Low Flicker-Noise DC Amplifier for 50 Ω Sources

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Abstract— **This article analyzes the design of a low-noise dc amplifier primarily intended as the post-detection front-end for phase noise measurements. Low residual flicker when the amplifier is follows a source with 50 Ω impedance is the most desired performance. This feature can only be appreciated if white noise is sufficiently low, and if an appropriate design ensures dc stability. An optimal solution is proposed, in which the low-noise and dc-stability are achieved at a reasonable complexity and without need of temperature stabilization. Gain is accurate in amplitude and phase up to more than 100 kHz. This is relevant to dual-channel Fourier transform measurements, that is, the measurement of transfer functions and of cross spectra. The proposed amplifier turs out to be a general-purpose laboratory tool, useful in a variety of measurements other than phase noise.**

I. Introduction

In the laoratory practice it is often needed a low-noise preamplifier for the analysis of low-frequency components (below 10 Hz) from a 50 Ω source. The desired amplifier chiefly exhibits low residual flicker and high thermal stability without need for temperature control, besides low white noise. The design of a low-noise amplifier may be regarded as an old subject, nonetheless innovation in analysis methods and in available parts provides insight and new design. The reader familiar with phase or frequency noise spectra, power-law, and Allan variance will find himself at home. The application we initially had in mind is the post-detection preamplifier for phase noise measurements [1], [2]. Yet, there resulted a general-purpose scheme, useful in a variety of applications in experimental electronics and physics.

II. Design Guidelines

Current technology offers two types of low-noise amplifier devices, the junction field-effect transistor (JFET) and the bipolar transistor (BJT), either as a standalone component or as a part of a ready-to-use amplifier. The white noise of these devices is well understood [3], [4], [5], [6]. Conversely, flicker noise is still elusive and relies upon models, the most accredited of which are due to McWhorter [7] and Hooge [8], or on smart narrow-domain analyses, like [9], [10], [11], rather than on a unified theory. Even worse, aging and thermal drift chiefly depend on proprietary technologies, thus scientific literature does not help. The JFET is appealing because of the inherently low noise temperature, which can be as low as a fraction of a degree Kelvin. Unfortunately, the low noise of the JFET derives from low input current, hence a high input resistance (some MΩ) is necessary. The JFET noise voltage is hardly lower than 5 $\mathrm{nV}/\sqrt{\mathrm{Hz}}$, some five to six times higher than the thermal noise of a 50 Ω resistor ($\sqrt{4kTR} = 0.89$ $\mathrm{nV}/\sqrt{\mathrm{Hz}}$). The JFET is therefore discarded in favor of the BJT. A feedback scheme, in which the gain is determined by a resistive network, is necessary for gain accuracy and flatness over frequency.

Table 1 compares a selection of low-noise bipolar amplifiers. The first columns are based on the specifications published on the manufacturer web sites [12], [13]. The right-hand colum reports our measurements. Noise is described in terms of a pair of random sources, voltage and current, which are assumed independent. This refers to the Rothe-Dahlke model[14]. Nonetheless, a correlation factor arises in measurements, due to the distributed base resistance $r_{bb'}$. This derives from the simple fact that a resistor in serie to the input converts current noise into voltage noise. Whether and how $r_{bb'}$ is accounted for in the specifications is often unclear. The noise spectra are approximated with the power-law $S(f) = \sum_\alpha h_\alpha f^\alpha$. This model fits to the observations and provides simple rules of transformation of spectra into two-sample (Allan) variance $\sigma_y(\tau)$. Reference [15] provides the background on this subject, and [16] reports on the and application to operational amplifiers.

The noise power spectrum $2\sqrt{h_v h_i}$ is the minimum noise of the device, i.e., the noise that we expect when the input is connected to a cold (0 K) resistor of value $R_b = \sqrt{h_v/h_i}$, still under the assumption that voltage and current are uncorrelated. When the input resistance takes the optimum value R_b, voltage and current contributions to noise are equal. As a matter of fact, the optimum resistance is not the same for white and flicker noise, thus R_b splits into $R_{b,w}$ for white noise and $R_{b,f}$ for flicker noise. Denoting by f_c the corner frequency at which flicker noise is equal to white noise, thus $f_{c,v}$ for voltage and $f_{c,i}$ for current, it holds that $R_{b,w}/R_{b,f} = \sqrt{f_{c,i}/f_{c,v}}$. Analyzing the specifications numerous bipolar operational amplifiers we find $f_{c,i}/f_{c,v} \approx 50{-}80$, hence $R_{b,w}/R_{b,f} \approx 7{-}9$. There is no explanation for this ratio [17], for we believe that it is due to technology. Anyway, the lower optimum resistance for flicker is a fortunate outcome.

The equivalent temperature is the noise power spectrum divided by the Boltzmann constant $k = 1.38{\times}10^{-23}$ J/K. A crucial parameter of Table 1 is the total noise when each input is connected to a 50 Ω resistor at room temperature, which includes noise voltage and current, and the thermal noise of the two resistors.

Not surprisingly, matched transistor pairs show lower noise than the operational amplifiers. Experience indicates that PNP transistors are not as good as NPN ones to most extents,

Table I: Selection of Low-noise Amplifiers.

	OP27[a]	LT1028[a]	MAT02[b]	MAT03[b]	unit	MAT03 meas.[c]
WHITE NOISE						
noise voltage[d] $\sqrt{h_{0,v}}$	3	0.9	0.9	0.7	$\mathrm{nV}/\sqrt{\mathrm{Hz}}$	0.8
noise current[d] $\sqrt{h_{0,i}}$	0.4	1	0.9	1.4 [e]	$\mathrm{pA}/\sqrt{\mathrm{Hz}}$	1.2
noise power $2\sqrt{h_{0,v}h_{0,i}}$	$2.4{\times}10^{-21}$	$1.8{\times}10^{-21}$	$1.6{\times}10^{-21}$	$2.0{\times}10^{-21}$	W/Hz	$1.9{\times}10^{-21}$
noise temperature T_w	174	130	117	142	K	139
optimum resistance $R_{b,w}$	7500	900	1000	500	Ω	667
2×50Ω-input noise	3.3	1.55	1.55	1.5	$\mathrm{nV}/\sqrt{\mathrm{Hz}}$	1.5 [6]
FLICKER NOISE						
noise voltage[4] $\sqrt{h_{-1,v}}$	4.3	1.7	1.6	1.2	$\mathrm{nV}/\sqrt{\mathrm{Hz}}$	(0.4)[g]
noise current[d] $\sqrt{h_{-1,i}}$	4.7	16	1.6	n. a.	$\mathrm{pA}/\sqrt{\mathrm{Hz}}$	11
noise power $2\sqrt{h_{-1,v}h_{-1,i}}$	$4.1{\times}10^{-20}$	$5.3{\times}10^{-20}$	$5.1{\times}10^{-21}$	–	W/Hz	(…)[h]
1-Hz noise temperature T_f	2950	3850	370	–	K	(…)[h]
optimum resistance $R_{b,f}$	910	106	1000	–	Ω	(…)[h]
2×50Ω-input noise	4.3	2.3	1.6	–	$\mathrm{nV}/\sqrt{\mathrm{Hz}}$	1.1 [f]
THERMAL DRIFT	200	250	100	300	nV/K	–

a Low-noise operational amplifier.
b Matched-transistor pair. MAT02 is NPN, MAT03 is PNP. Data refer to the pair, biased at $I_C = 1$ mA.
c Some MAT03 samples measured in our laboratory.
d Power-law model of the spectrum, voltage or current, $S(f) = h_0 + h_{-1}f^{-1} + h_{-2}f^{-2} + \ldots$
e Obtained from the total noise with 100 kΩ input resistance.
f Measured on the complete amplifier, independently of the measurement of the above S_v and S_i.
g Derives from the noise current converted into voltge by the distributed base resistance $r_{bb'}$.
h Can not be compared to other data because voltage and current are correlated.

but exhibit lower noise. For this reason we tried the design of a differential amplifier based on the MAT03, after the independent measurement of some samples.

III. Design of the Input Stage

The typical noise spectrum of the MAT03 reported in the data sheet, shows an anomalous slope at low frequencies (0.1–1 Hz), significantly steeper than f^{-1}. This is noticeable at $I_C = 1$ mA, and outstands at lower collector current (10–100 μA). In our opinion, the typical spectrum reflects the temperature fluctuation of the environment through the temperature coefficient of the offset voltage V_{OS} rather than providing information on the flicker noise inherent in the transistor pair. The measurement of a spectrum from 0.1 Hz takes some 5 min. At that time scale, we expect that the dominant environment fluctuation is a drift. The Fourier transform of a linear drift $v(t) = ct$ starting at $t = 0$ is $V(\omega) = j\pi c\delta(\omega) - c/\omega^2$. Discarding the term $\delta(\omega)$, not visible in a log-log scale, the power spectrum density, i.e., the squared Fourier transform, is $S_v(\omega) = \frac{c^2}{\omega^4}$, or $S_v(f) = \frac{(2\pi)^4c^2}{f^4}$. A parabolic drift, seldom encountered in practice, has a spectrum proportional to f^{-6}. A smoothly walking drift tends to be of the f^{-5} type. As a consequence, a thermal drift can be mistaken for a random process of slope f^{-4} to f^{-5}, hiding the inherent f^{-1} noise of the transistors. For this reason, the test circuit (Figure 1) is to be enclosed in an appropriate environment that provides passive temperature stabilization and eliminates convection. Sikula [18] reports on thermal effects with the slope f^{-5} observed on a low-noise JFET amplifiers.

Due to the low value of $r_{bb'}$ (15–20 Ω) the current measurement can be made independent of voltage noise, but not vice versa. Thus, we first measure the noise current setting

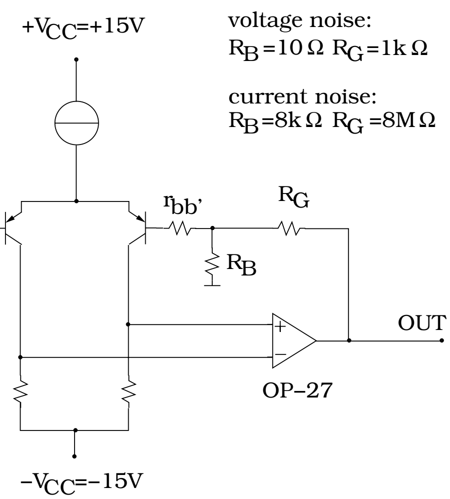

Fig. 1. Test circuit for the measurement of the matched transistor pair.

$R_B = 8$ kΩ, which is limited by the offset current; then we measure the noise voltage setting $R_B = 10$ Ω. The measured spectra are $S_i(f) = 1.45{\times}10^{-24} + 1.2{\times}10^{-22}f^{-1}$ A^2/Hz ($1.2\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$ white, and $11\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$ flicker), and $S_v(f) = 10^{-18} + 1.8{\times}10^{-19}f^{-1}$ V^2/Hz ($1\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ white, and $425\,\mathrm{pV}/\sqrt{\mathrm{Hz}}$ flicker). With our test circuit, the expected white noise is $h_{0,v} = 4kTR{+}2qI_BR \simeq 1.7{\times}10^{-20}R$ V^2/Hz, which is the sum of thermal noise and the shot noise of the base current I_B. $R = 2(R_B + r_{bb'})$ is the equivalent base resistance, while the shot noise of the collector current is neglected. Assuming $r_{bb'} = 16$ Ω (from the data sheet), the estimated noise is $h_{0,v} \simeq 9{\times}10^{-19}$ V^2/Hz. This is in

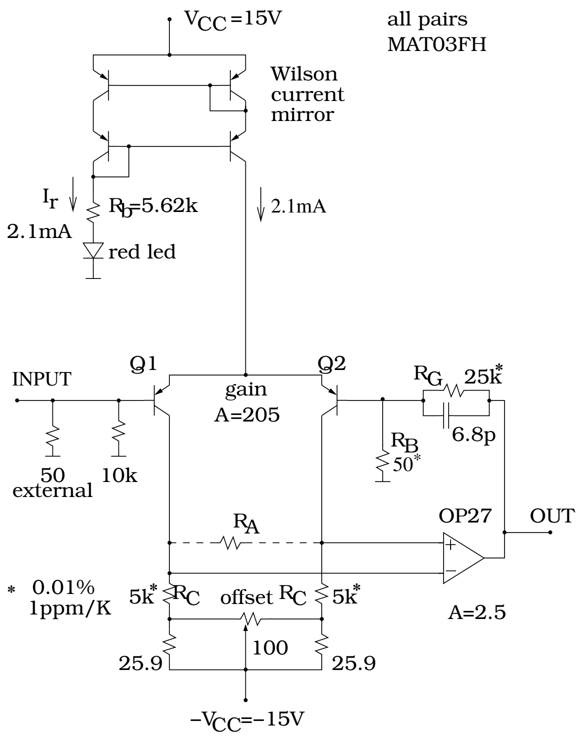

Fig. 2. Scheme of the complete amplifier.

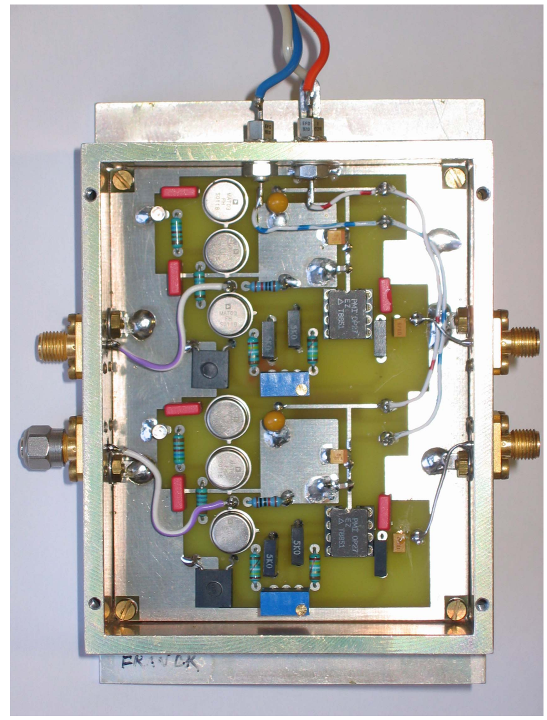

Fig. 3. Amplifier prototype.

agreement with the measured value of 10^{-18} $\mathrm{V^2/Hz}$. Then, we observe the effect of the current flickering on the test circuit is $R^2 h_{-1,i} \simeq 1.6{\times}10^{-19}$ $\mathrm{V^2/Hz}$. The latter is close to the measured value $1.8{\times}10^{-19}\,\mathrm{V^2/Hz}$. Hence, most of the observed voltage flickering derives from the current noise through the external resistors R_B and the internal distributed resistance $r_{bb'}$ of the transistors. Voltage and current are therefore highly correlated. As a further consequence, the product $2\sqrt{h_{-1,v} h_{-1,i}}$ is not the minimum noise power, and the ratio $\sqrt{h_{-1,v}/h_{-1,i}}$ is not the optimum resistance. The corresponding places in Table 1 are left blank. Due to the measurement uncertainty, we can only state that a true independent voltage flickering, if any, is not greater than $4{\times}10^{-20}$ $\mathrm{A^2/Hz}$. The same uncertainty affects the optimum resistance $R_{b,f}$, which is close to zero.

IV. Implementation

Figure 2 shows the scheme of the complete amplifier. This scheme is clearly inspired to the "super low-noise amplifier" proposed in Fig. 3a of the MAT03 data sheet, [12] and [19, p. 344]. Yet, the original circuit has three differential pairs connected in parallel, as it is designed for the lowest white noise with low impedance sources ($\ll 50\ \Omega$), like coil microphones. In our case, paralleling two or more differential pairs results in increased flickering because of current noise. The collector current $I_C = 1.05$ mA is a compromise between white noise (lower at high I_C), dc stability (better at low dissipated power), flicker, and practical convenience. The gain of the differential pair is $g_m R_C = 205$, where $g_m = I_C/V_T = 41$ mA/V is the transistor transconductance, and $R_C = 5\ \mathrm{k\Omega}$ is the collector resistance. The closed-loop gain is $1{+}R_G/R_B \simeq 500$, thus the gain of the OP27 is of 2.5. Given $g_m R_C = 205$, the closed-loop gain can not be safely lowered because the OP27 (as any operational amplifier) oscillates when used at a gain lower than one. If a lower gain is needed, the gain of the differential stage must be lowered by inserting R_A. The trick is that the midpoint of R_A is a ground for the dynamic signal, hence the equivalent collector resistance that sets the gain is R_C in parallel to $\frac{1}{2}R_A$. The bias current source is a cascode Wilson scheme, which includes a LED that provides some temperature compensation. The stability of the collector resistors R_C is a crucial point because the voltage across them is of 5 V. A fluctuation of 10^{-6} of each results in a worst-case fluctuation of 10 μV at the differential output, thus of 50 nV at the input. This is 1/6 of the thermal fluctuation of the the differential pair for an excursion of 1 degree Kelvin. Absolute accuracy is important in order to match the collector currents, necessary to take the full benefit from the symmetry of the transistor pair. The same requirements do not apply to R_A, which only sees the differential signal.

Two equal amplifiers are assembled on a printed circuit board, and inserted in a $10{\times}10{\times}2.8$ $\mathrm{cm^3}$, 4 mm thick aluminum box (Fig. 3). The box provides thermal coupling to the environment with a suitable time constant, breaks convection. LC filters, of the type commonly used in HF/VHF circuits, are inserted in series to the power supply, in addition to the usual bypass capacitors. For best stability, and also for mechanical compatibility with our equipment, input and output connector are of the SMA type.

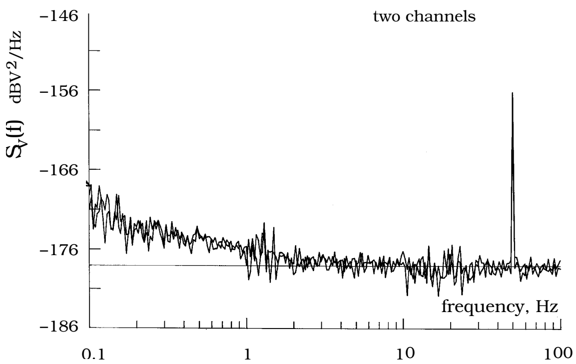

Fig. 4. Noise spectrum of the amplifier input-terminated to 50 Ω.

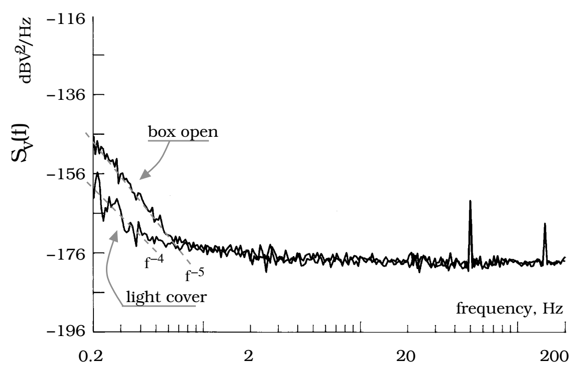

Fig. 5. Thermal effects when the aluminum cover is removed.

Figure 4 shows the noise spectrum of one prototype input-terminated to a 50 Ω resistor. The noise is $\sqrt{h_0} = 1.5$ $\mathrm{nV}/\sqrt{\mathrm{Hz}}$ (white) and $\sqrt{h_{-1}} = 1.1$ $\mathrm{nV}/\sqrt{\mathrm{Hz}}$ (flicker). The flicker noise exceeds the white floor at $f < f_c = 0.5$ Hz. Converting the flicker noise into two-sample (Allan) deviation, we get $\sigma_v(\tau) = 1.3$ nV, independent of the measurement time τ.

The importance of the assembly is shown by a simple and amazing experiment. The amplifier is tested in a quiet environment, far from doors, fans, etc. The aluminum cover is first removed, exposing the circuit to the air flow of the room, and then replaced with a sheet of 80 $\mathrm{g/m^2}$ paper (plain paper for photocopiers). The low-frequency spectrum (Fig. 5) is $5{\times}10^{-19} f^{-5}$ $\mathrm{V^2/Hz}$ in open air, and about $1.6{\times}10^{-19} f^{-4}$ $\mathrm{V^2/Hz}$ with the paper sheet. This indicates the presence of an irregular drift, smoothed by the paper protection.

HOW TO DUPLICATE OUR AMPLIFIERS.

Duplication is allowed. Hand-written sketch of the box and the gerber files of the printed circuit boards are available from the authors. Comments and feedback are welcome. E-mail `rubiola@esstin.uhp-nancy.fr` or `lardet@lpmo.edu`

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