeq 0.886/\sqrt{m} \ , \tag{6}$$

and standard deviation

$$\mathrm{dev} = \sqrt{\mathbb{E}\{|\,|S| - \mathbb{E}\{|S|\}|^2\}} \simeq 0.215/\sqrt{m} \ . \tag{7}$$

Interestingly, the deviation-to-average ratio is

$$\mathrm{dev}/\mathrm{avg} \simeq 0.523 \ , \tag{8}$$

independent of m. Ergodicity gives access to the ensemble by scanning the frequency axis. This means that white noise displayed in logarithmic scale is a band of random structure and constant thickness, shifting toward the bottom of the screen with law $1/\sqrt{m}$. Re-introducing the DUT noise, at some point the single channel noise becomes negligible, and the spectrum converges to the DUT noise. When this happens, the spectrum stops decreasing and shrinks. This fact, shown in Fig. 2, is the mathematical signature of the convergence, which validates the measurement.
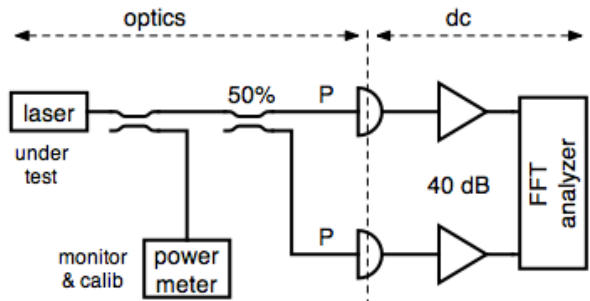
III. MEASUREMENT METHOD

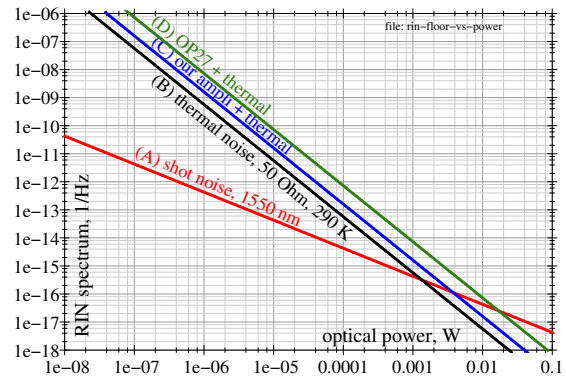Figure 3 shows the measurement scheme. Though often RIN measurements may not require wide bandwidth, for interoperability we want to use high-speed InGaAs p-i-n photodiodes. These diodes, commonly used in telecom, are already part of our optoelectronic oscillators and phase-noise test sets [3].

There are two basic choices for the amplifier, to load the photodetector with a resistor followed by a voltage amplifier, or a virtual-ground (VGND) amplifier. In principle the VGND amplifier can be designed with a single resistor, while the voltage amplifier has resistors at the input and in the feedback, and consequently higher thermal noise. Yet, noise in VGND amplifiers is difficult to control because the interplay between stray capacitances tends to produce large bumps, worse if the amplifier is not packaged with the detector [4, Ch. 5]. Out of our experience in microwave photonics, we abandoned the detectors with internally packaged amplifiers because of too large flicker as compared to good general-purpose external amplifiers. For the above reasons we decided to stick on our low-noise amplifiers in simple non-inverting configuration [5].

The background noise of most components, including amplifiers, is described by the power-law $S_v(f) = \sum_i h_i f^i$. Our amplifiers exhibit $\sqrt{h_{-1}} = 1.1$ nV/$\sqrt{\mathrm{Hz}}$ (flicker) and $\sqrt{h_0} = 1.5$ nV/$\sqrt{\mathrm{Hz}}$ (white), including the contribution of the external 50 Ω load resistor. Our amplifiers are optimized for low flicker in the presence of 50 Ω load. The corner frequency, at which flicker equals white noise, is of 0.75 Hz. Interestingly, the white noise is only 4.2 dB higher than the thermal noise
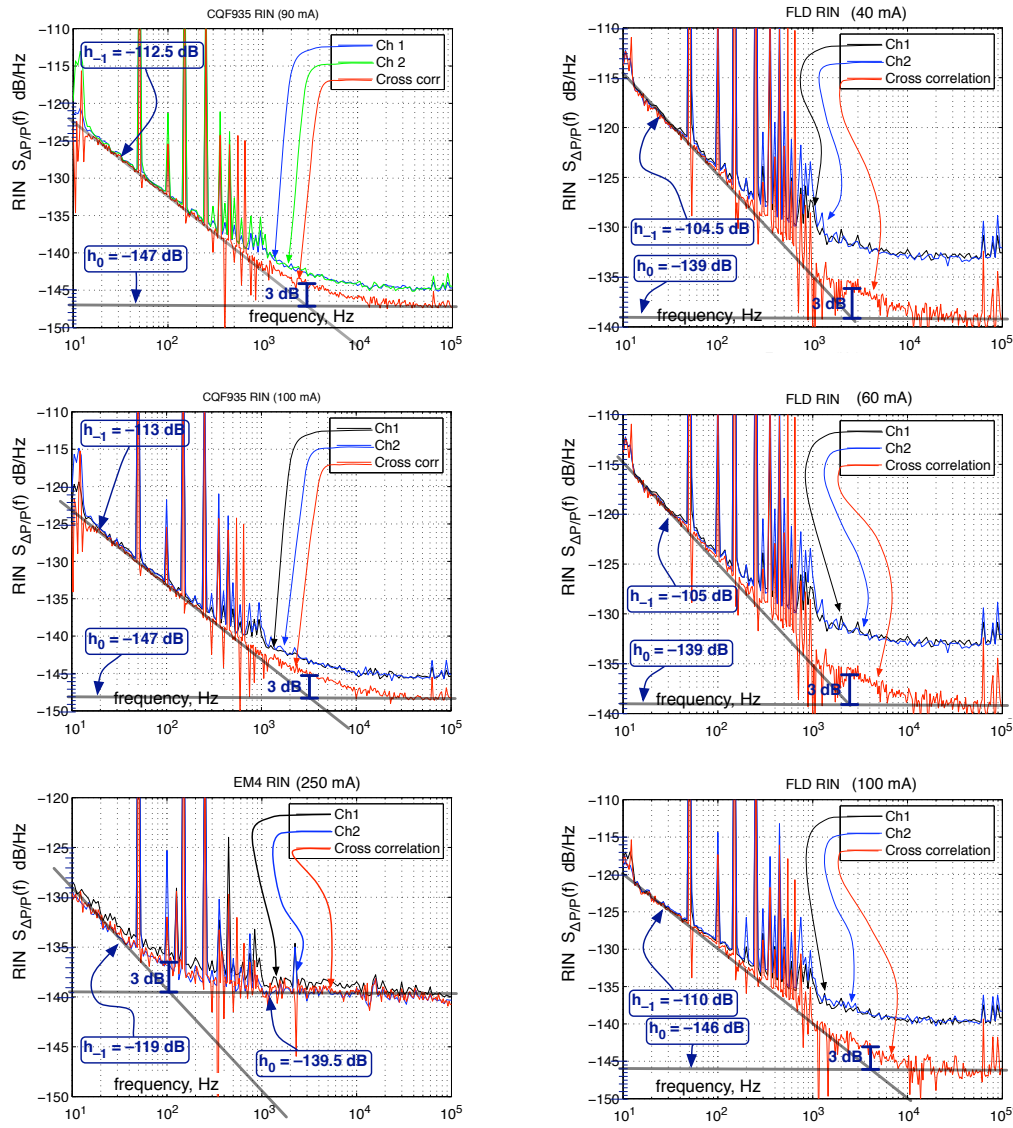
Fig. 5.   Example of RIN spectra.

of the 50 Ω load, so it can be rejected easily with correlation and averaging.

Figure 4 shows the white noises. The curve A is the shot noise $S_{\text{RIN}} = \frac{2}{\eta\Phi}$, where η is the quantum efficiency and $\Phi = \frac{P}{h\nu}$ is the photon flux. The curve B is the thermal noise $S_I = 4kT/R$ converted into RIN using $S_{\text{RIN}} = S_{\delta I/I}$ and the photocurrent $I = q\eta\Phi$, where q is the electron charge. The curve C is the white noise of our amplifiers, including the thermal noise of the load resistor at the input, converted into RIN as above. For reference, the curve D is the same but for the popular low-noise amplifier OP-27. The threshold power at which the noise of electronics equals the shot noise is of 1.3 mW for a single 50 Ω load at room temperature and no amplifier noise; of 3.7 mW using our amplifiers, and of 17 mW with the OP-27.

Though we know very little about flicker in this type of measurement, we believe that the amplifier is critical. Flicker comes from a parameter that fluctuates with $1/f$ spectrum, which modulates the current. The bias current is significantly larger than the signal, so the current fluctuations are proportionally large.

IV. RESULTS AND DISCUSSION

The RIN measurement with our method relies on some assumptions. The first is that there is no correlated environment effect on the two channels. Though quite reasonable in the frequency range of our experiments (from 10 Hz to 100 kHz), this may not to be true at lower frequencies, below 1 Hz. The second assumption is that the beam splitter is stable, so that there is no partition noise. Provisionally, we trust the mechanical construction of the commercial beam splitters. The third hypothesis is that the beam splitter works with the intensity, hence the Hanbury-Brown Twiss effect is absent. This effect is visible only in single-photon regime,

while practical RIN measurements are often made in the μW–mW region. A further hypothesis is that polarization noise and frequency noise have negligible effect on the power splitting. Honestly, this is taken as a hard hypothesis, to be checked further.

Figure 5 shows a few example of RIN spectra measured with the cross-spectrum method. The benefit of the correlation is clearly visible because in almost all cases the measured RIN is at least a few dB lower than the single-channel spectrum. The spectrum shows clearly white noise and flicker ($1/f$) noise. The $1/f$ noise is between -104.5 and -119 dB. The $1/f$ noise can be converted into the Allan variance floor, independent of the measurement time τ, using $\sigma^2 = 2\ln(2)\,h_{-1}$. For reference, -119 dB is equivalent to a deviation $\sigma = 1.18\times10^{-6}$.

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