Dual photonic-delay line cross correlation method for phase noise measurement

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***Abstract*— We describe the application cross-correlation method with dual photonic delay-line for homodyne phase noise measurement of the low-noise microwave oscillators. This method combines delay-line discriminator and cross correlation with use of low-loss kilometer-long photonic delay. By using 4.5 km optical fiber as photonic delay, we demonstrated a measurement noise floor of -158 dBc/Hz at 10 kHz offset without a reference oscillator.**

I. Introduction

Characterization of phase noise for microwave oscillators is very important as phase noise is the main limitation in many systems employing such oscillators. In general the oscillator phase noise can be measured by comparing it with a lower-phase-noise oscillator, a heterodyne measurement scheme depicted in Fig. 1 (a). Obviously such heterodyne measurement requires a reference oscillator operating exactly at the same frequency as the oscillator under test and a lower phase noise. At microwave frequencies sometimes it is hard to find reference oscillators exactly at the same frequency with the oscillator under test.

Fig. 1 (b) shows beat frequency measurement method, which relaxes the reference oscillator frequency requirement. Here the actual measurement is done at a HF frequency after downconversion, and many low-noise reference oscillators are available at HF frequencies. But still the first reference oscillator used for downconversion needs to be within 50 MHz of the oscillator to be measured.

Homodyne measurement with a delay-line discriminator (Fig. 1 (c)) is an alternative technique when a lower-noise reference oscillator at the exact frequency of the oscillator under test is not readily available. This method was used by Lance et al. [5] with electrical delay lines. The fact that electrical delay lines are lossy limits the practically achievable delay to some 100 ns at microwave frequencies, which is too short to characterize most oscillators. 100 ns delay can be achieved using about 25 m of semirigid coaxial cable. Typical loss for 25 m semirigid cable at 10 GHz is about 20 dB. On the other hand, standard telecommunication fiber SMF-28 has only 0.2 dB/km loss at 1.55 μm. A photonic link with 2 km of fiber provides about 10 μs of delay, long enough for a sensitive phase noise measurement system at microwave

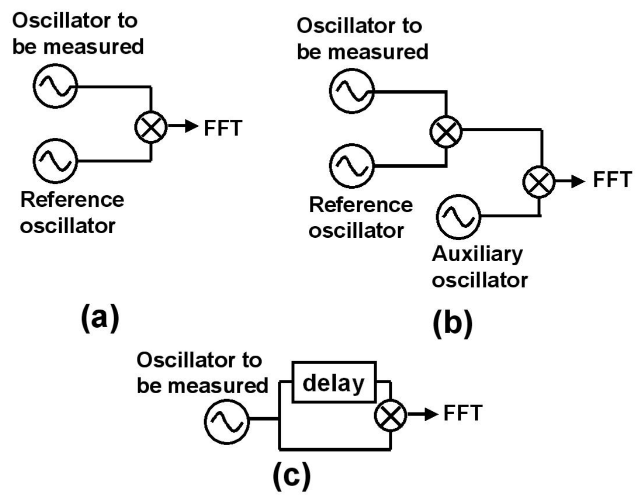

Fig. 1. Heterodyne phase noise measurement (a) requires a reference oscillator, beat frequency method (b) requires a reference oscillator, but not exactly at the same frequency of the oscillator to be measured, whereas in homodyne case (c) there is no need for a reference oscillator.

frequencies [6]. The homodyne delay line technique is also wide band, and works in any range of microwave frequencies.

In phase noise measurement, frequency mixers are commonly used as phase detectors. For high sensitivity to phase noise, and high AM noise rejection the mixer should be saturated. This requires 7 dBm or more power into LO and RF ports of the mixer. When using photonic delay lines, we are bound to use photodetectors to recover the microwave signals. Usually the photodetectors give about -20 dBm microwave power output. In order to deliver enough power into mixer ports, one needs to use high amplification, and this introduces a measurement limitation. Cross correlation method can be used to overcome this problem. As shown in Fig. 2, all components that are potential noise sources are placed in two different isolated paths or channels. This way most noises added by the components in each channel are uncorrelated. Only the oscillator noise is common to both channels. A dual channel FFT analyzer acquires the time signal from each channel simultaneously and computes the cross spectrum. Therefore, using enough averaging on cross spectrum of the separate

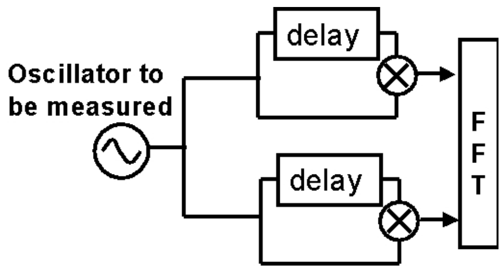

Fig. 2. Cross correlation of the signals in two separate channels, where only the oscillator signal is common to, is a way of removing the noise added by the components in the phase noise measurement system.

channels, one can remove the uncorrelated noise.

In this paper, we demonstrate the power of low-loss long photonic delay lines in combination with the cross correlation technique in a high performance phase noise measurement system.

II. Delay line discriminator theory

The delay line theory can be found in various references [7], [6]. Here we include a short summary as the context requires it. As seen in Fig. 1 (c) the signal is compared with its decorrelated replica through delay using a mixer, which converts the phase fluctuations into voltage fluctuations. If we start with a sine wave, introduce the delay, and multiply the original wave with its delayed replica, we arrive at the following delay transfer function:

$$|H_\varphi(jf)|^2 = 4\sin^2(\pi f \tau) \ , \quad (1)$$

where f is the offset frequency, and τ is the time delay introduced by the delay line. If we plot the transfer function vs. the offset frequency we see singularities when $f = 1/\tau$ (Fig. 3). The data around these points should be discarded, and the delay length should be selected accordingly. If we use a short delay then the singularities will appear at a larger frequency, so the range of the measurement will be extended. However, as indicated in Fig. 3, the signal strength will be lower, which means we have lower measurement sensitivity.

The oscillator phase noise power spectrum, $S_\varphi(f)$, is converted to $|H_\varphi(jf)|^2 S_\varphi$ through the delay line. If the mixer voltage gain coeffcient is K_φ (volts/radian), then the mixer output voltage can be expressed as [7]:

$$V^2_{out}(f) = K^2_\varphi |H_\varphi(jf)|^2 S_\varphi(f) \ , \quad (2)$$

and if we express single side band phase noise (SSB phase noise), $\mathcal{L}(f) = S_\varphi/2$ in terms of measured mixer output voltage:

$$\mathcal{L}(f) = \frac{V^2_{out}(f)}{2K^2_\varphi |H_\varphi(jf)|^2} \ . \quad (3)$$

Using $\sin(\theta) \approx \theta$ for $\theta < 1$ we can say that for $f < 1/4\tau$, $\mathcal{L}(f)$ is proportional to $1/\tau^2$, which means doubling the delay will improve the sensitivity of the measurement by 4 times (6 dB).

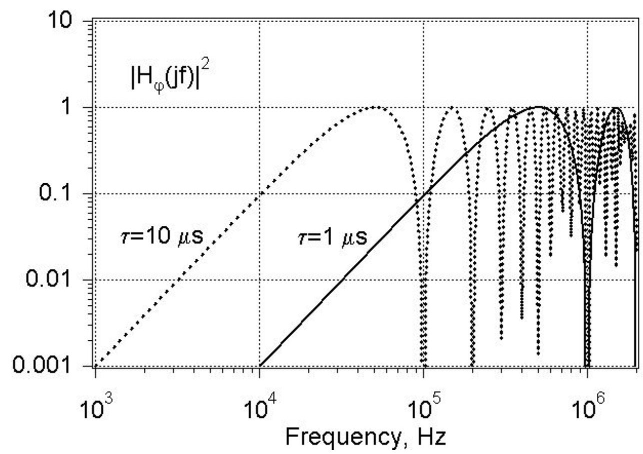

Fig. 3. Transfer function of the delay line. For 2 km optical fiber delay, the first singularity appears at 100 kHz, whereas for 200 m at 1 MHz, at the expense of losing sensitivity.

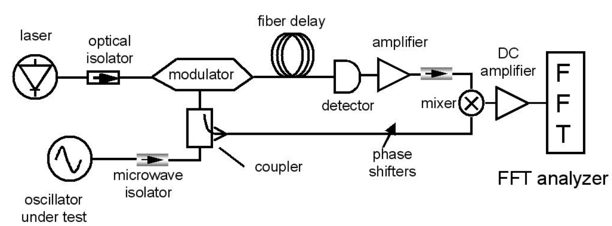

Fig. 4. Phase noise measurement setup employing photonic delay line, that is, a few km-long optical fiber.

III. Phase noise measurement with photonic delay line

Typical photonic delay phase noise measurement setup to measure phase noise of microwave oscillators is shown in Fig. 4. The essence of the system is that the microwave signal should be carried by an optical signal at a wavelength where fiber is most transparent, typically 1.3 μm or 1.55 μm. We used 1.55 μm semiconductor lasers. A lithium niobate ($LiNbO_3$) electro-optic Mach-Zehnder modulator is used to modulate the optical signal by the microwave signal from the oscillator under test. If the oscillator signal is weak, it needs to be amplified. For high modulation depth, which translates into high microwave power at photodetector output, more than 20 dBm power should be applied on the electro-optic modulator with about 27 dBm maximum power that can be applied.

We use typically 1–5 km fiber lengths. Mostly we use standard telecommunication fiber SMF-28 (manufactured by Corning, Inc.), which has 0.2 dB/km loss, and 17 ps/nm-km dispersion. Dispersion may be important if the optical beam is a pulsed output derived from a laser. In these cases the pulse may broaden as a result of dispersion, and this can severly effect the performance of the phase noise measurement system.

We use InGaAs *p-i-n* photodetectors, which are sensitive to light in the 0.9–1.8 μm region [8]. Although their specifi-

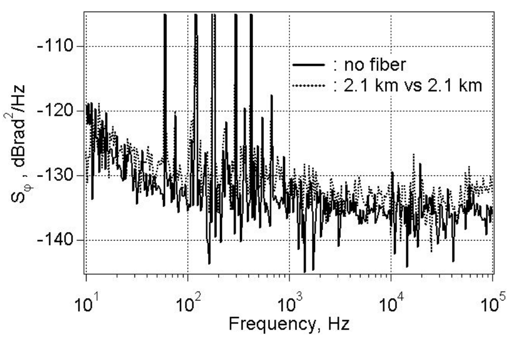

Fig. 5. Phase noise added by the optical fiber is not significant.

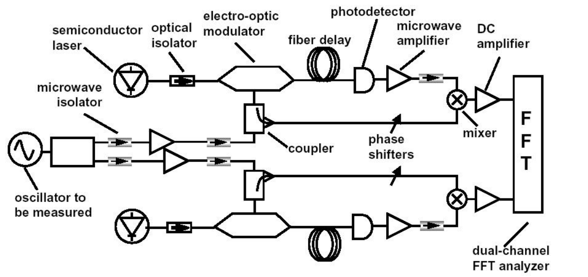

Fig. 6. The dual photonic delay line cross correlation phase noise measurement setup.

cations vary with manufacturer, such photodetectors are fast enough to detect at least 12 GHz modulation. Detectors as fast as 40 GHz and beyond are available. The photocurrent generated in the photodetector (i_p) upon absorption of photons is linearly related to the number of photons received per unit time, or photon flux (Φ), the quantum efficiency (η), and the electron charge (e):

$$i_p = \eta e \Phi = \eta e \frac{P}{h\nu} \ , \tag{4}$$

where P and ν is the power and frequency of the light beam respectively, and h is the Planck's constant. If we use $LiNbO_3$ Mach-Zehnder amplitude modulator, with modulation depth of m:

$$P(t) = P_{avg}[1 + m\cos(2\pi f t)], \tag{5}$$

where P_{avg} is the average optical power, and f is the microwave frequency. Microwave power at the photodetector output is given by:

$$P_\mu = i_p^2 R, \tag{6}$$

where R is the photodetector output impedance. We have photodetectors with 50 Ω and 1 $k\Omega$ resistors. Combining all these equations we arrive at the conclusion that the microwave output power of the photodetector increases quadratically with average optical power, and the modulation depth. However, typically 2-4 mW average optical power saturates the photodetectors. Assuming 70% quantum efficiency, 50% modulation depth, 3 mW average optical power, and 50 Ω resistor, we get about -14 dBm microwave output power, which is a typical value one would get in the lab. Of course, -14 dBm power is not enough to measure phase noise using double balanced mixers, which have to be saturated for better amplitude noise rejection. Therefore one has to use amplifiers.

The noise floor of a measurement system is determined by the components used, with amplifiers usually being the dominating noise source. Fibers and other microwave and opto-electronic components also contribute noise, but at a much lower level. For example, we have measured the noise added by the fiber using a carrier suppression measurement scheme, which suppresses the amplifier $1/f$ noise (flicker).When two equal lengths of fiber are used in both paths, then we only measure the noise contributed by the fiber. As seen in Fig. 5, there is no significant amount of noise added by the 2.1 km fibers used above the thermal noise of the amplifier. Therefore, the noise floor of our photonic delay line measurement system is mainly determined by the amplifier noise.

IV. Dual-photonic delay line with cross correlation

It is well established that the uncorrelated noise added by the components can be averaged out using the cross correlation technique [5], [9], [10]. Cross spectrum is defined as [11]

$$S_{AB}(f) \equiv B(f) \cdot A^*(f) \ , \tag{7}$$

where $A(f)$ and $B(f)$ are the Fourier transform of the signals in channel A and B. Each signal has the correlated noise, i.e., the oscillator noise, and the uncorrelated noise from all components in each channel.

By taking the average of the cross spectrum S_{AB}, the uncorrelated noise is removed. Averaging over m spectra, the single channel noise in Eq. 7 is reduced by a factor of $1/\sqrt{2m}$ [11].

The dual photonic delay-line cross-correlation phase noise measurement setup is shown in Fig. 6. The signal from an oscillator is first split into two channels by a power splitter. Note that the two channels are completely independent of each other except for the input power splitter, which is virtually free from noise (flicker). A dual channel FFT analyzer measures the cross spectrum. For this scheme to work effectively, the delay of the two channels must be the same.

We used a 10 μs long delay (2.0 km optical fiber) to measure the oscillator noise. Fig. 7 shows the phase noise floor measured in each channel, and their cross correlation in the absence of delay fibers. Clearly, the noise added by the amplifiers in each channel is removed. The noise floor measurement is done without any delay so that the oscillator noise is cancelled. Here we refer the noise floor to the oscillator noise using Eqs. 1 and 3 with $\tau = 10\ \mu s$. Plot B in Fig. 8 shows the single channel power spectrum for the 10 μs delay. Should we used a 100 ns long delay (typical microwave delay) the noise floor referred to the oscillator would be 40 dB higher (plot A).

A reduction of 13 dB is obtained by averaging $m = 200$ times (Fig. 8, Plot C), resulting in -151 dBc/Hz measurement

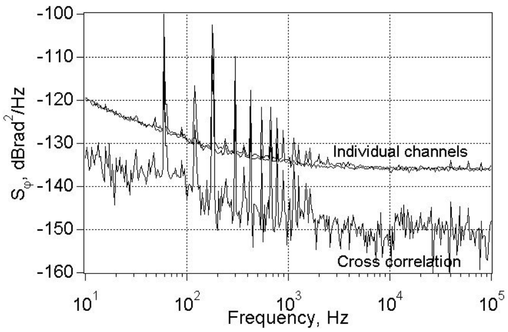

Fig. 7. The cross correlation improves the noise floor over single channel measurements. The amplifier noise limit is removed using cross correlation.

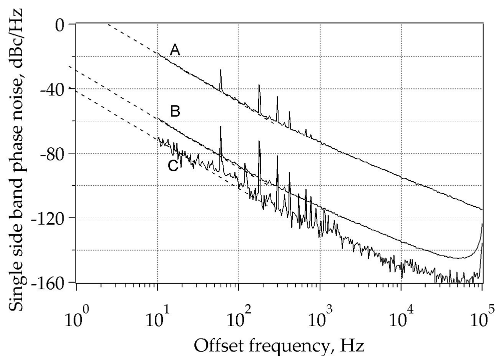

Fig. 8. The measurement noise floor for the single channel 100 ns delay for typical RF delay lines (A), single channel 10 μs (2.0 km) photonic delay (B), and the cross spectrum 10 μs (2.0 km) photonic delay with 200 averaging (C). The discontinuity in the cross spectrum around 10 kHz is the effect of the FFT analyzer and its averaging algorithm, not related to any components or the signal. The discrete lines are line frequency and its harmonics.

noise floor at 10 kHz offset frequency for 10 μs of delay (2.0 km fiber), and -158 dBc/Hz for a 22.5 μs long delay (4.5 km fiber). More averaging with longer measurement time may further improve the noise floor. Typically, for offset frequencies in the range 10 Hz-100 kHz, HP3562B dual channel FFT analyzer gives 100 times averaged data in about 14 minutes. When the delay lines are inserted.

As indicated before the delay transfer function given by Eq. 1 goes to zero at offset frequency $f = 1/\tau$, and its integer multiples. As a result, longer delay gives higher phase sensitivity, but limits the highest offset frequency at which the noise can be measured. Experimental results around the zeros are not reliable, although a few points between the first and second zeros can still be used with difficulty because of the resolution of the FFT analyzer and the stray signals. For the 2 km photonic delay line, the first zero is around 100 kHz, and our measurement limit is then about 90 kHz offset from the carrier.

The success of cross correlation measurement depends on the isolation between channels. Any cross talk shows up as correlated noise. In this regard, microwave and/or optical isolators should be used as needed. In addition, if there is ambient noise in the room that can couple to both channels, this might also mask the actual noise of the oscillator.

V. Conclusion

We have described the dual photonic delay line cross correlation method for measuring the phase noise of low-noise microwave oscillators. With use of low loss kilometer-long optical fiber as photonic delay line and cross correlation, this scheme offers the capability of ultra-low phase noise measurement of oscillators operating at a wide range of frequencies without the need of a reference oscillator. We have applied this scheme for measuring microwave oscillators at 9–10 GHz, such as opto-electronic oscillator[1], [2] and coupled opto-electronic oscillator[3], [4]. The measurement noise floor of -158 dBc/Hz at 10 kHz offset frequency has been demonstrated.

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