

## ELISA : an ultra-stable frequency reference for space mission ground segment

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## INTRODUCTION

Deep space tracking requires the use of a secondary frequency standard presenting an exceptional relative frequency instability. In the actual ground stations network of **ESA** (**E**uropean **S**pace **A**gency), a system combines a hydrogen maser and an ultra-stable quartz. The frequency stability (Allan Deviation) is limited by the quartz instability at  $1 \times 10^{-13}$  for short term (integration times up to 100 s) and  $1 \times 10^{-15}$  to  $1 \times 10^{-16}$  for long term (integration times greater than 100 s). In order to validate for example some new experiments in the field of radiosciences or of space navigation, it is necessary to improve the performances in the short term of these frequency standards. ESA asked the valuation of the **FEMTO-ST** Institute to realize the new system. This project is in collaboration with the **NPL** (**N**ational **P**hysical **L**aboratory) and the firm **Timetech** GmbH. The primary objective of this project is the design and realization of an oscillator whose frequency stability is equal to  $3 \times 10^{-15}$  over time intervals between 1 and 1000 s. Such a breakthrough in performance can today be achieved with oscillators which incorporate, instead of electromechanical quartz resonators, cryogenic sapphire whispering-gallery-mode electromagnetic oscillators. Moreover, the frequency stability of the oscillator has to be transferred to low frequencies (5-100 MHz) for a maximum phase noise equal to -120 dBc/Hz@1 Hz.

## WHISPERING GALLERY MODE RESONATOR

The sapphire resonator consists of a cylindrical resonator machined in a low defect  $\text{Al}_2\text{O}_3$  monocrystal. With the use of high order modes of the Whispering Gallery (WG) type, high Q-factor can be obtained. The electromagnetic field is localized near the cylindrical surface of the resonator. Thus, the Q-factor, essentially limited by the sapphire dielectric losses, is 200,000 at room temperature, higher than 10 millions at 77 K and up to  $10^9$  at the liquid helium range.

But, the frequency stability of the cryogenic sapphire oscillators is drastically limited by the resonator sensitivity to the thermal fluctuations. This sensitivity decreases slowly with temperature but remains of the order of few

ppm at cryogenic temperatures (-10 ppm@77 K). Considering the temperature can be hardly stabilized within a few  $10\mu\text{K}$ , there results in a frequency stability worst than  $10^{-11}$  which cannot compete with the quartz oscillator.

The resonator crystalline structure has then to be modified to introduce some perturbation which will compensate the sapphire permittivity at liquid helium temperatures. If the compensating perturbation is well designed, the resonator shows a turnover temperature  $T_0$  at which the thermal sensitivity is nulled at the first order.

However the HEMEX resonator shows a natural turnover temperature near 6 K. Fortunately even high quality crystals contain a small concentration ( $\lesssim 1\text{ppm}$  in weight) of paramagnetic impurities as  $\text{Mo}^{3+}$  ions as the consequence of the growth process. There results a natural thermal compensation at random turning point between 4K and 10K, depending on the actual impurities and on the mode [1].

Experience shows that the best results are obtained with WG modes between 13 and 18 because the energy confinement into the dielectric is improved as the mode order increases. This fact has two practical consequences: *i* Q is progressively degraded at low-order modes (less than 13) due to electromagnetic radiation, and *ii* high order modes (greater than 18) are difficult or impossible to exploit because the couplers needs too sharp adjustment. From our preceding experiences and litterature, we focused on the  $WGH_{15,0,0}$  mode with which high Q-factor have been already observed with 50x30 mm resonators.

The next issue is the resonance frequency. We chose 10 GHz due to the frequency transitions of the different impurities present in the sapphire and the facility of conception of the synthesizer (see Figure 1).

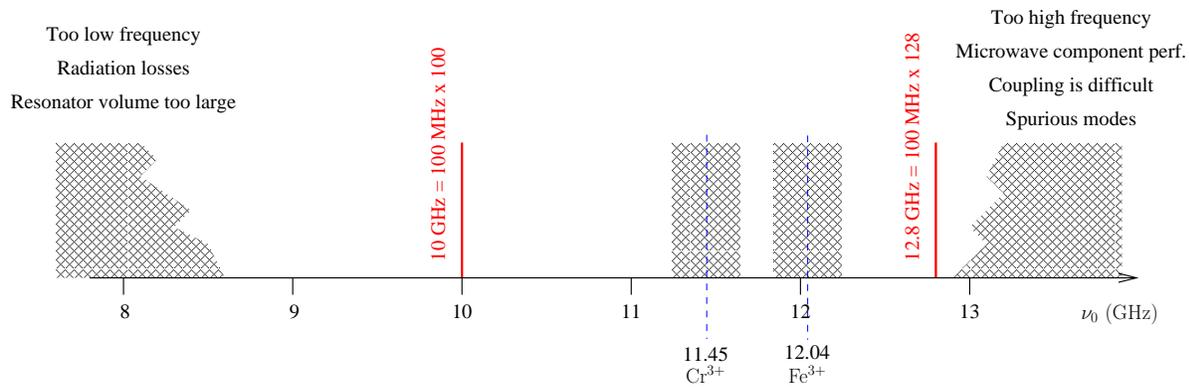


Fig. 1: Choice for the resonator frequency

The resonant frequency is mainly determined by the resonator diameter  $\phi$  and a for a smaller extent by its height  $H$ . The spindle diameter ( $d$ ) and height ( $h$ ) as well as the metallic cavity dimensions have a negligible influence on the resonator frequency. The spindle constitutes the thermal and mechanical interface with the cavity.

These conditions, 50x30 mm dimensions,  $WGH_{15,0,0}$  mode and 10 GHz resonance frequency have constituted our starting point for the calculations of Elisa and Alisee resonators geometry.

The objective is to obtain an operational resonance at  $10\text{GHz}-D$  where  $D$  is of the order of 10MHz.

Finite Elements analysis has been used to calculate precisely the resonator dimensions (see Figures 2 and 3):

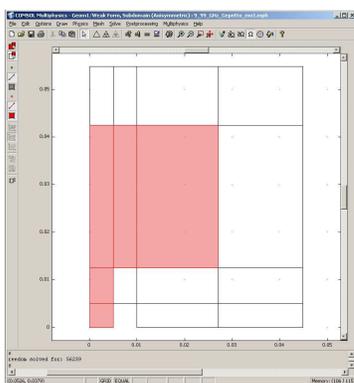


Fig. 2: Geometry used for F.E. simulation.

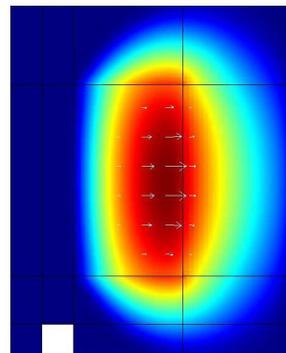


Fig. 3:  $WGH_{15,0,0}$  magnetic field configuration simulated by F.E..

The key parameters to accurately determine the modes frequencies are the values of the sapphire tensor permittivity components used for the calculation. By comparing experimental frequencies of available crystals and simulation results, we deduced :

$$\epsilon_{\perp} = 9.270,688 \quad \epsilon_{\parallel} = 11.340,286 \quad (1)$$

Eventually, we get a  $WGH_{15,0,0}$  mode at 9.989,89 GHz with  $\phi = 54.2$  mm and  $H = 30$  mm. The designed resonator is represented on the figure 4.

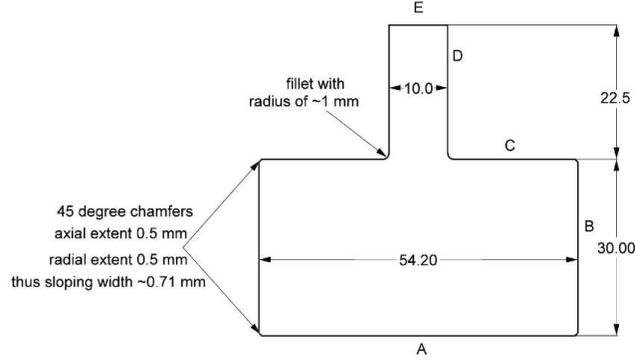


Fig. 4: Sapphire resonator geometry

The complete set of the resonator frequencies has been obtained through another (more simple) model based on the mode matching technique. Although less accurate than F.E. (but still better than 1% accuracy) model, the mode matching method enables to evaluate rapidly the frequency and Q-factor of the whispering gallery modes. We used a simulation program written few years ago by Di Monaco and available at Femto-ST. The mode matching method is based on the model represented in figure 5.

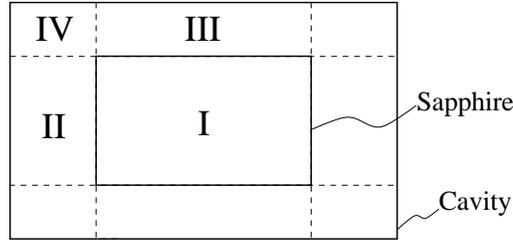


Fig. 5: *Sapphire Resonator Modeling for Mode Matching.*

In each region I,II and III, the electromagnetic field is expressed in term of classical solutions of the Maxwell equations in the cylindrical structure. It is also assumed that there is no field within the region IV. In region III the solution is assumed to be a pure transverse magnetic (for WGH mode) or pure transverse electric (for WGE mode). These assumption are necessary to separate the variables and solve the problem. For WGH modes with  $m > 5$ , the frequencies and Q-factor are determined with 1% and 10% accuracy respectively. The obtained results are given in the figures 6 and 7.

The machining tolerances have an impact on the final resonator frequency and of course on the resonator cost. Thus they should be specified with care. The sensitivity of a 54.2x30 resonator to the diameter and height tolerances are:

$$\frac{1}{\Delta\phi} \frac{\Delta\nu}{\nu} = 2 \times 10^{-2}/\text{mm} \quad \frac{1}{\Delta H} \frac{\Delta\nu}{\nu} = 1 \times 10^{-3}/\text{mm} \quad (2)$$

After some exchange with potential suppliers the settled tolerances are:

$$\Delta\phi = \pm 10\mu\text{m} \quad \text{and} \quad \Delta H = \pm 20\mu\text{m} : \quad (3)$$

The resonator frequency is then defined as 9.99 GHz  $\pm$ 3.5 MHz to fit our requirements.

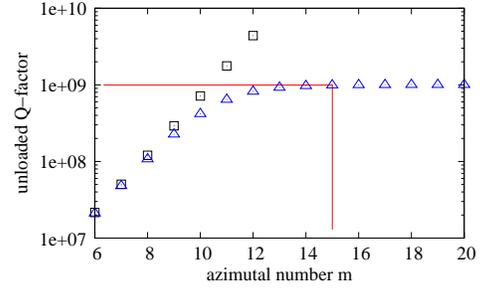
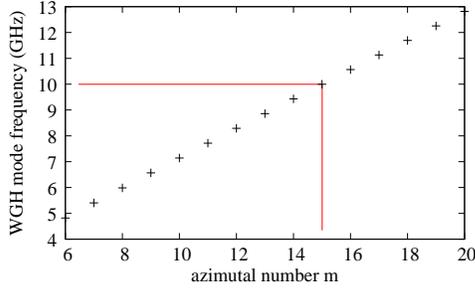


Fig. 6: *Sapphire resonator frequencies for  $WGH_{m,0,0}$  modes at 4 K.* Fig. 7:  $\triangle$ : *Q-factor for  $WGH_{m,0,0}$  modes;  $\square$ : contribution of metallic losses at 4 K*

In the following tables 1 and 2, we can observe the first frequencies measurements of the WG modes of Elisa and Alisee. These measures have been done at room temperature. Q-factors and insertion losses are also reported.

Table 1: Elisa's room temperature characteristics

mode $m$	$\nu_{\text{calc}}$ GHz	$\nu_{\text{meas}}$ GHz	Q-factor	Losses dB
10	7.061	7.065	225000	-26
11	7.629	7.631	227000	-24
12	8.194	8.195	194000	-28
13	8.758	8.758	215000	-32
14	9.321	9.320	205000	-35
15	9.881	9.880	192000	-40
16	10.441	10.439	185000	-43
17	10.999	10.997	194000	-47
18	11.556	11.554	186000	-43
19	12.112	12.109	168000	-45

Table 2: Alisee's room temperature characteristics

mode	$\nu_{\text{calc}}$ GHz	$\nu_{\text{meas}}$ GHz	Q-factor	Losses dB
10	7.061	7.065	223000	-21
11	7.629	7.631	227000	-24
12	8.194	8.195	211000	-27
13	8.758	8.758	216000	-33
14	9.321	9.320	199500	-36
15	9.881	9.880	195000	-40
16	10.441	10.439	185000	-41
17	10.999	10.997	213000	-46
18	11.556	11.554	197000	-40
19	12.112	12.109	173000	-43

We notice a frequency gap  $\Delta\nu \approx 300\text{kHz}$  (Figure 8) which is useful for frequency stability comparison.

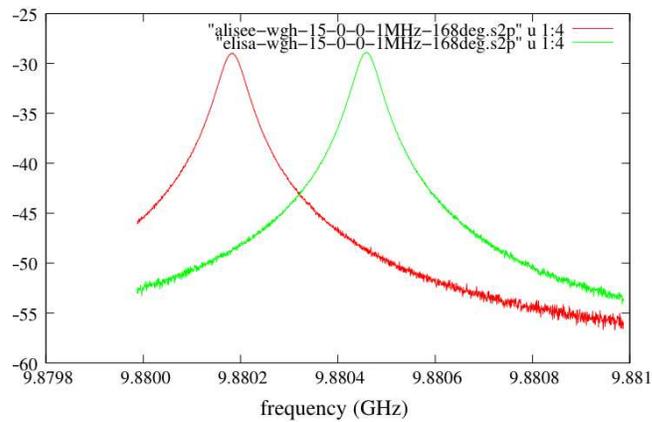


Fig. 8: *Measure of the  $WGH_{15,0,0}$*

## CRYOGENIC SAPPHIRE OSCILLATORS

### CRYOCOOLER

All the above thermal compensation techniques need to cool the sapphire resonator at a cryogenic temperature. The resonator can be immersed in a liquid nitrogen or helium bath (dewar) or can be mounted in a closed-cycle cryocooler. The resonator is thermally connected to the cold source and stabilized at its turnover temperature. For some autonomy reasons, the solution of the cryocooler has been chosen. Indeed, the dewar is a free vibration solution but it has a weak autonomy. It is also necessary to refill it frequently (depending on the size of the chosen cryostat) that disturb the right operation of the oscillator. Finally, liquid helium is expensive and difficult to be supplied.

Nevertheless the cryocooler presents mechanical vibrations that are prejudicial for the frequency stability requirement. The frequency of cryogenic sapphire oscillator (expressed conveniently as a fraction of the absolute frequency) is a function of its mechanical acceleration (the latter, in general, must be treated as a vector). There are essentially two sources of vibrational noise (*i.e.* acceleration) to which the resonator and peripheral microwave/electronic circuitry are exposed:

- background vibrations at the location where the resonator is installed
- mechanical vibrations induced by the cryocooler

By finite-element mechanical simulations in COMSOL Multiphysics, the sensitivity of (the frequencies of) a cryogenic sapphire X-band resonator's transverse-magnetic WG modes to a constant vertical acceleration of  $g = 9.8065\text{ms}^{-2}$ , was estimated to be  $-3.21 \times 10^{-10}/g$ . To meet ESA's fractional-frequency stability target of  $3 \times 10^{-15}$ , with a pulse-tube cooler shaking at its characteristics/fundamental frequency of 1 Hz, this sensitivity translates (naively) into demanding that the resonator's vertical amplitude of displacement (at 1 Hz) be less than  $2.3 \mu\text{m}$ . Indeed, if we assume that the displacement  $x(t)$  of the resonator takes the form

$$x(t) = A_0 \sin(\omega t) \quad (4)$$

then the acceleration is given by

$$\frac{d^2 x}{dt^2} = -\omega^2 A_0 \sin(\omega t) \quad (5)$$

With cryocoolers the main source of vibrational noise is at the drive frequency of cooling cycle, typically about 1 Hz. Therefore, equation 5 simplifies to

$$\frac{d^2 x}{dt^2} = -4\pi^2 A_0 \sin(\omega t) \quad (6)$$

Hence, for vibration levels of  $6 \times 10^{-6}$  g we require a maximum peak displacement  $A_0$  of

$$A_0 \leq \frac{9 \times 10^{-6} g}{4\pi^2} \quad (7)$$

$$A_0 \lesssim 2\mu\text{m} \quad (8)$$

For the moment, the type of cryocooler (pulse tube, Gifford-MacMahon, etc.) and the cooling power at 4 K are being studied by the colleagues of NPL.

### OSCILLATOR LOOP AND ADDITIONAL CONTROL

To get high frequency stability the power injected in the resonator as well as the phase of the oscillating loop have to be controlled. These controls are difficult to implement in the cryogenic environment. Generally only the resonator is cooled while the sustaining circuit is outside the cryostat. This hybrid solution permits more flexibility in the choice of the WG mode and in the oscillator parameters, as power, gain etc...Figure 9 shows the typical circuit implemented in our oscillators.

Two semi-rigid coaxial cables link the resonator to the oscillator circuit at room temperature. The coaxial cables undergo large temperature gradients. Their electrical length fluctuates due to the cryogenic fluid evaporation or to room temperature fluctuations. There results in a large phase drift along the oscillator loop which has to be compensated. This is realized thanks to a Pound servo which ensures the oscillation frequency to be equal to the resonator frequency [2]. The loop signal is phase modulated by a voltage phase shifter inserted in the loop. The modulation frequency  $\nu_m$  is higher than the resonator bandwidth. A circulator placed at the resonator input port enables to derive the signal reflected by the resonator which is sent to a tunnel diode operating as a quadratic detector. The latter delivers a voltage which is synchronously demodulated at  $\nu_m$  in

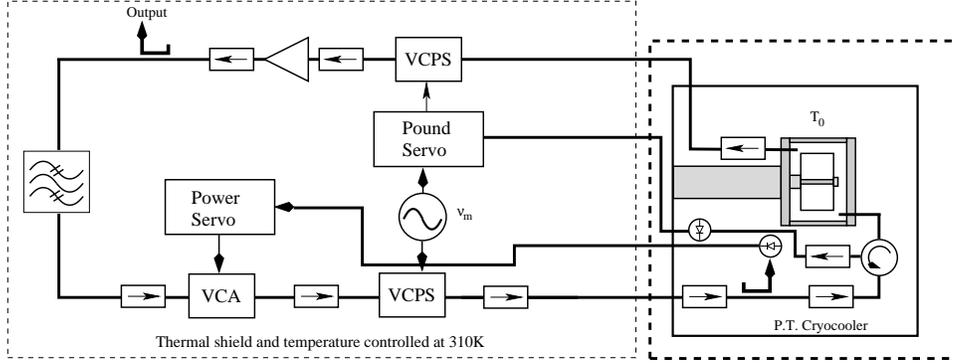


Fig. 9: Scheme of the oscillator circuit. The bold lines are the oscillator loop. The thin lines refer to the electronic controls required to get a high frequency stability

a locking amplifier. Let's assume that the resonator input port coupling is adjusted to the unity, *i.e.*  $\beta_1 = 1$ . The reflected signal contains the residual carrier at  $\nu_{osc}$  and two sidebands at  $\nu_{osc} - \nu_m$  and  $\nu_{osc} + \nu_m$ . When the oscillating frequency  $\nu_{osc}$  is just equal to the resonator frequency  $\nu_0$ , the residual carrier is absorbed by the resonator and only the two sidebands subsist. These sidebands are mixed in the diode which delivers a voltage whose a.c. component is at the frequency  $2\nu_m$ . The demodulated signal is then equal to zero. If  $\nu_{osc} \neq \nu_0$ , the diode voltage is modulated at  $\nu_m$  and the demodulated signal is proportional to  $\nu_{osc} - \nu_0$ . The locking amplifier output signal is integrated and sent back to the bias stage of the voltage controlled phase shifter. The phase fluctuations arising in the loop are corrected in real time in the loop bandwidth. The condition  $\beta_1 = 1$  is required to optimize the Pound servo operation otherwise the slope of the frequency discriminator is decreased. In practise the adjustment of the resonator coupling coefficients is a difficult task. Indeed as the coupling coefficients increase as the Q-factor,  $\beta_1$  is almost 100 times higher at cryogenic temperature than at 300 K. At room temperature, the resonator input port reflexion coefficient is hardly measurable with a network analyser. Multiple cooldown are generally needed to get a proper value for  $\beta_1$ .

The sapphire resonator is also sensitive to the injected power. This sensitivity arises from two effects: the microwave power dissipation and the radiation pressure inside the resonator [3]. In our experiments we stabilized the power injected in the resonator by a classical power control using a quadratic detector as the sensor and a voltage controlled attenuator (VCA) in the loop.

Finally all the room temperature electronics are stabilized at  $300\text{K} \pm 0.1\text{K}$ .

## CONCLUSION

The technology of the Sapphire Whispering Gallery Mode Resonator permits to surpass the frequency stability of traditional ultra-stable oscillators, and then can be exploited in number of applications requiring a high frequency stability like a frequency standard for the ground station of ESA. We have demonstrated through this article the preliminary results on the sapphire resonator and quick description of the cryocooler choice and the sustaining loop.

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